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ABSTRACT

The nonlinear interaction of a tenuous continuous electron beam with a traveling electromagnetic wave in the cyclotron resonance laser accelerator is analyzed using a single-particle model and multi-particle simulation. It is found that the maximum beam energy gain and acceleration distance obey certain scaling laws for optimized systems, and that the maximum energy gain is limited primarily by the degree of wave dispersion and is almost independent of the wave amplitude.

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There has been considerable interest recently in laser accelerators[1] with the potential for achieving high accelerating gradient. One of such systems is the electron cyclotron resonance laser (CRL) accelerator[2] that makes use of the resonant interaction of a coherent electromagnetic wave with a beam of electrons gyrating in a (guide) magnetic field. An intriguing feature of the CRL mechanism is autoresonance,[3,4] that is, the synchronism between the electrons and laser field is self-sustained in the course of acceleration in an ideal situation so long as the system is initially in synchronism. Although recent proof-of-principle experiments[5-7] have successfully demonstrated the acceleration of electrons to moderately relativistic energies using microwaves and the technique of tapering the guide field, there are severe limitations posed by laser field dispersion, synchrotron radiation losses, and the available strength of magnetic field in laboratories.

In this Letter, we examine the effects of laser field dispersion on the performance of CRL accelerators. In particular, the nonlinear interaction of a tenuous continuous electron beam with a traveling electromagnetic wave in a tapered guide field is analyzed using a one-dimensional single-particle model and three-dimensional, self-consistent, multi-particle simulation. The dynamical bunching of the beam electrons in a synchronous phase validates the use of the single-particle model. It is found in the single-particle model, and confirmed in the multi-particle simulation, that the maximum beam energy gain and acceleration distance obey certain scaling laws for optimized systems, and that the energy gain is limited primarily by the degree of laser field dispersion and is almost independent of the strength of the laser field.

We first consider the motion of an individual electron in the field configuration consisting of a right-hand circularly polarized electromagnetic wave $(\omega, k_{\parallel}\vec{e}_z)$ and a linearly tapered guide magnetic field (with $\nabla \cdot \vec{B} = 0 = \nabla \times \vec{B}$)

$$\vec{B}(\vec{x}) = B_0\Big(1 + \frac{z}{L}\Big)\vec{e}_z - \frac{B_0}{2L}(x\vec{e}_x + y\vec{e}_y) , \tag{1}$$

where $L = B_0^{-1} dB_z(z)/dz$ characterizes the degree of the taper. The normalized vector potential of the wave and guide fields can be expressed as

$$\frac{e\vec{A}(\vec{x})}{mc^2} = \left[-\frac{\Omega_{c0}yz}{2cL} + a\cos(k_{\parallel}z - \omega t)\right]\vec{e}_x + \left[\frac{\Omega_{c0}x}{c} + \frac{\Omega_{c0}yz}{2cL} - a\sin(k_{\parallel}z - \omega t)\right]\vec{e}_y \ . \quad (2)$$

In Eq. (2), m and $-e$ are the electron mass and charge, respectively; c is the speed of light *in vacuo*; $\Omega_{c0} = eB_0/mc = \text{const.}$ is the nonrelativistic cyclotron frequency; $k_{\parallel} = 2\pi/\lambda$ and $a = eE/\omega mc$ are the (axial) wave number and normalized wave amplitude of the laser field, respectively.

When synchrotron radiation losses are negligibly small, the dynamics of an individual electron can be described by the Hamiltonian

$$H(\vec{x}, \vec{P}, t) = [(cP_x + eA_x)^2 + (cP_y + eA_y)^2 + c^2P_z^2 + m^2c^4]^{1/2} \ . \quad (3)$$

By performing a time-dependent canonical transformation from $(\vec{x}, \vec{P})$ to $(\phi, \psi, z', P_\phi, P_\psi, P_{z'})$, and assuming the transverse drift of the electron guiding center be small (with $\psi \cong 0 \cong P_\psi$), it can be shown that the Hamiltonian in the new variables is an approximate constant of motion and can be expressed in the form[8]

$$H'(\phi, z', P_\phi, P_{z'}) = -\omega P_\phi + \gamma mc^2$$

$$= -\omega P_\phi + \{2mc^2\Omega_c(z')P_\phi + 2a[2m^3c^6\Omega_c(z')P_\phi]^{1/2}\cos\phi + c^2(P_{z'} + k_{\parallel}P_\phi)^2 + (a^2+1)m^2c^4\}^{1/2}. \quad (4)$$

