Design for a Ferris Wheel

Conrad May 1900
List of Drawings.

Sheet #1 - Graphical Statics

"  #2 - Detail drawing of segment of wheel.

"  #3 - Drawing showing parts assembled
Among others, the following references have been of use to the writer:

   Part II, Traffic Statics: pp. 205-211.

Engineering News: Mar 22nd 1894, p. 284
   April 26th 1894, pp. 349-350

"Gutehricht fùr Bauwesen" Vol XLIV (1894) pp. 586-596
   Plates: pp. 69-70


Wright, Ch. St. Railways: pp. 135-140

Sanzu - Friction Notes.

Scientific American, and Supplement.
Design for a Ferris Wheel

The original Ferris Wheel, so called from the name of its inventor, consisted essentially of two great wheels, fixed side by side on a great horizontal shaft, and with numerous cars suspended between the two wheels at the rim. Each wheel consists of an outer and inner rim, and a series of spokes connecting these rims with the main shaft at the centre of the wheel. While these constitute the chief parts, there are a great number of minor parts, such as struts and bracing connecting the two wheels, sprocket plates, etc. Since the construction of the original wheel, a number of smaller wheels have been erected, some of the same type as the original, and others of a different type, such as the one at Bière, Beach, etc.
Data for Wheel -

Diam = 100'-0"
Distance apart, c. to c. wheels = 14'-0"
Number of cars = 12
Capacity of each car = 20 persons.
Max Speed = 1 rev. in two min.

Wheel to be revolved by means of sprocket wheels fastened to the outer rim of the wheel; a cog wheel or chain placed at the base of the wheel being used to force the wheel around.
In the design of the wheel, the stresses were first found as though the wheel had but a single rim, and this rim was connected to the main shaft by means of tension spokes. In this case, considering the top spoke to carry no stress, that is, that none of the spokes are capable of bearing compression, the structure is statically determined and may be easily figured. The stresses were found for different kinds and combinations of loadings. Then when the maximum stresses were found, the outer rim and tension spokes were designed accordingly. To insure rigidity and stiffness, it was thought best to introduce an inner rim in each wheel. As soon as this was done, the structure became statically undetermined. However, as the outer rim was designed sufficiently strong enough to bear the whole load alone, the wheel is evidently not weakened by the introduction of the inner rim. The question arises now as to the
stresses in the inner rim. When the inner rim is introduced it is necessary to connect the outer and inner rims by means of compression spokes or struts, in order to stiffen the frame, and when a strut is used, part of the load will be born by the strut and part by the outer chord. The amount of each stress is not known, and hence it is necessary to make an assumption. A fair assumption is, that half the load at apex of wheel is born by the outer chord and half by the strut. Under this assumption, the stresses in the outer chord may be found. In finding the stress in the inner rim, the stresses found in the compression spokes were used as the load at each joint. In addition to the loading due to weight of structure, dead wt. of car, live load, and wind load, it is also necessary to consider the thrust necessary to be applied at the base of wheel in order to revolve the wheels; and also when the wheel is
unequally loaded, any force that is necessary to apply at the base of wheel, in order to balance the loads.

In finding the loads an approximate method was first used. Since no data could be had in regard to weight of wheels of this size, it was necessary to make a trial solution of stresses, using approximate weight for loaded cars, no account being taken of not of wheel itself in this solution of stresses. This method was used in order to get some idea of the size of the different members of the wheels. The max. stresses in both chords and rims were found, and then stresses due to wind pressure were computed and combined with the max. resulting from loaded cars and from the thrust on base of wheel. In this work, the inner rim was entirely neglected in the computation, but an allowance was made for it when the entire weight of wheel was figured (for the final computation). The stresses were all found graphically.
Weight of Car. (Approximate)

Car (12' x 7' x 8')

Weight:

Wood:
- Bottom: $12' \times 7' \times \frac{1}{2} x 8' = 1080^\#$
- Sides: $2 \times 12' \times 8' \times 8' = 2400$
- Roof: $12' \times 7' \times \frac{1}{2} x 8' = 1080$
- Ends: $2 \times \frac{1}{2} \times 7' \times \frac{1}{2} \times 8' = \frac{1210}{6} = 6670^\#

