ON FEEDBACK LAWS FOR ROBOTIC SYSTEMS

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INTRODUCTION

A variety of control problems arising from robotics applications can be restated as optimal control problems of minimum-time state transfer in the presence of state-space constraints and constraints of incomplete-state information. The traditional approaches to solving such problems are Pontryagin's Maximum Principle (Pontryagin et.al., 1962), in the case of open-loop control, and Bellman's Dynamic Programming method (Bellman, 1957). While a number of technical difficulties exist, approximate solutions of such problems can generally be computed off-line (see Kahn and Roth, 1971). Perturbation methods for obtaining local feedback laws are also available (Whitney, 1972, Hemami, 1980).

However, no currently operational robots are known to be based on solutions of such optimal control problems, nor is it likely that this will come about. Some of the reasons for this situation can be given: (a) complete equations of motion are extremely complex, and are often not available; (b) trajectories must usually be planned in a short time-period preceding execution--there is no time for detailed design studies or numerical analysis for every motion being performed; (c) reliability and repeatability or accuracy of motion are often more important than minimizing execution time; (d) optimal control laws often require too much storage or real-time computation during execution of the motion; (e) nonlinearities are often sufficiently severe that local linearization gives poor results (even if its heavy computational requirements are overlooked).

By contrast, current practice is often to determine a feasible open-loop position trajectory by a "teaching" procedure (e.g., Unimation, Inc., 1979). The trajectory recorded during this procedure is then "played-back" as a sequence of position commands to joints which are servo-actuated; the rate of playback may be increased in a sequence of preliminary trials, until the bandwidth or power limitations of the servos are encountered. This methodology is relativly quick, intuitive, and yields reliable performance when the disturbances to the robot and workspace are relatively small. Although this state-of-the-art approach to trajectory formation is very effective, it possesses inherent limitations and is already being superceded in more demanding applications such as locomotion and manipulation. One limitation is that a human controller cannot readily communicate commands to such a robot. The robot is also unable to anticipate or accommodate unexpected changes in workspace configuration; the teaching paradigm cannot be readily extended to allow for feedback from additional sensors (e.g., touch or machine vision). The objective of the present note is to extend and affirm the suggestion of Young (1978) that discontinuous feedback laws are naturally-suited to robotics problems, to describe two further examples of discontinuous feedback laws, and to explore further notions for the synthesis of such systems.

Rationale for Discontinuous Feedback Laws

Accepting the fact that optimal feedback laws for this class of problems generically exhibit discontinuous behavior (Athans and Falb, 1966, Kahn and Roth, 1971), one is motivated to seek simpler methods of determining loci of discontinuity. The theory of variable-structure controllers, developed originally by Emel'yanov (1967) and extended by his colleagues (see Utkin, 1978) has provided new design methods for certain classes of systems; it is a remarkable observation that the performance of such systems can be qualitatively quite robust, even though their precise trajectories may depend strongly on the initial state, disturbances, or modelling errors (Young, 1978).

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The author has previously suggested (Johnson, 1978) that there is a close relationship of this theory and the theory of control laws described in linguistic terms, e.g., as a digital computer program (Zadeh, 1973). Example 1, in the sequel, exhibits this relationship. Discontinuous control in robotics applications can thus arise from the nature of the task description as well as from discontinuities in the mechanical system and environment, as illustrated by Example 2. A third reason for developing discontinuous controls arises from implementation considerations. Discrete sensors and actuators are usually cheaper and more reliable than continuous ones; they arise naturally in discontinuous control law synthesis. Discrete signals are also preferred for signalling task initiation, completion, or interrupts to a control computer. Finally, digital computers typically perform binary operations faster than (approximations to) real number operations.

The following two examples illustrate the use discrete feedback control in two very simplified problems arising in robotics, which lie just beyond the current state-of-the-art. Since a general design theory for such cases is not yet available, each example is solved on its own merits.

Example 1: Catching a Ball

In this example, the "hand" is idealized as a cup-shaped weight of mass M which can be acted on by vertical and horizontal forces in order to catch a (vertically) falling ball of mass m. First, it is assumed that the hand is beneath the ball and the interception dynamics are analyzed. Then, a simple control law to achieve catching from an arbitrary initial position, using remote sensing of the position of the ball (a primitive form of vision), is given in algorithmic form.

