Hierarchical planning for multi-contact non-prehensile manipulation

by

Gilwoo Lee

Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of Master of Engineering in Electrical Engineering and Computer Science at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Abstract

Manipulation planning involves planning the combined motion of objects in the environment as well as the robot motions to achieve them. Ideally, we would like to be able to plan prehensile and non-prehensile manipulation in an integrated fashion, but despite much progress in planning motions in the robot’s configuration space in the presence of obstacles, progress in general manipulation planning has been more limited. The manipulation planning problem has a number of complicating factors, including the large dimensionality of the combined space. In this thesis, we explore a hierarchical approach to planning sequences of non-prehensile and prehensile actions. We subdivide the planning problem in three stages (object contacts, object poses and robot contacts) and thereby reduce the size of search space that is explored. We show that this approach is more efficient than earlier strategies that search in the combined robot-object configuration space directly.

Thesis Supervisor: Tomás Lozano-Pérez
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Chapter 1

Introduction

1.1 Motivation

Humans exploit both prehensile and non-prehensile manipulation when working with objects. For a small, light object, such as a cup, a simple grasp operation often suffices. For a large, heavy object, such as a piece of furniture, non-prehensile manipulation, such as pushing, may achieve the desired result. In other cases, it may be necessary to combine prehensile and multiple non-prehensile operations, including pushing, tilting, tumbling, etc., to achieve a desired result.

Manipulation planning involves planning the combined motion of objects in the environment as well as the robot motions to achieve them. We would like to be able to plan prehensile and non-prehensile manipulation in an integrated fashion. Ideally, a planner could pick from among a large repertoire of diverse action types to achieve a desired state in the most reliable and efficient manner. However, although great progress has been made in planning motions in the robot’s configuration space in the presence of obstacles, progress in general manipulation planning has been more limited.

While the state of the world and robot can be represented in a single combined configuration space, the manipulation planning problem has a number of complicating factors. The dimensionality of the combined space may be very large, and the domain is severely under-actuated: the robot can only affect the state of objects when
it is contact with them and the interactions are mediated by possibly complicated
dynamics constraints. As a result, there are few planners that address the problem
of combining prehensile and non-prehensile manipulation. Some exceptions are the
work of Hauser and Latombe [8], Dogar and Srinivasa [4], Barry et al. [2] and King
et al. [11], but they have mostly focused on pushing.

Manipulation planning is a form of multi-modal planning [8, 2]; that is, it is planning
in a hybrid space, in which the modes represent sub-manifolds where different
motion constraints, such as contact with a particular object face, hold. The presence
of these constraints mean that the classic sample-based algorithms for robot motion
planning do not apply directly, since samples need to be generated at the lower-
dimensional mode transitions. Extensions to this style of algorithm to multi-modal
planning exist; however, when the modes form continuous families (which is the case
in manipulation), planning in such multi-modal spaces generally requires some form
of hierarchical planning for search guidance. For example, planning for the manipu-
lated object ignoring the robot (and possibly obstacles) and then using the abstract
plan to guide search for a full plan for the robot [2].

In this thesis, we explore a hierarchical approach to planning sequences of non-
prehensile and prehensile actions. This hierarchical approach is more efficient than
earlier strategies that search in the combined robot-object configuration space directly.
We illustrate the approach through an initial implementation that manipulates a
single convex planar object among planar obstacles using point contacts with friction;
we leave robot shape, robot kinematics and grasp stability considerations for future
work.

1.2 Approach

Our planner operates hierarchically, first finding a sequence of qualitative “object
contact states” that characterize which parts of the moving object are in contact
with which parts of other objects, then finding a feasible sequence of poses for the
object, and finally finding a sequence of contact points for the manipulators on the
Figure 1-1: A contact state graph with poses connected through linear interpolation. Poses connecting two contact states are very close to each other.

object. This hierarchical structure provides significant search guidance, and divides the problem into three search problems that are much smaller than a search in the full combined configuration space of the object and manipulators.

The formulation of the first two search problems builds on existing work on contact-state graphs [21, 10]. We start by constructing a graph that represents the contacts between the moving object and other fixed objects in the world, not yet worrying about the exact pose of the object or the robot. A contact-state path provides a useful constraint for the actual path of the object. Figure 1-1 shows a contact-state graph of an object moving on a table, containing sample object poses in each contact state. To move the object from pose$_i$ to pose$_j$ without lifting it, we can first find a path from contact state$_k$ to contact state$_m$ and then find a pose sequence within the contact-state path. Finally, we search for a robot-contact plan, which is a sequence of robot contacts and forces on the object, which will cause the object’s motion to comply to the pose path found in the previous phase.

