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S. I. Krasheninnikov

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Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 USA

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Two Dimensional Effects in Plasma Radiation Fronts and Radiation Front Jumps in Tokamak Divertor Plasmas

S. I. Krasheninnikov†

Massachusetts Institute of Technology, Plasma Science and Fusion Center,

Cambridge, MA 02139, USA

Abstract

The impact of two dimensional (2D) effects of energy transport on impurity radiation fronts in a tokamak Scrape off Layer (SOL) plasma is considered. It is shown that 2D effects significantly alter both the physics of the fronts and the radiation loss from the SOL plasma, explain the experimental observations of impurity radiation region jumps from the target to the X-point after transition to a detached regime, and suggest an explanation for the easier access to a detached divertor regime in "vertical" target geometry in comparison with the "horizontal" case.

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Energy loss due to impurity radiation plays an important role in the physics of such interesting phenomena in laboratory and astrophysical plasmas as: regimes with Multifaceted Asymmetric Radiation from Edge (MARFE) [1] and radiative and detached divertor regimes [2, 3] in tokamak plasmas; and the dynamics of solar prominences and formation of interstellar clouds [4]. Even though the effects of energy radiation loss on plasma dynamics were investigated theoretically quite intensively [4], the nonlinear stages were mainly treated in one dimensional (1D) approximation. Often, however, two dimensional (2D) effects can qualitatively change the results of an analysis.

Here we consider analytically the impact of 2D effects on impurity radiation fronts in a tokamak Scrape off Layer (SOL) plasma. We show that 2D effects significantly alter the physics of fronts and the radiation loss from the SOL and allow us to explain the experimental observations of an impurity radiation region jump from the target to the X-point after transition to a detached regime, and the impact of divertor geometry on improved access to a detached divertor regime. A more detailed paper also containing numerical results confirming our analytic findings will be published elsewhere [5].

One can distinguish between two topologicaly different regions in a tokamak SOL: i) the divertor region (below the X-point), and ii) the rest of the SOL (see Fig. 1). The divertor region has no direct contact with the bulk plasma. When radiation loss is localized in the divertor, the heat flux from the bulk plasma first enters the SOL and then flows to the divertor region before radiating. In this case the radial broadening of the heat flux profile mainly occurs upstream from divertor, as determined by a competition between parallel and perpendicular heat conduction, and depends very weakly on the divertor plasma parameters. However, when the radiation region moves to the X-point, the heat flux from the bulk flows directly into the radiating region, preventing, as experiment suggests [2], further movement of the radiation region upstream. In what follows we focus on the factors affecting the localization of the radiation region within the divertor and assume that the front can not move above the X-point.

We start with an analysis of a 2D diffusion-reaction equation for a slab model of a tokamak divertor region assuming that plasma heat conduction is the dominant mechanism of energy transport:

$$\partial_x\left(\kappa_\perp \partial_x T\right)+\partial_y\left(\kappa_p \partial_y T\right)=R(T,x), \tag{1}$$

where T is the plasma temperature; x and y are the "radial" and "poloidal" (along the magnetic flux surface) coordinates; $\kappa_p=(B_p/B)^2\kappa_{\parallel}(T)$ and $\kappa_\perp(T)$ are the poloidal and radial heat conduction coefficients; $\kappa_{\parallel}(T)$ is the parallel heat conduction coefficient; and the projection factor B_p/B=constant is the ratio of poloidal to total magnetic field strength. The function $R(T,x)$ describes the plasma energy loss due to impurity radiation and models the peaked impurity emissivity at low temperatures in the coronal approximation [6] by assuming $R(T,x)>0$ in a small interval δT_R around the temperature T_R so that R behaves somewhat like a delta function.

In Eq. (1) we neglect the influence of perpendicular heat conduction on poloidal heat flux assuming that $\kappa_p(T)$ is much larger than $\kappa_\perp(T)$ in the temperature range of interest. However, we will see that high poloidal heat conductivity results in a strong poloidal extension of the front and in a sharp radial variation of the temperature within the front which can lead to a strong impact of radial transport on the radiation loss in spite of the inequality $\kappa_p(T)>>\kappa_\perp(T)$. We will find that the role of radial transport is sensitive to the temperature dependence of the function $\kappa(T)\equiv\kappa_p(T)/\kappa_\perp(T)$.

