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ON IMPURITY TRANSPORT IN TOKAMAKS

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ABSTRACT

We study theoretically the effects of diffusion, convection, and sawteeth on impurity profile evolution in tokamaks. Sawtooth effects are modeled after our experimental observations that internal disruptions flatten radial profiles of impurity densities in the Alcator-C plasma core. We present numerical simulations and analytic results for confinement times and profile shapes, and we predict that sawteeth may significantly decrease impurity confinement times under some circumstances.

Because impurities play a crucial role in determining plasma characteristics in tokamaks, it is important to understand impurity transport. Here we investigate theoretically some general consequences of diffusion, convection and sawtooth oscillations (periodic "internal disruptions"), using experimental observations as a guide to modeling effects of sawteeth. Internal disruptions are known to affect the transport of electrons and working-gas ions, flattening radial profiles of temperatures and densities in the plasma core (presumably by momentarily providing a radial magnetic field component and allowing rapid radial transport along field lines). We show here that disruptions also flatten radial profiles of individual charge states of impurities. A numerical code is then used to simulate impurity profile evolution in impurity injection experiments and in "steady-state" plasmas; and an analytic model, based on eigenfunctions of the transport equation, is used to explicitly relate confinement times and profile shapes to properties of the impurity flux and to sawtooth effects. Our earlier work[1] along these lines neglected convection, but experimental results[2,3] indicate that convection may be important in some tokamaks, and our theoretical results here indicate that the effects of sawteeth on impurity confinement can be much more important if convection is present.

Silicon has been injected[4] into the Alcator-C plasma for impurity transport experiments; it appears first as a shell at the outer surface of the plasma, then penetrates to the plasma core, and finally leaves the system with an exponential decay rate. Using a method we've described recently, we have followed the evolution of absolute radial density profiles of different silicon ionization states and of the sum of all states.[1,5] Figure 1a shows what happens to the helium-like and hydrogen-like ions as a consequence of an internal disruption during the inflow stage, when the silicon density profile

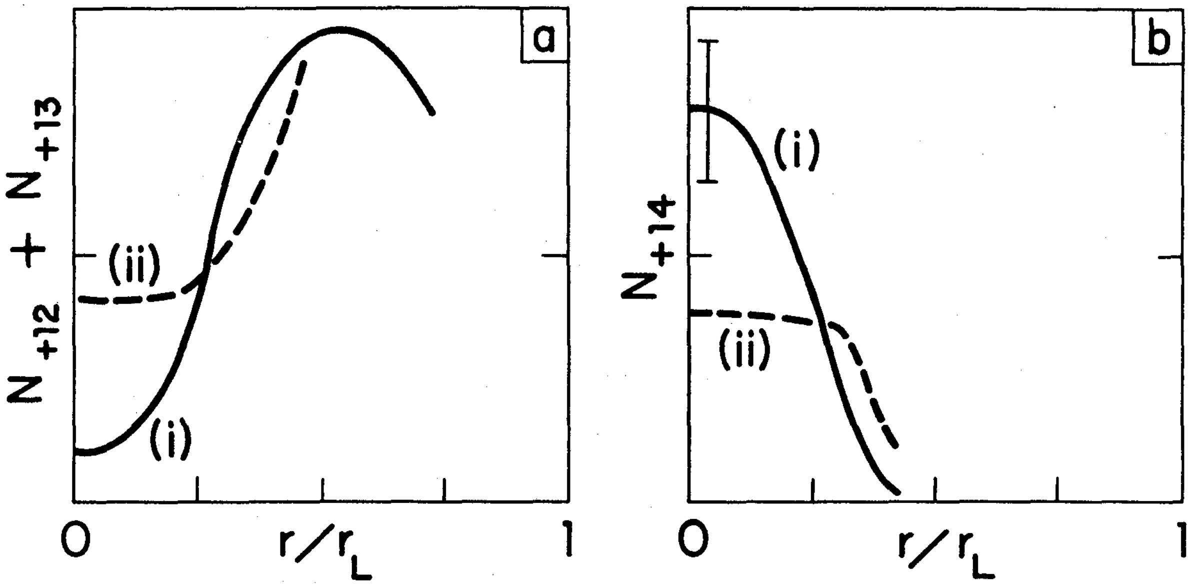

FIG. 1. Observed ion density profiles just before (i) and after (ii) internal disruptions, for silicon injected into an Alcator-C deuterium discharge. a) Sum of helium-like and hydrogen-like ion densities, about 6 ms after injection. b) Densities of the fully-stripped ion, about 15 ms after injection. r is the minor radius; r_L is the limiter radius. See Ref. [5] for experiment details.