To find a robot-contact plan, we begin by discretizing the object’s surface into a set of possible contact points and define a state to contain an object pose and a set of contacts of the robot’s manipulators on the object. We then identify states that are feasible: both accessible, meaning that the robot can reach all of the specified contacts
Figure 1-2: Robot contact space for $p = (0, 0, \pi/6)$. Each axis represents possible contact points along the object’s surface accessible by $\text{hand}_1$ and $\text{hand}_2$. The leftmost column and the bottom row represent no-contact for $\text{hand}_1$ and $\text{hand}_2$, respectively. Green cells represent feasible states with only one contact, i.e. where the object can be balanced by only one hand. If one hand can stabilize the object, the other hand can be placed on any accessible surface; these states are colored in red. For example, $(c_1, none)$ represents a state where $\text{hand}_1$ is lifting the object. Then, all $(c_1, c_2)$, e.g., $(c_1, c_3)$ and $(c_1, c_2)$, are stabilizable as long as $c_2$ is accessible. Some states require both hands; these are colored in blue. Grey cells represent invalid or inaccessible states, and white cells represent states that are accessible but not stabilizable. In this representation, a transit is a transition from one valid state to another within a robot contact space. The figure shows transits from $(c_1, c_2)$ to $(c_1, none)$ to $(c_1, c_3)$.

and stabilizable, meaning that there exists a set of contact forces between the object and the robot’s manipulators, as well as the fixed objects, that can stabilize the object against gravity. Throughout this thesis we will focus on the case of two point-manipulators with hard contacts. For example, if the object is at pose $p = (0, 0, \pi/6)$ and one manipulator is on $c_1$, as shown in figure 1-2, the manipulator can passively support the object. The range of feasible locations of $c_1$ depends on the angle and the coefficient of friction. Since the other manipulator is free, it can be placed on any accessible region of the object’s surface. For a particular object pose $p$, we can evaluate all combinations of $c_1$ and $c_2$ to find robot-contact states that are accessible and stabilizable. Any geometrically valid pose $p$ can be associated with a space of discretely sampled robot contact pairs. The left part of figure 1-2 shows such a space.

The decomposition of robot motions into transit and transfer motions [19] can
be viewed as transitions within and across robot-contact spaces. A *transit*, changing the manipulator contacts with the object in a fixed pose, is a transition between two feasible states within the same robot-contact space. For example, in figure 1-2, \((p, c_1, c_2), (p, c_1, \text{none})\), and \((p, c_1, c_3)\) are all feasible, and the manipulator can transit by from \((p, c_1, c_2)\) through \((p, c_1, \text{none})\) to \((p, c_1, c_3)\) by moving hand\(_2\) from \(c_2\) to \(c_3\).

In the robot contact space, this is a vertical transition among three states that share \(c_1\). A *transfer*, changing the object pose while maintaining the manipulator contacts, is a transition between \((p_1, c_1, c_2)\) and \((p_2, c_1, c_2)\), where \(p_1\) and \(p_2\) are neighboring poses in an object-pose path. This is a transition from a state in \(p_1\)'s robot contact space to the corresponding state in \(p_2\)'s robot contact space, as shown in figure 1-3. For a *transfer* to be valid, it must also be possible for the manipulator to induce the wrench necessary to move the object in the desired direction.
Figure 1-5: Object-contact state path, object-pose path, and robot-contact paths aligned together. With a fixed object-contact path and a fixed object-pose path, we can horizontally stack the corresponding robot contact spaces for each pose.

Figure 1-4 illustrates the connected search spaces: within the discrete contact states in the contact-state graph, there are individual object poses, and a path through object-contact space can be realized by a path through object pose space. Then, for each object pose, there is a set of robot contacts, and a path through object pose space can be realized by a path of transit and transfer motions through the combined space of robot contacts and object poses.

Figure 1-5 illustrates the hierarchical search process. At the top is a path in contact space; the next one is a realization of it in object pose space, and at the bottom is a realization of that path in the combined robot-contact and object-pose space. Because we are reducing the search space at each step, we also reduce the set of feasible paths at each step. Hence, if we fail to find a path in the final search, we can backtrack to find different solutions in the previous searches. Despite the possibility of having to repeat these steps multiple times, we find empirically that this hierarchical approach is much faster than performing a single search in the combined space directly.

Our planner can be extended to enable combinations of prehensile and nonprehensile manipulation by adding extra connections within the “free” contact state, in which the object is not in contact with any of the fixed objects. For any stable set of robot contacts, all collision-free transfers (changes to the object pose), possibly subject to additional constraints such as force closure, are allowed. This combination
of manipulation strategies is very rich: for example, we can move a box from a tight shelf to a detached table by first pulling it out, reorienting it, and lifting and placing it on the table, as shown in figure 1-6.

The main advantage of our approach is that the individual search spaces are relatively small. In section 3, we compare our approach with a single search in the combined space and show a major computational advantage.