Iron and Steel - assumed

- Bottom: 45 and 60
  - 3 L5 - 12' long
  - 3 L5 - 7' 9"
  - 600#

- Sides:
  - 6 L5 - 8' long
  - 4 L5 - 12' 6"
  - 400#

- Ends:
  - 2 L5 - 8' long
  - 100#

- Roof:
  - 3 L5 - 12' long
  - 3 L5 - 7' 9"
  - 400#

Steel pole fence
- 14' long, 2' 10" diam
- 18' 0"#

Bracing etc., pay
- 18' 0"#

Weight box = 1900#
Total weight:

Wood -  6670**
Iron -  1900
dead wt = 8500**
Dead live = 3000**
Live weight - 3000**

\[ \frac{12070}{12000} \]

Therefore weight of car as found approximately is 8500**, while the live weight is 3000**, making a total of 12000** when the car is fully loaded.

The thrust or driving force necessary to revolve the wheel was assumed at 12000**.

This assumption was made by comparison with the thrust shown graphically in figures in "Zeitschrift für Baumeister" (Page 76.)

In getting live load, the maximum live load was estimated at 25 persons at 140** each.

Wind load was taken at 40** per sq. ft on a vertical surface, none being allowed for on structure of wheel. The results are tabulated on the following two pages, but the stress diagram is not shown in accompanying drawings.
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<th>Stress Confin.</th>
<th>Spoke</th>
<th>Stress Tension</th>
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Max stresses resulting from assumed loading Dead and Wind combined.

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<th>Spoke</th>
<th>Stress Tens.</th>
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From the above table it can be seen that the max. stress in chord is 64700 # cont, and occurs in chord 15-16, and that the max. stress in spokes is 39000 # and occurs in spoke 15-16.
From these trial stresses, a fair idea could be formed of the real sizes of the required member and hence their weights found very closely.

The first thing was to find the weight of car as nearly as possible, then the weight of structure was computed. The thrust at the bottom of the car for full loading and for the most extreme case of vertical loading was then found. The stress diagram for each case was then drawn, that is, with car fully loaded, and with car partly loaded and the two compared to see which gave the greater stress. The wind pressure at the various joints was then found (account being taken of the pressure on wheel frame) and the diagram for this case drawn. Then the different combinations of these loads necessary to give a maximum stress was found, and the outer rim and spokes designed accordingly.

In all these computations, the inner rim is supposed to be out. The loadings, and weight of wheel are given in the next few pages.
Weight of Wheel (per sector)
The outer chord members and the flanges were computed from the results of the trial stresses. The other members were assumed. To each column 30% of detail was added for the extra weight due to single lattice and tie plates.

Chord (Outer Rim) - 2 - 10" L/s d 20" 26' long = 1040#
30% for lattice = 310
1850#

Inner Chord - 2 - 7" L/s d 9.75" 19' long = 370#
30% for lattice = 125
495#

Spoke Shunt - 2 - 7" L/s d 9.75" 19' long = 292#
108
300#

Tension Spoke - 5.5" 33' long d 17.5" = 610#

Tension Brace - 2 - 28' long d 1.8" = 84#

Cast Sprocket Plate (on one wheel only)
26' x 12" x 1" x 4/10 = 92.75#, say 100#

S truts connecting wheels
2 - 7" L/s d 9.75" 14' long = 970#
125 = 440#
Diagonal Bracing 1" rod
2 2.047" long = 2.047#
Twin shackle = 100#
Summary: Weight of one wheel and braiding
Chords = 19.50 + 4.95 = 18.45#
Spokes = 400 + 610 = 1010#
Tension force = 84#
Diagonal Bracing = 200#
Connecting Struts = 498#
Sprocket Plates = 1000#
4684#
Say 1000# = dead wt. of wheel per sector
or 4000# = wt. without sprocket plates.
Moment of Inertia -

Case I - Car fully loaded.

Cars \( 9000 \) ft. dead

Wheel \( 10000 \) ft. per apex or joint

Effect of Wind as regards Moment of Inertia is neglected.

The weight of wheel may be regarded as concentrated in the rim and the weight of cars at the apex or the weight of both wheel and cars may be regarded as concentrated at the apex. In either case, the result is approximate but as accurate as the case requires.

\[ I = \frac{Mr^2}{2}(r^2 - r'^2) \] - 1st method.

\[ I = mr^2 \] 2nd "

\[ I = 12(9^2 \times 50^2) = 18,000 \text{ in.}^2 \] - Moment Inertia of Wheel.

