Essays on Competition and Financial Intermediation

by

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A.B., Economics
University of California, Berkeley, 1990

Submitted to the Department of Economics
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Abstract

This thesis studies a number of different issues on the interaction between competition and financing, both from the lender as well as the borrower's point of view.

Chapter 2 looks at how information may act as a barrier to entry in banking markets. If banks obtain information about borrowers after lending to them, they are able to reject riskier borrowers when refinancing. Potential entrant banks face an adverse selection problem stemming from their inability to distinguish new borrowers from old borrowers which have been rejected by other banks. This chapter show that, under Bertrand competition, an equilibrium with more than two banks does not exist (blockaded entry for a third bank).

Chapter 3 extends the analysis of the previous chapter by focusing on how information affects competition among banks. Here I show that borrower-specific information, to the extent that it is proprietary to a lending bank, becomes more disperse in more competitive banking markets, as each bank becomes informed about a smaller pool of borrowers. This reduces banks' ability to screen borrowers, leading to an inefficiency as more low quality borrowers are able to obtain financing. This effect may actually lead to higher interest rates as the number of banks increases. I also find that entry should be easier in rapidly growing markets or markets with high turnover among borrowers, where the large number of new borrowers diminishes the incumbent banks' informational advantage. The model has implications for whether financial liberalization or deregulation in the banking industry can actually be expected to lead to increased competition in loan markets, and what patterns of entry, if any, might be observed.

Chapter 4 analyzes a firm's choice between an inside (informed) or an arm's length (uninformed) lender as a function of the product market in which the firm operates. A lender's acquisition of information about a firm may destroy managers' incentives to perform since it diminishes the lender's ability to commit to carry out ex-post inefficient liquidation policies. Arm's length finance provides managerial discipline through its reliance on performance-based measures, which allows creditors to commit to liquidation policies that would be renegotiated were they better informed. Competition increases the effectiveness of uninformed finance as a disciplinary device by making observable signals, such as profits, more sensitive to the underlying cost structure, and hence more informative. This makes uninformed finance and competition complements in the provision of managerial incentives. Conversely, informed financing may be more important when current profits are a poor indicator of future profitability, or when creditors' reliance on performance based measures makes these firms vulnerable to aggressive behavior by rivals.

Chapter 5 studies a Bertrand model with asymmetric fixed costs. I find that competition among low cost firms may prevent high cost firms from entering, even if the difference in
costs is small. Specifically, I show that the two firms with the lowest fixed costs compete ignoring the threat of potential entry as all other firms find entry blockaded.

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Chapter 1

Adverse Selection as a Barrier to Entry in the Banking Industry

1.1 Introduction

In this paper\textsuperscript{1}, we investigate the effects of adverse selection on the market structure of the banking industry. By endogenizing entry, we are able to focus on the equilibrium industry structure when information asymmetries play an important role in the market. Specifically, we ask whether informational asymmetries in the banking industry can create a barrier to entry for other banks. We have in mind a situation where existing (i.e. "incumbent") banks in a market have an informational advantage over other potential lenders (i.e. "entrants") by virtue of their established relationships with borrowers seeking credit, and are able to use this advantage to prevent other banks from entering their market. Potential entrants into a banking market suffer an adverse selection effect stemming from their inability to determine whether applicant borrowers are new borrowers seeking financing for their untested projects, or whether these are in fact borrowers that have been previously rejected by an incumbent bank, and are looking elsewhere for financing.\textsuperscript{2} This puts entrants into a worse position relative to the incumbents, and may lead to diminished or deterred entry.

Asymmetric information is a driving force in this kind of analysis. Banks considering offering funding to applicant borrowers are always faced with an information problem to

\textsuperscript{1}This chapter was written jointly with Giovanni Dell'Ariccia and Ezra Friedman.

