

ADIABATIC THEORY OF CURRENT GENERATION
BY NONLINEAR WAVES IN A VLASOV PLASMA*

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Adiabatic Theory of Current Generation by Nonlinear Waves in a Vlasov Plasma

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Abstract

We derive the one-dimensional Vlasov distribution function in terms of the adiabatic invariants of the Vlasov equation for a single, nonlinear, large amplitude travelling wave excited by an external source. The result takes into account the adiabatic turn-on of the pump and is valid to all orders in the electric field amplitude everywhere in phase-space, except for a narrow region near the separatrix. All electrons, including the trapped ones, are adiabatic and the processes are reversible. The nonlinearity is found to lead to the generation of a steady-state current whose properties are examined. The theory is applied to study the steady-state currents that could be driven by the lower hybrid wave in present day tokamak experiments. The trapped electrons play a very significant role in determining the nature of this current.

I. Introduction

In order to achieve fusion in a tokamak plasma, it is necessary that the plasma be confined for a reasonable period of time (\sim several seconds). This can be done, in part, by a poloidal magnetic field which is generated by the toroidal current. Furthermore, a steady state toroidal current is also essential for the steady state mode of operation of a fusion reactor. It has been suggested recently [1] that in a tokamak, this current may be achieved by an array of wave guides whose phases are adjusted to launch a nearly unidirectional (parallel to the toroidal magnetic field, B_T) high-powered lower-hybrid wave. The resonance cones formed away from the wave-guide mouth completely fill the interior of the tokamak. The ray trajectories with a group velocity along B_T close to the speed of light (c) make many cycles within the tokamak. The electromagnetic effects may also lead to the spreading of the wave packet [2]. If the spatial extent of the wave packet is L , and, T is the characteristic time related to the adiabatic turn-on of the pump and the temporal scale of change of the wave packet, then the particle velocities along B_T inside the plasma satisfy $|v_T| \ll L/T$.

Recently [3], the Vlasov equation has been solved for a single, large amplitude, nonlinear wave in terms of the adiabatic invariants of the equation. The distribution function was determined to all orders in the electric field amplitude, and is valid everywhere in phase space except for a small region near the separatrix. The solutions were found for both the homogeneous ($|v - v_g| \ll L/T$, v_g is the group velocity) and the boundary value ($|v - v_g| \gg L/T$) problems. In both cases, nonlinear normal modes other than the usual Langmuir modes were shown to exist. For these modes, the distribution function is significantly modified compared to a Maxwellian. The boundary value problem determines the nonlinear coupling of the LH wave near the plasma edge [4]. Here, one finds a "ponderomotive" density modification, but, no current is driven by the wave. However, for the homogeneous problem inside the plasma, the distribution function is significantly modified and the wave drives a current in the plasma. In this paper, we study the properties of this current.

It has been shown [1], that the high phase velocity waves drive a current as the wave momentum is irreversibly transferred to the electrons. It is assumed that the RF waves modify only the tail of the electron distribution function, while, the rest remains Maxwellian. A steady state is achieved by a balance between quasilinear diffusion and collisions of the resonant particles with the bulk of the distribution function. The resonant electrons, travelling at several times the thermal velocity, undergo collisions relatively infrequently and, thus, are able to drive a current. We do not treat the effect of collisions in this paper, but find that the nonlinear modification to the distribution function is sufficient to drive a current in the case of homogeneous problem.

This paper is organized as follows: in Section II, we solve the Vlasov equation to derive the electron

distribution function for the homogeneous case in terms of the adiabatic invariants. The dispersion relation is also set up. In Section III, the steady state current and its properties are examined, and the theory is applied to study the steady state currents that could be driven in present tokamak experiments. Related topics of ongoing and future research are discussed in Section IV.

II. The Adiabatic Theory for a Large Amplitude Wave

The Vlasov equation for the electrons in one-dimension, where the externally launched wave is slowly modulated in time, but remains homogeneous in space is:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \phi(t, x - \frac{\omega}{k}t) \frac{\partial f}{\partial v} \quad (1)$$

where the electric potential ϕ depends on the fast time scale of the oscillations, $kx/\omega - t$, and the slow time scale of the modulation, t . The scale of the characteristic time for the modulation, T , depends on the adiabatic turn-on of the pump, parametric processes and Landau damping. Here the ions are treated as forming a stationary, quasineutral background.

In the wave-frame: $\tilde{v} = v - \omega(t)/k$, $\tilde{t} = t$, $\tilde{x} = x - \int^t \omega(t')/k dt'$, Eq. (1) becomes:

$$\frac{\partial f}{\partial \tilde{t}} + \tilde{v} \frac{\partial f}{\partial \tilde{x}} - \frac{\partial}{\partial \tilde{x}} \phi(\tilde{t}, \tilde{x} - \Delta \tilde{x}) \frac{\partial f}{\partial \tilde{v}} = 0 \quad (2)$$

where $\Delta \tilde{x} = \omega \tilde{t}/k - \int^{\tilde{t}} \omega(t')/k dt'$.

The evolution of the system is then described by the Hamiltonian:

$$H = \frac{\tilde{v}^2}{2} + \phi(\tilde{t}, \tilde{x}) \quad (3)$$

where $\tilde{\tilde{x}} = \tilde{x} - \Delta \tilde{x}$ and \tilde{v} are the canonical coordinate and momentum, respectively.

f is then a constant of the motion, and Eq. (2) takes the canonical form:

$$\frac{df}{d\tilde{t}} = \frac{\partial f}{\partial \tilde{t}} + \{f, H\} = 0 \quad (4)$$

where the convention $\{\tilde{\tilde{x}}, \tilde{v}\} = 1$ is adopted for the Poisson brackets.

If the modulation of the wave packet was constant in time, i.e. $\phi = \phi(\tilde{x} - \Delta \tilde{x})$, then the Hamiltonian would be a conserved quantity for the system. The distribution function $f = f(H)$ would give the well-known

BGK [5] modes. But, in any realistic situation, the electric field amplitude is turned on as a function of time and, consequently, the Hamiltonian is not a conserved quantity. However, the adiabatic invariant [6]:

$$I = \frac{1}{2\pi} \int \bar{v} d\bar{x} \\ = \frac{1}{\sqrt{2\pi}} \int (H - \phi(\bar{t}, \bar{x}))^{1/2} d\bar{x} \quad (5)$$

where the integral is taken over one cycle of the motion, is conserved to all orders in the slow time variation. Then, the distribution function $f = f(I)$ is a solution of the Vlasov equation valid in all the phase space of $(\bar{x}, \bar{v}, \phi_0)$ [ϕ_0 is the amplitude of ϕ] where the nonlinear frequency $\Omega = dH/dI \gg 1/T$. For a large amplitude wave, the volume of phase space where this condition is violated is small and located near the separatrix.

The adiabatic invariant of equation (5) is different for the particles trapped in the potential well (I^{TR}) than for the free particles (I^{os}). The discontinuity occurs across the separatrix. With the initial conditions: $\phi(t_0) = 0, f(t_0) = f_M$ and using $I(t_0) = I(t)$, the distribution function for the free particles is [2]:

$$f^{os} = \frac{n_0}{\sqrt{\pi}v_t} \exp\left[-\left\{\frac{\omega}{kv_t} - \frac{k}{v_t} I^{os}(H, \phi) \text{sign}\left(\frac{\omega}{k} - v\right)\right\}^2\right] \quad (6)$$

where n_0 is the density of electrons, v_t is their thermal velocity. This result is written in the laboratory frame and $\phi = \phi(t, x - \omega t/k)$.