In finding moment of inertia of cars, it is best to take length of car from the center of gravity of the car, rather than from the apex of the wheel, because as all the cars are suspended from the top the lever arm of load
will vary. Taking the height of car as 9 from suspending point to bottom of car, assume centre of gravity of each car to be 5/16 or 0.3125 from bottom of car. Also assume this C.G. to remain constant whether car is loaded or unloaded.

Seven arms from centre of wheel to centre gravity of cars are as follows (approx.):

- \( a_0 = 4'4'' \)
- \( b_0 = 4'3'' \)
- \( c_0 = 4'7'' \)
- \( d_0 = 5'1'' \)
- \( e_0 = 5'3.5'' \)
- \( f_0 = 5'5.5'' \)
- \( g_0 = 5'6'' \)
- \( h_0 = 5'8'' \)
- \( i_0 = 5'3.8'' \)
- \( j_0 = 5'7'' \)
- \( k_0 = 4'7'' \)
- \( l_0 = 4'3'' \)
\[ I = \pi r^2 \] where \( r \) equals weight of car and \( r \) equals lever arm, that is, distance from axle to c.g. of car. Carefully loaded weigh five tons total.

\[ I = \pi X \frac{1}{4} X^2 = 9680 \]
\[ = \pi X \frac{1}{4} \pi^2 X^2 = 20250 \]
\[ = \pi X \frac{1}{4} \pi^2 X^2 = 22090 \]
\[ = \pi X \pi^2 X^2 = 26030 \]
\[ = \pi X \pi^2 X^2 = 28620 \]
\[ = \pi X \pi^2 X^2 = 30882 \]
\[ = \pi X \pi^2 = \frac{15580}{193139} \]

Mom. I. for wheel = 19'0000

Total Mom. Inertia = 808138

Now \( FL = \frac{dI}{d} \) where \( F \) equals force necessary to be applied at center of wheel, \( L \) the lever arm, \( I \) the Mom. of Inertia, \( d \) = ang. accel, and \( g = 32.16 \) ft. per sec.

Assuming 1 r.p.m. and full speed in two minutes, 
\[ \text{ang. vel} = 2 \pi \text{ rad.} \]
\[ \text{alg. vel} = \text{ang. accel} \times \text{time} \]
\[ d = \frac{2\pi}{120} = \frac{\pi}{60} \]
Now \( l \) (the beam arm) equals the radius plus \( \frac{1}{2} \) width of chord \& the width of sprocket plate (nearly), so \( l = (r + \frac{1}{2} \text{ width of chord} + \text{sprocket}) = 9^0.5' \) (about).

\[
F = \frac{Q I}{g d} = \frac{F o I}{(3216) I} = \frac{F o \times 0.03200}{3216 \times 0.08} = 0.78 \text{ tons}
\]

force necessary to be applied to rotate wheel when cars are all loaded.

Say \( F = 10 \text{ tons} \) or \( 20,000 \text{ lb} \).

Case II  
Cars partially loaded.

When the cars are partially loaded, in addition to the force necessary to rotate the wheel there will also be required another force in order to maintain equilibrium, or to balance the loaded cars. The greatest force necessary to maintain this equilibrium will be required when the cars are in the position shown in figure, all the loaded cars being
on one side, while the unloaded ones are on the other. The wheel is supposed to be revolving in a right-handed direction, and the loaded cars are suspended on the left side.

![Diagram of wheel with loaded and unloaded cars at different positions]

The loading as shown in fig. 5 page 7 of the script assumes a fully loaded car at a and an empty car at b, otherwise all the loaded cars are on left and unloaded cars on right side. The loading shown in above figure will give a greater force F, as when the wheel begins to revolve, the load at top (a) will tend to decrease the moment as soon as it moves a very small amount, and this decrease will be larger than any increase of moment caused by an empty car at b moving upward and to the left. Of course, this loading as shown above would rarely
if ever occur. Still it is best to find the force required, under the very worst possible condition.

The moment of inertia of the wheel will remain the same as in page 5. The moment of inertia of the car, however, will be different as the loads are different. For the fully loaded case the moment will be the same while the moment of inertia of the unloaded case will be decreased.

