PHYSICAL MECHANISMS CONTROLLING THE
STRENGTH AND DEFORMATION BEHAVIOR OF
FROZEN SAND

by

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B.S. Civil Engineering, Brigham Young University (1985)

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ABSTRACT

Frozen soil research at M.I.T. has as an ultimate objective the
development of physically based constitutive relationships for frozen soil behavior
which capture the major features of the intrinsic physical mechanisms of
deformation. This is a formidable task in light of the complex nature of the
frozen soil system and the poorly understood interaction between its various
components which include the soil skeleton, the pore ice, the unfrozen water
surrounding the silicate particles and possible air voids. Prior attempts at
modeling frozen soil behavior have been based largely on empirical "curve fitting"
procedures with little attention being paid to the underlying physical
mechanisms. The current research focus at M.I.T. is the identification of these
physical mechanisms and a quantitative assessment of their relative importance.

A complete characterization of the stress—strain behavior of frozen soil
from very small strains (10^-4) to very large strains (0.25), which is absent from
the literature, is a necessary component of such a study. A high—pressure
low—temperature triaxial testing system has been designed and built at M.I.T.
for this purpose. This system incorporates internal force and temperature
measurement, lubricated end platens, on—specimen axial strain measurement,
pressure control to 10 MPa, temperature regulation to ±0.35 °C at −10 °C and
volumetric strain measurement to ±0.2%.

An extensive testing program has been conducted on the stress—strain
behavior of frozen Manchester Fine Sand. The relative density has been varied
between 20 and 100%, the confining pressure has varied from 0.1 MPa to 10
MPa, the nominal testing temperature has been −10 °C and the strain rate has
been varied from 3x10^-6/sec to 5x10^-4/sec.

Experimental results indicate that the initial Young's Modulus for this
frozen sand is 26.6 ± 4.5 GPa. The behavior at the upper yield point is
insensitive to relative density, exhibits a slight sensitivity to confining pressure
and an extreme sensitivity to strain rate. The post upper yield behavior is
strongly affected by relative density and confining pressure and is somewhat
affected by the strain rate.
These results are compared with a dilatancy hardening model proposed by Ladanyi (1985). This model attempts to draw an analogy between the effective stresses acting on the frozen soil skeleton and those which occur on the same soil in an unfrozen state. A qualitative comparison with the current data indicates that the effective stresses acting on the frozen soil skeleton may not be similar to the effective stresses acting on the unfrozen soil skeleton, at least within the range of applicability of the original model.

A direct comparison of the strengths of unfrozen sand and normally tested polycrystalline ice to that of frozen sand indicate that the strength of frozen sand is much greater than the sum of the strength of the unfrozen sand and ice. Some promising results are shown in a comparison of these data with an isostrain model proposed by Counto (1964) to predict the initial modulus and creep of concrete. This model can be used to study the relative contributions of the soil and ice phases to better understand how they combine to generate the overall behavior of the frozen sand. Predictions of the initial modulus with the "cube-in-a-cube" model agree quite well with the measured values.

This research represents a significant contribution to the field of frozen soil mechanics by presenting for the first time a complete characterization of the stress—strain behavior of a frozen sand. The conclusions are: 1) a complete characterization of the stress—strain behavior is of fundamental importance in understanding the complex interaction between its basic constituents; 2) the modeling of frozen sand as a particulate material may have limited applicability; 3) the modeling of frozen sand as a composite material shows some promise; 4) the small strain behavior of frozen sand is unaffected by the frictional character of the sand skeleton up to the upper yield stress, at this point it exhibits the same rate sensitivity of polycrystalline ice; and 5) the large strain behavior depends both on the behavior of the pore ice and the sand, the greater the contribution of the frictional sand strength (which is rate insensitive), the greater the power law coefficient.

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Table of Contents

Abstract 2
Acknowledgements 5
Table of Contents 6
List of Tables 11
List of Figures 14

1. INTRODUCTION 24

1.1 Engineering Applications for Frozen Soils 25
1.2 Objective of Research 27
1.3 Organization of Thesis 31

2. LITERATURE REVIEW 33

2.1 Unfrozen Sand 33
   2.1.1 Description of a Sand Mass 33
   2.1.2 Stress Transfer and Definition
       of Stress in a Dry Particulate Medium 40
   2.1.3 Saturated Sand and Terzaghi's Principle of Effective Stress 42
   2.1.4 Nature of Forces Transmitted Between Sand Grains 44
   2.1.5 Overview of Sand Behavior 47
   2.1.6 The State Parameter for Sands 54
   2.1.7 The Existence of a Critical State 58
   2.1.8 Mechanisms of Deformation in Sand 61
   2.1.9 Mechanisms of Strength in Sand 66
   2.1.10 Other Factors Which Affect the Strength and Deformation of Sands 71
   2.1.11 Summary of Sand Behavior 77

2.2 Polycrystalline Ice 83
   2.2.1 Description of Isotropic Polycrystalline Ice Ih 83
   2.2.2 Some Aspect of the Mechanical Behavior of Polycrystalline Ice 87
   2.2.3 Mechanisms of Deformation in Polycrystalline Ice 102
   2.2.4 Mechanisms Affecting the Strength of Isotropic Polycrystalline Ice 116
   2.2.5 Summary of Isotropic Polycrystalline Ice Behavior 125
2.3 Frozen Sand

2.3.1 Classification and Description of Frozen Soil
2.3.2 Structure of Frozen Sand
2.3.3 Overview of the Behavior of Frozen Sand
2.3.4 The Mechanics of Frozen Sand
2.3.5 Mechanisms of Strength in Frozen Sand
2.3.6 Summary of Strength and Deformation Behavior of Frozen Sand

3. HIGH-PRESSURE LOW-TEMPERATURE TRIAXIAL TESTING SYSTEM

3.1 Temperature Control and Measurement System
3.1.1 Temperature Measurement
3.1.2 M.I.T. Low Temperature Testing Facility
3.1.3 Environmental Chamber and Thermoregulator System
3.1.4 Triaxial Cell and Silicone 510 Oil

3.2 Pressure Control and Volume Change Measuring System
3.2.1 Description of Pressure/Volume Control System
3.2.2 Cell Pressure Controller Performance
3.2.3 Specimen Volumetric Strain Measure

3.3 High Pressure Triaxial Cell and Loading Frame
3.3.1 Modified High Pressure Triaxial Cell
3.3.2 Internal Floating Load Cell
3.3.3 On-Specimen Axial Strain Measure
3.3.4 Internal Temperature Measure
3.3.5 Connections to Power Supplies and Data Acquisition System
3.3.6 Screw Driven Loading Frame

3.4 Calibration of Measuring Devices

4. MATERIAL, PROCEDURES, ERROR ANALYSIS AND SCOPE OF TESTS

4.1 Description of Manchester Fine Sand
4.1.1 Mechanical Sieve Analysis
4.1.2 Mineralogical Analysis
4.1.3 Soluble Salt Concentration
4.1.4 Specific Gravity
4.1.5 Maximum and Minimum Densities
4.1.6 Particle Shape and Weathering 325
4.1.7 Unfrozen Water Content 325
4.1.8 Classification of Frozen MFS
According to Sayles et al. (1987) 326

4.2 Specimen Preparation, Freezing and Storage 326
4.2.1 Specimen Preparation 327
4.2.2 Specimen Saturation and Freezing 330
4.2.3 Specimen Storage 332

4.3 Specimen Trimming, Set–up and Testing Procedures 333
4.3.1 Trimming Procedure 333
4.3.2 Specimen End Conditions 334
4.3.3 Setup of Specimen in Triaxial Cell 336
4.3.4 Testing Procedure 337

4.4 Density Computation 338
4.4.1 Preparation Density 339
4.4.2 Trimmings Density 339
4.4.3 Oily Density 340
4.4.4 Cleaned Density 340
4.4.5 Selected Density Determination 341

4.5 Data Acquisition System and Data Reduction Procedures 342
4.5.1 M.I.T. Geotechnical Data Acquisition System 342
4.5.2 Conversion of Raw Data to Engineering Units 343
4.5.3 Conversion of Data to Units of Stress and Strain 344

4.6 Error Analysis of Triaxial Tests 348
4.6.1 Definition of Parameters from Stress–Strain Curves 349
4.6.2 Repeatability of Test Results 354
4.6.3 Some Factors That May Affect the Repeatability 355
4.6.4 Best Estimate of Precision 358
4.6.5 Bias of Measurements 359
4.6.6 Modulus Measures with Aluminum Dummy Specimen 362

5. BEHAVIOR OF FROZEN MANCHESTER FINE SAND IN TRIAXIAL COMPRESSION 385

5.1 Scope of Testing Program 386
5.2 Qualitative Overview of Stress–Strain Behavior 386
5.3 Small Strain Behavior
  5.3.1 Young's Modulus
  5.3.2 Proportional Limit
  5.3.3 Yield Offset at 10–4 Strain
5.4 Large Strain Behavior
  5.4.1 Upper Yield Region
  5.4.2 Work Hardening/Softening Region
  5.4.3 Peak Strength Region
  5.4.4 Volumetric Strain Behavior
5.5 Other Factors That May Affect the Behavior
  5.5.1 Effect of End Conditions
  5.5.2 Effect of Specimen Preparation Technique
  5.5.3 Effect of Temperature
5.6 Summary of the Measured Behavior

6. ANALYSIS OF RESULTS AND DISCUSSION OF POSSIBLE MECHANISMS

6.1 Comparison with Prior Testing Programs
  6.1.1 Small Strain Behavior
  6.1.2 Upper Yield Behavior
  6.1.3 Peak Strength Behavior
6.2 Analysis of Frozen Sand as a Composite Material
  6.2.1 Counto's Model
  6.2.2 Composite Modulus Prediction
  6.2.3 Prediction of Initial Yielding
6.3 Ladanyi's Dilatancy Hardening Model
6.4 Discussion of Frozen Sand Behavior
  6.4.1 Initial Young's Modulus
  6.4.2 Yielding Behavior to Upper Yield Point
  6.4.3 Large Strain Behavior
  6.4.4 Discussion of Ting et al. (1983)

7. SUMMARY AND CONCLUSIONS

7.1 Summary
  7.1.1 Literature Review
  7.1.2 High-Pressure Low-Temperature Triaxial Testing System
  7.1.3 Materials, Procedures and Error Analysis
  7.1.4 Behavior of Frozen Manchester Fine in Triaxial Compression
7.1.5 Analysis of Results and Discussion of Possible Mechanisms 542
7.2 Conclusions 545
7.3 Recommendations 547
8. REFERENCES 548
List of Tables

| Table 2.1:                      | Effect of Grain Size and Strain Rate on the Unconfined Compressive Strength of Polycrystalline Ice @ $-10^\circ$ C | 194 |
| Table 2.2:                      | Possible Deformational Mechanisms in Polycrystalline Ice                                           | 195 |
| Table 2.3:                      | Possible Weakening Processes in Polycrystalline Ice                                                 | 197 |
| Table 2.4:                      | Possible Strengthening Processes in Polycrystalline Ice                                              | 198 |
| Table 2.5:                      | Classification and Description of Frozen Soils                                                       | 199 |
| Table 2.6:                      | Overview of Testing Programs to Measure the Small Strain Behavior of Frozen Sand                    | 201 |
| Table 2.7:                      | Summary of Observed Trends in Small Strain Behavior of Frozen Sand                                   | 202 |
| Table 2.8:                      | Overview of Testing Programs to Measure Large Strain Behavior of Frozen Sand                         | 203 |
| Table 2.9:                      | Summary of Possible Deformational Mechanisms in Frozen Sand                                          | 205 |
| Table 2.10:                     | Mechanisms of Strength in Frozen Sand (after Ting et al. 1983)                                       | 206 |
| Table 3.1:                      | Temperature Fluctuations Inside Cold Room                                                           | 289 |
| Table 3.2:                      | Temperature Fluctuations Inside Environmental Chamber                                                | 290 |
| Table 3.3:                      | Temperature Fluctuations Inside Triaxial Cell                                                       | 291 |
| Table 3.4:                      | Summary of Typical Temperatures and Fluctuations                                                    | 292 |
| Table 3.5:                      | Variation in Cell Pressure During Shear                                                            | 293 |
| Table 3.6:                      | Summary of Cell Pressure Control During Shear                                                       | 295 |
| Table 3.7: | Variable Corrections to Volume Change | 296 |
| Table 3.8: | Average of the Variable Corrections to the Volume Change | 298 |
| Table 3.9: | Piecewise Correction to Volume Change in Order to Account for the Nonuniform Diameter of the Cell Pressure Piston | 299 |
| Table 3.10: | Overview of Volume Change Corrections | 300 |
| Table 3.11: | Line Voltage Drops and Calibration Factors for Electronic Sensing Devices | 301 |
| Table 3.12: | Piecewise Linear Calibration Curve for LVDT on Cell Pressure Controller Piston | 302 |
| Table 4.1: | Mechanical Sieve Analysis on Manchester Fine Sand | 363 |
| Table 4.2: | Summary of Specimen Physical Properties | 364 |
| Table 4.3: | Summary of Specimen Relative Density Determinations | 366 |
| Table 4.4: | Computation Factors Used in Data Reduction | 368 |
| Table 4.5: | Range in Stress–strain Parameters from Repeatability Tests | 370 |
| Table 5.1: | Measured Strength and Deformation Properties | 418 |
| Table 5.2: | Scope of Testing Program on the Triaxial Compression Behavior of Frozen MFS | 420 |
| Table 5.3: | Combinations of Variables That Lead to Stress–Strain Types | 421 |
| Table 5.4: | Statistical Analysis of Small Strain Data | 422 |
| Table 5.5: | Summary of Small Strain Behavior | 423 |
| Table 5.6: | Statistical Analysis of Upper Yield Region Data | 424 |
| Table 5.7: | Summary of Upper Yield Behavior | 425 |
| Table 5.8: | Summary of Work Hardening Behavior | 426 |
Table 5.9: Summary of Peak Strength Behavior 427
Table 5.10: Summary of Volumetric Behavior 430
Table 5.11: Effect of End Conditions on Stress–Strain Behavior 434
Table 5.12: Effect of Specimen Preparation Technique on the Stress–Strain Behavior 435
Table 5.13: Effect of Temperature on Stress–Strain Behavior 436
Table 6.1: Comparison of Initial Modulus Data with Prior Testing Programs 516
Table 6.2: Computation of Ladanyi Strength Prediction 517
## List of Figures

| Figure 2.1: | Visual Roundness Classification  (from Pettijohn 1975) | 207 |
| Figure 2.2: | Stresses in a Dry Particulate Medium  (from Lambe and Whitman 1969) | 208 |
| Figure 2.3: | Behavior of Dry Sand in Direct Shear  (from Casagrande 1936) | 209 |
| Figure 2.4: | Stress–strain Curves for Sand in Drained Triaxial Compression  (from Lee and Seed 1967) | 210 |
| Figure 2.5: | Effect of Confining Pressure on the Undrained Behavior of Loose Banding Sand in Triaxial Compression  (from Mohamad and Dobry 1986, Data from Castro 1969) | 211 |
| Figure 2.6: | Effect of Relative Density on the Undrained Behavior of Loose Banding Sand  (from Mohamad and Dobry 1986, Data from Castro 1969) | 212 |
| Figure 2.7: | Definition of the State Parameter  (from Elghaib 1989) | 213 |
| Figure 2.8: | Comparison of Normalized Stress Paths for Samples with the Same State Parameter  (from Been and Jefferies 1985) | 214 |
| Figure 2.9: | Mohr’s Circles and Failure Envelope for Dense Sand at Confining Pressures up to 40ksc  (from Lee and Seed 1967) | 215 |
| Figure 2.10: | Pressure Sensitivity of Principal Stress Ratio from Drained Triaxial Compression Tests  (from Lee and Seed 1967) | 216 |
| Figure 2.11: | Components of Strength in Sands According to Rowe’s Equation  (after Rowe 1962) | 217 |
| Figure 2.12: | Effect of Intermediate Principal Stress on the Drained Friction Angle of Several Sands  (from Ladd et al. 1977) | 218 |
| Figure 2.13: | Basic Tetrahedron Forming Ice Ih | 219 |
| Figure 2.14: | Bases of Three Tetrahedrons Forming a Wrinkled Hexagonal Ring | 219 |
Figure 2.15: Structure of Ice Ih Showing the Basal Plane (from Shyam Sunder 1989 proposal to do research) 220

Figure 2.16: Idealized Creep and Stress–strain Curves for Polycrystalline Ice Tested in Compression (from Shyam Sunder 1989 proposal to do research) 221

Figure 2.17: Stress–strain Curves From Uniaxial Tension Tests on Polycrystalline Ice (from Hawkes and Mellor 1972) 222

Figure 2.18: Idealize Stress–strain Curve for "Moderate" Strain–rate Compression Test on Polycrystalline Ice (from Mellor and Cole 1982) 223

Figure 2.19: Temperature Dependence of Young's Modulus, Rigidity Modulus and Poisson's Ratio of Granular Ice (from Sinha 1989) 224

Figure 2.20: Stress/Strain–Rate for Uniaxial Tests on Ice at −7°C (from Mellor 1980) 225

Figure 2.21: Stress–Strain Curves for Fine–grained Polycrystalline Ice at −5°C and Various Strain Rates (from Mellor and Cole 1982) 226

Figure 2.22: Logarithm of Strain Rate as a Function of 1/T for Polycrystalline Ice and Single Crystals (from Mellor and Testa 1969) 227

Figure 2.23: Effect of Confining Pressure on the Yield Stress of Polycrystalline Ice (from Jones 1982) 228

Figure 2.24: Creep Data from Monocrystals in Easy and Hard Glide and for Polycrystalline Ice (from Duval et al 1983) 229

Figure 2.25: Stress–strain Curves for Single Ice Crystals in Basal and Nonbasal Glide (from Higashi 1969) 230

Figure 2.26: Proposed Structure of Frozen Sand (from Ting et al. 1983) 231

Figure 2.27: Schematic Diagram of Jellinek's Ice Adhesion Experiments (from Jellinek 1957) 232

Figure 2.28: Young's Modulus and Poisson's Ratio for Peabody Gravelly Sand (from Kaplar 1963) 233
Figure 2.29: Young’s Modulus and Poisson's Ratio for Ottawa 16–100 Sand in Freshwater (from Baker and Kurfurst 1985)

Figure 2.30: Young’s Modulus Versus Confining Pressure for Ottawa Sand and Sand–Ice Mixture (from Vinson 1978)

Figure 2.31: Effect of Salinity on the Modulus of Frozen Ottawa 16–100 Sand at Various Densities and Temperatures (from Baker and Kurfurst 1985)

Figure 2.32: Effect of Changing Salinity on Poisson’s Ratio (from Baker and Kurfurst 1985)

Figure 2.33: Variation in Upper Yield Stress with Confining Pressure for Uniform Ottawa Sand at -10°C

Figure 2.34: Effect of Strain Rate on the Unconfined Compressive Strength of Dense Sands at -10°C

Figure 2.35: Peak Deviator Stress Versus Confining Pressure for Tests on "Saturated" Dense Sands at Low to Moderate Confining Pressures

Figure 2.36: Peak Deviator Stress Versus Confining Pressure for Tests on "Saturated" Dense Sands at High Confining Pressures

Figure 2.37: Peak Deviator Stress Versus Volume Fraction of Sand

Figure 2.38: Peak Deviator Stress Versus Relative Density for Frozen Ottawa Sand (from Baker and Kurfurst 1985)

Figure 2.39: Uniaxial Compressive Strength Versus Temperature for Several Frozen Granular Materials (from Bourbonnais and Ladanyi 1985)

Figure 2.40: Peak Deviator Stress Versus Degree of Ice Saturation for Frozen Ottawa 20–30 Sand (from Alkire and Andersland 1973)

Figure 2.41: Effect of Degree of Saturation on the Minimum Creep Rate in Frozen Manchester Fine Sand (from Ting 1981)

Figure 2.42: Effect of Changing Salinity and Dry Density on the Unconfined Compressive Strength of Frozen Ottawa Sand (from Baker and Kurfurst 1985)
Figure 2.43: Effect of Confining Pressure on the Power Law Coefficient of Frozen Toyoura Sand (from Shibata et al. 1985) 248

Figure 2.44: Effect of Temperature on the Power Law Coefficient of Several Frozen Sands 249

Figure 2.45: Uniaxial Tensile Strength of Frozen Penn Sand (from Perkins and Reudrich 1973) 250

Figure 2.46: Failure Strain as a Function of Dry Density for Frozen Ottawa Sand (from Baker and Kurfurst 1985) 251

Figure 2.47: Failure Strain as a Function of Temperature for Coarse Silica Sand (from Bourbonnais and Ladanyi 1985) 252

Figure 2.48: Failure Strain as a Function of Strain Rate for Wedron Silica Sand (from Bragg and Andersland 1980) 253

Figure 2.49: Force Equilibrium of Sand Particle In Frozen Sand 254

Figure 2.50: Freebody Diagram for Kelvin Equation 255

Figure 2.51: Proposed Mechanism Map for Unconfined Compressive Strength of Frozen Ottawa Sand (from Ting et al. 1983) 256

Figure 2.52: Boundary Between Cavitation and Noncavitation Regimes in Unfrozen and Frozen Sand (from Ladanyi and Morel 1990) 257

Figure 2.53: M.I.T. q–p Plot for Ladanyi’s Dilatancy Hardening Model 258

Figure 3.1: Schematic Diagram of Thermistor Circuit 303

Figure 3.2: Plan View of M.I.T. Cold Room Facility 304

Figure 3.3: Temperature Inside Cold Room During NS13 Preshear (short term) 305

Figure 3.4: Temperature Inside Cold Room During NS13 Preshear (long term) 306

Figure 3.5: Schematic View of Environmental Chamber and Triaxial Cell 307
Figure 3.6: Temperature Inside Environmental Chamber During NS13 Preshear (short term) 308
Figure 3.7: Temperature Inside Environmental Chamber at Two Locations During NS10 Shear 309
Figure 3.8: Oil Temperature at Two Locations Near Specimen During FRS61 Shear 310
Figure 3.9: Comparison of Temperature Measurements in Cold Room, Environmental Chamber and Triaxial Cell During NS13 Preshear 311
Figure 3.10: Schematic Drawing of Mechanical Portion of Pressure/Volume Controller 312
Figure 3.11: Schematic Diagram of Pressure Controller System 313
Figure 3.12: Cell Pressure Control During FRS46 Shear 314
Figure 3.13: Cell Fluid Volume Change During FRS59 Preshear 315
Figure 3.14: Compressibility Determination of Cell Fluid and Triaxial Cell Prior to Shear of FRS59 316
Figure 3.15: Cell Fluid Volume Change During Aluminum Stiffness Test 317
Figure 3.16: Cell Fluid Volume Change With No Specimen Over Full Travel of Cell Pressure Piston 318
Figure 3.17: Details of the Modified High Pressure Triaxial Cell 319
Figure 3.18: Details of On-Specimen Axial Strain Yoke 320
Figure 3.19: Axial Strain Versus Time for Initial Shearing During FRS59 321
Figure 4.1: Mechanical Sieve Analysis on MFS and Comparison With Prior Testing Programs 371
Figure 4.2: Unfrozen Water Content Versus Temperature for MFS 372
Figure 4.3: Schematic Diagram of Specimen Preparation Molds (after Martin et al. 1981) 373
Figure 4.4: Schematic Diagram of Multiple Sieve Pluviation Frame  374
Figure 4.5: Comparison of ACDT Versus LVDT Axial Strain Measures  375
Figure 4.6: Over Lubricated Versus Under Lubricated Specimens  376
Figure 4.7: Idealized Stress–Strain Behavior and Graphical Construction Techniques  377
Figure 4.8: Example of Poor Agreement in Initial Strain Measurements  378
Figure 4.9: Example of Good Agreement in Initial Strain Measurements  379
Figure 4.10: Example of Specimen Stability Qualifiers  380
Figure 4.11: Cell Fluid Volume Change With No Specimen (21 tests)  381
Figure 4.12: Effect of $\alpha$ on Deviator Stress  382
Figure 4.13: Aluminum Stiffness Measure (first test)  383
Figure 4.14: Aluminum Stiffness Measure (second test)  384
Figure 5.1: Overview of Types of Stress–Strain Curves  437
Figure 5.2: Stress–Strain Curves Showing Small Strain Behavior  438
Figure 5.3: Young's Modulus Versus Relative Density  441
Figure 5.4: Effect of Confining on Young's Modulus  442
Figure 5.5: Effect of Strain Rate on Young's Modulus  443
Figure 5.6: Proportional Limit Versus Relative Density  444
Figure 5.7: Effect of Confining Pressure on Proportional Limit  445
Figure 5.8: Effect of Strain Rate on Proportional Limit  446
Figure 5.9: Yield Offset Stress at $10^{-4}$ Strain Versus Relative Density  447
Figure 5.10: Effect of Confining Pressure on Yield Offset Stress 448
Figure 5.11: Effect of Strain Rate on Yield Offset Stress 449
Figure 5.12: Stress–Strain Curves Showing Upper Yield Behavior 450
Figure 5.13: Measured Variation in Upper Yield Stress With Height of Specimen 453
Figure 5.14: Corrected Upper Yield Stress Versus Relative Density 454
Figure 5.15: Effect of Confining Pressure on Correct Upper Yield Stress 455
Figure 5.16: Effect of Strain Rate on Upper Yield Stress 456
Figure 5.17: Axial Strain at Upper Yield Stress Versus Relative Density 457
Figure 5.18: Effect of Confining Pressure on the Axial Strain at the Upper Yield Stress 458
Figure 5.19: Effect of Strain Rate on the Axial Strain at the Upper Yield Stress 459
Figure 5.20: Stress–Strain Curves to Show Effect of Relative Density on the Rate of Work Hardening 460
Figure 5.21: Rate of Work Hardening Versus Relative Density for a Moderate Strain Rate 461
Figure 5.22: Rate of Work Hardening Versus Relative Density for Range in Strain Rates 462
Figure 5.23: Effect of Strain Rate on the Rate of Work Hardening 463
Figure 5.24: Axial Strain at Beginning of Work Hardening Versus Relative Density for Moderate Strain Rate 464
Figure 5.25: Axial Strain for Beginning of Work Hardening Versus Relative Density for Range in Strain Rates 465
Figure 5.26: Normalized Stress–Strain Curves Showing the Effect of Relative Density for Moderate Strain Rate and Low Confining Pressure

Figure 5.27: Normalized Stress–Strain Curves Showing the Effect of Relative Density for Moderate Strain Rate and High Confining Pressure

Figure 5.28: Normalized Stress–Strain Curves Showing the Effect of Confining Pressure for Low Relative Density and Moderate Strain Rate

Figure 5.29: Normalized Stress–Strain Curves Showing the Effect of Confining Pressure for High Relative Density and Moderate Strain Rate

Figure 5.30: Stress–Strain Curves Showing the Effect of Strain Rate for Low Relative Density and Low Confining Pressure

Figure 5.31: Stress–Strain Curves Showing the Effect of Strain Rate for High Relative Density and High Confining Pressure

Figure 5.32: Stress–Strain Curves Showing the Effect of Strain Rate for Low Relative Density and High Confining Pressure

Figure 5.33: Peak Stress Minus Upper Yield Stress Versus Relative Density for Moderate Strain Rate for Range in Confining Pressures

Figure 5.34: Peak Stress Minus Upper Yield Stress Versus Relative Density for Range in Strain Rates and Confining Pressures

Figure 5.35: Peak Deviator Stress Versus Relative Density at Moderate Strain Rate

Figure 5.36: Peak Deviator Stress Versus Relative Density for Range in Strain Rates and Confining Pressures

Figure 5.37: Effect of Confining Pressure on Peak Deviator Stress for Moderate Strain Rate

Figure 5.38: Effect of Strain Rate on Peak Deviator Stress for Range in Relative Densities and Confining Pressures
Figure 5.39: Axial Strain at Peak Deviator Stress Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures
Figure 5.40: Axial Strain at Peak Deviator Stress Versus Relative Density for Range in Strain Rates and Confining Pressures
Figure 5.41: Rate of Dilation at Peak Deviator Stress Versus Relative Density for Moderate Stain Rate and Range in Confining Pressures
Figure 5.42: Rate of Dilation at Peak Deviator Stress Versus Relative Density for Range in Strain Rates and Confining Pressures
Figure 5.43: Maximum Measured Rate of Dilation Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures
Figure 5.44: Maximum Measured Rate of Dilation Versus Relative Density for Range in Strain Rates and Confining Pressures
Figure 5.45: Axial Strain at the Beginning of Dilation Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures
Figure 5.46: Axial Strain at the Beginning of Dilation Versus Relative Density for Range in Strain Rates and Confining Pressures
Figure 5.47: Volumetric Strain at 20% Axial Strain Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures
Figure 5.48: Volumetric Strain At 20% Axial Strain Versus Relative Density for Range in Strain Rates and Confining Pressures
Figure 5.49: Final Deformed Shape of Several Lubricated Specimens
Figure 5.50: Stress–Strain Curves to Show the Effect of Specimen Preparation Technique
Figure 5.51: Stress–Strain Curves to Show the Effect of Temperature
Figure 6.1: Comparison of Rate Sensitivity of Upper Yield Stress to that of Peak Strength of Polycrystalline Ice

Figure 6.2: Comparison of Rate Sensitivity of Peak Deviator Stress to Prior MIT Data

Figure 6.3: Simple Two–Phase Composite Model for Concrete (after Illston et al. (1979))

Figure 6.4: Predicted Composite Modulus Versus Volume Fraction of Sand

Figure 6.5: Comparison of Predicted Initial Stress–Strain Response to that of FRS34 and Polycrystalline Ice

Figure 6.6: Comparison of the Rate Sensitivities of the Upper Yield Stress and Peak Strength to that Predicted by Ladanyi's Dilatancy Hardening Model

Figure 6.7: Probable Structure for Frozen Sand
CHAPTER 1
INTRODUCTION

Engineering in Arctic Regions and the use of artificial ground freezing as a means of temporary support both require a fundamental understanding of the behavior of frozen soils if rational design practices are to be developed. This is a difficult proposition because frozen soils are perhaps the most complex of all geomaterials. Their mechanical behavior is due not only to the behavior of the soil skeleton and the behavior of the pore ice, but also to interaction between these two components.

In order to develop solutions to design problems on frozen soils, engineers have generally relied on empirically based behavioral models as a means of predicting material response. These have been based on correlations between various parameters such as temperature, strain rate or creep stress, and peak strength, minimum creep rate or time to minimum creep rate. Uniaxial compression and tension tests have been used most often as a means of calibrating the behavioral models. These tests are performed in the laboratory where the time scale is generally much shorter than that in the field. Accordingly, the measured laboratory behavior is extrapolated to the field behavior and this introduces a considerable amount of uncertainty in the predicted response.

In addition, uniaxial compression and tension tests apply a relatively simple state of stress to the specimen. In the field, where the actual states of stress and deformational constraints are very complex, behavior extrapolated from such simple element tests may lead to unconservative predictions unless a large factor of safety (factor of ignorance) is used. In order to model complex
field conditions, it would be necessary to conduct sophisticated laboratory testing programs where principal stress directions could be controlled and widely varying deformational constraints could be imposed. Such testing procedures are extremely costly and have not been developed for frozen soils applications.

Empirically based behavioral models, while popular for solving immediate engineering design problems, are thus limited in the scope of application. A lack of understanding of the underlying mechanisms of strength and deformation in frozen soils is generally masked by a larger factor of safety in order to decrease the probability of failure. If an understanding of these fundamental mechanisms can be achieved, the engineer will be able to make more reliable predictions and this will lead to safer and more cost efficient designs. An understanding of these mechanisms is therefore essential to the development of more rational design procedures using physically based behavioral models.

1.1 Engineering Applications for Frozen Soils

Engineering applications requiring an understanding of the stress–strain response of frozen soils include the design and construction of structures in Arctic regions, the use of artificially frozen structures such as gravel islands and causeways for natural resource exploration and the use of artificial ground freezing as a means of temporary support for unstable ground in deep excavations and tunneling applications.

Jones and Holden (1988) write "Artificial ground freezing has been used to form an impermeable barrier and/or temporary support for excavations, shafts and tunnels for over 100 years. Ingenious schemes have extended its application to such areas as underpinning, temporary roadways and bridging. Depending on the circumstances, chilled brine or liquid nitrogen is circulated through freeze
tubes to form 'ice walls'. The design of frozen structures to ensure adequate strength and tolerable deformations, during both the freezing and thawing stages, has received considerable impetus with the development of computer-based methods. These have required a better understanding of the creep characteristics of frozen ground...".

Artificial ground freezing is currently being used extensively in Europe and Japan where space is at a premium and where new construction often takes place very close to or underneath existing facilities. In such cases, the need to provide a strong but temporary support in order to prevent damage to existing structures often makes artificial ground freezing an attractive alternative.

Several such applications were presented recently at the 5th International Conference on Ground Freezing in 1988 held in Nottingham, England. The following is a brief overview of some of the ground freezing applications presented.

Deix and Braun (1988) described the use of artificial ground freezing during the expansion of the Vienna Subway System. This expansion required the placement of two tunnels under an existing telecommunications building. The proposed tunnel crown was 1.5 meters under the foundation. Strict ground settlement limitations were imposed on the design. The placement of the tunnel was accomplished by the use of artificial ground freezing. A pad was frozen under the building and the excavation took place underneath the frozen pad.

Numazawa et al.(1988) described the use of artificial ground freezing to connect the two shields of a large diameter tunnel in Japan. The tunnel was constructed as two shield tunnels driven from either end and meeting in the center. Most of the tunnel was located beneath a canal. The sea water depth was 7.5 m and the tunnel was placed under 15 m of sand and silt. The final
connection of the two sections was accomplished by artificial ground freezing.

Restelli and Volpe (1988) described the use of artificial ground freezing to solve a very difficult tunneling problem. During the construction of a tunnel using a fully automated shield, a section of tunnel collapsed behind the shield and filled the tunnel with a saturated sandy silt. A shell was frozen around the collapsed sections of the tunnel and this, in concert with grouting techniques, allowed for the recovery of the automated shield and completion of the tunneling operation.

Maishman et al. (1988) described the use of temporary ground freezing in the U.S. as a means of preparing a temporary roadway for the passage of a 3000 ton dragline over farm lands. Environmental authorities required that the farm lands be restored to their original condition after moving the dragline. Engineers opted for ground freezing as the most economical and environmentally friendly means of preparing a temporary road.

These examples indicate that the engineering of frozen soils has a broad range of applications, well beyond the scope of cold regions engineering. Accordingly, an understanding of the fundamental behavior of frozen soils is of great interest to the entire geotechnical community and forms the focus of this research.

1.2 Objective of Research

Long Term Research Objective

The long term objective of M.I.T. frozen soil research is to develop physically based constitutive relations for frozen soil behavior which will aid in the development of rational design techniques. Physically based constitutive relations are behavioral models which capture the major features of the
underlying physical mechanisms of strength and deformation. The development of rational design techniques will aid in producing more cost effective designs.

**Short Term Objective and Current Thrust**

In order to develop physically based constitutive relations, an understanding of the fundamental behavior of frozen soils is essential. The near term objective of frozen soil research at M.I.T. is the identification of the major mechanisms which control the behavior of frozen soils and a quantitative assessment of their relative importance. This forms the primary thrust of this thesis.

These mechanisms can be broadly classified into those which occur in the soil skeleton, those associated with the pore ice and those which are due to the interaction between the silicate particles and the pore ice. If it were possible to measure directly the behavior of the soil skeleton in a frozen soil or the behavior of the pore ice, such measurements would go along way towards an understanding of frozen soil behavior. A direct assessment of the behavior of the sand skeleton in terms of the effective stresses acting on it, and a direct measure of the behavior of the ice in the pores is not currently possible. Accordingly, the current research has adopted an indirect approach which can be described in three parts.

**Indirect Approach to Studying Physical Mechanisms**

The first part of this indirect approach has been to make high-quality measurements of the behavior of frozen soils. Following an extensive literature review, part of which will be presented subsequently, it was observed that a complete characterization of the stress-strain behavior of any frozen soil had not
been published. A complete characterization of the stress–strain behavior includes a measurement of the stress–strain response from very small strains (10^{-4}), which capture the initial elastic behavior, through very large strains which capture the yielding behavior and subsequent strain hardening or strain softening. It also includes a measurement of the volume change behavior and an investigation of the effect of changes in confining pressure, relative density, strain rate, and temperature. A major advance in this regard has been accomplished by the development of a sophisticated triaxial testing system and by the use of careful testing procedures. The characterization of the stress–strain behavior of a frozen soil is the most significant contribution of this research.

Manchester Fine Sand from the Merrimack River Valley in New Hampshire was chosen for the purpose of study in this research. This sand has been artificially frozen to form a homogeneous sand–ice system. Prior frozen sand research at M.I.T. was focused on the uniaxial compression creep behavior of this sand. The behavior under triaxial compression loading conditions with constant rate of deformation was selected for this current research.

The parameters affecting the behavior of frozen sand which were selected for this thesis include the relative density of the sand, the rate of strain and the confining pressure. Other variables included the method of preparation of the sand specimens, the effect of specimen end conditions and the effect of temperature (this was studied only in one test).

The entire stress–strain response was measured from strains on the order of 4\times10^{-5} to greater than 0.2. On–specimen axial strains were measured with specially designed displacement yokes and lubricated end platens were used to achieve uniform specimen strains throughout the course of the deformation.
The behavior of frozen Manchester Fine Sand as measured in this research forms the most complete set of data describing the triaxial compression behavior of any frozen soil which has been presented in the literature. While the actual development of a physically based constitutive model for frozen soils may be a long way off, the generation of this robust and high-quality data set, which can be used to validate and assess the importance of various mechanisms, has been a necessary and important step in the attainment of the ultimate objective.

The second part of the indirect approach has been to consider the measured behavior of frozen sand in terms of a behavioral model proposed by Ladanyi (1985). This model has the objective of estimating the relative contributions of the soil skeleton effective stresses and the pore ice stresses in an effort to understand the composite behavior. This model considers the particulate nature of the sand skeleton in a frozen sand system and attempts to describe its behavior in terms of the tendency of the sand skeleton to dilate. The essence of this model is a comparison between the unfrozen undrained behavior of a soil system to the behavior of an identical system in a frozen state.

Concurrent research is being carried out by another M.I.T. graduate student to characterize the behavior of unfrozen Manchester Fine Sand. A full comparison of the measured data with the Ladanyi behavioral model will await the results of this concurrent research. The writer is capable at this stage of making some comparisons between the predicted behavioral response and the measured behavioral response and these are presented herein.

The third part of this indirect approach has been the use of a behavioral model from the field of composite materials in an effort to understand the interaction between the sand skeleton and the pore ice at small strains. A composite material model can be particularly useful in the small strain range.
where the particulate nature of the sand skeleton may not be fully "activated". The predicted behavior of frozen sand from this model is compared directly with the measured behavior.

1.3 Organization of Thesis

Chapter 2 is a background which encompasses a detailed literature review of the behavior of unfrozen sand, polycrystalline ice and frozen sand from a mechanistic standpoint. This review is intended to give the reader an appreciation for the complexity of frozen sand behavior by considering the complex behavior of its constituents and how they might interact as a composite system.

Chapter 3 is a discussion of the equipment that was developed at M.I.T. for this testing program. It consists of a high-pressure low-temperature triaxial testing system. Each component of this system is described and its performance is evaluated. The method used to measure specimen volumetric strains is presented. A calibration of the individual electronic sensing devices is then outlined.

Chapter 4 is a discussion of the materials and procedures. This discussion includes the results of several index tests and a mineralogical analysis on Manchester Fine Sand. The experimental procedures used for preparing, freezing and testing the frozen sand specimens are then presented. This chapter also discusses the data acquisition system and the data reduction procedures that have been used to convert the raw data to engineering units and then to units of stress and strain. This chapter is concluded with an analysis of the errors in the measured data.
Chapter 5 is a presentation of the measured behavior of frozen Manchester Fine Sand in triaxial compression. It first gives an overview of the scope of the testing program. The effect of the relative density, confining pressure and strain rate on the stress–strain behavior are treated in detail. Summary plots of parameters taken directly from the stress–strain curves form the basis of this discussion. These are supplemented by several stress–strain and volumetric strain curves. The effect of end conditions, specimen preparation techniques and temperature on the behavior of frozen sand is also presented.

Chapter 6 is an analysis of the measured data. This is done in three separate sections. In the first section, the measured data is evaluated in terms of the existing data in the literature. In the second section, the measured behavior is analyzed in terms of a behavioral model from the field of composite materials presented by Counto (1964). In the next section, the frozen sand data is analyzed in terms of particulate soil mechanics by considering the work hardening model proposed by Ladanyi (1985). As a result of the measured behavior of frozen sand and its analysis in terms of these behavioral models, some insight is gained into the relative importance of various strength and deformational mechanisms. A summary and discussion of possible mechanisms which may govern the strength and deformation behavior of frozen sand is then presented following the framework first proposed by Ting at al. (1983).

Chapter 7 presents a summary of this research, conclusions and recommendations for continued research.
CHAPTER 2
LITERATURE REVIEW

2.1 UNFROZEN SAND

This literature review focuses on the behavioral aspects of sand from a mechanistic point of view. This review includes the following major topics: parameters used to describe a sand mass; definition of stresses and mechanisms of stress transfer in a particulate medium; Terzaghi's principle of effective stress and pore water–sand skeleton interaction; effects of density and confining stress on the stress–strain behavior and the concept of an ultimate (steady state, critical state) condition; and mechanisms of deformation and strength.

2.1.1 Description of a Sand Mass

According to the Unified Soil Classification System, a sand is composed of those particles fine enough to pass through the #4 sieve and coarse enough to be retained on the #200 sieve. This corresponds to particle diameters ranging from 4.76 mm to 0.074 mm. Other classification systems have slightly different limits. The M.I.T. system as presented in Lambe (1951) defines sand as having particles with diameters between 2.0 mm and 0.06 mm.

Mineral Composition

The mineral composition of a sand can vary widely and is affected by such factors as the composition of the parent rock material, method of formation, amount of weathering, and transportation distance. Soils are usually the products of rock weathering. Thus, the most abundant soil minerals are common rock forming minerals and those that are most resistant to chemical and physical
weathering such as framework silicate minerals (Lambe and Whitman 1969, pg. 45). Pettijohn (1975 pg. 197) lists quartz, feldspars and rock fragments as comprising the major fraction of particles in most sand deposits. Micas, other light minerals and heavy minerals comprise approximately 10% or less of the remaining grains. Light minerals and heavy minerals refer, respectively, to minerals existing in a sand mass whose specific gravity is much less than or much greater than the average specific gravity of the deposit. The actual percentages of these constituents will vary widely between sand masses. Although it may be important to know the mineral composition of a sand, there are very few general relationships which link the mineral composition of a sand to its measured behavior (Lambe and Whitman 1969, pg. 41). Other features of the sand are generally much more important in controlling its stress–strain behavior.

One notable exception is carbonitic sands. These sands are formed by the deposition of shell fragments and the formation of oolites (precipitation of dissolved minerals on nuclei) in a marine environment. The individual particles have a much lower crushing strength than quartz or feldspars. This results in a lower degree of interlocking during shear, which leads to a large decrease in the peak mobilized friction angle. See Lambe and Whitman (1969 pg. 148) for a similar discussion on the effect of particle strength in gravels.

Another exception is calcitic loess deposits found in the western United States and parts of Asia. These deposits are wind blow sediments with particle sizes from fine sand to silt. A small amount of clay may also be present. They generally contain large amounts of calcite, up to 30%. This calcite acts as a weak cement bond between the individual particles. This cementation can support a stable soil structure at extremely low densities so long as the deposit remains dry. But if water is introduced, the calcitic cement dissolves and the structure may
collapse. Thus the presence of a large amount of calcite can indicate the potential for a collapsible behavior [Abstracted from Mitchell (1976 pg. 76–77)].

**Roundness**

The roundness of individual particles can be affected by the mineral type and will certainly be affected by the amount and nature of weathering which has occurred. For example, wave action on beaches and transportation of sediments in rivers and streams lead to well rounded particles, while the breakdown of rocks by alternating cycles of freezing and thawing leads to angular particles. Various systems have been proposed in order to classify sand particles according to their roundness. Pettijohn (1975) reported one such system. Depending on the sharpness or roundness of the edges and corners of the particles, sands can be classified as angular, subangular, subrounded, rounded or well-rounded. This classification can be done from a visual inspection. Charts for visual classification and a table with descriptions and limits based on the average radius measure are given in Fig. 2.1.

**Shape**

Sand particles can also be classified according to shape. By assuming that the particles are roughly ellipsoidal or tubular in shape, various researchers have used a ratio between the length and the width as a measure of shape [Oda (1972), and Ochai and Lade (1983) among others]. Such measurements can be somewhat uncertain because of the difficulty of identifying the major and minor principal axis of irregularly shaped grains. Some judgment must be exercised. Oda (1972) suggests that this measure be done by projecting the image of the sand grain on a screen using a thin section of the sand and a projector. The apparent shortest
and longest axis of each image are then measured from the screen. Ochai and Lade (1983) suggest that the individual sand grain be placed in the most stable position on the edge of a table and the vertical and horizontal lengths be measured with microscopes. In both of these proposed methods the average ratio for a given sand is computed from a representative number of individual grain measures. The mean value of the ratio of axial length to width for a natural sand deposit can range from 1.4 to 2.0 according to Ochai and Lade (1983).

Gradation

The size distribution of particles in a sand is referred to as the gradation. A gradation analysis is performed by passing a representative quantity of sand through sieves with standardized square mesh openings and weighing the amount of material that is retained on each sieve. If the sand contains a wide range of grain sizes with substantial amounts of all intermediate sizes, then it is referred to as a well-graded sand. If there is predominantly one size of particles or a range in sizes with some intermediate sizes missing, then it is referred to as a poorly-graded sand or a gap-graded sand, respectively.

According to the Unified Soil Classification System (ASTM D2487-85), the coefficient of uniformity, $C_u$, is defined as

$$C_u = \frac{D_{60}}{D_{10}}$$

2.1.1

where $D_{60}$ and $D_{10}$ are, respectively, the grain sizes for which 60% and 10% of the particles are finer by weight. This is a measure of the uniformity or range in grain sizes for a given sand deposit.

The Unified Soil Classification System also defines the coefficient of curvature, $C_c$, as

$$C_c = \frac{(D_{30})^2}{D_{60}D_{10}}$$

2.1.2
where $D_{30}$ is the grain diameter for which 30% of the particles are finer by weight. This is a measure of the shape of the gradation curve between the $D_{60}$ and $D_{10}$ sizes. A sand with less than 5% fines and having $C_u \geq 6$ and $C_c$ between 1 and 3 is a well-graded sand. Sands that do not meet these criteria are poorly graded.

**Void Ratio**

The void ratio ($e$) of a sand equals the volume of the voids divided by the volume of the solids. In general the loosest (i.e., the maximum void ratio) and densest (i.e., the minimum void ratio) possible state that a sand mass can assume will depend on the shape, gradation and roundness of the grains comprising the sand deposit. Under low and moderate confining stresses, the behavior of a sand is very sensitive to the degree of packing of the particles. This will be illustrated in Section 2.1.5. Engineers have developed various experimental procedures to estimate the maximum and minimum void ratios in an effort to bracket sand behavior between these two limits.

The proposed methods for measuring the maximum ($e_{\text{max}}$) and minimum ($e_{\text{min}}$) void ratios of a given sand deposit have been the subject of some controversy in the literature. Hence the ASTM standards D4253–83 and D4254–83 allow several different methods for measuring the densest and loosest states. Riemer et. al. (1990) suggested that $e_{\text{min}}$ be defined as "the void ratio corresponding to the maximum density which can be achieved by any method, without significant particle breakage". They suggested that $e_{\text{max}}$ be defined as "the void ratio corresponding to the loosest stable density that can be achieved by any method in the absence of any capillary tension".
Relative Density

De Mello (1979) noted that the relative density of a sand was initially conceived as a means of correlating behavior of different sands and predicting strength. The denseness of packing was presumed to be the most important factor in determining strength and the relative density is a measure of this state of packing. The relative density describes (on a percentage scale) the degree of compactness of a sand and is defined as

\[ D_r = \left( \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}} \right) \times 100\% \]

where \( e \) is the existing void ratio. However, \( D_r \) alone cannot be used to predict the behavior of sand. First, it does not consider the effect of confining stress. As will be illustrated in Section 2.1.5, sands with the same \( D_r \) will exhibit marked differences in behavior at different confining stress levels. A parameter which incorporates both the effect of density and the effect of confining stress is the state parameter \( \psi \) (discussed in Section 2.1.5). Second, the \( D_r \) neglects the effect of "sand structure" (defined below). Various sand structures can exist at the same \( D_r \) and sand behavior can be affected by its structure (considered in Section 2.1.10).

Sand Structure

A sand mass can be classified according to its "structure" or "fabric". There is some disagreement in the literature as to the exact definition of the structure and fabric. The writer adopts the definition of soil structure presented in Lambe and Whitman (1969 pg. 71) who use the term to refer "to the orientation and distribution of particles in a soil mass (also called "fabric" and "architecture") and the forces between adjacent soil particles". First the "fabric" component and then the "force" component of the soil (sand) structure will be
Brewer (1964) defined the soil fabric as "The physical constitution of a soil material as expressed by the spatial arrangement of the solid particles and associated voids". Oda (1972) concurred with Brewer's definition and added: "Principally two types of fabric features, i.e., homogeneous fabric and heterogeneous fabric, can be distinguished. The granular mass of heterogeneous fabric is composed of component parts or submasses of homogeneous fabric having different kinds and degrees of particle configurations. Three-dimensional orientations of these homogeneous submasses and their mutual relationships must be the most important fabric feature prevailing in the heterogeneous granular mass." Oda further outlined that the homogeneous fabric should include the concepts of "the orientation of an individual particle and the position of the particle and its mutual relationship to other particles." Oda (1981) summarized his definition of fabric in the following manner "...the fabric of sand can be defined by the following two elements: (1) Orientation of nonspherical particles (orientation fabric); and (2) mutual relation of particles (packing)". The orientation fabric refers to the alignment of nonspherical particles (i.e., 3-D orientation of the particles or "particle distribution") and the packing refers to both the orientation of the unit normals at particle contact points and the number of contacts per particle (= "fabric" component of the soil structure). The average number of contacts per particle is referred to as the coordination number.

In order to distinguish between the differences in behavior due to the orientation of the individual particles and the direction and magnitude of the interparticle contact forces, Ladd et al. (1977) have discussed the concept of the "soil structure" in the following manner. "The term 'soil structure' will be
defined as having two components: the orientation and distribution of particles in soil mass, i.e., its fabric; and the nature and magnitude of the interparticle forces. With cohesionless soils, fabric and structure are closely related since interparticle contacts will be seen to be largely governed by the fabric”. This definition is consistent with the observation by Oda (1981) that the behavior of sand is affected both by the preferential alignment of nonspherical particles parallel to the horizontal and by the concentration of the unit normal vectors at the points of contact in preferred directions.

A homogeneous sand fabric is considered isotropic if it possesses a uniform distribution of long axis orientations and unit normals. In naturally occurring sands and laboratory specimens prepared by pluviation through air or water, the homogeneous sand fabric is usually cross–anisotropic. The sand particles tend to orient themselves with their long axis direction randomly distributed in the horizontal plane and their contact point unit normals closer to the vertical depositional direction [Oda (1981), Ochai and Lade (1983) and Mitchell (1976)]. Sand specimens which possess a pseudoisotropic homogeneous sand structure have been prepared in the laboratory by forming the dry sand specimen in layers and penetrating each layer with a hand plunger (Oda 1981).

2.1.2 Stress Transfer and Definition of Stress in a Dry Particulate Medium

Stress Transfer

Forces applied to the boundary of a dry sand mass are obviously transmitted through the sand mass by direct particle to particle contacts. However, the actual area of contact between adjacent particles is very small compared to the surface area of each sand grain and compared to the surface area
at the boundary. This results in a significant amplification of the imposed boundary stresses at these points of contact. The magnitude of these contact stresses can exceed the local strength of the silicate particles.

In considering the stress transferring mechanism in a particulate material, Gourves (1981) discussed the possibility that in a given sand mass there are some particles which do not actively participate in the stress transferring process. These are referred to as inactive particles that can be removed from the sand structure without any resulting change in the transfer of stresses.

Definition of Stress (abstracted from Lambe and Whitman 1969)

The variation of stress in a sand mass at the particle level is quite dramatic. In order to average out these locally high variations in stress, the following definition of stress in a dry particulate medium has been adopted, e.g., Lambe and Whitman (1969 pg. 98). This definition is based on an equilibrium condition considering each particle individually. The stresses being carried by a sand skeleton are defined by passing a plane through the sand mass (Fig. 2.2). The resultant internal forces, both normal and shear, within each particle cut by the plane are summed over a specific area and the total force, both normal and shear, is divided by the area of summation. The area of summation must be chosen large enough so as to not affect the magnitude of the stresses computed. Lambe and Whitman comment that the stresses in a dry soil may thus be thought of "as the force in the mineral skeleton per unit area of soil".

In practice it is impossible to measure the forces acting on each individual particle and to perform this type of summation. Instead, a pressure or force transducer is placed in contact with the sand mass and the total force or stress is measured directly. The presence of the transducer can alter the stress field. The
electronic transducer performs the integration of the forces directly. The stresses acting on the soil skeleton defined in this manner will be much smaller than the actual internal stresses acting in each "active" particle.

**Mechanics of Deformation**

A study of the mechanics of deformation in sand masses is dependent on the nature and magnitude of the interparticle contact forces. A significant portion of the total deformation experienced by a sand is usually accommodated by the rigid body motion of the individual grains. The rigid body motion of each particle is governed by the resultant of the contact forces acting on it. Thus, the total deformations experienced by the sand will be related to changes in the intergranular contact forces which must always form a force equilibrium condition.

2.1.3 Saturated Sand and Terzaghi's Principle of Effective Stress

Terzaghi (1925) presented a method of partitioning stresses in a saturated soil mass. This has come to form the basis of all analytical work in soil mechanics, and is known as the principle of effective stress. As a result of experimental measurements, Terzaghi proposed that the total stress acting on a soil element be partitioned into the effective stresses acting on the skeleton and the pore water pressure according to the following equation

\[ \sigma = \sigma' + u \]  \hspace{1cm} 2.1.4

where \( \sigma \) is the total stress, \( \sigma' \) is the effective soil skeleton stress and \( u \) is the pore water pressure.

After an extensive theoretical analysis of the stresses acting in a particulate medium, Skempton (1961) concluded that Terzaghi's empirical
equation is an excellent approximation in the case of saturated soils because the total intergranular contact area is very small. In other porous media such as concrete and rocks, where the intergranular contact area is much larger, this stress partitioning does not strictly apply.

When considering the interaction between the effective stresses acting on the soil skeleton and the pore water pressures, it is important to consider the case of an incremental change in total stress occurring under conditions where the pore water is not free to drain out of the soil. Skempton (1954) studied such a situation and introduced a pore pressure parameter to quantify the amount of stress sharing which occurs between the soil skeleton and the pore water under these conditions. This parameter is known as Skempton's B-value and is defined as the ratio of the increment in pore pressure to the increment in total applied (hydrostatic) stress. Skempton proposed that the magnitude of the B-value is dependent on the porosity of the soil (the ratio of the volume of the voids to the total volume), the bulk compressibility of the soil skeleton and the compressibility of the pore fluid (water). The magnitude of the B-value can range from zero to one. A B-value of one means that the entire increment of total stress is carried by the pore water.

Bishop (1973) summarized Skempton's earlier work and extended his analysis to include the effect of the compressibility of the individual sand grains. He concluded that the ratio of the increment of pore water pressure to increment of total stress could be expressed by the following equation

\[ B = \frac{\Delta u}{\Delta \sigma} = \frac{1}{1 + n(C_w - C_s)/(C - C_s)} \]

(2.1.5)

where \( n \) is the porosity, \( C_w \) is the bulk compressibility of the pore fluid, \( C_s \) is the bulk compressibility of the solid material forming the porous medium, and \( C \) is
the bulk compressibility of the porous medium (skeleton) under drained conditions. For saturated sand, Skempton (1961) reports: \( C_w = 48 \times 10^{-6}/ksc \), \( C_s = 2.7 \times 10^{-6}/ksc \), and \( C = 1,800 \times 10^{-6}/ksc \) for dense sand and \( C = 9,000 \times 10^{-6}/ksc \) for loose sand (Note: 1 ksc = 1 kgf/cm² = 98.1 kPa). These values lead to \( B \)-values of 0.990 and 0.998 for dense and loose sands, respectively. This means that essentially all of the increment in total stress applied to a saturated sand in an undrained mode is transferred to the pore water and that there is a negligible change in effective stresses. Even at very high effective confining stresses approaching 50 MPa, the compressibility of the soil skeleton is still relatively large compared to the compressibility of the individual soil particles and the \( B \)-value is still on the order of 0.9. Only in very dense particulate materials such as concrete does the \( B \)-value begin to decrease significantly from unity. In addition, a compliant measuring system can produce a measured \( B \)-value which is significantly less than unity (Wissa 1969).

2.1.4 Nature of Forces Transmitted Between Sand Grains

Frictional Forces

The forces transmitted between sand grains at points of contact can exhibit frictional behavior. The mechanism for frictional behavior can be explained in terms of the Adhesion Theory of Friction first proposed by Terzaghi (1925) and later used by Bowden and Tabor (1964) to describe the frictional nature of metals. This review has been abstracted from Section 6.2 of Lambe and Whitman (1969). On the submicroscopic scale, the surfaces of the sand particles are very rough. In the zone of apparent contact the two particles are actually touching only at the high points (termed asperities). The contact stresses at
these points are extremely high, even under light loading, and the contact stress reaches the yield strength of the silicate material. This is assumed to cause a plastic flow of the asperities. The actual area of contact will be proportional to the normal load acting at the junction and the yield stress of the material. This can be expressed in the following form

\[ A_c = \frac{N}{q_u} \]  

where \( A_c \) is the area of contact, \( N \) is the normal force at the point of contact and \( q_u \) is the yield stress of the material composing the two particles in contact. Because \( q_u \) is constant, the actual area of contact increases linearly with the normal force. The high contact stresses cause the two silicate surfaces to chemically bond to each other. This provides an adhesive shear strength \((s)\) which generates the shearing resistance of junction and is presumed to be constant. The maximum possible shear force, \( T_{\text{max}} \), which can be transmitted by this junction can be expressed by the following equation

\[ T_{\text{max}} = s A_c \]  

where \( s \) is the shear strength of this bonded junction and \( A_c \) is the actual area of contact. Combining equations 2.1.6 and 2.1.7 leads to the following expression

\[ T_{\text{max}} = N \frac{s}{q_u} \]  

The ratio \( s/q_u \) is referred to as the coefficient of friction and \( \arctan(s/q_u) \) is known as the interparticle friction angle. Thus, the maximum possible shear force that can be transmitted between particles increases linearly with the normal force at a rate equal to the coefficient of friction of the material.

The Adhesion Theory of Friction does not depend on the actual character of the adhesive bond formed by the particles in contact, nor does it depend on whether ductile plastic flow mechanisms or brittle fracture mechanisms operate at the points of contact. It only requires that the area of contact increases
linearly with normal force and that the adhesional shear strength of the contact junction be constant. In ductile metals, cold welding can occur at the junction points, which can result in significant adhesional strength even after unloading. However, such cold welding behavior probably does not occur at the contact junctions of brittle materials such as quartz sand particles (Lambe and Whitman 1969, pg.64). The release of elastic energy upon unloading in sands is probably sufficient to break any bonds which are formed under the high contact stresses. Bromwell (1966) considered that plastic flow type mechanisms were operative at the contact points in quartz sand. Brace (1963), while not ruling out the possibility of plastic flow type mechanisms occurring at the surface of quartz indented with a pyramidal shaped diamond indenter, believed that the observed "plastic" deformations were probably a result of extremely fine microfractures.

Cohesive Forces

Forces between sand grains can also exhibit a cohesive behavior. This can be due to the deposition of various cementing agents from the pore fluid at points of contact between adjacent grains. Pettijohn (1975 Chapter 7) explains that such cementation processes are operative during the diagenesis of sandstone. He states that silica is the most common cementing material. Carbonate minerals (i.e., calcite and dolomite), iron oxide and iron sulfide can also serve as cementing materials. If a sufficient amount of cementation occurs in a sand, eventually a "tight" structure is formed which can have an extremely low permeability.

Lee (1977) observed cohesive bonds between particles of Sacramento River Sand which had been sheared at high confining pressures, and suggested that these bonds might be explained by capillary forces at contact points due to small
amounts of water, by cold welding at contact points, or by the redeposition of silica solution from the pore water at the contact points.

In order to explain a time–dependent strength gain in densified sand, Mitchell and Solymar (1984) concluded that the formation of silica acid gels on the particle surfaces and the precipitation of silica or other materials from solution at the points of contact could act as a cement and probably serve as a "dominant mechanism" for cohesive type behavior which increases the strength of a sand mass.

2.1.5 Overview of Sand Behavior

Direct Shear Tests

Casagrande (1936) summarized the results of direct shear tests on sands with varying densities by stating that "during shearing tests, we find that dense sand expands and very loose sand reduces its volume". The expansion of dense sand during shear is referred to as dilation, a phrase that was coined by Reynolds (1886). Casagrande (1936) presented two sketches which illustrated qualitatively his observations. These have been reproduced in Fig. 2.3. He stated "in a dense sand the grains are so closely interlocked that deformation is not possible unless accompanied by a loosening up of the structure". This loosening is observed as a net volume increase. Casagrande further explains that "if dense sand is so confined that it cannot expand, then the shearing strength is determined by the resistance of the grains to crushing, and therefore it acts essentially like a rigid stone". In loose sands the structure collapses during shear to a more stable configuration. Due to this tendency for dense sands to expand and loose sands to contract during shear, Casagrande proposed the existence of an ultimate
condition at large strains where the shearing proceeds at constant volume and constant shear stress. He observed that both dense and loose sands tested in direct shear under identical vertical normal stresses eventually end up at the same density and shear stress. He called the void ratio at this ultimate state the critical void ratio and explained that sands in this state have reached a condition at which continuous deformation is possible at constant shear stress.

Casagrande's tests on sand were performed in a direct shear device. A complete characterization of the state of stress and reasonable measurements of deformational strains in such a device are impossible. For this reason, researchers moved away from using direct shear testing devices to study sand behavior in favor of more sophisticated devices that do not have these major limitations.

**Triaxial Compression Drained Tests**

One such device is the triaxial cell. Lee and Seed (1967) performed a series of drained triaxial compression tests on processed Sacramento River Sand. The grain size ranged from 0.149 to 0.297 mm and the sand was tested at initial relative densities ranging from 38 to 100%. The tests were performed with frictional ends. The authors initially started with lubricated ends. They compared these results with tests that had frictional ends and noted that the difference in the dilatancy effects at the maximum deviator stress was very small. Subsequently, they used frictional ends for the remainder of the testing program.

Lee and Seed (1967) observed that Sacramento River Sand tested under drained conditions in triaxial compression can exhibit either brittle type behavior with significant strain softening after peak, or ductile type behavior with no strain softening. This change in behavior depends both on the relative density
and on the magnitude of the effective consolidation pressure, as illustrated in Fig.2.4. The results are presented in terms of the principal stress ratio \((\sigma'_{1}/\sigma'_{3})\) and volumetric strain \((\varepsilon_v)\) versus axial strain \((\varepsilon_\text{a} = \varepsilon_1)\). Note that a negative volumetric strain represents a decrease in volume and the "\(\sigma_3\)" is the effective consolidation pressure \((\sigma'_c)\) for each test. The principal stress ratio is a dimensionless representation of the applied shear stress for drained compression tests since \(\sigma'_{3} = \sigma'_c\), and

\[
R = \frac{\sigma'_{1}/\sigma'_{3}}{\sigma'_{c}} = \frac{\sigma'_{c} + (\sigma'_{1} - \sigma'_{3})}{\sigma'_{c}} = 1 + \frac{(\sigma_{1} - \sigma_{3})}{\sigma'_{c}}
\]

where \((\sigma_{1} - \sigma_{3})\) is called the deviator stress or stress difference in geotechnical engineering. The principal stress ratio is also related to the friction angle in the following manner

\[
\phi' = \arcsin\left(\frac{R-1}{R+1}\right)
\]

In all cases the stress difference increased with effective consolidation stress, even though the principal stress ratio decreased.

The dense sand specimens were prepared with an initial (preconsolidation) void ratio of 0.61, which corresponds to a relative density of 100%. The test with the lowest confining pressure, \(\sigma'_c = 1 \text{ kg/cm}^2\), exhibited the largest principal stress ratio at peak and the most post peak strain softening. The principal stress ratio (hence \(\phi'\)) at peak decreased and the strain to peak increased with increasing confining stress. Thus, an increase in effective confining pressure led to a more ductile type behavior. The specimens tested with lower confining pressures also exhibited the greatest dilation. It is also interesting to note that for the specimens which exhibited some tendency for dilation (i.e., \(\sigma'_c < 20 \text{ kg/cm}^2\)), the maximum rate of dilation occurred near the peak strength.

The rate of dilation in triaxial compression tests is defined as \(d\varepsilon_v/d\varepsilon_1\) (for \(+\varepsilon_v = \text{expansion}\)). When a dense particulate material undergoes shear
deformations under drained conditions, the structure must be "loosened up" as observed by Casagrande (1936). This results in an increase in volume as the particles move up and over each other and contributes additional shear resistance. At the maximum rate of dilation, the maximum amount of energy is being expended in loosening up the structure and hence the maximum amount of strengthening is occurring due to dilation. This gives a qualitative argument for why the maximum rate of dilation in a dense sand should occur at the peak strength. Section 2.1.9 will discuss these concepts in greater detail.

The loose sand specimens were prepared at an initial void ratio of 0.87 corresponding to a relative density of 38%. These loose sands did not exhibit strain softening. The effect of increasing the effective confining pressure was the same as for the dense sand. The test with the lowest effective confining pressure exhibited the largest principal stress ratio a peak (largest \( \phi' \)) and the lowest strain to peak. All loose specimens with \( \sigma' c > 2 \text{kg/cm}^2 \) exhibited volume decreases during shear. The greatest decrease in volume occurred in the test with the largest effective confining stress. Thus, an increase in confining pressure caused a decrease in peak principal stress ratio, an increase in the strain to failure and a greater amount of volume decrease during shear.

The results in Fig. 2.4 also show that the stress-strain behavior of the dense sand sheared at a high effective confining pressure is qualitatively similar to a loose sand sheared at a low effective confining pressure. Thus, the effect of an increase in effective confining stress is similar to a decrease in density. Both lead to a more ductile response. Dense sand sheared at a high effective confining pressure obviously has a much larger peak deviator stress than loose sand sheared at a low effective confining pressure. But, the loose sand at \( \sigma' c = 1 \text{ksc} \) had a larger peak principal stress ratio, 3.75, compared to 3.4 for the dense sand at \( \sigma' c \).
= 120 ksc. Also note that the dense sand has a much larger peak deviator stress and peak principal stress ratio (peak $\phi'$) than a loose sand when both are sheared at the same effective confining pressure.

**Triaxial Compression Undrained Tests**

Castro (1969) presented the results of a series of undrained triaxial compression tests to study the effect of relative density and confining pressure on the undrained response of several sands. One of these sands was a uniform Ottawa Banding Sand with particle sizes ranging from 0.4 to 0.07 mm and preshear relative densities ranging from 16 to 96%. The effective confining pressure varied from 0.3 to 10 ksc. These tests were conducted by incrementally increasing the deviator stress and measuring the resulting pore pressures and axial strains. In an effort to achieve uniform specimen strains, lubricated end platens were used. Mohamad and Dobry (1986) summarized some of Castro's data. These summary plots have been reproduced as Fig. 2.5 and 2.6. On these plots, $q$ is defined as

$$q = (\sigma_1 - \sigma_3)/2$$

2.1.11

and $\overline{p} = p'$ is defined as

$$\overline{p} = (\overline{\sigma}_1 + \overline{\sigma}_3)/2$$

2.1.12

The effect of consolidation stress ($\overline{\sigma}_{3c} = \sigma_c'$) on the undrained response of a medium sand is illustrated in Fig. 2.5. These tests were conducted at a preshear relative density of 42% and effective confining pressures of 0.3, 4 and 10 kg/cm$^2$. Specimen #1 tested at $\sigma_c' = 10$ kg/cm$^2$ exhibited a brittle type behavior with the peak stress occurring at an axial strain level of between 1 and 2% and pronounced strain softening to a low constant strength at large strains. The $q-\overline{p}$ (stress path) diagram shows that positive pore pressures were generated
during the entire deformation process. Castro (1969) referred to this type of stress-strain behavior as liquefaction. Specimen #2 tested at $\sigma'_c = 4 \text{ kg/cm}^2$ exhibited a similar type behavior (similar stress path) up to axial strains on the order of 5%. Positive pore pressures were generated and the specimen began to strain soften after the initial peak stress which occurred at axial strains of 1 to 2%. But at 5% axial strain, the specimen exhibited strain hardening behavior as the pore pressures started to decrease. The effective stresses began to increase as the stress path climbed up the $K_f$-line. The $K_f$-line is the locus of points on the $q-p$ diagram representing the failure envelope (maximum achievable principal stress ratio). The test was terminated at an axial strain of 20% without reaching a peak undrained strength. Castro referred to this type of behavior as limited liquefaction. Specimen #3 tested at an effective confining stress of 0.3 kg/cm$^2$ exhibited continuous strain hardening behavior as the stress path reached the $K_f$-line very early during the deformation and climbed continuously up the $K_f$-line until the test was terminated at an axial strain of just over 15% without achieving a discernible peak strength.

Comparing the undrained stress-strain curves in Fig. 2.5 with the drained stress-strain curves in Fig. 2.4 illustrates that qualitatively the effect of confining pressure is opposite in undrained and drained tests. In undrained tests, the higher the effective confining pressure, the more brittle the behavior and the greater amount of strain softening. Note, however, that in undrained tests on specimens which tend to climb up the failure envelope, a discernible peak strength is not usually achieved. Such behavior is not observed in drained tests on sands.

Figure 2.6 presents the effect of relative density on the undrained behavior of Banding sand as measured by Castro (1969). All the specimens were
isotropically consolidated to 4 kg/cm², but at relative densities ranging from 37 to 96%. Specimen #6 tested at a preshear relative density of 96% exhibited continuous strain hardening behavior with the generation of negative pore pressures as the stress path climbed up the Kf-line. The test was terminated at an axial strain of only about 1% and a discernible peak strength was not achieved. Specimen #5, tested at a preshear relative density of 50%, exhibited a slightly more ductile response than specimen #6, with the initial generation of positive pore pressures. When the stress path reached the Kf-line, the pore pressure began to decrease and the strength increased continuously as the stress path climbed up the Kf-line. The test was terminated when the capacity of the load measuring system was approached and again no discernible peak strength was achieved. Specimen #2, tested at a relative density of 43%, has been previously described. It exhibited a limited liquefaction behavior with strain softening followed by strain hardening when the stress path reached the Kf-line. No discernible peak strength was achieved. Specimen #4, tested at a relative density of 37%, exhibited a brittle type behavior with a peak strength occurring at an axial strain level of approximately 1%. Beyond this peak, the strength decreased continuously to an ultimate strength which was much lower than the peak. This behavior was accompanied by the generation of large positive pore pressures.

The behavior of sand in undrained shear can be summarized as follows in terms of changes in the effective confining pressure and changes in the relative density.

Changes in effective confining pressure cause the following changes in the character of the deformation. For sands at a "moderate" relative density and "low" confining pressure, the behavior will be continuous strain hardening. The
stress path reaches the $K_f$-line early and then continuously climbs up the $K_f$-line. Negative pore pressures are generated throughout and it is difficult/impossible to reach the peak undrained strength of the specimen. The same sand with the same relative density tested at a "high" confining pressure will exhibit brittle type behavior with a peak undrained strength at very low axial strains, followed by a significant amount of strain softening to a very low strength at large strains. This strain softening is caused by the generation of large positive pore pressures.

Changes in relative density can cause the following changes in the character of the deformation. At a "moderate" confining stress, a "dense" sand will reach the $K_f$-line at very low axial strains and continuously climb up the $K_f$-line through the generation of negative pore pressures, while a "loose" sand will exhibit liquefaction and significant post peak strain softening caused by the generation of large positive pore pressures. Thus qualitatively for undrained shear, a decrease in relative density has the same effect as an increase in confining pressure.

2.1.6 The State Parameter for Sands

Been and Jefferies (1985) introduced a concept which helps to unify the effects of $D_r$ and $\sigma_c'$ on the behavior of sands. It is known as the state parameter $\Psi$ and is a measure of the physical condition (state) of a sand in terms of its initial void ratio and the initial state of stress with respect to the ultimate state or critical state. They propose the use of the first invariant ($I_1$) of the stress tensor as a suitable measure of the stress. $I_1$ is defined as

$$I_1 = (\sigma'_1 + \sigma'_2 + \sigma'_3)/3$$

which is the mean normal stress.
Been and Jefferies assume that all specimens of a given sand tend to approach the same ultimate state irrespective of the initial state. They adopt the concept of "The Steady State of Deformation" as proposed by Poulos (1981) as the ultimate condition of the sand. They explain, however, that their concept of the state parameter $\Psi$ "does not depend on the nature of the sand structure at the steady state. Rather it depends on there being a unique, repeatable particle arrangement at the steady state condition". The controversy over the uniqueness of the steady state line and the existence of a critical state will be covered in Section 2.1.7

Fig. 2.7 is a reproduction of a figure that Been and Jefferies (1985) used to define the state parameter. The ordinate is the void ratio and the abscissa is the mean normal stress ($I_1$) on a log scale. The $\Psi$ parameter is defined as the vertical distance from the initial state to the Steady State Line (SSL) expressed in units of void ratio. A sand which has an initial state that plots above the SSL has a $+\Psi$ and one that plots below the SSL has a $-\Psi$. Been and Jefferies (1985 and 1986) report that many sands with the same $\Psi$ exhibit the same general behavior (e.g., the same peak drained $\phi'$ for $-\Psi$ and the same peak undrained strength ratio for $+\Psi$) irregardless of the absolute magnitude of the $D_r$ or $\sigma'_c$. Thus $\Psi$ can apparently unify the effects of both $D_r$ and $\sigma'_c$.