1.3 Related Work

There has been substantial research on planning individual non-prehensile manipulation operations, such as pushing [16, 13] and toppling [12]. There has been less work on planning sequences of non-prehensile operations. Early work by Erdmann and Mason [6] considered planning a sequence of “sensorless” tilting operations to place an object at a desired final pose. Abell and Erdmann [1] developed a planner for rotating a 2D polygon by maintaining two contacts that can induce quasi-static rotations of the object. Erdmann [5] later presented a non-prehensile two-palm manipulation planner that reorients an object by calculating angles in which the object is held in equilibrium and angles in which the object would slide or rotate around a vertex.
The line of work most closely related to our own is by Maeda and Arai; they have successfully demonstrated non-prehensile manipulation planning with multiple robot contacts across multiple contact states [14, 15]. Initially, they assumed a planar environment and that the contact state graph was given; the discretized the combined configuration space of the object and the robot hands in each contact states, connecting poses for transits and transfers. They extended the approach to three dimensions and incrementally generated poses for a single contact states. They performed A* search in the combined object, robot configuration space, in this case, limited to a single contact state. In both cases, this approach resulted in a very large state space with many possible transition edges. Since checking for geometric constraints and manipulation feasibility is expensive, it resulted in large running times. More recently [18] they applied an RRT-variant to perform the search which resulted in some improvements in average running time but with high variance. Their work has also pursued a careful assessment of the stability of contacts, finding plans that are most stable under this measure, an important issue that we have not considered here.

In this thesis, we integrate contact-state compliant task-planning and force stability analysis to achieve non-prehensile manipulation planning. Our planner considers motions compliant to a sequence of contact states, as in Xiao and Ji [21][10], to connect the initial and goal poses of the object. Then, we identify manipulator contacts that can move the object along a chosen path. To do this we apply the force feasibility analysis introduced by Han et al. [7].
Chapter 2

Algorithms

The top-level structure of our planner is as follows:

- Construct a sequence $\pi_{oc}$ of connected abstract states representing contacts of the robot with the other objects in the world.

- Construct a sequence $\pi_{op}$ of connected object poses that moves through, while conforming to, the chosen elements of $\pi_{oc}$.

- Construct a sequence $\pi_{rcop}$ of the robot’s contacts on the object, that will allow the robot to apply wrenches that will move the object along the trajectory specified by $\pi_{op}$.

We then use a feedback controller to apply the robot contact forces needed to execute $\pi_{rcop}$ to move the target object to its goal pose.

Formally, we are given as input: a description of the convex polyhedral shape for a target object to be moved; initial and goal object poses at which the object is statically stable without any robot contacts; a description of the other objects $O_1, \ldots, O_m$ in the world, in the form of a union of convex polygons; a number $n$ of point contacts the robot can exert; coefficients of friction for all contact surfaces; and a maximum amount of force that can be applied by the robot. The following sections describe algorithms for solving each of the planning problems, given the input.
2.1 Object-contact planner

An object-contact graph (OCG) is a connected set of abstract states that are characterized by the contacts between one target object and other objects in its environment. Contact graphs have been suggested for planning compliant motions [9, 17] and used by Maeda and Arai [14] for planning non-prehensile manipulation. Xiao and Ji [21] provide algorithms for constructing an OCG given descriptions of the objects’ shapes. We construct an OCG and use it to find an abstract plan for moving the target object.

Each node in an OCG is a geometrically valid object-contact state. Define an object-contact state for target object $T$ to be a set of principal contacts between $T$ and fixed objects $O_1, \ldots, O_m$. Each principal contact (PC) has the form $(O_k, \phi_i, \phi_j)$, where $\phi_i$ is a feature (edge, or vertex) of $T$ and $\phi_j$ is a feature of $O_k$. So, for example, if edge $e_j$ of $T$ makes contact with vertex $v_j$ of $O_k$, the principal contact would be $(O_k, e_i, v_j)$. An object-contact state is geometrically valid if and only if there is at least one collision-free pose that satisfies the geometric constraints imposed by the state.

To define edges between object-contact states, we make use of the following properties, defined by Xiao and Ji [21] and illustrated in figure 2-1.

- A principal contact $(O_k, \phi_i, \phi_j)$ contains principal contact $(O_k, \phi_a, \phi_b)$ if and only if (a) $\phi_a \in \phi_i$ and $\phi_b \in \phi_j$ or (b) $\phi_a = \phi_i$ and $\phi_b \in \phi_j$ or (c) $\phi_a \in \phi_i$ and $\phi_b = \phi_j$.

- A contact state $S_1$ contains contact state $S_2$ if and only if (a) $|S_1| \geq |S_2|$ and (b) for every PC $z_2 \in S_2$, either it is in $S_1$ or there is a unique PC $z_1 \in S_1$ such that $z_1$ contains $z_2$ and (c) no two PCs in $S_2$ are contained by the same PC in $S_1$.

- If $S_1$ contains $S_2$, we call $S_2$ a less-constrained neighbor of $S_2$.