Here, $z = z'$ is the axial distance; $P_z = P_{z'} + k_{\parallel}P_\phi$ and γmc^2 are the axial momentum and total (rest plus kinetic) energy of the electron, respectively; $\Omega_c(z) = eB_z(z)/mc$ is a z-dependent nonrelativistic cyclotron frequency. Because Eq. (1) can be generalized to represent arbitrary piecewise linear taper, the Hamiltonian in Eq. (4) provides a good approximation to the problem for a continuous tapering of the guide field.

The constancy of H' can be interpreted as follows. For an untapered system, the Hamiltonian in Eq. (4) is independent of z'. Thus, $P_{z'}$ is a constant of motion, and the total axial momentum of the (electron plus photon) system is conserved, i.e., $P_z + n\hbar k_{\|} = P_{z'} + k_{\|}P_\phi + n\hbar k_{\|} =$ const., where n is the number of photons in the laser field and $h = 2\pi\hbar$ is the Planck constant. As δn photons are emitted or absorbed, from the conservation of axial momentum, it follows that the change in P_ϕ is given by $\delta P_\phi = -\delta n\hbar$. On the other hand, from the conservation of energy, we have $\delta\gamma mc^2 = -\delta n\hbar\omega$. This leads to $\delta H' = -\omega\delta P_\phi + \delta\gamma mc^2 = 0$. Therefore, the constancy of H' states simply the conservation of total energy of the system.

The electron phase ϕ and energy γmc^2 evolve according to the Hamilton equations

$$\frac{d\phi}{dt} = -\omega + \frac{1}{\gamma}\left[\Omega_c + \frac{k_{\|}}{m}(P_{z'} + k_{\|}P_\phi) + a\left(\frac{2mc^2\Omega_c}{P_\phi}\right)^{1/2}\cos\phi\right] \tag{5}$$

and

$$\frac{d\gamma}{dt} = \frac{\omega a}{\gamma}\left(\frac{2\Omega_c P_\phi}{mc^2}\right)^{1/2}\sin\phi\ . \tag{6}$$

At the cyclotron resonance, $\omega = \Omega_c/\gamma + k_{\|}v_z$, the electrons with different initial phases are rapidly bunched around a synchronous phase of $\phi_s \cong \pi/2$ by the sinusoidal forcing term in Eq. (5), and then they act like a macro-particle if the resonance is sustained. It is this bunching effect that justifies the validity of the single-particle approximation to the multi-particle problem.

For a non-dispersive laser field with $\beta_{ph} \equiv \omega/ck_{\|} = 1$ and a uniform magnetic field $\Omega_c(z) = \Omega_{c0}$, it can be easily shown from Eqs. (5) and (6) that for $\phi = \phi_s = \pi/2$,

$\gamma(\omega - kv_z) = $ const., corresponding to the autoresonance condition. Moreover, integrating Eq. (6) with $\phi = \pi/2$ and $\Omega_c(z) = \Omega_{c0}$ yields the well-known asymptotic scaling relation[3] $\gamma(t) \sim a^{2/3}(9\Omega_{c0}/2\omega)^{1/3}(\omega t)^{2/3}$, resulting in infinite acceleration as $t \to \infty$.

In practise, the driving laser field is dispersive with $\beta_{ph} > 1$, resulting in the violation of the autoresonance condition. In this case, magnetic field tapering can be used to maintain a stationary phase for the electron. By analyzing the Hamiltonian dynamics of the electron in various tapered guide field profiles, we find that the maximum electron energy gain can be obtained by choosing an optimal taper such that $\phi(t) \approx \phi_s = \pi/2$ for the entire acceleration process. Figure 1 shows typical results of such an analysis. In Fig. 1, the electron kinetic energy and optimal axial magnetic field are plotted as a function of acceleration distance, for the choice of $a = 0.05$, $\beta_{ph} = 1.001$, and the initial conditions: $z = 0$, $\Omega_{c0}/ck_{\|} = 0.595$, $\phi_0 = \pi/2$, $\gamma_i = 1.15$, and $v_{\perp 0}/v_{z0} = 0.2$ (initial pitch angle). The phase variation is found to be within $\pm 8^\circ$ for the normalized acceleration distance from $k_{\|}z = 0$ to 500. It is evident in Fig. 1 that the degree of the taper has to become increasingly large in order to maintain the synchronous phase at the end of acceleration. Such an abrupt termination of acceleration allows us to estimate accurately the values of maximum electron energy gain and maximum acceleration distance z_m. Typically, the axial magnetic field at the end of the acceleration is about twice its value at $z = 0$.

