Statistical Analysis of Acoustic Transmission Scintillation in the 2014 Nordic Seas Experiment

by
Guy Schory

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Signature of Author: __________________________

Department of Mechanical Engineering
Jan 20, 2015

Certified by: __________________________

Nicholas C. Makris
Director of the Laboratory for Undersea Remote Sensing

Accepted by: __________________________

David E. Hart
Chairman, Department Committee on Graduate Students
Abstract

The propagation of acoustic signals through an underwater waveguide adds randomness to the received signal, called scintillation. For a fully saturated field, this scintillation features statistical properties that can be measured and estimated. Many analyses were conducted in the past for finding and estimating the characteristics of these statistical properties.

In February 2014, the 2014 Nordic Seas Experiment was conducted in Norway, in which acoustic broadband signals were transmitted and received frequently, propagating through a waveguide on the continental shelf, and large sets of acoustic data were gathered.

The work presented here shows analysis of the statistical properties of the data recorded in the 2014 Nordic Seas Experiment. We show that the received signals indeed comply with a fully saturated field by analyzing the distribution of the intensity and of several representations of energy. We also show how the number of coherent cells can be estimated and that the statistical properties of it fit the theory and previous work.
Acknowledgements

First and foremost, I wish to thank my advisor, Nicholas Makris, for his guidance and mentoring as I worked on this thesis as well as throughout my studies at MIT. His attentiveness to my academic strengths and weaknesses, his teachings and explanations allowed me to advance my understanding in acoustics and made me a much better scientist altogether.

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1. Introduction

On February 2014, the 2014 Nordic Seas Experiment went underway, led by Prof. Nicholas C. Makris from MIT, with the participation of numerous scientists and engineers from MIT, Northeastern University, Penn State University, the Institute of Marine Research, the Naval Research Laboratory, BAE and Woods Hole Oceanographic Institute as well as the Institute of Marine Research and the Defense Research Establishment, Norway.

In the experiment, acoustic broadband signals were transmitted and received frequently, using the Ocean Acoustic Waveguide Remote Sensing system (OAWRS). The OAWRS system was used in several experiments in the past, and is an effective system for imaging large areas on the continental shelf instantaneously (1) (2). The acoustic signals propagated through a waveguide, and large sets of acoustic data were gathered, that allow for a thorough analysis, both for the detection of fish and other scatterers and for acoustic propagation properties. In certain times in the experiment, an auxiliary ship deployed a transmitter and a receiver, and the data collected using them allowed for one way propagation analysis. The analysis shown here is conducted on the data received on March 3rd, both by the OAWRS receiver array and by the receiver hydrophone deployed by the auxiliary ship.

The propagation of acoustic signals in an underwater waveguide adds randomness to the signals, known as scintillations. Knowing the properties of the scintillations is important both before conducting an experiment, for better planning of the experiment and when analyzing the results, for accounting for inherent randomness. Substantial research about the scintillation of fully saturated fields was conducted in the past, for example in (3) (4) (5) and (6).

One of the most important conclusions of the analysis of fully saturated fields is that the standard deviation of the TL can be estimated, and for large time-bandwidth products, is approximated by $4.34\sqrt{1/\mu}$ (3), where $\mu$ is the number of coherent cells – the number of independent fluctuations through the propagation (3) (5) (4). The number of coherent cells is dependent on the bandwidth
and the central frequency of the signal, as shown in (5). In chapter 7 we show that the number of coherent cells in the data used in this analysis comply with the conclusions reached in (5).

The statistical properties described above are relevant for a fully saturated field (6) (3) (7) (4), which means that the received signal is fully randomized, which in turn can be described by a Circular Complex Gaussian Random (CCGR) (3). Chapter 5 shows that the received signals used in this analysis is indeed CCGR, by showing that the distribution of the received intensity and energy comply with the analytical distribution of these values, which is the gamma distribution, with the number of coherent cells as the parameter.

Two representations of received energy are the Parseval’s Sum (PS) energy, which is the total energy received in the signal, and the Matched Filter (MF) energy, which is proportional to the correlation of the received signal with the transmitted signal. The MF energy is very important in sonar applications, as the matched filter allow for advanced signal processing (8) (9). The difference between the PS energy and the MF energy gives us a measure of the deterioration of the signal through the propagation (4), and in chapter 7 we analyze the dependence of this difference on the range.

