Integrated Real - Time Disruption Recovery Strategies:

A Model for Rail Transit Systems

by

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Abstract

Rail transit systems are subject to frequent disruptions caused by a variety of random disturbances, signal problems and door problems, for example. Such disruptions usually last for 10 to 20 minutes, which degrades the level of service significantly. To improve service reliability, transit agencies employ various real time control strategies, such as holding, expressing and short turning, to deal with these disruptions. The effectiveness of these control strategies relies upon the bird's-eye-view of the whole system. Unfortunately, it is difficult for human dispatchers to assess the situation and make good decisions in real time, even with the aid of advanced information technologies such as automatic vehicle location systems.

This thesis focuses upon the development of a real-time disruption control model for rail transit systems during disruptions. A deterministic model to representing the rail transit system is first introduced. In the model, the passenger flow rates and running time between stations are constant but station-specific. Assuming that the disruption duration is known, a formulation is developed that makes use of real time vehicle location information and considers holding, expressing and short turning strategies to reduce the impact of the disruption. The objective is to minimize the sum of total platform waiting time and weighted in-vehicle delay. The original formulation is transformed into a linear mixed integer problem, which can be solved by any linear optimizer. The formulation is applied to a disruption scenario on a simplified system based on the Massachusetts Bay Transportation Authority Red Line. The sensitivity of different control strategies to the disruption duration assumption is investigated.

The results showed that holding strategies combined with short turning strategies can reduce the weighted waiting time (the sum of platform waiting time and weighted in-vehicle delay) by about 10-60%, compared with not applying any control strategies. Expressing only provided modest additional benefits. For the deterministic disruption duration assumption, sensitivity analysis showed that holding and expressing strategies are fairly robust, but the effectiveness of short turning strategies is quite sensitive to the accuracy of the disruption duration estimate. Most problem instances of the formulation can be solved in real-time with the proposed branching sequence used in the branch-and-bound algorithm to solve this mixed integer problem.

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Chapter 1

Introduction

The high-frequency rail transit system has been playing and will play an important role in urban transportation because of the increasing concerns about environment, urban mobility and social equity. Rail transit service is usually not subject to urban congestion, indeed it helps to alleviate congestion by offering an alternative to driving. Therefore, service reliability is a major advantage of the rail transit service in the competition with the car. However, frequent minor disturbances can diminish this advantage and degrade the overall level of service.

The development of information technology has provided better computer and communication hardware for transit agencies to handle disturbances. Many transit agencies are starting to make use of automatic vehicle location (AVL) systems and automatic vehicle monitoring (AVM) systems. Surprisingly, there has been little research done to improve the control decision-making process and make the best use of the advanced technology.

This thesis reports on research on real-time control for rail transit systems to deal with disruptions. A model that considers holding, expressing, and short-turning strategies is presented with the objective of reducing passenger waiting time and on-board delay. The model is tested on a modified system based on the Massachusetts Bay Transportation Authority Red Line.

1.1 Background and Motivation

Disruptions frequently occur in transit systems. According to Song (1998), there were 323 significant incidents or disruptions on the MBTA Red Line during a recent two-year period, approximately three disruptions per week. Reasons for disruptions included fires on tracks, disabled trains, and door problems. Most of these disruptions cause delays of between 10 and 20 minutes. However, they greatly worsen the level of service provided by the transit system, especially during the peak periods when the system is operating near capacity. Moreover, due to the inherent instability of transit system headways, even a small delay can be amplified down the line (Welding, 1957). In the absence of appropriate and timely control intervention, a 10 to 20 minute disruption can affect the system long after the blockage has been cleared.

For short periods, such as 10 to 20 minutes, it is costly and non-responsive to change the operations plan, for example by introducing supplementary bus service. Therefore, transit agencies usually employ real time control strategies to deal with these disruptions. In both bus and rail transit, commonly used strategies include holding a vehicle at a station, expressing a vehicle over a segment, and short-turning a vehicle. Other strategies include speed control and splitting trains. (For detailed description of real time control strategies, refer to Wilson et al, 1992). Unfortunately, it is difficult, if not impossible, even for an experienced dispatcher to assess the situation and make the best decision from a system-wide perspective in a short time period. The difficulty is compounded, given that the dispatcher must be concerned about dealing with the problem causing the disruption as well as its ramifications.

With the development of new technologies, many transit agencies are starting to make use of automatic vehicle location (AVL) systems and automatic vehicle monitoring (AVM) systems. These technologies provide rich real time information and greatly improve the control environment. In contrast, there has been surprisingly little research and development capitalizing on these rapidly emerging technologies. Transit agencies still rely heavily on the experience and judgement of the dispatchers.

The research reported in this thesis is partly motivated by the implementation of a new Operation Control System (OCS) at the Massachusetts Bay Transportation Authority (MBTA). The OCS provides dispatchers with real time train location information. Hence, dispatchers are now in a better position to handle the disruptions. However, such a large amount of data may easily overwhelm the dispatcher, and the difficulty of assessing the system-wide implications of alternative control strategies still exists. Therefore, it remains difficult for the dispatchers to make consistent and appropriate control decisions.

The formulation developed in this research can be applied on any rail transit system with real time train location information available. The model can also be the base for an eventual fully automated real time control system.

1.2 Prior Research

Prior research was obviously subject to the limitation of the technologies available at the time of the research. In the 1970's and early 1980's, real time information was generally not available, and other data such as passenger flow rates were costly to collect and often dated and unreliable. Research on transit real time control tended to rely on as little information from transit agencies as possible. To make up for the lack of real time information, most researchers used probability density functions to describe the running times or vehicle headways. In terms of control strategies, holding attracted the most research interest. The reasons may be: (1) holding is easy to implement; (2) the cost structure of holding is simpler than expressing or short-turning because it does not involve cost associated with passengers being dumped or skipped; (3) the effectiveness of holding is less sensitive to errors than expressing or short-turning. That is, imprecise data may not reduce the effectiveness of holding very much. However, due to the probability density function, analysis was very difficulty even after major simplification of the problem. On the other hand, results were often not generally applicable because of these simplifications.

Since the late 1980's, real time information has become available to an increasing number of transit agencies. Research interest was then extended to expressing and short-turning strategies. At the same time, advances in large-scale optimization and computer technology make it possible to develop large-scale models and solve such models in real time.

The following sections will review the notable literature with and without the use of real time information.

1.2.1 Research without Real Time Information

(1) Holding

Osuna and Newell (1972) considered a system with only one service point at which passengers boarded the vehicles. The time horizon was infinite, and there were a given number of identical vehicles providing the same service. They assumed that the capacity of the vehicle was infinite, and the cycle times of the vehicles were independently and identically distributed. Their objective was to minimize the average waiting time of a randomly arriving passenger. They formulated the problem as a Markov decision problem. The action to be taken at any time was a two-valued (dispatch or hold) function of the past history of events and given properties of the system. They showed that the optimal strategy for the one-vehicle case and the two-vehicle case was to hold the vehicle until a certain threshold headway, and derived the corresponding solutions. They concluded that even very idealized problems were difficult to analyze, and more complex problems would require better intuition and perhaps less mathematics.

In order to study the vehicle-pairing effect, Newell (1974) considered a transit route with two vehicles and one control point. The decision variable was again the threshold headway at the control point. In his paper, Newell assumed that the passenger arrival rate was constant, but instead of a continuous vehicle headway distribution, he used diffusion approximations of the first and second moments of the vehicle inter-station running time and dwell time and derived an approximate solution. His results suggested that the optimal control was one in which the effects of pairing were kept well under control.

Barnett (1974) considered a simplified transit line with two terminals and one control point, and assumed that the passenger arrival rate was constant. The control scheme he considered was holding the vehicles at this point, and the decision variable was the optimal holding time at the control point. He used a simple two-point discrete distribution to approximate the distribution of the vehicle arrival headways. Arguing that the most effective way to reduce average waiting time for passengers beyond the control point was evening the headways at the control point, Barnett used an objective function to minimize the weighted expected waiting time for passengers entering the system at the control point and the expected delay for passengers aboard the vehicle at the control point. The optimal holding strategy depended upon the mean and variance of the headway distribution, the correlation between successive arrivals, and the ratio of passengers benefited to passengers disbenefited by holding. The correlation between successive arrivals represented the degree of the vehicle pairing effect.

In his 1978 paper, Barnett considered another rudimentary system with only one infinite-capacity vehicle, and all passengers boarding at the same stop and having the same travel cost structure. The interaction between passenger and transit company was modeled as a cooperative game. The passengers tried to minimize their waiting cost based upon the service characteristics, and the transit company, subject to economic constraint, used the vehicle dispatching strategy to minimize the time-related cost for passengers. Based on the rudimentary system, he obtained the optimal holding policy analytically. The policy was holding the vehicle until a threshold headway, which was consistent with the results from Osuna & Newell (1972). However, he suspected that strong analytic results for the multi-vehicle system would be rare.

Turnquist and Blume (1980) evaluated the potential effectiveness of holding strategies applied to headway control. They adopted the same objective function that Barnett used in his 1974 paper. They argued that the holding scheme would be most effective if successive arrival headways were perfect negatively correlated (i.e., a short headway always follows a long headway) because holding the vehicle when the headway was short always reduced the long following headway. In contrast, they argued that holding strategy would be least effective if the successive arrival headways were statistically independent. By defining the above two extremes, they derived upper and lower bounds on the effectiveness of holding. Those bounds depended upon the coefficient of variation of the arrival headway, and the ratio of passengers benefited to passengers disbenefited by holding.

Abkowitz, Eiger and Engelstein (1986) developed an empirical headway variation model based upon Monte Carlo simulation to examine the threshold-based holding strategy. The cost they examined was the total passenger wait and delay time along the route with control at a certain stop. The passenger waiting time at each stop was represented by a function of the mean and variance of headway at that stop from a simulation model. The optimal control stop location was obtained by selecting the stop at which the total cost was a minimum, and the optimal threshold headway was the associated optimal headway at that stop. Their results suggested that holding was more effective at stops that were closer to the control point and effectiveness was reduced at stops further downstream.

(2) Expressing & Short-turning

Macchi (1989) presented the first research on expressing strategies. He discussed the different groups impacted by expressing. Based upon the Green Line system in Boston, he proposed an expressing guideline based upon the preceding and following headways. To simplify the problem, he assumed that the express segment was pre-determined, and the headway downstream did not vary. His results showed that expressing was justifiable only if the following headway was short. However, his strategy was local instead of global due to the structure of the strategy and the limited information used, and the simplification made the strategy less applicable in the general context.

Adopting a similar approach as Macchi, Deckoff (1990) studied the short-turning strategy on the Green Line system. He again made the assumption that the headway after the control point did not vary down the line, and considered short-turning one train at the control point. With data from Monte Carlo simulation, he developed guidelines for field inspectors to make short-turning decision based upon headway series at the control point. His results suggested that the following two cases were where short-turning might be beneficial. First, short turn the second train when two consecutive trains had very short headway; Second, if the short-turning candidate had a fairly large headway and was close to the short-turn location, and, hence, it would have a large headway in the other direction as well. Like Macchi, Deckoff's research was closely based on the Green Line operations, and the short-turning strategies were local strategies.

Both Macchi and Deckoff found that the use of automatic vehicle location systems would help make better expressing and short-turning decisions.

1.2.2 Research with Real Time Information

In the early stage of applying real time information, Turnquist (1989) discussed the recovery of schedule deviations with real time vehicle location information. Instead of using holding strategies, he considered controlling vehicle speed on multiple segments. Focused upon the same problem posed by Turnquist, Furth (1995) considered using holding strategies to recover the schedule deviation after a delay occurred. Furth's objective function was the sum of passenger waiting time and delay, and the decision variables were the number of trains over which to spread the delay and the delay time for each train. He compared the results with that of the no-control case and concluded that the benefits were not very large, but appeared to be great enough to merit incorporation in automated train control. Furth suggested that the use of real time information could help make better holding decisions. Furth's strategies were reactionary in the sense that control occurred after the delay had occurred and the trains he chose to control were those behind the blockage. As shown by O'Dell (1997, 1999), large benefits come from control of the trains ahead of the blockage, and actions should be taken before the delay is over to achieve large benefits.

Eberlein (1995, 1999) presented the first important research on real time deadheading, expressing and holding strategies, applied both separately and in combination. She defined two types of deterministic models to describe alternative transit systems: a general system and a fixed system, which was based upon a simplified version of the general system with some parameters fixed. For example, in the fixed system, passenger flow and dwell times were fixed as the same level for each station. For the general system, running time between stations and passenger flow rates were deterministic but station-specific. Although it was a deterministic model, the assumptions made about running time between station and about passenger flow may not be too unrealistic.

Based upon the model, Eberlein formulated deadheading, expressing and holding problems in both systems, taking real time vehicle location information as input. Eberlein used total passenger waiting-time as the objective function to minimize. Therefore, there was no cost associated with holding. This is not strictly true because holding results in an increase in passenger on-board time. Eberlein's focus was routine control without significant disruptions. To simplify the problem, she used infinite train capacity and considered a single loop route structure. In terms of the formulations, they did not include short-turning strategies and used a non-linear objective function with non-linear constraints, which was costly to solve and not practical for real time control. For that reason, Eberlein developed heuristics to solve the problem. Nevertheless, Eberlein's formulation provided a solid foundation for further work on this problem area.

At the same time that Eberlein were doing her research, Li (1994) conducted related research on the real time bus dispatching problem. His problem context was a highly capacity constrained bus route with extremely short scheduled headways (1-2 minutes) with the terminus as the only control point. With different levels of real time bus location information, the arrival times of buses at the terminus were estimated. The objective was to minimize passenger-waiting time. The dispatching decision chose a dispatching time and a route operating pattern out of a small set of candidates. The pre-determined set of candidates included several short-turning patterns and several station-skipping patterns. Li developed two models. The first one was a deterministic model with similar assumptions to Eberlein's. The second was a stochastic model assuming that part of the travel time and the initial queues were stochastic. Li assumed the random part of the travel time was identical so that the headway was still deterministic. To simplify the stochastic model, Li assumed constant dwell times and the number of passengers travelling between each O-D pair for each pattern was pre-determined. Both of these two assumptions are significant. The specific decision structure and the significant simplifications limited the applicability of Li's models.

Another important step in this area was the work of O'Dell (1997,1999), who formulated holding and short-turning models based upon the fixed and general systems defined by Eberlein. In O'Dell's formulation, she included the train capacity, which was an important step towards reality. O'Dell focused upon disruption control. In addition to the deterministic assumptions made by Eberlein, O'Dell assumed the duration of the blockage to be known. Unlike Eberlein's non-linear formulation, O'Dell's formulation used piece-wise linear functions to approximate the non-linear objective function, and all the constraints she used were linear. Thus, her formulation could be solved by a linear optimizer and was more practical for real time use. Another feature of O'Dell's formulation was that it considered transit lines with two branches.

O'Dell formulated the holding and short-turning problems in the context of the MBTA Red Line, but did not include the expressing strategy. For the holding problem, O'Dell considered three different holding strategies, namely holding any train at any station, holding any train only at the first station the train arrives at after the blockage occurred, and holding each train at most once. Clearly, the optimal solution of being able to hold all trains at all stations was the best of the three, since the other two imposed additional constraints on the holding strategy. However, considering the limitations of the transit control system and implementation difficulty, we would like simple but effective control strategies. Therefore, we may prefer to hold several trains at several stations. Again, since O'Dell's formulation still only considered minimizing passenger waiting time, it could tend to hold trains for longer periods than desirable and to hold at multiple stations. For the short-turning problem in O'Dell's model, the train that was a candidate for short-turning and the short-turning location were both pre-determined.

After testing her models for different blockage duration in two scenarios, O'Dell concluded that holding by itself could achieve 15-40% reduction in passenger waiting time. However, this could overestimate the net benefit because it did not consider the increase of passenger on-board time. For short-turning, O'Dell's results showed that the longer the blockage, the more benefit short-turning could provide. Intuitively this makes sense because it takes some time to short-turn a train and if the duration of blockage is not long, the time spent on short-turning may not be justifiable.

O'Dell tested the formulations on a Sun SPARC 20 workstation with CPlex 3.0. It took less than 30 seconds to obtain the optimal solution to the formulations with small sets of integer variables and small sets of possible nodes (less than 150 for "Holding All" and "Holding First"), which was feasible for real time control in transit systems. However, for formulations with larger set of integer variables and possible nodes, it took much longer to get the solution. O'Dell did not develop an algorithm to increase the solution speed.

1.3 Thesis Content and Organization

In this thesis, a model to help dispatchers make decisions when disruptions occur in rail transit systems is presented. The model makes use of real time information and considers combinations of the holding, expressing, and short-turning strategies.

In the context of transit real time control, changes in operations costs such as vehicle and crew costs are usually negligible. Therefore, the weighted sum of passenger waiting time and in-vehicle delay is taken as the objective function to be minimized. The decision variables are the departure times of trains at each station, which determine the holding times and control decision variables such as whether to let a train skip a station or short-turn a train on a particular cross-over track.

In the holding strategy, any train can be held at any station. If we want to simplify and reduce control actions, holding at the first station that a train reaches after the blockage occurs can also be used as an alternative. In fact, if we consider the increases in on-board time as a result of holding, the trade-off between passenger waiting time and on-board delay will be evaluated, and the optimal solution will tend to include fewer holding actions. In the expressing strategy, trains can skip any station, or we can impose desired control constraints. For example, a train can be expressed at most once. In the short-turning strategy, any train can be selected to be turned back, and trains can be turned back at multiple locations if such options are available. In addition, the formulation can be used for transit systems with multiple branches.

The model is applied to two problem instances in a simplified transit system, which is based on the MBTA Red Line. The first instance is a 10-minute disruption, and the second is a 20-minute disruption. Moreover, to understand the impact of the deterministic disruption duration assumption, the sensitivity of the different control strategies is investigated. Since the formulation is used for real time control, practical issues to speed up the solution process are discussed and a simple empirical branch and bound algorithm specific for this formulation is presented.

In Chapter 2, the general model representing the transit system is introduced and the formulation that considers the holding, expressing and short-turning control strategies is presented. Since the initial formulation is nonlinear, linearization of the formulation is also discussed.

In Chapter 3, the formulation is applied to a modified system, which is a simplified representation of the MBTA Red Line. Two problem instances are tested and the effectiveness of various control strategies is compared. The sensitivity of the control strategies to the disruption duration is also investigated. Since the formulation is used for real time control, some practical approaches to reduce the solution time are discussed and a method is presented that increases the speed of the branch and bound process to solve this MIP formulation.

Finally, Chapter 4 summarizes the findings and offers suggestions for future research.

Chapter 2

Model and Formulation for Disruption Control

In this chapter, a general model is introduced to represent the transit system and related assumptions and features are discussed. Based on the general model, a disruption control formulation is presented, which includes holding, expressing, and short-turning strategies. Since the original formulation has a non-linear objective function and non-linear constraints, the linearization of the formulation is also addressed.

2.1 General Model of Transit Systems

Transit systems are inherently probabilistic. Many variables such as passenger arrival rates and inter-station running time are random in nature. However, a probabilistic approach to model transit systems can be complex and probabilistic models involve large solution effort, which may not be feasible for real time control especially for large-scale systems. With real time information available, we can apply a mathematical model to each specific scenario, and, hence, do not need to be concerned about making decisions that are statistically sound across all scenarios. In addition, the coefficients of variation for many of these random variables are not very large. Therefore, a deterministic simplification may not be too unrealistic. In this thesis, a deterministic model, referred to as System G, is used to represent transit systems.

2.1.1 Assumptions of System G

The following assumptions are made to define System G.

A(1) Passenger arrival rates and alighting fractions are deterministic constants and station-specific.

The passenger alighting fraction at a certain station is the portion of passengers on a train who alight at that station. Although the passenger arrival rate and alighting fraction vary by time of a day and day of a week, the variance in a particular time period is usually not large, and this variance is not expected to have significant impact if passenger flow rates specific to each time period are used. Further decrease of this variance may be possible by defining time periods as short as can be supported by the available data.

In addition, passenger arrivals are generally independent of the arrival of trains for transit service with headway less than 10 minutes. That is, passengers do not arrive at a station according to the scheduled train arrivals in frequent transit services. Hence, constant passenger flow rate can be assumed within a certain time period. If automatic passenger counter (APC) systems can provide reliable real time passenger flow information, different deterministic values by time of day and day of week may be used to reduce the impact of this assumption.

A(2) Running time between stations is approximated by the maximum running time under the non-inter-station-stopping condition, and the minimum separation of trains at different locations is approximated by a minimum departure-arrival interval at each station under the non-inter-station-stopping condition.

Under the non-inter-station-stopping condition, a train can depart from a station only if it can travel to the next station without stopping. In many cases, the speed of a train may be higher than that implied by this condition, up to the maximum allowable speed. On the other hand, since many existing train control systems allow a train to depart earlier than the time governed by the non-inter-station-stopping condition, trains may have to stop between stations. Therefore, the maximum non-inter-station-stopping running time may be a fair approximation of the real average running time, at least under congested running conditions.

This maximum running time is consistent with the minimum departure-arrival condition used for maintaining the minimum separation of trains. The departure-arrival interval is the time between a departure of a train and the arrival of the following train at a station. In most rail transit systems, the distances between trains are determined by control lines (Song, 1998). When two trains are close together, the maximum permitted speed of the second train is reduced. If the distance of the two trains continues to decrease, the second train will eventually be stopped. By *A(2)*, a train will depart a station only when it can run to the next station without any inter-station stop. In practice, the minimum separation may be smaller than the minimum departure-arrival interval used under this assumption because many train control systems permit a train to leave a station before this time. Under this assumption, instead of stopping between stations or running slowly, a train will stay at a station for more time and pick up more passengers and reduce their waiting time. Hence, the situation is slightly better than the real one. If signal systems exist at each station to control the departure of trains, the impact of this assumption will be reduced.

A(3) The duration of a blockage is a known deterministic value.

The duration of the blockage is the most unpredictable factor in reality. There are many types of disruptions, such as disabled trains, door problems, and fires on tracks. In each case, the blockage duration is usually unknown. Generally speaking, if the actual duration is larger than the estimate, the control strategies based upon the estimate will not be optimal, but at least will not make things worse. On the other hand, if the time to clearance is over-estimated, control actions may have negative impacts. This assumption is investigated later through sensitivity analysis.

A(4) The short-turning time is deterministic.

The time to short-turn a train is random, although compared with the duration of blockage, it has relatively low variability. If data on the mean and variance of the time to short-turn a train at a specific crossover track are available, the impact of this assumption can be better addressed. Unfortunately, such data are not yet available. Nevertheless, the impact of the uncertainty of short-turning time on the objective function value depends on the specific scenario. For example, if the train to be short-turned is close to the short-turning location when the disruption occurs, it may have to be held for some time at the short-turning destination station. Therefore, the objective function value may not change much if the actual short-turning time is longer than the deterministic value. In some other cases, short-turning may lose all its benefit if the actual short-turning time is much longer than the estimate.

A(5) Dwell time is approximated by linear functions with respect to the number of passengers boarding and alighting.

Dwell time is the major cause for the vehicle pairing effect. It is related to factors such as the number of passengers boarding and alighting and the train crowding condition. If the headway of a certain train is larger than that of its predecessor, it will pick up more passengers and take longer to unload and load the passengers. Therefore, its headway will keep increasing down the line.

The dwell time function is likely to be non-linear in nature. The marginal time for a passenger to board/alight the train increases with the level of crowding. However, according to the dwell time data collected, a linear function with respect to the number of passengers boarding and alighting fits the dwell time well even within the highly crowded range. Hence, the linearity assumption on the dwell time is not too unrealistic. In addition, two linear functions are used to approximate the dwell time, one for the uncrowded condition and the other for the crowded condition, which recognizes the non-linear characteristics to some extent.

2.1.2 Features of System G

System G has the following features.

1. The passenger arrival rate and alighting fraction, train running time and minimum headway are all station-specific parameters. Typically, the arrival rate and alighting fraction vary by time of a day, but the model only relies on data for a single time period.
2. The capacity of trains is considered. Passengers will be left behind if a train is full.
3. The transit line can have more than one branch.
4. Trains can be short-turned at multiple locations, based upon the available crossover tracks.
5. Many types of disruption can be considered, including a temporary closure or speed restriction on a track section.

2.1.3 Input Data

System G requires the following data as input.

1. Track configuration, including branches and available crossover tracks.
2. Dwell time function coefficients. These may be specific for each station having unusual characteristics.
3. Passenger arrival rate and alighting fraction at each station for the time period of interest.
4. Minimum departure-arrival interval at each station.
5. Running time between stations, which includes acceleration and deceleration time and should be consistent with the minimum departure-arrival interval used.
6. Train location.
7. Blockage location and estimated duration.
8. Estimated short-turning time for each possible location.

2.2 Notation and Basic Concepts

$a_{i,t}$ = Arrival time of train i at terminal t

$ah_{i,k}$ = Arrival headway for train i at station k

$al_{i,k}$ = The number of alighting passengers for train i at station k

$b_{i,k}$ = The number of boarding passengers for train i at station k

$d_{i,k}$ = Departure time for train i at station k

$dh_{i,k}$ = Departure headway for train i at station k

$dp_{i,k}$ = Departure time for the train preceding train i at station k

$dw_{i,k}$ = Dwell time of train i at station k

dw_k^0 = The typical dwell time at station k in that time period

$h_{i,k}$ = Maximum platform waiting time for train i at station k

$ht_{i,k}$ = Holding time of train i at station k

i_{BL} = Blocked or disabled train

$l_{i,k}$ = Passenger load on train i departing station k

$l_{i,k}^0$ = Approximate passenger load on train i departing station k

$late_{i,t}$ = 1 if the arrival time of train i is later than the departure time of its predecessor at terminal t, 0 otherwise

$p_{i,k}$ = The number of passengers left behind by train i at station k

$p_{i,k}^e$ = The number of passengers left by express train i at station k

$p_{i,k}^p$ = The number of passengers left behind by predecessor of train i at station k

$r_{i,k}'$ = The number of potential riders for train i at station k if it will not skip any later station

$r_{i,k}$ = The number of potential riders for train i at station k considering possible expressing

$se_{i,k}$ = 1 if train i starts expressing from station k, 0 otherwise

$sk_{i,k}$ = 1 if train i skips station k, 0 otherwise

$so_{i,m}$ = 1 if train i operates on segment m, 0 otherwise

$st_{i,m}$ = 1 if train i is short-turned on the crossover track of segment m, 0 otherwise

$t^{s}_{k,k'}$ = Short-turning time from station k to k'

t_{BL} = Earliest time at which the blocked train or disabled train can move

t_{mc} = Minimum recovery time at terminal

$v_{i,k}$ = 1 if train i is loaded to capacity at station k, 0 otherwise

$y_{j,i,m}$ = 1 if train j precedes train i on segment m, 0 otherwise

$z_{i,k}$ = Variable to approximate the quadratic term of platform waiting time for train i at station k

$zt_{i,k}$ = Variable to approximate the quadratic term of holding time for train i at station k

A_k = Passenger arrival rate at station k

C_k = Dwell time function parameter at station k

H_k = Minimum non-inter-station-stopping headway at station k

L = Train capacity

M = Sufficiently large number

Q_k = Passenger alighting fraction at station k

R_k = Non-inter-station-stopping running time from station (k-1) to station k, including acceleration and deceleration time

S = The set of stations in the impact set

$Sch_{i,t}$ = Scheduled dispatching time of train i at terminal t

S^t = The set of terminal stations in the impact set

G = The set of segments in the impact set

T = The set of trains in the impact set

$T^{p}_{i,m}$ = The set of trains that can be the predecessor of train i on segment m

$T^{s}_{i,m}$ = The set of trains that can be the successor of train i on segment m

U^{iw} = Weight for in-vehicle waiting time

Headway

Headway is a common concept used to represent the interval between two arrivals or two departures and is represented by $ah_{i,k}$ or $dh_{i,k}$ in Figure 2-1.

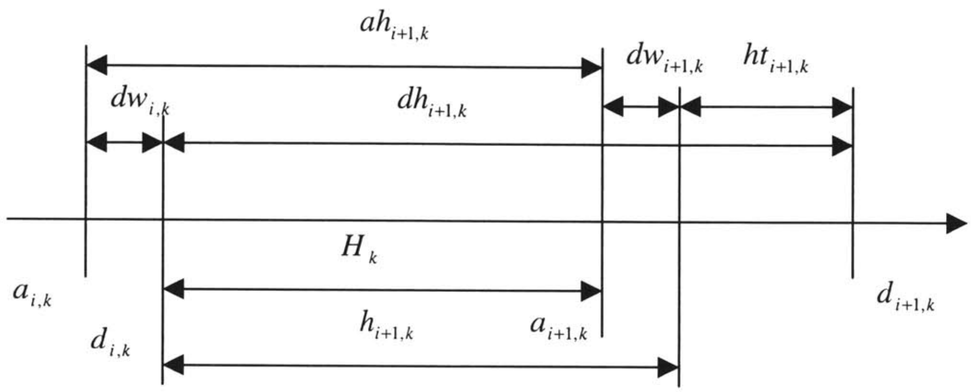


Figure 2-1 Headway, Platform and In-vehicle Waiting Time, and Minimum Separation

Platform Waiting Time and In-Vehicle Waiting Time

Platform waiting time is the time passengers spend on the platform waiting for the next train to arrive. In-vehicle waiting time is the time passengers spend on a train when the train sits at a station. This occurs when trains are held at stations to produce more even headways. It is likely that passengers may perceive in-vehicle waiting time to be less onerous than platform waiting time. Thus, it may be reasonable to apply different weights on in-vehicle and platform waiting time.

The time between the departure of a train and the arrival of the next train at a station is the maximum platform waiting time for passengers arriving during that interval. For dwell time, which is the time for a train to unload and load passengers at a station, precisely speaking, it is platform waiting time for some passengers and in-vehicle waiting time for other passengers, because some passengers may get on the train early and wait for others to board. To simplify the problem and maintain consistency when control strategies are not applied, dwell time is taken as platform waiting time for passengers arriving before the train arrives and during the dwell time. Therefore, platform waiting time includes the time passengers wait on platforms for the next train arrival and the dwell time. In this thesis, the maximum platform waiting time for train i at station k is denoted by $h_{i,k}$.

Passengers on-board the train when holding starts and passengers arriving during holding have to wait on the train until holding is finished. Therefore, holding time is taken as in-vehicle waiting time for these passengers.

Minimum Separation

Minimum separation at a station is denoted by H_k as shown in Figure 2-1. In this thesis, the minimum distance between trains is maintained through the minimum separation constraints at stations. It mandates that the next train can arrive at station k no earlier than H_k after the departure time of the preceding train.

2.3 Control Set and Impact Set

The control set is the set of trains and stations for which we can apply control strategies. The impact set is the set of trains and stations that are affected by the disruption and our control strategies. Obviously, the impact set must be at least as large as the control set.

