CREATING STABLE TOKAMAK REACTOR EQUILIBRIA BY SUPPLEMENTAL HEATING*

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ABSTRACT

We show that a temperature dependent supplemental heating can be used to prevent thermal runaway in a tokamak reactor by creating stable equilibria at desired operating points. We use HFCTR as an example, and show that we can obtain large Q (>20) at modest temperatures (< 13 Kev) using power inputs on the order of 3 - 30 MW.

Power balance studies [1-5] of tokamak reactors indicate the existence of thermal instability at the ignition point (for a given density, the lowest temperature at which the alpha-particle heating balances all of the losses.)

Various remedies to provide stable operation in the vicinity of the ignition temperature have been proposed, such as: rescaling the losses to improve stability at equilibrium [2]; magnetic ripple to deconfine the alpha particles [6]; periodic compression and decompression to form a cycle around an equilibrium point [3]; and using active burn control to avoid reaching ignition by on/off cycle of the supplementary heating source [4].

We propose in this Letter a novel method of heating a tokamak with temperature dependent supplemental power both to create new equilibria at temperatures below ignition and to serve as a control mechanism to obtain stability at the chosen operating point with desired value of Q (= power out divided by power in). We evaluate the power requirements for such steady state operation.

We perform our computations for the reactor design of HFCTR [6], though we note that the method is independent of the exact numbers, or even of the exact functions, used for the losses in the plasma. The tokamak has a major radius $R = 6$ m, a minor radius $a = 1.2$ m, an impurity $Z_{eff} = 1.2$, a safety factor $q = 3$, and a Coulomb logarithm [7] $\Lambda = 18$. The power density balance of an HFCTR reactor in the one temperature model comprises the following terms: alpha particle heating [8], *bremmstrahlung* loss [6], synchrotron loss [6], diffusion loss (empirical electron [6] and neoclassical ion [6]), and ohmic heating [7], representing the gains and losses to which a tokamak is subject. We take the following form for the power densities (all units are MKS except n in 10^{20} m^{-3}, T in Kev):

$$P_{DT} = 4.29\ 10^{8}\ \frac{n^2 \exp(-\ 17.7\ T^{(-.348)})}{1 - 0.05\,T + T^{\frac{1}{3}}\ (0.1554 - 0.1418\ T^{\frac{1}{3}} + 0.0364\ T)}\ , \tag{1}$$

$$P_{br} = 5.35\ 10^{3}\ Z_{eff}\, n^2 \sqrt{T} \tag{2}$$

$$P_{syn} = 6.4\ \sqrt{\frac{T^3\, B^5\, n}{\sqrt{R}}} \tag{3}$$

$$P_{diff} = \frac{2.4\ 10^4\ n\ T}{0.32\ n\ \sqrt{q}\ a^2} + \frac{2.4\ 10^4\ n\ T}{\dfrac{50\ B^2\ a^2\ \sqrt{T}}{q^2\ Z_{eff}\ n}\ (\dfrac{a}{R})^{\frac{3}{2}}} \tag{4}$$

$$P_{ohm} = 4.2\ 10^{3}\ \frac{B^2\ Z_{eff}\ \Lambda}{R^2\ q^2\ T^{\frac{3}{2}}} \tag{5}$$

The equation for the time rate of change of the energy density of the plasma is

$$\frac{dW}{dt} = P_{DT} + P_{ohm} - P_{br} - P_{syn} - P_{diff} = F(n,T)\ , \tag{6}$$

where

$$W = 4.8\ 10^4\ n\ T \tag{7}$$

is the internal energy of the plasma.

The equilibria of Eq. (6) are given by $F(n,T) = 0$. To determine their thermal stability we must evaluate $\frac{dF}{dT}$ at the equilibria; $\frac{dF}{dT} < 0$ is a necessary and sufficient condition to have stability.

There are three roots of F for T when n is in the range above 1. 10^{20} m^{-3}: the Ohmic equilibrium at around 0.4 Kev when there is negligible neutron production and the Ohmic heating balances the small losses; the ignition equilibrium in the range 5 - 25 Kev when the alpha particles produced by the fusion reaction balance all of the losses and make the tokamak self-sustaining; and the high temperature equilibrium above 25 Kev is due to the less steeply rising alpha particle heating being intercepted by the diffusion loss.