Throughout this work, we reproduce some of the analysis conducted in (3), (5), (4), we compare our results with the ones received there, and we show that results received here support the conclusions of these works.
2. Conducting the experiment and Data Collection

2.1. Overview

The experiment was conducted between February 16th and March 8th 2014, in several locations in the Nordic sea, and it consisted of transmissions of LFM signals in six different bands, with 50 Hz bandwidth and 1 sec pulse duration, and reception of echoes by the OAWRS system, both deployed by the WHOI RV Knorr. In addition to the research vessel, a second ship was used in the experiment, whose main purposes were to check points of interest using an echosounder, and assist with calibration of the sonars and the acoustic models.

On certain times during the experiment, a transmitter (Lubell single hydrophone) and a receiver (Reson single hydrophone) were deployed from the second ship, allowing for one way propagation, both by receiving the signals transmitted from the Knorr, and by transmitting signals received by the OAWRS system.

The Transmitted signal was an increasing 50Hz LFM over one second in one of the following bands: 825-875Hz, 930-980Hz, 1100-1150Hz, 1310-1360Hz, 1440-1490Hz and 1575-1625Hz.

In order to estimate the sound speed profiles, XBTs were deployed frequently, while CTDs were used occasionally to get a better estimation of the sound speed and to calibrate the salinity levels.

2.2. Data Collection

The data we used for most of the analysis in this work is the data collected on March 3rd, both by the OAWRS system and by the Reson hydrophone. The reasons that we used these sets of data is that on that date there are sufficient measurements to have a reliable statistical analysis. Furthermore, using the data from both sets allows us to compare the results from the two systems at similar environment characteristics.
3. TL Analysis

3.1. General

As sounds propagate through an underwater medium, the wave's energy decreases due to spreading and attenuation (10) (11) and is called Transmission Loss (TL). The TL can be estimated using different models, and the model used in this analysis is the Parabolic Equations (PE) model (11), implemented by the RAM procedure (12). An example for the output of the RAM model, can be seen in figure 3.1.

The PE model uses the sound speed profiles to calculate the TL. As mentioned above, the sound speed was estimated using frequently deployed XBTs and CTDs, and figure 3.2 shows the estimated sound speed profiles measured on March 3rd.
The thicker blue line is the mean sound speed at each depth, and is used for the TL calculations in this work.
4. Data Processing

4.1. Basic Analysis

Let the received pressure be \( \Psi(r, t) \), where \( r \) is the receiver location, denoting the transmitter location as the origin. In the frequency domain, the received pressure is \( \Phi(r, f) = \int_{T} \Psi(r, t) \exp(-2\pi jft) \, dt \), where \( T \) is the time window for which we perform the spectral analysis.

The intensity of the signal is then defined by \( I(r, t) = |\Psi(r, t)|^2 \), and the average intensity is then \( W(r) = \frac{1}{T} \int_{T} I(r, t) \, dt \), for a time window \( T \). The Source Pressure Level (SPL) is defined as \( 10\log(W) \), for a time window that contains the whole received signal, and in our case \( T=1 \) sec.

4.2. Methodology

The first stage in the analysis was to find the received signals out of all the data collected. In order to do that, for each frequency the total received signal was correlated with the source signal, and local peaks were found. Most of the data that is used in this work was taken at times when two sources transmitted simultaneously, with different Pulse Repetition Intervals (PRI), so in order to discern between peaks that are received from the relevant source and peaks that are from the second source or from noise, each peak was tested for: (a) time difference between the peak and other peaks, and (b) peak correlated level, to be between maximum and minimum values.

After the peak times were collected, the received pressure at a time window around each peak was obtained, as well as the timestamp of the peak and the positions and aspects of both ships. The nominal time window starts at the peak time, and ends after 1 second, which is the waveform duration, since the dispersion of the signal is very low, as will be shown later in this document.

Next, the received signal was filtered using a band-pass filter, around its central frequency, in order to omit noise, residue from previous signals of different frequencies and signals from the undesired transmitter. The filtering was done by performing an FFT, setting all frequencies outside of the bandwidth window as zero, and returning to time domain by applying an IFFT.
The result of the IFFT is referred to from this point on as the received signal, and will be denoted as $z$.

This procedure was done both for the data received on the Reson hydrophone, transmitted by the MIT OAWRS source array (this will be referred to as “MIT source array“ transmissions), and for the data received on the FORA receiver array, transmitted by the Northeastern University Lubell source (this will be referred to as “NU source” transmissions). Since the FORA array is consisted of multiple hydrophones, taking all received data as independent measurements may not be correct. In order to get a complete, unbiased results, the analysis was conducted for several sets of data: 1. All the MF hydrophones, as if the received signals are independent (denoted as “All Hydrophones”); 2. Only the desensitized hydrophone; 3. One random MF hydrophone.

