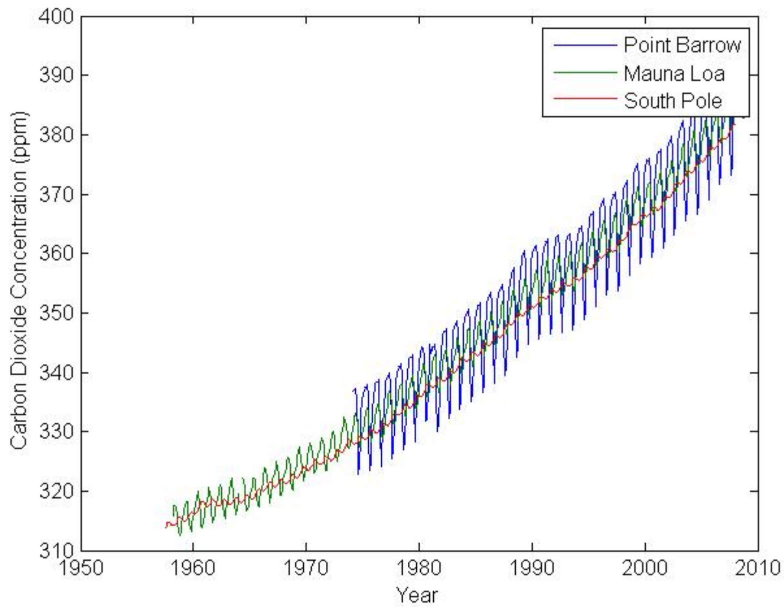
12.009 Problem Set 8: Climate Fluctuations and Random Graphs

Due Tuesday, May 3

1. *Modern Climate Change*: In a previous problem set, we looked at modern climate time series and thought about what caused the oscillations and phase shifts between the climate records in Point Barrow, Mauna Loa, and the South Pole.



For the analysis in this problem use the time-series for carbon dioxide at the Mauna Loa station. See problem set 3 for information on accessing the data.

(a) Make a plot of the power spectrum for the entire record and label the peak. Where is the spectral peak and what is its physical interpretation?

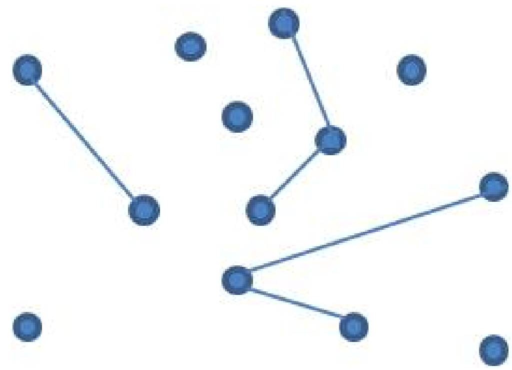
(b) Looking at the power spectrum more closely, what do you think the power spectrum of the noise is in this system?

(c) Repeat the above pair of calculations using the first 5, 10, 15, 20, 25 and 30 years of the record. How does the height of the 'noise floor' and spectral peak scale with the length of the time series?

(d) Show that this is consistent with the calculations done in class. Assuming Gausian white noise, what is the probability that the dominant peak in the record appeared there by random chance?

(e) Use the `fft` and `ifft` commands in MATLAB, along with some coding of your own, to remove the dominant periodic signal from the data. What part of the signal remains? How might these ideas be very useful for analyzing more complicated data sets? Verify that the imaginary part of the inverse fourier transform is within numerical noise. Include a plot of both the real and imaginary parts of the inverse transform in your answer.

2. *Food Webs: Random Networks?* In class we will explore how species are interconnected in predator-prey relationships and look at how energy is partitioned in organism classes. As a null model, we suppose that the inter-connections between organisms are completely random as in the graph below where the nodes represent organisms and the connecting lines interactions.



Here, as in the lecture notes, we will confine ourselves to a graph where the number of nodes, n, is fixed along with the probability, p, that any two nodes are connected. Knowing the rules for how the graph behaves allows us to use numerical simulations to explore its properties.

(a) Write a MATLAB script which builds a random network as a function of n and p. Make a plot of the degree distribution inside the network.

(b) On the same plot, show the theoretical prediction for the degree distribution. Under what conditions are the two consistent?

In addition, we might be interested in how the null model of a random network does in predicting properties relating to the connectivity of the entire system. Towards this end, we seek the largest cluster of interconnected nodes on a graph of size n and probability p. If the size of this characteristic cluster scales with the graph size we call it a giant cluster.

(a) Let u represent the probability that a vertex is not part of the giant cluster. Show that for the average network with large n

$$u = \left[1 - \frac{\langle k \rangle}{n-1}(1-u)\right]^{n-1}, \tag{1}$$

where $\langle k \rangle$ is the average number of links per node.

(b) Allowing $S = 1 - u$ to be the probability that a node is part of the giant cluster, use a Taylor series approximation to show that

$$S = 1 - e^{-\langle k \rangle S}. \tag{2}$$

(c) What is the trivial solution to this equation and what does it represent?

(d) For a given large n what is the critical probability, p_c, for which there is a giant cluster? Make a plot of the size of the cluster S as a function of the probability p. Label p_c on the x axis.

(e) How does the system behave differently on each side of p_c? How might this manifest itself in the natural world?

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