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Forces on a Small Grain in the Nonlinear Plasma Wake of Another

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Forces on a small grain in the nonlinear plasma wake of another

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The transverse force on a spherical charged grain lying in the plasma wake of another grain is analysed with a view to assessing the importance of ion-drag perturbation, in addition to the wake-potential-gradient. It is shown that the ion-drag perturbation is intrinsically one order smaller than the wake-potential force in the ratio of grain size (r_p) to Debye length (λ_{De}). So ion-drag perturbation is important only in nonlinear wakes. Rigorous particle-in-cell calculations of the force are performed in the non-linear regime with two interacting grains. It is found that even for quite large grains, $r_p/\lambda_{De} = 0.1$, the force is dominated by the wake-potential-gradient. The wake potential structure can then help explain the preferred alignment of floating dust grains.

INTRODUCTION

A finite-sized solid grain in the flowing-plasma wake of another, the 'upstream-grain', is known experimentally to experience forces that are substantially different from those in the unperturbed plasma flow. In addition to the mutually repulsive electric inter-grain force arising from both of them being negatively charged, the wake gives rise to additional electric fields, attributable to the perturbation of charge density in the plasma. This wake-field can act in transverse directions, and in various circumstances cause either alignment[1, 2] or misalignment[3–6]of the two grains in the flow. The linear theoretical potential structure has been used[7] to explain some of these observations. However, the ion drag forces on the grains, arising from the flow of ions, are also an important part of their force balance. A long-standing suggestion[8] is that the perturbation to the ion drag force may be more important than the potential structure of the wake, in determining the equilibrium and dynamics of the wake-grain. The purpose of the present investigation is to answer the question: under what circumstances are the forces on the wake grain adequately represented by the total wake electric-field, and conversely when is the ion-drag perturbation an important effect?

The qualitative answer to this question is shown to be that ion-drag perturbation is important only in the regime in which non-linearity of the wake is important. Particle simulation in the non-linear regime is then used to compare the actual total force experienced by the wake-grain with the electric field force that would arise purely from the wake potential structure of the upstream-grain by itself.

The extent to which the wake of a floating upstream-grain is nonlinear is determined by the ratio of its radius r_p (assumed spherical) to the electron Debye length λ_{De}. This is because the floating potential is generally $\phi_p \sim 2-4$ times $-T_e/e$ (with T_e the electron temperature), so that the charge on the grain $Q \approx 4\pi\epsilon_0 r_p \phi_p$ (when $r_p/\lambda_{De} \ll 1$) is proportional to its radius. One can therefore take the grain's normalized charge, $\bar{Q} \equiv Q/(4\pi\epsilon_0\lambda_{De}T_e/e) \approx (e\phi_p/T_e)(r_p/\lambda_{De})$, as a proxy for the normalized size of the grain. The ions will be considered singly charged so unperturbed electron and ion densities are equal: $n_e = n_i$. The external ion drift velocity past the grains will be expressed as a Mach number $M = v_d/c_s$ normalized to the (cold-ion) sound speed $c_s = \sqrt{T_e/m_i}$. We presume $M \sim 1$, as is the case in experiments near a plasma sheath. The ion temperature is by supposition $T_i \ll T_e$. Using the standard Coulomb cut-off integration, ignoring the thermal ion velocity, the ion drag (F_d) on a small grain is in normalized units

$$\frac{F_d}{T_e n_e \lambda_{De}^2} \approx \left(\frac{\bar{Q}^2}{M^2}\right) 4\pi \ln\Lambda. \tag{1}$$

In the regime of interest $\Lambda \sim 1 + M^2/|\bar{Q}|$ [9] is bigger than one but not a very large quantity; so $\ln\Lambda$ is at most 'a few'. This contrasts with the situation of classical Coulomb drag on plasma particles, in which Λ for elementary charges is very large. The charge, Q, on even a small dust grain can be thousands of times e. [Note that by definition $n_e T_e \lambda_{De}^2 = \epsilon_0(T_e/e)^2$. Justification of the precise $\ln\Lambda$ value in eq. (1) requires detailed discussion [10].]

