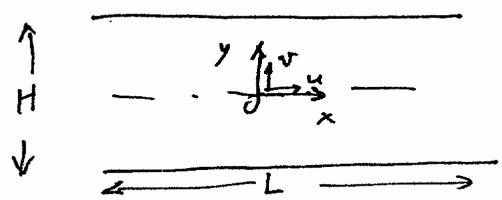
16.522 Space Propulsion
Homework no. 9

Handed 4/22/04; Due 4/29/04

In class we treated a limiting model of a Hall thruster in which all losses to the lateral ceramic walls were neglected. It was found, among other things, that the ionization kinetics ended up determining the electron temperature in the diffusion region, for a given length of this region. The ion losses are likely to be non-negligible, and this HWK explores the opposite limit, in which these lateral ion losses are dominant. This is a variation on a classical situation in plasma physics, called the "Glow Discharge" limit, in which the electron temperature is determined by ion losses to the container walls.

Consider a rectangular geometry, representing a section of the "diffusion region" of a ceramic lined Hall thruster, as shown:



Assume no collisions other than ionizing collisions. The lateral ion flux is $\Gamma_y = n_e v_i$, and the forward ion flux is $\Gamma_x = n_e u_i$. The particle conservation law then states

$$\frac{\partial \Gamma_x}{\partial x} + \frac{\partial \Gamma_y}{\partial y} = \dot{n}_e \qquad ; \qquad \dot{n}_e = R_i(T_e) n_n n_e$$

For a rough approximation, assume the fluxes are due to diffusion, and hence proportional to density gradients. If the longitudinal and lateral dimensions are L and H, respectively, we can approximate $\left|\frac{\partial \Gamma_x / \partial x}{\partial \Gamma_y / \partial y}\right| \approx \frac{H^2}{L^2}$. With this approximation, the conservation equation becomes one-dimensional, in the y direction only.

(a) Write the momentum conservation equation for ions and for electrons in the y-direction (in conservative form), and add them to produce the "ambipolar momentum equation". The sum of electron and ion temperatures should appear; neglect eth ion component here.

(b) Combine the particle conservation and ambipolar equations, plus the definition $\Gamma_y = n_e v_i$, to generate a single differential equation for the velocity v_i as a function of y. In doing this, assume constant neutral density and electron temperature.

(c) Integrate this equation using $v_i = 0$ at y=0, and then impose the proper velocity at the sheath edge (approximately at y=H/2). Justify this condition. It should be possible to arrange your result in the form

$$\frac{n_n H R_i(T_e)}{v_B(T_e)} = 2(\pi - 1)(1 + \frac{H^2}{L^2})$$

which is an equation for the electron temperature. and $v_B = \sqrt{\frac{kT_e}{m_i}}$ is the Bohm velocity.

(d) Use the approximate expression $R_i = \sigma_0 \bar{c}_e (1 + 2\frac{kT}{E_i}) e^{-\frac{E_i}{kT_e}}$, with $\sigma_0 = 3.6 \times 10^{-20} m^2$ and $E_i = 12.1 eV$ (for Xenon), and plot T_e vs. the non-dimensional group $(1 + \frac{H^2}{L^2})/(\sigma_0 n_n H)$. Apply to H=2 cm, L=4 cm, n_n=2e($\cancel{20}$ 19) m^{-3}.