The derivative of F with respect to T is positive for the ignition equilibrium and negative for the other two; so that T_{ohm} is where a plasma will go when below T_{ig}. If we heat the plasma beyond T_{ig} then it will go to T_{hi}.

Since the plasma starts off at T_{ohm}, it will be necessary to use some external source of energy to heat the plasma so that it will get hot enough to produce neutrons. We propose to exploit the holonomy of the system, that is, having assumed that P_{supp} is a function only of the state variable temperature, T, in order to obtain new equilibria with the property of stability at selected values of Q and with temperature below T_{ig}. The new equilibria in the T - n plane are those obtained by adding to F a supplemental heating P_{supp} that is dependent only on T.

To find the desired form of P_{supp}, we examine the contours of F (see Fig. 1) and note the various negative ones. The 0 contour is just the equilibrium curve of Eq. (6). Addition of a positive supplemental heating of the size to make $P = F + P_{supp} = 0$ at any given point on a negative contour of F makes an equilibrium at that point for the new function P.

If we put a definite power $P_{in} = E$ into the plasma at its operating point, we can create an equilibrium at any point (T,n) with

$$E + F(T,n) = 0 . \tag{8}$$

This is on one of the negative contours for F if we take $E > 0$. The neutron power that we obtain at the point (T,n) is then

$$P_{out} = \frac{14.1}{3.5} P_{DT}(T,n) , \tag{9}$$

and the Q of the operation is power out divided by power in, or

$$Q = \frac{P_{out}}{P_{in}} = \frac{\frac{14.1}{3.5} P_{DT}(T,n)}{E} = \frac{\frac{14.1}{3.5} P_{DT}(T,n)}{-F(T,n)} . \tag{10}$$

We thus obtain contours for constant values of Q by plotting the 0 contour of the function

$$F(T,n) + \frac{\frac{14.1}{3.5} P_{DT}(T,n)}{Q} . \tag{11}$$

The contours for moderate values of Q are shown in Fig. 2. Note that the equilibrium curve $F = 0$ represents an infinite value of Q since we get a finite output with no input.

On the contours of Fig. 1, F = const, so that the total differential of F is 0:

$$\frac{dn}{dT} = - \frac{\frac{\partial F(T,n)}{\partial T}}{\frac{\partial F(T,n)}{\partial n}} . \tag{12}$$

Above 5 Kev (see Fig. 3) $\frac{\partial F}{\partial n} > 0$, so that the system is thermally stable ($\frac{dF}{dT} < 0$) when $\frac{dn}{dT} > 0$. The same, of course, applies for the contours of $P = F + P_{supp}(T)$.

Hence, to make P have a stable equilibrium at the operating point (T_{st},n) we must have a positive slope to the curve $P = 0$ at that point. We thus draw a path in the $T - n$ plane on the contour plot for F which connects the 0 contour to itself after passing below the 0 contour; if the connection is made with a portion having a positive slope passing through the oerating point, then the function P which has this curve as its 0 contour has a stable equilibrium at the operating point.

The supplemental heating as a function of temperature can be read off this new curve by looking at the value of the contour of F at the temperature and then changing the sign to plus, since we must add a positive number to F to get the equilibrium for P. We see that our supplemental heating is 0 outside the interval that we pushed down, and that it increases to a maximum at some intermediate point; that is, it behaves something like a downward opening parabola where it is non-zero.

Fig. 4 shows a case where we take a parabola for P_{supp} and see the downward distortion of the 0 contour for F into the 0 contour for P. The supplemental heating is taken to be a quadratic polynomial from 5 to 13 Kev and 0 outside that region. The new equilibrium curve is below the old equilibrium curve in the range of 5 to 13 Kev, and coincides with the old equilibrium curve elsewhere. We use, in this example,

$$P_{supp}(T) = 10^4\,(T - 5)\,(13 - T)\,. \tag{13}$$

To operate the tokamak, we must first get past the low temperature equilibrium T_{lo} which we have moved by pushing the equilibrium curve for F down. Then the plasma will be in a region that is unstable and the temperature will rise to the next equilibrium, the stable one which we have created, and will make the approach in an asymptotic manner. We thus add a modest amount of energy to get over the barrier of T_{lo} and then get to operate at T_{st} by pumping in a limited amount of power. The less power that we pump in the closer we can get to the equilibrium curve for F, and thus close to unstable operation; we take .25 Kev as the closest that we permit to the zero of the supplemental heating, and note that we are still much further than .25 Kev from the equilibrium curve.