The geometry of the problem is shown in Figure 1. Suppose that $x_m(t) = x_M(t) = 0$ to analyze the catching process. According to Newtonian mechanics, the ball's motion is given approximately[2] by

$$m\ddot{z}_m = -mg \quad ; \; z_m(t_o) = z_{mo} \; ; \; \dot{z}_m(t_o)=0 \qquad (1)$$

where g is the acceleration due to gravity and z_{mo} is the initial position of the ball at t_o, the time it is dropped. The motion of the hand is given by

$$M\ddot{z}_M = -Mg + f_z(t); \; z_M(t_o) = z_{Mo}; \; \dot{z}_M(t_o)=0 \qquad (2)$$

where $f_z(t)$ represents the control force.

[2]In air, a viscous drag force depending on cross-sectional area is also present, and could be used in estimating the ball's mass. This digression is not pursued here.

Suppose that t_1 is the first time of contact between ball and hand, and let t_1^- denote a time just prior to t_1 while t_1^+ denotes a time just after t_1. Assuming $f_M(t)$ is approximately constant on the interval (t_1^-, t_1^+), it can be set to zero without affecting the conclusions of the following analysis; this is done for simplicity in the sequel. Either an elastic or inelastic collision may occur at t_1. If an inelastic collision occurs, and the ball is caught, the combined dynamics for $t>t_1$ are

$$(M+m)\ddot{z}_M = -(M+m)g + f_z(t); \; z_M(t) \equiv z_m(t), \quad t \geq t_1$$

$$z_M(t_1) = z_{M1} = z_{m1}, \text{ the location of impact}$$

$$\dot{z}_M(t_1) = ? \qquad (3)$$

If an elastic collision occurs, then

$$m\ddot{z}_m = -mg, \; z_m(t_1) = z_{m1}; \; \dot{z}_m(t_1) = ?$$

$$M\ddot{z}_M = -Mg + f_z(t); \; z_M(t_1) = z_{M1} = z_{m1};$$

$$\dot{z}_M(t_1) = ? \qquad (4)$$

Conservation of energy and momentum can be invoked (now taking $f_z(t)=0$) in order to deduce which of these situations will occur, and to find the missing velocites at $t=t_1$. Conservation of momentum is

$$P \doteq m\dot{z}_m(t_1^-) + M\dot{z}_m(t_1^-) = m\dot{z}_m(t_1^+) + M\dot{z}_M(t_1^+) \qquad (5)$$

while conservation of energy (omitting potential energy, which is approximately constant, from both sides of the equation) is

$$E \doteq m\dot{z}_m(t_1^-)^2 + M\dot{z}_M(t_1^-)^2 = m\dot{z}_m(t_1^+)^2 + M\dot{z}_M(t_1^+)^2 \qquad (6)$$

Viewing these as simultaneous equations for $\dot{z}_m(t_1^+)$ and $\dot{z}_M(t_1^+)$, bouncing is predicted when there is a solution with $\dot{z}_m(t_1^+) > z_M(t_1^+)$[3]. A simultaneous solution yields the possibilities

$$\dot{z}_m(t_1^+) = \{P \pm [mM(E(M+m)-P^2)]^{1/2}\}/m(M+m) \quad (7)$$

As a special case, suppose that $\dot{z}_M(t_1^-)=0$, i.e., the hand is at rest at the time of impact. Then it can be shown that a real-valued solution of (7) always exists and that

[3]Otherwise, in an inelastic collision, energy dissipation will occur at t_1 so that the physically realizable solution $\dot{z}_m(t_1^+) = \dot{z}_M(t_1^+)$ comes about. This is not explored further here.

$$\dot{z}_m(t_1^+) = \frac{(m-M)}{(m+M)} \dot{z}_m(t_1^-) \quad (8)$$

Thus bouncing will occur whenever $M>m$, which is typically the case. A further analysis shows that if $M>m$, a finite negative velocity of the hand prior to impact ($\dot{z}_M(t_1^-)<0$) will prevent bouncing; in the limit $m\approx 0, \dot{z}_M(t_1^-) \approx \dot{z}_m(t_1^-)$, i.e., perfect tracking will be required; if the ball is very heavy ($m>M$), or has a very large velocity at impact, then a catch can be made even if $\dot{z}_M(t_1^-)$ is positive, i.e., if the hand comes to meet it. Typically, one expects $m<M$ but not $m<<M$, so that a very small movement to produce a slightly negative hand velocity prior to impact will ensure a successful catch.

In a catch, the hand must merely intercept the ball's predicted trajectory before the ball arrives at the point of interception, and then wait to make a small final maneuver to avoid bouncing. If the ball is to be struck, (say, in the x-direction) quite a different strategy is required: The ball's trajectory must be intercepted precisely at the time the ball reaches the interception point, with a velocity which is approximately perpendicular to the trajectory.