The grain gives rise to a wake potential structure whose magnitude is proportional to charge Q in the linear regime, where in fact the peak wake potential (for $M \sim 1$) is [11] $\phi_{\max} \approx 2Q/(4\pi\epsilon_0\lambda_{De}) \approx 2\phi_p r_p/\lambda_{De}$. Fig. 1 illustrates the wake potential structure.

The wake scale length in the radial direction, transverse to the background flow, is approximately λ_{De}, so the (maximum) transverse force ($F_{w\phi}$) that would arise from the wake potential gradient acting on a grain of the same charge can be expressed in dimensionless form as

$$\frac{F_{w\phi}}{(T_e n_e \lambda_{De}^2)} = -Q\frac{\partial\phi}{\partial r}\frac{1}{(T_e n_e \lambda_{De}^2)} \sim \bar{Q}^2 8\pi. \tag{2}$$

Consequently, the ratio of the transverse wake-field-force on a wake-grain to the (total) drag force on the upstream-grain is

$$F_{w\phi}/F_d \sim 2M^2/\ln\Lambda, \tag{3}$$

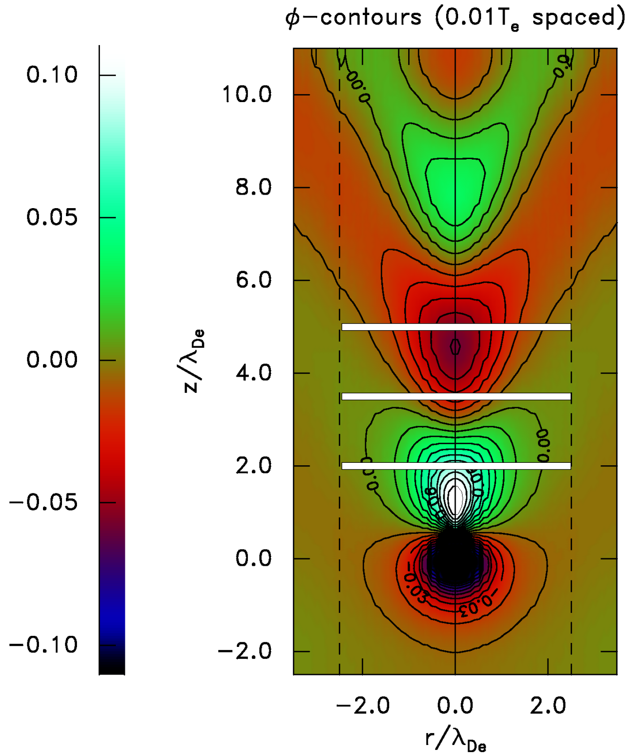

FIG. 1. Contours of the distribution of potential in units of T_e/e, in the wake of a point grain of normalized charge $\bar{Q} = 0.1$. The background flow is $M = 1$ in the $+z$-direction.

which is of order unity (in the regime of interest). The transverse wake field and the drag force are the same order of magnitude because the drag force can be considered to arise (mostly) from the force exerted upon the upstream-grain by the ions focussed in its wake. The drag force thus arises physically from the *same* source (the charge arising from wake densification of ions) as the transverse wake-field force, and is evaluated at roughly the same distance (for $M \sim 1$) from its source.

The *total* ion drag force on a wake grain is approximately the same as on the upstream-grain when they have equal charges. The transverse component of this ion drag force, thus rivals the transverse wake potential force only if the *direction* of the wake drag force is at a large angle to the original flow direction.

Of course, for a free grain in the unperturbed plasma, the force-balance *parallel* to the background flow requires ion drag to balance the background electric field, gravity and other forces if present. This balance is what determines the height at which a dust grain floats in a plasma sheath, for example. Then the *change* in the parallel drag force, arising from wake perturbation, is still intrinsically a factor (r_p/λ_{De}) smaller than the equilibrium value; whereas the wake potential force is of the same order as the drag force. [The enhancement of the ion density precisely in the wake focus can be very large[11], even when the wake potential is modest. So even though by ordering the change in parallel wake ion drag is smaller than the wake potential force, drag changes may still be significant precisely at the focus.]