\textsuperscript{2}Another way of stating this is that an incumbent's superior information about its own clients allows it to cream-skim the best borrowers and weakens a competitor's ability to offer low interest rates.
the extent that managers and/or owners are usually more informed about their own firms than the lenders, and hence banks will usually wind up making some loans that are ex-ante unprofitable (as opposed to loans that may not pay off ex-post, even though they have a positive expected return). This is a problem that applies to all lenders when faced with a credit prospect about whom they have limited information. However, this effect need not be of the same magnitude for all borrowers as some banks may have better knowledge about some borrowers than others, and may also differ across banks, as each bank may have superior information about different borrowers than its competitor banks. It is precisely this kind of informational asymmetry of banks vis-à-vis each other on which we focus. Banks obtain information about prospective clients from previous lending arrangements, and are therefore better able to distinguish the good and the bad risks among borrowers with whom they have established a relationship than among borrowers that are new and unknown to them. Hence, we argue that banks that have been around longer may have a greater informational advantage if they have made loans to more borrowers than younger banks have, and also that any existing banks should possess an advantage over entrant banks, which have no private knowledge about borrowers. Note that the incumbents’ comparative advantage is not driven by the informational asymmetry that exists between banks and borrowers, but rather by the asymmetry of information between banks, and all that we require for our results on entry is that some amount of heterogeneity among borrowers exist.

A large literature has recognized that, in a variety of settings, perfect competition is precluded by asymmetric information. The fact that search and screening are costly and that information may be obtained in the course of an exchange relationship implies that new firms will be imperfect substitutes for old firms. For example, Schmalensee (1982) analyzes the case where buyers are uncertain about the quality of the product of each particular producer, and can resolve that uncertainty only by buying and trying the product. He shows that, in this setting, an incumbent firm (a “pioneering brand”) may be immunized from competition by a potential entrant, by virtue of the superior information consumers have about the incumbent’s product. One of the main points of this paper is to explore a new channel through which imperfect information undermines competition.

A paper similar in spirit to ours is Broecker (1990), which analyzes a competitive credit market where banks have the ability to perform binary credit worthiness tests on applicant
firms, and offer credit conditional on the realization of this test. He finds that as long as this test is imperfect and independent across banks performing it, then increasing the number of banks can have adverse effects on the equilibrium interest rates that are obtained. In Broecker’s model, this effect arises because with more banks performing independent tests, the average credit worthiness of firms that pass at least one test is decreasing in the number of banks. Whenever a firm accepts the highest possible interest rate offer (the least attractive for it), it must have been rejected by all other banks, and therefore represents a very bad risk on average (banks face a “winner’s curse”). This is similar to the adverse selection effect that we envisage, where raising the interest rate a bank offers only leads to worse and worse risks on average, as a bank begins to attract only the most risky borrowers that are being held by other banks.\(^3\) Hoff and Stiglitz (1997) analyze the effects of credit subsidies in a banking market. Focusing on moral hazard rather than adverse selection, they obtain a result similar to Broecker’s that competition may have a negative and perverse effect on the equilibrium interest rate. However, in both of these papers the number of banks is exogenous, so that the effect of information asymmetries on the structure of the industry is not analyzed.

Another related paper is Riordan (1992), who offers an analysis similar to that performed by Broecker (1990). Using an application of auction theory, he finds that there is a cutoff value of the quality signal arising from the credit worthiness test (assuming the quality signal is continuous) above which borrowers are offered financing. Increasing the number of banks has the perverse effect of increasing this cutoff value, so that each borrower finds it more difficult to obtain financing from any particular bank. A somewhat different perspective is offered by Sharpe (1990), where information asymmetries among banks and competition between them leads to inefficiencies in lending. In particular, Sharpe (1990) finds that banks may offer borrowers lower introductory rates since they know that they will have an informational advantage vis-à-vis other banks and so will be able to extract surplus from good borrowers in future periods. Finally, Rajan (1992) considers the financing decision from the point of view of the firm, which has the incentive to prevent banks from obtaining private information regarding its projects and using that information to extract surplus from the firm. The main difference between our analysis and that found in most of the previous literature is that we focus on the organizational structure of the banking industry

\(^3\)This is also similar to the credit rationing model of Stiglitz and Weiss (1981).
itself, and the forces that might lead to deterred entry.