is hollow: the central part of the density profile is flattened, resulting in an abrupt inward movement of particles. After the silicon has penetrated to the plasma core, its distribution eventually becomes peaked on axis. Figure 1b shows how the fully-stripped state responds to an internal disruption at such a time: density gradients are once again reduced in the center, though the result of flattening this time is an outward movement of particles. These results are consistent with other experimental indications that impurity ions move in and out during sawtooth oscillations[6] and that impurity density profiles are flatter in the presence of sawteeth.[7]

This response to sawteeth has been incorporated into a numerical transport code. We assume that, apart from effects of sawteeth, transport is governed by

$$\frac{\partial N}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(D\frac{\partial}{\partial r} + V\right)N\right] \ ; \qquad N(r_L) \equiv 0 \ . \tag{1}$$

$N(r,t)$ is the total impurity density (summed over all ionization states), and r_L is the limiter radius. Since the diffusion coefficient $D(r)$ and/or the inward convection velocity $V(r)$ may be anomalous (and of unknown radial dependence) in many plasmas, we simplify this discussion by illustrating here results corresponding only to the simplest choices:

$$D \equiv D_a, \quad V \equiv V_a, \quad S \equiv \frac{r_L V_a}{2\, D_a} \ , \tag{2}$$

where D_a and V_a are constants and the dimensionless "convection parameter" S describes the relative importances of convection and diffusion. In neoclassical transport theory,[8] diffusion and convection coefficients vary with radius but can have ratios corresponding roughly to an S of order the impurity charge Z. Some impurity profiles observed[7,9] in discharges without sawteeth have shapes which can be shown to be consistent with $S \simeq 20$, while certain impurity injection experiments have been interpreted with convection corresponding to S in the range 0 to 4.[2,3,10] Outward convection ($S < 0$) can occur in some neoclassical fluxes and in certain predictions of anomalous transport arising from collective modes[11].

To simulate injection experiments, we assume that an impurity is contained in a narrow shell just inside $r = r_L$ at time $t = 0$. The density profile is then allowed to evolve according to equation (1); but at specified intervals (corresponding to the time t_d between disruptions) the profile is made to be flat inside one radius and to join the original profile at a slightly larger radius r_d, with continuous 1st derivatives everywhere and conserving the total number of particles. (Note that r_d is the <u>outer boundary</u> of the region affected by sawteeth, and not the singular surface radius.) Once values of r_L, D_a, and V_a are specified, behavior in the absence of sawteeth is fully determined and a "confinement" time τ_o, corresponding to the ultimate exponential decay rate, is found. When sawteeth are incorporated, a new confinement time $\tau_d < \tau_o$ always results (see Figure 2); τ_d/τ_o decreases with increasing S, increasing r_d, and decreasing t_d. In possible qualitative confirmation of this result, a substantial reduction of confinement time by sawteeth has recently been observed on PDX (with $r_d \simeq 0.9\ r_L$)[12].

We can simulate a "steady state" plasma situation by continuously injecting new impurities and waiting until a stationary profile is established (see Figure 3). Without convection, equation (1) has a time-independent solution only if $\partial N/\partial r = 0$ everywhere inside the source layer at the surface. Sawteeth have no effect on this flat profile of total impurity

