Characterization of a two-color magneto-optical trap
for a spin-squeezed optical lattice clock

by
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Submitted to the Department of Physics
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Abstract

We experimentally realize a magneto-optical trap (MOT) for ytterbium that simultaneously scatters light from two atomic transitions, in order to reduce the minimum magnetic field gradient required for trapping atoms. From the experimental data, we propose a model of the MOT where one transition is responsible for trapping the atoms the other transition is responsible for cooling the atoms. In order to test this model and to better understand the dynamics of this two-color MOT, we perform numerical simulations of the contributions of each transition to the steady-state number of trapped atoms. We then compare the results of the simulations with the experimental results and conclude that the model of the separated trapping and cooling functions is a good model for the two-color MOT. Finally, we examine other elements for which the two-color MOT is possible and derive a condition for the linewidths of the two transitions such that the atoms are trapped.

Thesis Supervisor: Professor Vladan Vuletić
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Chapter 1

Introduction

We often think of the magneto-optical trap (MOT) as one of the workhorses of atomic physics [1]. MOTs are used to cool and trap neutral atoms for experiments in metrology [2], quantum information, simulations of solid state systems, and more [3] [4]. Its versatility as a laser cooling method is illustrated by the 14 species of atoms that have been trapped in a MOT [5]. Being able to trap an atom with a MOT only requires that the atom has one closed atomic transition with a broad linewidth, with "closed" meaning that it is mainly coupled to only one other state. Typically, limitations on availability of lasers with the appropriate frequency and linewidth are what prevent an atomic species from being able to be trapped in a MOT. Of the atoms that have been trapped in a MOT, the majority of atoms have been from the alkali, alkali earth, alkali earth-like, and noble gas groups. This is mainly due to the simple outer-shell electronic structure of these atoms and the presence of strong closed transitions, as well as the availability of lasers.

A drawback of making a MOT for trapping alkali earth atoms is that it typically requires a high magnetic field gradient to both cool and trap atoms, due to the broad linewidth of the principal singlet-singlet transition. This is because in order to achieve both functions, the trapping potential must be both broad and deep. However, large magnetic fields can be impractical due to the high power consumed in generating the fields using current coils. In compact systems, such as airborne systems, using such high power is often impossible. Thus, we are motivated to develop a MOT for alkali
earth and earth-like atoms that can use a low magnetic field gradient and can still trap on the order of $10^5$ atoms.

These characteristics are necessary for our final atomic clock experiment, where we will use an ensemble of neutral atoms in an optical lattice to make a precise sin-squeezed clock. The clock only requires $10^5$ atoms and the geometry of our clock experiment requires that we can only use low magnetic field gradients. Even though the atomic clock experiment is the main goal, it is ongoing and not the focus of this thesis.

In this thesis, we study a method that separates the two functions of a MOT of trapping and cooling into two different atomic transitions for the alkali earth-like atom ytterbium. This allows us to trap on the order of $10^5$ ytterbium atoms using a magnetic field gradient of only 5 G/cm. We study this method both experimentally and theoretically. We use numerical simulations of the dynamics of an atom inside the MOT to calculate the critical velocity of trapped atoms and fraction of trapped atoms. This allows us to create a working model of this experimentally-observed two-color MOT.

1.1 Magneto-optical trap theory

In order to trap atoms in a MOT, we need (a) a magnetic field gradient and (b) tunable lasers that excite a transition of the atom [1]. As a broad summary, in a MOT, the momentum transferred from the laser photons to the atoms depends on the position and velocity of the atom; by tuning the polarization of the lasers and the magnetic field strength, the MOT is able to slow the atoms and drive exactly to its center.

We will start by understanding a system where the atoms are only allowed to move in 1 dimension. As we will see, this system is a good approximation of a 3-dimensional MOT and will give us good intuition for the dynamics of an atom inside a 3-D MOT. We will then move on to understand MOT dynamics in 3 dimensions.
1.1.1 1 dimensional MOT

Fundamental to any laser cooling technique is the force that a photon imparts on an atom when the atom absorbs the photon. The force \( \vec{F} \) imparted to an atom is proportional to the probability of the atom absorbing a photon of frequency \( \nu \) and the momentum of the photon \( \hbar \vec{k} \). Here, \( \vec{k} \) is the wave vector of the light; it points in the direction of the traveling photon and has magnitude \( |k| = \frac{2\pi}{\lambda} \), where \( \lambda = c/\nu \) is the wavelength of the light. We know that the probability of absorption is a Lorentzian centered on the resonance frequency of the transition \( \nu_0 \) as shown in Figure 1-1. Thus, the force is given by

\[
\vec{F} \propto \hbar \vec{k} \frac{\Gamma/2}{\Gamma/2 + (\Delta - k\nu)^2}
\]

where \( \Delta = \nu - \nu_0 \) is the frequency detuning of the light away from the resonance frequency, \( \nu \) is the velocity of the atom, and \( \Gamma \) is the atomic linewidth. Since the force will always point in the direction of motion of the photon, I will drop the vectors from here forward and just calculate the magnitudes of the forces.

![Figure 1-1: The force imparted on an atom from absorption of a photon takes the shape of a Lorentzian in frequency space.](image)

If we now send two counter-propagating laser beams and shift the frequency of the laser so that it is slightly below the resonance frequency, then the magnitude of the force imparted to the atoms is given simply by the sum of forces in 2 counter-propagating directions, and is shown in Figure 1-2:

\[
F \propto \hbar k \frac{\Gamma/2}{\Gamma/2 + (\Delta - k\nu)^2} - \hbar k \frac{\Gamma/2}{\Gamma/2 + (\Delta + k\nu)^2}
\]

As we see in the figure, this "optical molasses" now gives a force to the atom in the
positive direction when the atom is moving in the negative $x$ direction, and vice versa in the positive $x$ direction, which slows the atoms until they are at $v = 0$. Although this force can slow down the atoms, it cannot confine the atoms spatially, which would be a problem for many atomic physics experiments, where the atoms need to be confined spatially.

Figure 1-2: The force imparted on an atom from absorption of a photon reverses the atom’s velocity and takes the shape of a superposition of two Lorentzians in frequency space. Here, the frequency detuning of the laser away from resonance $\Delta$ has been set to 0. The force is a slowing force because when the atom is moving towards the negative $x$ direction, it receives a force in the positive $x$ direction; conversely, if it is moving the positive $x$ direction, it receives a force in the negative $x$ direction.