The main result we obtain in this paper is one of blockaded entry once there are already two banks serving a particular credit market. When banks actively compete for customers by offering them lower interest rates, an equilibrium with homogeneous borrowers or with symmetric information would be characterized by each bank's profits being reduced to zero, and with no further entry by new banks if entry must be preceded by a decision to incur a sunk cost. However, we find that if we allow banks to be asymmetrically informed regarding borrowers' credit worthiness, the limited entry result holds even in the absence of any sunk cost of entry or fixed cost of operation. Moreover, the equilibrium we obtain even for the two bank case is no longer characterized by a zero profit condition for each bank, but rather allows each bank's profit to depend on its informational advantage.

We find that with just two banks, competition will drive the profits of the smaller of the two banks down to zero. In the context of our model, we will interpret the smaller bank as the one who had granted loans to a smaller fraction of the population of borrowers in the past, and hence is more "informationally challenged". The profits to the larger bank will then be determined by the extent to which it has superior information about potential borrowers. The intuition for our result of blockaded entry can now be gleaned from this argument. A potential entrant bank is in a worse informational position than either of the incumbents. Any action by an entrant could of course always be mimicked by an incumbent to yield it strictly higher profits (since, by virtue of its tenure in the market, it suffers less from the adverse selection of having to finance bad borrowers). Since the only possible equilibrium will involve the second largest bank obtaining zero profits, this implies that the entrant would make negative profits.4 Thus, it can never be an equilibrium for another bank to enter a market where two rival banks are already active. In what follows, we formalize these arguments and demonstrate our result regarding blockaded entry.

The rest of the paper proceeds as follows. Section 2 presents the basic setup of the model. Section 3 characterizes the equilibrium that obtains with just two banks. Section 4 contains the main results of this paper. There we show that the equilibrium we obtained for just two banks is also an equilibrium to the game with three banks, thus leading to

---

4Hendricks, Porter and Wilson (1994) analyze auctions where an informed and an uninformed buyer bid for an object of unknown value. Using an argument similar to ours, they show that in equilibrium the uninformed buyer has to make zero profits, while the informed buyer makes positive profits thanks to his superior information.
deterred entry for the third bank. Moreover, we demonstrate that entry by a third bank is blockaded after two banks are already in the market. We provide an example where we explicitly compute the equilibrium strategies for each bank under the assumption that there is a uniform distribution of borrowers in section 5. Section 6 concludes with a discussion of the key results and their applicability to other contexts, as well as some possible extensions under consideration.

1.2 Model

We assume that there is a continuum of borrowers, each with an investment project that requires a capital inflow of $\mathcal{K}$, which we normalize to 1, but have no private resources, so that they must look to banks to obtain this financing. This project pays off an amount $R$ with probability $\theta$, and 0 with probability $1 - \theta$, and we assume that this outcome is perfectly observable and contractible by both parties, but that the parameter describing the probability of success, $\theta$, is unknown to either the borrower or the bank before entering into a credit relationship. Borrowers are heterogeneous in their probability of success $\theta$, with a distribution in the population given by the distribution function $G(\theta)$. We assume that once a borrower obtains a loan from a given bank, that bank learns the borrower's type $\theta$, but is unable to credibly communicate it to other banks (this information becomes private to the two parties in the relationship - the bank and the borrower).

The market is composed of two groups of borrowers: $\lambda$ new borrowers and $1 - \lambda$ old borrowers. Both of these groups have the same distribution over types given above.\(^5\) Essentially, we are assuming that a mass of size $1 - \lambda$ of borrowers is already in the market and seeking refinancing (and hence their type is already known by one bank) and a mass $\lambda$ of borrowers are seeking a loan for the first time.\(^6\)

As previously stated, we assume that banks compete in a Bertrand fashion over interest rates for the pool (of size $\lambda$) of new borrowers, and are able to charge differential rates to their pool of remaining old customers (for which the bank already knows the borrower's

\(^5\)While this is a static framework, it lends itself to a dynamic interpretation where, in each period, a share $\lambda$ of the population dies (exits the market), and is replaced by an equal mass of new borrowers that have the same distribution over types. In the appendix we present a simple example of a two-period game with these characteristics, and show that our main results continue to hold.

