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# SENSITIVITY OF ENERGY MARGIN AND COST FIGURES OF INTERNALLY COOLED CABLED SUPERCONDUCTORS (ICCS) TO PARAMETRIC VARIATIONS IN CONDUCTOR DESIGN

by J.H. Schultz and J.V. Minervini
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Sensitivity of Energy Margin and Cost Figures
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J.H. Schultz and J.V. Minervini
Massachusetts Institute of Technology, Plasma Fusion Center

Abstract–Length-independent models of internally cooled cabled superconductor (ICCS) recovery from rapid disturbances are described. The length independence makes these models sufficiently simple to allow systematic variations of parameters for a given design application. A worked example of parametric variations with a time-dependent model is presented for the mission of the fusion Multipurpose Coil. The time-dependent model suggests an analytic design criterion for the design of ICCS conductors, that is independent of both time and length, for the design of ICCS conductors, based on a simple power balance. An example, using NbTi, is used to illustrate the criterion and contrast with the results using $Nb_3Sn$.

## BACKGROUND

Traditionally, few methods have been available for the variation of parameters in order to improve or optimize cost figures for the design of internally cooled cabled superconductors (ICCS). A one-dimensional, time-dependent, finite-element code developed in 1978 by Arp has been widely used to analyze recovery from a fixed disturbance in a point design [1]. This code is also useful for long-time phenomena such as quench propagation. However, with present-day supercomputers, Arp's code is too slow to be used in parametric variations for the purpose of optimizing cost-performance parameters in the early stages of magnet design. In 1977, Hoenig made the experimental observation that the recovery of an ICCS was relatively independent of the Reynold's number of forced-flow supercritical helium in the conductor [2]. These results were duplicated by Miller, Lue and Shen [3] and were plotted against a curve of the enthalpy of helium in a conductor envelope from bath to current-sharing temperature, in order to illustrate that conductor stability exhibited a "well-cooled" regime, in which the energy margin of an ICCS was essentially equal to the available enthalpy to current-sharing in the envelope helium, and an "ill-cooled" regime in which the energy margin rapidly became more comparable to the enthalpy rise in the metal alone. This led to the perception that there was a broad range of design parameter space within which an ICCS would be well-cooled. An assumption of well-cooled design allowed Miller and others to generate design curves of equal energy margin, permitting optimization of conductor design [4]. Dresner [5] derived an expression for calculating the transition between the "well-cooled" and "ill-cooled" regime, based on the observation that the foot of the stability curve occurs when

$$\frac{I^2\rho}{A_{cu}} = Ph(T_c - T_b) \tag{1}$$

where $I$ is the conductor current (A), $\rho$ is the resistivity of copper $(\Omega - m)$, $A_{cu}$ is the cross-sectional area of copper (m$^2$), $P$ is the wetted perimeter (m), $h$ is the heat-transfer coefficient (W/m$^2$-K), $T_c$ is the critical temperature, and $T_b$ is the bath temperature (K).

Dresner postulates a transient cooling rate that implies

$$h \propto v^{4/5} D^{-1/5} \tag{2}$$

where $v$ is the heat flux induced flow velocity (m/s) and $D$ is the conductor hydraulic diameter, which leads to the prediction [6] of a transitional current density of:

$$J_{co} \propto \sqrt{[f(1-f_{co})/f_{co}]\,\frac{(T_c - T_b)}{\rho_e} t^{-1/5} l^{2/5} D^{-1}} \tag{3}$$

where $J_{co}$ is the current density in the metal strands of the conductor (A/m$^2$), $f$ is the volume fraction of copper in the metal, $f_{co}$ is the volume fraction of metal in the cable space, $T_c$ is the critical temperature (K), $T_b$ is the ambient helium temperature (K), $\rho_e$ is the resistivity of the copper $(\Omega - m)$, $t$ is the duration of the disturbance (s), $l$ is the length of the heated zone, and $D$ is the hydraulic diameter of the helium-filled part of the cable space (m). This design criterion is not very sensitive to the assumptions made about the duration or length of the disturbance. However, the length dependence is sufficiently significant that an uncertainty of a factor of ten in the length of a disturbance would cause a change of a factor of 2.5 in the design operating current, which is not an acceptable situation for a large, expensive magnet. This led to a design method, adopted by Miller, in which the conductor is designed to satisfy a specified energy margin, while remaining below the transition current. A variation of this is to assume that advanced superconductors will always have adequate energy margins in the well-cooled regime, and to impose only the second design constraint. This design method probably has a broad range of applicability, but also raises concerns. In the typical design case, neither the length nor the duration of a disturbance are well defined. The time dependence of Dresner's equation agrees with experiment [7], but does not exclude other time-dependent mechanisms, such as boundary layer formation or heat removal during a finite time pulse. The time dependence in Dresner's equation is also of a form that must break down at very long and very short times. The length dependence has not been successfully correlated with experiment [5] and must also break down at very long lengths. Two practical examples of suspected breakdowns of the model would be transients in tokamak poloidal field coils, in which hundreds of meters are heated fairly uniformly, very long tokamak start-up and ramp-down transients, and extremely short time stick-slip events.

In 1980, Hoenig proposed a length-independent numerical model to describe conductor recovery [8]. This model used a single equivalent heat transfer coefficient to simulate the transient properties modelled in the one-dimensional code, and is described below. In subsequent years, the authors have modified this code in order to increase its speed of execution and to automatically calculate the energy margin (energy deposition for marginal recovery) for pulses of different durations and shapes. The code has also been modified to include transient heat transfer due to Kapitza resistance and transient conduction, but the implications of these modifications are not reported here. The studies reported below are also restricted to the limiting case of instantaneous energy deposition in the conductor, which, for many applications, is the correct basis for conservative design. A determination of the absolute validity of different correlations for the heat transfer coefficient is beyond the scope of this paper. However, it should be pointed out in favor of Hoenig's suggestion, that if it is true or approximately true over a broad range of physical parameters, it is favored for design by being independent of assumptions about disturbances or boundary conditions.

