A Costing and Sizing Code for Highly Irradiated Normal Magnets

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ABSTRACT

A sizing and costing code has been written for highly irradiated normal magnets, such as those used in accelerators and fusion reactors. Organic and inorganic insulators have been modeled. Thermal, electrical and structural sizing are included. Costing includes the magnets, as well as there associated power supplies and cooling systems. Radiation effects modeled include leakage current, heating, and increased copper resistivity due to transmutations and lattice displacements. A trade study for typical reactor parameters indicates the desirability of lightly shielded, low impedance magnet designs.

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A. Introduction

In fusion reactors, employing magnetic confinement of thermonuclear plasmas, there is always a trade-off involved with nuclear shielding of the magnets. Lighter shielding allows less stored energy and less magnet current for any desired field pattern at the confined plasma, but also causes higher nuclear and gamma heating of the magnet as well as integrated damage to the magnet conductor and insulation. Because of the obvious desirability of limiting recirculating power in a fusion reactor, most designs attempt to incorporate sufficient shielding to permit operation of cryogenically cooled magnets. However, in some cases, the economies of light shielding favor magnets with normal conductors. For tokamaks, these applications include bundle divertor magnets, internal poloidal field coils, ripple trim coils, "throw-away" (Riggatron grade) toroidal field coils, and neutral beam field compensating coils. Relatively short-pulse experimental fusion reactors, such as TFTR, JET and ZEPHYR, also favor lightly shielded normal magnets.

In a heavily irradiated normal magnet, insulation integrity is usually considered to be the most limiting factor in determining magnet shielding requirements. Limitations of 10^9 rads for organic insulation and 10^{12} rads for inorganic insulations are frequently used as a basis for conceptual design, although neither design limit has much substantiation from magnet operating experience. Because of the importance of insulation selection in determining the overall magnet design, models of different insulation systems are included as an option in the code. At present, jacketed MgO conductors and unjacketed G-10 conductors are modeled. Leakage currents and temperature rises due to irradiation are modeled within the code. Lifetime radiation limits, however, are not included. Life limitations can be modeled implicitly by the specification of the external shield thickness. However, the integrated failure mechanisms, particularly for organic insulations, are not well enough characterized at this time to warrant imposing limits within the body of the code [SC80].

Another option in the code is a choice of coil shapes between circular coils and an l-shaped saddle coil. The l-shaped saddle coil is currently the favored shape for bundle divertors and ripple trim coils, while circular coils are the favored shape for internal poloidal field coils.

The sizing code includes a rudimentary sizing of the magnet case structure, based on allowable pulsed stresses. Radiation effects on thermal and electrical sizing are modeled by calculating direct nuclear heating, irradiation induced leakage currents and increased conductor resisitivity from transmutation, lattice displacements and elevated temperature. The costing code includes the magnet, its power supply and its cooling system. Each of these is costed as a stand-alone item, and the cooling system in particular should be integrated with the overall plant cooling system in a systems sizing code.

B. Description of Method

The basic method of the code is to input the required ampere-turns in the coil, the lengths which define its perimeter and the allowable temperatures at the copper and insulation hot-spots. The conductor size and the current density in the copper are input. These are considered to be free parameters, rather than allowables, and finding good values for the conductor size and the current density are among the principal goals of this code. The code then determines the size of the coolant channel needed to reach the temperature allowables. There is never any failure to converge, no matter how small the conductor or how high the curernt density in the conductor, because the code increases the channel area at the expense of the conductor area, until the conductor current is reduced to an acceptably small quantity.

The allowable hot-spot in the insulation can be significantly higher for inorganic insulations than for G-10. Typical values for the hot-spot temperature in the insulation (C) is given by:

$$T_{hotins} = T_{hotMgo} = 150 \tag{1}$$

where T_{hotMgO} is the hot-spot temperature in the MgO insulation (C), and

$$T_{hotins} = T_{hotG10} = 70 \tag{2}$$

when the insulation is G-10.

The hot-spot temperature in the copper (C) is given by:

$$T_{hotcu} = T_{hotins} - \Delta T_{ins} \tag{3}$$

where ΔT_{ins} is the design temperature drop across the insulation (C)

The first guess at the wall temperature on the outlet side (C) is given by:

$$T_{wallguess} = T_{hotcu} - 30 \tag{4}$$

The average temperature in the copper (C) is given by:

$$T_{av} = \frac{(T_{wallguess} + T_{hotcu})}{2} \tag{5}$$

The purpose of the following iteration is to self-consistently size the conductor and the case. We already know the forces on the case, but we don't know its moment of inertia and, therefore, its thickness, until we know the cross-section area of the conductor. Similarly, we don't know the achievable current density in the conductor until we know the attenuation of neutron and gamma irradiation by the case. Both the case thickness and the conductor cross-section are second-order dependent on each other, so a few iterations, substituting the latest calculated values of those two parameters, should relax to a self-consistent solution in all cases. The first guess is that the cross-sectional area of the conductor package is twice the conductor copper area. The area required by all the turns of the coil (m^2) is given by:

$$A_{condtotal} = \frac{2.0\, Amp_{turns}}{J_{cu}} \tag{6}$$

where J_{cu} is the current density in the copper (A/m^2), and Amp_{turns} is the number of ampere-turns in the coil (A-T).

Equations model case thickness for the option of a constant thickness case about the magnet. The most limiting clearance affecting the toroidal ripple on axis is the case thickness about the vertical leg closest to the plasma. Other case thicknesses are not calculated here. For modeling purposes, we are assuming that the force due to the interaction of the main toroidal field and the vertical current is dominant. The case is assumed to be rigidly supported at top and bottom. The radial force supported by the case (N) is given by:

$$Fr_{case} = Bt_{vleg}\, V_{leg}\, Amp_{turns} \tag{8}$$

where V_{leg} is the height of the vertical leg of the magnet (m), and Bt_{vleg} is the toroidal flux density at the vertical legs of the magnet (T). The design stress in the case (Pa) is given by:

$$\sigma_{design} = \frac{\sigma_{Yss}}{SM_{fatiguele5}} \tag{9}$$

where σ_{Yss}, the yield strength of the case steel (Pa), is 350×10^6 and $SM_{fatiguele5}$, the safety margin vs. yield strength for 10^5 cycles is 3.

The bending moment applied to the case (N-m) is given by:

$$M_{case} = \frac{Fr_{case} V_{leg}}{8} \tag{10}$$

We iterate a few times to solve self-consistently for the two equations for the moment of inertia of the case and the stress in the extreme fiber.

The width of the square conductor winding package (m) is given by:

$$W_{cond} = \sqrt{A_{condtotal}} \tag{12}$$

The overall width of the case (m) is given by:

$$W_{case} = W_{cond} + 2t_{case} \tag{13}$$

The moment of inertia of the case about its vertical axis (m^4) is given by:

$$I_{case} = \frac{(W_{case}^4 - W_{cond}^4)}{12.0} \tag{14}$$

The height of the extreme "fiber" from the neutral axis (m) is given by:

$$c_{fiber} = \frac{W_{cond}}{2.0} + t_{case} \tag{15}$$

The bending stress in the extreme fiber of the case (Pa) is given by:

$$\sigma_{bend} = \frac{M_{case}\, c_{fiber}}{I_{case}} \tag{16}$$

The iteration is ended with equation 19 by setting the case thickness equal to half the sum of the smallest guess that is too high and the largest guess that is too small. The case thickness (m) is given by:

$$t_{case} = \frac{(max_{small} + min_{big})}{2.0} \tag{19}$$

The neutron energy attenuation of the case ( ) is given by:

$$a_{case} = \exp^{(-13 t_{case})} \tag{20}$$

Notice that an inverse e-folding distance of 13 cm^{-1} is inserted on the assumption that the case is made of stainless steel. If the shield material is stainless steel, the attenuation coefficient ( ) is given by:

$$Atten = 13 \tag{21}$$

If the shield material is $WTiH_2$ or W, the attenuation coefficient ( ) is given by:

$$Atten = 11 \tag{22}$$

The attenuation of any additional shielding, surrounding the case ( ) is given by:

$$a_{shield} = \exp^{(-Atten\, t_{shield})} \tag{23}$$

where t_{shield} is the thickness of the shield (m). The neutron fluence at the surface of the coil (n/cm^2) is given by:

$$Fluence = .44 \times 10^{16} P_{wall}\, duty\, avail\ years\ a_{case}\, a_{shield} \tag{24}$$

where $years$ is the lifetime of the magnet ( ), $avail$ is the integrated availability of the reactor ( ), $duty$ is the local duty factor of the reactor ( ), and P_{wall} is the neutron wall-loading near the magnet (W/m^2).

The atomic density of copper (atoms/cc) is given by:

$$Atom_{dens} = \frac{6.023 \times 10^{23} \times 8}{64} \tag{25}$$

The e-folding distance of neutron capture in copper (cm) is given by:

$$efold_{Cu} = 10. \tag{26}$$

The density of transmuted atoms in copper (atoms/cc) is given by:

$$Transmu_{dens} = \frac{Fluence}{efold_{Cu}} \tag{27}$$

The atomic parts per million of transmuted atoms ( ) is given by:

$$Appm = \frac{Transmu_{dens} \times 10^6}{Atom_{dens}} \tag{28}$$

The volumetric nuclear and gamma heating (W/m^3) is given by:

$$Nuc_{heat} = 18.8\, P_{wall}\, a_{case}\, a_{shield} \tag{29}$$

Equations (34-120) are iterated to find the only self-consistent solution for the ratio of the coolant channel to the conductor height. The method is to take half the difference between a guess that is too low and a guess that is too high. The initializations of the incorrect guesses are:

$$a_{maxlow} = .001 \tag{30}$$

The minimum guess at a_{overh} that is too high ( ) is given by:

$$a_{minhigh} = 1.0 \tag{31}$$

The initial guess of the ratio of coolant channel to conductor height ( ) is given by:

$$a_{overh} = 0.5 \tag{32}$$

The flat-to-flat height of the coolant channel (m) is given by:

$$a = ha_{overh} \tag{34}$$

The thermal conductance of the thermal circuit (W/m-K) is given by:

$$G = 4\,k_{Cu}\left(\frac{2a}{(h - a)} + .54\right) \tag{35}$$

from McAdams text on Heat Transmission [Mc64], where k_{Cu}, the thermal conductivity of copper, is 350 (W/m-K).

The cross-section area of copper in the conductor (m^2) is given by:

$$A_{cu} = h^2 - a^2 \tag{36}$$

The jacket thickness (m) is given by:

$$T_{jack} = \frac{h}{(16 \times 1.63)} \tag{37}$$

The insulation thickness (m) is given by:

$$T_{ins} = \frac{h}{20.} \tag{38}$$

The flat-to-flat height of the jacket. (m) is given by:

$$H_{jack} = h + T_{ins} + T_{jack} \tag{39}$$

This is $2.07 \times h/1.63$ in todays conductors, where h is the flat-to-flat height of the conductor (m).

The area of the conductor insulation and jacket. (m^2) is given by:

$$A_{cond} = H_{jack}^2 \tag{40}$$

The dissipation per unit length due to nuclear heating (W/m) is given by:

$$Q_{nuc} = Nuc_{heat} A_{cond} \tag{41}$$

The overall packing factor, copper/conductor ( ) is given by:

$$Pac_{fac} = \frac{A_{cu}}{A_{cond}} \tag{42}$$

The overall current density in the conductor and jacket (A/m^2) is given by:

$$J_{cond} = J_{cu} Pac_{fac} \tag{43}$$

The current density in the copper (A/m^2) is given by:

$$J_{cu} = \frac{J_{cond}}{Pac_{fac}} \tag{44}$$

The conductor current (A) is given by:

$$I_{cond} = J_{cu} A_{cu} \tag{45}$$

The area required by all the turns of the coil (m^2) is given by:

$$A_{condtotal} = \frac{Amp_{turns}}{J_{cond}} \tag{46}$$

If the coil shape is circular, the length of a single turn (m) is given by:

$$L_{turn} = 2.\pi R_{coil} \tag{47}$$

where R_{coil} is the coil radius for the circular coil option (m).

If the coil is an l-shaped saddle, the length of a single turn (m) is given by:

$$L_{turn} = 2\left(V_{leg} + H_{legtor} + H_{legrad}\right) \tag{48}$$

where, for the example of a bundle divertor, H_{legrad} is the length of the horizontal leg in the radial direction (m), and H_{legtor} is the length of the horizontal leg in the toroidal direction (m).

The number of turns required ( ) is given by:

$$N_{turns} = \frac{Amp_{turns}}{I_{cond}} \tag{49}$$

The minimum permissible bending radius (m) is given by:

$$Minbend = 12\,H_{jack} \tag{50}$$

The expected displacements per neutron after annealing ( ) is given by:

$$dpn = 60 \tag{51}$$

The expected lattice displacements per atom (dpa) is given by:

$$dpa = dpn\,Appm 1.0 \times 10^{-6} \tag{52}$$

If the expected displacements per atom are less than 0.001, the displacements are unsaturated, and the electrical resistivity of the copper ($\Omega - m$) is proportional to the irradiation.

$$\rho_{lattice} = \frac{dpa}{.001}\,0.4 \times 10^{8} \tag{53}$$

At 0.001 displacements per atom, the amount of the displacements that copper can support saturates, and resistivity due to displacements no longer increases with irradiation.

$$\rho_{lattice} = 0.4 \times 10^{8} \tag{54}$$

The badness ratio of additional resistivity due to a single transmutation over that due to a single lattice displacement ( ) is given by:

$$Bad_{trans} = 3.5 \tag{55}$$

The Joule-Thomson coefficient -dT/dP at constant enthalpy. (C/Pa) is given by: The electrical resistivity of the copper due to transmutations of the copper into zinc and nickel ($\Omega - m$) is given by:

$$\rho_{transmu} = \frac{Bad_{trans}\,Appm\left(.0148 \times 10^{-8}\right)}{300} \tag{56}$$

The electrical resistivity of copper as a function of temperature. ($\Omega - m$) is given by:

$$\rho_{temp} = (1.48 + .00754T_{av})1.\times 10^{-8} \tag{57}$$

The total electrical resistivity of the copper ($\Omega - m$) is given by:

$$\rho_{cu} = \rho_{temp} + \rho_{lattice} + \rho_{transmu} \tag{58}$$

The power/volume dissipated in the conductor (W/m^3) is given by:

$$J_{squarep} = \rho_{cu} J_{cu}^2 \tag{59}$$

The resistance of one turn of the coil (Ω) is given by:

$$R_{turn} = \frac{\rho_{cu} L_{turn}}{A_{cu}} \tag{60}$$

The electrical power dissipated per turn (W) is given by:

$$Turn_{disse} = L_{turn}\, A_{cu}\, J_{squarep} \tag{61}$$

The electric power dissipated per coil (W) is given by:

$$Coil_{disse} = Turn_{disse}\, N_{turns} \tag{62}$$

The normalized average temperature in the MgO insulation (C) is given by:

$$T_{norm} = \frac{(T_{hotins} + T_{hotcu})}{200.} \tag{63}$$

The thermal conductivity of MgO. (W/m-K) is given by:

$$k_{MgO} = -.7728 + \frac{84.835}{T_{norm}} - \frac{62.696}{T_{norm}^2} + \frac{16.23}{T_{norm}^3} \tag{64}$$

The thermophysical properties of MgO are taken from Kingery,Bowen and Uhlmann [KI76], The correlation is by the author. The conversion factor, scaling from the RTPR design, for a ceramic facing combined neutron and gamma radiation from W/m^2 to Gray/s (Gray/s/W/m^2) is given by:

$$RTPR_{conv} = .05 \tag{65}$$

The maximum radiation absorption in the magnet insulation (Gray/s) is given by:

$$Mag_{rad} = RTPR_{conv} P_{wall} \tag{66}$$

The electrical conductivity in ceramic insulation due to irradiation (mho/m) is given by:

$$\sigma_{MgOrad} = 5. \times 10^{-6} \left(\frac{Mag_{rad}}{6.6} \right)^{.65} \tag{67}$$

The correlation is based on Clinard's contribution to the LASL 1979 Special Purpose Materials Annual Progress Report [CL79].