To begin, we make a qualitative estimate of the impact of radial and poloidal heat conduction on the radiation loss without taking into account any specific geometrical factors in the SOL. Assume that the radiation function does not depend on x, and the V-shaped radiation front (defined by $T(x,y)\approx T_R$) has a poloidal length length L and a radial extent w. Assuming that the radial broadening of the SOL mainly occurs in the hot upstream region which contacts the bulk plasma, the relation between L and w, found from Eq. (1), is

$$w/L\sim\left(\kappa(T_{up})\right)^{-1/2}<<1, \tag{2}$$

where $T_{up}>>T_R$ is the plasma temperature in that upstream hot region. The radiation loss from the front, W_R, can be written as

$$W_R \approx L_{tor} R(T_R) \int \Delta(\xi) d\xi, \tag{3}$$

where Δ and L_{tor} are the width and toroidal length of the radiation front ($\Delta << w,\ L$), and we introduce the ξ coordinate to integrate along the projection of the front in the (x, y) plane. We can estimate Δ and W_R for the cases when either poloidal or radial heat transport is dominant within the front and the radiation of the heat flux entering the front is nearly complete. From Eq. (1) we find the front width in the poloidal direction, $\delta_p \approx \left(\delta T_R \kappa_p(T_R)/R(T_R)\right)^{1/2}$, when poloidal transport is dominant; and the front width in the radial direction, $\delta_\perp \approx \left(\delta T_R \kappa_\perp(T_R)/R(T_R)\right)^{1/2}$, when radial transport dominates. Recall that the front is very extended along the poloidal coordinate ($L >> w$) so that it makes (on average) a small angle $\sim w/L$ to poloidal direction and is approximately perpendicular to radial direction. Therefore, we find $\Delta \rightarrow \Delta_p \sim (w/L)\delta_p$ for the case when poloidal transport is dominant within the front, and $\Delta \rightarrow \Delta_\perp \approx \delta_\perp$ for the radial case. Then, using relation (2), we find

$$W_R^{(\perp)}/W_R^{(p)} \sim \Delta_\perp/\Delta_p \sim \left(\kappa(T_{up})/\kappa(T_R)\right)^{1/2}, \tag{4}$$

where $W_R^{(p)}$ ($W_R^{(\perp)}$) is the radiation loss for the case when poloidal (radial) transport is dominant within the front.

Since the actual front width and radiation loss are determined by both poloidal and radial transport we can estimate $W_R \sim W_R^{(p)} + W_R^{(\perp)}$ and $\Delta \sim \Delta_p + \Delta_\perp$. Then, from Eq. (4) one sees that the impact of the poloidal and radial heat conduction on the radiation loss and front width is determined by the function $\kappa(T)$. When $\kappa(T)$ increases (decreases) with increasing temperature then both W_R and Δ are determined by radial (poloidal) heat transport. This behavior has a simple physical explanation. Fast poloidal heat conduction provides the heat transport from hot upstream region with $T \sim T_{up}$ to the front and, simultaneously, makes a small angle between the front and poloidal direction. In the case when $\kappa(T)$ is an increasing function of temperature this small angle has a strong impact on the radial heat conduction because of the strong radial gradient at $T \sim T_R$

and thereby on the radiation loss. As a result of the radial conduction induced widening of the front the radiation loss is much higher [7] than the estimate from Ref. [6] where only poloidal heat conduction was taken into account in the radiation front. For example, in tokamak SOL plasmas it is usually assumed [6] that $\kappa_{\parallel}$ is the Spitzer-Harm parallel heat conduction, $\kappa_p(T) \propto T^{5/2}$, and that the perpendicular heat diffusivity, $\chi_\perp$, ($\kappa_\perp = n\chi_\perp$, where n is the plasma density) is constant, resulting in $\kappa_\perp(T) \propto T^{-1}$ for a constant plasma pressure ($n \propto 1/T$). Therefore, in this case $\kappa(T) \propto T^{7/2}$. Then from Eq. (4) we find that for a reactor relevant upstream temperature of $T_{up} \sim 100$ eV and $T_R \sim 6$ eV (corresponding to the peak of carbon emissivity at low temperatures [6]) the effect of radial heat conduction on the radiation front width results in a more than 100 fold increase in the radiation loss from the front.

