FOKKER-PLANCK STUDY OF STEADY-STATE CURRENT GENERATION BY A LH WAVE*

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The one-dimensional adiabatic theory for current generation [1] is extended to include electron-ion collisions and to determine the corresponding change in the distribution function. The LH current generation can be described by this model when the auto-correlation time is greater than the trapping time and $\omega_{ce} > \omega_{pe}$. The Fokker–Planck equation is solved for the untrapped and the trapped electrons. Calculations for the ratio of the power dissipated in the plasma to the current generated are presented for Versator II parameters.

For the steady–state operation of a tokamak reactor and to provide particle confinement, a poloidal magnetic field generated by a toroidal current is necessary. It has been proposed that unidirectional lower–hybrid (LH) waves launched by a wave–guide array may generate such a steady–state current [2]. The high phase velocity waves drive a current as the wave momentum is irreversibly transferred to the electrons.

The single wave one–dimensional Vlasov equation for the electrons in a strongly magnetized plasma:

$$\frac{\partial f}{\partial t} + v\,\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\phi(t, z - \frac{\omega}{k_z}t)\,\frac{\partial f}{\partial v} \tag{1}$$

(where ϕ is the electric potential, t is the slow time modulation and $z - \omega\, t\,/\,k_z$ is the fast modulation) has been solved recently [1] for the distribution function. For the initial conditions: $\partial\phi(0, z - \omega\, t/k_z)\,/\partial z = 0$ and $f(t = 0) = f_m$, the Maxwellian distribution function, the solution for the untrapped electrons is :

$$f^{os} = \frac{n_o}{\sqrt{\pi} v_t}\,\exp\left[-\left\{\frac{\omega}{k_z\, v_t} - \frac{k_z}{v_t}\, I^{os}(H,\phi)\;\mathrm{sign}\left(\frac{\omega}{k_z} - v\right)\right\}^2\right] \tag{2}$$

and, for the trapped electrons:

$$f^{TR} = \frac{n_o}{\sqrt{\pi}\, v_t}\,\exp\left[-\left(\frac{\omega}{k_z\, v_t}\right)^2 - \left(\frac{k_z\, I^{TR}}{2\, v_t}\right)^2\right]\cosh\left(\frac{\omega\, I^{TR}}{v_t^2}\right). \tag{3}$$

H is the Hamiltonian corresponding to (1); I is the action, $I = \frac{1}{2\pi}\int d\tilde{z}\;\tilde{v}$, where $\tilde{z}$, $\tilde{v}$ are the coordinate and velocity in the wave frame, n_o is the density of the electrons and v_t is their thermal velocity. These solutions are valid everywhere in the phase space where $\Omega = \partial H/\partial I$, the nonlinear frequency, satisfies $\Omega \gg 1/T$ with T being the modulational time scale over which the electric field amplitude changes. For large amplitude waves this theory is valid everywhere except for a narrow region near the separatrix.

The dominant nonlinearity for the LH wave is along the tôroidal direction as $v_{ox} \ll v_{oz}$ and $v_{oy}/v_{oz} \approx \omega_{pe}/\omega_{ce} < 1$.

$$v_{ox} = \frac{e\,E_x\,\omega}{m\,(\omega_{ce}^2 - \omega^2)}\,, \quad v_{oy} = \frac{e\,E_x\,\omega_{ce}}{m\,(\omega_{ce}^2 - \omega^2)}\,, \quad v_{oz} = \frac{e\,E_z}{m\,\omega}$$

are the oscillating velocities in the three directions. Thus, f is assumed to be Maxwellian along the x and y directions and given by (2) and (3) in the z–direction. Further, for the above single wave theory to be valid for

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LH current generation, we also require that the auto-correlation time, $\tau_{AC} = k_z/\omega\,\Delta k_z$, is greater than the trapping time, $\tau_{TR} = \sqrt{m/e\,k_z E_z}$. This requires that $E_z > c\,m\,\omega\,(\Delta n_z)^2/e\,n_z^3$. For Versator II parameters: $\omega = 5.027 \times 10^9\ \mathrm{sec}^{-1}$, $n_z = 10$, $\Delta n_z = 2$, this implies that $E_z \geq 340$ V/cm.