The electrical conductivity of G-10 insulation due to irradiation is given by:

$$\sigma_{G10rad} = 2.0 \times 10^{-12} + \frac{300. \times 10^{-12} Mag_{rad}}{40.0} \tag{68}$$

If the insulation is MgO, the heat flux in the insulation (W/m^2) is given by:

$$Q_{ins} = \frac{2.0 \ \Delta T_{ins} \ k_{MgO}}{T_{ins}} \tag{71}$$

If the insulation is G-10, the heat flux in the insulation (W/m^2) is given by:

$$Q_{ins} = \frac{2. \Delta T_{ins} k_{G10}}{T_{ins}} \tag{72}$$

The power density in the MgO due to leakage currents (W/m^3) is given by:

$$P_{ins} = \frac{Q_{ins}}{T_{ins}} \tag{73}$$

The current density in the MgO (A/m^2) is given by:

$$J_{ins} = \sqrt{P_{ins} \sigma_{rad}} \tag{74}$$

The electric field in the MgO (V/m) is given by:

$$E_{ins} = \frac{J_{ins}}{\sigma_{rad}} \tag{75}$$

The ratio of peak charging voltage to steady-state voltage ( ) is given by:

$$V_{chgr} = 1.4 \tag{76}$$

since a ratio of 1.4 is typical of exponentially charged normal magnets.

The peak coil terminal voltage (V) is given by:

$$V_{term} = E_{ins} T_{ins} \tag{77}$$

The resistive terminal voltage at the coil (V) is given by:

$$V_{rterm} = \frac{V_{term}}{V_{chgr}} \tag{78}$$

The terminal voltage, if all the coil turns were in series (V) is given by:

$$V_{series} = N_{turns} R_{turn} V_{chgr} I_{cond} \tag{79}$$

The number of parallel lead pairs ( ) needed to achieve the design value of temperature drop in the insulation is given by:

$$N_{pairs} = \frac{V_{series}}{V_{term}} \tag{80}$$

rounded down to the nearest integer unless the ratio is less than one.

The power per unit length due to leakage currents (W/m) is given by:

$$Pow_{plins} = 4\, P_{ins} T_{ins} H_{jack} \tag{82}$$

The power dissipated per single turn (W) is given by:

$$Turn_{diss} = L_{turn} \left(A_{cu} J_{squarep} + A_{cond} Nuc_{heat} + Pow_{plins} \right) \tag{83}$$

The power dissipated per coil (W) is given by:

$$Coil_{diss} = Turn_{diss} N_{turns} \tag{84}$$

The heat flux into the water-cooling channel (W/m^2) is given by:

$$Q_h = \frac{Turn_{diss}}{(4\, L_{turn} a)} \tag{85}$$

The wetted perimeter (m) is given by:

$$P_w = 4\, a \tag{86}$$

The area of the coolant channel (m^2) is given by:

$$A_{chan} = a^2 \tag{87}$$

The hydraulic diameter (m) is given by:

$$D_h = \frac{4\, A_{chan}}{P_w} \tag{88}$$

The viscosity of water as a function of temperature (kg/s-m) is given by:

$$Visc = \frac{95}{3600\Big((T_{in} + 20)^{.9}\big(\frac{T_{in}}{400} + 1\big)\Big)} \tag{89}$$

where T_{in} is the inlet temperature of the coolant water (C).

The mass flow rate/unit area (kg/m^2-s) is given by:

$$G_m = \rho_{H2O} v \tag{90}$$

where ρ_{H2O}, the mass density of water, is 1000 (kg/m^3).

The Reynold's number ( ) is given by:

$$Re = \frac{G_m D_h}{Visc} \tag{91}$$

The Prandtl number ( ) is given by:

$$P_r = \frac{Visc\, Cp_{H2O}}{k_{H2O}} \tag{92}$$

where Cp_{H2O}, the specific heat of water, is 4178 (J/kg-C), and k_{H2O}, the thermal conductivity of water is 0.61 (W/m-K).

The initial guess at the wall drop ΔT_{wall} (C) is:

$$\Delta T_{wall} = 10 \tag{93}$$

Equations (95-96) are iterated to avoid having to find an analytic solution for ΔT_{wall}.

The heat-transfer coefficient according to the Dittus-Boelter correlation as modified by Giarratano (W/m^2-K) is given by:

$$H_{Giar} = 0.259\left(\frac{k_{H2O}}{D_h}\right)Re^{0.8}P_r^{.4}\left(\frac{T_{in}}{(T_{in} + \Delta T_{wall})}\right)^{.716} \tag{95}$$

The wall drop (C) is given by:

$$\Delta T_{wall} = \frac{Q_h}{H_{Giar}} \tag{96}$$

The difference between the outlet and inlet temperature due to Joule heating (C) is given by:

$$\Delta T_{joule} = \frac{Turn_{disse}}{(v\, \rho_{H2O}\, A_{chan}\, Cp_{H2O})} \tag{97}$$

The H2O temperature rise from nuclear heating of the conductor (C) is given by:

$$\Delta T_{nuc} = \frac{Nuc_{heat} A_{cond} L_{turn}}{(v \rho_{H2O} A_{chan} Cp_{H2O})} \tag{98}$$

The water temperature rise due to insulation leakage currents (C) is given by:

$$\Delta T_{ins} = \frac{Pow_{plins} L_{turn}}{(v \rho_{H2O} A_{chan} Cp_{H2O})} \tag{99}$$

The friction factor, good for clean steel pipe, when $Re < 10^5$. ( ) is given by: The Reynold's number ( ) is given by:

$$f_{lowRe} = \frac{0.04}{(Re^{0.16})} \qquad (100)$$

The friction factor, good for $Re > 2000$ in smooth tube ( ) is given by: The Reynold's number ( ) is given by:

$$f_{highRe} = .0014 + \frac{.125}{Re^{.32}} \qquad (101)$$

The friction factor ( ) is given by:

$$if(Re > 1.e4),\ f = f_{highRe} \qquad (102)$$

or

$$if(Re < 1.e4),\ f = f_{lowRe} \qquad (103)$$

The pressure drop per unit length (Pa/m) is given by:

$$\Delta p_l = \frac{2f\rho_{H2O}v^2}{D_h} \qquad (104)$$

The ideal pump power per unit length (W/m) is given by:

$$Pow_{pl} = vA_{chan}\Delta p_l \qquad (105)$$

The pressure drop per hydraulic channel (Pa) is given by:

$$P_{delt} = \Delta p_l L_{turn} \qquad (106)$$

The Joule-Thomson coefficient -dT/dP at constant enthalpy. (C/Pa) is given by:

$$\mu = \frac{k_{H2O}}{(\rho_{H2O}Cp_{H2O})} \qquad (107)$$

The temperature rise due to isenthalpic expansion (C) is given by:

$$\Delta T_{\mu} = P_{delt}\mu \tag{108}$$

The difference between the inlet and outlet water temperature (C) is given by:

$$\Delta T_{io} = \Delta T_{joule} + \Delta T_{\mu} + \Delta T_{nuc} + \Delta T_{ins} \tag{109}$$

The temperature difference across the copper (K) is given by:

$$T_{diffcu} = \frac{(2.Pow_{plins} + Nuc_{heat}A_{cond} + J_{square\rho}\mathbf{A_{cu}})}{G} \tag{110}$$

The outlet temperature of the water (C) is given by:

$$T_{out} = T_{in} + \Delta T_{io} \tag{111}$$

The wall temperature, calculated starting at the water inlet (C) is given by:

$$T_{wallH2O} = T_{out} + \Delta T_{wall} \tag{112}$$

The wall temperature, calculated starting at the insulation (C) is given by:

$$T_{wallcu} = T_{hotcu} - T_{diffcu} \tag{113}$$

If the temperature of the wall, calculated starting from the water inlet is higher than the temperature of the wall, calculated starting from the conductor hot-spot, then the coolant channel is too small. A revised guess is made of a_{overh}, the ratio of coolant channel height to conductor height, and equations (34-120) are repeated.

The copper wall temperature at the outlet (C) is given by:

$$T_{wall} = \frac{(T_{wallH2O} + T_{wallcu})}{2.0} \tag{117}$$

The average temperature in the copper (C) is given by:

$$T_{av} = \frac{(T_{wall} + T_{hotcu})}{2.0} \tag{118}$$

The copper hot spot temperature with the specified water velocity (C) is given **by:**

$$T_{hot} = T_{out} + T_{diffcu} + \Delta T_{wall} \tag{119}$$

The average temperature in the copper (C) is given by:

$$T_{av} = T_{hot} - \frac{T_{diffcu}}{2.} \tag{120}$$

The stress in the jacket due to the water pressure. (Pa) is given by:

$$\sigma_{pH2O} = \frac{P_{delt}\, a}{(h - a)} \tag{121}$$

The pump power per coil (W) is given by:

$$P_{pump} = Pow_{pl} L_{turn} N_{turns} \tag{122}$$

The efficiency of the pump motors ( ) is given by:

$$\eta_{pump} = 0.7 \tag{123}$$

The electrical power need for the pump motors (W) is given by:

$$Elec_{pump} = \frac{P_{pump}}{\eta_{pump}} \tag{124}$$

The efficiency of the bundle divertor power supply ( ) is given by:

$$\eta_{ps} = 0.8 \tag{125}$$

The total electrical power for the system (W) is given by:

$$P_{total} = Elec_{pump} + \frac{Coil_{disse}}{\eta_{ps}} \tag{126}$$

The overall packing factor, copper/conductor ( ) is given by:

$$Pac_{fac} = \frac{A_{cu}}{A_{cond}} \tag{127}$$

The mass of a conductor (kg) is given by:

$$M_{cond} = \rho_{mcu} L_{turn} N_{turns} A_{cu} \tag{128}$$

where ρ_{mcu}, the mass density of copper, is 8900 (kg/m^3).

The mass of the case (kg) is given by:

$$M_{case} = 4.0\ \rho_{mss}\ L_{turn}\ t_{case}\ (W_{case} - t_{case}) \tag{129}$$

where ρ_{mss}, the mass density of stainless steel, is 7800 (kg/m^3).

The mass of the magnet (kg) is given by:

$$M_{magnet} = M_{cond} + M_{case} \tag{130}$$

The cost of the magnet case ($) is given by:

$$Cost_{case} = 31.3\ M_{case} \tag{131}$$

For example, the fabrication of the various TF case pieces in TFTR cost $6.7 M and the pieces weighed 670 thousand pounds. If EDIA costs for the case were proportional to hardware costs as a fraction of total EDIA, then the EDIA charge was $1.5 M. Contingency is expected to be 10 to 20 % . Therefore, I estimate an approximate total cost of $9.5 M or $31.3/kg. The cost of magnet fabrication, including winding ($) is given by:

$$Cost_{magfab} = 11.3\ M_{magnet} \tag{132}$$

For example, fabrication of the TFTR TF coils cost $4.24 M. If EDIA was was split proportionately to hardware cost, the EDIA cost was $1 M. Contingency was 20 % . Since the case plus conductor for the system weighs 1.16 million pounds, this is a specific cost of \$11.3/kg. The cost of the magnet support pedestal (\$) is given by:

$$Cost_{ped} = 0.6\ M_{magnet} \tag{133}$$

Although the July, 1979 ETF bundle divertor magnet was supported through the TF case, there should still be some sort of magnet pedestal, connecting the magnet to the floor. The TFTR pedestal cost \$300 k and supported 1.16 million pounds. The cost of conductor insulated by MgO (\$) is given by:

$$Cost_{condMgO} = 15.7 M_{cond} \tag{134}$$

This is based on a 1979 quote from Pyrotenax of Canada [HA79]. The cost of G-10 insulated, internally cooled conductor is taken to be:

$$Cost_{condG10} = 6.94 M_{cond} \tag{135}$$

For example, the TFTR TF conductor cost \$1.578 M for 500,000 lb of conductor.

The volt-ampere requirement of the dc power supply (W) is given by:

$$P_{psva} = V_{term} N_{pairs} I_{cond} \tag{136}$$

The total cost of the magnet (\$) is given by:

$$Cost_{magnet} = Cost_{cond} + Cost_{case} + Cost_{magfab} + Cost_{ped} \tag{139}$$

The cost of the rectifier-transformer (\$) is given by:

$$Cost_{xfmr} = .00125\ P_{psva} \tag{140}$$

For example, most of the TFTR transformers were purchased at \$40,000 for transformer with a 2 kV x 24 kA, rectified output. However, the last four transformers cost \$60,000, so the higher price is used. If the terminal voltage is greater than 1 kV, the cost of the solid-state controlled rectifier (\$) is given by:

$$Cost_{rect} = .004\, P_{psva} \tag{141}$$

If the terminal voltage is less than 1 kV, the cost of the solid-state controlled rectifier ($) is given by:

$$Cost_{rect} = 4 N_{pairs} I_{cond} \tag{142}$$

The cost of rectifiers is a controversial term, which has engendered many arguments, ever since the LASL/Westinghouse evaluation of the EPR Ohmic Heating system [HE76]. The basis for the controversy is that large controlled rectifiers have been sold to industry and the fusion program with specific costs ranging from $3/kW to $100/kW, with no apparent explanation due to economies of scale. Most studies have assumed specific costs of about $30/kW, which eliminates the use of solid-state switches for ohmic heating circuits and makes ohmic heating power supplies the dominant cost item for the whole reactor, when the aspect ratio is made too small. However, I advocate the more optimistic specific cost of $4/kW for relatively low voltage supplies. All of the large 1,000 V supplies procured by the fusion program cost in the range of $4/kW, including the TF supplies for Alcator C and Ormak and the PF and TF modules for TFTR.

The cost of buswork, one magnet ($) is given by:

$$Cost_{bus} = 50.0\, N_{pairs}\, I_{cond} \tag{143}$$

The total cost of the power supply ($) is given by:

$$Cost_{ps} = Cost_{bus} + Cost_{xfmr} + Cost_{rect} \tag{144}$$

The cost of the water pump ($) is given by:

$$Cost_{H2Opump} = .007\, Coil_{diss} \tag{145}$$

The cost of the water purifier ($) is given by:

$$Cost_{purif} = .0015\, Coil_{diss} \tag{146}$$

The cost of the heat exchanger ($) is given by:

$$Cost_{hx} = .004\, Coil_{diss} \tag{147}$$

The cost of water piping, assumed to be SS-316, 2 cm OD ($) is given by:

$$Cost_{H2Opipe} = 100.0\, Bus_{run} \tag{148}$$

where Bus_{run} is the bus length from the magnet to the power supply (m).