A magneto-optical trap is able to make this spatial confinement. It can do this by exploiting the Zeeman effect and angular momentum of the transitions. Let's first consider a 2-level system with $l = 1$ in the excited state (see energy diagram in Figure 1-3a). Under a constant magnetic field $\vec{B} = B_c \hat{z}$, the magnetic sublevel energies are split by $\mu m B_c$ where $\mu$ is the magnetic moment associated with the transition and $m$ is the magnetic sublevel number. The energy level diagram at all points in space is shown in Figure 1-3b. Because of the angular momentum associated with each magnetic sublevel, right circularly polarized light (denoted by $\sigma^+$) with angular momentum $\hbar$ will excite the electron from the $m = 0$ ground state into the $m = +1$ state and left circularly polarized light ($\sigma^-$) with angular momentum $-\hbar$ is likely to excite the electron from the ground state into the $m = -1$ state. In this present situation, light incident on the atom from any direction is simply going to impart a force given in Equation 1.1 in the direction of the traveling photon. However, in order to effectively trap the atoms, the forces imparted on the atom must selectively point
towards the center of the trap and decrease in magnitude the closer towards the trap center.

\[ b \text{C} \]

\[ m = \pm 1 \]

\[ m = 0 \]

\[ B_0 \]

\[ \mu B \]

\[ m = \pm 1 \]

\[ m = 0 \]

\[ m = \pm 1 \]

\[ B = B_0 x \hat{x} \]

Figure 1-3: Energy level diagram of an atomic transition with \( l = 1 \) under (a) no magnetic field gradient, (b) constant magnetic field gradient, (c) linear magnetic field gradient.

The clever trick of a MOT is to have a spatially varying magnetic field, that causes the energy level configuration to depend on the position of the atom. This way, we can impart a selective direction and magnitude of a force on the atom depending on its spatial location. Let's consider a linear magnetic field gradient \( \vec{B} = B_0 x \hat{x} \). The energy level diagram is shown in Figure 1-3c. If instead of using light resonant with the transition, we now use light that is slightly red-detuned with the transition, then most of the light is absorbed on either side of the center of the trap (see Figure 1-4). If we now selectively send \( \sigma^- \) light towards the left, then it will mostly be absorbed at positions to the right of the center of the trap, where it imparts a force to the left, or in other words, to the center of the trap! We can send \( \sigma^+ \) in the opposite direction, to the right, and again we see that it is only absorbed to the left of the center of the trap, where it again imparts a force towards the center of the trap. Since the re-emission of the photon is in a random direction, it will not cancel out the effect of this force. Thus, the MOT in total imparts a force towards the center of the trap.

The force \( F \) inside the trap is a function of an atom’s position from the center \( x \)
Figure 1-4: Diagram of lasers in a MOT. By sending $\sigma^+$ light to the right and $\sigma^-$ light to the left, and red-detuning the laser beams from the atomic transition, the light will be selectively absorbed to give the atom a net force towards the center of the trap.

and velocity $v$ and is described by the following equation

$$F = \frac{\hbar k\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + 4\left(\frac{\Delta - kv - \mu B_x}{\Gamma}\right)^2} - \frac{\hbar k\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + 4\left(\frac{\Delta + kv + \mu B_x}{\Gamma}\right)^2}$$ (1.3)

where $k$ is the wavenumber of the light, $\Gamma$ is the linewidth of the transition, $I$ is the light intensity, $I_0$ is the saturation intensity, $\mu = \mu_g \gamma$ is the magnetic moment associated with the transition, $\mu_b$ is the Bohr magneton, and $B = \partial B/\partial x$ is the magnetic field gradient. This force is shown in Figure 1-5 and by numerically integrating the force with respect to position we can find the trap potential$^1$, which is also shown in the figure. Close to the center of the trap, the trap resembles a harmonic oscillator. In this region, any force imparted on the atom also decreases in magnitude linearly towards the center of the trap, which in combination with the damping provided by cooling prevents the atom from overshooting the center of the trap.

By inspecting Equation 1.3, we notice that using a transition with a larger linewidth $\Gamma$ produces both a larger maximum force and increases the range in momentum space over which the force acts. Figure 1-6 shows the force as a function of atom velocity.

$^1$The trap potential is given by $U(x) = \int_{-\infty}^{x} F(x')dx'$. 

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Figure 1-5: The force (green solid) on an atom in a MOT, as described by Equation 1.3. By numerically integrating the force, we find the trap potential (blue dashed). We have used the following sample parameters in the plot: $\lambda = 399$ nm, $\Gamma = 28 \times 10^6$ Hz, $I/I_0 = 0.13$, $B = 6.75$ G/cm, $\Delta = -20 \times 10^6$ Hz, $g = 1.04$.

for transitions with different linewidths. A transition with a larger linewidth can slow down faster atoms and give them a greater slowing force than a transition with a smaller linewidth. We can think of large linewidth transitions as having a larger slowing power.

1.1.2 3 dimensional MOT

In order to trap atoms in 3 dimensions, the atom needs to be confined from 3 orthogonal directions. This is possible through three counter-propagating laser beams and an anti-Helmholtz magnetic field, where the currents in the pair of coils are running in opposite directions (see Figure 1-7).

The theory for a 3-dimensional MOT is more complicated because the magnetic field is no longer parallel to the light beams at every point in space. As a result, the dipole matrix elements of multiple magnetic sub-levels are non-zero and there is a contribution to the force on the atom from scattering off of more than one magnetic sub-level. Furthermore, the light beam no longer has circular polarization from the point of view of the atoms. Thus, we must also calculate the reduced intensity of the circular polarized portion of the light for each magnetic sub-level. When taking into
Figure 1-6: The force on an atom as a function of its velocity at $x = 0$. A transition with a larger linewidth has a larger slowing power than a transition with a smaller linewidth. All other parameters were kept constant.

account all of the factors, the force of the MOT on the atom should be smaller in the 3-dimensional case than in the 1-dimensional case because of the superposition of forces from scattering off of multiple magnetic sub-levels.

1.1.3 A MOT for ytterbium

The energy level diagram for ytterbium is given in Figure 1-8. We have used the conventional notation $^{2S+1}L_J$ to label the energy levels, where $S$ is the spin quantum number, $J$ is the total angular momentum quantum number, and $L$ is the orbital angular momentum quantum number. The atomic transitions that can be used in a MOT are $^1P_1$ (singlet transition) and $^3P_1$ (triplet transition) because they have large linewidths and are coupled to the ground state ($^1S_0$). In making a MOT with each of these transitions, we can ignore the other transitions and treat the atom like a two-level system. This approximation is valid when neither transition is saturated by the intensity of the lasers, which is almost the case in our experiment.

Previous experiments have typically used both the singlet and triplet transitions for both cooling and trapping the atoms. However, the experiments have typically used a two-stage MOT where the atoms were loaded into a pure singlet MOT, the singlet transition was turned off, and then the triplet transition was turned on [7].
Although this method can trap a large number of atoms, the disadvantage is that the pure singlet MOT must cool and initially confine the atoms without the help of the triplet MOT, which requires a large magnetic field gradient of typically $\approx 45$ G/cm. This consumes a large amount of power which is impractical for portable and space-borne systems.