Fig. 5 verifies our analysis. It shows the integration for the temperature as a function of time, starting the integration from a little above T_{lo}. The temperature asymptotically approaches 12.75 Kev with time, and we can evaluate the ratio of the neutron energy at 12.75 Kev to the supplemental heating at 12.75 Kev to get the steady state $Q = 23$. Fig. 6 is a graph of $P_{supp}(t)$ and shows that about 28 MW must be input to the plasma at the peak, falling to 3.2 MW asymptotically.

To conclude, we have demonstrated the possibility, in principle, of high-Q steady-state operation of a reactor below the (unstable) ignition equilibrium, using supplementary heating as a control. The result is particularly relevant for continuous operation current-drive schemes [9], where the radio-frequency source serves not only to supplement Ohmic heating, but also drives a steady toroidal current.

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FIGURE CAPTIONS

Fig. 1 Density *vs*. temperature for the contours of $F(T,n)$.

Fig. 2 Density *vs*. temperature for the contours Q = constant.

Fig. 3 Density *vs*. temperature for $\frac{\partial F}{\partial n} = 0$, for a magnetic field of $B = 70$ KG. The curve is nearly independent of density and is at about $T = 4.8$ Kev.

Fig. 4 Density *vs.* temperature for the equilibrium curves of F and P; the magnetic field is 70 Kg.

Fig. 5 Temperature *vs.* time, when parabolic P_{supp} is used to create new stable equilibria. We have asymptotic operation at $T = 12.75$ Kev, with a magnetic field of $B = 70$ KG.

Fig. 6 Power *vs.* time that must be input to the plasma to have stable operation at $T = 12.75$ Kev, as in Fig. 5.