Now suppose that the ball's position, $x_m(t)$, $z_m(t)$ can be measured, that the hand position $x_M(t)$, $z_M(t)$ is available from internal measurement, and that forces $f_x(t)$ and $f_z(t)$ can be applied independently. Assume that accurate velocity estimates can be obtained from the position measurements. At $t=t_o$, the initial time, suppose $z_m(t_o)=z_{mo}$, $x_m(t_o)=0$, while $z_m(t_o) = z_{Mo} < z_{mo}, x_M(t_o) = x_{Mo}$. A simple implementation of the rendezvous strategy for catching the ball is the following pseudo-Pascal algorithm:

PROCEDURE CATCH

BEGIN

REPEAT

$$f_z(t) = 0$$
$$e_x(t) = x_M(t) - x_m(t)$$
$$f_x(t) = -K_{x1} e_x(t)$$

UNTIL $|e_x(t)| < E_x$

$i_x = 0$

REPEAT

$$fz_M(t) = -K_{\dot{z}}(\dot{z}_M(t) - \dot{z}_m(t))$$
$$e_x(t) = x_M(t) - x_m(t)$$
$$i_x = i_x + \Delta e_x(t)$$

[Δ is the sampling interval]

$$f_x(t) = -K_{x_2} i_x$$

UNTIL $z_m(t) \leq z_M(t) + E_z$

IF $|z_M(t)-z_m(t)| \leq E_z$ AND $|x_M(t)-x_m(t)| \leq E_x$

THEN RETURN

ELSE [MISSED THE BALL, GO TO ERROR RECOVERY]

END

The first REPEAT loop uses position feedback on the x-position error (intended with a "large" gain K_{x1}) to bring the hand below the ball as fast as possible. The second REPEAT loop uses integral control on the x-error to more accurately position the hand below the ball, and derivative feedback on the z-velocity error (intended with a "small" gain, $K_{\dot{z}}$) so that the hand has a small downward velocity when the ball strikes it. Although the details of this control law are essentially irrelevant, it is primarily intended to illustrate two points: (a) it is not necessary to explicitly predict the trajectory of the ball (i.e., to preplan the trajectory) or to know the precise mass of the ball; (b) The control strategy is discontinuous at the time between the two REPEAT loops, which is determined by the motion of the ball itself. In the second example, the control law discontinuity arises primarily from state-variable constraints rather than from the task description.

<u>Example 2: Converting Vertical Force to Horizontal Locomotion</u>

A single massless link of length ℓ_0 terminated at the upper end by a mass m_1 and at the lower end by a mass m_0, is considered in the example (Figure 2). A vertical force, $F(t)$, may be applied to the upper mass: When this force lifts the link above a horizontal surface at $z=0$, it is free to swing back and forth in one direction (defined as the x-direction); when mass m_0 is in contact with the surface, it "sticks" unless an upward vertical force component is subsequently applied to it. This assumption approximates the effect of a friction contact between m_0 and the surface.

The intriguing feature of this example is that there exist simple strategies whereby the purely vertical force $F(t)$ can be used to propel the link in a forward horizontal motion. These result from a proper combination of two motions:

F: The link falls down (like an inverted pendulum) when m_0 is on the surface and no vertical force is applied ($F(t)=0$).

S: The link swings back and forth in a stable pendulum motion when m_0 is off the surface and a vertical force is applied to counteract gravity ($F(t) \geq (m_0+m_1)g$).

The equations of motion are first derived in the two cases where m_0 is not in contact with the surface z=0 (Case S), and then when it is in contact (Case F).

Case S: Let F_{01} denote the force on m_0 exerted through the link by m_1, and F_{10} denote the force on m_1 exerted by m_0, defined in the direction of the link for each mass. Newton's equations for m_0 are

$$m_0\ddot{z}_0 = -m_0 g + F_{01}\sin\theta_0 \tag{9}$$

$$m_0\ddot{x}_0 = F_{01}\cos\theta_0 \tag{10}$$

And for m_1 they are

$$m_1\ddot{z}_1 = F - m_1 g - F_{10}\sin\theta_0 \tag{11}$$

$$m_1\ddot{x}_1 = -F_{10}\cos\theta_0 \tag{12}$$

where g denotes the acceleration due to gravity. The constraint of equal and opposite reactions (rigid link) is $F_{01}=F_{10}$. The link imposes constraints between (x_0,z_0) and (x_1,z_1) which are most readily expressed in terms of θ_0:

$$x_1 = x_0 + \ell_0\cos\theta_0 \tag{13}$$

and

$$z_1 = z_0 + \ell_0\sin\theta_0 \tag{14}$$

The time-derivatives of the constraints are used because the constraints must hold at each instant of time. Elementary algebra and trigonometry can be used to solve for F_{01} and F_{10} in (9) and (10). Further algebra yields the key equation for θ_0:

$$\ddot{\theta}_0 = F\cos\theta_0/m_1\ell_0 \tag{15}$$

In this example it is natural to assume that inertial measurements could be made only on m_1, and thus it is of interest to have equations of motion directly in terms of the inertially measured states (x_1,z_1) rather than (x_0,z_0). These equations are:

$$\ddot{x}_1 = -(m_0\ell_0/(m_0+m_1))\cos\theta_0\dot{\theta}_0^2 - (m_0/m_1(m_0+m_1))\sin\theta_0\cos\theta_0 F \tag{16}$$

$$\ddot{z}_1 = -(m_0\ell_0/(m_0+m_1))\sin\theta_0\dot{\theta}_0^2 - g + [\cos^2\theta_0/m_1+\sin^2\theta_0/(m_0+m_1)]F \tag{17}$$

Purely algebraic constraints (13)-(14) can be used to find (x_0,z_0) and to check that $z_0>0$; otherwise a transition to Case F may occur. Furthermore, note that (16), the foreward acceleration of m_1, is driven by the vertical force F, providing the possibility of locomotion.

Case F: Let F_{01} and F_{10} be defined as in Case S. During Case F, it is assumed that (x_0,z_0) remain fixed at their initial values, and that $z_0=0$. Newton's equations for m_1 are

$$m_1\ddot{z}_1 = F - m_1 g - F_{10}\sin\theta_0 \tag{18}$$

$$m_1\ddot{x}_1 = -F_{10}\cos\theta_0 \tag{19}$$

In differentiating the constraints (13) and (14), x_0 and z_0 are held constant. The equation for θ_0 is derived in a similar fashion to Case S:

$$\ddot{\theta}_0 = F\cos\theta_0/m_1\ell_0 - g\cos\theta_0/\ell_0 \tag{20}$$

Since x_0 and z_0 are fixed, (x_1,z_1) could be found directly from the algebraic constraints once (20) was solved. However, differential expressions analogous to (16) and (17) are more useful for guidance purposes:

$$\ddot{x}_1 = (-\ell_0\dot{\theta}_0^2 + g\sin\theta_0)\cos\theta_0 - F\sin\theta_0\cos\theta_0/m_1 \tag{21}$$

$$\ddot{z}_1 = (-\ell_0\dot{\theta}_0^2 + g\sin\theta_0)\sin\theta_0 - g - F\cos^2\theta_0/m_1 \tag{22}$$

As expected, (20)-(22) do not depend on m_0, because m_0 doesn't move in Case F.

Feedback law: Only the most simple form of feedback control strategy is described here, and it is shown that feedback from only θ_0 and $\dot{\theta}_0$, as illustrated by the solid feedback line of Figure 3, is sufficient to provide the features of useful locomotion described above. The discontinuous feedback law is most readily illustrated on the phase-plane plot of θ_0 vs. $\dot{\theta}_0$ of Figure 4.

The feedback law is:

Whenever $(\theta_0(t),\dot{\theta}_0(t))$ in Regions A,E or F

take $F(t) = 0$

Whenever $(\theta_0(t),\dot{\theta}_0(t))$ in Regions B,C or D

take $F(t) = (m_0+m_1)g$

For any initial condition inside the shaded area except the point $(\pi/2,0)$[4], the motion of the system will eventually settle into a periodic motion. Initial conditions outside the shaded regions cannot be corrected by this feedback law. Disturbances such as variation in surface height, friction, etc., result in perturbations to the trajectory, which are stable if the system remains inside the shaded region. Thus, one goal of accommodating small

[4] Certain additional constraints and assumptions, which may slightly decrease the size of this area, have been intentionally ignored in this simplified analysis.

obstacles has been met. A second goal, of varying the speed of locomotion, can be met by varying θ_{min} parametrically. The time per cycle is roughly related to the area enclosed by the periodic trajectory, while the horizontal distance is approximately $\ell_0(\theta_{max}-\theta_{min})$; the ratio of distance to time is an approximate measure of average forward velocity. The range of achievable velocities with this locomotion strategy is rather small, even though the corresponding range of step sizes (between 0 and $2\ell_0$) is rather large. The margin of stability of the larger step sizes is considerably decreased, however.