An obvious insight about time integration, neglecting end effects, is that once a constant heat transfer coefficient has been selected, it is probably no longer necessary to do a time integration in order to select conductor operating current. The fraction of critical current at which the transition between the well-cooled and ill-cooled regimes takes place should be:

$$f_{tr} = \frac{P\,h\,(T_c - T_b)}{\rho A_{cu} J_c^2} \tag{4}$$

One method of design then is to calculate the transition fraction analytically, and design to an operating current less than that current, for example, 0.8 $f_{tr}$. If $f_{tr}$ is greater than one, which is frequently the case with $Nb_3Sn$, the conductor will be in the well-cooled regime all the way out to the critical current, and one could design to a high fraction of critical current. If there is an independent specification for energy margin, it could be checked against the helium enthalpy to current sharing, and the current would be reduced further, if the specification is not satisfied. Similarly, in the case of very long time disturbances, the energy balance criterion in the helium would always be used. This simple analytic method may be adequate for specifications as complex as that of the Multipurpose Coil. However, a detailed point design should then be done to verify the specification.

A concern with the power balance criterion, proposed above, is that it has not been established that it really simulates time integration models over a broad range. In particular, it is clear that in the case of very poorly cooled conductors, the energy margin is only equal to the enthalpy of the metal to current sharing, $H_{metal}(T_{cs}) - H_{metal}(T_b)$, and it is thus not certain that the scaling with the difference between the bath and critical temperature will have a sharp relation to the lower knee of the transition, as stated by Dresner [5]. A numerical study over all of the examples reported below showed that the maximum ratio of initial heating to cooling with a fixed heat transfer coefficient always occurs at $T_c$, so that this intuition is apparently correct. The transitional currents were also calculated for all of the designs reported below. They were only less than one for four design cases with $Nb_3Sn$ and in all three designs for NbTi. In all designs modeled so far, $f_{tr}$ falls somewhere between the upper and lower knee of the transition. Furthermore, in all cases, a design to 0.8 $f_{tr}$ would place the conductor in a high energy margin regime with an adequate energy margin if quality control degraded the critical current by 10 %. It should be stressed that, so far, we are only calibrating a numerical model against its analytical consequences. However, it might also be stated that the results of experiments and analysis by more sophisticated techniques do not preclude the possibility that a length independent model can be the basis of a design method with broad applicability.

## METHOD

The model used in these studies was first proposed by Hoenig [8], and has been modified for additional speed and flexibility by the authors. The key assumption of the model is that heating and cooling occur uniformly along a length of conductor. The basic energy balance equation for the wire is given by:

$$\delta Q_{net} = Q_h + Q_{imp} - Q_c \tag{5}$$

where $\delta Q_{net}$ is the net heat flux into the wire (W/m), $Q_h$ is the rate of Joule heating (W/m), $Q_{imp}$ is the externally imposed heating (W/m), and $Q_c$ is the convective heat flux to the helium (W/m).

The differential change in wire temperature $dT_w$ is determined by equating the net heat flux to the change in enthalpy of the wire:

$$dT_w = \frac{\delta Q_{net}}{A_w C_v} dt \tag{6}$$

where $A_w$ is the cross-section area of the wire ($m^2$) and $C_v$ is the specific heat of the metal composite ($J/m^3$). The differential change in helium temperature can be determined from the change in enthalpy of the helium, either at constant pressure (open system) or constant density (closed system). The results presented in this paper assume helium at constant density. The change in helium enthalpy comes from the heat input to the fluid by cooling the wire:

$$dH_{he} = \frac{Q_c}{\rho_{he} A_{he}} dt \tag{7}$$

where $\rho_{he}$ is the helium density ($kg/m^3$) and $A_{he}$ is the helium cross-section area ($m^2$).

## RESULTS: MULTIPURPOSE COIL

A pulsed Multipurpose Coil, proposed jointly by the Lawrence Livermore Laboratory and M.I.T., is designed to advance superconducting magnet technology for tandem mirror choke coils and tokamak poloidal and toroidal field coils. It aims at achieving new benchmarks for all-superconducting solenoids with high fields, high current densities and exposed to both long and short field ramps.

The base conductor for the first four studies reported here is the Airco/Oxford bronze method $Nb_3Sn$ cable, used in the Westinghouse LCP coil and the M.I.T. 12 T coil. The most significant variation in these conductors from the previous applications is the use of an Incoloy 903 sheath, in order to decrease the compressive prestress on the $Nb_3Sn$. This conductor is a leading candidate for the Multipurpose Coil. With the present specifications, the outer coil would require an overall current density of about 50 $A/mm^2$, if it is designed for a peak field of 10 T or about 40 $A/mm^2$ at 12 T. Requirements for internal structural reinforcement within the winding pack are held at a constant fraction of total winding pack volume in these studies.

The first parametric study examines the effect of changing the background field to 8 T or 12 T, as shown in Figure 1. These curves show higher energy margins at low field and low current, corresponding to the higher critical and current-sharing temperatures. At high fractions of critical current, the energy margins are higher for high field conductors, because of the smaller Joule heating. Normalizing the results to the critical current creates an apparent paradox, by making the performance of a conductor look worse in low field, although the absolute value of the current at a given energy margin is always higher at low field, as shown in Figure 2. Figure 2 gives a better measure of the limits on coil performance, by taking the fractional space requirements for conduits, insulation and additional structure, in order to compare the overall current density of the winding pack, $\lambda J$, with the energy margin at different fields. The increase in achievable current density is faster than linear with inverse maximum field for fixed energy margin and even faster for a fixed fraction of critical current.