Another method that has been used is to first slow down the atoms with a Zeeman slower, which is composed of a large spatial magnetic field gradient and a fixed frequency laser beam [8]. As the atoms move across the slower, the magnetic field magnitude gets smaller, bringing progressively slower atoms into resonance and cooling them. Thus, it makes use of optical molasses to cool the atoms across the length of the slower. After being slowed by the Zeeman slower, the atoms are loaded into a triplet MOT [13]. However, a Zeeman slower also requires high magnetic fields of about 100 G, and still requires high operating power.
Figure 1-8: Energy level diagram of ytterbium. The singlet $^1P_1$ transition and triplet $^3P_1$ transition can be used to make a MOT. Taken from Ref. [6].
Chapter 2

Two-color MOT experiment

2.1 Motivation

Our final clock experiment aims to use neutral atoms in an optical lattice to create a precise clock through spin-squeezing [10]. By measuring the atomic transition frequency $\nu_0$ along the direction of lowest uncertainty, we have a possibility of surpassing the standard quantum fractional noise $\sigma$, which is given by

$$\sigma = \frac{1}{\nu_0} \frac{1}{\sqrt{T N \tau}}$$ (2.1)

where $T$ is the total measurement time of the entire experiment, $N$ is the number of atoms measured, and $\tau$ is the length of each experimental cycle [6]. The quantity $\frac{1}{\sqrt{T N \tau}}$ can be thought of as the uncertainty of the frequency measurement $\Delta \nu$. The frequency uncertainty $\Delta \nu$ can be minimized by using a narrow transition; we use the doubly forbidden transition in ytterbium-171 ($^1S_0$ to $^3P_0$). The number of atoms $N$ can be optimized by confining the neutral ytterbium atoms in a lattice made of standing waves of light, called an optical lattice. Using neutral atoms has the advantage of being unaffected by the Coulomb force, which would otherwise limit the number of atoms that can be trapped. An optical lattice can confine $\approx 10^4$ atoms, and clocks made in this way have been able to reach a precision limit of $\sigma = 10^{-17}$ [11].

This limit can be surpassed by decreasing the uncertainty of the spins of atoms
in one direction, while increasing it in the orthogonal direction. This is called “spin-
squeezing” [12]. If we pick to measure the transition frequency along the direction
that has decreased frequency uncertainty, then the uncertainty of the clock $\sigma$ also
decreases. The advantage is that for a fixed number of atoms and measurement
time, a spin-squeezed clock in an optical lattice is more precise than a classical clock.
We can achieve spin-squeezing by increasing light-mediated interactions between the
atoms, which is possible through a cavity with a high cooperativity that gives strong
coupling between the light and atoms.

We trap the atoms using a magneto-optical trap, and then plan to load them into
a high cooperativity cavity. In order to allow for a high level of cooperativity, we
strongly focus the light beam in the cavity by using one mirror with a small radius
of curvature (see Figure 2-1). We load the atoms in the region with the highest light
intensity to allow for the most interactions, but this region is extremely close to the
highly-curved mirror. As a result, there is not enough room geometrically for three
semi-orthogonal MOT beams.

![Figure 2-1: (a) Schematic of atoms in MOT region in the experiment. Taken from Ref. [6]. (b) Current coils in experiment. Due to the geometry of the mirror MOT, only the white and blue coil sets are used. The blue coils give a magnetic field gradient only up to 8 G/cm.](image)

To solve this problem, we embed the highly curved mirror into a planar mirror and
use the reflected MOT beams off of the planar mirror to make two sets of counter-
propoagating beams for a “mirror-MOT” (see Figure 2-1a) [6]. However, in order to
be able to see the atoms inside the cavity through the biggest viewport in our vacuum
chamber, we must place the cavity vertically with the mirrors parallel to the table,
making the MOT beams diagonal. This configuration prevents us from using the large current coils which can produce a large magnetic field (see Figure 2-1b). Thus, we are constrained to using smaller diagonal coils, which can achieve a maximum magnetic field gradient of $\approx 8\ \text{G/cm}$, and are insufficient for trapping atoms directly from the oven into a singlet MOT. This problem stimulated investigation of the two-color MOT.

In our experiment, we only need on the order of $10^3$ atoms to achieve spin-squeezing, which makes large magnetic fields and a Zeeman slower unnecessary. Achieving $10^5$ atoms in a MOT will allow for losses when we transfer the atoms to the optical lattice. Additionally, we would like to confine the atoms spatially to stay within the waist of the cavity laser beam in the final atomic clock experiment. This requires that the cloud size be a few hundred microns in diameter. Thus, our aim is to investigate whether it is possible to use small magnetic field gradients to trap on the order of $10^5$ atoms with an atom cloud size of a few hundred microns.

### 2.2 Setup and procedures

In order to first make a proof of principle that we can trap $10^5$ atoms with a small magnetic field gradient, we use a conventional MOT setup to make the two-color MOT (see Figure 2-2b) as opposed to the final more complicated mirror-MOT setup. The setup consists of the MOT, laser beams, and an oven containing ytterbium atoms. Our setup does not have a Zeeman slower, and thus Yb atoms are loaded directly to the MOT from the oven, which reaches a temperature of $\approx 700\text{K}$. The two-color MOT has 2 sets of 3 pairs of counter-propagating $\sigma^+$ and $\sigma^-$ beams that excite the Yb transitions at the 399 nm singlet transition and 556 nm triplet transition (see Figure 2-2a). The singlet transition has a transition linewidth $\Gamma_s/(2\pi)=28\ \text{MHz}$ and the triplet transition has a transition linewidth $\Gamma_s/(2\pi)=184\ \text{kHz}$, which makes the ratio of linewidths $\approx 200$. The singlet transition beam has an intensity of $I_s = 0.26I_{\text{sat},s}$, where $I_{\text{sat},s} = 58\ \text{mW/cm}^2$, and a detuning of $\Delta_s = -0.7\Gamma_s$. The triplet transition beam has an intensity of $I_t = 160I_{\text{sat},t}$, where $I_{\text{sat},t} = 0.138\ \text{mW/cm}^2$, and a detuning
of $-3.7\Gamma_t$.

![Diagram showing energy levels and MOT setup](image)

Figure 2-2: (a) Energy levels of ytterbium used in the MOT. (b) MOT setup. Taken from Ref. [11].

Additionally, we use a 399 nm longitudinal cooling beam that counter-propagates along the direction that atoms exit the oven, in order to cool down the atoms before entering the MOT region. The longitudinal cooling beam is detuned $-4\Gamma_s$ from 399 nm and has a power of 3mW. We use anti-Helmholtz coils that give maximum magnetic field gradients of (45, 22.5, 22.5) G/cm along the (x, y, z) axes (defined in Figure 2-2).

We use a CCD camera to take images of the atoms' fluorescence. Two sample images are shown in Figure 2-3. From these images we measure two quantities for every setup: number of atoms trapped and the atomic cloud size. We count the atom number by measuring the calibrated intensity of scattered light onto the CCD camera. We measure the cloud size by taking the root mean squared spatial spread of the scattered light. The atom number tells us the cooling and trapping power of the MOT, and the density tells us the compression of the MOT. For each of these quantities, we average 5 measurements and take the standard deviation as the uncertainty. The atom density can be extracted from these two quantities.

### 2.3 Results

Figure 2-4 shows the number of atoms trapped in the singlet and two-color MOTs for different magnetic field gradient values. We observe that at magnetic field gradients
below 4.5 G/cm, the singlet MOT is unable to trap any atoms, while the two-color MOT still traps on the order of $10^5$ atoms. This seems to suggest that the two-color MOT has additional functionality outside of that of the singlet MOT. We suggest a possible model for the MOT to explain this result in the next section.