2.3.1 Discussion

For several reasons, it is desirable to limit the size of the control set. First of all, it is unrealistic to project train movements far into the future with any deterministic model due to stochasticity. The uncertainty of benefits increases with the increase of projection range and uncertain benefits may not justify control actions. Instead, the formulation can be solved repeatedly with updated real time information. Secondly, the marginal benefit from controlling trains decreases with the increase of the number of trains controlled. For example, O'Dell(1997, 1999) showed that the benefit from holding additional trains ahead of the blockage became negligible beyond four trains. Furthermore, O'Dell's formulation did not consider the on-board delay associated with holding, hence, there will be even less benefit from holding additional trains if on-board delay is considered. Thirdly, the impact of the disruption will presumably be alleviated down the line with proper control actions. Considering the available technologies to control trains at intermediate stations, it is preferable to control a small set of trains while achieving substantial benefits. Finally, larger numbers of trains and stations in the control set will increase computation time, while the small benefit from considering controlling additional trains and stations may not justify the loss from a less responsive decision making process.

For the impact set, ideally, we want to include all passenger waiting costs caused by the disruption and the control strategies to avoid any possible bias, which means we should consider all the passengers affected by the disruption and the control strategies. The major computation cost comes from evaluating control decisions. Therefore, even if we include more trains and stations in the impact set in addition to the control set to evaluate the cost, the computation burden will not increase significantly. However, with more trains and stations included in the impact set, the uncertainty of the benefits increases.

There are several factors that influence the choice of control set and impact set.

- Disruption duration
- Crossover tracks
- Passenger flow profile
- Location of disruption

Disruption duration is the primary factor in determining the control set and impact set. The longer the blockage, the larger the number of affected trains and the farther down the line impacts will occur. If crossover tracks are available, short-turning can be considered and so short-turning candidates need to be included in the control set and impact on the reverse direction should be considered. Passenger arrival rates and alighting fractions also affect the choice of control/impact set. At stations where there are significant number of boarding passengers, the associated waiting costs are large, and the impact from the disruption and control strategies should be considered. If the disruption is close to the terminal station, the impact on the reverse direction may also need to be considered.

2.3.2 Control Set

For the control set, we choose potential candidates according to whether control strategies may provide significant benefits. Figure 2-2 shows an example of the possible control set after a disruption occurs.

In terms of the holding strategy, we can have both passive holding and active holding. Some trains behind the blockage must be (passively) held because they are blocked. We may also (actively) hold some trains at stations ahead of the blockage to reduce overall passenger waiting time. According to O'Dell's results (1997, 1999), large benefits can come from holding trains ahead of the blockage whereas actively holding trains behind the blockage provides only marginal benefits. If a train has not passed the stations with large passenger demand, holding may be beneficial. Otherwise, we may consider holding trains only when they reach the terminal. For trains behind the blockage, we have passive holding for those that are blocked.

For the expressing strategy, the only possible candidates are trains behind the blockage: the same trains that are subject to passive holding before the blockage is cleared. After the blockage is cleared, it may be beneficial to express one or two trains to avoid over-crowding and further delays due to increased dwell times at stations. The number of trains that may be beneficial to express is related to the duration of the blockage. However, it is less likely for the optimal solution to include much expressing because it can waste needed vehicle capacity. Moreover, passengers hate to be skipped by several consecutive trains. Therefore, it is most unlikely to be justifiable to express more than two trains immediately after the blockage clearance.

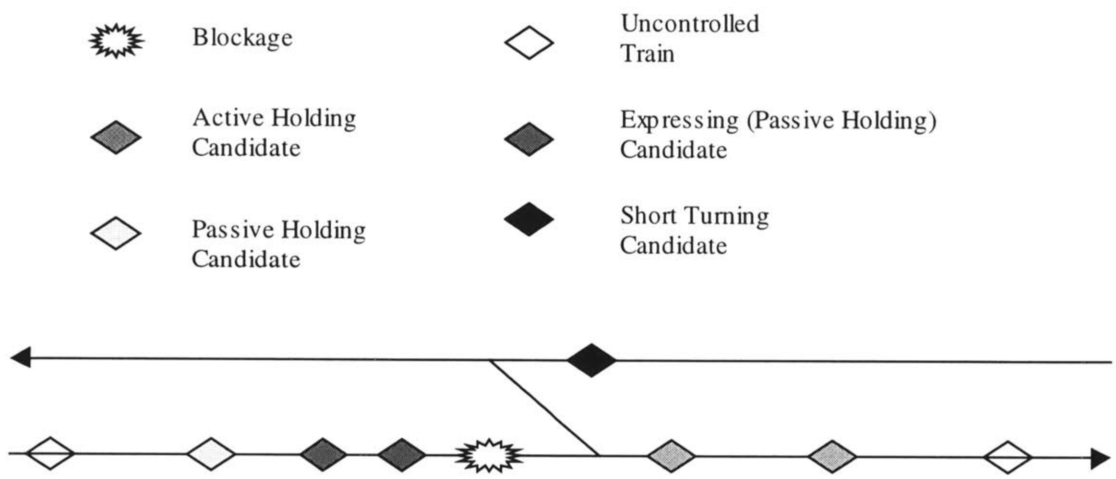


Figure 2-2 Control Set

For the short-turning strategy, given the availability of crossover tracks, the number of trains that may be beneficially short-turned is again related to the duration of the disruption, the short-turning time, and the normal headway. We want to short-turn trains to reduce the gap resulting from the disruption, but we also need to be concerned about the gaps we may be creating.

The number of short-turning candidates needs to recognize both the benefit and the feasibility. From the benefit perspective, suppose there is no significant demand in the reverse direction and, therefore, we do not need to be concerned about the gap in the reverse direction, we want to short-turn at most as many trains as required to achieve headways close to normal. For example, assume the disruption is 12 minutes and the normal headway is six minutes, the resulting gap will be 18 minutes. We can short-turn two trains into the gap and achieve six-minute headways ahead of the blockage in the ideal situation. That is, no passenger ahead of the blockage will perceive the disruption. Behind the blockage, if we hold all the trains behind the blockage for 12 minutes after they arrive at the first station after the blockage occurs, the headways after the blockage is cleared will also be six minutes in the ideal case. If we short-turn more than two trains into the gap, it is certain that we will create a gap in this direction in future, even if we do not need to be concerned about the gap in the reverse direction. Therefore, the number of trains required to fill the gap to achieve the normal headway can be an upper bound for the number of short-turning candidates. One exception may be if a long disruption occurs near the end of the peak period. Since the blocked trains may have been packed, to avoid many passengers being left behind by the packed trains ahead of the blockage after the blockage is cleared, we may want to short-turn more trains to achieve smaller than normal headways, and thus, leave less demand for the blocked trains. We may not be so concerned about the potential gap being created in this case since it will occur only after the peak period. However, it is still unlikely to short-turn more than one train beyond the upper bound considering the feasibility condition.

The normal headway in the reverse direction is usually the same as the normal headway in the blocked direction. Therefore, from the feasibility perspective, the maximum number of trains in the reverse direction that we may consider short-turning can also be based upon the number of trains that can reach the crossover track and that we are able to short-turn considering the short-turning time and the disruption duration. It is unlikely to be justifiable to hold the trains behind the blockage and short-turn a train after the blockage is cleared. Therefore, we can also heuristically determine the number of trains for which we considering short-turning following the above reasoning, which gives us a tighter bound.

Nevertheless, the exact number of trains to be short-turned within the set of short-turning candidates is ultimately based on the evaluation of the benefit and cost of the impact set, which includes passengers in both directions.

2.3.3 Impact Set

After we have determined the control set, we can determine the corresponding impact set. Obviously, control strategies affect the trains and stations within the control set. Therefore, the impact set must include the control set.

Ahead of the blockage and beyond the control set downstream, stations with significant boardings, and thus, significant waiting cost, should be included. Moreover, if the disruption occurs near the terminal, or if there are potential short-turns, we should also consider impacts in the reverse direction. To simplify computation, we can assume that headways beyond the control set do not vary downstream or in the other direction. Thus, we can include all the stations with significant boardings ahead of the blockage or in the reverse direction. To reflect the uncertainty of the costs, we can use smaller weights for waiting costs at stations far down the line in the impact set in both directions.

Behind the blockage, since it may require more trains than those in the control set to clear the passengers accumulated during the disruption, we may include a set of trains in addition to the control set behind the blockage as part of the impact set. From the passenger's perspective, if a passenger can board the first train to arrive without waiting for a longer than normal headway, he/she will not perceive a delay, although the train may be somewhat crowded. After the blockage clearance, trains behind the blockage usually have short headways. Hence, if no passenger needs to wait for a second train to board, we are guaranteed that no passenger will perceive a delay. That is, we just need to include enough trains so that the last train in this set will leave no passengers behind.

We can use the following heuristic method to determine the number of trains needed to clear all the affected passengers behind the blockage. Assuming we do not apply any control strategies, we can calculate the number of passengers left behind by each train behind the blockage at each station within the impact set, based on the length of blockage, the train capacity, the running time between stations, the minimum headway, and the upper bound on dwell time at each station. When a train leaves no passengers behind, it can be the last train behind the blockage to be included in the impact set.

Figure 2-3 shows the initial situation of a simple example of heuristic determination of the number of trains needed to be included in the impact set behind the blockage. We assume stations A and B are the only stations in the impact set. Alighting fractions are negligible. Other parameters are listed as follows. Suppose the normal headway is six minutes.

Passenger Arrival Rate:	40/min at A, 30/min at B
Minimum Separation:	2 minutes
Running Time between A and B:	5 minutes
Dwell Time Upper Bound:	1 minute at each station
Train Capacity:	900
Initial Load:	100 (T1, T2, T3)
Length of Disruption:	20 minutes

We assume that we do not apply any control strategy. With real time information on the initial train location, we can estimate the approximate departure time and load of each train at each station.

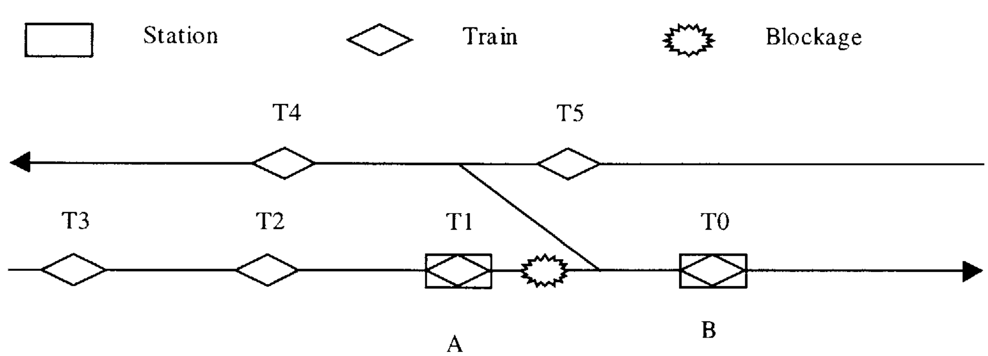


Figure 2-3 Heuristic Determination of the Trains to be Included in the Impact Set

The results are shown in Table 2-1. (d_k = departure time at station k, dh_k = departure headway at station k, l_k = departure load at station k, p_k = passenger left at station k) In this case, no passenger will be left behind after train T3, even if we do not apply any control strategy. Therefore, passengers boarding the trains following T3 will perceive no delay, or even less waiting time because of the reduced headway, and T3 can be the last train behind the blockage to be included in the impact set.

Table 2-1 Results of the Impact Set Example

Train	d_A	dh_k	l_k	p_k	d_B	dh_B	l_B	p_B
T0					0			
T1	20	26	900	240	26	26	900	780
T2	23	3	460	0	29	3	900	430
T3	26	3	220	0	32	3	740	0

If the control set includes more than that determined by this heuristic method, the impact set should include the same set of trains.

2.4 Disruption Control Formulation

2.4.1 Objective Function and Modeling Methods

A common cost function to be minimized is given by function (2-1).

$$\sum_{i\in T}\sum_{k\in S}\{\frac{A_k}{2}h_{i,k}^{\ 2} + p_{i,k}(d_{i+1,k} - d_{i,k}) + U^{iw}[\frac{A_k}{2}ht_{i,k}^{\ 2} + (l_{i,k} - A_k ht_{i,k})ht_{i,k}]\} \qquad (2\text{-}1)$$

We assume that passenger arrival rate at any station is constant over the time period of interest. The first term is the passenger platform waiting time for all trains and stations within the impact set. The second term is the additional platform waiting time for passengers who are left behind or skipped by a train and have to wait for another train. The third term is the in-vehicle waiting time for passengers who arrive during the holding interval. The last term is in-vehicle waiting time for passengers who are on-board when holding starts. Since passengers may perceive in-vehicle waiting time to be less onerous than platform waiting time, we use weight $U^{iw} \leq 1$ for these two terms.

In order to calculate the platform waiting time, we need to know the arrival time of the preceding train. However, the preceding train varies in different parts of the system. As shown in Figure 2-4, assume that there are only two crossover tracks, and there is a branch on the primary transit line. Trains can branch off the main line or be short-turned at crossover tracks. Hence, there can be different preceding trains for a certain train in different parts of the system, and some trains may not operate on parts of the transit line.

To determine the preceding train, we first define segments. A segment is a track section within which the order of trains can not change, while across segments the order of trains may change. The last station in a segment is usually either a terminal, where trains can be pulled out of or inserted into service, or is associated with a crossover track or a junction point, where the sequence of trains may change. We use m to denote a segment, and m' to denote the reverse segment. For example, segment 6 is the reverse segment of segment 3 in Figure 2-4.

As shown in Figure 2-4, the preceding train does not change within any of the 8 segments identified. However, from segment 1 to segment 2, trains may branch off the main line, and so the sequence of trains may change. From segment 5 to segment 6, if we decide to short-turn train T3 at crossover track 2, the predecessor of T3 will be T1 instead of T2 on segment 6, while the predecessor of T2 will be T3 instead of T1.

Therefore, segment occupancy binary variables are used by train and segment to determine whether a certain train operates on a certain segment. Across segments, a set of potential candidate predecessors for a train is defined if the predecessor of that train is undetermined. For example, the set of potential predecessors for train T2 on segment 6 in Figure 2-4 includes train T3 and T1. Within the set, predecessor binary variables are used for each potential candidate to determine which will be the predecessor. Obviously, both segment occupancy variables and predecessor variables are constrained by the control strategies.

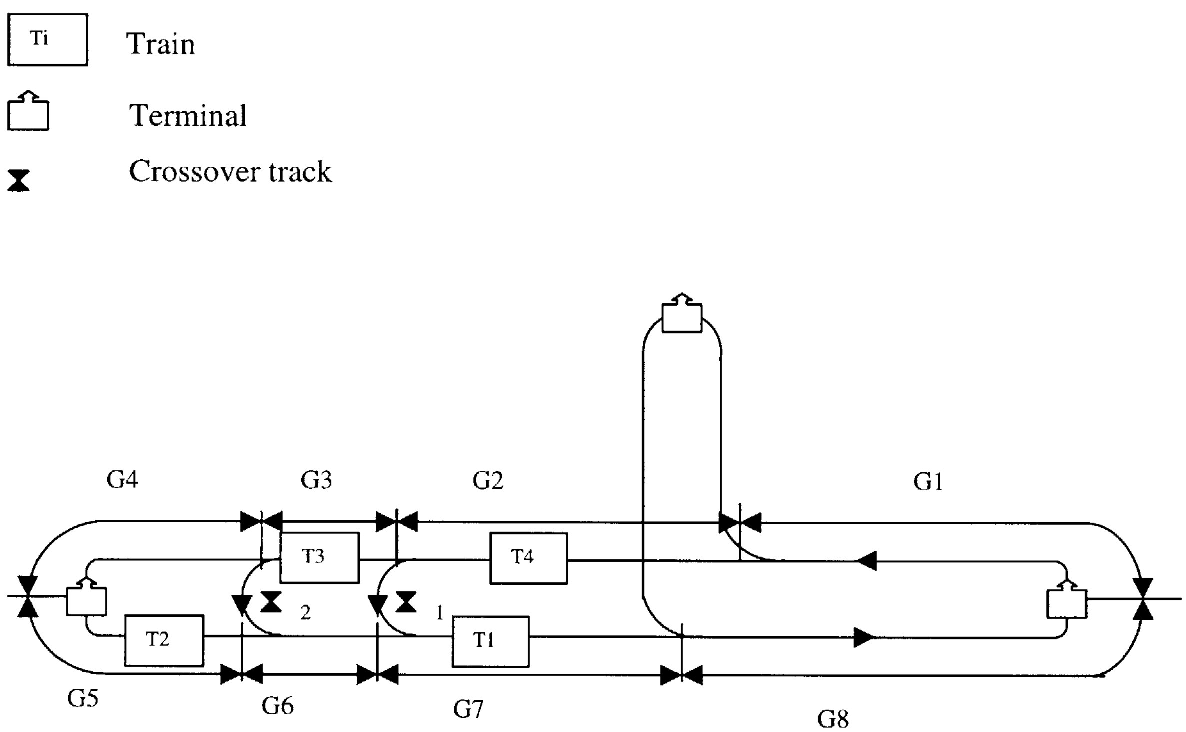


Figure 2-4 Definition of Segment

2.4.2 Dwell Time Function

Dwell time is important in determining the departure time for trains at stations. It is related to the number of passengers alighting and boarding and the crowding condition. As discussed in Section 2.1, the marginal time for a passenger to alight/board the train increases with the level of crowding, as shown in Figure 2-5. In Figure 2-5 (a), two linear functions are used to approximate the dwell time function. Since there were not enough data to estimate the linear function for the non-crowded range, the following function is used to approximate the dwell time, as shown in Figure 2-5 (b)

$$dw = Max\ (C_0, C_1 + C_2 \cdot Alightings + C_3 \cdot Boardings),$$

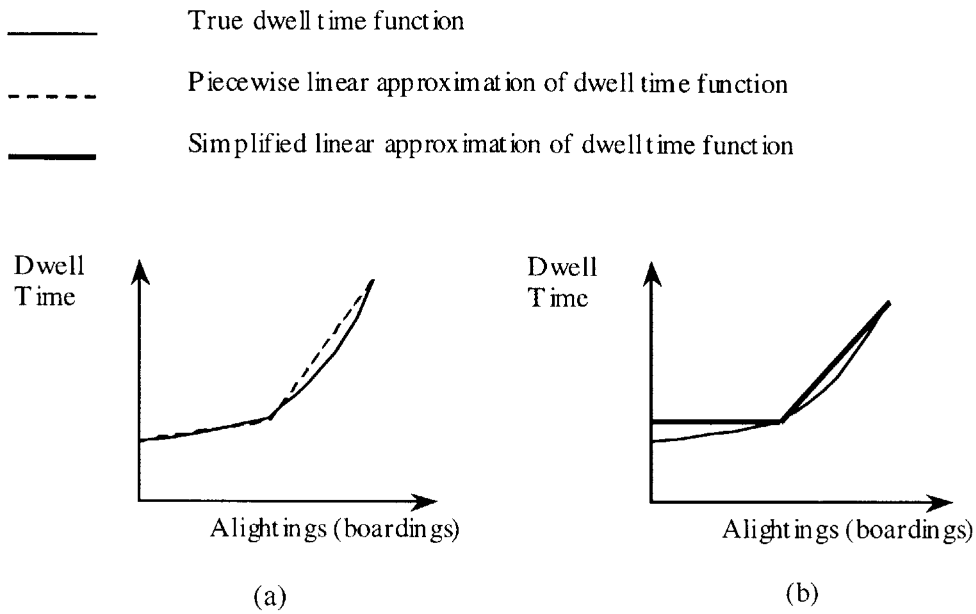


Figure 2-5 Dwell time functions

In addition, under non-crowded conditions, the number of alightings and boardings may not have a strong linear relation with the dwell time. For example, provided other conditions are the same, the dwell time when there are three passengers boarding may not be much different from that when there are six passengers boarding. For detailed discussion of dwell time function, see Lin & Wilson (1993), Song (1998).

2.4.3 Decision Variables

1. Short-turn decision variable

$$st_{i,m} = \begin{cases} 1, \text{ if train } i \text{ is short-turned on segment } m \text{ (the end of } m \text{ is a crossover track)} \\ 0, \text{ otherwise.} \end{cases}$$

2. Express decision variable

$$sk_{i,k} = \begin{cases} 1, \text{ if train } i \text{ skips station } k. \\ 0, \text{ otherwise.} \end{cases}$$

3. Departure time of train i at station k, $d_{i,k}$

   For holding, once the departure time at each station is determined, the holding time at each station is also determined.

4. Predecessor binary variable

$$y_{j,i,m} = \begin{cases} 1, \text{ if train } j \text{ precedes train } i \text{ on segment } m. \\ 0, \text{ otherwise.} \end{cases}$$

On segments defined by crossover tracks, the short-turn variables completely constrain the predecessor binary variables. However, at the junction point, control decision variables alone may not determine the sequence of trains. For example, if a disruption occurs on one of the branches, the optimal strategy to reduce the impact on that branch may not be feasible system-wide because there may be conflict at the junction point. The predecessor binary variables have to be combined with control decision variables to determine the optimal control strategies system-wide.

As discussed in Section 2.3.1, there are segment occupancy binary variables to determine whether trains operate on segments. However, unlike the predecessor binary variable, the segment occupancy variable is completely constrained by the short-turn decision variable. That is, once we determine the short-turn decision variables, the segment occupancy variables are also determined. Therefore, segment occupancy variables are equivalent to the short-turning decision variables. They are used to simplify presenting the formulation. Similarly, binary variables indicating starting of express section are also completely constrained by express decision variables, and, therefore, are not decision variables.

2.4.4 Real Time Disruption Control Model

The objective function for the disruption control model is show in (2-2). (2-3) to (2-44) are constraints.

Min

$$\sum_{i\in T}\sum_{m\in G} so_{i,m}\sum_{k\in m}\{\frac{A_k}{2}h_{i,k}{}^2 + p_{i,k}(d_{i+1,k}-d_{i,k}) + U^{iw}[\frac{A_k}{2}ht_{i,k}^2 + (l_{i,k}-A_k ht_{i,k})(ht_{i,k}+dw_{i,k})]\} \quad (2\text{-}2)$$

Subject to:

$d_{i,k}$ - $d_{i,k-1}$ - R_k - $dw_{i,k} \geq M\ (so_{i,m}$ -1), $\forall\ i\in T,\ k\in m,\ m\in G$. (2-3)

$d_{i,k}$ - $dp_{i,k+1}$ - H_{k+1} + $R_{k+1} \geq M\ (so_{i,m}$ -1), $\forall\ i\in T,\ k\in m, m\in G$. (2-4)

$dp_{i,k}$ - $d_{j,k} \geq M\ (y_{j,i,m}$ - 1), $\forall\ i\in T,\ j\in T^p_{i,m},\ k\in m$. (2-5)

$dp_{i,k}$ - $d_{j,k} \leq M$ (1- $y_{j,i,m}$), $\forall\ i\in T,\ j\in T^p_{i,m},\ k\in m$. (2-6)

$ht_{i,k}$ - $d_{i,k}$ + $d_{i,k-1}$ + R_k + $dw_{i,k} \geq M\ (so_{i,m}$ -1), $\forall\ i\in T, k\in m,\ k\notin S^t,\ m\in G$. (2-7)

$h_{i,k} \geq d_{i,k}$ - $dp_{i,k}$ - $ht_{i,k}$ + $M\ (so_{i,m}$ -1), $\forall\ i\in T, k\in m,\ k\notin S^t,\ m\in G$. (2-8)

$a_{i,t}$ - $d_{i,t-1}$ - $R_t \geq M\ (so_{i,m}$ -1), $\forall\ i\in T,\ t\in S^t,\ t\in m$. (2-9)

$d_{i,t}$ - $a_{i,t}$ - $t_{mc} \geq M\ (so_{i,m}$ -1), $\forall\ i\in T, t\in S^t,\ t\in m$. (2-10)

$d_{i,t}$ - $Sch_{i,t} \geq M\ (so_{i,m}$ -1), $\forall\ i\in T, t\in S^t,\ t\in m$. (2-11)

$a_{i,t}$ - $dp_{i,t}$ - $M\ late_{i,t} \leq 0$, $\forall\ i\in T, t\in S^t$. (2-12)

$ht_{i,t}$ - [($d_{i,t}$ - $a_{i,t}$) $late_{i,t}$ +($d_{i,t}$ - $dp_{i,t}$)(1- $late_{i,t}$)] $so_{i,m}$ = 0, $\forall\ i\in T, t\in S^t,\ t\in m$. (2-13)

$h_{i,t}$ - ($a_{i,t}$ - $dp_{i,t}$) $late_{i,t}\ so_{i,m}$ = 0, $\forall\ i\in T, t\in S^t,\ t\in m$. (2-14)

$d_{i,k'}$ - $d_{i,k}$ - $t^s_{k,k'}$ - $dw_{i,k'} \geq M\ (st_{i,m}$ -1),

$\forall\ i\in T,\ k\in m,\ m\in G$, a crossover track exists at the end of m. (2-15)

$ht_{i,k'}$ - $d_{i,k'}$ + $d_{i,k}$ + $t^s_{k,k'}$ + $dw_{i,k'} \geq M\ (st_{i,m}$ -1)

$\forall\ i\in T,\ k\in m,\ m\in G$, a crossover track exists at the end of m. (2-16)

$$dw_{i,k} \geq Max\{ C_0,\ C_1+C_2\, b_{i,k} + C_3\, al_{i,k} \}(1\text{-}sk_{i,k})\, so_{i,m}, \quad \forall\ i \in T,\ k \in m,\ m \in G. \quad (2\text{-}17)$$

$$r'_{i,k} - A_k\, h_{i,k} - (1\text{-}\ Q_k)\ l_{i,k-1} - p^{p}_{i,k} = 0, \quad \forall\ i \in T,\ k \in S. \quad (2\text{-}18)$$

$$p^{e}_{i,k} - r'_{i,k}\,[Q_{k+1}\, sk_{i,k+1} + \sum_{u=k+2}^{S} Q_u\,(\prod_{v=k+1}^{u-1}(1-Q_v))sk_{i,u}] = 0, \quad \forall\ i \in T,\ k \in S. \quad (2\text{-}19)$$

$$r_{i,k} - r'_{i,k} + p^{e}_{i,k} = 0, \quad \forall\ i \in T,\ k \in S. \quad (2\text{-}20)$$

$$r_{i,k} - L\ v_{i,k} \geq 0, \quad \forall\ i \in T,\ k \in S. \quad (2\text{-}21)$$

$$r_{i,k} - M\ v_{i,k} \leq L, \quad \forall\ i \in T,\ k \in S. \quad (2\text{-}22)$$

$$\{ b_{i,k} - [\, A_k\ h_{i,k}\,(1\text{-}v_{i,k}) + (L - (1 - Q_k)\, l_{i,k-1})\, v_{i,k}\,]\}\, sk_{i,k}\ so_{i,m} = 0, \quad \forall\ i \in T,\ k \in S. \quad (2\text{-}23)$$

$$(al_{i,k} - l_{i,k-1}\, Q_k)(1\text{-}sk_{i,k})\, so_{i,m} = 0, \quad \forall\ i \in T,\ k \in S. \quad (2\text{-}24)$$

$$p^{p}_{i,k} - p_{j,k} \geq M\,(y_{j,i,m} - 1), \quad \forall\ i \in T,\ j \in T^{p}_{i,m},\ k \in m. \quad (2\text{-}25)$$

$$p^{p}_{i,k} - p_{j,k} \leq M\,(1\text{-}\ y_{j,i,m}), \quad \forall\ i \in T,\ j \in T^{p}_{i,m},\ k \in m. \quad (2\text{-}26)$$

$$\{ l_{i,k} - [\, l_{i,k-1}\ sk_{i,k} + (L\ v_{i,k} + r_{i,k}\,(1\text{-}v_{i,k}))(1\text{-}sk_{i,k})]\}\ so_{i,m} = 0,$$

$$\forall\ i \in T,\ k \in m,\ m \in G. \quad (2\text{-}27)$$

$$p_{i,k} - [(\, p^{p}_{i,k} + A_k\ h_{i,k})\, sk_{i,k} + [(r_{i,k} - L)\, v_{i,k} + p^{e}_{i,k}\,(1\text{-}v_{i,k})](1\text{-}sk_{i,k})]\, so_{i,m} \geq 0,$$

$$\forall\ i \in T,\ k \in m,\ m \in G. \quad (2\text{-}28)$$

$$p_{i,k} - (\, p^{p}_{i,k} + A_k\ h_{i,k} + l_{i,k-1}\, Q_k)\, st_{i,m} \geq 0,$$

$\forall\ i \in T, k \in m, (k+1) \in (m+1), m \in G$, a crossover track exists at the end of m. (2-29)

$st_{i,m} - so_{i,m} \leq 0, \quad \forall\ i \in T,\ m \in G$, a crossover track exists at the end of m. (2-30)

$so_{i,m+1} + st_{i,m} \leq 1, \quad \forall\ i \in T,\ m \in G$, a crossover track exists at the end of m. (2-31)

$so_{i,m} - so_{i,m-1} \leq 0, \quad \forall\ i \in T,\ m \in G$, the margin between m and (m-1) is not a crossover track. (2-32)

$y_{i,i+1,m} + st_{i,m} + st_{i+1,m} \geq 1, \quad \forall\ i \in T,\ m \in G$, the end of m is a crossover track. (2-33)

$y_{i,j,m'} - st_{i,m} \leq 0, \quad \forall\ i \in T^{p}_{j,m},\ m \in G$, the end of m is a crossover track. (2-34)

$$\sum_{m \in G} st_{i,m} \leq 1, \quad \forall\ i \in T. \quad (2\text{-}35)$$

$$\sum_{j\in T^{P}_{i,m}} y_{j,i,m} = 1, \qquad \forall\, i\in T,\ m\in G. \qquad (2\text{-}36)$$

$$\sum_{j\in T^{S}_{i,m}} y_{i,j,m} = 1, \qquad \forall\, i\in T,\ m\in G. \qquad (2\text{-}37)$$

$$y_{i,j,m} + y_{j,i,m} \le 1, \qquad \forall\, i\in T^{P}_{j,m},\ j\in T^{P}_{i,m},\ m\in G. \qquad (2\text{-}38)$$

$$y_{j,i,m} - so_{i,m} \le 0, \qquad \forall\, j\in T^{P}_{i,m},\ m\in G. \qquad (2\text{-}39)$$

$$v_{i,k} - v_{i,k-1} - sk_{i,k} \ge -1, \qquad \forall\, i\in T,\ k\in S. \qquad (2\text{-}40)$$

$$v_{i,k} - v_{i,k-1} + sk_{i,k} \le 1, \qquad \forall\, i\in T,\ k\in S. \qquad (2\text{-}41)$$

$$se_{i,k} + sk_{i,k-1} - sk_{i,k} \ge 0, \qquad \forall\, i\in T,\ k\in S. \qquad (2\text{-}42)$$

$$\sum_{k\in S} se_{i,k} \le 1, \qquad \forall\, i\in T. \qquad (2\text{-}43)$$

$$d_{i_{BL},k_{BL}} \ge t_{BL}. \qquad (2\text{-}44)$$

$$y_{i,j,m},\ v_{i,k},\ v'_{i,k},\ sk_{i,k},\ se_{i,k},\ st_{i,m},\ so_{i,m} \in \{0,\ 1\} \qquad \forall\, i\in T,\ k\in S,\ m\in G.$$

$$d_{i,k},\ dw_{i,k},\ dp_{i,k},\ h_{i,k},\ p_{i,k},\ pp_{i,k},\ r_{i,k},\ r'_{i,k},\ p^{e}_{i,k},\ l_{i,k} \ge 0,\ \forall\, i\in T,\ k\in S,\ m\in G.$$

The objective function is an extension of function (2-1). The stations in the impact set are divided into segments. If a train does not operate on a segment, there will be no cost associated with it on that segment. In addition, dwell time is included in the cost function as explained below.