In the next study, the copper/noncopper ratio was varied at a fixed field of 10 T, as shown in Figures 3 and 4. At a copper/noncopper ratio of 1, the calculated transition between the ill-cooled and well-cooled regimes occurs at $f_{tr}$=0.67, as shown in Figure 3. For the higher ratios, $f_{tr}$ is greater than 1.0, which means that they are always in the well-cooled regime. Since the current-sharing temperatures are identical, the energy margins of copper/noncopper = 2 and 3 almost overlay each other. However, figure 4 shows that the low copper option has better performance at every energy margin. At 400 mJ/cm$^3$, the current density of the copper/noncopper = 1 option is not significantly better than that at copper/noncopper = 2. However, because of its higher amount of superconductor, it can also extend out to high $\lambda J$ without exceeding the critical current. Using the design criterion of $f = 0.8\ f_{tr}$, the achievable $\lambda J$ is 42, 41.5 and 31 A/mm$^2$ for copper/noncopper ratios of 1,2 and 3, respectively. Therefore, in order to really take advantage of the low copper design, it is necessary to adopt a more aggressive design strategy of not designing in the well-cooled regime.

Fixing the copper/noncopper ratio at 1.74 and the field at 10 T, the void or helium fraction of the inside conductor envelope was varied, as shown in Figure 5. In this case, there is a performance crossover, so that there is an optimum void fraction for any energy margin. If one is willing to design to low energy margins, very high overall current densities can be achieved with low helium void fractions, while design for high energy margins favours high void fractions. The crossovers occur in the region of 30-50 kA/mm$^2$, implying that performance cannot be greatly improved by optimizing the void fraction in that region.

Finally, a sensitivity study was done, using, as a thought experiment, a series of arbitrary multipliers on the critical current of the superconductor. Multipliers less than one might physically correspond to degraded conductor performance, due to poor quality assurance, while multipliers greater than one might correspond to the performance of more advanced conductors, such as internal tin. The purpose of this study is to assess the impact of degraded performance and the motivation for the development of more advanced conductors. Figure 6 shows the overall winding current densities as functions of energy margin for critical current density multipliers of $1/\sqrt{2}$, 1, $\sqrt{2}$, and 2. The effect of critical current density degradation is nearly linear. However, the effect of critical current density improvement appears to saturate rapidly, except at very low energy margins. Within the quantization error of our calculations, there is no significant improvement in the stability margin at 10 T for a conductor with a critical current density double that of the bronze method conductor over that of a conductor with 1.4 times the critical current density.

The performance of NbTi at 8.0 T was evaluated for copper/noncopper ratios of 1, 2 and 3, as shown in Figures 7 and 8. Figure 7 shows a transition between the well-cooled and ill-cooled regimes that is much sharper than with $Nb_3Sn$. Consequently, the estimate of $f_{tr}$, the fraction of critical current density at the transition between the well-cooled and ill-cooled regimes is much better defined. Figure 7 shows that the estimate of $f_{tr}$ in equation 4 falls towards the middle of the transition region. In all cases examined so far, the calculated value of $f_{tr}$ is at a higher energy margin than the lower knee of the energy margin curve. Also, in all cases examined so far, a design method that specified an operating current at 0.8 $f_{tr}$, when $f_{tr} < 1$ and 0.8 when $f_{tr} > 1$ would predict reasonable performance. Figure 8 shows the performance achievable with NbTi at 8 T. There is very little difference in performance between the three copper/noncopper ratios, except in the uninteresting, low energy margin regimes. These curves give some insight into why NbTi is generally not selected for advanced ICCS applications and why so many designs and design studies have limited NbTi ICCS to overall current densities of 20 A/mm$^2$.

## CONCLUSIONS

- A time-dependent model of ICCS recovery can be used to give insight into design options for specific magnet applications. Removal of length dependence allows for a broad range of options to be examined, without necessarily sacrificing accuracy in predicting conductor recovery.

- ICCS design might be simplified considerably by the application of an analytical power balance for short disturbances and energy balance for long disturbances of infinite length. An expression for the transitional fraction of critical current has been suggested.

- The ambitious goals of the Multipurpose Coil project would be aided by lower copper/noncopper ratios, lower helium void fractions, and higher critical current densities. In order to comfortably achieve the design goals with present technologoy, it will be useful to simultaneously adjust all the variable parameters, including the shape and disposition of structure [9].