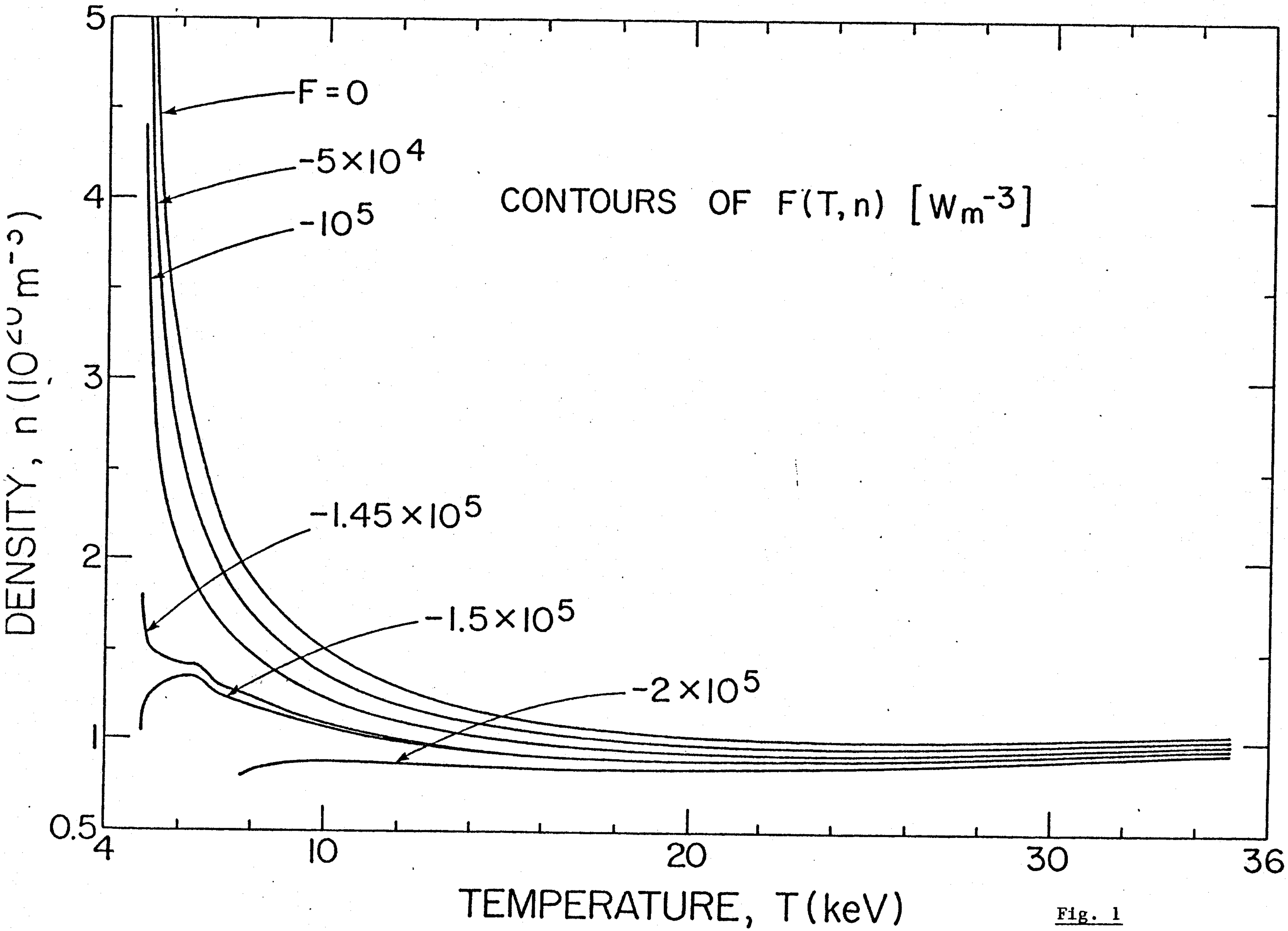
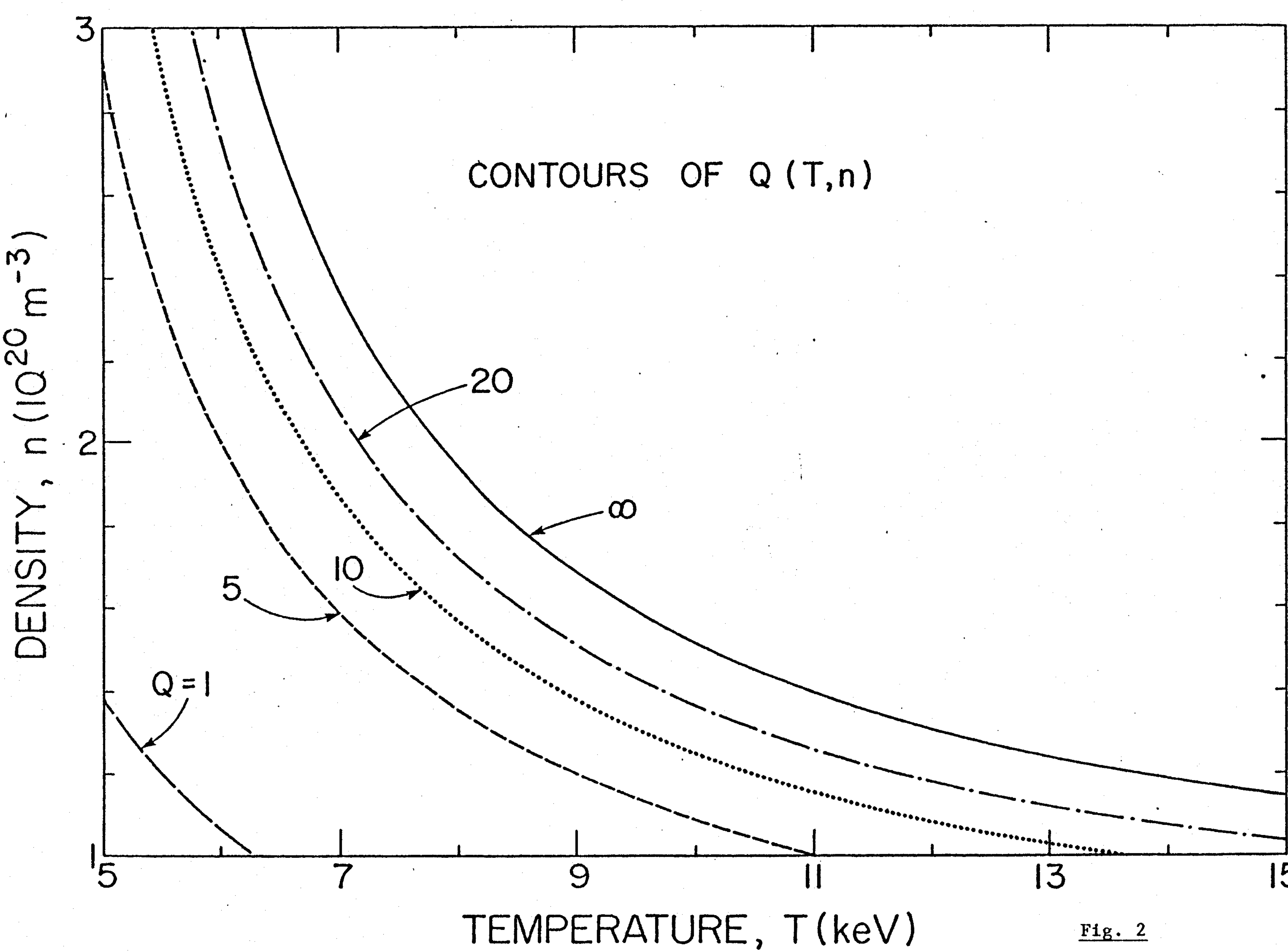