The trapped distribution function (f^{TR}) cannot be determined from the initial conditions since it vanishes as the electric field vanishes. So to find f^{TR} we impose the additional condition of the invariance of the norm of the total distribution function. This is the same as requiring that the total number of particles be conserved, i.e. [3]

$$N_0 = 2\pi \int_{v_I} f(I) dI = \int_{v_{ph}} f(\bar{x}, \bar{v}) d\bar{x} d\bar{v} \quad (7)$$

where the integral is over all the phase space. This implies that:

$$\int_0^B f^{TR}(I^{TR}) dI^{TR} = \frac{N_0}{2\pi} - \frac{n_0}{\sqrt{\pi}v_t} \left(\int_{-\infty}^{-A(B)} + \int_{A(B)}^{\infty} \right) \exp\left[-\left(\frac{\omega}{kv_t} - \frac{k}{v_T} I^{os}\right)^2\right] dI^{os} \quad (8)$$

where B is the value of I^{TR} at the separatrix, and A is the value of I^{os} at the separatrix written as a function of B . Differentiating the last equation with respect to B gives:

$$f^{TR}(B) = 2 \frac{n_0}{\sqrt{\pi} v_t} \frac{dA}{dB} \exp \left[-\left(\frac{\omega}{k v_t} \right)^2 - \left(\frac{kA}{v_t} \right)^2 \right] \cdot \cosh \left(\frac{2\omega A}{v_t^2} \right) \quad (9)$$

Finally, replacing B by I^{TR} gives the trapped distribution function. So the distribution functions have been determined over all phase space within the constraints for the validity of the adiabatic theory. In Appendix A, the specific example of a plane wave where $\phi = (\omega/k)v_0(t) \sin(kx - \omega t)$ is treated.

For a self-consistent one-dimensional theory, the distribution function has to satisfy the Maxwell's equations:

$$\frac{\partial E}{\partial t} = 4\pi e \int_{-\infty}^{\infty} v f(v, E) dv \quad (10)$$

where E is the electric field. The nonlinear dispersion relation is the condition for the existence of a solution to Eq. (10) for E . Let us assume that the source and the plasma allow for the excitation of a finite number of harmonics. Thus,

$$E = \sum_{n=1}^N E_n(t) e^{-in(\omega t - kx)} \quad (11)$$

$$f = \sum_{n=1}^N f_n(v, \{E_i\}) e^{-in(\omega t - kx)} \quad (12)$$

where $E_n(t)$ are the amplitudes modulated in time. Substituting Eqs. (11) and (12) into Eq. (10) gives a system of N equations:

$$E_n = \varphi_n(E_1, \dots, E_N), \quad n = 1, 2, \dots, N. \quad (13)$$

Then the nonlinear dispersion relation is the condition for the existence of a nontrivial solution to the above system. For small amplitudes this would be the determinant of the coefficients of E_i . In reality, the amplitude of a certain harmonic dominates the rest. Then, assuming that this is the first harmonic, the dispersion relation

$$-i\omega E_1 = 4\pi e \int_{-\infty}^{\infty} v f_1(v, E_1) dv \quad (14)$$

takes into account the self-consistent response of the pump on itself, neglecting the generation of higher harmonics.

This dispersion relation is worked out for the plane wave case [Appendix A] and the results plotted in Fig. 1. The upper normal mode is the nonlinear Langmuir mode, while the lower one is a purely nonlinear

mode that exists due to the effects of trapped particles. Recent experiments and computer simulations [7] have supported the existence of such a mode. The average distribution function, $\langle f \rangle$, plotted for points on the two branches of the dispersion curves (Figs. 2 and 3) show deviations from the Maxwellian distribution. Thus, the nonlinearity leads to the generation of a steady-state current:

$$\langle j \rangle = -e \int_{-\infty}^{\infty} v \langle f \rangle dv \quad (15)$$

III. Steady-State Current Generation in Tokamak Experiments

The feasibility of generating steady state currents in tokamaks by using an array of wave guides, whose phases have been adjusted to launch a unidirectional LH wave parallel to the magnetic field, has been studied by Fisch and Bers [1]. Two experiments have been carried out on small linear devices [8], and another experiment was recently carried out on the General Atomic Octopole-Tokamak [9]. The experiment on the M.I.T. Versator II tokamak is currently in progress.

The externally launched wave has its wave number parallel to the toroidal magnetic field, k_z , determined by the wave guides. The amplitude of the electric field is also externally determined. The plasma properties then determine k_x —the wave number in the direction of the various inhomogeneities (density, magnetic field, temperature). The nonlinearity parameter is the amplitude of the oscillating velocity, which is:

$$v_{0x} = \frac{eE_x \omega}{m(\Omega_{ce}^2 - \omega^2)} \quad (16)$$

$$v_{0y} = \frac{eE_x \Omega_{ce}}{m(\Omega_{ce}^2 - \omega^2)} \quad (17)$$

$$v_{0z} = \frac{eE_z}{m\omega} \quad (18)$$

with E_x, E_z being the electric field amplitudes in the x and z directions, respectively; and, Ω_{ce} being the electron cyclotron frequency. In the case of a lower-hybrid wave $E_y \ll E_z \ll E_x$. For the lower-hybrid range of frequencies [11]:

$$\frac{v_{0x}}{v_{0z}} \ll 1, \quad \frac{v_{0y}}{v_{0z}} \approx \frac{\omega_{pe}}{\Omega_{ce}} < 1 \quad (19)$$

so the dominant nonlinearity is along the toroidal direction. Consequently, the transverse effect is ignored which reduces the motion of the electrons to be one-dimensional along B_T . Thus, we can use our distribution function (6.9) to determine the steady-state current along B_T with E_z, k_z and ω being the externally determined independent parameters.

In Figs. (4) and (5), we have shown the properties of such a LH-wave generated current for Alcator A parameters. With the parameters:

$$n_0 = 3.0 \times 10^{14} \text{cm}^{-3}, \text{ temperature } T_0 = 1 \text{keV}, \omega = 1.54 \times 10^{10} \text{sec}^{-1},$$

we have plotted v_d/v_t versus $b = (eE_z/m\omega_{pe}v_t)^2$ for various n_z 's, where v_d is the drift velocity. A value of $b = 10^{-5}$ corresponds to an electric field amplitude of $E_z \approx 3.3 \text{ kV/cm}$. From Fig. (4) it is evident that $\langle j \rangle \propto n_z E_z^2$ for low values of n_z and b . This is the same result as one would get from nonresonant quasilinear theory [12]. However, the deviations from this scaling for higher n_z 's and b 's become significant as the trapped electrons start playing an important role. The current reaches a saturation level (Fig. 5) for large b 's when all the electrons are trapped in the potential wells. The saturation current is inversely proportional to n_z (Appendix B). The saturation level is reached at lower electric field amplitudes for higher n_z 's.

The theory has also been applied to steady state current generation in the Versator II experiment [10]. The relevant parameters used are:

$$n_0 = 2 \times 10^{13} \text{cm}^{-3}, T_0 = 300 \text{eV}, \omega = 5.027 \times 10^9 \text{sec}^{-1}$$

with the Alcator A profiles:

$$n = n_0 \left(1 - \left(\frac{\rho}{1.215} \right)^2 \right)^{3/2}$$

$$T = T_0 e^{-2\rho^2}$$

where ρ is the ratio of the distance from the center of the plasma (the toroidal axis) to the minor radius (13cm.). In Fig. 6, we have plotted $|\langle j \rangle|$ in amperes cm^{-2} versus ρ for different n_z 's. The contributions from the trapped and the untrapped electrons to the total current are shown in Fig. 7 (for $n_z = 10$) and Fig. 8 (for $n_z = 12$). For $n_z = 11$ the untrapped electrons have zero drift velocity at the center of the plasma. For $n_z > 11$ and after a certain position inside the plasma, the untrapped electrons start contributing a backward current (Fig. 8). However, the current from the trapped electrons becomes large enough to compensate for this so that the net drift is always in the direction of wave propagation.