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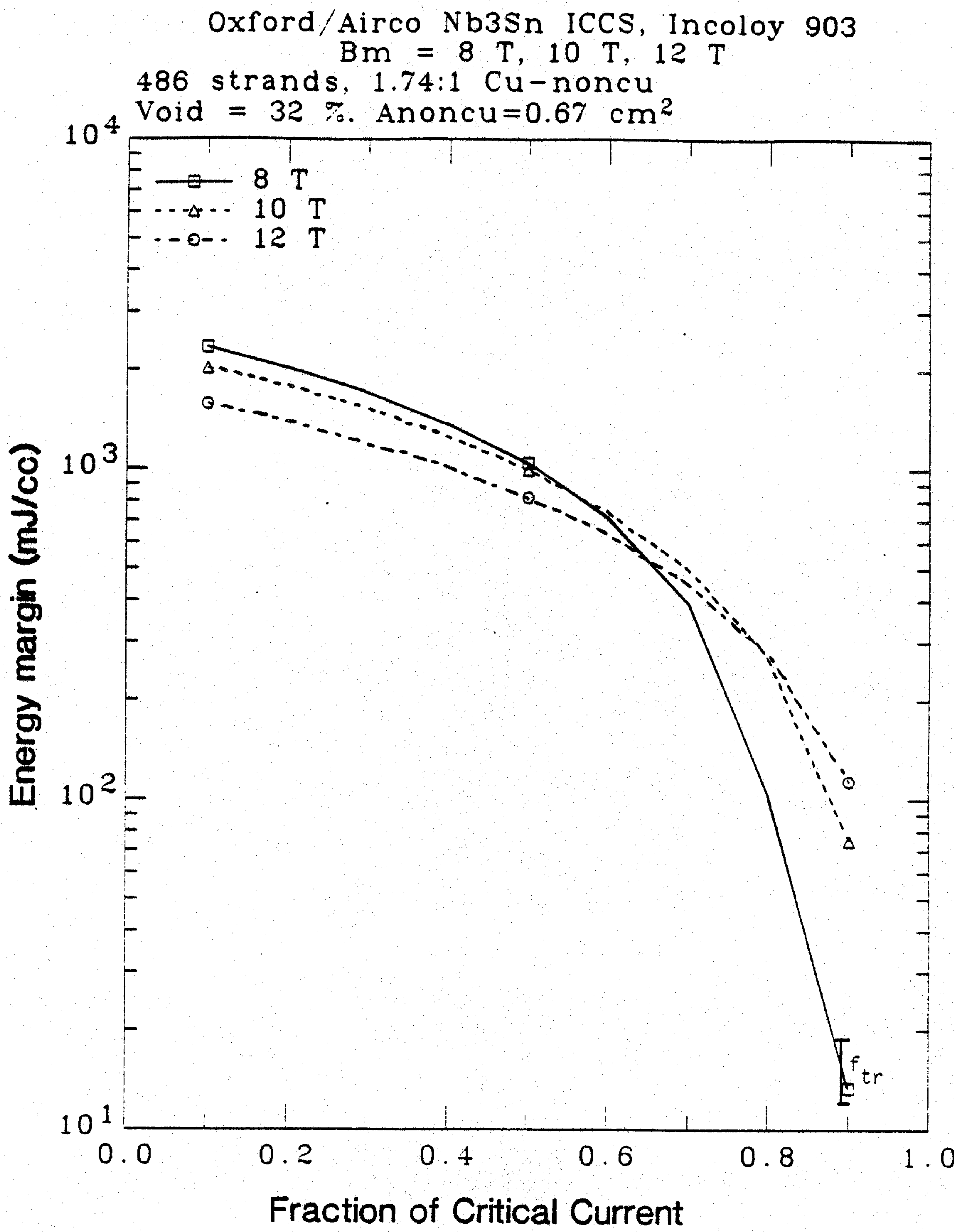