The above theory for LH current generation is valid for time scales shorter than the collisional time scale. Since for large amplitude waves Landau damping is not important, the collisional damping determines the power dissipated in the plasma and the feasibility of the current drive scheme. We assume that the dominant collsional damping is due to electron–ion collisions. Then from the Landau form of the collisional term $C^{e/i} = -\partial \overline{j}^{e/i}/\partial \overline{v}$ with

$$j_\alpha^{e/i} = \frac{2\,\pi\,\lambda\,Z\,e^4}{m} \int d\overline{v}'\; U_{\alpha\beta} \left(\frac{f(\overline{v})}{m_i} \frac{\partial f_i(\overline{v}')}{\partial v'_\beta} - \frac{f_i(\overline{v}')}{m} \frac{\partial f(\overline{v})}{\partial v_\beta} \right) \tag{4}$$

with $U_{\alpha\beta} = \delta_{\alpha\beta}/\overline{u} - u_\alpha\, u_\beta/\,u^3$, $\overline{u} = \overline{v} - \overline{v}'$, λ is the Coulomb logarithm, Z the ion charge number, f_i the ion distribution function and m (m_i) the electron (ion) mass. Assuming cold ions, $f_i = n_o\,\delta(\overline{v})$, and that f is Maxwellian in the x–y directions, the collisional term:

$$C^{e/i} = \frac{1}{2} \frac{\lambda}{v_t^3} \frac{Z\,e^2}{m} \omega_{pe}^2 \frac{\partial}{\partial w} \left[\eta(w) \left(2w + \frac{\partial}{\partial w} \right) \right] f \tag{5}$$

is added to the right side of (1) to get the Fokker–Planck equation. Here:

$$\eta(w) = \sqrt{\pi}\, e^{w^2} \{1 - \Phi(|w|)\}\,(1 + 2\,w^2) - 2\,|w| \tag{6}$$

is the e–i form factor ($w = v/v_t$, Φ is the error function). In Figure (A), $\eta(w)$ is compared with its asymptotic form $1/w^3$. Numerical integration of the Fokker-Planck equation indicates that for reasonable field amplitudes the untrapped electron distribution function is approximately the same as f^{os} of (2).

Assuming a sinusoidal field, then to order w_o^2 $(w_o = e\,E_z/m\,\omega\,v_t)$:

$$f^{os} \approx f_m + \frac{w_o^2}{4} \frac{\partial}{\partial w} \frac{1}{(1 - a\,w)^2} \frac{\partial}{\partial w} f_m \tag{7}$$

where $a = k_z\,v_t/\omega$. This gives a non-resonant steady–state current of:

$$< j > = -\frac{1}{2}\, e\, n_o\, v_t\, w_o^2\, a \tag{8}$$

and the power dissipated in the plasma to be

$$P_D = -\int dv\; \frac{1}{2}\, m v^2 C^{e/i} = \frac{4}{15\,\sqrt{\pi}}\, Z\, \frac{n_o}{v_t}\, \lambda\, e^2\, \omega_{pe}^2\, w_o^2\,. \tag{9}$$

We define "longitudinal RF resistivity" to be:

$$\eta_{RF} = \frac{P_D}{< j >^2} = 9.205 \times 10^{-15} \frac{\lambda\, Z}{(a\, w_o)^2} \frac{1}{T_e^{3/2}}\ \mathrm{sec.} \tag{10}$$

Compare this to the longitudinal Spitzer resistivity:

$$\eta_{SP} = 3.378 \times 10^{-15} \frac{\lambda Z}{T_e^{3/2}} \text{ sec.} \tag{11}$$

For $a w_o = 0.1$, $\eta_{RF} \approx 10^2 \eta_{SP}$. Defining $a_P = P_D / < j >$ we find for Versator II, with $n_o = 2 \times 10^{13}$ cm^{-3}, $T_e = 300$ eV, major radius $= 40$ cm, minor radius $= 13$ cm, that $a_P = 28.8$ for $n_z = 10$.