There is an undirected edge between two object-contact states $S_1, S_2$ in the OCG if $S_1$ is a less-constrained neighbor of $S_2$. Edges correspond to adding and removing
Figure 2-1: The left contact state is \((O_k, e_i, e_j)\), and the right contact state is \((O_B, v_i, e_j)\). Since \(e_i\) contains \(v_i\), the principal contact \((O_k, v_i, e_j)\) is contained by \((O_k, e_i, e_j)\), and therefore the right contact state is a less-constrained neighbor of the left contact state.

principal contacts, and to changing the nature of a principal contact (for example, changing an edge contact to a vertex contact on that edge).

The object-contact planner constructs an OCG and uses it to find a connected path \(\pi_{oc}\). It takes as input contact states \(S_0\) and \(S_g\) corresponding to the initial and goal poses of the object. It also takes seeds, a set of user-provided contact states representing the most constrained, geometrically valid contact states. The result depends on the shapes and poses of the permanent objects and the shape of the moving object.

The algorithm begins by constructing a sub-graph for each seed, by recursively generating the set of object-contact states that are less constrained than the seed, using the less-constrained neighbor relationships. In the absence of user-provided seeds, an additional pre-processing step to identify them from the models is required.

Some object-contact states generated in this process may be geometrically invalid, in which case we discard them. Testing for geometric validity can be difficult: it is necessary to construct a collision-free pose for the object that satisfies the contact constraints. Therefore, this is an instance of a geometrically constrained motion-planning problem that can be addressed (approximately) with a sampling-based motion planner [10]. In our implementation we use a hybrid approach. For states with single principal contact, we randomly sample poses that satisfy the principal contact, and check whether they are collision-free. For states with multiple principal contacts, we
formulate and solve an optimization problem. We begin by constructing a plausible solution pose, \( p_0 \). For each principal contact \( z_i \) in object-contact state \( S \), we find an arbitrary satisfying pose \( p_i \). Then we set \( p_0 \) to be the average of the \( p_i \). This pose is not likely to satisfy either the contact or collision constraints. So, we formulate the problem of finding \( p \) to minimize \((p_0 - p)T(p_0 - p)\) subject to the contact constraints in \( S \) and to the constraint of not colliding with any of the fixed objects. The optimization problem is non-convex, but in practice we have found that a satisfying solution can be found, possibly with a few re-starts using different values of \( p_0 \). If we find such a pose, then we consider \( S \) to be geometrically valid, otherwise, we discard \( S \). Algorithm 1 gives an overview of the construction process.

**Algorithm 1** Constructing a contact state graph.

1: function \textsc{Construct}(seeds)
2: \( G \leftarrow \emptyset \)
3: \textbf{for} \( cs_{seed} \in \text{seeds} \) \textbf{do}
4: \( G' \leftarrow \{cs_{seed}\} \)
5: \( CS \leftarrow \text{relax}(cs_{seed}) \)
6: \textbf{for} \( cs \in CS \) \textbf{do}
7: \textbf{if} \( \text{is\_valid}(cs) \) \textbf{then}
8: \( \text{connect}(cs_{seed}, cs) \)
9: \( G' \leftarrow G' \cup \{cs\} \)
10: \( G \leftarrow \text{merge}(G, G') \)
11: \textbf{return} \( G \)

Now, we have a graph grown from each seed. Generally, they will have some contact states in common, which will allow us to merge them into a single graph. If not, we identify more seed states and repeat the process until \( S_0 \) is connected to \( S_g \). Once these states become connected in the graph, we search using breadth-first search for the path with the fewest transitions and return the resulting path, \( \pi_{oc} \), through the OCG.

### 2.2 Object-pose planner

Given a qualitative plan for the object in term of its contacts with other objects, \( \pi_{oc} \), we search for a detailed plan for the object motion, in the form of a sequence of poses,
\( \pi_{op} \), such that the continuous object trajectory resulting from linear interpolation of these poses moves the object through the contact states \( \pi_{oc} \).

Again, we treat the problem as one of constructing a graph and then searching it for a path. The nodes in the \textit{object pose graph} (OPG) are \textit{object pose states}, which have the form \((p, S)\), where \(p\) is an object configuration \((p = (x, y, \theta))\) in the planar case, and \(S\) is an object-contact state. It is relatively simple to compute \(S\) from \(p\) but it is more efficient to represent it explicitly; we will only consider nodes for which the object placed at \(p\) satisfies the constraints in \(S\) and does not collide with the permanent objects.

Pairs of nodes in the object pose graph that are drawn from the same contact state, \((p_1, S_1), (p_2, S_1)\), are connected if a linear interpolation between \(p_1\) and \(p_2\) that continuously satisfies the non-collision constraint exists. The interpolation algorithm is described by Ji and Xiao [10].