Both singlet and two-color MOTs capture a maximum number of atoms, but the peak occurs at $\partial B/\partial x = 13.5$ G/cm for the singlet MOT and $\partial B/\partial x = 6.75$ G/cm for the two-color MOT. We can see already that the two-color MOT seems to have a better trapping ability than the singlet MOT at a lower magnetic field gradient. Additionally, we notice that the peak atom number for the singlet MOT occurs at a lower magnetic field gradient than for a conventional singlet MOT in other experiments, whose peak is at $\partial B/\partial x = 30-45$ G/cm \cite{13}. One possible explanation is that we tuned the intensities of the counter-propagating MOT beams to match well.

We also notice that the number of atoms trapped is unexpectedly lower for the two-color MOT than for the singlet MOT for $\partial B/\partial x > 8$ G/cm. We suspect that some experimental conditions were different for these data points, such as the detuning of singlet beams or the position that the singlet beams overlap. However, we are not sure of the specific cause because these conditions are hard to quantify.

Figure 2-5 shows a characterization of the two-color MOT. Specifically, we’ve measured atom number and cloud size as a function of the light intensities and frequency detunings of the triplet and singlet MOT lasers. By examining 2-5a and 2-5b, we notice that the atom number peaks at $-0.7\Gamma_s$ and $-22\Gamma_t$ and the frequency range

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**Figure 2-3**: Sample CCD images of the atomic cloud.
Figure 2-4: Experimentally measured number of atoms trapped as a function of magnetic field gradient. Green squares: two-color MOT. Blue triangles: singlet MOT. The error bars of the two-color MOT are smaller than the size of the markers.

over which the two-color MOT is able to trap atoms is $1.5 \Gamma_s$ and $35 \Gamma_t$, where $\Gamma_s$ and $\Gamma_t$ are the linewidths of the singlet and triplet transitions respectively.

From examining 2-5c and 2-5d, we observe that the atom number of the two-color MOT saturates for large light intensities, which we can explain by the atomic transition being saturated. As more photons excite the atoms into the two excited transitions, fewer atoms are into the ground state and thus fewer photons can scatter.

The atomic cloud size of the singlet MOT is 2.0 mm while the cloud size for the two-color MOT is 0.7 mm at $\partial B/\partial x = 6.75$ G/cm. The cloud size of the triplet MOT is 0.7 mm, which is similar to the cloud size of the two-color MOT. This suggests that the atoms are primarily trapped by the triplet MOT.

Finally, we were able to create a pure triplet MOT that trapped $(1.9 \pm 0.3) \times 10^3$ atoms by decreasing the singlet light intensity to zero (see Figure 2-5c). This is the first observed pure triplet MOT that has been loaded directly from the atomic beam [111].

2.4 A model for the MOT

One model to explain our results is that the blue and green transitions have separate functions of cooling and trapping the atoms respectively. We use the naive model that the forces from the blue and green light are added in superposition:
Figure 2-5: Characteristics of the two-color MOT. Here we show atom number and cloud size measurements of the two-color MOT for different light intensities and frequency detunings of the triplet and singlet MOT lasers. Taken from reference [14].

\[ \vec{F}_{\text{tot}} = \vec{F}_{\text{singlet}} + \vec{F}_{\text{triplet}} \]  

(2.2)

Figure 2-6 shows the trapping potential depth of the singlet and triplet MOTs as a function of the magnetic field gradient using this model. From the model, we see that this model predicts at magnetic field gradients less than 8 G/cm, the singlet MOT has a smaller trapping potential depth than the triplet MOT. However, since the singlet transition is broader than the triplet transition, the associated force acts over a larger range of velocities (see Figure 1-6). Thus, at low magnetic field gradients, the singlet MOT is broad and shallow, which allows cooling but not trapping, and the triplet MOT is narrow and deep, which allows trapping but not cooling. At magnetic field gradients greater than 8 G/cm, the trapping potential depth of the singlet MOT is larger than the triplet MOT, so we predict that the triplet MOT has little contribution to both cooling nor trapping at large magnetic field gradients.

Thus, by separating the functions of the MOT, we are able to exploit each function.
at a low magnetic field. Additionally, the peaks in the experimental data in Figure 2-4 occur at about the same magnetic field gradient as the intersection of the potential depths of the two MOTs in the model, approximately 8 G/cm.

Another interesting observation from Figure 2-6 is the effect of the imbalanced intensities of counter-propagating beams. Even when we carefully align the counter-propagating beams, the retro-reflected beam passes through more optics components than the forward propagating beam and has more loss. In total, the retro-reflected beam passes through the back window of the MOT chamber, one quarter wave plate, a mirror, back through the quarter wave plate, and back through the window before it reaches the atoms again. Each element contributed $\approx 1\%$ intensity loss, resulting in a $\approx 5\%$ total loss in the counter-propagated beam even when the beams are aligned carefully. From Figure 2-6, we see that a 5% imbalance causes the trap potential of the singlet MOT to be 1.5 times smaller than the trap potential of the triplet MOT at 8 G/cm, where the trap potentials were the same with no power imbalance. Furthermore, the triplet MOT is more robust against imbalanced beams than the singlet MOT.
Chapter 3

Simulations of cooling and trapping forces in two-color MOT

One way to test the model in Section 2.4 is by comparing numerical simulations of the atom number to the experimental atom numbers. Specifically, we would like to understand the cooling and trapping functions of the two-color MOT separately, and answer the question: Which transitions contribute to which functions? It is valid to treat the two functions separately because they are fundamentally two different physical processes. The cooling is due to confinement in velocity space, whereas the trapping is due to confinement in position space.

To figure out which transition or transitions contribute to the cooling force, we numerically simulate a single moving atom that experiences the forces of a two-color MOT and calculate the maximum initial velocity that the atom can have for it to be captured into the MOT. We call this velocity the "critical velocity" $v_{\text{crit}}$. The critical velocity tells us the magnitude of the cooling force. This method of calculating the critical velocity is useful because it allows us to adjust the parameters of the two-color MOT to determine which transition is more important in setting the critical velocity. Additionally, we can extrapolate the fraction of captured atoms from the critical velocity by integrating the Maxwell-Boltzmann distribution over velocity, from $v = 0$ to $v = v_{\text{crit}}$, and normalizing to the total number of atoms. We can neglect interactions between atoms because the atom density in the MOT is $10^9 \text{ cm}^{-3}$, which
is low [16].

Similarly, for the trapping force, the goal is to determine the contributions of each of the two transitions. Instead of performing numerical simulations, it is easier and just as enlightening to make theoretical calculations of the maximum depth of the potential well \( U_{\text{max}} \). This method tells us, if we begin with atoms confined inside the two-color MOT, what fraction of atoms leave the trap due to thermal collisions. Again, we can vary each parameters of the two-color MOT to determine which transition plays a bigger role in the trapping function. We can also extrapolate the fraction of trapped atoms by integrating the Maxwell-Boltzmann distribution over the energy, from \( E = 0 \) to \( E = U_{\text{max}} \). By combining the fraction of atoms that become confined from the cooling function and the fraction that stays trapped from the trapping function, we can get an estimate of the steady state fraction of atoms in the MOT (compared to the total number of incoming atoms) at different magnetic field gradients, and compare to the experimental results in Figure 2-4.