First find the force necessary to keep the loads in equilibrium. To do this take moments about the axis of wheel

\[ \Sigma M = (-1.5 \times 25 - 1.5 \times 48.8) x 1.5 + 1.5 x 50 + 50 \times 1.5 \times 1.5 = 0 \]

\[ F_1 = \frac{50 \times 1.5 + 5 \times 48.8 + 50 \times 1.5}{15} = 5.54 \text{ tons} \]

In taking moments as above, it was only necessary to consider the live load, as the dead loads were all the same, and being equally distributed, balanced each other.
The Moment of Inertia when car A, B, C, D are loaded only is as follows:

\[ I = 3.5 \times 4.5^2 = 67.76 \]
\[ = 3.5 \times 4.5^2 = 70.87 \]
\[ = 3.5 \times 4.5^2 = 77.91 \]
\[ = 3.5 \times 4.5^2 = 91.68 \]
\[ = 3.5 \times 4.5^2 = 100.15 \]
\[ = 3.5 \times 4.5^2 = 107.80 \]
\[ = 3.5 \times 4.5^2 = 126.0 \]
\[ = 3.5 \times 4.5^2 = 138.12 \]
\[ = 3.5 \times 4.5^2 = 130.56 \]
\[ = 3.5 \times 4.5^2 = 110.45 \]
\[ = 3.5 \times 4.5^2 = 101.28 \]

\[ \frac{191.062}{2.8162} \text{ Tons} \]

Moment of Wheel = \[ \frac{191.062}{2.8162} \]

\[ F = \frac{2I}{gR} = \frac{191.062}{8216 \times 30.5} \approx 9.063 \text{ tons} \]

Force necessary for equilibrium = \[ \frac{2}{I} \text{ rotation} = 9.063 \text{ tons} \]

14.605 Tons, Total

Say 29,000 lbs = total force F, force necessary to revolve wheel
Therefore, the horizontal thrust \( H \) for wheel fully loaded is 20,000#, and for wheel partially loaded 29,000#. Each case should be worked out in order to see which gives the most stress, whether the wheel is fully loaded and the thrust is 20,000#, or when the wheel is only partially loaded, and the thrust is 29,000#.

\[
\text{Loading of Wheel:} \\
\begin{align*}
\text{Per joint} & \quad \left\{ \\
& \quad \left. \begin{array}{l}
\text{1500# live} \cdot \text{one half wt. car} \\
3000# \text{ dead} \\
5000# \text{ dead - not on face of wheel} \\
4000# \text{ wet surface - wind pressure} \\
\text{Thrust of 20,000# for full load} \\
\text{29,000# for partial load}
\end{array} \right. \\
\end{align*}
\]
Wind Pressure -

Case I. — Wind blowing in direction of plane of wheel. Cars 12' long by 8' high. Sides of cars are always vertical. The roof is so nearly horizontal that the wind pressure on it may be neglected. Wind pressure taken as 45 lb. per sq. ft. on a vertical surface. In all probability, the wind pressure at the cars at the top of wheel will be much greater than at those at the bottom, especially if there are buildings surrounding the wheel so situated as to shield the lower part.

However, the safest and best assumption is to take the pressure equal, that is the same at the top as at the bottom. The cars on the side of the wheel toward the direction from which the wind is blowing will probably have a greater pressure than the cars on the opposite side, and the same will be true in regard to the pressure on the structure itself. This is so, because the cars and frame on one side may be
shielded to some extent by the cars on the other side. The difference, however, will be very small, especially if the wind should obviate from its direction in the plane of the wheel. Therefore, the pressure may be taken the same or both sides. The wind pressure on the lateral bracing, etc., may well be neglected, but the pressure on the outer rim, however, ought not to be disregarded. Although the outer rim is latticed, the pressure may safely be taken as though it had solid plat covering. The pressure on the different chords of the outer rim will vary, depending upon the inclination of the chord.