Figure 1

Energy Margin vs. Fraction of Critical Current at Different Fields

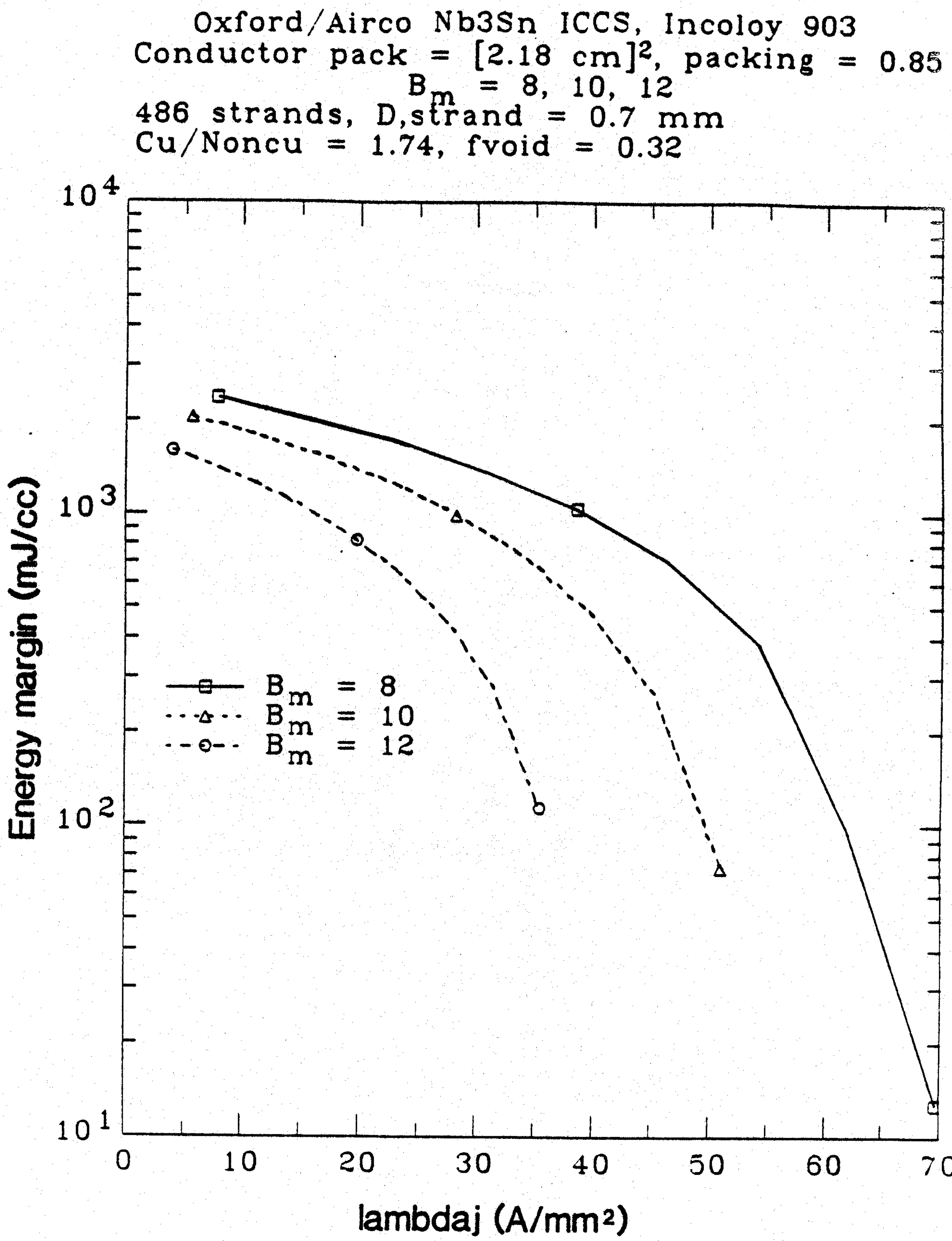


Figure 2

Energy Margin vs. Overall Current Density at Different Fields

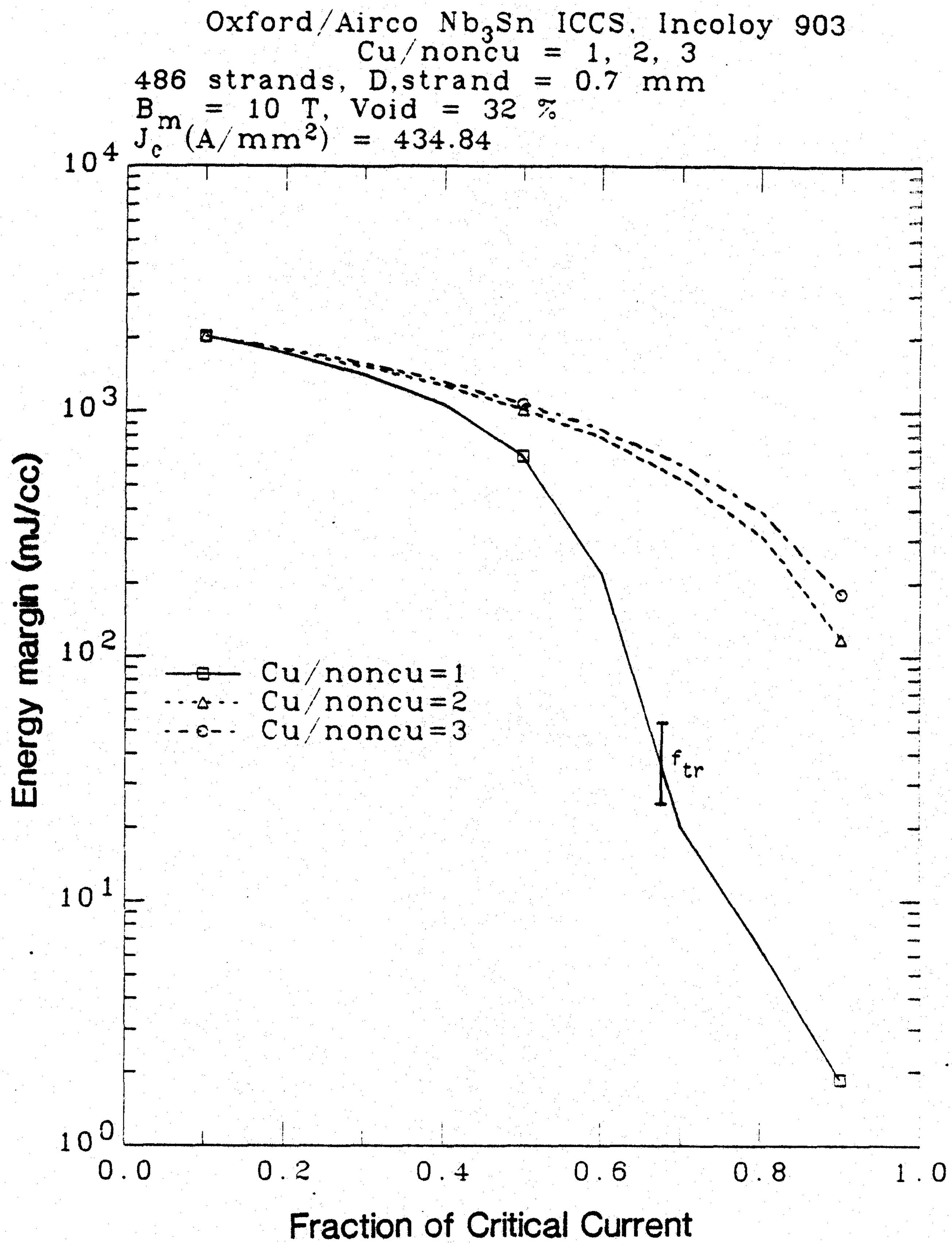


Figure 3

Energy Margin vs. Fraction of Critical Current at Different Copper/Noncopper Ratios