CONTOURS OF F(T,n) [Wm-3]
F=0
-5×10^4
-10^5
-1.45×10^5
-1.5×10^5
-2×10^5
DENSITY, n (10^20 m^-3)
TEMPERATURE, T(keV)
5
4
3
2
1
0.5
4
10
20
30
36

Fig. 1



CONTOURS OF Q (T,n)
20
∞
10
5
Q=1
3
2
1
5
7
9
11
13
15
DENSITY, n (10^20 m^-3)
TEMPERATURE, T (keV)

Fig. 2

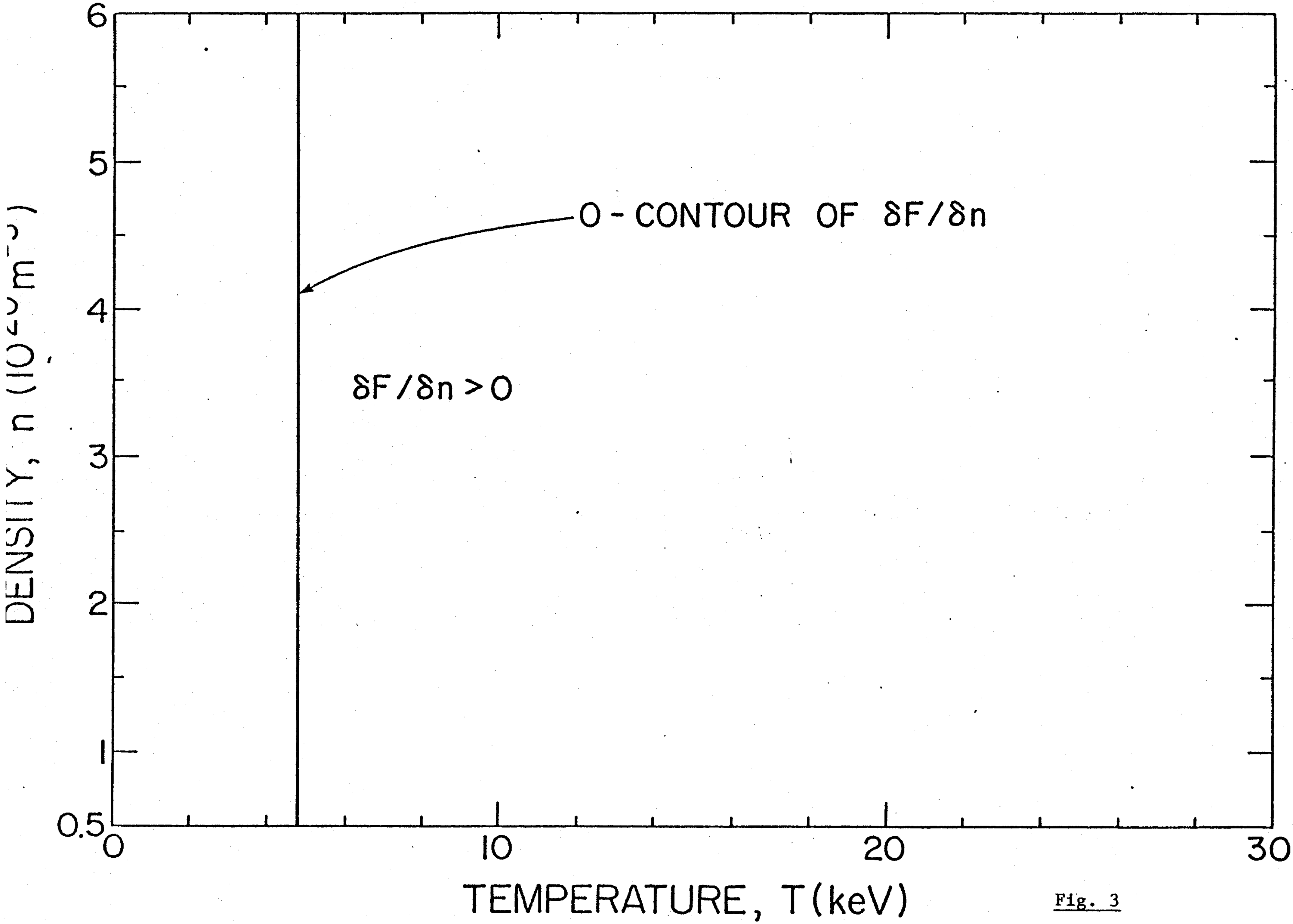
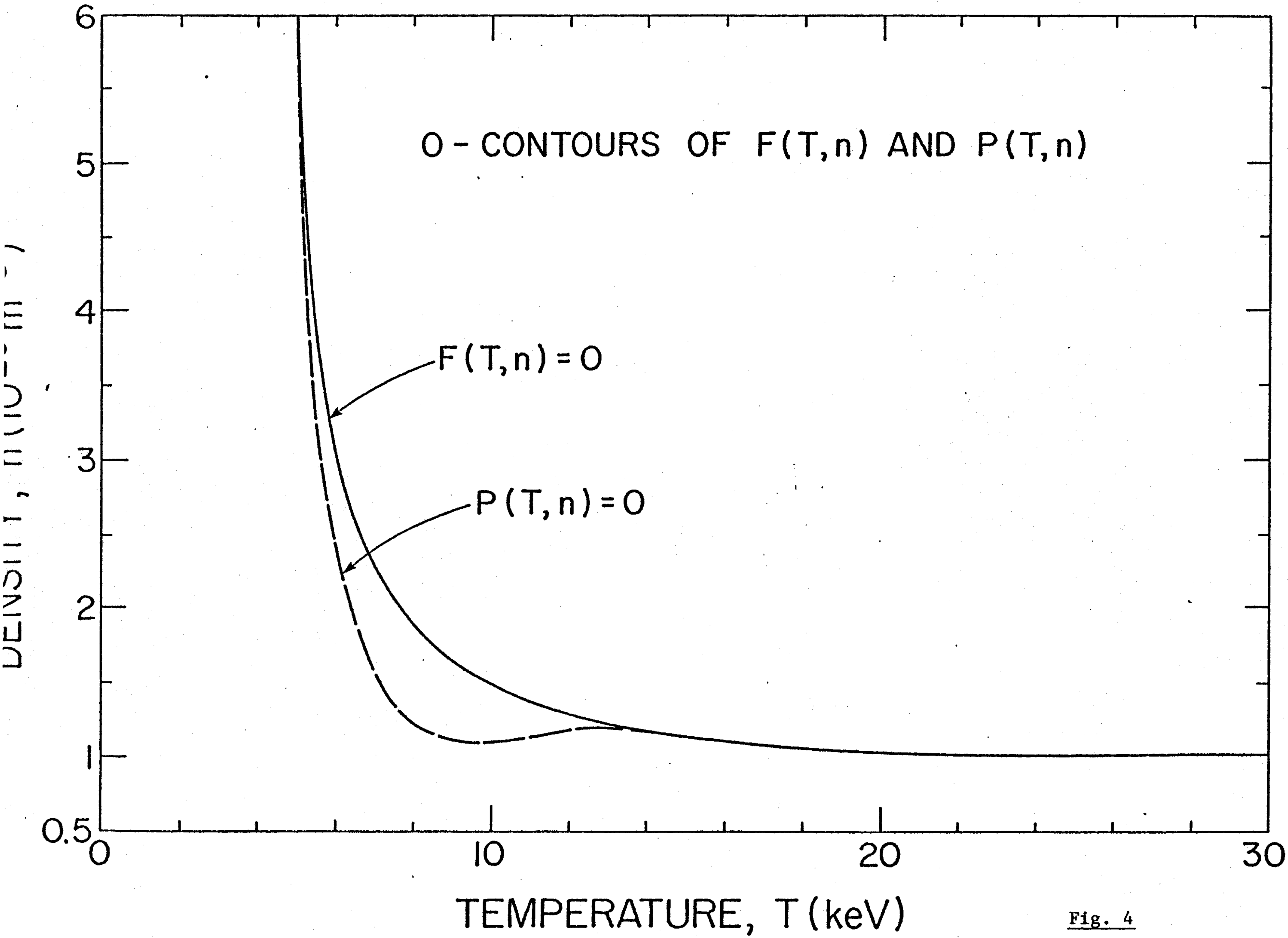
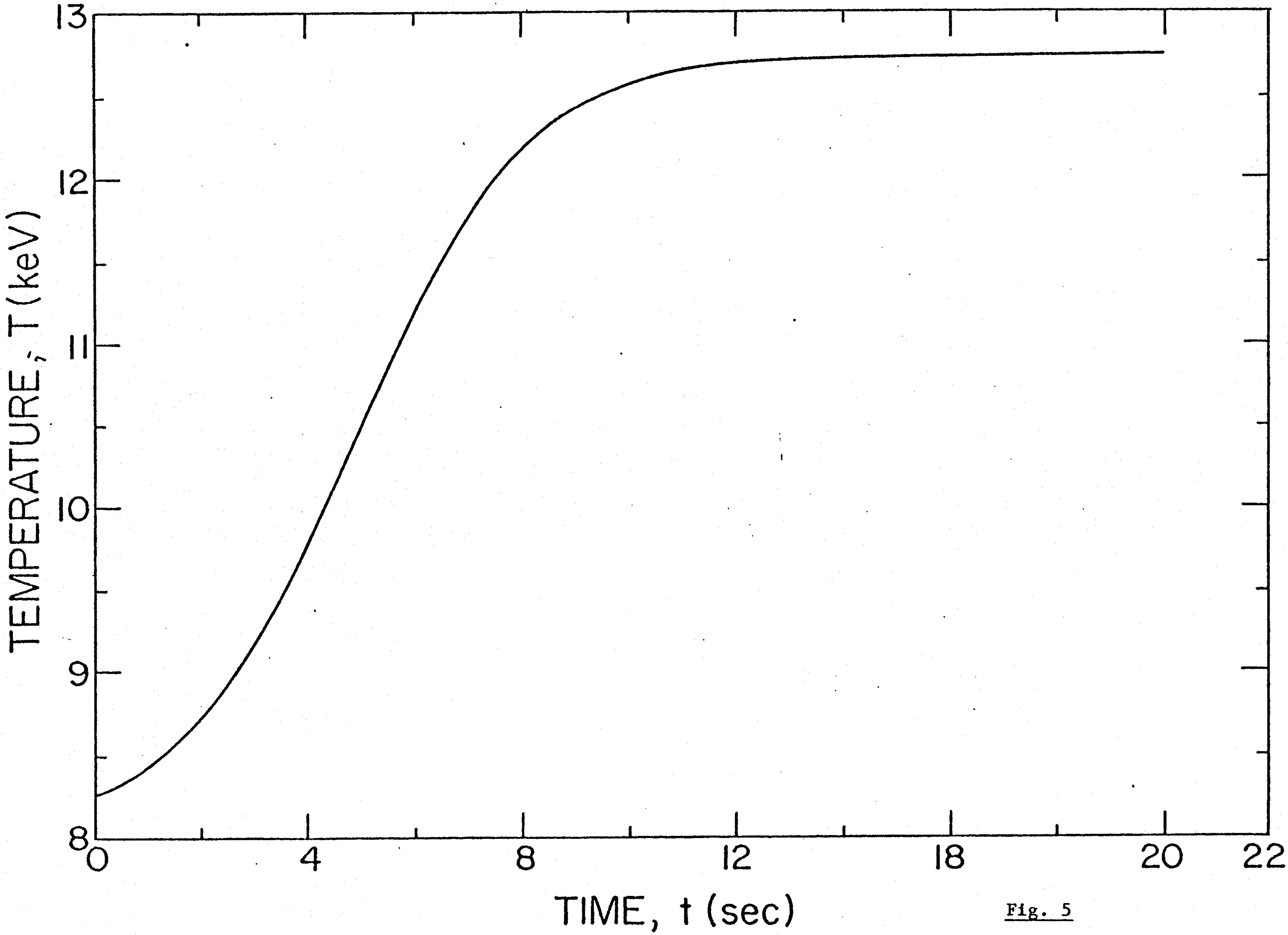