3.1 Cooling force

3.1.1 1 dimensional simulation

A 1-dimensional numerical simulation of the two-color MOT, as described in Section 1.1.1, is sufficient to get a first order approximation of the effect of the two transitions on the cooling and trapping forces. Additionally, since we are mainly concerned with comparing the two transitions, we can assume that increasing the simulation to higher dimensions will add similar non-linear effects to both transitions' functions, and thus will not affect the comparison.

To perform the 1D simulation, we numerically integrated the force on an atom in a two-color MOT, as given in Equation 3.3, using the Runge-Kutta method to obtain

\footnote{From the work of Lindquist, Stephens, and Wieman (1992), we can assume that the atom numbers calculated from 1-dimensional simulations are proportional to the atom numbers from 3-dimensional simulations. Since we are only comparing the general dependencies of the atom numbers on different parameters of the MOT and not absolute atom numbers, a 1-dimensional simulation will suffice [16].}
the velocity and position of the atom at all times for a range of positive initial velocities and an initial position of -5 cm. Sample results of the phase space trajectories for one set of sample parameters are shown in Figure 3-1. We see that starting below the critical velocity with a specific set of two-color MOT conditions causes the atom to converge to $v = 0$ and $x = 0$, but starting above the critical velocity will cause the atom to escape the trap. This is due to the insufficient slowing force from the two-color MOT.

![Figure 3-1: Position and velocity trajectories of a simulated atom with initial velocities 2 m/s (green dotted), 8.4 m/s (red solid), and 8.5 m/s (blue dashed) in a two-color MOT. For these sample parameters, the critical velocity is 8.4 m/s because it is the highest initial velocity an atom can have while still converging to $x = 0$, $v = 0$.](image)

To better understand the dynamics near the critical velocity, we plot in Figure 3-2 the forces from the left and right traveling beams, position, and velocity as a function of time when starting at 8.4 m/s, which is just below the critical velocity of this set of sample parameters. We see that at small times, the total force takes on a gaussian shape. This can be explained by the negligible change in the atom’s velocity, which causes the intensity of the light to be the dominating effect on the magnitude of the force; in our experiment and simulations, we have a gaussian beam profile. The atom moves through the gaussian spatial profile over time and thus experiences a force that varies as a gaussian. At times closer to when the atom is reaching an

2 We used the ode45 function in MATLAB, which has a variable order.
equilibrium position, the total force as a function of time takes on the Lorentzian shape of the transition resonance. This can be explained by the sharp change in the atom’s velocity, which puts the laser frequency in resonance with the Lorentzian shape of the atomic transition.

![Graph](image)

Figure 3-2: MOT dynamics for sample parameters and with an atom speed of 8.4 m/s, which is just below the critical velocity. (Left inset) The left and right forces take a gaussian shape at small times, when the forces are primarily affected by the gaussian shape of the laser beams. (Right inset) At large times, the force takes on a Lorenzian shape, which is characteristic of the Lorentzian shape of the transition.

In order to calculate the critical velocity, we begin by simulating the phase space trajectory of an atom with initial velocity 20 m/s, determine whether it is captured or not, decrease the initial velocity by $\Delta v$ (typically 0.0001 m/s to 0.1 m/s), and repeat the simulation until the atom is captured. The critical velocity is the largest velocity at which the atom is captured. The condition for the atom being captured is if the final speed of the atom is less than 0.1 m/s.

In order to probe each transition’s contribution on the critical velocity, we plot the simulated critical velocity as a function of the different parameters of the trap in Figure 3-3. The parameters that we explore are the frequency detuning and light intensity for each transition and the magnetic field gradient. Figures 3-3a and 3-3b show the critical velocity as a function of the singlet and triplet light detuning.
We see that at larger detunings of the singlet light frequency away from resonance, the critical velocity increases. This is due to the larger cooling power coming from a larger force on the atoms. As the detuning increases, there is less of an overlap between the forces from the left and right moving beams, which lowers the canceling effect of the two forces towards the center of the trap. On the other hand, changing the detuning of the triplet light has a negligible effect on the critical velocity. This supports our experimental result that the loading rates with and without the triplet light are approximately the same. Furthermore, since the critical velocity is directly related to the cooling force, we conclude that the triplet light provides no cooling force, which is consistent with our model in Section 2.4.

Figures 3-3c and 3-3d show the effect of the singlet and triplet light intensities on the simulated critical velocity. We see that as we increase the intensity of the singlet light, the critical velocity increases sharply from $I_s/I_{sat,s} = 0$ to $I_s/I_{sat,s} = 0.01$, after which there is a smaller steady increase in $v_{crit}$ towards higher light intensities. This is due to the saturation of the singlet transition. On the other hand, we observe again that the intensity of the triplet light has no effect on the critical velocity, which further supports our conclusion that the triplet light provides no cooling force. The value of the magnetic field gradient has no effect on the cooling force because it is independent of $v$.

To calculate the fraction of atoms $f$ from the oven that are captured into the MOT, we integrated the 3-dimensional Maxwell Boltzmann distribution over velocity, from $v = 0$ to $v = v_{crit}$ and normalized by the total number of atoms in the Maxwell-Boltzmann distribution.

$$f = \frac{\int_0^{v_{crit}} v^2 e^{-\frac{mv^2}{2kT}} dv}{\int_0^\infty v^2 e^{-\frac{mv^2}{2kT}} dv}. \quad (3.1)$$

where $v$ is the velocity of the atom, $k$ is Boltzmann’s constant, and $T$ is the temperature of the atoms’ environment which in this case is the temperature of the oven 700K. We noticed that $f$ follows the same graph shape as in Figure 3-3 because we were in the small $v$ region of the Maxwell-Boltzmann distribution where $v^2 e^{-\frac{mv^2}{2kT}} \approx v^2$, and $f \propto \int_0^{v_{crit}} v^2 = v_{crit}^3 \approx v_{crit}$ for small $v_{crit}$. Thus, $f$ had the same qualitative
Figure 3-3: Effect of (a) singlet light detuning, (b) triplet light detuning, (c) singlet light intensity, and (d) triplet light intensity on the critical velocity inside the two-color MOT. (a) Singlet light detuning. As the singlet light frequency approaches the resonance frequency, the critical velocity decreases, because the cooling power is smaller. This is due to the increased overlap of the forces from the left and right moving beams in velocity space. (c) Singlet light intensity. As the intensity is increased, the critical velocity increases until saturation. (b) and (d) show that the triplet light provides no cooling force. We used a step size of $\Delta v = 0.1m/s$ for the initial velocities in simulations (a) and (c) and $\Delta v = 0.0001$ for (b) and (d).

dependence on the two-color MOT parameters as the critical velocity.

3.1.2 Addition of longitudinal cooling beam

To simulate the MOT more closely to our experimental conditions, we examined how adding the longitudinal beam into the MOT region in our numerical simulation affects the critical velocity. The reason for including the longitudinal cooling beam is that in the realistic, 3-dimensional situation of our final experiment, the longitudinal cooling beam may partially intersect the MOT region, as illustrated in Figure 3-4. By contrast, in ideal experimental conditions, the longitudinal cooling beam only
slows down the atomic beam before it reaches the MOT region. We both wanted to implement the most realistic experimental conditions and we were curious whether the accidental intersection of the longitudinal cooling beam helps or hurts the critical velocity of the MOT. Our expectation is that adding the longitudinal beam restricts the possible captured velocities to a smaller range because the longitudinal cooling beam gives a large force that distorts the trapping potential and kicks the atom out of the trap.