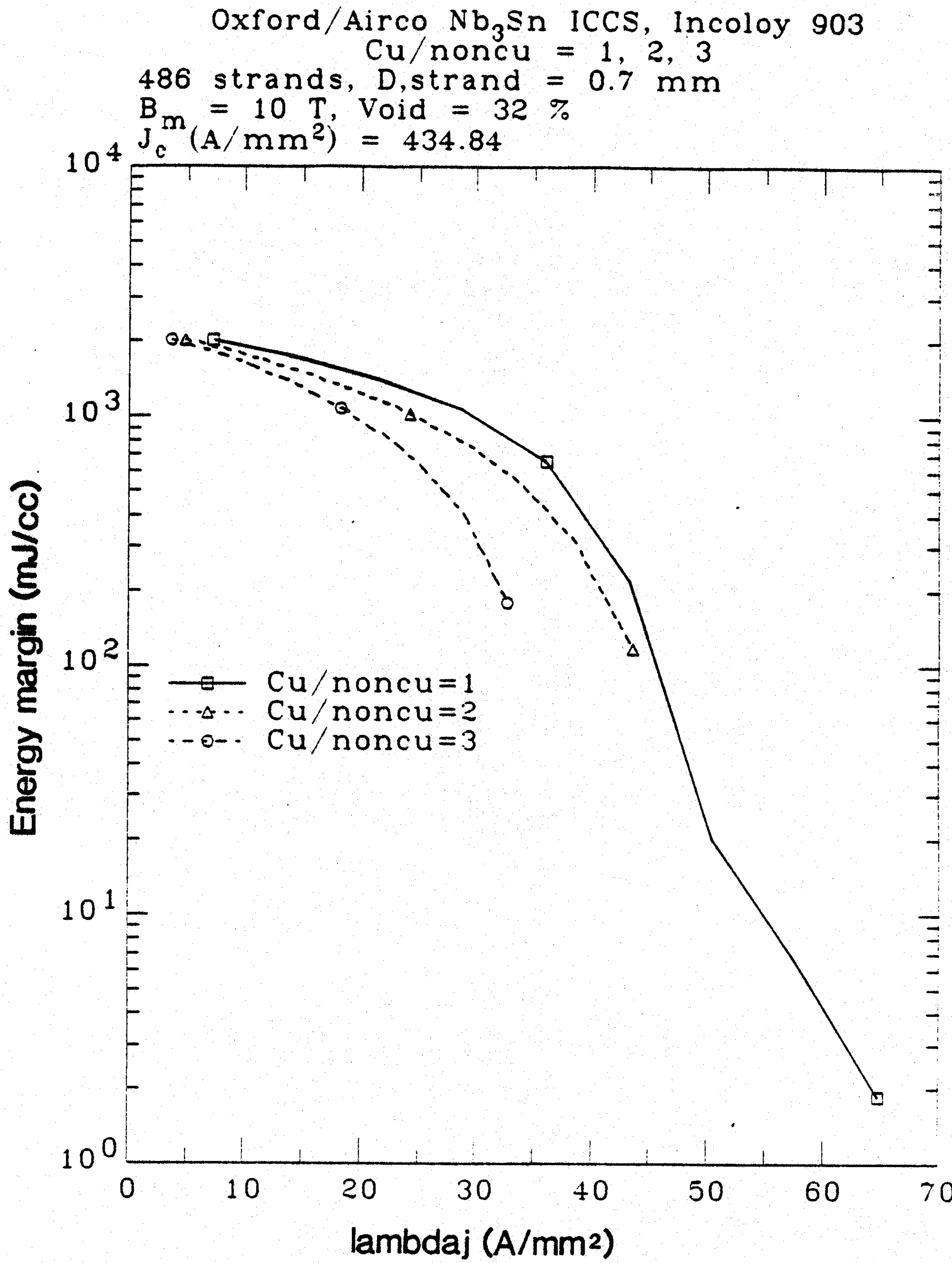


Figure 4

Energy Margin vs. Overall Current Density at Different Copper/Noncopper Ratios

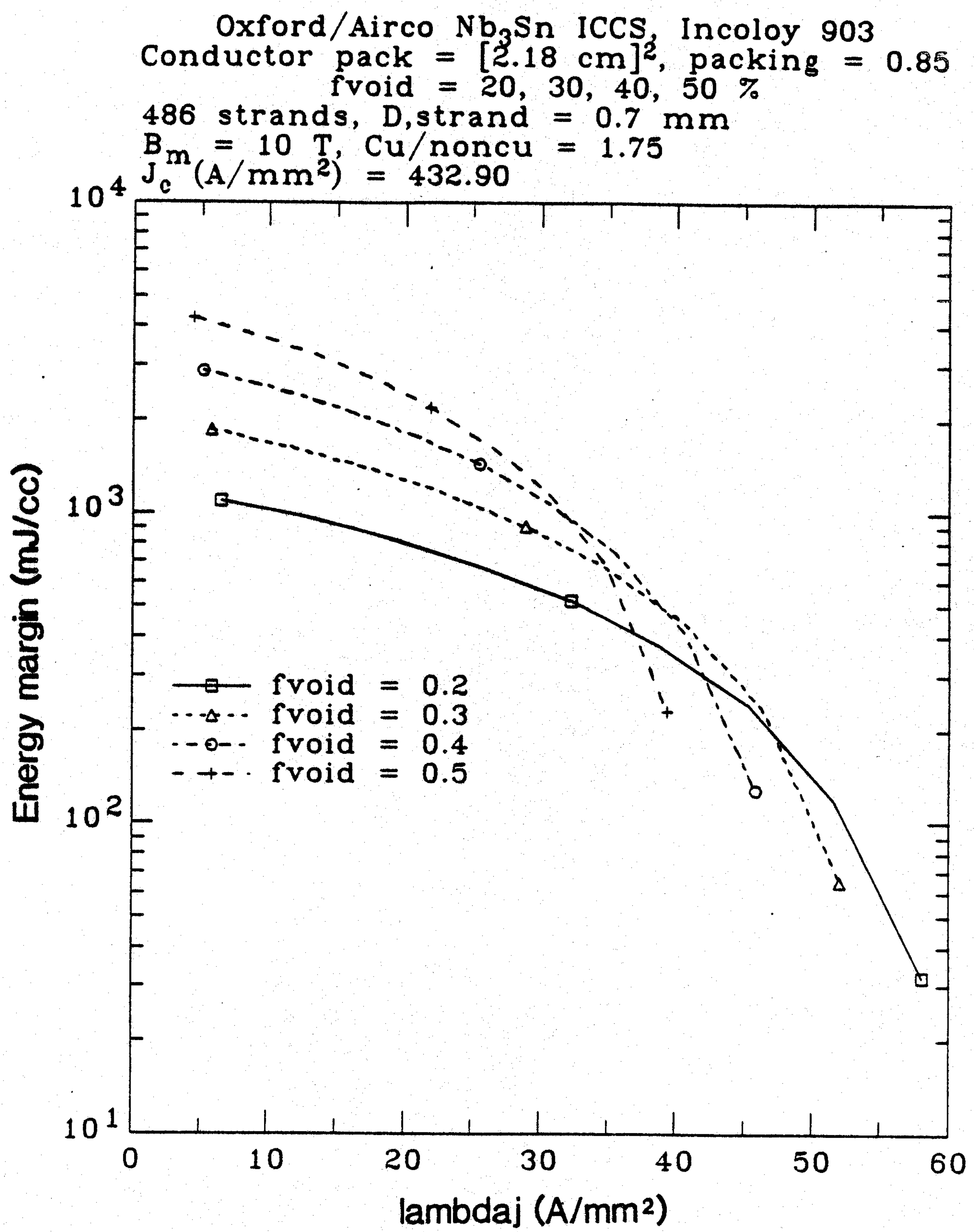


Figure 5

