

PSFC/JA-02-15

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OF PLASMAS IN SPHERICAL TORI**

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October 2002

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To appear in *Proceedings of the 12th Joint Workshop on Electron Cyclotron Emission and Electron Cyclotron Resonance Heating*, Aix-en-Provence, France, May 13–16, 2002.

Electron Cyclotron Resonance Heating of Plasmas in Spherical Tori

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ABSTRACT

The conventional ordinary and extraordinary modes in the electron cyclotron range of frequencies are not suitable for heating of and/or driving currents in spherical tori (ST) plasmas. However, electron Bernstein waves offer an attractive possibility for heating and current drive in this range of frequencies. In this paper, we summarize our theoretical and numerical results which describe the excitation and propagation of electron Bernstein waves in ST plasmas.

INTRODUCTION

Plasmas in spherical tori, e.g. in MAST and NSTX, present a special experimental challenge when considering heating and current drive by waves in the electron cyclotron range of frequencies. This is primarily due to such plasmas being overdense, i.e., $\omega_{pe}/\omega_{ce} \gg 1$ where ω_{pe} and ω_{ce} are the electron plasma and electron cyclotron frequencies, respectively. For fundamental or second harmonic heating in such plasmas, the conventional X mode and O mode are cutoff near the edge of the plasma and cannot access the core. For higher harmonics the plasma is essentially transparent to the X and O modes.

Mode conversion near the plasma edge allows X or O mode polarized power, incident from free space, to couple to electron Bernstein waves (EBW). The X mode couples to EBWs in the vicinity of the upper hybrid resonance (UHR). The O mode coupling to EBWs is via the slow X mode whereby power from the externally excited O mode is first mode converted to the slow X mode which subsequently mode converts to EBWs near the UHR. The propagation of EBWs is not density limited and the waves damp effectively on electrons in the vicinity of the Doppler shifted electron cyclotron resonance (or its harmonics) [1]. In this paper we discuss theoretical details of the constraints imposed on the mode conversion of X and O modes to the EBWs in spherical tori. We also demonstrate interesting features of EBWs as they

propagate in the high magnetic shear geometry of spherical tori. For ST plasmas, the location of the EBW energy deposition is significantly different for excitations on or away from the equatorial plane [1].

EXCITATION AND PROPAGATION OF EBWs

The excitation of EBWs is studied in a slab geometry model where the x coordinate is in the direction of the inhomogeneity, y is along the poloidal direction, and z along the toroidal direction. Assuming that the magnetic field is along z , the propagation of waves in an inhomogeneous cold and collisionless plasma is given by

$$K_{\perp} \frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2} \left(K_{\perp}^2 - K_X^2 - \frac{c^2}{\omega^2} k_{\parallel}^2 K_{\perp} \right) E_y = k_{\parallel} K_X F \quad (1)$$

$$\frac{d}{dx} \left[\frac{1}{K_{\parallel}} \frac{dF}{dx} \right] + \frac{\omega^2}{c^2} \left(1 - \frac{c^2}{\omega^2} \frac{k_{\parallel}^2}{K_{\perp}} \right) F = \frac{\omega^2}{c^2} \frac{K_X}{K_{\perp}} k_{\parallel} E_y \quad (2)$$

where $F = ik_{\parallel} E_x - dE_z/dx$, $dF/dx = (\omega^2/c^2) K_{\parallel} E_z$, c is the speed of light, ω is the wave frequency, k_{\parallel} is the component of the wave vector \vec{k} along the magnetic field $\vec{B} = B_0(x)\hat{z}$,

$$K_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}, \quad K_X = -\frac{\omega_{ce}}{\omega} \left(\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \right), \quad K_{\parallel} = 1 - \frac{\omega_{pe}^2}{\omega^2} \quad (3)$$

ω_{pe} and ω_{ce} are the electron plasma and cyclotron angular frequencies, respectively.