Figure 3-4: (top) Possible geometry for intersection of longitudinal beam with the MOT region. (bottom) The partial intersection of longitudinal beam is simulated by adding an intense singlet light beam counter-propagating against direction of atoms and clipping the force at $x_{clip}$.

We introduce the longitudinal cooling beam into our 1-dimensional simulation by creating a singlet light beam that has high intensity and propagates in the opposite direction as the initial velocity of the atom. To best approximate the partial intersection of the longitudinal beam with the MOT region, we cut off the intensity at...
varying positions $x_{\text{clip}}$, as shown in the bottom of Figure 3-4. The total force on the atom inside the MOT is:

$$F_{\text{tot}}(x, x_{\text{clip}}) = F_{\text{singlet}}(x) + F_{\text{triplet}}(x) + F_{\text{long}}(x) \times u(x - x_{\text{clip}})$$  \hspace{1cm} (3.2)$$

where $u(x - x_{\text{clip}}) = 0$ for $x < x_{\text{clip}}$ and $u(x) = 1$ for $x > x_{\text{clip}}$. We then perform the numerical simulations as described in section 3.1.1.

In Figure 3-5, we show the simulated captured velocities as a function of the clipping position of the longitudinal cooling beam. We notice that as the atoms are exposed to the longitudinal cooling beam for a longer portion of their path, the velocity of atoms that can be captured increases. However, the range of all possible captured velocities decreases. On the other hand, the total number of captured velocities goes as $v^4_{\text{crit}}$, which gives a maximum number of atoms captured around $x_{\text{crit}} = -0.4$.

In Figure 3-6, we simulate the capture velocity as a function of the intensity of the longitudinal beam. As the intensity increases, the critical velocity first rapidly increases from $I/I_{\text{sat},s} = 0$ to $I/I_{\text{sat},s} = 0.2$, and then slowly for $I/I_{\text{sat},s} > 0.2$ until saturation. This makes sense because as the intensity of the beam increases, the cooling force increases until the transition is saturated. We also observed some intensity intervals where no atoms are captured, especially at high intensities and when the longitudinal beam is exposed to a larger portion of the atom’s path ($x_{\text{clip}} > -0.4$). We believe is due to the finite step size of the velocity $\Delta v = 0.1$ when simulating atom trajectories. As the intensity of the longitudinal beam increases, the range of possible captured velocities decreases. Thus, the gaps where no atoms seem to be captured in Figure 3-6 occur because $\Delta v$ did not include the range of possible captured velocities.

### 3.2 Trapping force

By contrast to the cooling force, the trapping force is what confines the atoms spatially after they are cooled. Our goal is to check which transition has the primary
Figure 3-5: (top) Critical velocity as a function of position that longitudinal beam is clipped. Here, the MOT is centered at $x = 0$ and the atoms are approaching the MOT from negative $x$. We have used and intensity of $I / I_{\text{sat}} = 1.4$ for the longitudinal beam. (bottom) The total number of captured atoms given by $v_{\text{max}}^4 - v_{\text{min}}^4$. We see that the maximum atom number is achieved at a longitudinal cooling beam clipping of -0.4 cm.

contribution to the trapping force, and again compare with the model in Section 2.4. Instead of performing numerical simulations, we make theoretical calculations of the trapping force, because it is more simple and just as accurate as a numerical simulation. We expect from our model in Section 2.4 that the trapping force primarily comes from scattering off of the narrow transition.

We have one main motivation for investigating the trapping force. We would like to explain the discrepancy between the negligible effect of the triplet MOT on the simulated critical velocity of the two-color MOT with the experimental results of differing atom numbers between the singlet and two-color MOTs. We hypothesize that this is due to some cooled atoms exiting the trap at some rate because their energy is too large to be trapped by depth of the potential. Qualitatively, at magnetic field gradients less than 8 G/cm, the triplet MOT has a deeper maximum trapping
Figure 3-6: Critical velocity as a function of intensity of the longitudinal beam for different clipping positions. For clipping positions closer to $x = 0$ and large intensities, the intervals where there seem to be no atoms captured are due to the finite velocity step size in the simulation.

The potential $U_{\text{max}}$ than the singlet MOT, as shown in Figure 2-6, and thus the two-color MOT should trap more atoms than the singlet MOT alone. This is our hypothesis for why even when the number of atoms cooled into the MOT in the simulation is the same for the singlet and two-color MOTs, we can still explain why the two-color MOT experimentally captures more atoms than the singlet MOT at low magnetic fields.

Because it is difficult to calculate absolute numbers of trapped atoms due to unknown experimental conditions, we instead calculate the fraction of the atoms that are cooled into the MOT region that become lost. We can think of this fraction as the equilibrium fraction of lost atoms in the MOT. This is also one minus the fraction of atoms that are trapped. We calculate the equilibrium fraction of lost atoms in the singlet, triplet, and two-color MOTs by first calculating the theoretical potential depth from integrating the magnitude of the force in Equation 1.3.

$$U_{\text{tot}}(x) = \int_{-\infty}^{x} F_{\text{tot}}(x')dx'$$  \hspace{1cm} (3.3)
We take the maximum potential depth $U_{\text{max}}$, which is at $x = 0$, and integrate the 1-dimensional Maxwell-Boltzmann distribution over energy from $E = U_{\text{max}}$ to $E = \infty$ to determine the number of atoms that have too high energy and are lost from the trap. We then normalize this number to the maximum number of cooled atoms by dividing by the integral of the Maxwell-Boltzmann distribution over energy from $E = 0$ to $E = \infty$.

Using this method, we calculated the equilibrium fractions of lost atoms of each MOT and the two-color MOT as a function of the magnetic field gradient. The result is shown in Figure 3-7. If we compare Figure 3-7 with the experimentally measured atom number in Figure 2-4, we see that at magnetic fields below $\approx 10$ G/cm, the simulation accurately predicts that the two-color MOT loses fewer atoms than the singlet MOT alone. However, at magnetic fields above 10 G/cm, it seems that the calculation fails to explain why the singlet MOT would lose fewer atoms than the two-color MOT. From the calculation, the singlet MOT's trapping potential is deep enough at high magnetic fields to trap almost all of the atoms. Thus, summing the singlet and triplet MOT potentials does not give a noticeable effect, leading to the saturation that we see in Figure 3-7. This discrepancy between the calculation and the experiment could be explained by either different experimental conditions while taking data for the singlet and two-color MOTs or saturation in the transitions.
Figure 3-7: Fraction of atoms that are lost from the MOT after being cooled by MOT beams. We see that for $\partial B/\partial x < 10$ G/cm, the hybrid MOT loses fewer atoms than the singlet MOT alone, which supports our experimental observations. For $\partial B/\partial x > 10$ G/cm, the calculation does not explain our experimental results.
Chapter 4

Additional experiments

In addition to the numerical simulations, we also performed further experimental tests to confirm that the two-color MOT is a simple superposition of the singlet and triplet transition MOTs. In the first test, we alternately turned on and off the singlet and triplet MOT lasers to rapidly switch between what we think to be the separate slowing and trapping functions (see Figure 4-1a). We expect that for high pulse frequencies $\nu_{\text{pulse}}$, the pulsed MOT acts like the two-color MOT, but at low frequencies, it acts like two separate singlet and triplet MOTs.