Cooling system costs are taken from schedule 22.03.02 of the fusion reactor standard costing document [SC79]. The cost of the water cooling system ($) is given by:

$$Cost_{cool} = Cost_{H2Opump} + Cost_{purif} + Cost_{hx} + Cost_{H2Opipe} \tag{149}$$

The total cost of the bundle divertor magnet system ($) is given by:

$$Cost_{bdmagsys} = Cost_{ps} + Cost_{magnet} + Cost_{cool} \tag{150}$$

The annual on-time of the magnet (s) is given by:

$$t_{on,annual} = duty\ avail \times 3600 \times 24 \times 365 \tag{151}$$

The annual electrical energy usage (J) is given by:

$$E_{annual} = P_{total}\, t_{on,annual} \tag{152}$$

The annual electricity usage in kW-hr (kW-hr) is given by:

$$E_{yr,kwhr} = \frac{E_{annual}}{3.6 \times 10^6} \tag{153}$$

The annual cost of electricity ($) is given by:

$$Cost_{elecyr} = 0.06\, E_{yr,kwhr} \tag{154}$$

C. Results

Of all the fusion applications which might require high magnet irradiation, the bundle divertor appears to benefit the most from the elimination of shielding, because of the high order dependence of magnetic field ripple enhanced transport on the magnetic moment of the divertor magnet. The use of ceramic insulation to reduced shielding requirements has been considered previously by Schultz [SC80]. Typical magnet requirements for an INTOR or ETF scale bundle divertor magnet in a four magnet system might be 5 million ampere-turns, 4 tesla toroidal field at the front vertical leg and front leg heights and lengths of 1.2 m. Keeping allowable stresses and temperatures fixed, we examined the effect of varying conductor size, copper current density, shield thickness, and the allowable temperature rise in the insulation. In order to look at the overall tradeoff of current density, recirculating power and system code, we assumed that a current density over the envelope of 2 kA/cm^2, a recirculating power of 25 MW in one front magnet and a cost of 25 M$ were each equally intolerable and adopted the sum of the ratioes of the actual design values to those three "equal badness" values as a normalized overall cost figure.

As seen in figure 1, the overall economics of the system appear to improve as water velocity is increased in the range of 2 to 15 m/s, While the cost of the system is increased by increased water velocity, due to increased water pump costs and increased recirculating power to drive the pumps, the overall conductor current density increases by 50 % , as the water velocity is increased from 2 to 16 m/s. However, lifetime limitations caused by magnet erosion and vibration have not been modeled. Since improvements in magnet performance are very small in the velocity range of 7 to 15 m/s, we selected 7 m/s as the reference velocity.

For a water velocity of 7 m/s, the optimum conductor size is about 5 cm, as shown in Figure 2. In fact, the optimum is broad between 5 and 10 cm. Since this size range is somewhat beyond the limits of what is now commercially available, a value at the low end of the optimum size range was selected. Unlike my previous study [SC80], which suggested that many electrically parallel turns should be employed (as well as hydraulically paralleling every turn), in order to avoid thermal runaway with reasonably small conductors, this trade study suggests that it is better to use larger conductors that avoid the need for paralleling. This is apparently caused by the relatively high heat removal and thus high pumping and electric power requirements of the water pumping system, which are a nontrivial fraction both of total system cost and recirculating power. Since pump power increases with the cube of the water velocity, these costs are reduced when larger conductors are used, instead of paralleling.

Up to a flat-to-flat height of 4 cm, the current density over the envelope of the magnet and its case improves, as the increased temperatue drop in the copper is more than balanced by the improved cooling effectiveness of the larger cooling channels. Above 4 cm, there is no significant improvement. The total electrical power also declines as the flat-to-flat height increases because of the decrease in pumping power, but the improvement is not significant over a broad range of conductor sizes. The total cost of the bundle divertor magnet system has a minimum at a conductor height of 4.2 cm. This minimum is fairly sharp, since above 4.2 cm, bus and power supply costs increase linearly with current (or with the square of the height), and these are dominant costs of the whole system, as shown in Figure 3. The sharp downward transitions in cost represent decision points, where the number of parallel electrical connections is reduced by one and there is a quantum drop in the bus and power supply costs.

With a coolant velocity of 7 m/s and a flat-to-flat height of the conductor of 5 cm, an insulation temperature drop of about 5 C is about optimum, as shown in figure 4. A higher temperature leads to a significant fraction of the total power dissipation in the conductor being caused by leakage currents in the insulation. Because the insulation is thin and has a relatively large surface area, being on the outside of the conductor, a proportionally large amount of power can be dissipated without a very large change in the temperature drop across the insulation. This justifies the code's use of average values of temperature dependent physical properties, such as thermal conductivity. As shown in figure 4, once it is possible to use a single pair of leads at ΔT_{ins} above 4 C, there is no advantage to allowing the temperature across the insulation to increase further. As shown in Figure 5, the cost trade-off is totally dominated by the high bus currents needed to achieve a very low temperature drop across the insulation. Once the number of parallel leads has been reduced to a single pair, further increasing the temperature drop just increases the thermal load on the system, with concomitant increases in the cost of the water cooling system, as shown in Figure 5. Another reason favoring the selection of a lower temperature drop is the lower terminal voltage, as shown in Figure 5. Although low voltage per se does not save any money, the high degree of uncertainty about the breakdown behaviour of highly irradiated magnets favors designs with low enough voltages that it may be difficult to sustain an arc under any circumstances ($<< 1,000V$). At a temperature drop of 4 C, the terminal voltage is 300 V, which is not guaranteed safe, but is less risky than the 700 V corresponding to a drop of 20 C.

The most controversial and significant conclusion of the trade study is shown in figure 6. In this study, copper current density and conductor size were held constant, while the shield thickness was decreased. Despite insulation leakage, increased copper resistivity due to lattice displacements and transmutations and increased neutron and gamma heating of the conductor, there is an obvious improvement in the overall "goodness" of the bundle divertor, down to zero shielding. The overall current density over the envelope improves by more than a factor of three as the shield is decreased from 30 cm to nothing more than the structural case, and it cannot be improved that much by any other parameter, such as increasing the currrent density in the copper, as shown in Figure 8. The total electrical power for the system actually declines a little bit with decreased shielding. Presumably the increased nuclear and gamma heating opens up the coolant channels, in order to maintain constant allowable temperatures, and the decrease in pump power slightly outweighs the increase in Joule heating. As shown in Figure 7, this same effect also leads to counterintuitive increases in the costs of the power supply and the magnet, as the shielding is increased. The cost of the power supply increases by 15 % and the cost of the magnet by 6 % as the shield thickness increases from 0 to 30 cm.

Reducing the shielding is the only parameter change that causes a dramatic improvement in system performance. This conclusion has perhaps been obvious to plasma designers [HI80] for some time, but is certainly not obvious to magnet designers. We will explore a magnet design in the region of overall optimum goodnesss in more detail, in order to gain more insight about the validity of the conclusion that all shielding other than that provided by the magnet case should be eliminated.

With conductor size fixed, the most desirable value of the current density in the copper is shown to be approximately 2 kA/cm^2, as shown in figure 8. Higher current density decreases ripple, while increasing recirculating power. Because of the need for a case and a shield, the overall current density over the magnet envelope does not increase nearly as rapidly as the current density of the copper. For example, with a 20 cm shield, as the copper current density increases a factor of ten from 5 to 50 MA/m^2, the envelope current density only increases by a factor of 3. However, with no shield other the the structural case, the current density over the envelope increases by a factor of 5 from 3 MA/m^2 to 15 MA/m^2, over the same range of copper current density. The recirculating power is also a first-order function of the current density in the copper and increases from 8 MW to 65 MW in a single coil, over the same range. The system cost showns an optimum at about 1.5 kA/cm^2, but it is not as important a factor as the current density and the recirculating power. Even with a copper current density of 5 kA/cm^2, the divertor system cost is only 11.5 M$ for a front coil. The power supply is only 2 million dollars at the optimum current density of 2 kA/cm^2 in the copper, as shown in the Figure 9. Again the paradoxical result that the cost of the coolant system declines at higher current densities is seen in Figure 9, again probably a function of the larger coolant channels.

D. Worked Example with Low Overall Cost Figure

We look at an example with a low cost figure in Tables I through XXIX. The overall coil dimensions are shown in Figure 10. In this example, the inputs include a conductor copper height of 5 cm, a water velocity of 7 m/s, current density in the copper of 1.38 kA/cm^2 and a temperature drop across the insulation of 5 C. There is no shield, other than the structural case. The structural case is 4.4 cm thick and provides slightly less than a factor of 2 attenuation of the neutron and gamma flux. These dimensions give an overall conductor current density of 1.38 kA/cm^2 and a current density over the coil envelope, including the case, of 1.1 kA/cm^2. The case must be considerably thicker on the top and bottom than around the vertical leg.

Because of the high coil irradiation, the code selected a low impedance design. Two parallel conductors carry 41 kA apiece, while the peak voltage during charging is only 356 V, with a peak resistive voltage of 254 V. Despite the low impedance and the limitation of the temperature rise in the insulation, the power dissipation density in the MgO is 53 MW/m^3 vs. 21 MW/m^3 due to nuclear and gamma heating and only 11.6 MW/m^3 due to Joule heating. The electric field in the insulation is 1.42×10^5 V/m, or 3 V/mil, which would represent a conservative design in an unirradiated environment.

The conductor dimensions are shown in Figure 11. The 5.44 cm flat-to-flat dimension of the outer jacket is larger than any conductor manufactured by Pyrotenax of Canada with MgO insulation, but does not represent any significant extrapolation in copper conductor technology as such, being smaller than any of the three toroidal field coil conductors in the next generation of tokamaks, TFTR, JET and JT-60. With the manufacturer's recommendation that the radius of curvature should be no smaller than 12 times the flat-to-flat height, the radius of curvature would be restricted to 60 cm, which would leave the 1.2 m long legs rather more circular than square. However, in the field at Los Alamos, the larger conductors are routinely bent to 6 times the flat-to-flat height, so hopefully the conductor could be bent to 30 cm or better, if desired.

A major caveat is that the sizing code, as it is now written, does not take into account that a noneroding sleeve may be necessary between the water and the copper to prevent rapid erosion of the copper by radiolysis products. Steel cladding of thinness down to 1 mil is routinely used in the nuclear industry [SC81], and may be useful here without serious degradation of heat removal or achievable bending radius.

The inventory of temperature drops through the system is shown in Table III. The water inlet temperature is 25 C. It rises 13 C due to Joule heating, 34 C due to nuclear heating and 16 C due to leakage currents in the insulation, giving an outlet temperature of 88 C, which will require a modest amount of pressurization to avoid any cavitation. These temperature rises are achieved with a Reynold's number of 184,000, a heat flux into the water channel of 135 W/cm^2 and a pressure drop of 1.3 atmosphere. The temperature rise across the copper at the hot outlet end is 57 C, giving a copper hot-spot temperature of 145 C, 5 C below the manufacturer's recommendation. The rise of 5 C across the insulation brings the system hot-spot temperature to 150 C. This is considerably higher than the insulation hot-spot temperature of 70 C permitted with G-10 insulation. The resistivity of the copper has risen from the nominal room temperature value of $1.67 \times 10^{-8}\Omega - m$ to $2.9 \times 10^{-8}\Omega - m$. Two-thirds of the rise is due to the elevated temperature and the rest due to lattice displacements. Transmutations add only a little, because of the low integrated duty factor of the FED application.

The electrical dissipation of a single coil is 24 MW, while the total electrical and thermal dissipation is 102 MW. With the addition of the electrical power needed for the cooling system, the system recirculating electrical power is 26.4 MW. Since the rear coils of the bundle divertor can have a considerably lower current density than the front coils, without adversely affecting the toroidal ripple, the coil system recirculating power should be significantly more than 50 MW, but significantly less than 100 MW. Because of the low duty factor of FED, the additional annual cost of electricity should only be $278 K.

The largest and dominant component of the magnet system cost is electrical buswork, costing $4 M. The total cost of the electrical power supplies is only $4.46 M, using the low cost of rectifiers, justified above. The cooling system is costed at $1.3 M, while the magnet, despite its comparatively expensive conductor, is only $725 K. This implies a total cost of the four magnet system somewhere in the neighbourhood of $15 M.

The use of optimization techniques, using weighted cost figures or figures of merit, generally has no justification in real designs. However, in this case, the technique seems to have selected a reasonably sensible design. Having examined the "optimum" design, I would opt for an even lower impedance design. If we doubled the parallel current, the cost of the bus and power supply would increase by about $10 M, but the heat generated by leakage currents would decrease markedly. What is more important, designing in the face of uncertainty, we would be much closer to achieving a conservative design. If dielectric breakdown were now to occur, it would have to occur with only 150 V to sustain an arc and only 1.5 V/mil to drive leakage currents.

The principle reason for using MgO insulation in FED is in order to some possibility of a conservative design for a reasonable service life. While there are currently uncertainties with all candidate technologies, I believe that the development of a larger MgO insulated conductor and the characterization of leakage current in MgO under neutron and gamma irradation represents an RDAC program of moderate cost and duration, while the debates over the lifetime of organic insulation will never be resolved without considerable field experience. Over the short run, organic insulations, such as G-10, will always outperform MgO with any shielding thickness, because they have much less leakage current under irradiation and they have good mechanical properties without the need for a jacket. It is only with a high integrated fluence that the superior parametric stability of the MgO insulation dominates. Therefore, I believe that the low electric field design approach is the most consistent with this conservative philosophy. If one is willing to take risks with either the ripple in the plasma or the life of the insulation, then an organic insulation with a normal or superconducting conductor should be selected.

E. Conclusions

• A self-consistent code has been developed which can be a powerful tool in the design and analysis of highly-irradiated copper magnets.

• The overall performance of a bundle divertor magnet can be improved by eliminating the shielding, without imposing severe lifetime limitations on the magnet.

• For a typical bundle divertor, application the optimum current density in the copper is about 2 kA/cm^2.

• Designing with a low electric field across MgO insulation is a conservative approach to near-term magnet design under intense irradiation.

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INPUT VARIABLES

Parameter	Description	Units
Amp_{turns}	number of ampere-turns in the coil	(A-T)
Bt_{vleg}	toroidal flux density at the vertical legs of the magnet	(T)
Bus_{run}	bus length from the magnet to the power supply	(m)
ΔT_{ins}	design temperature drop across the insulation	(C)
$G10$	is G-10 insulation	()
H_{legrad}	length of the horizontal leg in the radial direction	(m)
H_{legtor}	length of the horizontal leg in the toroidal direction	(m)
J_{cu}	current density in the copper	(A/m^2)
MgO	is MgO insulation	()
P_{wall}	neutron wall-loading near the magnet	(W/m^2)
R_{coil}	coil radius for the circular coil option	(m)
T_{hotG10}	hot-spot temperature in G-10 insulation	(C)
T_{hotMgO}	hot-spot temperature in the MgO insulation	(C)
T_{in}	inlet temperature of the coolant water	(C)
V_{leg}	height of the vertical leg of the magnet	(m)
avail	integrated availability of the reactor	()
circle	is a circular coil	()

INPUT VARIABLES - Continued

Parameter	Description	Units
$duty$	local duty factor of the reactor	()
h	flat-to-flat height of the conductor	(m)
insulation	specifies the insulation technology selected	()
l-coil	is an l-shaped coil	(plan view)
$shape$	specifies the shape of the magnet	()
shield	shield material	()
t_{shield}	thickness of the shield	(m)
v	velocity of the coolant water	(m/s)
$years$	lifetime of the magnet	()

ASSIGNED VARIABLES

Parameter	Description	Units
A_{chan}	area of the coolant channel	(m^2)
A_{cond}	area of the conductor insulation and jacket.	(m^2)
$A_{condtotal}$	area required by all the turns of the coil	(m^2)
A_{cu}	cross-section area of copper in the conductor	(m^2)
$Appm$	atomic parts per million of transmuted atoms	( )
$Atom_{dens}$	atomic density of copper	(atoms/cc)
$Atten$	shield material attenuation coefficient	( )
Bad_{trans}	resistivity badness ratio of transmutation vs lattice displacement	( )
$Coil_{diss}$	power dissipated per coil	(W)
$Coil_{disse}$	electric power dissipated per coil	(W)
$Cost_{H2Opipe}$	cost of water piping, assumed to be SS-316, 2 cm OD	($)
$Cost_{H2Opump}$	cost of the water pump	($)
$Cost_{bdmagsys}$	total cost of the bundle divertor magnet system	($)
$Cost_{bus}$	cost of buswork, one magnet	($)
$Cost_{case}$	cost of the magnet case	($)
$Cost_{cond}$	cost of the conductor	($)
$Cost_{condG10}$	cost of conductor insulated by G-10	($)