We now look for steady-state solutions to the Fokker-Planck equation with a sinusoidal field. In the wave frame, the steady-state equation is:

$$\frac{\partial f}{\partial \xi} = \mu \frac{\partial}{\partial \epsilon} \left[\eta(\pm\sqrt{2(\epsilon + \phi)} + a) \left\{ \pm\sqrt{2(\epsilon + \phi)} \left(2 + \frac{\partial}{\partial \epsilon} \right) + 2a \right\} \right] f \tag{12}$$

where $\epsilon = w - a$, $\xi = k_z z - \omega t$, $a = \omega / k_z v_t$, $\phi = -\phi_o \sin \xi = -w_o a \, sin\xi$, $\mu = \lambda Z e^2 \omega_{pe}^2 / 2 v_t^4 m k_z$ with the $+$ sign for velocities $w > a$ and the $-$ sign for $w < a$. For the untrapped electrons, $\epsilon > \phi_o$, we look for a solution of the form:

$$f^{os}(\epsilon, \xi) = f_o^{os}(\epsilon) + \mu f_1^{os}(\epsilon, \xi). \tag{13}$$

Substituting this into (12) and solving for $f_o^{os}(\epsilon)$, we get:

$$f_o^{os}(\epsilon) = f_c \exp \left[-2(\epsilon - \epsilon_c) - 2a \int_{\epsilon_c}^{\epsilon} d\epsilon' \frac{b(\epsilon')}{a(\epsilon')} \right] \tag{14}$$

where f_c is a constant of integration and,

$$a(\epsilon) = \frac{1}{2\pi} \int_0^{2\pi} d\xi \;\; \eta(\pm\sqrt{2(\epsilon + \phi)} + a) \tag{15}$$

$$b(\epsilon) = \frac{1}{2\pi} \int_0^{2\pi} d\xi \; (\pm\sqrt{2(\epsilon + \phi)}) \;\; \eta(\pm\sqrt{2(\epsilon + \phi)} + a). \tag{16}$$

In Figure (B), we compare $< f_o^{os}(\epsilon) >$ (averaged over ξ) with the Maxwellian for $a = 4$ and $v_{TR}/v_t = \sqrt{a w_o} = 1$. It is clear that the untrapped Fokker-Planck distribution function is significantly modified relative to the Maxwellian. This is in contradiction to the assumption usually made that the bulk remains Maxwellian. This modification will clearly effect the trapped electron distribution function when the conditions of continuity at the separatrix and the conservation of particles is enforced. For the trapped electrons, $\epsilon < \phi_o$, one cannot, in general, require that $f_+^{TR} = f_-^{TR}$, unless η is a constant, as this is inconsistent with the steady-state Fokker-Planck equation. Taking the asymptotic form of $\eta(w) \sim 1/w^3$ and expanding in powers of $1/a$ we get:

$$f_{\pm}^{TR} = c_1 \exp\left(4\epsilon + \frac{8\mu\xi}{a^2} \right). \tag{17}$$

The constant c_1 is determined by requiring that f^{TR}, f^{os} together conserve the number of particles. Further analytical and numerical work is currently in progress to determine the power dissipated and the steady-state current generated.