Fig. 2.8 plots several normalized stress paths for specimens tested undrained at various relative densities and effective confining pressures, but with similar $\Psi$ parameters. The deviator stress and the mean stress ($I_1$) have both been normalized by the mean stress at the steady state ($I_{ss}$). Sands with $-\Psi$ parameters tested in undrained shear tend to strain harden while generating negative pore pressures as they climb up the $K_F$-line. Specimen #103 with a $\sigma'_c = 50$ kPa and $D_r = 33\%$ and specimen #108 with a $\sigma'_c = 300$ kPa and $D_r = 50\%$
both have the same $\Psi$ parameter and both have almost identical normalized stress paths. Specimens 3 from Fig. 2.5 and 2, 5 and 6 from Fig. 2.6 of Castro's data had similar behaviors to these.

Sands with large $+\psi$ parameters exhibit strain softening type behavior in undrained shear caused by the generation of large positive pore pressures. Specimen #45 with $\sigma'_c = 350$ kPa and $D_r = 56\%$ and specimen #112 with $\sigma'_c = 1100$ kPa and $D_r = 75\%$ both have the same $\Psi$ parameter and exhibit similar behaviors. Specimens 1, 2 from Fig. 2.5 and 4 from Fig. 2.6 of Castro's data are examples of similar behavior.

In the writer's opinion, the quantitative use of the $\Psi$ parameter to combine the effects of confining pressure and $D_r$ can be helpful so long as the user is aware of some of the possible limitations. These are as follows:

1) The $\Psi$ parameter relies on the uniqueness of the Steady State Line for a given sand and the assumption that all specimens of a given sand tend to approach that condition irregardless of the initial conditions or the method of loading. This has not been demonstrated conclusively. Castro (1969) was unable to observe a peak or steady state undrained strength in sands that exhibited a strong dilational tendency (i.e., large $-\Psi$). Researchers have tended to rely on undrained testing of sands with $+\psi$ parameters to locate the SSL. The difficulty in reaching the same SSL with specimens that have $-\psi$ parameters can be a major limitation.

2) The $\Psi$ parameter relies on an accurate measure of the void ratio in the sand. There can be a significant redistribution in the void ratio during shear, especially if frictional end platens are used, because significant bulging may occur or a failure plane may develop. Most researchers report the void ratio of drained tests from an overall measure of the volume change and they implicitly assume that in undrained tests that the void ratio remains constant throughout shear. This can lead to significant errors. An accurate measure of the void ratio at the steady state is important and perhaps only achievable through careful freezing and sectioning procedures similar to those outlined by Gilbert and Marcuson (1988).

3) The effects of membrane compliance on the behavior of both drained and especially undrained tests can be significant for coarse
to medium sands. Changes in the amount of membrane penetration occurring during shear can significantly influence the accuracy of the volume change measured in drained tests and the "zero volume change" condition in undrained tests, e.g., Ramana and Raju (1980) and Nicholson et al. (1989). In drained tests with a constant back pressure, if the sand particles supporting the membrane move closer together, the amount of membrane penetration will decrease and the measured volume change will be less than the actual volume change. Conversely, if the particles move further apart during shear the membrane penetration will increase and the measured volume change will be greater than the actual volume change. In an undrained test if the specimen has a tendency to dilate (i.e., \(-\Psi\) parameter) negative pore pressures will be generated and the membrane will be "sucked" into the pore spaces. This will be balanced by an increase in the void ratio of the specimen because the total volume must remain constant (neglecting the effects of compressibility of the pore water and the individual sand particles). A similar effect will occur if positive pore pressures are generated during undrained shear (i.e., \(+\Psi\) parameter). In this case the void ratio of the specimen will decrease to compensate. In neither case is the behavior truly "undrained".

In the writer's opinion, there is another fundamental difference between the behavior of a sand in drained versus undrained shear which might limit the universality of the state parameter. In drained shear the volume changes necessary to accommodate the required deformations occur unimpeded as pore fluid flows in or out of the specimen. The specimen deforms in a manner consistent with the imposed stresses. In undrained tests volume changes are prevented, ideally by correcting for membrane compliance and taking precautions to ensure uniform specimen deformations. It is not clear how the deformations can proceed under such conditions without a local redistribution of void ratio, except in the case of the loosest specimens where some of the particle groupings can collapse while others expand to maintain a constant volume relationship. Casagrande (1936) postulated that in the very densest state if volume changes were prevented, the measured strength would be due to cleavage of the individual
particles. This would seem to indicate that for sand with a large $-\Psi$ sheared undrained, the strength might be less dependent on the effective stress level and more dependent on the cleavage strength of the sand particles. This would violate the fundamental assumption that the behavior is dependent on the effective stress level and perhaps explain why dense sands do not reach a peak strength under normal undrained testing conditions.

2.1.7 The Existence of a Critical State

The actual state of a sand at large deformations has been a disputed topic in the literature. This is perhaps due to the difficulty of measuring sand behavior at large deformations with conventional testing equipment as was discussed in the previous section. Various researchers have developed methods for predicting sand behavior based on the concept of a critical state. Castro (1969) developed a qualitative argument for predicting the undrained response of a sand, in triaxial compression under load control, based on the initial state and its position relative to the ultimate state. Poulos et al. (1985) developed a method to predict the liquefaction susceptibility of a given sand by comparing the in situ state to the steady state. Been and Jefferies (1985) and Been et al. (1991) have proposed the use of a state parameter $\Psi$ to predict the response of a sand as was explained in Section 2.1.6. All of these researchers have used Casagrande’s 1936 hypothesis that sands tend to approach a condition of continuous deformation at large strains as a basis for their studies, but they disagree as to the structure which must be present in this condition, the methods which should be used to measure this state and whether or not it is unique and independent of the initial conditions.

Roscoe et al. (1958) discuss the concept of critical (ultimate) state in soils
which is reached only after large straining of the sample. They propose that in a
drained test, the critical void ratio state be characterized by a condition that
"any arbitrary further increment of shear distortion will not result in any change
in voids ratio". This state is defined by both the void ratio and the effective
stresses acting on the soil skeleton and supposedly represents a unique condition
independent of the initial conditions. The set of critical void ratio points will lie
on or near the drained failure surface. In undrained tests, they state that "the
sample remains at a constant voids ratio, but the effective stress $\bar{p}$ will alter to
bring the sample into an ultimate state such that the particular voids ratio, at
which it is compelled to remain during shear, becomes the critical voids ratio". They state that these two conditions in the drained and undrained tests may or
may not be identical and lie on a unique critical void ratio line.

Rowe (1962) postulated the existence of an ultimate condition in a
particulate medium. This would be achieved at large strains where there would
be a continual rearranging of the particle assembly, a process which he called
remoulding. He envisioned a condition where half of the particles are effectively
resisting the applied shear stress and are about to break contact while the other
half have failed and are making new contacts. If this process is kept in motion,
then there will be no tendency for volume change and the shearing will proceed at
a constant normal and shear stress.

Poulos (1981), Poulos et al. (1985) and Poulos et al. (1988) have
hypothesized that an unique ultimate condition of sand exists which can be
described as "The Steady State of Deformation". They define this condition in
the following manner. "The steady state of deformation for any mass of particles
is that state in which the mass is continuously deforming at constant volume,
constant normal effective stress, constant shear stress, and constant velocity.
The steady state of deformation is achieved only after all particle orientation has reached a statistically steady-state condition and after all particle breakage, if any, is complete so that the shear stress needed to continue deformation and the velocity of deformation remain constant. " Been and Jefferies (1985) and Been et al. (1991) also ascribe to this notion of ultimate state.

This steady state condition is assumed to be unique for a given sand and is characterized in e, q, \( \sigma' \) space by a steady state surface. Here e is the void ratio, q is one half of the deviator stress and \( \sigma' \) is the minor principal effective stress. Poulos et al. (1985) claim that not only is this steady state condition identical for both drained and undrained tests, but also that the position of the steady state line is extremely sensitive to the gradation and mineralogy of the sand and that the steady state condition cannot be achieved for dense sands in triaxial compression. It is only possible to achieve this state for clean narrowly graded sands that exhibit initially contractive behavior and are strained in excess of 20% to 30%.

Konrad (1990a and 1990b) accepts the notion of "The Steady State of Deformation" but proposes that the steady state line for a given sand is not unique but depends on the initial state of the sand prior to shear. An upper and a lower steady state line are described and the actual final state of the sand at large deformations will end up between one of these two conditions depending on its initial position relative to the two lines. In isotropically consolidated–undrained triaxial compression tests on a subangular dune sand and a well–rounded silica sand, Konrad concluded that the actual location of the SSL, in terms of the mean stress at a constant void ratio, could be determined to \( \pm 50\% \) and \( \pm 63\% \) respectively. All of these tests were conducted on sand with \( +\psi \) parameters.
In tests on Erksak 330/0.7 sand, a uniformly graded medium to find sand, Been et al. (1991) were able to locate the SSL to ± 25% of the mean stress at constant void ratio for mean stresses > 1 MPa. Most of these tests were isotropically consolidated undrained triaxial compression tests with +ψ parameters.

Recent low stress experiments carried out by Hardin (1989) seem to indicate that the ultimate condition of a sand at very low stresses is not independent of the initial density. He designed a special plane strain device which is capable of applying large shear deformations to sands at very low confining stresses. This device was not capable of measuring the stress, but he used a seating load of approximately 0.1 kPa for all tests. He observed that the void ratio at major principal strain levels in excess of 30% was highly dependent on the initial void ratio.

2.1.8 Mechanisms of Deformation in Sand

The mechanics of the deformation of a sand mass is intimately related to the motion of the individual particles comprising the sand. The rigid body motions of these particles are governed by the location and magnitude of the interparticle contact forces. If the resulting deformations do not involve the crushing or fracturing of the individual particles, then the overall deformations experienced by the sand mass can, in principle, be described by force equilibrium equations for each particle.

Rowe (1971) has suggested that the total strain experienced by a deforming sand mass be decomposed into two parts; that due to slip of the individual particles which is irrecoverable and that due to elastic deformation of the individual particles which is recoverable. The slip strains can be due to
either particle crushing or to rigid particle motions. Rigid particle motions may involve both sliding and rolling.

**Crushing**

Slip strains can occur due to the crushing of particles. Crushing observed in laboratory testing is usually quantified by comparison of grain size distributions before and after shearing. The result of crushing is a decrease in volume of the sand during shear and a flattening of the Mohr failure envelope (refer to Section 2.1.9). Bishop (1966) explains that crushing is initially concentrated at the particle contact points. At higher shear stresses and confining pressures, complete particles will ultimately shatter. Crushing of grains is more predominant in sands that have undergone shear deformations as compared to those which have undergone only isotropic consolidation, Lee (1977). Hardin (1985) explains "the amount of particle crushing in a soil element under stress depends on particle size distribution, particle shape, state of effective stress, effective stress path, void ratio, particle hardness, and the presence or absence of water". Only particles larger than silt size, i.e., diameters greater than 0.074 mm, are very susceptible to breakage.

**Rigid Body Motions**

Another component of slip strain is the rigid body motion of particles relative to each other. This causes a geometrical rearrangement of the soil skeleton. It can be accomplished by sliding and rolling. The relative importance of sliding as compared to rolling has been debated repeatedly in the literature. Horne (1965) studied the mechanisms of deformation occurring in an assembly of round, rigid, cohesionless particles. He proposed that sliding at any one instant
is not occurring at the majority of the contacts, but rather that it occurs as the relative motion between instantaneously rigid groups of particles. These groups form and reform continuously by division and coalescence. He stated "Whereas sliding allows contact between any adjacent pair of groups at three points, rolling allows contact at no more than two points. Sliding is thus more stable than rolling, and implies a lower rate of volume increase. If all sliding contacts between two groups had exactly the same orientation, the mechanism would involve translation only with no components of rotation. In practice, such a mechanism might not be possible, and some rotation would then be involved. The above argument shows, however, that the stability of the assembly requires a minimization of rolling in relation to sliding."

Direct observation of the motions of individual particles in a sand mass, which is necessary to confirm the mechanisms of deformation such as proposed by Horne (1965), is difficult. To identify an individual particle in a sand mass and track its motion during shear deformations would be a formidable task. Some researchers have attempted such direct observations of both real and idealized granular media, while others have made indirect observations and still others have used the principles of discrete particle analysis to make theoretical studies of these mechanisms.

In an effort to observe directly the motion of particles in a granular mass during shear deformations, Rowe (1971) reported the use of a direct shear device with transparent sides to study the motion of marked glass beads during shear deformations using a magnifying glass. Precise details of these experiments were not given. The tests were conducted on both dry and water saturated specimens. No rolling was observed except in one case where one of the glass beads was found to move freely without load.
Skinner (1969) conducted a series of direct shear tests on 1 and 3mm diameter glass beads, 3.2mm diameter steel ball bearings and 3 mm diameter lead shot to study the effect of the true physical friction angle ($\phi_{\mu}$) on the peak strength and volumetric behavior. He observed that changes in the $\phi_{\mu}$ resulted in no change of the peak ($\phi_{p}^{'}$) or constant volume ($\phi_{cv}^{'}$) friction angles. He reasoned that at low $\phi_{\mu}$, the deformation occurs primarily by sliding but that at high $\phi_{\mu}$, the mechanism of deformation must change from sliding to rolling in order to balance the energy so that the strength will remain the same. The validity of these conclusions has been debated in the literature, Rowe (1971).

Athanasiou-Grivas and Harr (1980) investigated rigid body motions of spherical glass marbles subjected to triaxial compression loading. The marbles were coated with paint as they sat in the triaxial cell prior to loading. The excess paint was drained off and the marbles were allowed to dry. During loading, a video camera was used to record the motion of the particles on the surface of the specimen. The porosity of these specimens ranged from 30 to 45%. By studying the pattern of paint chipping on the surface of the individual grains, which gave a history of the locations of the interparticle contact forces during deformation, and by studying the video tape of deformations occurring at the surface of the specimens, they were able to conclude that both sliding and rolling of the grains had occurred. They observed that sliding predominated at the upper and lower surfaces in contact with the end platens where the rotation was a minimum. They also observed that rotation was more likely to occur along the curved surface of the specimen rather than inside the specimen.

Oda and Konishi (1974) worked with an idealized granular medium composed of a 2-dimensional array of photoelastic rods in an effort to study deformational mechanisms and stress transferring processes. These rods were
right circular cylinders with radii of 0.3, 0.4, and 0.5 cm and lengths of 1.9 cm. They were all placed together in loose and dense packings. These assemblies were deformed in simple shear. By photographing the ends of the rods at various stages during the imposed shear displacements, both the rigid body motions of the rods and the magnitude and direction of the contact forces acting between adjacent rods were observed. The magnitude and direction of the contact forces were deduced from the photoelastic isochromes. They observed that sliding at the contact points was not occurring at the majority of the contacts at any given instant in time, but was confined to some preferred contacts. They also observed that the contact normals tended to concentrate themselves in the directions of the maximum principal stress axis during shear. They did not report the observation of any rolling type motions. These observations are in agreement with Horne's (1965) proposed deformational mechanisms.

Cundall et al. (1982) studied the mechanisms of deformation and force transfer in granular materials by creating a discrete element model of a 2-dimensional granular medium composed of disk shaped particles of varying diameters in a random arrangement. No attempt was made to study the effect of the degree of packing on the behavior. In terms of stress transferring mechanisms, this analysis showed that forces are not transmitted through a particulate medium in a uniform way, but are concentrated in chains of particles extending between the boundaries. Particles not associated with these force chains may carry little or no load. This observation is in agreement with Gourves (1981) [See Section 2.1.2]. This analysis also indicated that the motion of the individual particles could be described in terms of both sliding and rotation. They stated that "Sliding contacts are almost never observed between particles comprising the major force chains; slip takes place predominantly in the
relatively unloaded regions between the major chains." The contact points that 
slide are ones that are subsequently lost during increased deviatoric loads.

2.1.9 **Mechanisms of Strength in Sand**

The shear strength of a granular material is related to the strength of the 
contact points between the individual particles and the amount of energy 
necessary to rearrange these particles. The nature of the forces that can be 
transmitted particle to particle in a sand was discussed in Section 2.1.4. These 
forces can be both frictional and cohesive. Any mechanism which affects the 
magnitude of these contact forces will also affect the strength of the sand mass.

Mohr–Coulomb Failure Envelope

A convenient method for studying the strength of a sand is by the use of a 
shear stress/normal stress plot and Mohr's circles to represent the state of stress 
at failure. Such a plot depicting the results of a series of drained triaxial 
compression tests on the Sacramento River Sand previously discussed is shown in 
Fig. 2.9. Mohr's circles representing the state of stress at peak strength for 
confining pressures up to 40 kg/cm² are drawn and an envelope tangent to these 
circles is superimposed. This envelope is termed the Mohr–Coulomb failure 
envelope.

The failure envelope in Fig. 2.9 is not linear, but exhibits a decreasing 
slope as the effective stress increases. This means that the peak friction angle 
changes as a function of effective stress. Figure 2.10 is a plot of the principal 
stress ratio \( R = \sigma'_1/\sigma'_3 \) as a function of confining pressure for both the 
initially loose and dense specimens of Sacramento River Sand. See Eq. 2.1.10 to 
convert \( R \) to \( \phi \). The principal stress ratio is shown to decrease with confining
pressures up to approximately 40 kg/cm². In the dense sand the R thereafter is constant up to confining pressures of 140 kg/cm². In the loose sand, the R increases slightly with confining stresses above 40 kg/cm². At $\sigma' = 140$ kg/cm², the peak friction angle for both initially loose and dense sand is approximately the same.

**Effective Stresses and Intergranular Friction**

It is apparent that the strength of sand increases dramatically with effective normal stress. As was discussed in Section 2.1.2, the effective stresses acting on the soil skeleton are directly related to the interparticle contact forces through equilibrium equations. As the effective stress increases, the normal forces acting at contact points increase and as a result the maximum shear force which can be transmitted particle to particle also increases. This results in an increase in strength of the sand mass. Thus an increase in effective normal stress increases the strength of the sand through the mechanism of interparticle contact friction.

**Rowe's Components of Strength**

Rowe (1962) proposed that the peak friction angle can be decomposed into three separate components: interparticle (true physical) friction, particle interference and dilation. In order to assess the relative importance of each of these three mechanisms in contributing to the peak strength of granular materials, Rowe derived an equation by considering the mechanical behavior of uniform rods and uniform spheres in various packing arrangements. By assuming that a random mass of irregular particles behaved in a similar manner to his idealized particulate masses, and that the rate of internal work done by frictional
forces responsible for the strength would be a minimum, he developed the following equation for the peak stress ratio, \( R_p \),

\[
R_p = \left[ \frac{\sigma'_\text{I}}{\sigma'_\text{III}} \right]_p = (1 + \left. \frac{d\sigma'_\text{II}}{d\epsilon_\text{I}} \right) \tan^2(45 + \frac{\phi'_\text{f}}{2}) = (1 + D)R_f
\]

2.1.13

In this equation, \( \sigma'_\text{I} \) and \( \sigma'_\text{III} \) are the major and minor principal effective stresses, \( D = \frac{d\sigma'}{d\epsilon_\text{I}} \) is the rate of dilation, and \( \phi_f \) and \( R_f \) represent the combined frictional-interference components of strength.

Rowe (1971) states that his stress dilatancy relation is valid throughout the entire stress path, i.e., up to the peak strength and also during the strain softening phase as the sand tends towards the critical state. By measuring the effective principal stress ratio and the rate of dilation at any stage in the test, it is possible to compute \( \phi'_\text{f} \) as a function of stress or strain level. Figure 2.11 presents hypothetical results of a series of drained triaxial compression tests on a sand which plots the \( \phi_{\text{peak}}, \phi_\text{I} \) and \( \phi_\text{f} \) according to Rowe's stress dilatancy equation. The difference between \( \phi_p \) and \( \phi_\text{f} \) is the dilatancy component of strength.

The dilatancy component of strength is due to the expansion of the soil skeleton as it "loosens up" to accommodate the imposed deformations. This results in a greater expenditure of energy to accommodate the deformation and thus leads to an increase in strength. This strengthening component is the greatest at very high densities (large \( -\Psi \)) and only operates in sands which exhibit changes in volume. As the density decreases (i.e., as \( \Psi \) increases), the amount of dilation decreases until at low densities the sands begin to exhibit contraction (negative dilation). Thus, the dilatancy component of strength decreases with decreasing relative density.

Bishop (1954) proposed a slightly different equation to account for dilatancy effects (in Lee and Seed 1967). His equation is
\[ R_p = \frac{d\varepsilon_v}{df} + \tan^2(45 + \frac{\phi' f}{2}) = D + R_f \]  

2.1.14

Lee and Seed (1967) corrected their data using both the Rowe (1962) and the Bishop (1954) equations. The Rowe equation always leads to a larger effect of dilation. Therefore, if the sand is dilating (+D and -\(\Psi\)) the predicted \(\phi' f\) is smaller and if the sand is contracting (-D and -\(\Psi\)) the predicted \(\phi' f\) is larger than the Bishop equation.

The friction between particles (\(\phi_\mu\)), which Rowe termed true physical friction, depends on the nature of the mineral forming the sand, the surface properties and the roughness and size of the particles. This component of strength is constant for all \(D_r\), but will decrease with increasing pressure and particle size according to Rowe (1971). He proposed that this component of strength be measured by conducting a direct shear test where particles of the sand are dragged across a solid block of the same material. The amount of force necessary to overcome the frictional resistance of the sand particles against the solid material is measured as a function of the normal force being applied to the sand mass. The true physical friction angle \(\phi_\mu\) of the sand particles is computed from the following relation

\[ \phi = \arctan(T/N) \]  

2.1.15

where \(T\) is the maximum measured shear force and \(N\) is the applied normal force in the direct shear test. Whether or not this method of measuring the true physical friction angle includes both sliding and rolling friction is still controversial (see Section 2.1.8 for a discussion of rolling versus sliding mechanisms).

If the points of contact between adjacent particles behave elastically, the ratio \(T/N\) is not constant. It depends on the magnitude of \(N\) (because the contact area increases as the two thirds power of \(N\) according to the Hertzian
contact equation) and this causes $\phi_\mu$ to decrease with increasing normal stress. If, on the other hand, the points of contact behave plastically, the ratio $T/N$ is constant and $\phi_\mu$ is not pressure dependent. Rowe (1971) explained that "Soil minerals subject to the nominal applied pressure ranges common in practice and testing, 0–7000 kN/m², behave intermediate between the perfectly elastic and perfectly plastic conditions. Since an increase in particle size decreases the number of particles subject to a given average pressure and therefore increases the load per contact, the value of $\phi_\mu$ decreases with both pressure increase and particle size increase". Rowe (1962) reported that the measured true physical friction angle of a fine quartz sand in water (diameter = 0.06mm) was approximately 29°, while that for a coarse quartz sand in water (diameter = 1.0mm) 22°. The writer has been unable to find other data in the literature which substantiate this pressure dependency as explained by Rowe.

The interference component of strength, $\phi_i$, equals the difference between $\phi_\mu$ (measured "true" friction) and $\phi_f$ (computed from Eq. 2.1.13). Rowe (1962) proposed that the interference component is associated with the amount of energy necessary to rearrange the particle assembly. For very dense sand (large $-\Psi$), the peak strength occurs at a low strain that involves very little particle rearrangement. Hence, $\phi_i$ equals zero (i.e., $\phi_f = \phi_\mu$ as per Fig. 2.11). With decreasing density (increasing $\Psi$), the strain to peak strength increases, thus leading to a larger contribution of interference to the total strength. Finally, for all sands at the large strain constant volume condition, $\phi$ reaches its maximum value (i.e., $\phi_i = \phi_{cv} - \phi_\mu$).

Rowe's (1962) hypothesis leads to the condition that the peak strength should occur simultaneously with the maximum rate of dilation, i.e., when the $d\epsilon_v/d\epsilon_1$ is a maximum. This has been substantiated in the literature, e.g., Lambe
and Whitman (1969 pg. 131, 133) and the Lee and Seed (1967) data previously presented.

**Weakening Processes**

There is a large decrease in the peak friction angle with increasing effective confining pressure as demonstrated by the data from Lee and Seed (1967) presented in Fig. 2.10. This trend seems to stabilize at very high confining pressures. Bishop (1966) attributed the decrease in the peak friction angle with confining pressure to the crushing of the particles and suppression of the dilation. At very high stresses where the $\phi_p$ is constant, he expected that the interparticle friction ($\phi_\mu$) will decrease due to the increase in the particle contact forces, but that significant particle crushing also decreases the average grain size and increase the number of interparticle contact points (which then decreases the average force acting on the contact points). These two effects have a tendency to counter balance each other so that at high confining pressures $\phi_p$ is relatively unaffected by further increases in effective stress.

2.1.10 Other Factors Which Affect the Strength and Deformation of Sands

The other factors to be considered include the effects of drainage, anisotropy, intermediate principal stress $\sigma_2$ and rate effects. In this discussion, unless otherwise noted, the shearing is restricted to increasing axial stress in triaxial compression.

**Effect of Drainage**

The strength of a saturated sand mass is strongly influenced by the drainage conditions. In a saturated dense sand with a large $-\Psi$ (i.e., relatively
low $\sigma'_c$) constrained to deform under constant volume (undrained) conditions, the tendency of the skeleton to dilate will cause a decrease in the pore water pressure. This decrease will cause an increase in the effective stresses acting on the soil skeleton. An increase in effective skeletal stresses will increase the fictional strength of the sand by increasing the normal forces at the particle contact points. Under ideal testing conditions, this strengthening effect will continue during undrained shear until one of two conditions occurs. Either the pore water will cavitate or the sand skeleton will cease to tend to dilate. The former can be prevented by application of a sufficiently large back pressure. The latter will occur when the sand reaches the critical state. During this process, the effective stresses often become large enough to crush the individual sand particles.

Under less than ideal conditions, the peak undrained strength in a dense sand might be controlled by the formation of a slip surface due to a redistribution of void ratio in the specimen. If this occurs the specimen is not deforming under constant volume conditions. In the authors opinion, this can lead to a strength much less than the true undrained strength (refer to Section 2.1.5). In some cases a peak strength may not be reached during the course of the test, e.g. curves #5 and #6 in Fig. 2.6.

Thus, in dense sands with large negative $\Psi$ parameters, a strengthening mechanism due to the generation of negative pore pressures will cause the undrained strength to be much larger than the drained shear strength.

A saturated loose sand with a large $+\Psi$ sheared in an undrained test will generate positive pore pressures as the skeleton tends to collapse. The same behavior will occur for dense sand tested at a very high $\sigma'_c$. The generation of positive pore pressures decreases the effective stresses acting on the skeleton,
which results in a decrease in the strength and a significant amount of strain softening, as illustrated by curve #1 and #4 in Fig. 2.5 and 2.6. Thus a loose sand with a large $+$Ψ sheared in an undrained mode will exhibit a lower strength than the same loose sand sheared in a drained test at the same effective consolidation stress.

**Anisotropy**

Sands can exhibit an anisotropic response under applied loads due to the effects of sand structure. This anisotropic response is evidenced by changes in the initial stiffness, changes in the peak strength, changes in the strain to peak and differences in the volumetric behavior or pore pressure behavior. Wong and Arthur (1985) explain that the anisotropy of a sand can be due to either inherent anisotropy or induced anisotropy. Inherent anisotropy is due to the depositional process. Induced anisotropy comes about as a result of shearing when the sand structure is changed as a result of imposed plastic deformations. They explain "that the directional distribution of particle contact tangents alters radically during strain. This alteration is initially a gain in particle contacts with normals approximately parallel to the major principal stress direction and then a loss of particle contacts with normals nearly parallel to the direction of the minor principal stress. The consequences of this systematic alteration are hidden in nearly all stress–strain data. In fact it is this very redistribution of contacts that determines the stress–strain behavior and leads to failure".

Symes et al. (1984) conducted a series of hydrostatically consolidated undrained tests on a medium loose saturated Ham River Sand in a Torsional Shear Hollow Cylinder device. In this device the effect of anisotropy can be studied in terms of the angle ($\delta$) between the depositional direction and the major
principal stress. Tests were run at $\delta = 0^\circ$ (which corresponds to the major principal stress being aligned with the depositional direction), $\delta = 24.5^\circ$ and $\delta = 45^\circ$. They observed that the initial modulus and yielding behavior was very dependent on the direction of the major principal stress. For $\delta = 0^\circ$ the response is much stiffer and the yield point much higher than for $\delta = 45^\circ$. There was also a large difference in $\phi'$ at maximum stress obliquity as a function of $\delta$. The $\phi'$ at maximum obliquity was largest for $\delta = 0^\circ$ and decreased monotonically for $\delta$ approaching $45^\circ$. Post yield strain hardening was observed in all of these tests, but these were terminated before reaching a maximum undrained strength. The final strengths at the end of shear were similar for all $\delta$ angles. However, the reliability of the data from Torsional Shear Hollow Cylinder devices at large strains is questionable.

Wong and Arthur (1985) present the results from a series of drained tests on a dense sand in the Directional Shear Cell (DSC). This device is capable of continuous rotation of the principal stress directions, $\sigma_1$ and $\sigma_3$, and has the capacity to investigate the effects of both inherent and induced anisotropy. By placing the specimen in the DSC so that the depositional direction lies in the $\sigma_1-\sigma_3$ plane, the effect of inherent anisotropy can be studied. As before, the angle between the depositional direction and the major principal stress is defined as $\delta$. Wong and Arthur tested a Leighton Buzzard 20–30 sand placed with a relative density of 90%. They observed marked differences in the stress–strain behavior of sand as a function of $\delta$. In monotonic loading tests, the maximum principal stress ratio (directly related to the friction angle) was found to decrease as $\delta$ increased from 0 to $90^\circ$. A much stiffer response was observed in tests where $\delta = 0^\circ$ and a much softer response with larger strains occurred as $\delta$ approached $90^\circ$. They also observed that the amount of dilation during shear decreased as $\delta$
increased from $0^\circ$ to $90^\circ$.

Induced anisotropy can be studied in the DSC by placing the specimen such that the depositional direction is normal to the $\sigma_1-\sigma_3$ plane. These tests are conducted in two stages. First, the sand is loaded and unloaded with $\sigma_1$ in a fixed direction. The $\sigma_1$ direction is then rotated and the specimen is sheared again. They observed a monotonic decrease in stiffness as the angle between $\sigma_1$ on the first loading and $\sigma_1$ on the second loading, designated $\Delta\Psi$, increased from $0$ to $70^\circ$. They also observed that the $\Delta\Psi = 0^\circ$ tests exhibited a much stiffer response and the $\Delta\Psi = 70^\circ$ exhibited a much softer response when the first loading was carried to higher levels of strain. This shows that the amount of induced anisotropy depends on the level of strain. They also observed that $\Delta\Psi = 0^\circ$ tests exhibited much greater dilation than $\Delta\Psi = 70^\circ$, and that for $\Delta\Psi = 0^\circ$ the amount of dilation increased with the level of strain on the first loading while the opposite effect occurred for $\Delta\Psi = 70^\circ$. These tests were terminated at 3 to 5% axial strain.

Arthur et al. (1981) reported similar results on Leighton Buzzard 20–30 sand tested in a Directional Shear Cell located at M.I.T.. They studied the effects of both inherent and induced anisotropy. For inherent anisotropy they observed that the softest response occurred at a $\delta = 90^\circ$, and for induced anisotropy they observed that the softest response occurred at $\Delta\Psi = 70 \pm 10^\circ$.

All of these experimental testing programs to investigate the anisotropy of sand have illustrated that the "structure" can have a significant influence on the deformation behavior of sand and to a lesser extent the strength behavior. The effects of inherent and induced anisotropy can both be significant.
Effect of Intermediate Principal Stress

The strength and deformation behavior of sand is affected by the magnitude of the intermediate principal stress, $\sigma'_2$. This effect is normally expressed in terms of the $b$ parameter defined as

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3} \quad 2.1.16$$

where $\sigma'_1$, $\sigma'_2$, and $\sigma'_3$ are the major, intermediate and minor principal stresses, respectively. The magnitude of $b$ ranges from $b = 0$ in triaxial compression where $\sigma'_2 = \sigma'_3$, to $b = 1$ in triaxial extension where $\sigma'_1 = \sigma'_2$. Ladd et al. (1977) summarized the results of several testing programs that investigated the effect of $b$ on the drained strength of sands. Figure 2.12 is a plot of peak friction angle as a function of $b$ for several sands. In all cases the peak friction angle increases as $b$ increases from 0 in triaxial compression to 0.2 to 0.4 for plane strain conditions, and is more pronounced in dense than in loose sands. There is, however, some disagreement as to whether the friction angle increases or decreases as $b$ increases beyond plane strain to triaxial extension where $b = 1$. Ladd et al. (1977) state that comparisons of behavior at $b = 1$ are less reliable due to possible experimental problems.

Effect of Strain Rate

The behavior of sand shows slight strain rate effects. Seed and Lundgren (1954) presented the results from a series of triaxial compression loading tests on saturated Monterey Sand and Sacramento River Sand tested at strain rates of $2 \times 10^{-4}$, $2.5 \times 10^{-2}$ and 10/sec and an effective confining pressure of 2ksc. The Sacramento River Sand was a fine sand with 100% passing the #100 sieve and the Monterey Sand was a coarse sand passing the #4 sieve and retained on the #50 sieve. They tested sand densities ranging from loose to dense in both
drained and undrained shearing.

They observed for both the coarse and fine sands in a loose packing tested in undrained shear that there was no change in strength with strain rate. For both sands tested undrained in dense packings, the strengths were unaffected by strain rate in the slow and moderate range, but at the fastest strain rate the strength was 10 to 15% greater. Thus, the effect of strain rate on undrained strength seems to be important only for dense sand at extremely high strain rates, such as those experienced during blast loadings.

Whitman and Healy (1962) reported similar results on the undrained triaxial compression behavior of saturated sand as a function of the rate of loading. They observed that, in a Ottawa 20–30 sand, the peak friction angle changed less than 1° as the time to failure decreased from 300 to 0.005 seconds. Due to changes in the amount of excess pore pressure generation, they observed a 40% increase in the undrained compressive strength of loose sand as the time to failure decreased from 5 to 0.025 seconds.

These results indicate that rate effects in sands are insignificant in the range of strain rates used for this current testing program when comparing the undrained behavior.

2.1.11 Summary of Sand Behavior

In the Unified Soil Classification System, sands are defined as those particles with diameters between 4.76 mm and 0.074 mm. The mineral composition of the sand generally does not significantly influence sand behavior. Two notable exceptions are carbonitic sands and calcitic loess deposits. Sands are always classified according to their particle size distribution (gradation) and may be classified according to their roundness and shape (Fig. 2.1). Particle size
and shape affect the types of packing arrangements that are possible.

The void ratio ($e$) and relative density ($D_r$) give indications of the type of packing of the sand particles. Various procedures have been proposed to measure the maximum and minimum void ratio. The diversity of these procedures and the resulting variations in $e_{\text{max}}$ and $e_{\text{min}}$ can cause variations in the measured $D_r$. $D_r$ should not be viewed as an absolute measure. Reimer et al. (1990) suggest that $e_{\text{max}}$ be determined as the void ratio corresponding to the loosest possible density achievable by any method without bulking due to capillary tension. They suggest that $e_{\text{min}}$ be determined as the void ratio corresponding to the maximum density achievable by any method in the absence of significant particle crushing.

The structure of a sand is characterized by two components: the fabric, which refers to both the distribution and orientation of the particles; and the magnitude and direction of the contact forces. The direction of the contact normals generally has more influence on the behavior of sands than particle orientation.

Normal and shear stresses are transferred through dry soil masses through points of contact between grains. Some particles may not be actively participating in the stress transferring process. The contact stresses at contact junctions are large enough to cause local yielding on the mineral surface. Normal and shear stresses acting on the soil skeleton are defined as average stresses over a large area in order to smooth out the local high variations in contact stresses at the particle level (see Figure 2.2). These stresses are related directly to the contact forces acting between the particles and thus represent a state of equilibrium of particle contact forces. This may not be the case in frozen sands where the particle contact forces may not be in equilibrium.
For saturated sands, the principle of effective stress as presented by Terzaghi (1925) provides a partitioning of total normal stress \( (\sigma) \) between those being carried by the sand skeleton \( (\sigma') \) and those being carried by the pore water \( (u) \). The effective stress principle can be expressed by the equation

\[
\sigma = \sigma' + u
\]

The legitimacy of Terzaghi’s effective stress equation lies in the very small area of contact that exists between sand particles. In undrained conditions, an increment of isotropic total stress applied to a saturated sand will be transferred almost entirely to the pore water pressure.

The forces transmitted between sand particles can exhibit both frictional and cohesive behavior. Frictional behavior can be explained in terms of the Adhesion Theory of Friction, where the maximum shear force which can be transmitted between two particles is directly related to the normal force acting at the point of contact through the coefficient of friction of the material. The validity of the Adhesional Theory of Friction does not depend on the actual character of the adhesional bond at the contact point. Cohesive behavior of sand can be caused by the deposition of silica, carbonate minerals, or other cementing agents between adjacent particles at points of contact.

The stress–strain–strength behavior of sand depends both on its \( D_r \) and preshear consolidation stress \( (\sigma' c) \). Quantitatively these two variables have very similar effects. Drained triaxial compression tests on sands show that an increase in confinement or a decrease in \( D_r \) leads to lower peak friction angles, larger failure strains, the suppression of dilation (or enhancement of contraction) and less post peak strain softening. Dense sands at high confining pressures respond similarly to loose sands at low confining pressures (Fig. 2.4).

Been and Jefferies (1985) introduced the state parameter \( \Psi \) to account for
the combined effect of $\sigma'_{c}$ and $D_r$ on stress–strain behavior (Fig. 2.7). By referencing the initial state (initial void ratio and mean effective stress) to the ultimate "steady state" (critical state) line, they propose that the general nature of the stress–strain behavior and certain soil parameters (e.g., drained peak friction angle and maximum rate of dilation) are uniquely related to the magnitude of the state parameter $\Psi$. A negative state parameter predicts a dilative type behavior and a positive state parameter predicts a contractive type behavior. Figure 2.8 illustrated "unique" normalized behavior from isotropically consolidated undrained compression (CIUC) tests on a sand at varying $D_r$ and $\sigma'_{c}$. However, the universality of this approach has not been clearly demonstrated in the literature. Various experimental difficulties in the testing of sands must be overcome, such as the ability to measure the true strength of sands with a large negative state parameter tested in undrained compression. Attempts to compensate for membrane compliance effects and nonuniform specimen deformations show promise in this regard. In addition, the definition of the critical state (ultimate) condition in sands remains controversial. All researchers agree that an ultimate condition exists, but they disagree about the structure in this ultimate condition and whether or not it is unique and independent of the initial conditions. Part of the disagreement may stem from the difficulties associated with the triaxial testing of sands at large strains.

Sands deform by various mechanisms. These include the elastic and plastic straining of individual particles at points of contact, crushing of particles and the rearranging of particles within the sand mass. The rearranging of the sand particles may be accomplished by both sliding and rolling motions. There is debate as the relative importance of sliding versus rolling although most researchers believe that sliding is predominant. Some researchers postulate that
deformations occur due to the relative motions of instantaneously "rigid groups" of particles, such that slip (and rolling) occurs at a relatively few locations at any instant in time and are not proceeding generally throughout the granular mass. Discrete element modeling of idealized granular media support this hypothesis that deformations in granular materials are accommodated by the rigid body motion of groups of particles which continuously group and regroup during shear.

Rowe (1962) proposed that the drained strength of a sand can be divided into three components: that due to sliding friction ($\phi_\mu$) between particles, that due to interference of particles as they move around each other during large deformations ($\phi_i$), and that due to the dilation of the skeleton ($\phi_d$) (Fig. 2.11). Rowe (1971) states that sand grain contacts behave intermediate between perfectly elastic and perfectly plastic conditions so that the sliding coefficient of friction decreases somewhat with an increase in the load per contact. Hence the value of the $\phi_\mu$ should decrease with increases in the particle size and increase in the effective stress level. However, definitive data on these trends appear to be lacking. In a dense sand sheared at a low confining pressure ($-\psi$), the peak strength is reached at small strains and the peak friction angle is due to dilatancy and interparticle friction without any particle interference. Sand specimens sheared to large strains (i.e., that reach the critical state condition) have the same constant volume friction angle ($\phi_{cv} = \phi_\mu + \phi_i$), independent of the initial density. For a loose sand ($+\psi$), the peak friction angle equals $\phi_{cv} (=\phi_\mu + \phi_i$). At intermediate $\psi$ parameters, the peak strength is due to dilatancy, interference and sliding friction.

The Mohr failure envelope for a sand is not linear, but exhibits a decreasing slope at low to moderate confining stress and tending to a constant value at high confining pressures (Fig. 2.10). This is caused mainly by the
suppression of dilation, and at high stresses is also accompanied by particulate crushing and increases in the number of grain to grain contact points.

The drainage conditions strongly affect the strength of sand. In a sand with a large $-\Psi$, if drainage is prevented during shear, pore water pressures will decrease and the effective stresses acting on the skeleton will increase, resulting in an increase in strength over that of the same sand sheared drained. Frequently the peak strength under such conditions is not achieved using conventional testing methods (Fig. 2.6). Sands with a large $+\Psi$ sheared undrained will exhibit softening behavior as positive pore pressures are generated and effective skeletal stresses decrease (Fig. 2.5 and 2.6).

Sands exhibit anisotropic response to applied loads. This anisotropy can be inherent as a result of the depositional process or induced as a result of prior straining. Generally the stiffness and strength is much greater if the major principal stress is oriented in the direction of deposition or in the direction of the major principal stress of prior loadings.

The magnitude of the intermediate principal stress influences the behavior of sand. Strengths in plane strain are higher than those in triaxial compression, especially for dilative specimens.

The strength of sand is insensitive to the rate of loading in the range of strain rates being investigated in this research.
2.2 POLYCRYSTALLINE ICE

2.2.1 Description of Isotropic Polycrystalline Ice Ih

Ice Ih, normal hexagonal ice, is the stable form of solid water at temperatures to approximately \(-110^\circ C\) and pressures up to 200 MPa. It is normally encountered at temperatures very close to its melting point. The homologous temperature of ice can typically be in the range of 0.9 to 0.95 or greater. The homologous temperature is the actual temperature divided by the melting temperature expressed in absolute temperature units.

The basic building structure of ice Ih is a tetrahedron formed by the five oxygen atoms of five water molecules. These oxygen atoms arrange themselves at the vertices and geometric center of the tetrahedron. The interatomic spacing of the oxygen atoms is approximately 2.76 Å. See Fig. 2.13. The basic tetrahedron combines with two others at three coplanar vertices to form a wrinkled hexagonal ring of oxygen atoms with three atoms located at the common vertices and three oxygen atoms located at the geometric center of each tetrahedron. See Fig. 2.14. The hexagonal ring of oxygen atoms is in a slightly distorted plane due to the fact that the oxygens at the vertices and the oxygens at the centers are not all coplanar. This distorted plane forms the basal plane. The axis normal to the basal plane is referred to as the c-axis. The density of ice in this arrangement is 0.917 Mg/m\(^3\) at 0 \(^\circ\)C (Hobbs 1974). The oxygen atoms are packed more densely along the basal plane than they are along the c-axis direction. This forms a naturally anisotropic structure. See Fig. 2.15. Crystallographic slip along the basal plane is referred to as easy glide and crystallographic slip in other directions is referred to as hard glide. The yield stress of a single ice crystal is approximately 0.4 MPa in easy glide at a strain
rate of $2.7 \times 10^{-6}$/sec and temperature of $-15^\circ$C, while at a temperature of $-19^\circ$C and a strain rate of $3 \times 10^{-6}$/sec is approximately 9 MPa in hard glide (Higashi 1967). This is greater than a 20 fold increase in yield stress from easy glide to hard glide.

The hydrogen atoms for each water molecule in the ice crystal lattice are located such that each hydrogen is normally shared by the two oxygens in adjacent molecules. This forms the hydrogen bond which holds the water molecules in the lattice. In considering the location of the protons in the lattice, Goodman et al. (1981) explain "The proton lattice is disordered, even at 0*K, so that the selection of the occupied site on a bond is random". Occasionally this random disorder of protons between adjacent oxygen atoms results in an ionization defect where either two protons or no protons occupy the position between adjacent oxygens. These are referred to as Bjerrum defects. A Bjerrum D—defect is when two protons are located between adjacent oxygens and a Bjerrum L—defect is when no protons are located between adjacent oxygens. Various researchers have proposed models to explain how the presence of these ionization defects might be responsible for creating drag forces which affect the velocity of dislocations along the basal plane.

If ions are present in the water during the freezing process, these can be substituted into the crystal at lattice positions, be trapped in the lattice at interstitial sites, or concentrated at grain boundaries. Ions can also diffuse into the lattice after it has been formed. The presence of these ions can dramatically affect the mechanical behavior of ice by influencing the defect population and hence the mobility of dislocations. This will also be discussed in Section 2.2.4. Gases can also be accommodated into the ice crystal lattice if they are present during crystallization. They can form clathrate hydrates (Hobbs 1974). The
water molecules arrange themselves around the gas molecules in a structure that differs from ice Ih and as a result the mechanical properties can change. Jones (1982) suggested that air clathrates were formed in his isotropic ice specimens at confining pressures in excess of 10.5 MPa from small air voids that had been present in the ice when it was formed. He stated that "If the mechanical properties of this air clathrate had been very different from those of ice, we might have seen a significant change in the yield stress at a confining pressure of about 10 MN/m². However, no such change was noticed...from which we conclude that a small amount of air clathrate does not significantly affect the flow of ice".

Ting et al. (1983) postulated that the ice in the pores of a frozen sand is polycrystalline ice. This has been the subject of some debate in the literature and will be treated in Section 2.3.2. Current work being carried out at CRREL on thin sectioning of frozen sands seems to support this idea (Cole 1990 personal communication). In isotropic equiaxed polycrystalline ice, the c–axis of the individual grains (i.e., the individual crystals) is randomly oriented and the grain size is uniform. This type of ice is also referred to as granular ice in the literature.

One probable difference between the ice existing in the pores of frozen sand and the granular ice commonly encountered in nature is the grain size. Ice crystals in nature can have grain diameters which range from just under 1 mm to 10's of cm or larger. The grain sizes of granular ice which have been routinely tested in the laboratory range from just under 1 mm to 10 mm. Assuming that the ice is polycrystalline the maximum grain size is possibly limited to the maximum pore size between adjacent sand grains. In order to estimate the maximum ice grain diameter which probably exists in a frozen Manchester Fine Sand, Martin et al. (1981) assumed that the sand grains were equisized spherical
particles in regular packings. They estimated that the maximum grain diameter would range from 0.031 mm to 0.083 mm, or more than one order of magnitude smaller than normally tested in the laboratory.

Mellor (1980) discusses the structure of a polycrystalline ice in terms of its texture and fabric. The texture refers to the shape and assembly of the grains and the fabric refers to the orientation of the crystallographic axis in the constituent crystals. The texture of a polycrystalline ice can evolve with time and the fabric can evolve under different loading conditions as the grains change shape, the boundaries between the grains migrate and recrystallization takes place [Glen (1963); Mellor and Testa (1969a); Wilson and Russell—Head (1982); Jacka (1984); Cole (1986, 1987)]. Under a given applied stress state there is a tendency for the crystals to reorient themselves into positions which favor deformation in easy glide. Some researchers have observed that under a given set of loading conditions a fabric can evolve which will be dependent only on the temperature and applied stress and will be independent of the initial grain size (Jacka 1984 and Cole 1987). Changes in texture can occur during periods of storage (Cole 1986). Because the material response of polycrystalline ice is extremely dependent on both its texture and fabric, it is necessary to fully characterize these both initially and as they evolve during the deformation process. This topic will be discussed in more detail in Section 2.2.3.

The grain boundaries in polycrystalline ice represent a significant feature of the structure. These zones represent areas where the orderly arrangement of water molecules breaks down. Grain boundaries serve to transfer forces between adjacent crystals and present a mechanism by which stronger grains, i.e., those oriented in hard glide directions, can reinforce weaker grains, i.e., those oriented in easy glide directions. In addition to the boundaries between individual
crystals, small subgrain boundaries can form during the course of deformation inside individual grains that are experiencing bending moments (Gold 1963).

2.2.2 Some Aspects of the Mechanical Behavior of Polycrystalline Ice

General Stress–Strain Behavior

Polycrystalline ice can exhibit a wide range of behaviors from ductile flow to depending on the applied stress and strain rate, temperature, confining pressure, texture and fabric and presence of impurities. Uniaxial compression and uniaxial tension tests have been used most frequently to study ice behavior.

In these tests either the axial stress is maintained constant during the deformation (termed a creep test) or the axial strain rate is kept constant (termed a strength test). Creep tests are usually conducted at low stresses in the flow regime and the strength tests are conducted at much higher strain rates. At low strain rates, ice exhibits ductile behavior whereas at high strain rates ice exhibits brittle behavior due to cracking and fracture.

Correspondence Principle

Researchers have noted the probability of a correspondence between creep and strength tests in the form of a limiting stress/strain rate relationship. Figure 2.16 is a summary plot of idealized creep and strength curves to illustrate this correspondence. Figure 2.16 (a) plots idealized creep curves in $e$–$t$ space from two compression tests. In these tests, a constant axial stress is applied to the specimen and the resulting deformation (strain) is measured over time. This strain–time behavior can be decomposed into four parts: an initial elastic deformation, a region of decelerating creep rate called "primary creep", a
transition region between decelerating and accelerating creep called "secondary creep" and finally a region of accelerating creep called "tertiary creep". The minimum slope of the $\varepsilon$–t curve in the secondary creep region is referred to as the minimum creep rate.

Figure 2.16 (b) plots of several idealized stress–strain curves from constant strain rate compression tests. These curves illustrate four types of behaviors as a function of the applied strain rate (other factors such as grain size can cause similar changes in behavior). These are: brittle fracture in the high strain rate region (10^{-2}/sec), a peak strength followed by significant strain softening in the moderate to high strain rate region (10^{-3}/sec), a peak strength followed by a slight amount of strain softening in the moderate to low strain rate region (10^{-4} to 10^{-5}/sec) and finally continuous strain hardening in the low strain rate region ($\dot{\varepsilon}$<10^{-4}/sec). Brittle fracture is thought to dominate the deformational mechanisms at the very fast strain rates, microcracking processes dominate at the moderate to fast strain rates, flow mechanisms dominate at the moderate to low strain rates and recrystallization dominates at very low strain rates.

The limiting stress/strain rate correspondence seeks to unify the behaviors just outlined by assuming that the relationship between the $\sigma_c/\dot{\varepsilon}_{\text{min}}$ in a creep test is identical to the $\sigma_p/\dot{\varepsilon}$ in a strength test. If the results of both creep and strength tests are plotted on a log$\sigma$–log$\dot{\varepsilon}$ plot in terms of the peak or applied stress and the minimum or applied strain rate, these will fall along the same curve if the behavioral mechanisms are identical. The region of power law creep is defined as that portion of the log$\sigma$–log$\dot{\varepsilon}$ where the correspondence can be described by a simple power law equation of the form

$$\dot{\varepsilon} = b\sigma^n$$ 2.2.1
where \( \dot{\varepsilon} \) is the strain rate, \( \sigma \) is the stress, \( b \) is a constant and \( n \) is the power law coefficient. The power law coefficient ranges from 3 to 4.5 in the range of strain rates of general interest to engineers \((10^{-7} \text{ to } 10^{-4}/\text{sec})\). At lower strain rates, the power law coefficient tends to decrease to near 1.0 and at very high strain rates, the peak strength tends to become strain rate insensitive. This region is often referred to as power-law breakdown.

In discussing the possibility of using either creep or strength tests to study the fundamental behavior of ice, Mellor and Cole (1982) stated "If the constitutive equations and failure criteria are properly formulated, the constants which express material properties ought to be the same for any type of loading condition, and therefore they should be measurable from a variety of tests. More explicitly, ice should deform and fail in essentially the same way, irrespective of whether it is subject to constant stress or constant strain-rate.... However, there is a complication in that the material properties actually change with strain and with time....". Thus, if the behavioral regimes are different and the material properties are changing with time, creep tests and strength tests will give different results. Clearly this would be the case if the results from a creep test at very low stress levels where recrystallization processes are dominant (see Fig. 2.16 (a)) were compared to the results of a strength test at very high strain rates where brittle fracture was the dominant mechanism.

The writer has chosen to focus this discussion of the general stress-strain behavior on the results from constant strain rate strength tests as these are most germane to the current testing program.

**Stress–Strain Behavior in Tension Versus Compression**

Hawkes and Mellor (1972) performed a series of constant rate of strain
tensile tests on fine grained polycrystalline ice specimens at a temperature of 
\(-7\pm1^\circ\text{C}\). They observed roughly the same stress–strain behavior over strain rates 
ranging from \(9\times10^{-7}/\text{sec}\) to \(1/\text{sec}\). Figure 2.17 is a stress–strain plot from two of 
their tests. The fastest test exhibited essentially a linear response to the peak 
strength. The slowest test exhibited a significant amount of yielding, followed by 
continuous strain hardening to a peak strength. These specimens gave no visible 
signs of internal cracking prior to failure and they all failed by fracture and 
separation of the specimen without exhibiting any strain softening.

Figure 2.18 presents an idealized stress–strain curve from a constant 
strain rate test on a polycrystalline ice in uniaxial compression as presented by 
Mellor and Cole (1982). This behavior is significantly different from the tensile 
behavior of Hawkes and Mellor (1972). Three significant regions are identified on 
this idealized curve: a linear behavior to an initial yield point, which occurs at 
very low strains; a period of strain hardening to a peak stress or maximum 
deviator stress; and post peak strain softening to an ultimate strength which 
occurs at large strains. The initial yield point represents the onset of significant 
internal cracking. The actual shape of the stress–strain curve will deviate from 
this idealized version under changes in temperature, strain rate, confining 
pressure or grain size and these will be discussed subsequently in this review.

**Small Strain Behavior**

The most reliable method of measuring the small strain properties of ice is 
by the use of ultrasonic pulse–echo and pulse interference techniques. Dantl 
(1969) reported the results of such measurements on the elastic properties of 
single ice crystals. The frequencies which he used ranged from 5–190 MHz and 
the temperature ranged from 0 to \(-140^\circ\text{C}\). Over this range of frequencies, Dantl
observed no frequency dependence on the elastic properties. Sinha (1989) used Dantl’s results along with an averaging technique in order to compute the Young’s modulus, shear modulus and Poisson’s ratio of a granular ice with a random orientation of the c-axis for a temperature range of 0 to −50°C. Figure 2.19 plots these elastic properties as a function of temperature. All three quantities exhibit a tendency to increase with decreasing temperature. At a temperature of −10°C, the Young’s modulus for granular ice is 9.05 GPa, the shear modulus is 3.46 GPa and the Poisson’s ratio is 0.309. Sinha’s empirical equations for estimating these quantities as a function of temperature are also included on the plots.

Effect of Strain Rate

Hawkes and Mellor (1972) conducted a series of uniaxial tension and compression strength tests to study the behavior of polycrystalline ice over a wide range of strain rates. They tested a fine-grained isotropic polycrystalline ice of approximately 0.7 mm diameter at a temperature of −7°C and a strain rate from 10⁻⁵ to 10⁻²/sec in compression and from 10⁻⁵ to 1/sec in tension. In the compression tests they observed an initial yield point or knee in the stress–strain curve and noted that it corresponded to the onset of internal cracking. This initial yield was followed by strain hardening to a peak strength. The peak compressive strength increased with strain rate. There was quite a bit of scatter in the data at the highest strain rates. At the highest strain rates the initial yield knee disappeared. They also observed that the peak strength in tension was insensitive to the strain rate over the range of strain rates investigated. By combining their results with prior creep data and assuming a correspondence based on the limiting stress/strain rate relation, they constructed a plot of axial
stress versus axial strain rate for strain rates from $10^{-11}$ to $1/sec$ (Fig. 2.20). They noted that at a strain rate of $10^{-6}$/sec, the strength in compression and tension were the same. They assumed that at strain rates below $10^{-6}$/sec, the power law coefficient (slope of the line in loge-loga space) was the same for tension and compression.

Mellor and Cole (1982) reported the results of a series of uniaxial compression creep and strength tests on fine-grained isotropic polycrystalline ice with a mean grain diameter of 1.2 mm at a temperature of $-5^\circ C$. The strength tests were performed with strain rates in the range of $10^{-7}$ to $10^{-3}$/sec. Figure 2.21 presents a summary plot of several stress-strain curves for various strain rates. In the tests conducted between $10^{-6}$ to $10^{-5}$/sec, the stress-strain curves were similar to the idealized curve shown in Fig. 2.18. As the strain rate was lowered to $10^{-7}$/sec, they observed that the amount of strain hardening after the initial yield point began to decrease until eventually, at the lowest strain rate, the initial yield point was the peak strength. These slow strain rate tests exhibited some strain softening after the peak strength, but the ultimate strength was very close to the peak (e.g., test #117). In the tests at higher strain rates, ($>10^{-8} - 10^{-7}$), the amount of stress drop after the initial yield decreased and the amount of subsequent strain hardening after the initial yield also decreased. Eventually there was no strain hardening, but only post peak strain softening (e.g., test #16). Thus, at either extremely high or extremely low strain rates, the initial yield point and the maximum strength occur at the same strain. However, the degree of post peak strain softening becomes much more pronounced at the faster strain rates.

The magnitude of the peak strength was strongly dependent on the strain rate. At $10^{-7}$/sec the peak strength was approximately 0.7 MPa and at $10^{-3}$/sec
the peak strength was approximately 10 MPa. In the range of strain rates from $10^{-7}$ to $10^{-4}$/sec, the axial strain at the peak strength was approximately 1% and appeared to be unaffected by strain rate. At strain rates above $10^{-4}$/sec, the axial strain at peak strength began to decrease to approximately 0.5%. The magnitude of the ultimate strength was also influenced by strain rate. At $10^{-7}$/sec the ultimate strength was approximately 0.5 MPa, while at $10^{-3}$/sec the ultimate strength was approximately 3.5 MPa.

These testing programs demonstrate that the magnitude of the initial yield stress, the peak stress and the ultimate strength are extremely dependent on the applied strain rate. As the strain rate increases, the initial yield stress, the peak strength and the ultimate strength all increase. At very high strain rates the difference between the initial yield stress and the peak strength decreases, until the initial yield stress eventually becomes the maximum strength and this is followed by significant amounts of strain softening. At very low strain rates the amount of strain hardening after the initial yield stress decreases until again the initial yield stress becomes the maximum strength with little variation in the stress after the initial yield.

The flow curve for ice is not linear. If creep and strength data are plotted on a log–log plot of stress versus strain rate for a wide range of strain rates, as illustrated in Fig. 2.20, the relationship between stress and strain rate is more complicated than a simple power law formulation. In discussing this point, Mellor (1980) stated "At high stresses, the log–log plot takes on curvature in the sense that $n$ increases with increasing stress. At low stresses, the curvature appears to be in the sense that $n$ decreases with decreasing stress...". In the range of 0.2 to 2 MPa (2 to 20 bar) and at temperatures above $-20^\circ$C, the power law coefficient appears to be between 3 and 4 (Mellor 1980). At very low stresses
or strain rates, the power law coefficient can approach $n = 1$. At very high strain rates or stresses, the power law coefficient can increase to infinity (i.e., the strength no longer increases with strain rate) or become negative (i.e., the strength decreases with increasing strain rate). Increases in the power law coefficient at high strain rates means that the ice strength becomes less strain rate dependent (e.g., Cole 1987). This deformational regime is referred to as power law breakdown or as the transition zone between ductile and brittle behavior.

**Effect of Temperature**

Mellor and Testa (1969b) investigated the effect of temperature on the behavior of isotropic polycrystalline ice by conducting a series of creep tests on specimens of approximately 1mm grain size at temperatures ranging from 0°C to −60°C and an axial stress of 12 ksc. The tests at 0°C were performed by surrounding the specimen and loading platens with cans of wet crushed ice. The authors observed a decrease in the minimum strain rate with a decrease in temperature. At 0°C the minimum strain rate was $4 \times 10^{-6}$/sec decreasing to approximately $10^{-10}$/sec at −60°C. A decrease in minimum strain rate in a constant stress creep test corresponds to an increase in strength in a constant strain rate test. Thus a decrease in temperature led to an increase in strength.

In discussing the effect of temperature on the behavior of ice, Mellor (1980) stated "The effect of temperature on the creep of ice has usually been described in terms of activation energy $Q$, i.e., it has been assumed that creep is a thermally activated process such that the $\dot{\varepsilon}$ varies with absolute temperature $T$ according to the Arrhenius equation $\dot{\varepsilon} = A\exp(-Q/RT)$". If this is true, then a plot of the log of the strain rate versus $1/T$ would generate a straight line.
indicating a constant activation energy. If the activation energy is constant, this is consistent with a similar deformational mechanism for all temperatures.

Figure 2.22 plots the $\log e$ versus $1/T$ for the Mellor and Testa (1969) data on both monocrystals and polycrystals. The plot indicates that in the range of temperatures from $-10^\circ C$ to $-60^\circ C$, the temperature dependency could well be described by the Arrhenius equation. Above $-10^\circ C$, the curve becomes extremely nonlinear and hence the activation energy changes.

Weertman (1983) reviewed prior data on measured activation energies in polycrystalline ice. In the range of temperatures less than $-10^\circ C$, the activation energies are generally in the range of 55 to 85 kJ/mole. Above $-10^\circ C$, there is a wide range in activation energies ranging from 122 to 200 kJ/mole. Thus for temperatures less than $-10^\circ C$, the temperature dependency of polycrystalline ice behavior can be described by the Arrhenius equation with a constant activation energy and this is consistent with a similar deformational mechanism for this range. For temperatures warmer than $-10^\circ C$, the Arrhenius equation breaks down because of changes in the activation energy and this indicates that the deformational mechanisms are changing.

**Combined Effects of Strain Rate and Temperature**

The combined effects of strain rate and temperature, in terms of the minimum creep rate or peak strength, can be estimated by a combination of the power law and the Arrhenius equation as suggested by Glen (1955). So long as these estimates are made within the range of applicability for the power law coefficient and the activation energy of the Arrhenius equation, they can be reasonable. The form of the combined equation is

$$\sigma = V_0 \exp\left(\frac{Q}{RT}\right)\varepsilon^{1/n}$$

2.2.2
where $V_o$ is a constant equal to $6.59 \times 10^{-3}$ MPa, $Q$ is the activation energy for ice ($67 \times 10^3$ J/mole), $R$ is the universal gas constant ($8.31$ J/mol °K), $T$ is the absolute temperature, $\dot{\varepsilon}$ is the strain rate (1/sec), $n$ is the power law coefficient (ranges from 3 to 4.5 in power law region) and $\sigma$ is the predicted peak strength (MPa). This equation is not a complete constitutive relationship, but rather it describes only the peak point on the stress-strain curve. However, it does quantify the relationship between temperature and strain rate as long as the deformational regime is not changing.

Recently, Ashby and Duval (1985) have proposed three dimensionless variables which can be used to normalize the results of constant stress creep data. Hence, for a given structure and grain size, a single curve can describe the creep response (in terms of pairs of these variables) for a wide range in temperatures and creep stresses. The normalization for strain and strain rate and time used the elastic strain ($\varepsilon_e$), the minimum creep rate ($\dot{\varepsilon}_{\text{min}}$) and the time to reach $\varepsilon_e$ at $\dot{\varepsilon} = \varepsilon_{\text{min}}$, respectively.

**Effect of Confining Pressure**

Jones (1978, 1982) reported the results from a series of triaxial compression tests with constant rate of deformation on isotropic equiaxed (i.e., grains have approximately the same size) polycrystalline ice of approximately 1mm grain size to study the effects of confining pressure. The confining pressure was varied from 0 to 85 MPa. The temperature was $-12 \pm 1^\circ$C and the strain rate was varied from $10^{-8}$ to $4 \times 10^{-2}$/sec. Figure 2.23 presents these results. In the range of confining pressures from 0 to 10 MPa, he observed an increase in strength with confining pressure for strain rates greater than $10^{-5}$/sec. Below this strain rate, the confining pressure had little effect.
Jones (1982) also observed that the beneficial effect of confining pressure was lost at higher confining pressures (i.e., at confining pressures greater than 10 to 40 MPa, further increases in confining pressure led to decreases in the peak deviator stress). Section 2.2.4 explains the weakening mechanism to which Jones attributed this loss in strength.

**Effect of Texture and Fabric**

**Texture**

Cole (1987) reported the results of an extensive series of uniaxial compression creep tests designed to study the effect of grain size on the behavior of isotropic equiaxed polycrystalline ice. The grain size was varied from 1.5 to 6.0 mm, the applied axial stress was 2.0 MPa and the testing temperature was −5°C. He observed that varying the grain size over this range significantly changed the character of the deformation. The finest grained specimens exhibited no internal cracking and had larger strains to the minimum creep rate than the coarsest grained specimens which exhibited severe cracking and much smaller strains to minimum creep rate. For a grain diameter of 1.5 mm, the strain to minimum strain rate was 0.01 and this decreased to approximately 0.004 at a grain diameter of 6 mm. Cole observed some subtle trends in the minimum creep rate with grain size, but concluded that from these tests the minimum creep rate was not clearly affected by grain size. The lack of sensitivity to grain size could be due to the formation of a texture and fabric during the straining which is independent of the initial texture and fabric in a manner similar to that observed by Jacka (1984). The minimum creep rate for all of Cole's tests ranged from $1 \times 10^{-6}$ to $6 \times 10^{-6}$/sec.

Schulson and Cannon (1984) reported the results from a series of
unconfined compression tests at \(-10^\circ C\) on isotropic equiaxed polycrystalline ice with grain sizes ranging from 0.8 to 9 mm and constant strain rates varying from \(10^{-6}/\text{sec}\) to \(10^{-3}/\text{sec}\). They observed a decrease in peak stress with an increase in grain diameter for all strain rates. The specimens tested at the fastest strain rates exhibited the greatest decrease in strength as summarized in Table 2.1. Schulson (1987) presents similar results of tests on isotropic polycrystalline ice in uniaxial tension and compression at constant rates of deformation. These all indicate a decrease in strength with increase in grain size. The authors also observed that the strain at peak strength decreased with increasing grain size. At a strain rate of \(10^{-4}/\text{sec}\), the strain was approximately 0.012 for a grain diameter of 1 mm and this decreased to approximately 0.002 for a grain diameter of 9 mm.

The writer notes the apparent contradiction between the Cole (1987) data and the Schulson and Cannon (1984) data with respect to the grain size dependency on the peak strength and minimum strain rate, i.e., if these two points correspond to the same behavior, then the grain size dependency should be identical. This contradiction might be explained in terms of differences in the deformational regimes between the two testing programs. The Schulson and Cannon data was obtained from strength tests at much higher strain rates and lower temperatures (\(-20^\circ C\)) than the Cole creep data (\(-5^\circ C\)). If the Cole data were affected by the development of a texture and fabric which was independent of the initial conditions, as was alluded to by the writer, and this were a time dependent process, it might be possible that the shorter duration tests of Schulson and Cannon did not have sufficient time to develop such a "changed" structure. Hence, their data might more clearly reflect the grain size effect on the peak strength while Cole's data may not. The writer also notes that in
virtually all tests where the grain size effects are observed, microcracking occurs as well.

Fabric (Orientation of the c-axis)

Duval et al. (1983) made a comparison with some of the reported creep data on polycrystalline ice at $-10^\circ$C and the creep behavior of single ice crystals in basal and nonbasal glide. A summary of the creep stress versus minimum strain rate on a log–log plot with this comparison is presented as Fig. 2.24. This indicates that for single ice crystals in basal glide, the power law coefficient was approximately 2, while that for nonbasal glide and polycrystalline ice was approximately 3. This figure also indicates that for a given level of creep stress, the creep rate of polycrystalline ice is intermediate between that of a single ice crystal in basal glide and that of a single ice crystal in hard glide. For a given creep stress, the creep rate of a single crystal in hard glide can be as much as $10^3$ times lower than a single crystal in easy glide (or for a given creep rate, nonbasal slip requires a level of stress that is at least 60 times greater than that for basal slip).

Glen and Jones (1967) and Jones and Glen (1969a) reported results from a series of constant stress creep tests on single crystals of pure ice. In tensile tests performed at $-50^\circ$C where slip was allowed along the basal plane, they observed a continuously increasing creep rate over time for applied stresses ranging from 0.8 to 1.6 MPa (i.e., they observed continuous strain softening type behavior).

Similar results have been presented by Higashi (1967, 1969). He described results from a series of constant strain rate tension tests on single ice crystals tested at $-19^\circ$C and a strain rate of $3\times10^{-6}$/sec and constrained to deform in both basal and non–basal glide. Two of these stress–strain curves are reproduced in
Fig. 2.25. Note that the tensile stress scale for basal glide is one order of magnitude smaller than for non-basal glide. The shapes of the stress-strain curves are also dissimilar. In basal glide, the stress builds up rapidly to a magnitude of just over 1 MPa and then falls off rapidly to only about 0.1 MPa. In non-basal glide, the stress builds up rapidly to just over 10 MPa. This is followed by a slight stress drop and thereafter strain hardening to a peak stress at large strains.

Effect of Ionic Impurities

Ionic impurities can have either a strengthening or weakening effect on the behavior of ice. Jones and Glen (1969 b and c) reported results from a series of strength and creep tests on single ice crystals doped with hydrogen fluoride (HF), ammonia (NH₃) and ammonium fluoride (NH₄F) at temperatures ranging from −20 to −70°C. In constant stress creep tests on ice crystals doped with HF and resolved shear stresses in the basal plane ranging from 0.4 to 0.8 MPa, they observed a considerable increase in the creep strain rate over that of pure ice. In constant strain rate strength tests at −70°C and a nominal strain rate of 2.7 x 10⁻⁷/sec, they observed a significant decrease in peak strength from 3.2 MPa in pure ice to 0.8 MPa in ice doped with 3p.p.m. HF. For the same testing conditions, they observed an increase in strength from 3.2 MPa in pure ice to 5.0 MPa for an ice crystal doped with NH₃. Ice crystals doped with NH₄F exhibited no change compared to pure ice behavior.

Nakamura and Jones (1973) reported results from tensile strength tests on single ice crystals doped with various impurities tested in the range of temperatures from −5 to −46°C. At an applied strain rate of 1.6 x 10⁻⁶/sec and a temperature of −11°C, they observed a dramatic decrease in the strength of ice
crystals doped with HCl and HF. The peak resolved shear stress decreased from 0.6 MPa in pure ice to 0.2 MPa for ice doped with 2.5 p.p.m. HCl and to 0.19 MPa for ice doped with 8 p.p.m. HF.

Explanations for the differences in the effect of some ionic impurities on the behavior of ice are presented in Section 2.2.4.

Preface to Sections on Deformational and Strengthening Mechanisms

As a preface to the sections which will describe various deformational and strengthening mechanisms which may account for the behavior of polycrystalline ice, the writer would like to quote a portion of the preface of W.T. Read’s book "Dislocations in Crystals" published in 1953. The writer feels that this gives an insightful look at the development of the science of dislocation theory and in many ways may be relevant to the current state-of-the-art in ice behavioral theories.

"The history of dislocation theory is divided into three overlapping stages. In the first stage ingenious simple ideas emerged and gave natural qualitative explanations for many of the mechanical properties of crystals—for example, the low mechanical strength [as compared to the theoretical strength of a perfect crystal]. In the second stage, the explanations became more detailed and speculative and were extended to a wider range of observations; it became the fashion to invent a dislocation theory for almost every experimental result in plastic deformation. Finally, it became apparent that dislocations could explain not only any actual result but virtually any conceivable result, usually in several different ways. This led to the third stage which is a critical step-by-step development of basic theory from first principles and a search for clear-cut experimental checks of the theory;...".
In performing this literature review, the writer has been amazed at the number of possible explanations of various aspects of ice behavior that have been proposed in the literature, e.g., Langdon (1973) reviewed six theories which seek to explain the power law creep behavior of ice. In the writer's opinion, a detailed treatment of each of the proposed ice behavioral theories would be unfruitful and unwarranted, especially since in many cases the jury is still out (i.e., clear-cut experimental checks of the theories have not been made).

Accordingly, this review will focus on general classes of theories without presenting the details of each proposed theory and will attempt to present a broad overview of possible deformational and strength mechanisms in order to give the reader an appreciation for the complexity of ice behavior and to highlight the necessity for continued basic research.

2.2.3 Mechanisms of Deformation in Polycrystalline Ice

In this review, the writer will extract material from both the polycrystalline ice literature and the more general crystalline solids literature in an effort to present an overview of the possible mechanisms. This summary is not intended to be comprehensive. The writer has chosen to highlight nine classes of deformational mechanisms which can roughly be placed into four groups. Wherever possible, experimental evidence for the given class of mechanisms will be reviewed. Table 2.2 summarizes each of the mechanisms which are discussed. It contains a brief description, a summary of the probable range of applicability and a list of references.

The writer has relied extensively on the interpretations of the ice literature from Langdon (1973), Goodman et al. (1981) and Duval et al. (1983). Deformational mechanisms in polycrystalline ice can be broadly categorized into
four groups: elastic mechanisms, anelastic mechanisms, plastic mechanisms (including an evolving microstructure) and mechanisms of internal fracturing.

**Elastic Deformation Mechanisms**

The elastic straining of a polycrystalline ice may be explained in terms of the elastic straining of the hydrogen bonds holding the water molecules in the crystalline lattice. Cottrell (1953) described this elastic straining mechanism for crystalline solids in the following manner "The mechanical stability of [crystalline] solids is due to the existence of balanced forces between their constituent atoms. Attractive forces cause the atoms to cohere to one another while short—range repulsive forces prevent the atoms from approaching too closely. Normally the atoms occupy positions where these forces are balanced, but when an external force is applied this balance is upset. To restore equilibrium, the atoms move slightly to nearby positions where there is a balance between the attractive, repulsive, and external forces". Upon removal of the external force, the water molecules move back to their original equilibrium positions and the deformation is fully recovered.