Case being 12' x 8' give 96", this at 40" gives 9840", or say 4000# wind pressure per each car. The car being suspended between the two wheels, half the pressure will be transmitted to each wheel, or 2000# pressure at each outer joint will result from the wind pressure on the car.
\[ p = 40^\circ 5' \text{ to vertical surface} \]

\[ bc = 25.1' \]

\[ \log \theta = 1.698970 \]
\[ \log \cos 30 = 9.937931 \]
\[ 1.698970 \]
\[ cd = 40.31 \]

\[ \log \theta = 1.698970 \]
\[ \log \sin 30 = 9.9698970 \]
\[ 1.697940 \]
\[ cd = 25.0' \]}
Pressure, \( 40^\circ \) on vertical surface.

\[
P = 6.7 \times 40 = 268 \text{#}
\]

\[
R = 134 \text{#}
\]

\[
R_1 = 134 \text{#}
\]

\[
P = 18.3 \times 40 = 732 \text{#}
\]

\[
R = 366 \text{#}
\]

\[
R_1 = 366 \text{#}
\]

\[
P = 28 \times 40 = 1080 \text{#}
\]

\[
R_2 = 800 \text{#}
\]

\[
R_3 = 800 \text{#}
\]

\[\text{Pressure Summary}\]

At joint a:

\[
134 \times 2 = 268 \text{#, say 300 #}
\]

\[
\text{m} = 366 + 134 = 500 \text{#, say 500 #}
\]

\[
\text{m} = 366 + 800 = 866, \text{say 900 #}
\]

\[
\text{m} = 866 + 800 = 1080 \text{#}
\]

\[
\rho = 900 \text{#}
\]

\[
\phi = 500 \text{#}
\]

\[
f = 300 \text{#}
\]
At joint 5, \( 134 + 366 = 500 \) #

" " \( t \) = 900 #

" " \( v \) = 1000 #

" " \( v \) = 900 #

" " \( w \) = 700 #

In addition, the wind pressure of each cone amounting to 2000 # in joint must be added, making total pressures as follows:

Joint - a = \( (2 - 24) \) = 2800 # pressure.

" m = \( (2 - 4) \) = 2500 # "

" n = \( (4 - 6) \) = 2900 # "

" o = \( (6 - 8) \) = 3000 # "

" p = \( (8 - 10) \) = 2900 # "

" q = \( (10 - 12) \) = 2800 # "

" r = \( (12 - 14) \) = 2800 # "

" s = \( (14 - 16) \) = 2800 # "

" t = \( (16 - 18) \) = 2900 # "

" u = \( (18 - 20) \) = 3000 # "

" v = \( (20 - 22) \) = 2900 # "

" w = \( (22 - 24) \) = 2500 # "

The above values were used in drawing stress diagram for joint.
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<th>Half Load</th>
<th>Wind Load</th>
<th>Wind Load</th>
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Max. = 90800# (comp.) in chord 13-14.
Design of Chute Chord.

Chord 28.88, c. to c. pine, say 26' long.
Assume $f = 8000$#

Max. stress = $90800$#

Then \( \frac{90800}{8000} = 11.35" \) necessary.

Tiy 2 - 12" 20.80# E's @ 6.03" c = 4.61

\( \frac{90800}{12.06} = 7530" \) column does stand.

\( \frac{26}{4.61} = 5.64 \quad \frac{39500}{3} = 7970" \) can stand

Therefore the 12" E's are O.K.

\[
\begin{align*}
\frac{K_x}{K_y} & = I, \\
I & = c = 4.61" \quad \text{(approximately)}
\end{align*}
\]

\( x - y = 7.0 " \)

Then \( y = 4.61 - 7.0 = 3.21 \)

or the E's must be at least 3.52" apart

lock to lock, in order to be as strong in

one direction as another. That is, to be

as strong along I, as along I, (against

buckling or bending)
**Stresses for Tension Spoke**

See drawing #1 for companion.

**Sheet 1**

<table>
<thead>
<tr>
<th>Fig 1</th>
<th>Fig 2</th>
<th>Fig 3</th>
<th>Fig 4</th>
<th>Max. Stress due to Full Load + Thrust from RR</th>
<th>Max. Stress due to Half Load + Thrust from RR</th>
<th>Max. Stress due to Full Load + Thrust from LT</th>
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Max. Stress due to Full Load in spoke (13-15)
Tension Spoke -

Allow 10,000* for tension -
Then as the max. stress is 3,900#, the necessary area is \[ \frac{3,900 \times 10,000}{10,000} = 390" \text{ sq }, \text{ say } 3.6" \]

Take 2 - 1" x 3"

Compression Spoke -

Must be able to carry the max. tension of 3,900#, also a compression of 7,000# that may come on to it. In addition, must be designed to resist a bending moment due to wind pressure.