![Figure 4-1: (a) Experimental sequence for pulsing the singlet and triplet MOT lasers. When the singlet light is turned on, the triplet light is turned off, and vice versa. (b) Setup for blocking the center of the singlet MOT beams.](image)

At a pulse frequency of 25 kHz, we trapped a maximum $(25 \pm 3) \times 10^3$ atoms. This is a factor of 10 smaller than the number of atoms trapped with the two-color MOT at the same parameter settings, which was $(243 \pm 10) \times 10^3$. The lowest pulse
frequency that still trapped atoms was 1 kHz. However, we also trapped atoms from 1 Hz to 40 Hz, which we believe is due purely to the triplet MOT. We hypothesized that one cause of fewer atoms being trapped in the pulsed case was that the average power of the laser beams was cut by half, since the beams were only on half the time. To test this hypothesis, we turned all the lasers of the two-color MOT to half the power and did not pulse them. We observed that the number of atoms trapped was $(60 \pm 8) \times 10^3$ atoms. This only accounts for a factor of 4 difference in the number of atoms trapped between the pulsed and non-pulsed two-color MOT. The additional factor of 2.5 for the pulsing case comes from the atoms in the Maxwell-Boltzmann distribution that are not slowed enough by the singlet MOT in $1/25 \text{kHz} = 40 \mu s$ to be loaded into the triplet MOT. We can predict what percentage of atoms in the Maxwell-Boltzmann distribution have energy larger than the trap potential depth of the triplet MOT $U_t$ by taking the difference $(60 \times 10^3 - 25 \times 10^3)/(60 \times 10^3) = 58\%$.

However, the pulsed two-color MOT still trapped more atoms compared to a pure triplet MOT, which trapped $(1.1 \pm 0.2) \times 10^3$ atoms and a pure singlet MOT, which trapped $(5.4 \pm 0.7) \times 10^3$ atoms. This helps to support our hypothesis that there is an interaction between the singlet and triplet MOTs that helps to capture more atoms than each MOT alone.

In the second test, we wanted to isolate the cooling function of the singlet MOT beams in order to probe to what extent the singlet and triplet MOT beams were solely responsible for cooling and trapping. In order to do this, we blocked the center of the 1cm $1/e^2$ diameter singlet MOT beams with a 3mm diameter disk (see Figure 4-1b). We chose a 3mm disk because we wanted to block off the region of the singlet MOT that was responsible for trapping, which we can measure by the atomic cloud size of 2.04 mm for the pure singlet MOT (see Table 4.1). We observed $(16 \pm 1) \times 10^3$ atoms trapped in the blocked two-color MOT, which is more than the number of atoms trapped in a pure triplet MOT and a pure singlet MOT, but smaller than the number of atoms trapped in a two-color MOT or the pulsed two-color MOT. We hypothesize that this is blocking too much of the singlet MOT beam such that some of the slow atoms are not arriving at the triplet MOT region. We checked in a
numerical simulation that blocking 3mm of the force allowed 0.24 of the atoms to be trapped, which is on the same order as the 0.07 of the atoms that were experimentally trapped. More importantly, the atomic cloud size of the blocked two-color MOT is about the same as the size of the full two-color MOT, which suggests that the triplet light is primarily providing the trapping force.

Table 4.1: Number of atoms trapped and atomic cloud size for additional experimental tests. The parameters used for the measurements were $\Delta_s = -0.7\Gamma_s$, $\Delta_t = -3.7\Gamma_t$, $I_s = 0.26I_{sat,s}$, $I_t = 160I_{sat,t}$, and $\partial B/\partial x = 6.75\text{G/cm}$.

<table>
<thead>
<tr>
<th>Experimental setup</th>
<th>N ($\times 10^3$)</th>
<th>$r_{rms}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full power two-color MOT</td>
<td>243 ± 10</td>
<td>0.69 ± 0.01</td>
</tr>
<tr>
<td>1/2 power two-color MOT</td>
<td>60 ± 8</td>
<td>0.69 ± 0.01</td>
</tr>
<tr>
<td>25 kHz pulsed two-color MOT</td>
<td>25 ± 3</td>
<td>0.80 ± 0.09</td>
</tr>
<tr>
<td>Blocked two-color MOT</td>
<td>16 ± 1</td>
<td>0.67 ± 0.01</td>
</tr>
<tr>
<td>Pure singlet MOT</td>
<td>5.4 ± 0.7</td>
<td>2.04 ± 0.03</td>
</tr>
<tr>
<td>Pure triplet MOT</td>
<td>1.1 ± 0.2</td>
<td>0.61 ± 0.02</td>
</tr>
</tbody>
</table>
Chapter 5

Two-color MOT for other elements

The two-color MOT can be used for other elements than ytterbium where the energy of the atoms that can be trapped by the narrow triplet transition is larger than the energy of the atoms after being cooled by the broad singlet transition. The energy of the atoms that are cooled by the singlet transition is $E_s = k_B T_s$ and it is proportional to the doppler temperature $\hbar \Gamma_s$ of the singlet MOT. In order for these atoms to be trapped in the triplet MOT, the triplet MOT’s maximum potential depth $U_t$ must be greater than this energy $U_t > k_B T_s \sim \hbar \Gamma_s$.

Table 5.1 shows the ratio of linewidths between singlet and triplet transitions for different elements and Figure 5-1 compares the maximum potential depth of the singlet and triplet transitions of these atoms. The curves were obtained by numerically integrating the force due to each transition and taking the largest potential depth. From here, we will refer to the maximum potential depth as just the potential depth. The doppler temperature of each singlet MOT is indicated by the horizontal dashed black line. We notice that for ytterbium, cadmium, and dysprosium, the potential depth of the triplet MOT is larger than the singlet MOT doppler temperature, which means that a two-color MOT is possible. However, for strontium, the ratio of linewidths is too high and the triplet MOT potential depth is always lower than the singlet MOT doppler temperature; as a result, a two-color MOT is not possible for strontium atoms with natural linewidths. It could be possible to make a strontium two-color MOT if the triplet line is broadened.
Table 5.1: Linewidths and linewidth ratios for other atoms that have been trapped in a MOT. For elements with $\Gamma_s/\Gamma_t < 1500$, the two-color MOT is possible.

<table>
<thead>
<tr>
<th>Element</th>
<th>$^{1}P_{1}$ $\Gamma_s$</th>
<th>$^{3}P_{1}$ $\Gamma_t$</th>
<th>$\Gamma_s/\Gamma_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ytterbium</td>
<td>28 MHz</td>
<td>184 kHz</td>
<td>207</td>
</tr>
<tr>
<td>Dysprosium</td>
<td>32.2 MHz</td>
<td>135 kHz</td>
<td>239</td>
</tr>
<tr>
<td>Cadmium</td>
<td>91 MHz</td>
<td>70 kHz</td>
<td>1300</td>
</tr>
<tr>
<td>Strontium</td>
<td>32 MHz</td>
<td>7.5 kHz</td>
<td>4267</td>
</tr>
</tbody>
</table>

Figure 5-1: Trap potential depth of singlet (blue solid) and triplet (green dashed) MOTs as a function of the magnetic field gradient for different elements (a) ytterbium, (b) dysprosium, (c) cadmium, and (d) strontium. See Table 5.1 for the linewidth ratios of these elements. The doppler temperature of the atoms cooled by each singlet MOT is represented by the dashed black line.