As shown in Figure 2-6, $dp_{i,k}$ is the departure time of the train preceding train i at station k, $a_{i,k}$ is the arrival time of train i at station k, $dw_{i,k}$ is the dwell time of train i at station k, $h_{i,k}$ is the term used to measure the platform waiting time, and $ht_{i,k}$ is the holding time of train i at station k, which we count as in-vehicle waiting time. Due to the structure of the dwell time function used, it is difficult to impose constraints to ensure that the dwell time is exactly equal to the time for the passengers to alight and board. Hence, the solution may potentially over-estimate the dwell time in exchange for underestimating holding time. The overestimate would be reflected in platform waiting time. Although platform waiting time has larger weight than in-vehicle waiting time, the overall cost may be smaller.

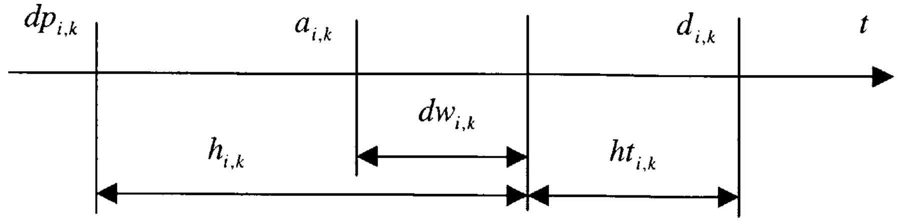


Figure 2-6 Headway and Holding Time

As shown in Figure 2-6, suppose we want to increase the departure time to $(d_{i,k}+\Delta)$. This can be achieved either by increasing holding time $ht_{i,k}$ to $(ht_{i,k}+\Delta)$ or increasing dwell time $dw_{i,k}$ to $(dw_{i,k}+\Delta)$. (In this case, the dwell time is larger than the actual time for passengers to alight and board the train). In the following part, we would like to compare the changes in objective function (2-1) due to the change of holding time or dwell time. Since the departure time is $(d_{i,k}+\Delta)$ in either case, the change of terms only related to the departure times are not considered.

Suppose the holding time increases by Δ, (2-45) gives the corresponding change in objective function (2-1) (The second term is only related to departure time, and, hence, is not considered.)

$$U^{iw}\{[\frac{A_k}{2}(ht_{i,k}+\Delta)^2 - \frac{A_k}{2}ht_{i,k}^2]+[(l_{i,k}-A_k(ht_{i,k}+\Delta))(ht_{i,k}+\Delta)-(l_{i,k}-A_k ht_{i,k})ht_{i,k}]\}$$
$$\approx U^{iw}(l_{i,k}-A_k ht_{i,k})\Delta. \qquad (2\text{-}45)$$

In contrast, if the dwell time is over-estimated by Δ, since the platform-waiting time $h_{i,k}$ includes the dwell time $dw_{i,k}$, the change is

$$\frac{A_k}{2}(h_{i,k}+\Delta)^2 - \frac{A_k}{2}h_{i,k}^{\ 2}$$
$$\approx A_k\, h_{i,k}\,\Delta. \qquad (2\text{-}46)$$

The value of (2-46) can be smaller than that of (2-45) if there is significant load on the train after unloading. For this reason, the same cost for dwell time is imposed as that for holding time.

We can show that the overall cost by increasing $dw_{i,k}$ to $(dw_{i,k}+\Delta)$ is larger than that by increasing $ht_{i,k}$ to $(ht_{i,k}+\Delta)$ under the cost structure of objective function (2-2).

By increasing $dw_{i,k}$ by Δ, we increase $h_{i,k}$ by Δ. The cost change is

$$A_k h_{i,k}\,\Delta + (l_{i,k} - A_k\, ht_{i,k})\Delta. \qquad (2\text{-}47)$$

The cost increase due to increasing $ht_{i,k}$ to $(ht_{i,k}+\Delta)$ is given by

$$U^{iw}\,(l_{i,k} - A_k\, ht_{i,k})\,\Delta. \qquad (2\text{-}48)$$

The value of (2-47) is no smaller than that of (2-48) because U^{iw} is no larger than one. Therefore, the solution will never over-count the dwell time if holding is an option.

If we use objective function (2-1), we in fact impose an assumption that the on-board delay is only introduced by the holding time. That is, running time between stations and dwell time at stations are the same as that passengers incur under normal conditions. However, during a disruption, the dwell time is larger than normal. Therefore, it theoretically makes sense to considering the dwell time cost for passengers on-board even if we do not need to ensure exact dwell time. The problem of objective function (2-2) is that it double-counts the dwell time for passengers boarding the train before holding starts. The platform waiting time of these passengers, the first term of (2-2), has included the dwell time, but the last term of (2-2) also included the dwell time as in-vehicle delay. However, since the dwell time is short, the over-counting is not significant.

Constraints (2-3) to (2-44) are explained below.

Constraint (2-3) ensures that the departure time of train i at station k is no earlier than its departure time from previous station $(k-1)$ plus running time between those two stations and dwell time at station k.

In the general system model, we use minimum separation to ensure a minimum distance between trains. Constraint (2-4) ensures that only if train i can run without stopping from station $k-1$ to k and the arrival headway at k is no less than the minimum separation at k, can it depart station $k-1$. There is a variation of (2-4) near at the terminus. Let the queuing station be defined as the last station before the terminal station. There are usually two platforms at the terminal. If both platforms are occupied, the train departing from the queuing station has to stop at some intermediate point until one of the platforms clears. The minimum separation can be used to model such a queuing phenomenon. That is, the departure time of train i from the queuing station plus the running time from the queuing station to the terminal should be no earlier than the departure time of the second preceding train from the terminal station. Constraint (2-3) and (2-4) apply only if train i operates on segment m, that is if $so_{i,m} = 1$.

Constraints (2-5) and (2-6) determine the departure time of the train preceding train i with the predecessor indicator $y_{i,j,m}$. Constraint (2-7) determines the holding time and (2-8) determines the platform waiting time. Both of them apply only if $so_{i,m} = 1$.

Constraints (2-9) to (2-14) are terminal constraints. For a terminal station, the following two scenarios are identified.

(1) There is no train at the platform when train i arrives at the terminal (Figure 2-7).

At the terminal, the departure time can not be earlier than the arrival time plus the minimum recovery time. Since the minimum recovery time is enough for all passengers to alight from and board the train, we can assume that all passengers board the train during the recovery time. In this case, platform waiting time is from the departure time of the predecessor of train i to the arrival of train i, which is represented by $h_{i,k}$. The in-vehicle waiting time is from the arrival time to the departure time of train i, which is taken as the holding time $ht_{i,k}$ at the terminal.

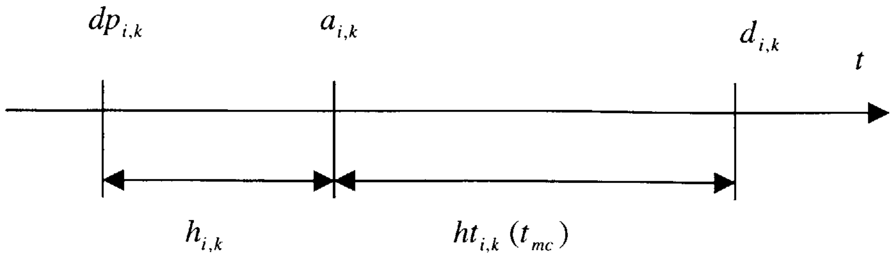


Figure 2-7 Terminal Scenario 1

(2) There is a train at the platform when train i arrives at the terminal (Figure 2-8).

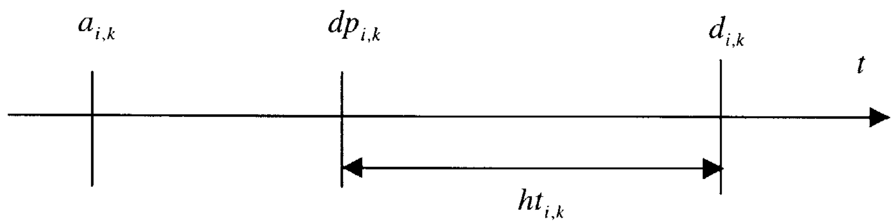


Figure 2-8 Terminal Scenario 2

In this case, there is no platform waiting time for train i between the arrival of train i and the departure of its predecessor. The in-vehicle waiting time is from the departure time of the predecessor of train i to the departure time of train i.

The above two scenarios are identified by the *late* binary variable, which identifies whether the arrival time of train i is later than the departure time of its predecessor. Constraint (2-9) ensures that the arrival time of train i at terminal t is no earlier than the departure time from the station prior to the terminal plus the running time. Constraint (2-10) ensures that the departure time of train i at the terminal is no earlier than the arrival time plus the minimum recovery time. Constraint (2-11) ensures that the departure time of train i is no earlier than the scheduled departure time. All these constraints apply only if train i operates on the terminal segment. Constraint (2-12) sets the *late* index to 1 if the arrival time of train i is later than the departure time of its predecessor from the terminal. Constraints (2-13) and (2-14) determine the holding time and platform and in-vehicle waiting time based on the *late* and $so_{i,m}$ variables.

Constraint (2-15) is a short-turning constraint. If a train is to be short-turned via the crossover track on segment m, its departure time at the short-turning destination station can be no earlier than its departure time at the short-turning origin station plus the constant short-turning time and the dwell time at the destination station. Constraint (2-16) determines the corresponding holding time after short-turning.

Constraint (2-17) is the dwell time constraint. Only if a train operates on the segment and does not skip the station, does it incur a dwell time, which has the structure discussed in the Section 2.4.2.

Constraint (2-18) determines the number of potential riders for a train at a station assuming it will not skip any station. It includes the passengers on the train and the passengers on the platform. Constraint (2-19) determines the number of passengers that will be left because the train will skip their destinations. It uses the alighting ratio at each station to estimate the potential riders whose destinations are at each station, then uses the expressing decision variables to determine whether those people have to wait for the next train. For example, suppose there are r' potential riders at station k if the train does not skip any station. Based upon the alighting ratio, we can estimate how many of those r' potential riders will stay on the train beyond station $(k+q-1)$, and how many people will alight at station $(k+q)$. Then, if the train skips station $(k+q)$, those people whose destination is $(k+q)$ will have to wait at k for the next train. Constraint (2-20) determines the potential riders excluding those who can not board the train due to expressing.

Constraints (2-21) and (2-22) determine the binary load variable value, so that if and only if the train is loaded to capacity $v_{i,k} = 1$. Constraint (2-23) determines the boarding passengers based on the binary load variable, expressing decision variable and segment occupancy variable. Constraint (2-24) determines the alighting passengers if the train operates on the segment and stops at the station. Constraint (2-25) and (2-26) use the predecessor index to determine the number of passengers left by the preceding train at a station. Constraint (2-27) is the load constraint. If the train skips station k, its departure load at k will be the same as that at $(k-1)$. If the train stops at the station, and if it is loaded to capacity, the load will be equal to the maximum load, otherwise, it will be equal to the number of riders. This constraint is valid only if the train operates on segment m.

It is assumed that the train to be short-turned must stop at the station immediately before the crossover track. Constraint (2-28) determines the number of passengers left by a train. It includes those people who can not board the train due to either expressing or train capacity. Constraint (2-29) determines the number of passengers left by the short-turned train at the station immediately before the crossover track. All passengers on-board have to alight the train and wait for the next train.

Constraint (2-30) ensures that a train can be short-turned on segment m only if it operates on segment m. Constraint (2-31) ensures that a train will not operate on segment $(m+1)$ if it is short-turned on segment m. Constraint (2-32) ensures that any train not operating on segment $(m-1)$, will not operating on the following segment m, either. It does not apply to the situation where two segments are formed by a crossover track because trains may be short-turned from the other direction. At the merge point, this constraint means that if any train from one of the branches does not operate on the last segment on that branch (trunk portion), it will not operate on the first segment of the trunk portion (that branch). Constraint (2-33) ensures that if neither train i and nor its initial successor train $(i+1)$ is short-turned on segment m, train i will be the predecessor of train $(i+1)$ on segment m. Constraint (2-34) states that only if train i is turned back on segment m, may it be the predecessor of train j on the opposite segment m' in the reverse direction. Constraint (2-35) ensures that a train can be turned back at only one of the available crossover tracks. Constraint (2-36) ensures that there can be only one predecessor of a train on a certain segment. Constraint (2-37) ensures a train to be the predecessor of only one train. Constraint (2-38) prevents any train that is the predecessor of another train also being the successor of that train. Constraint (2-39) states that any train not operating on a segment can have no predecessor on that segment. Constraint (2-40) and (2-41) state that if a train skips a station, its load must be the same as the load before that station, since there is no boarding/alighting at the station.

Constraint (2-42) determines the express starting binary variables, which is used to show the number of the express segments. If the train stops at station $k-1$ but skips station k, there must be an express segment starting from station k. Generally speaking, multiple express segments may cause confusion to the passengers, complicate the announcements, and waste needed capacity. Therefore, constraint (2-43) ensures there is only one express segment. Constraint (2-44) ensures that the blocked train can depart from the blocked point only after the blockage has been cleared.

2.5 Model Implementation

The above model has a non-linear objective function and non-linear constraints. To solve it using a linear solver, we need to transform it into a linear formulation.

2.5.1 Approximation of the Objective Function

The objective function contains quadratic functions with respect to the out-of-vehicle waiting time and holding time. Piece-wise linear functions can be used to approximate these quadratic functions. The additional waiting time for passengers left by trains, which is the product of the number of passengers left and the departure interval, and the in-vehicle delay for passengers on-board, which is the product of the passenger load when holding starts and the holding time, are non-separable terms. In other words, we can not separate the two variables and can not use linear functions to approximate them. Therefore, approximate values are applied to one of the two variables.

1. Piece-wise Linear Approximation

As shown in Figure 2-9, a quadratic function can be approximated by piece-wise linear functions.

Suppose $x \in [x_i, x_{i+1}]$, then

$x = \lambda x_i + (1-\lambda) x_{i+1}$, $x^2 = [\lambda x_i + (1-\lambda) x_{i+1}]^2$, $0 \le \lambda \le 1$.

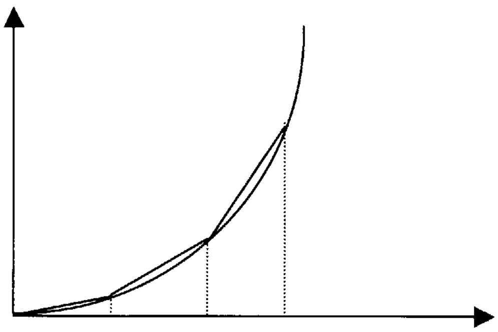


Figure 2-9 Piece-wise Approximation

Let $y_i'(x) = \lambda x_i^2 + (1-\lambda) x_{i+1}^2$ denote the piece-wise linear function to approximate x^2 on $[x_i, x_{i+1}]$. Obviously, $y_i'(x)$ is no less than x^2 because the quadratic function is convex. The residual is:

$$y_i'(x) - x^2 = \lambda x_i^2 + (1-\lambda) x_{i+1}^2 - [\lambda x_i + (1-\lambda) x_{i+1})]^2$$
$$= \lambda (1-\lambda)(x_{i+1} - x_i)^2$$

Therefore, the maximum residual is achieved when $\lambda = 0.5$, and the maximum residual is:

$$0.25\,(x_{i+1} - x_i)^2$$

Clearly, the smaller the interval we use the more precise the approximation.

Let $y_i(x) = x_i^2 + k_i(x - x_i)$ denote the extended line that contains linear piece-wise function $y_i'(x)$ over $[x_i, x_{i+1}]$, k_i = the slope of piece i, b_i = the intercept of extended line.

Suppose $x \in [x_i,\ x_{i+1}]$, the following function gives the approximate value from $y_i'(x)$, the piece-wise function to approximate the quadratic function over $[x_i,\ x_{i+1}]$, because the quadratic function is convex.

$$y = \underset{n}{Max}\ y_n(x) = \underset{n}{Max}\{k_n\ x + b_n\} = y_i'(x) \approx x^2$$

The following is an example of using piece-wise linear functions to approximate the first quadratic term in (2-2).

$$h_{i,k}{}^2 \approx z_{i,k} = \underset{n=1,\dots,N}{Max}\{a_n\ h_{i,k} + b_n\}.$$

Or

$z_{i,k} \geq a_n\ h_{i,k} + b_n$, for $n = 1, 2, \dots$

Similarly, we have

$$ht_{i,k}^2 \approx zt_{i,k} = \underset{n=1,\dots,N}{Max}\{a_n'\ ht_{i,k} + b_n'\}.$$

Or

$zt_{i,k} \geq a_n'\ ht_{i,k} + b_n'$, for $n = 1, 2, \dots$

2. Approximation of the Non-Separable Terms

The additional waiting time for passengers left by trains and the in-vehicle delay for passengers on-board trains are non-separable terms because they are the products of two variables that are related to the decision variables. To linearize these terms, we use approximate values for one of the variables in each case.

Trains initially ahead of the blockage may be held to even out headways. However, it will never make sense to hold these trains for so long that they will leave passengers behind at stations. If a train is to leave passengers behind after holding, a better solution can always be achieved by holding the train for less time with reduced in-vehicle delay. Therefore, if the system meets the capacity requirement under normal operating conditions, no passengers would be left behind by trains ahead of the blockage. We only need to consider passengers left by trains initially behind the blockage. Since the headways of trains behind the blockage are usually constrained by the minimum safe separation condition, the minimum headway, which is the sum of minimum safe separation plus dwell time, is a good approximation of the additional waiting time for passengers left behind at a certain station. That is, the product of the actual number of passengers left behind and the minimum headway can be used to approximate the waiting cost of these passengers.

The second term, in-vehicle delay, is the product of the passenger load when holding starts and the holding time. For trains initially ahead of the blockage, normal operation is interrupted only when holding strategies are applied. Hence, the typical passenger load in that time period can be used as an approximation for the number of passengers on-board when holding starts. However, once trains are held, the typical load will be an underestimate of passengers on-board at following stations. Nevertheless, it is less likely to hold the same train at multiple stations ahead of the blockage because there are likely to be more passengers on-board, and the total in-vehicle delay per unit holding time becomes larger and larger. Therefore, the product of the typical passenger load in that time period and the holding time can be used to approximate the in-vehicle delay for passengers on-board trains initially ahead of the blockage when holding starts.

Unlike trains ahead of the blockage, passenger load for trains behind the blockage is usually high. Therefore, typical passenger load in that time period will be an underestimate of the true load. However, before the blockage clearance, passengers who are on-board blocked trains at stations behind the blockage have to wait for the whole blockage period no matter what holding strategies are applied. Hence, their in-vehicle delay is a constant and does not affect the decisions. After the blockage is cleared, trains behind the blockage are clustered together and there is a gap ahead of the first blocked train. Thus, there is no inherent reason to hold these trains unless there is another gap behind, and blocked trains should move ahead as quickly as possible. Therefore, any underestimate of the passenger load should not have a significant impact.

After these approximations, the objective function (2-2) becomes:

$$\sum_{T}\sum_{S}\{\frac{A_k}{2}z_{i,k}+p_{i,k}(H_k+dw_k^0)+U^{iw}[\frac{A_k}{2}zt_{i,k}+l_k^0(ht_{i,k}+dw_{i,k})]\} \qquad (2\text{-}49)$$

where $z_{i,k}$ is used to approximate the quadratic term of normal waiting time for train i at station k, $zt_{i,k}$ is used to approximate the quadratic term of holding time for train i at station k, l_k^0 is the typical passenger load at station k in that time period, H_k is the minimum safe separation (minimum departure-arrival interval) between trains at station k, and dw_k^0 is the typical dwell time at station k in that period.

2.5.2 Transformation of Non-linear Constraints

The formulation also includes numerous non-linear constraints ((2-13), (2-14), (2-17), (2-18), (2-19), (2-23), (2-24), (2-27), (2-28), (2-29)) to ensure that variables have different values under different control interventions. For this reason, integer programming techniques are used to transform those constraints into linear constraints with binary variables and sufficient large numbers M.

These large numbers are the upper bound for different variables and can be determined heuristically. For example, for the load constraint (2-27), train capacity is the upper bound of the load and can be used as the large number. For departure times, assuming we do not apply any control strategies, we can obtain the heuristic bounds of departure times for trains at different stations based upon the running time between stations, maximum dwell time and the duration of blockage.

Constraint (2-19) determines the number of passengers left behind by the express trains. To simplify this problem, the following assumption is made.

A(7) Express announcement is made at the station immediately before the express segment.

Under this assumption, the station immediately before the express segment is the only station where the express train dumps passengers. For disruption control, the possible trains that we may want to express are trains behind the blockage after the blockage is cleared. Since most of the benefit from expressing is the reduced waiting time at stations down the line, we want to express the train as early as possible. Hence, the express announcement can be made before the blockage is cleared at stations behind the blockage to save time, and the express segment can begin with the next station the train arrives at after the blockage is cleared. Therefore, the above assumption is realistic.

In addition to this assumption, a predetermined value is used to approximate the proportion of passengers dumped by the express train and the portion of passengers whose destinations are within the express segment, and, therefore, can not board the express train at the station immediately before the express segment. This group of passengers is usually a small portion of all passengers, and the train following the express train usually has a very small headway. Hence, this simplification does not have significant impact on the control strategies. Intuitively, it is "expensive" to skip stations with large alighting fraction because large numbers of passengers will have to wait for the next train. Hence, it makes sense to limit the express segment according to the alighting ratio at stations. In another word, the maximum express segment starting from each station can be predefined based on the alighting fractions.

Corresponding to this approximation, the stations that can be skipped should be restricted so that the portion of passengers whose destinations are within the express segment should not exceed the assumed approximate portion of passengers dumped by the express train. Otherwise, the cost of leaving passengers due to expressing may be underestimated.

Chapter 3

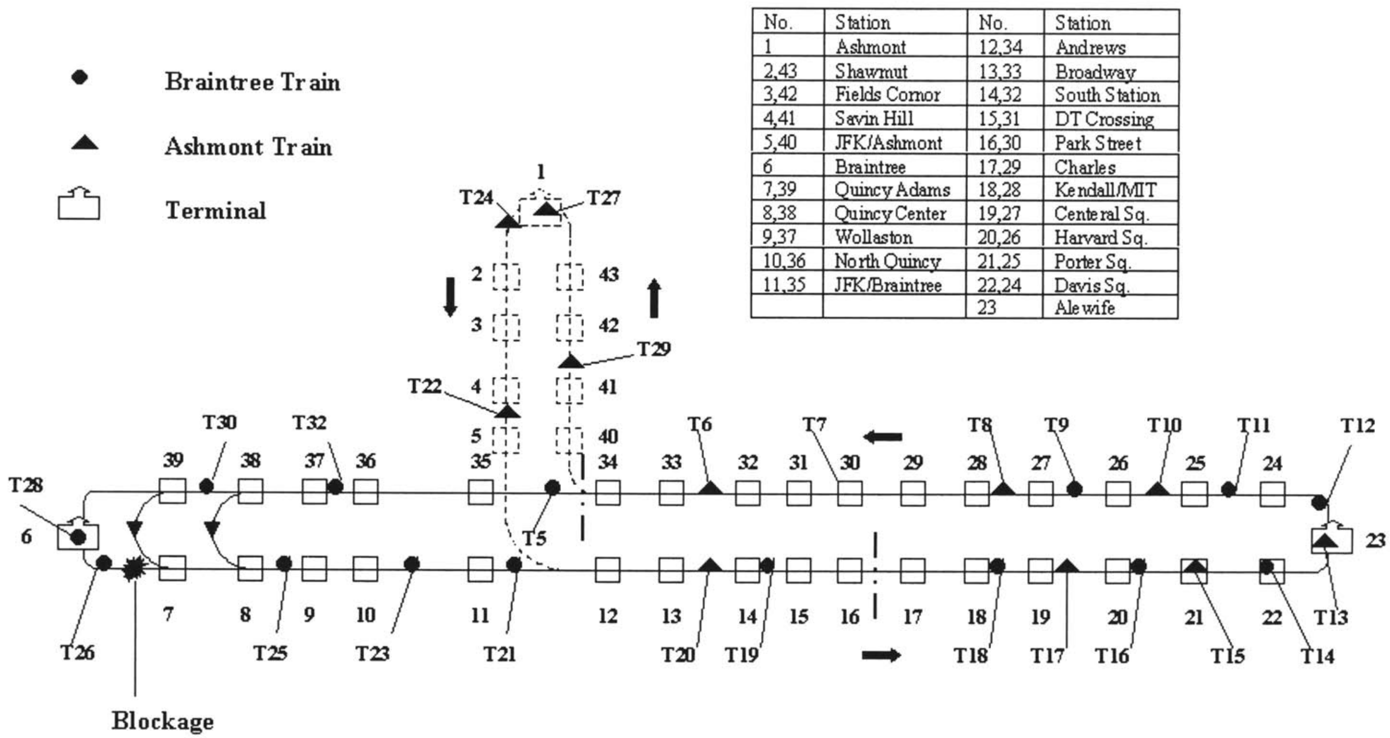
Model Application

In this chapter, the objectives are to test the model, compare the effectiveness of different control strategies, and investigate the sensitivity of the control strategies to the deterministic disruption duration assumption. First, the Massachusetts Bay Transportation Authority Red Line is described and important simplifications are made to the original system to make it easier to interpret the solution and to highlight the effectiveness of different control strategies. Then, the linearized model described in Chapter 2 is applied to a disruption scenario on the modified system. Disruptions of two different lengths are tested: 10-minutes and 20-minutes. Both disruptions occur at 8:15AM during the morning peak period. To look at the impact of the assumption that the disruption duration is known, sensitivity analysis is then applied in order to estimate the change in benefits if the true disruption duration is different from the estimate.

3.1 Problem Description

3.1.1 Description of the MBTA Red Line

The MBTA Red Line system is shown in Figure 3-1. It is a heavy rail system with two branches and a common trunk portion. Each branch is named according to the terminal station at the end of the branch, Ashmont and Braintree. The junction point is effectively at JFK station, where passengers can board trains from (for) either branch, although there are separate tracks for Braintree and Ashmont trains. Alewife is the terminal station at the other end of the trunk portion. There are five stations on the Ashmont Branch, six stations on the Braintree branch, and twelve stations on the trunk portion. For modeling purpose, platforms at the same location but serving different directions are considered to be different stations except for terminal stations. At terminal stations, two trains can be accommodated at the same time.



No.	Station	No.	Station
1	Ashmont	12,34	Andrews
2,43	Shawmut	13,33	Broadway
3,42	Fields Cornor	14,32	South Station
4,41	Savin Hill	15,31	DT Crossing
5,40	JFK/Ashmont	16,30	Park Street
6	Braintree	17,29	Charles
7,39	Quincy Adams	18,28	Kendall/MIT
8,38	Quincy Center	19,27	Centeral Sq.
9,37	Wollaston	20,26	Harvard Sq.
10,36	North Quincy	21,25	Porter Sq.
11,35	JFK/Braintree	22,24	Davis Sq.
		23	Alewife

Figure 3-1 MBTA Red Line and Disruption Location 8:15 AM

Trains are typically dispatched onto the line from Braintree and Ashmont. During the AM peak, there are 11 Ashmont trains and 16 Braintree trains. All trains consist of 6 cars, and each train has a capacity of approximately 960 passengers. The Ashmont and Braintree trains are dispatched at headways of approximately 8 and 6 minutes, respectively. The resulting mean headway on the trunk portion is approximately 3-4 minutes.

3.1.2 Simplified System

Since the blockage in the scenario occurs on one of the two Red Line branches, there are several associated difficulties. First, there may be required (forced) control actions at the junction point. On occasion, trains must be held to ensure that they enter the trunk portion with a safe separation. Second, since a blockage on one of the two branches does not completely disable the whole system, benefits from different control strategies may be confounded due to interaction effects with the other branch at the junction point.

To filter out the "noise" due to the junction point and highlight the performance of the control strategies, the following modifications were made to the Red Line system in this test of the model system. The modified system is shown in Figure 3-1 with solid lines.

1. Trains and stations on the Ashmont branch are not considered.

2. The passenger arrival rate at each station on the trunk portion is scaled by the ratio of Braintree trains to all trains. During peak periods, the ratio of Ashmont and Braintree trains is about 3:4. Therefore, 4/7 is used to scale the passenger arrival rate for stations on the trunk portion. The scaled arrival rates are also shown in Appendix A.

3. At the same time, the minimum headway at each station on the trunk portion is scaled by 7/4 to accommodate trains from the Ashmont branch. Therefore, a solution feasible to the modified system is also feasible to the original system.

After these modifications, the solution to the modified system will not be optimal but will be feasible for the original system. In terms of the solution speed, since the order of trains entering the trunk portion is not considered, fewer binary variables are needed to determine the preceding train on the trunk portion. Therefore, it potentially takes less time to obtain the optimal solution for the modified system.

3.1.3 Disruption Description

The problem analyzed is a blockage between Braintree (Station 6) and Quincy Adams (Station 7) in the inbound direction on the Braintree branch. Train T26 is blocked about half way between Braintree and Quincy Adams. Two instances with different lengths of blockage are investigated: 10 minutes and 20 minutes.

Figure 3-1 shows the Red Line track configuration as well as the train locations at the time of the hypothetical disruption. These train locations were originally taken from the MBTA Operation Control System and modified so that the headways of trains are close to the 6-minute scheduled headway. There are crossover tracks at both Quincy Adams and Quincy Center with an assumed short-turning time of 6 minutes at either location.

3.1.4 Input Data

The following input data are assumed:

1. Dwell time

As discussed in Chapter 2, the following dwell time function form is used:

$$dw = Max\ (C_0, C_1 + C_2 \cdot Alightings + C_3 \cdot Boardings). \qquad (3\text{-}1)$$

Dwell time data was collected at South Station in the outbound direction on the Red Line during the PM peak period with the resulting raw data shown in Appendix B.

The estimated linear dwell time function (in seconds) is

$$dw = \underset{(1.48)}{5.52} + \underset{(3.32)}{0.12} \cdot Alightings + \underset{(5.69)}{0.12} \cdot Boardings. \quad (R^2_{adj} = 0.79) \qquad (3\text{-}2)$$

The constant term has a small value and the t-statistic is only marginally significant. The reason for this result may be that the data collected are most concentrated in the area where the volume of boarding and alighting passengers is high. Little data was collected in the low passenger volume range. Therefore, a constant value is used to approximate the dwell time in the range where the volume of boardings and alightings is low. Based on the data, a value of 20 seconds was selected as C_0, which is close to the lowest dwell time in the data set. The final dwell time function (in seconds) is as follows:

$$dw = Max\ (20, 5.52 + 0.12 \cdot Alightings + 0.12 \cdot Boardings). \qquad (3\text{-}3)$$

Such approximation in the low passenger boarding/alighting range can potentially result in overestimates of the dwell time. However, as discussed in Chapter 2, the slope of the dwell time function is small in the low passenger volume range. Therefore, even though this simplification may not be precise, it should not have a significant impact on system performance.