Pairs of nodes in different contact states, \((p_1, S_1), (p_2, S_2)\) where \(S_1\) and \(S_2\) occur in sequence in \(\pi_{oc}\), could be connected in the same manner; however we have found that this strategy produces non-smooth pose transitions. For smoother pose transitions, we start with poses in the more constrained contact state and attempt a small translation or rotation to obtain a pose in the neighbor contact state [21]. If this results in a valid pose, we add it to the pose graph and connect the poses. Figure 2-2 illustrates the steps.

The object-pose planner constructs an OPG and uses it to find a connected path \(\pi_{op}\). It takes as input the object-contact state path \(\pi_{oc}\), the initial object pose \(p_0\) and the goal object pose \(p_g\). It begins by generating sample states, \((p^j_i, S_i)\), for each of the \(S_i\) in \(\pi_{oc}\), that satisfy contact constraints of \(S_i\) and are collision-free. Next, for each transition between object contact states, \(S_i, S_{i+1}\) in \(\pi_{oc}\), and for each sampled pose \(p^j_i\) for \(S_i\) we generate new poses by applying small translations and rotations, as described by Xiao and Ji [21]: If the resulting object pose, \(p'\) satisfies the conditions of object contact state \(S_{i+1}\), then we add node \((p', S_{i+1})\) to the graph, connect it to the \((p^j_i, S_i)\) from which it was constructed, and attempt to linearly connect \((p', S_{i+1})\) to other object-pose states in object-contact state \(S_{i+1}\). Finally, we add nodes for the initial
Figure 2-2: Once we connect poses within each contact state (step 1), we connect poses across neighbor contact states. We take the poses in the more constrained contact state and apply a small translation/rotation. If this results in a valid pose in the other contact state, we add them to the graph and connect them with the original poses (step 2). Finally, we connect the newly added poses with other poses in the contact state.

and final poses, \((p_0, S_0)\) and \((p_g, S_g)\), and connect them to directly reachable nodes in the graph with matching \(S\) values. Algorithm 2 and 3 describe this procedure. Now, we search the graph to find a path \(\pi_{op}\) through the object-pose graph from the initial to the goal pose.

Note that in the construction of this pose graph, we evaluate and connect poses only within a contact state and across neighboring contact states. The number of evaluations that must be performed for connecting poses is \(O(n^2)\), where \(n\) is the number of sampled poses per contact state. In practice, the size of \(n\) can be very small relative to the total number of sampled poses as the number of contact states gets large.

### 2.3 Robot-contact planner

The last level of planning is to construct a sequence \(\pi_{rcop}\) of the robot’s contacts on the object, that will allow the robot to apply wrenches that will move the object
Algorithm 2 Constructing an object-pose graph. CONNECTPoses connects poses in the list if they can be linearly interpolated without collision.

1: function CONSTRUCTOPG(csPath)
2: \[ G \leftarrow \emptyset \]
3: \[ G_{prev} \leftarrow \text{null} \]
4: for \( cs \in \text{csPath} \) do
5: \[ P \leftarrow \text{RANDOMVALIDPoses}(cs) \]
6: \[ G' \leftarrow \text{CONNECTPoses}(P) \]
7: \[ G \leftarrow G \cup \text{CONNECTGraphs}(G_{prev}, G') \]
8: \[ G_{prev} \leftarrow G' \]
9: return \( G \)

Algorithm 3 Connecting object-pose graphs. PERTURB translate/rotates the pose and check if the perturbation makes a valid pose in \( G' \).

1: function CONNECTGraphs(\( G, G' \))
2: for \( pose \in G \) do
3: \[ pose' \leftarrow \text{PERTURB}(pose, G') \]
4: if \( \text{VALID}(pose') \) then
5: \[ G' \leftarrow G' \cup \{pose'\} \]
6: \[ G' \leftarrow \text{CONNECTPoses}(poses \in G') \]
7: return \( G' \)

along the pose trajectory specified by \( \pi_{op} \). Once again, we construct and then search a graph.

Nodes The robot contact graph (RCG) has robot contact states as nodes. A robot contact state (RCS) has the form \((p, S, C)\) where \( p \) is an object pose, \( S \) is an object contact state and \( C \) is a vector of contacts, corresponding to contacts the robot makes on the object. Each robot contact \( c \) is either none or a point of contact between the robot and the object. For a two-fingered robot and planar objects, when both fingers are in contact and there is one object contact, e.g. with the table, the entire state is described by: \(((x, y, \theta), \{(O^1, \phi^1_i, \phi^1_j), \ldots, (O^m, \phi^m_i, \phi^m_j)\}, (c^1, c^2))\). The contact points \( c^i \), can be represented as displacements along the movable object’s surface, in our planar examples, it is sufficient to represent a one-dimensional displacement from a fixed vertex on the polygon. This can be mapped to \( x, y \) coordinates as necessary.