To obtain quantitative solutions of Eq. (1) in the divertor region in a slab approximation (see Fig. 2) we use as boundary conditions fixed temperature T_t at the target (y=0), and a prescribed radial profile of the poloidal component of the heat flux, $q_y(x,y) = -\kappa_p \partial_y T$, at the upstream boundary: $q_y(x, y = L_d) = -q_L(x) \leq 0$, where L_d is the poloidal extent of the divertor region. We will assume that $T_t << T_R << q_L(x) L_d / \kappa_p(T_R)$, so that the radiation region is always located within the slab $0 < y < L_d$.

First, we consider the case when $q_L(x) = q_L$=constant and $R(T,x) = R(T)$. Neglecting the radial derivatives in Eq. (1) and assuming $\kappa_p \propto T^{\alpha_{\parallel}}$, where $\alpha_{\parallel} > 0$ is constant, we write energy balance as (see for example [6]) $(q_t)^2 = (q_L)^2 - 2\int_{T_t}^{T_L} \kappa_p(T) R(T)\, dT$, where T_L is the temperature at the upstream boundary, $q_t \approx \kappa_p(T_R) T_R / \{(\alpha_{\parallel} + 1) y_R\}$ is the heat flux to the target, y_R is the poloidal coordinate of the radiation front $T(y_R) = T_R$. Recalling the inequalities $T_t << T_R << q_L(x) L_d / \kappa_p(T_R)$ we note that how completely the heat flux q_L is radiated is determined by the size of Q_p, where $(Q_p)^2 = 2\int_0^{\infty} \kappa_p(T) R(T)\, dT$. For $q_L >> Q_p$ the impact of the radiation loss on the heat flux is small ($q_t \approx q_L$), the radiation front is localized near the target ($y_R \approx 0$), and $T_L >> T_R$. In the opposite case, when $q_L < Q_p$, the radiation of the entering heat flux is almost complete ($q_t << q_L$) and to maintain the energy balance the radiation front is shifted close to the upstream boundary ($y_R \approx L_d$) so that $T_L \sim T_R$. Consequently, the transition of the radiation front from near target to the upstream boundary occurs for a rather small variation of q_L about Q_p. Thus, in the radial homogeneous case complete radiation is only possible for $q_L \lesssim Q_p$.

Next, we consider the case when the incoming heat flux $q_L(x)$ and radiation function $R(T,x)$ vary slowly in the radial direction and any re-distribution of the poloidal heat flux profile due to radial heat conduction between upstream and the radiation front is weak, $q_y(x,y) \approx -q_L(x)$. Assuming that radial broadening of poloidal heat flux mainly occurs at high temperatures, the corresponding requirement of weak radial re-distribution of $q_y(x,y)$ is

$$w/\ell_q(x) \gtrsim \left(\kappa(T_L(x))\right)^{-1/2}, \tag{5}$$

where w is the radial scale length of $q_L(x)$ and $R(T,x)$, and $\ell_q(x)$ is the distance from the front to upstream boundary.

Consider the situation when the radiation of the heat flux $q_L(x)$ is practically complete ($q_t << q_L$). Then assuming a weak radial variation of $q_L(x)$ and $R(T,x)$ we can treat the radiation front $y_R(x)$ (corresponding to the solution $T(x,y_R)=T_R$) in a local approximation as a straight line making an angle ψ to y direction as shown in Fig. 2. Notice that radial heat conduction affects re-distribution of the heat flux only at the radiation front where the radial gradients are strong. Taking into account the effects of both poloidal and radial heat conduction and integrating Eq. (1) normal to the radiation front we find that complete radiation occurs when

$$q_L(x) = Q(\psi,x) \equiv \left\{\left(Q_p(x)\right)^2 + \left(Q_\perp(x)/\tan\psi\right)^2\right\}^{1/2}, \tag{6}$$

where $\left(Q_p(x)\right)^2 = 2\int_0^\infty \kappa_p(T)R(T,x)dT$ and $\left(Q_\perp(x)\right)^2 = 2\int_0^\infty \kappa_\perp(T)R(T,x)dT$. However, from Fig. 2 we expect the radial extent of the front ($\ell_q \tan\psi$) to be less than the radial scale of $q_L(x)$: $\tan\psi \lesssim w/\ell_q(x)$. Then inequality (5) restricts the allowed values of $\tan\psi$ to $\tan\psi \gtrsim \left(\kappa(T_L)\right)^{-1/2}$ and imposes the constraint

$$Q(\psi,x) \lesssim Q_p(x)\left\{1+\kappa(T_L)/\kappa(T_R)\right\}^{1/2}. \tag{7}$$

Since T_L can be much higher that T_R, from Eq. (7) one sees that for the case when $\kappa(T)$ increases with increasing temperature and the front makes a small angle to poloidal direction, the impact of radial heat conduction on the radiation of the heat flux entering the divertor region can be substantial, in agreement with our qualitative estimate (4).