**Observations of Elastic Deformational Mechanism**

The elastic straining of polycrystalline ice can be observed under all types of loading conditions so long as sufficient care is taken to make high quality stress and strain measurements. Experimental programs have included the use of both static and dynamic measurement techniques. Hobbs (1974) reviews some of these measurements. More recently, Cole (1990) has used cyclic compression and tension tests with on—specimen strain measurements. He observed that at a temperature of −50°C, a frequency of 1 Hz and a load amplitude of ±1 MPa, a
specimen of polycrystalline ice with grain sizes ranging from 1 to 5mm deformed elastically in uniaxial compression and tension.

A complete characterization of this elastic deformational mechanism will await a measurement of the elastic properties of the hydrogen bonds holding the water molecules in the lattice. This has not yet been accomplished (Hertzberg 1989 pg. 55).

Range of Applicability of Elastic Deformational Mechanism

Elastic straining occurs under all loading conditions and under certain circumstances it may be the dominant deformational mechanism if anelastic, plastic and internal fracturing mechanisms are not activated.

Anelastic Deformation Mechanisms

Anelasticity refers to a time dependent elastic recovery of some of the deformation experienced by the crystals. This is considered to be caused by the motion of a certain population of mobile dislocations under the influence of an internal stress field and is observed after removal of the external forces. Note also that proton rearrangement can give rise to anelastic effects, but this is of a much lower magnitude. Anelasticity due to the development of an internal stress field can be explained in the following manner. The dislocations move under applied external stresses and their motion is opposed by gradually increasing internal stresses. These internal stresses can be both long range and short range. The long range internal stresses may be caused by, but is not limited to, the pile up of dislocations against grain boundaries. These forces can be highly directional or kinematic in nature. The short range internal stress may be caused by interactions between dislocations and these are nondirectional or isotropic in
nature. After removal of the external stresses, these built up internal stresses seek to restore the crystal to its original configuration by driving the motion of some of the mobile dislocations in the opposite direction (Duval et al. 1983).

Observations of Anelasticity in Ice

The anelasticity of ice results in a phase lag between the applied stress and the resulting strains and has the effect of dissipating energy in the form of internal damping or friction. In monotonic loading and unloading tests, this shows up as a delayed elastic recovery (Duval 1978). Under conditions of cyclic loading, anelasticity has been demonstrated by a hysteretic behavior, (Cole 1990).

Ranges of Applicability of Anelastic Deformational Mechanisms

Anelastic behavior can occur under all loading conditions so long as there is sufficient time for the build up of the internal short range or long range stresses. In creep tests, the anelastic strain component can be several times larger than the elastic strain component. In discussing the results of uniaxial creep tests, Duval et al. (1983) state "the anelastic strain can be more than 10 times the expected elastic strain....negative creep [creep in the opposite direction] will result if the directional internal stress is greater than the applied stress".

Plastic Deformational Mechanisms

Diffusional Flow

A possible plastic deformational mechanism in polycrystalline ice is self diffusion, which has been shown to occur in ice by the intersstitial movement of
intact water molecules. Possible diffusion paths for these interstitials can be either along grain boundaries or through the crystal lattice. Coble (1963) proposed the former mechanism and Nabarro (1947) and Herring (1950) proposed the latter.

The main feature of these types of diffusional mechanisms is that they result in a linear relationship between the applied stress and the resulting strain rate. This is tantamount to assuming that the ice is deforming as a Newtonian viscous fluid. In terms of power law creep, this results in a power law coefficient of $n = 1$.

Grain Boundary Sliding with Diffusional Accommodation

Another possible mechanism of deformation is the sliding of grain boundaries. Raj and Ashby (1971), in considering grain boundary sliding as a deformational mechanism in crystalline materials, have suggested that the diffusion of point defects is critical for this mechanism to operate in order to conserve the integrity of the continuum (i.e., if the grain boundaries slide between polycrystals, a diffusional mechanism must operate to fill in the discontinuities which would otherwise be produced). They propose a model for grain boundary sliding with diffusional accommodation which incorporates both the Nabarro–Herring and the Coble models. This grain boundary sliding with diffusional accommodation also leads to a Newtonian viscosity for the polycrystalline ice with a power law coefficient of $1$.

Observations of Diffusional Mechanisms

Weertman (1973) summarizes the results of several testing programs which have measured the self diffusion of tritium, deuterium and oxygen through
ice. These validate the possibility of a diffusion based deformational mechanism.

Ignat and Frost (1987) presented some preliminary results of creep tests conducted on bicrystals of ice in an effort to observe grain boundary sliding. They applied shear stresses of 0.1 and 0.4 MPa on the boundary between the two crystals and observed the resulting deformations. The testing temperature was $-3.5 \pm 0.5^\circ\text{C}$. They observed that grain boundary sliding does not always occur and that it probably depends on the relative orientations of the crystals.

**Range of Applicability of Diffusional Type Mechanisms**

In considering the probability that diffusional type mechanisms might be the dominant form of deformation under certain loading conditions in ice, Langdon (1973) summarized prior low stress creep data from laboratory tests and measurements of the creep of glaciers and ice sheets. He concluded that at very low effective shear stresses, i.e., $< 0.1$ MPa (effective shear stress here is roughly equivalent to a $0.17$ MPa creep stress in a uniaxial compression test), there is a tendency for the power law coefficient to approach $n = 1$. The strain rate at this transition was roughly $10^{-9}$/sec for all of the measurements in his summary. This indicates that at very low stresses and strain rates, the behavior of polycrystalline ice might be dominated by diffusional types mechanisms.

**Dislocation Gliding and Climbing Mechanisms**

Dislocations are linear defects in a crystalline lattice. The motions of dislocations in a crystalline lattice allows the relative slip of adjacent sections of the same crystal. It is through the slipping of adjacent sections that macroscopic deformations are accommodated. Hertzberg (1989) presents a useful summary of dislocation theory.
Digression on Dislocation Theory

By assuming that plastic straining in crystalline materials occurs by gliding of dislocations along slip planes, researchers have been able to relate the overall plastic strain rate experienced by the crystal to the velocity of the dislocations in their glide planes, the density of the dislocations and the length of the Burger's vector in the following manner

\[ \dot{\varepsilon} = \alpha \rho b \nu \]  

where \( \dot{\varepsilon} \) is the rate of plastic straining, \( \alpha \) is a geometric factor which depends on the orientation of the slip plane, \( \rho \) is the dislocation density, \( b \) is the length of the Burger's vector, and \( \nu \) is the dislocation velocity. This equation has been referred to in the literature as the classical equation for dislocation theory. A derivation of this equation is detailed in Cottrell (1953).

Dislocations in Ice

Duval et al. (1983) explain that in order for a crystal to undergo an arbitrary change in shape "each crystal in the polycrystalline body must have at least four independent deformation systems available to it. Basal slip in ice provides only two independent systems. Polycrystal plasticity must involve slip or climb of dislocations on nonbasal systems....". They present different combinations of possible slip systems which will produce the required amount of independent deformation systems.

Slip along the basal plane is the preferred mode of deformation in ice Ih. As explained in Section 2.2.2, easy glide along the basal plane can occur much more easily than nonbasal slip (e.g., Fig. 2.24). Nonbasal slip systems might include prismatic planes or pyramidal planes and be supplemented by the
climbing motion of dislocations out of their basal planes.

**Velocities of Dislocations**

Dislocations move along the basal plane with a velocity that is dependent on the resolved shear stresses acting on the basal plane. Weertman (1983) summarized the results of various research projects which measured dislocation velocities in ice. The data all seem to indicate an approximately linear relation between resolved shear stress in the basal plane and the dislocation velocity. At a temperature of $-10^\circ$C, the dislocation velocity in ice ranges from roughly $10^{-5}$mm/sec at a resolved shear stress of 0.01 MPa to roughly $5\times10^{-2}$mm/sec at a resolved shear stress of 10 MPa.

**Observations of Dislocation Controlled Deformation Mechanisms**

Dislocations in the basal planes of ice Ih have been observed by various researchers (Jones and Gilra 1973, Fukuda et al. 1987 and Sinha 1978). Gold (1963) observed the formation of slip bands on the face of columnar ice as a result of the gliding motion of dislocations. The climbing motion of dislocations out of basal planes and the activation of nonbasal slip systems have been rarely observed in ice (Duval et al. 1983).

**Range in Applicability of Dislocation Glide and Climbing Mechanisms**

Theoretical models which are based on dislocation glide and dislocation climbing mechanisms tend to predict power law coefficients in the range of $n = 3$ to 4.5 (Langdon 1973). This has lead researchers to conclude that such mechanisms are probably dominant in the power law creep regime, i.e., for strain rates from approximately $10^{-7}$/sec to $10^{-4}$/sec, Duval et al. (1983).
Evolving Texture and Fabric

Plastic deformations in polycrystalline ice Ih can also be accompanied by dynamic recrystallization which can result in changes in the texture and fabric. This mechanism does not accommodate deformation per se, but is a response of the ice to internal strain energy gradients which are built up during the deformation.

Grain boundary migration and the accompanying grain growth can take place either under externally applied stresses or during periods of storage as was observed by Cole (1986, 1987). This occurs as the boundaries of some crystals in the polycrystalline lattice move and consume adjacent crystals and can be the result of internal strain energy gradients or can be driven by the surface tension forces associated with the curvature of the crystal boundaries, i.e., fine-grained polycrystalline ice tends to coarsen over periods of storage. Duval et al. (1983) state "It is a matter of common experience that ice with a grain size of the order 1mm shows rapid grain coarsening just below its melting point".

Duval et al. (1983) explain "Dynamic recrystallization starts when the difference in energy between adjacent grains is sufficient to nucleate a 'bulge' in the boundary which separates them". They go on to explain that in order for this process to influence the creep rate, the grain boundary of this nucleated bulge must be able to migrate fast enough so that the "new" crystal can grow to a size equal to the grain size of the other crystals before the steady state condition is reach (i.e., the minimum creep rate). Thus dynamic recrystallization must be accompanied by grain boundary migration in order to affect the behavior of the polycrystalline ice.
Experimental Observations of Grain Growth and Recrystallization

Gold (1963) tested columnar grained polycrystalline ice in biaxial shear with grain sizes between 1.5 and 7.0 mm at a temperature of $-9.5 \pm 0.5^\circ C$ and compressive creep stresses of 5 to 9 kg/cm$^2$. He noted that practically every grain boundary which he observed showed evidence of migration. Wilson and Russell-Head (1982) conducted a series of plane strain compression creep tests on isotropic polycrystalline ice with grain sizes of 0.7 and 1.2 mm, a testing temperature of $-1 \pm 0.01^\circ C$ and an axial stress of 0.6 MPa. These specimens were sheared to various levels of strain and extensive thin-sectioning revealed an evolving microstructure as a function of axial strain. At lower levels of strain, they observed the nucleation of new grains near grain boundaries and in the more deformed regions of the microstructure. As the levels of strain increased, they observed marked increases in grain sizes and a tendency for the basal planes to become more closely oriented in directions of maximum shear strains, (i.e., in easy glide directions).

Cole (1987) also reported an evolving microstructure in polycrystalline ice specimens tested at $-5^\circ C$ in uniaxial compression at a strain rate of $10^{-6}$/sec. The grain size was varied from 1.5 to 5.8 mm. He observed that in fine-grained specimens there was a significant increase in the average grain size and a distortion of grain boundaries which he attributed to dynamic recrystallization. These fine-grained specimens showed no evidence of internal cracking. In coarse-grained specimens, he observed that most of the deformation was localized in the grain boundary regions and that these boundaries became granulated, i.e., the average grain size decreased during the deformation. He noted that this grain boundary granulation was accompanied by a slight degree of visible cracking. For specimens tested at much higher strain rates, he did not
observe the dynamic recrystallization processes.

**Range of Applicability of Grain Growth and Dynamic Recrystallization Mechanisms**

As noted in the above discussion, grain growth is a time dependent process. During periods of storage under no externally applied stress, the grain size can increase dramatically, especially for fine-grained specimens stored near their melting point. During periods of shear when certain grains in easy glide orientations begin to grow and bear the brunt of the imposed deformations, the resulting texture and fabric will evolve. Dynamic recrystallization and accompanying grain growth is both a strain energy dependent and time dependent process. The nucleation of new grains along grain boundary "bulges" requires a given amount of strain energy and the growth of these grains requires time. Clearly the smaller the grain size, the greater the number of potential nucleation sites and the shorter the time required to recrystallize a significant portion of the polycrystalline ice mass. Using simple estimates based on the data presented by Duval et al. (1983), strain rates in the range of $10^{-7}$ to $10^{-2}$/sec might be sufficient to allow for dynamic recrystallization and grain growth to dominate the behavior of fine grained polycrystalline ice, (i.e., at the lower end of the power law region near the transition to Newtonian viscous type behavior. Cole (1987) comments that above a strain rate of $4\times10^{-8}$/sec, the dynamic recrystallization effects are greatly reduced.

**Deformation by Internal Fracturing**

If the deformation of the polycrystalline ice proceeds at a strain rate that is too fast or a temperature that is too low for the previously explained
mechanisms and processes to be dominant, then internal fracturing mechanisms will be activated. Internal fracturing mechanisms in crystalline materials are characterized by the nucleation of cracks in the continuum and the propagation of these cracks. A summary of the micromechanisms of fracture in crystalline materials has been presented by Ashby et al. (1979).

Internal fracturing mechanisms result in volume changes in the crystalline solid. This leads to a strong pressure dependence for cracking activity. In polycrystalline ice, a pressure increase will suppress the cracking activity and alter the behavior. This will be discussed in greater detail in Section 2.2.4.

Types of Internal Cracking

Cole (1988) explained that "the total cracking activity of ice appears to consist of the superposition of two distinct distributions: one is strain—dependent and associated with the mechanism [of dislocation pile up at grain boundaries]..., the other is strain—-independent and associated with an elastic mechanism stemming from the anisotropy of the lattice structure". Both of these mechanisms have been used to develop crack nucleation theories. The dislocation pile up mechanism for crack nucleation in crystalline solids was first proposed by Zener (1948) and the elastic anisotropy mechanism for crack nucleation was first suggested by Gold (1972), further developed by Cole (1988) and subsequently by Sunder and Wu (1990).

Observations of Cracking Activity in Polycrystalline Ice

Observations of the cracking activity in ice have been made by various researchers. The first stages of brittle fracturing mechanism can be small fractures localized at the grain boundaries like those observed by Cole (1987) is
his coarse grained specimens. Gold (1960) also observed this type of localized fracturing at the grain boundaries and described them as appearing like stair cases.

At higher strain rates, lower temperatures or in coarser grained specimens, small localized fractures at the grain boundaries may not be sufficient to accommodate the imposed deformations and large cracks can form either at the grain boundaries or inside the crystals due to the possible nucleation mechanisms just discussed. These cracks may extend through several crystals or be confined to a single crystal.

The cracking activity in ice has been studied both by visual observation and by the use of acoustic emission detection. Cole (1986) reported the results of a careful post-test optical analysis of isotropic equiaxed polycrystalline ice specimens tested under constant stress conditions at $-5^\circ$ C, an axial stress of 2.0 MPa and grain sizes ranging from 1.5 to 6.0mm. The author detected the presence of both grain boundary cracks and transcrystalline cracks. He observed that the amount of cracking was both a function of the grain size and the level of strain. In general as the level of strain increased the number of cracks also increased. Cole noted that there was significantly more cracking activity experienced by the coarsest grained specimens.

Cole and St.Lawrence (1981) reported acoustic emission measurements on constant rate of strain compression tests on equiaxed polycrystalline ice 1.2mm diameter, a temperature of $-5^\circ$ C and strain rates that varied from $1.14 \times 10^{-3}$ to $10^{-7}$/sec. They observed that the peak rate of acoustic emissions, which corresponds to the peak rate of cracking activity, occurred near the initial yield point (this was also the peak strength) for tests at a strain rate of $10^{-3}$/sec. At a strain rate $10^{-5}$/sec, they observed that the maximum rate of acoustic emissions
occurred after the initial yield point during strain hardening but before the peak strength was reached.

**Ranges of Applicability of Brittle Fracturing Mechanisms**

Cole (1988) suggested that a difference between the Zener dislocation pile up model and his own elastic anisotropy model is the amount of time (strain) necessary for the nucleation of a crack. In the former model, sufficient time must be available for the dislocations to move to the pile up. Cole suggested that a range of 40 sec to 400 sec for the achievement of the maximum cracking rate would be a sufficient amount of time for the activation of the pile up mechanism. Assuming that a peak maximum cracking rate occurs somewhere between the first yield point and the peak strength, this leads to an upper limit of applicability of this model of approximately $10^{-4}$/sec strain rate. Cole stated in contrast, that the elastic anisotropy mechanism is time independent and "is expected to account for an increasing fraction of the cracking as the rate of deformation increases. At strain rates resulting in purely brittle behavior, this mechanism can completely dominate the behavior".

Due to the volumetric component of the fracturing process, it is possible to suppress cracking by the application of a sufficiently high confining pressure. Jones (1982) observed in a isotropic polycrystalline ice specimen tested at a strain rate of $5.5 \times 10^{-3}$/sec and a temperature of $-11^\circ C$, that "the unconfined test gave an elastic deformation followed by brittle failure....The application of a 10 MPa confining pressure suppressed this brittle failure and the associated cracking, and allowed the stress to rise much higher than in the unconfined test, leading to plastic yielding and flow".
2.2.4 Mechanisms Affecting the Strength in Isotropic Polycrystalline Ice

Mechanisms which affect the deformational processes discussed in Section 2.2.3 will directly affect the strength of ice. This section will review several such mechanisms which have been proposed in the literature. These will be drawn from both the ice literature and the more general crystalline solids literature. As in Section 2.2.3, this review is not intended to be exhaustive but rather to present a general summary of classes of proposed theories.

These mechanisms can be broadly placed into two categories: those which tend to weaken the ice, and those which tend to strengthen the ice. Weakening mechanisms can include a decrease in grain size (from the fine to ultra-fine range), pressure melting at grain boundaries and triple point junctions, the concentration of impurities at the grain boundaries, the increased mobility of dislocations and an evolving fabric with a preferred orientation of c-axis. Strengthening mechanisms may include an increase in the internal stresses opposing the motion of dislocations, the decreased mobility of dislocations, a decrease in grain size (from the coarse to the fine range), the suppression of dilation (cracking activity) and the existence of a random fabric. Table 2.3 is a summary of the mechanisms which might tend to weaken ice and Table 2.4 is a summary of those which may tend to strengthen ice. Each table contains a brief description of the mechanism, a list of deformational mechanisms which are involved and some references.

Possible Weakening Processes in Polycrystalline Ice

Decreasing Grain Size from the Fine (1mm) to Ultrafine (<<1mm) Range

Armstrong (1970) explains how a decreasing grain size may serve to
weaken a polycrystalline metal. "At low stresses or high temperatures, grain boundaries are normally regarded as potentially weakening a material for a number of reasons. The weakening may be principally attributed to two major grain boundary processes: (1) grain boundary sliding or shearing which occurs to an increasing degree as the grain size decreases; and (2) the grain boundary enhancement of diffusion–controlled deformation mechanisms. The grain boundaries contribute to increasing diffusion by offering short–circuited routes for mass transport and by acting as sources of sinks for point defects [vacancies]. It may also occur that certain grain boundaries are intrinsically weak at high temperatures because dislocations or cracks are easily nucleated at the boundaries,. Grain boundaries may contribute to weakening a material through serving as nucleation points for new grains of low shear strength" [i.e., dynamic recrystallization and associated grain boundary migration].

Cole (1987) observed that in finer grained polycrystalline ice specimens which he tested were much more susceptible to dynamic recrystallization. Below a strain rate of 4x10^{-6}/sec, the finer grained specimens were weaker than the coarser grained specimens. He explained "Since strain–induced boundary migration initiates by the bulging of existing grain boundaries, a fine grained material has many more potential nucleation sites than does a coarse grained material". Thus, he attributed the lower strengths to dynamic recrystallization.

Although the weakening mechanisms described by Armstrong were used to explain the strength of ultra fine–grained metals, in the writer's opinion, they might represent significant factors in the strength of ice in the pores of frozen sand where the grain size is probably an order of magnitude smaller than normally tested polycrystalline ice.
Pressure Melting at Grain Boundaries

Weakening affects due to pressure melting have been associated with the decrease in strength measured at very large confining pressures. Jones (1982) explained that the "decrease in strength with increasing confining pressure, after the maximum has been reached, is probably due to an increase in the amount of liquid at the grain-boundary junctions". He observed that at a temperature of $-11\pm1^\circ C$, increases in confining pressure above 10 to 40 MPa resulted in decreasing peak strength. He explained that the presence of liquid at the boundaries due to pressure melting serves to accommodate the extremely high stresses between grains caused by basal gliding mechanisms and as a result minimizes the magnitude of the stress concentrations and allow deformations to proceed at a lower applied stress.

Clearly an increase in the amount of unfrozen water between ice crystals will influence the magnitude of the shear stresses which can be transmitted between crystals and hence decrease the strength of the polycrystalline matrix by enhancing grain boundary sliding. Jones (1982) considered that this grain boundary weakening mechanism would probably not be significant at temperatures below $-10^\circ C$.

Concentration of Impurities at the Grain Boundaries

Mellor (1980) explains that even the purest forms of natural ice contain impurities. These can concentrate at the grain boundaries and at grain boundary intersections (triple points). As the ice crystals freeze, these impurities are not easily accommodated into the lattice. Clearly this can lead to a local decrease in the melting point at the grain boundaries and hence to the formation of a liquid-like layer between adjacent crystals which can result in a significant
decrease in the shear strength of the polycrystalline ice as explained in the prior paragraph. This liquid-like layer can also serve to decrease the magnitude of the stress concentrations due to the pile up of dislocations by acting as a "weaker" barrier.

Increased Mobility of Dislocations

As was illustrated in Section 2.2.2, various ionic impurities can have a weakening effect on the strength of single ice crystals. Glen (1968) proposed that the motions of dislocations in ice is controlled by the reorientation of protons in the lattice. This constitutes a dislocation drag mechanism or a process that restricts the mobility of dislocations. As explained in Section 2.2.1, the random orientation of protons in the ice lattice occasionally produces an ionization defect where either two protons are located between adjacent oxygen atoms (Bjerrum D-defect) or no protons are located between adjacent oxygen atoms (Bjerrum L-defect). In Glen’s theory, these ionization defects migrate through the lattice and reorient the protons ahead of the dislocation allowing it to pass without creating an ionization defect. Thus, if the addition of an ionic impurity increases the number of mobile ionization defects, it will increase the mobility of the dislocations and decrease the strength.

Evolving Fabric

During shear, the processes of dynamic recrystallization and subsequent grain boundary migration or simply grain boundary migration can significantly alter the fabric of polycrystalline ice. Either "new" favorably oriented crystals (i.e., oriented in easy glide directions) are nucleated at grain boundaries and grow to dominate the behavior or grain boundary migration favors existing favorably
oriented crystals which eventually overcome other crystals. Duval et al. (1987) explain that these fabric changes have been associated with the strain softening observed in creep tests during the accelerating creep after the minimum creep rate, i.e., tertiary creep. They further explain that "dynamic recrystallization causes a major reorientation of certain grains. This can induce the development of a preferred c-axis orientation in ice deformed near its melting point".

Jacka (1984) observed changes in the fabric of initially isotropic polycrystalline ice tested in uniaxial compression and tension. The c-axis of the individual grains tended to be oriented at approximately 27° from the vertical at very large strains for the tests in compression. This is close to the 45° orientation which corresponds to the maximum shear stress on the basal plane.

Cole (1987) explains how these weakening processes can affect the behavior of polycrystalline ice in the following manner "As a high-angle grain boundary migrates from a region of low strain energy (dislocation density) through a region of high strain energy, it leaves behind it a relatively dislocation free lattice. The new or recrystallized lattice can deform more easily than the work-hardened parent material. The macroscopic flow stress is affected by this process when a significant fraction of the specimen has undergone recrystallization".

Possible Strengthening Processes in Polycrystalline Ice

Isotropic and Kinematic Hardening

The essence of the isotropic and kinematic hardening processes in polycrystalline ice is the formation of an internal stress field which opposes the
motions of mobile dislocations in their glide planes. Hertzberg (1989) explains that dislocations can interact with each other in one of two ways. Either they will combine and annihilate each other (in the case where their Burger's vectors are equal and opposite) or they will repel each other and increase the level of shear stress required to cause them to move in their glide planes (in the case where their Burger's vectors are identical).

Researchers agree that the motions of dislocations are controlled by the presence of internal stress fields but they disagree on how to describe these mathematically. Duval et al. (1983) have adopted the principle of an "effective stress" defined as

\[ \sigma_e = \sigma - \sigma_i \]

where \( \sigma_e \) is the effective stress driving the gliding motions of dislocations, \( \sigma \) is the total applied stress and \( \sigma_i \) is an appropriate sum of isotropic and kinematic components. If the velocities of the dislocations are assumed to depend on this effective stress, then an increase in the magnitude of the \( \sigma_i \) leads directly to a decrease in the velocity of the dislocations for a given total applied stress.

Note that recently Shyam Sunder and Wu (1990) have assumed that the dislocation velocity does not depend on this effective stress but rather on a "reduced" stress which is formed as a multiplication rather than an addition of the isotropic and kinematic hardening components in the following form

\[ \sigma_r = (\sigma - R)/D \]

where \( \sigma \) is the reduced stress, \( \sigma \) is the applied stress, \( R \) and \( D \) are internal variables that describe the kinematic and isotropic hardening, respectively.

The isotropic component of the internal stress "is caused by short-range interactions between dislocations on parallel planes or on intersecting planes, or between moving dislocations and arrays of dislocations forming cell walls (walls
resulting from the dense tangling of dislocations surrounding regions that are relatively free of dislocations). The kinematic component of the internal stress field is due to the "great difference in resistance to creep between the basal plane and other planes". This can be due to the pile up of dislocations in the basal plane at grain boundaries or due to the formation of kink bands (concentrations of dislocations inside the crystal which form a boundary and can accommodate bending deformations). Shyam Sunder and Wu (1990) concur with these definitions.

Shyam Sunder and Wu (1990) explain that "The differences in the creep-rate equations, ..., stem particularly from deficiencies in the knowledge of underlying microstructural processes governing the creep of polycrystalline ice and how such processes influence the macroscopic deformation behavior of the material". Thus, the jury is still out and the use of such formulations must be done with some caution.

**Decreased Mobility of Dislocations**

In an effort to explain the strengthening effect that some impurities can have on the behavior of single ice crystals, Jones and Glen (1969 b and c) proposed that if the dislocation core attracts Bjerrum D-defects, then the reorientation of protons ahead of the dislocation line would have to be accomplished largely by Bjerrum L-defects. An impurity which decreases the number of mobile L-defects (NH$_3$ in their experiments) will as a result decrease the mobility of the dislocations by decreasing the probability of proton reorientation ahead of the dislocation and as a result increase the strength of the crystal.
Decrease in Grain Size from the Coarse (10mm) to the Fine (1mm) Range

A decrease in the size of the grains in a polycrystalline material can have a strengthening effect on the behavior. This can be explained either by a grain boundary dislocation source theory or by crack nucleation theories depending on the regime of behavior.

Li (1963) proposed that the grain boundaries in a polycrystalline material can act as sources for dislocations. If the number of dislocations generated at the boundary is directly related to the surface area of the crystal, then the density of the dislocations within a crystal will be inversely related to the grain diameter (d) because as the grain size (d) decreases, the volume decreases as d^3 while the surface area decreases as d^2. Thus, the smaller the crystal, the greater the density of the dislocations. This will result in more interactions between dislocations that will cause an isotropic hardening. This mechanism was proposed for polycrystalline materials in general.

In discussing the cracking behavior of polycrystalline ice, Cole (1988) compares the dislocation pile up model with an elastic anisotropy model for predicting the crack nucleation stress and observes that they both result in an inverse square root dependency on the grain size, i.e., as the grain size decreases the crack nucleation stress increases. Cole's elastic anisotropy model predicts that the nucleation stress in tension and compression are identical.

Shyam Sunder and Wu (1990) have extended Cole’s elastic anisotropy model to include the effect of several neighboring grains. They predict that the nucleation stress in compression is 2.5 times that in tension, which is in better agreement with the experimental data.
Increase in Confining Pressure (low to moderate pressures)

In deformational regimes where cracking is a dominant mechanism, increases in confining pressure lead to dramatic increases in strength. Jones (1982) postulated that this increase in strength was due to the suppression of the cracking activity. Refer to Fig. 2.23. As the confining pressure increases, the local stresses near the potential crack nucleation site become more compressive. A greater tensile stress increment is therefore required to overcome this increased compressive stress in order to nucleate the crack. This allows the ice to undergo greater plastic yielding and flow behavior prior to the crack nucleation and results in an increase in strength.

Orientation of Crystals

A fundamental strengthening process in polycrystalline ice as compared to a single ice crystal in easy glide is the random orientation of the individual crystals and the strain compatibility which must exist between adjacent crystals. In considering plastic deformational mechanisms in polycrystalline materials, Johnston and Feltner (1970) stated that "strain compatibility induces the operation of many slip systems [dislocation glide mechanisms], the latter causes the rate of strain hardening to increase".

As previously discussed, the individual ice grains in an isotropic polycrystalline ice are anisotropic due to the preferred slip direction along the basal plane. In a matrix where there are randomly oriented crystals, some grains will be oriented in hard glide directions and others in easy glide directions relative to the applied stresses. There will be nonuniform straining of adjacent grains as the softer grains in easy glide tend to transfer their load to stiffer grains in hard glide. The stiffer grains act to reinforce the softer grains and as a result
the matrix is stronger than a single crystal in easy glide.

2.2.5 Summary of Isotropic Polycrystalline Ice Behavior

Description

Ice Ih is the stable form of solid water encountered in most engineering applications. It is normally encountered at high homologous temperatures. It is formed by the tetrahedral coordination of water molecules arranged in a hexagonal structure. Individual ice crystals are highly anisotropic with respect to flow properties due to the preferred slip of dislocations along basal planes. The proton lattice is disordered in ice. This gives rise to ionization defects when either two or no protons are located between adjacent oxygens atoms. A Bjerrum L—defect is an ionization defect when no protons are located between two adjacent water molecules and a D—defect is when two protons are located between adjacent water molecules. Some researchers have attributed these ionization defects to dislocation drag mechanisms in ice.

Impurities and gases can be found in ice. These can significantly influence the behavior by affecting the mobility of dislocations and changing the orderly arrangement of the water molecules (i.e., forming clathrate hydrates). The grain size of polycrystalline ice normally encountered in nature and tested in the laboratory are probably much larger than that found in the pores of frozen sand. The texture and fabric of the ice can significantly affect its behavior and can change dramatically during shear and long storage periods. The grain boundaries are an important feature of the structure of polycrystalline ice with respect to its mechanical properties. They function to transfer forces between grains and can serve to either strengthen or weakening the polycrystalline ice depending on the
Mechanical Behavior

A correspondence has been established experimentally between the results of constant load (creep) tests and constant strain rate (strength) tests. The stress/strain-rate correspondence is a relationship between the applied stress and minimum creep rate in a creep test and the peak strength and applied strain rate in a strength test. So long as the mechanical properties of the ice do not change under these two loading conditions, then the results of these two tests in terms of the minimum or applied strain rate and the peak or applied stress will plot along the same line on the flow curve. The flow curve is a log-log plot of these two quantities. Refer to Fig. 2.20.

The behavior of polycrystalline ice in tension is generally different from compression. Tensile specimens fail by fracture and separation except at low strain rates. Depending on the deformational regime, compressive specimens can exhibit yielding behavior, strain hardening and strain softening after the peak. Fig. 2.18 illustrated the idealized compression behavior observed at moderate strain rates. The initial yield point coincides with the onset of internal cracking. After the initial yield there is strain hardening to the peak strength and thereafter strain softening to an ultimate condition of constant stress at large strains.

The small strain behavior of polycrystalline ice can be described in terms of the Young's modulus and Poisson's ratio. Values of these parameters have been computed from the measured elastic properties of single ice crystals using ultrasonic measurement techniques. These show a slight tendency to increase with decreasing temperature (Fig. 2.19).
The strain rate has a significant effect on the behavior of ice. The flow curve indicates that in the range of strain rates from approximately $10^{-8}$ to $10^{-5}$/sec, the power law coefficient is between 3 and 4.5. Below this range in strain rates, the coefficient decrease towards 1 and above this range it increases to infinity and in some cases can become negative. In compression, as the strain rate changes, the character of the stress–strain curve also changes (Fig. 2.21). At very fast strain rates, the initial yield stress and peak strength coincide and is followed by very pronounced strain softening (e.g., test #16, 22 and 23 in Fig. 2.21) Moderate strain rates lead to the behavior described earlier, i.e., strain hardening after initial yielding up to the peak strength and then strain softening. At very low strain rates the amount of strain hardening after the initial yield decreases until the initial yield stress again represents the maximum strength. But unlike the behavior at high strain rates, there is little strain softening.

At strain rates greater than $10^{-6}$/sec, the tensile strength of polycrystalline ice is insensitive to strain rate. Below $10^{-6}$/sec the strength in compression and tension are identical (i.e., they both follow the same flow curve with the same power law coefficient as per Fig. 2.20.

Researchers have used the Arrhenius equation to describe the effect of temperature on the strength of ice at slow to moderate strain rates. This is based on the assumption that the creep of ice is a thermally activated process. For temperatures below $-10^\circ$C, this equation gives a good approximation to the behavior (fig. 2.22). Above $-10^\circ$C, the activation energy is not constant but increases dramatically.

It is possible to use a combination of the power law creep equation and the Arrhenius equation to obtain good approximations to the minimum creep rate or peak strength as a function of temperature within the range of applicability of
both equations.

The effect of an increase in the temperature for constant load creep tests has the same effect as an increase in the creep stress provided that all of the behavior is in the same deformational mechanism regime. This has been illustrated by dimensionless variables which allow the results of creep tests on similar ice specimens at different creep stresses and temperatures to be normalized (Ashby and Duval 1985).

Confining pressure can have both a strengthening and weakening effect on isotropic polycrystalline ice depending on the strain rate and stress level (Fig. 2.23). At very low strain rates there is no effect. For strain rates greater than about $10^{-5}$/sec, an increase in confining pressure from 0 to 10 MPa leads to an increase in the peak strength and this effect increases with strain rate. For confining pressures greater than 10 to 40 MPa, the beneficial effect of confinement at high strain rates is lost and increasing the confining pressure can actually lead to a decrease in strength.

The texture and fabric of isotropic polycrystalline ice dramatically influences its behavior. The character of the deformation is strongly influence by grain size. Polycrystalline ice with larger grain sizes has a tendency to exhibit more internal cracking. From the results of compression tests, there is a tendency for the strength of polycrystalline ice to increase with decreasing grain size at moderate to fast strain rates, whereas the reverse may occur at very slow strain rates (Cole 1987).

The fabric (orientation of the c–axis) in polycrystalline ice can have a dramatic effect on the behavior. This is evidenced by the highly anisotropic qualities of a single ice crystal (Fig. 2.24 and 2.25). For a given creep rate, a single crystal constrained to deform in hard glide requires a creep stress that is at
least 60 times larger than for easy glide. If constrained to deform in easy glide under constant stress, the creep rate will continuously accelerate. Under constant rates of strains, easy glide crystal exhibit strain softening after the first yield while hard glide crystals exhibit strain hardening after the first yield (fig. 2.25).

The presence of ionic impurities can have either a strengthening or weakening effect on the behavior of ice for reasons that are explained in subsequent sections.

Mechanisms of Deformation

Mechanisms of deformation in polycrystalline ice can be broadly classified into four groups: those involved with elastic processes; those involved with anelastic processes; those involved with plastic deformational processes and those concerned with internal fracturing processes. A summary of each proposed mechanism is presented in Table 2.2.

Elastic deformational processes involve the elastic straining of the hydrogen bonds holding the H$_2$O molecules in the ice lattice. The molecules move to slightly distorted positions under the influence of external forces. Upon removal of these forces they return to their original positions. Elastic deformations can be observed under all loading conditions so long as sufficient care is taken to make high quality measurements.

Anelastic deformations in ice refer to time dependent "elastic" behavior that produces decreasing strain rates during primary (transient) creep and strain recover during unloading. These are caused by the motion of a certain population of mobile dislocations under the influence of two types of internal stress fields. One type leads to isotropic hardening and the other leads to
kinematic hardening. Anelastic deformations can be readily observed in monotonic creep or strength tests upon removal of the load and in cyclic tests as a hysteretic behavior. Anelasticity will occur under all loading conditions so long as there is sufficient time for the build up of the internal stress field. In monotonic loading and unloading creep tests, the anelastic strain can be more than an order of magnitude greater than the corresponding elastic strain.

Plastic deformational mechanisms can include those associated with diffusional flow, grain boundary sliding and dislocation gliding and climbing. Diffusional flow is associated with the motions of point vacancies through the crystal lattice or along grain boundaries. Grain boundary sliding occurs in conjunction with diffusion of point defects. The main feature of these diffusional type mechanisms is a power law coefficient of $n=1$, i.e., they predict a Newtonian fluid type behavior for ice. These diffusional type mechanisms are considered to dominate the behavior of polycrystalline ice only at very low stresses or strain rates such as occur during the flow of glaciers.

Dislocation gliding and climbing mechanisms are associated with the slipping of adjacent sections of a given crystal. The velocity of the dislocations along their glide planes can be directly related to the plastic strain rate of the crystal through the classical equation of dislocation theory. In order for a crystal of ice to undergo an arbitrary change in shape, four independent slip systems must be available. The gliding motions of dislocations in the basal plane, which is the preferred slip plane in ice, provides two independent slip systems. This indicates that the motion of dislocations on prismatic or pyramidal planes or the climbing motions of dislocations out of their slip planes must accompany basal glide. Dislocations glide with a velocity which is linearly proportional to the effective stresses acting on them. Researchers have made observations of
dislocations gliding along the basal plane in ice. Nonbasal glide and climbing motions have been much more difficult to observe. These gliding and climbing motions of dislocations are considered to dominate the behavior of polycrystalline ice in the power law creep region where the coefficient is between 3 and 4.5, i.e., for strain rates between approximately $10^{-7}$ to $10^{-4}$/sec.

The texture and fabric of polycrystalline ice can evolve over time or under applied stresses. These are not deformational mechanisms per se, but rather they are responses of the polycrystal to strain energy gradients. Grain boundary migration and grain growth can take place during periods of storage or under the effect of applied stresses. Dynamic recrystallization and accompanying grain boundary migration occurs as the difference in the strain energy between adjacent grains is sufficient to nucleate a bulge at the boundary. The "new" crystals can grow and dominate the behavior of the polycrystal. Observations of these processes have been made through careful thin sectioning studies after shear and at various levels of imposed deformations. Dynamic recrystallization and grain growth are both strain and time dependent processes. These might dominate the behavior at strain rates in the range of $10^{-9}$ to $10^{-7}$/sec and are more important for finer grained specimens where there are a larger number of potential nucleation sites and a relatively short amount of time is necessary for the grain boundary of these "new" crystals to migrate so that they occupy most of the polycrystalline mass.

Internal fracturing processes dominate the behavior of polycrystalline ice when the strain rates are too fast or the temperatures are too low to allow for significant contributions from the other deformational mechanisms. Cracking results in a volumetric component to the straining of the crystal and hence exhibits a strong pressure sensitivity. Two mechanisms are responsible for
cracking. One is strain (and hence time) dependent and is associated with the pile up of dislocations at grain boundaries. The second is strain independent and is associated with elastic stresses developed from the anisotropy of the individual crystals. Both types of cracks have been observed directly through careful sectioning techniques after deformations. Acoustic emission detection has also been used to study the onset and rate of cracking. Internal cracking mechanisms are considered to operate at high strain rates or creep stresses. The dislocation pile up mechanism requires a certain amount of time for the dislocations to travel to the pile up. An upper limit of strain rate for which this mechanism may apply is approximately $10^{-4}$/sec. Confining pressures in excess of 10 MPa appear to be sufficient to suppress internal fracturing activity.

**Mechanisms Influencing the Strength**

Factors influencing the strength of ice can be broadly classified into two groups: those which tend to weaken ice; and those which tend to strengthen ice. Tables 2.3 and 2.4 present a summary of several possible weakening and strengthening mechanisms.

Weakening mechanisms include a decreasing grain size from the fine to the ultra fine range, pressure melting at the grain boundaries, concentration of impurities at the grain boundaries, increased mobility of dislocations and an evolving fabric.

A decrease in grain size from the fine (1mm) to the ultra fine (<<1mm) range may result in a significant decrease in the strength based on theory applicable to metals. This could be due to enhanced grain boundary sliding, greater diffusional processes along grain boundaries and a greater number of potential nucleation sites for dynamic recrystallization processes. A treatment
(either theoretically or experimentally) of this potentially important weakening mechanism (when considering the probable size of ice crystals in frozen sand) has not been presented in the ice literature.

Pressure melting at grain boundaries and triple point junctions can occur at very high confining pressures (> 10 to 40 MPa) and warm temperatures (> -10°C). This will serve to decrease the high stresses between grains which result from the pile up of dislocations and enhance grain boundary sliding mechanisms by decreasing the shear resistance of the boundaries.

Impurities present in the water before freezing will tend to concentrate at the boundaries of the grains during the freezing process because they are not easily accommodated into the lattice. High concentrations of impurities at the grain boundaries can serve to decrease the melting point and result in a liquid like layer between adjacent crystals. This will decrease the strength of polycrystalline ice by decreasing the magnitude of the stress concentrations which may be due to the pileup of dislocations and by decreasing the shear strength of the interface. This might also result in enhanced grain boundary sliding and diffusional processes.

Increased mobility of dislocations may result from the effect of various ionic impurities which can be incorporated into the ice lattice. If the presence of these impurities increases the number of mobile ionization defects (Bjerrum L and D defects), this might serve to increase the dislocation mobility if a proton reorientation mechanism is envisioned as the prominent dislocation drag process. An increase in the dislocation mobility leads to a decrease in the strength.

The fabric of polycrystalline ice (orientation of the c-axis) can evolve during deformations. The tendency is for crystals oriented in easy glide directions to begin to dominate. This can be due either to the preferred growth
of favorably oriented crystals (easy glide) at the expense of unfavorably oriented crystals (hard glide) or the nucleation of new crystals at grain boundaries (dynamic recrystallization) and subsequent grain boundary migration. When a larger population of crystals are oriented in easy glide directions due to these processes, the polycrystalline mass weakens.

**Possible Strengthening Mechanisms**

Possible strengthening processes in polycrystalline ice include isotropic and kinematic hardening, decreased mobility of dislocations, decreases in grain size from coarse (10 mm) to fine (1 mm) and increases in confining pressures up to 10 to 40 MPa.

Isotropic hardening in ice occurs as a result of the short range interactions of dislocations on parallel or intersecting planes or with tangled masses of dislocations on the same plane. These interactions contribute to one internal stress which can evolve during shear. As this internal stress field builds up it causes a decrease in the mobility of the dislocations.

Kinematic hardening can occur in polycrystalline ice as the dislocations interact with effective barriers such as grain boundaries or internal kink bands. These interactions develop a second internal stress field which also serves to retard the motions of dislocations.

The actual mathematical formulation of these internal stress fields has been a debated topic in the literature. The primary difficulty is the lack of a fundamental understanding of the intrinsic mechanisms which are affecting the motions of dislocations. Until this is resolved, the actual formulation of these internal stress fields is still debatable.

Decreased mobility of dislocations may also contribute to strengthening.
processes. Researchers have observed that some ionic impurities which are substituted into the ice lattice can actually decrease the mobility of dislocations. They have postulated that this is due to a decrease in the number of mobile Bjerrum L–defects and an affinity of the dislocation core for Bjerrum D–defects. The result is a decrease in the number of proton reorientations ahead of the dislocation line and hence an increase in the drag stress and decrease in dislocation velocity. This leads to an increase in strength.

A decrease in grain size from the coarse (10mm) to the fine (1mm) range can lead to an increase in strength. One theory which explains this strengthening is a grain boundary source theory. This assumes that the grain boundaries act as sources of dislocations and as a result, a smaller grain with a larger surface area to volume ratio will have a higher dislocation density. This leads to an increase in the internal stresses and a reduction in the effective stress acting on the dislocations causing an increase in strength.

Internal cracking theories also predict an increase in strength with decrease in grain size. Cracking due to both dislocation pileups and due to elastic anisotropy both predict an inverse square root relationship between the crack nucleation stress and the grain size.

Increases in confining pressure in the range of 0 to 10 MPa serve to strengthen polycrystalline ice primarily due to a suppression of the cracking activity. This strengthening effect is only prominent in deformational regimes where internal cracking is a dominant mechanism. Increases in confining pressure increase the stress required to nucleate a crack and increase the frictional resistance along the crack and hence lead to an increase in the strength.
2.3 FROZEN SAND

Frozen sands exhibit extremely complex behavior. This is due not only to the complex behavior of the sand skeleton and the pore ice, which has been illustrated in Sections 2.1 and 2.2, but also due to the interaction of the sand skeleton and the pore ice matrix and to the effect of unfrozen water and air voids. Researchers have been able to demonstrate experimentally most of the salient aspects of frozen sand behavior as a function of important variables such as the strain rate, confining pressure, temperature, sand density, degree of saturation and sand type. However, the interaction between the sand and pore ice is still poorly understood and a comprehensive knowledge of the physical mechanisms controlling the strength and deformation behavior of frozen sand has not been achieved.

This review of the behavior of frozen sand is from a mechanistic viewpoint (to the extent possible). It is divided into five parts: 1) presentation of the classification and description of frozen sands; 2) discussion of the probable structure for a frozen sand, along with supporting experimental evidence; 3) review of the important variables affecting the measured behavior of frozen sand; 4) an overview of possible mechanisms of deformation; and 5) discussion of possible strengthening and weakening mechanisms. This format follows the general pattern presented by Ting et al. (1983).

2.3.1 Classification and Description of Frozen Soil

Classification System

Sayles et al. (1987) outline a method for the classification of frozen ground which is based on a system originally proposed by Pihlainen and Johnston (1963) and Linell and Kaplar (1966) and summarized by Andersland and Anderson
A reproduction of the description and classification table presented by Sayles et al. (1987) is given as Table 2.5. This is based on a separate classification of the soil phase and a description of the amount and structure of the ice. The soil phase and the ice phase are classified separately because the behavior of frozen soils is highly dependent on the composition and structure of both of these phases.

In the first part of the description procedure, the soil phase is classified according to the Unified Soil Classification System (USCS). For sands, this classification system is based mainly on the size and gradation of the particles and the nature of the fines that are present. In addition to the USCS system, the shape of the particles can be described and additional information can include the relative density of the sand and its structure. Refer to Section 2.1.1 for further information on the description and classification of sands.

In the second part of the description and classification procedure, the frozen soil is categorized into two broad groups, i.e., whether or not the ice is visible or not visible to the naked eye and is less than 1 inch thick. If the pore ice is not visible to the naked eye, then it is further classified according to whether or not it is well-bonded. The ice is well-bonded if the soil particles are held tightly together by the ice crystals and the frozen soil is resistant to chipping or breaking. If the ice is not well-bonded, the frozen soil is classified as poorly bonded or friable. If the ice is well-bonded, it is also further classified according to whether or not there is "excess" ice present. Excess ice refers to the condition where the amount of ice in the pores is in excess of the original pore volume, i.e., during the freezing process, the soil skeleton expands but not enough to produce visible ice lenses.

If the pore ice is visible to the naked eye but is less than 1 inch thick, it is
classified as existing as individual ice crystals or inclusions, ice coatings on the particles, random or irregularly oriented ice formations, or stratified or distinctly oriented ice formations. This ice is further classified according to its location, orientation, thickness, length, spacing, hardness, structure and color. The volume of the visible ice is estimated as a percentage of the total specimen volume. If the visible ice is greater than 1 inch thick, then the specimen is classified as an ice with or without soil inclusions. The ice is then further classified according to the hardness of the ice mass (not the hardness of the individual crystals), the structure (granular or stratified), the color and the presence of admixtures (soil inclusions). Each of these cases is denoted either by letter qualifiers or one word descriptors. These are shown on Table 2.5.

The presence of large ice inclusions can have a dramatic effect on the behavior of the frozen soil and a complete characterization of the macrostructure of such a frozen soil is essential to understanding its behavior. Although a thorough treatment of frozen soil behavior would necessarily include a study of the effect of such large ice inclusions, these features will not be considered in this research.

Unfrozen Water Content

Frozen soils can be described in terms of the amount of unfrozen water which is present. This is usually described in gravimetric terms as the weight of unfrozen water divided by the weight of the soil particles. The unfrozen water content is denoted by the symbol $w_u$.

Anderson and Morgenstern (1973) have reviewed the various direct and indirect experimental methods which have been used to measure the unfrozen water content. A few of the direct methods are dilatometry, adiabatic
calorimetry, X-ray diffraction, heat capacity, nuclear magnetic resonance, differential thermal analysis and isothermal calorimetry.

Degree of Saturation

Frozen soils are described in terms of the degree of saturation. The degree of saturation is defined as the ratio of the volume of unfrozen water and ice in the pores to the total volume of the pores, and usually is expressed as a percentage. The condition of 100% saturation means that there is no air in the voids and a condition of 0% saturation means that there is no unfrozen water or ice in the pores. The degree of saturation can be computed from the following equation

\[ S = \frac{\gamma_i \left( w_u + \frac{w_i}{G_i} \right)}{\gamma_w \left( w + \frac{1}{G_s} \right)} - \frac{\gamma_i}{G_s} \times 100\% \]

where \( S \) is the degree of saturation expressed as a percentage, \( \gamma_t \) is the total frozen density, \( \gamma_w \) is the density of water (998.1 kg/m\(^3\) at \(-10^\circ\)C, Weast 1975), \( w \) is the total water content (both ice and unfrozen water), \( w_u \) is the unfrozen water content, \( w_i \) is the ice content (equal to the difference between the total water content and the unfrozen water content), \( G_i \) is the specific gravity of ice (0.9187 at \(-10^\circ\)C, Hobbs 1974 pg. 348) and \( G_s \) is the specific gravity of the soil particles.

2.3.2 Structure of Frozen Sand

A conceptual model for the structure of a frozen sand is presented in Fig. 2.26. According to Ting et al. (1983), a frozen sand structure can be summarized as follows:

- Solid contacts exist between most sand particles.
- A continuous unfrozen water film exists between the ice—soil interface and between ice grains at the boundaries (even at very low temperatures).
The unfrozen water is mobile parallel to the mineral surface, but strong tensile and significant shear adhesional forces can be transmitted between the ice and silicate surfaces.

No direct ice to soil particle contacts exist.

The ice in the pores of the frozen soil is polycrystalline with the maximum size of the grains limited to the pore size.

Each of these postulates will be reviewed in light of the experimental evidence and discussion in the literature. The writer recognizes the unrealistic assumption in Fig. 2.26 that a single plane passing through the frozen sand contains seven contact points in one pore. A more realistic structure might be one of sand particles suspended in an ice matrix (presented in Fig. 6.7). This type of a structure is treated in Section 6.2 in a discussion of a cube-in-a-cube model for composite materials.

Solid Contacts Between Sand Particles

Evidence that solid contacts exist between most soil particles in a frozen sand is illustrated through a series of experimental results concerning the effect of freezing on the structure of sands by Singh et al. (1982). The principal aim of this research was to assess the feasibility of using temporary freezing as a means of collecting undisturbed samples of sand for testing in the laboratory. The researchers reasoned that if freezing proceeded unidirectionally and drainage was provided to accommodate the expansion of the pore water during freezing, the structure of the sand might be preserved.

Unfrozen specimens of Monterey #0 sand with a mean grain size of 0.4 mm were placed in a triaxial cell and consolidated under an effective isotropic confining pressure of 0.56 kg/cm² (with a back pressure of 1.5 kg/cm² to achieve
S = 100%). These specimens were gradually frozen from the bottom to the top and corresponding volume of water flowing out of the specimen was measured.

When the specimens were totally frozen (the total freezing time was approximately 10 hours and the length of the specimens was approximately 15 cm), the cell pressure was released and the triaxial cell was disassembled. Subsequent diameter and height measurements indicated that no specimen volume changes had occurred. The volume of water expelled from the specimen during freezing was approximately equal to the theoretical volume change that would be expected as a result of the phase change in the water.

These specimens were reintroduced into the triaxial cell under the same effective confining pressure and allowed to thaw uniaxially in the opposite direction. An approximately equal amount of water was drawn back into the specimen during this thawing. Singh et al. (1982) concluded that uniaxial freezing under the confining pressure and back pressure resulted in no net volume change of the specimen.

In order to investigate the effect of freezing and thawing on the structure of the sand, Singh et al. (1982) subjected several unfrozen virgin specimens to a similar "seismic" history by cyclically loading these in a triaxial cell. Some of these "prestrained" specimens were then cyclically loaded to a pore pressure ratio of 100%, (i.e., liquefaction), and a reference line was established in terms of the cyclic stress ratio and the number of cycles to achieve liquefaction. Other specimens that had been subjected to the same "seismic" history were frozen and subsequently thawed and subjected to the same cyclic testing program. These frozen and thawed specimens exhibited identical cyclic strength behavior to the nonfrozen specimens. This indicated that the structure of the sand that had been produced by the prior stress history was preserved during the freeze-thaw cycle.
These experimental results are strong evidence that the uniaxial freezing of a clean sand (i.e., a sand with few nonplastic fines such that there is no formation of ice lenses during freezing) does not disturb the structure of the sand so long as drainage is provided and no net volume change occurs. The structure of the frozen sand skeleton is therefore very similar to the structure of the unfrozen sand so long as these conditions are met. It is reasonable to assume that the particle contacts that exist prior to freezing are probably intact after freezing.

Unfrozen Water Film

The unfrozen water film in frozen soils acts as a transition layer between the water molecules adsorbed on the surface of the silicate particles and the water molecules held in a hexagonal closed pack structure in the pore ice. The following is a summary of some experimental evidence for the presence of this unfrozen water film.

Hoekstra (1965) performed a series of conductance experiments on frozen bentonite pastes with total water contents ranging from 56 to 350%. He reported conductances in the range of $10^{-4}$/ohm cm for temperatures down to $-5^\circ$C. He compared these to conductances of $10^{-15}$/ohm cm for clay mineral particles and $10^{-8}$ to $10^{-9}$/ohm cm for ice and concluded that the higher conductance of frozen soils could best be explained by the presence of a continuous and mobile unfrozen water film surrounding the silicate particles in which the ions had a mobility similar to that of the bulk unfrozen solution.

Hoekstra and Chamberlain (1964) conducted a series of electro-osmosis experiments on frozen Wyoming bentonite and New Hampshire silt. They observed that significant amounts of water were transferred through the frozen
soil from the anode to the cathode. At the end of the test, large bodies of ice were observed in the cathode region and shrinkage cracks formed in the anode region. They explained that "When water freezes to ice most of the ions are expelled from the ice phase and remain in the water phase....The exchangeable cations surrounding the soil particles are crowded in a film of unfrozen water on the surface. In this unfrozen film the conditions for electro-osmosis are satisfied; there is an excess of cations over anions near the solid surface". As the cations are attracted to the cathode, an osmotic potential is established and water is drawn from the anode region towards the cathode. This indicates the presence of a continuous unfrozen water film in which water transport by electro-osmosis is possible.

Corte (1962) conducted a series of experiments to study the mobility of silicate particles on an advancing freezing front. The experiments consisted of generating a slowing advancing freezing front of ice and placing silicate particles of various types and sizes on top of the freezing front. The subsequent motion of the particles was observed as the freezing front advanced. Corte observed that regardless of the size or shape of the silicate particles, if the rate of advancement of the freezing front were slow enough, the particles would float upon it and advance with it. He explained, "For a particle to migrate, a layer of water must be continuously present between the particle and the ice front. This layer must be replenished".

Factors that are influential in affecting the amount of unfrozen water in a given soil include the temperature and the specific surface area of the soil particles, the surface chemistry, the presence of impurities and changes in confining pressure. Anderson and Morgenstern (1973) present a review of these factors and several empirical equations which can be used to estimate the
unfrozen water content.

**Strength of Ice–Silicate Interface**

The strength of the ice–silicate interface in a frozen sand was inferred by Ting et al. (1983) using the results of a series of experiments reported by Jellinek (1962). Jellinek measured the adhesive strength of snow ice frozen on a optically flat (to within 10 Å) fused quartz surface in both direct tension and direct shear. At a temperature of −4.5°C, the adhesional shear strength of the ice quartz interface was found to increase linearly with rate of deformation from 7 kPa at 5.3x10⁻³ mm/sec to 69 kPa at 0.41 mm/sec. Jellinek postulated the existence of an adhesive strength at zero deformation rate and stated that it "might possibly represent a yield value due to the structure of the liquid–like (unfrozen water) layer and the residual roughness of the.... quartz....".

In the direct tension tests at the same temperature, Jellinek (1962) observed that all of the failures were cohesive through the ice. The mean direct tensile strength was approximately 1.06 MPa. The tensile adhesional strength must have been greater than this. No deformation rate was reported for these tension tests. The tensile adhesional strength of the interface was thus at least one to two orders of magnitude greater than the adhesional shear strength.

In an effort to explain the large difference between the tensile and shear strengths, Jellinek (1962) considered the surface tension in the unfrozen water exposed to the atmosphere at the edges of the ice and quartz interface. Figure 2.27 is a schematic diagram of his proposed model. A precise estimate of the magnitude of this effect would have required a measurement of the thickness of the unfrozen water film which Jellinek did not make. Clearly the geometry of his testing setup is not similar to the ice–silicate interface in frozen sand. In a fully
saturated frozen sand, there are no surface tension forces due to the presence of a water–air interface that could act to generate the tensile strength of the interface. This casts some doubt on the applicability of Jellinek’s results to describe the behavior of the ice–silicate interface in a frozen sand.

Ting (1981) recognized this deficiency and suggested that perhaps a ball–bearing–on–magnetic–surface type model might explain the tensile strength of the ice–water–silicate interface in the absence of air–water surface tension forces. This two–dimensional model was explained by Martin (1960). In this model, the adsorbed water is assumed to behave much like a steel ball attracted to a magnetic surface, i.e., no shear strength but very large tensile strength.

The actual structure of the ice–silicate interface has been debated in the literature. Anderson and Morgenstern (1973) present an informative review of various theories and experimental evidence concerning the nature of interfaces in frozen soils. They propose their own conceptualized model for the silicate–water–ice interface. While conceptual models are useful in understanding the behavior, they are not capable of making a quantitative prediction of the behavior which is necessary for understanding the nature of the ice–sand interaction in frozen sands. In the writer’s opinion, the behavior of the ice–silicate interface is one of the most poorly understood aspects of frozen soil behavior. Validation of the Ting et al. (1983) conceptualized structure for frozen sand with regard to the nature of the ice–water–silicate interface will require continued theoretical and experimental research.

Direct Ice to Sand Contact

Anderson and Morgenstern (1973) considered the possibility of direct mineral to ice contacts in frozen soils and stated: "The fact that electrical and
thermal osmosis of water is easily accomplished has established the high molecular mobility and continuity of the interfacial water; and the fact that soil particles suspended in ice migrate through the ice along thermal gradients shows that the mineral particles are free of any but the most feeble and transitory connections to the ice phase".

Various mechanisms which might account for the adsorption of water to silicate surfaces are: hydrogen bonding, cation hydration, orientation of H₂O dipoles in an electric field, osmotic pressure and van der Waal's forces. These adsorption forces are responsible for the "structure" of the water next to the silicate surface and are responsible for the freezing point depression and supercooling that has been observed in clay–water systems [Mitchell (1976) pg. 108]. Temperatures above 100°C are required to remove all of the water from the silicate surfaces [Mitchell (1976) pg. 104]. Steinfink and Gebhart (1962) concluded that a normal stress in excess of 200 MPa (30,000 psi) was necessary to remove the next to last layer of water molecules in the interlayer zone between particles of sodium vermiculite. The strength of ice is much less than 200 MPa. In addition, a pressure of 200 MPa is more than sufficient to melt ice at −10°C (≈ 110 MPa, Chamberlain et al. 1972), and hence it would probably crush or melt before actually making direct contact with the silicate surface.

Structure of Pore Ice

Evidence that the pore ice in frozen soils has a normal hexagonal structure was presented by Anderson and Hoekstra (1965). They used x–ray diffraction to study the structure of nucleated ice in pastes of various clay minerals. They detected only the diffraction peaks corresponding to normal hexagonal ice structure. Later studies using nuclear magnetic resonance have reaffirmed the
presence of a hexagonal structure for the ice in frozen soils, (Wu 1964).

Colbeck (1982) conducted a series of experiments to study the macroscopic structure of the pore ice in granular materials. He prepared several partially saturated mixtures of glass beads and mixtures of silica sand with initial water contents of approximately 5% by volume. The sand had a particle diameter of 0.8 mm and the glass beads had diameters of 0.8 mm and 3.0 mm. These specimens were frozen in an open container at $-3{}^\circ C$ and the resulting structure of the pore ice was studied using a microscope. Colbeck observed both monocrytsalline and polycrystalline ice. Only single crystals were observed in the pores of the 0.8 mm glass beads. In the pores of the 3.0 mm glass beads both single crystals and polycrystals were observed. Occasionally single crystals extended into more than one pore. Colbeck concluded that within the pores of a frozen granular material "a wide variety of crystal structures is possible, even within one sample. The structure may depend on the freezing rate, grain size and other variables....".

These observed structures may have been influenced by the limited amount of water that was present in the soil pores prior to freezing. The polycrystalline ice observed in some pores may have been due to the existence of "islands" of beaded water on the surface that were not hydraulically interconnected prior to freezing and thus nucleated independently.

Colbeck (1985) extended these results to include fully saturated glass beads with sizes ranging from 0.4 to 3mm. He used a microscope to follow the progression of the freezing front as it propagated unidirectionally. The ice was nucleated with many small crystals at the top of the specimens. At the freezing front, Colbeck observed much larger crystals than those at the nucleation site. This seems to suggest that the freezing front was advancing due to the growth of
existing crystals rather than the nucleation of new crystals. This is consistent with the presence of long columnar ice crystals which grow following a tortuous path through the pores.

Sayles (1989) in a state-of-the-art report on the mechanical properties of frozen soil reviewed the probable structures for the pore ice in a frozen soil and stated that "In considering the ice component in frozen soil with a massive structure (i.e., soil without segregated ice), it has not been shown conclusively that the polycrystalline ice is randomly oriented in all three directions.... However, in soils with a massive structure the ice could be columnar, following a tortuous path through the soil pores in the direction of heat flow, or it could be small-grained with the c-axis randomly oriented in three directions. Clearly, the two different ice structures would influence the soil behavior differently".

A complete characterization of the structure of the ice in the pores of a given frozen sand would require careful sectioning techniques and etching and replicating procedures. Current efforts in this regard are ongoing at CRREL. The structure of the pore ice in the frozen Manchester Fine Sand tested in this research was not studied.

2.3.3 Overview of the Behavior of Frozen Sand

The behavior of frozen sand has been most frequently studied by uniaxial compression or tension tests either at constant strain rate (strength test) or constant stress (creep test). These two types of tests should yield the same information on the basic behavior of frozen soils so long as the material properties do not vary as a result of the loading conditions and a correspondence is establish along the lines of Mellor and Cole (1982, 1983).

The strength and deformation behavior of frozen sands has been shown to
depend on experimental variables that include the strain rate, confining pressure, temperature, relative density, degree of saturation, and the presence of ionic impurities. Each of these is illustrated by reviewing the results of several testing programs. In general, each of these testing programs has measured the behavior of frozen sands as a function of several variables. This review will be broadly divided into two parts, small strain behavior and large strain behavior. The review will present the effects of the experimental variables and focus on those testing series considered by the writer to be most significant.

Small Strain Behavior

The small strain behavior in this context refers to the behavior which can be described by the elastic constants of Young's modulus (E) and Poisson's ratio (μ), assuming that they are not time dependent. The behavior of frozen sands is very time dependent and permanent deformations can occur even under very small stresses. If, however, the duration and magnitude of the loading are small, the behavior might be approximated using elastic constants.

Overview of Testing Programs

The writer has selected three testing programs for this review of the measured small strain behavior of frozen sand. Table 2.6 summarizes these three programs and includes a description of the material tested, the types of tests employed and the variables which were studied. The materials represented in this table range from a gravelly sand to a medium to fine sand. The testing temperatures ranged from \(-1.1^\circ C\) to \(-27.8^\circ C\). The densities ranged from sand–ice mixtures where none of the particles were in contact to dense sands prepared at 100% relative density. The confining pressures ranged from
unconfined to 1.3 MPa and the salinities ranged from freshwater to 40 g/l salt water.

The testing techniques employed to measure the small strain response have varied from resonant beam vibration tests, (Kaplar 1963), to cyclic triaxial tests, (Vinson 1978), to unconfined compression tests with on-specimen axial strain measures, to acoustic wave propagation tests, (Baker and Kurfurst 1985).

Some uncertainty is associated with a direct comparison of these testing programs because of differences between these techniques. This can be particularly significant in comparing the results from the cyclic triaxial tests by Vinson with the other programs because the former involved considerably lower frequencies of excitation and much larger strain amplitudes. These both have a tendency to increase the amount of plastic deformation occurring during testing. As a result, the measured "elastic" modulus involves more "plastic" straining and is much lower.

**Effect of Temperature**

The testing results from Kaplar (1963) most clearly illustrate the effect of temperature on the small strain behavior of frozen sand. Figure 2.28 plots the Young's modulus (in longitudinal vibration) and Poisson's ratio for the Peabody gravelly sand as a function of temperature. This figure indicates that both of these quantities increased with decreasing temperature. The majority of the increase in Young's modulus occurred in the range from 0 to \(-10^\circ C\), thereafter increasing only slightly to a temperature of \(-28^\circ C\).

The data from Baker and Kurfurst (1985) also illustrate the effect of temperature. Figure 2.29 plots the Young's modulus and Poisson's ratio versus dry density from acoustic wave propagation measurements for temperatures of
-3.2°C and -10.0°C. The Young's modulus was apparently unaffected by temperature, whereas Poisson's ratio increased for a decrease in temperature.

**Effect of Density**

The effect of the density of the frozen sand on the small strain behavior is most clearly demonstrated from the results by Baker and Kurfurst (1985). Refer again to Fig. 2.29. For both temperatures (-3.2°C and -10.0°C), the Young's modulus increased with increasing dry density of the sand. The Poisson's ratio also showed a slight tendency to increase with dry density.

In their unconfined compression tests with on-specimen axial strain measurements, Baker and Kurfurst (1985) measured a Young's modulus that was within the same range as the acoustic wave propagation tests and also exhibited a tendency to increase with increasing dry density. The average Young's modulus from the acoustic wave propagation tests was 27.2±2.2 GPa and the average from the unconfined compression tests during unloading (considered by the authors to be most representative) was 31.5±7.1 GPa. The trend to increase with increasing dry density was not consistent over the full range of densities in the unconfined compression tests.

The data from Vinson (1978) also show this same trend as a function of sand density. Figure 2.30 is a plot of the Young's modulus versus confining pressure for both a dense sand (e = 0.49) and a sand–ice mixture (e = 3.73). The sand particles were not in contact in the "OSL" sand–ice mixture. The dense sand had a modulus 4 to 6 GPa higher than the sand–ice mixture over all confining pressures (but both have low E values compared to other data due to the "plastic" straining previously noted).
Effect of Confining Pressure

The data from Vinson (1978) illustrate the effect of confining pressure on the Young's modulus. In cyclic triaxial experiments, the maximum confining pressure was 1.3 MPa. Refer again to Fig. 2.30. For both the dense sand and the sand–ice mixture, the Young's modulus increased uniformly with confining pressure. The amount of increase was approximately 5 GPa for the dense sand and 2 GPa for the sand–ice mixture.

Effect of Salinity

Baker and Kurfurst (1985) investigated the effect of pore fluid salinity on the small strain response of frozen sand. Figures 2.31 and 2.32 respectively are plots of the Young’s modulus and Poisson’s ratio for frozen sand in a loose and dense state at temperatures of -3.2°C and -10.0°C for pore fluid salinities ranging from 0.5 to 40 g/l. These figures indicate that increasing the salinity of the pore fluid has a dramatic effect on the Young’s modulus and only a slight effect on the Poisson’s ratio. This was true for all densities and temperatures. The measured Young’s modulus decreased dramatically with increasing salinity from 0.5 g/l to 5 g/l. Beyond that point, the decrease was less dramatic with increases in salinity. The Poisson’s ratio increased slightly with increasing salinity.

Figure 2.31 also indicates that in the range of salinities from 0.5 to 2.0 g/l, changes in temperature had a more dramatic effect on the magnitude of the Young’s modulus than had occurred in the freshwater tests (Fig. 2.29). The specimens at -3.2°C had Young’s moduli which were much less than the specimens at -10.0°C.
Summary of Measured Small Strain Behavior

Table 2.7 is a summary of the measured small strain behavior as reported in these testing programs, both in terms of the trends and the ranges in the measured values. According to these results, the Young's modulus for soil without salt appears to:

- increase moderately with decreasing temperature, and then level off (Fig. 2.28);
- increase slightly with increasing sand density (Fig. 2.29);
- and perhaps increase with confining pressure (although the results in Fig. 2.30 include plastic strains). All of these trends are small compared to the effect of increasing salinity, which causes a dramatic decrease in modulus (Fig. 2.31).

Poisson's ratio appears to follow similar trends regarding temperature and density, but the reverse of the effects of salinity appears to increase with increasing density of the sand, decreasing temperature and decreasing salinity. The Poisson's ratio appears to increase with increasing sand density, increasing salinity and increasing temperature.

At a temperature of $-10^\circ C$, a representative Young's modulus for a frozen freshwater sand appears to range from 24 to 40 GPa and a representative Poisson's ratio is in the range of 0.26 to 0.32.

Large Strain Behavior

Figure 4.7 in this thesis presents idealized stress–strain curves for a frozen sand tested in triaxial compression. It includes curves for various levels of axial strain to depict the full range in behavior. In frozen sands, a nonlinear response begins at very small strains, circa 0.0001 (Fig. 4.7a). There is a significant amount of nonlinear behavior up to axial strains in the range of 0.005, when the most significant amount of yielding occurs (Fig. 4.7b). The writer has named this region of most significant yielding as the upper yield region. Note that in
the frozen soil literature this has been referred to as the first yield or lower yield point. Depending on the relative density, confining pressure, strain rate, and temperature among other factors, this upper yield region is followed by either strain hardening or strain softening (Fig. 4.7b). In some cases, particularly at higher confining pressures, the strain hardening may continue to very large strains. At low confining pressures the post upper yield strain hardening reaches a peak and is followed by strain softening to large strains (Fig. 4.7c).

The large strain behavior which is reviewed in this section considers the magnitude of the upper yield stress, the peak strength and the axial strain at the peak strength. These are presented, in so far as is possible, in terms of the effect of confining pressure, strain rate, relative density or volume fraction of sand, temperature, salinity and degree of saturation. The literature is still inconclusive as to the effect of each of these on the stress–strain behavior. A significant advance in this regard is presented in Chapter 5. Table 2.8 is a summary of the principal testing programs used by the writer to evaluate and illustrate the large strain behavior of frozen sand. For convenience, each program has a letter designation.

**Upper Yield Stress** (effect of confining pressure)

Very few of the published data on frozen sands exhibit a distinct upper yield stress. Data presented by Parameswaran (1980), Parameswaran and Jones (1981) and Chamberlain et al. (1972) are notable exceptions. These indicate a sensitivity of upper yield stress to confining pressure.

Inadequate axial strain measuring techniques in the Parameswaran et al. data negated the possibility of making a study of the level of axial strain at the upper yield stress. The axial strain at the upper yield stress was between 1 and
2% up to confining pressures of 138 MPa in the Chamberlain et al. data. At higher confining pressures, the axial strain at the upper yield stress increased to between 2 and 5%. However, these data are also subject to some uncertainty because the axial strains were not made directly on the specimen.

Figure 2.33 summarizes two testing programs run on uniform, medium dense Ottawa sand in terms of the magnitude of the upper yield stress as a function of confining pressure. This plot indicates that the upper yield stress increased with confining pressure in the range of 0 to 40 MPa (certainly for program G and less consistent for program L). Above that level, the upper yield stress decreased with confining pressure. The Chamberlain et al. (1972) data, which encompass a very large range in confining pressures, show that the upper yield stress decreases dramatically from a confining pressure of near 40 MPa to 100 MPa. At approximately 100 MPa, they reasoned that a phase change took place and all of the ice changed to water. Above 100 MPa, the upper yield stress, which was the maximum stress, increased again with increases in confining pressure.

Peak Deviator Stress (effect of strain rate)

Figure 2.34 is a summary of the effect of strain rate on the magnitude of the peak deviator stress for six testing programs. The points on the plot are not actual data points but rather they define the linear portion of the data. These are all unconfined compression strength tests run at a temperature of $-10^\circ$C on dense to very dense sands. This figure shows that in the range of strain rates from roughly $10^{-6}$ to $10^{-3}$/sec, the peak strength increases uniformly with strain rate for most of the testing programs. The power law coefficients range from 7.4 to 16. For two of the testing programs, namely Yuanlin et al. (1989) and Bragg
and Andersland (1980), the power law coefficient became infinite above a certain level of strain rate, i.e., an insensitivity to strain rate. Although seemingly inconsistent with the other testing programs, this does not necessarily surprise the writer. As was illustrated in Fig. 2.20, the power law coefficient in ice approaches infinity at high strain rates ($10^{-3}$ to $10^{-2}$/sec). It may also become negative in some cases (Section 2.2.2). Since frozen soils inherit their rate sensitivity from the pore ice, then a power law coefficient of infinity at very high strain rates would seem to be a reasonable observation. However, it is not clear why similar dense sands exhibit such different behaviors at the same strain rate and temperature.

**Peak Deviator Stress (effect of confining pressure)**

Figure 2.35 is a summary of the effect of confining pressure on the magnitude of the peak strength for three testing programs on "saturated" dense sands. The range in confining pressures is 0 to 10 MPa, the temperatures range from $-3.85$°C to $-12$°C and the strain rates vary from $4.5 \times 10^{-6}$ to $1.1 \times 10^{-2}$/sec. This figure indicates that for all of these programs, the peak strength increased with confining pressure in this range. Several of the programs showed a linear increase while others exhibited a pronounced curvature. Total stress friction angles for the data presented ranged from 0° in the Sayles data at the highest strain rate and confining pressures above 3 MPa to approximately 28° in the Alkire and Andersland (1973) data.