### 4.3. Signal Degradation

The propagation of the signal in the ocean causes it to disperse and deteriorate, due to spreading, bottom attenuation, multipath arrivals and random scatterers. Figures 4.1-4.2 show examples of the signal pressure, the matched filter energy and the signal spectrum at the source and at the receiver, at a frequency of 1600Hz, both for the MIT source array and the NU source.
signal deterioration, f=1600, MIT source

Figure 4.1: (a) The normalized pressure, (b) the normalized matched filter energy and (c) the signal spectrum at the transmitter and at the receiver for the 1600 center frequency, MIT source array.
signal deterioration, f=1600. 162 data points, sensitized hydrophone

We can see that the matched filter energy has several peaks, due to multipath arrivals, and that the received pressure and its spectrum are deteriorated compared to the transmitted signal.

Received Energy as a Function of Time Window

The total energy of a received signal is calculated by \( \int_{T} |\Psi(r, t)|^2 dt \). In order to view the dependence of the energy on the time window used, we show in figures 4.3-4.4 the total energy of all received signals as a function of the time window, starting 0.5 seconds before the beginning of the received signal and ending 0.5 seconds after the 1 second duration of the signal, both for the MIT source array and the NU source.
Figure 4.3: The normalized energy of all received signals at 955Hz, MIT source array. The mean energy is shown by the thick black curve.
Figure 4.4: The normalized energy of all received signals at 955Hz, NU source. The mean energy is shown by the thick black curve.

We can see that about 95% of the energy for the MIT source array and about 80% of the energy for the NU source are within the one second window. This percentage is smaller for the NU source than for the MIT source array, but still amounts for almost all of the energy. We can see that the slope is highest within the 1 second window, so it may be deducted that the signals' energies are within the 1 second window, and the reason for the large energy outside of the window is high noise, probably due to the proximity of the receiver to the main source and random scattering and echoes.
5. Fully Saturated Field Analysis

5.1. General

In order to conduct a meaningful analysis for the experiment, it is important to discern whether the received pressure fields are fully saturated. A fully saturated field is a field that can be described by a Circular Complex Gaussian Random, which means that both the real and the imaginary parts of the received signal has a Gaussian distribution (3). This section reproduces the analyses conducted in (3) (4) (7) (5), in order to show that the received signals of the 2014 Nordic Seas experiment are indeed CCGR and that the field is fully saturated.

5.2. The distribution of the received field

For a CCGR field, the received signal has a Gaussian distribution, which can be written as \( \text{Pr}(x) = \frac{1}{2\pi(x^2)} \exp\left(-\frac{(x-(\mu))^2}{2(x^2)}\right) \) (3). Figure 5.1 shows the distribution of all the received pressure measurements for the MIT source array, for the 955Hz central frequency band, along with the analytical distribution.
Figure 5.1: Measured (blue histogram) and calculated (red line) distribution of the real part of the received pressure, for 955Hz center frequency, MIT source array.

We can see that the distribution indeed seems to be normally distributed. However, since the distribution shown uses all measurements in all received signals, one may question whether taking 8000 not independent measurements from each received signal is correct. For that reason, we performed the same analysis for several subsets of measurements, that use only a small number of random measurements from each signal. Figure 5.2 shows the results for four sample sizes, for the NU source for the 955Hz frequency band.
We can see that the distribution indeed resembles a Gaussian distribution for all numbers of random measurements, with increasing fit for larger sample size, as was expected of a CCGR field.

5.3. The distribution of the average intensity

As shown in (3), for a CCGR signal, the power $W$ has a gamma distribution, such that $Pr(W) = \frac{(\mu/W)^\mu W^{-\mu-1} \exp(-\mu W/W)}{\Gamma(\mu)}$, where $\mu$ is the number of coherence cells, calculated by $\mu = \frac{\langle W^2 \rangle - \langle W \rangle^2}{\langle W \rangle}$, and $\langle W \rangle$ is the mean value of $W$. The number of coherence cells is “the number of independent intensity fluctuations averaged during the measurement time $T$” (3), and is dependent on the inverse of the standard deviation of the averaged intensity. A further discussion about the properties of the number of coherent cells appears later in this paper.
Figures 5.3-5.5 show the calculated and the measured distribution of $W$, for both the MIT and the NU sources (a random hydrophone and all hydrophones), for the 955Hz frequency band.