\(^6\)For new borrowers we mean untested ones that are applying for credit for the first time. This is not necessarily related to age, as it could be firms that until that moment have auto-financed their operations, or firms that have just relocated into the area, etc.
probability of success). Banks choose their interest rates from the set \([0, R] \cup \{D\}\), where \(D\) represents not offering a loan (denying credit). We assume that borrowers are free and able to switch banks in order to obtain the “market rate” that is being offered to potential new borrowers by a competitor bank, and will do so if that rate is lower than the one currently being offered to them by their current bank (so we are inherently assuming that borrowers have the last move in this setup).

We assume that banks are unable to distinguish between new borrowers and borrowers that are being rejected by a competitor bank or who are simply switching banks to take advantage of lower rates. This assumption may seem somewhat extreme in light of the fact that generally borrowers carry with them any kind of credit history they have earned, and that this history is usually publicly available to any new bank, so that in particular a bank should be able to tell whether a borrower has had a previous banking relationship. We defend our assumption by arguing that it captures the stylized notion that a borrower’s old bank may know more than what is available on a credit record, either from monitoring or having access to books or by simply being able to better observe the kind of projects in which a borrower invests.\(^7\) In this sense, the borrower’s old bank has an informational advantage, and a new bank is only able to less precisely determine an applicant borrower’s type, and may not have much more information about that borrower than about one for which it knows nothing.\(^8\)

In modeling the extensive form of this game, we have focused on a two-stage game in which all banks first simultaneously choose an interest rate for the free market. Then, after observing the realized rates for all banks, they simultaneously choose interest rates for their old customers. The story we have in mind is that borrowers are able to observe the “market rate” that they could obtain if they go elsewhere, and use that to bargain for lower rates from their banks if their bank wants to keep them.\(^9\) Note that we assume that the borrower

\(^7\)See also the explanation at the end of section 3.

\(^8\)We assume that banks are not limited by capacity (on this market at least). We have in mind generally small markets that may be subject to entry by competitor banks.

\(^9\)Greenwald (1986) uses a similar extensive form in his analysis of adverse selection in the labor market. This setup can be regarded as the result of a traditional market mechanism. The free market rate can be thought of as the rate offered by Bertrand competitors to borrowers seeking financing. In Greenwald, it is the wage that would be offered to workers in a traditional Walrasian auction market. Borrowers behave competitively by simply taking the lowest rate offered to them. However, banks faced with these conditions are able to use their “inside” knowledge of borrowers’ qualities to maximize profits. As we will see, this will lead them to offer “competitive” rates to borrowers they wish to keep, and deny credit to low quality borrowers.
acts last by choosing the lowest available interest rate.\(^{10}\)

1.3 Equilibrium with two banks

As described above, we assume that banks have two sequential moves, where they first all simultaneously choose an interest rate to charge to the free market, and then, after observing everyone's market rate, they all simultaneously choose an interest rate to charge to their old customers. Borrowers act last by choosing the lowest interest rate offered to them. Again, the justification for this is simply that any borrower \(\alpha_i\) use the threat of leaving and obtaining the free market rate in order to bargain for a lower rate (or at least a rate that is no higher) by their current bank. In this section we provide a characterization of the equilibrium for an arbitrary distribution of borrowers' types, \(G(\theta)\). In section 5, we provide an explicit example for the case where \(\theta\) is distributed uniformly between [0, 1].

We begin by stating some specifications that will be used throughout the analysis. Suppose that there are two banks in the market, each with a share of the \(1 - \lambda\) old borrowers equal to \(\alpha_i, i = 1, 2\), and assume WLOG that \(\alpha_1 > \alpha_2\) (where \(\alpha_2 \equiv 1 - \alpha_1\)). The banks must now compete for the \(\lambda\) new borrowers and try to hang on to their old existing clients if they are good.