The continued forward motion of m_1 does not violate conservation of momentum; the initial forward momentum is conserved during motion S, but during motion F, it is augmented by momentum exchange, which occurs due to the constraint that m_0 remain fixed on the surface. Thus, the energy expended in lifting during motion S can in fact be converted into forward acceleration during motion F, and the forward motion will not die out (e.g., due to friction effects). No laws of physics are violated by this strategy.

CONCLUSIONS

The examples, drawn from two different areas of robotics, illustrate that discontinuous feedback laws are readily devised for a variety of applications. In both examples, the feedback law could be viewed as a finite set of mutually exclusive continuous-control subtasks. In the first example, two different linear control laws were used, while in the second example, two different constant values of control were used. Furthermore, the transitions between tasks were closely tied to events in the (full) state space which were readily detectable, e.g. interception and rendezvous in the first example, and contact with the surface z=0 in the second. These examples illustrate that a generalization of the methods employed by Young (1978) may be useful in future robotics applications. A set of control values or continuous-control feedback laws sufficiently rich to control the motion of the system in each of its known or desired states is chosen. The trajectories of the system under these forms of feedback are computed. The switching loci between the control laws are then taken along the loci of intersection of these trajectories, or along a physical constraint locus of the system motion, in such a way that the desired combination of movements is obtained. This has the effect of assuring a well-defined mode of sliding along the discontinuities of the closed-loop system; otherwise the nature of sliding might change markedly and unpredictably within a discontinuity surface due to trigonometric-type nonlinearities of the equations of motion.

The selection of a finite number of candidate control strategies and the choice of switching loci defined by intersections of natural motions of the system under these candidate control laws appear to be primary requirements for a practical design theory of discontinuous control for robotic systems. Presently, the greatest difficulties in the development of such a theory are the relationship of linguistically-described goals to feedback law selection, the lack of analytic methods for characterizing controlled motions of the system, and the inherent difficulties of stability analysis for discontinuous systems (Johnson, 1980).

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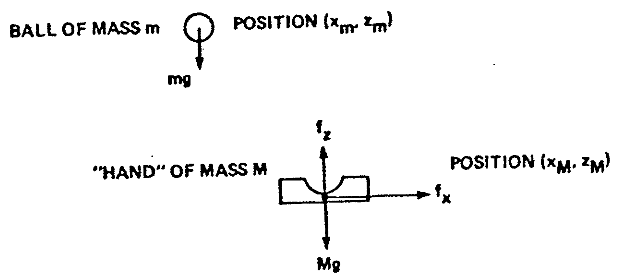

Figure 1. Geometry and Definition of Variables for Example 1

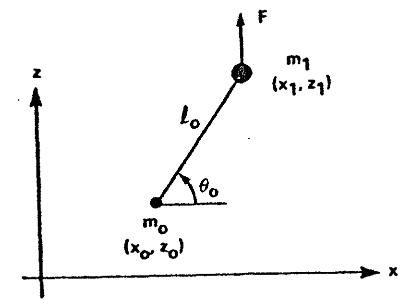

Figure 2. Geometry and Definition of Variables for Example 2

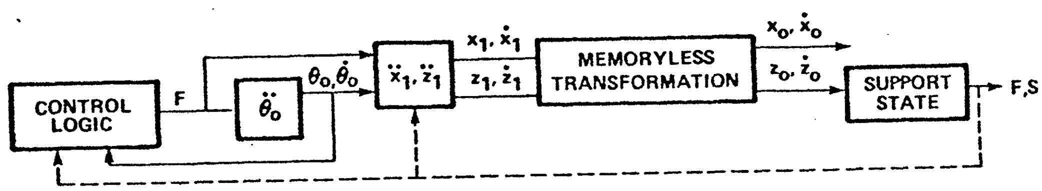

Figure 3. Block Diagram of System Dynamics and Control for Example 2

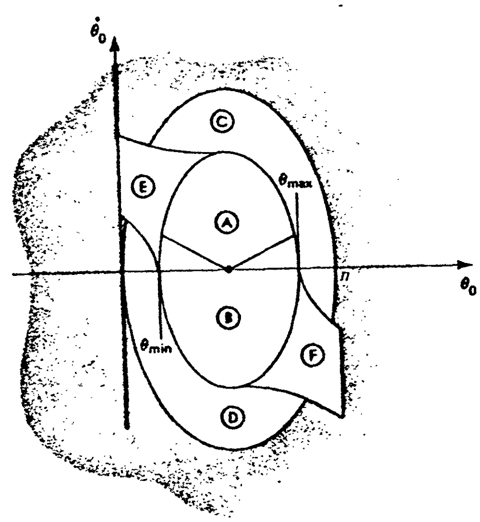

Figure 4. Phase-plane Plot of Feedback Law in Example 2