V. Conclusion

A recent study [13] designed to investigate the feasibility of a steady state tokamak power reactor driven by LH waves concludes that such a reactor is possible. It has shown the various advantages a steady state current

driven by LH waves would have for reactor operation and design. In this paper, we have studied the properties of the steady state current generated by large amplitude, high frequency waves. The kinetic theory used to describe the reversible nonlinear processes in a plasma takes the time evolution of the electric field amplitude into account. The equilibrium is then sufficiently distorted from the Maxwellian by the nonlinearities so that a steady state current is generated in the plasma. We show significant deviations from the nonresonant quasilinear theory of the steady state current even for the present experiments on tokamak plasmas.

However, the theory does not include some basic phenomena. The electrons near the separatrix in phase space do not obey the assumption of adiabaticity. These electrons lead to processes, such as Landau damping, that irreversibly transfer energy from the wave to the particles, or vice-versa. So the theory has to be extended to include the time evolution of the adiabatic invariant. But what we have presented is valid for $\omega_B \gg \gamma_L$.

The theory is valid for time scales shorter than the collisional time scale. To determine the effect of collisions on the distribution function the Fokker-Planck equation has to be solved. This would lead to further modifications to the distribution function. For large amplitude waves the Landau damping does not play an important role. The collisional damping determines the power dissipated in the plasma which is crucial for evaluating the feasibility of the steady-state current drive scheme. This problem is currently under investigation.

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Appendix A

For a plane wave:

$$\phi = \frac{\omega}{k} v_0(t) \sin(kx - \omega t) \quad (A1)$$

the action for the free particles is:

$$I^{os} = \frac{2\sqrt{2}}{\pi k} (H + \frac{\omega v_0}{k})^{1/2} E(p) \quad (A2)$$

and for the trapped particles is:

$$I^{TR} = \frac{4\sqrt{2}}{\pi k} (H + \frac{\omega v_0}{k})^{1/2} \left[pE\left(\frac{1}{p}\right) - \frac{p^2 - 1}{p} K\left(\frac{1}{p}\right) \right] \quad (A3)$$

where

$$H = \frac{1}{2} \left(v - \frac{\omega}{k} \right)^2 + \phi \quad (A4)$$

is the energy,

$$p^2 = \frac{2(\omega v_0/k)}{H + (\omega v_0/k)} \quad (A5)$$

and, K and E are complete elliptic integrals of the first and second kinds, respectively. For the free particles $p^2 < 1$ and for the trapped particles $p^2 > 1$. The free particle distribution function is then:

$$f^{os} = \frac{n_0}{\sqrt{\pi} v_t} \exp \left[- \left\{ \frac{\omega}{k v_t} - \frac{k}{v_t} I^{os} \operatorname{sign} \left(\frac{\omega}{k} - v \right) \right\}^2 \right] \quad (A6)$$

which in the limit $t \rightarrow 0$, $v_0 \rightarrow 0$ reduces to a Maxwellian.

The quantities A and B discussed in Eqs. (8) and (9) are given by:

$$A = I^{os}|_{p \rightarrow 1} = \frac{4}{\pi} \sqrt{\frac{v_0 \omega}{k^3}} \quad (A7)$$

$$\begin{aligned} B &= I^{TR}|_{p \rightarrow 1} = \frac{8}{\pi} \sqrt{\frac{v_0 \omega}{k^3}} \\ &= 2A \end{aligned} \quad (A8)$$

Then, following Eq. (9), the trapped particle distribution function is:

$$f^{TR} = \frac{n_0}{\sqrt{\pi}v_t} \exp \left[-\left(\frac{\omega}{kv_t}\right)^2 - \left(\frac{kI^{TR}}{2v_t}\right)^2 \right] \cosh \left(\frac{\omega I^{TR}}{v_t^2} \right) \quad (A9)$$

Equations (A6) and (A9) give the total distribution function everywhere in phase space. The adiabatic theory is valid where the nonlinear frequency:

$$\Omega = \frac{dH}{dI} = \frac{\pi\omega_B}{K(p)} \gg \frac{1}{T} \quad (A10)$$

where T is the characteristic time discussed earlier, and, $\omega_B = \sqrt{eEk/m}$ is the bounce frequency. The corresponding dispersion relation of Eq. (14) is plotted in Fig. 1. There exist two normal modes. The upper one is the Langmuir wave, while the lower one is the nonlinear mode which obeys the dispersion relation: $\omega \sim \omega_B$.

Appendix B

It is easy to show that the steady state current density is given by:

$$\begin{aligned}
 \langle j \rangle &= \int_{-\infty}^{\infty} dv v \langle f \rangle \\
 &= -\frac{4en_0v_t}{\pi\sqrt{\pi}} \frac{w_0}{a} \left[\int_0^1 \frac{dp}{p^2} \left\{ \left(\frac{1}{\sqrt{aw_0}} K(p) - \frac{\pi}{p} \right) \right. \right. \\
 &\quad \exp\left[-\left(\frac{1}{a} - \frac{4}{\pi p} \sqrt{\frac{w_0}{a}} E(p)\right)^2\right] + \left(\frac{1}{\sqrt{aw_0}} K(p) + \frac{\pi}{p} \right) \cdot \\
 &\quad \left. \left. \exp\left[-\left(\frac{1}{a} + \frac{4}{\pi p} \sqrt{\frac{w_0}{a}} E(p)\right)^2\right] \right\} \right] \\
 &\quad - \frac{ev_i^2}{a} \int_{I^{TR}(p \rightarrow \infty)}^{I^{TR}(p \rightarrow 1)} dI^{TR} f^{TR}(I^{TR})
 \end{aligned} \tag{B1}$$

where $w_0 = v_0/v_t$, $a = kv_t/\omega$

$$I^{TR}(p \rightarrow 1) \rightarrow \frac{8}{\pi} \sqrt{\frac{w_0}{a}} \tag{B2}$$

and

$$I^{TR}(p \rightarrow \infty) \rightarrow 0 \tag{B3}$$

The first integral in Eq. (B1) is the contribution to the current from the free particles. The second integral is due to the trapped particles, which can also be written as:

$$-\frac{ev_i^2}{a} \frac{8}{\pi} \sqrt{\frac{w_0}{a}} \int_1^{\infty} dp \frac{K(1/p)}{p^3} f^{TR}(p). \tag{B4}$$

It is then clear that the $p = 1$ singularity from the free particles exactly cancels the same singularity from the trapped particles.

The contribution of the free particles to $\langle j \rangle$ becomes small when $|1/a - (4/\pi p) \cdot \sqrt{w_0/a} E(p)| \gg 1$ for all $p > 1$. In particular,

$$\left| \frac{1}{a} - \frac{4}{\pi} \sqrt{\frac{w_0}{a}} \right| \gg 1. \tag{B5}$$

This condition can be written as:

$$|v_{ph} - \frac{\sqrt{2}}{\pi} \Delta v_{tr}| \gg 1. \quad (B6)$$

where $v_{ph} = \omega/k$ and $\Delta v_{tr} = 2\sqrt{2}\sqrt{v_0 v_{ph}}$ is the trapping width.