O - CONTOUR OF δF/δn
δF/δn > O
DENSITY, n (10
TEMPERATURE, T(keV)
6
5
4
3
2
1
0.5
0
10
20
30

Fig. 3



O - CONTOURS OF F(T,n) AND P(T,n)
F(T,n) = O
P(T,n) = O
6
5
4
3
2
1
0.5
0
10
20
30
TEMPERATURE, T (keV)

Fig. 4



<u>Fig. 5</u>

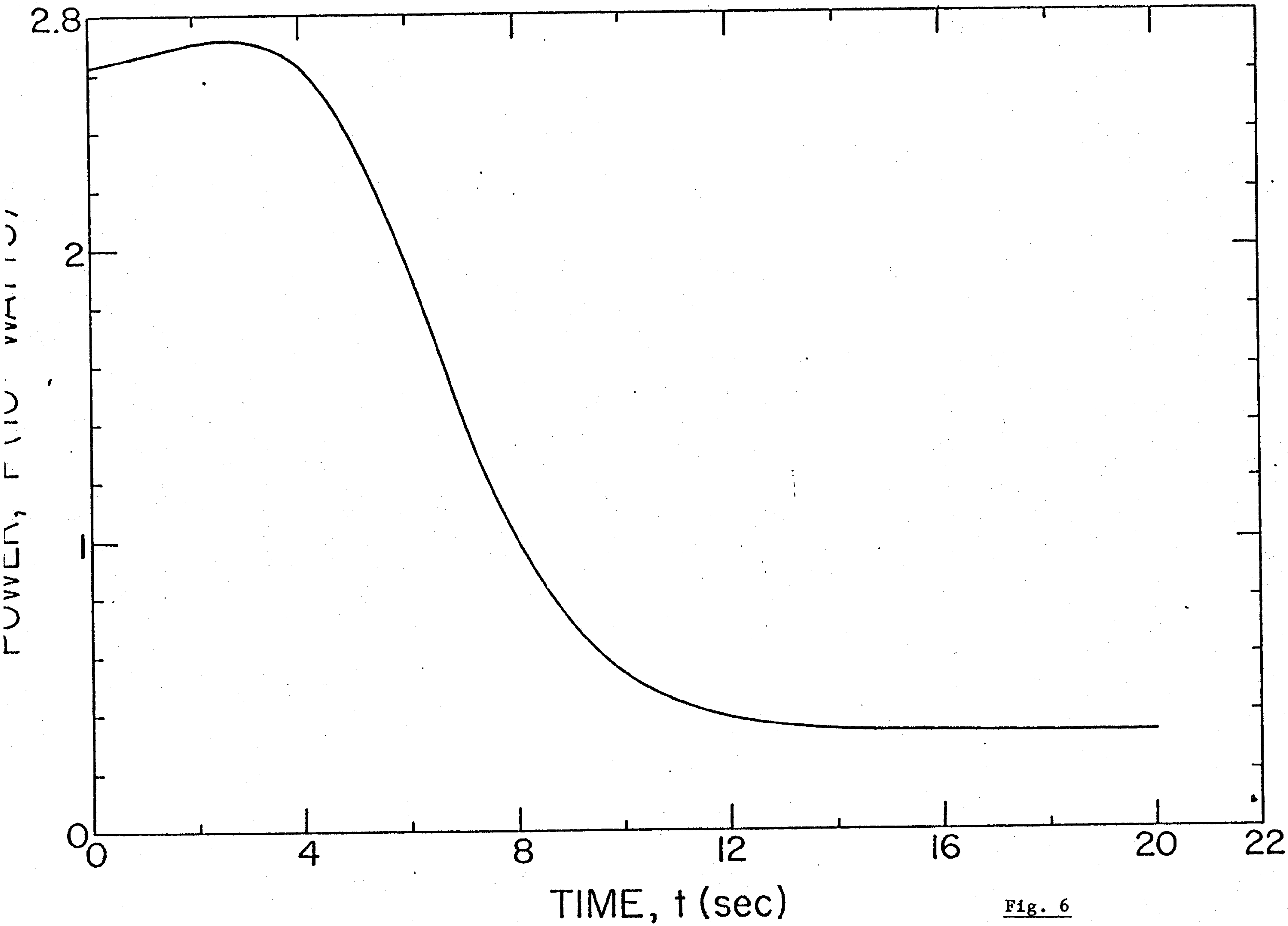


Fig. 6