Figure 5.3: measured (blue histogram) and calculated (red line) distribution of the received power for the 955Hz frequency band, MIT source array.
Figure 5.4: measured (blue histogram) and calculated (red line) distribution of the received power for a random sensitized hydrophone for the 955 Hz frequency band, NU source.
distribution of $W$, $f=955, 9792$ data points, all hydrophones

$0.05$

$0.04$

$0.03$

$0.02$

$0.01$

$0.005$

$0$

$0.5 1 1.5 2 2.5 3 3.5$

$W$ (Pa$^2$)

Figure 5.5: measured (blue histogram) and calculated (red line) distribution of the received power for all hydrophones for the 955Hz frequency band, NU source.

We can see a fairly good fit between the analytical distribution and the measured distribution. Results for all frequency bands are shown in appendix A.

5.4. The distribution of the energy

Next we will analyze the distributions of three representations of energy: (a) the total received energy, which is equal to the Parseval’s sum (PS) energy, (b) the matched filter (MF) energy, and (c) the central frequency (CF) energy.

(a) As was shown above, the total received energy is $E_{ps} = \int_{T} |\Psi(r, t)|^2 dt$, and it is equal to the Parseval’s sum energy $E_{ps} = \int_{f} |\Phi(r, f)|^2 df$, where $\Phi(r, f) = \int_{T} \Psi(r, t) \exp(-2\pi jft) dt$ is the Fourier transform of the received pressure. In our case, since the time window is 1 sec, $E_{ps}$ is also equal to the received power, $W$. 

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(b) The matched filter energy is the most important property for signal processing purposes (8), as it uses the similarity of the received signal to the transmitted signal. The matched filter energy is calculated by
\[ E_{mf}^T(r) = |h(t|t_m) * \Psi(t, t)|^2 = \left| \int_{f_1}^{f_2} \phi(r, f) H(f|t_m) \exp(j2\pi f t) df \right|^2 = \left| \int_{f_1}^{f_2} \phi(r, f) KQ^*(f) \exp(j2\pi f (t - t_m)) df \right|^2 \]

where \( h(t|t_m) = Kq(t - t_m) \) is the filter, with the Fourier transform \( H(f|t_m) = KQ^*(f) \exp(-j2\pi f t_m) \), \( K = (\int_{f_1}^{f_2} |Q(f)|^2 df)^{-0.5} \) and \( t_m \) is the delay time. The result of the integral is a function of time, which has a strong peak, as shown in figure 5.6. This peak is the matched filter energy, and is given by \( E_{mf} = \max_{t_m} E_{mf}^T \).

![Figure 5.6: Matched filter output for the 955Hz central frequency band, MIT source array.](image)

We can see that other than the main peak, there are lower peaks before and after the main peak, that result from early and multipath arrivals.
(c) The central frequency energy is the power at the central frequency, multiplied by the bandwidth: $E_{cf} = |\phi(r, f_c)|^2 \cdot BW$, where BW is the bandwidth, 50Hz in our case. This energy has no special significance, but is important for comparison and for coherence cells number analysis.

For a CCGR signal, the distributions of these energies follow the gamma distribution as well, but the number of coherence cells is calculated differently. Let $\sigma(L) = std(L(dB))$, where $L = 10\log(E)$ all energy levels in dB, then $\sigma(L) = 10\log(e) \sqrt{\sum_{k=0}^{\infty} \left( \frac{1}{\mu+k} \right)^2}$ (4) (7). For $\mu = 1$, the case for instantaneous signals (time-bandwidth product of 1), the standard deviation $\sigma(L) = 5.6\, dB$, which is the value received by Dyer (6).

The distribution of each energy is given by $Pr(E) = \frac{(\mu/\bar{E_n})^\mu E_n^{\mu-1} \exp(-\mu E_n/\bar{E_n})}{\Gamma(\mu)}$, where $E_n$ is the energy of each type normalized by the energy of the same type of the transmitted signal, and $\bar{E_n}$ is the mean energy (4) (7). Figures 5.7-5.9 show the calculated and measured distributions for the three types of energy for the 955Hz frequency band, for both the MIT source array and the NU source (one random hydrophone and all hydrophones).
Distribution of the energy, f=955, 106 data points, MIT source

Figure 5.7: The measured (blue histogram) and calculated (red line) distributions of the (a) PS, (b) MF and (c) CF energies for the 955Hz frequency band, MIT source array.
Figure 5.8: The measured (blue histogram) and calculated (red line) distributions of the (a) PS, (b) MF and (c) CF energies for a random hydrophone for the 955Hz central frequency, NU source.
We can see that the analytical calculations fit the measurements well for all energy representations, both for the MIT and the NU source.