In the case $v_{ph} \gg \Delta v_{tr}$, the number of trapped particles is very small and the distribution function is essentially a Maxwellian. Thus, the current is very small. However, in the limit $\Delta v_{tr} \gg v_{ph}$, most of the particles are trapped. Then,

$$\int_0^{(8/\pi)\sqrt{u_0/a}} dI^{TR} f^{TR}(I^{TR}) \approx 1 \quad (B7)$$

so that the current reaches a saturation level, $\langle j \rangle_s$. Clearly,

$$\langle j \rangle_s \propto \frac{1}{a} \propto \frac{1}{k} \quad (B8)$$

for fixed frequency.

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Figure Captions

- Fig. 1. Dispersion curves for different electric field amplitudes, $b = (eE/m\omega_{pe}v_t)^2$.
- Fig. 2. The average distribution function (normalized to 2π) on the upper branch. $b = 2.5$, $(k\lambda_{De})^2 = 0.0285$, $(\omega/\omega_{pe})^2 = 1.1$.
- Fig. 3. The average distribution function (normalized to 2π) on the lower branch. $b = 1.5$, $(k\lambda_{De})^2 = 0.0735$, $(\omega/\omega_{pe})^2 = 0.5$.
- Fig. 4. v_d/v_t as a function of b for various n_z with Alcator A parameters: $\omega = 1.54 \times 10^{10} \text{sec}^{-1}$, $n_0 = 3 \times 10^{14} \text{cm}^{-3}$, $T_0 = 1 \text{keV}$. v_d is the drift velocity.
- Fig. 5. Same as Fig. 4. except that the range of b has been increased.
- Fig. 6. Current density (in amperes cm^{-2}) for Versator II as a function of n_z and ρ [the ratio of the distance from the center of the plasma to the minor radius (13 cm.)] $\omega = 5.027 \times 10^9 \text{sec}^{-1}$, $n_0 = 2 \times 10^{13} \text{cm}^{-3}$, $T_0 = 300 \text{eV}$, $E_z = 600 \text{V/cm}$, $n(\rho) = n_0 \{1 - (\frac{\rho}{1.215})^2\}^{3/2}$, $T(\rho) = T_0 e^{-2\rho^2}$.
- Fig. 7. The total current density (—), untrapped electron current density (.....) and the trapped electron current density (- - - -) for Versator II with $n_z = 10$.
- Fig. 8. Same as Fig. 7 except now $n_z = 12$.
- Fig. 9. Total current (in kAmps.) in the Versator II as a function of n_z for different electric field amplitudes.