The maximum beam energy gain and acceleration distance for optimized systems are calculated using the single-particle model and there-dimensional, self-consistent, multi-particle simulation, for a wide range of system parameters and initial conditions. The simulation has been performed in the microwave regime using a code[9] that has been developed initially for the study of the cyclotron autoresonance maser (CARM) amplifier and described in our earlier work.[9] (Because the equations of motion are the same for the CARM amplifier and CRL accelerator, the code can be used to study both.) We find that the maximum beam energy gain is limited primarily by the degree of laser field dispersion, while there is no substantial increase in energy gain as the wave amplitude is increased. We also find that the maximum acceleration distance scales as $k_{\|} z_m \propto 1/a$, that is, the acceleration gradient is proportional to the wave amplitude.

Figure 2 shows logarithmic plots of the maximum kinetic energy gain, $(\gamma_f - \gamma_i)/(\gamma_i - 1)$, and the maximum (normalized) acceleration distance, $k_{\|} z_m$, as a function of $\beta_{ph} - 1$, a measure of microwave dispersion. Here, $\gamma_i mc^2$ and $\gamma_f mc^2$ are the average initial and final energy for the beam electrons, respectively. In Fig. 2, the choice of system parameters corresponds to $a = 0.01$ and $\omega/2\pi = 9.55$ GHz. The initial conditions in the single-particle analysis are $\gamma_i = 1.15$, $\phi_0 = \pi/2$, $v_{\perp 0}/v_{z0} = 0.05$, and $\omega = k_{\|} v_{z0} + \Omega_{c0}/\gamma_i$; the initial conditions in the simulation are the same as these in the single-particle analysis, except that the initial phases of 1024 particles, ϕ_0, are distributed uniformly from 0 to 2π; In the simulation, the TE_{11} mode of cylindrical waveguide is used, and the microwave power is adjusted to yield a normalized wave amplitude of $a = 0.01$ on the z axis as the waveguide radius (or β_{ph}) is varied. (The inclusion of waveguide effects results in somewhat longer acceleration distances than those from the one-dimensional single-particle analysis.) The best fit of data obtained from the one-dimensional, single-particle analysis shows that the maximum kinetic energy gain and normalized acceleration distance satisfy the scaling relations (for $\beta_{ph} > 1$)

$$\frac{\gamma_f - \gamma_i}{\gamma_i - 1} \propto (\beta_{ph} - 1)^{-\mu} , \tag{7}$$

$$k_{\|} z_m \propto (\beta_{ph} - 1)^{-\nu} , \tag{8}$$

with $\mu \cong 0.5$ and $\nu \cong 0.6$, which are in agreement with the results from the multi-particle simulation.

In summary, it was found that there exist a maximum energy threshold and a maximum acceleration distance, which have not been reported in earlier studies of CRL accelerators. It was shown in a one-dimensional single-particle model, and confirmed in three-dimensional multi-particle simulations, that the maximum energy gain and the maximum acceleration distance obey certain scaling laws for optimized systems, and that the maximum energy gain is limited primarily by the degree of laser field dispersion and is almost independent of the strength of the driving wave field. These scaling laws are important for future CRL accelerator experiments.