Only at Park Street Station can passengers board and alight simultaneously through doors on both sides of the train. Operators first open doors on one side, then doors on the other side, with a similar procedure for door closing. Since dwell time data at Park Street Station were not sufficient to estimate an independent dwell time function, the dwell time function coefficients are derived from (3-3) (in seconds) as follows:

$$dw = Max\ (20, 5.52 + 0.06 \cdot Alightings + 0.06 \cdot Boardings). \qquad (3\text{-}4)$$

Although the door opening and closing time at Park Street Station is twice that at other stations, it is usually a small portion of the constant time in the dwell time function. As discussed in Section 2.3.2, the dwell time may be similar within small passenger flow ranges. That is, a large portion of the dwell time constant may be the "buffering" time for a small number of passengers. For example, the dwell time for 10 passenger boardings and 20 passenger boardings may be similar. Linearity may become obvious only beyond a certain range. At Park Street Station, since passengers can alight and board through doors on both sides of the train, the buffering time can be expected to be smaller than at other stations. The combined effect of door opening/closing time and the buffering time is uncertain. Therefore, the constant terms are kept the same as those in (3-3).

Coefficients for alightings and boardings for the dwell time function at Park Street Station are taken as half the corresponding coefficients in (3-3). This implies that the passenger flows on both sides are similar. In other words, the doors on both sides are equally used, which is almost certainly not true in practice. Therefore, this simplification may lead to underestimating the marginal alighting/boarding times.

Since we do not yet have sufficient accurate data, such approximation is inevitable. Nevertheless, the dwell time at Park Street consists of only a small part of the total train travel time and Park Street station is the last station in the impact set of the test scenario. Therefore, the approximation should not have significant impact on system performance.

2. Train location

In the model, train location information is reflected in the latest departure time at stations when a disruption occurs. In this model application, this data item is estimated according to the running time between stations and the location of trains when the disruption occurs. In a real implementation, train arrivals at stations will be monitored and the latest train arrival time recorded.

3. Passenger arrival rate and alighting fraction

Both disruptions tested occur at 8:15 AM during the height of the morning peak period. The corresponding passenger arrival rate and alighting fraction at each station in that time period must be provided as input.

To maintain comparability with the analysis done by O'Dell (1997, 1999), the same passenger arrival rates and alighting fractions are used. The data are shown in Appendix A. This data set was processed by O'Dell based on a data set collected by the Massachusetts' Central Transportation Planning Staff (1991). The original data set consists of detailed counts of passengers arriving and alighting at each station, for 15-minute intervals throughout the day. The counts were collected on a train-by-train basis, but not all were collected on the same day. O'Dell smoothed the data by averaging the counts for the one hour period around 8:15 AM. For more detailed discussion of the data processing, see O'Dell (1997, 1999).

According to the data, a large number of passengers arrive and board in-bound trains at stations 6 through 10, but only a few passengers alight at these stations. There is not much passenger activity at stations 11 through 13. At stations 14 through 16, both the fraction of passengers alighting and passenger arrival rates are high.

4. Normal load at each station in the time period of interest

As discussed in Section 2.5.1, to approximate the non-separable passenger on-board delay for holding candidates ahead of the blockage, which is the product of the number of passengers on-board when holding starts and the holding time, the true holding time is multiplied by the estimated number of passengers on-board when holding starts.

The number of passengers on-board trains ahead of the blockage when holding starts is similar to the normal load at each station during the time period of interest if no control intervention occurs before holding starts. This condition is valid for trains held for the first time, but not valid for trains held multiple times. If holding cost is considered, holding at multiple stations is less likely. Therefore, normal load at each station during the time period of interest should be a good approximation of the number of passengers on-board when holding starts.

Based on the normal headway, arrival rates and alighting fractions, the normal load at each station in that time period is calculated. In a real implementation, such normal load at each station for the time period of interest must be determined prior to the application of the model and can be verified based on observations.

The current load of all trains that are operating when a disruption occurs is also required and is estimated as for the normal load. If the model is implemented on a transit line with an automatic vehicle monitoring system, these estimates can be improved by updating the load on a train based on the headway when it arrives at a station and arrival rates and alighting fractions.

5. Train running time and minimum separation

With the control line information for the MBTA Red Line, Heimburger et al (1999) developed a simulation model to determine the minimum separation of trains at each station and the corresponding maximum running time under the non-inter-station-stopping condition. These data are shown in Appendix C. Since Heimburger did not estimate the maximum running time between Braintree (Station 6) and Quincy Adams (Station 7), the mean running time for that section obtained from the MBTA operations control system was used.

6. Weight for in-vehicle delay

As discussed in Chapter 2, since passengers may perceive in-vehicle delay to be less onerous than platform waiting time, a weight less than 1 is used for in-vehicle delay. According to the research by Abdel-Aty, Kitamura and Jovanis (1995), 0.5 is a reasonable weight for in-vehicle travel time. In-vehicle delay should be more onerous than in-vehicle travel time but less onerous than platform waiting time. Therefore, a weight between 0.5 and 1 might be reasonable for in-vehicle delay. In this research, 0.5 is taken as the weight for in-vehicle delay.

7. Portion of passengers who have to leave the express train

As discussed in Section 2.5.2, a predetermined value is used to approximate the proportion of passengers dumped by the express train and the portion of passengers whose destinations are within the express segment, and therefore, can not board or have to alight from the train at the station immediately before the express segment.

Based on the location of the disruption and passenger arrival rates and alighting fractions, a beneficial express segment is most likely to be a set of stations between station 7 to 13. At stations 14 to 16, passenger alighting fractions are high, and it is not likely to be optimal to skip any of these stations. So trains are not allowed to skip these stations and the maximum express segment is thus from stations 7 to 13. In this research, the proportion of passengers whose destinations are within the maximum expressing segment, which is 0.1, is used to approximate the proportion of passengers dumped by the express train. Obviously, this may result in overestimates of the number of passengers dumped. However, since this value is not large and the train following the express train has a very small headway, the simplification should not have significant impact on the control strategies and the objective function value.

3.2 Model Results

3.2.1 Description

The first results to be presented are for the "No-Disturbance" case. This is used to establish normal conditions on the line. After this, two different blockage lengths are analyzed: 10 minutes and 20 minutes. In each scenario, "No-Control", "Holding Only", "Holding and Expressing", and "Holding, Expressing and Short-turning Strategies" are investigated. In the "No-Control" case, no active control strategies are applied. In the "Holding Only" case, only holding strategies are allowed. In the "Holding and Expressing" case, both holding and expressing strategies are allowed. In the "Holding, Expressing and Short-turning Strategies" case, holding, expressing, and short-turning strategies are all allowed.

3.2.2 Evaluation Time Window

To compare the benefits from different control strategies, ideally, we would like to look only at passengers that are affected by the disruption or the control strategies. Ideally, passengers arriving before the system has fully recovered to normal operations would be considered affected. However, after the disruption is cleared, there are usually many trains clustered behind the blockage (trains with smaller than normal headway). It provides little benefits to control these trains (see O'Dell (1997, 1999)) while increasing the control set and impact set. Therefore, not all clustered trains behind the blockage are considered. As discussed in Section 2.3.3, the last train included in the impact set behind the blockage is the first train behind the blockage that does not leave passengers behind. The total capacity of the clustered trains should exceed the accumulated volume of passengers arriving before the departure of the last clustered train if the system has enough capacity under normal conditions. Thus, the number of trains included in the impact set behind the blockage is no larger than the number of clustered trains. Since passengers boarding the clustered trains are positively affected, the benefits evaluated are a lower bound of the true benefits. However, since the number of clustered trains should not differ much from the number of trains included in the impact set behind the blockage, the underestimate of the benefits should not be significant.

In this research, passengers initially on-board controlled trains and trains behind the blockage in the impact set when the disruption occurs and passengers boarding the controlled or blocked trains at intermediate stations after the blockage occurs are taken as a common group of passengers to evaluate. To compare impacts across different control schemes, we would like to establish a common group of passengers, over which the impacts of the disruption and control strategies are evaluated.

As shown in Figure 3-2, at each station, there is an evaluation time window with starting time t_s and ending time t_e, within which the impacts on passengers arriving due to the disruption or control strategies are evaluated. Passengers initially on-board blocked or controlled trains incur normal platform waiting time but extra in-vehicle delay. Since they board trains before the disruption occurs, their platform waiting time is not affected. However, if they are left behind due to either expressing or short-turning, additional platform waiting time is incurred. Since both platform waiting time and in-vehicle delay are to be considered, the platform waiting time for passengers initially on-board the blocked or controlled trains before the blockage occurs are assumed to be the typical mean platform waiting time, which is half the normal headway. For passengers arriving at each station within the evaluation time window, their platform waiting time and in-vehicle delay at that station and following stations may both be affected by the disruption and control strategies.

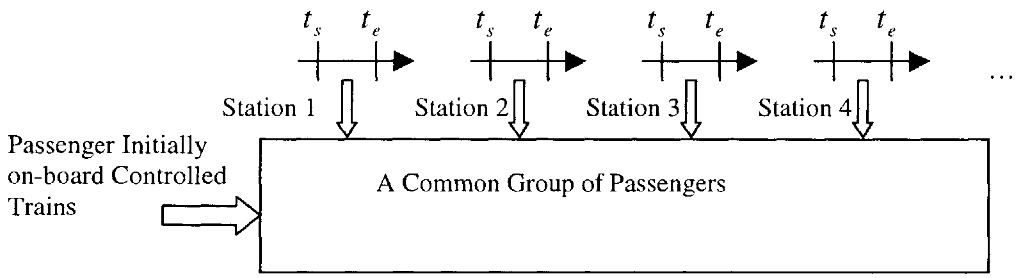


Figure 3-2 Passengers Groups to be Isolated

Precisely speaking, the starting time of the evaluation time window at each station should be the time after which arriving passengers will board a blocked or a controlled train, or the latest departure time of an uncontrolled train. However, if the starting time of the evaluation time window is defined this way, the group of passengers initially on-board the controlled trains may overlap with the group of passenger arriving at stations. For example, consider Figure 3-3, which shows a scenario when a disruption occurs. Suppose the impact set includes stations A through I and A' through C', and trains T_1 through T_6 and T_1' through T_3'. The control set includes T_1 through T_6 and T_2'. T_2' is the only short-turning candidate. Suppose the first train controlled is train T_2, which is held at station E for a certain time. The latest departure time of an uncontrolled train at station D is then the departure time of train T_1. That is, after this time, passengers arriving at station D may board a controlled train, for example, train T_2. However, passengers who arrive within the evaluation time window at station D and board train T_2 are also part of the passengers initially on-board T_2 when the disruption occurs and they have been already included in the group of passengers to be evaluated. To eliminate such overlap, at stations where the last train arrives before the disruption occurs is a train that is controlled at later stations, the starting time of the time window is the departure time of this train. This train may not be controlled under all control schemes. To have a common starting time, if this train is held under some control schemes, its departure time before the blockage occurrence is taken as the starting time of the time window. Since this time is earlier than the disruption occurrence, it is the same across different control schemes.

At stations where the last train is not a controlled train, the starting time of the time window is defined as the departure time of the last uncontrolled train. To establish a common starting time, the departure time of the last common uncontrolled train in all cases at each station is used as the starting time of the time window.

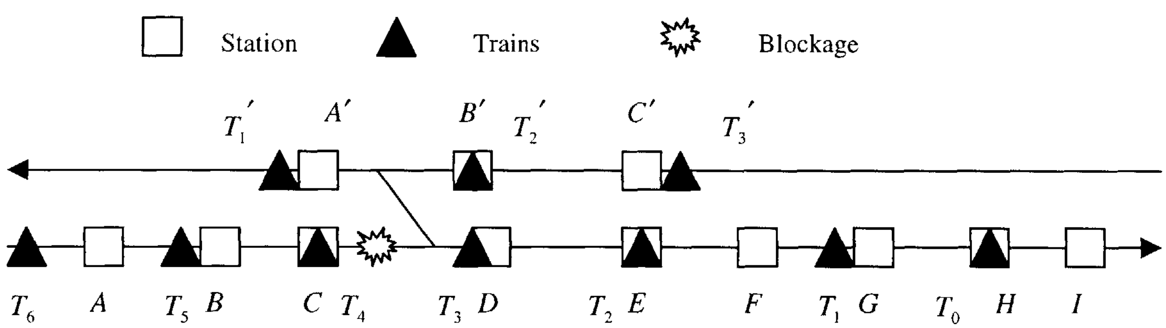


Figure 3-3 Determination of Evaluation Time Window

To demonstrate this concept, suppose in Figure 3-3, the first train that is controlled in the "Holding Only" and "Holding and Expressing" cases is train T_2, and is train T_3 in the "Holding, Express and Short-turning" case because train T_2' is short-turned and less holding is required. In the blocked direction at station A through E, since the last train arriving before the disruption occurs may be a controlled train, for example, T_2 at station D, the departure time of the last controlled train past the station before the blockage occurrence is used as the starting time of the window. For example, the starting time is the departure time of train T_5 at station A, is the departure time of train T_4 at station B, is the departure time of train T_3 at station C, and is the departure time of train T_2 at stations D and E. At station F through I, the first train arriving after the blockage occurs is an uncontrolled train, either T_1 or T_0. Therefore, T_1 is the last common uncontrolled train, and the departure time of T_1 at station F through I is taken as the starting time of the window. In the reverse direction, the evaluation starting times at stations A' and B' are the departure times of train T_1', and is the departure time of T_2' at station C'. In addition, passengers initially on-board trains T_2 through T_6 and T_2' are included in the common group of passengers to be evaluated.

The ending time of the time window, should be a time after which no passenger would perceive a waiting time that is longer than the normal headway. That is, no passenger would perceive a delay. Therefore, the departure time of the first train behind the blockage that does not leave passengers in the "No-Control" case at each station is used. We define this train as the margin train. Under active control schemes, the corresponding departure time of this margin train may be earlier than that in the "No-Control" case. The waiting time for passengers arriving between the departure time of the margin train and the ending time of the time window is assumed to be the mean typical waiting time, which is half the normal headway. If a disruption occurs, the headways for trains behind the blockage are usually smaller than the normal headway. Therefore, this assumption should give us a conservative estimate of benefits.

In the above example, suppose the last train included in the impact set in the blocked direction is train T_6. That is, T_6 is the first train behind the blockage that does not leave passengers in the "No-Control" case. Therefore, the departure time of train T_6 at each station in the impact set is taken as the evaluation ending time at that station. Under active control schemes, the departure time of train T_6 may be earlier than that in the "No-Control" case. In such cases, passengers arriving after the departure time of train T_6 and before the ending time of the window at each station are assumed to incur mean waiting time under normal operating conditions. In fact, the train following T_6 may be close to T_6 after the disruption clearance, and the headway may be less than the normal headway. The headway of the second following train may also be no larger than six minutes, unless there is a delay or disruption. Therefore, the mean waiting time assumption provides a conservative estimate of the benefits of active control.

As shown in the above, the starting time and ending time of the evaluation time window are determined only after the results from all control schemes have been obtained. Therefore, the size of the group of passengers over which we evaluate the impact may not be the same as that of the initial impact set. After the evaluation time window is determined at each station, the number of passengers chosen to evaluate the waiting time is the same under different control schemes applied to the same disruption. Total passenger waiting time and on-board delay time are then post-processed with passenger arrival rates, headway, passengers left and holding time. There may be a difference between the objective function value and the weighted sum of total passenger waiting time and on-board delay because the group of passengers arriving within the evaluation time window may not be identical to the group of passengers in the impact set. In addition, there are approximation errors.

Since the number of passengers in the evaluation time window is the same, the total weighted waiting time and mean weighted waiting time can be used equivalently as evaluation measures. There is consistency between the objective function value being minimized and the evaluation measure. Although there are passengers included in the evaluation time window whose waiting time may not be considered during optimization under the active control schemes, these passengers only comprise a small portion of the passengers in the time window. In terms of objective function value to optimize, total passenger waiting time is preferable than mean passenger waiting time. If the objective function value is mean passenger waiting time, trains behind the blockage may be held for more time than desirable to pick up more passengers. Since the mean waiting time of these passengers is smaller than the overall mean waiting time, each marginal passenger will help to reduce the overall mean waiting time. However, in practice, it is usually preferable to let trains behind the blockage move ahead as quickly as possible to avoid additional delay for clustered trains behind the blockage. Therefore, total waiting time is a better objective function value to minimize.

3.2.3 No-Disturbance Case

The model is first applied to the "No-Disturbance" case to obtain several important parameters used in the subsequent evaluation. The disruption length is zero in this case. The results are shown in Table 3-1. Above the table the objective function value (in passenger minutes), the solution time (in seconds), the number of iterations, and the number of nodes are all shown. The first part of the table comprises out-bound stations on the Braintree branch, including stations 36 to 39. The second part comprises in-bound stations 6 through 10 on the Braintree branch. The third part comprises in-bound stations 11 through 16 on the trunk portion. At each station, departure time, dwell time, departure headway, holding time, and the passenger load on the train are presented by train according to the departure sequence at that station. Each row corresponds to a single train.

In the "No-Control" case, no control strategies are allowed. Expressing and short-turning strategies can be restricted with binary variables. For the holding strategy, it is not enough just to constrain the holding time for trains ahead of the blockage to be zero. Since there is no upper bound on the dwell time, even if the holding time is constrained to be zero, the solution may overestimate the dwell time, which is equivalent to holding trains at stations as discussed in Chapter 2. To avoid over-estimating the dwell time, exact dwell times are developed from the "No-Disturbance" case. In the "No-Disturbance" case, headways are close to the scheduled headway and should be even, hence there is no benefit from holding a train. In the "No-Control" case, trains ahead of the blockage have the same headway, dwell time and load conditions as those in the "No Disturbance" case because no control strategies are applied to them and they are not affected by the disruption behind them. Therefore, the dwell times of trains ahead of the blockage are constrained to be the same as the dwell times obtained from the "No-Disturbance" case and the holding time can be constrained to be zero so that no active holding can occur.

Table 3-1 “No Disturbance” Case

OBJ Value =18607 passenger minutes Solution Time = 3.15 sec
Iterations = 2215 Nodes = 41

	North Quincy (36)					Wollaston (37)					Quincy Center (38)					Quincy Adams (39)				
	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load
T30																1.38	0.3		0	
T32	1.01	0.33	7.1	0		2.93	0.33	7.03	0		5.6	0.33	6.9	0		8.29	0.3	6.9	0	

	Braintree (6)					Quincy Adams (7)					Quincy Center (8)					Wollaston (9)					North Quincy (10)				
		DT	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load
T25															352	2.28	0.33	6	0	442	4	0.37	6	0	535
T26					75	2.93	0.4	6	0	227	5.8	0.35	6	0	352	8.29	0.33	6.01	0	442	10	0.38	6	0	535
T28		5.4	6	0	75	8.94	0.4	6	0	227	12	0.35	6	0	352	14.3	0.33	6	0	442	16	0.38	6	0	535
T30		11	6	0	75	14.9	0.4	6	0	227	18	0.35	6	0	352	20.3	0.33	6	-0	442	22	0.38	6	0	534
T32		17	6	0	75	20.9	0.4	6	0	226	24	0.35	6	0	351	26.3	0.33	5.99	0	441	28	0.38	6	0	534

	JFK (11)					Andrew (12)					Broadway (13)					South Station (14)					Downtown Crossing (15)					Park ST. (16)				
	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load	DT	DW	DH	HT	Load
T19																				504	2.02	0.65	6	0	344	3.38	0.5	6.2	0	214
T21					532	0.38	0.3	6.2	0	564	2.9	0.33	6.2	0	568	6.21	0.54	6.21	0	510	8.24	0.66	6.2	0	349	9.6	0.5	6.2	0	217
T23	3.45	0.3	6	0	532	6.39	0.3	6	0	563	8.9	0.33	6	0	567	12.2	0.54	6	0	506	14.2	0.65	6	0	345	15.6	0.5	6	0	212
T25	9.45	0.3	6	0	532	12.4	0.3	6	0	562	15	0.33	6	0	566	18.2	0.53	6	0	505	20.2	0.65	6	0	345	21.6	0.5	6	0	212
T26	15.5	0.3	6	0	532	18.4	0.3	6	0	563	21	0.33	6	0	565	24.2	0.55	6.04	0	505	26.3	0.67	6.1	0	345	27.6	0.5	6.1	0	213
T28	21.5	0.3	6	0	532	24.4	0.3	6	0	562	27	0.33	6	0	565	30.2	0.55	6	0	504	32.3	0.66	6	0	344	33.6	0.5	6	0	212
T30	27.5	0.3	6	0	532	30.4	0.3	6	-0	562	33	0.33	6	0	564	36.2	0.55	6	0	504	38.3	0.66	6	0	344	39.6	0.5	6	0	212
T32	33.5	0.3	6	0	531	36.4	0.3	6	0	561	39	0.33	6	0	565	42.2	0.55	5.99	0	504	44.3	0.66	6	0	344	45.6	0.5	6	0	212

DT = departure time DW = dwell time DH = departure headway HT = holding time

3.2.4 10-Minute Disruption Scenario

1. Problem Setting

In the 10 minute disruption scenario, the control set includes trains T19, T21, T23, T25, T26, T28, T30. The potential holding candidates include T25, T23, T21 and T19. The potential expressing candidates include T26 and T28. Based on the running time and the short-turning time, train T30 can be short-turned at Quincy Adams.

The impact set includes Station 38, 39 and 6 (Braintree) through 16 (Park Street). Beyond Park Street through Alewife in the AM peak period, passenger arrival rates are low, hence, the impact set does not need to include these stations. At stations in the outbound direction on the Braintree branch, passenger arrival rates are also insignificant. Therefore, the impact set does not include those stations, except Station 38 and 39. At Station 38 and 39, the passenger arrivals are negligible, so the waiting time for potential arriving passengers is not considered. However, since trains may be short-turned at Station 38 and 39, passengers on-board may have to alight and wait for the next train. Therefore, the waiting time for passengers who can potentially be dumped by short-turning trains is considered. With the heuristic methods described in Section 2.2 with 1 minute as the dwell time, the impact set should include at least train T30 behind the blockage to ensure that no passenger would be left behind by the last train in the impact set. Since T30 may be short-turned, the train behind T30 , train T32, is also included in the impact set.

2. Results

Table 3-2 shows the results of the "No-Control" case for the 10 minute blockage. The table format is identical to Table 3-1 except that we now include a new column, P, which shows passengers left behind by any train at any station. Only T26 leaves passengers behind. This is because the dwell time upper bound is used to estimate the load for train T26 and T28 in the heuristic method to determine the impact set, while real dwell time is shorter. Therefore, the real departure headways of T26 and T28 are shorter, and few passengers board T26 and T28.

Table 3-2 10 – Minute Scenario "No Control" Case

OBJ Value = 32722 passenger minutes

Iterations = 1481

Solution Time = 1.54 sec

Nodes = 72

	North Quincy (36)						Wollaston (37)						Quincy Center (38)						Quincy Adams (39)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T30																			1.38	0.33		0		
T32	1.0	0.33	7.1	0	0		2.9	0.33	7.0	0.0	0		5.6	0.33	6.9	0.0	0		10.6	0.33	9.2	2.3	0	

	Braintree (6)						Quincy Adams (7)						Quincy Center (8)						Wollaston (9)						North Quincy (10)					
		DT	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	Load	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T25																		352	2.3	0.33	6.0	0.0	0	442	4.0	0.37	6.0	0	0	535
T26						75	13.5	0.93	16.5	0	0	493	16.8	0.81	17.0	0	0	848	19.3	0.33	17.0	0.0	144	960	21.0	0.33	17.0	0	279	960
T28		11.7	12	6.3	0	154	16.5	0.33	3.0	1.3	0	230	19.5	0.33	2.7	0.2	0	285	22.1	0.47	2.8	0.0	0	470	24.3	0.84	3.3	0	0	787
T30		14.7	3	3.3	0	38	18.6	0.33	2.1	0.4	0	91	22.5	0.33	3.0	1	0	154	25.1	0.33	3.0	0.1	0	199	26.8	0.33	2.5	0	0	236
T32		17.7	3	0.3	0	38	21.6	0.33	3.0	0.4	0	113	25.5	0.33	3.0	1	0	176	28.1	0.33	3.0	0.1	0	221	29.8	0.33	3.0	0	0	267

	JFK (11)						Andrew (12)						Broadway (13)						South Station (14)						Downtown Crossing (15)						Park ST. (16)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T19																								504	2.0	0.65	6.0	0	0	344	3.4	0.5	6.2	0	0	214
T21						532	0.38	0.33	6.2	0	0	564	2.9	0.33	6.2	0	0	568	6.2	0.54	6.2	0.0	0	510	8.2	0.66	6.2	0	0	349	9.6	0.5	6.2	0	0	217
T23	3.5	0.33	6	0	0	532	6.39	0.33	6.0	0	0	563	8.9	0.33	6.0	0	0	567	12.2	0.54	6.0	0.0	0	506	14.2	0.65	6.0	0	0	345	15.6	0.5	6.0	0	0	212
T25	9.5	0.33	6	0	0	532	12.4	0.33	6.0	0	0	562	15.0	0.33	6.0	0	0	566	18.2	0.54	6.0	0.0	0	505	20.2	0.65	6.0	0	0	345	21.6	0.5	6.0	0	0	212
T26	26.4	0.33	17	0	10	960	29.4	0.33	17.0	0	90	960	31.9	0.33	17.0	0	19	960	35.7	1.07	17.5	0.0	0	959	38.4	1.32	18.2	0	0	725	40.0	0.71	18.4	0	0	510
T28	29.9	0.33	3.4	0.1	0	780	32.8	0.33	3.4	0	0	883	36.5	0.33	4.6	1.1	0	895	40.8	0.69	5.1	0.9	0	737	43.0	0.84	4.6	0	0	459	44.3	0.5	4.4	0	0	242
T30	33.0	0.33	3.2	0.8	0	236	37.2	0.33	4.5	1.3	0	260	41.5	0.33	5.1	1.7	0	266	45.3	0.36	4.6	0.7	0	262	47.1	0.41	4.1	0	0	190	48.5	0.5	4.1	0	0	126
T32	37.5	0.33	4.5	2.3	0	269	42.3	0.33	5.1	1.9	0	296	46.1	0.33	4.6	1.3	0	302	49.5	0.36	4.1	0.3	0	282	51.3	0.43	4.2	0	0	201	52.6	0.5	4.2	0	0	131

DT = departure time   DW = dwell time   DH = departure headway   HT = holding time   P = passengers left   L = departure passenger load

Table 3-3 shows the results of the "Holding Only" case. There is a 10% decrease in the objective function value compared with the "No-Control" case. However, since the group of passengers included in the impact set in the "No-Control" case is not the same as in the "Holding Only" case, the value may not represent the true savings.

In this case, train T19, T21, T23 and T25 can be (actively) held, but expressing or short-turning are not allowed. The results show that train T25, which is the first train immediately ahead of the blockage, is held for 5.8 minutes at Station 9, which is the first station it arrives at after the blockage occurs, and is held for another 1.9 minutes at Station 10. Station 10 (North Quincy) has a larger arrival rate than Station 9 (Wollaston). Since the platform waiting time for newly arrived passengers is quadratic with respect to the arrival headway and the weight for on-board delay is one half, the overall waiting cost may be smaller to hold T25 for 2 minutes at North Quincy instead of holding it for 2 more minutes at Wollaston, even if the number of on-board passengers is larger at North Quincy. Nevertheless, the holding time at the second holding station is considerably shorter than at the first holding station. Train T23, which is the second train ahead of the blockage, is held for 2.5 minutes at South Station and 2 more minutes at Park Street. Passenger arrival rate is not significant at stations 11 through 13. In addition, many passengers are on-board at those stations. Therefore, it makes sense to postpone holding until train T23 arrives at station 14, 15 and 16, where many passengers alight the train and arrival rate is relatively high. Trains T19 and T21 are not held. A reasonable explanation is that this disruption is not too severe, and so the benefit from holding T19 and T21 is not significant.

Trains behind the blockages are also held at multiple stations. On the branch, due to the disruption and the increased dwell time of train T26, trains T28, T30 and T32 have to be held to maintain minimum separations. Since the minimum separations are scaled by 7/4 on the trunk portion, there is a significant increase in minimum separation, after trains enter the trunk portion. In addition, minimum separation varies by station. So trains T28, T30 and T32 are also held at multiple stations to maintain minimum separations. Holding these trains due to increases in the minimum separation does not

Table 3-3 10 – Minute Scenario "Holding Only" Case

OBJ Value = 29611 passenger minutes
Iterations = 2895

Solution Time = 2.91 sec
Nodes = 79

	North Quincy (36)						Wollaston (37)						Quincy Center (38)						Quincy Adams (39)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T30																			1.4	0.33		0.0		
T32	1.0	0.33	7.1	0.0	0		2.9	0.33	7.0	0.0	0		5.6	0.33	6.9	0.0	0		10.6	0.33	9.2	2.3	0	

	Braintree (6)						Quincy Adams (7)						Quincy Center (8)						Wollaston (9)						North Quincy (10)					
		DT	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T25																		352	8.0	0.33	11.8	5.8	0	529	11.9	0.61	13.9	1.9	0	774
T26						75	13.5	0.93	16.5	0.0	0	493	16.8	0.81	17.0	0.0	0	848	19.3	0.33	11.2	0.0	57	960	21.0	0.33	9.0	0.0	122	960
T28		11.7	12.3	6.3	0	154	16.5	0.33	3.0	1.3	0	230	19.6	0.33	2.8	0.3	0	288	22.1	0.33	2.8	0.0	0	387	24.0	0.50	3.0	0.0	0	545
T30		14.7	3.0	3.3	0	38	18.7	0.33	2.3	0.6	0	95	22.5	0.33	2.8	0.9	0	154	25.1	0.33	3.0	0.2	0	199	26.8	0.33	2.8	0.0	0	244
T32		17.7	3.0	0.3	0	38	21.6	0.33	2.8	0.4	0	109	25.5	0.33	3.0	1.0	0	172	28.1	0.33	3.0	0.2	0	217	29.8	0.33	3.0	0.0	0	263

	JFK (12)						Andrew (12)						Broadway (13)						South Station (14)						Downtown Crossing (15)						Park ST. (16)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T19																								504	2.0	0.65	6.0	0.0	0	344	3.4	0.50	6.2	0.0	0	214
T21						532	0.4	0.33	6.2	0.0	0	564	2.9	0.33	6.2	0.0	0	568	6.2	0.54	6.2	0.0	0	510	8.2	0.66	6.2	0.0	0	349	9.6	0.50	6.2	0.0	0	217
T23	3.5	0.33	6.0	0.0	0	532	6.4	0.33	6.0	0.0	0	563	8.9	0.33	6.0	0.0	0	567	14.8	0.54	8.5	2.5	0	541	16.9	0.74	8.6	0.0	0	392	20.2	0.50	10.6	2.0	0	283
T25	17.4	0.33	13.9	0.0	0	783	20.3	0.33	13.9	0.0	0	857	22.9	0.33	13.9	0.0	0	874	26.4	0.84	11.7	0.0	0	814	28.9	1.04	12.0	0.0	0	580	30.2	0.53	10.0	0.0	0	356
T26	26.4	0.33	9.0	0.0	0	951	29.4	0.33	9.0	0.0	35	960	31.9	0.33	9.0	0.0	1	960	35.5	0.84	9.0	0.0	0	841	37.9	1.03	9.0	0.0	0	563	39.2	0.52	9.0	0.0	0	338
T28	29.9	0.33	3.5	0.5	0	536	32.8	0.33	3.5	0.0	0	586	36.3	0.33	4.3	0.9	0	585	40.2	0.52	4.8	0.7	0	502	42.2	0.63	4.4	0.0	0	326	44.1	0.50	4.9	0.5	0	192
T30	33.1	0.33	3.2	0.8	0	243	37.0	0.33	4.2	1.0	0	266	41.0	0.33	4.8	1.4	0	271	44.6	0.35	4.4	0.5	0	263	46.4	0.41	4.1	0.0	0	190	47.7	0.50	3.6	0.0	0	120
T32	37.2	0.33	4.2	2.0	0	265	41.8	0.33	4.8	1.6	0	290	45.4	0.33	4.4	1.0	0	295	48.7	0.36	4.1	0.3	0	278	50.5	0.43	4.2	0.0	0	198	51.9	0.50	4.2	0.0	0	130

DT = departure time DW = dwell time DH = departure headway HT = holding time P = passengers left L = departure load make much sense. In a real system, minimum separations at stations should be consistent. Therefore, we do not expect similar holding to occur in a real system.