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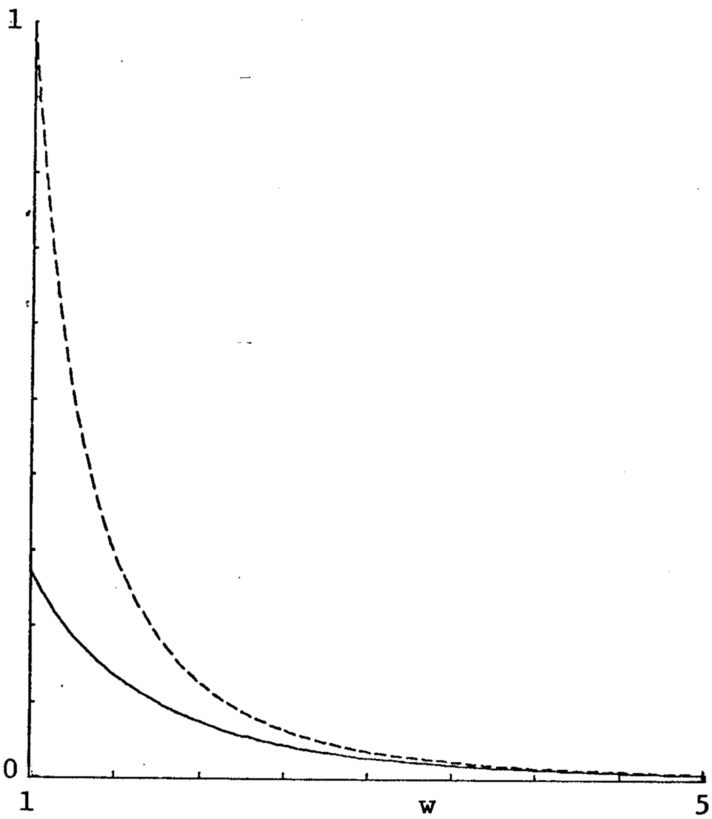

FIGURE (A)

Electron-ion form factor, $\eta(w)$, compared with its asymptotic value, $\frac{1}{w^3}$ (dashed line).

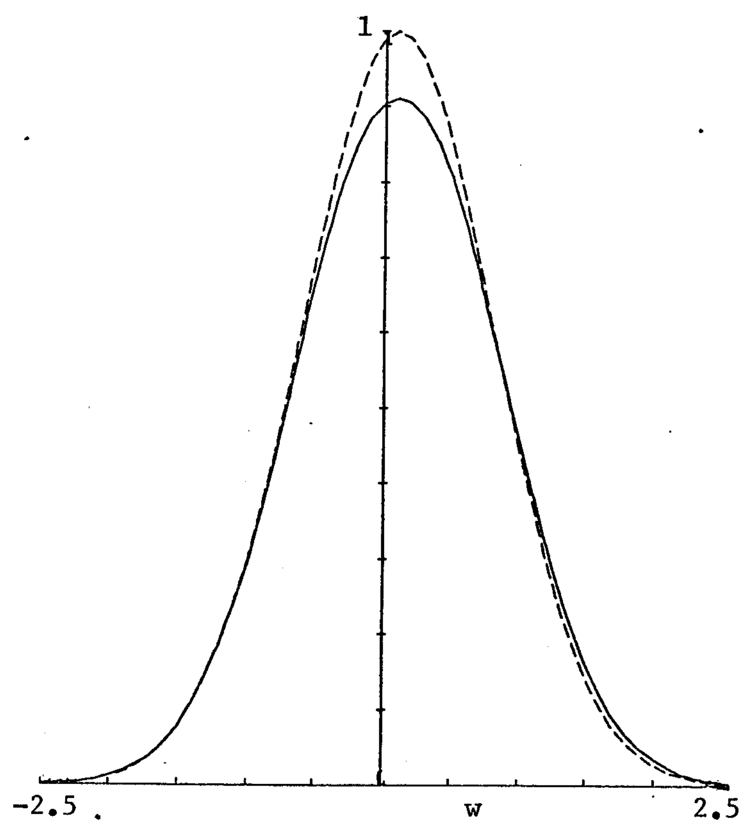

FIGURE (B)

Average Fokker-Planck distribution function compared with the Maxwellian (dashed line).

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Modes of DT and SCD-T Operation In A Compact Ignition Test Reactor

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Modes of DT and SCD-T Operation
In A Compact Ignition Test Reactor †

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Abstract

The use of high performance copper magnets makes possible the design of a compact ignition test reactor (CITR) which would have a wide range of operational capability in DT plasmas. Since it may be possible to obtain high values of $n\tau_e (\sim 10^{15}\ cm^{-3}\ sec)$, operation in the tritium assisted, semi-catalyzed deuterium (SCD-T) advanced fuel cycle might also be investigated. Modes of DT and SCD-T operation for illustrative CITR parameters are described.