Recalling that $\tan\psi$ is related to the shape of radiation front by $\tan\psi = (dy_R/dx)^{-1}$, we find from Eq. (6) for $q_L(x) > Q_p(x)$

$$\left(\frac{dy_R}{dx}\right)^2 = \left\{(q_L(x))^2 - \left(Q_p(x)\right)^2\right\} \Big/ (Q_\perp(x))^2. \tag{8}$$

Interestingly, Eq. (8) allows a sign switch in dy_R/dx which may occur at either a maximum or minimum of $y_R(x)$. In practice such a sign switch in dy_R/dx is only possible at a minimum of $y_R(x)$. We can analyze the heat transport at the minima and maxima of the radiation front by assuming that boundary effects can be neglected and $q_L(x) > Q_p(x)$. There is no problem with the energy balance at the minima of $y_R(x)$ where the heat flux arriving at the tip of $y_R(x)$ from upstream is re-distributed (by radial heat conduction) to the side regions of the front where $dy_R/dx \neq 0$. The re-distribution results in a sharp bending of $y_R(x)$ which on the scale length of $q_L(x)$ profile can be described by a sign switch in dy_R/dx with the value of $(dy_R/dx)^2$ determined by Eq. (8).

At a maximum of $y_R(x)$ there is no depletion of the incoming heat flux caused by a re-distribution to the side regions with $dy_R/dx \neq 0$ since there is no radiation in the upstream vicinity of a maximum of $y_R(x)$. Therefore, at a maximum of $y_R(x)$ complete radiation of the heat flux is only possible when the heat flux entering the radiation front, is smaller than Q_p. Analysis of the location of a maximum of $y_R(x)$ at $q_L \approx Q_p$ shows [5] that for a very weak radial dependence of $q_L(x)$ and $R(T,x)$ the maximum of $y_R(x)$ can be located between the target and the upstream boundary, but a rapid transition of the maximum of $y_R(x)$ from near target to the upstream boundary occurs for small variation of q_L about Q_p (as in radial homogeneous case considered before) and looks similar to a bifurcation. For stronger radial dependencies of $q_L(x)$ and $R(T,x)$, but still compatible with Eq. (5), the maximum of the radiation front can only be located near the target or at the upstream boundary. In this case the transition of the maximum of $y_R(x)$ between these two locations occurs as a real bifurcation (caused by radial re-distribution of the poloidal heat flux) and results in a jump of the maximum of $y_R(x)$. In what follows we do not distinguish between these two cases and refer to them as to a bifurcation of the maximum of $y_R(x)$.

We can find a constraint on the existence of the radiation front with complete radiation of the heat flux $q_L(x) > Q_p(x)$ which is imposed by the poloidal length of the divertor and because of the finite value of $(dy_R/dx)^2$. Consider two close points $x_1 < x_2$ corresponding to solutions of the equation $q_L(x_{1,2}) = Q_p(x_{1,2})$ such that $q_L(x) > Q_p(x)$ for $x_1 < x < x_2$. There can be only one minimum of $y_R(x)$ between points x_1 and x_2, and it should be located above the target. Then from Eq. (8) we find the condition for the existence of such minimum to be

$$\int_{x_1}^{x_2} \left\{ \left[(q_L(x))^2 - (Q_p(x))^2 \right] \Big/ (Q_\perp(x))^2 \right\}^{1/2} dx \le 2L_d. \tag{9}$$

When inequality (9) is satisfied a V-shaped, radial heat conduction widened (even though $\kappa_\perp << \kappa_p$) radiation front can be formed in this region of the slab resulting in complete radiation of even high heat flux $q_L(x) >> Q_p(x)$. In the opposite case the heat flux from upstream hits the target.