Energy Margin vs. Overall Current Density at Different Helium Void Fractions

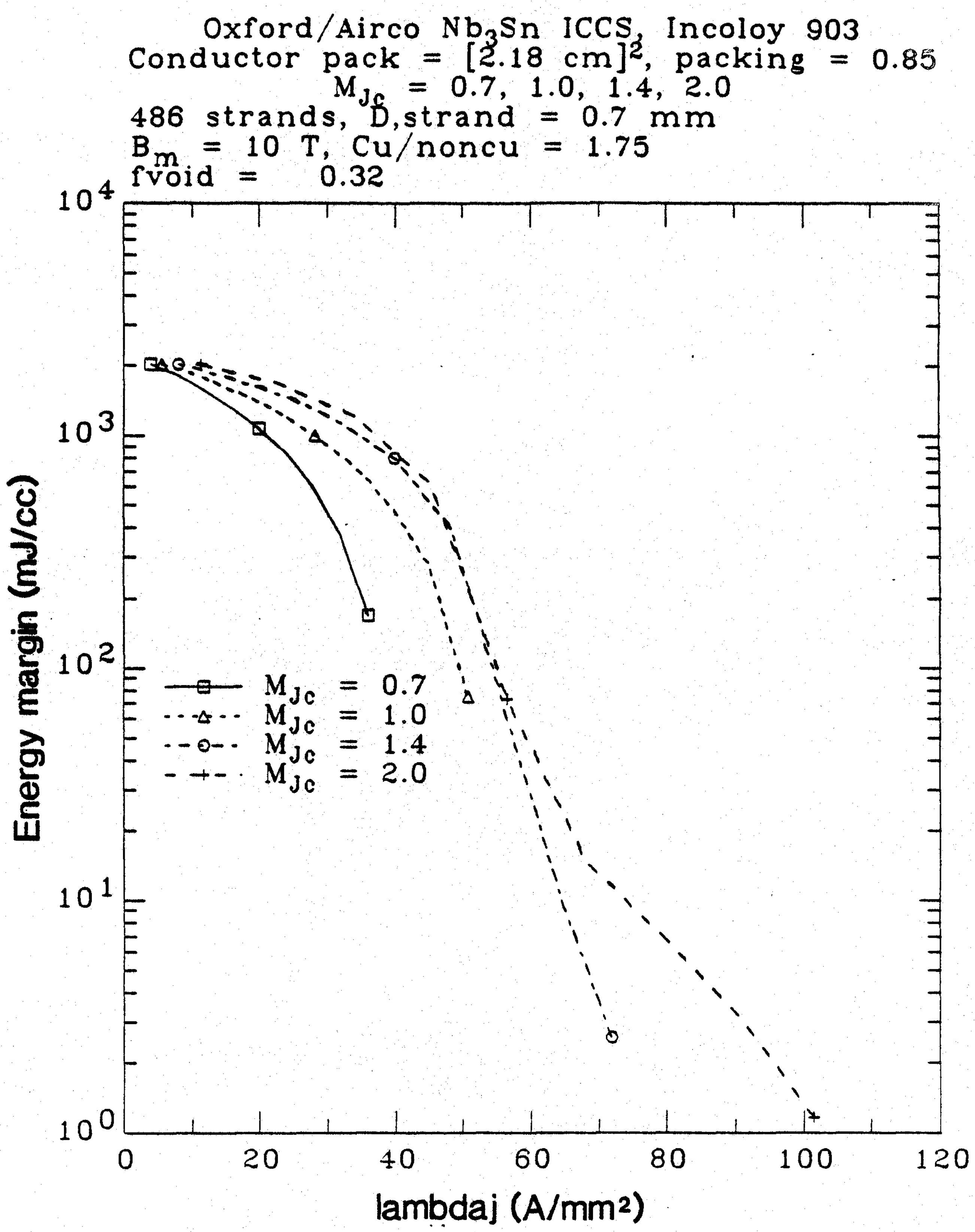


Figure 6

Energy Margin vs. Overall Current Density with Different Critical Current Density Multipliers