We solve the game by backward induction. We first need to characterize the equilibrium of the subgame after banks submit a bid to the free market, when banks are bidding for their old customers. Then we solve the first stage, where banks compete for new borrowers. Let \(S_i\) be the interest rate charged by bank \(i\) to the free market, and let \(S_{i\theta}\) be the interest rate charged by bank \(i\) to an old customer of type \(\theta\). \(S_i\) denotes a gross interest rate, that is, net interest plus principal (here equal to one). Without loss of generality, we normalize the net interest rate on bank funds (or the bank's opportunity cost) to zero. We then have the following result:

Claim 1 (1) All old borrowers (i.e., already known by bank \(i\)) for whom \(\theta \geq \frac{1}{S_j}\) will be charged \(S_{i\theta} = S_j\) (good borrowers are charged a rate equal to the rate charged by the competitor bank to the free market); and (2) borrowers known by bank \(i\) for whom \(\theta < \frac{1}{S_j}\) will

\(^{10}\)If an old borrower can get the same interest rate from both banks, we assume that it stays with its current bank, while if there is a tie in the free market, all banks split the market equally. The idea is that if there are (epsilon) positive switching costs for the borrowers to change banks, they prefer to borrow from their old bank for the same interest rates.
Claim 1 holds because $S_j$ is the maximum bank $i$ can charge its old customers without losing them to its rival bank, bank $j$. For borrowers whose quality parameter $\theta$ is less than $\frac{1}{S_j}$, bank $i$ clearly loses money by offering them credit at an interest rate of $S_j$, and any other higher offer will be rejected by these borrowers since they can always obtain $S_j$ elsewhere. Therefore bank $i$ simply does not provide credit to any of these low quality borrowers.

Using this simple claim which effectively characterizes the equilibrium of the subgame, we can now characterize the equilibrium of the whole game. Notice that claim 1 gives us the result that high quality old borrowers are charged at least as much as new borrowers, or, conversely, that new borrowers are offered a lower introductory rate in order to attract their business. In a dynamic setting, Sharpe (1990) finds that asymmetric information and competition leads banks to charge borrowers a lower interest in the first period of their relationship, even though they are able to extract surplus from the good borrowers in future periods. Our result, while derived from a purely static model, goes in the same direction.

From the discussion above, it is clear that the profits each bank makes from its old clients are a function of its own market share and of the interest rate charged by the competitor bank to new borrowers. These profits (for bank $i$) are given by:

$$\pi_i^{old}(S_j) = \alpha_i(1 - \lambda) \int_{\frac{1}{S_j}}^{1} (S_j \theta - 1)g(\theta)d\theta$$  \hspace{1cm} (1.1)

From equation (1.1) we see that the profit each bank makes from its old borrowers does not depend on the rate that they charge their own new customers. Therefore, this component of profits does not play a role in each bank's optimal choice of interest rate on the free market, as profits on old clients do not appear in the first order conditions for profit maximization in the first stage of the game. For this reason, we drop the term reflecting profits from old borrowers in the subsequent analysis, and only consider the component of each bank's

---

11The fact that all old borrowers that are "good enough" are charged the same (high) interest rate $S_\theta = S_j$ is an artifact of the fact that outside banks have no information about borrowers, and that we are only in one period. With multiple periods (an extended horizon for the borrowers), we would obtain that banks need to offer their good borrowers a discount early on in order to keep them, and that the size of this discount would depend on the type of the borrower.

12In the appendix we provide a two-period model with precisely this characteristic, that banks offer low introductory rates to customers in order to be in a position to extract rents in the future.
profits on the free market.

Therefore, consider the competition for the free market. In this market, banks compete for the new borrowers (of mass \( \lambda \)), and they always get the rejects from the other bank. Suppose that bank \( i \) charges \( S_i \). Define \( m(\theta) \) as the average quality of borrowers with quality parameter less than \( \theta \): 
\[
m(\theta) = \int_{0}^{\theta} \frac{tg(t)}{G(\theta)} dt.
\]
Then, using claim 1 above, its free market profits are:
\[
\pi_i(S_i|w) = \lambda \int_{0}^{1} (S_i \theta - 1)g(\theta)d\theta + \int_{0}^{\frac{1}{S_i}} \alpha_j(1 - \lambda)(S_i \theta - 1)g(\theta)d\theta
\]
\[
= \lambda(S_i \bar{\theta} - 1) + \alpha_j(1 - \lambda)G \left( \frac{1}{S_i} \right) \left( S_i m \left( \frac{1}{S_i} \right) - 1 \right)
\]
(1.2)