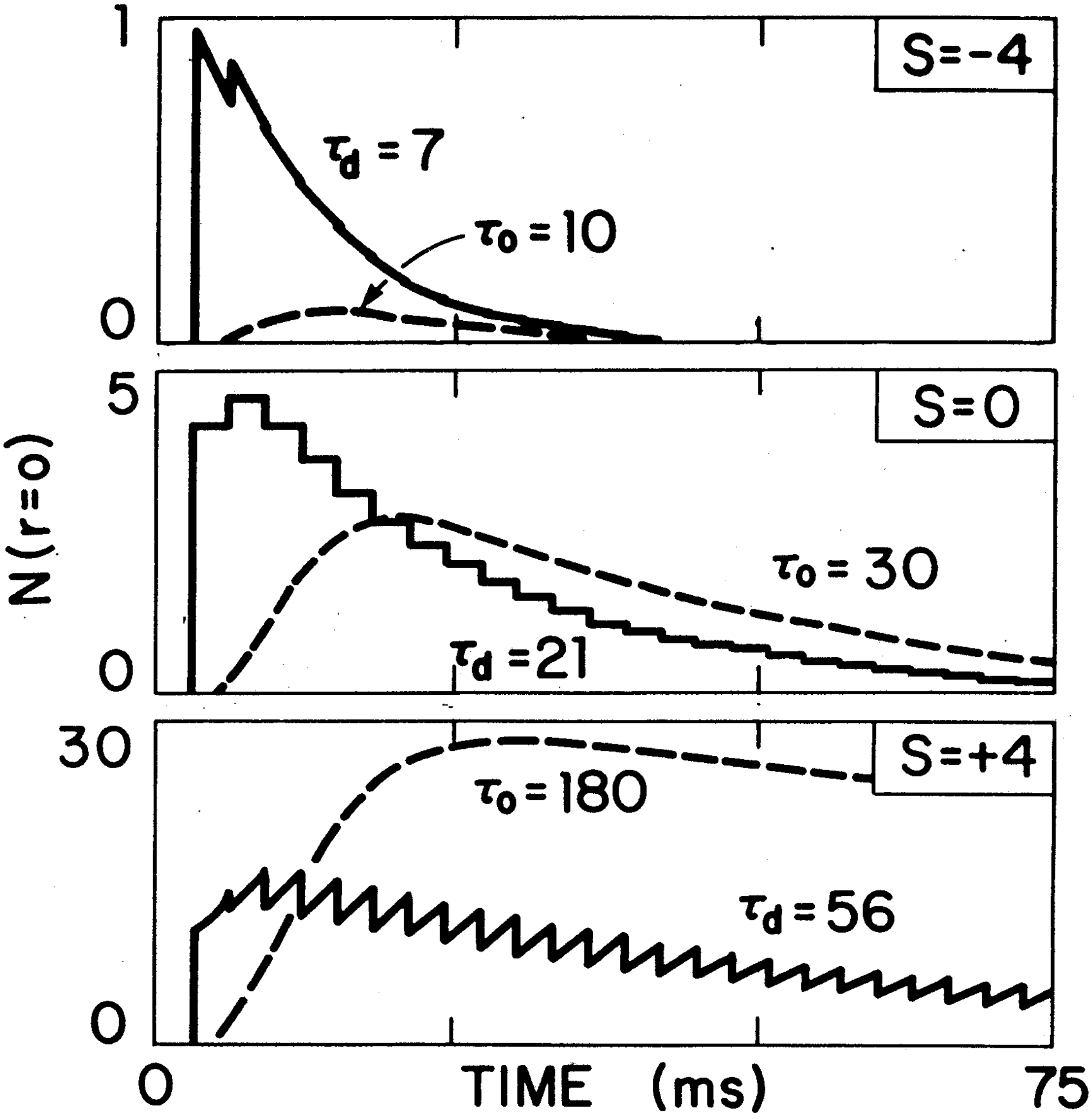

FIG. 2. Central impurity densities from sample numerical simulations of injection experiments, with (solid lines) and without (broken lines) sawteeth. $D_a = 1.48 \times 10^3$ cm^2/sec, r_L = 16 cm, $r_d = 0.75r_L$, and t_d = 3 ms.

density (although the profiles of the individual ionization states, which are not flat, will be periodically flattened by disruptions). The situation is quite different when S > 0, and we contrast 3 cases: (i) Without sawteeth, the equilibrium profile is peaked on axis. (ii) If we add sawteeth, holding fixed the rate of inflow of new material, then the time-average of the new "steady state" profile is flatter in the center, while the gradient of N at the edge remains unchanged since it determines the loss rate that must balance the specified inflow. The result can be substantial reductions in central impurity density and radiation losses. (iii) If, while sawteeth are added to case (i), we hold fixed the total number of particles in the system (to correspond to instantaneous recycling), then the resulting steady-state profile has the same shape as in case (ii) but a larger amplitude. Behavior consistent with the transition between cases (i) and (iii) has been observed in Doublet III (see Figure 6 of Ref. [7], in which measured profiles of intrinsic nickel, with and without sawteeth, can be modeled with $S \simeq 18$ and $r_d \simeq 0.4\ r_L$).