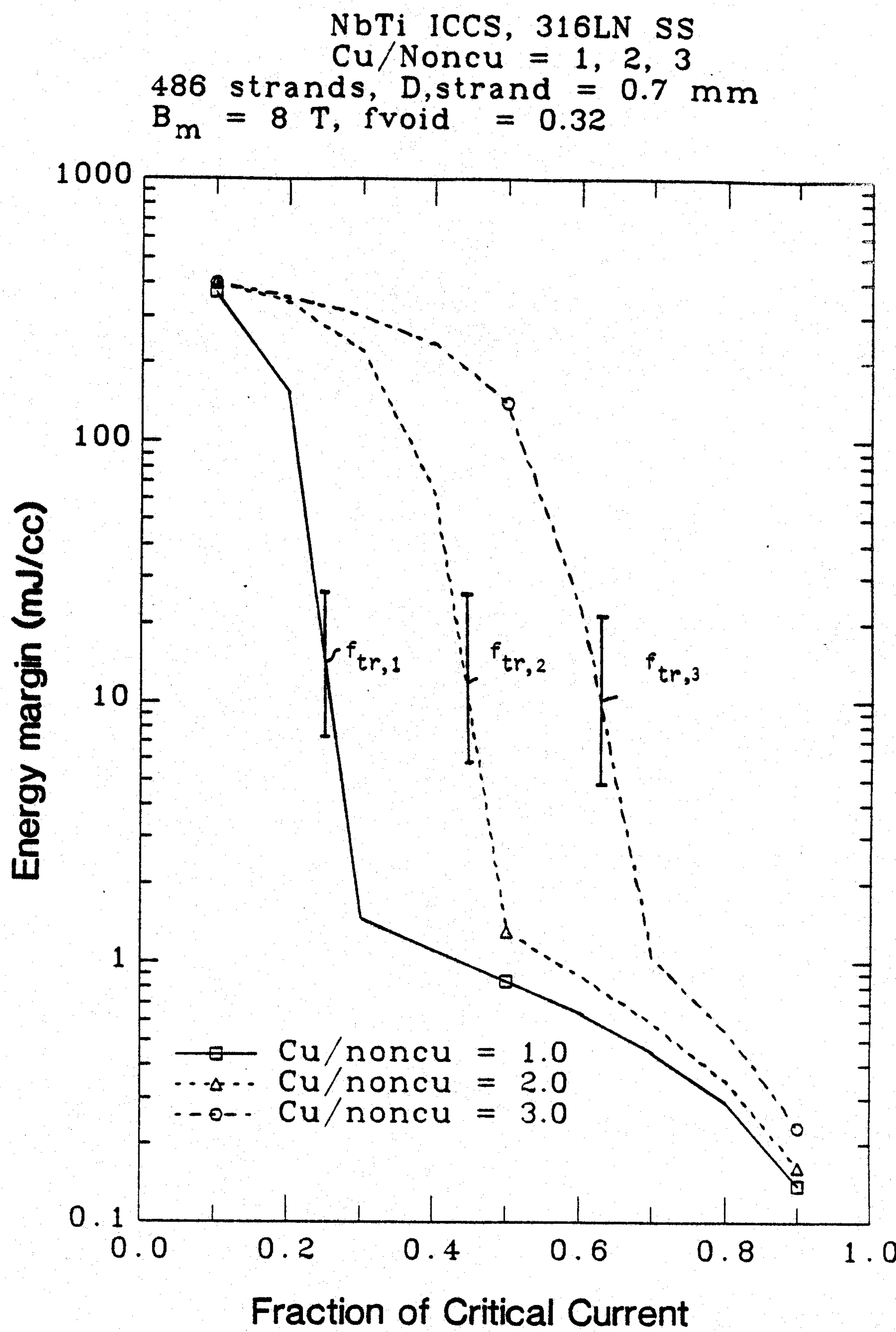


Figure 7

Energy Margin vs. Fraction of Critical for NbTi at Different Copper/Noncopper Ratios

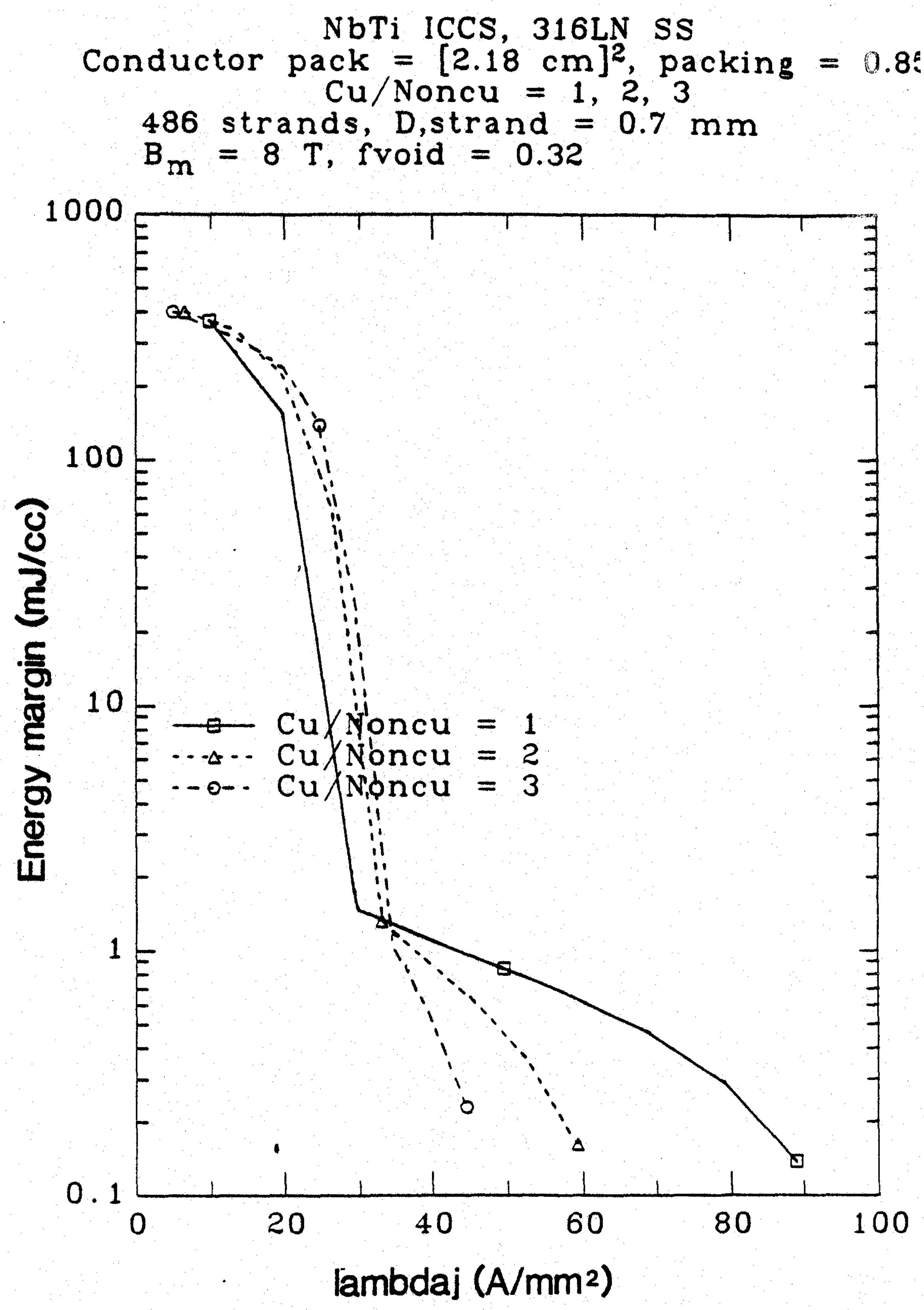


Figure 8

Energy Margin vs. Overall Current Density for NbTi at Different Copper/Noncopper Ratios