Figure 2.36 is a summary of the peak deviator stress versus confining pressure for confining pressures up to approximately 280 MPa. Both the Chamberlain et al. (1972) and the Parameswaran and Jones (1981) data exhibit the same general trend, i.e., an increase in peak deviator stress with confining
pressure up to $\sigma_c = 40$ to $50$ MPa and then a decrease. The initial total stress friction angle for the Chamberlain et al. (1972) data was approximately $19^\circ$ up to $40$ MPa, while that for the Parameswaran and Jones data was approximately $13^\circ$ over the same range in confining pressure. The Chamberlain et al. (1972) data exhibited a second increase with confining pressure above $\sigma_c = 110$ MPa.

The decrease in strength with confining pressure in the range of $40$ to $100$ MPa, that was evidenced in both of these testing programs, can be attributable to a gradual pressure melting of the pore ice. The strengthening in the Chamberlain et al. (1972) data above $110$ MPa is probably due to increases in the effective stresses with confinement because of a B-value less than 1 for the now unfrozen sand (Section 2.1.3).

**Peak Deviator Stress** (effect of sand content or relative density)

Figure 2.37 is a summary plot of the effect of sand content on the peak strength of sand–ice mixtures and frozen sand from testing programs reported by Goughnour and Andersland (1968) and Jones and Parameswaran (1983) for various temperatures and strain rates on coarse and medium Ottawa sand, respectively. All tests in this figure are for unconfined compression. These data exhibit the same trend, i.e., an approximately linear increase with increasing sand volume fraction up to about 40% sand. Above this level, the rate of strengthening increases dramatically but is still linear. At 40% sand by volume, the particles are just beginning to come into contact. It is important to note that not only do the particles begin to come into contact at 40% sand, but also the specimen preparation technique changes from freezing a mixture of snow and sand to unidirectional freezing of saturated sand. This might have a dramatic effect on the structure of the ice in the pores (Cole 1990 personal
Baker and Kurfurst (1985) present unconfined compressive strength data for frozen Ottawa 16–100 sand at relative densities ranging from 26% to 100%. These tests were conducted at a temperature of $-10^\circ C$ and a strain rate of $1.67 \times 10^{-4}$/sec. Figure 2.38 is a summary of these data which shows that for relative densities up to 50%, there was very little change in peak strength (the peak strength occurred at the upper yield stress for these tests). Between 50% and 80% relative density, the strength increased with increasing relative density and from 80% to 100% relative density, a greater rate of strengthening occurred. The authors note that for relative densities up to 50%, failure occurred at the upper yield point (knee of the stress–strain curve) and that for higher relative densities, failure occurred at the second peak. Section 5.4.1 presents data that also indicate that the magnitude of the deviator stress at the knee of the stress–strain curve is insensitive to changes in relative density.

**Peak Deviator Stress** (effect of temperature)

Figure 2.39 summarizes the effect of temperature on the uniaxial compressive strength of frozen sands by Bourbonnais and Ladanyi (1985). In general, the variation of strength with temperature over the full range of temperatures is not linear, but in the range of temperatures from 0 to $-30^\circ C$ it might be approximated as linear. These data have a similar trend that indicates that the increase in strength with decrease in temperature becomes less significant at very low temperatures. In the case of Bourbonnais and Ladanyi (1985) and Biarez et al. (1967), there was a weakening effect with decreasing temperature at extremely low temperatures. In some of these tests, most of the strength gain with decrease in temperature had occurred by a temperature of
—40°C, i.e., Sayles (1966) and Bourbonnais and Ladanyi (1985). Most of the data indicate that the strength gain with decreasing temperature in the range from 0 to —20°C is 0.7±0.3 MPa/°C for strain rates of 3±2x10⁻⁴/sec.

**Peak Deviator Stress (effect of degree of saturation)**

Figure 2.40 plots the peak deviator stress versus degree of saturation for triaxial compression tests on frozen sand reported by Alkire and Andersland (1973). This demonstrates that the degree of saturation has a dramatic effect on the peak strength. Over all confining pressures, a decrease in saturation from 97 to 55% lead to a decrease of approximately 5 MPa in the peak strength.

Ting (1981) reported similar results from uniaxial compression creep tests on frozen Manchester Fine Sand. Figure 2.41 is a plot of the minimum creep rate versus applied axial stress for various degrees of saturation. Assuming a correspondence between the minimum strain rate and the peak strength in a constant strain rate test (Section 2.2.2), this figure shows that a decrease in saturation lead to a significant decrease in strength.

**Peak Deviator Stress (effect of salinity)**

Baker and Kurfurst (1985) measured the unconfined compressive strength of frozen Ottawa sand as a function of the salinity of the pore fluid. Figure 2.42 plots the peak unconfined compressive strength as a function of pore fluid salinity for both a dense and a medium dense sand. This figure demonstrates that the introduction of salt into the pore fluid dramatically decreases the peak strength. This effect is most pronounced in the range from 0 to 2 g/l. The dense specimen tested with fresh water had a peak strength of approximately 20 MPa and a specimen with 0.5 g/l salinity had a peak strength of approximately 8 MPa. As
the salinity increased, the strength continuously decreased. At a salinity of 30 ppt the peak strength was approximately 2 MPa. Baker and Kurfurst also observed that for salinities greater than approximately 1 ppt, the strengths of both sands were almost identical. The presence of salt will increase the amount of unfrozen water at the ice-silicate interface and between the pore ice crystals and lead to the formation of brine pockets in the ice. This can lead to a dramatic decrease in the strength of the ice and the ice-silicate interface and can enhance grain boundary sliding and diffusional mechanisms (Section 2.2.4).

**Peak Deviator Stress (effect of confining pressure on power law coefficient)**

Figure 2.43 is a plot of the power law coefficient versus confining pressure from the Shibata et al. (1985) data. To the writer's knowledge, this is the only data set in the literature which can illustrate this effect. There is quite a bit of scatter in the data. Most of the data fall between the \( n = 10 \) to 16 range. The data at \(-30^\circ C\) appear anomalously high. There is no consistent trend in the power law coefficient with confining pressure. This will be discussed in greater detail in Chapter's 5 and 6 in light of the data presented in this thesis.

**Peak Deviator Stress (effect of temperature on the power law coefficient)**

Figure 2.44 is a presentation of the effect of temperature on the magnitude of the power law coefficient for six testing programs in the literature. In the majority of the testing programs, the power law coefficient increased with decreasing temperature to \(-10^\circ C\); the Sayles and Epanchin (1966) and the Shibata et al. (1985) data are exceptions. For temperatures lower than \(-10^\circ C\), three of the programs indicated a decrease in power law coefficient with decreasing temperature. In four of the six programs, the power law coefficient at
-2°C began to approach that of ice (n=3) for the ductile flow regime.

**Peak Deviator Stress (behavior in tension)**

Perkins and Ruedrich (1973) report the behavior of a frozen sand in tension. The strain rate was varied from $1.67 \times 10^{-6}$/sec to $1.67 \times 10^{-3}$/sec and temperature ranged from $-1.1^\circ C$ to $-13.3^\circ C$. Figure 2.45 plots these results. For strain rates greater than $1.67 \times 10^{-4}$/sec, the tensile strength was independent of strain rate. At slower strain rates, the tensile strength decreased with decreasing strain rate. The power law coefficient for the peak tensile strength at a temperature of $-13.3^\circ C$ was 7.98, while at a temperature of $-2.2^\circ C$ it was 3.06. For temperatures between $-6.7^\circ C$ and $-13.3^\circ C$, the tensile strength was almost independent of temperature, while at warmer temperatures the tensile strength increased with decreasing temperature. These general trends appear to follow that for granular ice, e.g., the results in Fig. 2.20.

**Axial Strain at Peak Deviator Stress**

Changes in the axial strain at the peak deviator stress can indicate a change in the behavioral regime. In tests on frozen sands where there is a tendency for a double peak behavior (i.e., an upper yield point followed by strain hardening to a peak strength), a dramatic decrease in the failure strain indicates a transition from a ductile behavior to a brittle behavior where the peak strength occurs at the upper yield stress. Of the strain data reported in the literature, only that of Baker and Kurfurst (1985) was obtained with on–specimen axial strain measurements. Thus, in the writer's opinion, the magnitudes of the axial strain at the peak strength from their data are the most reliable. The other data presented in this section may contain significant seating errors and are discussed
only to support the general trends. The reader should not rely on the magnitudes of the axial strain levels from the other programs.

**Axial Strain at Peak Deviator Stress (effect of sand density)**

Baker and Kurfurst (1985) present data on the unconfined compression strength testing on frozen Ottawa sand over a wide range of dry densities. These tests were conducted with on-specimen axial strain measurements. Figure 2.46 plots the axial strain at peak strength versus dry density for their tests. This figure demonstrates that in the range of dry densities up to 1630 kg/m\(^3\) (\(D_r = 58\%\)), the axial strain at peak was approximately 0.25\%. They noted that in this region, the upper yield stress was the peak strength. Above this dry density, the axial strain at peak strength increased with dry density up to approximately 3.3\% at 1780 kg/m\(^3\) (\(D_r = 100\%\)).

**Axial Strain at Peak Deviator Stress (effect of temperature)**

The axial strain at peak strength in unconfined compression as a function of temperature from Bourbonnais and Ladanyi (1985) is presented in Fig. 2.47. This indicates that for a temperature of \(-6.7^\circ C\) (the warmest temperature investigated), the axial strain at peak was approximately 2\% (the strain at the upper yield point was 0.5\%). For all cooler temperatures, the axial strain at peak strength was constant at 0.5\% and independent of strain rate in the range of \(3\times10^{-3}/\text{sec}\) to \(3\times10^{-5}/\text{sec}\).

**Axial Strain at Peak Strength (effect of strain rate)**

The effect of strain rate of the axial strain at the peak unconfined strength can be clearly indicated by the Bragg and Andersland (1980) data. Figure 2.48
plots the failure strain versus strain rate from these tests over temperatures from $-2^\circ$ to $-15^\circ$ C. This demonstrates that in the range of strain rates from $5 \times 10^{-7}$ to $10^{-4}$/sec, the axial strain at failure was approximately constant at $4\pm1\%$ as a function of strain rate and temperature. At a strain rate of $10^{-4}$/sec, there was an abrupt transition and the failure strain decreased to between 0.9 and 0.5% again with no trend in strain rate or temperature.

A summary of the large strain behavior is given in Section 2.3.6.

2.3.4 The Mechanics of Frozen Sand

This section discusses the mechanisms of deformation that enable a frozen sand to accommodate an arbitrary change in shape. These include those associated with the sand skeleton and the pore ice matrix and those that occur as a result of composite material behavior, i.e., sand–ice interaction. The deformational mechanisms in unfrozen sands were treated in Section 2.1.8 and the deformational mechanisms in granular ice were treated in Section 2.2.3.

The primary focus of this review is to discuss possible deformation mechanisms that are a result of the interaction between the pore ice and the sand particles. A qualitative treatment of the mechanics of particle motion in frozen sands will first be presented. This presentation highlights what, in the writer's opinion, is the major difference between the mechanics of deformation in the particle skeleton for frozen and unfrozen sands. This is followed by a review of the various mechanisms that have been previously proposed in the literature.

Stresses and the Equilibrium of an Individual Sand Particle

The mechanics of particle deformation in a frozen sand are different from those in unfrozen granular materials. As discussed in Sections 2.1.2 and 2.1.8,
the mechanics of deformation in unfrozen sands that experience no particle crushing is governed by the magnitude and direction of the intergranular contact forces. These contact forces, when integrated over a large area as explained in Section 2.1.2, describe an equilibrium stress state in terms of both normal and shear stress. In contrast to an unfrozen sand, a saturated frozen sand is a continuous medium (i.e., the pore contain and ice matrix that has a significant shear strength). This means that both the sand particles and pore ice participate actively in the stress transferring process. This results in significant differences between the types of forces that are carried by the sand skeleton.

In a frozen sand, the portion of the total applied stress which is transmitted by the sand skeleton via intergranular contact forces may not form an equilibrium stress state as is the case in unfrozen sands. This is illustrated qualitatively in Fig. 2.49. This figure is a two-dimensional free-body diagram of an individual sand particle in a frozen sand matrix. The particle is idealized as a five-sided polygon. One set of forces acting on this particle are due to the intergranular contacts with adjacent sand grains and these are depicted as solid vectors. Another set of forces acting on this particle are those transmitted across the ice–silicate interface, that may have both normal and shear components, Jellinek (1962). These are depicted as broken arrows. The writer recognizes the inaccuracy associated with this schematic diagram in the assumption that five intergranular forces are coplanar. In actuality, there are probably not even five particles in contact with an individual grain considering all three dimensions. This presentation is only given to discuss the idea of particle equilibrium and the 2-D simplification is made for convenience.

Since ice can support shear stresses, this means that in general, neither the intergranular contact forces when considered separately, nor the ice–silicate
interface forces when considered separately, will form an equilibrium force condition on the particle. But when these are considered jointly and if the particle is not accelerating, the net resultant of all of the forces acting on the particle must be zero (neglecting the weight of the sand particle), i.e., the total forces must constitute an equilibrium condition.

As explained in Section 2.2.3, ice will creep even under extremely small shear stresses. This means that the magnitude of the forces transmitted across the ice-silicate interface are highly time dependent and the state of stress in the pore ice will eventually tend towards hydrostatic (this neglects the small shear stresses associated with the curvature of the individual ice crystals due to surface tension forces as will be presented subsequently). When external stresses are applied to a frozen sand, a portion of that stress will be carried by the pore ice. Some fraction of these pore ice stresses must be transmitted across the ice-silicate interface. Over time, if the strain rate goes to zero, the shear stresses being carried by the ice matrix decrease towards zero and a hydrostatic state of stress will be approached in the pore ice. For this limiting condition, both the ice-silicate interface forces (now hydrostatic) and the intergranular contact forces, when considered separately, will each be in equilibrium and the state of stress determined from the intergranular contact forces will be similar to that in an unfrozen sand under the sand total applied stresses. Such a condition may only apply under isotropic stresses after consolidation when the strain rate is very close to zero. In general however, under the action of external stresses, the strain rate does not go to zero and there will always be some shear stresses acting on the pore ice and hence the intergranular contact forces will not be in equilibrium when considered separately.

If the intergranular contact forces do not represent a state of equilibrium
at the particle level, then integrating these forces over a large area as described in Section 2.1.2 will generate a state of stress that is also not in equilibrium. Thus, unless the intergranular contact forces are in equilibrium, the state of stress determined from these will not be similar to the state of stress acting on the same sand skeleton in an unfrozen state. This means that it might be possible for a frozen sand to undergo a strain history which would be impossible for the same system in an unfrozen state. In the writer's opinion, this represents a significant difference between the mechanics of particle deformation in frozen sands as opposed to unfrozen sands.

Nature of Stresses in Ice and Unfrozen Water in Frozen Sand

(portions abstracted from Ladanyi 1985)

Under isothermal conditions, when the frozen sand system is in equilibrium, there will be a pressure differential between the hydrostatic (mean = octahedral normal) pore ice pressure and the hydrostatic pressure in the unfrozen water film surrounding the silicate particles due to the curvature of the ice grains and the surface tension of the ice–water interface. This pressure differential can be described by the Kelvin equation,

\[(p_i - p_w) = 2\sigma_{iw}/r_c\]  \hspace{1cm} (2.3.2)

where \(p_i\) is the hydrostatic pressure in the ice, \(p_w\) is the hydrostatic pressure in the unfrozen water film, \(\sigma_{iw}\) is the surface tension of the ice–water interface and \(r_c\) is the radius of curvature of the ice–water interface. This equation is derived by considering the equilibrium of a hemisphere of ice surrounded by unfrozen water. Figure 2.50 is a free body diagram of such a condition which illustrates how the surface tension of the ice–water interface increases the pressure in the ice above that in the unfrozen water. Using \(\sigma_{iw} = 3.31\times10^{-3} \text{ N/m}\) (Miller 1973) and
radii of curvature of 0.5mm and 0.05 mm yields a range of \( p_i - p_w \) from 13.2 Pa to 132 Pa. This pressure differential is very small compared to the strength of ice or frozen sand. Miller (1973) suggested that "the ice–water interface will alter its shape by melting or freezing to achieve an equilibrium interface which will obey the Kelvin equation". The Kelvin equation represents an equilibrium condition. In a frozen sand where the pore spaces are irregularly shaped and the curvature of the surface of the ice crystals may also be irregular, these pressure differentials and the corresponding induced shear stresses on the pore ice may drive grain boundary migration processes. Thus, the structure of the pore ice in a frozen sand may be continuously changing over time.

Ladanyi (1985) explains that the Kelvin equation "has been used extensively in some earlier studies for explaining the water suction phenomena at the freezing front. It also implies that, for mechanical stress changes, it is often sufficient to consider what happens in only one of the two phases".

If the deformational process is not isothermal, the pressure differential between the pore ice and the unfrozen water film can be described by the Clausius–Clapeyron equation

\[
\frac{dp_w}{\rho_w} - \frac{dp_i}{\rho_i} = L \frac{dT}{T}
\]

where \( p_w \) and \( p_i \) are again the pressures in the water and ice, \( \rho_w \) and \( \rho_i \) are the densities of water and ice, respectively, \( L \) is the latent heat of fusion of water, \( dT \) is the difference between the normal freezing temperature of water and the actual temperature of the system and \( T \) is the temperature of the system. This means that if phase changes are occurring during the deformation of frozen soils, these will be associated with pressure differentials between the ice and unfrozen water.
Deformational Processes in Frozen Sand

As a frozen sand mass undergoes an arbitrary change in shape, clearly this must be accommodated by motions of the sand grains and the pore ice. A direct comparison between the motion of the grains in a frozen and unfrozen sand may not be possible because the nature of the intergranular contact forces that control these motions in an unfrozen sand may not be related to those in a frozen sand as was explained in the previous discussion. Table 2.9 presents a summary of the possible deformational processes in frozen sands.

Deformation of Sand Skeleton

In Section 2.1.8, the deformational processes in an unfrozen sand mass were reviewed in terms of recoverable and irrecoverable strains. The recoverable strains were associated with the elastic deformation of the individual particles under the action of the intergranular contact forces. The irrecoverable strains were associated with slip mechanisms which could be due either to particle crushing or rigid body particle motions. The published research on rigid particle motions tends to favor sliding over rolling mechanisms in unfrozen sand. These sliding motions might occur as the relative motions of instantaneously rigid groups of particles.

Whether or not these same deformational mechanisms may be occurring in the skeleton of a frozen sand is somewhat debatable in the writer's opinion. Clearly, elastic deformations of the individual sand grains can occur under the action of both the intergranular contact forces and the forces transmitted between the ice–water–silicate interface. Measurements of crushing occurring during the deformation of a frozen sand have been presented by Chamberlain et al. (1972). In the writer's opinion, the sliding motion due to the relative motions
of instantaneously rigid groups of particles is probably not occurring in frozen sand. In fact, due to the nature of the stress transferring mechanisms, the actual motions of the individual particles may not even resemble those of an unfrozen sand. For example, Shibata et al. (1985) reported an approximately 2% increase in volume at an axial strain of 10% for a dense frozen sand at a temperature of \(-10^\circ\) C and a confining pressure of 4.9 MPa, while Lee and Seed (1967) reported a 4% decrease in volume at 10% axial strain for a dense sand in drained triaxial compression at an effective confining pressure of 4.0 MPa.

Although the actual deformational mechanisms that are experienced by the sand skeleton in frozen sand are uncertain, they clearly can be much different from those occurring in an unfrozen sand.

**Deformation of the Pore Ice**

The deformational processes that can occur in a polycrystalline ice were reviewed in Section 2.2.3. From Table 2.2, these included: the elastic straining of the lattice; anelastic deformations due to the development of internal stress fields acting on the dislocations; plastic deformation due to diffusional mechanisms and dislocation mechanics; evolving microstructure due to dynamic recrystallization and associated grain boundary migration; and internal cracking and fracturing mechanisms. The writer assumes that the same deformational processes apply to the pore ice of a frozen sand. However, the relative importance of the various processes may be different from that for normally tested polycrystalline ice (at the same temperature and strain rate) due to uncertainties in the structure of the pore ice, the complicated states of stress that exist in the pore ice, the effect of the unfrozen water film, and the presence of the silicate particles.

The following discussion of possible mechanisms represents the opinions of
the writer (some are supported by quotes from the literature). These mechanisms and under what conditions they operate in a frozen sand are still very poorly understood. A considerable amount of both theoretical and experimental research must be performed in order to develop these ideas from first principles and to develop definitive experimental checks (see Section 2.2.3).

The high values of Young’s modulus for frozen sand (Section 2.3.3) which have been reported in the literature (e.g., several times higher than that of polycrystalline ice) is consistent with the notion that both the ice crystals and silicate particles are experiencing elastic deformations and behaving as a composite material. Section 6.2 will consider these elastic deformations in greater detail.

It is not clear to the writer how the presence of significant amounts of unfrozen water at the ice-silicate interface affects the motions of dislocations in ice. If the ice-water-silicate interface is less effective in acting as a barrier for the pileups of dislocations, then the internal stress fields which are built up due to these pileups may be much less that in a polycrystalline ice. This might not only decrease the amount of anelastic straining attributable to the ice matrix, but might also serve to increase the mobility of dislocations in their glide planes by decreasing the magnitude of the internal stress field.

The presence of unfrozen water may also significantly influence the diffusional type mechanisms in the pore ice. In discussing this possibility, Parameswaran (1980) stated that "In frozen sand, the fluid phase simply replaces the grain boundary as a diffusive path". This diffusional flow mechanism is similar to that proposed by Coble (1963). The unfrozen water may also enhance grain boundary sliding processes. Researchers have argued that the function of the unfrozen water film as a diffusive path can be used to explain the creep
behavior of frozen soils (Hoekstra 1969, Andersland 1989, Sayles 1968 among others). As discussed in Section 2.2.3, diffusional type mechanisms lead to a predicted power law coefficient of 1 and generally are thought to operate in ice only at very low shear stresses or strain rates. That these types of mechanisms are dominant in frozen soils is somewhat debatable. To the writer's knowledge, there are no reported measurements of power law coefficients in frozen soils that begin to approach 1. Thus, diffusional mechanisms cannot be the dominant deformational process, but may serve to weaken the frozen soil when considered with other deformational processes.

The deformation of the pore ice in frozen sand may also be accompanied by recrystallization. As explained in Section 2.2.3, these processes depend on the rate of migration of the grain boundaries. Smaller size crystals are more susceptible to these processes because less time is required for the grain boundaries of the new or favored crystals to grow and dominate the polycrystalline mass. The unfrozen water film may also aid in the enhancement of these processes. The extent to which these processes operate in frozen sand has not been determined.

Internal cracking and fracturing mechanisms must be operating during the deformation of frozen sands when a net volume increase is occurring. The nature of the cracks, i.e., along ice grain boundaries, through individual ice crystals, along the ice–water–silicate interface or a combination of these, has not been determined. In the writer's opinion, this represents a significant source of uncertainty in understanding frozen sand behavior.

Theories to Describe Deformational Processes

Various researchers have attempted to describe the deformation of frozen
soils as a thermally activated process. Bourbonnais and Ladanyi (1985) review several prior attempts to use the rate process theory. They conclude that "The use of the rate process theory for modeling the temperature-dependence of the strength of frozen sands.... does not seem.... to be justified.... an empirical relationship giving the best fit to the results, like a simple power law, is more convenient for that purpose".

Martin et al. (1981) conducted an extensive experimental testing program to check the validity of rate process theory (RPT) and concluded that although their data were generally consistent with the rate process theory, there might be other possibilities that better describe the deformational processes in ice. The temperature dependency might adequately be described by an Arrehenius type equation. Martin et al. (1981) also concluded that "there are multiple mechanisms of approximately equal importance operating during creep deformation of frozen sand".

### 2.3.5 Mechanisms of Strength in Frozen Sand

Ting et al. (1983) proposed a series of possible mechanisms which might explain the strength of frozen sands. Their paper is one of the most thorough treatments of the subject in the literature and is based on the modeling of a frozen sand as a particulate material. Their proposed mechanisms can be divided into three broad categories: those that are involved with the pore ice strength; those that are concerned with the sand strength; and those that are due to the interaction between the pore ice and the sand skeleton.

Ting et al. (1983) presented a mechanism map in an effort to delineate the relative contributions of their proposed strengthening components as a function of the volume fraction of sand. This figure has been reproduced as Fig. 2.51. As
the volume fraction of the sand increases from 0% to 40%, the linear increase in strength observed by Goughnour and Andersland (1968) is assumed to be due entirely to ice strengthening mechanisms. These continue to strengthen the sand–ice mixture in a linear fashion up to a relative density of 100%. At a volume fraction of 40% when the sand particles are just coming into contact, there is an enhanced strengthening effect due to the interaction between the ice and the sand. Table 2.10 is a summary of their proposed mechanisms and conclusions. These are each discussed in this section.

Researchers have concluded that the behavior of the pore ice tends to dominate the behavior of frozen soils at small strains up to the first peak (i.e., the writer’s upper yield stress) and the behavior of the soil skeleton tends to dominate at large strains [Parameswaran (1980), Bragg and Andersland (1980), Andersland (1989) among others].

Ice Strength and Strengthening 1, 3a

Mechanisms that are responsible for the strength of granular ice were treated in detail in Section 2.2.4. According to Hertzberg (1989), strengthening mechanisms in crystalline materials can be broadly classified into intrinsic and extrinsic mechanisms. Intrinsic mechanisms are those associated with the processes occurring in the crystalline material. For polycrystalline ice, these were treated in Section 2.2.4. The addition of high strength fibers to metals can significantly increase their strength. This type of strengthening can be viewed as extrinsic because "the load of the matrix is transferred to the high strength fibers" (Hertzberg 1989). In the case of frozen sands, the addition of silicate particles to the ice matrix can constitute such an extrinsic strengthening mechanism.
Ting et al. (1983) suggested that the ice strengthening depicted in Fig. 2.51 could be due to three possible mechanisms: strengthening due to a change in structure in the pore ice; strengthening due to differing states of stress and deformational constraints; and strengthening due to differing strain rates in the pore ice.

**Strengthening Due to a Change in Structure 3ai**

The structure of the ice in the pores of a frozen may be different from that of normally tested granular ice. A proposed structure for a frozen sand was discussed in Section 2.3.2. There is general agreement in the literature that the molecular structure of the ice in frozen soil is not affected by the presence of the silicate particles. The oxygens are coordinated in a hexagonal structure. Ting et al. (1983) hypothesized that the macrostructure was that of a granular ice, but having a much smaller grain size (i.e., the maximum grain size is less than the maximum pore diameter, Martin 1981). As was explained in Section 2.3.1, the texture and fabric of the pore ice may be very different from the hypothesized granular ice. It may consist of long tortuous crystals which run through several pores (Colbeck 1985 and Sayles 1989).

If the structure of the pore ice is polycrystalline and the grain size is much smaller than the pores, in frozen MFS this means that the average grain size is approximately one order of magnitude smaller than the finest grained isotropic ice that has been reported in the literature. As explained in Section 2.2.4, the effect of a decreasing grain size, into the range of probable grain sizes in frozen sand, on the strength of polycrystalline ice has not been reported in the literature. It is not clear whether the strength increases or decreases. Comparison of the results of unconfined testing on frozen sands reported by Orth
(1985), $D_{50} = 0.6\text{mm}$, and Yuanlin et al. (1988), $D_{50} = 0.3\text{mm}$, at a temperature of $-10^\circ\text{C}$ and a strain rate of approximately $1\times10^{-6}/\text{sec}$ indicates that coarser sands have higher strengths for the same coefficient of uniformity, $9\text{ MPa}$ compared to $5.2\text{ MPa}$. But this trend is not consistent over all reported data.

If the structure of the ice in the pores of a frozen sand is that of single crystals of ice extending through several pores, the behavior of the frozen sand may be strongly affected (Sayles 1989). This is due to the anisotropic nature of the ice crystal as was explained in Sections 2.2.1 and 2.2.2.

**Differing Stress States and Deformational Constraints, 3aii**

Ting et al. (1983) suggested that "Ice strengthening could be due to.... stress states and deformational constraints imposed on ice in the pores differing from those commonly used in ice testing". This refers to the fact that the actual state of stress and imposed boundary conditions acting on the pore ice are very complex and probably do not resemble the state of stress imposed on an ice specimen in triaxial compression or tension testing.

**Effect of Differing Strain Rates in the Pore Ice, 3aiii**

Ting et al. (1983) refer to Goughnour and Andersland (1968) who postulated that one of the strengthening mechanisms which they observed in their sand–ice systems over that of pure ice was due to a higher effective strain rate in the pore ice. They explained that "The addition of a few sand particles requires the sample deformation to take place in a smaller volume of ice matrix because the sand particles do not deform under these loads. For the same axial strain–rate this requires a higher deformation rate in the ice matrix, hence a higher strength". 
A simple computation based on an power law coefficient for ice of $n = 3$ indicates that such a mechanism is reasonable based on the observed strength increase (Fig. 2.37) by Goughnour and Andersland (1968). But, this mechanism neglects the possible composite interaction between the sand and ice in which the stiffer sand particles may also serve to reinforce the ice matrix. This interaction is discussed in more detail in Section 6.2.

**Sand Strength, 2**

Mechanisms of sand strength were discussed in Section 2.1.9. The magnitude of sand strength is very dependent on the magnitude of the effective stresses acting on the sand skeleton. The strength of a sand was presented in terms of the Rowe (1962) decomposition, i.e., three components associated with the interparticle (true physical) friction, particle interference and dilation. An assessment of the contribution of these components on the strength of a frozen sand would rely on a measure of the effective (intergranular) stresses acting on the sand skeleton and the volume strain. Such a direct measure of the effective stresses is impossible. Moreover, according to the discussion in Section 2.3.4, these "effective" stresses may not even be related to those acting in the unfrozen sand system. Despite these difficulties, there is clearly an effect of the sand structure on the strength of a frozen sand as illustrated by Goughnour and Andersland (1968), Jones and Parameswaran (1983) and Baker and Kurfurst (1985). Refer to Figs. 2.37 and 2.38.

**Interaction Between Ice and Sand, 3**

**Tension in the Unfrozen Water Film, 3c**

Ting et al. (1983) suggested that a possible strengthening mechanism in
frozen sands could be due to a tension in the unfrozen water film as a result of a physico-chemical interaction between the ice and silicate surface. They stated that the magnitude of this tension is dependent on the thickness of the unfrozen water film which surrounds the silicate particles and is therefore determined by the mineralogy of the silicate particles, the presence on impurities and the temperature of the sand–ice system. The greater the tension in the unfrozen water film, the greater will be the strengthening effect.

Measurements of the induced tension in pore water of a soil during freezing have been reported by Lunardini et al. (1982) and many others. Ting et al. (1983) comment that "While the extensive literature dealing with ice nucleation and frost heaving indicate that a substantial suction force can be generated in the adsorbed water film around a silicate particle during the freezing process under certain conditions, no data exist to substantiate its existence after the sample has equilibrated to the ambient temperature and applied stress".

Ting et al. (1983) reasoned that such a decrease in pressure in the unfrozen water film could lead to a strengthening effect due to a resulting positive increment in the effective confining stress acting on the sand skeleton. If this mechanism were dominant, they argued that the strengths of partially and fully saturated frozen sands should be similar. They concluded that "this mechanism probably does not contribute significantly to the overall behavior of frozen soil, except possibly for conditions approaching the long term limiting strength. Clearly, any mechanism that relies on changes in the effective stresses acting on the sand skeleton also inherits the uncertainties associated with these effective stresses as described in Section 2.3.4."
Dilatancy Effects, 3bi

Ting et al. (1983) postulated that an additional mechanism of strength in frozen sand is due to the dilation of the sand skeleton and to the resistance that the pore ice provides to this dilation. They explained that "For dense sands, the dilational behavior of the soil skeleton during undrained shearing should create an increment of tension in the ice in the pores. Assuming sufficient tensile adhesional strength between the ice and soil, the ice reacts against the tension with an effect analogous to pore water in unfrozen soils. A decreased pore pressure results, translating into a positive increment of effective confining stress, causing a strength increase similar to that observed during the undrained shear of unfrozen saturated dense sands...".

Dilatancy Hardening Model

Ladanyi (1985) and Ladanyi and Morel (1990) propose a behavioral model for dense frozen sands which attempts to show analytically this dilatancy effect. They refer to this model as a dilatancy hardening model and they draw an analogy between the undrained behavior of an unfrozen sand and the shearing at constant volume of a frozen sand. Following a model presented by Seed and Lee (1967) for unfrozen sands, they reasoned that the strength of the frozen dense sand is controlled by either cavitation in the pore ice or by the effective stresses acting on the sand skeleton.

This dilatancy hardening model makes the following assumptions:

1) All of the pore water in the sand is considered to be frozen.

2) The behavior of the sand is a function of the state parameter $\Psi$ as introduced by Been and Jefferies (1985) and described in Section 2.1.5.
3) No consolidation occurs prior to shear, i.e., the tests are considered as "unconsolidated". The shear starts from a known density and confining pressure.

4) So long as the pore ice is continuous and unbroken during shear, the sand will behave as "undrained". When the pore ice fails, i.e., breaks up, the sand behaves as "drained".

5) The model attempts to predict the strength at the point when the pore ice is fracturing. In the literature, this has been equated with the first peak on the stress strain curve (the writer's upper yield point).

The shearing of a dense frozen sand up to the point where the pore ice begins to fail is assumed to be identical to the undrained shearing of a dense unfrozen sand, the only difference being that in the frozen sand the pore fluid (ice) can support significantly larger tensile stresses and also large shear stresses.

Seed and Lee (1967) and others have observed that if a dense sand is sheared in an undrained mode with no back pressure in a state where it exhibits a tendency to dilate, i.e., it has a large $-\Psi$ parameter, the pore water pressure decreases and the effective stresses increase on the sand skeleton until one of two conditions occurs. Either the pore water pressure reaches the "tensile strength" of water, i.e., it decreases sufficiently to cause cavitation, or the effective stresses increase until the "state point" reaches the Critical Void Ratio Line (CVRL), which represents a condition of $\Delta \varepsilon_v = 0$ at the peak strength.

The transition between the regime where the test terminates in cavitation of the pore water and where the state of the sand reaches the CVRL is determined by the critical confining pressure and the magnitude of the tensile stress increment which can be supported by the pore fluid. The critical confining pressure was defined by Seed and Lee (1967) as that confining pressure for a given void ratio which results in no net volume change at peak strength in a drained triaxial compression test on unfrozen sand. Other researchers have adopted a more stringent requirement that the rate of dilation at the peak
strength is zero for drained shearing at the critical confining pressure.

A representation of the Ladanyi et al. dilatancy hardening model in terms of the total stress and effective stress envelopes is illustrated in Figure 2.52 [Note: here \( q = (\sigma_1 - \sigma_2) \) and \( p = \sigma_{oc} \) in contrast to the M.I.T. \( q \) and \( p \)]. The strength of the unfrozen soil (US) is first considered. The effective stress envelope starts at the origin and has a slope equal to \( M = (6\sin\phi')/(3-\sin\phi') \). The cavitation domain is represented by a total stress failure envelope with an initial slope identical to the effective stress envelope, but translated to the left by an amount equal to the tensile strength of the pore fluid. The transition point between the cavitating and noncavitating regimes occurs for a confining pressure of

\[
\sigma_c = \sigma_{3\text{crit}} - T
\]

where \( \sigma_{3\text{crit}} \) is the critical confining pressure and \( T \) is the tensile strength of the pore fluid (\( T = T_w \) for water = 1 atm and \( T = T_i \) for ice >> 1 atm). For a confining pressure greater than the critical confining pressure minus the pore fluid tensile strength (\( P_o \) greater than \( O'P' \)), the total stress envelope is horizontal with a value of \( q = q_{us} \). The predicted strength of the soil skeleton in this region is therefore solely controlled by the magnitude of the critical confining pressure.

The effective stresses acting on the sand skeleton are assumed to be identical for both the frozen and unfrozen systems. The only difference is that in the unfrozen system, these are limited by the lower tensile strength of the pore water. If sufficient back pressure is applied so that the water cavitates when a net negative increment of pore water pressure is equal to the tensile strength of the ice, then the effective stresses acting on the two skeletons would be identical. Thus, the magnitude of the effective stresses acting on the frozen sand skeleton is enhanced (as is its frictional shear strength) by the greater ability of the pore ice
to support tensile stresses. In terms of the total stresses, these are drawn as TSL (FS) on Fig. 2.52.

Since the pore ice can support both tensile and shear stresses, Ladanyi proposed that the shear strength of the ice be added directly to the enhanced shear strength of the sand skeleton. Thus, the total stress line (TSL) for the frozen sand plus ice is offset vertically a distance equal to \( q_i \) above the TSL (FS) line. The total shear strength of the frozen "system" can be described by the following equations. In the case when the confining pressure is less than the critical confining pressure minus the tensile strength of the ice \( (T_i) \), the shear strength of the frozen "system" is governed by

\[
q_{fs} = q_{soil sk} + q_i = (\sigma_{3cell} + T_i)(K_p' - 1) + q_i
\]

where \( \sigma_{3cell} \) is the cell pressure, \( T_i \) is the tensile strength of ice, \( K_p' \) is the passive earth pressure coefficient \( (\sigma'_i/\sigma'_3) = R_f = \tan(45 + \phi/2) \), and \( q_i \) is the shear strength of the pore ice. Hence the shear strength in this region increases linearly at slope \( M \) with the confining pressure.

In the noncavitating region, the shear strength of the frozen "system" is governed by

\[
q_{fs} = \sigma_{3crit}(K_p' - 1) + q_i
\]

where \( \sigma_{3crit} \) is the critical confining pressure for the given initial state of the sand skeleton. This equation predicts that the strength of the frozen "system" is independent of the confining pressure.

An analysis of the data in this research in terms of this dilatancy hardening model will be presented in Chapter 6. In the writer's opinion there are several potential deficiencies with this model which should be discussed.
1) This model assumes that the effective stresses acting on the frozen sand skeleton are the same as those acting on an unfrozen sand skeleton. The writer has discussed the differences between the effective (intergranular) stresses acting on the sand skeleton in the frozen and unfrozen systems in Section 2.3.4. Any model that equates these effective stress systems inherits the uncertainties which were discussed.

2) This model adds directly the shear strength of pore ice to that of the unfrozen sand skeleton. In most cases, this will result in a predicted total stress which does not lie on the total stress path, i.e., the total stresses will not be in equilibrium. This is illustrated Fig. 2.53 which is an M.I.T. q–p diagram, i.e., q = (σ₁–σ₃)/2 and p = (σ₁+σ₃)/2. The total stress envelope has been drawn with the assumption that qᵢ = 2 MPa (Fig. 2.52 qᵢ = Qᵢ = 2qᵢ), Tᵢ = 2 MPa, σ₃crit = 4 MPa and $\phi'$ = 32°.

In the case where qᵢ = Tᵢ, and $\sigma_c \leq (\sigma_{3\text{crit}} - Tᵢ)$, the predicted strength lies on the total stress path, e.g., $\sigma_c = P₄$ and predicted strength is $T₄$. If however, qᵢ $\neq$ Tᵢ, then $T₄$ will not fall on the total stress path.

A similar argument applies using the Ladanyi and Morel (1990) q–p space, i.e., q = (σ₁–σ₃) and p = (σ₁+σ₂+σ₃)/3, except that in this instance, in order to ensure total stress equilibrium in the "cavitating region", the ratio of $Q₁$ to $Tᵢ$ must be 3 to 1.

3) This model relies on a direct equality between the levels of strain necessary to mobilize the peak strength in the frozen and unfrozen sand. In unfrozen undrained tests on sands with a large $\psi$ parameter, the stress path climbs up the $K_f$ line and significantly larger levels of axial strain may be necessary to mobilize the level of effective stresses which are assumed to occur in the frozen sand.

In light of these possible deficiencies, the dilatancy hardening model presented by Ladanyi and Morel (1990) may not accurately predict the actual amount of strengthening in dense frozen sands. They do, however, present some data which shows excellent agreement with their predictions. These are discussed in Section 6.3, along with a comparison of the writer's data.

**Structural Hindrance, 3bii**

Ting et al. (1983) proposed another mechanism that might enhance the strength of a frozen sand. They referred to this mechanism as a structural hindrance and explained "An analogy to this proposed mechanism is that of an
axially loaded slender column embedded in a matrix with a finite lateral resistance. A small resistance by the matrix greatly inhibits potential column buckling. Similarly, only a relatively small resistance (compared with the overall applied stress) may be required in the ice to stabilize the soil skeleton from 'collapse". They state that this interaction can be synergistic in nature.

In the writer's opinion, this structure hindrance might be better described in terms of a composite interaction between the sand particles and the ice matrix. A model representing such an interaction is discussed in Section 6.2.

2.3.6 Summary of Strength and Deformation Behavior of Frozen Sand

Classification and Description

Table 2.5 presents a proposed classification and description system for frozen soils. In this system, the soil phase in a frozen soil is classified according to the Unified Soil Classification System (Section 2.1.1). The ice in a frozen soil is classified according to whether it is visible to the naked eye, whether it holds the soil particles tightly together, i.e., it is well-bonded, and whether or not there is "excess" ice. If the pore ice is visible, then it is described in terms of its macroscopic features. If there are ice inclusions greater than 1 inch thick, the frozen ground is classified as an ice and the nature and amount of soil in the ice is described.

The presence of significant amount of excess ice and large inclusions can have a dramatic effect on the behavior of frozen sands. Such a heterogeneous structure and its effect on the behavior is not treated in this research.

Frozen soils are described in terms of the amount of unfrozen water that is present and the degree of saturation. The amount of unfrozen water is expressed
as a gravimetric ratio of the unfrozen water to the weight of the soil particles. The degree of saturation is a measure of the amount of ice and water that is present in the pore spaces as a percentage of the total void space. If no air voids are present, the degree of saturation is 100%.

**Structure of Frozen Sand**

A schematic diagram of an idealized structure for frozen sand is presented in Fig. 2.26 (a more probable structure is presented in Section 6.4.4). Ting et al. (1983) discussed this structure and made the following postulates. Solid contacts exist between most of the sand particles. A continuous unfrozen water film surrounds the silicate particles and is present to very low temperatures. The unfrozen water in this film is mobile parallel to the surface of the particles, but strong tensile and moderate shear adhesional forces can be transmitted between the pore ice and the silicate particles. There are no direct ice to soil contacts. The macroscopic structure of the ice in the pores is polycrystalline and the maximum grain size is limited to the size of the individual pores.

It can be concluded that solid contacts exist between most adjacent particles in a frozen sand if the freezing process does not alter the structure of the sand. In a careful experimental program, Singh et al. (1982) measured the effect of freezing and thawing on the structure of a clean sand (no fines) by performing a series of cyclic triaxial tests on specimens with prior "seismic histories". All of the specimens were subjected to the same stress history. Some were subsequently frozen uniaxially in an open system and thawed while other were tested directly. They observed that the freezing and thawing cycle had no effect on the cyclic response of the sand. They concluded that so long as drainage is provide and no net volume change occurs during freezing and subsequent thawing, the structure
of the sand is preserved.

The presence of a continuous unfrozen water film in frozen soils has been demonstrated conclusively by a series of electrical conductance experiments (Hoekstra 1965) and electro-osmosis experiments (Hoekstra and Chamberlain 1964) on frozen soils materials. Corte (1962) observed that silicate particles placed on top of a slowly advancing freezing front "floated" on top of the ice. He concluded that this could only be possible if a continuous unfrozen water film were present between the ice and the sand particles and that this film was continually replenished as freezing occurred at the advancing front.

The amount of unfrozen water present in a frozen soil is strongly affected by the temperature, the specific surface area of the soil particles, the surface chemistry, the presence of ionic impurities, and changes in confining pressure.

The strength of the ice–silicate interface in a frozen soil has not been measured directly. This strength was inferred by Ting et al. (1983) from a series of experiments reported by Jellinek (1962). In these experiments, Jellinek measured the shear and tensile adhesional strength of an ice silicate interface by performing direct shear and tension experiments on snow ice frozen to a fused quartz surface. He observed that the shear adhesional strength was one to two orders of magnitude less than the tensile adhesional strength and that this tensile adhesional strength of the ice–silicate interface was greater than the tensile strength of the snow ice (> 1 MPa).

Jellinek (1962) attributed the large tensile strength of the interface to surface tension forces. It is the writer’s opinion, a direct application of Jellinek’s interfacial tensile strengths to a fully saturated frozen soil system might be somewhat misleading because these surface tension forces are not present. Various qualitative models have been proposed to explain the behavior of the
ice–water–silicate interface. The actual structure of this interface and its strength and deformation behavior are still poorly understood.

The postulate that there is probably no direct ice to soil contact in frozen sand can be explained by the strong forces that are necessary to remove the next to last molecular layer of adsorbed water from a silicate surface. Steinfink and Gebhart (1962) concluded that a normal stress in excess of 200 MPa was required. This is an order of magnitude larger than the unconfined compressive strength of ice and is considerable higher than the pressure needed to melt ice at $-10^\circ$C ($\approx 110$ MPa). Thus, the ice crystals would probably crush or melt prior to making a direct contact with the silicate surface.

The molecular structure of ice in frozen soils is normal hexagonal ice Ih. The macroscopic structure, i.e., the fabric and texture, is still being debated in the literature. Ting et al. (1983) proposed that it is polycrystalline with the maximum grain size being limited by the size of the pores. Colbeck (1982, 1985) and Sayles (1989) argue that the structure could be either that of single crystals extending in a tortuous path through several pores or fine grained polycrystalline with random orientations of the c-axis. CRREL is currently studying the structure of pore ice.

**Overview of Frozen Sand Behavior**

**Small Strain Behavior**

The small strain "elastic" properties of frozen sands have been most reliably measured using acoustic wave propagation techniques (Baker and Kurfurst 1985), resonant beam tests (Kaplar 1963), and on–specimen axial strain measurements (Baker and Kurfurst 1985). Vinson (1978) used cyclic triaxial tests to assess the effect of confining pressure on Young's modulus, but his results
probably involve some plastic straining. A review of these programs indicates that the Young's modulus increases with decreasing temperature and then levels off (Fig. 2.28); increases moderately with increasing dry density (Fig. 2.29); perhaps increases with increasing confining pressure (Fig. 2.30); and decreases dramatically with increasing salinity (Fig. 2.31). The Poisson's ratio increases slightly with decreasing temperature (Fig. 2.28); increases slightly with increasing dry density (Fig. 2.29); and increases slightly with increasing salinity (Fig. 2.32).

Large Strain Behavior

Figure 4.5 in this thesis gives idealized stress–strain curves from triaxial compression testing of frozen sand to show the major features of the behavior. The initial curve becomes nonlinear beginning at axial strain levels on the order of 0.0001 (Fig. 4.5a). The second knee in the stress–strain curve (Fig. 4.5b) has been designated as the upper yield point in recognition of the yielding which occurs prior to that point. Frozen sands can exhibit either strain hardening or strain softening after the upper yield point (Fig. 4.5b) depending on the relative density, confining pressure, strain rate and temperature for freshwater frozen sand. It should be emphasized that the above idealized behavior is primarily based on data obtained in this thesis since most published stress–strain curves do not exhibit distinct upper yield behavior due to inadequate axial strain measurement techniques. Hence, little information exists in the literature regarding factors affecting the upper yield behavior (Table 2.8 lists the testing programs that were reviewed).

Limited data indicate that the magnitude of the upper yield stress increases with confining pressure in the range of 0 to 40 MPa, decreases with
confining pressure in the range of 40 to 100 MPa, and increases again with confining pressures greater than 100 MPa (Fig. 2.33).

The peak strength is usually strongly affected by strain rate and can be described by the simple power law relationship commonly used in ice mechanics. Power law coefficients in unconfined compression generally range from 7 to 16 (Fig. 2.34). Some testing programs show that above a certain strain rate, the strength of frozen sand becomes strain rate insensitive (infinite power law coefficient). This observation, although not surprising to the writer, is not uniform throughout the reported data (i.e., observed in only two of the six programs in Fig. 2.34).

The peak strength of saturated frozen sands generally increases with confining pressure in the range of 0 to 10 MPa (Fig. 2.35). Some of the testing programs exhibit a linear increase with confinement while others show a pronounced curvature. For the results in Fig. 2.35, the total stress friction angle ranged from 0° to 28°. For larger confining pressures, the peak strength of frozen sand shows a behavior similar to that of the upper yield point (Fig. 2.36), i.e., increases with confining stress up to 40 MPa, decreases with confining stress from 40 to 100 MPa and increases with confining pressure above 100 MPa.

The peak deviator stress is strongly affected by the sand content (Fig. 2.37). In the range of sand volume fractions from 0 to 40%, there is a linear increase in strength above that of pure ice. At 40% sand by volume, the particles begin to come into contact and the rate of strengthening increases dramatically but is still approximately linear. Baker and Kurfurst (1985) present the only detailed study of the effect of sand relative density on the peak strength (Fig. 2.38). Their results indicate that at the strain rate investigated and low relative densities, the upper yield stress is the peak strength and is unaffected by the
relative density. For higher relative densities where there is strain hardening after the upper yield stress, the strength is strongly affected by sand density and follows a bilinear relationship.

The peak deviator stress increases with decreasing temperature (Fig. 2.39). This is most dramatic in the range from 0 to $-30^\circ$C, with a typical rate of strengthening of $0.7\pm0.3$ MPa/$^\circ$C. In some testing programs, the strengthening effect of decreasing temperature was lost at lower temperatures (i.e., some programs actually measured a decrease in strength with decreasing temperatures over some range of temperatures).

A decrease in the degree of saturation will dramatically decrease the strength of frozen sand (Fig. 2.40 and 2.41).

The presence of salts in frozen sands dramatically decreases the peak unconfined strength (Fig. 2.42). This decrease occurs primarily in the range from 0 to 2 g/l.

The power law coefficient ($n$) for peak strength shows no consistent trend with increasing confining pressure (Fig. 2.43). It does, however, appear to be affected by changes in temperature (Fig. 2.44). Most of the testing programs reviewed indicated an increase in "$n$" from near that of polycrystalline ice at $-2^\circ$C to about 10 to 15 at temperatures of $-10^\circ$C. But, this trend was not consistent for all testing programs.

Perkins and Reudrich (1973) reported the results of tensile strength tests on frozen sand. These data indicate that the tensile strength of frozen sand is independent of strain rate above $1.67\times10^{-4}$/sec, but exhibits a power law dependency for lower strain rates (Fig. 2.45). At $-13.3^\circ$C the power law coefficient was approximately 8 and it decreased to approximately 3 at $-2.3^\circ$C. In the range from $-13.3$ to $-6.7^\circ$C, the tensile strength is independent of
At warmer temperatures, the tensile strength decreased with increasing temperature.

The axial strain at the peak deviator stress is strongly dependent of the deformational regime (Figs. 2.46, 2.47 and 2.48). If the peak strength of the frozen sand occurs at the upper yield stress, the testing programs indicate that it is insensitive to relative density, temperature and strain rate. If the frozen sand exhibits post upper yield strain hardening behavior, the axial strain at the peak strength increases with relative density but is apparently unaffected by changes in strain rate.

**Mechanics of Frozen Sand**

Consideration of the equilibrium of an individual sand particle in a frozen sand matrix indicates that in all but the most specialized cases, the intergranular contact forces due not constitute an equilibrium stress state when considered separately from the ice–silicate interface forces (Fig. 2.49). This means that a quantitative evaluation of the strength and deformation behavior of frozen sands in terms of the intergranular effective stresses may not be appropriate.

The nature of the stresses transmitted across the ice–water interface can be described by the Kelvin equation under isothermal (equilibrium) conditions (Eq. 2.3.2 and Fig. 2.50). If the deformational process is not isothermal, then the Clausius–Clapeyron equation is used (Eq. 2.3.3). These equations indicate that the differences in the ice pressure and unfrozen water pressure are due to curvature in the ice water interface and phase changes in the ice, respectively.

Deformational processes which may be occurring in frozen sands are summarized in Table 2.9. In terms of those associated with the sand skeleton: elastic deformations of the individual sand grains are probably occurring;
crushing of the sand grains has been measured (Chamberlain et al. 1972); rolling and sliding motions are probably occurring but not along the same trajectories as in unfrozen sand. In terms of those mechanisms occurring in the pore ice, these are probably similar to those described in Section 2.2.3 and summarized in Table 2.2. A detailed study of how these mechanisms are affected by the presence of the silicate particles has not been presented in the literature. The presence of the unfrozen water film may enhance diffusional type mechanisms and limit the magnitude of the internal stresses caused by the pileups of dislocations. Dynamic recrystallization processes may be occurring and be enhanced by the presence of unfrozen water and the extremely small crystal sizes. Internal cracking and fracturing mechanisms must be occurring when the frozen sand specimens are dilating.

**Mechanisms of Strength**

Mechanisms of strength in frozen sand were reviewed in terms of the Ting et al. (1983) hypothesis as summarized in Table 2.10 and Fig. 2.51. Their frame of reference was to consider frozen sands as a particulate material. They divided their strengthening mechanisms into three categories: those associated with the pore ice; those associated with the sand skeleton; and those associated with the ice–sand interaction.

The pore ice strength may be different from that of normally tested isotropic ice due to a decrease in the grain size or a change in structure from that of polycrystalline to single tortuous crystals filling several pore spaces (3ai). The magnitude of this effect has not been determined. Also, there may be differences in the structure of the pore ice for specimens with greater than 40% sand by volume as opposed to those less than 40% sand.
The state of stress and deformational constraints in the pore ice (3aii) are different from those imposed during uniaxial compression or tension tests. In addition, the stress concentrations occurring between the ice and silicate particles may be complicated by the presence of the unfrozen water film. Because of these complications, a linear extrapolation of $\sigma_{\text{ice}}$ versus $V_s$ to all $D_r$ may not be valid as proposed in Fig. 2.51 by Ting et al. (1983).

The pore ice should be experiencing enhanced strain rates (3aiii) compared to the overall imposed strain rate on the frozen sand system, i.e., most of the imposed deformations may be transferred to the ice while the silicate particles remain relatively undeformed.

The tension in the unfrozen water film (3c) may serve to strengthen the frozen sand. Ting et al. (1983) considered that this effect was minimal.

Dilatancy of the sand skeleton and the resultant pore ice tensile stress increments can serve as a strengthening mechanism in frozen sand (3bi). Ladanyi (1985) and Ladanyi and Morel (1990) have attempted to quantify this mechanism by developing a dilatancy hardening model (Fig. 2.52). This relies on the assumption that the effective stresses acting on the sand skeleton in a frozen sand are identical to those acting on an unfrozen sand skeleton in an undrained test. They develop two equations to predict the strength of the frozen sand skeleton at the point where the pore ice begins to cavitate (pull apart in tension). These were presented as Eqs. 2.3.4 and 2.3.5. Several potential deficiencies of this model were discussed. These included: 1) the application of the effective stress principle to predict the frictional strength of the frozen sand skeleton may not be reasonable since the state of stress in the pore ice is not hydrostatic; 2) the total stresses predicted by this model do not satisfy total stress equilibrium except under special circumstances; and 3) the model relies on strain compatibility.
between the frozen and unfrozen systems (a much larger level of axial strain may be required in the unfrozen sand to mobilize the level of effective stresses predicted by model). Further analysis of this model is presented in Section 6.3.

The structural hindrance component of strength (3bii) might best be described in terms of the composite interaction between the sand and ice. This is considered in Section 6.2.
Table 2.1: Effect of Grain Size and Strain Rate on the Unconfined Compressive Strength of Polycrystalline Ice @ -10°C

<table>
<thead>
<tr>
<th>STRAIN RATE ($s^{-1}$)</th>
<th>GRAIN DIAMETER (mm)</th>
<th>PEAK STRENGTH (MPa)</th>
<th>PERCENT DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-6}$</td>
<td>1</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.7</td>
<td>43</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>6.0</td>
<td>50</td>
</tr>
</tbody>
</table>

Data from Schulson and Cannon (1984)
<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>NAME</th>
<th>DESCRIPTION</th>
<th>RANGE OF APPLICABILITY</th>
<th>REFERENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>Elastic Straining of Crystal Lattice</td>
<td>Distortion of lattice due to elastic straining of hydrogen bonds</td>
<td>Occurs under all loading conditions. May dominate behavior in the absence of other mechanisms.</td>
<td>Cottrell (1953)</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Hobbs (1974)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Cole (1990)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Hertzberg (1980)</td>
</tr>
<tr>
<td>Anelastic</td>
<td>Anelasticity</td>
<td>Motion of a population of mobile dislocations under internal stresses</td>
<td>Under all loading conditions where sufficient time to build up internal stresses</td>
<td>Duval et al. (1983)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Duval (1978)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Cole (1990)</td>
</tr>
<tr>
<td>Plastic</td>
<td>Diffusional Flow</td>
<td>Diffusion of vacancies either the lattice or along the grain boundaries.</td>
<td>At very low stresses or strain rates or high homologous temperatures (n=1)</td>
<td>Coble (1963)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Nabarro (1947)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Herring (1950)</td>
</tr>
<tr>
<td></td>
<td>Grain Boundary Sliding with Diffusional Accommodation</td>
<td>Combined diffusion along boundaries or through lattice to accommodate discontinuities during sliding.</td>
<td>At low stresses or strain rates or high homologous temperatures (n=1)</td>
<td>Raj and Ashby (1971)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Ignat and Frost (1987)</td>
</tr>
<tr>
<td></td>
<td>Dislocation Gliding and Climbing</td>
<td>Gliding and climbing motions of dislocations either along or between slip planes</td>
<td>Considered dominant mechanism controlling power law creep for n = 3 to 4.5</td>
<td>Hertzberg (1989)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Duval et al. (1983)</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Veertman (1983)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Langdon (1973)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sinha (1978)</td>
</tr>
</tbody>
</table>
Table 2.2: Possible Deformational Mechanisms in Polycrystalline Ice

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>NAME</th>
<th>DESCRIPTION</th>
<th>RANGE OF APPLICABILITY</th>
<th>REFERENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolving Micro-Structure</td>
<td>Grain Boundary Migration</td>
<td>Motion of grain boundaries as individual crystals grow and consume adjacent crystals</td>
<td>During periods of storage or under low stresses and strain rates at the lower end of the power law regime</td>
<td>Cole (1986, 1987) Duval et al. (1983) Gold (1963)</td>
</tr>
<tr>
<td></td>
<td>Dynamic Recrystallization and Grain Boundary Migration</td>
<td>Nucleation and growth of new grains along existing grain boundaries due to strain energy gradients</td>
<td>Lower end of power law regime as coefficient approaches 1</td>
<td>same as above</td>
</tr>
<tr>
<td>Internal Cracking and Fracture</td>
<td>Transcrystalline Fracture</td>
<td>Nucleated cracking due to the pileup of dislocations against grain boundaries</td>
<td>Higher end of power law regime up to strain rates of approximately $10^{-4}$/sec</td>
<td>Ashby et al. (1979) Cole (1987, 1988)</td>
</tr>
<tr>
<td></td>
<td>Grain Boundary Fracture</td>
<td>Cracking along boundaries between adjacent grains due to elastic anisotropy</td>
<td>Higher end of power law regime and in the region of power law breakdown</td>
<td>Cole (1988) Zener (1948) Sunder and Wu (1990)</td>
</tr>
</tbody>
</table>
Table 2.3: Possible Weakening Processes in Polycrystalline Ice

<table>
<thead>
<tr>
<th>NAME OF PROCESS</th>
<th>DESCRIPTION</th>
<th>DEFORMATIONAL MECHANISM(S) AFFECTED</th>
<th>REFERENCE(S)</th>
</tr>
</thead>
</table>
| Decrease in Grain Size (fine to Ultra-fine range)     | Increased grain boundary sliding less enhanced diffusional processes along grain boundaries Greater number of nucleation sites for dynamic recrystallization | a) Diffusional Flow  
               b) Grain Boundary Sliding  
               c) Evolving Fabric                                                        | Armstrong (1970)  
                                                                                   Cole (1987)                |
| Pressure Melting at Grain Boundaries                  | Increases in the amount of unfrozen water at grain boundaries and triple point junctions due to increased confining stress causing decreases in the magnitude of the stress concentrations caused by basal slip mechanisms and decreases in the shear strength of the grain boundary interface | a) Dislocation Gliding and Climbing  
               b) Grain Boundary Sliding                                                 | Jones (1982)               |
| Concentration of Impurities at Grain Boundaries       | Impurities rejected from the lattice during freezing are concentrated in the grain boundary regions. These can lower the melting point of the ice and create a liquid like layer with the same weakening effect as localized pressure melting. | a) Dislocation Gliding and Climbing  
               b) Grain Boundary Sliding  
               c) Diffusion                                                              | Mellor (1980)              |
| Increased Mobility of Dislocations                    | The introduction of ionic impurities into the ice crystal lattice may increase the number of point ionization defects (Bjerrum defects) which may in turn increase the mobility of the dislocations and decrease the strength. | a) Dislocation Gliding and Climbing  
                                                                                     | Glen (1968)  
                                                                                     Jones and Glen (1969) |
| Evolving Fabric                                       | Processes of dynamic recrystallization and grain boundary migration can change the texture and fabric which can decrease the strength of the polycrystalline ice mass. | a) Diffusional Flow  
               b) Grain Boundary Sliding  
               c) Dislocation Climbing and Sliding                                         | Cole (1987)  
                                                                                       Duval et al. (1983)  
                                                                                     Mellor (1980) |
### Possible Strengthening Processes in Polycrystalline Ice

<table>
<thead>
<tr>
<th>NAME OF PROCESS</th>
<th>DESCRIPTION</th>
<th>DEFORMATIONAL MECHANISM(S) AFFECTED</th>
<th>REFERENCE(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotropic Hardening</td>
<td>Interactions between dislocations on parallel or intersecting slip planes increase the amount of externally applied force necessary to cause them to move by increasing the internal stresses. (Short range)</td>
<td>a) Dislocation Gliding and Climbing</td>
<td>Duval et al. (1983)</td>
</tr>
<tr>
<td>Kinematic Hardening</td>
<td>Pile ups of dislocations in slip planes along grain boundaries or formation of internal kink bands increase the level of internal stress and decrease the mobility of the dislocations.</td>
<td>a) Dislocation Gliding and Climbing</td>
<td>Duval et al. (1983)</td>
</tr>
<tr>
<td>Decreased Mobility of Dislocations</td>
<td>The presence of some ionic impurities may serve to decrease the mobility of the dislocations in their glide planes.</td>
<td>a) Dislocation Gliding and Climbing</td>
<td>Jones and Glen (1969)</td>
</tr>
<tr>
<td>Decrease in Grain Size (Coarse to fine range)</td>
<td>Leads to an increase in the dislocation density according to grain boundary source theories and hence isotropic hardening. Leads to an increase in the stress required to nucleate a crack and hence an increase in strength.</td>
<td>a) Dislocation Climbing and Gliding b) Internal Fracturing</td>
<td>Li (1963) Cole (1988) Sunder and Wu (1990)</td>
</tr>
<tr>
<td>Increase in Confining Stress</td>
<td>Leads to a suppression of cracking processes by increasing the crack nucleation stress leading to more ductile type behavior.</td>
<td>a) Internal Fracturing</td>
<td>Jones (1982)</td>
</tr>
</tbody>
</table>
Table 2.5: Classification and Description of Frozen Soils

<table>
<thead>
<tr>
<th>Pertinent properties of frozen materials which may be measured by physical tests to supplement field identification</th>
<th>Guide for Construction on Soils Subject to Freezing and Thawing</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-Place Temperature</td>
<td>Thaw Characteristics</td>
</tr>
<tr>
<td>Density and Void Ratio</td>
<td>(3)</td>
</tr>
<tr>
<td>Water Content (total H2O, including ice)</td>
<td>(4)</td>
</tr>
<tr>
<td>Strength</td>
<td>(5)</td>
</tr>
<tr>
<td>Elastic Properties</td>
<td>(6)</td>
</tr>
<tr>
<td>Plastic Properties</td>
<td>(7)</td>
</tr>
<tr>
<td>Thermal Properties</td>
<td>(8)</td>
</tr>
<tr>
<td>Ice Crystal Structure (using optical instruments)</td>
<td>(9)</td>
</tr>
<tr>
<td>a. Orientation of Axes</td>
<td>(1)</td>
</tr>
<tr>
<td>b. Crystal Size</td>
<td>(2)</td>
</tr>
<tr>
<td>c. Crystal Shape</td>
<td>(3)</td>
</tr>
</tbody>
</table>

Part I—Description of Soil Phase

**Segregated** ice is not visible by eye*

<table>
<thead>
<tr>
<th>Major Group</th>
<th>Sub-Group</th>
<th>Field Identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Designation</td>
<td>Description</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Part I—Description of Soil Phase (Independent of Frozen State)</td>
<td>Segregated ice is not visible by eye*</td>
<td>-</td>
</tr>
<tr>
<td>Part II—Description of Frozen Soil</td>
<td>Segregated ice is visible by eye (ice 1 inch or less in thickness)*</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>Ice coatings on particles</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>Thickness</td>
<td>Pattern of</td>
</tr>
<tr>
<td></td>
<td>Spacing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hardness</td>
<td>Structure</td>
</tr>
<tr>
<td></td>
<td>Stratified or distinctly oriented ice formations</td>
<td>Vs</td>
</tr>
</tbody>
</table>

*Segregated ice is not visible by eye.

**Guidelines for Construction on Soils Subject to Freezing and Thawing**

- The potential intensity of ice segregation in a soil is dependent on a large degree on its void sizes and for pavement design purposes may be expressed as an empirical function of grain size as follows:

  Most inorganic soils containing 3% or more of grains finer than 0.002 mm in diameter by weight are frost-susceptible for pavement design purposes. Gravels, well-graded sands and silty sands, especially those approaching the theoretical maximum density curve, which contain 1-1/2 to 3% finer by weight than 0.02 mm size should be considered as possibly frost susceptible and should be subjected to a standard laboratory frost susceptibility test to evaluate actual behavior during freezing. Uniform sandy soils may have as high as 10% of grains finer than 0.02 mm by weight without being frost susceptible. However, their tendency to occur interbedded
### Part II - Designation of Substantial Ice Strata

| Ice (Greater than 1 in. in thickness) | Ice with soil inclusions | Ice without soil inclusions | Designate material as ICE and use descriptive terms as follows, usually one item from each group, as applicable: 

- Hardness
- Structure
- Aeration
- Color
- Ice with other soils usually makes it impractical to consider them separately.

Soils classed as frost-susceptible under the above pavement design criteria are likely to develop significant ice segregation and frost heave if frozen at normal rates with free water readily available. Soils so frozen will fall into the thaw-unstable category. However, they may also be classed as thaw-stable if frozen with insufficient water to permit ice segregation.

Soils classified as non-frost-susceptible (NFS) under the above criteria usually occur without significant ice segregation and are usually thaw-stable for pavement applications. However, the criteria are not exact and may be inadequate for some structure applications: exceptions may also result from minor soil variations.

In permafrost areas, ice wedges, pockets, veins, or other ice bodies may be found whose mode of origin is different from that described above. Such ice may be the result of long-term surface expansion and contraction phenomena or may be glacial or other ice which has been buried under a protective earth cover.

### Table 2.5: Classification and Description of Frozen Soils (page 2/2)

<table>
<thead>
<tr>
<th>Ice Type</th>
<th>ICE</th>
<th>ICE with soil inclusions</th>
<th>ICE without soil inclusions</th>
<th>ICE Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice</td>
<td>ICE</td>
<td>ICE</td>
<td>ICE</td>
<td>ICE</td>
</tr>
<tr>
<td>Classify soil phase by the Unified Soil Classification System.</td>
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</tr>
<tr>
<td>When rock is encountered, standard rock classification terminology should be used.</td>
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</tr>
<tr>
<td>Frozen soils in the N group may, on close examination, indicate presence of ice within the voids of the material by crystalline reflections or by a sheen on fractured or trimmed surfaces. However, the impression to the unaided eye is that none of the frozen water occupies space in excess of the original voids in the soil. The opposite is true of frozen soils in the V group.</td>
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</tr>
<tr>
<td>When visual methods may be inadequate, a simple field test to aid evaluation of volume of excess ice can be made by placing some frozen soil in a small jar, allowing it to melt and observing the quantity of supernatant water as a percent of total volume.</td>
<td></td>
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</tr>
</tbody>
</table>
Table 2.6: Overview of Testing Programs to Measure the Small Strain Behavior of Frozen Sand

<table>
<thead>
<tr>
<th>INVESTIGATOR(S)</th>
<th>MATERIAL TESTED</th>
<th>TYPE OF TEST</th>
<th>TEMP (°C)</th>
<th>RANGES IN TESTING VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaplar (1963)</td>
<td>Peabody Gravelly Sand (SP)</td>
<td>Resonant Beam</td>
<td>-1.1 to -27.8</td>
<td>1805-1861 kg/m³ ≥85% of max from Providence Vibrated Density</td>
</tr>
<tr>
<td></td>
<td>D₅₀ = 3.8mm</td>
<td></td>
<td></td>
<td>Unconfined</td>
</tr>
<tr>
<td></td>
<td>Cᵤ = 16</td>
<td></td>
<td></td>
<td>Not Measured</td>
</tr>
<tr>
<td></td>
<td>Cₑ = 0.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vinson (1978)</td>
<td>Ottawa 20-30 Sand</td>
<td>Cyclic Triaxial</td>
<td>-4</td>
<td>Dense Sand (e = 0.49)</td>
</tr>
<tr>
<td></td>
<td>D₅₀ = 0.7mm</td>
<td></td>
<td></td>
<td>Sand-Ice Mix. (e = 3.73)</td>
</tr>
<tr>
<td></td>
<td>Cᵤ = 1.1</td>
<td></td>
<td></td>
<td>0 - 1.3 MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Not Measured</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baker and Kurfurst (1985)*</td>
<td>Ottawa 16-100 Sand</td>
<td>Wave Propogation</td>
<td>-3.2 to -10</td>
<td>Dₑ from 30% to 100%</td>
</tr>
<tr>
<td></td>
<td>D₅₀ = 0.4mm</td>
<td>Unconfined Compresson with on-specimen axial strain measurements</td>
<td></td>
<td>Unconfined</td>
</tr>
<tr>
<td></td>
<td>Cᵤ = 2</td>
<td></td>
<td></td>
<td>Freshwater and Salt Water with 0.5 to 40g/l</td>
</tr>
</tbody>
</table>

* Also presented large strain data
### Table 2.7: Summary of Observed Trends in Small Strain Behavior of Frozen Sand

<table>
<thead>
<tr>
<th>INVESTIGATOR(S)</th>
<th>OBSERVED TRENDS</th>
<th>RANGES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kaplar (1963)</strong></td>
<td>Increases with decreasing temp. particularly from -1 to -10°C</td>
<td>Slight increase with decreasing temp.</td>
</tr>
<tr>
<td><strong>Vinson (1978)</strong></td>
<td>Increases with sand density and confining pressure</td>
<td>Not measured</td>
</tr>
<tr>
<td><strong>Baker and Kurfurst (1985)</strong></td>
<td>Increases with $D_r$, Decreases with increasing $D_r$, increase with temperature, and increase with salinity</td>
<td>Slight increase with Freshwater (all densities)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Baker and Kurfurst (1985)</strong></td>
<td>Increases with $D_r$</td>
<td>---</td>
</tr>
<tr>
<td>INVESTIGATOR(S) (PROGRAM LETTER)</td>
<td>MATERIAL TESTED</td>
<td>TYPE OF TEST</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>----------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Bragg and Andersland (1980) (A)</td>
<td>Medron Silica Sand</td>
<td>Unconfined Compression</td>
</tr>
<tr>
<td>Parameswaran (1980) (B)</td>
<td>Ottawa 30 - 100 Sand</td>
<td>Unconfined Compression</td>
</tr>
<tr>
<td>Parameswaran and Roy (1982) (C)</td>
<td>Ottawa 30 - 100 Sand, Dₜₜ = 0.37mm, Cₜₜ = 1.9</td>
<td>Unconfined Compression</td>
</tr>
<tr>
<td>Orth (1985) (D)</td>
<td>Karlsruhe Sand</td>
<td>Unconfined Compression</td>
</tr>
<tr>
<td>Yuanlin et al. (1988) (E)</td>
<td>Lanzhou Medium Sand</td>
<td>Unconfined Compression</td>
</tr>
<tr>
<td>Barbounais and Ladany (1985) (F)</td>
<td>LeSueur 16.30 rounded silica sand, Dₜₜ = 1mm, Cₜₜ &lt; 1.5</td>
<td>Unconfined Compression</td>
</tr>
<tr>
<td>Chamberlain et al. (1972) (G)</td>
<td>Ottawa 100 - 200 Sand</td>
<td>Triaxial Compression</td>
</tr>
<tr>
<td>Sayles (1973, 1974) (H)</td>
<td>Ottawa 20 - 30 Sand, Dₜₜ = 0.7mm, Cₜₜ &lt; 1.5</td>
<td>Triaxial Compression</td>
</tr>
</tbody>
</table>
### Table 2.8: Overview of Testing Programs to Measure Large Strain Behavior of Frozen Sand (page 2/2)

<table>
<thead>
<tr>
<th>INVESTIGATOR(S)</th>
<th>MATERIAL TESTED</th>
<th>TYPE OF TEST</th>
<th>STRAIN RATE ($a^{-1}$)</th>
<th>TEMP (°C)</th>
<th>void</th>
<th>DENSITY</th>
<th>SATURATION (%)</th>
<th>SALINITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shibata et al. (1985)</td>
<td>Toyoura Sand</td>
<td>Triaxial Compression</td>
<td>4.5x10^{-4} to 4.5x10^{-8}</td>
<td>-2 to -10</td>
<td>0</td>
<td>Dense</td>
<td>c=0.65, D=942</td>
<td>Not Reported (100?)</td>
</tr>
<tr>
<td>Alkire and Andersland (1973)</td>
<td>Ottawa 20-30 $D_5=0.7$ $C_u=1.0$</td>
<td>Triaxial Compression</td>
<td>4.4x10^{-8}</td>
<td>-12 to -6.89</td>
<td>0 to 76</td>
<td>Medium</td>
<td>e=0.58</td>
<td>97 and 55</td>
</tr>
<tr>
<td>Perkins and Revdrich (1973)</td>
<td>Penn 80-200 Sand</td>
<td>Triaxial Compression</td>
<td>1.1x10^{-8}</td>
<td>-7.8 to 11.9</td>
<td>0 to 6.89</td>
<td>Dense</td>
<td>Not Reported (90-100?)</td>
<td>Distilled Water</td>
</tr>
<tr>
<td>Parameswaran and Jones (1981)</td>
<td>Ottawa 30-100 $D_5=0.37$ $C_u=1.0$</td>
<td>Triaxial Compression</td>
<td>7.7x10^{-8}</td>
<td>-10 to 76</td>
<td>0 to 6.89</td>
<td>Ice, Sand, Ice, Dense Sand(60%)</td>
<td>Not Reported (90-100?)</td>
<td>Distilled Water</td>
</tr>
<tr>
<td>Goughnour and Andersland (1968)</td>
<td>Ottawa 20-30 $D_5=0.7$ $C_u&lt;1.5$</td>
<td>Triaxial Compression</td>
<td>2.2x10^{-6} and 4.4x10^{-6}</td>
<td>-4 to 12</td>
<td>0 to 0.69</td>
<td>Ice, Sand, Ice, Dense Sand(60%)</td>
<td>Not Reported (90-100?)</td>
<td>Distilled Water</td>
</tr>
<tr>
<td>Jones and Parameswaran (1983)</td>
<td>Ottawa 30-100 $D_5=0.37$ $C_u=1.0$</td>
<td>Triaxial Compression</td>
<td>7.7x10^{-8}</td>
<td>-11 to 0.1</td>
<td>0.1 to 82</td>
<td>10-67% Sand by Weight e=24.156</td>
<td>Not Reported (90-100?)</td>
<td>Distilled Water</td>
</tr>
<tr>
<td>Baker (1979)</td>
<td>Ottawa ASTM C-109 Fine Sand 10-100; $D_5=0.4$ $C_u=2$</td>
<td>Uniaxial Compression</td>
<td>1.0x10^{-2} to 1.0x10^{-7}</td>
<td>-5.5 to --</td>
<td>--</td>
<td>Dense</td>
<td>D=74.8%, Dense</td>
<td>97</td>
</tr>
<tr>
<td>Sayles and Epanchin (1966)</td>
<td>Ottawa ASTM C-109, 100 $D_5=0.4$ $C_u=2$</td>
<td>Uniaxial Compression</td>
<td>1.37x10^{-3} to 3.83x10^{-5}</td>
<td>-3 to -6.5</td>
<td>3 to 6.5</td>
<td>Dense to Very Dense</td>
<td>D=60-100%, 88.4-100</td>
<td>?</td>
</tr>
</tbody>
</table>
Table 2.9: Summary of Possible Deformational Mechanisms in Frozen Sand

<table>
<thead>
<tr>
<th>MAJOR GROUP</th>
<th>PROCESS DESCRIPTION</th>
<th>APPLICABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformation of Sand Skeleton</td>
<td>Elastic Deformation of Sand Grains</td>
<td>Occurs in frozen sand</td>
</tr>
<tr>
<td></td>
<td>Particle Crushing</td>
<td>Measured by Chamberlain et al. (1972)</td>
</tr>
<tr>
<td></td>
<td>Rolling or Sliding</td>
<td>Probably occurs but particles do not follow same trajectories</td>
</tr>
<tr>
<td>Deformation of Pore Ice</td>
<td>Diffusional Mechanisms</td>
<td>May be enhanced by unfrozen water film</td>
</tr>
<tr>
<td></td>
<td>Motions of Dislocations</td>
<td>Ice–water–silicate interface may not act as a strong barrier for dislocation pile ups</td>
</tr>
<tr>
<td></td>
<td>Dynamic Recrystallization</td>
<td>May be enhanced by unfrozen water film and smaller grain size</td>
</tr>
<tr>
<td></td>
<td>Internal Fracturing</td>
<td>Must occur during changes in volume</td>
</tr>
</tbody>
</table>
Table 2.10: Mechanisms of Strength in Frozen Sand (after Ting et al. 1983)

**POSSIBLE MECHANISMS FROM TING et al. (1983)**

1. **Ice Strength**
   - Function of \( \dot{\varepsilon}, T, \sigma_c, \) grain fabric

2. **Soil Strength**
   - Cited Rowe (1962) for drained strength of cohesionless soil having three components:
     - Sliding between grains
     - Particle inference
     - Dilatancy (volume increase against \( \sigma'_c \))

3. **Interaction Between Ice and Soil**
   - (a) Ice strengthening—due to altered
     - Structure
     - State of stress & deformational constraints
     - Strain rate (higher)
   - (b) Soil strengthening
     - Increased dilatancy
     - Structural hindrance
   - (c) Tension in unfrozen water film

**CONCLUSIONS FROM TING et al. (1983)**

- Thought to dominate at low strains.
- Thought to dominate at large strains.
- Thought to dominate at low strains.
- Thought to dominate at large strains.
- Increased \( \sigma' \) due to strong tensile adhensional soil–ice interface forces resisting expansion of soil skeleton.
- Synergistic mechanical interaction due to strong bonding between the ice matrix and soil skeleton.
- Probably not very significant (e.g., decreasing ice saturation leads to much lower strength).
Angular (0.00-0.15): very little or no evidence of wear; edges and corners sharp; secondary corners (the minor convexities of the grain profile, not the principal interfacial angles) numerous and sharp.