In summary, for *transverse* force balance or changes to *parallel* force on the wake-grain, the ion drag component is important only in the non-linear regime, i.e. only to the extent that (r_p/λ_{De}) is not negligible.

Direct numerical calculation of the dynamics of a wake in a collisionless plasma requires the full non-thermal ion distribution function to be tracked. It lends itself to Particle in Cell (PIC) simulation. The results here are obtained with the fully 3-dimensional "Cartesian-mesh, Oblique-boundary Particles and Thermals In Cell" (COPTIC) code, described in [11]. Electron density is presumed to be governed by Boltzmann distribution $n_e = n_{e\infty}\exp(e\phi/T_e)$ and ions are represented by particles accelerated by the self-consistent electric field and advanced in time and space by a standard leap-frog scheme, until a steady state is reached. This typically involves moving 30-200 million ions for about 1500 timesteps. The calculations here are collisionless.

To compare the wake potential gradient force with the total force, the wake potential from a single grain is first calculated. Then the code is run with two grains having the same charge, with the second located at some fixed position in the wake of the first. The result of a single-grain simulation is illustrated in Fig. 1. The regions in which the transverse wake force will be explored by introducing the second charge are marked with white bands.

The force on the grains is calculated in the manner described previously [10]. The Maxwell stress tensor and the electron pressure are integrated over a spherical surface, surrounding the grain whose force is to be obtained. Also the momentum flux of the ions crossing that sphere (both inward and outward) is accumulated and time-averaged in steady state. The total force is the sum of the three components: Maxwell stress, electron pressure, ion momentum flux. We perform this calculation for different (nested) spheres and observe that the totals (though not the individual components) are the same (when the simulation is converged).

Other PIC simulations of multiple grains have recently been published. Ikkurthi et al[12] report the transverse force only when the the grains are adjacent to one another, not in each other's wake. They appear also to omit the electron pressure force. Miloch et al[13] treat a grain truly in the wake of another but obtain the field force from a heuristic integration of inter-particle forces over a somewhat ill-defined region. Their results show some of the qualitative features of the present calculations, but do not appear to be quantitatively rigorous. Both of these other works address changes in the floating potential of the wake-grain arising by ion focusing caused by the upstream grain. The present work, rather than requiring the grains to float, simply prescribes their potential (or equivalently charge). To the best of my knowledge these are the first rigorous calculations of the wake forces in this regime directly from multiple-grain PIC simulation.

The calculations use uniform external drifting-Maxwellian ion distributions with $T_i/T_e = 0.01$. COPTIC can accommodate finite-sized objects (that absorb ions) or fixed point charges treated by a PPPM technique[14] to retain orbital accuracy, for which there is therefore no "direct ion collection" flux. Fig. 2 shows

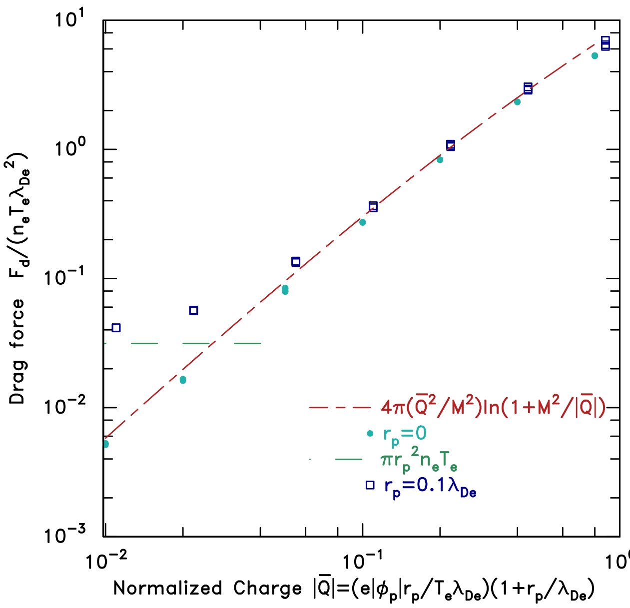

FIG. 2. Longitudinal force on a single isolated grain as a function of charge magnitude obtained by the COPTIC code. Two sets of simulations are shown, both with drift velocity $M = 1$. One has finite grain radius $r_p = 0.1\lambda_{De}$ (squares) and one has point charges (points, $r_p = 0$). The normalized charge is taken as $|\bar{Q}| = |\phi_p|r_p(1 + r_p/\lambda_{De})$.