Assuming the wind pressure on the end of a car to be equally divided between the two supports, that is, each bearing taking half, the load on a joint would be as follows.

\[ \frac{4 \times 7 = 28 \text{#}}{2 \times 14,285.7 = 8.57} \text{ chord stress} \]
\[ \frac{54}{5.4} \]
2,600# - say 2,500# load at joint.
Undoubtedly, the car axle passing from A to C will carry the load at A into C, but at present consider total load of 2600# at A.

Moments at E1 (considering lift of E only)

\[12 \times 2600 \times 12 = 31200 \times 12 = 374400 \text{ in. lbs.}\]

Bending moment at brake is 374400 in. lbs.

The section of wheel as shown above would be unstable were it not for the fact that the rim of the wheel act somewhat like an umbrella frame, and owing to their stiffness, keep the frame up.

Now the max. tension = 99900# and the compression = 9000#. Taking \(\frac{E}{10000}\) of 9000 and adding it to the tension we have 99900 + 44000

\[= 99900\# \quad \frac{59900}{10000} = 59.9\#\]
Try 2 - 10" E's @ 2000# \( a = 0.88 \) \( p = 366 \)
These E's were assumed in order to take account of bending moment.

\[
\frac{9900}{1176} = 428 \text{#} \text{ does stand in compression}
\]

For bending:

\[
f = \frac{M y}{I} = \frac{374000 \times 6}{I}
\]

\[
\frac{3.25}{2.74} \text{ # pg 6.}
\]

The value of \( f \) was found from above and then added to the stress caused by the 94900#. The value found being consistent with stress desired in column, the 10" E's were taken O.K.
See Fig 5, Sheet 1. of accompanying drawings.

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From annotation: the hell under the earth. See figure 5, sheet 1.
Stresses for Spokes resulting from assumed loading. See Fig 5, Sheet 1.

<table>
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<tr>
<th>Fig 1.</th>
<th>Fig 5 Stress</th>
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Design of Inner Cord

Must be able to carry stress of 9700# (cont.)

Chord length = 19.6' (c. To c. Fine at end)

Assume f = 7000#

Then \( \frac{9700}{7000} = 1.4\)’

But the inner rim is connected to the
tension spokes by pin joints, and in order
to have a good connection we must have a
sufficiently large channel. For the sized
pins to be used in this structure, a 6” C
is about the minimum size to be used.

Therefore take 2 - 6’ 8 00’ E’s at 2 88” p = 234

\( \frac{9700}{4.76} = 2100#”\) does stand.

\( \frac{19.6}{2.34} = 6.7 \)

\( \frac{33000}{y'} = 6600#”\) can stand.

Therefore column is properly stiff.

\( y = \rho = 2.34” \) (approx)

\( y - y' = .52”\)

\( y = 2.34 - .52 = 1.82”\)

The E’s must be at least 3.64” apart (back to
back).
Splice Plates of Chord

Hole assumed 4"

Compression

\[
\frac{45400}{7000} = 6.5\text{"} \quad \text{necessary}
\]

Take \(\frac{3}{8}\) " plate on outside

\[
\frac{3}{8} \times (9.5 - 4) = 2.1\text{"
}

Take one \(\frac{1}{2}\) " plate on inside

\[
\frac{1}{2} (9.5 - 4) = 4.75\text{"
}

Not area enough

Therefore take an additional plate on inside (\(\frac{3}{8}\))

\[
\frac{3}{8} (12 - 4) = 3\text{"}
\]

\[
\frac{2}{11}
\]

9.11" area. O.K.

Tension

\[
\frac{25000}{8000} = 3.125\text{"} \quad \text{necessary}
\]

\[
\frac{3.125}{2} = 1.57\text{" thick}
\]
For value of rivets use

\begin{align*}
\text{Shear} & - 7000 \text{ lb} \\
\text{Bearing} & - 12000 \text{ lb}
\end{align*}

\text{drive}

\begin{align*}
\text{Shear} & - 7000 \text{ lb} \\
\text{Bearing} & - 12000 \text{ lb}
\end{align*}

\text{lift}

Rivets \frac{7}{8}" - Hard drive

Double Shear - \(2 \times 6 \times 6000 = 7200 \text{ lb}\)

Bearing - \(\frac{25}{16} \times \frac{7}{8} \times 12000 = 2900 \text{ lb}\)

\[
\frac{7200}{2900} = 18.6 \\
16 \text{ rivets required}
\]

---

Pin connection in Chord.