Subangular (0.15-0.25): definite wear; edges and corners rounded off to some extent; secondary corners numerous (10-20), although less so than in the angular class.

Subrounded (0.25-0.40): considerable wear; edges and corners rounded; secondary corners much rounded and reduced in numbers (5-10). Area of the original faces reduced, original interfacial angles, though rounded, still distinct.

Rounded (0.40-0.60): original faces almost completely destroyed; some comparatively flat surfaces may be present. May be broad re-entrant angles between remnant faces; all original edges and corners smoothed off to rather broad curves; secondary corners greatly subdued and few (0-5). At roundness 0.60 all secondary corners disappear. Original shape still apparent.

Well-rounded (0.60-1.00): no original faces, edges, or corners left; entire surface consists of broad convexities; flat areas absent; no secondary corners present. Original shape is suggested by the present form of the grain.
Cross sections through particles

Point of contact between particles lying below and above plane

\[ \sigma = \frac{\Sigma N}{a \times a} \quad \tau_x = \frac{\Sigma T_x}{a \times a} \quad \tau_y = \frac{\Sigma T_y}{a \times a} \]

Figure 2.2: Stresses in a Dry Particulate Medium (from Lambe and Whitman 1969)
Figure 2.3: Behavior of Dry Sand in Direct Shear (from Casagrande 1936)
Figure 2.4: Stress-strain Curves for Sand in Drained Triaxial Compression (from Lee and Seed 1967)
Figure 2.5: Effect of Confining Pressure on the Undrained Behavior of Loose Banding Sand in Triaxial Compression (from Mohamad and Dobry 1986, Data from Castro 1969)
Figure 2.6: Effect of Relative Density on the Undrained Behavior of Loose Banding Sand (from Mohamad and Dobry 1986, Data from Castro 1969)
Definition of the State Parameter (from Elghaib 1989)

Figure 2.7:
Figure 2.8: Comparison of Normalized Stress Paths for Samples with the Same State Parameter (from Been and Jefferies 1985)
Figure 2.9: Mohr's Circles and Failure Envelope for Dense Sand at Confining Pressures up to 40ksc (from Lee and Seed 1967)
Figure 2.10: Pressure Sensitivity of Principal Stress Ratio from Drained Triaxial Compression Tests (from Lee and Seed 1967)
Increasing Particle Interference

\[ R_p = \tan^2 (45 + \phi_f/2)(1 + d\epsilon_v/d\epsilon_a)_{\text{max}}. \]

where \( \phi_f = \phi_p - \phi_d \)

Figure 2.11: Components of Strength in Sands According to Rowe's Equation (after Rowe 1962)
Figure 2.12: Effect of Intermediate Principal Stress on the Drained Friction Angle of Several Sands (from Ladd et al. 1977)
Figure 2.13: Basic Tetrahedron Forming Ice Ih

Figure 2.14: Bases of Three Tetrahedrons Forming a wrinkled Hexagonal Ring
Figure 2.15: Structure of Ice Ih Showing the Basal Plane (from Shyam Sunder 1989 proposal to do research)
Figure 2.16: Idealized Creep and Stress–strain Curves for Polycrystalline Ice Tested in Compression (from Shyam Sunder 1989 proposal to do research)
Figure 2.17: Stress-strain Curves From Uniaxial Tension Tests on Polycrystalline Ice (from Hawkes and Mellor 1972)
Figure 2.18: Idealize Stress-strain Curve for "Moderate" Strain-rate Compression Test on Polycrystalline Ice (from Mellor and Cole 1982)
Figure 2.19: Temperature Dependence of Young's Modulus, Rigidity modulus and Poisson's Ratio of Granular Ice (from Sinha 1989)

\[
E = 8.93 + 1.2 \times 10^{-2} (T_m - T)
\]

\[
G = 3.41 + 4.5 \times 10^{-3} (T_m - T)
\]

\[
\mu = 0.308 + 7 \times 10^{-5} (T_m - T)
\]
Figure 2.20: Stress/strain-rate for Uniaxial Tests on Ice at -7°C (from Mellor 1980)
Figure 2.21: Stress–strain Curves for Fine–grained Polycrystalline Ice at 
-5°C and various Strain Rates (from Mellor and Cole 1982)
Figure 2.22: Logarithm of Strain rate as a Function of 1/T for Polycrystalline Ice and Single Crystals (from Mellor and Testa 1969)
Figure 2.23: Effect of Confining Pressure on the Yield Stress of Polycrystalline Ice (from Jones 1982)
Figure 2.24: Creep Data from Monocrystals in Easy and Hard Glide and for Polycrystalline Ice (from Duval et al 1983)
Figure 2.25  Stress–strain Curves for Single Ice Crystals in Basal and Nonbasal Glide (from Higashi 1969)
Figure 2.26: Proposed Structure of Frozen Sand (from Ting et al. 1983)
Figure 2.27: Schematic Diagram of Jellinek's Ice Adhesion Experiments (from Jellinek 1957)
Figure 2.28: Young's Modulus and Poisson's Ratio for Peabody Gravelly Sand (from Kaplar 1963)
Figure 2.29: Young's Modulus and Poisson's Ratio for Ottawa 16-100 Sand in Freshwater (from Baker and Kurfurst 1985)
Figure 2.30: Young's Modulus Versus Confining Pressure for Ottawa Sand and Sand-Ice Mixture (from Vinson 1978)
Figure 2.31: Effect of Salinity on the Modulus of Frozen Ottawa 16–100 Sand at Various Densities and Temperatures (from Baker and Kurfurst 1985)
Figure 2.32: Effect of Changing Salinity on Poisson's Ratio (from Baker and Kurfurst 1985)
Figure 2.33: Variation in Upper Yield Stress with Confining Pressure for Uniform Ottawa Sand at \(-10^\circ C\)
Figure 2.34: Effect of Strain Rate on the Unconfined Compressive Strength of Dense Sands at -10°C
Figure 2.35: Peak Deviator Stress Versus Confining Pressure for Tests on "Saturated" Dense Sands at Low to Moderate Confining Pressures
**Figure 2.36:** Peak Deviator Stress Versus Confining Pressure for Tests on "Saturated" Dense Sands at High Confining Pressures
Figure 2.37: Peak Deviator Stress Versus Volume Fraction of Sand
Unconfined Compression -10C  1.67x10^-4/s

Figure 2.38: Peak Deviator Stress Versus Relative Density for Frozen Ottawa Sand (from Baker and Kurfurst 1985)
Sayles (1966)
- Bieriez et al. (1967), $S_{pc} = 100$
- Bieriez et al. (1967), $S_{pc} = 60$
- Mellor and Smith (1966)
- Sayles and Epanchin (1966)
- Siarez et al. (1967), Srt - 100%
- Siarez et al. (1967), Srt - 60%
- Mellor and Smith (1966)
- Bragg (1983)
- Present study (Bourbonnais, 1984)

Figure 2.39: Uniaxial Compressive Strength Versus Temperature for Several Frozen Granular Materials (from Bourbonnais and Ladanyi 1985)
Figure 2.40: Peak Deviator Stress Versus Degree of Ice Saturation for Frozen Ottawa 20–30 Sand (from Alkire and Andersland 1973)
Figure 2.41: Effect of Degree of Saturation on the Minimum Creep Rate in Frozen Manchester Fine Sand (from Ting 1981)
Figure 2.42: Effect of Changing Salinity and Dry Density on the Unconfined Compressive Strength of Frozen Ottawa Sand (from Baker and Kurfurst 1985)
Figure 2.43: Effect of Confining Pressure on the Power Law Coefficient of Frozen Toyoura Sand (from Shibata et al. 1985)
Figure 2.44: Effect of Temperature on the Power Law Coefficient of Several Frozen Sands
Figure 2.45: Uniaxial Tensile Strength of Frozen Penn Sand (from Perkins and Reudrich 1973)
NOMINAL STRAIN RATE: $1.67 \times 10^{-4} \text{ s}^{-1}$ (1% min $^{-1}$)

TEMPERATURE: $-10^\circ \text{C}$

SALINITY: 0 ppt

$\sigma_c = 0$

Figure 2.46: Failure Strain as a Function of Dry Density for Frozen Ottawa Sand (from Baker and Kurfurst 1985)
Figure 2.47: Failure Strain as a Function of Temperature for Coarse Silica Sand (from Bourbonnais and Ladanyi 1985)
Figure 2.48: Failure Strain as a Function of Strain Rate for Wedron Silica Sand (from Bragg and Andersland 1980)
Force Polygons

Sand Intergranular Contact Forces

Ice-Silicate Interface Forces

Figure 2.49: Force Equilibrium of Sand Particle In Frozen Sand
Figure 2.50: Freebody Diagram for Kelvin Equation
Figure 2.51: Proposed Mechanism Map for Unconfined Compressive Strength of Frozen Ottawa Sand (from Ting et al. 1983)
Figure 2.52: Boundary Between Cavitation and Noncavitation Regimes in Unfrozen and Frozen Sand (from Ladanyi and Morel 1990)
ASSUMPTIONS

\[ \sigma_{\text{crit}} = 4 \text{ MPa} \]
\[ \phi' = 32^\circ \]
M.I.T. \( q_1 = T_1 \)
\[ T_1 = 2 \text{ MPa} \]
\[ \sigma'_3 = 0.1 \text{ MPa} \]

\[ p' = \frac{\sigma'_1 + \sigma'_3}{2} \]
\[ p = \frac{\sigma_1 + \sigma_3}{2} \quad \text{(MPa)} \]

Figure 2.53: M.I.T. \( q-p \) Plot for Ladanyi's Dilatancy Hardening Model

Failure occurs when effective stresses reach \( B \) or when the pore ice cavitates.
CHAPTER 3
HIGH-PRESSURE LOW-TEMPERATURE TRIAXIAL TESTING SYSTEM

This chapter describes the high-pressure low-temperature triaxial testing system built at M.I.T. for this research. The purpose for this system is to evaluate the strength and deformation behavior of frozen Manchester Fine Sand. The objectives of the system are: temperature control capable of covering a range from $-5$ to $-20^\circ C$, although in this thesis the temperature was maintained at $-10^\circ C$ (except for one test); pressure control capable of reaching 10 MPa; accurate on-specimen axial strain measurements; and accurate volumetric strain measurements. The temperature control and measurement system is first described. This is followed by a discussion of the pressure/volume system. Finally, the high pressure triaxial cell and loading frame are described.

3.1 TEMPERATURE CONTROL AND MEASUREMENT SYSTEM

This section describes the temperature control and measurement system designed and built at M.I.T.. The design objective was to regulate the temperature of the frozen specimens for a variable testing period that would range from several minutes up to 22 hours. The system consists of the existing M.I.T. low-temperature testing facility (Fig. 3.2), an environmental control chamber with a thermostator system, and silicone 510 fluid in a steel high-pressure triaxial cell (Fig. 3.5). The technique used to measure the temperature is presented and the system performance is evaluated in terms of the temperature fluctuations in the testing room, in the environmental chamber which encloses the triaxial cell, and inside the triaxial cell next to the specimen.
3.1.1 Temperature Measurement

Temperatures were measured by wafer thermistors, part number WM222C, manufactured by Sensor Scientific Inc. of Fairfield, New Jersey. These are composed of a semi-conductor material whose resistance changes as a function of temperature. These sensors have a negative temperature coefficient, i.e., as the temperature decreases their resistance increases. By connecting one of these thermistors in series with a fixed resistance and providing a known input voltage, it is possible to measure the resistance of the thermistor. The resistance is then converted to a temperature by the use of a calibration curve.

Interchangeable wafer thermistors were used in the first portion of the testing series. The manufacturer reports that the interchangeable thermistors have an accuracy of ±0.2°C under "zero power" conditions, i.e., very low self heating. One of these thermistors was suspended inside the triaxial cell near the top of the frozen specimen. Refer to Fig. 3.17 for location. In the final testing series individually calibrated wafer thermistors were used. The individual calibrations were traceable to the National Bureau of Standards and thus represented a more accurate measurement of temperature. Two of these were mounted inside the triaxial cell, one near the bottom of the specimen and one near the top of the specimen. Refer again to Fig. 3.17. Two other thermistors were located outside of the triaxial cell at various locations to sense the air temperature in the environmental chamber or the testing room. Refer to Fig. 3.5.

Figure 3.1 presents a schematic diagram of the circuit used to measure temperature with these thermistors. A known voltage was applied across the fixed resistor and thermistor in series. The voltage drop across the fixed resistor
was measured. The following equation was used to compute the thermistor resistance,

\[ R_t = R_f \left[ \frac{V_o}{V_f} - 1 \right] \]  \hspace{1cm} 3.1.1

where \( V_o \) is the input voltage, \( V_f \) is the measured voltage drop across the fixed resistor, \( R_f \) is the fixed resistance (which also includes the line resistance), and \( R_t \) is the thermistor resistance.

The temperature as measured by the interchangeable thermistors used in the first portion of the testing series was computed from the Steinhart–Hart Equation,

\[ \frac{1}{T} = A + B \ln(R_t) + C (\ln(R_t))^3 \]  \hspace{1cm} 3.1.2

where \( T \) is the temperature in °K, \( R_t \) is the thermistor resistance from Eq. 3.1.1, and \( A, B \) and \( C \) are constants determined from a three point calibration. The values obtained for these constants were \( A = 1.3147 \times 10^{-3}/°K \), \( B = 2.3186 \times 10^{-4}/°K \), and \( C = 1.0625 \times 10^{-7}/°K \) using the interchangeable thermistors.

The individually calibrated thermistors used in the final series of tests were provided with calibration tables of resistance versus temperature at increments of 0.005°C. A linear interpolation scheme, between calibration points at a spacing of 0.5°C, was used to compute the temperature. These temperatures from the interpolation scheme varied by less than 0.01°C from the reported calibration values read directly from the calibration tables. The temperature as computed by this interpolation scheme was rounded to the nearest 0.01°C.

Due to their small size, both the interchangeable thermistors and the individually calibrated thermistors can self-heat and cause inaccuracies in the measured temperature. These thermistors had a self-heating coefficient of 2.4 \( \times 10^{-3} \) Watts/°C. The self-heating error associated with each type of thermistor was estimated by computing the power dissipated. The interchangeable
thermistors used in the first part of the testing series were used with an input voltage of 5.5 Volts and were placed in series with a 10 kΩ resistor. At −10°C the resistance of these thermistors is approximately 12 kΩ. This leads to an estimated self-heating error of 0.6°C, due primarily to the very large input voltage used. For this reason, temperature as measured by these thermistors was not relied upon and will not be reported in this Chapter. The individually calibrated thermistors were used with a much lower input voltage (0.25 Volts) in order to cut down on the self-heating error. These were placed in series with a 10 kΩ resistor. Their resistance at −10°C is also approximately 12 kΩ. This leads to an estimated self-heating error of less than 0.001°C, which is on the order of the accuracy of the calibration as reported by the manufacturer and well below the accuracy of the linear interpolation scheme used to convert the resistances to temperatures.

Note that in the following discussion where temperatures are reported, these represent the measured temperatures. The writer recognizes that since these are sampled at various time intervals, they represent a discontinuous data set and hence may not accurately reflect the actual maximum, minimum and mean values for the temperatures experienced. The reader should consider the presented data accordingly.

3.1.2 M.I.T. Low Temperature Testing Facility

The M.I.T. Low-Temperature Testing Facility was completed in 1988 with a combination of grants from NSF and two M.I.T. projects, ARO/PACT and the Center for Scientific Excellence in Offshore Engineering. It has three separate rooms with independent temperature control: the vestibule; the growth room; and the testing room. Figure 3.2 is a plan view of the facility. It is
located inside of a regular room—temperature laboratory. The outer room or vestibule is used as a temperature buffer between the outside laboratory and the inner cold rooms. It cuts down on the amount of humidity that reaches the two inner rooms and also serves as a staging area for preparing frozen soil specimens. It normally has a temperature of -4 to -5°C. The growth room, located to the rear of the vestibule and maintained at a temperature of 0°C, is used for specimen preparation and freezing. The testing room contains the loading frame and the triaxial testing system. It can be regulated to -40°C, and is normally maintained at a temperature a few degrees colder than the desired set point temperature for triaxial testing. Temperature control of the triaxial cell is accomplished by heating a small environmental chamber (Fig. 3.5) located inside the cold room.

Each of these rooms is composed of sheet metal walls with styrofoam insulation. The testing room has a double-paned glass window to allow for observation of the testing equipment from outside of the cold room. This window is heated to prevent icing. The testing and growth rooms are fitted with feed-through ports to allow for the passage of instrumentation cables and power lines. These are used to connect the thermoregulator system, load cell, displacement transducers, pressure transducer and temperature sensors to the controller circuits, power supplies and data acquisition system located outside the harsh environment of the testing room. Only the actual sensing devices, the circulating fan and heat source of the thermoregulator system and the pressure/volume controller system are located inside the cold room.

The temperature inside the testing room fluctuates about a control point. Figure 3.3 shows the measured air temperature inside the testing room over a one hour period sampled at ten second intervals during test NS13ps. Here,
NS stands for a test that was conducted without a specimen (NS = no specimen) and ps stands for that portion of the test after the application of the cell pressure but prior to the shearing phase of the test (ps = preshear). Testing procedures are discussed in Section 4.3.4. The mean temperature (control point) for NS13ps is $-15.00^\circ$C. The air temperature cycles with a period of 410 seconds with a maximum observed temperature of $-13.42^\circ$C and a minimum observed temperature of $-16.19^\circ$C. Thus, the fluctuation is roughly $\pm 1.5^\circ$C about the set point. Figure 3.4 plots the temperature fluctuation for the same test over a 22 hour period sampled at four minute intervals. The mean temperature over this period is approximately $-14.95^\circ$C, with a maximum of $-11.95^\circ$C and a minimum of $-16.17^\circ$C.

The testing room has a preprogrammed defrost cycle that operates twice a day for a period of approximately 20 minutes. The two largest temperature spikes in Fig. 3.4 reflect this defrost cycle. Table 3.1 summarizes the measured temperature inside the testing room for five tests run over a four day period (including the test presented in Fig. 3.4). The maximum observed temperature over this entire period was $-10.06^\circ$C and the minimum was $-16.17^\circ$C. The average value of the mean temperatures of all of these observations was $-14.93^\circ$C. The average fluctuation in temperature for all these observations was $3.5 \pm 1.3^\circ$C. The mean temperature computation for the measurements lasting only 500 seconds (0.14hr) are subject to some error because they include data for just over one time cycle. The desired set point for testing the frozen sands in this program was $-10^\circ$C. Over the four day period of these observations, the measured cold room temperature never rose above that desired set point.
3.1.3 **Environmental Chamber and Thermoregulator System**

The environmental chamber is a double-walled container composed of sheets of styrofoam mounted on a wooden frame. Figure 3.5 is a drawing of this chamber and illustrates the location of all of the devices within it. The front door of the chamber is constructed of two sheets of plexiglas mounted on an aluminum frame to allow for direct observation of the testing equipment during operation. It is mounted on top of the loading frame and completely encloses the triaxial cell, the loading platen, the crosshead of the loading frame and the oil reservoir for the pressure/volume controller. The chamber creates an environment in which the temperature can be regulated by a heating source to a few degrees warmer than the ambient temperature of the testing room.

The temperature inside of the environmental chamber is controlled by a thermoregulator system composed of a continuously circulating fan (flow rate = 45cfm), a heat source mounted inside of a metal box and a mercury contact switch preset at the desired testing temperature. This system operates on the following principle. A fan mounted on the front of the metal box that houses the heat source continuously circulates the air in the chamber. A mercury contact switch is suspended in the chamber over the top of the metal blower box. When the temperature in the environmental chamber at the position of the mercury contact switch drops below the set point, the heat source (a light bulb) turns on and increases the temperature. When the temperature inside the environmental chamber rises above the set point, the light turns off and the temperature begins to decrease as heat is lost to the colder testing room. The temperature in the chamber is thus fluctuating near the desired set point. The mercury contact switch used for these tests, made by Princo Instruments of New Jersey, was preset in the factory to switch at $-10 \pm 0.05^\circ C$. The controller circuit that does
the actual switching of the heat source in response to the switching of the mercury contact switch is located outside of the cold room.

Figure 3.6 shows the fluctuation of temperature recorded every ten seconds at a point in the environmental chamber close to the mercury contact switch (labeled the upper thermistor in Fig. 3.5) during a one hour interval of test NS13ps. The temperature cycles with a mean of $-10.01^\circ C$, a maximum of $-9.91^\circ C$ and a minimum of $-10.13^\circ C$. The period of this cycle is approximately 400 seconds, i.e., similar to the period of the temperature fluctuation in the testing room.

In order to assess the distribution of temperatures inside the chamber, two thermistors were suspended in different locations along the air circulation path during several tests. One was placed near the base of the triaxial cell (labeled lower thermistor in Fig. 3.5) and another was placed near the top close to the mercury contact switch (labeled upper thermistor).

Table 3.2 presents the mean temperature and maximum and minimum observed temperature at these two locations for various times during tests FRS59 through FRS67 (FRS here refers to a test where there was a frozen sand specimen in the triaxial cell) and NS10 through NS18. Examination of Table 3.2 indicates that, in general, the difference between the maximum and minimum observed temperature at a specific location inside the environmental chamber during a given test is approximately $\pm 0.125^\circ C$, with the temperature near the base of the triaxial cell warmer than near the top. The average difference between the mean upper and lower temperatures is $0.2 \pm 0.15^\circ C$. The average range in temperature at any one point is $0.29 \pm 0.09^\circ C$.

Warm air circulates out of the metal blower box, under the triaxial cell, up the face of the plexiglas door, across the top of the triaxial cell, down the back
of the environmental chamber, passed the thermoregulator mercury contact switch and back into the top of the metal blower box. After each of the FRS tests, the triaxial cell was completely removed from the environmental chamber and then replaced as a part of the setup procedure. The thermistors were located each time in approximately the same position. Slight variations in the location of the thermistor could have some effect on the measured temperature. In the NS series, the thermistors were not moved between tests. These reflect more clearly the stability of the temperature in the environmental chamber over time at one location.

Figure 3.7 is a time history of the measured temperature sampled every 4 minutes at two locations in the environmental chamber over a 22 hour period during test NS10sh (the sh refers to that portion of the test when the loading frame was activated and the loading piston was forced into the triaxial cell, i.e., sh = shear). The temperature near the base of the cell had a mean value of \(-9.75^\circ C \pm 0.19^\circ C\). The temperature near the top of the triaxial cell had a mean value of \(-10.04^\circ C \pm 0.15^\circ C\).

All of these measurements indicate that the air temperature in the environmental chamber is fairly stable and reproducible and that the environmental chamber and thermoregulator system have been successful in decreasing the temperature fluctuations in the testing room by roughly one order of magnitude, i.e., in the cold room the temperature fluctuates by roughly \(3^\circ C\) compared to a \(0.29^\circ C\) fluctuation in the environmental chamber. These measurements also show that the temperature is warmer near the base of the triaxial cell where the air blows out of the metal box and colder near the top of the triaxial cell after it has traveled along the face of the plexiglas door. The greater amount of scatter in the measurements during the FRS series is probably
due to placing the thermistor in slightly different positions with each setup. The fact that the measured temperature near the base of the triaxial cell is warmer than that at the top, with the exception of FRS65, can be explained by the heat produced by the continuously circulating fan and the cooling that occurs as the air passes along the circulation path and back into the blower fan box.

3.1.4 Triaxial Cell and Silicone 510 Oil

The wall of the triaxial cell is 0.5 inch thick steel and has a high thermal conductivity. A buffer is needed to damp out the temperature fluctuations in the environmental chamber that might be readily transmitted through the triaxial cell wall. The silicone 510 oil manufactured by the Dow—Corning Company which was used in the triaxial cell as a pressurizing medium also functions as this buffer. It was selected for its low viscosity at low temperatures, its nonconducting properties (necessary for internal force and temperature measurements with electronic devices) and its compatibility with the prophylactic membranes used to surround the frozen sand specimens. The manufacturer recommended its use for temperatures down to $-51^\circ$ C.

Temperature measurements inside the triaxial cell indicate that the silicone oil acts as an efficient temperature buffer. Figure 3.8 shows a time history of the fluctuation in the temperature of the silicone oil near the base and top of the frozen soil specimen over a 22 hour period during the shear of FRS61. The temperature was sampled every 30 seconds for the first 3 hours, and then every 11 minutes for the remainder of the test. The mean value of the temperature observed near the top of the specimen was $-9.41^\circ$ C, and varied less than $0.11^\circ$ C over the entire 22 hour period. The mean value of the temperature observed near the base of the specimen was $-9.72^\circ$ C, and also varied less than
0.1°C over the entire 22 hour period. This amount of temperature variation is representative of most of the tests in the FRS series. The cyclic nature of the temperature was not apparent in these data. This is probably due to the infrequent reading rate and the limited bit precision of the temperature measure. A more precise measure at a higher frequency would probably demonstrate a cyclic nature.

Table 3.3 presents the mean, and maximum and minimum observed temperatures in the oil surrounding the frozen soil specimen for tests FRS59 to FRS67. The average of the mean values over all these tests for the temperature near the bottom of the specimen was $-9.72°C$, with a maximum of $-9.62°C$ and a minimum of $-9.86°C$. The average of the mean values of the temperature near the top of the specimen was $-9.38°C$, with a maximum of $-9.17°C$ and a minimum of $-9.53°C$. The maximum measured range in oil temperature at a given location during the FRS series was 0.22°C. This occurred near the top of the specimen in test FRS65. The average range in temperatures at a given location during shear was $0.10 \pm 0.04°C$ standard deviation. For each of these tests, the thermistors were removed and then replaced as a part of the new specimen setup procedure. They were placed in nearly the same position. Some of the variation in the mean temperature from test to test is possibly due to slight changes in the positions of the sensors.

Table 3.3 also presents the mean value of the oil temperature near the bottom of the pedestal during tests NS10 through NS18. These NS tests were conducted without a specimen in order to evaluate the behavior of the volume change system. In these tests the observed temperature of the oil over a 10 day period had a maximum of $-9.58°C$ and a minimum of $-9.92°C$. The average of the mean values of the temperature near the base of the pedestal was $-9.80°C$. 
The oil temperature measurements for the FRS tests indicate a considerable temperature gradient in the triaxial cell along the length of the specimen. The oil tends to be approximately 0.3 to 0.4°C warmer near the top of the specimen. This is opposite to the temperature gradient observed in the environmental chamber outside the triaxial cell. This gradient may be caused by the heat generated by the ACDT’s mounted on the displacement yoke attached to the specimen, the heat generated by the load cell located inside the triaxial above the specimen, and/or the stagnant condition of the oil inside the cell, favoring the warmer oil to float on top of the cooler oil. Note that in the NS testing series, both the ACDT yoke and the load cell were present inside the triaxial cell just as during the FRS series. The results from both the FRS and the NS series indicate that, although an internal temperature gradient exists, the oil temperature at a given location is very stable and reproducible. These oil temperature measurements also indicate that the specimen was slightly warmer than the target temperature. The target temperature was \(-10 \pm 0.3^\circ\text{C}\). The mean oil temperature near the base of the specimen was \(-9.72 \pm 0.04^\circ\text{C}\) and near the top of the specimen it was \(-9.36 \pm 0.07^\circ\text{C}\).

Figure 3.9 summarizes the temperature control system performance by plotting the temperature fluctuations in the cold room, environmental chamber and triaxial cell over a 22 hour period to show the efficiency of the system in damping out the fluctuations. The data in this figure were presented in Figs. 3.4, 3.7 and 3.8. Table 3.4 summarizes the typical temperatures and temperature fluctuations in the cold room, environmental chamber and triaxial cell.

3.2 PRESSURE CONTROL AND VOLUME CHANGE MEASURING SYSTEM

This section describes the pressure/volume control system designed at
M.I.T.. This system was intended to regulate the cell pressure up to 10 MPa with an accuracy of ±1% and to measure specimen volumetric strain to an accuracy of ±0.1% at axial strain levels in excess of 20%. The pressure volume controller is a closed-loop analog feedback control system. This section first describes the system and its various sensing devices. Next, the measured performance of the pressure control is evaluated. The method for measuring volumetric strains is then described and evaluated.

3.2.1 Description of Pressure/Volume Control System

This pressure/volume control system is a DC servomotor based controller system which continuously senses and adjusts the fluid pressure to maintain it at the required set point. Figure 3.10 shows a schematic diagram of the mechanical portion of the pressure/volume control system. The various components of the system are as follows:

1) DC servomotor and controller, models E352-MGHD and E-352-BU, respectively, manufactured by Robbins and Myers Electrocraft of Hopkins, Minnesota

2) Semiconductor pressure transducers, models ABHP 200 and ABHP 2000 manufactured by Data Instruments of Acton, Massachusetts

3) Ball screw actuator, model 28631 manufactured by Duff Norton of Charlotte, North Carolina

4) 3/4 in. diameter cylindrical stainless steel piston

5) Brass and stainless steel cylindrical pressurizing assembly

6) DC–DC type Linear Variable Differential Transformers manufactured by Trans–Tek Inc. of Ellington, Connecticut.

The DC–servomotor is attached to the ball–screw actuator. The ball screw actuator drives a piston into or out of a reservoir of silicone oil. By controlling
the motion of the piston, the fluid pressure in the reservoir can be either increased or decreased.

The pressure control system operates on the following principle. See Fig. 3.11 for schematic diagram of the pressure controller system. The semiconductor pressure transducer continuously senses the pressure of the cell fluid. A 2000 psi transducer was used for tests with cell pressures of 2, 5 and 10 MPa and a 200 psi transducer was used for tests with a cell pressure of 0.1 MPa. The DC output signal from these transducers is amplified 100 times and compared electronically to a reference voltage using an instrumentation amplifier, model AD524 AD manufactured by Analog Devices of Norwood, Massachusetts. The difference between the reference voltage and the amplified pressure transducer signal (called the error signal) is sent directly to the DC—servomotor controller and this causes the motor to either advance or retract. The error signal can be selectively amplified to increase the sensitivity of the system.

If, during the course of the test, there is a tendency for the cell pressure transducer signal to decrease below the reference voltage (due to leakage of the oil, a decrease in volume of the frozen soil specimen or a decrease in temperature of the oil), the electronic comparator circuit will sense this difference and will send a positive voltage signal to the the DC—servomotor, which will advance the piston and increase the fluid pressure. If, during the course of the test, there is a tendency for the cell pressure transducer signal to increase above the reference voltage signal(due to advancement of the triaxial loading piston into the triaxial cell, an increase in the volume of the frozen soil specimen or an increase in the temperature of the pressurizing oil), the electronic comparator circuit will sense this difference and send a negative voltage signal to the DC—servomotor, which will retract the pressurizing piston from the cylinder and decrease the pressure.
These adjustments take place continuously during the test.

The sensitivity of this pressure control circuit can be increased by increasing the amplification of the error signal. The higher the amplification, the faster the circuit will respond to small deviations in cell pressure. This becomes important in fast strain rate tests when the triaxial cell loading piston is entering the chamber rapidly, thus causing the cell pressure to increase. This error amplification was changed for various testing conditions, but was not recorded in each test. In general, the largest amplification was used for the fastest tests and the smallest error amplification was used for the slowest tests.

### 3.2.2 Cell Pressure Controller Performance

Figure 3.12 shows a time history of the measured cell fluid pressure \((\sigma_c)\) during shear at \(\sigma_c = 0.1\) MPa and the slowest strain rate \((3 \times 10^{-4}/\text{sec})\) in test FRS46. The total duration of this test (approximately 22 hours) is representative of the longest tests in this testing program. The point at the start of shear is labeled with a square. The magnitude of the cell pressure increased during shear. The major cause for this pressure increase was the motion of the triaxial cell loading piston into the triaxial cell. The pressure started off at approximately 0.0993 MPa and within several hours, the pressure increased to approximately 0.1013 MPa. The target pressure in this case was 0.100 MPa. The observed pressure jump represented a 2% increase in the magnitude of the cell pressure. After the shift to the higher cell pressure, the pressure remained stable over the rest of the test. The behavior in this test is representative of the majority of the tests regardless of the total duration, i.e., there was a pressure increase after the start of shear and thereafter the pressure was stable.

Table 3.5 presents the cell pressure control for test FRS15 and tests
FRS18–FRS67. The maximum, minimum and mean values of the cell pressure for each reported test are tabulated. The deviation of the mean pressure from the target pressure is also presented as a percentage of the target pressure. The range in pressure is also tabulated as a percentage of the target pressure.

Table 3.6 summarizes of the cell pressure control during shear for the cell pressures tested in this research. It indicates that in general the mean cell pressures were slightly above the target values and that as the cell pressure increased, the deviation of the mean pressure from the target decreased. The 0.1 MPa tests exhibited the largest variation in cell pressure (based on a percentage of the target pressure).

The reason for the differences in the performance of the pressure controller system when using the same transducer, but at different target pressures, was that a different amplification factor was used for various tests. The magnitude of the error amplification was not recorded and may have been different for various tests with identical cell pressures and strain rates.

Variations in the cell pressure during shear, of the magnitude reported in Table 3.5 and 3.6, should have a negligible effect on the measured stress–strain behavior of frozen Manchester Fine Sand as presented in Chapter 5. In these tests, a fluctuation in the magnitude of the cell pressure did not affect the output of the load cell because it was mounted inside the triaxial cell. Refer to Section 3.3. The difference between the major and minor principal stresses was measured directly by the load cell and it was insensitive to small fluctuations in the cell pressure.
3.2.3 Specimen Volumetric Strain Measure

The volume changes of the frozen sand specimens were inferred from changes in volume of the cell fluid. These were measured directly from the motions of the cell pressure piston and the triaxial cell loading piston. These were measured by DC–DC type Linear Variable Differential Transformers (LVDT’s) mounted on each piston. The corresponding volume changes were computed from the cross-sectional area of each piston. Various "corrections" were then applied to these measurements in order to account for leakage of the cell fluid, compressibility of the cell fluid and triaxial chamber, flexure of the base of the triaxial cell, and nonuniformities in the diameter of the cell pressure piston. Each of these corrections will be discussed in turn.

The equation used to compute the volume change of the frozen soil specimen is as follows

\[ V_s = -\Delta L_{cp} A_{cp} - \Delta L_{tp} A_{tp} + LR \Delta t + \Delta F_{ax} C_{ax} + \Delta \sigma_c C_p + \Delta V_{cp} \]  

3.2.1

where

\[ \Delta V_s = \text{the change in volume of the specimen (positive means dilation)} \]
\[ \Delta L_{cp} = \text{the distance that the cell pressure piston travels into the cell fluid reservoir} \]
\[ A_{cp} = \text{the area of the cell pressure piston} \]
\[ \Delta L_{tp} = \text{the distances that the triaxial cell piston travels into the triaxial cell} \]
\[ A_{tp} = \text{the area of the triaxial cell piston} \]
\[ LR = \text{the leakage rate of fluid out of the triaxial cell} \]
\[ \Delta t = \text{the time from the start of the test} \]
\[ \Delta F_{ax} = \text{the change in axial force acting on the base of the triaxial cell through the specimen} \]
C_{ax} = the coefficient relating the amount of base flexure of the triaxial cell to the corresponding volume change

\Delta \sigma_c = the change in cell fluid pressure from the start of the test

C_p = the measured compressibility of the cell fluid and triaxial cell including transmission lines

\Delta V_{cp} = a term to account for nonuniformities in the diameter of the cell fluid piston along its length.

**Leakage Rate**

The leakage rate (due to the flow of oil between the loading piston and its bushings and around the O-rings in the triaxial cell) correction was computed for each test in the following manner. Prior to the shearing phase of each test, the cell pressure was applied to the specimen and sufficient time was allowed to stabilize the temperature and to obtain a stable leakage rate of oil out of the triaxial cell. This preshear time was at least as long as the shear time for each test. The leakage rate of oil out of the triaxial cell was computed using Eq. 3.2.1 by assuming that \( \Delta V_s = 0 \) (i.e., \( LR \Delta t = \Delta L_{cp} A_{cp} - \Delta \sigma_c C_p - \Delta V_{cp} \)). Figure 3.13 is a time history of the volume change during the preshear phase of test FRS59. This test was run at \( \sigma_c = 5.0 \) MPa with a moderate strain rate (2 hour shear time). As observed in the figure, the leakage rate of oil out of the triaxial cell was fairly stable for a time period almost five times larger than the shear time. In this test the leakage rate was \( 6.9 \times 10^{-6} \) cc/sec (computed from a linear regression analysis). The duration of the shear portion of this test was 7200 seconds. The absolute magnitude of the final leakage rate correction (at the end of the test) was 0.07% of the specimen volume.

The leakage rate was sensitive to the confining pressure and to the amount of time over which the observation was made, i.e., the leakage rate
decreased slowly with time and a shorter observation period would generally yield a larger leakage rate. In general, tests at the lowest confining pressures (0.1 MPa) with the longest shear times, 22 hours, exhibited the smallest leakage rates and tests at the highest cell pressures (10 MPa) and the shortest shear times exhibited the greatest leakage rates. Table 3.7 lists the measured leakage rate for each reported test and the maximum magnitude of the leakage rate correction (typically 0.07 to 2.1 cc) for each test as a percentage of the specimen volume. Table 3.8 summarizes the cell fluid leakage rate behavior by presenting the average maximum leakage rate correction for each of the testing conditions. This table indicates that the slowest tests (total shear time 22 hours) at the highest confining pressure ($\sigma_c = 10$ MPa) had the largest average leakage rate correction, 2.4% of the specimen volume, and the fastest tests (total shear time 5 minutes) at the lowest confining pressures ($\sigma_c = 0.1$ MPa) had the smallest average leakage rate correction, 0.01% of the specimen volume. The largest leakage rate correction in any test was 2.94% of the specimen volume in FRS48 and the smallest leakage rate correction in any test was < 0.01% of the specimen volume in FRS34, 40, 55 and 65.

Compressibility of Cell Fluid and Triaxial Cell

The compressibility of the cell fluid and triaxial cell was obtained directly for FRS25 through FRS67. In FRS15 and FRS18 through FRS24, the cell fluid compressibility was not measured. The average compressibility from all later tests at the same cell pressure was used to correct the data in these tests.

After the preshear period, when the leakage rate and temperature were stable, the cell fluid compressibility was determined by manually cycling the cell pressure over a small range and measuring the resulting volume changes. The
magnitude of these cell pressure cycles was selected to be roughly within the range of the expected cell pressure change caused by shearing (see Section 3.2.2). Figure 3.14 plots the volume change versus cell pressure for FRS59 during the compressibility check. The pressure was cycled twice between 4.92 and 5.08 MPa over a period of 162 seconds. This pressure cycle encompassed the measured range in cell pressure for this test (see Table 3.5). The response of the system during this compressibility check was almost linear over the range of cell pressures investigated and the volume change over this range was on the order of 0.3 cc. This was a typical response for all of the compressibility checks made during this testing series. A linear regression was performed on the data and the resulting compressibility was 1.9cc/MPa with an $r^2$ of 0.990.

Table 3.7 reports the cell fluid compressibility for each test and the magnitude of the correction as percentage of the specimen volume. Table 3.8 summarizes the average compressibility correction for each of the testing conditions. In the actual data reduction program, the cell pressure was smoothed with a 3-point moving window prior to making the compressibility correction to the volume change.

**Base Flexure**

A series of calibration tests were conducted using an aluminum dummy specimen in place of a frozen sand specimen. During these tests a measured increase in volume of the cell fluid with an increase in axial force was observed (thus leading to an apparent decrease in the specimen volume). Such an observation could be explained by flexure of the base of the triaxial cell as a diaphragm. A correction to the volume change was made for this flexure. This correction was assumed to be a linear function of the axial force acting on the
base of the triaxial cell over the entire range of axial forces experienced in these tests. The magnitude of this correction was deduced in the following manner.

An aluminum dummy specimen was placed in the triaxial cell and the cell pressure was increased to 10 MPa. The specimen was loaded manually to a deviator stress of 13 MPa (13.2kN) and unloaded. The corresponding volume change was computed from Eq. 3.3. Figure 3.15 plots the measured volume change versus the axial force for two loading and unloading cycles in one such test. The slope of these volume change curves was greater in loading than unloading. The correction was taken from the average slope of the measured volume change curve from the loading portion of four such cycles. This correction was 10.2 cc/MN. The maximum axial force measured during the test program was 23.8kN and occurred in FRS39. This corresponded to a correction of 0.35% of the specimen volume.

In addition to causing changes in the volume of the cell fluid, flexure of the base of the triaxial cell also caused errors in the axial strain measure of the specimen by the DCDT mounted outside of the triaxial cell. The magnitude of this compliance error was measured using the aluminum dummy specimen without grease as explained in the prior paragraph. The strain of the aluminum, as measured by the ACDT's, was subtracted from the total measured compliance. The compliance due to base flexure was 0.862 cm/MN. This correction was applied to all the DCDT data and corresponds to approximately 0.3% $\varepsilon_a$ based on the maximum measured axial force of 23.8 kN.

**Diameter of Cell Pressure Piston**

The diameter of the cell pressure piston was not uniform along its length as originally assumed. Figure 3.16 plots the measured volume change versus the
normalized cell pressure piston voltage (a measure of the absolute position of the piston) for NS31, a test with no specimen carried out over the entire range of travel of the pressure piston. The target pressure for this particular test was 0.1 MPa. The cell pressure piston was set at its maximum possible extension into the pressure reservoir (Fig. 3.10) and the axial loading piston was advanced into the triaxial cell at the rate corresponding to the fastest strain rate. This caused the cell pressure piston to retract. The test was terminated when the cell pressure piston had fully retracted from the pressure reservoir. The total length of travel for the cell pressure piston was approximately 15 cm. The test lasted 2580 seconds. The corresponding volume change was computed from Eq. 3.3.

Figure 3.16 indicates that along the first 5.5 cm travel of the piston out of the pressure reservoir (normalized voltage from 0.5 to 0.1), the diameter of the piston was fairly constant. Along the final 9.5 cm of travel (normalized voltage from 0.1 to -0.5) there was an apparent increase in the cell fluid volume. This indicated a slight tapering (decrease in the diameter) of the rod. The total error in volume measure was 1.3cc (ideally it should have been 0cc with no specimen), and assuming that all other errors are fully accounted for (i.e., the leakage rate and cell compressibility), this corresponds to a change in the diameter of the rod of 0.045cm along the 9.5cm length, assuming that the change is linear. An actual measurement of this diameter was not performed to validate this assumption.

In order to correct for the nonuniformity in the diameter of the piston, a piecewise linear correction was applied to the data. This has been summarized in Table 3.9. The normalized position of the piston was divided into seven regions and the volume measure was corrected within each of these seven regions using a linear calibration. Several tests were conducted without a specimen (NS series) in order to assess the variation in the "no volume change" condition. The
correction was made in each test by converting travel of the piston to a normalized voltage, computing the corresponding error in the volume measure and adding or subtracting this error volume to the measured volume change. Section 4.6.5 presents the results of these tests in a discussion of the accuracy of the volume change measure. The maximum magnitude of the correction due to the nonuniformity of the cell pressure piston diameter was approximately 1.2 cc or 1.8% of the average specimen volume.

**Summary to Volume Change Corrections**

Corrections to the measured volume change were made to all specimens in tests FRS18 through FRS67. In the case of FRS15, the pressure controller was located outside of the cold room and the volume change was severely affected by temperature fluctuations. Additionally, the triaxial cell loading piston had only one O-ring and this caused an enormous leakage rate. The confining pressure was 10 MPa in this test and the strain rate was moderate. Based on results from similar tests with reliable volume change data, the \( \epsilon_v \) for this test was assumed to be zero.

Table 3.10 summarizes the ranges in the maximum magnitude of these volume change corrections for all reported tests. This indicates that in a given test under the least favorable conditions, i.e., where the maximum of each correction were applied, the magnitude of the correction in terms of the specimen volume could have been as much as 4.2% of the specimen volume. The majority of this is due to the leakage correction in the slow tests. A discussion of the accuracy of the volume change measure is presented in Section 4.6.5.
3.3 HIGH PRESSURE TRIAXIAL CELL AND LOADING FRAME

The triaxial cell used for this testing program was manufactured by Wykeham Farrance Engineering Ltd. of Slough, England. It was first used at M.I.T. 25 years ago for investigating the strength–deformation properties of stabilized soils (sponsored by the Army Material Command). The cell was modified to accommodate lubricated end platens, internal force measurement, on–specimen axial strain measurement and internal temperature measurement. This section describes the cell and most of these modifications.

3.3.1 Modified High Pressure Triaxial Cell

Figure 3.17 is a side view of the triaxial cell. As originally designed, this triaxial cell had a fixed bottom pedestal and floating top cap (i.e., not rigidly attached to the loading ram) which were both 1.4 inch diameter and were intended for use with porous stones for drainage. In the redesign the bottom pedestal and top cap were replaced by enlarged "lubricated" ends that could accommodate radial deformation of the frozen soil specimen during shear. The specimens tested measured 1.4 inch diameter and the end platens were 1.6 inch diameter. The modified top cap was also designed as a floating top cap. The cylinder wall of the triaxial cell was replaced with a much longer one to accommodate the length of the new top cap and internal load cell.

3.3.2 Internal Floating Load Cell

The load cell was located inside the triaxial chamber at the end of the loading piston to avoid measurement errors due to friction between the loading piston and its brass bushing guide. The load cell was model JP–10,000 manufactured by Data Instruments of Acton, Massachusetts. The load cell
contacted the top cap with a cylindrical seating piece. This piece was in contact with the entire flat end of the cylindrical top cap and had a small circular lip with an inside taper which provided a guide during set up when the load cell and piston were lowered onto the top cap and that restricted lateral motions of the top cap during shear.

3.3.3 On-Specimen Axial Strain Measure

On-specimen axial strains were measured by two Alternating Current Differential Transformers (ACDT's) mounted in diametrically opposite positions around the circumference of the frozen sand specimen on a specially designed yoke. The ACDT's were model 100 MHR manufactured by Schaevitz Engineering of Pennsauken, New Jersey. Figure 3.18 shows a side view of this yoke. The upper piece holds the barrels of the displacement transducers and the cores rest on the lower yoke. Each yoke is spring loaded and contacts the specimen at third points around the circumference. The face of each contact foot is a flat rectangle approximately 0.125 inches wide and 0.25 inches long.

During set up, three alignment rods run between the upper and lower yoke. These are held in position with 0.063 inch diameter hardened steel pins. The function of these rods is to aid in the placement of the yoke on the specimen and to fix the appropriate gage length. After placing the yoke on the specimen, these small pins are removed and the alignment rods drop freely away from the yoke allowing it to be free floating and move unimpeded as the specimen deforms.

3.3.4 Internal Temperature Measure

Thermistors were suspended in the oil near the specimen to measure the temperature of the oil. Details of these thermistors and the oil temperature
measurements were covered in Section 3.1.

3.3.5 Connections to Power Supplies and Data Acquisition System

The load cell, ACDT's and thermistors are connected to the power supplies and the Geotechnical Laboratories central data acquisition system through feed-through connectors in the base and top of the triaxial cell. Refer to Fig. 3.17. These connectors were made by epoxying 7-pin Amphenol hexagonal connectors into machined holes through the base and top of the cell. The load cell was connected through the top cap, and the thermistors and ACDT's were connected through the base of the cell.

3.3.6 Screw Driven Loading Frame

The loading frame was a Wykeham Farrance T-57 screw driven loading frame having a nominal capacity of 10,000 lbs. An AC motor and gear box advances the loading platen at a constant rate of deformation. Due to the flexibility of the loading frame and the use of lubricated end platens, the actual rate of deformation experienced by the specimen was not constant. Figure 3.19 plots axial strain versus time as experienced by FRS59 during the initial portion of straining. The actual strain rate increased slowly to a maximum, but did not achieve this maximum until about 0.5% $\varepsilon_a$. This was typical of all the tests. There was roughly one order of magnitude increase in the actual strain rate between start of shearing and the strain at the upper yield stress regardless of the nominal strain rate. Thus the tests were not really sheared at a constant strain rate until reaching the "large strain" region (Fig. 4.5).
3.4 CALIBRATION OF MEASURING DEVICES

Line Voltage Drops

All devices except the LVDT on the cell pressure piston were calibrated inside the cold room using the electrical transmission lines. The central data acquisition system is located well outside of the cold room. The DC voltage that energized these sensing devices was transmitted over more than 40 feet of electrical line from the power supplies outside the cold room to the sensing devices inside the cold room. This resulted in a line voltage drop such that the input voltage sensed by the central data acquisition system outside the cold room did not accurately reflect the actual input voltage \( (V_0) \) as seen by the individual devices. The voltage drop along the transmission line to each sensor was measured and subtracted from the input voltage as measured by the data acquisition system. Table 3.11 presents a summary of these measured line voltage drops for each device.

Cell Pressure Piston LVDT

The DC–DC type Linear Variable Differential Transformer (LVDT) used to measure the motion of the cell pressure piston was manufactured by Trans–Tek, model 0246 00000 D–9. This transducer had a ±3 inch active range and covered the full stroke of the cell pressure piston. This displacement transducer was calibrated at 0.25 inch increments on a digital stage manufactured by Sony, model number SR–741 100. These 0.25 inch increments formed a piecewise linear calibration curve that was used to reduce the data. The calibration curve was divided into 24 segments and the displacements of the piston were computed using a linear interpolation scheme between the end points of these segments. Table 3.12 presents the endpoints of these line segments in
terms of the normalized voltage \((V/V_0)\) and the corresponding calibration factors. The calibration of this transducer was done at room temperature.

**Triaxial Cell Piston LVDT**

The motion of the loading piston on the triaxial cell was measured by a 0.5 inch DC–DC type LVDT manufactured by Trans–Tek, model number 0244–0000 H–5. This LVDT was calibrated inside the cold room at a temperature of approximately \(-23^\circ C\) with a veneer micrometer sensitive to 0.0001 of an inch. The calibration covered one complete cycle of displacement. The output was recorded at 0.1 of an inch intervals and a linear regression was performed on the resulting data. The calibration factor was 3.454 cm/(V/V_0) with an \(r^2 = 0.999994\).

**On–Specimen ACDT's**

The two ACDT's used to measure the specimen axial strains were both calibrated inside the cold room at approximately \(-23^\circ C\). These each had an active range of \(\pm 0.1\) inch and were calibrated using a veneer micrometer with a sensitivity of 0.0001 inch. The output of each transducer was recorded at 0.005 inch intervals. These were each calibrated with one full cycle of displacement and a linear regression was performed on the resulting data. The calibration for ACDT1 was 0.02528 cm/V with an \(r^2 = 0.9998\) and the calibration factor for ACDT2 was 0.02585 cm/V with and \(r^2 = 0.99998\). Note that the input voltage to these displacement transducers was not measured directly. A separate excitation, modulation and demodulation device was used for each transducer, model LPM–210 manufactured by Schaevitz. These were assumed to produce a constant calibration over the entire testing series. Subsequent calibration checks
during the course of the testing program indicated a maximum deviation in the calibration factor of <0.5% for both transducers.

10k Load Cell

The 10k load cell was calibrated in a static loading frame model PR-500 manufactured by Soiltest of Chicago, Illinois. The maximum load range for this loading frame was 1000 lbs. The calibration was performed by two cycles of loading and unloading. The output was recorded at 200 lb intervals and a linear regression analysis was performed on the data. This calibration was performed in the cold room at -23°C. The resulting calibration factor was 1.4698 MN/(V/V₀) with an r² = 0.999997. This calibration factor was used for the full 10k range of the transducer although it was obtained for only 1/10th of the range (i.e., the capacity of the calibration frame).

Cell Pressure Transducers

The cell pressure in these tests was measured by two high-performance semiconductor pressure transducers manufactured by Data Instruments of Acton, Massachusetts. In the high pressure tests (σC ≥ 2 MPa), a 2000 psi transducer model ABHP 2000 was used. In the low pressure tests (σC = 0.1 MPa), a 200 psi transducer model ABHP 200 was used. These were both calibrated inside the cold room at -23°C using a static pressure calibration device manufactured by Refinery Supply Company of Tulsa, Oklahoma. This calibration device had a maximum capacity of 500 psi. In each calibration the pressure was cycled at least twice between the seating pressure of 5 psi and the maximum pressure. The output was recorded for at least five pressure levels up to the maximum and a linear regression was performed on the measured data. The calibration factor for
The 2000 psi transducer was based on a maximum calibration pressure of 500 psi. This factor was assumed to be constant for all pressures in this testing series (maximum = 1500 psi). The calibration factor for the 2000 psi transducer was 698.98 MPa/(V/V₀) with an r² = 0.999998, and the calibration factor for the 200 psi transducer was 67.951 MPa/(V/V₀) with an r² = 0.999993.

Table 3.11 summarizes all of the electronic sensing devices with their calibration factors and r² values.
Temperature Fluctuations Inside Cold Room

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Start Date</th>
<th>Duration (hrs)</th>
<th>MEASURED TEMPERATURE (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NS13ps</td>
<td>9/13/90</td>
<td>22.5</td>
<td>-11.95 -16.17 -14.95</td>
</tr>
<tr>
<td>NS13sh</td>
<td>9/14/90</td>
<td>2.0</td>
<td>-12.47 -16.10 -14.90</td>
</tr>
<tr>
<td>NS14ps</td>
<td>9/14/90</td>
<td>22.0</td>
<td>-11.93 -16.02 -14.88</td>
</tr>
<tr>
<td>NS14sh</td>
<td>9/15/90</td>
<td>0.14</td>
<td>-13.34 -15.98 -14.75</td>
</tr>
<tr>
<td>NS15sh</td>
<td>9/15/90</td>
<td>0.14</td>
<td>-14.20 -16.00 -14.95</td>
</tr>
<tr>
<td>NS16sh</td>
<td>9/15/90</td>
<td>0.14</td>
<td>-13.90 -16.11 -14.98</td>
</tr>
<tr>
<td>NS17ps</td>
<td>9/15/90</td>
<td>44.3</td>
<td>-10.06 -16.08 -15.03</td>
</tr>
<tr>
<td>NS17sh</td>
<td>9/17/90</td>
<td>2.0</td>
<td>-12.77 -16.12 -14.98</td>
</tr>
</tbody>
</table>

ps = preshear
sh = shear
Table 3.2: Temperature Fluctuations Inside Environmental Chamber

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Start Date</th>
<th>Duration (hrs)</th>
<th>Temperature Near Base of Triaxial Cell (°C)</th>
<th>Temperature Near Top of Triaxial Cell (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>max.</td>
<td>min.</td>
</tr>
<tr>
<td>FRS62sh</td>
<td>8/22/90</td>
<td>2.3</td>
<td>-9.70</td>
<td>-9.96</td>
</tr>
<tr>
<td>FRS65ps2</td>
<td>8/27/90</td>
<td>3.3</td>
<td>-9.84</td>
<td>-10.04</td>
</tr>
<tr>
<td>FRS65sh</td>
<td>8/27/90</td>
<td>0.4</td>
<td>-9.85</td>
<td>-10.02</td>
</tr>
<tr>
<td>FRS66sh</td>
<td>8/29/90</td>
<td>2.1</td>
<td>-9.70</td>
<td>-10.00</td>
</tr>
<tr>
<td>NS13pse</td>
<td>9/13/90</td>
<td>22.5</td>
<td>-9.90</td>
<td>-10.13</td>
</tr>
<tr>
<td>NS13sh</td>
<td>9/14/90</td>
<td>2.0</td>
<td>-9.87</td>
<td>-10.13</td>
</tr>
<tr>
<td>NS14ps</td>
<td>9/14/90</td>
<td>22.0</td>
<td>-9.90</td>
<td>-10.14</td>
</tr>
<tr>
<td>NS14sh</td>
<td>9/15/90</td>
<td>0.14</td>
<td>-9.92</td>
<td>-10.11</td>
</tr>
<tr>
<td>NS15sh</td>
<td>9/15/90</td>
<td>0.14</td>
<td>-9.91</td>
<td>-10.13</td>
</tr>
<tr>
<td>NS16sh</td>
<td>9/15/90</td>
<td>0.14</td>
<td>-9.87</td>
<td>-10.11</td>
</tr>
<tr>
<td>NS17ps</td>
<td>9/15/90</td>
<td>44.3</td>
<td>-9.91</td>
<td>-10.15</td>
</tr>
<tr>
<td>NS18ps</td>
<td>9/17/90</td>
<td>23.4</td>
<td>-9.17</td>
<td>-9.77</td>
</tr>
<tr>
<td>NS18sh</td>
<td>9/18/90</td>
<td>2.0</td>
<td>-9.27</td>
<td>-9.70</td>
</tr>
</tbody>
</table>
Table 3.3: Temperature Fluctuations Inside Triaxial Cell

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Start Date</th>
<th>Duration (hrs)</th>
<th>TEMPERATURE NEAR BASE °C</th>
<th>TEMPERATURE NEAR TOP °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>OF SPECIMEN max.</td>
<td>min.</td>
</tr>
<tr>
<td>NS11ps</td>
<td>9/8/90</td>
<td>40.2</td>
<td>-9.80</td>
<td>-9.84</td>
</tr>
<tr>
<td>NS12ps</td>
<td>9/11/90</td>
<td>24.2</td>
<td>-9.78</td>
<td>-9.84</td>
</tr>
<tr>
<td>NS12sh</td>
<td>9/12/90</td>
<td>23.1</td>
<td>-9.75</td>
<td>-9.92</td>
</tr>
<tr>
<td>NS13ps</td>
<td>9/13/90</td>
<td>22.5</td>
<td>-9.73</td>
<td>-9.86</td>
</tr>
<tr>
<td>NS13sh</td>
<td>9/14/90</td>
<td>2.0</td>
<td>-9.79</td>
<td>-9.82</td>
</tr>
<tr>
<td>NS14ps</td>
<td>9/14/90</td>
<td>22.0</td>
<td>-9.77</td>
<td>-9.87</td>
</tr>
<tr>
<td>NS15sh</td>
<td>9/15/90</td>
<td>0.14</td>
<td>-9.77</td>
<td>-9.84</td>
</tr>
<tr>
<td>NS17ps</td>
<td>9/15/90</td>
<td>44.3</td>
<td>-9.78</td>
<td>-9.81</td>
</tr>
<tr>
<td>NS17sh</td>
<td>9/17/90</td>
<td>2.0</td>
<td>-9.76</td>
<td>-9.84</td>
</tr>
<tr>
<td>NS18sh</td>
<td>9/18/90</td>
<td>2.0</td>
<td>-9.70</td>
<td>-9.87</td>
</tr>
</tbody>
</table>

* Near bottom pedestal for NS tests
Table 3.4: Summary of Typical Temperatures and Fluctuations

<table>
<thead>
<tr>
<th>Location</th>
<th>Position</th>
<th>Mean</th>
<th>Fluctuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold Room</td>
<td>Near Center</td>
<td>-14.9</td>
<td>± 1.5°C</td>
</tr>
<tr>
<td>Environmental Chamber</td>
<td>Top of Cell</td>
<td>-9.9</td>
<td>± 0.1°C</td>
</tr>
<tr>
<td></td>
<td>Bottom of Cell</td>
<td>-9.8</td>
<td></td>
</tr>
<tr>
<td>Triaxial Cell (FRS tests)</td>
<td>Top of Specimen</td>
<td>-9.4</td>
<td>± 0.05°C</td>
</tr>
<tr>
<td></td>
<td>Bottom of Specimen</td>
<td>-9.7</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.5: Variation in Cell Pressure During Shear

<table>
<thead>
<tr>
<th>TEST NAME</th>
<th>Strain Rate *</th>
<th>Target Cell Press. (MPa)</th>
<th>MEASURED RANGES IN CELL PRESSURE (MPa) (during shear)</th>
<th>Deviation of Mean from Target(%)</th>
<th>Range of Pressure (%Target)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRS15</td>
<td>m</td>
<td>10.0</td>
<td>10.170 10.127 10.152</td>
<td>1.5</td>
<td>0.4</td>
</tr>
<tr>
<td>FRS18</td>
<td>m</td>
<td>10.0</td>
<td>10.181 10.135 10.173</td>
<td>1.8</td>
<td>0.5</td>
</tr>
<tr>
<td>FRS19</td>
<td>m</td>
<td>10.0</td>
<td>10.185 10.135 10.178</td>
<td>1.8</td>
<td>0.5</td>
</tr>
<tr>
<td>FRS20</td>
<td>m</td>
<td>10.0</td>
<td>10.190 10.142 10.182</td>
<td>1.8</td>
<td>0.5</td>
</tr>
<tr>
<td>FRS21</td>
<td>m</td>
<td>10.0</td>
<td>10.191 10.140 10.180</td>
<td>1.8</td>
<td>0.5</td>
</tr>
<tr>
<td>FRS22</td>
<td>m</td>
<td>10.0</td>
<td>10.153 10.104 10.145</td>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td>FRS23</td>
<td>m</td>
<td>5.0</td>
<td>5.131 5.064 5.115</td>
<td>2.3</td>
<td>1.3</td>
</tr>
<tr>
<td>FRS24</td>
<td>m</td>
<td>5.0</td>
<td>5.143 5.058 5.122</td>
<td>2.4</td>
<td>1.7</td>
</tr>
<tr>
<td>FRS25</td>
<td>m</td>
<td>2.0</td>
<td>2.072 1.988 2.054</td>
<td>2.7</td>
<td>3.7</td>
</tr>
<tr>
<td>FRS26</td>
<td>m</td>
<td>2.0</td>
<td>2.106 2.026 2.075</td>
<td>3.3</td>
<td>4.0</td>
</tr>
<tr>
<td>FRS27</td>
<td>m</td>
<td>2.0</td>
<td>2.100 2.023 2.068</td>
<td>3.4</td>
<td>3.9</td>
</tr>
<tr>
<td>FRS28</td>
<td>m</td>
<td>0.1</td>
<td>0.1063 0.0993 0.1031</td>
<td>3.1</td>
<td>7.0</td>
</tr>
<tr>
<td>FRS29</td>
<td>m</td>
<td>0.1</td>
<td>0.1074 0.0991 0.1057</td>
<td>5.7</td>
<td>8.3</td>
</tr>
<tr>
<td>FRS30</td>
<td>m</td>
<td>0.1</td>
<td>0.1076 0.1005 0.1063</td>
<td>6.3</td>
<td>7.1</td>
</tr>
<tr>
<td>FRS31</td>
<td>m</td>
<td>0.1</td>
<td>0.1078 0.0995 0.1060</td>
<td>6.0</td>
<td>8.3</td>
</tr>
<tr>
<td>FRS32</td>
<td>m</td>
<td>0.1</td>
<td>0.1037 0.0992 0.1024</td>
<td>2.4</td>
<td>4.5</td>
</tr>
<tr>
<td>FRS33</td>
<td>m</td>
<td>0.1</td>
<td>0.1036 0.1000 0.1026</td>
<td>2.6</td>
<td>3.6</td>
</tr>
<tr>
<td>FRS34</td>
<td>m</td>
<td>0.1</td>
<td>0.1029 0.1000 0.1018</td>
<td>1.3</td>
<td>2.9</td>
</tr>
<tr>
<td>FRS35</td>
<td>m</td>
<td>0.1</td>
<td>0.1013 0.1005 0.1008</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>FRS36</td>
<td>f</td>
<td>0.1</td>
<td>0.1020 0.1008 0.1014</td>
<td>1.4</td>
<td>1.2</td>
</tr>
<tr>
<td>FRS37</td>
<td>f</td>
<td>0.1</td>
<td>0.1016 0.1004 0.1008</td>
<td>0.8</td>
<td>1.2</td>
</tr>
<tr>
<td>FRS38</td>
<td>m</td>
<td>0.1</td>
<td>0.1021 0.0993 0.1007</td>
<td>0.7</td>
<td>2.8</td>
</tr>
<tr>
<td>FRS39</td>
<td>f</td>
<td>10.0</td>
<td>10.110 10.100 10.105</td>
<td>1.1</td>
<td>0.1</td>
</tr>
<tr>
<td>FRS40</td>
<td>f</td>
<td>10.0</td>
<td>10.141 10.123 10.133</td>
<td>1.3</td>
<td>0.2</td>
</tr>
<tr>
<td>FRS41</td>
<td>f</td>
<td>0.1</td>
<td>0.1023 0.1012 0.1020</td>
<td>2.0</td>
<td>1.6</td>
</tr>
<tr>
<td>FRS42</td>
<td>f</td>
<td>10.0</td>
<td>10.150 10.089 10.125</td>
<td>1.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

* See Section 5.1 for description of strain rates
## Table 3.5: Variation in Cell Pressure During Shear

<table>
<thead>
<tr>
<th>TEST NAME</th>
<th>Strain Rate</th>
<th>Target Cell Press. (MPa)</th>
<th>MEASURED RANGES IN CELL PRESSURE (MPa)</th>
<th>Deviation of Mean from Target (%)</th>
<th>Range of Pressure (%Target)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRS43</td>
<td>m</td>
<td>0.1</td>
<td>0.1038 0.1012 0.1024</td>
<td>2.4</td>
<td>2.6</td>
</tr>
<tr>
<td>FRS44</td>
<td>m</td>
<td>0.1</td>
<td>0.1033 0.1003 0.1015</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>FRS45</td>
<td>s</td>
<td>0.1</td>
<td>0.1031 0.1015 0.1023</td>
<td>2.3</td>
<td>1.6</td>
</tr>
<tr>
<td>FRS46</td>
<td>s</td>
<td>0.1</td>
<td>0.1015 0.0986 0.1011</td>
<td>1.1</td>
<td>2.9</td>
</tr>
<tr>
<td>FRS47</td>
<td>s</td>
<td>10.0</td>
<td>10.390 9.986 10.027</td>
<td>0.3</td>
<td>4.0</td>
</tr>
<tr>
<td>FRS48</td>
<td>s</td>
<td>10.0</td>
<td>10.057 9.923 10.042</td>
<td>0.4</td>
<td>1.3</td>
</tr>
<tr>
<td>FRS49</td>
<td>f</td>
<td>10.0</td>
<td>10.012 9.997 10.005</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>FRS50</td>
<td>m</td>
<td>10.0</td>
<td>10.033 10.002 10.027</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>FRS51</td>
<td>m</td>
<td>10.0</td>
<td>10.030 9.995 10.015</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>FRS52</td>
<td>m</td>
<td>10.0</td>
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* See Section 5.1 for description of strain rates
Table 3.6: Summary of Cell Pressure Control During Shear

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Table 3.7: Variable Corrections to Volume Change

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* Compressibilities from average value of all future tests at the same σc.
Table 3.7: Variable Corrections to Volume Change

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Table 3.8: Average of the Variable Corrections to the Volume Change

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<td>0.12</td>
</tr>
<tr>
<td>moderate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 MPa</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>moderate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 MPa</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>fast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 MPa</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>moderate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 MPa</td>
<td>0.32</td>
<td>0.03</td>
</tr>
<tr>
<td>slow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Includes anomalous test FRS47
** Average without FRS47
Table 3.9: Piecewise Correction to Volume Change in Order to Account for the Nonuniform Diameter of the Cell Pressure Piston

<table>
<thead>
<tr>
<th>SEGMENT NORMALIZED VOLTAGE ((V/V_0))</th>
<th>SEGMENT CALIBRATION FACTOR ((cc/(V/V_0)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0.2</td>
<td>0.21</td>
</tr>
<tr>
<td>0.2 to 0.15</td>
<td>1.8</td>
</tr>
<tr>
<td>0.15 to 0</td>
<td>-1.6</td>
</tr>
<tr>
<td>0 to -0.14</td>
<td>-3.4</td>
</tr>
<tr>
<td>-0.14 to -0.28</td>
<td>-1.1</td>
</tr>
<tr>
<td>-0.28 to -0.44</td>
<td>-1.3</td>
</tr>
<tr>
<td>&lt; -0.44</td>
<td>-0.78</td>
</tr>
</tbody>
</table>
Table 3.10: Overview of Volume Change Corrections

<table>
<thead>
<tr>
<th>Component</th>
<th>Range of Corr. % Vol</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Compressibility of cell fluid</td>
<td>0.01 - 0.34</td>
<td>FRS47 0.95 anomalously high</td>
</tr>
<tr>
<td>2) Leakage Rate</td>
<td>0.01 - 2.94</td>
<td></td>
</tr>
<tr>
<td>3) Base Flexure</td>
<td>0.35 maximum</td>
<td></td>
</tr>
<tr>
<td>4) Diameter of Cell Pressure Piston</td>
<td>0.6 maximum</td>
<td></td>
</tr>
<tr>
<td>5) Sum of All Corrections</td>
<td>4.2 maximum</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.11: Line Voltage Drops and Calibration Factors for Electronic Sensing Devices

<table>
<thead>
<tr>
<th>DEVICE TYPE</th>
<th>MAKE AND MODEL NUMBER</th>
<th>VOLTAGE DROP (Volts)</th>
<th>CALIBRATION FACTOR</th>
<th>CORRELATION COEFFICIENT ($r^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10k load cell</td>
<td>Data Instruments JP-10000</td>
<td>0.0442</td>
<td>1.4698 MN/(V/V₀)</td>
<td>0.999996</td>
</tr>
<tr>
<td>DC-DC type LVDT</td>
<td>Trans-Tek 0244-0000 H-5</td>
<td>0.0094</td>
<td>3.454 cm/(V/V₀)</td>
<td>0.999994</td>
</tr>
<tr>
<td>Pressure Trans.</td>
<td>Hewlett Packard ABHP 2000</td>
<td>0.0590</td>
<td>698.98 MPa/(V/V₀)</td>
<td>0.999996</td>
</tr>
<tr>
<td>Pressure Trans.</td>
<td>Hewlett Packard ABHP 200</td>
<td>0.0520</td>
<td>67.951 MPa/(V/V₀)</td>
<td>0.999993</td>
</tr>
<tr>
<td>AC-DC type LVDT</td>
<td>Schaevitz 100 MHR</td>
<td>N.A.</td>
<td>2.5276x10⁻² cm/V</td>
<td>0.99998</td>
</tr>
<tr>
<td>AC-DC type LVDT</td>
<td>Schaevitz 100 MHR</td>
<td>N.A.</td>
<td>2.5845x10⁻² cm/V</td>
<td>0.99998</td>
</tr>
</tbody>
</table>
Table 3.12: Piecewise Linear Calibration Curve for LVDT on Cell Pressure Controller Piston

<table>
<thead>
<tr>
<th>NORMALIZED VOLTAGE OF SEGMENT ENDPOINTS ($V/V_o$)</th>
<th>SEGMENT CALIBRATION FACTOR ($cm/(V/V_o)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; -0.4920</td>
<td>13.282</td>
</tr>
<tr>
<td>-0.4920 to -0.4446</td>
<td>13.385</td>
</tr>
<tr>
<td>-0.4446 to -0.3974</td>
<td>13.459</td>
</tr>
<tr>
<td>-0.3974 to -0.3520</td>
<td>13.971</td>
</tr>
<tr>
<td>-0.3520 to -0.3068</td>
<td>14.052</td>
</tr>
<tr>
<td>-0.3068 to -0.2627</td>
<td>14.402</td>
</tr>
<tr>
<td>-0.2627 to -0.2189</td>
<td>14.518</td>
</tr>
<tr>
<td>-0.2189 to -0.1745</td>
<td>14.283</td>
</tr>
<tr>
<td>-0.1745 to -0.1303</td>
<td>14.373</td>
</tr>
<tr>
<td>-0.1303 to -0.0835</td>
<td>13.563</td>
</tr>
<tr>
<td>-0.0835 to -0.0355</td>
<td>13.232</td>
</tr>
<tr>
<td>-0.0355 to 0.0139</td>
<td>12.846</td>
</tr>
<tr>
<td>0.0139 to 0.0613</td>
<td>13.408</td>
</tr>
<tr>
<td>0.0613 to 0.1062</td>
<td>14.139</td>
</tr>
<tr>
<td>0.1062 to 0.1501</td>
<td>14.458</td>
</tr>
<tr>
<td>0.1501 to 0.1925</td>
<td>14.987</td>
</tr>
<tr>
<td>0.1925 to 0.2361</td>
<td>14.578</td>
</tr>
<tr>
<td>0.2361 to 0.2797</td>
<td>14.551</td>
</tr>
<tr>
<td>0.2797 to 0.3243</td>
<td>14.254</td>
</tr>
<tr>
<td>0.3243 to 0.3697</td>
<td>13.971</td>
</tr>
<tr>
<td>0.3697 to 0.4148</td>
<td>14.080</td>
</tr>
<tr>
<td>0.4148 to 0.4613</td>
<td>13.671</td>
</tr>
<tr>
<td>0.4613 to 0.5075</td>
<td>13.724</td>
</tr>
<tr>
<td>&gt; 0.5075</td>
<td>13.697</td>
</tr>
</tbody>
</table>
\[ V_o = \text{Input Voltage} \]
\[ V_f = \text{Voltage Drop Across Fixed Resistor} \]
\[ R_f = \text{Fixed Resistance} \]
\[ R_t = \text{Thermistor Resistance} \]

Figure 3.1: Schematic Diagram of Thermistor Circuit
Figure 3.2: Plan View of M.I.T. Cold Room Facility
Figure 3.3: Temperature Inside Cold Room During NS13 Preshear (short term)
Figure 3.4: Temperature Inside Cold Room During NS13 Preshear (long term)
Figure 3.5: Schematic View of Environmental Chamber and Triaxial Cell
Figure 3.6: Temperature Inside Environmental Chamber During NS13 Preshear (short term)
Figure 3.7: Temperature Inside Environmental Chamber at Two Locations During NS10 Shear
Figure 3.8: Oil Temperature at Two Locations Near Specimen During FRS61 Shear
Figure 3.9: Comparison of Temperature Measurements in Cold Room, Environmental Chamber and Triaxial Cell During NS13 Preshear
Figure 3.10: Schematic Drawing of Mechanical Portion of Pressure/Volume Controller
Pressure Transducer Signal \( A \) to \( N \), then negative error signal
If \( A = B \), then no new signal
If \( A < B \), then positive error signal

Figure 3.11: Schematic Diagram of Pressure Controller System
Figure 3.12: Cell Pressure Control During FRS46 Shear
Figure 3.13: Cell Fluid Volume Change During FRS59 Preshear
Figure 3.14: Compressibility Determination of Cell Fluid and Triaxial Cell Prior to Shear of FRS59
Figure 3.15: Cell Fluid Volume Change During Aluminum Stiffness Test
Figure 3.16: Cell Fluid Volume Change With No Specimen Over Full Travel of Cell Pressure Piston
Figure 3.17: Details of the Modified High Pressure Triaxial Cell
Figure 3.18: Details of On-Specimen Axial Strain Yoke
Figure 3.19: Axial Strain Versus Time for Initial Shearing During FRS59
CHAPTER 4
MATERIAL, PROCEDURES, AND ERROR ANALYSIS

This chapter describes the material tested in this research, summarizes the experimental procedures, discusses the data reduction methods and presents an analysis of possible errors associated with the measured results.