According to the cost structure of holding, holding is most appropriate at stations with large passenger demand but not many passengers on-board. Therefore, the first few stations in a large passenger demand section are attractive places for holding. Since the number of passengers on-board increases down the line, holding time is expected to decrease down the line in multiple-holding situation. In addition, since the benefit group, passengers waiting down the line become smaller, the benefit from holding trains decreases down the line.

Overall, the above holding scheme is reasonable. The headways for trains T23, T25 and T26 out of Park Street are 11, 10 and 9 minutes, which is fairly even. Behind the blockage, compared with the "No-Control" case, train T26 left far fewer passengers behind, 215 versus 542.

Table 3-4 shows the results of the "Holding and Expressing" case. In this case, the table includes a new column S, which takes a value of 1 if a train skips a station. There is a 12% decrease in the objective function value compared with the "No-Control" case. The additional benefits from expressing seem to be modest. In addition to holding, trains T26 and T28 can skip stations 7 through 13. The results show that train T26, which is the train immediately behind the blockage, skips station 7, which is the first station it arrives at after the blockage is cleared. Since most passengers benefited from expressing are passengers ahead of the blockage, the first few stations the express candidate arrives at are the most likely stations to be skipped.

Train T25 is held for 4.2 minutes at station 9 and 0.3 minutes at station 10. Train T23 is held for 0.2 minute at South Station and 2 minutes at Park Street. There is a decrease in total holding time for both trains. Since expressing can help reduce the gap between train T26 and T25, less holding is needed ahead of the blockage.

Table 3-4 10 - Minute Scenario "Holding and Expressing" Case

OBJ Value = 28719 passenger minutes
Iterations =5056

Solution Time = 5.60 sec
Nodes = 253

	North Quincy (36)							Wollaston (36)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30														
T32	1.0	0.3	7.1	0		0		2.9	0.3	7.0	0.0		0	

	Quincy Center (37)							Quincy Adams (38)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30								1.4	0.3		0.0			
T32	5.6	0.3	6.9	0.0		0		9.7	0.3	8.3	1.4		0	

	Braintree (6)							Quincy Adams (7)							Quincy Center (8)						
	AT	DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																					352
T26							75	12.5	0.0	15.6	0.0	1	395	75	15.8	0.8	16.0	0.0	0	0	412
T28		10.7	11.4	5.4		0	142	15.5	1.0	3.0	0.6	0	0	613	18.8	0.3	3.0	0.4	0	0	673
T30		13.7	3.0	2.4		0	38	18.5	0.3	3.0	1.3		0	113	21.8	0.3	3.0	0.4		0	176
T32		17.4	3.6	0.0		0	45	21.5	0.3	3.0	0.7		0	121	24.8	0.3	3.0	0.4		0	184

	Wollaston (9)							North Quincy (10)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25	6.4	0.3	10.2	4.2		0	505	8.7	0.5	10.7	0.3		0	686
T26	18.4	0.5	12.0	0.0	0	0	592	20.4	0.6	11.8	0	0	0	790
T28	21.4	0.3	3.0	0.1	0	0	717	23.4	0.3	3.0	0	0	0	733
T30	24.4	0.3	3.0	0.1		0	221	26.1	0.3	2.7	0		0	261
T32	27.4	0.3	3.0	0.1		0	229	29.1	0.3	3.0	0		0	274

	JFK (11)							Andrew (12)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19														
T21							532	0.4	0.3	6.2	0.0		0	564
T23	3.5	0.3	6.0	0.0		0	532	6.4	0.3	6.0	0.0		0	563
T25	14.1	0.3	10.7	0.0		0	690	17.0	0.3	10.7	0.0		0	746
T26	25.9	0.3	11.8	0.0	0	0	793	28.8	0.3	11.8	0.0	0	0	855
T28	29.2	0.3	3.3	0.3	0	0	718	32.1	0.3	3.3	0.0	0	0	731
T30	32.4	0.3	3.2	0.8		0	261	36.5	0.3	4.4	1.2		0	284
T32	36.7	0.3	4.4	2.1		0	276	41.3	0.3	4.8	1.6		0	302

	Broadway (13)							South Station (14)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19														504
T21	2.9	0.3	6.2	0.0		0	568	6.2	0.5	6.2	0.0		0	510
T23	8.9	0.3	6.0	0.0		0	567	12.4	0.5	6.1	0.2		0	508
T25	19.6	0.3	10.7	0.0		0	757	23.1	0.8	10.7	0.0		0	713
T26	31.4	0.3	11.8	0.0	0	0	864	35.0	0.9	11.9	0.0	0	0	809
T28	35.7	0.3	4.4	1.0	0	0	726	39.7	0.6	4.8	0.7	0	0	607
T30	40.5	0.3	4.8	1.5		0	289	44.2	0.4	4.5	0.6		0	277
T32	45.0	0.3	4.5	1.1		0	307	48.4	0.4	4.2	0.3		0	286

	Downtown Crossing (15)							Park ST. (16)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19	2.0	0.7	6.0	0		0	344	3.4	0.5	6.2	0.0		0	214
T21	8.2	0.7	6.2	0		0	349	9.6	0.5	6.2	0.0		0	217
T23	14.4	0.7	6.1	0		0	348	17.7	0.5	8.1	2.0		0	237
T25	25.4	0.9	11.0	0		0	513	28.7	0.5	11.0	2.0		0	339
T26	37.4	1.1	12.0	0	0	0	577	38.8	0.5	10.0	0.0	0	0	355
T28	41.8	0.7	4.5	0	0	0	385	43.8	0.5	5.0	0.6	0	0	218
T30	46.0	0.4	4.2	0		0	198	47.9	0.5	4.2	0.6		0	130
T32	50.2	0.4	4.2	0		0	203	51.5	0.5	3.6	0.0		0	126

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip station P = passengers left L = departure passnger load

Table 3-5 10 - Minute Scenario “Holding, Expressing and Short Turning” Case

OBJ Value = 22166 passenger minutes
Iterations = 4760

Solution Time = 7.39 sec
Nodes = 169

	North Quincy (36)							Wollaston (37)							Quincy Center (38)							Quincy Adams (39)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30																						1.4	0.3		0.0		10	
T32	1.0	0.3	7.1	0.0		0		2.9	0.3	7.0	0.0		0		5.6	0.3	6.9	0.0		0		8.3	0.3	6.9	0.0		0	

	Braintree (6)							Quincy Adams (7)							Quincy Center (8)							Wollaston (9)							North Quincy (10)						
		DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																					352	4.9	0.3	8.6	2.6		0	481	6.7	0.5	8.7	0.0		0	625
T30								8.0	0.7	11.1	0.0		0	281	11.1	0.6	11.3	0.0		0	518	13.6	0.4	8.8	0.0		0	650	15.5	0.5	8.8	0.0		0	786
T26		0.0	0.6	0.0		0	75	12.9	0.3	4.8	0.0	0	0	197	16.3	0.9	5.2	0.0	0	0	305	18.8	0.3	5.2	0.0	0	0	382	20.5	0.3	5.0	0.0	0	0	458
T28		11.1	11.1	5.7		0	147	17.6	0.3	4.7	3.0	0	0	265	20.4	0.3	4.1	0.0	0	0	350	22.9	0.3	4.1	0.0	0	0	411	24.6	0.3	4.1	0.0	0	0	468
T32		17.4	6.3	0.0		0	79	20.9	0.3	3.3	0.0		0	162	23.7	0.3	3.3	0.0		0	231	26.2	0.3	3.3	0.0		0	281	27.9	0.3	3.3	0.0		0	330

	JFK (11)							Andrew (12)							Broadway (13)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19																					
T21							532	0.4	0.3	6.2	0.0		0	564	2.9	0.3	6.2	0.0		0	568
T23	3.5	0.3	6.0	0.0		0	532	6.4	0.3	6.0	0.0		0	563	8.9	0.3	6.0	0.0		0	567
T25	12.2	0.3	8.7	0.0		0	626	15.1	0.3	8.7	0.0		0	671	17.6	0.3	8.7	0.0		0	679
T30	21.0	0.3	8.8	0.0		0	782	23.9	0.3	8.8	0.0		0	826	26.5	0.3	8.8	0.0		0	830
T26	25.9	0.3	5.0	0.0	0	0	455	28.9	0.3	5.0	0.0	0	0	480	31.4	0.3	5.0	0.0	0	0	482
T28	30.0	0.3	4.1	0.0	0	0	463	33.0	0.3	4.1	0.0	0	0	483	35.5	0.3	4.1	0.0	0	0	482
T32	33.4	0.3	3.3	0.0		0	327	36.3	0.3	3.3	0.0		0	343	39.7	0.3	4.2	0.8		0	347

	South Station (14)							Downtown Crossing (15)							Park ST. (16)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19							504	2.0	0.7	6.0	0.0		0	344	3.4	0.5	6.2	0.0		0	214
T21	6.2	0.5	6.2	0.0		0	510	8.2	0.7	6.2	0.0		0	349	9.6	0.5	6.2	0.0		0	217
T23	12.2	0.5	6.0	0.0		0	506	14.2	0.7	6.0	0.0		0	345	15.6	0.5	6.0	0.0		0	212
T25	21.0	0.7	8.8	0.0		0	629	23.2	0.8	9.0	0.0		0	445	24.6	0.5	9.0	0.0		0	288
T30	30.0	0.8	8.9	0.0		0	742	32.3	0.9	9.0	0.0		0	509	33.6	0.5	9.0	0.0		0	315
T26	34.6	0.5	4.7	0.0	0	0	424	36.5	0.6	4.3	0.0	0	0	281	37.9	0.5	4.3	0.0	0	0	166
T28	38.9	0.5	4.3	0.2	0	0	419	40.8	0.6	4.3	0.0	0	0	279	42.8	0.5	4.9	0.6	0	0	172
T32	43.2	0.4	4.3	0.4		0	318	45.0	0.5	4.2	0.0		0	221	46.4	0.5	3.6	0.0		0	133

DT = departure time   DW = dwell time   DH = departure headway   HT = holding time   S=1, skip station   P = passengers left   L = departure load

There are about 400 passengers skipped by T26 at Station 7. Compared with the "Holding Only" case, the passenger loads on trains T26 and T28 are more balanced. Since T26 skips only one station, the time saved is only about 1 minute, T28 still needs to be held for 1 minute to maintain distance with T26, even though it takes 1 minute to load the passengers skipped by T26 at Station 7 and T26 saves dwell time at Station 7. The reasons are: (1) T26 still has large dwell times at following stations; (2) Trains have to be held because of the scaled minimum separation on the trunk portion. The total holding time for T28 at Stations 6 and 7 is one minute less than that in the "Holding Only" case. The departure time for T28 at Park Street is marginally earlier (0.3 minute) than in previous case. Since trains bunch together after the blockage clearance, moving trains quickly and transporting delayed passengers quickly to their destinations are desirable. Moreover, moving trains quickly also helps to reduce schedule deviation.

Table 3-5 shows the results of the "Holding, Expressing and Short-Turning" case. There is a 32% decrease in the objective function value compared with the "No Control" case. Train T30 is short-turned from Station 39 to Station 7. Train T25 is held for 2.6 minutes at Station 9. Train T23 is not held. Only passengers initially on train T30 have to wait for the next train, which is only a small number of passengers. Although expressing is allowed, it is not used in the optimal solution. This is probably because the headway of T26 is already small. The departure time of T28 at Park Street is about 1.5 minutes earlier than in the "No-Control" case, and about 1.3 minutes earlier than in the "Holding Only" case. This is because headways of trains T26 and T28 are shorter and there are fewer passenger boardings and alightings for T26 and T28. Therefore, dwell time is reduced and further delays due to long dwell times after blockage clearance are avoided.

3. Comparison of Control Strategies

The evaluation time windows by station are shown in Table 3-6. The evaluation window starting time at stations 11 through 16 is the corresponding departure time of train T21 at these stations, at stations 9 and 10 is the departure time of T23, at stations 7 and 8 is the departure time of T25, and at station 6 is the departure time of T26. Although the last train in the impact set that leaves no passenger behind in the "No-Control" case is train T28, the departure time of train T30 at each station in the "No-Control" case is chosen as the evaluation ending time because T30 is short-turned. In addition, passengers initially on-board trains T23 (535), T25 (352) and T26 (75) are also included, their platform waiting time is assumed to be 3 minutes, half of the normal headway. There are in total 4962 passengers included in the evaluation time window.

Table 3-6 Evaluation Time Window by Station for the 10 Minute Disruption Scenario

Station	t_s	t_e	Passengers	Station	t_s	t_e	Passengers
6	-0.64	14.67	192	11	-2.55	33.03	86
7	-3.07	18.58	548	12	0.38	37.24	215
8	-0.23	22.45	477	13	2.95	41.55	87
9	-3.72	28.10	436	14	6.21	45.34	548
10	-2	26.78	571	15	8.24	47.12	410
				16	9.60	48.48	430

t_s **= starting time,** t_e **= ending time.**

The objective function value, mean platform-waiting times and in-vehicle delays for different cases are shown in Table 3-7. We can see the savings in terms of the total weighted waiting time are consistent with the decreases of the objective function values.

Table 3-7 Comparison of Strategy Effectiveness for 10 Minute Disruption Scenario

Control Scheme	Objective Function Value	Change	Platform Waiting Time	In-vehicle Delay	Total Weighted Waiting Time	Saving	Passengers Left at Stations
ND	18607	-	3.00	0.00	3.00	-	0
NC	32722	-	5.70	0.15	5.78	-	542
H	29611	10%	4.53	1.39	5.23	10%	215
HE	28719	12%	4.59	0.83	5.00	13%	395
HET	22166	32%	3.55	0.39	3.74	35%	10

ND = No Disturbance, NC = No Control, H = Holding Only, HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning. Saving is in terms of total weighted waiting time, and is compared with that of the "No-Control" case. The weight for in-vehicle delay time is 0.5.

The headway in the simplified system is the same as that of the Braintree Branch in the MBTA Red Line in the AM Peak, which is 6 minutes. If each headway is maintained at exactly 6 minutes, the ideal mean platform waiting time should be 3 minutes, the same as the mean weighted waiting time because no holding is required. The mean total weighted waiting time is 3 minutes in the "No-Disturbance" case, which is expected. In the "No-Control" case, the mean total weighted waiting time is 5.78 minutes, about twice that in the "No-Disturbance" case. There are 542 passengers left behind because of train capacity constraints.

In the "Holding Only" case, the mean platform waiting time decreases to 4.53 minutes, but the mean in-vehicle delay increases to about 1.39 minutes. There are significantly fewer passengers left behind (215 versus 542). Compared with the "No Control" case, the saving in terms of the total weighted waiting time is about 10%.

In the "Holding and Expressing" case, there are 395 passengers skipped by express train T26 at station 7. The mean platform waiting time is increased somewhat, because more passengers have to wait for the next train, but the mean in-vehicle delay is reduced because of less holding time. There is a modest increase in savings from expressing.

In the "Holding, Expressing and Short-Turning" case, there is a significant decrease in both mean platform waiting time and in-vehicle delay with a total savings of about 35%. There are only 10 passengers dumped by train T30 at Station 39. As shown in Table 3-5, the departure time of train T32 at Station 16 is very close to that in the "No-Disturbance" case, which demonstrates that schedule deviation is not significant.

According to the results in this 10-minute disruption scenario, expressing only provides marginal benefit over holding. Short-turning can potentially provide a large benefit. However, the savings will depend on the specific problem; in this scenario, the short-turned train T30 is very close to the terminus, and there are few passengers travelling in the reverse direction, therefore, few passengers are negatively affected by short-turning. This is a perfect situation for short-turning. In other situations, short-turning may not provide nearly such significant savings.

The control strategies under different control schemes are shown in Table 3-8. The holding time for trains in the "Holding Only" case is the longest among the three. Since expressing helps to reduce the gap, the holding time for train T25 and T23 is reduced in the "Holding and Expressing" case. In the "Holding, Expressing and Short-turning case, only train T25 is held and for a still shorter time.

Table 3-8 Control Strategies for the 10-minute Disruption Scenario

Control Schemes	Control Strategies
H	Hold T25 for 5.8 minutes at station 9, and 1.9 minutes at station 10
	Hold T23 for 2.5 minutes at station 14 and 2 minutes at station 16
HE	Hold T25 for 4.2 min at Station 9 and 0.3 min at station 10
	Hold T23 for 2 minutes at station 16
	Let T26 skip station 7
HET	Hold T25 for 2.6 min at station 9
	Short-turn T30 from station 39 to 7

H = Holding Only, HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning

3.2.5 20-Minute Disruption Scenario

In the 20-minute disruption scenario, train T32 is added into the control set and it can be turned back at either station 38 or station 39.

1. Results

Table 3-9 shows the results of the "No-Control" case for the 20-minute blockage. Trains T26, T28, and T30 would leave passengers behind without active control. At station 9 (Wollaston) and 10 (North Quincy), a large number of passengers may not be able to board the first two trains arriving after the blockage clearance because of binding train capacity. At station 12 (Andrew), some passengers might have to wait for the fourth train to board after the blockage is cleared.

Table 3-9 20 - Minute Scenario "No Control" Case

OBJ Value = 65784 passenger minutes
Iterations = 714

Solution Time = 0.90 sec
Nodes =27

	North Quincy (36)						Wollaston (37)						Quincy Center (38)						Quincy Adams (39)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T30																			1.4	0.3		0.0		
T32	1.0	0.3	7.1	0.0	0		2.9	0.3	7.0	0.0	0		5.6	0.3	6.9	0.0	0		21.1	0.3	19.7	12.8	0	

	Braintree (6)						Quincy Adams (7)						Quincy Center (8)						Wollaston (9)						North Quincy (10)					
		DT	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T25																		352	2.3	0.3	6.0	0.0	0	442	4.0	0.4	6.0	0.0	0	535
T26						75	24.0	1.5	27.1	0.0	0	760	27.0	0.5	27.2	0.0	368	960	29.5	0.3	27.2	0.0	410	960	31.2	0.3	27.2	0.0	482	960
T28		22.2	22.8	16.8	0	286	26.1	0.3	2.1	0.4	0	339	29.6	0.9	2.6	0.0	0	760	32.3	0.5	2.7	0.0	250	960	33.9	0.3	2.7	0.0	478	960
T30		24.3	2.1	13.0	0	27	28.7	0.3	2.6	0.9	0	92	31.7	0.3	2.2	0.2	0	137	34.6	0.7	2.3	0.0	0	422	37.2	1.2	3.2	0.0	0	939
T32		27.3	3.0	10.0	0	38	30.9	0.3	2.2	0.0	0	92	34.7	0.3	3.0	1.0	0	155	37.7	0.3	3.1	0.4	0	201	39.9	0.3	2.7	0.5	0	243

	JFK (11)						Andrew (12)						Broadway (13)						South Station (14)						Downtown Crossing (15)						Park ST. (16)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T19																								504	2.0	0.7	6.0	0.0	0	344	3.4	0.5	6.2	0.0	0	214
T21						532	0.4	0.3	6.2	0.0	0	564	2.9	0.3	6.2	0.0	0	568	6.2	0.5	6.2	0.0	0	510	8.2	0.7	6.2	0.0	0	349	9.6	0.5	6.2	0.0	0	217
T23	3.5	0.3	6.0	0.0	0	532	6.4	0.3	6.0	0.0	0	563	8.9	0.3	6.0	0.0	0	567	12.2	0.5	6.0	0.0	0	506	14.2	0.7	6.0	0.0	0	345	15.6	0.5	6.0	0.0	0	212
T25	9.5	0.3	6.0	0.0	0	532	12.4	0.3	6.0	0.0	0	562	15.0	0.3	6.0	0.0	0	566	18.2	0.5	6.0	0.0	0	505	20.2	0.7	6.0	0.0	0	345	21.6	0.5	6.0	0.0	0	212
T26	36.7	0.3	27.2	0.0	35	960	39.6	0.3	27.2	0.0	150	960	42.1	0.3	27.2	0.0	42	960	45.9	1.1	27.7	0.0	143	960	48.9	1.5	28.6	0.0	0	835	50.6	0.9	29.0	0.0	0	674
T28	39.8	0.3	3.2	0.4	12	960	42.9	0.3	3.3	0.1	161	960	46.7	0.3	4.6	1.3	33	960	51.2	1.0	5.3	0.8	0	931	53.6	1.0	4.7	0.0	0	569	56.0	0.5	5.4	1.0	0	300
T30	43.1	0.3	3.3	0.5	0	928	47.5	0.3	4.6	1.4	148	960	52.0	0.3	5.3	2.0	26	960	56.0	0.7	4.7	0.5	0	781	58.2	0.9	4.6	0.0	0	484	60.2	0.5	4.3	0.7	0	252
T32	47.7	0.3	4.6	2.4	0	247	52.7	0.5	5.3	2.0	0	423	56.7	0.5	4.7	1.3	0	453	60.6	0.5	4.6	0.7	0	401	62.5	0.5	4.3	0.0	0	268	63.8	0.5	3.6	0.0	0	153

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left Load = departure load

Table 3-10 shows the results of the "Holding Only" case. There is a 14% decrease in the objective function value compared with the "No-Control" case, slightly greater than in the 10 minute "Holding Only" case. Trains T19, T21, T23 and T25 can be (actively) held in this case. The results show that train T25 is held for 6 minutes at station 9, 5.6 minutes at station 10, 1.3 minutes at station 14 and 2.2 minutes at station 16. Train T23 is held for 3 minutes at station 11, 2.8 minutes at station 14, and 2 minutes at station 16. Train T21 is held for 2 minutes at station 16. Train T19 is not held. Since the blockage is more severe than that in the 10-minute disruption scenario, it makes sense that more trains are held and holding time is longer. The headways for trains T21, T23, T25 and T26 out of station 16 are 8.3, 12, 14, 12 minutes, respectively. Again, some passengers at stations 9 and 10 might have to wait for the third train to board after the blockage clearance, but no passenger has to wait for the fourth train after blockage clearance.

In this case, train T25 reaches capacity at Station 13, but leaves no passengers behind, which is consistent with the argument made in Chapter 2. This demonstrates a capacity-driven holding pattern. That is, because the blockage time is long and passengers may potentially wait for more than two trains after the blockage clearance, the solution tries to use the maximum capacity of the trains immediately ahead of the blockage. That is, trains are held as long as it is more beneficial than to leave the passenger potentially wait for two or more consecutive trains, and tries to leave as few passengers as possible. On one hand, the held train can pick up passengers and reduce their waiting time. On the other hand, it can save the capacity for the blocked trains, and thus, fewer passengers would be left due to train capacity.

Nevertheless, it is never good to leave passengers after holding trains ahead of the blockage. Therefore, holding trains further downstream to utilize their capacity can increase the potential holding time of the following trains. However, beyond a certain holding time threshold, a train may reach capacity no matter how long the preceding train is held. That is, holding the preceding train may not lead to greater holding for the

Table 3-10 20 - Minute Scenario "Holding Only" Case

OBJ Value = 56343 passenger minutes
Iterations =11143

Solution Time = 12.10 sec
Nodes = 551

	North Quincy (36)						Wollaston (37)						Quincy Center (38)						Quincy Adams (39)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T30																			1.4	0.3		0.0		
T32	1.0	0.3	7.1	0.0	0		2.9	0.3	7.0	0.0	0		5.6	0.3	6.9	0.0	0		21.1	0.3	19.7	12.8	0	

	Braintree (6)					Quincy Adams (7)						Quincy Center (8)						Wollaston (9)						North Quincy (10)					
	DT	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T25																	352	8.3	0.3	12.0	6.0	0	533	15.9	0.6	17.9	5.6	0	855
T26					75	24.0	1.5	27.1	0.0	0	760	27.0	0.5	27.2	0.0	368	960	29.5	0.3	21.2	0.0	319	960	31.2	0.3	15.3	0.0	247	960
T28	22.2	22.8	16.8	0	286	26.1	0.3	2.1	0.4	0	339	29.6	0.9	2.6	0.0	0	760	32.3	0.5	2.7	0.0	159	960	33.9	0.3	2.7	0.0	243	960
T30	24.4	2.2	13.0	0	27	28.7	0.3	2.6	0.8	0	92	31.7	0.3	2.2	0.2	0	137	34.8	0.5	2.6	0.4	0	335	36.9	0.7	3.0	0.0	0	618
T32	27.4	3.0	10.0	0	38	30.9	0.3	2.2	0.0	0	92	34.7	0.3	3.0	1.0	0	155	37.8	0.3	3.0	0.6	0	200	39.9	0.3	3.0	0.4	0	247

	JFK (11)						Andrew (12)						Broadway (13)						South Station (14)						Downtown Crossing (15)						Park ST. (16)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T19																								504	2.0	0.7	6.0	0.0	0	344	3.4	0.5	6.2	0.0	0	214
T21						532	0.4	0.3	6.2	0.0	0	564	2.9	0.3	6.2	0.0	0	568	6.3	0.5	6.3	0.1	0	511	8.3	0.7	6.3	0.0	0	351	11.7	0.5	8.3	2.0	0	240
T23	6.5	0.3	9.0	3.0	0	540	9.4	0.3	9.0	0.0	0	587	11.9	0.3	9.0	0.0	0	598	18.1	0.6	11.8	2.8	0	610	20.3	0.9	12.0	0.0	0	466	23.7	0.5	12.0	2.0	0	330
T25	21.3	0.3	14.9	0.0	0	863	24.2	0.3	14.9	0.0	0	942	26.8	0.3	14.9	0.0	0	960	31.7	0.9	13.6	1.3	0	905	34.2	1.2	13.9	0.0	0	650	37.9	0.6	14.2	2.2	0	432
T26	36.7	0.3	15.3	0.0	6	960	39.6	0.3	15.3	0.0	81	960	42.1	0.3	15.3	0.0	15	960	45.8	1.0	14.1	0.0	0	912	48.4	1.2	14.2	0.0	0	657	49.9	0.6	12.0	0.0	0	411
T28	39.8	0.3	3.2	0.4	0	943	42.9	0.3	3.3	0.1	75	960	46.6	0.3	4.5	1.2	6	960	50.8	0.7	4.9	0.7	0	783	53.0	0.9	4.6	0.0	0	485	54.4	0.5	4.5	0.0	0	255
T30	43.1	0.3	3.3	0.8	0	606	47.4	0.3	4.5	1.3	0	702	51.6	0.3	4.9	1.6	0	705	55.4	0.6	4.6	0.5	0	589	57.5	0.7	4.4	0.0	0	375	58.8	0.5	4.4	0.0	0	208
T32	47.6	0.3	4.5	2.3	0	249	52.3	0.3	4.9	1.8	0	276	56.2	0.3	4.6	1.3	0	282	59.8	0.4	4.4	0.6	0	272	61.6	0.4	4.1	0.0	0	195	63.0	0.5	4.1	0.0	0	128

DT = departure time DW = dwell time DH = departure headway HT = holding time P = passengers left L = departure Load following train. For example, in this scenario, even if train T23, which is not at capacity, is held for more time, so that train T25 has the minimum possible headway at station 11, the possible holding time for T25 may not increase much. There may already be many passengers on-board when T25 reaches station 11, and it may reach capacity even with the minimum possible headway. Therefore, the further from the blockage, the less appropriate it is to hold trains to reach capacity.

In addition, holding trains ahead of the blockage occurs at the expense of passengers on-board. Suppose two trains are held to reach capacity, for example in this case, T25 and T23, and it is possible to hold the third train, T21, for 5 more seconds to pick up one more passenger and save space for one additional passenger for T25. To obtain this one extra passenger served by T25, T23 has to be held for an additional 5 seconds, as does T25. There would be many passengers on-board T23 and T25 when holding ends under the current condition, say 600 passengers each. To hold T23 and T25 for an additional 5 seconds, these passengers will incur 5 additional seconds in-vehicle delay, totaling 100 minutes, not including the delay for train T21. Suppose the passenger picked up would have waited for 20 minutes to board the third train after blockage clearance if T25 were not held for these 5 extra seconds. The holding cost is much higher than the saving. Therefore, it is unlikely that many trains ahead of the blockage will be held to reach capacity. This will, of course, depend on the passenger demand profile on the line and the location of the blockage.

Since the solution uses the full capacity of T25 and some extra capacity of T23, there are 1,000 fewer passengers left behind due to train capacity after the blockage clearance compared with the "No Control" case.

Table 3-11 shows the results of the "Holding and Expressing" case. There is an 18% decrease in the objective function value compared with the "No Control" case. Again, the additional benefits obtained from expressing are modest.

In this case, trains T26 and T28 can skip stations 7 through 13. Train T26 skips station 7 and 8, and T28 skips station 7 in the solution. Train T25 is held for 6.7 minutes at station 9 and 4 minutes at station 10. Train T23 is held for 1 minute at Station 11 and 3.9 minutes at Station 14. Train T21 is also held for 2 minutes at Station 16. There is a decrease in total holding time for both T25 and T23. The reason is that expressing reduces the gap between trains T26 and T25, therefore, less holding is needed ahead of the blockage.

Train T26 skips 648 passengers at Station 7 and 531 passengers at Station 8. Train T28 skips 689 passengers at Station 7. T26 and T28 do not reach capacity at any station. T26 has about 200 passengers' unused capacity and T28 has about 100 passengers' unused capacity. However, it is obvious that T26 and T28 will reach capacity if they stop at any of the stations they skip. In addition, the dwell time will be more than one minute because of the large number of possible boardings and the crowding condition. Considering the increased gap, it may make sense to express both T26 and T28. The resulting passenger loads for trains T26, T28 and T30 are fairly balanced. The departure time for T26 and T28 at Park Street is 1.6 minutes earlier than that in the "Holding Only" case.