Results for all frequency bands are shown in appendix A.

Since in our case the PS energy and the average intensity are in fact the same (due to the 1 second averaging), but the number of coherence cells is calculated differently, it is interesting to compare the analytical distribution of both. Figures 5.10-5.12 show this comparison for the 955Hz frequency band, for both the MIT source array and the NU source (one random hydrophone and all hydrophones).
Figure 5.10: Comparison between the probability distributions of the mean intensity \( W \) and the PS energy for the 955Hz frequency band, MIT source array.
Figure 5.11: Comparison between the probability distributions of the mean intensity $W$ and the PS energy for a random hydrophone for the 955Hz frequency band, NU source.
Figure 5.12: Comparison between the probability distributions of the mean intensity $W$ and the PS energy for all hydrophones for the 955Hz frequency band, NU source.

This comparison shows us that the PS energy and the averaged intensity result are very similar, such that both methods seem to be equal in merit.
6. NU source and FORA array Properties

6.1. SPL measurements

Figure 6.1 shows the NU source Source Pressure Level (SPL) measurements as a function of range for all hydrophones.

Figure 6.2 shows the measures SPL as a function of hydrophone number, for three random transmissions for the 955Hz frequency band.
6.2. OAWRS array desensitized hydrophone

As stated above, the OAWRS receiver array consists of several sensitized hydrophones as well as one desensitized hydrophone. Since the gains used for the sensitized and the desensitized hydrophones are not equal, we checked if the SPL results for both types of hydrophones are the same. Figure 6.3 shows the measured SPL for all hydrophones for the 955Hz frequency bands, where the green markers are the sensitized hydrophones and the red markers are the desensitized hydrophone.
Figure 6.3: Measured SPL for the sensitized (green) and the desensitized (red) hydrophones as a function of range.

We can see that the mean value of the sensitized hydrophones SPL is higher by more than 2dB than the mean value of the desensitized hydrophone SPL. For that reason, we based our results and conclusions on the sensitized hydrophones’ data. Figure 6.4 shows the mean SPL as a function of the central frequency band.
We see that the sensitized hydrophones are as much as 2dB higher, but as the frequency increases, the difference becomes smaller.

Further results for the desensitized hydrophones are shown in appendix A.

6.3. Signal correlation by sensor

It may be interesting to see the correlation between the sensors, as a signal is received by all of them. The correlation coefficient is defined by $\rho_{xy} = \frac{(x\cdot y)}{\sqrt{(x^2)(y^2)}}$, the covariance between two signals divided by the product of the standard deviation of each signal. We took the first sensor to be the reference signal, and plotted the absolute correlation coefficient as a function of sensor number in figure 6.5.
Figure 6.5: Correlation coefficient as a function of sensor number for all hydrophones for the 955Hz frequency band, NU source. The red line marks the 1/e value.

We can see that the correlation is very high for the closer sensor, but decreases rapidly as the sensors grows farther from the reference sensor.
7. Additional Analyses

7.1. Matched filter degradation

As we've shown above, the matched filter energy is the correlation between the transmitted and the received signals, whereas the Parseval's sum energy is simply the received signal energy. The difference between these energies (normalized to the respectful source energies), then, gives us a measure of the deterioration of the signal through propagation. We define this difference as the Matched Filter Degradation, \( D_{mf} = L_{PS} - L_{MF} \) (4) (7). Figures 7.1-7.3 show the matched filter degradation as a function of range for both the MIT and the NU sources (one random hydrophone and all hydrophones), for the 955Hz frequency band, with two trend lines – one for linear dependence of the degradation on the range and one for exponential dependence.

![Matched Filter Degradation vs range, f=955, MIT source](image)

Figure 7.1: Matched filter degradation as a function of range, for the 955Hz frequency band, MIT source array.
Figure 7.2: Matched filter degradation as a function of range for a random hydrophone, for the 955Hz frequency band, NU source.
The degradation of the MIT source array is lower than that of the NU source by about 1 dB, probably due to the fact that the MIT source array is directional and the NU source transmitted from closer to the sea surface, which probably caused the signal to degrade more.