calculations of the longitudinal (z-direction) drag force for isolated single grains with a range of charge and both finite-radius and infinitesimal radius. For each case, three points are actually plotted, corresponding to the force measurement on different spheres of radius between 0.2 and 0.5 times λ_{De}. These points coincide on the plot to within the size of the marker, giving an indication of how well the simulation is converged, and uncertainty from measurement noise (a few percent).

At large charge, the finite size of the grain is relatively unimportant. A force depending only on charge is obtained. This is consistent with the studies[11] which show that there is little difference in the wake fields between a point charge and a spherical object of finite radius even as large as $\lambda_{De}/10$. As the potential on the finite-radius grain drops below T_e/e, however, a saturation of the force is observed at a level approximately equal to the unperturbed momentum flux to the grain ($\pi r_p^2 n_e T_e$ at this velocity). For point grains this saturation does not occur, the force is found to be about 10% below the value of eq. (1), which is within the theoretical uncertainty in that expression. Simulation systematic uncertainty may also be up to 10%.

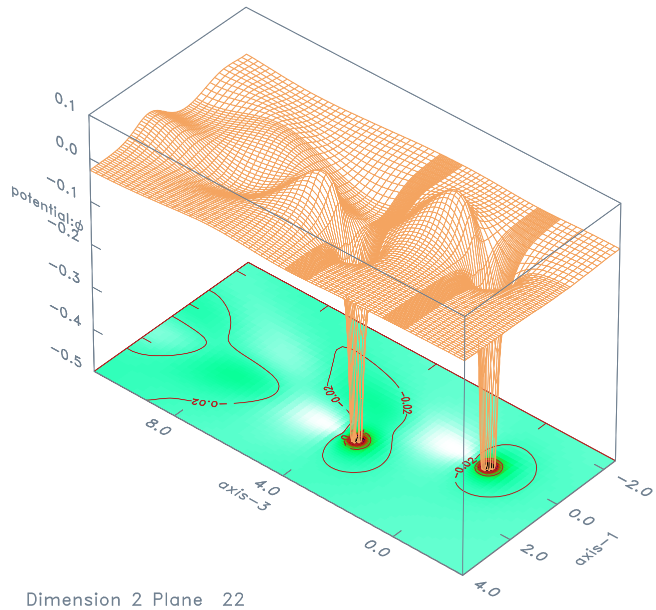

FIG. 3. Potential of two point charges of normalized value $Q/(4\pi\epsilon_0\lambda_{De}T_e/e) = -0.1$, in a flow $M = 1$ in the positive z-direction (axis-3). A two-dimensional slice through the 3-D domain at $y = 0$ is contoured and rendered. Charges are at positions (0,0,0) and (1.5,0,3.5) in units of λ_{De}. The deep negative potential wells around each charge are truncated only for visibility. Potential is in normalized units of T_e/e.

The potential obtained in a simulation with two charges is illustrated in Fig. 3. Non-uniform mesh spacing is used so as to obtain good resolution of the potential in the important regions without excessive mesh size ($64 \times 42 \times 100$ over a total domain of $6.5 \times 5 \times 13.5\lambda_{De}$). Both grains must be a distance of at least $2\lambda_{De}$ away from the mesh boundary to prevent spurious image forces caused by the potential boundary condition. In this case the wake grain is far enough downstream from the leading grain that it is adjacent to an axial potential valley, rather than the potential peak that immediately trails the leading grain.