For the case when $k_{\parallel} = 0$, Eqs. (1) and (2) are uncoupled and describe the propagation of the X mode and the O mode, respectively, across the magnetic field. The X mode equation (1) has a resonance at $K_{\perp} = 0$ corresponding to the UHR. Also Eq. (1) has two cutoffs corresponding to the zeros of $K_{\perp}^2 - K_X^2$. The resonance absorption of X mode power at the UHR corresponds to the power mode converted to the EBWs [1]. Since the O mode is completely decoupled from the X mode, if an O mode is excited at the edge there will be no resonance absorption. The power will be reflected back out at the O mode cutoff located at the position where $K_{\parallel} = 0$. In this analysis we have ignored the effect of temperature on the propagation of waves and assumed that there is no electron cyclotron resonance between the edge of the plasma and the cutoffs of the X and O modes. Thus, the only absorption is at the UHR.

If we assume that, for sufficiently small $n_{\parallel} = ck_{\parallel}/\omega$, only the X mode is excited at the plasma boundary, then the equation for X mode propagation is approximately given by

$$\left(K_{\perp} - n_{\parallel}^2\right) \frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2} \left[\left(K_{\perp} - n_{\parallel}^2\right)^2 - K_X^2 \right] E_y = 0. \quad (4)$$

This equation is obtained from (1) and (2) by assuming that $E_z \approx 0$. In this approximation the X mode resonance is no longer at $K_{\perp} = 0$ but at $K_{\perp} - n_{\parallel}^2 = 0$ which, for small n_{\parallel} , is a small correction. From an analogous analysis carried out in [2], we find that the fraction of the power on the X mode that is converted to EBWs is given by

$$C_{XB}(\eta, \phi) = 4e^{-\pi\eta} (1 - e^{-\pi\eta}) \cos^2(\phi) \quad (5)$$

where

$$\phi = \frac{\pi}{2} + \frac{1}{2} \cos^{-1} \left(\frac{N_R^2 - N_I^2}{N_R^2 + N_I^2} \right) \quad (6)$$

$$N = N_R + iN_I,$$

$$\begin{aligned} N &= \ln \left(\frac{i\eta}{\eta_1} \right) + \pi \left[i \coth \left(\frac{\pi}{2} \eta \right) + \cot \left(\frac{\pi}{2} \eta_1 \right) \right] - \frac{1}{\eta_1} - \frac{i}{\eta} \\ &+ \psi \left(1 + \frac{\eta_1}{2} \right) - \psi \left(1 + \frac{i\eta}{2} \right) \end{aligned} \quad (7)$$

ψ is the Psi function,

$$\eta = (1 - n_{\parallel}^2)^{3/2} \left[\frac{\omega_{ce} L_n}{c\alpha} (\sqrt{1 + \alpha^2} - 1)^2 \right]_{UHR} \quad (8)$$

$$\eta_1 = (1 - n_{\parallel}^2)^{3/2} \left[\frac{\omega_{ce} L_n}{c\alpha} (\sqrt{1 + \alpha^2} + 1)^2 \right]_{UHR}. \quad (9)$$

In Eqs. (8) and (9) the right-hand sides are evaluated at the UHR ($K_{\perp} = 0$), L_n is the density scalelength, and $\alpha = \omega_{pe}/\omega_{ce}$.

In Fig. 1(a) we plot

$$C_{XB}^{(max)} = 4e^{-\pi\eta} (1 - e^{-\pi\eta}) \quad (10)$$

as a function of η . This gives the envelope of (5), i.e., the maximum possible fractional power of the X mode that can be mode converted to EBWs.

Alternatively, from (5), $C_{XB}^{(max)} = C_{XB}(\eta, n\pi)$ where n is any integer including zero. For $\eta \approx 0.22$ $C_{XB}^{(max)} = 1$ which implies that all of the X mode power is converted to EBW. From Fig. 1(a) we note that $C_{XB}^{(max)} \geq 0.5$ if $0.05 \lesssim \eta \lesssim 0.6$. For $n_{\parallel} = 0$, from (8) we find that the necessary condition for the X-B mode conversion to be better than 50% requires

$$0.013 \text{ T cm} \lesssim |BL_n|_{UHR} \lesssim 0.16 \text{ T cm} \quad (11)$$

This shows that the X-B mode conversion process is efficient for sharp density gradients, i.e., short density scalelengths. As n_{\parallel} is increased from zero, the fraction of the X mode power that is coupled to EBWs decreases.