There is seemingly a linear dependence of the matched filter degradation on the range. According to (4), this dependence is given by $D_{MF}(r) = c + mr$, where $r$ is the range, $c$ is the offset and $m$ is the dispersion per unit distance. Figures 7.4-7.6 show the values of $c$ and $m$, as a function of the central frequency for both the MIT and the NU (one random hydrophone and all hydrophones) sources.
Figure 7.4: The coefficients for the linear dependence $D_{MF}(r) = c + mr$, MIT source array.
Figure 7.5: The coefficients for the linear dependence $D_{MF}(r) = c + mr$, one hydrophone, NU source.
Figure 7.6: The coefficients for the linear dependence $D_{MF}(r) = c + mr$, all hydrophones, NU source.

We can see that the degradation in our case is much higher than the results shown in (4), of $c=0.6-0.7\text{dB}$ and $m=0.05-0.17\text{ dB/km}$. There may be several reasons for that: a. the experiments were conducted in very different environments, with different parameters $c$ and $m$; b. the ranges in this analysis are higher than the ranges shown in (4); c. the range window in this analysis is small and as such is susceptible to inaccuracies; and d. the model described in (4) may not be correct in our case.

If we assume that the model given in (4) is inaccurate, we can offer a different model, which is an exponential dependence of $D_{MF}$ on the range, such that $D_{MF}(r) = c \cdot \exp(mr/c)$. For small $mr/c$, $c \cdot \exp\left(\frac{mr}{c}\right) \approx c + mr$, the first order of the Taylor series expansion, which fits the linear dependence shown in (4), and it can explain the degradation difference between our results and the ones in (4).
When comparing the R squared values of the two models for all frequencies and all data sets, we see that both models explain the data quite well, at similar compliance. Figures 7.7-7.9 show the c and m values for the MIT source array and the NU source (one hydrophone and all hydrophones) for the linear and for the exponential dependence.

\[ D_M(r) = c \cdot \exp(mr/c) \]

Figure 7.7: The coefficients for the exponential dependence \( D_M(r) = c \cdot \exp(mr/c) \), MIT source array.
Figure 7.8: The coefficients for the exponential dependence $D_{MF}(r) = c \cdot \exp(mr/c)$ for one hydrophone, NU source.
We can see that the coefficients are much closer to the values received in (4). Table 7.1 shows the coefficients values received by both models and the respective R squared values.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>c (dB)</th>
<th>m (dB/km)</th>
<th>R squared</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
<td>Exponential</td>
<td>Linear</td>
</tr>
<tr>
<td>MIT</td>
<td>-28-(-15)</td>
<td>0.02-0.15</td>
<td>1.5-2.7</td>
</tr>
<tr>
<td>NU – one hydrophone</td>
<td>-48-(-34)</td>
<td>0.002-0.026</td>
<td>3.2-4.2</td>
</tr>
<tr>
<td>NU – all hydrophones</td>
<td>-46-(-36)</td>
<td>0.003-0.025</td>
<td>3.3-4</td>
</tr>
</tbody>
</table>

Figure 7.9: The coefficients for the exponential dependence $D_{MF}(r) = c \cdot \exp(mr/c)$ for all hydrophones, NU source.
Table 7.1: $c$ and $m$ coefficients for the dependence of the matched filter degradation on the range.

We can see that the values of $c$ and $m$ are much closer to the values shown in (4), a fact that may suggest that the exponential model has merit over the linear model.

## 7.2. Number of coherence cells

As mentioned above, the number of coherence cells is an important property of the field, as it signifies the number of independent statistical fluctuations of the signal. The number of coherence cells as a function of frequency for the MIT and the NU (one random hydrophone and all hydrophones) sources are shown in figures 7.10-7.12.

![Number of coherence cells for all frequencies, MIT source](image)
Figure 7.10: Number of coherence cells for the received power (blue), Parseval’s sum energy (red), matched filter energy (black) and central frequency energy (green), MIT source array.

Figure 7.11: Number of coherence cells for the received power (blue), Parseval’s sum energy (red), matched filter energy (black) and central frequency energy (green) for a random hydrophone, NU source.
We can see that the number of coherent cells for the averaged intensity and for the PS energy are very similar, but in some occasions with a difference of up to 1.5dB. We can also see that the number of coherence cells for the CF energy is very close to one, an expected result, as the central frequency energy is an instantaneous measurement. These results comply with the results shown in (4), but a little higher for PS energy.