An RCS is a legal node in an RCG if it is both accessible and stabilizable. An RCS is accessible if the contact points \( c^1 \) and \( c^2 \) are on a surface of the movable object that
is not in contact with any other object. This is a very simplified condition that should be extended to require the existence of a collision-free path for a particular physical robot, as well. An RCS is stabilizable if there exists a solution to its corresponding force feasibility problem [7].

Consider an object (assumed not to be in motion) that has contacts \( c_1, \ldots, c_n \), including robot contacts as well as contacts with other objects (implied by the contact state \( S \)); contact between two edges is approximated as two point contacts at the two endpoints of the edge contact. We wish to determine whether there is a combination of forces that can be exerted by these contacts that will balance the force of gravity.

Let \( f^i = (f_x^i, f_y^i) \) be the force exerted at \( c^i \) in the coordinate frame defined by \( c^i \). For each contact \( c^i \), let \( G^i \) be a transformation matrix that maps points in the contact’s frame of reference to points in a frame of reference fixed at the object’s center of mass. Let \( f \) be the vector \((f_1, \ldots, f_n)\) of forces applied at each of the contacts. Then the total wrench exerted on the object is

\[
Gf = (G^1, \ldots, G^n)f
\]

in the frame of the object. To maintain equilibrium, this wrench must balance the wrench caused by gravity:

\[
Gf + G' \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix} = 0 \tag{2.1}
\]

where \( G' \) is a transformation matrix that maps vectors in a world coordinate frame to the object’s coordinate frame. In addition, we also have the following frictional constraints for each contact:

\[
f_y^i \geq 0
\]

\[
|f_x^i| \leq \mu_i f_y^i \tag{2.2}
\]

where \( \mu_i \) is the coefficient of friction for \( c_i \). If there is an upper bound on the amount of force applied by the robot, we add this constraint as well. Given the set of robot
contacts and contacts with other objects, it is possible to determine whether there exists a set of forces at these contacts that can balance the force of gravity. The exact distribution of forces is statically indeterminate, but the minimum norm solution (that minimizes $f^T f$), provides a reasonable prediction of the actual force distribution [20].

Given the same set of contacts, there may be other solutions that balances gravity, but we always take the minimal net force solution. Hence, each state is unique in the RCG.

**Edges** There are two qualitatively different types of transitions between RCSs: transit and transfer. In a *transit*, the object’s pose stays constant but the robot contacts change; in a *transfer*, the robot’s contacts on the object stay constant with respect to the object, and the object’s pose changes.

A transit between two stabilizable RCSs $(p, S, (c^1, \ldots, c^n))$ and $(p, S, (d^1, \ldots, d^n))$ is an edge in the RCG if for every contact $i$, at least one of the following conditions holds: $c^i = d^i$, $c^i = \text{none}$ or $d^i = \text{none}$. This means that the object will be stable during the execution of an interpolated path between those states.

For a *transfer* to be a legal edge, it is necessary but not sufficient for the two end states to be stabilizable. For example, a box on a table can be in equilibrium at two distinct poses with no robot contact, but without robot contact, there is no single *transfer* between the two states. In order for a pose change to be legal, there must be a combination of contact forces that induces the necessary motion. If the transfer involves a combined translation and rotation, we break it into two steps, one pure translation and one pure rotation. We also assume that a rotation always pivots around a contact with the environment.

**Algorithm** The robot-contact planner constructs an RCG and uses it to find a connected path $\pi_{oprc}$. It takes as input the object-pose state path $\pi_{op}$, the number of $n_c$ of contacts to sample per object edge, the initial RCS $(p_0, S_0, C_0)$ and the goal RCS $(p_g, S_g, C_g)$. We assume that the object should be statically stable at initial and goal poses, so both $C_0$ and $C_g$ are $(\text{none}, \text{none})$.

We begin by constructing nodes in the graph. We generate a discrete set of possible contact points $\mathcal{C}$ on the object, $n_c$ per edge; also included in $\mathcal{C}$ is the element $\text{none}$.
representing no contact. For each \((p_j, S_j)\) in \(\pi_{op}\), we consider RCS \((p_j, S_j, (c^1, \ldots, c^n))\) for all \(c^1, \ldots, c^n \in C^n\). Any of these RCSs that are inaccessible are immediately discarded.

For the remaining candidate RCSs, we attempt to find a set of forces \(f\) that make the state stabilizable. We use the gravitational and friction constraints to form a quadratic program that seeks to minimize total force \((f^T f)\), as in [20]. If a solution exists, with forces that can be exerted by the robot, it is added to the graph.

Transit edges are added between any pair of RCSs that differ only by adding or deleting contacts. For any transition between OPSs \((p_i, S_i), (p_{i+1}, S_{i+1})\) in \(\pi_{op}\), we consider transfer edges between \((p_i, S_i, \mathcal{C})\) and \((p_{i+1}, S_{i+1}, \mathcal{C})\), for all \(\mathcal{C} \in C^n\). We must check to see whether the contacts \(\mathcal{C}\) can generate a wrench that would cause the necessary rotation or translation.