ASSIGNED VARIABLES - Continued

Parameter	Description	Units
$Cost_{condMgO}$	cost of conductor insulated by MgO	($)
$Cost_{cool}$	cost of the water cooling system	($)
$Cost_{elecyr}$	annual cost of electricity	($)
$Cost_{hx}$	cost of the heat exchanger	($)
$Cost_{magfab}$	cost of magnet fabrication, including winding	($)
$Cost_{magnet}$	total cost of the magnet	($)
$Cost_{ped}$	cost of the magnet support pedestal	($)
$Cost_{ps}$	total cost of the power supply	($)
$Cost_{purif}$	cost of the water purifier	($)
$Cost_{rect}$	cost of the solid-state controlled rectifier	($)
$Cost_{xfmr}$	cost of the rectifier-transformer	($)
Cp_{H2O}	specific heat of water	(J/kg-C)
Δp_l	pressure drop per unit length	(Pa/m)
ΔT_{wall}	wall drop	(C)
ΔT_{ins}	water temperature rise due to MgO leakage currents	(C)
ΔT_{io}	difference between the inlet and outlet water temperature.	(K)
ΔT_{joule}	difference between the outlet and inlet temperature due to Joule heating.	(C)
ΔT_{μ}	temperature rise due to isenthalpic expansion.	(K)

ASSIGNED VARIABLES - Continued

Parameter	Description	Units
ΔT_{nuc}	H2O temperature rise from nuclear heating of the conductor	(C)
D_h	hydraulic diameter	(m)
E_{annual}	annual electrical energy usage	(J)
E_{ins}	electric field in the MgO	(V/m)
$Elec_{pump}$	electrical power need for the pump motors	(W)
$E_{yr,kwhr}$	annual electricity usage in kW-hr	(kW-hr)
$Fluence$	neutron fluence at the surface of the coil	(n/cm^2)
Fr_{case}	radial force supported by the case	(N)
G	thermal conductance of the thermal circuit	(W/m-K)
G_m	mass flow rate/unit area	(kg/m^2-s)
H_{Giar}	Giarratano's heat-transfer coefficient	(W/m^2-K)
H_{jack}	flat-to-flat height of the jacket.	(m)
I_{case}	moment of inertia of the case about its vertical axis	(m^4)
I_{cond}	conductor current	(A)
J_{cond}	overall current density in the conductor and jacket	(A/m^2)
J_{cu}	current density in the copper	(A/m^2)
J_{ins}	current density in the MgO	(A/m^2)
$J_{squarep}$	power/volume dissipated in the conductor	(W/m^3)

Parameter	Description	Units
L_{turn}	length of a single turn	(m)
Mag_{rad}	maximum radiation absorption in the magnet insulation	(Gray/s)
M_{case}	bending moment applied to the case	(N-m)
M_{cond}	mass of a conductor	(kg)
$Minbend$	minimum permissible bending radius	(m)
M_{magnet}	mass of the magnet	(kg)
N_{pairs}	number of parallel lead pairs	( )
N_{turns}	number of turns required	( )
Nuc_{heat}	volumetric nuclear and gamma heating	(W/m^3)
$t_{on,annual}$	annual on-time of the magnet	(s)
Pac_{fac}	overall packing factor, copper/conductor	( )
P_{delt}	pressure drop per hydraulic channel	(Pa)
P_{ins}	power density in the MgO due to leakage currents	(W/m^3)
Pow_{pl}	ideal pump power per unit length	(W/m)
Pow_{plins}	power per unit length due to leakage currents	(W/m)
P_{psva}	volt-ampere requirement of the dc power supply	(W)
P_{pump}	pump power per coil	(W)
P_r	Prandtl number	( )

ASSIGNED VARIABLES - Continued

Parameter	Description	Units
P_w	wetted perimeter	(m)
Q_h	heat flux into the water-cooling channel	(W/m^2)
Q_{ins}	heat flux in the insulation	(W/m^2)
Q_{nuc}	dissipation per unit length due to nuclear heating	(W/m)
$RTPR_{conv}$	conversion factor from W/m^2 to Gray/s.	($Gray/s/W/m^2$)
Re	Reynold's number	()
R_{turn}	resistance of one turn of the coil	(Ω)
$SM_{fatiguele5}$	safety margin vs. yield strength for 10^5 cycles	()
σ_{G10rad}	electrical conductivity of G-10 due to irradiation	(mho/m)
σ_{MgOrad}	electrical conductitivity in ceramic insulation due to irradiation	(mho/m)
Σ_{pH2O}	stress in the jacket due to the water pressure.	(Pa)
σ_{rad}	insulator electrical conductivity due to irradiation	(mho/m)
T_{av}	average temperature in the copper	(C)
T_{diffcu}	temperature difference across the copper	(K)
T_{hot}	copper hot spot temperature with the specified water velocity	(C)
T_{hotcu}	hot-spot temperature in the copper	(C)
T_{hotins}	hot-spot temperature in the insulation	(C)
T_{ins}	insulation thickness	(m)

ASSIGNED VARIABLES - Continued

Parameter	Description	Units
T_{jack}	jacket thickness	(m)
T_{norm}	normalized average temperature in the MgO insulation	(C)
P_{total}	total electrical power for the system	(W)
T_{out}	outlet temperature of the water	(C)
$Transmu_{dens}$	density of transmuted atoms in copper	(atoms/cc)
$Turn_{diss}$	power dissipated per single turn	(W)
$Turn_{disse}$	electrical power dissipated per turn	(W)
T_{wall}	copper wall temperature at the outlet	(C)
$T_{wallH2O}$	wall temperature, calculated starting at the water inlet	(C)
T_{wallcu}	wall temperature, calculated starting at the insulation	(C)
$T_{wallguess}$	first guess at the wall temperature, outlet side	(C)
V_{chgr}	ratio of peak charging voltage to steady-state voltage	()
$Visc$	viscosity of water as a function of temperature	(kg/s-m)
V_{rterm}	resistive terminal voltage at the coil	(V)
V_{series}	terminal voltage, if all the coil turns were in series	(V)
V_{term}	peak coil terminal voltage	(V)
W_{case}	overall width of the case	(m)
W_{cond}	width of the square conductor winding package	(m)

Parameter	Description	Units
a	flat-to-flat height of the coolant channel	(m)
a_{case}	neutron energy attenuation of the case	()
a_{shield}	attenuation of any additional shielding, surrounding the case	()
a_{maxlow}	maximum guess at a_{overh} that is too low	()
$a_{minhigh}$	minimum guess at a_{overh} that is too high	()
a_{overh}	ratio of coolant channel to conductor height	()
c_{fiber}	height of the extreme "fiber" from the neutral axis	(m)
dpa	expected lattice displacements per atom	(dpa)
dpn	expected displacements per neutron after annealing	()
$efold_{cu}$	efolding distance of neutron capture in copper	(cm)
η_{ps}	efficiency of the bundle divertor power supply	()
η_{pump}	efficiency of the pump motors	()
f	friction factor	()
f_{highRe}	friction factor, good for Re >2000 in smooth tube	()
f_{lowRe}	friction factor, good for clean steel pipe, when $Re < 10^5$.	()
k_{Cu}	thermal conductivity of copper	(W/m-K)
k_{G10}	thermal conducitivity of G-10	(W/m-K)
k_{H2O}	thermal conductivity of water	(W/m-K)

ASSIGNED VARIABLES - Continued

Parameter	Description	Units
k_{MgO}	thermal conductivity of MgO.	(W/m-K)
μ	Joule-Thomson coefficient -dT/dP at constant enthalpy.	(C/Pa)
ρ_{H2O}	mass density of water	(kg/m^3)
ρ_{cu}	total electrical resistivity of the copper	$(\Omega - m)$
$\rho_{lattice}$	electrical resistivity of the copper	$(\Omega - m)$
ρ_{mcu}	mass density of copper	(kg/m^3)
ρ_{mss}	mass density of stainless steel	(kg/m^3)
ρ_{temp}	electrical resistivity of copper as a function of temperature.	$(\Omega - m)$
$\rho_{transmu}$	electrical resistivity of the copper due to transmutations	$(\Omega - m)$
σ_{Yss}	yield strength of the case steel	(Pa)
σ_{bend}	bending stress in the extreme fiber of the case	(Pa)
σ_{design}	design stress in the case	(Pa)
t_{case}	case thickness	(m)

TABLE I

LENGTHS

Length	Description	Value
R_{coil}	coil radius for the circular coil option	1.000 m
V_{leg}	height of the vertical leg of the magnet	1.200 m
H_{legtor}	length of the horizontal leg in the toroidal direction	1.200 m
H_{legrad}	length of the horizontal leg in the radial direction	1.200 m
t_{shield}	thickness of the shield	0.000 m
Bus_{run}	bus length from the magnet to the power supply	100.0 m
e_{foldcu}	efolding distance of neutron capture in copper	100.0 mm
h	flat-to-flat height of the conductor	50.00 mm
t_{case}	case thickness	44.21 mm
W_{cond}	width of the square conductor winding package	604.8 mm
W_{case}	overall width of the case	692.7 mm
a	flat-to-flat height of the coolant channel	21.25 mm
T_{jack}	jacket thickness	1.917 mm
T_{ins}	insulation thickness	2.500 mm
H_{jack}	flat-to-flat height of the jacket.	54.42 mm
L_{turn}	length of a single turn	7.200 m
Min_{bend}	minimum permissible bending radius	653.0 mm
P_w	wetted perimeter	84.99 mm
D_h	hydraulic diameter	21.25 mm

TABLE II

DIMENSIONLESS-VARIABLES

Dimensionless Variable	Description	Value
$duty$	local duty factor of the reactor	0.0400
$avail$	integrated availability of the reactor	0.500
$years$	lifetime of the magnet	5.000
$SM_{fatiguele5}$	safety margin vs. yield strength for 10^5 cycles	3.000
a_{case}	neutron energy attenuation of the case	0.528
a_{shield}	attenuation of any additional shielding, surrounding the case	1.000
a_{overh}	ratio of coolant channel to conductor height	0.454
Pac_{fac}	overall packing factor, copper/conductor	0.618
N_{turns}	number of turns required	1233.
N_{pairs}	number of parallel lead pairs	2
Re	Reynolds number	184200.
Pr	Prandtl number	5.531
f	friction factor	0.00383
η_{pump}	efficiency of the pump motors	0.700
η_{ps}	efficiency of the bundle divertor power supply	0.800

TABLE III

Temperatures

Temperature	Description	Value
T_{diffcu}	temperature difference across the copper	50.91 K
T_{in}	inlet temperature of the coolant water	25.00 C
ΔT_{ins}	design temperature drop across the insulation	5.000 C
T_{hotins}	hot-spot temperature in the insulation	150.0 C
T_{hotcu}	hot-spot temperature in the copper	145.0 C
T_{av}	average temperature in the copper	119.9 C
ΔT_{μ}	temperature rise due to isenthalpic expansion.	0.019 C
ΔT_{wall}	wall drop	6.691 C
ΔT_{io}	difference between the inlet and outlet water temperature.	62.79 K
ΔT_{joule}	inlet-outlet temperature difference due to Joule heating.	12.94 C
ΔT_{nuc}	H2O temperature rise from nuclear heating of the conductor	34.17 C
ΔT_{ins}	water temperature rise due to MgO leakage currents	15.66 C
T_{out}	outlet temperature of the water	87.79 C
T_{wall}	copper wall temperature at the outlet	94.29 C

TABLE IV

COSTS

Costs	Description	Value
$Cost_{case}$	cost of the magnet case	201.6 k$
$Cost_{magfab}$	cost of magnet fabrication, including winding	255.6 k$
$Cost_{ped}$	cost of the magnet support pedestal	13.57 k$
$Cost_{condMgO}$	cost of conductor insulated by MgO	254.0 k$
$Cost_{condG10}$	cost of conductor insulated by G-10	112.3 k$
$Cost_{cond}$	cost of the conductor	254.0 k$
$Cost_{magnet}$	total cost of the magnet	724.8 k$
$Cost_{xfmr}$	cost of the rectifier-transformer	36.43 k$
$Cost_{rect}$	cost of the solid-state controlled rectifier	327.8 k$
$Cost_{bus}$	cost of buswork, one magnet	4.097 M$
$Cost_{ps}$	total cost of the power supply	4.461 M$
$Cost_{H2Opump}$	cost of the water pump	715.1 k$
$Cost_{purif}$	cost of the water purifier	153.2 k$
$Cost_{hx}$	cost of the heat exchanger	408.6 k$
$Cost_{H2Opipe}$	cost of water piping, assumed to be SS-316, 2 cm OD	10.00 k$
$Cost_{cool}$	cost of the water cooling system	1.287 M$
$Cost_{bdmagsys}$	total cost of the bundle divertor magnet system	6.473 M$
$Cost_{elecyr}$	annual cost of electricity	277.5 k$

TABLE V

HEAT-FLUXES

Heat Flux	Description	Value
P_{wall}	neutron wall-loading near the magnet	2.000 MW/m^2
Q_{ins}	heat flux in the insulation	131.9 kW/m^2
Q_h	heat flux into the water-cooling channel	1.354 MW/m^2

TABLE VI

CURRENT DENSITIES

Current Density	Description	Value
J_{cu}	current density in the copper	2.00 kA/cm^2
J_{cond}	overall current density in the conductor and jacket	1.38 kA/m^2
J_{ins}	current density in the MgO	371.0 A/m^2

TABLE VII

AREAS

Area	Description	Value
$A_{condtotal}$	area required by all the turns of the coil	0.365 m^2
A_{cu}	cross-section area of copper in the conductor	20.49 cm^2
A_{cond}	area of the conductor insulation and jacket.	29.61 cm^2
A_{chan}	area of the coolant channel	4.515 cm^2

TABLE VIII

PRESSURES

Pressure	Description	Value
σ_{design}	design stress in the case	116.7 MPa
σ_{bend}	bending stress in the extreme fiber of the case	117.5 MPa
Δp	pressure drop per hydraulic channel	132.3 kPa
σ_{pH2O}	stress in the jacket due to the water pressure.	97.73 kPa

TABLE IX

POWER DENSITIES

Power Density	Description	Value
Nuc_{heat}	volumetric nuclear and gamma heating	21.16 MW/m^3
$J^2\rho$	power/volume dissipated in the conductor	11.58 MW/m^3
P_{ins}	power density in the MgO due to leakage currents	52.77 MW/m^3

TABLE X

THERMAL-CONDUCTIVITIES

Thermal Conductivity	Description	Value
G	thermal conductance of the thermal circuit	2.825 kW/K
k_{MgO}	thermal conductivity of MgO.	32.98 W/m-K

TABLE XI

LOSS/UNIT LENGTH

Loss/Length	Description	Value
Q_{nuc}	dissipation per unit length due to nuclear heating	62.67 kW/m
Pow_{plins}	power per unit length due to leakage currents	28.72 kW/m
Pow_{pl}	ideal pump power per unit length	58.05 W/m

TABLE XII

RESISTIVITIES

Resistivity	Description	Value
$\rho_{lattice}$	copper resistivity due to lattice displacements	4.000 n$\Omega - m$
$\rho_{transmu}$	copper resistivity due to transmutations	1.136 n$\Omega - m$
ρ_{temp}	copper resistivity as a function of temperature.	23.82 n$\Omega - m$
ρ_{cu}	total electrical resistivity of the copper	28.96 n$\Omega - m$

TABLE XIII

POWERS

Power	Description	Value
$Turn_{disse}$	electrical power dissipated per turn	170.9 kW
$Coil_{disse}$	electric power dissipated per coil	21.06 MW
$Turn_{diss}$	power dissipated per single turn	828.8 kW
$Coil_{diss}$	power dissipated per coil	102.2 MW
P_{pump}	pump power per coil	51.52 kW
P_{epump}	electrical power need for the pump motors	73.59 kW
P_{total}	total electrical power for the system	26.40 MW
P_{psva}	volt-ampere requirement of the dc power supply	29.14 MW