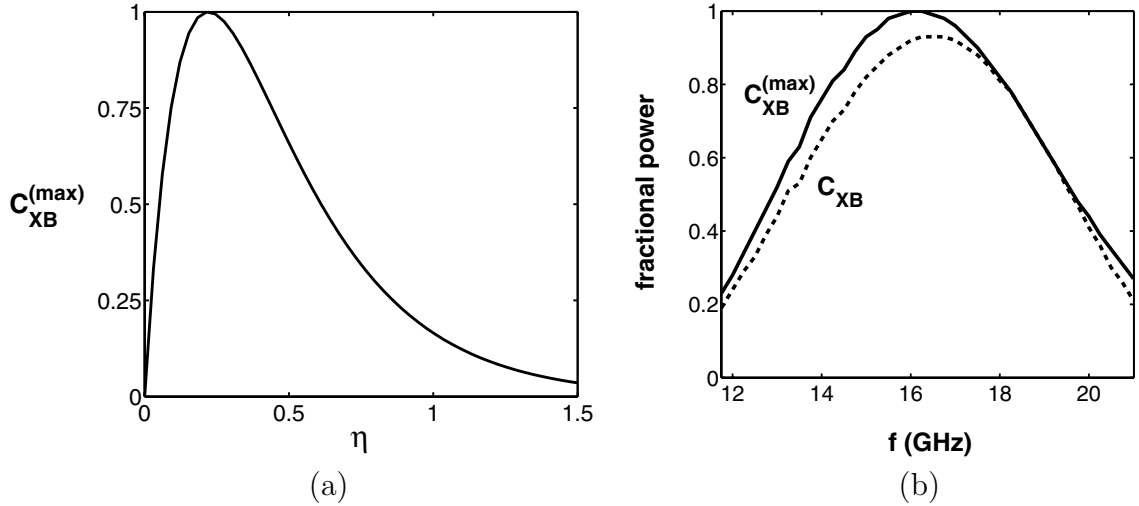


Figure 1: (a) $C_{XB}^{(max)}$ as a function of η ; (b) a comparison of $C_{XB}^{(max)}$ (solid line) and C_{XB} (dashed line) as a function of the X mode frequency from (5) for NSTX-type parameters [1, 4].

From Eq. (2) it is clear that for $k_{\parallel} = 0$ the O mode does not couple power to the EBW. For the O mode power to couple to EBW it is necessary to have oblique propagation of the O mode, i.e., $k_{\parallel} \neq 0$. Then the O mode power can couple to the X mode which in turn can undergo resonance absorption. The necessary condition for optimum coupling of the O mode power to the X mode is [3]

$$n_{\parallel,c} = \frac{ck_{\parallel}}{\omega} = \left[\left(1 + \frac{\omega}{\omega_{ce}} \right)^{-1/2} \right]_c \quad (12)$$

where the right-hand side is to be evaluated at the spatial location where $\omega = \omega_{pe}$. However, in order for the O mode power not to get reflected back out on the X mode requires that $\eta > 1$ [1, 4]. From (8) we find that this condition is satisfied when the density scalelength is long.

From the above discussion we conclude that the X-B and the O-X-B mode conversion processes are optimized in different regions of the two-dimensional parameter space spanned by wave frequency and parallel wavelength, and at different locations inside the plasma. The X-B mode conversion process is optimized for $k_{\parallel} \approx 0$ and wave frequencies which place the UHR in the short density scalelength region. The O-X-B mode conversion process is optimized for k_{\parallel} given in (12) and for wave frequencies which place the UHR in a longer density scalelength region of the plasma. The optimum X-B process occurs closer to the edge of the plasma than the optimum O-X-B conversion. A more detailed analysis based on a kinetic treatment of the EBW in sheared magnetic fields confirms the basic results of the analysis given above [1, 4].

