12.009 Problem Set 6: Transform Away...

Due Thursday, April 14

1. *The Cosine*: In order to help you gain an intuitive understanding for the Fourier Transform and the influence of discrete sampling we will begin with the time domain function

$$f(t) = \cos \omega t + \phi, \tag{1}$$

which has an especially simple power spectrum.

   (a) What is the continuous power spectrum of $f(t)$?

   (b) How does it depend on ϕ? Explain your answer.

   Now imagine that we have discretely sampled the space so that $f(t)$ gets replaced by

$$x_j = \cos \omega t_j + \phi. \tag{2}$$

   (a) Calculate the discrete power spectrum, $|\hat{x}_j|^2$, for x_j. Remember to include Δt, $t_{\max}$, and n in your calculation.

   (b) Use MATLAB to find $|\hat{x}_j|^2$ for $\omega = 2\pi s^{-1}$, $t_{\max} = 20s$, and $\Delta t = .04s$. Use the `fft` function and make a plot of $|\hat{x}_j|^2$ vs frequency f. Make sure to carefully label your axes. What is the numerical value for Δf?

2. *The Square Wave Revisited*: Imagine that we have the following square wave:

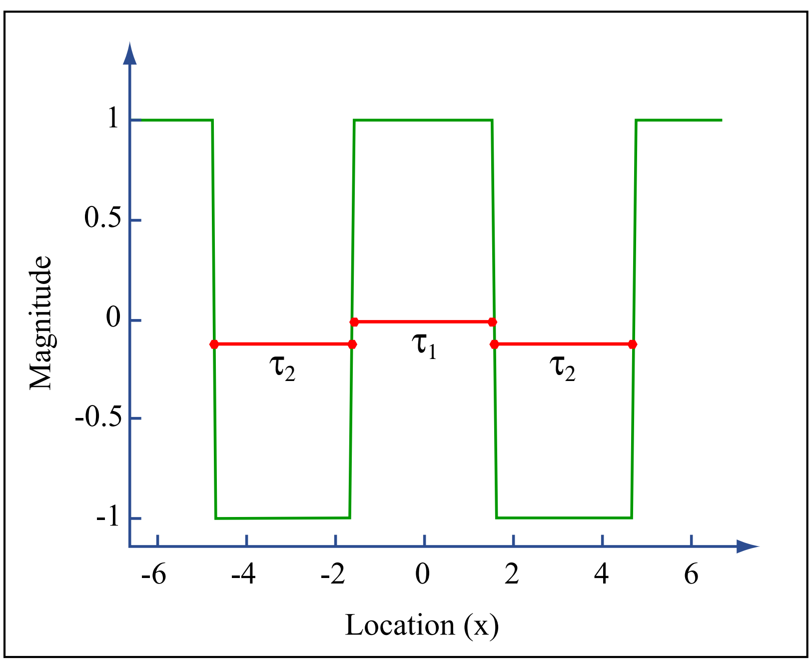

Image by MIT OpenCourseWare.

where τ_1 and τ_2 represent the width of the pulses. We will keep the height of the pulses equal to 1.

(a) For the case where $\tau_1 = \tau_2$, calculate the discrete power spectrum of the square wave. You may use intuition from problem set 2 to derive the answer for the discrete case if this calculation proves difficult.

(b) Using MATLAB, calculate and plot the power spectrum of a square wave where $\tau_1 = \tau_2 = \pi$. Let the maximum value equal 1 and the minimum value be -1. The `square` function in MATLAB might be useful. Let $t_{\text{max}} = 10\pi$ and $\Delta t = .01\pi$. Label the first few dominant peaks with their theoretical values.

We can take the inverse Fourier transform in MATLAB using the `ifft` function. Prove to yourself that this works by plotting the inverse Fourier Transform of the Fourier Transform of your favorite function. I don't need to see the plot.

(c) Returning to the square wave case, use MATLAB to take the Fourier Transform of the above square wave. Now, on the same plot, graph the inverse Fourier transform after removing the top $5, 10, 15 \ldots 95\%$ of frequencies from the Fourier Transform (setting them to 0).

(d) What is the result of removing high frequencies for the time-domain?

(e) In problem set 2 we used Fourier Analysis to help us solve the heat equation in a slab. How do your observations regarding high frequencies relate to what we learned in that problem?

3. *Remember the Geysers?* Way back when on problem set 1 we looked at the distribution of wait times between eruption events for one of the geysers in Yellowstone Park. By plotting the data in a strange mapping we got at the alternating nature of the long and short eruption times. To refresh your memory, the histogram of wait times looked like this

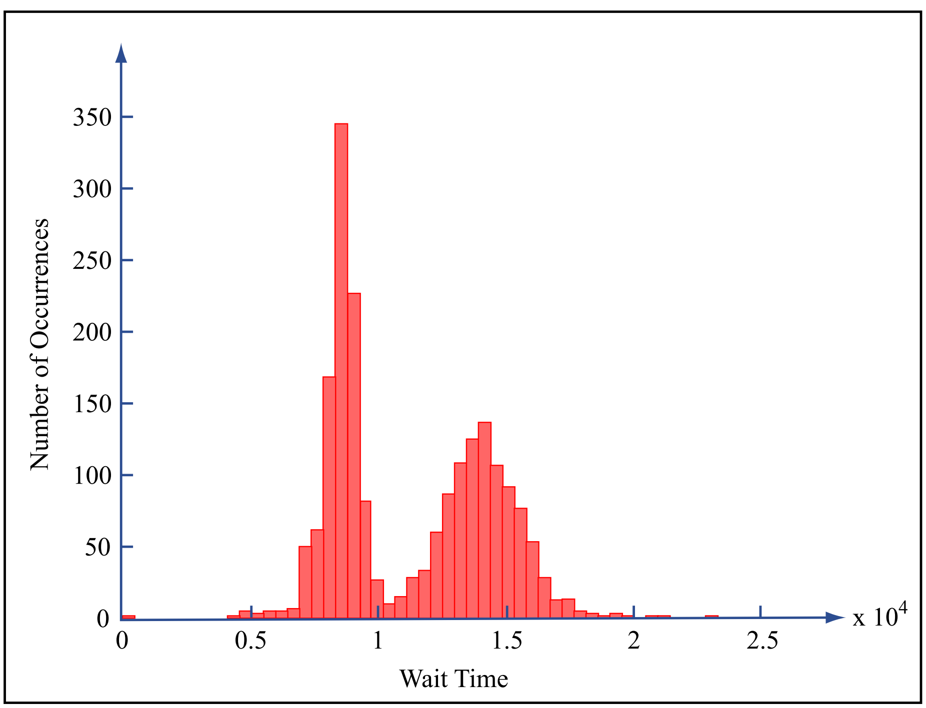

Image by MIT OpenCourseWare.

where the wait times are in seconds. We have transformed this into a binary time-series where 1 represents an eruption and 0 no eruption. To keep things sane each 0 or 1 represents a Δt of one hour. This data can be found in `geysertimeseries.mat`.

(a) Using the information in the histogram and your results from problem set 1, explain where you would expect two different peaks in the power spectrum of the time series.

(b) Using the `fft` function in MATLAB, find the power spectrum of the time series. Plot the power-spectra. Make sure to carefully label your axes. Also, make sure to scale the plot so that the interesting peaks are visible. (Hint: you may find the `fftshift` function in MATLAB useful.)

(c) Looking at your plot of the power spectra, explain what causes the peaks you did not explain in part 3a