Next, we consider the evolution of the radiation front when the magnitude, R_0, of the radiation function increases (here for simplicity we take $R(T,x) = R(T) \propto R_0$). As an example we analyze the evolution of the radiation front with increasing R_0 for a smooth, periodic (in the radial direction) profile of heat flux $q_L(x)$ as shown in Fig. 3 ($q_{min} \le q_L(x) \le q_{max}$, and $q_{min} << q_{max}$). At low R_0, radiation of the heat flux is incomplete even for $q_L(x) \approx q_{min}$ and the radiation front stays very close to the target (curve a). When R_0 reaches the level where $q_{max} >> Q_p \gtrsim q_{min}$, bifurcation of the front in the regions with $q_L(x) \approx q_{min}$ occurs and two scenarios of the subsequent front evolution are possible. The first scenario ("jump") corresponds to the case when inequality (9), written as an integral over the period of $q_L(x)$, is satisfied. Then, bifurcation in the regions with $q_L(x) \approx q_{min}$ triggers formation of a strongly shaped, radial conduction widened radiation front leading to complete radiation of the heat flux $q_L(x) \approx q_{max} >> Q_p$. As a result, the entire radiation front jumps to the upstream region (curve b). The second scenario ("gradual") corresponds to the case when inequality (9) is not satisfied. Then bifurcation of the radiation front occurs only in regions with $q_{max} >> q_L(x) \sim q_{min}$. Outside these regions the radiation is weak and $q_t \approx q_L(x)$. With a further increase of R_0 the regions that incompletely radiate the heat flux shrink and gradually disappear resulting in the formation of a front with a strongly modulated shape which radiates the heat flux $q_L(x) \approx q_{max} >> Q_p$ (curve c).

Finally, we discuss how our results are related to tokamak experiments. The experiments show that during the transition from a radiative to a detached divertor regime the impurity radiation region shifts abruptly away from the target to the X-point [2]. Recall now that the radiation function is proportional to the plasma and impurity density denoted by $n(x)$. The typical radial structure of the poloidal heat flux $q_L(x)$ and $n(x)$ profiles in the divertor region is as follows. Near the separatrix ($x=0$) both $q_L(x)$ and $n(x)$ reach their maximum values and decrease as one moves away from the separatrix to the "wings" of the profiles, $x \sim x_w$. Now assume that the magnitude of the radiation function is an increasing function of plasma density of the form $R(T,x) \equiv R_0(n(x))R(T)$ and recall $Q_p(x) \propto R_0(n(x))$. For typical experimental conditions the heat flux is very high near the separatrix where most of the heat flux streams to the target. As a result, $F(0) >> 1$, where $F(x) \equiv q_L(x)/Q_p(n(x))$. This high heat flux can only be radiated by forming a V shaped radiation front with radial heat conduction setting the front width. However, our analysis shows that a V shaped front can only be formed when: i) it is triggered somewhere outside separatrix region (which means $q_L(x) \lesssim Q_p$ at the wings) and ii) the inequality (9) is satisfied. Assuming that the main contribution to the integral (9) is from the region near the separatrix having a width w, the two requirements for the formation of a V shaped front in the divertor become: i) $F(x_w) \lesssim 1$ and ii) $F(0) < L_d/\left(w\sqrt{\kappa(T_R)}\right)$ (we assume that $L_d/\left(w\sqrt{\kappa(T_R)}\right) >> 1$, which can only be true for $\kappa(T)$ an increasing function of T, recall the estimate (5)). Then, as in the preceding example, two scenarios of the front abruptly jumping to the X-point and the gradual formation of a V shaped radiation front can be envisioned depending on whether (i) or (ii) is satisfied first as R_0 increases. A "jump" scenario, similar to experimental observations, occurs when $F(0)<L_d/\left(w\sqrt{\kappa(T_R)}\right)$ (inequality (9) is satisfied) and then $F(x_w)\rightarrow 1$, corresponding to relatively high heat flux at the "wings", $F(x_w)/F(0)> \left(w\sqrt{\kappa(T_R)}\right)/L_d$. In this case the radiation front jumps towards the X-point causing strong radiation loss, which leads to the plasma detaching from the target [8]. The "gradual" scenario corresponds to the case when $F(x_w)<1$ before inequality (9) is satisfied, causing a gradual formation of a V shaped front, and plasma detachment if R_0 is increased further. Presumably, the "jump" scenario can be avoided by intensive neutral gas puffing into the divertor from the sidewalls over a large poloidal distance to reduce the heat flux at the "wings" and switch the front evolution to the "gradual" scenario. Thus we see that the plasma parameters at the "wings", where the heat flux is very small, play a crucial role in the dynamics of the radiation front in a tokamak divertor!