While the numerical code can be used to predict transport details, an analytic model can be used to easily predict essential transport features and provide physical insight. Equations (1) and (2) have discrete solutions $N_n(r)e^{-t/\tau_n}$, and in the absence of sawteeth any evolution can be described through a sum

$$N\ (r,t) = \sum_n A_n N_n\ (r)\ e^{-t/\tau_n}\ . \qquad (3)$$

The eigenfunctions $N_n(r)$ and eigenvalues τ_n have been found for different values of S, and some of them are shown in Figure 4 (see Ref. [13] for details). The "slowest" eigenvalue τ_o corresponds to the confinement time. Its values can be approximated by the formula

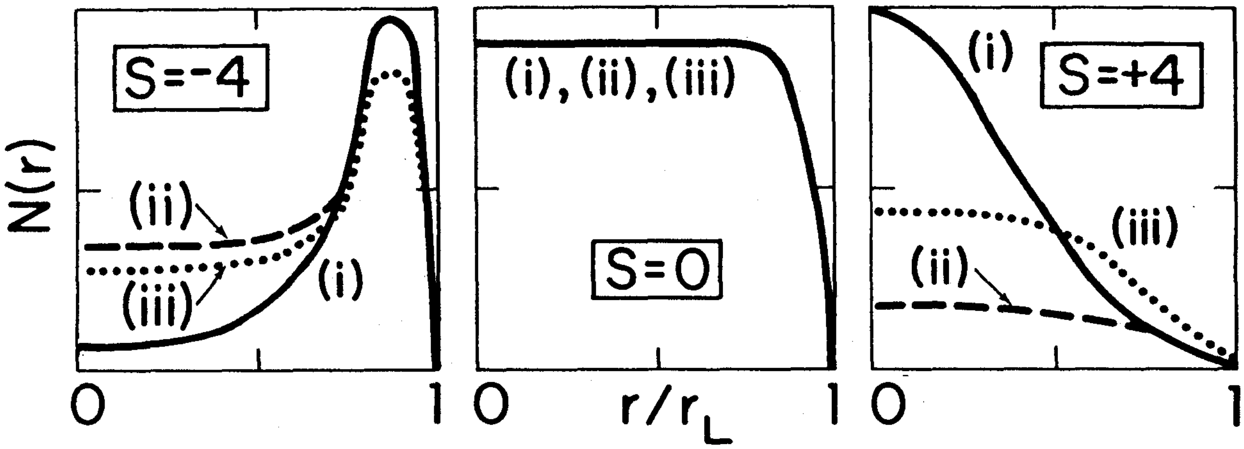

FIG. 3. Sample numerical simulations of steady-state impurity profiles; D_a, r_L, r_d, and t_d are as in Figure 2. (i): No sawteeth; (ii) and (iii): with sawteeth, as described in text.

$$\tau_o \approx \frac{77 + S^2}{56 + S^2} \, \frac{e^S - S - 1}{4S^2} \, \frac{r_L^2}{D_a} , \tag{4}$$

which is useful for determining parameters for numerical simulations when a specific confinement time is desired. The "second slowest" eigenvalue τ_1 represents the time scale for relaxation of an arbitrary initial profile toward the shape $N_o(r)$. τ_1 determines the inflow time in an injection experiment (without sawteeth), and also the repeaking time after flattening by a disruption. We can approximate the effects of sawteeth in the limiting case $t_d \ll \tau_1$ by constraining a profile to stay flat inside r_d; outside r_d, any profile will eventually adopt the shape of $N_o(r)$. The confinement time is then $\tau_d = \alpha \, \tau_o$, where

$$\alpha \equiv \frac{\int_0^{r_d} r \, dr \, N_o(r_d) + \int_{r_d}^{r_L} r \, dr \, N_o(r)}{\int_0^{r_L} r \, dr \, N_o(r)} . \tag{5}$$

$\alpha(S, r_d)$ is plotted in Figure 4; it decreases rapidly as S and r_d increase. If $t_d \gtrsim \tau_1$, a flattened profile has time to relax toward $N_o(r)$ between disruptions, and the time-averaged shape of the profile leads us to the more general expression

$$\tau_d \approx (\alpha + \sigma - \alpha\sigma) \, \tau_o , \tag{6}$$

$$\sigma \equiv 1 - t_d^{-1} \int_0^{t_d} dt \, e^{-t/\tau_1} . \tag{7}$$

Equations (4) and (6) give results in good agreement with numerical simulations.