TABLE XIV

CONDUCTIVITIES

Conductivity	Description	Value
σ_{MgOrad}	MgO conductivity due to irradiation	2.608 mmho/m
σ_{G10rad}	G-10 conductivity due to irradiation	750.0 nmho/m

TABLE XV

VOLTAGES

Voltage	Description	Value
V_{term}	peak coil terminal voltage	355.6 V
V_{rterm}	resistive terminal voltage at the coil	254.0 V
V_{series}	terminal voltage, if all the coil turns were in series	719.6 V

TABLE XVI

MASSES

nil	Description	Value
M_{cond}	mass of conductor	16.18 Mg
M_{magnet}	mass of the magnet	22.62 Mg

TABLE XVII

ENERGIES

Energy	Description	Value
E_{annual}	annual electrical energy usage	166.5 GJ
E_{yrkwhr}	annual electricity usage in kW-hr	4.625 MkW-hr

TABLE XVIII

AMPERE-TURNS

Ampere-Turns	Description	Value
$Ampturns$	number of ampere-turns in the coil	5.050 MA-T

TABLE XIX

FLUX-DENSITIES

Flux Density	Description	Value
B_{tvleg}	toroidal flux density at the vertical legs of the magnet	3.000 T

TABLE XX

VELOCITIES

Velocity	Description	Value
v	velocity of the coolant water	7.000 m/s

TABLE XXI

FORCES

Force	Description	Value
F_{rcase}	radial force supported by the case	18.18 MN

TABLE XXII

MOMENTS

Moment	Description	Value
M_{case}	bending moment applied to the case	6.441 kN-m

TABLE XXIII

CURRENTS

Current	Description	Value
I_{cond}	conductor current	40.97 kA

TABLE XXIV

MOMENTS OF INERTIA

Moment of Inertia	Description	Value
I_{case}	moment of inertia of the case about its vertical **axis**	8.041 mm^4

TABLE XXV

FLUENCES

Fluence	Description	**Value**
Fluence	neutron fluence at the surface of the coil	$4.953 \times 10^{16} n/cm^2$

TABLE XXVI

RESISTANCES

Resistance	Description	**Value**
R_{turn}	resistance of one turn of the coil	**101.8 $\mu\Omega$**

TABLE XXVII

RADIATION ABSORPTION

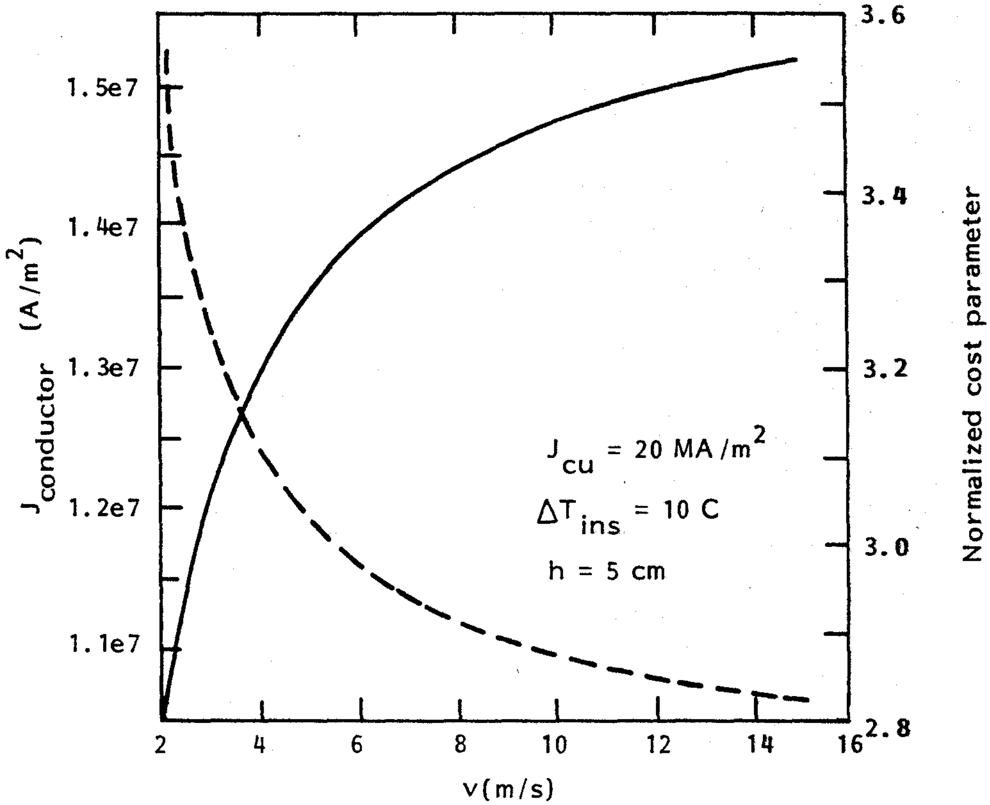
Absorption	Description	Value
Mag_{rad}	maximum radiation absorption in the magnet insulation	100.0 kGray/s

TABLE XXIX

ELECTRIC FIELDS

Electric Field	Description	**Value**
E_{ins}	electric field in the MgO	142.3 kV/m

Overall current density in the conductor and jacket vs. velocity of the coolant ———
Total badness of the system vs. velocity of the coolant water - - - -



Total cost of the bundle diverter magnet system vs. velocity of the coolant

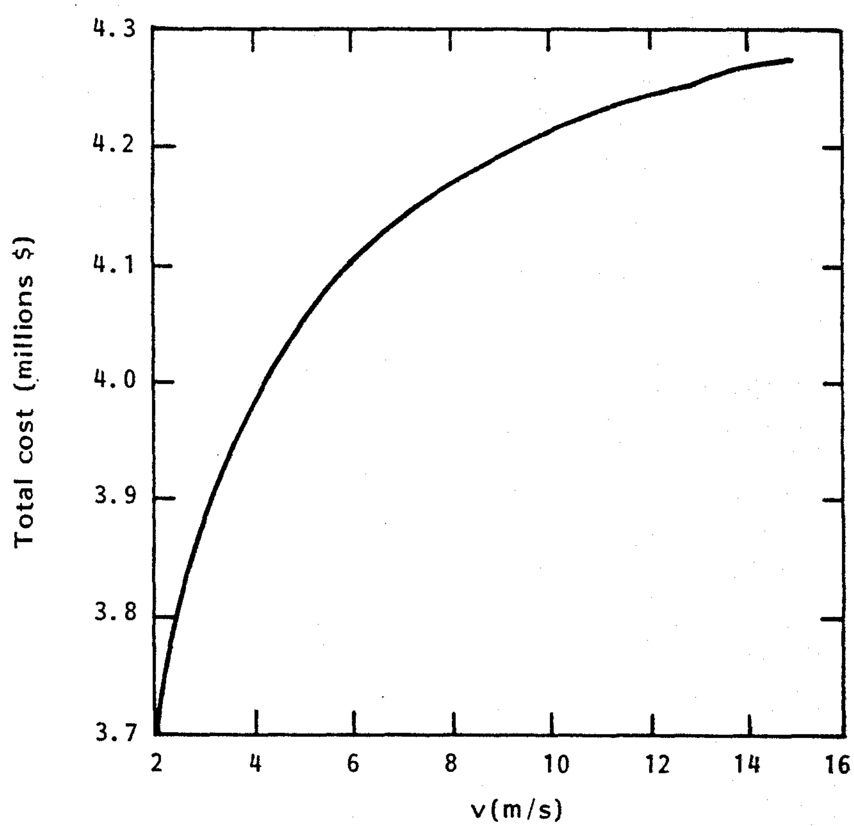
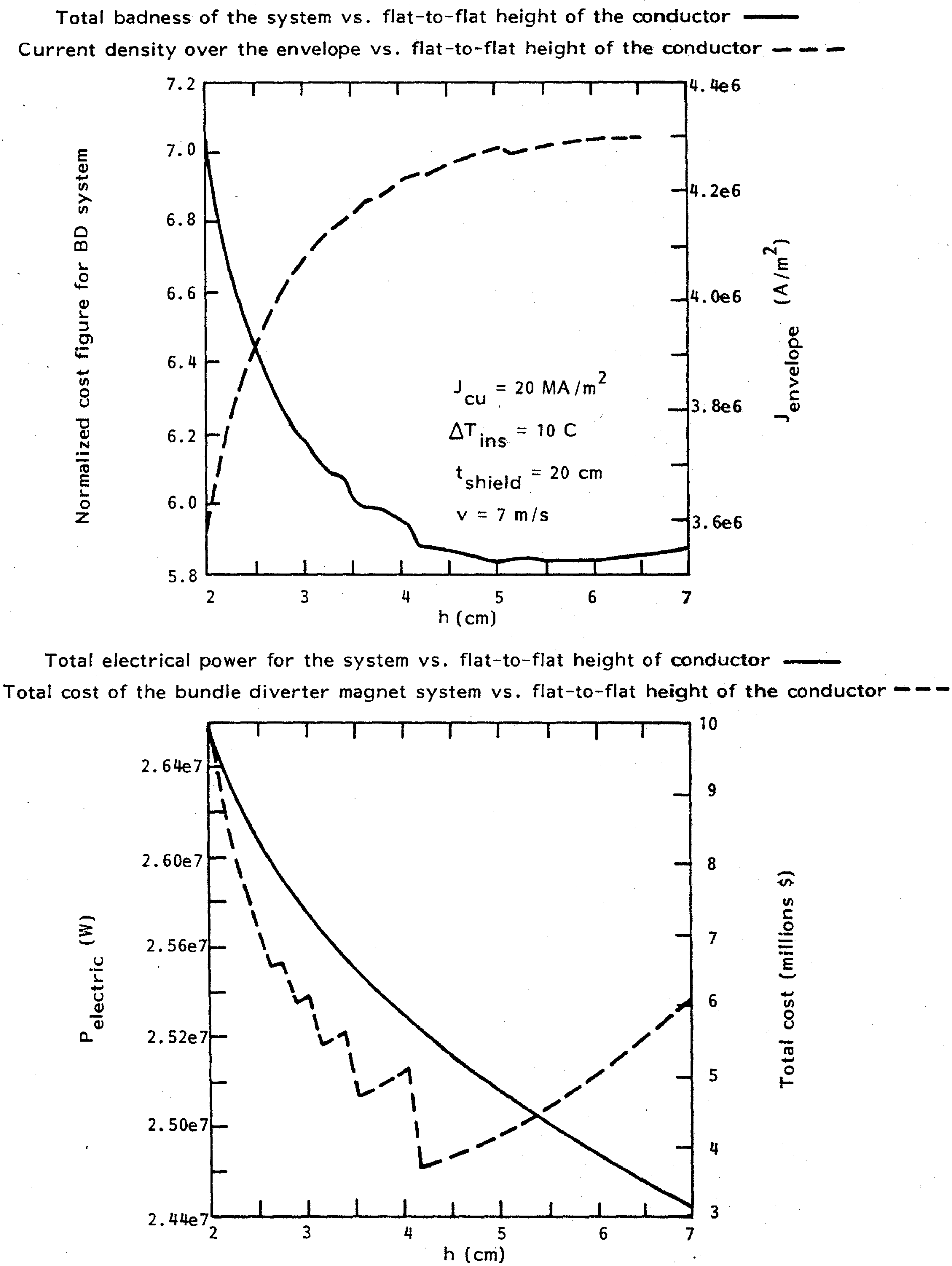


Figure 1

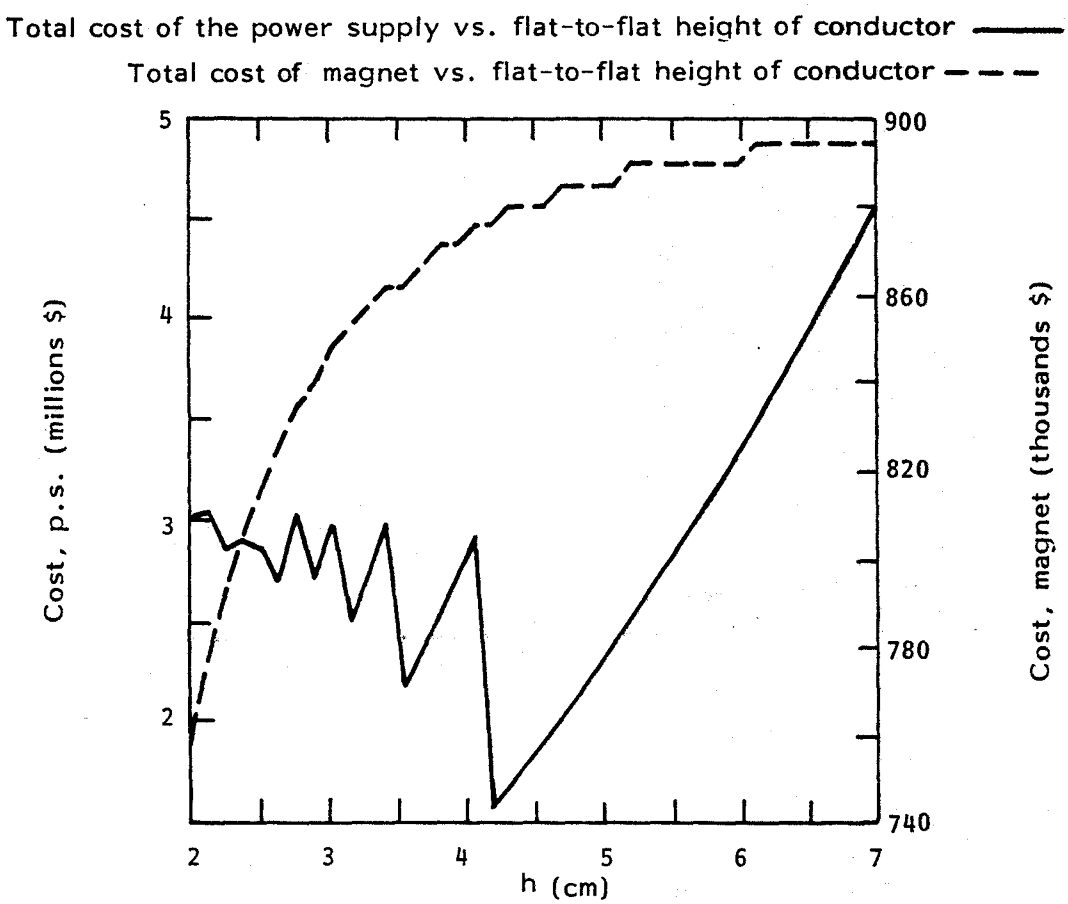


Total badness of the system vs. flat-to-flat height of the conductor
Current density over the envelope vs. flat-to-flat height of the conductor
Normalized cost figure for BD system
J envelope (A/m2)
7.2
7.0
6.8
6.6
6.4
6.2
6.0
5.8
4.4e6
4.2e6
4.0e6
3.8e6
3.6e6
J cu = 20 MA/m2
ΔT ins = 10 C
t shield = 20 cm
v = 7 m/s
2
3
4
5
6
7
h (cm)
Total electrical power for the system vs. flat-to-flat height of conductor
Total cost of the bundle diverter magnet system vs. flat-to-flat height of the conductor
P electric (W)
Total cost (millions $)
2.64e7
2.60e7
2.56e7
2.52e7
2.50e7
2.44e7
10
9
8
7
6
5
4
3
2
3
4
5
6
7
h (cm)

Figure 2

2 3 4 5 6 7
h (cm)

Figure 2


Total cost of the power supply vs. flat-to-flat height of conductor
Total cost of magnet vs. flat-to-flat height of conductor
Cost, p.s. (millions $)
Cost, magnet (thousands $)
h (cm)
5
4
3
2
900
860
820
780
740
2
3
4
5
6
7