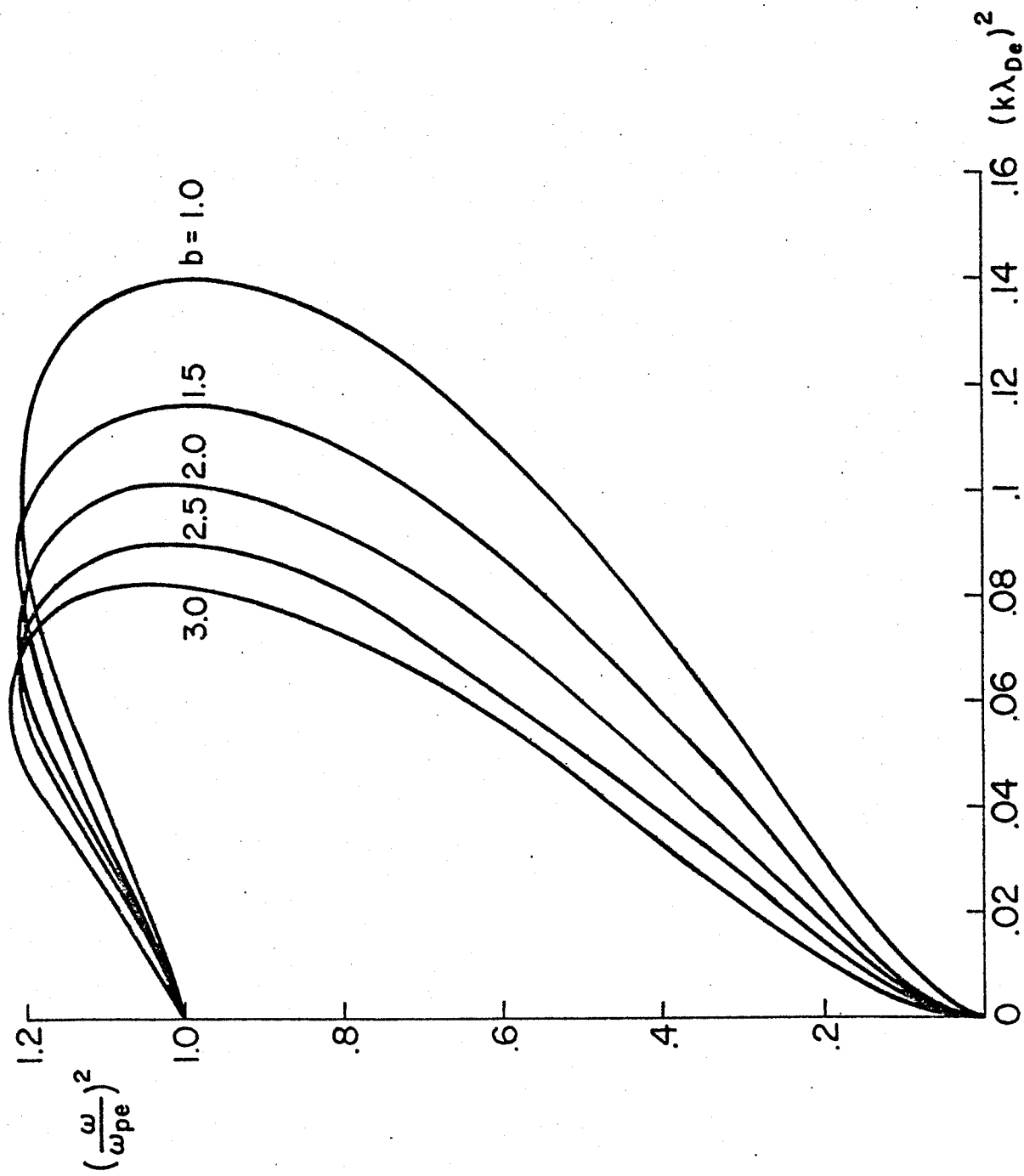


Figure 1

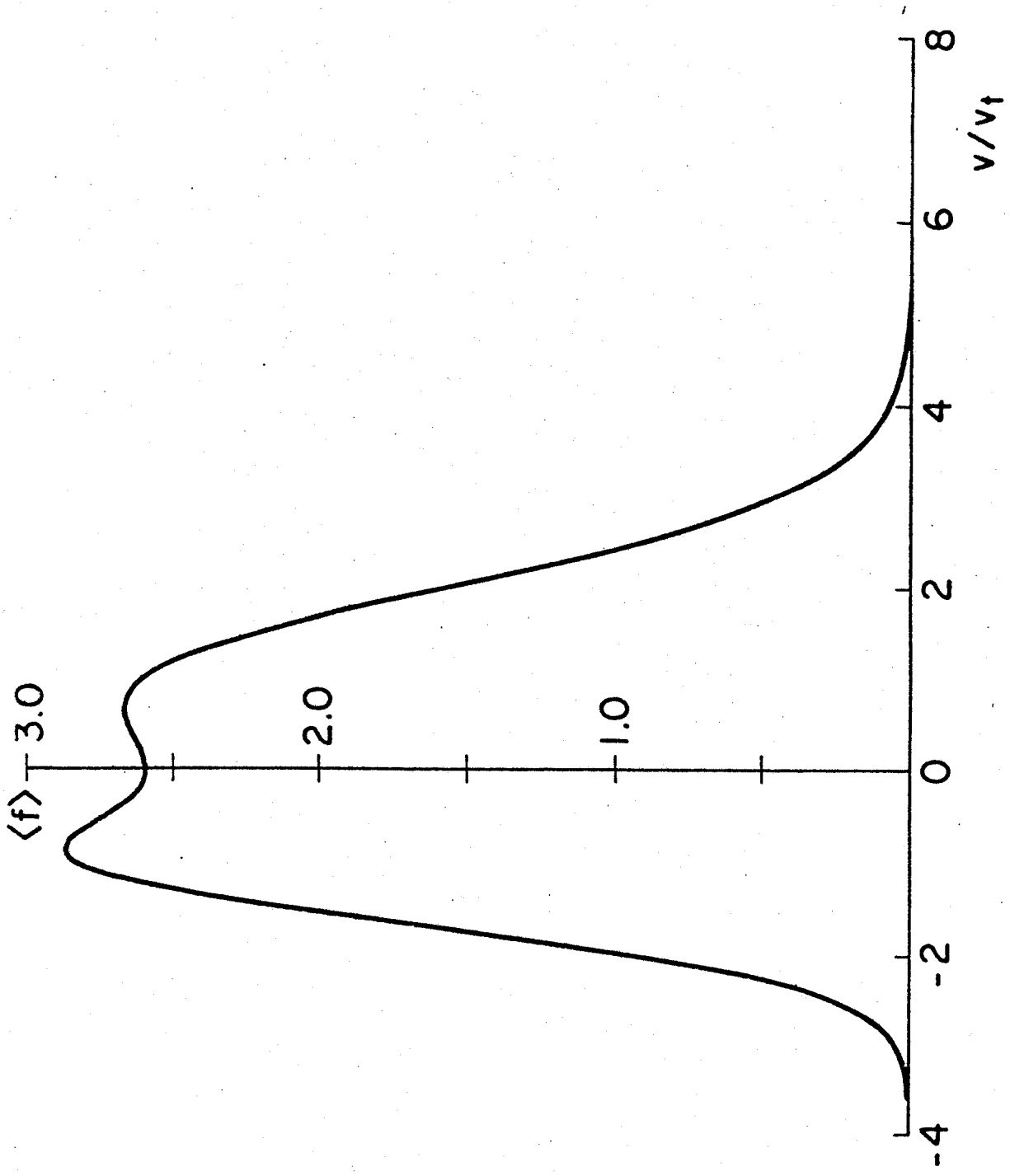


Figure 2

