12.009 Problem Set 1

Due Thursday, 10 February 2011

Welcome to 12.009 and to Problem Set 1. The objective of this problem set is to provide a set of straightforward problems to exercise your MATLAB skills, or lack thereof. These problems should provide a nice warm-up to some of the things you will see later in the course. If you have never seen MATLAB before we are happy to assist you in your journey. Many recourses exist to help you along your way including your TA, Professor and various online resources. You might look at OCW's Matlab Introduction or the MIT MATLAB page. The software itself can be downloaded from IST if you don't want to use Athena.* Note that the blue text and boxes on the electronic form of this pdf are active links to the suggested resources. Because we are always trying to make the course and problem sets better we ask that when you turn in your problem set if you could please rank each question on a difficulty scale from 1 (too easy) to 10 (too hard). Good luck.

1. *Radiocarbon dating.*

   As part of this class we will take a look at a series of problems all relating to the cycling of Earth's most important element, Carbon. As you may remember, the chemistry of an element depends on the number of protons it has (six for carbon). How the atom interacts, then, is independent of the number of neutrons it carries around. Carbon has three relevant isotopic forms on earth: carbon-12, carbon-13, and carbon-14 which are traditionally written as ^{12}C, ^{13}C, and ^{14}C. These three isotopes carry around 6, 7, and 8 neutrons respectively. Of these three isotopes ^{12}C, and ^{13}C are stable whereas ^{14}C is unstable and decays away with a half life of 5730 years (the time it takes for half of a group of ^{14}C atoms to decay). Historically, the concentration of ^{14}C in the atmosphere is roughly constant in time because decay is balanced by generation from cosmic rays. However, once the carbon is removed from the atmosphere either through transport (uptake into the ocean) or consumption (the formation organic matter by plants) the radiocarbon decays away until none remains. Thus, by measuring the ratio of ^{14}C atoms to ^{12}C atoms we can estimate the age of the material.

   We measure this ratio using an accelerator mass spectrometer to count the ^{14}C and ^{12}C atoms directly. We seek to understand how this random counting effects our age measurement. (This will become very useful to us when we look at real data.) The MATLAB function `randtime.m` was written to simulate the counting process. Given a probability p for getting a ^{14}C atom (the ^{14}C/^{12}C ratio) and the total number of atoms collected, n, it will produce a series of 1's and 0's representing ^{14}C and ^{12}C atoms respectively. It also produces a vector of the intervals between ^{14}C atoms.

   (a) Use `randtime.m` or any other MATLAB function to simulate the collection of carbon atoms. Generate a plot of the histogram of wait times (number of ^{12}C atoms) between each ^{14}C atom. What is the functional form of this distribution and how does it depend on the probability p? Use a p value which is small (say 0.005), and a very large number of counts (say 10^6).

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*Athena is MIT's UNIX-based computing environment. OCW does not provide access to it.

(b) We now ask a different, but related question. How does the measured isotopic ratio vary with count number? Modify `randtime.m` or write your own function to generate the measured isotope ratio $^{14}C/^{12}C$ given sample size s. Repeat this numerical experiment many times for any given sample size. The measured variance, σ_s^2 in a data set of size s is given by

$$\sigma_s^2 = \frac{1}{s} \sum_{i=1}^{s} (x_i - \bar{x}_s)^2$$

where

$$\bar{x}_s = \frac{1}{s} \sum_{i=1}^{s} x_i.$$

is the sample mean. Plot the variance as a function of the sample size s. What is the functional form of this relationship? How does it depend on p?

(c) What does this suggest about the age of a sample and the amount of material needed to get a similar precision measurement?

2. *A few geyser thoughts...*

If you ever have a chance to go to Yellowstone in the winter you should do it. The geysers not only shoot out jets of water hundreds of feet in the air, but release giant towers of frozen steam. The favored mechanism for these marvels is as follows. An empty tube deep in the earth is surrounded by super-heated rock. Water from the surface fills the tube at a roughly constant rate where it is then heated by the surrounding rock. When the water at the surface of the column reaches the boiling point it triggers a cascade of boiling from the surface to the bottom creating a jet of water and steam.

(a) Based on the above mechanism what would you expect the wait time distribution between eruptions to look like?

(b) How would you expect random variability to express itself?

Yellowstone National Park has a monitoring program where they record the duration between eruption events. Load `geyserdat.mat` into MATLAB. The `Waitimes` variable is a list of consecutive interval durations between eruptions for the Narcissus geyser.

(c) Plot a histogram (see MATLAB help for function `hist.m`) of the recorded wait times. Is this consistent with the above mechanism? Why or why not?

(d) Define the interval time series as I_i, $i = 1, 2, \ldots$ Plot I_i versus I_{i+2}. What does the structure in this plot suggest about the dynamics of this geyser?

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