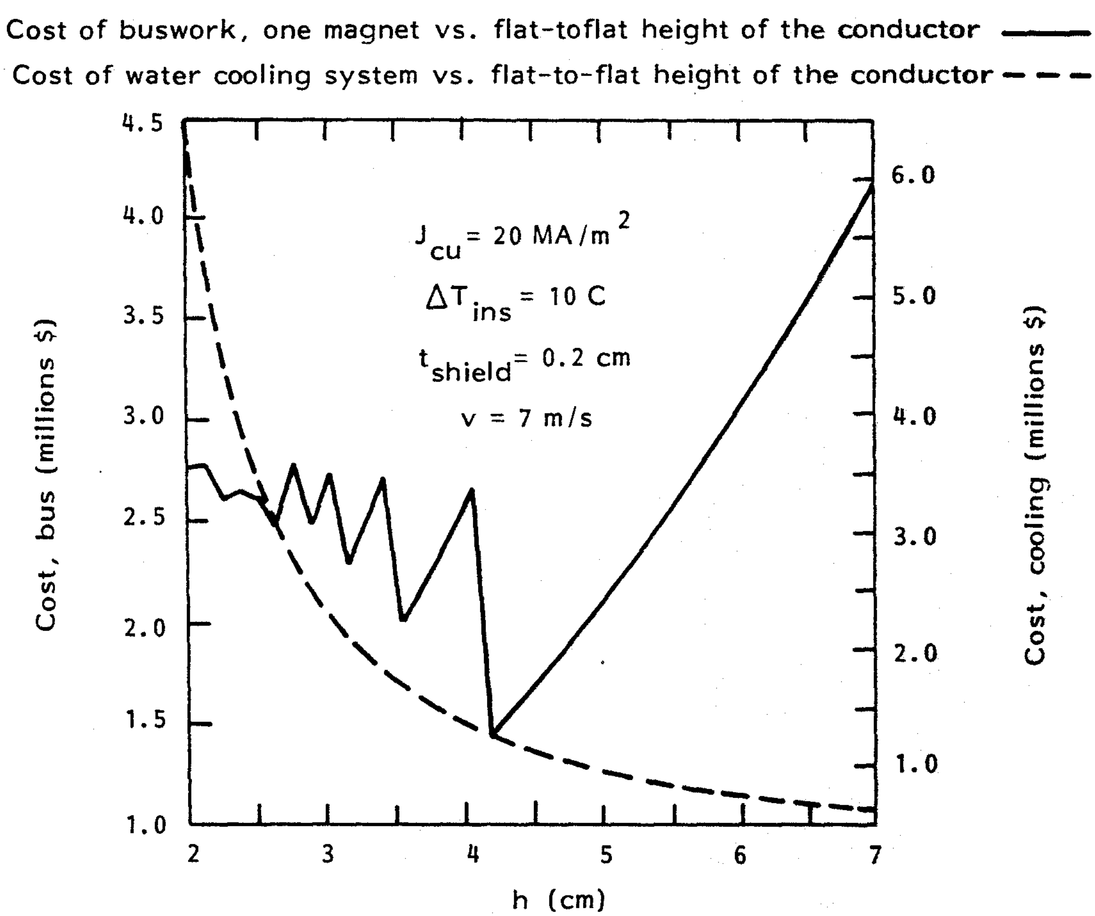
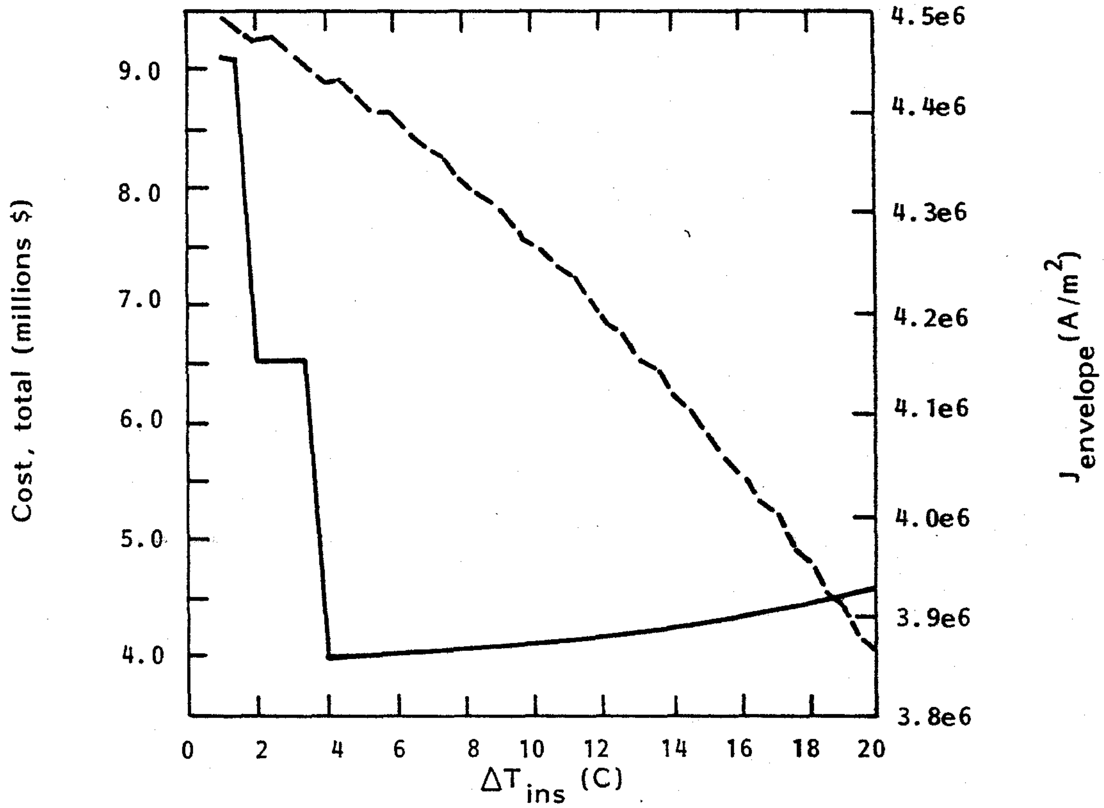

Cost of buswork, one magnet vs. flat-toflat height of the conductor
Cost of water cooling system vs. flat-to-flat height of the conductor
J_{cu} = 20 MA/m^2
ΔT_{ins} = 10 C
t_{shield} = 0.2 cm
v = 7 m/s
Cost, bus (millions $)
Cost, cooling (millions $)
h (cm)
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
6.0
5.0
4.0
3.0
2.0
1.0
2
3
4
5
6
7

Figure 3

Total cost of the bundle diverter magnet system vs. design temperature drop across the insulation ——
Current density over the envelope vs. design temperature drop across the insulation – – –



Total badness of the system vs. design temperature drop across the insulation ——
Total electrical power for the system vs. design temperature drop across the insulation – – –

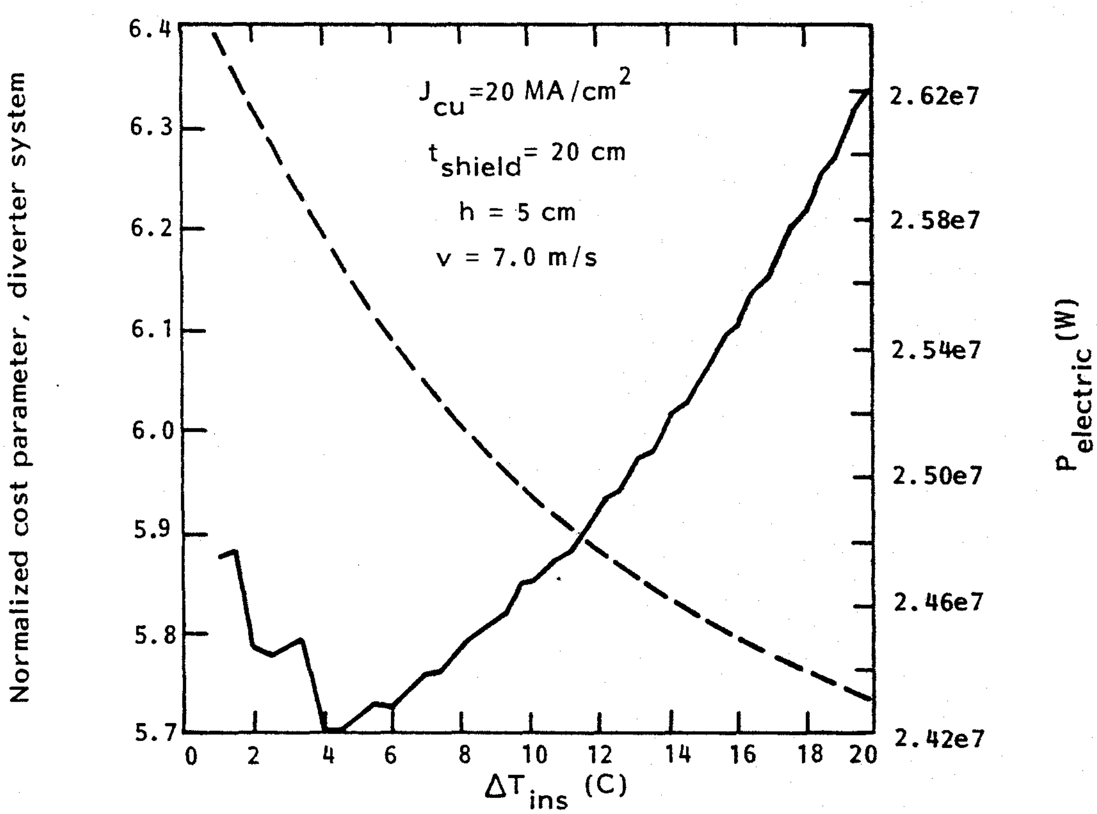
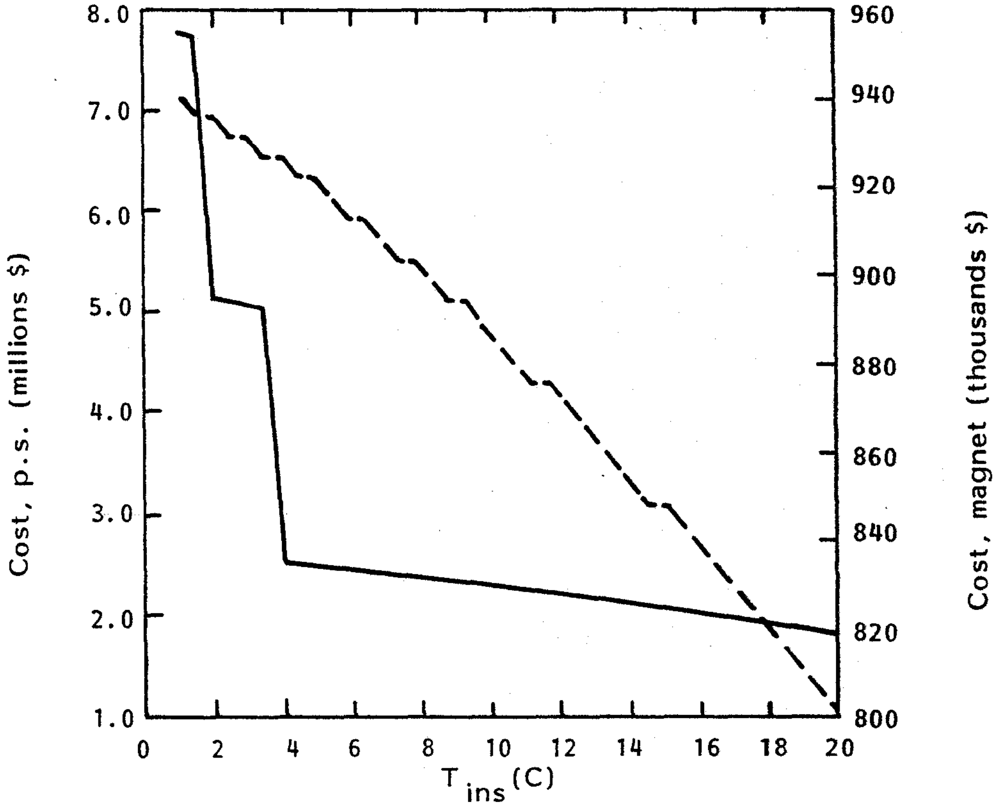


Figure 4

Total cost of the power supply vs. design temperature drop across the insulation

Total cost of the magnet vs. design temperature drop across the insulation



Peak coil terminal voltage vs. design temperature drop across the insulation

Cost of the water cooling system vs. design temperature drop across the insulation

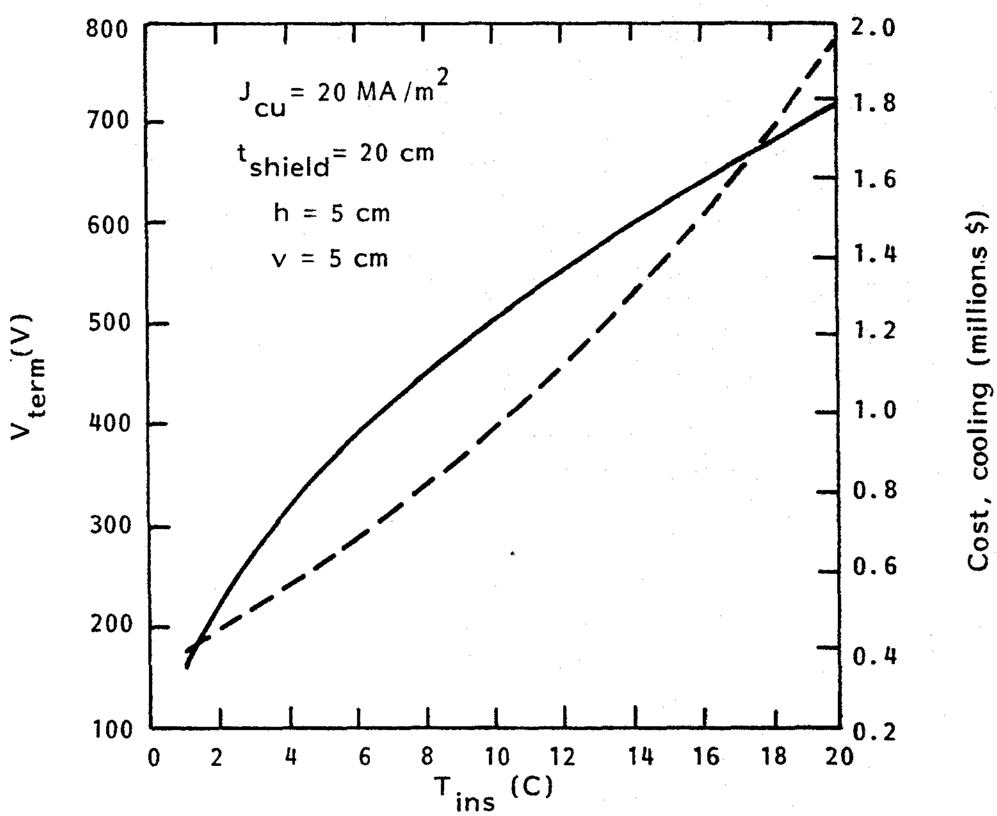
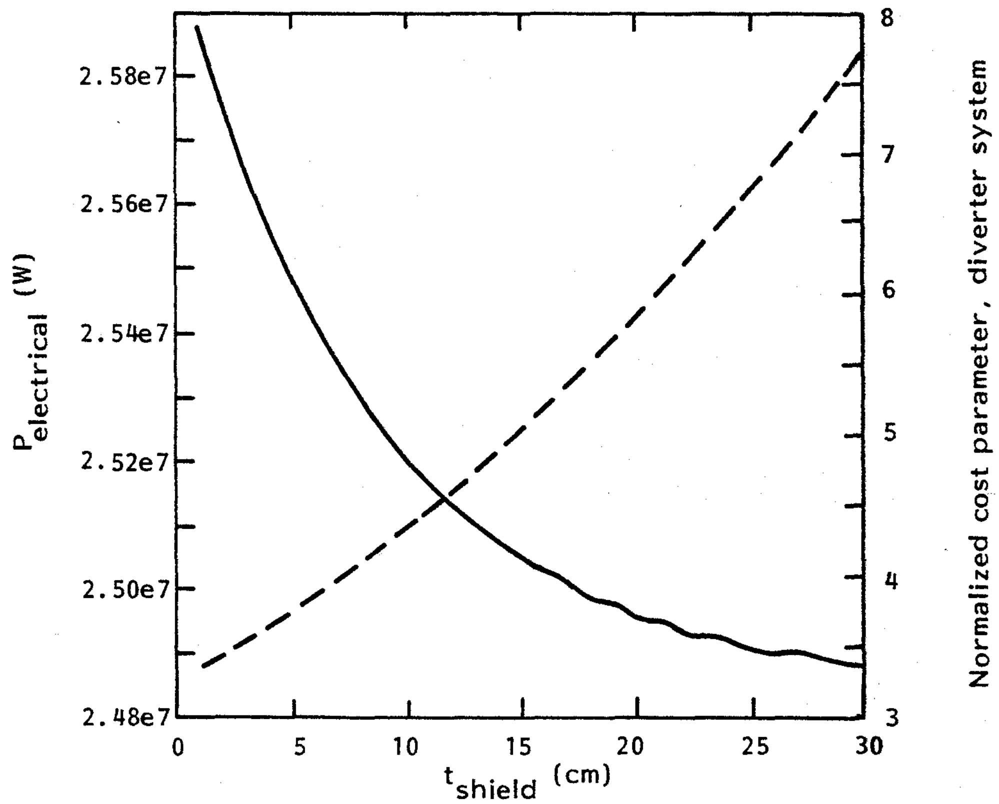