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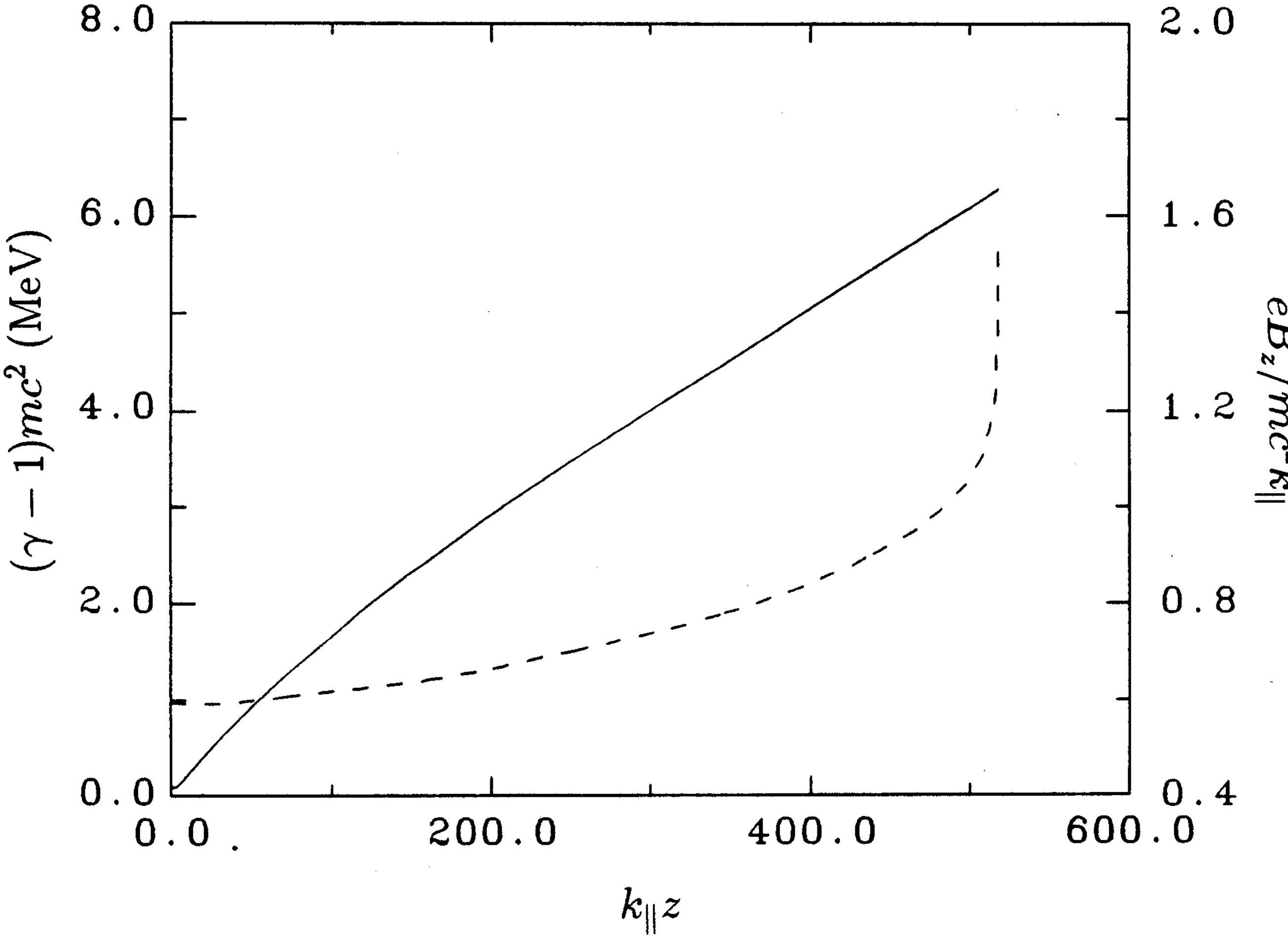
FIGURE CAPTIONS

Fig. 1 The kinetic energy of an electron (solid curve) and optimal (normalized) axial magnetic field (dashed curve) as a function of the normalized acceleration distance $k_{\|}z$, for $a = 0.05$, $\beta_{ph} = 1.001$, and the initial conditions $z = 0$, $\Omega_{c0}/ck_{\|} = 0.595$, $\gamma_i = 1.15$, $\phi_0 = \pi/2$, and $v_{\perp 0}/v_{z0} = 0.2$.