As shown in (3), (4), (5), the number of coherence cells depends on the time-bandwidth product. In order to view this dependence, we further processed the data by taking only some of the measurements of each 1 second received signal, to form shortened signals of duration $\tau$. Since the signals are LFM signals, the bandwidth of a subset of duration $\tau$ is then $\tau/T \cdot 50Hz$. Figures 7.13-7.15 show the number of coherence cells and the standard deviation for the PS energy as a function of bandwidth for all central frequencies, for the MIT source array and the NU source (one random hydrophone).
Figure 7.13: Number of coherence cells and standard deviation as a function of BW for all frequency bands, MIT source array.
number of coherence cells vs. $B/f_c$ sensitized hydrophone

- 850 Hz
- 955 Hz
- 1125 Hz
- 1335 Hz
- 1465 Hz
- 1600 Hz

Relative BW $B/f_c$

number of coherence cells

0 0.01 0.02 0.03 0.04 0.05 0.06
Figure 7.14: Number of coherence cells and standard deviation as a function of BW for one hydrophone for all frequency bands, NU source
Figure 7.15: Number of coherence cells and standard deviation as a function of BW for all hydrophones for all frequency bands, NU source.

We can see that the number of coherence cells fits the dependence presented in (5), of \( \mu = A - (A - 1) \exp(-k \frac{B}{f_c}) \), for A and k constants, B the bandwidth and \( f_c \) the central frequency. The values for A and k that optimize the least means squares cost function are shown in figures 7.16-7.18.
Figure 7.16: the constants $k, A$ for the dependence of the number of coherence cells on the bandwidth, as a function of frequency, MIT source array
Figure 7.17: the constants $k, A$ for the dependence of the number of coherence cells on the bandwidth, as a function of frequency for one hydrophone, NU source.
Figure 7.18: the constants k, A for the dependence of the number of coherence cells on the bandwidth, as a function of frequency for all hydrophones, NU source.

The results for A and k shown here are slightly different than the results shown in (5), and we cannot specify a clear trend for the parameters.

7.3. Correlation between signals

Next in our analysis we show the correlation of the received signal as a function of distance, in order to have a measure of the distance in which the signal becomes uncorrelated. Figures 7.19-7.20 show the correlation coefficient as a function of distance from the closest signal, both for the MIT source array and for the NU source (one random hydrophone).
Figure 7.19: Abs correlation coefficient as a function of distance from the reference location, MIT source array. The red line marks the $1/e$ value.
We can see that for the MIT source array, the correlation coefficient is relatively high and decreases with distance, and for our relevant distances it is always above the folding value of $1/e$. For the MIT source array, on the other hand, the correlation is much lower, around the folding value, so it is difficult to deduce if they are indeed correlated.
8. Conclusions

The propagation of acoustic waves in an underwater medium has several effects on the signal, mainly transmission loss and added randomness to the received signals. This randomness, called scintillation, was thoroughly researched in the past for fully saturated fields, and it was shown that it is statistical in nature and is dependent on the number of independent statistical fluctuations, which in turn depends on the range, the central frequency and the bandwidth of the signal. A fully saturated field can be described by a Circular Complex Gaussian Random variable, which means that both the real and the imaginary parts have Gaussian distribution.

In February 2014, the 2014 Nordic Seas Experiment was conducted in Norway, in which LFM signals at central frequencies of 850Hz, 955Hz, 1125Hz, 1335Hz, 1465Hz and 1600Hz with 50Hz bandwidth were transmitted and received repeatedly, using the OAWRS system, and large sets of measurements of the returned echoes as well as the one way propagation were collected. These measurements allowed us to conduct an analysis of the scintillation in this experiment, and to compare this analysis to previous ones (3) (4) (5).

First, we showed that the received field is fully randomized by showing several properties of a CCGR field:

1. The distribution of the field is circular complex Gaussian.
2. The distribution of the averaged intensity is the gamma distribution with a parameter $\mu$, such that $\mu = \frac{\langle W \rangle^2}{\langle W^2 \rangle - \langle W \rangle^2}$.
3. The distribution of energy representations, for three representations of the energy, is the gamma distribution with a parameter $\mu$, such that $\sigma(L) = 10\log(e) \sqrt{\sum_{k=0}^{\infty} \frac{1}{\mu+k}}$, where $\sigma(L)$ is the standard deviation of the respective energy.

After showing that the field is indeed fully saturated, we analyzed the matched filter degradation, a value that signifies the deterioration of the signal through propagation, and we showed that it increases with range with a seemingly linear or exponential dependence on the range, and that the results that we received comply with the results received in (4) better for the exponential model.
Lastly, we showed that the number of coherence cells is dependent on the frequency and on the bandwidth, and that our results fit relatively well with the results received in (5).