We will now do a more quantitative analysis to calculate the allowed ratio of linewidths of the singlet and triplet transitions to determine which other elements can be used in a two-color MOT. We already have the condition of $U_t > \hbar \Gamma_b$. In order to calculate the allowed linewidths, we must relate the triplet MOT potential depth $U_t$ to the linewidth of the triplet transition $\Gamma_t$. We can approximate $U_t$ as a
function of the linewidth of the triplet transition $\Gamma_t$ by multiplying the maximum force $F_{\text{max}}$ due to photon scattering by the spatial range over which the force acts $x_{\text{range}}$. We know $F_{\text{max}} \sim \hbar \Gamma_t$ because the linewidth of the transition gives the scattering rate, and thus the maximum force (see Equation 1.2).

To calculate $x_{\text{range}}$, we must remember that we operated the experiment with light intensities $I$ that saturated the excited triplet transition. The Lorentzian shape of the scattering rate as a function of frequency is broadened by $\sqrt{I/I_{\text{sat,t}}}$, where $I_{\text{sat,t}}$ is the saturation intensity of the transition. The saturation intensity is proportional to $\Gamma_t$, because the rate of decays from the excited state will determine the saturation parameter. Thus, $\sqrt{I/I_{\text{sat,t}}} \sim \sqrt{I/I_t}$ is the broadening factor in frequency space. Finally, $x_{\text{range}}$ is the width of the Lorentzian in position space times the broadening factor in position space. We can approximate the position space parameters as being directly proportional to the frequency space parameters by assuming a roughly linear magnetic field gradient, and thus a linear Zeeman energy splitting as a function of space. Thus, $x_{\text{range}} \sim \Gamma_t \times \sqrt{I/I_t} \sim \sqrt{\Gamma_t}$. Now we can find $U_t$:

$$U_t \sim F_{\text{max}} \times x_{\text{range}} \sim \Gamma_t \times \sqrt{\Gamma_t} \sim \Gamma_t^{3/2}$$ (5.1)

$$E_s \sim k_B T_s \sim \hbar \Gamma_b$$ (5.2)

In order to calculate the critical ratio of the linewidths that gives $U_t \approx E_s$, we consider the magnetic field gradient that gives the maximum $U_t$ and we also consider a fictional narrow linewidth $\Gamma_{t,\text{fict}}$ for the atom ytterbium. From Figure 5-1 (d), we see that for ytterbium, $U_t/E_s = 85$ at the magnetic field gradient that gives the largest $U_t$. By calculating backwards from this observation, we can extract $\Gamma_s/\Gamma_t$ that gives $U_b \approx E_s$, which is $(\frac{1}{85} U_t)/E_s = 1$. From Equation 5.2, $\frac{1}{85} U_t = \frac{1}{85} \Gamma_t^{3/2} = (\frac{1}{20} \Gamma_t)^{3/2}$. Thus, to achieve the same trapping potential depth for the two transitions, $\Gamma_{t,\text{fict}} = \frac{1}{20} \times 184\text{kHz} \approx 9.2\text{kHz}$. Finally, the maximum ratio of linewidths needed for the two-color MOT when allowed to choose any magnetic field gradient is $\Gamma_s/\Gamma_{t,\text{fict}} > 30 \text{ MHz}/9.2 \text{ kHz} \approx 3200$. In this discussion, we've assumed that the lasers addressing the two transitions have constant power and that the singlet transition is on the order
of a few tens of Megahertz.

We check this critical ratio by simulating an atomic species with a linewidth ratio of 3200. However, we notice that the triplet MOT has a higher potential depth than the doppler temperature; for that, we found that a linewidth ratio of 1500 is more appropriate, as shown in Figure 5-2. We see that the triplet transition is equal to the doppler temperature at the magnetic field that gives the deepest trapping potential depth for the triplet transition. The difference between the simulated and actual critical linewidth ratios may be due to the breakdown of the $U_t \sim \Gamma_t^{3/2}$ approximation.

![Graph](image)

**Figure 5-2:** Trap potential depth as a function of the magnetic field gradient for an element with $\Gamma_s/\Gamma_t = 1500$, which is the maximum ratio for using the two-color MOT. The maximum triplet potential depth is equal to the Doppler temperature.
Chapter 6

Conclusions and future work

We have demonstrated that a magneto-optical trap that uses two atomic transitions is able to trap more atoms at a magnetic field gradient less than 8 G/cm than a conventional MOT. This is due to the cooling and trapping forces being separated into the singlet and triplet transitions, respectively. To confirm this theory, we’ve simulated the cooling and trapping forces separately and confirmed that the cooling force is primarily due to the singlet transition and the triplet force is primarily due to the triplet transition. Our simulations of the cooling force through the critical velocity show that it is entirely dependent on the singlet transition and independent of the triplet transition. Our calculations of the trapping force show that the fraction of atoms lost from the MOT is mostly dependent on the triplet MOT from 0 G/cm until about 2 G/cm, after which it is more dependent on the singlet MOT. Thus, we have predicted that the magnetic field gradient below which the two-color MOT traps more atoms than the singlet MOT is about 2 G/cm, which is on the same order as our experimental results.

We have trapped on the order of $10^5$ atoms using the two-color MOT, which we plan to implement in a spin-squeezed atomic clock experiment. Using the two-color MOT allows us create a cavity where the atom-light interactions are large, while still allowing us to look inside the cavity from the side window of the vacuum chamber. We are currently in the process of recreating the two-color MOT after having opened the vacuum chamber to put the cavity inside.
We have also calculated that the maximum linewidth ratio between the singlet and triplet transitions that will still allow a two-color MOT is $\approx 3200$. This means that the following atoms can benefit from a two-color MOT: ytterbium, radium, dysprosium, erbium, and others. Additionally, broadening the triplet linewidth for some species such as cadmium and strontium may also make the two-color MOT feasible.

In the future, it may be interesting to repeat the simulations in 2 and 3 dimensions, which is closer to reality, but also more difficult to simulate. In higher dimensions, the magnetic field does not always point along the direction of the laser beam. This causes mixing of the magnetic sub-levels, and the superposition of the forces transitioning to each sub-level creates smaller cooling and trapping forces, which may change the number of atoms trapped and the size of the atomic cloud.

There is still much to be explored with the two-color MOT. We can implement additional changes to the simulations that more closely approximates our experimental setup in 3 dimensions, such as taking into account the finite angle of atoms from the atomic beam that become captured by the MOT. This changes the velocity distribution of the atoms inside the MOT because only the slowest atoms in the transverse direction are captured. It may also be interesting to create an alternative model for the MOT that is not a simple superposition of the forces from the singlet and triplet MOTs. This model can be compared to our simple model to determine the uniqueness of our model’s relation to the experimental data.
Bibliography