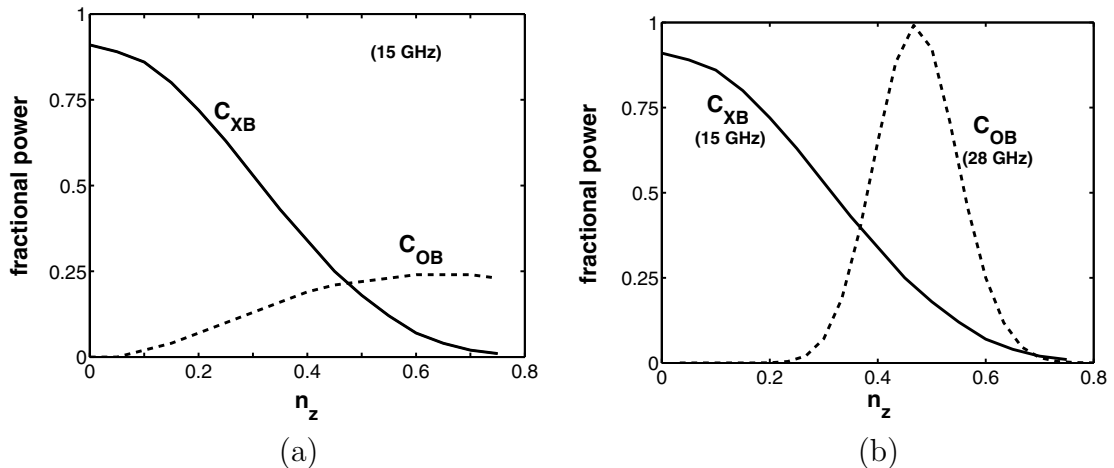


Figure 2: Fractional power mode converted to EBWs for an externally launched X mode, C_{XB} , or externally launched O mode, C_{OB} , as a function of n_z .

Figure 2 shows numerical results obtained from the kinetic formulation for NSTX-type parameters [1, 4]. Figure 2(a) shows the case when both the X mode and the O mode are excited at the same frequency of 15 GHz. For small n_{\parallel} ($n_{\parallel} \lesssim 0.1$) the results for C_{XB} are in good agreement with

the analytical result given by (5). However, for larger n_{\parallel} the agreement breaks down because the assumptions leading to (4) are no longer valid. The analytical result leads to a larger mode conversion fraction than the numerical results because the coupling to O mode, which occurs for non-zero k_{\parallel} , has not been properly accounted for in the analytical model. For the NSTX-type parameters used to carry out these simulations, the critical $n_{\parallel,c}$ from (12) is approximately 0.58. At this value of $n_{\parallel,c}$ the O-X-B mode conversion coefficient C_{OB} is approximately 0.25. This is well below the maximum of 100% conversion efficiency because for these parameters $\eta \approx 0.08$ and does not satisfy the additional condition requiring $\eta > 1$ for optimum O-X-B mode conversion. Figure 2(b) compares the case of mode conversion from the X mode, for a wave frequency of 15 GHz, to the mode conversion from the O mode for a wave frequency of 28 GHz. It is evident from these figures that the mode conversion to EBWs of the X mode and of the O mode are optimized for different ranges of the parameter space spanned by the wave frequency and parallel wave numbers. In the case of O-X-B conversion for a frequency of 28 GHz, the critical $n_{\parallel,c} \approx 0.48$. From (8) we find, that for this case, $\eta \approx 2$.

PROPAGATION AND DAMPING OF EBWs

We have also studied the propagation of EBW rays in model NSTX profiles. The ray trajectories are obtained in a fully kinetic (Maxwellian), non-relativistic, plasma. We find that the EBW damps locally at the Doppler-shifted electron cyclotron resonance or its harmonics. The k_{\parallel} along the ray trajectories vary over a large range if the rays are launched off the equatorial plane. This is due to the effect of a significant poloidal magnetic field [5].