Assume 3" pin. Allow 14000# for bearing value pins.

\[
\frac{25000}{14000} = 2.0 " \text{ necessary}
\]

\[
\frac{2}{0} = 1.67 " \text{ thick}
\]

Say one plate \(\frac{9}{8}" \text{ thick.}

Thickness web 10" channel = .95"

Plate \(\frac{3.75}{.75} " \text{ thick} \text{ O.K.}

Since the inside splice plates (to which 10" channel is connected) have a combined thickness of .875", they have sufficient bearing surface.
The web and bearing plate are nearly equal in thickness (0.38 in. 0.375), therefore assuming each to carry one half the stress of 14,000#. Then number of rivets necessary is as follows.

Machine Shear: 7800 x 0.6 = 4,680#.

Bearing: \(0.375 \times \frac{3}{8} \times 18,000 = 4,100#\).

\(\frac{14,000}{4,100} = 3.4\) rivets necessary.

Pin for Chord Joint.

\[ M = 37,100\]
2 3/4 " pin will do.

Above pin was figured as though inner chord were not in action. The moment was also figured for the case of inner chord in action, and, a smaller moment resulting, a smaller pin could be used. The max case however, should rule.
Since Cloud splice plate
assume 3" hole

\[
\frac{9700}{7000} = 1.38" \text{ needed.}
\]

\[
\frac{1.38}{2} = 0.69" \text{ thick.}
\]

Take \(\frac{1}{2}\)" plate on inside of channel

Rivet \(\frac{7}{8}\"\), Single shear.

\[
.6 \times 6000 = 3600
\]

Bear. \(0.20 \times \frac{7}{8} \times 12000 = 2100\) **

\[
\frac{4880}{2100} = 2.3 \text{ rivets necessary.}
\]

The ends of channels should also be flamed to fit.
Pins connecting tension spokes to main axle
Max stress = \( \frac{3 \times 3000}{2} \) #
Two spokes \( \frac{3 \times 3000}{2} = 2500 \) # about.

Mom: 12900 in. lb.
2" pin O.K.

Thickness of plates for beam
\[
\frac{14000}{12900} = 10 " \text{ necessary}
\]
\[
t = \frac{10}{2} = 5" \text{ thickness necessary}
\]
As \( \frac{3}{4} " \) plates were used, they are O.K.
Plate or axle for tension spokes

Say 6600# for shearing value

\[
\frac{14000}{6600} = 2.125'' \text{ necessary}
\]

\[
\frac{2.125}{.75} = 2.82'' \quad \frac{2.82}{2} = 1.41'' \text{ dist. necessary to edge of plate. Say 2''}
\]

Say 10000# for tension

\[
\frac{14000}{10000} = 1.4'' \text{ necessary}
\]

\[
\frac{1.4}{.75} = 1.86'' \text{ dist. required.} \quad \therefore 2'' \text{ O.A.}
\]

For tension (between pin holes)

\[
\frac{1.4}{.75} = 1.86'' \text{ necessary}
\]

Real dist = 2.5'' O.A.
Diameter of Plate.

\[ d = \frac{\text{3.5 ft}}{\sin 140^\circ} = 13.9^\circ \]

\[ 13.9 + 3.5 = 17^\circ = \alpha. \]

Say \( \alpha = 18^\circ \)

\( 0\)\( \circ \) diameter = 0' - 0".

In order to find whether the plate was strong enough to resist tearing apart, the max. tension possible was found graphically. This amounted to 35,000 lb and was in the direction of stroke (19-18°).

Assumed 12"

\[ \sqrt{35,000^*} \text{ max. tension in plate.} \]
\[
\frac{9.5000}{10000} = 9.5 \text{ net area needed}
\]

\[
\frac{3.5}{.75} = 4.7'' \text{ required}
\]

\[
36 - 20 = 16'' \text{ actual net section in plate OK.}
\]

Shear plate on outer chord.

\[
\frac{28000}{6666} = 41.24'' \text{ area}
\]

\[
21.2'' \text{ on each side}
\]

\[
\frac{2.12}{\frac{7}{8}} = 2.42'' \text{ depth necessary}
\]

Tension
\[
\frac{28000}{10000} = 2.85'' \text{ area needed}
\]

\[
\frac{2.8}{\frac{7}{8}} = 28.9''
\]

\[
4 + 1.8 = 5.8'' \text{ c. h. hole}
\]

Say 7'' to edge plate

\[
\frac{8.8}{\frac{7}{8}} = 4.0'' \text{ depth necessary}
\]
Compression spoke.