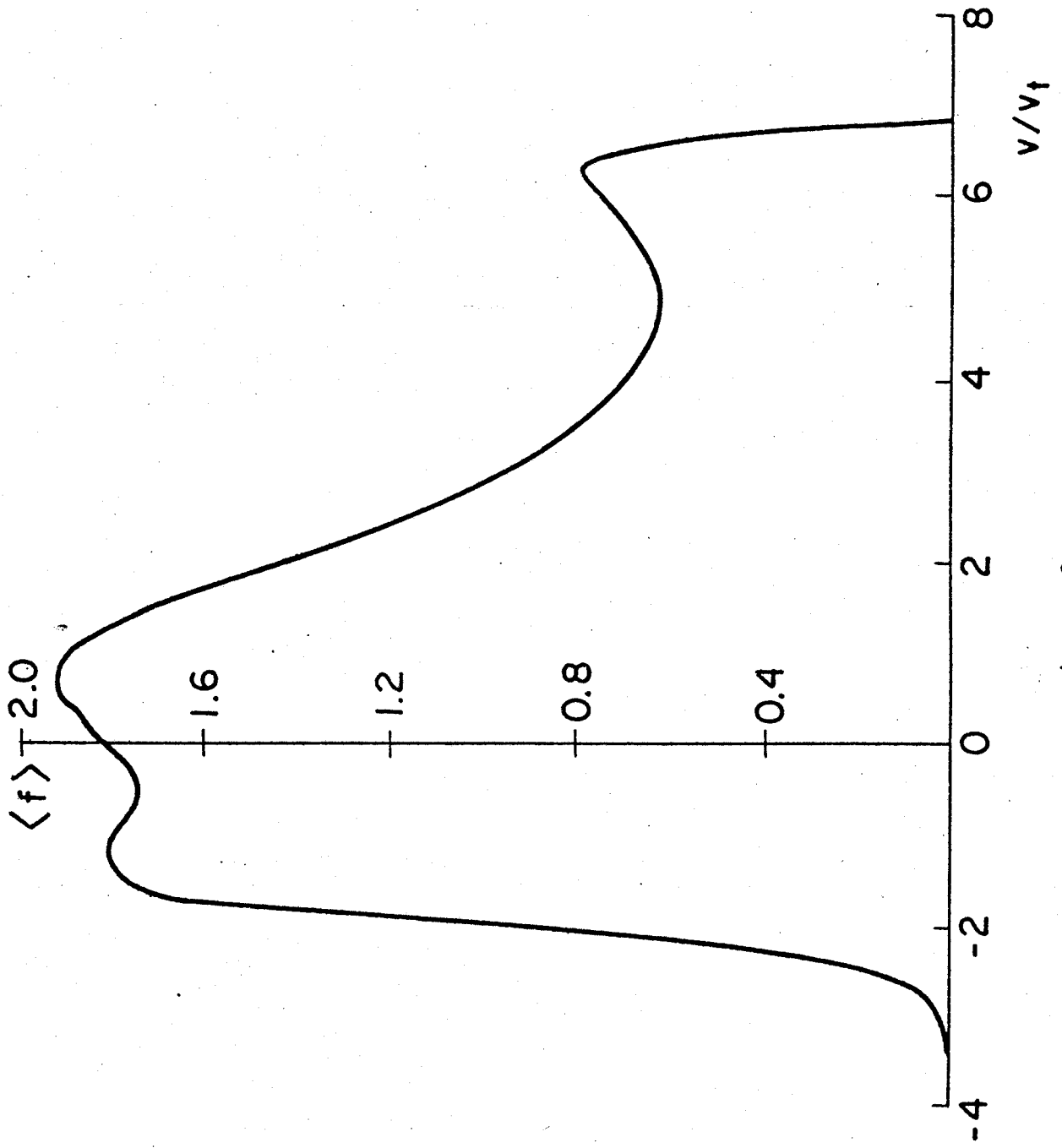


Figure 3

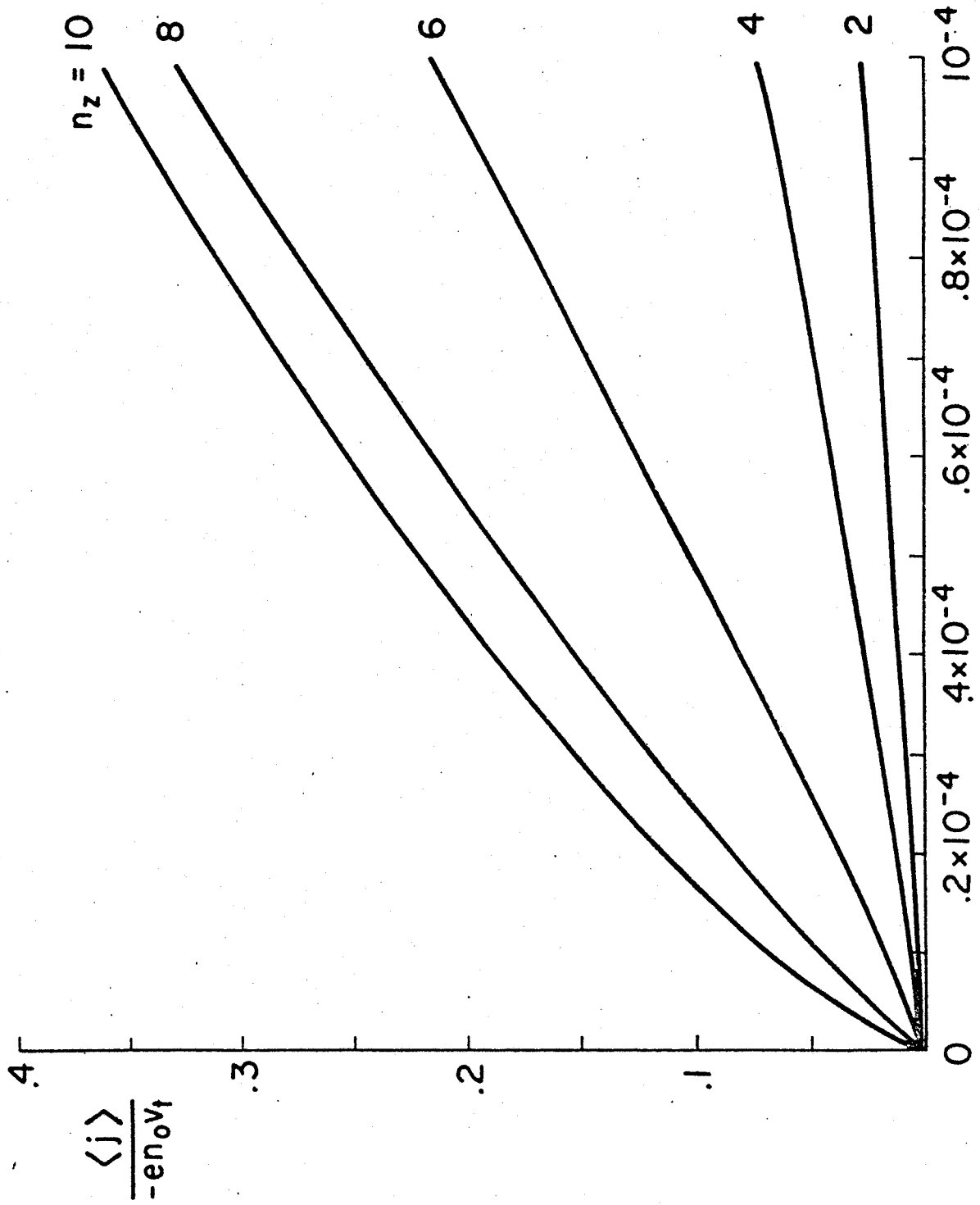


Figure 4
b

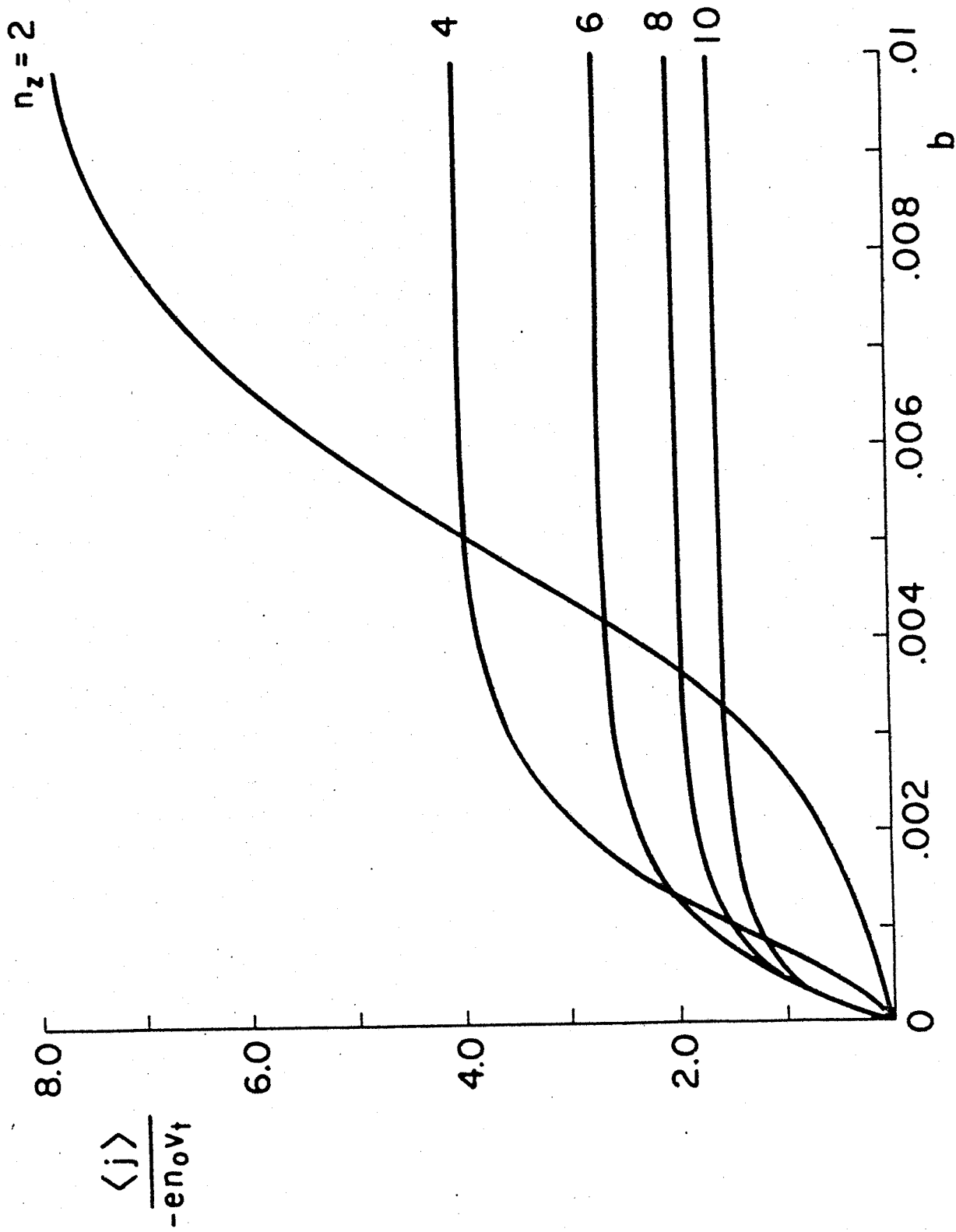