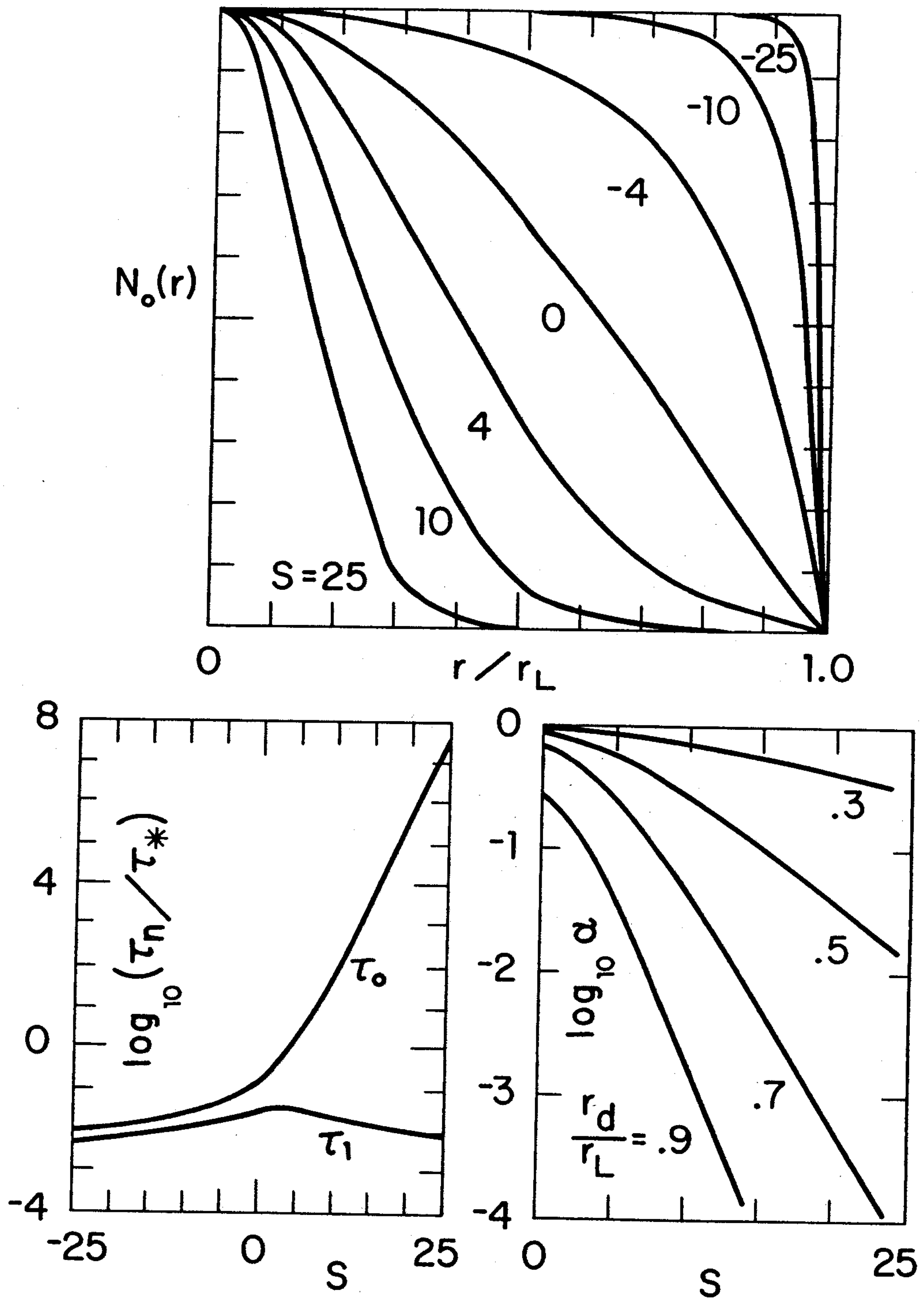

FIG. 4. $N_o(r)$; τ_o and τ_1, in units of $\tau_* \equiv r_L^2/D_a$; and $\alpha(S, r_d)$.

The quantitative results presented here are useful because they correspond to a form for the impurity flux (defined through equations (2)) which is commonly used in modeling,[2,3] and because they illustrate the general nature of impurity profile evolution when diffusion, convection, and sawtooth effects are present in various proportions. We've predicted that sawteeth can modify inflow times, profile shapes, confinement times, and radiative loss rates; these effects can be modest under some circumstances, but they can be quite significant if the impurity flux contains a significant inward convection term and/or if sawteeth affect a sufficiently large fraction of the plasma volume. When analyzing experimental data, it is important to remember that details of the above results can be altered if D and V have radial variations (see Ref. [13]). In particular, radial variations can change the ratio τ_1/τ_0, which means that a comparison of inflow time to decay time in an injection experiment is not by itself sufficient for determining whether convection is present; measurements of time scales <u>and</u> radial profiles are necessary. An additional complication for interpretation of real experiements is that the convection velocity may fluctuate drastically with <u>time</u> if it behaves anything like the neoclassical convection velocity; this velocity is proportional to <u>gradients</u> of background plasma quantities (such as temperature), and these gradients are strongly affected by sawteeth.

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