4.1 DESCRIPTION OF MANCHESTER FINE SAND

The sand used in this testing program was collected at the Plourde Sand and Gravel Company located in Hooksett, New Hampshire approximately 10 miles north of Manchester along the banks of the Merrimack River. This sand is known as Manchester Fine Sand (MFS) and is a river bed deposit with nearly horizontal bedding planes. The gradation can vary widely between adjacent bedding planes and the sand for this testing series was collected from several of these with an effort to obtain the finest grained material.

4.1.1 Mechanical Sieve Analysis

An initial mechanical sieve analysis of the collected material indicated that the fines content, i.e., the material finer than 0.074mm, was approximately 20%. The sand was subsequently processed by dry sieving with a #200 sieve to remove a portion of the fines. The processed batch was thoroughly remixed and subsequent sieve analyses indicated that the fines content ranged from 6 to 8%. Table 4.1 presents a summary of five dry mechanical sieve analyses that were performed on this processed material to establish a base line. Note that in the first three sieve analyses, the #200 sieve was not used. The data from the 2nd through 5th sieve analysis are all fairly consistent. The first analysis gave a
slightly coarser gradation. The coefficient of uniformity (Eq. 2.1.1) for this sand is approximately 2.4 and the coefficient of concavity (Eq. 2.1.2) is approximately 1.1, based on the average from #2 through #5. The material passing the #200 sieve (fines) is nonplastic. The Manchester Fine Sand (MFS) tested in this research is classified as SP–SM, a poorly graded fine sand with 6 to 8% nonplastic silt size particles according to the Unified Soil Classification System.

Also presented in Table 4.1 are gradations reported for three prior testing programs on MFS: Sayles (1968); Perrone (1978); and Martin et al. (1981). These have similar gradations, containing from 5 to 12% fines. The coefficients of uniformity ranged from 2.2 to 2.6 and coefficients of concavity ranged from 0.9 to 1.0. Assuming that the fines in the previously tested material were similar to the fines in the material tested in this research, these can also be classified as SP–SM, poorly graded fine sand with 5 to 12% nonplastic silt size particles. Figure 4.1 plots the gradation curves of these prior testing programs and an average gradation curve from the 2nd through 5th sieve analyses performed on the processed material used in this research. The MFS tested in this research is slightly coarser than the material tested by Perrone (1978) and Martin et al. (1981).

4.1.2 Mineralogical Analysis

Martin et al. (1981) performed a mineralogical analysis of Manchester Fine Sand and reported that: "The mineralogy of the sand material was predominantly quartz and feldspars. Potash feldspar was the most abundant feldspar species. The clay minerals present were mica, illite, vermiculite and chlorite. The sand contained 0.48% magnetics, primarily magnetite."
4.1.3 Soluble Salt Concentration

A series of soluble salt concentration determinations were performed both on the virgin processed material and on the material after testing in this research using the conductivity method described by Martin (1968). This uses KCl as a reference salt and hence yields an equivalent to KCl concentration. Other salts or conductive materials that are in solution will therefore cause some errors in the measurement. The processed virgin material yielded soluble salt concentrations ranging from 0.30 to 0.37 gm/kg. Several soluble salt concentration determinations were made on the material after testing and oven drying. The resulting soluble salt concentrations ranged from 0.018 to 0.060 gm/kg. A total of fourteen such determinations were made. The difference in the soluble salts concentration between the tested and untested material is attributed to the procedure used to saturate the sand specimens. This involved flooding of the specimens with deionized distilled water and then flushing the specimens with several pore volumes. This procedure most likely leached out a portion of the readily soluble salts. The saturation procedure is discussed in Section 4.2.2.

4.1.4 Specific Gravity

Four specific gravity determinations were made on the processed MFS in accordance to ASTM D854–83. These were conducted at a bath temperature of 25°C. The specific gravities as determined by this method equaled 2.688 ±0.003. A value of 2.688 was used to compute the void ratios and degree of saturation in this research.
4.1.5 Maximum and Minimum Densities

Maximum and minimum densities were determined for the processed material by Haley and Aldrich, Inc. of Cambridge, Massachusetts. The maximum density was determined in general conformance with ASTM standard D4253, Method 1.A. The sand was oven dried before testing. The minimum density was determined in general conformance with ASTM standard D4254–83, Method A. The maximum and minimum dry densities were 1701 kg/m\(^3\) \((e_{\text{min}} = 0.508)\) and 1408 kg/m\(^3\) \((e_{\text{max}} = 0.909)\), respectively. All relative densities reported in this research are computed from these quantities.

4.1.6 Particle Shape and Weathering

A Scanning Electron Microscope (SEM) was used to study the surface texture and shape of the individual particles. The shape of the particles is classified as sub-angular according to the visual classification system proposed by Pettijohn (1975). In a similar SEM study on Manchester Fine Sand for a prior research program at M.I.T., Martin et al. (1981) observed the presence of micaceous flakes with visible split edges and concluded that there had been considerable weathering.

4.1.7 Unfrozen Water Content

The unfrozen water content was determined by Dr. Pat Black of the U.S. Army Cold Regions Research and Engineering Laboratory (CRREL) using nuclear magnetic resonance (NMR). The processed batch sand was mixed with distilled water to achieve a water content of approximately 25.5%. The temperature was gradually lowered in increments and the corresponding unfrozen water content was measured with NMR. The measured unfrozen water content
decreased from 1.8% at \(-0.1^\circ C\) to 0.14% at \(-4.89^\circ C\). Figure 4.2 plots the measured unfrozen water content versus temperature. The amount of unfrozen water at \(-5^\circ C\) is almost an order of magnitude lower than that reported by Martin et al. (1981). They estimated that the salt concentration in their unfrozen MFS was no greater than 0.06M (3.5g/l assuming KCl as the salt). A soluble salt concentration of 0.33 gm/kg, prior to flushing with distilled water, is approximately 1.3 g/l when fully saturated. Thus, the difference in the unfrozen water content between these two programs may be due to differences in the salt concentration. Martin et al. (1981) also removed all of the magnetics prior to determining the unfrozen water content and this was not done in the current research.

4.1.8 Classification of Frozen MFS According to Sayles et al. (1987)

The frozen Manchester Fine Sand tested in this research can be classified as a poorly graded fine sand with 6 to 8% nonplastic silt size particles. The segregated ice is not visible by eye and the ice is well-bonded with no excess ice. This leads to a SP-SM, Nbn classification according to Sayles et al. (1987).

4.2 SPECIMEN PREPARATION, FREEZING AND STORAGE

This section describes the two techniques used to prepare specimens, the saturation and freezing techniques and storage procedures. Specimens were prepared with relative densities ranging from 20 to 100% and with computed degrees of saturation from 96.1 to 100.2%.
4.2.1 Specimen Preparation

Description of Molds

The molds used for the preparation and freezing of the Manchester Fine Sand specimens were obtained from CRREL and were the same molds used previously by Martin et al. (1981). Figure 4.3 presents a series of schematic drawings of one of these molds in different modes of operation. Figure 4.3(a) is the mold prepared for the wet tamping procedure, Fig. 4.3(b) is the mold prepared for multiple sieve pluviation, Fig. 4.3(c) is the mold prepared for saturation and Fig. 4.3(d) is the mold prepared for freezing. Each of these modes are described subsequently.

The molds consist of a plexiglas split sleeve which rests inside a plexiglas hollow cylinder. The sand specimen (nominal dimensions $D = 3.54\,\text{cm}$, $A = 9.84\,\text{cm}^2$, and $L = 8.1\,\text{cm}$) rests inside the split sleeve. Refer to Figure 4.3 (c). Two porous stones in the base and top cap allow for drainage. Filter papers are placed in direct contact with these two stones. The base of the mold is a piece of brass which contains connections for drainage lines which allow for the saturation of the specimen. A brass collar can be placed on top of the hollow plexiglas cylinder and bolted firmly to the base of the mold. A top cap can be placed inside the upper collar and firmly bolted to it. This top cap also provides for drainage. O-rings between the top cap and upper brass collar and between the plexiglas hollow cylinder and upper collar and base allowed for a vacuum tight seal.

Two techniques were used to prepare the sand specimens at various densities. These were wet tamping and multiple sieve pluviation. The prepared sand relative densities ranged from 21.8% to 99.2%. Table 4.2 reports the preparation technique used for each specimen.
Preparation by Wet Tamping

Wet tamping was used to prepare only dense to very dense specimens. This procedure did not follow the undercompaction method used by Martin et al. (1981). For this process, a compaction collar was placed inside the upper brass collar resting on top of the inner plexiglas split sleeve. This collar acted as a guide for the compaction rod. Refer to Figure 4.3 (a). A predetermined quantity of oven dried MFS was thoroughly mixed with water to achieve approximately 10% moisture content. The mixture was then divided into five equal portions. Each portion was placed successively into the compaction mold and compacted with a metal tamping rod to a predetermined height, i.e., the thickness of each lift was 20% of the total length of the specimen. The rod had a flat face that came into contact with the sand. The compaction was performed in a circular pattern until the required lift thickness was achieved. Between each lift the compacted surface of the prior lift was scarified to improve bonding between adjacent lifts. A metal sliding collar on the tamping rod was preset to the correct lift height and came into contact with the compaction collar to control the lift height.

Specimens prepared by wet tamping contained distinct nonuniformities in the density. Several of the "wet tamped" specimens were studied after freezing using X-ray photography. These X-ray photographs indicated that the density of the material in each lift was nonuniform, with the bottom of the lifts being less dense than the top of the lifts. This was deemed undesirable, and an alternative method of sand specimen preparation was introduced in order to eliminate these nonuniformities.
Preparation by Multiple Sieve Pluviation

The second specimen preparation technique used in this research was a multiple sieve pluviation (MSP) technique outlined by Miura and Toki (1982). This was used to produce specimens of uniform density with relative densities ranging from 20 to 100%. For this procedure, the sand was dried in an oven at 100°C and stored in a desiccator prior to preparation. The mold was carefully cleaned, the upper brass collar was removed and the entire mold was weighed. See Figure 4.3 (b). A knife edged aluminum guide was placed directly on top of the plexiglas hollow cylinder and split sleeve. The purpose of the knife edge was to prevent sand particles from bouncing off the top of the plexiglas sleeve and falling into the mold at a velocity different from that of the raining cloud. Dry sand was then poured into a funnel mounted in a frame above the mold. See Fig. 4.4. A cork in the base of the funnel with a variable size hole regulated the flow rate of the sand into the mold. The sand fell out of the funnel, through a vertical plexiglas tube and across four 3 inch diameter sieves (two with #20 mesh sizes and two with #10 mesh sizes). The sieves acted to disperse the raining cloud of sand. The sand then fell passed the knife edge aluminum guide and into the mold. The sand was allowed to pluviate until it filled the mold. After pluviation, the upper knife edge guide was carefully removed and the excess sand was cleaned off the mold. The mold was then reweighed with the sand.

The pouring density of the sand was estimated from the measured volume of the mold and the difference between the empty and full weight of the mold. The density of the sand achieved during the pouring could be roughly regulated by changing the rate of pouring. This was accomplished by adjusting the size of the opening in the bottom of the funnel. A larger opening led to a faster rate of pluviation and produced a lower density. If the sand as poured was
denser than the target preparation density, the mold was emptied and the entire process was repeated. If the sand was looser than the target density, the mold was gently tapped with a small knife around the circumference and extra material was added until the desired preparation density was achieved.

X-ray photography was used to study the uniformity of the density of specimens prepared by MSP. Several of the specimens were X-ray photographed after freezing and these indicated that the density was uniform throughout the entire specimen.

4.2.2 Specimen Saturation and Freezing

After achieving the desired density, a piece of filter paper was placed on the top of the specimen and a brass top cap fitted with a porous stone was placed inside the upper brass collar of the preparation mold. See Fig. 4.3(c). O-rings between the top cap and the upper brass collar and between the plexiglas hollow cylinder and the base and upper collar allowed for a vacuum tight seal. The dry sand specimens were taken into the growth room maintained at 0°C. The upper side of each mold was connected to one reservoir (referred to as the upper reservoir) and the lower side was connected to another reservoir (referred to as the lower reservoir). A 28 in.Hg (95 kPa) vacuum was drawn on the specimens and the system was checked for vacuum leaks. Deaired deionized distilled water under the same vacuum was passed thorough the molds from the base to the top by gravity feed under an initial head of approximately 5 feet of water. After several pore volumes of water had passed through the specimens (the final head of water was approximately 2.5 feet), the upper and lower reservoirs were raised to the same elevation above the specimens and these were maintained under water and the vacuum over night.
The following morning, the vacuum was slowly removed and the lower reservoir was placed at the elevation of the top of the specimens. The upper reservoir was disconnected. The brass top caps of the specimen preparation molds were removed and replaced with freezing caps. The filter papers were retained on the top of each specimen and freezing caps were placed in direct contact with these. Refer to Figure 4.3(d). Antifreeze fluid at $-15^\circ$C was circulated through the freezing caps and the specimens were frozen from the top down, while allowing for drainage through the base of the molds and into the lower reservoir. The antifreeze was maintained at $-15^\circ$C during the course of the freezing by a Lauda constant temperature circulating bath located outside of the 0°C growth room. The entire freezing process lasted approximately four hours. Completion of the freezing process was indicated by the freezing of small beads of water placed on the lower brass base of each mold.

Table 4.2 presents the physical properties for each of the specimens reported. The measured degree of saturation as achieved by this process ranged from approximately 96% to 100% as computed by Eq. 2.3.1 assuming $w_u = 0.001$, $w_i = (w - w_u)$, $G_i = 0.9187 \times 10^{-10}$ C, $\gamma_w = 998.1$ kg/m$^3$, and $G_s = 2.688$. The frozen density was measured during the trimming process from a direct measure of the diameter, length and weight of the specimen. The total moisture content was determined after the test from the dry density.

In order to estimate the accuracy of this degree of saturation measure, the writer used a second-order approximation to the variance using a procedure outlined by Ang and Tang (1975, pg 199). All of the variables in Eq. 2.3.1 were assumed to be independent and the $\gamma_w$ and $G_i$ were assumed to be deterministic with values of 998.1 kg/m$^3$ and 0.9187 at $-10^\circ$C, respectively. The mean values and standard deviations used in the analysis were:
\( \gamma_t = 1940 \pm 1.98 \text{kg/m}^3 \) (obtained from a second-order analysis with the following mean values and standard deviations:

- Mass = 0.143 \pm 0.0001 kg
- Diameter = 0.0354 \pm 2.54 \times 10^{-5} \text{m}
- Length = 0.0737 \pm 2.54 \times 10^{-5} \text{m}

\( w = 0.247 \pm 0.00249 \) obtained from a second-order analysis with the mean and standard deviation for \( \gamma_t \) as reported, \( \gamma_d = 1560 \pm 3 \text{kg/m}^3 \) (the standard deviation here was taken from 26 of the reported tests computed as the difference between the oily and cleaned dry densities for tests where this difference was less than 7 \text{ kg/m}^3). Refer to Section 4.4.

\( w_u = 0.001 \pm 0.0001 \)
\( G_s = 2.688 \pm 0.0029 \)
\( w_i = 0.246 \pm 0.00249 \)

The estimated standard deviation for the degree of saturation from this analysis is 0.55%.

**4.2.3 Specimen Storage**

After freezing was completed, the freezing caps and the upper brass collars were removed from the molds and the lower reservoir was disconnected. Porous stones were removed but the filter paper was left in contact with the face of the specimen. The inner split sleeves containing the frozen sand specimens were extruded from the plexiglas cylinders using a hand operated hydraulic press located in the 0°C growth room. The specimens were stored inside the plexiglas split sleeves. High-vacuum silicone grease was smeared along the seam of the split sleeve and across the top and bottom of the specimen over the filter paper. The entire split sleeve was then wrapped in Saran Wrap and placed in a ziplock freezer bag with crushed ice. This was taken to a freezer located outside of the cold room and stored at a temperature of -20°C until testing. Table 4.2 presents the dates of freezing and dates of testing for each specimen. Most of the
specimens were tested within a 2 to 3 week period. The longest storage time was approximately 4.5 months and was for FRS59.

4.3 SPECIMEN TRIMMING, SETUP AND TESTING PROCEDURES

4.3.1 Trimming Procedure

The specimen to be tested was brought inside the testing room from the external storage freezer, unwrapped and the silicone grease was carefully removed with a paper towel. The actual trimming of the specimen took place in the vestibule room of the cold room facility. This room was maintained at −4 to −5°C. The specimens were taken from the testing room at −15°C to the vestibule for 5 minutes intervals of trimming. They were returned to the colder testing room every five minutes in order to prevent possible melting of the specimen during handling in the trimming process. Care was taken throughout the entire set up procedure to maintain the proper orientation of the specimen, i.e., top end up (as established during the preparation phase).

One end of the specimen was extruded approximately 0.25 inches out of the split sleeve and a hose clamp was placed on the split sleeve to hold the specimen firmly in this position. The trimming was done by hand with sharpened steel knives. Small amounts of frozen sand were systematically removed from the exposed end by cutting with the knife and working from the outer edges towards the center. All of the trimmed material was carefully collected. This material was later dried and weighed in order to estimate the dry density of the specimen.

The parallelism of the ends of the frozen sand specimen was established by using the edge of the split sleeve as a rough guide, i.e., each end was initially
trimmed to be parallel to the ends of the split sleeve. After trimming one end, the specimen was partially extruded from the other end and the process was repeated. In order to measure the parallelism of the ends, the specimen was taken out of the split sleeve and placed on a metal stand with a spring loaded depth gage sensitive to 0.0001 of an inch. The height of the specimen was indicated across the diameter and around the circumference. If the height varied by more than 0.001 of an inch, it was selectively hand trimmed with a knife to decrease it to within the 0.001 of an inch tolerance.

The diameter of the specimen was measured with a hand held micrometer with a direct readout to 0.001 of an inch at eight locations along the length (two at each end and third point both at 90° to each other). The height of the specimen was measured at four locations spaced at roughly 90° around the circumference. The averages of the diameter and height measures were used subsequently to estimate the specimen volume. The entire trimmed specimen was then weighed on an electronic scale sensitive to ±0.01 g.

4.3.2 Specimen End Conditions

For those specimens tested with "lubricated" ends, an ice cap was frozen onto each end in the following manner. One end of the trimmed specimen was pushed back into the split sleeve approximately 0.25 inches. The specimen was placed in the testing room at −15°C and deionized distilled water at 0°C was placed on top of the frozen sand and allowed to freeze. The end of the frozen sand specimen was then pushed back out of the split sleeve exposing all but approximately 0.063 of an inch of the ice cap. A hose clamp was tightened around the split sleeve to hold the specimen firmly in this position and a portion of the cap was trimmed off with a steel knife in a manner similar to the trimming
of the frozen sand. The edge of the split sleeve was again used as a rough guide for parallelism. The specimen was inverted and the process repeated on the other end. Final trimming of the ice caps was accomplished by the use of the metal stand and spring loaded depth gage. The specimen was trimmed by hand until the two ends were parallel to within 0.001 of an inch across the diameter and around the circumference. The length with ice caps was then measured at four locations around the circumference with a veneer micrometer.

Table 4.2 contains a summary of the end conditions for each specimen and the total measured ice cap length for those tested with "lubricated" ends. This measure includes the thickness of both ice caps and was deduced from the difference between the total length measured with and without the caps. The total ice cap length in these tests ranged from 0.47mm to 2.41mm. The average total thickness of the ice caps was 1.7mm.

In order to properly align the frozen sand specimen between the base pedestal and floating top cap of the triaxial cell, 0.063 inch diameter holes were drilled through the center of each ice cap and into the frozen sand using a drill bit mounted in a plexiglas boring tool which fit around the outside of the plexiglas split sleeve. These holes were bored approximately 0.125 of an inch deep and mated with 0.063 of an inch diameter hardened steel pins mounted in the center of the base pedestal and top cap. In addition to providing alignment during setup, these pins also helped to improve the stability of the specimen at large deformations where some of the specimens exhibited a tendency to slide.

A few specimens were tested without ice caps in order to simulate "frictional" end conditions. These were trimmed as explained in Section 4.2.1. The alignment holes were drilled directly into the frozen sand and a layer of high vacuum silicone grease was placed between the frozen sand. In FRS31 a piece of
180 grit emory cloth was placed with the rough side towards the specimen. A small amount of grease was placed between the backside of the emory cloth and the brass ends caps. The results of these tests with varying end conditions will be discussed in Chapter 5.

4.3.3 Setup of Specimen in Triaxial Cell

Prior to placement of the trimmed frozen sand specimen in the triaxial cell, the base pedestal and top cap were covered with high-vacuum silicone grease and a prophylactic membrane was placed on the base pedestal with two O-rings. The specimen was placed on the base pedestal bottom end down and the top cap was lowered onto the upper surface of the specimen using a specially designed alignment jig mounted directly on the base of the triaxial cell. An axial force was applied by hand to the specimen through the alignment jig in order to force the excess grease from between the specimen and the brass surfaces of the base pedestal and top cap and to bring the frozen sand specimen into vertical alignment. The grease also acted to hold the specimen and top cap on the base pedestal during the set up procedure. The extruded grease was wiped off and the rubber membrane was carefully stretched from the base pedestal over the specimen and onto the top cap. One of the O-rings on the base pedestal was raised to the top cap to secure the membrane in place.

The axial displacement yoke was placed on the outside of the membrane at roughly the third points along the specimen so that the active gage length included the central portion of the specimen. Section 3.3.3 contains a description of this yoke. The gage length of the displacement yoke was preset by alignment rods which run between the upper and lower yoke. After placement of the yoke on the specimen and removal of these alignment posts, the gage length was
measured twice for each ACDT using a height measuring scope with a vertical veneer that had a direct read out of 0.002 of an inch. The average of the two measurements of the gage length for each ACDT was used subsequently as the gage length in the computation of axial strain. The gage length for these tests was either 1.5 or 2 inches. An error in the gage length of 0.002 inches is thus approximately a 0.1% error in the gage length. This is not considered to be substantial in terms of the computed engineering strain.

For some of the tests, FRS59–67, thermistors were placed on the axial displacement yoke in order to sense the temperature of the oil in the triaxial cell near the specimen. These were suspended roughly near the top and bottom of the specimen. The measurement of the oil temperature was discussed in Section 3.1.4.

With the axial displacement yoke in position, the alignment jig was removed from the base of the triaxial cell and the base was placed on a guide stand. The loading ram and load cell was fully withdrawn into the top of the triaxial cell and the top of the triaxial cell was carefully lowered onto the base using the guide stand to prevent accidental disturbance of the axial displacement yokes. The top and base of the triaxial cell were bolted together and the load cell was gently lowered onto the top cap and locked into position. The entire triaxial cell was then placed in the loading frame. Each of the electronic devices were connected to the power supplies and data acquisition system and the output level of each device was checked to verify that it was within the normal range.

4.3.4 Testing Procedure

A slight preload was applied to the specimen through a manual crank on the loading frame. This preload was intended to help with the seating of the
specimen and to improve its alignment during subsequent shearing. The magnitude of the preload varied from 0.1 to 0.3 MPa. It was reapplied at various times throughout the entire preshear stage of the test. After each application, the preload gradually reduced over time.

The triaxial cell was filled with silicone oil using a diaphragm pressure reservoir located inside the cold room. The diaphragm was activated by air pressure generated by a compressor located outside of the cold room. The cell pressure was applied and the specimen was allowed to sit in the triaxial cell under pressure with the slight preload until both the temperature and the leakage rate of oil out of the triaxial cell had stabilized. This preshear pressurization phase lasted longer than the expected shearing time of each test.

After the preshear period, the compressibility of the cell fluid was checked using the procedure outlined in Section 3.2.3. Just prior to shearing, the small preload was removed and the zero of the load cell was recorded. The deviator stress was applied to the specimen with the screw driven loading frame described in Section 3.3.6. In general, the specimens were sheared to axial strains in excess of 20%. After shearing, the cell pressure was relieved and the triaxial cell was removed from the loading frame and disassembled. The specimen was photographed and then removed from the base pedestal. The final diameter was measured near the ends and at third points along the length. The specimen was then placed in an oven and dried.

4.4 DENSITY COMPUTATION

The dry density of the sand was computed at four different stages during each test for the majority of the specimens. This section describes each of these density computations and some of the inherent uncertainties. The density
measure that was chosen as most representative for each test is presented. As explained in Section 4.2, the frozen density was measured only once during the trimming procedure from a direct measure of the diameter, length and weight of the specimen.

4.4.1 Preparation Density

The first measure of density is referred to as the preparation density and was made for specimens prepared by MSP. The volume of the mold was measured, and the dry weight of the sand placed into the mold was measured using a Mettler BB2400 scale with an accuracy of ±0.01g over a 2.5kg range. The weight of the sand placed in the mold was computed as the difference between the weight of the mold empty and full.

This initial density measure was subject to some uncertainty due to difficulties in measuring the inner height of each mold. This was due not only to the irregular surface of the porous stones on the base of the mold, but also due to the incomplete compression of the O-ring between the plexiglas hollow cylinder and the lower brass base. See Figure 4.3(b). This density measure also relied upon a thorough cleaning of each mold after pluviation and before weighing. If some sand particles remained on the outside of the mold, these caused errors in the computed densities. Therefore, this first density measure was used only as a preparation index and was not representative of the test density.

4.4.2 Trimmings Density

The second measure of specimen density was performed during the setup procedure for specimens prepared by MSP and is referred to as the trimmings density. The weight of the material removed from the specimen during the
trimming procedure was subtracted from the weight of the material which had been placed in the mold during the preparation procedure. The volume of the trimmed specimen was estimated from a direct measure of the specimen height and diameter as explained in Section 4.3.2. The trimmings density was more accurate than the preparation density because the volume measure was taken directly from the specimen.

4.4.3 Oily Density

The third measure of the density was obtained after shearing and is referred to as the oily density. The specimen was removed from the triaxial cell and carefully cleaned to remove excess high vacuum silicone grease. It was then oven dried. The resulting dry weight was divided by the volume as measured during the set up procedure. Sources of error in this measure include the presence of small amounts of silicone grease and silicone oil on some of the sand particles. In FRS48 the membrane ruptured and allowed the cell fluid to completely impregnate the specimen. This significantly affected the oily density measure for this test.

4.4.4 Cleaned Density

The fourth measure of density is referred to as the cleaned density and was obtained by carefully washing the dried sand specimen with methyl ethyl ketone (MEK) in order to remove the excess silicone grease and silicone oil which remained after testing. The density was computed as the dry weight of the washed material divided by the volume as computed during the set up procedure. One possible source of error in this density measure was the possible decanting of suspended fines in the supernatant fluid above the sand.
4.4.5 Selected Density Determination

Table 4.3 summarizes the relative densities measured for each of the reported tests. The relative density was computed from the best estimate of the dry density of the specimen using the equation

\[ D_r = \frac{\gamma_{d_{\text{max}}}}{\gamma_d} \times \frac{\left(\gamma_d - \gamma_{d_{\text{min}}}\right)}{\left(\gamma_{d_{\text{max}}} - \gamma_{d_{\text{min}}}\right)} \times 100\% \]

where \( \gamma_d \) is the dry density. The maximum and minimum dry densities were presented in Section 4.1.5.

In FRS15 and FRS18 through FRS29 the preparation density, the trimmings density and the cleaned density were not measured. In FRS30 through FRS35 and FRS65 through FRS67, the cleaned density was not measured. In FRS65, a weighing error negated the oily and cleaned densities.

The preparation density for FRS30–FRS36 and FRS56 were measured on a scale that was accurate to ±1g. The specimens had a dry weight in the range of 100 to 110g and a volume of approximately 70cc. A 1g error in the weight corresponds to roughly a 15kg/m³ error on the density, which corresponds roughly to a 5% error on the relative density. This could account for the large discrepancies between the preparation relative density and the other relative density measure in some of these tests. Additionally, FRS30, 31 and 34 that were prepared at \( D_r < 20\% \) probably experienced some densification during saturation.

Examination of Table 4.3 indicates that for FRS37–FRS64, the trimmings relative density, the oily relative density and the cleaned relative density are all in close agreement (except where the specimen was oil soaked, i.e., FRS48). The majority of these relative density measures differ by only 1 to 2%. The oily density determined after shearing was selected as representative for
most of the tests. This was chosen because it tended to fall in between the trimmings and clean relative density and it represented the only density measure for FRS15 and FRS18 through FRS29. Table 4.3 indicates which density measure was selected as most representative for each test.

An estimate of the accuracy of this relative density measure was made by performing a second—order approximation to the variance using Eq. 4.1 with the following mean values and standard deviations:

\[ \gamma_{\text{dmax}} = 1701 \pm 2 \, \text{kg/m}^3 \]
\[ \gamma_{\text{dmin}} = 1408 \pm 2 \, \text{kg/m}^3 \]
\[ \gamma_d = 1560 \pm 3 \, \text{kg/m}^3 \] (based on scatter between the oily and cleaned densities of 26 tests where this difference was less than 7 kg/m^3.

This analysis yielded an estimated standard deviation in the reported relative density of 2.0%, which is in the range of variation in relative density between the oily and cleaned specimens.

4.5 DATA ACQUISITION SYSTEM AND DATA REDUCTION PROCEDURES

This section describes the central data acquisition system that was used to collect and store the data from these tests. The method of data reduction is also outlined. This includes the reduction of the raw data to engineering units and the conversion of the engineering units data into the stress—strain data using various corrections.

4.5.1 M.I.T. Geotechnical Data Acquisition System

The raw voltages from the electronic sensing devices were measured with a Hewlett Packard 3497A Data Acquisition/Control Unit. This unit is controlled
by a 286 based microcomputer that runs a user friendly data acquisition program
developed by Dr. J.T. Germaine of M.I.T. in collaboration with R.S. Ladd of
Woodward–Clyde Consultants. This data acquisition unit is centrally located in
the Geotechnical Laboratory. It has a 300 reading/sec conversion rate. The
software has a 1 second scan limit per task. At present, the system has 120
channels connected. The voltmeter has a 100 Volt capacity with an autoranging
feature on a 17 bit precision, 1 microvolt sensitivity. As the data are read, they
are stored on the hard disk of the 286 based computer. After testing, the data
are converted into ASCII format and down loaded onto floppy disk for
subsequent storage and manipulation. All data reduction was accomplished on a
286 based microcomputer.

4.5.2 Conversion of Raw Data to Engineering Units

The electronic sensing devices that measure force, pressure and
displacement have output signals that are proportional to the magnitude of the
input excitation voltage and the magnitude of the force, pressure or
displacement. The general form of the equation used to convert the raw output
voltages to engineering units was

\[ \Delta P = \left( \frac{V}{V_0} - \frac{V_i}{V_{oi}} \right) C_f \]

where \( \Delta P \) is the measured force, pressure or displacement in appropriate units, \( V \)
is the measured output voltage from the transducer, \( V_i \) is the initial "zero"
voltage for zero load, pressure or displacement, \( V_o \) is the input voltage supplied
to the sensing devices (an appropriate line voltage drop was subtracted for this
voltage as explained in Section 3.4), \( V_{oi} \) is the input voltage corresponding with
the initial "zero" voltage, and \( C_f \) is the calibration factor as determined by the
procedures outlined in Section 3.4.
The zero voltages for the displacement transducers mounted on the cell pressure piston, the triaxial loading ram and the on-specimen displacement yokes were taken as the initial readings at the beginning of each test. The pressure transducer zero was determined before pressurizing by venting the triaxial cell to the atmosphere and measuring the output of the transducer. The slight difference in elevation between the midheight of the specimen and the mounting height of the transducer was neglected. The zero of the load cell was determined just prior to shearing by lifting the load cell off the top of the specimen.

A computer program was written to perform the conversion of the raw data to engineering units. This program also computed a correction to the cell fluid volume change measure due to the nonuniform diameter of the cell pressure piston along its length. This correction was a function of the position of the cell pressure piston in the pressurizing reservoir as described in Section 3.2.3.

4.5.3 Conversion of Data to Units of Stress and Strain

The measured data in engineering units and the correction for nonuniformities of the diameter of the cell pressure piston were converted to stresses and strains in a separate computer program. The following is an overview of the main operations or functions of this program.

Axial Strain Measure

The axial strain of the specimen was measured from the Alternating Current Displacement Transducers (ACDT's) mounted on the specimen and from a Linear Variable Differential Transformer (LVDT) sensing the motion of the loading piston outside of the triaxial cell. One of the axial strain measures was computed as the average of the two ACDT's mounted on the specimen. Each
transducer displacement was converted to engineering strain by dividing by the initial gage length. The second axial strain from the external LDVT was computed by dividing the measured displacement by the total initial length of the frozen sand specimen, including the ice caps (i.e., the compliance of the grease between the ice caps and brass platens was not considered). This measure was corrected for the flexure of the base of the triaxial cell as explained in Section 3.2.3.

The axial strain reported in this research is a combination of both the ACDT and LVDT measures. The ACDT's mounted on the specimen had an active length of ±0.1 of an inch. Comparison of the measured displacements from these two ACDT's indicated that they measured similar strains for most tests up to axial strain levels in excess of 5%. Occasionally however, the two ACDT strain measures would not be similar. This reflected a nonuniform straining of the specimen or possible seating problems with the displacement yokes.

The strain measure from the ACDT's was relied upon for \( \epsilon_a < 2.5\% \) or up to a level of strain for which both ACDT's were in close agreement. At strain levels above this point, the strain measure was computed from the displacement of the loading piston. The computer program made this switch at a predetermined axial strain level as input by the user. Table 4.4 presents this level of axial strain for each reported test. For most tests the level of axial strain at which this switch took place was 2.5%. It was assumed that at strain levels beyond this transition point, all of the grease had squeezed out between the ice caps and brass pedestal and top cap, thus making the LVDT (piston) measure of the axial strain reasonably accurate.

Fig. 4.5 plots the axial strain versus time for both the ACDT and the LVDT strain measures in FRS30. This behavior is typical. The figure shows a
very large seating error in the LDVT data (on the order of 0.0015%) compared to the behavior of the ACDT's. A linear regression analysis on these data from 2900 seconds to 3350 seconds (up to approximately 2.5% axial strain) indicated that the external LVDT measured a slightly higher strain rate than the on-specimen ACDT's.

This behavior is typical of most of the tests conducted, i.e., there was a large seating error associated with the external strain measure and the external strain rate was generally greater than the on-specimen axial strain rate. This could be due in part to squeezing out of the grease used as a lubrication. In the case of specimens with ice caps, this could also be due in part to larger strains in the weaker and more compliant ice caps and a flaring or spreading of the ends, which results in a lower effective axial strain rate near the center of the specimen where the ACDT's were located.

**Deviator Stress Measure**

The axial force measured by the internal load cell was divided by the specimen cross-sectional area in order to compute the deviator stress. The load cell was fully immersed in the triaxial cell fluid and was therefore insensitive to the cell pressure. The area of the specimen was estimated by a modified right cylindrical correction procedure. At each stage of the test the cross-sectional area was computed by the following equation

\[ A_c = A_i \frac{(1+\epsilon_v)}{(1-\epsilon_a)} \]

where \( A_c \) is the current cross-sectional area, \( A_i \) is the initial cross-sectional area measured during the set up procedure, \( \epsilon_v \) is the specimen volumetric strain (a volume increase in this equation is positive in order to be consistent with the plots of volumetric strain which are presented in Chapter 5), \( \epsilon_a \) is the axial strain.
defined as the engineering strain (compression is positive), and $\alpha$ is a factor to account for bulging of the center of the specimen or flaring of the lubricated ends. If $\alpha = 1.0$, this area correction is that for a perfect right cylinder. An $\alpha$ less than 1.0 is indicative of some flaring or spreading at the ends of the specimen, and an $\alpha$ parameter greater than 1.0 is indicative of some barreling near the center of the specimen.

The effect of $\alpha$ in equation 4.3 is to distribute the total volume of the specimen over either a smaller or a larger effective length of the specimen. In the case of specimens which exhibited lateral spreading or flaring at the ends, an effective length was chosen as somewhat larger than the actual specimen length. This decreased the computed cross-sectional area from that of a perfect right cylinder (based on the actual deformed length of the specimen) in an attempt to match the actual cross-sectional area near the center of the specimen. In the case of a specimen that experienced bulging, a shorter than actual effective length was chosen. Distributing the total volume over the shorter specimen had the effect of increasing the cross-sectional area over that which would have occurred in a perfect right circular cylinder undergoing the same axial and volumetric strains. Figure 4.6 presents the profiles of FRS41 and FRS46 which represent two extreme cases. For FRS41, the $\alpha$ was 1.50 and for FRS46 it was 0.58.

The magnitude of the $\alpha$ was determined after each test by a trial and error method. The diameter of the deformed frozen sand specimen was measured at eight separate locations along the specimen length, two near each end and two at each third point. The final area was estimated from the four diameter measures made at the third points. An $\alpha$ was selected for the reduction (assumed to be constant throughout the test) and the final area was generated by the data reduction program. This area was compared to the final measured area. If the
area generated by the data reduction program did not match the measured area, a new \( \alpha \) was chosen and the process was repeated until a suitable \( \alpha \) was determined to the nearest 0.01.

Table 4.4 presents the \( \alpha \) for each of the reported tests from this trial and error method. The value of \( \alpha \) indicates the quality of the test or effectiveness of the lubrication. If \( \alpha \) is close to 1.0, the final deformed shape was very close to a right circular cylinder. If \( \alpha \) is close to 1.5, the specimen deformed as a barrel, and if \( \alpha \) is close to 0.5 the specimen deformed as a dumbbell with flared ends. Of the 51 tests reported in this research, all but 15 of them had an \( \alpha \) in the range of 0.90 to 1.10. Of the remaining 15, only 7 of them had an \( \alpha \) greater than 1.2 or less than 0.8.

### 4.6 ERROR ANALYSIS OF TRIAXIAL TESTS

This section presents an analysis of the errors that may be associated with the triaxial testing portion of this research. An analysis of the errors associated with the determination of the phase relations has been presented in Sections 4.1 and 4.4. An estimate of the errors inherent in these results can be achieved by consideration of the precision and bias of the measurement techniques. The precision is the degree of mutual agreement among individual measurements made under similar conditions and the bias is a constant or systematic error in test results (ASTM E111-82).

The discussion of the precision of these measurements focuses on the repeatability of the test results and factors which affect the repeatability. The bias of these measurements cannot be evaluated directly because this requires standard reference values for frozen sand and these do not exist (see ASTM E111-82, Section 10.2). In place of this direct comparison, a discussion of the
calibration procedures of the electronic measuring devices, the area correction procedure, and a presentation of the results of a Young's Modulus measure on an aluminum dummy specimens, give an indication of the overall bias which may be associated with this measurement system.

In preparing this Section, the writer has attempted to identify those aspects which might potentially have the greatest influence on the accuracy of the reported data. This however does not discount the possibility that other factors not considered herein may have an effect on the overall accuracy of the data.

4.6.1 Definition of Parameters from Stress–Strain Curves

In order to consider the repeatability of the test results, it is necessary to define various quantities from the stress–strain curves. The following quantities have been obtained from graphical constructions on the actual stress–strain curves. Fig. 4.7 shows an idealized version of the stress–strain behavior of frozen Manchester Fine Sand in triaxial compression loading. Each of the stress–strain quantities is presented in this figure.

Figure 4.7 consists of four parts: (a) the initial stress–strain behavior up to 0.1% $\varepsilon_a$; (b) the stress–strain behavior up to 2.6% $\varepsilon_a$; (c) the stress–strain behavior to large strains; and (d) the volumetric strain behavior. The vertical axis of the stress–strain curves is the "stress difference" or deviator stress defined as $Q = (\sigma_1 - \sigma_3)$ in units of MPa. The strain axis is defined as the engineering axial strain $\varepsilon_a = \Delta L/L_0$, in absolute units. The volumetric strain in (d) is defined as $\varepsilon_v = \Delta V/V_0$, with positive representing expansion or dilation.
Young’s Modulus

The initial Young’s modulus is obtained from a best fit straight line, using graphical construction, through the initial portion of the stress–strain curve in Fig. 4.7(a). The modulus is computed as the slope of that line. The Young’s modulus values are given to the nearest 0.1 GPa.

Proportional Limit

The proportional limit is defined as that point on the initial stress–strain curve in Fig. 4.7(a) where the behavior begins to deviate from linear. This is read on the vertical stress axis to the nearest 0.1 MPa.

Yield Offset Stress at 10⁻⁴ Strain

The 10⁻⁴ yield offset stress gives an indication of the level of stress at which significant initial yielding occurs on the initial stress–strain curve in Fig. 4.7(a). It is obtained by translating the initial slope of the stress–strain curve, i.e., using Young’s modulus, by 10⁻⁴ strain and reading the intersection of this translated slope with the stress–strain curve on the vertical axis to the nearest 0.1 MPa.

Upper Yield Region

The upper yield region is that portion of the stress–strain curve where the most significant amount of yielding is occurring (i.e., the knee before the peak strength). See Fig. 4.7 (c). An expanded view of this region is depicted in Fig. 4.7(b).
Upper Yield Point

The upper yield point on the stress–strain curve is defined as the point in the upper yield region where the slope \( \frac{dQ}{d\varepsilon_a} \) either becomes zero or decreases to its minimum value before significant strain hardening or strain softening behavior occurs. Refer to Fig. 4.7(b) There are four types of upper yield behavior depicted on this plot.

Curve I represents a condition where the upper yield point represents the peak strength and there was continuous strain softening. The upper yield point in this case is the peak deviator stress. Curve II represents a behavior where a local peak deviator stress \( \frac{dQ}{d\varepsilon} = 0 \) occurs in the upper yield region. This is followed by a slight amount of strain softening and then strain hardening to the peak strength. The upper yield point is determined as the point at the local maximum stress. Curve III represents a behavior where there is not a local maximum but rather the slope of the stress–strain curve decreases to a minimum positive value and then increases during subsequent strain hardening (to be defined subsequently). The upper yield point is determined as the point at the beginning of the local minimum slope prior to strain hardening. Curve IV represents a behavior where the minimum slope in the upper yield region corresponds to the maximum slope of the subsequent strain hardening region, i.e., no upward concavity as in the case of Curve II. The upper yield point for this case is defined as the point where the slope reaches a minimum value. For all four types of curves, the upper yield point is reported both in terms of deviator stress to the nearest 0.1 MPa and axial strain to the nearest 0.0001.
Post Upper Yield Strain Hardening or Softening

Axial Strain at the Beginning of Strain Hardening or Softening

For curve I, the axial strain at the beginning of strain softening is defined as the point of intersection between tangents 1 and 2 representing the maximum negative slope after the upper yield point and the minimum negative slope during the subsequent strain softening.

The axial strain at the beginning of strain hardening is defined for Curve II as the intersection of the tangent lines 3 and 4 representing the slopes of the stress–strain curve in the local region of strain softening after upper yield and the maximum slope of the stress–strain curve during subsequent strain hardening. For Curve III, the axial strain at the beginning of strain hardening is the point of intersection between the tangents 5 and 6 representing the minimum slope after upper yield and the subsequent maximum rate of strain hardening. In the case of curve IV, the axial strain at the beginning of strain hardening corresponds to the axial strain at the upper yield stress, i.e., the point of tangency of line 7. In all of these cases, the level of axial strain is reported to the nearest 0.0001.

Rate of Strain Hardening or Strain Softening

The rate of strain hardening after the upper yield point is taken as the slope of the stress–strain curve \( \frac{dQ}{d\varepsilon_a} \). In the case of continuous strain softening after the upper yield point, it is defined as the minimum negative slope, i.e., tangent line #2 on Curve I in Fig. 4.7(b). In the case of localized strain softening after the upper yield point, it is defined as the subsequent maximum slope, i.e., tangent #4 on Curve II. In the case of Curve III with a slight bend after the upper yield point, it is defined as the slope of the tangent #6. In the case of Curve IV with strong and continuous strain hardening after the upper...
yield, it is defined as the maximum slope, i.e., the slope of tangent line 7.

When the maximum rate of strain hardening occurred after an axial strain of 0.026, i.e., it was off the scale of Figure 4.7(b), the complete stress–strain curve was used in the construction. The maximum rate of strain hardening is reported to the nearest 1 MPa, although slight deviations in the placement of the tangent curve can cause variations of ± 5 MPa or more.

**Peak Strength**

The peak strength and the axial strain to the peak have been read directly from the reduced data sets for each test. The peak strength is reported to the nearest 0.1 MPa and the axial strain to the peak is reported to the nearest 0.001, except in the case where the upper yield stress was the peak strength and the axial strain measure was provided by the ACDT's. For such cases, the axial strain is reported to the nearest 0.0001.

**Rate of Dilation**

The maximum rate of dilation \((\frac{d\epsilon_v}{d\epsilon_a})\) for each test was computed directly from the volumetric strain plot as the tangent line with the maximum slope. This is reported to the nearest 0.01. The rate of dilation at the peak strength was similarly obtained and reported.

**Volumetric Strains**

The volumetric strain at the beginning of dilation was identified as that point on the volumetric strain plot where the \(\epsilon_v\) increases above zero. The volumetric strain at 20% axial strain is also reported.
4.6.2 Repeatability of Test Results

In order to assess the repeatability of the measurements, seven of the test conditions were repeated twice and two were repeated three times. These conditions ranged from low to high confining pressure, slow to moderate strain rates and moderate to high density. The writer recognizes that this is a relatively small set of data from which to derive an assessment of the repeatability of the test results. These data are analyzed accordingly and conclusions are drawn in so far as is possible.

Table 4.5 is a summary table of all of these repeatability tests. This table presents the values in the measured stress–strain quantities for each series. The coefficient of variation (COV) has been computed for each of these quantities even though it was based on only two or three data points. Also included in this table is the mean and standard deviation of the COV for each stress–strain parameter over all repeatability tests. A direct comparison of the variability of these quantities for the wide range in testing conditions may not be entirely valid because the variability may be a function not only of the inherent variability of the measure but also a function of the absolute magnitude of each parameter.

The scatter in these stress–strain quantities as evidenced in Table 4.5 may be due to several factors including natural variations in the intrinsic properties of the frozen sand, unobserved external variables, and observable external variables. Two of the observed external factors which may have affected the measured stress–strain response were the behavior of the ACDT’s and the stability of the specimens on the lubricated end platens. These are discussed and a best estimate of the precision of the measured stress–strain behavior is presented.
4.6.3 Some Factors That May Affect the Repeatability

ADCT Behavior

The measured small strain behavior was affected by the behavior of the individual ACDT's. As explained in Section 4.5.3, the initial axial strain was computed as the average of the strains measured by each of the two ACDT's. In some tests, one of the ACDT's registered initial compressive strains, while the other registered initial tensile strains. This could have been caused by slight eccentricities in the applied axial force (due to nonparallel specimen ends) or slight density nonuniformities producing bending of the specimen. Another cause could have been seating problems with the on-specimen displacement yoke. The axial strain as measured by the ACDT's were compared for each test. In some tests both of the outputs agreed closely throughout the entire initial portion of the stress–strain curve and in other cases the two outputs deviated dramatically.

Figure 4.8 is a plot of the initial stress–strain behavior of FRS58. In this test there was poor agreement between the two ACDT's. Stress–strain curves have been drawn with the axial strains based on the average of the two ACDT's and from each individually. The axial strain measured from ADCT2 indicated an apparent lengthening of the specimen under the initial compressive deviator stress, and the axial strain measured from ACDT1 indicated axial shortening. This test gave an anomalously low $E$ value of only 14.3 GPa (Table 4.5). Figure 4.9 plots the initial stress–strain behavior for FRS66, which represents one of the best tests where the ACDT strain measures agree. This plot shows that the axial strains as measured by each of the ACDT's were almost identical.

A qualitative assessment of the behavior of the ACDT's has been made for each test, with the qualifiers "good", "fair" and "poor" being assigned to
indicate the agreement between the ACDT's. A test in which one of the ACDT's recorded an initial negative strain received a "poor" qualifier. A test for which the ACDT's were in good agreement received a "good" qualifier. Tests that were intermediate received a "fair" qualifier. The writer realizes that this is a rather subjective classification system. Clearly a different observer might assign different qualifiers to the same test. Of the 51 tests reported in this thesis, 30 of them showed good agreement, 5 of them showed fair agreement and 16 of them showed poor agreement. These qualifiers have been included both in Table 4.5 and 5.1.

Examination of Table 4.5 indicates that the coefficients of variation for the Young's modulus, proportional limit, $10^{-4}$ yield offset stress, the axial strain at the upper yield stress, and the rate of strain hardening were generally lower for those series where the ACDT's were in either "good" or "fair" agreement. Although these trends are not entirely consistent, they lead the writer to conclude that only those tests with either good or fair agreement between the ACDT's should be used to present the stress–strain behavior in terms of these parameters. Also, although the COV of the axial strain at the beginning of strain hardening was not consistently affected by the behavior of the ACDT's, the same conclusion was applied to this parameter, i.e., "good" or "fair" ACDT's lead to the most representative measure.

**Stability of Specimens**

The use of lubricated end platens and tall specimens for this testing program compromised the stability of some to the specimens at very large strains. Due to the low level of lateral confinement between the specimens ends and the loading platens, slight eccentricities in load tended to cause the
specimens to slide on the pedestal and top cap. The careful trimming procedures outlined in Sections 4.2.1 and 4.2.2, along with the use of alignment pins between the platens and the frozen sand specimens, were not always sufficient to insure stability. Each specimen was photographed after shear while it was still on the base pedestal and a qualitative assessment of the stability was made from the photograph in terms of the qualifiers "good", "fair", "poor" and "very poor".

These qualifiers are included in both Table 4.5 and 5.1. A specimen with good stability exhibited no sliding at large deformations. A specimen with very poor stability exhibited excessive sliding after shear. Figure 4.10 shows photographs of four specimens after shear that illustrate very stable = "good", slight instability = "fair", unstable = "poor", and very unstable = "very poor" assessments. Again, these are subjective and are presented only to give the reader a feel for the stability of the specimens. Of all reported tests, 19 specimens showed no signs of instability "good", 18 showed slight signs of instability "fair", 11 exhibited instability "poor" and two were extremely unstable "very poor".

Comparison of the stress–strain parameters in Table 4.5 shows that in some cases, the stability of the specimens does affect the repeatability of the large strain behavior. In the case of the peak strength, the two series which had specimens with very poor stability also had the largest COV's, while for all other stability conditions, the COV's were fairly similar. The failure strain however did not behave in a similar manner, i.e., some series which had specimens with very poor stability had a lower a COV than series with specimens that had good or fair stability. The maximum rate of strain hardening was not clearly affected by either the stability or the agreement between the ACDT's.

The magnitude of the volume change quantities varied dramatically with
test conditions. This makes a complete comparison with all of the repeatability data very difficult because of the limited number of observations. Focusing on the quantities with the largest magnitude (d$\varepsilon_v/\varepsilon_a > 0.4$, and $\varepsilon_v @ 20\% \varepsilon_a > 0.7$), the stability of the specimen did have some affect on the volumetric behavior, i.e., those series with very poor stability specimens had the largest COV for the maximum rate of dilation and volumetric strain at 20% axial strain.

In the subsequent analysis of the data presented in Chapter 5, the writer has not considered the large strain and volumetric parameters for those tests which exhibited "very poor" stability.

4.6.4 Best Estimate of Precision

Considering the effects of both the stability of the specimens and the behavior of the ACDT's at small strains the writer made a best estimate of the repeatability of these test results. For small strain parameters (Young's modulus, proportional limit and $10^{-4}$ yield offset stress), only those specimens for which the ACDT's were in "good" or "fair" agreement were considered. For large strains, only those specimens which exhibited "good" or "fair" or "poor" stability were considered. In the case of the maximum rate of strain hardening and the axial strain at the beginning of strain hardening, the writer adopted both the ACDT and stability criteria. The COV for each of the stress–strain quantities in terms of these criteria and the standard deviation of the COV are presented in Table 4.5. These give an idea of the precision of the reported test results.
4.6.5 Bias of Measurements

Calibration of Devices

The calibration procedures for each of the electronic sensing devices have been thoroughly explained in Section 3.4. The displacement transducers that measured the on-specimen axial strains and the displacement of the triaxial cell loading piston were calibrated directly with a veneer micrometer accurate to 0.0001 of an inch. The large displacement transducer that measured the position of the cell pressure piston was calibrated on a Sony digital stage with direct readout to 0.0001 of an inch. The pressure transducers that measured the cell pressure were calibrated with a static pressure calibration device, i.e., dead weights placed on a closed fluid reservoir with a low friction piston. The load cell measuring the axial force acting on top of the specimen was calibrated using a dead weight loading frame. The use of these calibration procedures with dead weights and micrometers ensure a high degree of accuracy in the calibration factors.

One source of error in the axial force measurement was the possibility of eccentric loading, especially with unstable specimens. The load cell used in this research was designed with an S-beam construction. The strain gages were located at the center of the load cell on the neutral axis for lateral bending. This design helps to minimize the sensitivity to lateral loads. A direct measure of the sensitivity of the load cell to lateral loads was not made. The manufacturer, Data Instruments of Acton Massachusetts, estimated that roughly 20% of an applied side load might show up as a fictitious axial force. Thus, if the magnitude of the side load were 5% of the axial force, the error associated with the axial load would be roughly 1%. This effect is probably greatest in the
specimens that exhibited sliding at large deformations.

**Volume Change Measure**

The volume change measurement system has been thoroughly discussed in Section 3.2.3. An estimate of the accuracy of the volume change measure was obtained by conducting a series of mock tests with no specimen. It was assumed that under these conditions, if Eq. 3.3 correctly captures the volume changes in the system, then a test without a specimen should exhibit no volume change. Measurements of the volume change under such conditions when no volume change can occur give an indication of the accuracy of the measure. Repeating this no specimen test several times gives an indication of the precision of the volume change measure.

Figure 4.11 is a summary plot for 21 no—specimen tests conducted over the full range in cell pressures and strain rates investigated in this research. Eq. 3.3 was used to compute the volume change and the only term that had no effect was $\Delta F_{ax}$. The vertical scale on the plot is the measured volume change and the horizontal scale is the normalized voltage of the LVDT on the cell pressure piston. With a normalized voltage of approximately 0.5, the piston is fully extended into the reservoir and with a normalized voltage of approximately −0.5 the piston is fully withdrawn. This figure shows that the mean volume change was approximately zero over the entire length of travel of the piston for all cell pressures and strain rates. The maximum error in the volume change measure was on the order of ±0.15 cc. This corresponds with a volumetric strain of ±0.2% for the $V = 75cc$ specimens and gives an estimate of the precision of the measure. At an axial strain of 20%, this error in the volumetric strain would correspond to an error in the measured rate of dilation of ±0.01.
Specimen Cross-Sectional Area Correction

The correction made to the specimen cross-sectional area in order to account for bulging or flaring was described in Section 4.5.3. The final diameter of the specimen at third points was measured directly with a micrometer to the nearest 0.001 of an inch. An $\alpha$ was selected for each test from a trial and error procedure in order to distribute the total volume of each specimen over an effective length so that the final area closely matched the final measured area. Table 4.4 reports the magnitude of the error between the final computed and measured area as a percentage of the final measured area. These errors have been reported to the nearest 0.1%. This table indicates that the maximum error in the computed versus measured final area was approximately 0.2% and the minimum error was less than 0.1%. The majority of the tests had errors on the order of 0.1%.

The effect of $\alpha$ on the computed stress-strain response is illustrated in Figure 4.12. This is a plot of several "hypothetical" stress-strain curves for FRS56 with $\alpha = 0.58$, 1.0 and 1.50. The actual $\alpha$ for FRS56 was 1.0. The curve for $\alpha = 0.58$ has the highest deviator stress and the curve for $\alpha = 1.50$ has the lowest deviator stress. These stress-strain curves indicate that the deviator stress is relatively unaffected by the magnitude of the $\alpha$ up to axial strains of about 2%. Beyond that level of axial strain, the magnitude of the correction increases with level of strain. At an axial strain of 10%, the maximum correction was on the order of $\pm$ 5% of the deviator stress and at an axial strain of 20% the maximum correction was on the order of $\pm$ 11% of the deviator stress.
4.6.7 Modulus Measure With Aluminum Dummy Specimen

A check on the accuracy of the axial displacement measuring system was performed by conducting a series of compression tests on an aluminum dummy specimen in the triaxial cell. These tests were mentioned in Section 3.2.2 in conjunction with a measurement of flexure of the base of the triaxial cell. The aluminum dummy specimen was placed in the triaxial cell without a rubber membrane. The axial displacement yoke was placed on the aluminum dummy and the cell pressure increased to 10 MPa. The nominal temperature of $-10^\circ$C was allowed to stabilize and the deviator stress was cycled manually between 0 and 13 MPa using the hand crank on the loading frame. Figures 4.13 and 4.14 are the resulting stress–strain curves for two separate tests each with two loading and unloading cycles. A linear regression analysis through these data yielded a Young's modulus of 74.3 GPa and 74.6 GPa, respectively, with an $r^2$ value for each of 0.99. This corresponds well with reported modulus values for aluminum which range from 69 to 73 GPa (Popov 1968 pg. 554).
Table 4.1:  Mechanical Sieve Analysis on Manchester Fine Sand

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Table 4.2: Summary of Specimen Physical Properties
(page 1/2)

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<th>Date Tested</th>
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<th>Dry Density (kg/m³)</th>
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<th>Water Content (%)</th>
<th>Deg. of Sat. (%)</th>
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* Based on final washed weight
** Based on total weight from trimmings and pouring
Table 4.2: Summary of Specimen Physical Properties

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<th>Test Name</th>
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<th>Frozen Density (kg/m³)</th>
<th>Dry Density (kg/m³)</th>
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<th>Water Content (%)</th>
<th>Deg. of Sat. (%)</th>
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<th>Ice Cap Length (mm)</th>
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* Based on total weight from trimmings and pouring
### Table 4.3: Summary of Specimen Relative Density Determinations

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* Density selected as most representative
Table 4.3: Summary of Specimen Relative Density Determinations (page 2/2)

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### Table 4.4: Computation Factors Used in Data Reduction (page 1/2)

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Table 4.4: Computation Factors Used in Data Reduction (page 2/2)

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Table 4.5: Range in Stress–strain Parameters from Repeatability Tests

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<th>Prop. Limit (MPa)</th>
<th>10(^{-4}) Yield (MPa)</th>
<th>ACOT</th>
<th>LARGE STRAIN PARAMETERS (COV)</th>
<th>Strain Hardening</th>
<th>VOLUMETRIC PARAMETERS</th>
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<td>g,f</td>
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<td>±12.7</td>
<td>g,f</td>
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<td>±5.6</td>
<td>±29.8</td>
<td>±2.6</td>
<td>±9.1</td>
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* Includes very poor stability specimens also
Figure 4.1: Mechanical Sieve Analysis on MFS and Comparison With Prior Testing Programs
Figure 4.2: Unfrozen Water Content Versus Temperature for MFS

Tested by Pat Black 8/2/90
Figure 4.3: Schematic Diagram of Specimen Preparation Molds (after Martin et al. 1981)
Figure 4.4: Schematic Diagram of Multiple Sieve Pluviation Frame
Figure 4.5: Comparison of ACDT Versus LVDT Axial Strain Measures

LVDT Strain Measure

\[ \varepsilon = 3.60 \times 10^{-5} \text{/sec} \]

\[ r^2 = 0.99998 \]

ACDT Strain Measure

\[ \varepsilon = 3.39 \times 10^{-5} \text{/sec} \]

\[ r^2 = 0.99997 \]
Figure 4.6: Over Lubricated Versus Under Lubricated Specimens

FRS46
Over Lubricated

FRS41
Under Lubricated

\[ \alpha = 0.58 \]

\[ \alpha = 1.50 \]
a. Initial Stress–Strain Response

b. Upper Yield Behavior

c. Full Stress–Strain Curve

d. Volumetric Behavior

Figure 4.7: Idealized Stress–Strain Construction Techniques Behavior and Graphical
Figure 4.8: Example of Poor Agreement in Initial Strain Measurements
Figure 4.9: Example of Good Agreement in Initial Strain Measurements
Figure 4.10: Example of Specimen Stability Qualifiers
Figure 4.11: Cell Fluid Volume Change With No Specimen (21 tests)
Figure 4.12: Effect of $\alpha$ on Deviator Stress
Figure 4.13: Aluminum Stiffness Measure (first test)
Figure 4.14: Aluminum Stiffness Measure (second test)
CHAPTER 5
BEHAVIOR OF FROZEN MANCHESTER FINE SAND
IN TRIAXIAL COMPRESSION

This Chapter presents the results of the triaxial compression tests conducted on frozen Manchester Fine Sand. The discussion focuses on the effects of relative density \((D_r)\), strain rate \((\dot{\varepsilon})\) and confining pressure \((\sigma_c)\) on the stress–strain behavior. After outlining the scope of the testing program, a qualitative overview of the stress–strain behavior follows. The various types of stress–strain curves observed in this research are presented along with combinations of variables that cause each type. A quantitative treatment of the stress–strain behavior is then presented. This is divided into two major sections, small strain behavior and large strain behavior. The small strain behavior is considered in terms of the Young’s modulus, the proportional limit and the yield offset stress at \(10^{-4}\) strain. The large strain behavior is considered in terms of the upper yield region, the strain hardening or strain softening region, the peak strength region and the volumetric behavior.

The writer has chosen to present these data primarily through summary plots for each of the measured stress–strain parameters. These give a concise overview of the effect of \(D_r\), \(\dot{\varepsilon}\), and \(\sigma_c\). Composite stress–strain curves are presented to reinforce the observed trends. Simple statistical analyses are performed on various portions of these data.

All of the parameters used to describe the stress–strain response in this Chapter were defined previously in Section 4.6.1. The graphical construction techniques used to obtain these parameters were described. Table 5.1 presents all of these parameters. All summary plots presented in this chapter have been
constructed from the values in this table.

In addition to the main presentation of the data, the effects of end conditions, specimen preparation technique and temperature are also discussed. The final section presents a summary of the measured behavior of frozen Manchester Fine Sand in triaxial compression.

5.1 SCOPE OF TESTING PROGRAM

Table 5.2 summarizes the scope of the testing program. The relative density ($D_r$) varied from 20 to 100% and the confining pressure ($\sigma_c$) varied from 0.1 to 10.0 MPa. Three nominal strain rates ($\dot{\varepsilon}$) were used: $3\times10^{-6}$/sec, $3\times10^{-5}$/sec and $4\times10^{-4}$/sec. These are referred to as slow, moderate and fast tests, respectively. One specimen, FRS36, was tested at a strain rate of $1.2\times10^{-4}$/sec (referred to as "intermediate"). The testing temperature was $-9.55\pm0.3^\circ$C. One test, FRS50, was performed at approximately $-15^\circ$C.

The specimens that were tested at the moderate strain rate and confining pressures of 0.1 and 10.0 MPa formed the principal basis of the study of the effect of relative density. The effect of strain rate was studied at relative densities of 20 to 40% and 80 to 100% and confining pressures of 0.1 and 10.0 MPa. The effect of confining pressure was studied most thoroughly at the moderate strain rate and relative densities of 20 to 40%, 40 to 60% and 80 to 100%. Several other tests were performed to investigate the effect of end conditions and specimen preparation technique.

5.2 QUALITATIVE OVERVIEW OF STRESS–STRAIN BEHAVIOR

The general shape of the stress–strain curves as a function of the relative density, confining pressure and strain rate can be classified into seven different
types. Figure 5.1 presents several normalized stress–strain curves that serve to illustrate visually these different types of behavior. Each curve has been normalized with respect to the magnitude of the deviator stress at the upper yield point in order to facilitate comparison (Note: as will be seen, the upper yield stress does not vary with relative density and only varies slightly with confining stress). Types I through IV are associated with low confining pressures and Types V through VII are associated with high confining pressures. The low pressure tests all exhibit some strain softening, while the high pressure tests generally do not.

A qualitative description of each type of stress–strain behavior follows.

**Low Confining Pressure**

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>The peak strength occurs at the upper yield point. This is followed by pronounced strain softening.</td>
</tr>
<tr>
<td>Type II</td>
<td>The upper yield stress (knee in the stress–strain curve) is followed by a yield drop. A slight amount of strain hardening occurs after this yield drop to a second peak having a strength similar to the upper yield stress. Some strain softening occurs at very large strains.</td>
</tr>
<tr>
<td>Type III</td>
<td>The upper yield stress is followed by a plateau in the stress–strain curve. After this plateau, there is a significant amount of strain hardening to the peak strength, followed by strain softening.</td>
</tr>
<tr>
<td>Type IV</td>
<td>The upper yield stress is followed by immediate strain hardening to the peak strength. After the peak, there is a considerable amount of strain softening.</td>
</tr>
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</table>

For the low confining pressure Type I–IV curves, strain softening causes the large strain post peak resistance to be generally less than the upper yield stress.
High Confining Pressure

Type V The upper yield stress is followed by a yield drop. After the yield drop, there is a slight amount of strain hardening. The deviator stress is almost constant to very large strains.

Type VI The upper yield stress is followed by a small plateau. After the plateau, there is continuous strain hardening to a peak strength at very large strains.

Type VII The upper yield stress is followed by an immediate and significant amount of strain hardening to the peak strength at very large strains.

For the high confining pressure Type V–VII curves, the large strain resistance will generally be larger than the upper yield stress. Table 5.3 summarizes the various combinations of testing conditions which produced these different types of stress–strain curves. These are discussed in greater detail in the following sections.

5.3 SMALL STRAIN BEHAVIOR

The small strain behavior can be described in terms of the measured Young's modulus (initial slope of the stress–strain curve), the proportional limit (point on the stress–strain curve where the response departs significantly from linear), and the yield offset stress at $10^{-4}$ strain (intersection point of the stress–strain curve and a line with a slope of the Young's modulus translated by $10^{-4}$ strain). These quantities were defined in Fig. 4.7(a).

The summary plots in this section contain only those tests for which the ACDT's were in either "good" or "fair" agreement as per Section 4.6.4. Of the 51 tests reported, only 16 had poor agreement for the ACDT's. Specimens are included that were prepared by either multiple sieve pluviation (most tests) or
wet tamping and that had either lubricated ends (most tests) or frictional ends. The strain rates considered for the small strain behavior are nominal strain rates computed from the start of the test to an axial strain of $10^{-4}$ as listed in the fourth column of Table 5.1. The actual strain rate is increasing continuously in this region of the stress–strain behavior (see Section 3.3.6 for a discussion of this behavior).

Table 5.4 summarizes the initial stress–strain behavior as a function of confining pressure and strain rate by presenting the mean values and standard deviations of the Young's modulus, proportional limit and yield offset stress. Table 5.5 presents an overview of the effect of each variable on the small strain parameters and Fig. 5.2 presents the initial stress–strain curves (i.e., up to 0.03% axial strain for a wide range of testing conditions to illustrate the effects of relative density, confining pressure and strain rate. Refer to these tables and figure during the following discussion.

5.3.1 Young's Modulus

Effect of Relative Density

The Young's modulus calculated for all the experiments is presented in Fig. 5.3 as a function of relative density. The majority of the measured Young's modulus data fall in the range of 20 GPa to 30 GPa. The mean value over all testing conditions is $26.6 \pm 4.5$ GPa. The data exhibit no consistent trend with relative density. There are, however, six specimens with relative densities in the range of 50% to 100% that have Young's moduli in the range of 30 GPa and 40 GPa.
Effect of Confining Pressure

Figure 5.4 presents the effect of confining pressure on the measured Young's modulus. This figure includes the data at the moderate strain rate and clearly shows the Young's modulus decreasing with increasing confining pressure. This trend is consistent for all strain rates, refer to Table 5.4.

Effect of Strain Rate

The measured Young's modulus is not affected by changes in strain rate over the range investigated (see Table 5.4 with similar confining pressures). Figure 5.5 shows this insensitivity for tests conducted at a confining pressure of 10 MPa.

5.3.2 Proportional Limit

Effect of Relative Density

Figure 5.6 plots the proportional limit as a function of relative density. The magnitude of the proportional limit ranges from 1 MPa to 3.1 MPa over all the testing conditions and has no consistent trend as a function of relative density.