Table 3-12 shows the results of the "Holding, Expressing and Short-Turning" case. There is a 58% decreases in the objective function value compared with the "No-Control" case, which is significant. Trains T30 and T32 are short-turned from station 39 to station 7. Train T25 is held for 2.5 minutes at Station 9. Train T23 is not held. Only passengers initially on trains T30 and T32 have to wait for the next train, totaling 30 passengers. Expressing is not used in the optimal solution. In this case, the situation is as if trains T30 and T32 switch places with trains T26 and T28 in the train series. The departure time of T28 at Park Street is about 12 minutes earlier than the departure time of T32 in the "No-Control" case, and is more than 10 minutes earlier than in the "Holding Only" and "Holding and Expressing" cases. It is only about 6.5 minutes later than the departure time of train T32 in the "No-Disturbance" case. Therefore, the schedule deviation is much less severe.

Table 3-11 20 - Minute Scenario "Holding and Expressing" Case

OBJ Value = 54242 passenger minutes
Iterations = 63853

Solution Time = 82.07 sec
Nodes = 3598

	North Quincy (36)							Wollaston (37)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30														
T32	1.0	0.3	7.1	0.0		0		2.9	0.3	7.0	0.0		0	

	Quincy Center (38)							Quincy Adams (39)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30								1.4	0.3		0.0			
T32	5.6	0.3	6.9	0.0		0		19.7	0.3	18.3	11.4		0	

	Braintree (6)							Quincy Adams (7)							Quincy Center (8)						
	AT	DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																					352
T26							75	22.5	0.0	25.6	0.0	1	648	75	25.0	0.0	25.3	0.0	1	531	75
T28		20.7	21.4	15.4		27	241	24.2	0.0	1.6	0.3	1	689	241	28.0	1.3	3.0	0.1	0	0	833
T30		22.4	1.6	11.0		0	47	27.2	1.6	3.0	0.0		0	812	30.0	0.3	2.0	0.0		0	850
T32		25.4	3.0	8.0		0	38	29.1	0.3	2.0	0.3		0	87	33.0	0.3	3.0	1.0		0	150

	Wollaston (9)							North Quincy (10)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25	9.0	0.3	12.7	6.7		0	543	15.0	0.6	17.0	4.0		0	847
T26	27.9	0.7	18.9	0.0	0	0	361	30.0	0.7	15.0	0.0	0	0	637
T28	30.5	0.3	2.6	0.0	0	0	871	33.0	0.3	3.0	0.8	0	0	878
T30	33.5	0.3	3.0	1.0		0	894	35.6	0.3	2.7	0.5		0	893
T32	36.5	0.3	3.0	1.0		0	195	38.6	0.3	3.0	0.5		0	243

	JFK (11)							Andrew (12)							Broadway (13)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19																					
T21							532	0.4	0.3	6.2	0.0		0	564	2.9	0.3	6.2	0.0		0	568
T23	4.8	0.3	7.4	1.4		0	536	7.7	0.3	7.4	0.0		0	574	10.3	0.3	7.4	0.0		0	581
T25	20.4	0.3	15.6	0.0		0	857	23.3	0.3	15.6	0.0		0	941	25.9	0.3	15.6	0.0		0	960
T26	35.4	0.3	15.0	0.0	0	0	653	38.3	0.3	15.0	0.0	0	0	734	40.9	0.3	15.0	0.0	0	0	754
T28	38.7	0.3	3.3	0.3	0	0	858	41.7	0.3	3.3	0.0	0	0	870	45.3	0.3	4.4	1.1	0	0	862
T30	41.9	0.3	3.2	0.8		0	872	46.0	0.3	4.4	1.2		0	890	50.1	0.3	4.8	1.5		0	883
T32	46.3	0.3	4.4	2.2		0	245	50.8	0.3	4.8	1.6		0	271	54.5	0.3	4.4	1.1		0	277

	South Station (14)							Downtown Crossing (15)							Park ST. (16)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19							504	2.0	0.7	6.0	0.0		0	344	3.4	0.5	6.2	0.0		0	214
T21	6.2	0.5	6.2	0.0		0	510	8.2	0.7	6.2	0.0		0	349	11.6	0.5	8.2	2.0		0	239
T23	17.5	0.6	11.3	3.9		0	590	19.7	0.8	11.5	0.0		0	450	23.1	0.5	11.5	2.0		0	317
T25	30.4	0.9	12.9	0.9		0	894	32.9	1.1	13.2	0.0		0	637	36.3	0.6	13.3	2.0		0	416
T26	44.5	0.9	14.1	0.0	0	0	758	46.9	1.1	14.0	0.0	0	0	570	48.3	0.6	12.0	0.0	0	0	374
T28	49.3	0.6	4.8	0.7	0	0	709	51.5	0.7	4.5	0.0	0	0	443	52.8	0.5	4.5	0.0	0	0	237
T30	53.7	0.7	4.4	0.3		0	719	55.9	0.8	4.5	0.0		0	449	57.8	0.5	5.0	0.5		0	246
T32	58.3	0.4	4.6	0.7		0	270	60.1	0.4	4.1	0.0		0	194	61.4	0.5	3.6	0.0		0	122

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left L = departure load

Table 3-12 20 – Minute Scenario "Holding, Expressing and Short-turning" Case

OBJ Value = 27913 passenger minutes
Iterations = 24066

Solution Time = 68.32 sec
Nodes = 1162

	North Quincy (36)							Quincy Center (37)							Wollaston (38)							North Quincy (39)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30																						1.4	0.3		0.0		10	
T32	1.0	0.3	7.1	0.0		0		2.9	0.3	7.0	0.0		0		5.6	0.3	6.9	0.0		0		8.3	0.3	7.0	0.1		20	

	Braintree (6)							Quincy Adams (7)							Quincy Center (8)						
		DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																					352
T30								8.0	0.4	11.1	0.0		0	281	11.1	0.4	11.3	0.0		0	518
T32								14.8	0.3	6.7	0.0		0	171	19.7	0.3	8.5	0.0		0	307
T26							75	23.0	0.5	8.3	0.0	0	0	284	25.9	0.4	6.2	0.0	0	0	414
T28		21.3	21.9	15.9		0	274	25.0	0.7	2.0	0.3	0	0	324	28.9	0.6	3.0	1.0	0	0	385

	Wollaston (9)							North Quincy (10)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25	4.8	0.3	8.5	2.5		0	480	6.6	0.5	8.6	0.0		0	623
T30	13.7	0.4	8.8	0.0		0	651	15.5	0.5	8.9	0.0		0	788
T32	22.2	0.3	8.5	0.0		0	436	24.0	0.3	8.5	0.0		0	578
T26	28.4	0.3	6.2	0.0	0	0	507	30.2	0.4	6.1	0.0	0	0	599
T28	31.4	0.4	3.0	0.0	0	0	430	33.1	0.5	2.9	0.0	0	0	462

	JFK (11)							Andrew (12)							Broadway (13)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19																					
T21							532	0.4	0.3	6.2	0.0		0	564	2.9	0.3	6.2	0.0		0	568
T23	3.5	0.3	6.0	0.0		0	532	6.4	0.3	6.0	0.0		0	563	8.9	0.3	6.0	0.0		0	567
T25	12.1	0.3	8.6	0.0		0	623	15.0	0.3	8.6	0.0		0	668	17.6	0.3	8.6	0.0		0	676
T30	21.0	0.3	8.9	0.0		0	784	23.9	0.3	8.9	0.0		0	829	26.5	0.3	8.9	0.0		0	833
T32	29.5	0.3	8.5	0.0		0	580	32.4	0.3	8.5	0.0		0	624	35.0	0.3	8.5	0.0		0	633
T26	35.6	0.3	6.1	0.0	0	0	594	38.5	0.3	6.1	0.0	0	0	625	41.1	0.3	6.1	0.0	0	0	626
T28	38.8	0.3	3.2	0.2	0	0	455	41.9	0.3	3.3	0.1	0	0	470	45.2	0.3	4.1	0.8	0	0	470

	South Station (14)							Downtown Crossing (15)							Park ST. (16)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19							504	2.0	0.7	6.0	0.0		0	344	3.4	0.5	6.2	0.0		0	214
T21	6.2	0.5	6.2	0.0		0	510	8.2	0.7	6.2	0.0		0	349	9.6	0.5	6.2	0.0		0	217
T23	12.2	0.5	6.0	0.0		0	506	14.2	0.7	6.0	0.0		0	345	15.6	0.5	6.0	0.0		0	212
T25	21.0	0.7	8.8	0.0		0	626	23.1	0.8	8.9	0.0		0	442	24.6	0.5	9.1	0.1		0	287
T30	30.0	0.7	9.0	0.0		0	746	32.3	0.8	9.1	0.0		0	511	33.6	0.5	9.0	0.0		0	316
T32	38.3	0.5	8.4	0.0		0	588	40.5	0.5	8.2	0.0		0	414	41.8	0.5	8.2	0.0		0	266
T26	44.4	0.6	6.1	0.0	0	0	551	46.5	0.7	6.0	0.0	0	0	370	47.8	0.5	6.0	0.0	0	0	223
T28	48.8	0.8	4.4	0.5	0	0	412	50.8	0.9	4.3	0.0	0	0	274	52.1	0.5	4.3	0.0	0	0	163

DT = departure time DW = dwell time DH = departure headway HT = holding time S =1, skip P = passengers left L = departure load

1. Comparison of Optimal Control Strategies

The evaluation time windows by station are shown in Table 3-13. The evaluation window starting time is the same as that used in the 10-minute instance except at stations 15 and 16, where the departure time of train T19 instead of train T21 is used because train T21 is held at station 14. The departure time of train T32 at each station in the "No-Control" case is chosen as the evaluation ending time since it is the first train that does not leave passengers behind after the blockage clearance. Passengers initially on-board trains T21 (532), T23 (535), T25 (352) and T26 (75) are included, making a total of 7779 passengers included in the common group of passengers evaluated.

Table 3-13 Evaluation Time Window by Station for the 20-Minute Scenario

Station	t_s	t_e	Passengers	Station	t_s	t_e	Passengers
6	-0.64	27.34	350	11	-2.55	47.71	121
7	-3.07	18.58	859	12	0.38	52.75	342
8	-0.23	34.74	735	13	2.95	56.74	136
9	-3.72	37.68	626	14	6.21	60.57	848
10	-2	39.89	831	15	2.02	62.49	700
				16	3.38	63.84	737

t_s **= starting time,** t_e **= ending time.**

The objective function value, mean platform-waiting times and in-vehicle delays for the different cases are shown in Table 3-14.

Table 3-14 Comparison of Strategy Effectiveness for 20-Minute Disruption Scenario

Control Scheme	Objective Function Value	Change	Platform Waiting Time	In-vehicle Delay	Total Weighted Waiting Time	Saving	Passengers Left
NC	65784	-	9.11	0.19	9.20	-	2691
H	56343	14%	6.57	1.98	7.56	18%	1514
HE	54222	18%	6.23	1.75	7.10	23%	1896
HET	27913	58%	3.79	0.35	3.97	57%	30

ND = No Disturbance, NC = No Control, H = Holding Only, HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning. Saving is in terms of total weighted waiting time, and is compared with that of the "No-Control" case. The weight for in-vehicle delay time is 0.5.

The decreases in the objective function values and the savings in the total weighted waiting time are consistent. In all three active control cases, the ending times of the impact set used at stations in the optimization, the departure time of trains T32, are all earlier than in the "No Control" case. They are modest earlier in the "Holding Only" and "Holding and Expressing" cases, but significantly earlier in the "Holding, Expressing and Short-turning" case. That is, the number of passengers included in the impact set is smaller than that in the "No Control" case, especially in the "Holding, Expressing and Short-turning" case. Therefore, in addition to the benefits from active control strategies, fewer passengers in the impact set is another reason for the decrease in the objective function values.

However, in the post-evaluation, the ending times are the same across all cases, passengers arriving after the departure of train T32 are assumed to have typical mean waiting time under normal operating conditions, 3 minutes. The portion of the passengers having waiting time around 3 minutes in the active control cases, thus, is higher in the post-evaluation than in the optimization. Therefore, the percentage savings in terms of mean weighted waiting time in the post-evaluation should be more significant than the decrease of the objective function value, if the departure times of the last train in the impact set in the active control cases are not significantly earlier than in the "No-Control" case after optimization, or equivalently, if not significantly fewer passengers are included in the impact set used in the optimization.

Since the departure times of train T32 in the "Holding Only" and "Holding and Expressing" are only modest earlier than in the "No Control" case, the savings in term of mean weighted waiting time are larger than the decreases in the objective function value. Due to that the 20-minute disruption is more severe than the 10-minute disruption, the differences are more obvious in the 20-minute case. In the "Holding, Expressing and Short-turning" case, since significantly fewer passengers are included in the impact set, which results in significantly lower total waiting time, the decrease in the objective function value is larger than the saving in terms of mean weighted waiting time.

In the "No-Control" case, the mean platform time is 9.20 minutes, more than three times that in the "No-Disturbance" case. About 2,691 passengers are left behind because of binding train capacity.

In the "Holding Only" case, the mean in-vehicle delay is 1.98 minutes because there is substantial holding involved. About 1,000 fewer passengers are left behind. Compared with the "No Control" case, the savings in terms of the total weighted waiting time are about 18%.

In the "Holding and Expressing" case, the number of passengers left is increased because express trains skip many passengers. The mean platform time is reduced, as is the mean in-vehicle delay time because of reduced holding time. There is a modest increase in savings from expressing.

In the "Holding, Expressing and Short-Turning" case, the results are much like the normal situation, except that the train sequence has changed. The total saving is about 57%. There are only 30 passengers dumped by trains T30 and T32 at Station 39. Due to the highly favorable characteristics of this specific problem, the benefit of short-turning is clearly highly significant.

According to the results, savings from the holding and expressing strategies are comparable with expressing providing only marginal additional benefits beyond holding. Short-turning can dramatically reduce the gap, and is specially efficient in dealing with large disruptions if crossover tracks are conveniently located.

The different optimal control strategies under different control schemes are shown in Table 3-15. The holding time for trains in the "Holding Only" case is longer than that in the 10-minute scenario, and more trains are held. Again, the holding time in the "Holding Only" case is the longest. In the "Holding and Expressing" case, two trains are expressed. One of them skips two stations, the other skips one. In the "Holding, Expressing and Short-turning" case, both trains T30 and T32 are short-turned.

Table 3-15 Control Strategies for the 20-minute Disruption Scenario

Control Schemes	Control Strategies
H	Hold T25 for 6.0 minutes at station 9, 5.6 minutes at station 10, 1.3 minutes at station 14, and 2.2 minutes at station 16
	Hold T23 for 3.0 minutes at station 11, 2.8 minutes at station 14, and 2.0 minutes at station 16
	Hold T21 for 2.0 minutes at station 16
HE	Hold T25 for 6.7 min at station 9 and 4.0 min at station 10, 0.9 minute at station 14, and 2 minutes at station 16
	Hold T23 for 1.4 minutes at station 11, 3.9 minutes at station 14, and 2.0 minutes at station 16
	Hold T21 for 2.0 minutes at station 16
	Let T26 skip station 7 and 8
	Let T28 skip station 7
HET	Hold T25 for 2.5 minutes at station 9
	Short-turn T30 from station 39 to 7
	Short-turn T32 from station 39 to 7

H = Holding Only, HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning

3.3 Sensitivity Analysis

The assumption that blockage duration is known will not be true in practice. To explore the potential changes in benefits arising from uncertainty in blockage duration for different control strategies, sensitivity analysis is done for the (estimated) 10 minutes disruption scenario. Errors of $\pm 50\%$ in the estimate are examined. In one case, the actual disruption is 5 minutes. In the other case, it is 15 minutes.

3.3.1 5-Minute Scenario

1. Optimal Control Strategies under Correct Disruption Duration Estimate

Table 3-16 gives the optimal solution of the 5-minute instance, provided the disruption duration estimate is exact. Neither expressing nor short-turning strategies are selected in the optimal solution. Only train T25 is held for 2.6 minutes at station 9.

2. Control Strategies under Incorrect Disruption Estimate and Re-optimization

In the case that the actual disruption is 5 minutes instead of the estimated 10 minutes, we assume that the initial control strategies based on 10-minute estimate are carried out and control strategies are re-optimized after 5 minutes, when the blockage is cleared.

Table 3-17 gives the results of the re-optimized "Holding Only" case. In the "Holding and Expressing" case, holding strategies completed when the disruption is cleared 5 minutes after its occurrence is the same as that completed in the "Holding Only" case, and the re-optimized solution is also the same as that in the "Holding Only" case. Table 3-18 gives the results of the "Holding, Expressing and Short-turning" case in which the initial blockage estimate is 10 minutes and the control strategies is re-optimized 5 minutes later when the blockage is cleared.

Table 3-19 gives optimal control strategies based on the correct blockage duration estimate for the 5-minute scenario, as well as control strategies based on an initial 10 minute estimate, control actions implemented within 5 minutes, and new control strategies after re-optimization. In the "Holding Only" and "Holding and Expressing" cases, T25 is initially chosen to be held for 5.76 minutes and 4.2 minutes, respectively, while the optimal strategy suggest holding for 2.6 minutes. At 5 minute when the disruption is cleared, T25 has been held for 2.72 minutes, so it can depart station 9 immediately. The realized control action is therefore similar to the optimal strategy.

In the "Holding, Expressing and Short-turning" case, train T30 is in the process of short-turning at 5 minute. Therefore, train T-26 is held for another 3 minutes after the blockage clearance. Table 3-20 gives results of the objective function value, platform-waiting time, in-vehicle waiting time and weighted waiting time correspond to the control strategies under the correct estimate and 10-minute estimate.

Table 3-16 5 - Minute Scenario "Holding, Expressing and Short-turning" Results

Blockage Duration Estimate = 5 min

OBJ Value = 21508 passenger minutes Solution Time = 4.83 sec

Iterations = 3328 Nodes = 78

	North Quincy (36)						
	DT	DW	DH	HT	S	P	L
T30							
T32	1.0	0.3	7.1	0.0		0	

	Wollaston (37)						
	DT	DW	DH	HT	S	P	L
T30							
T32	2.9	0.3	7.0	0.0		0	

	Quincy Center (38)						
	DT	DW	DH	HT	S	P	L
T30							
T32	5.6	0.3	6.9	0.0		0	

	Quincy Adams (39)						
	DT	DW	DH	HT	S	P	L
T30	1.4	0.3		0.0			
T32	8.3	0.3	6.9	0.0		0	

	Braintree (6)						
		DT	DH	HT	S	P	L
T25							
T26							75
T28		6.4	7.0	1.0		0	88
T30		11.9	5.5	0.6		0	69
T32		17.4	5.4	0.0		0	68

	Quincy Adams (7)						
	DT	DW	DH	HT	S	P	L
T25							
T26	8.2	0.7	11.3	0.0	0	0	360
T28	13.7	0.4	5.5	3.8	0	0	228
T30	16.7	0.3	3.0	1.3		0	145
T32	20.9	0.3	4.1	0.0		0	173

	Quincy Center (8)						
	DT	DW	DH	HT	S	P	L
T25							352
T26	11.3	0.6	11.5	0.0	0	0	600
T28	16.6	0.3	5.3	0.0	0	0	338
T30	19.6	0.3	3.0	0.0		0	207
T32	23.7	0.3	4.1	0.0		0	259

	Wollaston (9)						
	DT	DW	DH	HT	S	P	L
T25	4.9	0.3	8.6	2.6			481
T26	13.8	0.4	8.9	0.0	0	0	734
T28	19.1	0.3	5.3	0.0	0	0	416
T30	22.1	0.3	3.0	0.0		0	252
T32	26.2	0.3	4.1	0.0		0	321

	North Quincy (10)						
	DT	DW	DH	HT	S	P	L
T25	6.7	0.5	8.7	0.0			625
T26	15.7	0.5	9.0	0.0	0	0	868
T28	20.8	0.3	5.1	0.0	0	0	492
T30	23.8	0.3	3.0	0.0		0	297
T32	27.9	0.3	4.1	0.0		0	384

	JFK (11)						
	DT	DW	DH	HT	S	P	L
T19							
T21							532
T23	3.5	0.3	6.0	0.0		0	532
T25	12.2	0.3	8.7	0.0		0	626
T26	21.2	0.3	9.0	0.0	0	0	862
T28	26.2	0.3	5.1	0.0	0	0	488
T30	29.4	0.3	3.2	0.2		0	295
T32	33.4	0.3	4.0	0.0		0	381

	Andrew (12)						
	DT	DW	DH	HT	S	P	L
T19							
T21	0.4	0.3	6.2	0.0		0	564
T23	6.4	0.3	6.0	0.0		0	563
T25	15.1	0.3	8.7	0.0		0	671
T26	24.1	0.3	9.0	0.0	0	0	907
T28	29.1	0.3	5.1	0.0	0	0	513
T30	32.5	0.3	3.3	0.1		0	311
T32	36.5	0.3	4.0	0.2		0	401

	Broadway (13)						
	DT	DW	DH	HT	S	P	L
T19							
T21	2.9	0.3	6.2	0.0		0	568
T23	8.9	0.3	6.0	0.0		0	567
T25	17.7	0.3	8.7	0.0		0	680
T26	26.7	0.3	9.0	0.0	0	0	909
T28	31.7	0.3	5.1	0.0	0	0	514
T30	35.7	0.3	4.0	0.7		0	314
T32	40.0	0.3	4.3	1.0		0	404

	South Station (14)						
	DT	DW	DH	HT	S	P	L
T19							504
T21	6.2	0.5	6.2	0.0		0	510
T23	12.2	0.5	6.0	0.0		0	506
T25	21.0	0.7	8.8	0.0		0	629
T26	30.2	0.8	9.1	0.0	0	0	805
T28	34.9	0.5	4.7	0.0	0	0	449
T30	39.2	0.4	4.3	0.4		0	294
T32	43.4	0.4	4.2	0.3		0	359

	Downtown Crossing (15)						
	DT	DW	DH	HT	S	P	L
T19	2.0	0.7	6.0	0.0		0	344
T21	8.2	0.7	6.2	0.0		0	349
T23	14.2	0.7	6.0	0.0		0	345
T25	23.2	0.8	9.0	0.0		0	445
T26	32.6	1.0	9.3	0.0	0	0	546
T28	36.9	0.6	4.3	0.0	0	0	296
T30	41.0	0.4	4.2	0.0		0	208
T32	45.3	0.5	4.2	0.0		0	245

	Park ST. (16)						
	DT	DW	DH	HT	S	P	L
T19	3.4	0.5	6.2	0.0		0	214
T21	9.6	0.5	6.2	0.0		0	217
T23	15.6	0.5	6.0	0.0		0	212
T25	24.9	0.5	9.3	0.3		0	292
T26	33.9	0.5	9.0	0.0	0	0	331
T28	38.8	0.5	4.9	0.6	0	0	179
T30	42.4	0.5	3.6	0.0		0	128
T32	46.6	0.5	4.2	0.0		0	150

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left L = departure load

Table 3-17 5 - Minute Disruption "Holding Only" Case Re-optimized Results

Blokage Duration Estimate = 10 min
OBJ Value = 39501 passenger minutes
Iterations = 3769

Solution Time = 5.82 sec
Nodes = 83

	North Quincy (36)						Wollaston (37)						Quincy Center (38)						Quincy Adams (39)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T30																			1.4	0.3		0.0		
T32	1.0	0.3	7.1	0.0	0		2.9	0.3	7.0	0.0	0		5.6	0.3	6.9	0.0	0		8.3	0.3	6.9	0.0	0	

	Braintree (6)						Quincy Adams (7)						Quincy Center (8)						Wollaston (9)						North Quincy (10)					
		DT	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T25																		352	5.0	0.3	8.7	2.7	0	483	6.8	0.5	8.8	0.0	0	629
T26						75	8.2	0.7	11.3	0.0	0	360	11.3	0.6	11.5	0.0	0	600	13.8	0.4	8.8	0.0	0	732	15.7	0.5	8.9	0.0	0	864
T28		6.4	7.0	1.0	0	88	13.7	0.4	5.5	3.8	0	227	16.5	0.3	5.3	0.0	0	337	19.0	0.3	5.2	0.0	0	415	20.7	0.3	5.0	0.0	0	490
T30		11.9	5.5	0.5	0	69	16.7	0.3	3.0	1.3	0	145	19.5	0.3	3.0	0.0	0	207	22.0	0.3	3.0	0.0	0	252	23.7	0.3	3.0	0.0	0	296
T32		17.4	5.5	0.0	0	68	20.9	0.3	4.2	0.0	0	174	23.7	0.3	4.2	0.0	0	261	26.2	0.3	4.2	0.0	0	323	27.9	0.3	4.2	0.0	0	386

	JFK (11)						Andrew (12)						Broadway (13)						South Station (14)						Downtown Crossing (15)						Park ST. (16)						
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	S	P	L
T19																								504	2.0	0.7	6.0	0.0	0	344	3.4	0.5	6.2	0.0	0	0	214
T21						532	0.4	0.3	6.2	0.0	0	564	2.9	0.3	6.2	0.0	0	568	6.2	0.5	6.2	0.0	0	510	8.2	0.7	6.2	0.0	0	349	9.6	0.5	6.2	0.0	0	0	217
T23	3.5	0.3	6.0	0.0	0	532	6.4	0.3	6.0	0.0	0	563	8.9	0.3	6.0	0.0	0	567	12.2	0.5	6.0	0.0	0	506	14.2	0.7	6.0	0.0	0	345	15.7	0.5	6.1	0.1	0	0	214
T25	12.3	0.3	8.8	0.0	0	631	15.2	0.3	8.8	0.0	0	677	17.8	0.3	8.8	0.0	0	685	21.2	0.7	9.0	0.0	0	635	23.4	0.8	9.1	0.0	0	450	24.9	0.5	9.2	0.2	0	0	292
T26	21.2	0.3	8.9	0.0	0	858	24.1	0.3	8.9	0.0	0	902	26.6	0.3	8.9	0.0	0	904	30.2	0.8	9.0	0.0	0	798	32.5	1.0	9.2	0.0	0	541	33.9	0.5	9.0	0.0	0	0	329
T28	26.2	0.3	5.0	0.0	0	487	29.1	0.3	5.0	0.0	0	512	31.7	0.3	5.0	0.0	0	513	34.9	0.5	4.7	0.0	0	448	36.8	0.6	4.3	0.0	0	295	38.8	0.5	4.9	0.6	0	0	179
T30	29.4	0.3	3.2	0.2	0	294	32.4	0.3	3.3	0.1	0	311	35.7	0.3	4.0	0.7	0	314	39.2	0.4	4.3	0.4	0	294	41.0	0.4	4.2	0.0	0	208	42.4	0.5	3.6	0.0	0	0	128
T32	33.4	0.3	4.0	0.0	0	384	36.4	0.3	4.0	0.1	0	403	40.0	0.3	4.3	1.0	0	406	43.4	0.4	4.2	0.3	0	361	45.2	0.5	4.2	0.0	0	245	46.6	0.5	4.2	0.0	0	0	151

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left L = departure load

Table 3-18 5 - Minute Disruption "Holding, Expressing and Short-turning" Case Re-optimized Results

Blockage Duration Estimate = 10 min
OBJ Value = 39501 passenger minutes   Solution Time = 4.48 sec
Iterations = 3054   Nodes = 38

	North Quincy (36)							Wollaston (37)							Quincy Center (38)							Quincy Adams (39)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30																						1.4	0.3		0.0		10	
T32	1.0	0.3	7.1	0.0		0		2.9	0.3	7.0	0.0		0		5.6	0.3	6.9	0.0		0		8.3	0.3	6.9	0.0		0	

	Braintree (6)						Quincy Adams (7)							Quincy Center (8)							Wollaston (9)							North Quincy (10)						
	DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																				352	4.9	0.3	8.6	2.6		0	481	6.7	0.5	8.7	0.0		0	625
T30						75	8.0	0.7	11.1	0.0		0	281	11.1	0.6	11.3	0.0		0	518	13.6	0.4	8.8	0.0		0	650	15.5	0.5	8.8	0.0		0	785
T26	0.0	0.6	0.0		0	0	11.0	0.3	3.0	0.0	0	0	151	16.3	2.8	5.2	0.0	0	0	259	18.8	0.3	5.2	0.0	0	0	337	20.5	0.3	5.0	0.0	0	0	416
T28	9.2	9.2	3.9		0	124	17.0	0.4	6.0	4.2	0	0	275	20.4	0.3	4.1	0.5	0	0	360	22.9	0.3	4.1	0.0	0	0	421	24.6	0.3	4.1	0.0	0	0	477
T32	17.4	8.1	0.0		0	102	20.9	0.3	3.8	0.0		0	198	23.7	0.3	3.3	0.0		0	267	26.2	0.3	3.3	0.0		0	317	27.9	0.3	3.3	0.0		0	363

	JFK (11)							Andrew (12)							Broadway (13)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19																					
T21							532	0.4	0.3	6.2	0.0		0	564	2.9	0.3	6.2	0.0		0	568
T23	3.5	0.3	6.0	0.0		0	532	6.4	0.3	6.0	0.0		0	563	8.9	0.3	6.0	0.0		0	567
T25	12.2	0.3	8.7	0.0		0	626	15.1	0.3	8.7	0.0		0	671	17.7	0.3	8.7	0.0		0	680
T30	21.0	0.3	8.8	0.0		0	781	23.9	0.3	8.8	0.0		0	826	26.5	0.3	8.8	0.0		0	829
T26	26.0	0.3	5.0	0.0	0	0	414	28.9	0.3	5.0	0.0	0	0	440	31.5	0.3	5.0	0.0	0	0	442
T28	30.0	0.3	4.1	0.0	0	0	471	33.0	0.3	4.1	0.0	0	0	491	35.5	0.3	4.1	0.0	0	0	490
T32	33.4	0.3	3.3	0.0		0	360	36.3	0.3	3.3	0.0		0	376	39.7	0.3	4.1	0.8		0	379

	South Station (14)							Downtown Crossing (15)							Park ST. (16)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19							504	2.0	0.7	6.0	0.0		0	344	3.4	0.5	6.2	0.0		0	214
T21	6.2	0.5	6.2	0.0		0	510	8.2	0.7	6.2	0.0		0	349	9.6	0.5	6.2	0.0		0	217
T23	12.2	0.5	6.0	0.0		0	506	14.2	0.7	6.0	0.0		0	345	15.6	0.5	6.0	0.0		0	212
T25	21.0	0.7	8.8	0.0		0	629	23.2	0.8	9.0	0.0		0	445	24.6	0.5	9.0	0.0		0	288
T30	30.0	0.8	8.9	0.0		0	742	32.3	0.9	9.0	0.0		0	508	33.6	0.5	9.0	0.0		0	315
T26	34.6	0.4	4.7	0.0	0	0	394	36.5	0.5	4.3	0.0	0	0	265	38.6	0.5	4.9	0.0	0	0	167
T28	38.9	0.5	4.3	0.2	0	0	424	40.8	0.6	4.3	0.0	0	0	282	42.2	0.5	3.6	0.0	0	0	159
T32	43.2	0.4	4.3	0.4		0	342	45.0	0.5	4.2	0.0		0	235	46.4	0.5	4.2	0.0		0	146

DT = departure time   DW = dwell time   DH = departure headway   HT = holding time   S=1, skip   P = passengers left   L = departure load

Table 3-19 Control Strategy Comparison with Over-estimated Disruption Duration

Blockage Duration Estimate	5 Minutes	10 Minutes		
	Optimal Action	Initial Action	Action Completed after 5 minute	New Action
H	Hold T25 for 2.6 min at Station 9	Hold T25 for 5.76 min at Station 9, and 1.9 min at Station 10	T25 has been held for 2.72 min	Release T25 immediately
HE		Hold T25 for 4.2 min at Station 9 and 0.3 min at Station 10 T26 skips Station 7	T25 has been held for 2.72 min	Release T25 immediately Do not express T26
HET		Hold T25 for 2.6 min at Station 9 Short-turn T30 from Station 39 to 7	T25 has been held for 2.6 min T30 has started short-turning process	Hold T26 until T30 has completed short-turning and left Station 7

H = Holding Only, HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning

Table 3-20 Effect of Over-estimating the Disruption Duration

Blockage Duration Estimate	5 Minutes	10 Minutes	
Control Schemes	**H, HE, HET**	**H & HE**	**HET**
Objective Function Value	21508	21512	22106
Platform waiting Time	17761	17760	17800
In-vehicle Delay Time	2315	2318	2402
Total Weighted Waiting Time	14875	18884	14968
Increase due to Wrong Estimate		+0.0006%	+0.6%

H = Holding Only, HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning. Increase is in terms of total weighted waiting time, and is compared with that in the correct estimate case. The weight for in-vehicle delay time is 0.5.