Figure 5

Total electrical power for the system vs. thickness of the shield ———
Total badness of the system vs. thickness of the shield — — —



Current density over the envelope vs. thickness of the shield

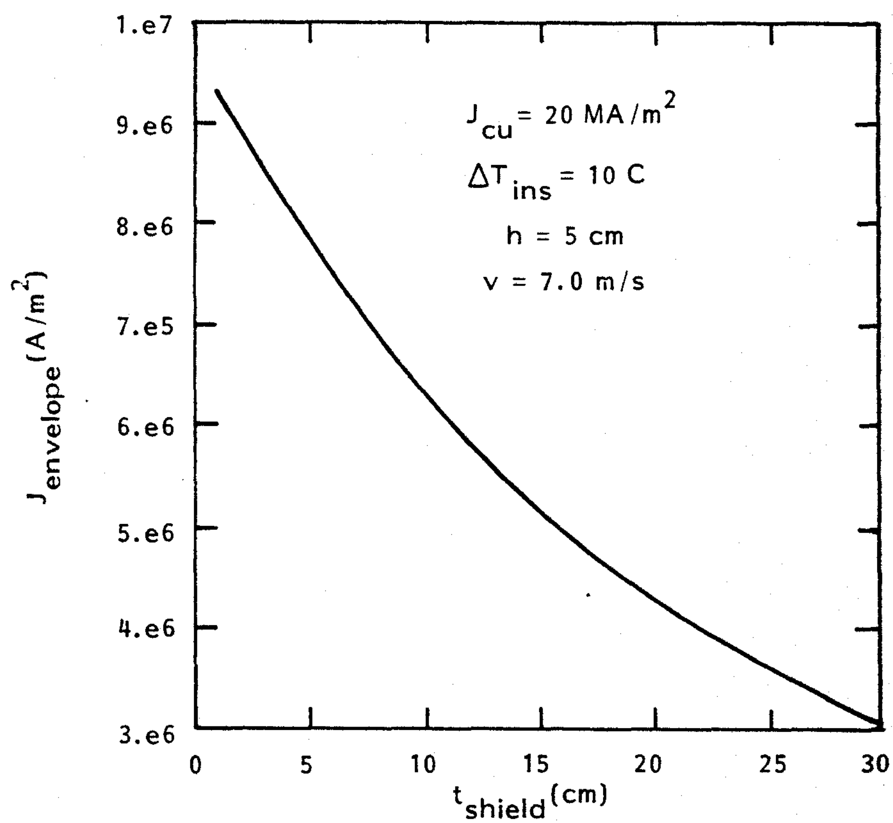
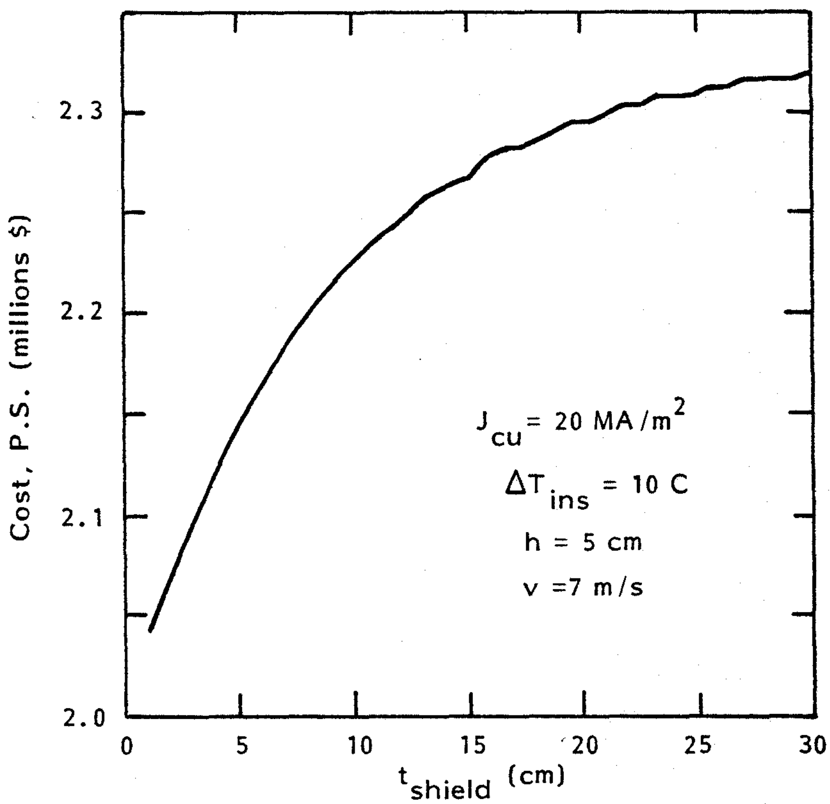


Figure 6

Total cost of the power supply vs. thickness of the shield



Total cost of the magnet vs. thickness of the shield

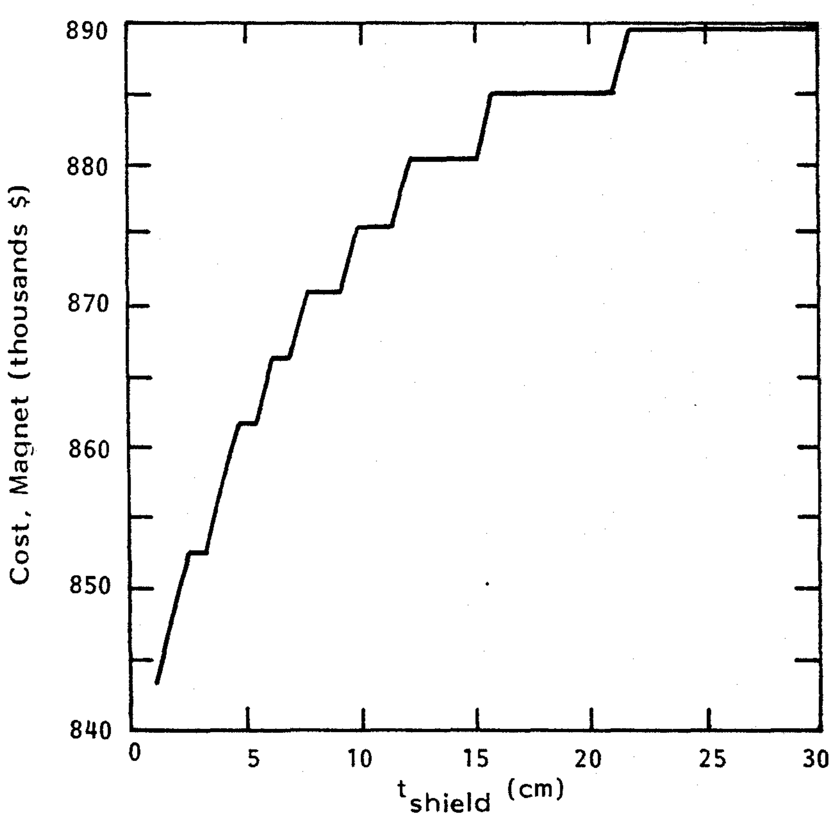
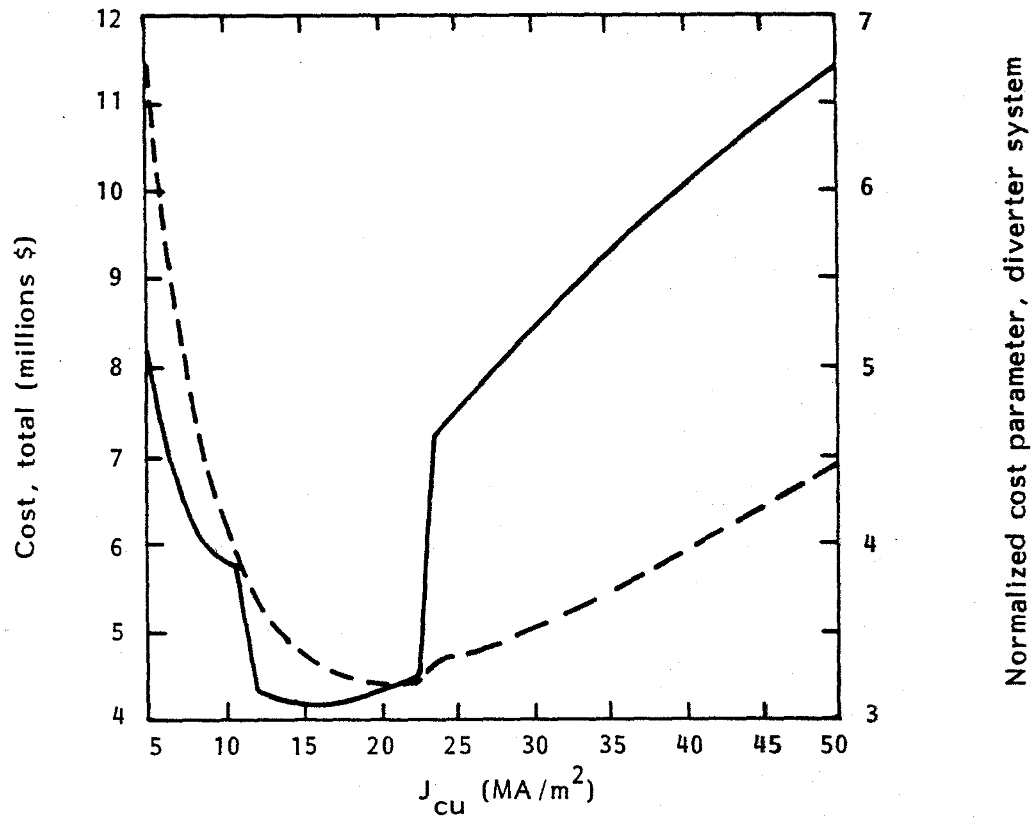


Figure 7

Total cost of the bundle diverter magnet system vs. current density in the copper ——