To check whether the robot contacts can generate the necessary wrench, we formulate the force feasibility problem once again. First, we identify the contact mode change for contacts with other objects during the motion. During a transfer between \((p_i, S_i, \mathcal{C})\) and \((p_{i+1}, S_{i+1}, \mathcal{C})\), all robot contacts remain in contact, but some of the contacts with other objects may change their modes. Some of them may be removed, and some of the contacts may become sliding contacts, depending on the nature of the motion. If a contact \(c^k\) is removed during the transfer, we remove the relevant terms from \(G\) and \(f\). If \(c^k\) becomes a sliding contact, we modify its corresponding inequality constraints in equation 2.2 to an equality constraint that opposes the direction of the motion.

Then, given the contacts and their contact modes, we must check whether these contacts can generate a wrench that cause the necessary rotation or translation. Under the quasi-static assumption, the desired wrench direction can be approximated with the desired motion direction of the object’s center of mass. Let \(\begin{bmatrix} \delta x & \delta y & \delta \omega \end{bmatrix}^T\) be the desired motion direction. In case of translation, this can be found by taking the offset between the two poses, with \(\delta \omega = 0\). In case of rotation, this can be found by identifying the center of rotation and finding the motion direction of the center of mass, with \(\delta \omega\) indicating the sign of rotation. Once we identify \(\begin{bmatrix} \delta x & \delta y & \delta \omega \end{bmatrix}^T\), we
can modify equation 2.1 of the force feasibility problem to be the following:

\[
G f = G' \left( \begin{bmatrix}
0 \\
m g \\
0
\end{bmatrix} + k \begin{bmatrix}
\delta x \\
\delta y \\
\delta \omega
\end{bmatrix} \right)
\]

(2.3)

with \( k \) being constrained to be positive. If there is a solution to this problem, it implies that the contacts can generate wrench in the direction of the object’s desired motion.

Once the graph has been constructed, we search it for a path from the initial RCS to the goal. If there is no such path, the overall search backtracks and tries to find paths at the previous levels.

### 2.4 Plan execution system

Given the final plan \( \pi_{rcop} \), we use a simple closed-loop controller to execute it on the robot.

During execution, we position-control the robot’s contacts on the object. Some contacts require active application of force while others passively support the object to resist gravity. In order to move the object in the desired direction, we need to do active "pushing" only when the robot contact’s motion direction, determined by its force vector, is in the same direction (within some predetermined range) as the object’s motion direction.

At time \( t \), the desired pose, contact point, and contact forces, \( p^t, c^t_1, f^t_1, c^t_2, f^t_2 \), are provided. If the desired motion direction at a contact point is within \( \pi/2 \) of the force direction, we move the robot contact, \( r_i \), in the direction of the contact force, \( f_i \). If it is in the opposite direction, the hand is slightly detached from the object to prevent applying extra force. The motion direction for the robot manipulator is defined by

\[
dr^t_i = \text{sign}(dc^t_i \cdot f^t_i) \delta f^t_i
\]
where $dc_i^t$ is the motion direction for the contact point and $\delta$ is a small constant.

Transit motions with the robot not in contact with the object are executed with pure position control.

If our goal were to track the designed trajectory very precisely, we would need a more sophisticated controller. However, since we only desire to move the object through the sequence of object contact states to a final pose, this simple approach suffices.
Chapter 3

Implementation and Results

We have implemented a version of this planner for planar objects and two robot contacts, without any further kinematic or collision constraints introduced to model the robot performing the manipulation. The implementation is in Python, with collision-checking done by Box2D [3]. The geometric validity of poses is checked using MATLAB libraries; this call to MATLAB is currently the bottleneck in our implementation. The results were tested in simulations that were constructed in Box2D. We compared the performance of our approach to that of a “combined” planner (similar in structure to those of Maeda et al. [14, 15]) that follows an object contact-state path, but plans for both and robot contacts at the same time.

We tested these approaches on two problems. The first, shown in figure 3-1, focuses on sequencing non-prehensile manipulation steps. There is an obstacle in the middle of the table, and the goal in this problem is to move the box to the other side of the table. Allowing only nonprehensile manipulation, the planner is able to find a solution.

Tables 3.1a and 3.1b summarize the running times for the hierarchical method on this problem. In this example, the OCG path always contains the same sequence of 9 OCSs. Given this OCG path, we varied the number of sampled poses per OCS in table 3.1a. Although the number of nodes and the search time increases, the size of the resulting OPG remains around 15 nodes. Hence, the impact on the next stage, robot-contact planning, is minimal. Using the 15 OPS nodes, we varied the
discretization of the object surface (the potential robot contacts); the results are in table 3.1b. The increased discretization leads to a rapid rise in the size of the graph and the search time. However, relatively small values are often sufficient.