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I. Introduction

The use of high performance copper magnets makes possible the design of a compact ignition test reactor (CITR) which would have a wide range of operational capability. This tokamak device could be used to study both ignited operation and long pulse, $Q > 1$ operation in DT plasmas[1]. The CITR would have considerable physics margin for meeting these goals. With this large margin it may be possible to obtain very large values of $n\tau_e$($\sim 10^{15}$ cm^{-3} sec); hence, operation in the tritium-assisted, semi-catalyzed deuterium (SCD-T) advanced fuel cycle[2] might also be investigated. In this paper we discuss the possible modes of DT and SCD-T operation that might be attainable in a CITR device.

II. Machine Design

The CITR would utilize a toroidal field (TF) magnet of Bitter construction which is inertially cooled with liquid nitrogen. The use of this type of magnet makes possible a compact, high performance reactor design since it does not require neutron shielding and can be operated at high fields, high stress and high current density. Illustrative CITR design parameters have been developed by scaling ZEPHYR ignition test reactor design parameters[3,4] to those of a somewhat larger, more versatile device[1]. An engineering conceptual design has not yet been performed. Table 1 lists major machine parameters for the illustrative CITR design. The stored energy and the stress in the TF magnet are given at values of the maximum magnet field at the plasma axis, B_t. These values will decrease with magnetic field as B_t^2. When the peak magnet temperature is allowed to rise to 350°C during a pulse, about one hour will be required to cool down to -190°C with liquid nitrogen. Shorter cool down times can be obtained for smaller temperature changes.

III. DT Operation

Table 2 shows possible DT modes of operation for the illustrative CITR design. In order to make some projection about the energy confinement time, τ_e, it is assumed that $\tau_e \sim Cna^2$ where n is the plasma density, a is the minor radius and the coefficient C is determined by results from PLT[1]. It is also assumed that $\tau_e << \tau_i$ where τ_i is the ion energy confinement time. A margin of ignition, $MI = \frac{n\tau_e}{(n\tau_e)_{ign}}$, is defined where $(n\tau_e)_{ign}$ is the value of $n\tau_e$ required for ignition. Parabolic temperature and density profiles and $Z_{eff} = 1$ are assumed. The density averaged ion temperature $\langle T_i \rangle_n$ is 10 keV. The CITR would operate with both circular and D shaped plasmas. Average values of toroidal beta, $\langle \beta_t \rangle$, of 0.58 are based upon the assumption of moderately elongated D shaped plasmas.

The burn pulse length, τ_b, given in Table 1 is determined by requiring that the increase in temperature of the inertially cooled magnet be less than 350°C. This temperature rise is mainly determined by resistive heating at low values of MI and by neutron heating at high values of MI. Table 2 shows that by reducing the magnetic field relatively long pulse lengths can be obtained. At B = 3.5 T, corresponding to a MI = 0.2 and a value of thermonuclear $Q = \left(\frac{\text{fusion power}}{\text{heating power}}\right)$ of ~ 1, the burn pulse length is 100 seconds

For MI = 1, the average fusion power density is 5 MW/m^3 and the total fusion power produced is 120 MW.

The primary option for heating the CITR would be ICRH. Approximately 15 MW of ICRH should be sufficient to heat to ignition in ~ 1 second if the energy loss is given by the $\tau_e \sim na^2$ scaling law described earlier and the heating power is centrally deposited. The CITR could be designed to accomodate up to 40 MW of ICRH heating power. Compression boosted start-up with neutral beams could be used as a backup option in the CITR[1,5].