Another way to assist formation of a V shaped front having strong radiation loss is to use geometrical effects due to the sidewalls and the target. So far, we have considered a so-called "horizontal" target where the separatrix magnetic flux surface (SMS) makes a 90^o angle with the target. However, the target may be turned in a such a way that it will make a grazing angle with SMS (a so-called "vertical" target [3]). For a "vertical" target radial heat conduction enhancement of radiation loss is automatically switched on. The grazing angle between the SMS and the target ensures a V shaped radiation front allowing radial heat conduction to increase the radiation loss. This mechanism may explain the easier access to detached divertor regimes for a "vertical" target configuration as compared to a "horizontal" one as observed in experiments [3].

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†Also at: I.V. Kurchatov Institute of Atomic Energy, Moscow, Russia

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Figure Captions

Fig. 1. Poloidal cross section of tokamak core and SOL-divertor regions.

Fig. 2. Geometry of the radiation front $y_R(x)$ in the divertor region (x and y are the "radial" and "poloidal" coordinates, $q_L(x)$ is the poloidal heat flux at the entrance to the divertor).

Fig. 3. The evolution of the radiation front with increasing radiation function for periodic radial profile of the poloidal heat flux $q_L(x)$.

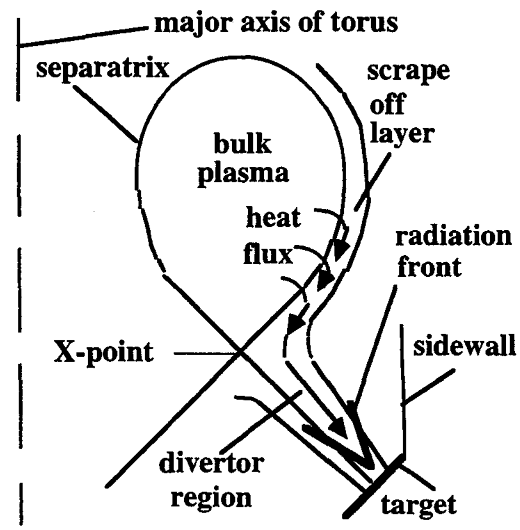
major axis of torus
separatrix
bulk
plasma
scrape
off
layer
heat
flux
radiation
front
X-point
sidewall
divertor
region
target

Fig. 1.

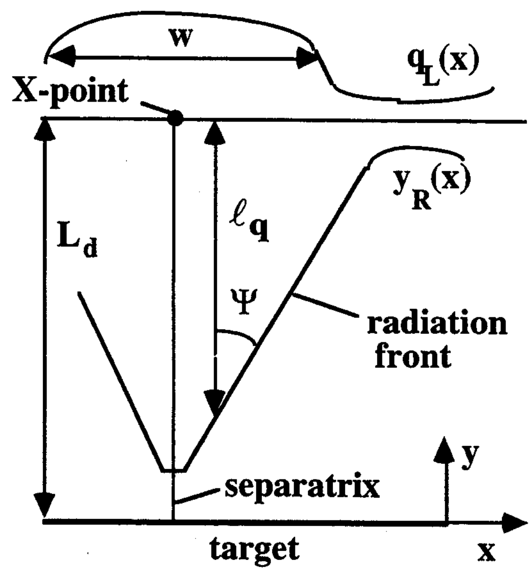
w
$q_L(x)$
X-point
$y_R(x)$
L_d
ℓ_q
Ψ
radiation
front
y
separatrix
target
x

Fig. 2

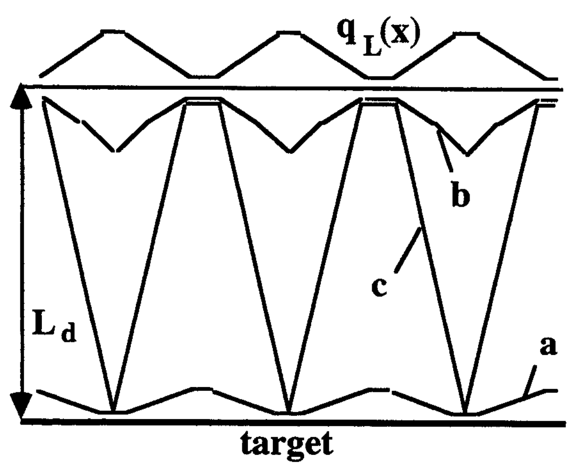

Fig. 3.