Effect of Confining Pressure

Figure 5.7 plots the proportional limit as a function of confining pressure for all tests conducted at moderate strain rate. This figure clearly indicates a decrease with increasing confining pressure. This trend is consistent for all strain rates investigated. See Table 5.4.
Effect of Strain Rate

Figure 5.8 shows the effect of strain rate on the proportional limit for tests conducted at $\sigma_c = 0.1$ and 10 MPa. There is a clear tendency for the proportional limit to increase with strain rate. A least squares regression analysis on these data yielded power law coefficients ($n$) of 15.6 and 17.8 for the 0.1 and 10 MPa confining pressures, respectively.

5.3.3 Yield Offset at 10^{-4} Strain

Effect of Relative Density

Figure 5.9 plots the deviator stress at the 10^{-4} yield offset as function of relative density. Over all testing conditions, the yield point ranges from 2.7 MPa to 5.2 MPa with no apparent trend as a function of relative density. For a confining pressure of 0.1 MPa and a moderate strain rate, the mean value is $4.0 \pm 0.5$ MPa. It is $3.0 \pm 0.2$ MPa for a confining pressure of 10 MPa at the same strain rate.

Effect of Confining Pressure

Figure 5.10 plots the yield offset stress as a function of confining pressure for the moderate strain rate tests. There is a clear trend for the offset stress to decrease with increasing confining pressure. This trend is uniform for all strain rates investigated. See Table 5.4. and Fig. 5.11.

Effect of Strain Rate

Figure 5.11 plots the yield offset stress as a function of strain rate. This clearly indicates a trend to increase with strain rate. A linear regression analysis on these data yielded a power law coefficient of 11.2 for both the 0.1 and 10 MPa
confining pressure tests.

5.4 LARGE STRAIN BEHAVIOR

Various parameters which describe the large strain behavior of frozen Manchester Fine Sand were defined in Fig. 4.7 (b–d). These include the upper yield stress and corresponding strain, the rate of post upper yield strain hardening or softening, the peak deviator stress and corresponding strain, and the volumetric behavior. This section considers each of these in terms of the effect of relative density, confining pressure and strain rate.

Specimens Included in Presentation

Only specimens prepared by multiple sieve pluviation are considered in this section. The effect of specimen preparation technique on the large strain behavior is presented in Section 5.5.2.

In the discussion of the upper yield region, both specimens with lubricated and frictional end conditions are presented. All specimen stabilities are considered for the magnitude of the upper yield stress, but only "good" and "fair" ACDT’s are considered for the axial strain at the upper yield stress.

Specimens tested with both lubricated and frictional end conditions are used to present the rate of work hardening/softening (shown later in Section 5.5.1 to be unaffected by end conditions). Only those tests with either "good" or "fair" ACDT agreement and with "good", "fair" or "poor" stability are used to discuss the rate of work hardening/softening and the axial strain at which it begins.

The peak deviator stress and the corresponding axial strain are presented for tests with lubricated end conditions. Only specimens with "good", "fair" and
"poor" stability are considered for the magnitude of the peak deviator stress. The specimens with "poor" stability are excluded from the presentation of the axial strain at the peak deviator stress.

The volumetric behavior is presented in terms of those specimens with lubricated end platens that exhibited "good", "fair" and "poor" stability.

5.4.1 Upper Yield Region

The upper yield region was defined in Section 4.6.1 (see Fig. 4.7(b)). This region represents the knee of the stress-strain curve where very significant amounts of plastic deformation begin to occur. In the frozen sand literature, this region has commonly been referred to as the first peak or first yield. The writer chose not to adopt the "first yield" terminology in recognition of the fact that there is a significant amount of plastic deformation in these tests well before the attainment of this yield stress.

Table 5.7 and Fig. 5.12 summarize the observed upper yield behavior as a function of the relative density, confining pressure and strain rate. Refer to these during the subsequent discussion.

Effect of Specimen Height on Upper Yield Stress

These data were corrected for differences in the nominally applied strain rate experienced by each specimen due to variations in the trimmed height of the specimen. As explained in Section 3.3, the loading frame used for this testing program was screw driven. This means that the rate of advancement of the piston into the triaxial cell was fixed for a given nominal strain rate. If one specimen was trimmed to a length slightly shorter than another, it would experience a slightly higher effective strain rate due to the fixed rate of
displacement of the loading ram. In order to unify these results, the upper yield stress for each specimen was corrected to a standard height of 7.5 cm. This correction assumes that the strain rate was the most important variable affecting the upper yield stress.

Figure 5.13 plots the measured upper yield stress \( (Q=\sigma_1-\sigma_3) \) versus specimen height for all specimens presented in this research. This figure shows a clear tendency for the upper yield stress to decrease with increasing specimen height. A linear regression analysis was performed on the data for each strain rate. The resulting empirical equations are presented on Fig. 5.13. Table 5.1 gives both the measured upper yield stress and the corrected upper yield stress for each test.

**Effect of Relative Density on the Upper Yield Stress**

Figure 5.14 presents the magnitude of the corrected upper yield stress as a function of relative density. This figure shows that the magnitude of the upper yield stress is not affected by changes in relative density. It ranges from \( 5.0 \pm 0.1 \) MPa for the slow strain rate tests, to \( 8.1 \pm 0.5 \) MPa for the moderate strain rate tests, to \( 15.3 \pm 0.4 \) MPa for the fast strain rate tests.

**Effect of Confining Pressure on Upper Yield Stress**

Figure 5.15 presents the corrected upper yield stress as a function of confining pressure for the moderate strain rate tests. This shows a clear tendency for the upper yield stress to decrease with increasing confining pressure. Table 5.6 presents the mean values for upper yield stress at each strain rate and confining pressure. This trend is consistent for all of the strain rates investigated.
Effect of Strain Rate on Upper Yield Stress

Figure 5.16 shows the measured upper yield stress (not corrected) as a function of strain rate. The strain rate selected for this comparison is the maximum strain rate measured from the on-specimen axial strain yoke. These are presented in Table 5.1. This figure shows a strong tendency for the upper yield stress to increase with increasing strain rate. The results of linear regression analyses on these data are presented directly on the figure. The power law coefficients are 4.7 and 4.6 for confining pressures of 0.1 and 10 MPa, respectively.

Effect of Relative Density on Upper Yield Strain

Figure 5.17 plots the axial strain at the upper yield stress versus relative density for all strain rates and confining pressures. This figure shows that the axial strain at upper yield is unaffected by changes in the relative density. It ranges from 0.35 ± 0.7% for the slow tests, to 0.46 ± 0.06% for the moderate tests, to 0.81 ± 0.04% for the fast strain rate tests.

Effect of Confining Pressure on Upper Yield Strain

Figure 5.18 plots the upper yield strain versus confining pressure for tests conducted at a moderate strain rate. This figure indicates that the upper yield strain is unaffected by changes in the confining pressure. Table 5.6 summarizes mean value of the upper yield strain for all strain rates and confining pressures. For the slower and faster tests, there is a tendency for the upper yield strain to increase with increasing confining pressure. The writer believes that this is probably due to the relatively few number of observations at these rates.
and that the upper yield strain shows little or no sensitivity to changes in the confining pressure.

**Effect of Strain Rate on Upper Yield Strain**

Figure 5.19 presents the upper yield strain versus strain rate for all confining pressures and densities. This shows a strong tendency for the upper yield strain to increase with increasing strain rate. The result of a linear regression analysis on these data is also presented in Fig. 5.19. This gives a power law coefficient of 5.5. This increase parallels that of the magnitude of the upper yield stress.

### 5.4.2 Work Hardening/Softening Region

The work hardening/softening region was defined in Section 4.6.1. See Figure 4.7(b). The behavior in this region is described in terms of the rate of work hardening/softening (i.e., $dQ/d\varepsilon_a$, slope of the stress–strain curve) and the axial strain at which it begins. In defining these parameters, the writer has chosen to neglect the localized strain softening that is observed in Type I, II and V curves and the localized plateaus of Type III and VI curves and rather to focus on the response after these localized zones.

In all but one case (i.e., Type I curve in Fig. 5.1), the specimens exhibit some amount of work hardening after the upper yield region. Rather than constantly repeat the term work hardening/softening in this section, the writer has chosen to refer to it as work hardening. In the Type I case of continuous strain softening after the upper yield region, this will be highlighted specifically in the text.

Table 5.8 summarizes the effect of relative density, confining pressure
and strain rate on the behavior in the work hardening region. Figure 5.20 presents four stress-strain curves at a moderate strain rate and low confining pressure to show the effect of changes in relative density. Refer to these during the subsequent discussion.

**Effect of Relative Density on the Rate of Work Hardening**

Figure 5.21 presents the maximum rate of work hardening as a function of relative density ($D_r$) for the moderate strain rate tests. In all these tests, there was work hardening. This figure shows a strong tendency for the rate of work hardening to increase linearly with relative density. Considering the low confining pressure tests ($\sigma_c = 0.1$ MPa), an increase in the rate of work hardening with relative density corresponds in a shift from a Type II to a Type IV curve. For high confining pressure tests ($\sigma_c = 5$ to 10 MPa), an increase in the rate of work hardening corresponds to a shift from a Type VI to a Type VII curve.

Figure 5.22 presents the rate of work hardening versus relative density for the slow and fast strain rate tests. This figure shows that at different strain rates, there is still an apparently linear increase in the rate of work hardening with increasing relative density.

For tests at the fast strain rate and low confining pressure (0.1 MPa), an increase in relative density causes a change from strain softening to strain hardening behavior. This corresponds to a change in stress-strain curve from Type I at low density to Type III at high density. At a high confining pressure and fast strain rate, increases in relative density lead to changes in the stress-strain curve from a Type V at low relative densities to a Type VI at high relative densities.
For tests at a slow strain rate with both low and high confining pressures, increases in relative density lead to changes in the stress–strain curve. For low confining pressures, the curve changes from a Type III to a Type IV and for high confining pressures it changes from Type VI to Type VII.

Effect of Confining Pressure on Rate of Work Hardening

Figure 5.21 shows little or no effect of confining pressure on the rate of work hardening for the moderate strain rate tests. Figure 5.22 shows that it increases significantly with confining pressure at both the slow and fast strain rates (Note that only one data point is presented for the slow strain rate at low confining pressure). It is apparent that the rate of work hardening varies in a complex manner with confining pressure.

Effect of Strain Rate on the Rate of Work Hardening

Figure 5.23 presents the rate of work hardening versus axial strain rate for three tests conducted at 10 MPa confining pressure and a relative density at or near 95%. This figure suggests that the rate of work hardening increases from slow to moderate strain rates and then decreases. However, for a confining pressure of 10 MPa at a slow strain rate and low relative density, the rate of work hardening is identical to that of the moderate strain rate tests and greater than that of the fast tests (see Fig. 5.22). Thus the limited data can be viewed as either being inconsistent or indicating a complex influence of strain rate.

Effect of Relative Density on the Strain at Beginning of Work Hardening

Figure 5.24 plots the axial strain at the beginning of work hardening versus relative density for tests conducted at a moderate strain rate. This figure
shows that over all relative densities and confining pressures, it ranges from approximately 0.3 to 1.4%. At constant confining pressure, the axial strain decreases with increasing relative density. For tests with $\sigma_c = 0.1$ MPa, this corresponds to the loss of the upper yield drop and yield plateau in Type II and III curves in favor of continuous strain hardening in the Type IV curve. Refer to Fig. 5.1.

Figure 5.25 presents the axial strain at the beginning of work hardening for all strain rates and relative densities. This shows that it decreases with increasing relative density at all strain rates, indicating the same type of transition in the shape of the stress–strain curves as experienced in the moderate strain rate tests, i.e., a loss of the yield drop and yield plateau after the upper yield stress in favor of continuous strain hardening.

**Effect of Confining Pressure on the Strain at the Beginning of Work Hardening**

Figures 5.24 and 5.25 both show an increase in the axial strain at the beginning of work hardening with increasing confining pressure for specimens at slow and moderate strain rates. Note, however, that no data are reported for a slow strain rate, low relative density and low confining pressure. The data at the fast strain rate indicate a complex behavior (see Fig. 5.25). At the lowest relative density, the test at the high confining pressure (10 MPa) has a slightly larger axial strain than the specimen at the low confining pressure (0.1 MPa). This coincides with a change in the behavior from one of strain hardening to strain softening after the upper yield. At high relative density, this trend is reversed. These data suggest that the behavior of the axial strain at the upper yield stress as a function of confining pressure is complex. However, the relatively few number of observations at the slow and fast strain rates may affect
these trends.

Effect of Strain Rate on the Axial Strain at the Beginning of Work Hardening

Figure 5.25 shows that the data at the slow strain rate for both the low and high confining pressures fall within the scatter band of the moderate strain rate tests for the corresponding confining pressures, thus indicating an insensitivity to strain rate in this region. The data from the fast strain rates exhibit a complex behavior. At the low confining pressure (0.1 MPa) and overall relative densities, the axial strain increases with increasing strain rate from moderate to fast. Note however, that at low relative densities, an increase in the strain rate from moderate to fast changes the behavior from a Type II to a Type I. At the high confining pressure (10 MPa) and low relative densities, the axial strain increases with strain rate from moderate to fast, while at high relative densities, the data from the fast tests falls within the scatter band of the moderate tests.

5.4.3 Peak Strength Region

The writer has chosen to describe the behavior of the peak strength region in terms of the magnitude of the peak strength, the axial strain at peak strength and the difference between the upper yield stress and peak strength. The representation of the peak strength in terms of the strength gain after the upper yield stress eliminates some of the scatter in the peak strength data due to variations in the upper yield stress and allows one to focus more clearly on the effect of relative density (since the upper yield stress was shown in Section 5.4.1 to be independent of the relative density).

Table 5.9 summarizes the effect of relative density, confining pressure
and strain rate on the behavior in the peak strength region. Additionally, Fig. 5.26 through 5.32 illustrate the stress-strain (volumetric strain curves are also included and will be discussed in the next section). Figures 5.26 and 5.27 present the effects of relative density at a moderate strain rate and for confining pressures of 0.1 and 10 MPa, respectively. Figures 5.28 and 5.29 present the effects of confining pressure at a moderate strain rate and relative densities of 35% and 95%, respectively. Figures 5.30 through 5.32 present the effects of strain rate for 35% Dr and $\sigma_c = 0.1$ MPa, 95% Dr and $\sigma_c = 10$ MPa, and 35% Dr and $\sigma_c = 10$ MPa, respectively. Refer to this table and these figures during the subsequent discussion. Note that the specimen stability qualifiers have been placed on each stress-strain curve and that the stress-strain curves illustrating the effect of relative density and confining pressure have been normalized with respect to the upper yield stress (which was shown to be independent of Dr and only slightly affected by confining pressure).

**Effect of Relative Density on the Peak Strength**

Figure 5.33 plots the peak stress minus the upper yield stress versus relative density for tests conducted at a moderate strain rate. Also included on this plot are the results of a linear regression analysis of the data at the various confining pressures. This figure shows a linear increase in peak strength with relative density. Note that the upper yield stress is unaffected by changes in relative density. Examination of this figure also shows that the rate of increase in strength with relative density is enhanced at higher confining pressures.

Increases in the magnitude of the peak stress minus the upper yield stress correspond to changes in the character of the stress-strain curves. For the 0.1 MPa confining pressure tests, an increase in the relative density changes the
stress–strain curve from a Type II to a Type IV. For the 2.0 MPa confining pressure tests, the curves change from a Type III to a Type IV with increasing relative density. For the 5.0 and 10.0 MPa tests, the curves change from a Type VI to a Type VII. See Figure 5.1 and Table 5.3.

Figure 5.34 plots the peak stress minus the upper yield stress for fast and slow strain rate tests. This figure also indicates a linear increase in peak strength with relative density for all strain rates. Note, however, that the magnitude of the upper yield stress is dramatically influenced by changes in the strain rate so that in this case, these quantities do not reflect the peak strength behavior directly. This figure also indicates that the rate of strengthening is similar for the high pressure tests over all strain rates and for the low pressure tests at the moderate and slow strain rates.

For the fast strain rate and low confining pressure (0.1 MPa) tests, the lowest relative density specimen exhibits continuous strain softening after the upper yield stress and hence plots on the 0 MPa line. In representing the behavior from 35% $D_r$ and 95% $D_r$, the writer has drawn two possible strengthening curves on Fig. 5.27. In the writers opinion, it is likely that from 35% to 75% relative density, the peak strength is the upper yield strength and hence there is no strengthening. Above 75% $D_r$, it is possible that the strengthening occurs with the same rate as the moderate and slow strain rate tests. This is speculation and must be born out by additional testing.

Figures 5.35 and 5.36 plot the peak deviator stress versus relative density for the moderate strain rate tests and the fast and slow strain rate tests, respectively. These are presented to give the reader a clear picture of the effect of relative density on the peak strength. A bilinear behavior is again assumed for the fast tests at low confining pressures to reflect the hypothesized transition
between strain softening and strain hardening behavior. In this case the estimated transition point is approximately 65% \( D_r \) rather than 75% \( D_r \) because the regression line on the moderate strain rate data for the same confining pressure is slightly flatter than that of the peak stress minus upper yield stress data.

**Effect of Confining Pressure on Peak Strength**

Figures 5.33 through 5.36 all show a strong trend for the peak strength to increase with confining pressure over all strain rates and relative densities. In order to quantify this effect, the writer constructed a plot of peak deviator stress versus confining pressure for relative densities of 35, 60 and 95% at a moderate strain rate using the results from linear regression analyses on the data presented in Fig. 5.35. Figure 5.37 presents the results of this analysis. This figure shows the strong trend for the peak strength to increase with both relative density and confining pressure and also shows distinct changes in the total stress friction angle behavior. In this plot, the total stress friction angle is computed from the following equation

\[
\phi = \sin^{-1}\left(\frac{dQ}{dQ + 2d\sigma_c}\right)
\]

5.1

where \( dQ \) and \( d\sigma_c \) are the corresponding changes in the peak deviator stress and confining pressure over a line segment drawn tangent to the curves at the point where the total stress friction angle is to be determined.

For the tests at 95% \( D_r \), the total stress friction angle is approximately constant at a value of 15.6° for all confining pressures. This corresponds to a linear failure envelope. For tests at 60% \( D_r \), the envelope shows a marked curvature and the total stress friction angle decreases from approximately 13.5° to 7.9° over the range of confining pressures. For the tests at 35% \( D_r \), the total
stress friction angle decreases even more dramatically from approximately 11.2° to 1.6°. Thus, these data show that the total stress failure envelope changes from linear to curved as the relative density decreases from 95 to 35%.

Changes in the confining pressure cause changes in the character of the stress–strain curves by affecting the amount of post peak strain softening. As illustrated in Fig. 5.1, the tests at the lowest confining pressure all eventually exhibit strain softening. Increasing the confining pressure causes a reduction in the amount of strain softening and at the highest confining pressures there is continuous strain hardening.

The scope of the test program does not enable a detailed evaluation of the influence of strain rate on the total stress friction angle. However, the data from tests run with $\sigma_c = 0.1$ and 10 MPa on loose ($D_r=35\%$) and dense ($D_r=95\%$) specimens suggest the following compared to the results plotted in Fig. 5.30 for the moderate strain rate:

1) A similar strength increase occurs at both densities for shearing at the slow strain rate

2) For shearing at the fast strain rate, added confinement did not increase the strength of the loose specimens (e.g., same $Q=15$ MPa in Fig. 5.29), but did cause a significant increase ($\phi=12°$) for the dense specimens.

Hence the increase in peak strength with confining pressure varies in a complex manner as a function of the level of confinement, the sand density and the rate of shearing.

**Effect of Strain Rate on the Peak Deviator Stress**

Figure 5.38 plots the peak deviator stress versus strain rate for low and high relative densities and low and high confining pressures. The data points for
this plot were obtained from linear regression analyses through the peak strength data presented in Figs. 5.35 and 5.36. The corresponding strain rates were obtained as the averages for all tests conducted in the given range. This figure shows that the power law coefficients for peak strength range from 6.1 to 16.7. The plot also demonstrates that increasing either the confining pressure or relative density increases the power law coefficient.

**Effect of Relative Density on the Axial Strain at Peak Deviator Stress**

Figures 5.39 and 5.40 present the axial strain at peak strength versus relative density for tests at moderate strain rates, and slow and fast strain rates, respectively. These show a complex behavior for various testing conditions. At the moderate strain rate and a confining pressure of 0.1 MPa, the axial strain at peak is independent of relative density with a mean of 5.2 ± 0.9%.

For fast tests at a 0.1 MPa confining pressure, the axial strain at peak increased with relative density from 35% to 95%. This coincides with a change in the character of the stress–strain curve from strain softening to strain hardening. The writer assumes that when the upper yield stress represents the peak strength, i.e., for relative densities up to 65 to 75%, the axial strain at peak strength should be independent of relative density as per Fig. 5.17 and Table 5.7. The behavior during the transition zone from strain softening to strain hardening may be abrupt or gradual. Clearly, at the transition point where the stress–strain curve is flat, a determination of the axial strain at the peak deviator stress may be somewhat uncertain.

For all other testing conditions, i.e., moderate strain rate and confining pressures from 2.0 to 10 MPa, slow strain rate at all confining pressures and fast strain rate at high confining pressures, the axial strain at the peak stress
decreased with increasing relative density. Moreover, the decrease in strain with increasing $D_r$ tends to be more pronounced at high confining pressures, e.g., the results in Fig. 5.40.

**Effect of Confining Pressure on Axial Strain at Peak Deviator Stress**

Figures 5.39 and 5.40 both show that the axial strain at peak deviator stress increases dramatically for increasing confining pressure. Under all testing conditions, this corresponds to a change from post peak strain softening to continuous strain hardening after the upper yield region. At the highest confining pressures, some of the specimens did not reach a peak deviator stress until axial strains in excess of 20%. See Fig. 5.1.

**Effect of Strain Rate on Axial Strain at Peak Deviator Stress**

Figure 5.40 illustrates the behavior of the axial strain at peak stress for changes in strain rate. At a confining pressure of 0.1 MPa, the axial strain decreases with increasing strain rate, i.e., the behavior becomes more brittle. At a confining pressure of 10 MPa, the axial strain at peak is apparently unaffected by changes in strain rate from the slow to the moderate tests (the slow test data are in or near the scatter band of the moderate strain rate data). There is however, a dramatic decrease in the axial strain at peak between the moderate and fast tests, again indicating a transition to a more brittle type behavior.

### 5.4.4 Volumetric Strain Behavior

Figures 5.26 through 5.32 present several volumetric strain curves along with the corresponding stress–strain curves. Note that on these curves, a positive volume change corresponds to dilation. These indicate that the behavior
ranges from essentially no volume change at high cell pressures to very large volume increases at low cell pressures. Significant volume decreases are not observed under any conditions (e.g., minimum $\varepsilon_v$ is greater than $-0.2\%$).

This behavior can be contrasted with that of unfrozen sand tested in drained triaxial compression. Figure 2.4 presented data from tests on Sacramento River Sand tested in both a dense and loose packing. This figure showed that for low effective confining pressures and high relative densities (large $-\psi$ parameter), the largest amounts of volume increase were observed and for low relative densities and high confining pressures (large $+\psi$ parameter), the greatest amounts of volume decrease were observed. Thus, the major difference between frozen and unfrozen volumetric strain behavior is that the former exhibits little or no volume decrease, even for low relative densities and high confining pressures (see Fig. 5.35).

Table 5.10 summarizes the effect of relative density, confining pressure and strain rate on the volumetric behavior. Refer to this table and Fig. 5.26 through 5.32 during the subsequent discussion.

**Rate of Dilation at Peak Strength**

Figures 5.41 and 5.42 plot the rate of dilation ($d\varepsilon_v/d\varepsilon_a$) at peak strength for moderate strain rate tests, and for fast and slow strain rate tests, respectively. Linear regression lines for the moderate strain rate at low and high confining pressures are also included. These figures show that over all testing conditions, the rate of dilation at peak ranges from 0 to 0.35. There is a strong tendency for the rate of dilation to increase with increasing relative density and decrease with increasing confining pressure. This is true for all strain rates. Figure 5.42 indicates that the effect of strain rate on the rate of dilation at peak strength
may be complex. For $\sigma_c = 0.1$ MPa and low relative densities, there is apparently no effect, i.e., the data for the fast and slow tests fall within the scatter band of the moderate strain rate data. For $\sigma_c = 0.1$ MPa and high relative densities, the data from the slow to the moderate strain rates indicate a decrease in rate of dilation, while that from the moderate to the fast rates indicate little or no effect. For $\sigma_c = 10$ MPa and all relative densities, the data from the slow to the moderate strain rates show no effect, while that from the moderate to the fast strain rates show an increase.

**Maximum Rate of Dilation**

Figures 5.43 and 5.44 present the maximum rate of dilation versus relative density for moderate strain rate tests, and fast and slow strain rate tests, respectively. Linear regression lines for the moderate strain rate data are also included. Over all testing conditions, the maximum measured rate of dilation ranges from 0 to approximately 0.7. Note that these occur at very large strains well beyond the peak deviator stress for the low confining pressure tests. In some cases where the test was terminated before reaching 20% axial strain, the peak deviator stress may not have been reached. Inspection of Figs. 5.34 to 5.39 shows that the frozen sand specimens are continuously dilating at the end of the test, even at $\sigma_c = 10$ MPa.

This behavior is in direct contrast with the volumetric strain behavior of unfrozen sand presented in Fig. 2.4. For unfrozen sands with a strong tendency for dilation (large $-\psi$ parameter), the peak rate of dilation occurs near the peak strength. After the peak strength, the rate of dilation decreases and the shearing begins to approach that of a constant volume condition.

Figures 5.43 and 5.44 show that the maximum rate of dilation decreases
dramatically with increasing confining pressure for all relative densities and strain rates, but is apparently only slightly affected by changes in strain rate. At low confining pressures, all of the data for the slow and fast strain rate tests plot in or near the scatter band of the data for the moderate strain rate tests. At high confining pressures, the slow and moderate strain rate data show no rate effect, while the fast strain rate data exhibit a slight increase in maximum rate of dilation for all relative densities.

Axial Strain at the Beginning of Dilation

Figures 5.45 and 5.46 present the axial strain at the beginning of dilation versus relative density for moderate strain rate, and fast and slow strain rate tests, respectively. For a confining pressure of 0.1 MPa and all strain rates, the axial strain at the start of dilation is insensitive to changes in relative density and has a mean value of 1.9 ± 0.6%. Data for higher confining pressures and all strain rates exhibit a tendency to decrease with increasing relative density although, there is significant scatter (S.D. ± 4.5% at the moderate strain rate).

These figures also show a dramatic increase in the axial strain at the beginning of dilation with increasing confining pressure. There is a subtle trend for this axial strain to decrease with increasing strain rate. This is most noticeable for a confining pressure of 0.1 MPa and a relative density of approximately 95% (Fig. 5.46).

Volumetric Strain at 20\% Axial Strain

Figures 5.47 and 5.48 present the measured volumetric strain at 20\% axial strain versus relative density for moderate strain rate tests, and fast and slow tests, respectively. These show that over all testing conditions, the amount
of volumetric straining ranges from 0 to 10%. All specimens were either not changing volume or still dilating at 20% axial strain.

Increases in the relative density cause linear increases in the amount of volume change for all cell pressures and strain rates. The sensitivity of the volume change to relative density decreases with increasing confining pressure (compare slopes of the linear regression lines presented in Fig. 5.48). Increases in the cell pressure cause substantial reductions in the amount of volume increase during shear. Increases in strain rate apparently cause no change in the volumetric straining at low confining pressures (0.1 MPa) and high \( D_r \), and no change over all \( D_r \) at high confining pressures (10 MPa) in the slow to moderate range. Increases in the strain rate cause an increase in the volumetric straining at low confining pressures and low relative densities in the slow to fast range, at low confining pressures and high relative densities in the moderate to fast range, and at high confining pressures and all relative densities in the moderate to fast range.

The writer observed that for some specimens tested at low relative density (\( \approx 20\% \)), and low confining pressure (0.1MPa), continuous dilation during shear led to the condition of less than zero relative density at large strains. Clearly, this would be impossible for unfrozen sand in the same condition.

5.5 OTHER VARIABLES THAT MAY AFFECT THE BEHAVIOR

This section presents the effect of end conditions, specimen preparation technique and temperature on the measured behavior of frozen Manchester Fine Sand.
5.5.1 Effect of End Conditions

Several tests were conducted to investigate the effect of end conditions on the measured behavior. Testing conditions were investigated with both frictional and lubricated end conditions. As explained in Section 4.3.2, three types of end conditions were used in this research: 1) an ice cap with grease, (I); 2) frozen sand with grease, (G); and 3) emery cloth with grease, (E). Table 5.11 summarizes the measured stress–strain behavior for six pairs of tests with I versus G or E end conditions. Also included in this table are the ACDT and stability performance qualifiers.

Chapter 4 discussed the various factors that affect the measured behavior of frozen MFS. Table 4.5 presented the ranges in the stress–strain parameters for several repeatability tests. Comparison of the ranges presented in Table 5.11 with those in Table 4.5 indicates that most of the differences between the stress–strain parameters for the frictional versus lubricated end conditions fall either in or close to the ranges observed in the repeatability tests (with the exception of the axial strain at peak strength and the maximum rate of dilation). Most the changes in the parameters between the frictional and lubricated end tests are probably mainly due to nonagreement between the ACDT's, problems with the stability of the specimens or differences in the relative density.

The axial strain at the peak strength for the frictional ends was lower than for the lubricated ends in five of the six pairs. There was a greater maximum rate of dilation for the frictional compared to the lubricated ends. One explanation for this may be the decreased stability of the lubricated specimens as they tend to slide off the base pedestal.

The specimen end conditions did affect the deformed shape of the specimens. In some cases, the lubricated end platens created an "overlubricated"
condition where lateral spreading or flaring of the ends occurred. FRS46 is an example of such lateral flaring, as shown in Fig. 4.6. Specimens tested with frictional ends deformed laterally by bulging near the center. These were termed "underlubricated". FRS41 is an example of such an underlubricated specimen as shown in Fig. 5.49.

In most cases, the lubrication of the ends did significantly improve the deformed shape of the specimen at large strains. Figure 5.50 presents the final deformed shape of three specimens with lubricated ends that were strained axially to greater than 20%. These were all essentially right circular cylinders throughout the deformation. This means that smaller adjustments are made to the large strain data to account for nonuniform lateral straining and hence the large strain behavior is much more reliable.

5.5.2 Effect of Specimen Preparation Technique

Two techniques were used to prepare the frozen sand specimens in this research, wet tamping (WT) and multiple sieve pluviation (MSP). These were thoroughly explained in Section 4.2.1. Three series were performed to investigate the differences between the stress–strain response for the two preparation techniques. Table 5.12 presents a summary of the stress–strain parameters from these testing pairs. All three involved fairly dense specimens and moderate to high confining pressures. For most of the stress–strain parameters, the difference was close to or within that of the repeatability tests presented in Table 4.5. Two notable exceptions are those of the rate of work hardening after upper yield and the axial strain at peak strength. In all cases, the rate of work hardening for the specimens prepared by tamping was around twice that for the MSP specimens. The axial strain at the peak was always much less for the wet tamped specimens.
Figure 5.51 presents the stress–strain curves for these tests to illustrate the differences in the behavior. In general, wet tamping tends to obscure the upper yield behavior, i.e., it causes a much stiffer post upper yield response. It also tends to increase the amount of post peak strain softening.

5.5.3 Effect of Temperature

The effect of temperature on the behavior of frozen Manchester Fine Sand was inadvertently studied with one test. FRS50 was tested without temperature control. The actual temperature during shear was not measured, but assuming that it was near the mean value of that in the cold room, it was probably close to \(-15^\circ C\). Refer to Section 3.1.2. Table 5.13 compares the stress–strain parameters for FRS50 and FRS52 which were conducted under identical conditions except for temperature. Figure 5.52 plots the stress–strain curves for these two tests. As discussed in Section 2.2.2, a change in temperature can cause an effect similar to a change in strain rate for ice. Thus, those stress–strain parameters that are strongly influenced by strain rate should also exhibit a temperature sensitivity. Table 5.13 indicates that this is the case. The proportional limit, yield offset stress at 10\(^{-4}\) strain, upper yield stress, peak strength, axial strain at the beginning of dilation, and volumetric strain all exhibit a temperature sensitivity that is similar to the observed strain rate sensitivity.

5.6 SUMMARY OF THE MEASURED BEHAVIOR

Scope

Table 5.2 summarizes the scope of the testing program. The three most
important variables that were studied are the relative density, confining pressure and strain rate.

**Stress—Strain Curve Type**

Figure 5.1 presents several normalized stress—strain curve types that were measured in this research. Table 5.3 summarizes various combinations of testing variables that lead to each curve type. Low confining pressures lead to curve Type I through IV. Increases in relative density or decreases in strain rate shift the stress—strain response from Types I or II to Types III or IV. High confining pressures lead to Types V through VII. Increases in relative density or decreases in strain rate shift the curves to higher type numbers.

**Small Strain Behavior**

Figure 5.2 presents initial stress—strain curves (i.e., up to 0.03% axial strain) for several specimens tested over a wide range of testing conditions in order to illustrate the behaviors just presented. Table 5.5 summarizes the observed small strain behavior in terms of the trends, the pertinent figures that illustrate the trends and the results of statistical analyses of the data.

The Young’s modulus is independent of the relative density (Fig. 5.3), decreases slightly with confining pressure (Fig. 5.4), and is independent of strain rate (Fig. 5.5). The proportional limit and yield offset at 10⁻⁴ strain are both independent of relative density (Figs. 5.6 and 5.9, respectively), decrease slightly with confining pressure (Figs. 5.7 and 5.10, respectively) and increase with strain rate (Figs. 5.8 and 5.11).
Large Strain Behavior

The observed stress strain behavior in the upper yield region is summarized in Table 5.7 in terms of the trends, pertinent figures and empirical equations from regression analysis. Figure 5.12 presents stress–strain curves for several specimens to show the effect of changes in relative density (Fig. 5.12a), changes in confining pressure (Fig. 5.12b) and changes in strain rate (Fig. 5.12c).

Upper Yield Region

Table 5.7 summarizes the behavior in the upper yield region. The upper yield stress is independent of the relative density (Fig. 5.14), exhibits a slight decrease with confining pressure (Fig. 5.15) and is very dependent on the strain rate (Fig. 5.16). It exhibits a rate dependency very similar to that of polycrystalline ice. The axial strain at the upper yield stress is independent of the relative density (Fig. 5.17), independent of the confining pressure (Fig. 5.18) and increases with strain rate having a sensitivity similar to that of the upper yield stress. It is linearly proportional to the magnitude of the upper yield stress.

Work Hardening Region

Table 5.8 summarizes the behavior in the work hardening region. The rate of work hardening after the upper yield point increase linearly with relative density and exhibits a complex response as a function of confining pressure and strain rate. See Fig. 5.21 and 5.22. The axial strain at the beginning of work hardening decreases with increasing relative density but shows a complex behavior as a function of confining pressure and strain rate.
Peak Strength Region

Table 5.9 summarizes the behavior in the peak strength region. The peak strength increases linearly with relative density. This is illustrated in Figs. 5.33 through 5.36. It increases in a mostly nonlinear fashion with confining pressure (Fig. 5.37) and shows a power law dependency with strain rate (Fig. 5.38). The strain rate dependency is less than that for polycrystalline ice and decreases with relative density and confining pressure.

The axial strain at the peak deviator stress exhibits a complex behavior as a function of relative density, increases uniformly with confining pressure and shows a complex behavior as a function of strain rate. See Figs. 5.39 and 5.40.

Volumetric Behavior

Table 5.10 summarizes the observed trends in the volumetric behavior, refers to pertinent figures and presents the empirical equations obtained from linear regression analysis of the data.

The volumetric behavior of frozen sand is different than that of unfrozen sand in two basic aspects: 1) there is never a significant amount of volume decrease even at very high confining pressures where an unfrozen sand would experience significant volume decreases; 2) the maximum rate of dilation occurs at very large strains well beyond the peak strength for low confining pressure tests. See Figs. 5.26 through 5.32.

The rate of dilation at peak strength (Figs. 5.41 and 5.42), the maximum rate of dilation (Figs. 5.43 and 5.44 and the volumetric strain at 20% axial strain (Figs. 5.47 and 5.48) increased with relative density, decreased with confining pressure and showed very little effect of strain rate. The axial strain at the beginning of dilation showed a complex behavior with relative density,
increased with confining pressure and decreased with strain rate (Figs. 5.45 and 5.46).

The end conditions were shown to have little or no effect on most stress-strain parameters (Table 5.11). The axial strain at the peak strength decreased and the maximum rate of dilation increased for the frictional ends. The final deformed shape of the specimens was strongly affected by the end conditions. The lubricated specimens deformed either as right cylinders or with a slight amount of flaring at the ends while the specimens with frictional ends deformed by barreling (Fig. 4.6). The lubricated ends lead to more reliable large strain measures because of the uniform strain field and the lower amount of area correction required.

Most of the stress-strain parameters were unaffected by the specimen preparation technique. The wet tamped specimens, however, exhibited a significant increase in the rate of work hardening and a large decrease in the axial strain at peak strength (Fig. 5.50 and Table 5.12).

A decrease in the temperature led to a dramatic increase in the stress-strain parameters that exhibited the strongest strain rate sensitivity, i.e., the upper yield point and the peak strength (Fig. 5.51 and Table 5.13).
Table 5.1: Measured Strength and Deformation Properties (Page 1/2)
### Table 5.1: Measured Strength and Deformation Properties (page 2/2)

<table>
<thead>
<tr>
<th>Test</th>
<th>$D_\text{r}$</th>
<th>$\sigma_c$ ave</th>
<th>STRAIN RATE/ $\mathbf{\sigma_c}$ at $\mathbf{Y_i}_{\text{max}}$</th>
<th>INITIAL RESPONSE Pro Yi E $\mathbf{Y_i}_{\text{off}}$ Off</th>
<th>VOLUMETRIC S*</th>
<th>A*</th>
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<td>49</td>
<td>96.9</td>
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<tr>
<td>50</td>
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<td>10.03</td>
<td>18 212 28.0 1.9 3.3</td>
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<td>5.01</td>
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<td>28 335 24.0 2.0 3.0</td>
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<tr>
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<td>290 4990 26.3 2.6 5.0</td>
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<tr>
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<tr>
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<td>2.01</td>
<td>26 339 25.0 1.9 3.6</td>
<td>8.0 7.9</td>
<td>4.2</td>
<td>4.2</td>
</tr>
</tbody>
</table>

* Test temperature $\approx -15^\circ$C  
$\mathbf{Y_i}_{\text{Off}}$ = Yield Offset at $10^{-4}$ Strain  
Pro Lim = Proportional Limit  
$Q_{\text{cor}}$ = Corrected Upper Yield Stress  
$\epsilon_a$ = Axial Strain at the Beginning of Dilation  
$\epsilon_v$ = Volumetric Strain at 20% Axial Strain  
$S^*$ = Stability Qualifiers  
$A^*$ = ACDT Qualifiers

The table above provides a comprehensive overview of measured strength and deformation properties, including strain rates, initial and upper strain peaks, volumetric behaviors, and stability and ACDT qualifiers. Each entry reflects critical characteristics such as yield offset, proportional limit, corrected upper yield stress, and various strain measurements, complemented by test conditions and temperatures.
Table 5.2: Scope of Testing Program on the Triaxial Compression Behavior of Frozen MFS

<table>
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<tr>
<th>RANGE IN RELATIVE DENSITY (%)</th>
<th>CONFINING PRESSURE (MPa)</th>
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<td>Slow</td>
<td>Fast</td>
<td>Mod.</td>
<td>Slow</td>
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<tr>
<td>20-40</td>
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<td>▶</td>
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<tr>
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<tr>
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</table>

**STRAIN RATES TESTED**
- Fast ≈ \(4 \times 10^{-4}/\text{sec}\)
- Moderate ≈ \(3 \times 10^{-5}/\text{sec}\)
- Slow ≈ \(3 \times 10^{-6}/\text{sec}\)
- One specimen tested at \(1.2 \times 10^{-4}/\text{sec}\)

**TESTING TEMPERATURE**
- \(-9.55 \pm 0.3^\circ\text{C}\)
- One test at \(-15^\circ\text{C}\)

**SPECIMEN PREPARATION TECHNIQUES**
- Most tests prepared by Multiple Sieve Pluviation
- Seven tests prepared by Wet Tamping

**SPECIMEN END CONDITIONS**
- Most tests had ice cap and grease ends (low friction)
- Six tests had frozen sand and grease (higher friction)
- One test had emery cloth
Table 5.3: Combinations of Variables That Lead to Stress–Strain Types

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<thead>
<tr>
<th>CURVE TYPE</th>
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<td>M</td>
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<td>VD</td>
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<tr>
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<td>INT.</td>
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<tr>
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<tr>
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<tr>
<td>VII</td>
<td>FAST</td>
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<tr>
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<tr>
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<td>SLOW</td>
<td></td>
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</table>

Relative Density:

- L (loose 15-35%)
- M (medium 35-65%)
- D (dense 65-85%)
- VD (very dense 85-100%)

Strain Rate:

- FAST (4x10⁻⁴/sec)
- INT. (1x10⁻⁴/sec)
- MOD. (3x10⁻⁵/sec)
- SLOW (3x10⁻⁶/sec)
# Table 5.4: Statistical Analysis of Small Strain Data

<table>
<thead>
<tr>
<th>Stress Strain Quantity</th>
<th>Strain Rate</th>
<th>Confining Pressure (MPa)</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
<th># of Observations</th>
<th>Comments</th>
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<td><strong>Yield Offset Stress at $10^{-4}$ Strain (MPa)</strong></td>
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<td>5.1</td>
<td>0.1</td>
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## Table 5.5: Summary of Small Strain Behavior

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<td>Figure</td>
<td>5.2(a), 5.3</td>
<td>5.2(b), 5.4</td>
<td>5.2(c) 5.5</td>
</tr>
<tr>
<td>Empirical Equations</td>
<td>- All ( i, \sigma_c ) and ( D ) -</td>
<td>- moderate strain rate-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mean = 26.6( \pm ) 4.5 GPa</td>
<td>( E(\text{GPa}) = 28.5 - 0.454\sigma_c(\text{MPa}) )</td>
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<tr>
<td></td>
<td></td>
<td>( r^2 = 0.22 )</td>
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<tr>
<td>Proportional Limit</td>
<td></td>
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<tr>
<td>Trend</td>
<td>None</td>
<td>Decrease</td>
<td>Increase</td>
</tr>
<tr>
<td>Figure</td>
<td>5.2(a) 5.6</td>
<td>5.2(b) 5.4</td>
<td>5.2(c) 5.8</td>
</tr>
<tr>
<td>Empirical Equations</td>
<td>- moderate strain rate-</td>
<td>- ( \sigma_c = 0.1 \text{ MPa} - )</td>
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<tr>
<td></td>
<td>P.L. (MPa) = 2.3 - 0.067 ( \sigma_c ) (MPa)</td>
<td>( \text{PL(\text{MPa})=5.33 (i)0.044} )</td>
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<tr>
<td></td>
<td>( r^2 = 0.37 )</td>
<td>( r^2 = 0.19 ) (( n=15.6 ))</td>
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<tr>
<td></td>
<td></td>
<td>(-\sigma_c = 10 \text{ MPa} -)</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>( \text{PL(\text{MPa})=3.45 (i)0.044} )</td>
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<tr>
<td></td>
<td></td>
<td>( r^2 = 0.18 ) (( n=17.9 ) )</td>
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<tr>
<td>Yield Offset Stress at 10(^{-4}) Strain</td>
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<td></td>
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</tr>
<tr>
<td>Trend</td>
<td>None</td>
<td>Decrease</td>
<td>Increase</td>
</tr>
<tr>
<td>Figure</td>
<td>5.2(a) 5.9</td>
<td>5.2(b) 5.10</td>
<td>5.2(c) 5.11</td>
</tr>
<tr>
<td>Empirical Equations</td>
<td>- mod. ( i, \sigma_c = 0.1 \text{ MPa} -)</td>
<td>- Moderate strain rate-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mean = 4.0( \pm ) 0.5 MPa</td>
<td>( YO(\text{MPa})=3.9 - 0.095\sigma_c(\text{MPa}) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r^2 = 0.58 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- mod. ( i, \sigma_c = 10 \text{ MPa} -)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>mean = 3.0( \pm ) 0.2 MPa</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(-\sigma_c = 10 \text{ MPa} -)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>( YO(\text{MPa})=9.96 (i)0.044 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( r^2 = 0.66 ) (( n=11.2 ) )</td>
</tr>
</tbody>
</table>
Table 5.6: Statistical Analysis of Upper Yield Region Data

<table>
<thead>
<tr>
<th>STRESS-STRAIN QUANTITY</th>
<th>STRAIN RATE</th>
<th>CONFINING PRESSURE (MPa)</th>
<th>MEAN VALUE (MPa)</th>
<th>STANDARD DEVIATION</th>
<th># OF OBSERVATIONS</th>
<th>COMMENTS</th>
</tr>
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<tbody>
<tr>
<td>Corrected Upper Yield Stress</td>
<td>Slow</td>
<td>0.1</td>
<td>5.1</td>
<td>0.1</td>
<td>3</td>
<td>-Over All Confining Pressures- mean = 5.0 ± 0.1 MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.0</td>
<td>4.9</td>
<td>0.13</td>
<td>4</td>
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</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>0.1</td>
<td>8.38</td>
<td>0.27</td>
<td>12</td>
<td>-Over All Confining Pressures- mean = 8.1 ± 0.5 MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.0</td>
<td>8.0</td>
<td>0.76</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.0</td>
<td>8.18</td>
<td>0.54</td>
<td>4</td>
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<tr>
<td></td>
<td></td>
<td>10.0</td>
<td>7.75</td>
<td>0.42</td>
<td>8</td>
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<tr>
<td>Fast</td>
<td></td>
<td>0.1</td>
<td>15.4</td>
<td>0.48</td>
<td>4</td>
<td>-Over All Confining Pressures- mean = 15.3 ± 0.3 MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.0</td>
<td>15.2</td>
<td>0.1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Axial Strain at Upper Yield Stress</td>
<td>Slow</td>
<td>0.1</td>
<td>0.0026</td>
<td>0</td>
<td>1</td>
<td>-Over All Confining Pressures- mean = 0.0035 ± 0.0006</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.0</td>
<td>0.0037</td>
<td>0.0005</td>
<td>3</td>
<td></td>
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<tr>
<td></td>
<td>Moderate</td>
<td>0.1</td>
<td>0.0047</td>
<td>0.0008</td>
<td>7</td>
<td>-Over All Confining Pressures- mean = 0.0046 ± 0.0006</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.0</td>
<td>0.0044</td>
<td>0.0003</td>
<td>3</td>
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<td></td>
<td></td>
<td>5.0</td>
<td>0.0046</td>
<td>0.0003</td>
<td>4</td>
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<td>10.0</td>
<td>0.0047</td>
<td>0.0003</td>
<td>5</td>
<td></td>
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<tr>
<td>Fast</td>
<td></td>
<td>0.1</td>
<td>0.0080</td>
<td>0.0004</td>
<td>3</td>
<td>-Over All Confining Pressures- mean = 0.0081 ± 0.0004</td>
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<td>10.0</td>
<td>0.0047</td>
<td>0.0001</td>
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### Table 5.7: Summary of Upper Yield Behavior

<table>
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<tr>
<th>PARAMETER</th>
<th>EFFECT OF INCREASING</th>
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<tbody>
<tr>
<td></td>
<td>RELATIVE DENSITY</td>
</tr>
<tr>
<td>Upper Yield Stress</td>
<td>Trend</td>
</tr>
<tr>
<td>Figure</td>
<td>5.12(a) 5.14</td>
</tr>
<tr>
<td>Empirical Equations</td>
<td>- Fast ( \dot{\epsilon} ), all ( \sigma_c ) &amp; ( D_r )-mean = 15.3 ± 0.4 MPa</td>
</tr>
<tr>
<td></td>
<td>- Mod ( \dot{\epsilon} ), all ( \sigma_c ) &amp; ( D_r )-mean = 8.1 ± 0.5 MPa</td>
</tr>
<tr>
<td></td>
<td>- Slow ( \dot{\epsilon} ), all ( \sigma_c ) &amp; ( D_r )-mean = 5.0 ± 0.1 MPa</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Strain at Upper Yield Stress</td>
<td>Trend</td>
</tr>
<tr>
<td>Figure</td>
<td>5.17</td>
</tr>
<tr>
<td>Empirical Equations</td>
<td>- Fast ( \dot{\epsilon} ), all ( \sigma_c ) &amp; ( D_r )-mean = 0.81 ± 0.04%</td>
</tr>
<tr>
<td></td>
<td>- Mod ( \dot{\epsilon} ), all ( \sigma_c ) &amp; ( D_r )-mean = 0.46 ± 0.06%</td>
</tr>
<tr>
<td></td>
<td>- Slow ( \dot{\epsilon} ), all ( \sigma_c ) &amp; ( D_r )-mean = 0.35 ± 0.07%</td>
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### Table 5.8: Summary of Work Hardening Behavior

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>EFFECT OF INCREASING</th>
<th>RELATIVE DENSITY</th>
<th>CONFINING PRESSURE</th>
<th>STRAIN RATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Work Hardening</td>
<td>Trend</td>
<td>Figure</td>
<td>Figure</td>
<td>Figure</td>
</tr>
<tr>
<td></td>
<td>Increase</td>
<td>5.21, 5.22, 5.20</td>
<td>Complex</td>
<td>5.21, 5.22</td>
</tr>
<tr>
<td></td>
<td>Empirical Equations</td>
<td>- Moderate ( \dot{\varepsilon} ), All ( \sigma_c )( \frac{d\sigma}{d\varepsilon_a} ) (MPa) = 31.3 + 2.2Dr(%)</td>
<td>- For Moderate ( \dot{\varepsilon} ) - No Effect</td>
<td>- ( \sigma_c = 0.1 )MPa, ( D_r = 95% ) - Slow to mod. ( \dot{\varepsilon} ) \rightarrow Increase</td>
</tr>
<tr>
<td></td>
<td>and Comments</td>
<td>( r^2 = 0.91 \pm 15.4 )</td>
<td>- For Slow and Fast ( \dot{\varepsilon} ) - Increase</td>
<td>Mod ( \dot{\varepsilon} ) to fast ( \dot{\varepsilon} ) \rightarrow Decrease</td>
</tr>
<tr>
<td>Axial Strain at Beginning of Work Hardening</td>
<td>Trend</td>
<td>Figure</td>
<td>Figure</td>
<td>Figure</td>
</tr>
<tr>
<td></td>
<td>Decrease</td>
<td>5.24, 5.25, 5.20</td>
<td>Complex</td>
<td>5.24, 5.25</td>
</tr>
<tr>
<td></td>
<td>Empirical Equations</td>
<td>- ( \sigma_c = 0.1 ), mod ( \dot{\varepsilon} )( \varepsilon_a(%) ) = 1.43 - 1.32x10(^{-2})Dr(%)</td>
<td>- Slow and mod ( \dot{\varepsilon} ) \rightarrow Increase</td>
<td>- ( D_r = 35% ), ( \sigma_c = 10 )MPa - Slow-mod. No change (within Scatter band)</td>
</tr>
<tr>
<td></td>
<td>and Comments</td>
<td>( r^2 = 0.57 )</td>
<td>- Fast ( \dot{\varepsilon} ) low ( D_r ) \rightarrow Increase</td>
<td>Mod-fast Increase</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- ( \sigma_c = 0.1 ), mod ( \dot{\varepsilon} )( \varepsilon_a(%) ) = 1.61 - 9.56 x 10(^{-3})Dr(%)</td>
<td>- Fast ( \dot{\varepsilon} ) high ( D_r ) \rightarrow Decrease</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r^2 = 0.62 \pm 0.25 )</td>
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Table 5.9: Summary of Peak Strength Behavior (page 1/3)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>EFFECT OF INCREASING</th>
<th>RELATIVE DENSITY</th>
<th>CONFINING PRESSURE</th>
<th>STRAIN RATE</th>
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</thead>
<tbody>
<tr>
<td>$Q_p - Q_{uy}$</td>
<td>Trend</td>
<td>Increase</td>
<td>Increase</td>
<td>Complex</td>
</tr>
<tr>
<td>Figure</td>
<td></td>
<td>5.26, 5.27, 5.33</td>
<td>5.28, 5.29, 5.3</td>
<td>5.30, 5.31, 5.32, 5.34</td>
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<tr>
<td>Empirical</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equations</td>
<td>- Moderate $\dot{\varepsilon}$ -</td>
<td></td>
<td>See Discussion of $Q_p$</td>
<td></td>
</tr>
<tr>
<td>and Comments</td>
<td>$\sigma_c = 10$ MPa</td>
<td>$Q_p - Q_{uy}$ (MPa) = 0.74 + 0.109$D_r$ (%)</td>
<td></td>
<td>Slow to mod $\dot{\varepsilon}$ all $\sigma_c$</td>
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<tr>
<td></td>
<td></td>
<td>$r^2 = 0.99$</td>
<td></td>
<td>No change</td>
</tr>
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</tr>
<tr>
<td></td>
<td>$\sigma_c = 5$ MPa</td>
<td>$Q_p - Q_{uy}$ (MPa) = 0.03 + 0.087$D_r$ (%)</td>
<td></td>
<td>Mod to fast $\dot{\varepsilon}$ all $\sigma_c$</td>
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<tr>
<td></td>
<td></td>
<td>$r^2 = 0.99$</td>
<td></td>
<td>Decrease</td>
</tr>
<tr>
<td></td>
<td>$\sigma_c = 2$ MPa</td>
<td>$Q_p - Q_{uy}$ (MPa) = 0.057$D_r$ (%)</td>
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<td></td>
<td></td>
<td>$r^2 = 0.83$</td>
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<tr>
<td></td>
<td>$\sigma_c = 0.1$ MPa</td>
<td>$Q_p - Q_{uy}$ (MPa) = 0.95 + 0.049$D_r$ (%)</td>
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<tr>
<td></td>
<td></td>
<td>$r^2 = 0.95$</td>
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Table 5.9: Summary of Peak Strength Behavior (page 2/3)

<table>
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<th>PARAMETER</th>
<th>RELATIVE DENSITY</th>
<th>CONFINING PRESSURE</th>
<th>STRAIN RATE</th>
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</thead>
<tbody>
<tr>
<td>Peak Deviator Stress ($Q_p$)</td>
<td>EFFECT OF INCREASING</td>
<td>EFFECT OF INCREASING</td>
<td>EFFECT OF INCREASING</td>
</tr>
<tr>
<td>Trend</td>
<td>Figure</td>
<td>Increase</td>
<td>Increase</td>
</tr>
<tr>
<td>Relative Density</td>
<td></td>
<td>$Q_p$ (MPa)</td>
<td>$Q_p$ (MPa)</td>
</tr>
<tr>
<td>Empirical Equations and Comments</td>
<td>- Moderate $\dot{\varepsilon}$</td>
<td>$\sigma_c = 10$ MPa</td>
<td>$\sigma_c = 2$ MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Q_p$ (MPa) = 7.36 + 0.128 $D_r$ (%)</td>
<td>$Q_p$ (MPa) = 8.55 + 0.082 $D_r$ (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r^2 = 0.98$</td>
<td>$r^2 = 0.98$</td>
</tr>
<tr>
<td></td>
<td>- Slow $\dot{\varepsilon}$</td>
<td>$\sigma_c = 0.1$ MPa</td>
<td>$\sigma_c = 0.1$ MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Q_p$ (MPa) = 4.98 + 0.042 $D_r$ (%)</td>
<td>$Q_p$ (MPa) = 4.40 + 0.122 $D_r$ (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r^2 = 0.93$</td>
<td>$r^2 = 0.93$</td>
</tr>
<tr>
<td></td>
<td>- Fast $\dot{\varepsilon}$</td>
<td>$\sigma_c = 0.1$ MPa</td>
<td>$\sigma_c = 0.1$ MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Assume $Q_p = Q_{uv}$ to 75% $D_r$ thereafter same slope as moderate $\dot{\varepsilon}$</td>
<td>$\sigma_c = 10$ MPa</td>
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<tr>
<td></td>
<td></td>
<td>$Q_p = 11.25 + 0.111 D_r$ (%)</td>
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</table>
Table 5.9: Summary of Peak Strength Behavior (page 3/3)

<table>
<thead>
<tr>
<th>PARAMETER</th>
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<tbody>
<tr>
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<td>RELATIVE DENSITY</td>
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<tr>
<td>Axial Strain at Peak Deviator Stress</td>
<td>Complex</td>
</tr>
<tr>
<td>Trend</td>
<td>Figures</td>
</tr>
<tr>
<td>Empirical Equations and Comments</td>
<td>- Mod $\dot{\varepsilon}$, $\sigma_c = 0.1$ MPa-</td>
</tr>
<tr>
<td></td>
<td>No Effect</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_a = 5.2 \pm 0.9%$</td>
</tr>
<tr>
<td></td>
<td>$-\text{mod. } \dot{\varepsilon}, \sigma_c = 2.0 - 0.10$ MPa-</td>
</tr>
<tr>
<td></td>
<td>Slow $\dot{\varepsilon}$ all $\sigma_c$ and</td>
</tr>
<tr>
<td></td>
<td>Fast $\dot{\varepsilon}, \sigma_c = 10$ MPa</td>
</tr>
<tr>
<td></td>
<td>Linear Decrease</td>
</tr>
<tr>
<td></td>
<td>$-\text{mod. } \dot{\varepsilon}, \sigma_c = 10$ MPa-</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_a(%) = 31.3 - 1.09 \times 10^{-3}D_r(%)$</td>
</tr>
<tr>
<td></td>
<td>$r^2 = 0.69$</td>
</tr>
<tr>
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<td>- Fast $\dot{\varepsilon}, \sigma_c = 0.1$ MPa-</td>
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<td>Increase</td>
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</table>
Table 5.10: Summary of Volumetric Behavior (page 1/4)

| PARAMETER | EFFECT OF INCREASING
|------------|------------------------
<p>|            | RELATIVE DENSITY | CONFINING PRESSURE | STRAIN RATE |
| Rate of Dilation at Peak Strength | Increase | Increase | Complex |
| Trend | 5.41, 5.42, 5.26, 5.27 | 5.41, 5.42, 5.28, 5.29 | 5.42, 5.30, 5.31, 5.32 |
| Figure | | | |
| Empirical Equations and Comments | - Moderate ( \dot{\varepsilon} ), ( \sigma_c = 0.1 \text{ MPa} ) ( \frac{d\varepsilon_y}{d\varepsilon_a} = 0.07 + 1.9 \times 10^{-3} \dot{\varepsilon}_r ) (%) ( r^2 = 0.79 ) | For All Strain Rates and relative densities | - ( \sigma_c = 0.1 \text{ MPa}, \text{ low } D_r ) - No effect (Slow and fast data within scatterband of moderate data) |
| | - Moderate ( \dot{\varepsilon} ), ( \sigma_c = 10 \text{ MPa} ) ( \frac{d\varepsilon_y}{d\varepsilon_a} = 0.04 + 1.1 \times 10^{-3} \dot{\varepsilon}_r ) (%) ( r^2 = 0.65 ) | | - ( \sigma_c = 0.1 \text{ MPa}, \text{ high } D_r ) - Slow to moderate Decrease Moderate to fast Little or no change |
| | | - ( \sigma_c = 10 \text{ MPa}, \text{ all } D_r ) - Slow to moderate No effect Moderate to fast Increase | |</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Maximum Rate of Dilation</th>
<th>Figure</th>
<th>RELATIVE DENSITY</th>
<th>Confining Pressure</th>
<th>Strain Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend</td>
<td>Empirical Equations and Comments</td>
<td>dεy/dεa = 0.16 + 4.5 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 0.08 + 2.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 1.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 0.04 + 1.1 × 10^{-3} D_r (%)</td>
</tr>
<tr>
<td></td>
<td>dεy/dεa = 0.16 + 4.5 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 0.08 + 2.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 1.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 0.04 + 1.1 × 10^{-3} D_r (%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>dεy/dεa = 0.16 + 4.5 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 0.08 + 2.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 1.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 0.04 + 1.1 × 10^{-3} D_r (%)</td>
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</tr>
<tr>
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<td>dεy/dεa = 0.08 + 2.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 1.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 0.04 + 1.1 × 10^{-3} D_r (%)</td>
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<td>dεy/dεa = 0.04 + 1.1 × 10^{-3} D_r (%)</td>
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<td>dεy/dεa = 0.08 + 2.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 1.2 × 10^{-3} D_r (%)</td>
<td>dεy/dεa = 0.04 + 1.1 × 10^{-3} D_r (%)</td>
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Table 5.10: Summary of Volumetric Behavior (page 3/4)

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<tr>
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<td>Complex</td>
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<tr>
<td>Axial Strain</td>
<td>Trend</td>
</tr>
<tr>
<td>at the Beginning of Dilation</td>
<td>Empirical Equations and Comments</td>
</tr>
<tr>
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<td>$-\sigma_c = 10$ MPa, all $\dot{\varepsilon}$ - Tendency to Decrease (Large scatter in data)</td>
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Table 5.10: Summary of Volumetric Behavior (page 4/4)

<table>
<thead>
<tr>
<th>PARAMETER</th>
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<th>RELATIVE DENSITY</th>
<th>CONFINING PRESSURE</th>
<th>STRAIN RATE</th>
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<td>Volumetric Strain at 20% Axial Strain</td>
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<td>Trend Figure</td>
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<td>Increase</td>
<td>Decrease</td>
<td>Increases</td>
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<tr>
<td>Empirical Equations and Comments</td>
<td></td>
<td>5.48, 5.49, 5.26, 5.27</td>
<td>5.48, 5.49, 5.28, 5.29</td>
<td>5.49, 5.30, 5.31, 5.32</td>
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<tr>
<td></td>
<td>Uniform over all (\dot{\varepsilon}) and (\sigma_c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Moderate (\dot{\varepsilon}) (\sigma_c = 0.1) MPa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\epsilon_v(%) = 1.5 + 7.0 \times 10^{-2}D_r(%))</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(r^2 = 0.94)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(\sigma_c = 2.0) MPa</td>
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<tr>
<td></td>
<td>(\epsilon_v(%) = 1.1 + 2.4 \times 10^{-2}D_r(%))</td>
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</tr>
<tr>
<td></td>
<td>(r^2 = 0.75)</td>
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<td></td>
<td>(\sigma_c = 5.0) MPa</td>
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<tr>
<td></td>
<td>(\epsilon_v(%) = 0.4 + 2.1 \times 10^{-2}D_r(%))</td>
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<tr>
<td></td>
<td>(r^2 = 0.84)</td>
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<tr>
<td></td>
<td>(\sigma_c = 10) MPa</td>
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<tr>
<td></td>
<td>(\epsilon_v(%) = -0.3 + 0.97 \times 10^{-3}D_r(%))</td>
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<tr>
<td></td>
<td>(r^2 = 0.47)</td>
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</table>

\(- \sigma_c = 0.1\) MPa low \(D_r\) - Slow to fast Increase
\(- \sigma_c = 0.1\) MPa high \(D_r\) - Slow to moderate No change Moderate to fast Increase
\(- \sigma_c = 10\) MPa - Slow to moderate No change Moderate to fast Increase
Table 5.11: Effect of End Conditions on Stress–Strain Behavior

<table>
<thead>
<tr>
<th>TEST CONDITIONS</th>
<th>TEST#</th>
<th>D_r (%)</th>
<th>END COND.</th>
<th>α PARAMETER</th>
<th>UPPER YIELD</th>
<th>WORK HARD.</th>
<th>PEAK</th>
<th>dε_x</th>
<th>dε_n @ 20%</th>
<th>ε_y</th>
<th>STAD</th>
<th>ACDTS</th>
<th>COMMENTS</th>
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<tr>
<td>Moderate ć</td>
<td>52</td>
<td>90.8</td>
<td>I</td>
<td>0.96</td>
<td>8.2</td>
<td>4.8</td>
<td>158</td>
<td>19.0</td>
<td>0.06</td>
<td>0.7</td>
<td>f</td>
<td>p</td>
<td></td>
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<td>0.8</td>
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<td>8.3</td>
<td>54</td>
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<td>9.5</td>
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<td>g</td>
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<td>92.8</td>
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<td>16.1</td>
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<td>21.8</td>
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<td>8.4</td>
<td>5.0</td>
<td>12</td>
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<td>23.0</td>
<td>E</td>
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<td>5.3</td>
<td>278</td>
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<td>0.31</td>
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<td>g</td>
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* Final area not measured
Table 5.12: Effect of Specimen Preparation Technique on the Stress–Strain Behavior

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<tr>
<th>FRS Test #</th>
<th>D_r %</th>
<th>σ_c ave MPa</th>
<th>Prep Tech</th>
<th>STRAIN RATE /sec @ Yi max</th>
<th>INITIAL RESPONSE</th>
<th>UPPER YIELD</th>
<th>STRAIN H/S</th>
<th>PEAK STRENGTH</th>
<th>VOLUMETRIC BEHAVIOR</th>
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<td>20</td>
<td>76.3</td>
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<td>VT</td>
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<td>82.7</td>
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<td>MSP</td>
<td>23 338 24.3 2.2 3.3 7.9 7.8 4.8 10.4 127</td>
<td>2 2 103 2</td>
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<td>24</td>
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<td>5.12</td>
<td>VT</td>
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Note: All specimens tested with lubricated ends
Table 5.13: Effect of Temperature on Stress–Strain Behavior

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<th>FRS Test #</th>
<th>Dr</th>
<th>$\sigma_{e ave}$ MPa</th>
<th>TEMP °C</th>
<th>STRAIN RATE /sec</th>
<th>INITIAL RESPONSE</th>
<th>UPPER YIELD</th>
<th>STRAIN H/S</th>
<th>PEAK STRENGTH</th>
<th>VOLUMETRIC BEHAVIOR</th>
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<td>Pro Yi</td>
<td>Q$_{uy}$</td>
<td>Q$_{cor}$</td>
<td>dQ</td>
<td>Q$_{p}$ Max$\epsilon_a$</td>
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<td></td>
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<td>E Lim Off MPa</td>
<td>MPa</td>
<td>MPa x10$^3$</td>
<td>d$\epsilon_a$ MPa</td>
<td>x10$^2$</td>
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<td>-15*</td>
<td>18</td>
<td>212</td>
<td>28.0</td>
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* Estimated temperature
a. Low Confining Pressure

b. High Confining Pressure

Figure 5.1: Overview of Types of Stress–Strain Curves
a. Various Relative Densities

![Graph showing stress-strain curves for different relative densities](image)

- **FRS64**: $D_r = 82.7\%$
- **FRS63**: $D_r = 51.1\%$
- **FRS62**: $D_r = 90.8\%$
- **FRS15**: $D_r = 31.6\%$

$t = $Moderate

$\sigma_c = 10$ MPa

**Figure 5.2:** Stress–Strain Curves Showing Small Strain Behavior (page 1/3)
b. Various Confining Pressures

\[ P_{\text{c}} = \begin{align*}
0.1 & \text{ MPa} \\
2.0 & \text{ MPa} \\
5.0 & \text{ MPa} \\
10 & \text{ MPa}
\end{align*} \]

\[ \dot{\varepsilon} = \text{Moderate} \]

\[ D_r = 35\% \]

Figure 5.2: Stress–Strain Curves Showing Small Strain Behavior (page 2/3)
c. Various Strain Rates

\[ \varepsilon = 5 \times 10^{-4} / \text{sec} \]

\[ \varepsilon = 3 \times 10^{-5} / \text{sec} \]

\[ \varepsilon = 3 \times 10^{-4} / \text{sec} \]

\[ \varepsilon = 3 \times 10^{-3} / \text{sec} \]

Deviator Stress (MPa)

Figure 5.2: Stress–Strain Curves Showing Small Strain Behavior (page 3/3)

\[ D_r = 35\% \]

\[ \sigma_c = 0.1 \text{ MPa} \]
Figure 5.3: Young's Modulus Versus Relative Density

MSP and Wet Tamp, gf ACDT's lub and fric ends
Young's Modulus (GPa)

Moderate Strain Rate
MSP and Wet Tamp, gf ACDT's
Lub and Fric Ends

Figure 5.4: Effect of Confining on Young's Modulus

\[ E(\text{GPa}) = 28.5 - 0.45\sigma_c(\text{MPa}) \]

\[ r^2 = 0.22 \]
Figure 5.5: Effect of Strain Rate on Young's Modulus

Mean = 23.8 ± 2.5 GPa

10 MPa Confining Pressure
MSP and Wet Tamp, gf ACDT’s
Lub and Fric Ends
Figure 5.6: Proportional Limit Versus Relative Density

LEGEND

Cell Pressure and Strain Rate

- 0.1 MPa Moderate
+ 2.0 MPa Moderate
* 5.0 MPa Moderate
□ 10.0 MPa Moderate
× 0.1 MPa Slow
◊ 10.0 MPa Slow
Δ 0.1 MPa Fast
× 10.0 MPa Fast

MSP and Wet Tamp, gf AGDT's
Lub and Fric ends
Proportional Limit (MPa)

\[ PL(MPa) = 2.3 - 0.67\sigma_c(MPa) \]

\[ r^2 = 0.37 \]

Figure 5.7: Effect of Confining Pressure on Proportional Limit
Figure 5.8: Effect of Strain Rate on Proportional Limit

Proportional Limit (MPa)

Nominal Strain Rate (/sec)

LEGEND
Cell Pressure

× 0.1 MPa
□ 10.0 MPa

MSP and Wet Tamp, gf ACDT's
Lub and Fric Ends

PL(MPa) = 5.33(\epsilon)^{0.044}

r^2 = 0.19

n=15.6

PL(MPa) = 3.45(\epsilon)^{0.056}

r^2 = 0.18

n=17.8
Figure 5.9: Yield Offset Stress at $10^{-4}$ Strain Versus Relative Density
Yield Offset Stress (MPa)

Confining Pressure (MPa)

$Q_{yo}(\text{MPa}) = 3.9 - 0.095\sigma_c(\text{MPa})$

$r^2 = 0.58$

Figure 5.10: Effect of Confining Pressure on Yield Offset Stress

Moderate Strain Rate
MSP and Wet Tamp, gf AC DT’s
Lub and Fric Ends
Figure 5.11: Effect of Strain Rate on Yield Offset Stress
a. Various Relative Densities

Figure 5.12: Stress–Strain Curves Showing Upper Yield Behavior (page 1/3)
b. Various Confining Pressures

Figure 5.12: Stress–Strain Curves Showing Upper Yield Behavior (page 2/3)
c. Various Strain Rates

Figure 5.12: Stress–Strain Curves Showing Upper Yield Behavior (page 3/3)
Figure 5.13: Measured Variation in Upper Yield Stress With Height of Specimen
Figure 5.14: Corrected Upper Yield Stress Versus Relative Density

**Legend**

- • 0.1 MPa Moderate
- † 2.0 MPa Moderate
- ★ 5.0 MPa Moderate
- □ 10.0 MPa Moderate
- × 0.1 MPa Slow
- ◇ 10.0 MPa Slow
- △ 0.1 MPa Fast
- ☆ 10.0 MPa Fast

Upper Yield Stress (MPa)

- Mean = 15.3 ± 0.4 MPa
- Mean = 8.1 ± 0.5 MPa
- Mean = 5.0 ± 0.1 MPa

MSP, gfpvp Stability, gfp ADCT's
Lub and Fric Ends
Corrected Upper Yield Stress (MPa)

\[ Q_{uy}(\text{MPa}) = 8.3 - 0.059\sigma_c(\text{MPa}) \]
\[ r^2 = 0.26 \]

Moderate Strain Rate
MSP, gpvvp Stability, gpv ACDT's
Lub and Fric Ends

Figure 5.15: Effect of Confining Pressure on Correct Upper Yield Stress
Upper Yield Stress (MPa)

$\sigma_c = 0.1 \text{ MPa}$
$Q_{uy}(\text{MPa}) = 77.8(1)^{0.214}$
$r^2 = 0.98$

$n = 4.7$
$n = 4.6$

$\sigma_c = 10 \text{ MPa}$
$Q_{uy}(\text{MPa}) = 77.3(1)^{0.214}$
$r^2 = 0.90$

LEGEND

Cell Pressure

+ 0.1 MPa
□ 10 MPa

MSP, gpvvp Stability, gpv ACDT’s
Lub and Fric Ends

Figure 5.16: Effect of Strain Rate on Upper Yield Stress
Axial Strain at Upper Yield Stress Versus Relative Density

**LEGEND**

- □ 0.1 MPa Slow
- * 10.0 MPa Slow
- × 0.1 MPa Fast
- ♦ 10.0 MPa Fast
- • 0.1 MPa Moderate
- ± 2.0 MPa Moderate
- ○ 5.0 MPa Moderate
- □ 10.0 MPa Moderate

**Figure 5.17:** Axial Strain at Upper Yield Stress Versus Relative Density

- Fast $\varepsilon$, Mean = 0.8 ± 0.04
- Moderate $\varepsilon$, Mean = 0.46 ± 0.06
- Slow $\varepsilon$, Mean = 0.35 ± 0.07

MSP, gfvp, Stability, gf ACĐT's
Lub and Fric Ends
MSP, gfvp Stability, gf ACDT's
Lub and Fric Ends

Figure 5.18: Effect of Confining Pressure on the Axial Strain at the Upper Yield Stress
Figure 5.19: Effect of Strain Rate on the Axial Strain at the Upper Yield Stress

All Confining Pressures and Dr MSP, gf,pvp Stability, gf ACDT's Lub and Fric Ends

\[ \varepsilon_a(\%) = 3.07(1)^{0.181} \]

\[ r^2 = 0.78 \]

\( n = 5.5 \)
Figure 5.20: Stress–Strain Curves to Show Effect of Relative Density on the Rate of Work Hardening
Rate of Strain Hardening (MPa)

\[ \frac{dQ}{dt_d} (\text{MPa}) = -31.3 + 2.2D_s(\%) \]

\[ r^2 = 0.92 \]

LEGEND

Cell Pressure

- 0.1 MPa
- 2.0 MPa
- 5.0 MPa
- 10.0 MPa

MSP, lub and fric, gfp stab, gf ACDT

Figure 5.21: Rate of Work Hardening Versus Relative Density for a Moderate Strain Rate
Figure 5.22: Rate of Work Hardening Versus Relative Density for Range in Strain Rates

LEGEND
Cell Pressure and Strain Rate
- X 0.1 MPa Slow
- O 10.0 MPa Slow
- △ 0.1 MPa Fast
- X 10.0 MPa Fast

MSP, lub and fric, gfp stab, gf ACDT
Figure 5.23: Effect of Strain Rate on the Rate of Work Hardening
Figure 5.24: Axial Strain at Beginning of Work Hardening Versus Relative Density for Moderate Strain Rate
Figure 5.25: Axial Strain for Beginning of Work Hardening Versus Relative Density for Range in Strain Rates
Figure 5.26: Normalized Stress–Strain Curves Showing the Effect of Relative Density for Moderate Strain Rate and Low Confining Pressure
Figure 5.27: Normalized Stress–Strain Curves Showing the Effect of Relative Density for Moderate Strain Rate and High Confining Pressure
Figure 5.28: Normalized Stress–Strain Curves Showing the Effect of Confining Pressure for Low Relative Density and Moderate Strain Rate
Figure 5.29: Normalized Stress–Strain Curves Showing the Effect of Confining Pressure for High Relative Density and Moderate Strain Rate
Figure 5.30: Stress-Strain Curves Showing the Effect of Strain Rate for Low Relative Density and Low Confining Pressure
Figure 5.31: Stress–Strain Curves Showing the Effect of Strain Rate for High Relative Density and High Confining Pressure
Figure 5.32: Stress–Strain Curves Showing the Effect of Strain Rate for Low Relative Density and High Confining Pressure
Figure 5.33: Peak Stress Minus Upper Yield Stress Versus Relative Density for Moderate Strain Rate for Range in Confining Pressures
Figure 5.34: Peak Stress Minus Upper Yield Stress Versus Relative Density for Range in Strain Rates and Confining Pressures

LEGEND

Cell Pressure and Strain Rate

- $0.1 \text{ MPa}$ Slow
- $10.0 \text{ MPa}$ Slow
- $0.1 \text{ MPa}$ Fast
- $10.0 \text{ MPa}$ Fast
Figure 5.35: Peak Deviator Stress Versus Relative Density at Moderate Strain Rate
Figure 5.36: Peak Deviator Stress Versus Relative Density for Range in Strain Rates and Confining Pressures

LEGEND

Cell Pressure and Strain Rate

- × 0.1 MPa Slow
- ◊ 10.0 MPa Slow
- △ 0.1 MPa Fast
- □ 10.0 MPa Fast

MSP, lub ends, gfp stability
Figure 5.37: Effect of Confining Pressure on Peak Deviator Stress for Moderate Strain Rate
Figure 5.38: Effect of Strain Rate on Peak Deviator Stress for Range in Relative Densities and Confining Pressures
Figure 5.39: Axial Strain at Peak Deviator Stress Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures
Figure 5.40: Axial Strain at Peak Deviator Stress Versus Relative Density for Range in Strain Rates and Confining Pressures
Figure 5.41: Rate of Dilation at Peak Deviator Stress Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures
Figure 5.42: Rate of Dilation at Peak Deviator Stress Versus Relative Density for Range in Strain Rates and Confining Pressures
Figure 5.43: Maximum Measured Rate of Dilation Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures
Figure 5.44: Maximum Measured Rate of Dilation Versus Relative Density for Range in Strain Rates and Confining Pressures
Figure 5.45: Axial Strain at the Beginning of Dilation Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures

\[ \sigma_c = 10 \text{ MPa} \]

\[ \epsilon_a(\%) = 13.2 - 7.0 \times 10^{-2} D_r(\%) \]

\[ r^2 = 0.12 \]

LEGEND

Cell Pressure

- 0.1 MPa
- 2.0 MPa
- 5.0 MPa
- 10.0 MPa

MSP, gfp Stability, Lub Ends
Figure 5.46: Axial Strain at the Beginning of Dilation Versus Relative Density for Range in Strain Rates and Confining Pressures
Volumetric Strain (%) \[ \epsilon_v(\%) = 1.5 + 7.0 \times 10^{-2} D_r(\%) \]

\[ r^2 = 0.94 \]

\[ \epsilon_v(\%) = 1.1 + 2.4 \times 10^{-2} D_r(\%) \]

\[ r^2 = 0.75 \]

\[ \epsilon_v(\%) = -0.4 + 2.1 \times 10^{-2} D_r(\%) \]

\[ r^2 = 0.84 \]

\[ \epsilon_v(\%) = -0.3 + 9.7 \times 10^{-3} D_r(\%) \]

\[ r^2 = 0.47 \]

Figure 5.47: Volumetric Strain at 20% Axial Strain Versus Relative Density for Moderate Strain Rate and Range in Confining Pressures
Figure 5.48: Volumetric Strain At 20% Axial Strain Versus Relative Density for Range in Strain Rates and Confining Pressures
FRS52

\[ \varepsilon = \text{Moderate} \]
\[ \sigma_c = 10 \text{ MPa} \]
\[ D_r = 90.8\% \]
\[ \alpha = 0.96 \]

FRS58

\[ \varepsilon = \text{Slow} \]
\[ \sigma_c = 0.1 \text{ MPa} \]
\[ D_r = 93.7\% \]
\[ \alpha = 0.91 \]

FRS64

\[ \varepsilon = \text{Moderate} \]
\[ \sigma_c = 10 \text{ MPa} \]
\[ D_r = 82.7\% \]
\[ \alpha = 0.97 \]

Figure 5.49: Final Deformed Shape of Several Lubricated Specimens
Figure 5.50: Stress–Strain Curves to Show the Effect of Specimen Preparation Technique (page 1/3)
Figure 5.50: Stress–Strain Curves to Show the Effect of Specimen Preparation Technique (page 2/3)
Figure 5.50: Stress–Strain Curves to Show the Effect of Specimen Preparation Technique (page 3/3)
Figure 5.51: Stress–Strain Curves to Show the Effect of Temperature
CHAPTER 6 ANALYSIS OF RESULTS AND DISCUSSION OF POSSIBLE MECHANISMS

This chapter gives a preliminary analysis of the results presented in Chapter 5 and focuses on a comparison of the present experimental results with prior testing programs including that by Martin et al. (1981) on the same frozen sand. This is considered in terms of the both the small strain and large strain behavior. This research represents the most comprehensive set of data on the behavior of a frozen soil ever presented. Many of the observations are unique and have no parallel in the frozen soil literature.