In the "Holding Only" and "Holding and Expressing" cases, since there is little difference between the optimal strategies with the correct disruption duration estimate and with initial incorrect estimate and re-optimized control strategies, the objective function value based on the wrong estimate is virtually identical to the optimal one. The post processed in-vehicle delay in the re-optimized scenario is a little greater than in the optimal scenario because of longer realized holding time, but the resulting platform waiting time is shorter and the total weighted passenger wait are similar.

In the “Holding, Expressing and Short-turning” case, train T26 is held for about 3 minutes at station 8 and train T28 is also held at station 6 for an extra 3 minutes after the blockage is cleared. However, the headway for T30 in the 10-minute estimate case is similar to that of T26 in the optimal solution. In addition, there are not many passengers on-board T26 and T28, and the resulting holding cost is low. Therefore, there is only a slight increase in the objective function value and the post-processed weighted waiting time. In other situations when there are many passengers on-board, the situation may be very different.

3.3.2 15-Minute Scenario

1. <u>Optimal Control Strategies under Correct Disruption Duration Estimate</u>

Tables 3-21, 3-22, 3-23 give the optimal solution to the “Holding Only”, “Holding and Expressing”, and “Holding, Expressing and Short-turning” cases of the 15-minute scenario, respectively.

In the 15-minute “Holding Only” case, T25 is held for 7.3 minutes at Station 9 and 2.6 minutes at Station 10. The total holding time is longer than in the 10-minute “Holding Only” case, but shorter than in the 20-minute case, which is understandable. The holding time of T25 at Station 9 is larger than that in the 20-minute case, but the holding time at Station 10 is smaller.

This gives another reason for multiple holding. Intuitively, the closer to the blockage the holding station is, the more benefit holding can provide. However, considering the capacity, trains may be held at later stations for longer time, especially when maximum usage of capacity is important. In the 20-minute instance, the optimal solution tries to hold T25 as long as possible. At the same time, T25 should leave no passengers behind. Therefore, T25 is held for only 6 minutes at Station 9 but 5.6 minutes

Table 3-21 15 - Minute Scenario "Holding Only" Case

Blockage Duration Estimate = 15 minutes
OBJ Value = 41381 passenger minutes
Iterations = 5104

Solution Time = 6.40 sec
Nodes = 339

	North Quincy (36)						Wollaston (37)						Quincy Center (38)						Quincy Adams (39)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T30																			1.4	0.3		0.0		
T32	1.0	0.3	7.1	0.0	0		2.9	0.3	7.0	0.0	0		5.6	0.3	6.9	0.0	0		15.9	0.3	14.5	7.6	0	

	Braintree (6)						Quincy Adams (7)						Quincy Center (8)						Wollaston (9)						North Quincy (10)					
	AT	DT	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T25																		352	9.6	0.3	13.3	7.3	0	552	14.2	0.7	16.2	2.6	0	840
T26						75	18.7	1.2	21.8	0.0	0	626	22.0	0.8	22.2	0.0	131	960	24.5	0.3	15.0	0.0	224	960	26.2	0.3	12.0	0.0	180	960
T28		16.9	17.6	11.6	0	220	21.1	0.3	2.4	0.7	0	280	24.1	0.4	2.1	0.0	0	453	26.9	0.6	2.4	0.0	0	712	29.2	0.7	3.0	0.3	0	909
T30		19.3	2.4	8.0	0	30	23.2	0.3	2.1	0.4	0	82	26.4	0.3	2.3	0.3	0	130	29.9	0.3	3.0	1.0	0	175	31.9	0.3	2.7	0.3	0	218
T32		22.0	2.6	4.6	0	33	25.5	0.3	2.3	0.0	0	91	29.4	0.3	3.0	1.0	0	153	32.9	0.3	3.0	1.0	0	198	34.9	0.3	3.0	0.3	0	246

	JFK (11)						Andrew (12)						Broadway (13)						South Station (14)						Downtown Crossing (15)						Park ST. (16)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T19																								504	2.0	0.7	6.0	0.0	0	344	3.4	0.5	6.2	0.0	0	214
T21						532	0.4	0.3	6.2	0.0	0	564	2.9	0.3	6.2	0.0	0	568	6.2	0.5	6.2	0.0	0	510	8.2	0.7	6.2	0.0	0	349	10.7	0.5	7.3	1.1	0	229
T23	3.5	0.3	6.0	0.0	0	532	6.4	0.3	6.0	0.0	0	563	9.0	0.3	6.0	0.0	0	567	16.2	0.5	10.0	4.0	0	562	18.4	0.8	10.1	0.0	0	419	21.7	0.5	11.0	2.0	0	299
T25	19.7	0.3	16.2	0.0	0	852	22.6	0.3	16.2	0.0	0	939	25.1	0.3	16.2	0.0	0	960	28.8	0.9	12.6	0.0	0	890	31.3	1.1	12.9	0.0	0	632	34.7	0.6	13.0	2.0	0	411
T26	31.7	0.3	12.0	0.0	0	958	34.6	0.3	12.0	0.0	60	960	37.1	0.3	12.0	0.0	8	960	40.8	0.9	12.0	0.0	0	882	43.3	1.1	12.0	0.0	0	618	44.7	0.6	10.0	0.0	0	372
T28	35.0	0.3	3.3	0.3	0	888	37.9	0.3	3.3	0.0	0	959	41.6	0.3	4.4	1.1	0	958	45.6	0.7	4.9	0.6	0	780	47.9	0.9	4.6	0.0	0	484	49.8	0.5	5.1	0.5	0	261
T30	38.1	0.3	3.2	0.8	0	219	42.3	0.3	4.4	1.2	0	242	46.4	0.3	4.9	1.5	0	249	50.3	0.3	4.6	0.8	0	249	52.0	0.4	4.1	0.0	0	182	53.9	0.5	4.1	0.5	0	123
T32	42.6	0.3	4.4	2.2	0	249	47.2	0.3	4.9	1.7	0	275	51.0	0.3	4.6	1.3	0	281	54.4	0.4	4.1	0.3	0	267	56.2	0.4	4.1	0.0	0	192	57.5	0.5	3.6	0.0	0	121

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left L = departure load

Table 3-22 15 - Minute Scenario "Holding and Expressing" Case

Blockage Duration Estimate = 15 minutes
OBJ Value = 39501 passenger minutes
Iterations = 16958

Solution Time = 21.97 sec
Nodes = 934

	North Quincy (36)							Wollaston (37)							Quincy Center (38)							Quincy Adams (39)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30																						1.4	0.3		0.0			
T32	1.0	0.3	7.1	0.0		0		2.9	0.3	7.0	0.0		0		5.6	0.3	6.9	0.0		0		14.7	0.3	13.3	6.4		0	

	Braintree (6)							Quincy Adams (7)							Quincy Center (8)							Wollaston (9)							North Quincy (10)						
		DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																					352	10.7	0.3	14.4	8.4		0	569	13.7	0.7	15.7	1.0		0	846
T26							75	17.5	0.0	20.6	0.0	1	521	75	21.0	1.0	21.3	0.0	0	0	521	23.7	0.5	13.0	0.0	0	0	717	25.7	0.7	12.0	0.0	0	0	912
T28		15.7	16.4	10.4		0	205	20.5	1.3	3.0	0.3	0	0	802	23.7	0.3	2.7	0.3	0	0	854	26.2	0.3	2.5	0.0	0	0	890	28.7	0.3	3.0	0.8	0	0	897
T30		18.7	3.0	7.4		0	38	22.8	0.3	2.3	0.6		0	96	25.7	0.3	2.0	0.0		0	137	29.2	0.3	3.0	1.0		0	182	31.4	0.3	2.7	0.5		0	225
T32		21.3	2.6	4.0		0	32	24.8	0.3	2.0	0.0		0	83	28.7	0.3	3.0	1.0		0	146	32.2	0.3	3.0	1.0		0	191	34.4	0.3	3.0	0.5		0	239

	JFK (11)							Andrew (12)							Broadway (13)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19																					
T21							532	0.4	0.3	6.2	0.0		0	564	2.9	0.3	6.2	0.0		0	568
T23	3.5	0.3	6.0	0.0		0	532	6.4	0.3	6.0	0.0		0	563	9.0	0.3	6.0	0.0		0	567
T25	19.1	0.3	15.7	0.0		0	857	22.1	0.3	15.7	0.0		0	941	24.6	0.3	15.7	0.0		0	960
T26	31.1	0.3	12.0	0.0	0	0	912	34.1	0.3	12.0	0.0	0	14	960	36.6	0.3	12.0	0.0	0	8	960
T28	34.5	0.3	3.3	0.3	0	0	876	37.4	0.3	3.3	0.0	0	0	901	41.1	0.3	4.4	1.1	0	0	901
T30	37.6	0.3	3.2	0.8		0	225	41.8	0.3	4.4	1.2		0	249	45.9	0.3	4.9	1.5		0	255
T32	42.1	0.3	4.4	2.2		0	242	46.7	0.3	4.9	1.7		0	268	50.5	0.3	4.6	1.3		0	274

	South Station (14)							Downtown Crossing (15)							Park ST. (16)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19							504	2.0	0.7	6.0	0.0		0	344	3.4	0.5	6.2	0.0		0	214
T21	6.2	0.5	6.2	0.0		0	510	8.2	0.7	6.2	0.0		0	349	10.7	0.5	7.3	1.1		0	229
T23	16.2	0.5	10.0	4.0		0	562	18.4	0.8	10.1	0.0		0	419	21.7	0.5	11.0	2.0		0	299
T25	28.3	0.9	12.0	0.0		0	883	30.7	1.1	12.4	0.0		0	622	34.2	0.6	12.4	2.0		0	401
T26	40.3	0.9	12.0	0.0	0	0	883	42.8	1.1	12.0	0.0	0	0	618	44.2	0.6	10.0	0.0	0	0	373
T28	45.1	0.7	4.9	0.7	0	0	738	47.3	0.8	4.6	0.0	0	0	460	49.2	0.5	5.0	0.5	0	0	250
T30	49.7	0.4	4.6	0.7		0	254	51.5	0.4	4.1	0.0		0	185	53.4	0.5	4.1	0.5		0	124
T32	53.8	0.3	4.1	0.3		0	262	55.6	0.4	4.1	0.0		0	189	57.0	0.5	3.6	0.0		0	120

DT = departure time   DW = dwell time   DH = departure headway   HT = holding time   S=1, skip   P = passengers left   L = departure load

Table 3-23 15 - Minute Scenario "Holding, Expressing and Short-turning" Case

Blockage Duration Estimate = 15 min
OBJ Value = 39501 passenger minutes
Iterations = 19768

Solution Time = 57.77 sec
Nodes = 999

	North Quincy (36)							Wollaston (37)							Quincy Center (38)							Quincy Adams (39)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30																						1.4	0.3		0.0		10	
T32	1.0	0.3	7.1	0.0		0		2.9	0.3	7.0	0.0		0		5.6	0.3	6.9	0.0		0		8.3	0.3	6.9	0.0		10	

	Braintree (6)							Quincy Adams (7)							Quincy Center (8)							Wollaston (9)							North Quincy (10)						
	AT	DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																					352	4.8	0.3	8.5	2.5		0	480	6.6	0.5	8.6	0.0		0	623
T30								8.0	0.7	11.1	0.0		0	281	11.1	0.6	11.3	0.0		0	518	13.7	0.4	8.8	0.0		0	651	15.5	0.5	8.9	0.0		0	788
T32								14.7	0.4	6.7	0.0		0	169	17.6	0.4	6.5	0.0		0	304	20.1	0.3	6.5	0.0		0	401	21.8	0.4	6.3	0.0		0	503
T26							75	17.9	0.3	3.1	0.0	0	0	154	20.7	0.3	3.1	0.0	0	0	219	23.2	0.3	3.1	0.0	0	0	266	24.9	0.3	3.1	0.0	0	0	310
T28		16.1	16.7	10.7		0	209	20.9	0.3	3.0	1.3	0	0	285	23.7	0.3	3.0	0.0	0	0	346	26.2	0.3	3.0	0.0	0	0	391	27.9	0.3	3.0	0.0	0	0	427

	JFK (11)							Andrew (12)							Broadway (13)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19																					
T21							532	0.4	0.3	6.2	0.0		0	564	2.9	0.3	6.2	0.0		0	568
T23	3.5	0.3	6.0	0.0		0	532	6.4	0.3	6.0	0.0		0	563	8.9	0.3	6.0	0.0		0	567
T25	12.1	0.3	8.6	0.0		0	623	15.0	0.3	8.6	0.0		0	668	17.6	0.3	8.6	0.0		0	676
T30	21.0	0.3	8.9	0.0		0	784	23.9	0.3	8.9	0.0		0	829	26.5	0.3	8.9	0.0		0	833
T32	27.3	0.3	6.3	0.0		0	502	30.2	0.3	6.3	0.0		0	534	32.8	0.3	6.3	0.0		0	539
T26	30.5	0.3	3.2	0.1	0	0	308	33.5	0.3	3.3	0.1	0	0	325	36.8	0.3	4.0	0.7	0	0	327
T28	33.8	0.3	3.3	0.4	0	0	422	37.6	0.3	4.0	0.9	0	0	441	41.2	0.3	4.4	1.1	0	0	443

	South Station (14)							Downtown Crossing (15)							Park ST. (16)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19							504	2.0	0.7	6.0	0.0		0	344	3.4	0.5	6.2	0.0		0	214
T21	6.2	0.5	6.2	0.0		0	510	8.2	0.7	6.2	0.0		0	349	9.6	0.5	6.2	0.0		0	217
T23	12.2	0.5	6.0	0.0		0	506	14.2	0.7	6.0	0.0		0	345	15.6	0.5	6.0	0.0		0	212
T25	21.0	0.7	8.8	0.0		0	626	23.1	0.8	8.9	0.0		0	442	24.6	0.5	9.1	0.1		0	287
T30	30.0	0.8	9.0	0.0		0	746	32.3	0.9	9.1	0.0		0	511	33.6	0.5	9.0	0.0		0	316
T32	36.0	0.5	6.1	0.0		0	487	38.1	0.6	5.8	0.0		0	332	39.4	0.5	5.8	0.0		0	205
T26	40.4	0.4	4.4	0.5	0	0	305	42.2	0.4	4.2	0.0	0	0	214	43.6	0.5	4.2	0.0	0	0	144
T28	44.6	0.4	4.2	0.2	0	0	388	46.5	0.5	4.3	0.0	0	0	261	47.9	0.5	4.3	0.0	0	0	150

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left L = departure load

at Station 10 with a total holding time of 11.6 minutes. In the 15-minute instance, the total holding time is less (9.9 minutes), therefore, T25 can be held at Station 9 for longer. T25 reaches capacity at Station 13 in both cases.

In the 15-minute "Holding and Expressing" case, only T26 is expressed, while both T26 and T28 are expressed in the 20-minute instance. It is expected that the longer the disruption, the more beneficial it will be to express multiple trains. For holding time, T25 is held for 8.4 minutes at Station 9 and 1 minute at Station 10. The total holding time is less than that in the "Holding Only" case. Hence, T25 can be held longer at Station 9.

In the 15-minute "Holding, Expressing and Short-turning" case, the optimal solution is to short-turn both trains T30 and T32. Even though the headway of T26 is quite small after the blockage clearance, T26 is only held at some stations on the trunk portion, which is largely due to the scaled minimum separation. That is, there is enough distance between trains T32 and T26. There is a 7 minute gap behind train T28 due to short-turning of train T32. However, if T32 is not short-turned, the headway for T26 is about 9 minutes. Therefore, instead of leaving a 9-minutes gap ahead of T26, it is better to leave a smaller 7 minutes gap behind train T28.

2. <u>Control Strategies under Incorrect Disruption Duration Estimate and Re-optimization</u>

We assume the actual disruption length is known 10 minutes after the disruption occurs. That is, the second estimate is made after 10 minutes and the disruption duration is estimated correctly the second time. On one hand, the disruption duration can be estimated frequently and the situation can be re-optimized according to the latest estimate and actual train locations. On the other hand, the second estimate of the disruption duration may not be correct, and therefore, the resulting strategies may still be sub-optimal. Nevertheless, if the disruption duration is longer than expected, the first step control strategies should not have negative impacts. Therefore, later strategies are only to achieve further benefits. Tables 3-24, 3-25, 3-26 give the re-optimized solution to the "Holding Only", "Holding and Expressing", and " Holding, Expressing and Short-turning" cases of the 15-minute scenario, respectively.

Table 3-24 15 - Minute Disruption "Holding Only" Case Re-optimization Results

Blockage Duration Estimate = 10 min
OBJ Value = 39501 passenger minutes
Iterations = 20231

Solution Time = 20.95 sec
Nodes = 544

	North Quincy (36)						Wollaston (37)						Quincy Center (38)						Quincy Adams (39)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T30																			1.4	0.3		0.0		
T32	1.0	0.3	7.1	0.0	0		2.9	0.3	7.0	0.0	0		5.6	0.3	6.9	0.0	0		15.9	0.3	14.5	7.6	0	

	Braintree (6)						Quincy Adams (7)						Quincy Center (8)						Wollaston (9)						North Quincy (10)					
		DT	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T25																		352	8.0	0.3	11.8	5.8	0	529	14.2	0.6	16.2	4.2	0	819
T26						75	18.7	1.2	21.8	0.0	0	626	22.0	0.8	22.2	0.0	130	960	24.5	0.3	16.5	0.0	247	960	26.2	0.3	12.0	0.0	180	960
T28		16.9	17.6	11.6	0	220	21.1	0.3	2.4	0.7	0	280	24.1	0.4	2.1	0.0	0	453	26.9	0.7	2.4	0.0	0	736	28.9	0.6	2.7	0.0	0	925
T30		19.3	2.4	8.0	0	30	23.2	0.3	2.1	0.4	0	82	26.4	0.3	2.3	0.4	0	131	29.9	0.3	3.0	1.0	0	176	31.9	0.3	3.0	0.3	0	225
T32		22.0	2.7	4.7	0	34	25.5	0.3	2.3	0.0	0	93	29.4	0.3	3.0	1.0	0	155	32.9	0.3	3.0	1.0	0	200	34.9	0.3	3.0	0.3	0	248

	JFK (11)						Andrew (12)						Broadway (13)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T19																		
T21						532	0.4	0.3	6.2	0.0	0	564	2.9	0.3	6.2	0.0	0	568
T23	3.5	0.3	6.0	0.0	0	532	6.4	0.3	6.0	0.0	0	563	9.0	0.3	6.0	0.0	0	567
T25	19.7	0.3	16.2	0.0	0	832	22.6	0.3	16.2	0.0	0	919	25.2	0.3	16.2	0.0	0	940
T26	31.7	0.3	12.0	0.0	0	958	34.6	0.3	12.0	0.0	59	960	37.1	0.3	12.0	0.0	8	960
T28	34.8	0.3	3.2	0.5	0	903	37.9	0.3	3.3	0.1	14	960	41.6	0.3	4.4	1.1	0	959
T30	38.1	0.3	3.3	0.8	0	225	42.3	0.3	4.4	1.2	0	263	46.4	0.3	4.9	1.5	0	269
T32	42.6	0.3	4.4	2.2	0	250	47.2	0.3	4.9	1.7	0	277	51.0	0.3	4.6	1.3	0	282

	South Station (14)						Downtown Crossing (15)						Park ST. (16)					
	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L	DT	DW	DH	HT	P	L
T19						504	2.0	0.7	6.0	0.0	0	344	3.4	0.5	6.2	0.0	0	214
T21	6.2	0.5	6.2	0.0	0	510	8.2	0.7	6.2	0.0	0	349	10.7	0.5	7.3	1.1	0	229
T23	16.2	0.5	10.0	4.0	0	562	18.4	0.8	10.1	0.0	0	419	21.7	0.5	11.0	2.0	0	299
T25	28.8	0.9	12.6	0.0	0	876	31.3	1.1	12.9	0.0	0	624	34.7	0.6	13.0	2.0	0	408
T26	40.8	0.9	12.0	0.0	0	882	43.3	1.1	12.0	0.0	0	618	44.7	0.6	10.0	0.0	0	372
T28	45.6	0.7	4.9	0.6	0	781	47.9	0.9	4.6	0.0	0	484	49.3	0.5	4.6	0.0	0	255
T30	50.3	0.4	4.6	0.8	0	264	52.0	0.4	4.1	0.0	0	191	53.9	0.5	4.7	0.5	0	133
T32	54.4	0.4	4.1	0.3	0	268	56.2	0.4	4.1	0.0	0	193	57.5	0.5	3.6	0.0	0	121

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left L = departure load

Table 3-25 15 - Minute Disruption "Holding and Expressing" Case Re-optimization Results

Blockage Duration Estimate = 10 min
OBJ Value = 39501 passenger minutes
Iterations = 17874

Solution Time = 22.33 sec
Nodes = 900

	North Quincy (36)							Wollaston (37)							Quincy Center (38)							Quincy Adams (39)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30																						1.4	0.3		0.0			
T32	1.0	0.3	7.1	0.0		0		2.9	0.3	7.0	0.0		0		5.6	0.3	6.9	0.0		0		14.7	0.3	13.3	6.4		0	

	Braintree (6)							Quincy Adams (7)							Quincy Center (8)						
		DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																					352
T26							75	17.5	0.0	20.6	15.0	1	521	75	21.0	1.0	21.3	0.0	0	0	521
T28		15.7	16.4	10.4		0	205	20.2	1.3	2.6	0.0	0	0	793	23.3	0.3	2.3	0.3	0	0	837
T30		18.4	2.7	7.1		0	34	22.4	0.3	2.3	0.5		0	91	25.8	0.3	2.5	0.5		0	143
T32		21.4	3.0	4.1		0	38	24.9	0.3	2.5	0.0		0	101	28.8	0.3	3.0	1.0		0	163

	Wollaston (9)							North Quincy (10)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25	6.5	0.3	10.2	4.2		0	506	8.9	0.7	10.9	0.3		0	691
T26	23.8	0.6	17.3	0.0	0	0	783	25.8	0.6	17.0	0.0	0	112	960
T28	26.3	0.3	2.5	0.5	0	0	873	28.1	0.5	2.3	0.0	0	0	960
T30	29.3	0.3	3.0	1.0		0	188	31.2	0.3	3.0	0.2		0	256
T32	32.3	0.3	3.0	1.0		0	208	34.5	0.3	3.3	0.5		0	262

	JFK (11)							Andrew (12)							Broadway (13)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19																					
T21							532	0.4	0.3	6.2	0.0		0	564	2.9	0.3	6.2	0.0		0	568
T23	3.5	0.3	6.0	0.0		0	532	6.4	0.3	6.0	0.0		0	563	8.9	0.3	6.0	0.0		0	567
T25	14.3	0.3	10.9	0.0		0	695	17.2	0.3	10.9	0.0		0	752	19.8	0.3	10.9	0.0		0	763
T26	31.3	0.3	17.0	0.0	0	10	960	34.2	0.3	17.0	0.0	0	90	960	36.8	0.3	17.0	0.0	0	19	960
T28	34.4	0.3	3.2	0.8	0	19	947	37.5	0.3	3.3	0.1	0	88	960	41.2	0.3	4.5	1.2	0	10	960
T30	37.7	0.3	3.3	1.1		0	256	42.0	0.3	4.5	1.3		0	368	46.2	0.3	4.9	1.6		0	382
T32	42.2	0.3	4.5	2.3		0	264	46.9	0.3	4.9	1.8		0	291	50.8	0.3	4.6	1.3		0	296

	South Station (14)							Downtown Crossing (15)							Park ST. (16)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19							504	2.0	0.7	6.0	0.0		0	344	3.4	0.5	6.2	0.0		0	214
T21	6.2	0.5	6.2	0.0		0	510	8.2	0.7	6.2	0.0		0	349	9.6	0.5	6.2	0.0		0	217
T23	14.2	0.5	8.0	2.0		0	534	16.3	0.7	8.1	0.0		0	382	19.7	0.5	10.1	2.0		0	273
T25	26.0	0.7	11.7	2.7		0	732	28.3	1.0	12.0	0.0		0	534	32.5	0.5	12.8	2.8		0	368
T26	40.5	1.0	14.5	0.0	0	0	917	43.0	1.2	14.7	0.0	0	0	665	44.5	0.6	12.0	0.0	0	0	414
T28	45.4	0.7	4.9	0.7	0	0	783	47.7	0.9	4.6	0.0	0	0	485	49.6	0.5	5.1	0.6	0	0	262
T30	50.0	0.4	4.6	0.7		0	349	51.9	0.5	4.2	0.0		0	239	53.8	0.5	4.2	0.6		0	147
T32	54.2	0.4	4.2	0.4		0	279	56.0	0.4	4.2	0.0		0	199	57.4	0.5	3.6	0.0		0	124

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left L = departure load

Table 3-26 15 - Minute Disruption "Holding, Expressing and Short-turning" Case Re-optimization Results

Blokage Duration Estimate = 10 min
OBJ Value = 39501 passenger minutes
Iterations = 16755

Solution Time = 30.28 sec
Nodes = 1346

	North Quincy (36)							Wollaston (37)							Quincy Center (38)						Quincy Adams (39)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	HT	S	P	L	DT	DW	DH	HT	S	P	L
T30																					1.4	0.3		0.0		10	
T32	1.0	0.3	7.1	0.0		0		2.9	0.3	7.0	0.0		0		5.6	0.3	0.0		0		8.3	0.3	6.9	0.0		0	

	Braintree (6)							Quincy Adams (7)							Quincy Center (8)							Wollaston (9)						North Quincy (10)						
		DT	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	HT	S	P	L	DT	DW	DH	HT	S	P	L
T25																					352	4.9	0.3	2.6		0	481	6.7	0.5	8.7	0.0		0	625
T30								8.5	0.7	11.6	0.5		0	281	11.6	0.6	11.9	0.0		0	529	14.2	0.4	0.0		0	668	16.1	0.5	9.4	0.0		0	814
T26		0.0	0.6	0.0		0	75	17.5	0.0	9.0	15.0	1	228	75	20.5	0.5	8.9	0.0	0	0	261	23.0	0.4	0.0	0	0	395	24.9	0.5	8.8	0.0	0	0	546
T28		15.7	15.7	10.4		0	205	19.7	0.7	2.2	0.1	0	0	487	22.5	0.3	2.0	0.0	0	0	527	26.0	0.3	1.0	0	0	572	27.9	0.3	3.0	0.2	0	0	597
T32		18.2	2.4	0.8		0	30	21.7	0.3	2.0	0.0		0	80	25.5	0.3	3.0	1.0		0	142	29.0	0.3	1.0		0	188	30.7	0.3	2.8	0.0		0	233

	JFK (11)							Andrew (12)							Broadway (13)						
	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19																					
T21							532	0.4	0.3	6.2	0.0		0	564	2.9	0.3	6.2	0.0		0	568
T23	3.5	0.3	6.0	0.0		0	532	6.4	0.3	6.0	0.0		0	563	8.9	0.3	6.0	0.0		0	567
T25	12.2	0.3	8.7	0.0		0	626	15.1	0.3	8.7	0.0		0	671	17.7	0.3	8.7	0.0		0	680
T30	21.5	0.3	9.4	0.0		0	811	24.5	0.3	9.4	0.0		0	858	27.0	0.3	9.4	0.0		0	862
T26	30.3	0.3	8.8	0.0	0	0	549	33.3	0.3	8.8	0.0	0	0	596	35.8	0.3	8.8	0.0	0	0	604
T28	33.6	0.3	3.3	0.3	0	0	586	36.6	0.3	3.3	0.0	0	0	600	40.0	0.3	4.1	0.8	0	0	597
T32	36.8	0.3	3.2	0.6		0	233	40.7	0.3	4.1	1.0		0	255	44.5	0.3	4.5	1.2		0	261

	South Station (14)						Downtown Crossing (15)							Park ST. (16)						
	DT	DW	HT	S	P	L	DT	DW	DH	HT	S	P	L	DT	DW	DH	HT	S	P	L
T19						504	2.0	0.7	6.0	0.0		0	344	3.4	0.5	6.2	0.0		0	214
T21	6.2	0.5	0.0		0	510	8.2	0.7	6.2	0.0		0	349	9.6	0.5	6.2	0.0		0	217
T23	12.2	0.5	0.0		0	506	14.2	0.7	6.0	0.0		0	345	15.6	0.5	6.0	0.0		0	212
T25	21.0	0.7	0.0		0	629	23.2	0.8	9.0	0.0		0	445	25.3	0.5	9.7	0.7		0	295
T30	30.5	0.8	0.0		0	774	32.9	1.0	9.7	0.0		0	533	34.3	0.5	9.0	0.0		0	325
T26	39.2	0.6	0.0	0	0	570	41.3	0.8	8.4	0.0	0	0	406	42.7	0.5	8.4	0.0	0	0	265
T28	43.7	0.5	0.5	0	0	507	45.7	0.6	4.4	0.0	0	0	329	47.1	0.5	4.4	0.0	0	0	187
T32	48.1	0.3	0.5		0	255	49.8	0.4	4.1	0.0		0	186	51.2	0.5	4.1	0.0		0	124

DT = departure time DW = dwell time DH = departure headway HT = holding time S=1, skip P = passengers left L = departure load

Table 3-27 gives optimal control strategies under different control schemes based on correct blockage duration estimate for the 15-minute scenario, control strategies based on 10 minutes estimate, control actions completed 10 minutes after the blockage occurs, and new control strategies after re-optimization.