In Fig. 3(a) we plot the poloidal projection of two electron Bernstein rays which differ, initially, only in their starting location. One ray (solid line) starts off on the equatorial plane and the other 18° above the equatorial plane. Both rays are followed until their powers are fully damped on the electrons. The ray started on the equatorial plane propagates close to the second electron cyclotron resonance before damping. The n_{\parallel} along this ray does not change significantly from its initial value as shown in Fig. 3(b). However, the ray started above the equatorial plane does not propagate very far into the plasma before damping on electrons. This, as is clear from Fig. 3(b) is due to the significant upshift in $|n_{\parallel}|$ leading to a large Doppler shift of

the second harmonic of the electron cyclotron resonance. Another aspect to keep in mind is that these ray trajectories are for a high- β scenario of a NSTX plasma [1]. The total magnetic field profile does not vary considerably in the outer half of the plasma. In spite of the large increases in k_{\parallel} the damping on electrons is still due to the Doppler-shifted cyclotron resonance rather than Landau damping. The change in n_{\parallel} is proportional to $\sin(\theta)B_{\theta}/|B|$ where θ is the poloidal angle ($\theta = 0$ being the outside on the equatorial plane) and B_{θ} is the poloidal magnetic field [5]. In an ST plasma B_{θ} and $|B|$ are comparable towards the outer edge of the plasma. This leads to significant changes in n_{\parallel} for EBW ray trajectories started off the equatorial plane. From Fig. 3 it is evident that the plasma region in which the damping of the EBW is desired, either for heating or for driving plasma currents, can be controlled by an appropriate launching angle in the poloidal plane. This will be investigated further in the near future.

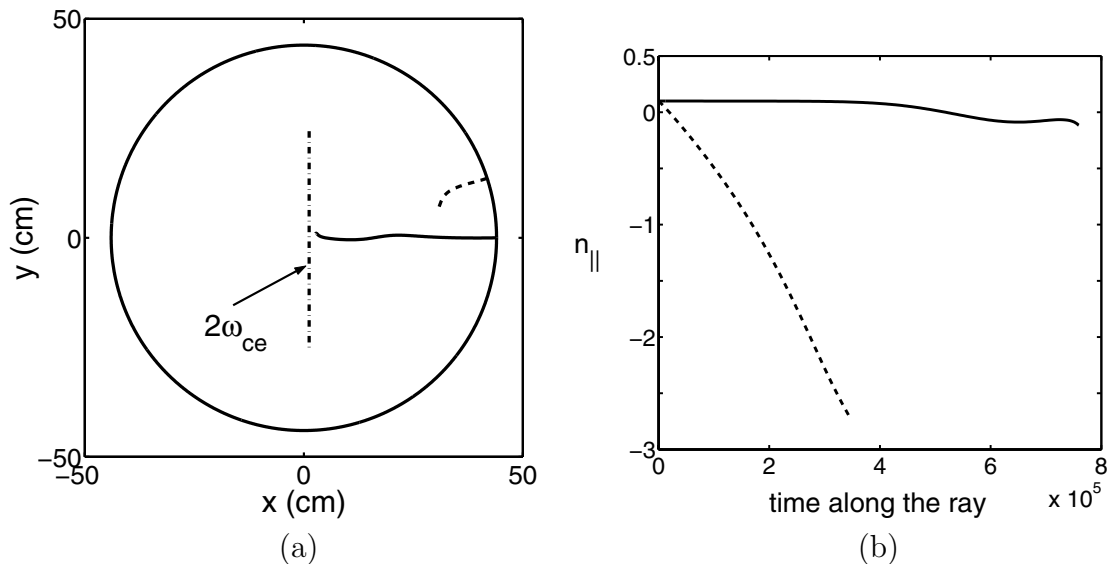


Figure 3: Ray characteristics for NSTX-type parameters and wave frequency of 15 GHz; (a) the poloidal projection of two EB rays started with the same initial conditions except that one ray is started on the equatorial plane (solid line) and the other 18° above the equatorial plane (dashed line); (b) the evolution of n_{\parallel} along the two rays which initially were started with $n_{\parallel 0} = 0.1$.

ACKNOWLEDGEMENTS

This work is supported by Department of Energy Contracts DE-FG02-91ER-54109 and DE-FG02-99ER-54521.

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