Accordingly, this comparison is limited and will focus on the initial Young's modulus, the behavior in the upper yield region, and the behavior in the peak strength region. The effect of relative density, confining pressure and strain rate, and temperature will be compared insofar as possible. After these comparisons, the measured behavior of frozen Manchester Fine Sand is analyzed as a composite material using a simple behavioral model presented by Counto (1963) and as a particulate material using the dilatancy hardening model proposed by Ladanyi (1985). In light of these analyses and the literature review presented in Chapter 2, a discussion is finally presented on the behavior of frozen sand which focuses on possible mechanisms which may be controlling this behavior.

6.1 COMPARISON WITH PRIOR TESTING PROGRAMS

6.1.1 Small Strain Behavior

Table 6.1 presents a comparison of the measured Young's modulus in
this research with three different frozen granular materials presented in Section 2.3. The effect of density, confining pressure and temperature are highlighted. The effect of strain rate is not considered because in this research, it has been shown to have an negligible effect on modulus (Fig. 5.5). Relevant figures or tables are also presented. This table indicates that the magnitude of the Young’s modulus measured in this research is within the same range as that measured by Kaplar (1963) and Baker and Kurfurst (1985) who used resonant beam and acoustic wave propagation techniques, respectively. Note that the MFS is the finest grained material presented in this table.

The effect of strain rate on the Young’s modulus is not considered in Table 6.1 because in this research, it has had a negligible effect on the measured modulus (Fig. 5.5). If the mechanisms which govern its behavior are truly elastic, it should be independent of strain rate. The writer recognizes however that ice will creep under extremely small stresses and that some plastic deformations are probably occurring during the initial stress—strain behavior of frozen Manchester Fine Sand (MFS) under the conditions investigated in this research (See Fig. 5.2). A thorough investigation of the initial stress—strain behavior has yet to be performed and hence definitive statement as to the effect of strain rate is not possible.

**Effect of Relative Density**

The trend for the Young’s modulus to increase with increasing density, as observed both in the Baker and Kurfurst (1985) and the Vinson (1978) data, was not clearly apparent in the current research data. The Baker and Kurfurst data show a 20% increase in modulus for an increase in relative density from 20% to 100%. The Vinson data show a much larger percent increase but as mentioned
in Section 2.3, these data are suspect because they probably include some plastic straining.

**Effect of Confining Pressure**

The trend for the Young's modulus to decrease with confining pressure (16% reduction for an increase from 0 to 10 MPa) is opposite that observed by Vinson (1978) and illustrated in Fig. 2.30. Vinson attributed the increase in modulus with confining pressure to closure of microfissures in the pore ice, a possible densification of the pore ice due to closure of air voids and a possible increase in the stress at the contact points between the sand particles. His tests were conducted at warmer temperatures and the maximum confining pressure was 1.3 MPa. In the current tests, the maximum confining pressure was much higher. This might have resulted in a more significant amount of pressure melting in the ice and a larger unfrozen water content, both of which might explain the decrease in modulus with confining pressure.

**Effect of Temperature**

The effect of temperature on the Young's modulus was not systematically studied in this research. Table 5.13 presented the results of two tests under identical conditions except for a change in temperature. The estimated temperature difference was approximately 5°C and over this range, the difference in modulus was within the scatter in the data as reported in Table 5.5. Thus a definitive statement cannot be made. However, the measured modulus did increase with a decrease in temperature for these two tests in a manner consistent with the Kaplar (1963) data.
6.1.2 Upper Yield Behavior

The results of this current research present the most detailed treatment of the upper yield behavior of any testing program in the literature. Of all the reviewed programs, the Chamberlain et al (1972) and Parameswaran and Jones (1981) data report clear and distinct upper yield behavior and these were conducted in either a dense or medium dense state. The use of careful preparation techniques and on-specimen axial strain measuring devices have allowed the current data to form a truly unique set.

The behavior of the upper yield stress (both stress magnitude and axial strain) in MFS as a function of relative density, strain rate and temperature (presented in Chapter 5), has no parallel in the literature.

Effect of Confining Pressure in the Upper Yield Stress

The trend for the upper yield stress to decrease slightly with confining pressure (Fig. 5.15) is opposite to the trend in the data presented by both Chamberlain et al (1972) and Parameswaran and Jones (1981) (Fig. 2.33). These, however, encompass much higher levels of confining pressure and either have very few observations in the 0 to 10 MPa range (Chamberlain et al. 1972) or show some scatter in that range (Parameswaran and Jones 1981). Hence a direct comparison with the current data is probably not justified.

Upper Yield Stress Versus Polycrystalline Ice Behavior

According to the results presented in Figs. 5.14 through 5.16, the upper yield behavior of frozen MFS is strongly affected by variables which affect ice behavior (i.e., strain rate) and is either weakly or not affected by variables which affect sand behavior (relative density and confining pressure). This leads the
writer to make a direct comparison between the rate sensitivity of the upper yield point in frozen sand and the peak strength strength in polycrystalline ice.

Figure 6.1 makes this comparison between the current data (at a confining pressure of 0.1 MPa) and strength data for fine grained (0.7mm) isotropic polycrystalline ice in unconfined compression, at a temperature of $-7\pm1^\circ$C presented by Hawkes and Mellor (1972) over roughly the same range in strain rates. This figure indicates that both the ice and the upper yield stress of the frozen sand have roughly the same strain rate sensitivity ($n = 4.5$ and $n = 4.7$ respectively. This figure also shows that the magnitude of the upper yield stress was approximately double that of isotropic polycrystalline ice.

In the writer's opinion, the fact that the power law coefficients are similar while the magnitude of the deviator stresses are very different is a significant observation. This suggests that the rate controlling processes at the MFS upper yield point might be similar to those controlling the strength of isotropic polycrystalline ice. Furthermore, whatever strengthening process is operating to increase the frozen sand strength above that of the ice has either an activation energy similar to that of the ice or it is not a rate dependent process (i.e., if it were rate dependent with a different rate sensitivity, then the combined rate sensitivity would be distinct from polycrystalline ice).

6.1.3 Peak Strength Behavior

Effect of Relative Density on Peak Strength

Figures 2.37 and 2.38 summarized the prior testing programs aimed at studying the effect of relative density (or volume fraction of sand) on the peak strength. The results presented in Figs. 5.33 through 5.36 for the current data compare with the prior results and indicate a linear relationship between the
relative density and peak strength, except in the case where the deformational regime is changing (i.e., the Baker and Kurfurst 1985 data where the upper yield stress represented the peak strength for relative densities up to 50% and there after a bilinear behavior).

**Effect of Confining Pressure on Peak Strength**

Figure 2.35 presented the effect of confining pressure on the peak strength for prior testing programs and Fig. 5.37 summarized the current results. The peak strength is mostly a nonlinear function of the confining pressure. The total stress friction angles in the current research range from 1.6 to 15.6° and fall within the ranges presented in Fig. 2.35. Both the linear increase in strength observed at the highest relative densities and the nonlinear increase in strength at the lower relative densities are similar to some of the trends presented in Fig. 2.35.

A detailed evaluation of the pressure sensitivity of the peak strength in terms of the effect of relative density over various strain rates has yet to be accomplished. Hence, definitive statements as to why frozen sands exhibit widely varying pressure sensitivities are not possible at this time. Clearly, however, the current research shows that decreasing the relative density leads to a more pronounced curvature in the failure envelope.

**Effect of Strain Rate on Peak Strength**

The rate sensitivity of several frozen dense sands was presented in Fig. 2.34. The current data is summarized in Fig. 5.38. The power law coefficients ranged from 6 to 17 for all conditions in the current research and compare well with the prior data. The current data did not indicate a rate insensitive region
as was observed in the Yuanlin et al. (1989) and Bragg and Andersland (1980) data.

This program presents a more complete picture of the rate sensitivity as influenced by changes in relative density. There is a consistent trend in the current data for the power law coefficient to decrease with decreasing relative density. The tests at $\sigma_c = 0.1$ MPa and $D_r = 30\%$ had a power law coefficient that was close to that of isotropic polycrystalline ice. Note that in the current data, there are a relatively few number of observations for each condition and this might lead to some scatter in the data.

The increase in power law coefficient with relative density might be explained in terms of the increasing contribution of the frictional sand strength, which is rate insensitive in the range of strain rates investigated in this research (see Section 2.1.10). If the upper yield stress inherits its strain rate sensitivity from the pore ice, and the gain in strength above the upper yield point is less rate sensitive, it follows naturally that the peak strength, which is a combination of the upper yield and post upper yield behavior, would become less strain rate sensitive as the contribution of the post upper yield strength becomes more significant.

These data also present the most complete picture of the effect of confining pressure on the rate sensitivity (Fig. 5.38). Unlike the Shibata et al. (1985) data (Fig. 2.43) which shows no clear trend in rate sensitivity as a function of confining pressure, the current data show a consistent increase in power law coefficient (decrease in rate sensitivity) with increasing confinement. This might be explained in terms of an increased contribution of frictional strength (rate insensitive) to the total strength making the composite strength less rate sensitive.
Comparison of Strength Data to Prior MIT Data

Figure 6.2 compares the current data with the Martin et al. (1981) data. The Martin et al. data are for slightly colder temperatures and a wider range of strain rates. In general, their data have the same slope (power law coefficient) with relatively little scatter. The power law coefficient for both the \(-14.5^\circ C\) and the \(-18^\circ C\) tests, which were at a relative density of approximately 55%, was 8.8. For comparable conditions, i.e., at a confining pressure of 0.1 MPa, the current data range from \(n = 5.1\) to \(7.0\). This is slightly lower than the Martin data and might be due to the larger range in strain rates covered in this prior testing program, i.e., if a portion of the strain rate sensitivity is due to the ice behavior, then the writer would expect an increasing power law coefficient at higher strain rates in frozen sands as is observed in the ice literature. Although such an increase was not conclusively demonstrated in the current research, Fig. 5.38 does indicate a tendency towards a flatter slope at higher strain rates. This will require additional investigation.

6.2 ANALYSIS OF FROZEN SAND AS A COMPOSITE MATERIAL

Counto (1964) proposed a simple two phase model to predict the elastic modulus and the creep behavior of a concrete which can be easily modified to predict the initial stress–strain response of a frozen sand. This model neglects the "structure" of the sand skeleton and considers only the stiffness of the sand particles and the pore ice and the relative volume fraction of the sand.
6.2.1 Counto's Model

Counto's model can best be described as a cube-in-a-cube and is a member of a general class of isostrain models. In this model, a prism of solid aggregate material (silicate mineral) representing the sand phase is embedded in a cube of matrix material which represents the ice phase. The presence of air or unfrozen water is neglected. Figure 6.3 is a schematic diagram of the model structure. Counto recommended that the ratio of height to area of the aggregate prism be unity. If, on the other hand, a cubic geometry is adopted, the model predicts a slightly higher elastic modulus.

Counto (1964) outlined several of the assumptions upon which this model is based. The major assumptions are:

1) The composite material is considered as having only two phases, the aggregate (sand particles) and the matrix (pore ice).

2) The aggregate (sand particles) does not exhibit time dependent deformations, only elastic deformations.

3) To determine the yielding or plastic response, the matrix is assumed to exhibit all of the yielding behavior. The elastic modulus of the matrix is replaced with an effective modulus which is a function of the level of strain.

4) The effect of the bond between the aggregate (sand particles) and the matrix (pore ice) is neglected, i.e. whatever shear stresses are required to insure strain compatibility between the silicate prism and the ice matrix, these are assumed to exist (perfect bonding).

The principle of the model is to cut the composite cube into three horizontal sections, i.e., two pure ice ends and the central section which includes all of the sand prism and the surrounding ice (see Fig. 6.3). An axial force is applied to the entire cube and the corresponding deformation of each section is computed from the "known" elastic modulii of the pore ice and sand particles. In
the middle composite section, an isostrain condition is imposed wherein both the silicate prism and the ice surrounding it must deform by the same amount. This isostrain condition allows for a partitioning of the stresses between the sand and ice.

The composite modulus is computed from the total deformation of all three sections and the magnitude of the applied force. The resulting equation for the composite modulus using the height to area assumption of Counto (1964) is

\[
\frac{1}{E_c} = \frac{1 - \sqrt{V_s}}{E_i} + \frac{1}{\left[ \frac{1 - \sqrt{V_s}}{\sqrt{V_s}} \right] E_i + E_s}
\]

6.1

where \( E_c \) is the composite Young's Modulus, \( E_i \) is the Young's Modulus of the ice, \( E_s \) is the Young's Modulus of the sand particles (not the sand skeleton), and \( V_s \) is the volume fraction of the sand particles.

Using the assumption that the geometry of the silicate prism is that of a perfect cube, the equation for the composite Young’s Modulus is

\[
\frac{1}{E_c} = \frac{1 - 3\sqrt{V_s}}{E_i} + \frac{1}{\left[ \frac{1 - 3\sqrt{V_s}^2}{3\sqrt{V_s}} \right] E_i + E_s 3\sqrt{V_s}}
\]

6.2

where \( E_c, E_i, E_s \) and \( V_s \) have the same meaning as previously defined.

6.2.2 Composite Modulus Prediction

Using Equation 6.1 based on Counto's assumption of a height to area ratio of unity with a Young's Modulus for the sand particles of 90 GPa (from Table 12.5 of Lambe and Whitman 1969, assuming an average value for quartzite) and 7.5 GPa for polycrystalline ice (measured in a test conducted at M.I.T. on the same testing system at a moderate strain rate and a confining pressure of 0.1 MPa), the computed composite Young's Modulus ranges from 20.9
GPa to 26.1 GPa for volume fractions ranging from 0.52 ($D_r = 0\%$) to 0.63 ($D_r = 100\%$). Using Equation 6.2 based on the assumption of a sand cube with the same sand and ice moduli and volume fractions, the computed Young’s Modulus ranges from 25.4 GPa to 31.6 GPa.

Figure 6.4 plots the predicted composite Young’s modulus as a function of volume fraction of the sand from 0% to 100% sand. This indicates that the stiffening effect of the sand particles becomes more dramatic as the volume fraction increases. The composite modulus starts out at a value equal to that assumed for the pore ice (7.5 GPA) and increases to that assumed for the sand grains (90 GPa).

6.2.3 Prediction of the Initial Yielding

Counto (1964) proposed that his model be extended to predict the initial yielding behavior of a composite material by assuming that all of the plastic deformation occurs in the matrix. The assumption that a composite model may represent the initial yielding behavior of frozen sand may be reasonable due to the fact that the upper yield stress is independent of relative density (Fig. 5.14) and almost unaffected by confining pressure (Fig. 5.15). Thus the interaction between the sand particles and the pore ice might not be influenced in this region by the frictional particle to particle contact forces.

Counto’s proposed extension is accomplished by replacing the Young’s modulus for the ice in Equation 6.1 or 6.2 with a "degraded" modulus or a modulus which decreases as a function of the level of strain, reflecting the plastic straining in the ice. This modulus can be input either as an analytic expression or directly as data points from a stress–strain curve which has been converted to a modulus versus strain curve. The composite modulus is then computed as a
function of strain. The resulting composite modulus versus strain curve is converted back into a stress–strain curve by multiplying by the corresponding axial of strain.

Figure 6.5 is a composite plot of the initial stress–strain curves from a polycrystalline ice with a grain size ranging from 1mm to 6mm (tested at M.I.T.), FRS34 and a predicted composite curve using equation 6.1. The predicted composite curve was generated using a sand grain modulus of 90 GPa and a volume fraction of 0.56 for the sand. This plot indicates that the initial modulus of the frozen sand in FRS34, is well–predicted by the cube–in–a–cube formulation. The measured modulus of FRS34 was 23.5 GPa and the model prediction is 23.3 GPa.

The model prediction breaks down in the region of the proportional limit. The deviator stress at a $10^{-4}$ yield offset is approximately 9.6 MPa for the predicted composite curve as compared to 4.9 MPa for FRS34. Fig. 6.5 indicates that the deviator stress in the upper yield region for the predicted curve is approximately double that of the frozen sand.

The writer recognizes several possible deficiencies associated with this extension on the model. The geometry of a frozen sand is much more complicated than a simple cube–in–a–cube. The actual state of stress acting on the pore ice is extremely complicated, even for the cube–in–a–cube geometry, and cannot simply be expressed as linear elastic. The levels of strain in the ice vary widely, even for the simple model, and there will be significant nonlinear behavior. Therefore, the assumption that at a given level of total strain, the states of stress in the material above the sand prism and next to the sand are linearly related through an elastic constant is highly questionable. In light of these difficulties, definitive conclusions as to the behavior of frozen sand in the
upper yield region as inferred from this model are not possible.

If, however, we overlook these deficiencies, the model predicted strength is consistent with the notion that either: 1) the actual strength of the ice in the pores of the frozen sand may be weaker than that of normally tested polycrystalline ice in conformance to some of the possible weakening mechanisms explained in Section 2.2.4; or 2) the actual strength of the ice–silicate interface (assumed to be infinite for this prediction) may be insufficient to adequately mobilize the strengthening effect of the stiffer sand particles.

6.3 LADANYI'S DILATANCY HARDENING MODEL

A complete evaluation of the current data in terms of the Ladanyi dilatancy hardening model outlined in Section 2.3.4, (Ladanyi 1985 and Ladanyi and Morel 1990) is not possible at this stage of the research. As mentioned in Chapter 1, this research has been carried out concurrently with the testing of unfrozen MFS. The writer anticipates a more thorough treatment of this model when the unfrozen testing program is complete. The model will be evaluated in terms of its rate sensitivity in unconfined compression and how this compares to the measured data at the lowest confining pressure.

In this comparison, the writer has assumed that the tensile strength of fine-grained polycrystalline ice as presented by Hawkes and Mellor (1972) is representative of the tensile strength of the pore ice (approximately 2 MPa). Note that the temperature of the Hawkes and Mellor data was \(-7\pm1^\circ C\) and the temperature of the current data was \(-9.55\pm0.2^\circ C\), but Mellor (1980) states that there should be very little temperature sensitivity for the tensile strength over this range. The writer has also assumed that the unconfined compressive strength of polycrystalline ice as reported by Jones (1982) at a temperature of
−11±1°C is a representative pore ice strength.

The sand strength data was provided by Mr Chris Swan from tests conducted at M.I.T. The friction angle used for the sand was from a test at a relative density of 95% and an effective confining pressure of 0.1 MPa. The value was selected as a secant angle to the failure envelope for a test with a sufficient back pressure to allow for a negative pore pressure increment of 2 MPa (i.e., the assumed tensile strength of the pore ice). At this pore pressure increment, the secant friction angle was 32°. In order to mobilize the shear stress associated with this negative pore pressure increment, the sand had to experience axial strains in excess of 15%.

Under these conditions (i.e., large relative density and low confining pressure), the frozen sand should have a large −Ψ parameter and exhibit a dilational tendency. The writer assumes in this case that Ladanyi’s equation for the cavitational regime is valid (i.e., Eq. 2.3.4). Table 6.2 presents the computation of the Ladanyi prediction over a range of strain rates comparable to current testing program using a ϕ′ of 32° and 38° (given for comparison).

Figure 6.6 presents the Ladanyi predicted strength and the measured strengths as a function of strain rate. Included on this figure are the upper yield strength, the strength at the beginning of dilation and the peak strength for specimens at Dr = 95% and a σc = 0.1 MPa. This figure indicates the although the power law coefficient from the Ladanyi prediction for ϕ = 32° (n = 10.2) is within the range of those from the current research (Fig. 5.38), the prediction leads to a significantly lower strength compared to the peak strength and the strength at the beginning of dilation.

The Ladanyi model is designed to predict the strength of the frozen sand at the point where the ice matrix begins to break apart, i.e., the frozen sand
begins to dilate. Prior to this research, this was assumed to occur at the upper yield point (Sayles 1989). The actual point at which the frozen sand skeleton begins to dilate is well beyond the upper yield stress (Compare Figs. 5.17 with 5.45 and 5.46).

For strain rates less than $2 \times 10^{-5}$/sec, the Ladanyi model over predicts the upper yield stress and for strain rates greater than $2 \times 10^{-5}$/sec it under predicts the upper yield stress, i.e., the model is less sensitive to changes in strain rate than the measured upper yield stress. This is inherent in the model because it assumes a frictional contribution to the strength which is almost totally insensitive to strain rate, except for the slight sensitivity of the ice tensile strength. Note that using a total stress friction angle of $38^\circ$ improves the model predictions and increases the power coefficient to $n = 12.1$, but still overpredicts the measured peak strength except at the lowest strain rate.

Another significant observation is the fact that in the unfrozen tests, a significant amount of straining ($>15\%$) was necessary in order to mobilize the frictional strength assumed for the prediction. The dilatancy hardening model is designed to predict the strength at the point where the ice matrix begins to fail in tension. If this corresponds to the point where dilation begins, the current test results indicate that this occurs at a level of axial strain of approximately $1.9\%$ (Fig. 5.45) for unconfined compression at a moderate strain rate. This is much lower then that necessary to mobilize the frictional strength of the sand.

### 6.4 DISCUSSION OF FROZEN SAND BEHAVIOR

#### 6.4.1 Initial Young's Modulus

The initial modulus for MFS ranged from $26.6 \pm 4.5$ GPa and is within
the range for other elastic modulii reported for frozen granular materials. This is approximately three times greater than that for granular ice (9 GPa, Fig. 2.19) and two orders of magnitude greater than that for an unfrozen sand (0.3 to 0.4 GPa Jardine et al. 1984). The Counto isostrain model gives valuable insight as to why the modulus is so much larger than either of its components. In frozen sand at small strains, the behavior of the sand skeleton is not important. The initial stiffness properties of the frozen sand are inherited from the stiffness of the individual silicate particles ($E = 90$ GPa) in concert with the pore ice and not from the sand skeleton. Thus, the initial stiffness should not be considered in terms of the sand skeleton stiffness and the ice matrix stiffness, but rather as a composite material into which a high stiffness material (silicate particles) has been added to a lower stiffness material (ice) to produce a modulus which is intermediate between these two components (Fig. 6.4).

**6.4.2 Yielding Behavior to Upper Yield Point**

The strong dependence of the initial yielding behavior on the applied strain rate, independence on the relative density and slight dependence on the confining pressure all indicate that the structure of the sand (i.e., its behavior and frictional resistance) is probably not important in this region. The agreement of the power law coefficients for the peak strength of isotropic ice and the upper yield stress for frozen sand over a wide range in relative densities (Fig. 6.1) suggests that the rate controlling processes are probably due to the ice behavior.

The higher yield stress of the frozen sand compared with granular ice is probably due to some composite material interaction similar to that responsible for the initial stiffness properties. The higher stress cannot be attributed to the
frictional contribution of to the sand skeleton.

6.4.3 Large Strain Behavior

The behavior of frozen MFS after the upper yield region is due not only to the frictional nature of the sand skeleton but is also affected to a certain extent by the pore ice behavior. Some of the stress–strain parameters presented in Chapter 5 exhibited a partial strain rate sensitivity. Refer to Tables 5.8 and 5.9. These are: the rate of strain hardening and the axial strain at which it begins; the strength gain above the upper yield stress; the peak strength; the axial strain at peak strength; and the volumetric behavior. The complex behavior after the upper yield stress is thus not solely attributable to the frictional characteristics of the sand skeleton but must involve a complex interaction between the ice and sand.

The volumetric behavior of the frozen sand is not similar to that of unfrozen sand. As described in Section 5.4.4, all of the frozen sand specimens exhibited a tendency for dilation (i.e., there were no significant volume decreases even for tests at high confining pressures and low relative densities, Figs. 5.28 5.47 and 5.48). Additionally, the maximum rate of dilation does not occur at the peak strength for tests at low confining pressures as is the case for unfrozen sands in drained compression (Fig. 2.4). Thus some of the volumetric behavior must be attributed to the pore ice and the sand–ice interaction.

6.4.4 Discussion of Ting et al. (1983)

Structure of Frozen Sand

Figure 2.26 presented the proposed structure of a frozen sand originally presented by Ting et al. (1983). As mentioned in Section 2.3.2, this is unrealistic
because it overemphasizes the number of particle contacts and does not truly represent the actual structure. Figure 6.7 presents a more probable cross section through a frozen sand. The frozen sand may be viewed more appropriately as that of silicate particles suspended in an ice matrix. For these tests, the actual volume fraction of the sand ranged from 52 to 63% for loose and very dense specimens, respectively. Note that this type of a structure is very similar to that of Counto's model presented in Section 6.2.

Strengthening Mechanisms in Frozen Sand

Ting et al. (1983) proposed that the possible mechanisms of strength in frozen sand can be divided into three broad categories: ice strength, sand strength, and the interaction between the ice and the sand (how the ice affects the sand and how the sand affects the ice). These are summarized in Table 2.10.

Ice Strength

In the writer's opinion, one of the most significant unknowns in understanding the behavior of frozen sand is the structure of the pore ice and its strength. Ting et al. (1983) down played the importance of the ice strengthening component by referring to the small observed strength gain (28%) from pure ice to a sand–ice mixture with 40% sand by volume (at 40% sand by volume the particles are barely in contact) reported by Goughnour and Andersland (1968) and suggesting that all of the possible ice strengthening mechanisms should be represented in that range. By extrapolating the observed strength gain to higher densities, they concluded that "the net effect of the ice strengthening, while significant, is still relatively small compared to the total effect of other mechanisms at high densities".
One item not included in the Ting et al. (1983) reasoning is the fact that in performing their experiments, Goughnour and Andersland (1968) had to change the specimen preparation technique from mixing snow and sand grains to preparing saturated sand specimens and freezing unidirectionally. The unidirectional freezing of a saturated sand, as opposed to the unidirectional freezing of sand–snow mixtures, could result in a significant difference in the pore ice structure due to differences in the number of nucleation sites. Hence a direct extrapolation of the results from one preparation regime to the next may not lead to a valid representation of the ice strengthening component (Cole 1989 personal communication).

It is also not clear what effect the large decrease in grain size, from normally tested fine-grained granular ice to the possible grain sizes which may exist in the frozen sand pores, has on the strength of the ice. There is some indication, by comparing the results of research on ultra fine-grained metals, that there can be a significant decrease in the strength due to enhanced grain boundary sliding and diffusional mechanisms. The analysis of the initial yielding behavior in terms of the cube-in-the-cube model is consistent with this notion that the pore ice strength may be substantially less than normally tested polycrystalline ice strength, although the extension of this model to the plastic yielding regime is highly questionable.

Sand Strength

The writer agrees with the statement by Ting et al. (1983) that it is difficult to assess the contribution of the sand strength without knowing the effective stresses acting on the sand skeleton. As was explained in Section 2.3, these effective stresses may not even be related to those in an unfrozen sand
system. Taken by themselves, without considering the forces being transmitted from the pore ice, they probably do not represent an equilibrium condition. Any assessment of the contribution to the frozen sand strength which refers to these effective stresses is thus uncertain.

It may not be reasonable to consider a frozen sand as a particulate material at all, but rather it may be more advantageous to think of it in terms of a composite material. The sand skeleton is not important during initial straining. It is the individual stiffness properties of the sand particles which seem to be more important. In a sense, the silicate particles can contribute to the strength of a frozen sand due to their individual stiffness properties rather than their frictional resistance as a particulate mass. Thus, the concept of sand strength as proposed by Ting et al. (1983) should be expanded to include the extrinsic hardening effect of adding a higher strength aggregate (silicate particles) to a lower strength matrix (ice).

**Ice—Sand Interaction**

Ting et al. (1983) refer to the ice—sand interaction which results in an increase in the strength of the frozen sand in terms of the effect of the sand on the ice and the effect of the ice on the sand. In the writer's opinion, this may prove misleading because as a composite material, neither one may behave in a manner which is consistent with its behavior as a separate material. In other words when considering the sand skeleton and identifying various processes which either increase or decrease the effective stresses acting on it or which activate or deactivate certain strengthening mechanisms, such mechanisms and processes may not at all represent the true composite behavior.

The dilatancy effect, mechanism 3bi from Ting et al. (1983) and
quantified by Ladanyi (1985) might be an example of one such process. The author's equate the dilational tendency of a dense sand at low effective confining pressures (−Ψ) to a possible strengthening mechanism in frozen sand. They reason that as the sand begins to dilate, an increment of tension is carried by the pore ice and is transmitted back to the sand skeleton as a positive increment of effective stress which leads to a strengthening of the sand skeleton. There are various factors which cast some doubt as to the direct correlation between the dilation of the sand skeleton and increment of pore ice tension.

The strength gain with relative density is linear for all testing conditions. This is not consistent with the proposed mechanism map for the strength of frozen sand presented by Ting et al. (1983) (Fig. 2.51). This indicates that the enhanced strengthening assumed to be attributed to dilatancy of the sand skeleton as proposed by Ting et al. (3bi) does not adequately reflect the nature of the ice–sand interaction.

First, this dilatancy effect assumes that all of the dilatancy of a frozen sand is directly attributable to the sand skeleton. This is clearly not the case. Even very loose frozen sands exhibit a tendency to expand during shear under moderate to high confining pressures where unfrozen sands exhibit contraction. Second, due to the complex geometry of frozen sand, it is not clear that all of the ice–water–silicate interfaces associated with a given pore are experiencing a tensile stress increment as assumed by the dilatancy argument. Locally, a portion of the pore space could be expanding while another may be contracting. Hence, the actual state of stress in a sand pore undergoing dilation may be quite complicated rather than that of a simple tensile increment in the pore ice.

It may prove more fruitful, when discussing the interaction between the sand and ice (dilatancy and structural hindrance) to consider the frozen sand as a
composite material rather than a sand skeleton with ice filling the pores. It may not be reasonable to consider the strengths of the two materials as separate entities and to develop a list of synergistic interaction mechanisms to explain the strength. It may prove more beneficial to consider how the components of a composite material interact and how this might be responsible for the observed strength and deformation behavior of a frozen sand.
Table 6.1: Comparison of Initial Modulus Data with Prior Testing Programs

<table>
<thead>
<tr>
<th>INVESTIGATORS</th>
<th>MATERIAL</th>
<th>MEASUREMENT TECHNIQUE</th>
<th>TEMP (°C)</th>
<th>YOUNG'S MODULUS (GPa)</th>
<th>OBSERVED TRENDS WITH</th>
<th>INCREASING DENSITY</th>
<th>CONFINING PRESSURE</th>
<th>TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaplan (1963)</td>
<td>Peabody Gravelly Sand (SP)</td>
<td>Resonant Beam (Longitudinal Vibration)</td>
<td>-27.8</td>
<td>34.5 to 47.1</td>
<td>Not Measured</td>
<td></td>
<td>Not measured</td>
<td>Increases</td>
</tr>
<tr>
<td></td>
<td>D₅₀ = 3.8 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Fig. 2.28</td>
</tr>
<tr>
<td></td>
<td>Cₙ = 16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cₑ = 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vinson* (1978)</td>
<td>Ottawa 20 - 30 Sand</td>
<td>Cyclic Triaxial</td>
<td>-4</td>
<td>5 to 13.5</td>
<td>Increases by 60%</td>
<td></td>
<td>Increases by 69%</td>
<td>Increases</td>
</tr>
<tr>
<td></td>
<td>D₅₀ = 0.7 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.3 MPa</td>
</tr>
<tr>
<td></td>
<td>Cₙ = 1.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Fig. 2.30</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baker and Kurfurst (1985)</td>
<td>Ottawa 16-100 Sand</td>
<td>Acoustic Wave Propagation</td>
<td>-10</td>
<td>24 to 30</td>
<td>Increases by 20%</td>
<td></td>
<td>Unconfined</td>
<td>Unaffected</td>
</tr>
<tr>
<td></td>
<td>D₅₀ = 0.4 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Fig. 2.29</td>
</tr>
<tr>
<td></td>
<td>Cₙ = 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sinha (1989) from Dantl (1969) experiments on monocrystals</td>
<td>Granular Ice</td>
<td>Pulse Echo and Pulse Interference</td>
<td>0 to -50</td>
<td>8.9 to 9.5</td>
<td></td>
<td></td>
<td>Unconfined</td>
<td>Increases</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Fig. 2.19</td>
</tr>
<tr>
<td>Current Program</td>
<td>Manchester Fine Sand</td>
<td>On-specimen axial strain in triaxial compression</td>
<td>-9.5</td>
<td>26.6 ± 4.50</td>
<td>No Clear Trend</td>
<td></td>
<td>Slight decrease; σₑ = 0.1-10 MPa</td>
<td>Possible Increase</td>
</tr>
<tr>
<td></td>
<td>D₅₀ = 0.18 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16% decrease</td>
<td>Table 5.13</td>
</tr>
<tr>
<td></td>
<td>Cₙ = 2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

* significant amounts of plastic deformation
### Table 6.2: Computation of Ladanyi Strength Prediction

<table>
<thead>
<tr>
<th>STRAIN RATE ($\text{sec}^{-1}$)</th>
<th>ICE$^1$ TENSILE STRENGTH (MPa)</th>
<th>ICE SHEAR$^2$ STRENGTH (MPa)</th>
<th>PREDICTION FOR $\phi = 32^\circ$ (MPa)</th>
<th>PREDICTION FOR $\phi = 38^\circ$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.6 \times 10^{-7}$</td>
<td>1.9</td>
<td>2.1</td>
<td>6.6</td>
<td>8.5</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>2.0</td>
<td>2.2</td>
<td>6.9</td>
<td>8.9</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>2.1</td>
<td>3.4</td>
<td>8.4</td>
<td>10.5</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>2.1</td>
<td>5.4</td>
<td>10.4</td>
<td>12.5</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>2.1</td>
<td>8.5</td>
<td>13.5</td>
<td>15.6</td>
</tr>
</tbody>
</table>

$$n = 10.2 \quad n = 12.1$$

$$r^2 = 0.996 \quad r^2 = 0.996$$

$^1$Tensile Strength from Hawkes and Mellor (1972)

$^2$Shear Strength from Jones (1982)
Deviator Stress (MPa)

Upper Yield Stress
MFS All Densities
\( \sigma_c = 0.1 \) MPa

\[ Q(\text{MPa}) = 77.8(\dot{\varepsilon})^{0.214} \]
\( n = 4.7 \)

\( r^2 = 0.98 \)

Peak Strength
Granular Ice

\[ Q(\text{MPa}) = 39.7(\dot{\varepsilon})^{0.214} \]
\( \sigma_c = 0 \)

\( r^2 = 0.96 \)

Data from Hawkes and Mellor (1972)

Figure 6.1: Comparison of Rate Sensitivity of Upper Yield Stress to that of Peak Strength of Polycrystalline Ice
From linear regression through peak strength data

Figure 6.2: Comparison of Rate Sensitivity of Peak Deviator Stress to Prior MIT Data
Figure 6.3: Simple Two-Phase Composite Model for Concrete (after Illston et al. (1979))
Figure 6.4: Predicted Composite Modulus Versus Volume Fraction of Sand
Figure 6.5: Comparison of Predicted Initial Stress–Strain Response to that of FRS34 and Polycrystalline Ice
Comparison of the Rate Sensitivities of the Upper Yield Stress and Peak Strength to that Predicted by Ladanyi's Dilatancy Hardening Model
Figure 6.7: Probable Structure for Frozen Sand
CHAPTER 7
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 SUMMARY

7.1.1 Literature Review

Chapter 2 presented an extensive review of the literature on frozen sand. This was broadly divided into three sections: 1) unfrozen sand; 2) polycrystalline ice; and 3) frozen sand. Each of these attempted to characterize the material, to present an overview of its behavior in terms of the most influential variables, and to describe mechanisms that may dominate its strength and deformational properties.

Unfrozen Sand

The first section on sand behavior described a method for describing the state of stress acting on a particulate material (Fig. 2.2). The stresses are averaged over a representative area. It was emphasized that the state of stress defined in this manner represents an equilibrium condition on the particle level, i.e., the interparticle contact forces relate directly to the state of stress. The principle of effective stresses was outlined. This provides a method of partitioning the total normal stress in a saturated sand system between that being carried by the sand skeleton and that being carried by the pore water.

The two most important factors that influence unfrozen sand behavior are the relative density (D_r) and the effective confining pressure (\sigma_c). Figures 2.5 through 2.7 presented the effect of each of these on the behavior. The state parameter developed by Been and Jefferies (1985) unifies these two parameters and was defined by Fig. 2.7 as the difference between the initial void ratio and
the steady state void ratio on an e-logp' plot. Sands with a state point above
the steady state line (+\psi) exhibit contractive behavior during shear (i.e.,
collapse of the structure during a drained test or generation of positive pore
pressures during an undrained test) and sands with a state point below the steady
state line (-\psi) exhibit dilative behavior during shear (i.e., expansion of the
skeleton during drained tests or generation of negative pore pressures during
undrained tests).

Sands deform by various mechanisms including: the elastic and plastic
straining of the individual particles at the contact points; the crushing of
particles; and the rearranging of the particles. Both rolling and sliding processes
may occur, but most researchers believe that sliding is predominant. These are
probably occurring as the motion of rigid groups of particle such that the actual
slip at any given instance is happening at relatively few locations.

The strength of unfrozen sands can be considered to occur as the result of
sliding friction between particles, the interference of particles as they move
around each other, and the dilation of the skeleton. These were proposed by
Rowe (1962) and were summarized in Fig. 2.11.

The pressure sensitivity of the strength of sands is not linear, i.e., the
Mohr failure envelope is curved and shows a decrease in slope with increasing
confining pressure. This may be caused by the suppression of dilation and at
high stresses may be accompanied by the crushing of sand grains and an increase
in the number of interparticle contact points.

The drainage conditions have a dramatic effect on the behavior of sands.
For sands with a large -\psi parameter, decreases in pore pressure during
undrained shear allow the effective stress path to climb up the failure envelope
and attain a strength greater than that of the same sand during drained shear.
For sands with a large $+\Psi$ parameter, increases in pore pressure during undrained shear result in a strain softening behavior and strengths that are much less than drained tests starting at the same state point.

Other factors which influence the strength of sands include inherent and induced anisotropy and the magnitude of the intermediate principle stress. Rate effects are minimal over the range of strain rates investigated in this research.

**Polycrystalline Ice**

The crystalline structure of ice Ih is formed by the tetrahedral coordination of water molecules in a hexagonal structure. The oxygens are packed most densely along basal planes and as a result, ice has a naturally anisotropic structure.

A correspondence has been established experimentally between the results of constant load "creep" tests and constant strain rate "strength" tests. This can be most readily observed in a plot of the log(applied or measured stress) versus log(minimum or applied strain rate), respectively. Figure 2.20 is one such plot that contains the results of both types of tests. This type of curve is termed a flow curve and the quasilinear relationship in the range of strain rates up to $10^{-4}$/sec is described by a simple power law with a variable coefficient. For very low strain rates or applied stresses (i.e., $>10^{-8}$ or the range occurring in glacier flow), the power law coefficient $(n)$ appears to approach unity. In the range of strain rates for engineering applications (i.e., $10^{-8}$ to $10^{-5}$/sec), the power law coefficient is between 3 and 4.5. As the strain rate increases above $10^{-5}$/sec, the behavior becomes less and less rate sensitive (i.e., the power law coefficient approaches infinity and in some cases may become negative).

Small strain parameters (Young's modulus, shear modulus and Poisson's
ratio) for ice have been most reliably measured using ultrasonic wave propagation
techniques. Sinha (1989) took the results of one such study and made estimates
of the small strain parameters for granular ice. These were reproduced in Fig.
2.19. These all show a slight tendency to increase with decreasing temperature.

An idealized stress–strain curve for the behavior of polycrystalline ice in
uniaxial compression at moderate strain rates was presented in Figure 2.18. It
shows a distinct double yield behavior. The initial yield or knee corresponds with
the onset of internal cracking and this is followed by strain hardening to the peak
strength. Figure 2.21 presented actual stress–strain curves from tests reported
by Mellor and Cole (1982). These show a complex behavior for both the knee
and the peak strength. At very high strain rates or very low strain rates, the
first yield point coincides to the peak strength. At high strain rates, there is a
significant amount of strain softening after the peak strength. The tensile
strength of polycrystalline ice is rate insensitive for strain rates greater than
$10^{-6}$/sec (Fig. 2.20). Below this strain rate, both the compressive and tensile
strengths are identical.

The temperature dependency of the strength in ice can be described as a
thermally activated process using an Arrhenius type equation for temperatures
below $-10^\circ$C. Above $-10^\circ$C, the activation energy increases and cannot be
approximated by the Arrhenius equation. Researchers have used a combination
of the power law formulation and Arrhenius equation to approximate the
minimum creep rate or peak strength. For constant load creep tests, the effect of
an increase in temperature is the same as an increase in creep stress so long as
the deformational regimes remain similar.

The texture and fabric of polycrystalline ice dramatically influence its
behavior. In constant strain rate tests where the deformation is accompanied
with cracking, a decrease in the grain size leads to an increase in strength. At slower strain rates where dynamic recrystallization and grain boundary migration processes may be occurring, there is an apparent reversal of this trend (Cole 1987).

The orientation of an individual grain in relation to the applied stresses has a significant effect on the behavior. Figure 2.24 presented the minimum creep rate for single crystals oriented in easy and hard glide and for polycrystalline ice. The strength of polycrystalline ice lies between that for easy and hard glide.

Mechanisms of deformation in polycrystalline ice were summarized in Table 2.2. These include: 1) the elastic straining of the hydrogen bonds.; 2) the anelastic motions of dislocations in response to internal stress fields; 3) diffusional flow of point vacancies along grain boundaries or through the lattice (these can accommodate grain boundary sliding processes; 4) plastic deformations due to dislocations gliding along slip planes or climbing between adjacent slip planes; and 5) internal fracturing processes that may be associated with either intracrystalline or transcrystalline cracking. In addition to these mechanisms, grain boundary migration and dynamic recrystallization processes occur in response to internal strain energy gradients.

Some processes which affect the strength of polycrystalline ice were summarized in Tables 2.3 and 2.4. Possible weakening processes include: 1) a decrease in grain size from the fine (1mm) to the ultrafine (<1mm) range; 2) pressure melting at the grain boundaries; 3) the concentration of impurities at grain boundaries; 4) an increase in the mobility of dislocations (due possibly to increases in the number of mobile ionization defects); and 5) an evolving fabric which favors crystals oriented in easy glide directions.
Possible strengthening processes include: 1) isotropic and kinematic hardening due to short range and long range stresses that decrease the mobility of dislocations; 2) decreases in the mobility of dislocations due to decreases in the number of mobile ionization defects; 3) decreases in grain size from the coarse (10mm) to fine (1mm) in deformational regimes where cracking processes are occurring; and 4) increases in confining pressure up to 10 to 40 MPa in deformational regimes where cracking is operating.

**Frozen Sand**

Ting et al. (1983) proposed a structure for frozen sand that was presented schematically in Fig. 2.26. A more realistic schematic diagram was presented in Fig. 6.8. Elements of their proposed structure include: 1) solid contacts exist between most adjacent sand particles; 2) continuous unfrozen water film surrounds the silicate particles; 3) strong tensile and significant shear adhesional forces can be transmitted across the ice–silicate interface; 4) no direct ice to soil contacts exist; and 5) the macrostructure of the pore ice is polycrystalline with the maximum grain size limited to the pore size. Each of these postulates were reviewed in light of evidence from the literature. Of these, the nature of the ice–silicate interface (i.e., its strength and deformational properties) and the macroscopic structure (i.e., whether or not the actual pore ice is polycrystalline or exists as long tortuous crystals extending through several pores) are both being debated in the literature.

The small strain behavior of frozen sand has been measured most reliably using acoustic wave propagation techniques, resonant vibration tests and on-specimen axial strain devices. These programs have indicated that the Young's modulus increases with decreasing temperature (Fig. 2.28), with dry
density (moderately, Fig. 2.29), perhaps increases with confining pressure (Fig. 2.31), and decreases dramatically with increasing salinity (Fig. 2.31). The Poisson's ratio increases with decreasing temperature (Fig. 2.28), increases slightly with dry density (Fig. 2.29), and increases slightly with increasing salinity (Fig. 2.32).

An idealized set of stress–strain curves illustrating the large strain behavior were presented in Fig. 4.5 from the results of this research. In general, frozen sand also exhibits a double yield behavior similar to polycrystalline ice. The writer identified the knee of the stress–strain curve as the upper yield point in recognition of the large amount of plastic deformation that occurs at very small strains prior to the knee.

Limited data on the effect of confining pressure on the stress at the upper yield point indicate that it increases in the range of 10 to 40 MPa, decreases for confining pressures between 40 and 100 MPa and then increases again for confining pressures greater than 100 MPa (Fig. 2.33).

The peak strength is strongly affected by strain rate. Figure 2.34 presented some of the prior results. Power law coefficients for these tests ranged from 7 to 16. Two of the testing programs showed strain rate insensitivity at high strain rates, but this observation was not uniform throughout the reported data.

The peak strength generally increases with confining pressure in the range of 0 to 10 MPa. This was illustrated in Fig. 2.35. In some tests, the increase is linear, while in other tests the failure envelope exhibits a pronounced curvature. Total stress friction angles for these tests range from 0 to 28°. Two programs that investigated the behavior at much larger confining pressures show a linear increase with confining pressure in the range of 0 to 40 MPa, a linear
decrease in peak strength with confining pressure in the range of 40 to 100 MPa and thereafter a linear increase (Fig. 2.36).

The sand content strongly affects the peak strength. Figure 2.37 illustrated that there is a bilinear relationship between sand content and strength. At approximately 40% sand by volume when the particles begin to interact, a greater rate of strengthening is observed. A separate study on the effect of relative density on the peak strength (Baker and Kurfurst 1985) indicated a slightly different behavior with a constant strength for relative densities from 25 to 50%, a linear increase in strength for 50 to 80% D_r and a steeper linear increase above 80% (Figure 2.38). These results were affected by changes in the deformational regime from one of failure at the upper yield stress (shown to be insensitive to relative density in this research) to failure at a peak strength.

The peak deviator stress increases with decreasing temperature (Fig. 2.39). The increase is most dramatic in the 0 to –30°C range with typical rates of strengthening on the order of 0.7±0.3 MPa/°C. Some of the testing programs have demonstrated a decrease in strength at very large negative temperatures.

A decrease in saturation (Figs. 2.40 and 2.41) and an increase in salt concentration (Fig. 2.42) both dramatically decrease the strength.

Limited data on the tensile strength of frozen sand indicates a strain rate insensitivity above 1.67x10^-4/sec (Fig. 2.45). Below this strain rate, the power law coefficients range from 3 to 8.

The axial strain at the peak deviator stress is strongly dependent on the deformational regime (Figs. 2.46, 2.47, and 2.48). If the upper yield stress coincides with the peak strength, then the axial strain at the peak is apparently insensitive to changes in relative density, temperature and strain rate. If the
upper yield is followed by strain hardening, the axial strain at peak strength increases with relative density but is apparently unaffected by changes in strain rate.

The mechanics of frozen sand were presented. This indicated that in all but the most specialized cases, the intergranular contact forces do not constitute a state of equilibrium (Fig. 2.49). The nature of the stresses transmitted across the ice–water interface can be described by the Kelvin equation under isothermal conditions and the Clausius–Clapeyron equation when the process is not isothermal.

Table 2.9 summarized possible deformational mechanisms which may be occurring in frozen sand. In terms of the sand skeleton, the individual grains are probably undergoing some elastic and plastic deformations at the interparticle contact points. Additionally some crushing probably occurs. The rolling and sliding mechanisms discussed in the section on unfrozen sand may be occurring in frozen sand but the individual particles are probably not following the same trajectories (i.e., a frozen sand may not be able to follow the same strain history because of differences in the mechanics of deformation). In terms of the pore ice, the deformational mechanisms outlined in Table 2.2 are probably operating to some extent in the frozen sand system, but are modified by the presence of the silicate particles. A complete understanding of these mechanisms will require a significant amount of research. The presence of the unfrozen water film may enhance diffusional type mechanisms, limit the internal stresses caused by the pileup of dislocations and enhance dynamic recrystallization and grain boundary migration processes. Internal fracturing processes must be occurring when the frozen sand is dilating.

Possible mechanisms of strength were presented in terms of Ting et al.
These include those associated with the pore ice, the sand skeleton and the ice–sand interaction. The interaction between the ice and sand as presented by these authors focuses primarily on the particulate nature of the sand skeleton.

The pore ice strength may be different from that of normally tested polycrystalline ice due to the decrease in grain size and the possibly complex macrostructure (long tortuous crystals). This is further complicated by the indeterminate state of stress in the pores and the widely varying deformational constraints which may be imposed on the pore ice.

Enhanced strain rates in the pore ice due to the presence of essentially rigid sand grains may serve to increase the strength.

The interaction between the ice and sand was described in terms of the dilatancy of the sand skeleton. Ladanyi (1985) attempted to quantify this mechanism by predicting the strength of a frozen sand at the point where the pore ice begins to fracture (cavitate). The assumption is that the pore ice resists the dilatancy of the sand skeleton through tensile stresses. These are transferred back to the sand skeleton as increased effective stresses and hence result in a hardening effect. The effective stresses acting on the frozen sand skeleton are assumed to be identical to those acting on an unfrozen sand skeleton at the same state in undrained shear. Several potential deficiencies were regarding this method of characterizing the interaction between the sand particles and ice matrix. These included: 1) the effective stresses acting on the frozen sand skeleton are not in a state of equilibrium and are not similar to the effective stresses in an unfrozen system; 2) the Ladanyi model does not satisfy total stress equilibrium except under special circumstances; and 3) the level of strain required to mobilized the effective stress levels assumed to occur in the frozen sand is several times larger than that which actually occurs.
7.1.2 High—Pressure Low—Temperature Triaxial Testing System

Chapter 3 described the high—pressure low—temperature triaxial testing system constructed. It consists of a temperature control and measurement system, a pressure control and volume change measurement system and a high pressure triaxial cell and loading frame. Each of these were discussed along with their evaluated performance. Figure 3.5 presented a schematic diagram of the environmental chamber and triaxial cell. The ability of the system to control the temperature was demonstrated in Fig. 3.9 which shows the temperature fluctuations inside the cold room, inside the environmental chamber and inside the triaxial cell next to the specimen. The temperature near the top of the specimen was typically $-9.4 \pm 0.05^\circ C$ and $-9.7 \pm 0.05^\circ C$ near the bottom of the specimen.

The pressure volume control system was presented schematically in Figs. 3.10 and 3.11. By sensing the changes in the volume of the cell fluid and incorporating corrections to account for the leakage of oil, the triaxial cell and fluid compressibility, flexure of the base of the triaxial cell, and nonuniformities in the diameter of the cell pressure piston, the volume changes of the specimens were measured. Table 3.10 presented a summary of the range in the various corrections as a percentage of the specimen volume.

The triaxial cells used in this research were modified to accommodate internal force measurements, on—specimen axial strain measurements (the first time such have been used in the triaxial compression testing of frozen soils), internal temperature measurement, and lubricated end platens. These modifications have permitted a measurement of the behavior of frozen sand to a degree unparalleled in the literature. The loading frame was screw driven.
Figure 3.19 presented the axial strain versus time for a typical test. The strain rate increased slowly to a maximum and did not achieve this maximum until greater than 0.5% $\varepsilon_a$. There was roughly a one order of magnitude increase in strain rate from the beginning of the test to the maximum achieved rate.

The electronic sensing devices used to measure forces, pressures and displacements were individually calibrated using standard procedures. The displacement transducers were calibrated with either a veneer micrometer (small stroke) or a digital stage (large stroke). The load cell was calibrated in a dead weight frame up to an axial load of 1000lbs ($\frac{1}{10}$th) of its capacity. The corresponding calibration factor was assumed to be linear for the full range. The pressure transducers were calibrated using a static pressure device. With the exception of the $\pm 3''$ LVDT used to measure the position of the cell pressure piston, all devices were calibrated in the cold room at negative temperatures in the range of those investigated in this research. Table 3.11 summarized all of the calibration factors for these devices.

7.1.3 Materials, Procedures and Error Analysis

The material tested in this research was Manchester Fine Sand (MFS) obtained just north of Manchester, New Hampshire along the banks of the Merrimack River. Table 4.1 and Fig. 4.1 presented the results of gradation analyses of this material and compared them to prior testing programs. The current material was processed to remove a portion of the fines. The resulting processed batch was classified as SP-SM according to the Unified Soil Classification System. A mineralogical analysis on MFS performed by Martin et al. (1981) indicated that it is mostly quartz and feldspar with Potash Feldspar being the most abundant feldspar series. Clay minerals were present in the form
of mica, illite, vermiculite and chlorite and some magnetics were also present. The readily soluble salt concentration ranged from 0.018 to 0.06 gm/kg. The specific gravity was 2.688 ± 0.003. The maximum and minimum densities were determined according to ASTM D4253, Method 1.A and D4254-83 Method A. These were 1701 kg/m³ \( (e_{\text{min}} = 0.508) \) and 1408 kg/m³ \( (e_{\text{max}} = 0.909) \). The particles had a subangular shape. The unfrozen water content was approximately 0.14% at -4.89°C (Fig. 4.2).

These specimens were prepared by either wet tamping (seven tests) or multiple sieve pluviation. Each of these procedures were thoroughly discussed. The specimens were saturated under a vacuum and frozen unidirectionally in an open system from the top down. The bottom of the specimens were hydraulically connected to reservoir to allow for expulsion of water during the freezing process. The degree of saturation ranged from 96 to 100% using these procedures. Table 4.2 presented the physical properties measured for each specimen. A second order approximation to the variance of the saturation measure indicated a standard deviation of 0.55%.

The specimens were stored in plexiglas split sleeves covered with high vacuum silicone grease wrapped in Saran Wrap placed in a Ziplock bag with crushed ice and placed in an external freezer maintained at -20°C until testing. The maximum storage time was 4.5 months. The average storage time was 2 to 3 weeks.

Procedures for trimming the specimens and setting them up in the triaxial cell were outlined. The ends were trimmed with a stainless steel knife and checked for parallelism with a dial indicator. These were trimmed until parallel to within 0.001". The end conditions were varied from ice caps with grease (lubricated), to frozen sand and grease (frictional) to emory cloth
The density of the specimens was measured at various stages during the testing procedure. Table 4.3 summarized these measures for each specimen and presented the one selected as most representative. A second order errors analysis indicated that the standard deviation of the density measure was 2%.

The data acquisition and reduction procedures were outlined. Axial strains were inferred both from the on-specimen ACDT's (for small strains) and the LVDT sensing the position of the triaxial cell piston (for large strains). The specimen cross-sectional area was computed using a modified right cylinder correction in order to account for bulging in the center for under lubricated specimens and flaring of the ends for over lubricated specimens.

Several tests were repeated (some twice) in order to assess the precision of the measuring system. Table 4.5 summarized these tests in terms of parameters taken from the stress-strain curves and coefficients of variation. The stress-strain parameters were defined in detail in Section 4.6.1. Some of the factors that affected the repeatability of the test results were the behavior of the ACDT's and the stability of the specimens. In some cases, the ACDT's were in close agreement while in other cases they diverged during shear. Some of the specimens with lubricated ends slid off the base pedestal during shear. This resulted in spurious large strain measurements, particularly in terms of the axial strain at peak strength. In light of these factors, a best estimate of the precision of the measurement system was presented in Table 4.5.

Careful calibration procedures for the electronic sensing devices helped to ensure a high degree of accuracy in the calibration factors. A check on the volume change measure was performed by conducting a series of tests without a specimen and measuring the resulting volume change. Figure 4.21 presented the
results of these tests and indicated that the error in the volume change measure was probably in the range of ±0.2% of the specimen volume.

The area correction to account for nonuniform lateral deformations (i.e., bulging or flaring) ranged from ±5% at peak strength to ±11% at 20% axial strain for the worst cases. In the majority of the tests, the amount of the correction was much lower.

The modulus of an aluminum dummy specimen was measured using the triaxial testing system. This provided an independent check of the measuring system. The resulting stress–strain curves were presented in Figs. 4.13 and 4.14. The measured modulus was very close to that reported in the literature.

7.1.4 Behavior of Frozen Manchester Fine Sand in Triaxial Compression

Chapter 5 presented and evaluated the experimental results obtained in this research on the triaxial compression behavior of frozen Manchester Fine Sand. This represents the most extensive presentation of the stress–strain properties of a frozen soil ever compiled.

The primary variables studied were the relative density, the confining pressure and the strain rate. Several tests were performed to investigated the effect of end conditions, several to investigate the effect of specimen preparation techniques, and one to illustrate the effect of temperature. Table 5.2 summarized the scope of the program. The relative density varied from 20 to 100% and the confining pressure varied from 0.1 to 10 MPa. The strain rates tested were: $3 \times 10^{-6}$/sec (slow); $3 \times 10^{-5}$/sec (moderate); $1 \times 10^{-4}$/sec (intermediate, one test); and $4 \times 10^{-4}$/sec (fast).

An overview of the types of stress–strain curves was presented in Fig. 5.1. Most of these exhibited a distinct double yield behavior similar to that of
polycrystalline ice, i.e., the knee in the stress–strain curve was followed by strain hardening. Some showed continuous yielding at almost constant deviator stress after the knee. One showed continuous strain softening after the knee. In general, the specimens tested at low confining pressures experienced some strain softening after the peak while at the highest confining pressures there was no strain softening.

The small strain behavior was described in terms of the Young’s modulus, the proportional limit, and the deviator stress at a yield offset of $10^{-4}$ strain. The measured Young’s modulus had no clear trend with changes in relative density (Fig. 5.3), decreased slightly with confining pressure (Fig. 5.4) and was independent of the strain rate (Fig. 5.5). Both the proportional limit and yield offset stress at $10^{-4}$ strain were independent of relative density (Figs. 5.6 and 5.9, respectively), decreased with confining pressure (Fig. 5.4 and 5.7, respectively), and increased with strain rate (Figs. 5.8 and 5.11).

The large strain behavior was described in terms of the upper yield region, the strain hardening region, the peak strength region and the volumetric strain behavior.

The behavior in the upper yield region was summarized in Table 5.7. The upper yield stress was shown to be independent of relative density (Fig 5.14), decrease slightly with confining pressure (Fig. 5.15) and exhibit a strong sensitivity to strain rate with a power law coefficient of approximately 4.6. The axial strain at the upper yield stress was independent of the relative density and confining pressure (Figs. 5.17 and 5.18, respectively). It had a strong rate sensitivity with a power law coefficient similar to that of the upper yield stress (Fig. 5.18).

The behavior in the work hardening region was summarized in Table 5.8.
The rate of work hardening after the upper yield region increased linearly with relative density and exhibited a complex behavior as a function of confining pressure and strain rate (Figs. 5.21 and 5.22). The axial strain at the beginning of work hardening decreased with relative density and exhibited a complex behavior as a function of confining pressure and strain rate.

The behavior in the peak strength region was summarized in Table 5.9. The peak strength increased linearly with relative density (Figs. 5.33 through 5.36), increased in a mostly nonlinear fashion with confining pressure (Fig. 5.37) with total stress friction angles ranging from 1.6 to $15.6^\circ$ and showed a strain rate sensitivity with power law coefficients ranging from 6.1 to 16.7 (Fig. 5.38). The rate sensitivity decreased with increasing relative density and confining pressure as a greater component of frictional strength was mobilized. The axial strain at peak strength exhibited a complex behavior as a function of strain rate, increased uniformly with confining pressure and showed a complex behavior as a function of strain rate (Fig. 5.39 and 5.40).

The volumetric behavior was summarized in Table 5.10. It was different from that of frozen sands in that: 1) there was never a significant amount of volume decrease even at the highest confining pressures; and 2) the maximum rate of dilation occurred at very large strains well beyond the peak deviator stress for low confining pressure tests. See Figs. 5.26 through 5.32.

The rate of dilation at peak strength (Figs. 5.41 and 5.42), the maximum rate of dilation (Figs. 5.43 and 5.44) and the volumetric strain at 20% axial strain (Figs. 5.47 and 5.48) increased with relative density, decreased with confining pressure and showed very little effect of strain rate. The axial strain at the beginning of dilation showed a complex behavior with relative density, increased with confining pressure and decreased with strain rate (Figs. 5.45 and
The end conditions were shown to have little or no effect on most stress-strain parameters (Table 5.11). The axial strain at the peak strength decreased and the maximum rate of dilation increased for the frictional ends. The final deformed shape of the specimens was strongly affected by the end conditions. The lubricated specimens deformed either as right cylinders or with a slight amount of flaring at the ends while the specimens with frictional ends deformed by barreling (Figs. 4.6 and 5.49). The lubricated ends lead to more reliable large strain measures because of the uniform strain field and the lower amount of area correction required.

Most of the stress-strain parameters were unaffected by the specimen preparation technique. The wet tamped specimens, however, exhibited a significant increase in the rate of work hardening and a large decrease in the axial strain at peak strength (Fig. 5.50 and Table 5.12).

A decrease in the temperature lead to a dramatic increase in the stress-strain parameters that exhibited the strongest strain rate sensitivity, i.e., the upper yield point and the peak strength (Fig. 5.51 and Table 5.13).

7.1.5 Analysis of Results and Discussion of Possible Mechanisms

The measured small strain behavior was compared with prior testing programs in Table 6.1. The magnitude of the Young's modulus was within the range of the prior research programs. These a slight sensitivity of the Young's modulus with changes in relative density whereas this research showed no clear trend. The decrease in Young's modulus with confining pressure contradicts the Vinson (1978) trend but this may be due to plastic straining occurring in the latter.
The decrease in upper yield stress with confining pressure was opposite that observed in two prior testing programs and presented in Fig. 2.33. This may be due, however, to the relatively few number of prior observations that were in the range of confining pressures covered by this research.

The magnitude of the upper yield stress is about twice the peak strength of a normally tested polycrystalline ice (Fig. 6.1) but exhibits the same rate sensitivity. This indicates that the rate controlling processes for the upper yield behavior in frozen sand may be similar to that governing the peak strength of polycrystalline ice in the same range of strain rates. Thus the process(es) which increases the strength in the frozen sand over that of polycrystalline ice may not be rate dependent.

The linear increase in peak strength with relative density (Figs. 5.33 through 5.36) agrees well with prior published results (Fig. 2.37) except in the case where the increase caused a change in the deformational regime (Fig. 2.38).

The effect of confining pressure on the peak strength agrees well with the prior data in that it can exhibit both a linear and nonlinear behavior depending on the testing conditions. The current data is the first to present the effect of relative density on the total stress friction angle.

The rate sensitivity of the peak strength compares well with prior data, i.e., the power law coefficients are in or near the range of those reported. A comparison of these data with the prior M.I.T. program showed that in general they had the similar rate sensitivities (Fig. 6.2).

These results were compared with the results of an isostrain model proposed by Counto (1964) to predict the modulus and creep straining of concrete. In this model, the composite material is idealized as a cube of aggregate (sand grains) embedded in a cube of matrix material (ice) (Fig. 6.3).
An isostrain condition is imposed between the silicate cube and the ice and the corresponding "composite" modulus is computed from the assumed stiffnesses of the ice and silicate mineral. The predicted Young's modulus was essentially identical to the measured modulus and this illustrates the value of considering the frozen sand system as a composite material.

The results were also compared to the Ladanyi (1985) dilatancy hardening model. This model attempts to predict the strength of the frozen sand at the point where the pore ice begins to fracture. It focuses on the particulate nature of the sand skeleton and uses dilatancy principles in unfrozen soil mechanics. A complete evaluation of this model was not attempted, but Fig. 6.7 showed that it did not predict well the upper yield stress (except at one strain rate) nor the peak stress, both in terms of the magnitude and the rate sensitivity. In the writer's opinion, this is due to the natural limitations of attempting to model a frozen sand as a particulate material.

A discussion of the results and analysis focused on the possibility of understanding the behavior of frozen sand as a composite material. The stiffness of the frozen sand system (20 to 30 GPa) is much greater than that of polycrystalline ice (9 GPa) because the ice is reinforced by the stiffer silicate particles (90 GPa). This assumes that shear stresses are transmitted across the ice—silicate interface.

The strong dependence of the initial yielding behavior on the strain rate, its independence on the relative density and its slight dependence on confining pressure all indicate that the structure of the sand is probably not important in this region. The large strain behavior is due not only to the frictional nature of the sand skeleton but is also dependent on the behavior of the pore ice.

The idealized structure for a frozen sand present by Ting et al. (1983)
may not be appropriate because it overemphasizes the number of particle contacts. A cross section through a frozen sand would look more like Fig. 6.7 with sand grains suspended in an ice matrix.

There are significant unknowns in the behavior of frozen sand. One of these is the actual structure of the pore ice and another is the effect of decreasing grain size in the ice. These are potentially significant areas.

Consideration of the sand strength in terms of the particulate nature of the skeleton is fraught with uncertainty due to the indeterminate nature of the interparticle contact forces. Consideration of the ice–sand interaction in terms of the particulate nature of the sand skeleton does not seem appropriate because it attempts to draw an analogy between the effective stresses acting on the frozen sand skeleton and an unfrozen sand skeleton and these are different.

7.2 CONCLUSIONS

Based on this research, the writer makes the following conclusions:

1) A complete characterization of the stress–strain behavior of frozen soils is essential to understanding the mechanisms controlling its strength and deformational properties. The use of state–of–the–art testing equipment and procedures including: on–specimen axial strain measurements; lubricated end platens; internal force measurements; and careful specimen trimming and set up procedures have allowed the writer to observe behaviors never reported in the literature.

2) The modeling of a frozen sand as a particulate material may have limited applicability. Any formulation that relies on drawing an analogy between the effective stresses acting on similar frozen and unfrozen systems is fraught with
uncertainty. The mechanics of deformation in frozen sands is different than unfrozen sands due to the presence of the pore ice that can support shear stresses. The actual interparticle contact forces do not represent an equilibrium condition and hence are not related to effective stresses in unfrozen systems.

3) The modeling of a frozen sand as a composite material shows some promise. This provides another frame of reference for understanding the interaction between the sand particles and ice matrix. The simple cube—in—a—cube model proposed by Counto (1964) is able to make an excellent prediction of the initial modulus. The stiffness of the individual sand grain, not the sand skeleton, is important. The pore ice transfers a portion of its load to the stiffer silicate particles and this results in a composite modulus much larger than that for polycrystalline ice.

4) The small strain behavior of frozen sand is unaffected by the frictional behavior of the sand skeleton. The strength of the frozen sand system, at the upper yield point, is nearly double that of ice yet it exhibits the same rate sensitivity. The strengthening mechanism controlling the behavior in this region is probably not rate sensitive.

5) The large strain behavior depends both on the properties of the sand skeleton and pore ice. The frictional contribution to the total strength is not rate sensitive and hence, as this is mobilized, the rate sensitivity decreases (i.e., the power law coefficient increases). This explains why the power law coefficients for the peak strength of frozen sand are larger than that for ice. Processes that increase the amount of frictional strength (i.e., increases in the relative density or confining pressure) lead to increases in the power law coefficient.
7.3 RECOMMENDATIONS

The following are some recommendations for further research:

1) Continue testing frozen soils. Improve the testing system to eliminate the temperature gradient along the specimen in the triaxial cell. Increase the capacity of the loading system to allow for investigation of behavior at higher strain rates where the transition occurred between strain hardening and strain softening behavior after the upper yield point. Investigate the behavior of other frozen soils to observe the effect of soil type (i.e., the amount of unfrozen water, the size of the soil particles etc.). Varying the sand particle size may prove helpful in understanding the grain size effect of the pore ice, particularly its effect on the upper yield behavior. Investigate a range in temperatures from near zero to large negative temperatures.

2) Attempt to address fundamental questions such as: a) What is the behavior of the ice–silicate interface?; b) What is the behavior of the pore ice and how is this modified by the presence of the silicate particles?; c) What is the structure of the pore ice in frozen soils?; d) How are the ice and sand interacting at large strains?.

3) Pursue modeling studies of frozen soils as composite materials. Review prior literature on composite materials. Apply some of these to the frozen sand system and observe which ones appear to best capture the ice–sand interaction.

4) Investigate the applicability of Ladanyi’s particulate model at high strain rates.
CHAPTER 8

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IAHR = International Association for Hydraulics Research
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JGED = Journal of the Geotechnical Engineering Division
JSMFD = Journal of the Soil Mechanics and Foundations Division
SPE-AIME = American Institute of Mining, Metallurgical, and Petroleum Engineers


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