Table 3-27 Control Strategy Comparison with Under-estimated Disruption Duration

Blockage Duration Estimate	15 Minutes	10 Minutes		
	Optimal Strategies	Initial Strategies	Completed after 10 minute	Revised strategies
H	Hold T25 for 7.3 min at Station 9 and 2.6 min at Station 10	Hold T25 for 5.76 min at Station 9, and 1.9 min at Station 10	T25 has been held for 5.76 min and has left Station 9	Hold T25 for 4.2 min at Station 10
HE	Hold T25 for 8.4 min at Station 9 and 1 min at Station 10 Let T26 skip Station 7	Hold T25 for 4.2 min at Station 9 and 0.3 min at Station 10 T26 skips Station 7	T25 has been held for 4.2 min at Station 9 and 0.3 min at Station 10, and has left Station 10	Let T26 skip Station 7
HET	Hold T25 for 2.6 min at Station 9 Short-turn T30 from Station 39 to Station 7 Short-turn T32 from Station 39 to Station 7	Hold T25 for 2.6 min at Station 9 Short-turn T30 from Station 39 to Station 7	T25 has been held for 2.6 min at Station 9 T30 has started short-turning T32 has passed station 39	Short-turn T30 from Station 39 to Station 7 Let T26 skip Station 9

H = Holding Only, HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning

In the "Holding Only" case, the total holding time for train T25 is about 11 minutes in the solution under the correct estimate and is about 10 minutes under the 10-minute estimate. Under the correct estimate, train T25 is primarily held at station 9, while the holding time is about the same at stations 9 and 10 under 10-minute estimate. Since benefit from holding comes from reduced passenger waiting time at downstream station and T25 does not have capacity issue even if T25 is primarily held at station 9, the holding strategy under the correct estimate is better than under the 10-minute estimate.

In the "Holding and Expressing" case, train T25 should be held for 8.4 minutes at station 9 and 1 minute at station 10 under correct disruption duration estimate. Under the 10 minutes estimate, T25 is only held for 5.76 minutes at station 9 and 0.3 minute at station 10. By the end of the 10-minutes estimated duration, it has left station 10 and holding it at following station does not do much good.

In the "Holding, Expressing and Short-turning" case, the optimal strategies are to short-turn both trains T30 and T32. Under the 10-minute estimate, only T30 is short-turned. By the end of the estimated 10-minute duration, T32 has passed station 39 and can not be short-turned. To smooth the gap ahead of train T26, T26 is expressed and skips station 7 after the blockage clearance.

Table 3-28 shows the comparison of the objective function values, platform waiting time, in-vehicle delay and weighted passenger wait correspond to the control strategies under the correct 15 minutes estimate and 10-minutes estimate. Except in the "Holding Only" case, there are significant differences between the optimal strategies and the realized strategies. The control strategies under the correct estimate provide significantly more benefits than those under the 10-minute estimate. This is particularly true for the strategy involving short-turning.

Table 3-28 Effect of Under-estimating Disruption Duration

Blockage Duration Estimate	15 Minutes			10 Minutes		
Control Schemes	**H**	**HE**	**HET**	**H**	**HE**	**HET**
Objective Function Value	41381	39501	23632	41589	41334	26481
Mean Platform waiting Time	5.57	5.26	3.62	5.57	5.67	4.02
Mean In-vehicle Delay Time	1.55	1.41	0.29	1.6	1.08	0.58
Mean Weighted Passenger Wait	6.34	5.97	3.77	6.37	6.21	4.31
Increase due to Wrong Estimate				+0.5%	+4.0%	+14.3%

H = Holding Only, HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning. Increase is in terms of total weighted waiting time, and is compared with that in the correct estimate case. The weight for in-vehicle delay time is 0.5.

According to the results, since holding decisions can be made incrementally according to the latest disruption duration estimate and can be terminated quickly, the benefit from holding does not vary much when the estimate of disruption duration is not accurate. For expressing, since the expressing strategy can be modified after the blockage clearance, the only impact of inaccurate estimate is the holding time for trains ahead of the blockage. Therefore, when expressing strategies are combined with holding strategies, the magnitude of sensitivity should be similar to that when applying holding strategies alone. For short-turning, however, once a short-turning movement has started, it has to be completed even if the blockage duration turns out to be different from the estimate. In addition, once a train has passed the crossover track, it can not be short-turned. Due to the limited number of crossover locations, the number of trains short-turned is highly sensitive to the estimate of the disruption duration. Moreover, when short-turning is used, holding time is usually not large. Consequently, if the changed strategy requires holding trains for more time, trains may have already passed optimal holding points. Therefore, the sensitivity of short-turning control strategies is the largest of the three. However, the overall impact of inaccurate estimates does not appear to be significant according to these very limited test results.

3.4 Solution Time

Since the linearized formulation is a mixed integer program (MIP) and this formulation is used for real time control, solution time is one of the important concerns. In this thesis, several practical techniques have been used to speed up the solution process.

3.4.1 Large Number

The formulation uses large numbers to transform nonlinear constraints into linear constraints. To reduce the feasible space, these large numbers should be as small as possible. Upper bounds for different variables are therefore used as the large numbers as discussed below.

1. Load

   Obviously, the capacity of the train can be used as the upper bound.

2. Passengers left behind

   In the worst case, all of the passengers accumulating at a station after the blockage can be used to approximate the number of passengers left behind.

3. Dwell time

   The upper bound of dwell time can be selected based on real data. Since dwell time is a small part of total time, a loose upper bound should not greatly affect the solution process.

4. Departure time at each station

   For trains behind the blockage, since departure times for trains behind the blockage in the "No-Control" case are always the latest, departure time upper bounds can be calculated heuristically by assuming no control strategies are applied, based on blockage duration, the upper bound on the dwell time, and available running time data. For trains ahead of the blockage, trains will almost unlikely be held for the whole blockage duration. Therefore, if we assume trains within the control set are held for blockage duration, departure time upper bounds can also be obtained heuristically.

3.4.2 Branch and Bound Process

The integer formulation is solved with CPlex 4.0 MIP Solver on a Micron 300M CPU PC. CPlex 4.0 MIP Solver uses a branch and bound approach to solve integer programs. This thesis will not describe the branch and bound method in detail, but only some features relevant to this specific problem.

The priority of variables selected for branching can be controlled through priority files in CPlex 4.0. The priority of different variables is predefined. To speed up the solution process, the priority file should guide the solution process towards potentially good solutions. There are two aspects in defining priorities: first, the feasibility of the solution; second, potential optimality of the solution.

If the priority file can guide the program to search for feasible solutions in the correct direction, potentially fewer irrelevant nodes will be kept and computation cost to check these nodes will be reduced. In addition, the program can potentially find a good feasible solution quickly and obtain a good bound. For example, suppose there are three binary variables x, y, z to be branched on, and feasible solutions demand that z equal to 1 while x and y can be either 1 or 0. In the first case, suppose we first branch on x, then on y, and finally on z. After branching on x, there are four potential directions to explore. After branching on y, there are eight directions left. Although four of the eight directions do not provide feasible solutions, each direction may have to be explored and cutting of one direction may not help cut the other three infeasible directions. In addition, all the nodes generated during branching on x, y have to be kept. Alternatively, suppose z is branched on first, then only one direction is left. There are at most additional four additional directions to explore branching on x and y.

Therefore, the order of binary variables to select for branching affects the solution speed significantly. It is important to define the priority file in such a way that major variables that affect feasibility and total cost are branched on first. Hence, variables determining the order of trains should be branched on first, especially for multiple-branch systems. Thus, variables such as predecessor binary variables and short-turn decision variables should have high priorities in branch selection.

During the branch and bound process, a binary variable is either set to one or zero and the potential change of objective function value for each direction is estimated. Then, promising directions are further explored. Since the number of nodes increases exponentially, the fewer the potential nodes and directions, the quicker the solution process. If there is potentially great difference between the two directions of a branched node, the direction with higher potential cost is less likely to be explored. If nodes from which the two directions potentially have large cost differences have high priority in the branching process, potentially fewer nodes and directions will need to be explored.

In this specific problem, short-turning decision variables completely determine the order of trains on a branch and largely determine the possible order at the junction point. Moreover, short-turning strategy may potentially provide large benefit. Hence, there may be large cost difference between short-turning and non-short-turning strategies. Therefore, it makes sense to branch on the short-turning decision variables first.

There are no branches in the simplified system of the test scenarios. If multiple branches exist, the next group of binary variables on which to branch should be the predecessor binary variables. These variables can affect the feasibility of the solution and usually the two directions will have a fairly large cost difference. For example, suppose train A potentially arrives at the junction point earlier than train B. It may be better to let train A enter the trunk portion first. The increase of objective function value may be substantial for the direction in which train B precedes train A entering the trunk portion.

Following short-turning variables and predecessor variables, express decision variables and load binary variables are the next ones to branch on. As discussed, the earlier the express train starts expressing, the larger the benefit may be. This suggests a logical way to branch on the station-skipping binary variables is: first, branch on variables for the first express candidate following the sequence of stations it arrives at; after branching on the first candidate, branch on the second and third and so on. Trains blocked behind the blockage are less likely to be held. Therefore, after the arrival headway and the control strategies are determined, the departure load of the train will also be determined. Hence, after branching on a station-skipping variable, the corresponding load variable can also be branched on.

Table 3-29 and 3-30 compare solution times with and without the priority file defined based on the above ideas.

Table 3-29 Solution Times under Different Control Schemes in the 10-Minute Disruption

Control Scheme	H		HE		HET	
	With Priority	Without Priority	With Priority	Without Priority	With Priority	Without Priority
Solution Time (sec)	2.91	3.17	5.60	16.81	11.28	47.01
Nodes	79	82	253	643	84	489
Iterations	2895	3061	5056	14260	6121	26756

Table 3-30 Solution Times under Different Control Schemes in the 10-Minute Disruption

Control Scheme	H		HE		HET	
	With Priority	Without Priority	With Priority	Without Priority	With Priority	Without Priority
Solution Time (sec)	12.10	21.60	155.01	793.47	68.32	>50000
Nodes	551	787	5085	28241	1162	
Iterations	11143	19918	122590	614187	24066	

In both the 10 minute and 20 minute scenarios, the "Hold Only" problem takes the least time to solve. This is because the problem does not involve expressing and short-turning decision variables. Without defining priority, the 20 minute "Holding, Expressing and Short-turning" problem can not be solved on the given platform due to insufficient memory. 50000 seconds is the time at which the system runs out of memory. The solution time of the 20-minute "Holding, Expressing and Short-turning" problem with priority file is shorter than for the "Holding and Expressing" problem. This is because short-turning provides large benefits in this specific scenario and good bounds can be obtained at an early stage. Since the disruption occurs near the terminus, and few passengers are on the trains in the reverse direction, short-turning can provide significant benefits. If disruption occurs in the middle of the line, branching on the short-turning decision variable may not provide us with nearly as sharp a bound.

3.4.3 Express Strategy

According to the results of different control strategies, the expressing strategy only provides marginal benefit beyond holding. In addition, skipping one more station or choosing another station to skip provides only a small change in the objective value. Table 3-31 gives the objective function value under different expressing strategies in the 20-minute "Holding and Expressing" case.

Table 3-31 Objective Function Value (in passenger minutes) under Different Expressing Strategies

Objective Function Value	Expressing Strategy
54241*	T26 skips 7 and 8, T28 skips 7
55392	T26 skips 7 and 8
54468	T26 skips 7
55401	T26 skips 7, T28 skips 7
55333	T26 skips 7, 8 and 9, T28 skips 7

According to the results, the two directions of an expressing decision potentially have little difference and few directions of expressing variables may be cut during branching. This explains why the solution time for the "Holding and Expressing" case is much longer than that of the "Holding Only" case. For the "Holding, Expressing and Short-turning" case, since short-turning provides significant benefit in this specific problem, a good bound may be obtained when first branching on short-turning binary variables and many station skipping directions may be cut with the bound. However, in cases where short-turning may not provide much benefit, the solution time of "Holding, Expressing and Short-turning" case may be longer than that of the "Holding and Expressing" case.

Considering that expressing only provides marginal benefit while affecting solution time substantially, it may be practical to exclude expressing as a primary strategy. Table 3-32 provides comparison of the solution time of the "Holding and Expressing", "Holding, Expressing and Short-turning", and "Holding and Short-turning" case with priority file controlling branch and bound process. We can see there is substantial improvement in solution time when fixing expressing variables in the 20-

minute instances. In the 10 minutes scenario, since the disruption is not very long, it is less likely to be beneficial to skip many stations or express two trains. Therefore, it does not take much time to solve the "Holding and Expressing" and "Holding, Expressing, and Short-turning" cases. In the 20 minutes scenario, expressing multiple trains and skipping multiple stations may be beneficial. In addition, more trains are likely to be loaded to capacity. Therefore, more nodes need to be explored.

Table 3-32 Solution Times with and without Expressing (in seconds)

Scenario	**HE**	**HET**	**HT**
10-Minute	5.60	11.28	12.06
20-Minute	155.01	68.32	24.72

HE = Holding and Expressing, HET = Holding, Expressing and Short-Turning. HT = Holding and Short-turning

3.5 Summary

1. Holding is most used at stations immediately ahead of the blockage. It is likely to be beneficial to hold a train at multiple stations, depending upon the passenger flow profile. However, the more passengers on-board, the higher the holding cost and the less likely for holding to occur. According to the results, holding strategies may provide a 10-20% benefit over no control.

2. Expressing is most used at stations immediately following the blockage. It can reduce further delay after blockage clearance and balance passenger load. However, expressing only provides modest additional benefits over holding.

3. Short-turning strategy may provide substantial additional benefits, up to 57%, especially when disruption duration is long.

4. The holding strategy is not sensitive to the estimate of disruption duration. The expressing strategy is essentially unaffected by the disruption duration assumption because expressing strategies are implemented after the blockage is cleared. The

sensitivity of the short-turning strategy can be significant especially when the blockage duration is not very long. In such cases, short-turning strategy may even create additional delay after the blockage is cleared.

5. One of the bottlenecks of the solution process is to determine expressing decision variables. Since expressing may only provide marginal benefit over holding, expressing strategy may be taken as a secondary strategy in order to save solution time.

Chapter 4

Summary and Conclusions

In this thesis, a deterministic model for real-time disruption recovery has been presented, which includes holding, expressing and short-turning strategies. The model is applied to a scenario with different disruption duration on a simplified version (non-branch) of the MBTA Red Line. Sensitivity analysis is conducted to investigate the impact of one of the assumptions that the disruption duration is known in advance. In this chapter, the findings are summarized and future research directions are recommended.

4.1 Summary and Conclusions

4.1.1 Disruption Control Model

In Chapter 2, the general model representing the transit system was introduced and a formulation that considers the holding, expressing and short-turning control strategies presented. Since the initial formulation is nonlinear, linearization of the disruption control model is also described.

The general model is a deterministic model. It takes train capacity into consideration, which is essential for analyzing disruption control strategies. In the general model, passenger arrival rates and alighting fractions, train running times and minimum safe separations are all deterministic station-specific parameters. These parameters are estimated from stored data, or alternatively can potentially be based on real time information. For disruption control, the general model assumes that the disruption duration is known once a disruption occurs, which is the most questionable simplification made.

Dwell time is approximated by two linear functions. One applies to the low passenger activity (boarding and alighting) range; the other applies to the high passenger activity range. Potentially both functions can be related to the number of passenger boardings and alightings and the crowding conditions. Due to lack of data in this research, the function applied to the low passenger activity range is assumed to be a constant.

In this research, the general model is extended to representing a transit line at the segment level. A segment of the transit line is a track section on which the order of trains can not change, but across segments, the order may change. Branch points, terminals and crossover track can all function to define the start or end of segments. By defining segments, the difficult problem of change in the train order is addressed. Thus, the general model can be applied to transit systems with multiple branches, and can model control strategies which affects the order of trains such as short-turning, as well as introducing trains into service, or indeed removing them from service.

Based on this extended model, a disruption control formulation that includes holding, expressing and short-turning strategies is developed. The objective function to be minimized is the weighted sum of the passenger waiting time and in-vehicle delay. Intuitively, minimization of passenger waiting time alone ignores the cost of holding trains. By considering resulting in-vehicle delay, the trade-offs among control strategies can be better evaluated. To accommodate the expectation that passengers may perceive in-vehicle delay to be less onerous than platform waiting time, a weight less than one can be used for in-vehicle delay.

The disruption control formulation includes holding, expressing and short-turning strategies. The decision variables are departure time by train and station, station-skipping binary variables by train and station, which model the expressing behavior, short-turning binary variables by train and crossover track, and predecessor binary variables, which determine the order of trains on different segments. There is no restriction on holding strategies. That is, holding can occur at any station. Since in-vehicle delay is considered, holding at multiple stations is expected to be less likely, especially if there are a significant number of passengers on-board. For expressing, the assumption that expressing is announced immediately before the expressing begins is made in the linearized model. In addition, the linearized model pre-defines a maximum express segment and assumes the number of passengers who have to alight the express train based on the maximum express segment. This potentially can over-estimate the number of dumped passengers and degrade the performance of expressing. For short-turning, trains can be short-turned at multiple crossover tracks.

Since the objective function of the initial formulation contains quadratic and non-separable terms, approximations are made to linearize them. Piece-wise linear functions are used to approximate the quadratic terms. The non-separable terms include the additional waiting cost of passengers left behind, which is product of the number of passengers left behind by trains and their additional waiting time, and the in-vehicle delay for passengers on-board the held trains when holding starts, which is the product of the number of passengers on-board when holding starts and the holding time. We show that we can reasonably approximate one of the two variables in each of the non-separable terms. For the additional waiting cost for passengers left behind, we show that only trains initially behind the blockage may leave passengers behind. Since trains initially behind the blockage are governed by the minimum headway, minimum headway is used to approximated the waiting time for passengers left behind. For the in-vehicle delay, we show that only in-vehicle delay of passengers on-board trains initially ahead of the blockage significantly affects the control decisions. Thus, typical passenger load in that time period can be used to approximate the number of passengers on-board when holding starts because holding is likely to be the first intervention for those trains. If holding at multiple stations occurs, the in-vehicle delay may be under-estimated. Nevertheless, since holding at multiple stations is not favored, the approximation should not have significant impact.

Since it is unrealistic to project train movements far into future with such a deterministic model, only a set of trains and stations ahead of and behind the blockage are selected to apply the control strategies (control set). In addition, the objective function only considers a set of trains and stations, known as the impact set. By this means, we can also ensure real-time solvability of the formulation. The sizes of the control set and impact set are related to the disruption duration and location, track configuration and passenger flow profile. Although it is not determined entirely quantitatively, heuristics are developed to partially define the size of impact set quantitatively.

4.1.2 Model Application

In Chapter 3, the disruption control model is applied to a scenario on a simplified version of the MBTA Red Line. Only one of the two branches on the Red Line, the Braintree branch, is considered. In addition, passenger arrival rates and minimum separation on the trunk portion are modified according to the portion of all trains operating on the Braintree branch.

The "No-Disturbance" case is first tested to check the reasonability of the model and generate some data to be used later. Then, disruptions of two different lengths are tested: 10 minutes and 20 minutes. In each scenario, the "No-Control" case, in which no active control strategies are applied, the "Holding Only" case, in which only holding strategies are allowed, "Holding and Expressing" case, in which only holding and expressing strategies are allowed, and "Holding, Expressing and Short-turning" case, in which holding, expressing and short-turning strategies are all allowed, are tested. To compare the benefits from different control schemes, we define an evaluation time window at each station, and identify a common group of passengers to evaluate the impact across control schemes. Benefits are compared in terms of mean weighted waiting time, which is the sum of the mean platform waiting time and mean weighted in-vehicle delay. The weight for the in-vehicle delay is 0.5. Mean weighted waiting times from the control strategies are compared with that from the "No-Control" case.

In the 10-minute disruption scenario, holding alone can reduce the weighted waiting time by 10%. Compared with "No Control" case, platform waiting time is significantly reduced, but at the expense of increased in-vehicle delay. Expressing only provides marginal additional benefit. Combined with holding, it reduces the weighted waiting time by 13%. The additional benefits are largely achieved through reduced in-vehicle delay. Short-turning combined with holding achieves the highest benefits, 35%. Both platform waiting time and in-vehicle delay are reduced significantly. However, since the specifics of the test scenario are highly favorable to short-turning, the benefits could be significantly lower in other cases.

In the 20-minute scenario, holding alone achieves a 18% reduction in weighted waiting time, expressing combined with holding achieves a 23% reduction, and short-turning combined with holding achieves a 57% reduction. In such long disruption scenario, the solution shows a capacity-driven holding pattern. That is, the capacity of trains ahead of the blockage is full used, especially the first train ahead of the blockage. Except in the "Holding, Expressing and Short-turning" case, the first train ahead of the blockage is loaded to capacity at some station.

We notice that:

1. <u>Holding combined with short-turning can achieve significant benefits in disruption control. Expressing only provides marginal additional benefits.</u>

2. <u>Active holding strategies are primarily applied to trains ahead of the blockage. Holding of trains behind the blockage is largely due to the minimum separation requirement between trains.</u>

In order to investigate the impact of the assumption that the disruption duration is known in the formulation, sensitivity analysis are conducted. The initial estimate of the disruption to be input into the formulation is fixed: 10 minutes. Errors of ±50% in the estimate are examined. That is, the actual disruption in one case is 5 minutes and 15 minutes in the other. The weighted waiting times from different control schemes with the incorrect estimate are compared with those with the correct estimate.

In the 5-minute disruption scenario, the initial disruption duration estimate is 10 minutes. The true duration is known when the disruption is cleared at 5 minute, and the scenario is re-optimized with updated train location information at that point. The incorrect estimate of the disruption duration does not have a significant impact in this case. Since the difference is very small between the weighted waiting times with the incorrect estimate and those with the correct estimate, the total weighted waiting times instead of mean weighted waiting times are compared. There is only 0.005% increase due to the incorrect estimate under the "Holding Only" and "Holding and Expressing" control schemes. The impact on short-turning strategies is slightly greater, +0.4%. This is because short-turning strategies are difficult to change. Once a short-turn is started, it is fully committed.

In the 15-minute scenario, the initial disruption duration is estimated to be 10 minute. At the end of the initial disruption duration estimate, we assume the disruption duration is known to be 15 minutes. The mean weighted waiting time is only increased by 0.4% due to incorrect estimate in the "Holding Only" case, which again shows that holding is fairly robust. The mean weighted waiting time is increased by 4% in the "Holding and Expressing" case. This is because holding time is less when expressing is applied. Thus, when the disruption turns out to be longer, holding candidates may have passed the best stations to be held at. Short-turning strategies seems the most sensitive to the estimate of the disruption duration with the mean weighted waiting time increasing by 13%. This is because the short-turning candidate has passed the crossover tracks and can not be short-turned even with the updated disruption duration estimate. In addition, holding time is the least in the "Holding, Expressing and Short-turning" case. Therefore, holding candidates have passed the appropriate stations to be held at.

3. The sensitivity analysis shows that short-turning is relatively sensitive to the estimate of disruption duration, while holding and expressing are not particularly sensitive.

In the last section of Chapter 3, we discuss several practical issues in solving this mixed integer problem. Possible bounds for the large numbers used in the integer constraints are developed to keep the feasible space small. The order of variables to branch on in the branch-and-bound algorithm is also discussed. Test results show that the proposed branching sequence significantly reduces the solution time. With the proposed sequence, the solution times of most problem instances are acceptable in transit real-time control. To further reduce the solution time, we propose that the expressing strategy is not taken as a primary strategy, considering the solution complexity involved and the marginal additional benefits achieved.

4.2 Future Research

With the continuous advances in information technology, real-time decision support systems have great potential, although there is still much work to be done before such systems could be fully implemented. This thesis suggest the following areas of further research into this topic:

1. Stochastic models. The model presented here is a deterministic model. The most vital assumption made is that the disruption duration is known with certainty. As shown in the sensitivity analysis, the effectiveness of some control strategies is sensitive to the estimate of the disruption duration. A stochastic formulation that relaxes this assumption could largely resolve this problem.

2. Robust control scheme. For disruption control, it is reasonable to expect that station controlling methods such as holding, short-turning and expressing will continue to function as the primary control strategies. As shown in the sensitivity analysis, once a train has passed an appropriate station (or point) for control, there could be significant loss of benefits from control strategies, especially for short-turning strategies. Therefore, a sequential decision making process aimed at maximizing the expected

benefits from control strategies seems more appropriate than a once-and-for-all process. That is, in addition to the disruption triggering mechanism, events such as train arrivals can also trigger decision-making and key control candidates can be held at appropriate points to maintain potential control options to maximize expected benefits.

3. Better expressing model. This research shows that expressing only provides little additional benefits over holding. However, due to the simplifications made in linearization of the model, the negative impacts of expressing may be amplified. It is possible to model holding and expressing strategies without fewer simplifications. Thus, more insight of the effectiveness of expressing can be provided.

4. Quick-response routine control model. The model presented deals with disruption control specifically, although it can be used for routine control. For routine control, the model has to be solved frequently and the solution time should be less due to the limited response time. Severe control strategies, such as short-turning and expressing, may not be included in the routine control model. Other control methods, particularly speed control could be an option. Compared with holding, speed control may have far less negative impacts on passengers.

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Appendix A

Passenger Arrival Rates and Alighting Fractions

Table A.1 Estimated Passenger Arrival Rate and Alighting Fraction for the MBTA Red Line. (IB = Inbound, OB = Outbound)

Station #	Station Name	Arrival Rate (A1)	Arrival Rate for Braintree Branch (A2)	Scaled Arrival Rate	Alighting Fraction
6	Braintree	12.52			1.000
7	Quincy Adams (IB)	25.30			0.003
8	Quincy Center (IB)	21.02			0.005
9	Wollaston (IB)	15.13			0.002
10	North Quincy (IB)	19.83			0.060
11	JFK/UMASS (IB)	4.22		*2.41	0.032
12	Andrew (IB)	10.22		*5.84	0.009
13	Broadway (IB)	3.95		*2.26	0.017
14	South Station (IB)	24.50		*14.00	0.256
15	Downtown Crossing (IB)	18.45		*10.54	0.442
16	Park Street (OB)	19.37		*11.07	0.577
17	MGH (OB)	5.43		*3.10	0.166
18	Kendall Square/MIT (OB)	1.05		*0.60	0.420
19	Central Square (OB)	2.70		*1.54	0.268
20	Harvard Square (OB)	1.67		*0.95	0.766
21	Porter Square (OB)	0.50		*0.29	0.239
22	Davis Square (OB)	0.77		*0.44	0.386
23	Alewife	31.33		*17.90	1.000
24	Davis Square (IB)	29.77		*17.01	0.002
25	Porter Square (IB)	23.85		*13.63	0.005
26	Harvard Square (IB)	31.67		*18.10	0.125
27	Central Square (IB)	21.17		*12.10	0.038
28	Kendall Square/MIT (IB)	3.83		*2.19	0.105
29	MGH (IB)	3.77		*2.15	0.060
30	Park Street (OB)	17.38	1.20	† 11.13	0.378
31	Downtown Crossing (OB)	14.30	2.08	† 10.25	0.439
32	South Station (OB)	0.49	0.82	† 1.10	0.645
33	Broadway (OB)	0.10	0.52	† 0.58	0.113
34	Andrew (OB)	0.00	0.37	† 0.37	0.065
35	JFK/UMASS (OB)	1.15			0.282
36	North Quincy (OB)	0.55			0.494
37	Wollaston (OB)	0.38			0.096
38	Quincy Center (OB)	0.10			0.626
39	Quincy Adams (OB)				0.660

* Arrival rates of column A1 scaled by 4/7.

† Arrival rates of column A1 scaled by 4/7 plus column A2.

Appendix B

Dwell Time Data

The data is collected during PM peak period (4:15-5:45) at South Station Outbound on the MBTA Red Line on Thursday, March 11, 1999.

Table B.1 Number of Alightings, Boardings and Dwell Times

Observation #	# Alightings	# Boardings	Dwell Time
1	132	53	35
2	89	59	32
3	138	37	18
4	93	85	23
5	103	79	24
6	229	212	60
7	103	86	34
8	101	78	27
9	119	61	26
10	101	109	21
11	86	88	29
12	153	201	50
13	91	125	32
14	102	150	26
15	140	227	50
16	193	160	44
17	112	163	40
18	133	133	36
19	117	47	28
20	101	122	30
21	114	76	28
22	147	206	52
23	80	71	24
24	94	166	40

Appendix C

Non-Stopping Running Time and Minimum Safe Separation

Table C.1 Running Times and Minimum Separations (in seconds)

From		To		Running Time	Minimum Separation	Scaled Minimum Separation
6	Braintree	7	Quincy Adams (IB)	200	83	
7	Quincy Adams (IB)	8	Quincy Center (IB)	150	92	
8	Quincy Center (IB)	9	Wollaston (IB)	131	100	
9	Wollaston (IB)	10	North Quincy (IB)	81	112	
8	North Quincy (IB)	11	JFK/UMASS (IB)	307	142	
11	JFK/UMASS (IB)	12	Andrew (IB)	156	97	*170
9	Andrew (IB)	13	Broadway (IB)	134	102	*179
13	Broadway (IB)	14	South Station (IB)	163	120	*210
10	South Station (IB)	15	Downtown Crossing (IB)	82	128	*224
15	Downtown Crossing (IB)	16	Park Street (OB)	52	106	*186
11	Park Street (OB)	17	MGH (OB)	80	86	*151
17	MGH (OB)	18	Kendall Square/MIT (OB)	101	91	*159
12	Kendall Square/MIT (OB)	19	Central Square (OB)	153	95	*166
19	Central Square (OB)	20	Harvard Square (OB)	162	134	*235
13	Harvard Square (OB)	21	Porter Square (OB)	136	81	*142
21	Porter Square (OB)	22	Davis Square (OB)	103	88	*154
14	Davis Square (OB)	23	Alewife	145	97	*170
23	Alewife	24	Davis Square (IB)	160	93	*163
15	Davis Square (IB)	25	Porter Square (IB)	121	84	*147
25	Porter Square (IB)	26	Harvard Square (IB)	124	103	*180
16	Harvard Square (IB)	27	Central Square (IB)	143	85	*149
27	Central Square (IB)	28	Kendall Square/MIT (IB)	156	85	*149
17	Kendall Square/MIT (IB)	29	MGH (IB)	102	102	*179
29	MGH (IB)	30	Park Street (OB)	128	180	*315
18	Park Street (OB)	31	Downtown Crossing (OB)	71	140	*245
31	Downtown Crossing (OB)	32	South Station (OB)	62	118	*207
19	South Station (OB)	33	Broadway (OB)	116	87	*152
33	Broadway (OB)	34	Andrew (OB)	114	88	*154
20	Andrew (OB)	35	JFK/UMASS (OB)	148	130	
35	JFK/UMASS (OB)	36	North Quincy (OB)	341	86	
21	North Quincy (OB)	37	Wollaston (OB)	95	86	
37	Wollaston (OB)	38	Quincy Center (OB)	141	98	
22	Quincy Center (OB)	39	Quincy Adams (OB)	141	82	
39	Quincy Adams (OB)	6	Braintree	189	124	

* Scaled by 7/4.