Figure 5

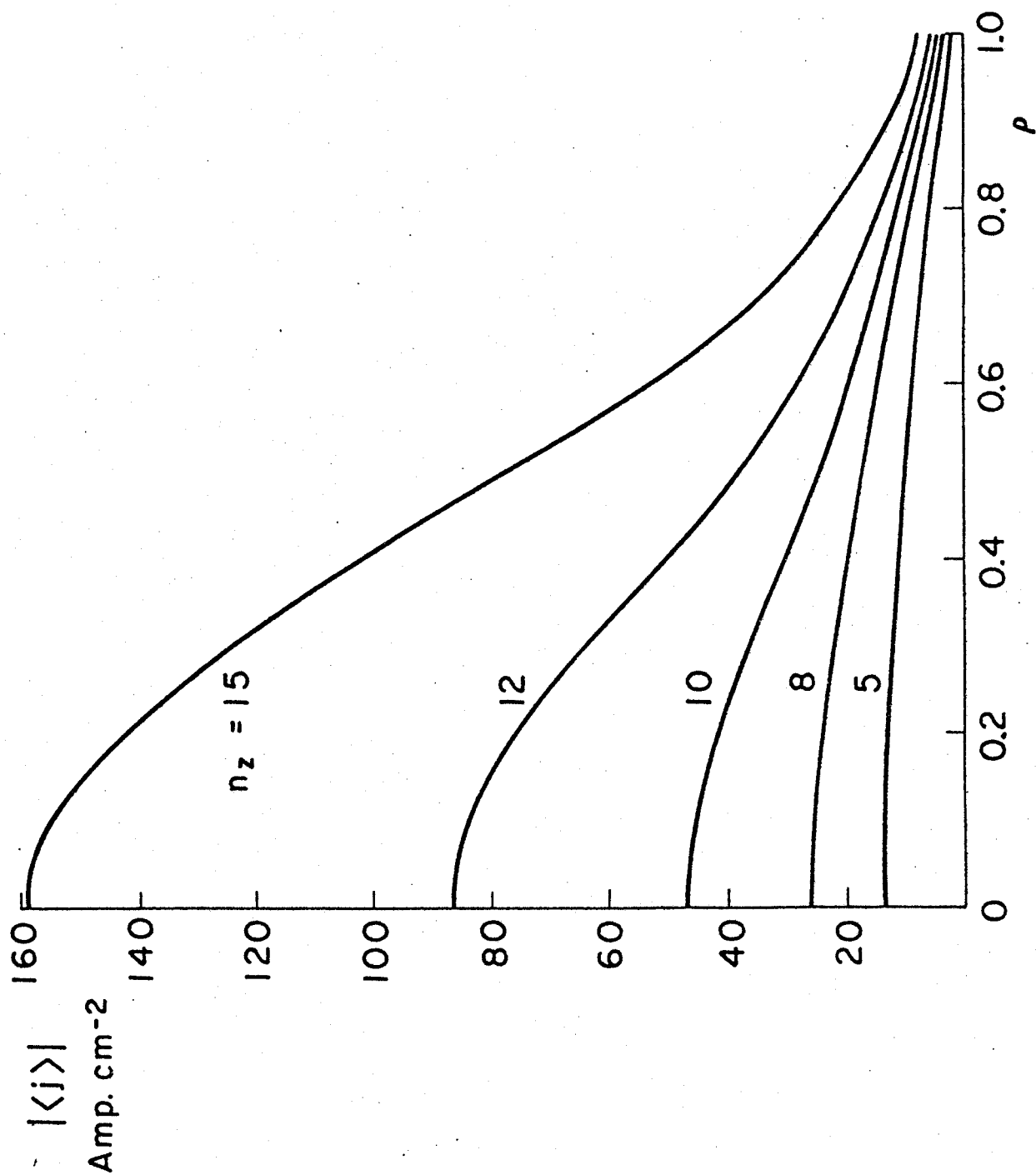


Figure 6

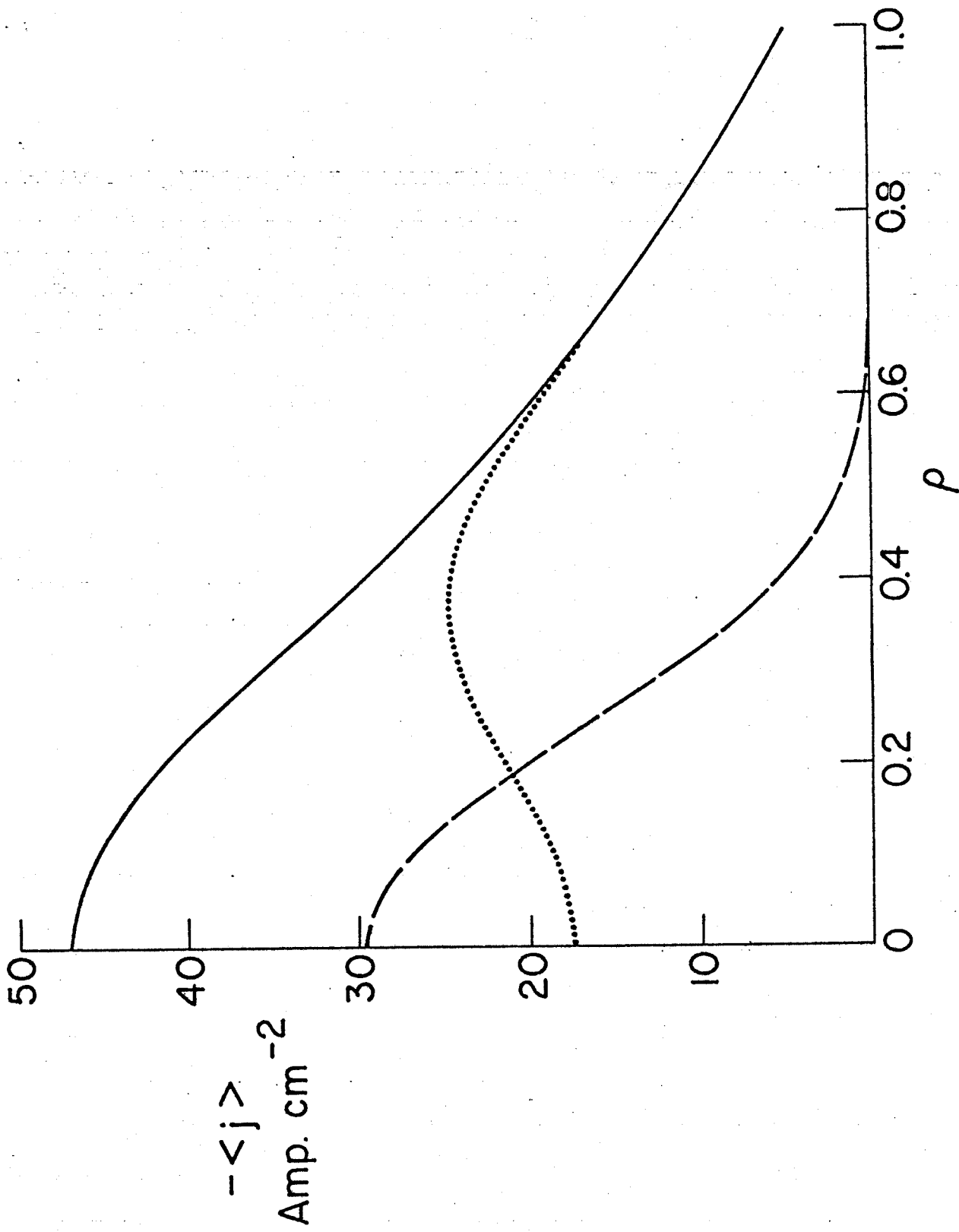