Fig. 2 Shown in (a) the maximum kinetic energy gain and (b) the maximum normalized acceleration distance as a function of $\beta_{ph} - 1$, for $a = 0.01$, $\omega/2\pi = 9.55$ GHz, and an initial kinetic energy of 67 keV ($\gamma_i = 1.15$) for the beam electrons. The open squares (solid circles) are the results from the single-particle model (simulation), while the solid lines are the best fit of data from the single-particle analysis.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6.0
4.0
2.0
0.0
2.0
1.6
1.2
0.8
0.4
0.0
200.0
400.0
600.0
$(\gamma-1)mc^2$ (MeV)
$eB_z/mc^2k_\parallel$
$k_\parallel z$

Fig. 1

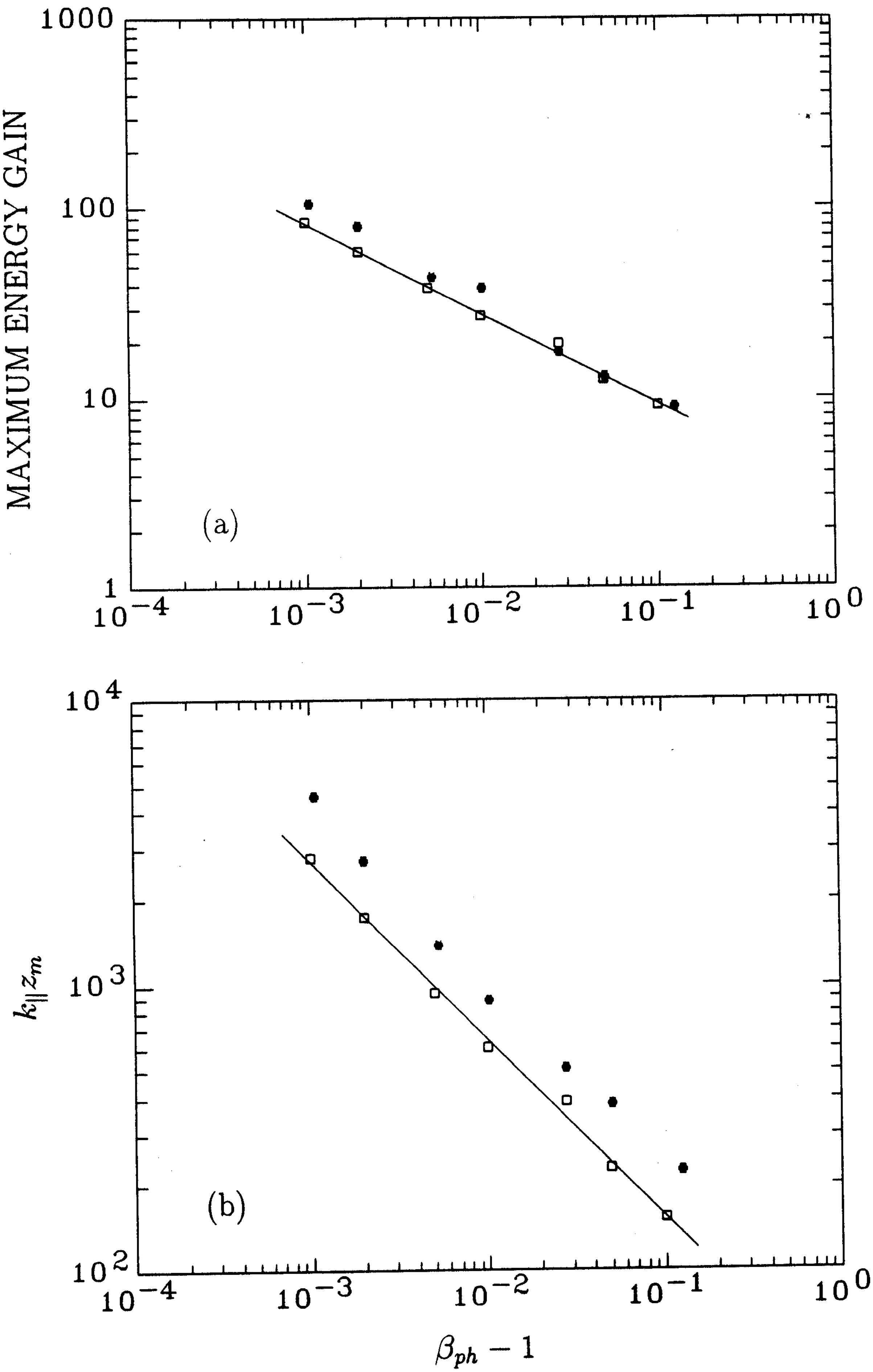
1000
100
10
1
MAXIMUM ENERGY GAIN
(a)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{4}
10^{3}
10^{2}
$k_{\|} z_m$
(b)
$\beta_{ph} - 1$

Fig. 2