Rivets - Machine Union

Single Shear: \( 0.6 \times 7800 = 4680 \) **

Bearing: \( \frac{3}{8} \times \frac{3}{8} \times 10000 = 9900 \) **

\[ \frac{14000}{4500} = 3.1 \] of rivets necessary.

Lower end, or inner chord joint, of compression spoke.

Shearing: 6600

14000 ** on each plate.

\[ \frac{14000}{6600} = 2.12 \] necessary

\[ \frac{1.06}{0.38} = 2.8 \] depth needed.
Tension = 10,000#

\[
\frac{28000}{10000} = 2.8''
\]
necessary
\[
\frac{2.8}{2.6} = 1.075
\]
additional
3.0'' needed.

Area channel = 3.88

" pin hole = 1.14

Net area = 2.74''

" area one side = 2.37''

\[
\frac{3.5}{2.37} = \text{area needed additionally}
\]

Take plate 10'' wide 3.0" thick.

Then area = 3.88''

" PL = 3.75

area pin hole = 2.26

7.37'' net area

3.69 = " one side OK.

Washer, or outer rim joint, of spoke.
Rivets same as in lower end.

Area channel (cut) = 3.80''

\[
\frac{3.4}{2.17} \text{ additional area needed}
\]

\[
\frac{3.33}{1.33} = 2.50
\]

Net = 2.66

Take plate 3/8 X 1.5'' wide.
Then area channel = 3.80

\[ \frac{PL}{9.80} = 6.00 \]

area hole = 2.26

7.94 = net area

3.77 = " one side. O.K.
Shaft or axle for cage.

Loads: 10,000# vertical (due to rot. of cage people)
4,000# horizontal (due to wind)

Bearing for cage shaft.

M = 97,000 in-lbs.

Max. shaft 3/8" dia.
Main shaft, or axle of wheel

Loads: Vertical: 12 x 10,000 = 120,000 # put care.
       12 x 10,000 = 120,000 # wheel.
       \[\frac{240,000 #}{240,000 #}\]

Horizontal - Wind Pressure.

\[40,000 \times \frac{1}{12} = 4800 #\] pressure on care.
3000 x 2 = 6000
2900 x 4 = 11600
2500 x 4 = 10000
2300 x 2 = 4600
\[80200 #\] for 80500 # total wind pressure (plane of wheel)

Main shaft bearing

\[\text{Table 12" shaft}\]
When wind pressure is against side of wheel, there will also be additional stress caused in the main axle, resulting in a bending moment. However, in this case, provided the wind pressure is evenly distributed, the shaft will act more like a strut, or column under compression. However, the 12" shaft is sufficiently large, as when the wind is blowing directly against side of wheel, there is no wind pressure in plane of wheel (the case for which shaft was designed). The pressure in plane of wheel is much greater than the wind pressure against side of wheel. For the bending moment of 2,325,000 in-lb, a shaft somewhat smaller might answer, but an allowance was made for any twisting, etc., that might result, hence the 12" size was selected.
Cast Steel Spool for car shaft.

6 bolt holes arranged in circumference of \( \frac{91}{2} \) inch.

Shaft \( \frac{3}{4} \) in diam.

Hole: Spool for shaft = 4".

Babbit metal \( \frac{1}{6} " \) for bearing.

Six bolts hold each end of spool to channel. Assume the stress on the bolts to be equally divided.

Total load = 10,000 lb. This is carried by two spools, \( \frac{1}{4} \) each. Each end of spool takes one half, or 2,500 lb. come on the six bolts at each end.

\[
\frac{2,500}{6} = 417 \text{ lb} \text{ on each bolt.}
\]

\[
\frac{417}{6000} = 0.07 \text{ in.} \text{ area required. Wind load not taken account of in above. Bolts assumed.}
\]
to carry none of the stress of the Mi-Ax plates, but merely to fasten upper to main chord. Take $\frac{3}{8}''$ threaded bolts.