Figure 7

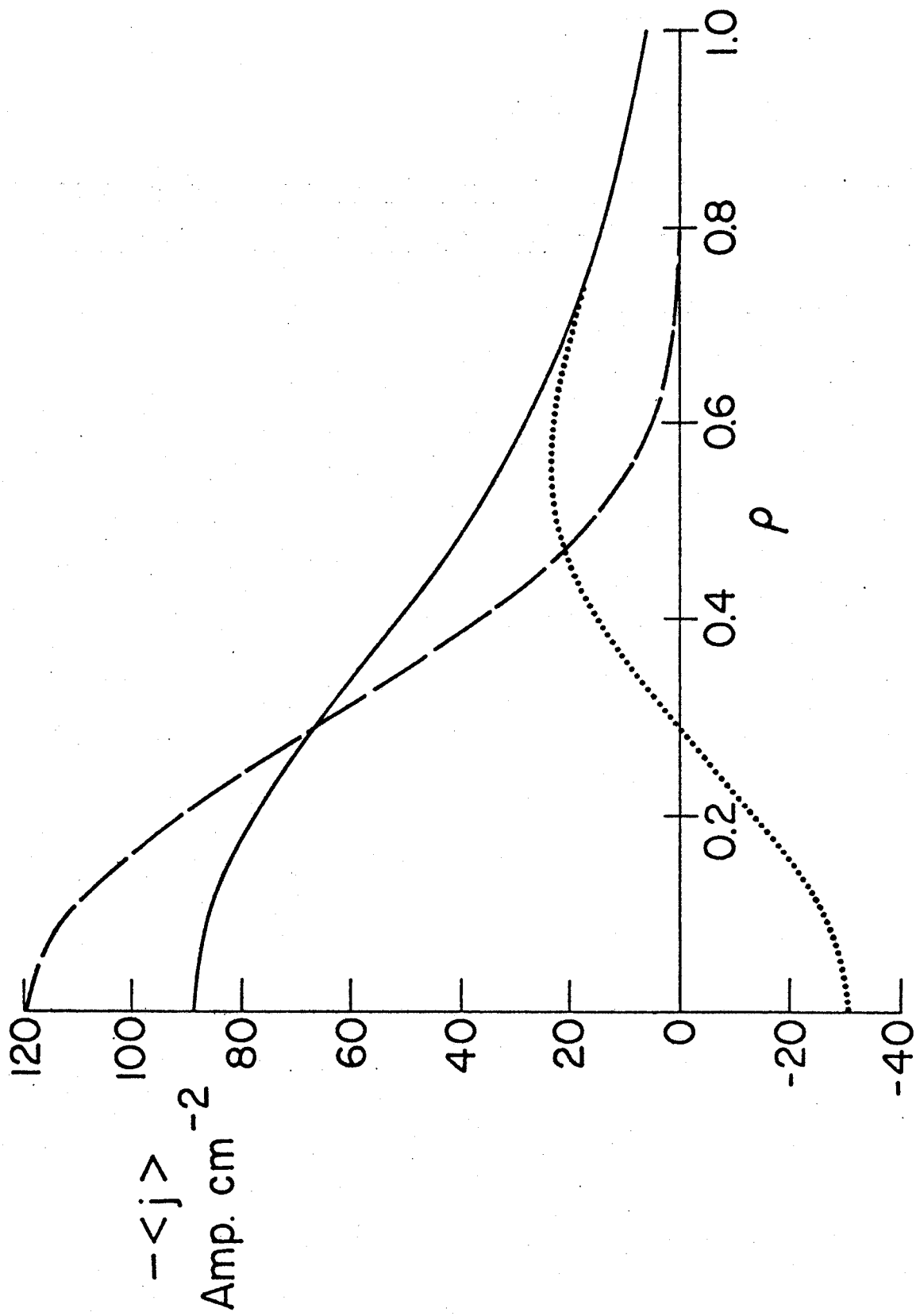


Figure 8

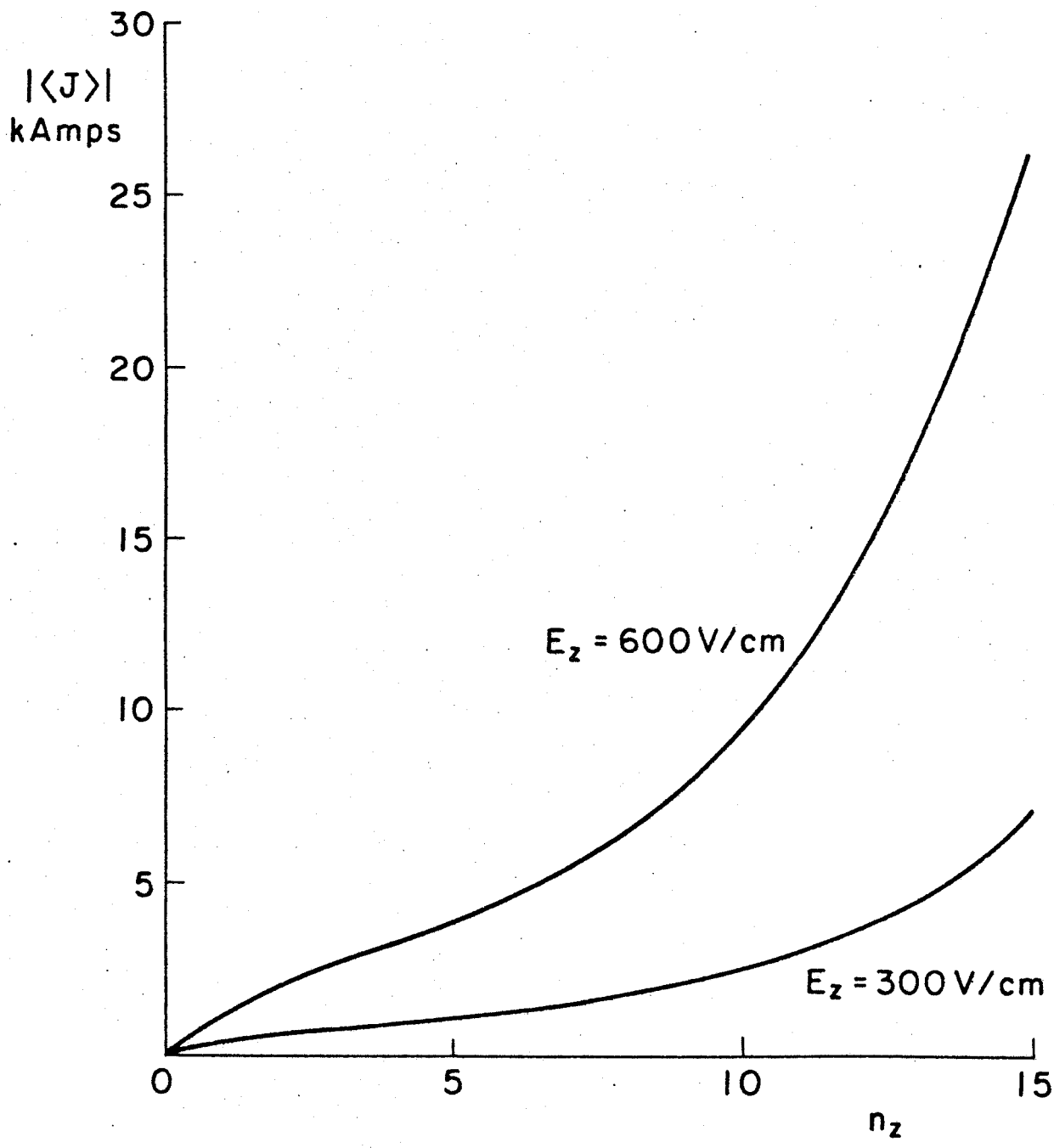


Figure 9