The hierarchical approach is much more efficient than the combined approach that considers both pose and robot contacts at the same time. Table 3.2 shows a comparison between the two approaches, given the same OCS path. The leftmost column indicates the discretization of the object surface. In the combined approach, increases in either the number of pose samples or the discretization compound, with significantly effect on both time and space. Even with 5 poses per object-contact state, the running time increased so dramatically that we could not complete the graph construction when the object surface discretization was 40. Likewise, we could not complete the graph construction for 10 poses per OCS case with surface discretization of 20 or 40. In the hierarchical approach, increasing the number of sample poses generally affect only the object-pose planner, and its impact on the robot-contact planner is small.
Table 3.1: Results for example in figure 3-1

(a) OPG construction and search with different number of pose samples; values are averaged over 3 trials. Used 9 OCS.

<table>
<thead>
<tr>
<th>Poses / OCS</th>
<th>Nodes in OPG</th>
<th>Nodes in path</th>
<th>Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>60</td>
<td>14</td>
<td>109.4</td>
</tr>
<tr>
<td>10</td>
<td>84</td>
<td>15</td>
<td>145.4</td>
</tr>
<tr>
<td>15</td>
<td>110</td>
<td>15</td>
<td>203.3</td>
</tr>
</tbody>
</table>

(b) RCG construction and search with different discretization of object surface. Used 15 OPS.

<table>
<thead>
<tr>
<th>Discretization</th>
<th>Nodes in RCG</th>
<th>Nodes in path</th>
<th>Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>452</td>
<td>24</td>
<td>19.7</td>
</tr>
<tr>
<td>20</td>
<td>1,157</td>
<td>22</td>
<td>58.5</td>
</tr>
<tr>
<td>40</td>
<td>4,095</td>
<td>21</td>
<td>278.5</td>
</tr>
</tbody>
</table>

Table 3.2: Combined approach compared with the hierarchical approach. Leftmost column indicates discretization of object surface. For hierarchical approach, the sums of OPG and RCG for corresponding columns are taken from table 3.1a and 3.1b. Blanks cells indicate those that could not be finished in three hours.

<table>
<thead>
<tr>
<th></th>
<th>5 pose per OCS</th>
<th>10 pose per OCS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Combined</td>
<td>Hierarchical</td>
</tr>
<tr>
<td></td>
<td>Nodes</td>
<td>Time(s)</td>
</tr>
<tr>
<td>12</td>
<td>1,492</td>
<td>362</td>
</tr>
<tr>
<td>20</td>
<td>3,315</td>
<td>1,666</td>
</tr>
<tr>
<td>40</td>
<td>&gt; 3hrs</td>
<td>4,154</td>
</tr>
</tbody>
</table>
Figure 3-2: Key frames from a sample solution trajectory for sliding a box out from underneath a shelf. Green lines indicate the force direction of the robot contact pushing the object.

In order to find a feasible plan, we need to be able to sample a large number of poses and some discretization of the object surface. In this example, we needed at least

The second example, shown in figure 3-2, sequences non-prehensile manipulation with prehensile manipulation. Here, a portion of the object is inaccessible because of a shelf and so the robot cannot grasp it. If the shelf is slippery, that is, the friction coefficient between with the object is lower than that with the robot, the robot can apply tangential force to slide the box from the shelf, and then grasp it to move it to another table. This is a simpler problem and the running times are substantially smaller, as shown in tables 3.3b and 3.3a.
Table 3.3: Results for the example in figure 3-2

(a) OPG construction and search with different number of pose samples.

<table>
<thead>
<tr>
<th>Poses / OCS</th>
<th>Nodes in OPG</th>
<th>Nodes in path</th>
<th>Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>3</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>3</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>17</td>
<td>3</td>
<td>3.7</td>
</tr>
</tbody>
</table>

(b) RCG construction and search, with different discretization of object surface.

<table>
<thead>
<tr>
<th>Discretization</th>
<th>Nodes in RCG</th>
<th>Nodes in path</th>
<th>Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>86</td>
<td>7</td>
<td>1.3</td>
</tr>
<tr>
<td>20</td>
<td>206</td>
<td>7</td>
<td>4.6</td>
</tr>
<tr>
<td>40</td>
<td>612</td>
<td>7</td>
<td>28.9</td>
</tr>
</tbody>
</table>
Chapter 4

Conclusion

We have presented a hierarchical approach to planning sequences of non-prehensile and prehensile actions. We have showed that we could reduce the search space significantly by planning and committing to a path in higher level searches. We have compared it with a less hierarchical approach and showed that our approach is significantly more efficient for both time and space. We have also showed its potential for combining non-prehensile and prehensile actions, which we hope to further explore. Current areas for improvement of our approach include assessment of the stability of contacts and running time for calculating geometric feasibility. In future work, we plan to resolve these issues and extend our approach to belief space planning, to make it more robust to uncertainty in estimation and in control.
Bibliography