IV. SCD-T Operation

In semi-catalyzed deuterium (SCD) operation the triton produced by the D(D,p)T reaction is assumed to be completely burned up by a DT reaction. The ^{3}He produced in the equally probable D(D,n) ^{3}He reaction leaves the plasma before it is burned. In tritium-assisted, semi-catalyzed deterium (SCD-T) operation the triton to deuteron ratio, n_T/n_D, is greater than the equilibrium value in SCD but is considerably less than one. Some amount of tritium must be provided by an external source, such as a tritium breeding blanket. This amount of tritium is determined by the parameters γ_{DT} and γ_{DD}, these parameters represent the ratios of tritons from the external source to DT and DD fusion neutrons generated in the plasma. For illustrative purposes it will be assumed that $\gamma_{DT} = \gamma_{DD} = \gamma$. γ ranges from 1 for DT operation to 0 for SCD operation.

SCD-T operation facilitates a continuum of possibilities for reduction in reactor blanket tritium breeding requirements by improvement in plasma physics performance[2]. The reduction in breeding requirements can significantly increase the range of blanket design options relative to DT operation. In addition the availability of fusion neutrons for nonelectrical applications can be considerably increased. If the ^{3}He produced in the D(D,n) ^{3}He reaction could be reinjected into the plasma performance could be further improved. Another possible use of ^{3}He might be as fuel for reactor operation on the D(^{3}He,p)^{4}He cycle.

Table 3 shows an example of possible modes of SCD-T operation which might be obtained for the illustrative CITR design. These modes of operation are based upon the assumptions that the $\tau_e \sim na^2$ scaling holds, that $<\beta_t> \approx .06$ can be obtained and that impurity radiation loss is negligible. For fixed values of $n\tau_e$, $<T_i>_n$ and $<\beta_t>$ there is a continum of tradeoffs between n_T/n_D, Q and average fusion power denstiy, P_f. The auxiliary heating power requirements, P_{aux}, represent the minimum start-up power requirement for ignited ($Q = \infty$) operation and the steady state heating requirement for driven operation.

V. Conclusions

A compact tokamak device using high field, high performance copper magnets can be designed to provide considerable margin to achieve ignited operation and long pulse, Q > 1, pulse operation in DT plasmas. If confinement physics and MHD stability limits are favorable, this large margin can be utilized to investigate various modes of SCD-T operation.

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Table 1

Illustrative CITR Parameters

Plasma Major Radius:	2 m
Plasma Minor Radius:	0.85 m
Possible Plasma elongation: (D shape plasma)	1.4
Weight of Toroidal Field Magnet:	800 tonnes
TF Magnet Plate Dimensions:	3.0 m × 3.7 m
Maximum Magnetic Field at Plasma:	8.8 T
TF Magnet Stored Energy at Max. Magnetic Field:	2.6 GJ
Max TF Magnet Stress at Max. Magnetic Field: (in copper plates)	40 kpsi

Table 2

DT Operation

$\langle T_i \rangle_n$ = 10 keV, $\langle T_e \rangle_n$ = 8 keV

B_t(T)	$\langle \beta_t \rangle$ (%)	$\bar{n}$ (10^{14}cm^{-3})	$\bar{n}\tau_e$ (10^{14}cm^{-3}sec)	MI	τ_b(sec)
8.8	5.8	5.6	9	9	3
8.8	4.5	4.2	5	5	5
7.6	2.6	1.8	1	1	17
5.1	5.8	1.8	1	1	30
4.0	4.5	0.8	0.2	0.2 (Q = 1)	75
3.5	5.8	0.8	0.2	0.2 (Q = 1)	100

Table 3

SCD-T Operation

$B_t = 8.8$ T $\quad \langle\beta_t\rangle = .058 \quad \tau_b \simeq 14$ sec

$\bar{n}\tau_e \simeq 9.5 \times 10^{14} cm^{-3} \quad \langle T_i\rangle_n = 14$ keV

n_T/n_D	γ	P_{aux}(MW)	Q	P_f(MW/m^3)
.04	.88	13	∞ (ignited)	7.2
.02	0.75	25	5	3.6
.01	0.65	30	3	2.8
.003	0.35	40	1	1.3