Throughout our analysis we showed that the data from the MIT source array features slightly less signal degradation than the data from the NU source, but other than that, both data sets show very good fit to the theory.
9. Appendix A

This appendix shows the complete results for the averaged intensity and the energy distributions, the matched filter degradation and the Parseval's sum energy for all frequencies and all datasets.

9.1. Averaged intensity distribution
Distribution of the averaged intensity, Lubell source, sensitized hydrophone

- $f=850$
- $f=955$
- $f=1125$
- $f=1335$
- $f=1465$
- $f=1600$
Distribution of the averaged intensity, Lubell source, all hydrophones

- $f=650$
- $f=955$
- $f=1125$
- $f=1335$
- $f=1465$
- $f=1600$

Occurrences / normalized distribution

$W (Pa^2) \times 10^1$

$W (Pa^2) \times 10^1$

$W (Pa^2) \times 10^1$

$W (Pa^2) \times 10^1$

$W (Pa^2) \times 10^1$
9.2. Distribution of the PS energy
Distribution of the PS energy, MIT source

- $f=950$
- $f=1125$
- $f=1335$
- $f=1465$
- $f=1600$

Occurrences / distribution, PS

Eps (Pa$^2$*sec)

x 10$^9$
Distribution of the PS energy. Lubell source, sensitized hydrophone
Distribution of the PS energy, Lubell source, all hydrophones

- $f = 850$
- $f = 955$
- $f = 1125$
- $f = 1335$
- $f = 1465$
- $f = 1600$

$\epsilon_{\text{PS}}$ (Pa$^2$*sec) $\times 10^{-6}$
9.3. Distribution of the MF energy
Distribution of the MF energy, MIT source

- $f = 950$
- $f = 955$
- $f = 1125$
- $f = 1335$
- $f = 1465$
- $f = 1600$

Emf (Pa² * sec)
Distribution of the MF energy, Lubell source, sensitized hydrophone
Distribution of the MF energy, Lubell source, all hydrophones

- $f = 850$
- $f = 955$
- $f = 1125$
- $f = 1335$
- $f = 1465$
- $f = 1600$

Emf (Pa^2 sec) vs. frequency (Hz)
Distribution of the MF energy, Lubell source, desensitized hydrophone

Figure 9.3: Distribution of the matched filter energy.

9.4. Distribution of the CF energy
Distribution of the CF energy, MIT source

\begin{tabular}{cc}
\begin{minipage}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{f=850}
\end{minipage} & \begin{minipage}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{f=955}
\end{minipage}
\\
\begin{minipage}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{f=1125}
\end{minipage} & \begin{minipage}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{f=1335}
\end{minipage}
\\
\begin{minipage}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{f=1465}
\end{minipage} & \begin{minipage}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{f=1600}
\end{minipage}
\end{tabular}
Distribution of the CF energy, sensitized hydrophone

- $f=850$, 167 data points
- $f=955$, 151 data points
- $f=1125$, 159 data points
- $f=1335$, 146 data points
- $f=1465$, 157 data points
- $f=1600$, 162 data points

$E_{cf} (\text{Pa}^2\text{sec})$ vs. occurrence / distribution, CF
Distribution of the CF energy, all hydrophones

- $f=950$, 10743 data points
- $f=955$, 9792 data points
- $f=1125$, 10255 data points
- $f=1335$, 9573 data points
- $f=1465$, 10253 data points
- $f=1600$, 10594 data points

$Ecf$ (Pa$^2$*sec)
Distribution of the CF energy, desensitized hydrophone

Figure 9.4: Distribution of the central frequency energy.

9.5. Matched filter degradation
Matched filter degradation vs range, MIT source
Matched filter degradation vs range, Lubell source, sensitized hydrophone
Matched filter degradation vs range, Lubell source, all hydrophones

\[ D_{\text{filt}} (\text{dB}) \]

- **f=950**
- **f=955**
- **f=1125**
- **f=1335**
- **f=1465**
- **f=1600**

range (m) \[ x 10^3 \]

- 1.2
- 1.25
- 1.3
- 1.35
- 1.4
9.6. Parseval’s sum energy as a function of time window
Parseval Sum vs. Time, MIT source

- $f = 850\text{Hz}$
- $f = 955\text{Hz}$
- $f = 1125\text{Hz}$
- $f = 1335\text{Hz}$
- $f = 1465\text{Hz}$
- $f = 1600\text{Hz}$

Normalized Parseval Sum vs. Time from beginning of signal (sec)
Parseval Sum vs. Time, Lubell source

Figure 9.6: Parseval’s sum energy as a function of time window.
10. Bibliography