Fig. 4 shows results, from a large number of runs like Fig. 3, for the transverse (x-direction) force on a point charge of normalized magnitude $Q/(4\pi\epsilon_0\lambda_{De}T_e/e) = -0.1$. This corresponds to the central point in Fig. 2 whose total (z) drag force is $0.27n_eT_e\lambda_{De}^2$. The individual points are obtained directly from code runs with two charges of equal magnitude. One charge is at the origin and is the cause of the wake. The second, whose transverse force is measured by integration over a sphere surrounding it, is placed in the wake of the upstream charge, at different distances z downstream, plotted against its

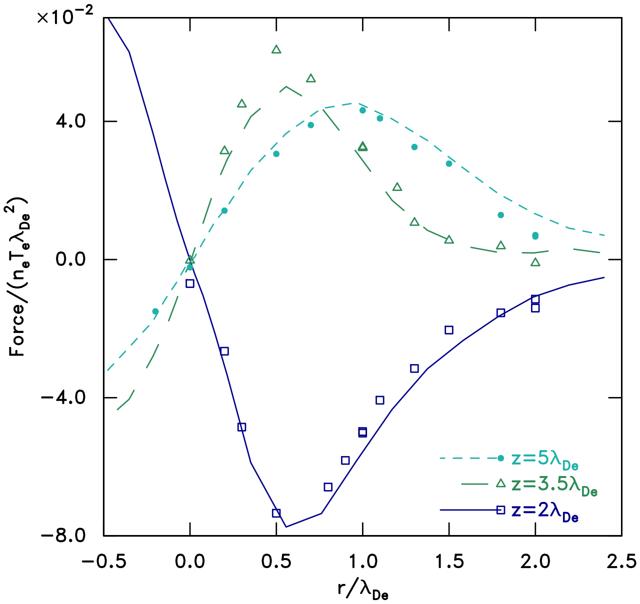

FIG. 4. Transverse force on a charge of normalized value -0.1 in the wake of an equal charge. The downstream distance is $z = 2,\ 3.5,\ 5\lambda_{De}$ and the transverse position is r. Points: 2-grain direct simulation. Lines: pure electric wake-field-force from 1-grain simulation.

radial offset distance (r, actually equal to x) from the axis of the wake. In addition, the lines on this plot, show for the corresponding cases the pure electric field force $F = -Q\nabla\phi$ that would be exerted on a charge of this magnitude by the potential gradient, at the position of the downstream charge, if the potential were just what exists with a single upstream charge present. The lines are thus the electric wake-field-force ignoring the perturbation by the downstream grain and any transverse ion flow drag it experiences.

At $z = 2\lambda_{De}$ the wake grain is adjacent to the primary potential peak of the leading grain's wake. It is consequently *attracted* (since its charge is negative) in the transverse direction towards the axis of the wake, the potential peak. It experiences a negative force at positive r and a stable equilibrium at $r = 0$. (We here are supposing that other forces constrain the grain's z-position to remain fixed.)

By contrast at $z = 3.5\lambda_{De}$ and $z = 5\lambda_{De}$ the wake grain is adjacent to a potential valley, and is *repelled* from the wake axis. The equilibrium at $r = 0$ is then unstable and the grain would be expected to find a stable equilibrium position where the force again crosses zero. This occurs near $r = 2\lambda_{De}$, although the simulation is not accurate enough to detect that zero-crossing unambiguously. Such observations appear to be capable of explaining the oblique alignment of grains at sufficient vertical separation[4, 11].

The total force obtained with two grain simulations (points) is slightly larger than the one-grain wake-field-force (lines) for $z/\lambda_{De} = 3.5$, by of order 20% at the peak. But differences are within uncertainties otherwise. These results thus contradict the factor-of-ten discrepancies reported in [8], even though the present simulations are at comparable parameters, substantially into the nonlinear regime. [Forces in Ref [8] were not derived from direct measurement in the code. They were obtained by applying analytic formulas using the code's flow.]

For smaller values of the non-linearity parameter $\phi_p r_p/\lambda_{De}$, as argued above, the discrepancy between total force and wake-field-force should be proportionally even smaller. The rapid decrease of the total force approximately $\propto Q^2$ causes the transverse force in the code to be overwhelmed by numerical noise before the fully linear regime is reached. So comprehensive computational demonstration of the transition to linearity of the force has not yet been obtained.

In summary, the force arising from flow perturbation is at most a small fraction of the total transverse force even in regimes with quite strong non-linear saturation of the wake field. Therefore in experiments with grains much smaller than the Debye length, it is a reasonable approximation to take the transverse force to be given by the wake potential gradient alone.

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