Total badness of the system vs. current density in the copper - - -



Current density over the envelope vs. current density in the copper ——

Total electrical power for the system vs. thickness of the shield - - -

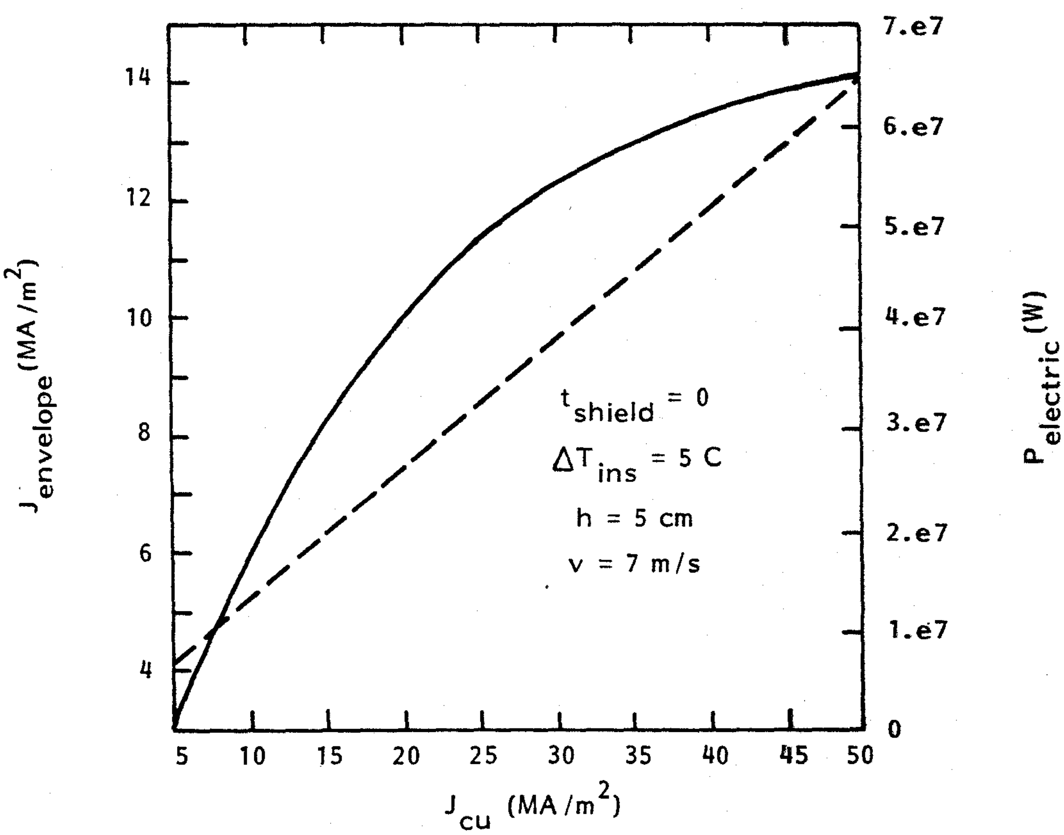
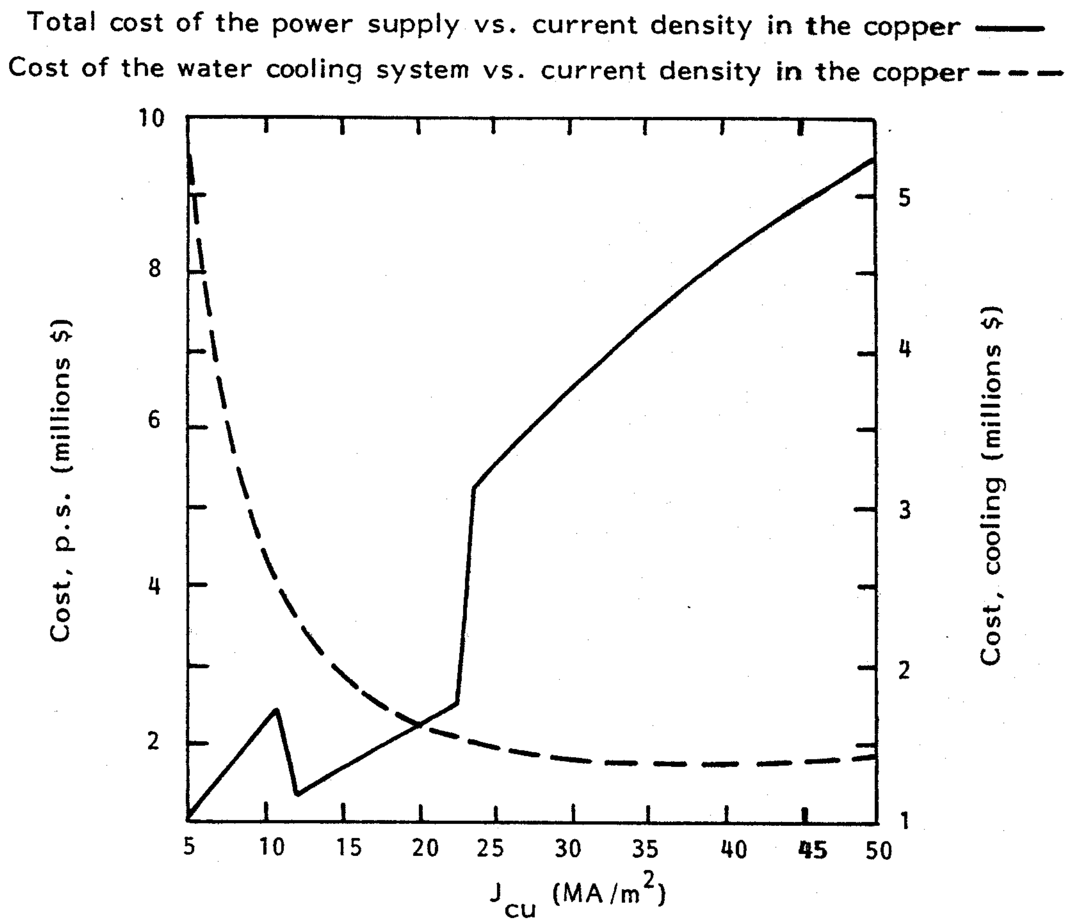
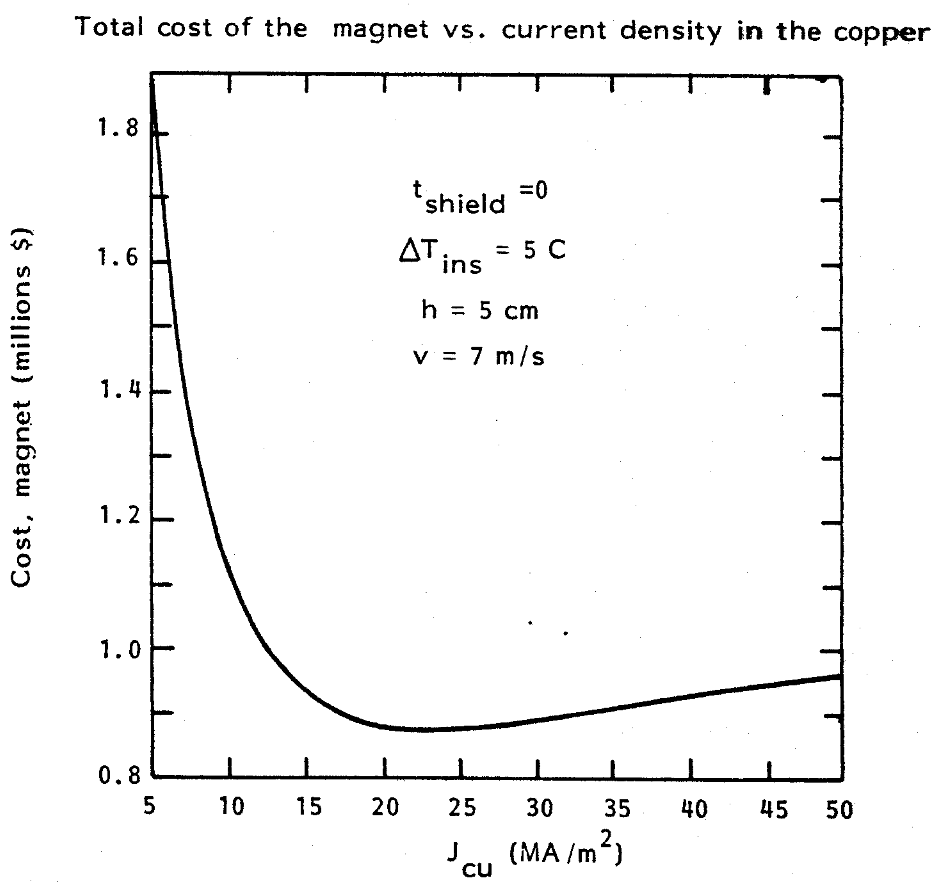


Figure 8



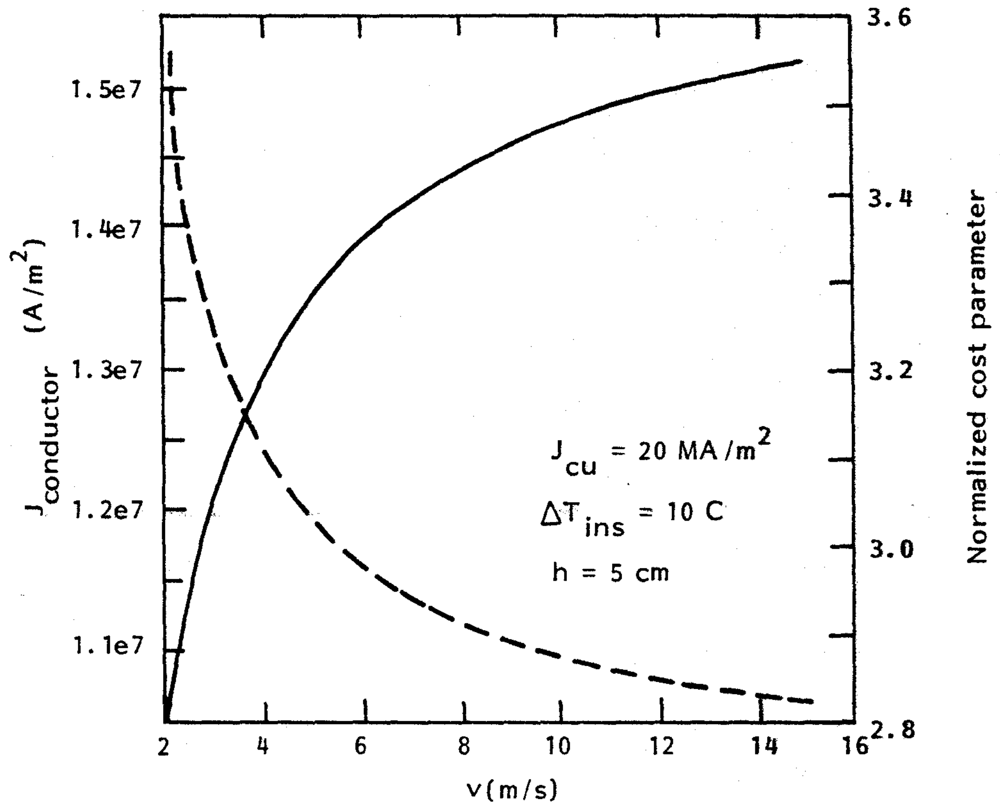
Total cost of the power supply vs. current density in the copper ——
Cost of the water cooling system vs. current density in the copper – – –
Cost, p.s. (millions $)
Cost, cooling (millions $)
2
4
6
8
10
1
2
3
4
5
5
10
15
20
25
30
35
40
45
50
J_{cu} (MA/m^2)



Total cost of the magnet vs. current density in the copper
Cost, magnet (millions $)
0.8
1.0
1.2
1.4
1.6
1.8
t_{shield} = 0
ΔT_{ins} = 5 C
h = 5 cm
v = 7 m/s
5
10
15
20
25
30
35
40
45
50
J_{cu} (MA/m^2)

Figure 9

Overall current density in the conductor and jacket vs. velocity of the coolant ——
Total badness of the system vs. velocity of the coolant water - - - -



Total cost of the bundle diverter magnet system vs. velocity of the coolant

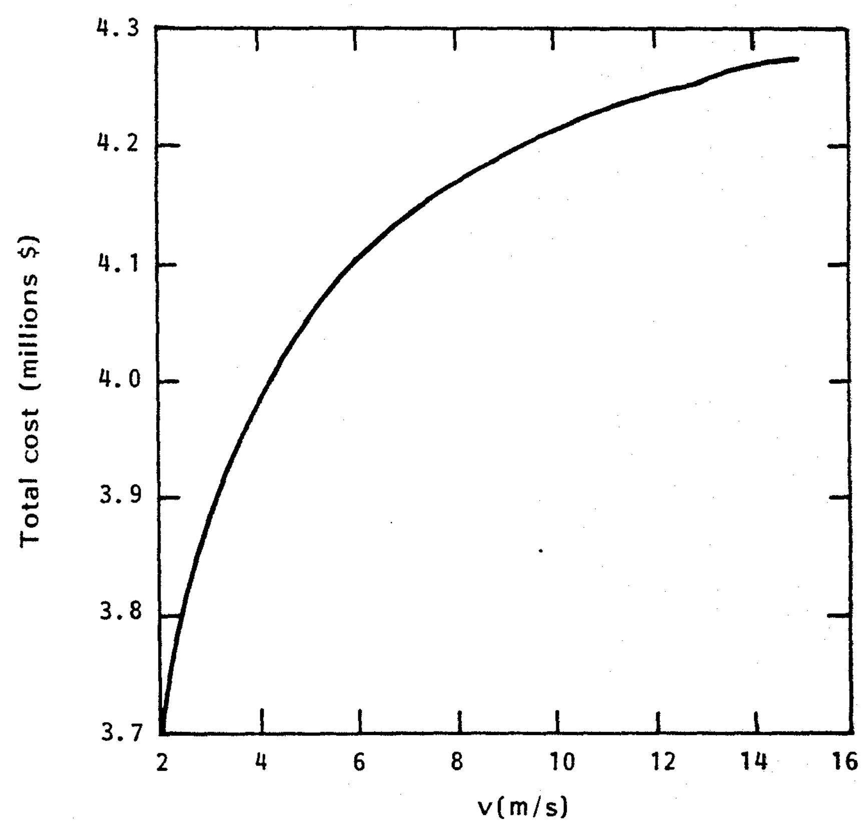


Figure 1

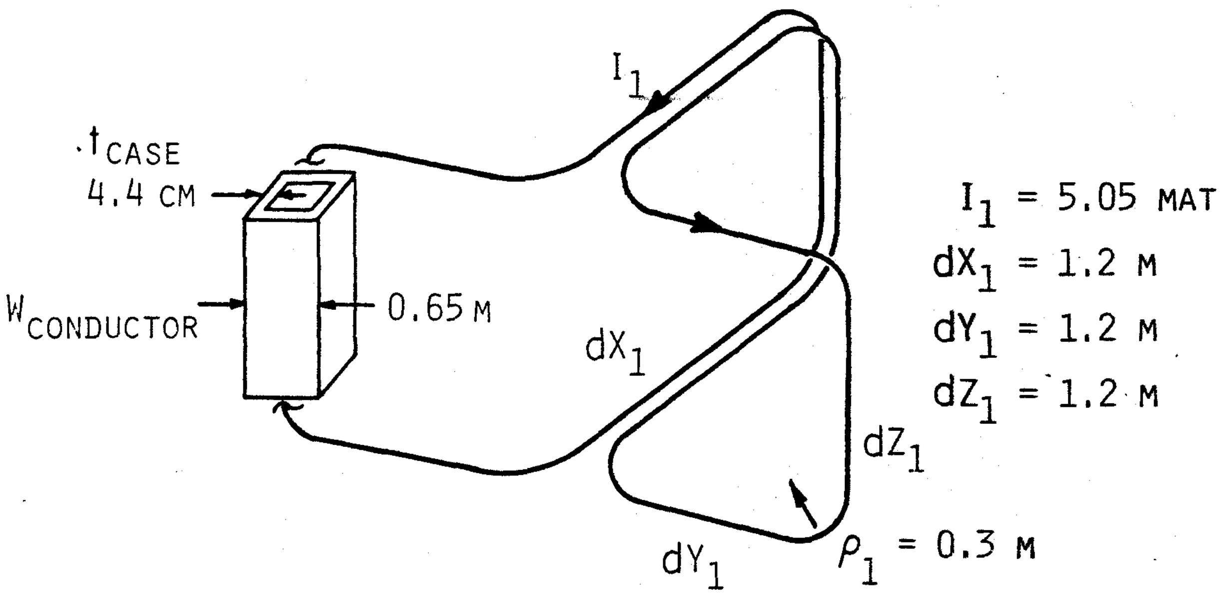


Figure 10
Front Bundle Diverter Coils in a Tokomak Reactor Example

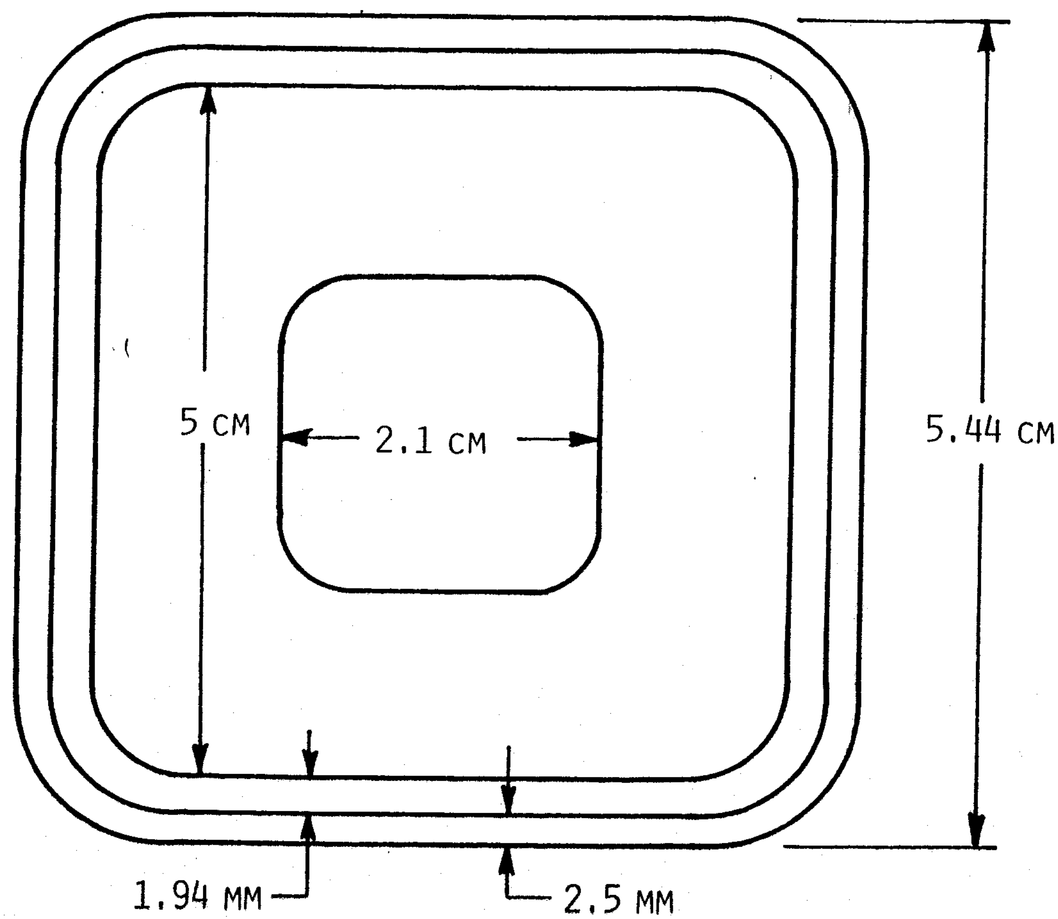


Figure 11
MgO Insulated, Jacketed Conductor for Reference Bundle Diverter Design