\[
\pi_i(S_i|l) = \int_{0}^{\frac{1}{S_i}} \alpha_j(1 - \lambda)(S_i \theta - 1)g(\theta)d\theta = \alpha_j(1 - \lambda)G \left( \frac{1}{S_i} \right) \left( S_i m \left( \frac{1}{S_i} \right) - 1 \right),
\]
(1.3)
where \( g = G' \) and \( \bar{\theta} = \int_{0}^{1} \theta g(\theta)d\theta = m(1). \) \( \pi_i(S_i|w) \) represents the bank's expected profits conditional on winning the free market, and \( \pi_i(S_i|l) \) is the bank's profit conditional on not obtaining (“losing”) the free market. Note that the equation (1.3) is negative by the result that competitor banks only cast out those borrowers for which \( \theta S_i < 1 \). We can now state the following result.

**Lemma 1** There does not exist an equilibrium with banks playing a pure strategy on the free market as long as \( \pi_i(R|w) > 0, i = 1, 2. \)

**Proof:** Clearly, the strategies where neither bank bids on new market cannot be an equilibrium as long as \( \pi_i(R|w) > 0 \). Similarly, any set of strategies where one bank, say bank \( i \), does not bid on free market and where \( S_j < R \) cannot be an equilibrium, since given bank \( i \) does not bid, bank \( j \) can always increase profits by increasing \( S_j \) up to \( R \). Consider the strategies where bank \( i \) does not bid, and \( S_j = R \). Bank \( i \) then makes zero profits on this market, and \( \pi_j(R) > 0 \). But then bank \( i \) could charge \( S_i = R - \epsilon \) obtain \( \pi_i(R - \epsilon|w) > 0 \), and do strictly better for small enough \( \epsilon \). Finally, consider the possible equilibrium \( (S_i, S_j) \), and WLOG assume that \( S_i > S_j \). As shown above, \( \pi_i(S_i|l) < 0 \). If \( \pi_j(S_j|w) > 0 \), and if \( \pi_i(S_j|w) > 0 \), then bank \( i \) prefers to charge \( S_i = S_j - \epsilon \). Otherwise, bank \( i \) prefers to not enter market at all. Therefore no equilibrium exists in pure strategies. □
From the above it can be verified that it is precisely the adverse selection in the competition for borrowers that leads to the non-existence of a pure strategy Nash equilibrium. Because the bank with the higher interest rate winds up making loans only to low quality borrowers and making negative profits, no pure strategy equilibrium could specify a different interest rate for each bank. At the same time, both banks bidding the same interest rate also cannot be an equilibrium. In that case, each bank would have an incentive to undercut slightly and obtain all the new borrowers (instead of dividing them up among the tying banks) and hence improve its distribution of borrowers to which it grants loans.

The non-existence of a pure strategy equilibrium is a standard feature of Bertrand games with fixed costs. The interpretation of mixed strategies is not always straightforward. In this context we can imagine that the randomization is the result of a process in which banks bargain over the interest rate separately with each individual client, where clients are heterogeneous in their bargaining skills, and these skills are uncorrelated with their type.\footnote{This is similar also to other results obtained in the literature, such as in Broecker (1990), or to the result obtained by Rajan (1992) when he allows firms to seek competitive bids from many banks. He argues that it is not uncommon to see firms seeking sealed bids from a number of different banks, and that this kind of behavior leads to mixed strategy equilibria.}

However, while no equilibrium exists in pure strategies, we do find an equilibrium in mixed strategies where both banks either mix continuously over some range \([\underline{r}, R]\), or simply do not bid at all. Establishing the details of the equilibrium will provide a stepping stone for the analysis that is central to this paper, the case where we allow for a third possible entrant bank into this market.

**Proposition 1** A unique equilibrium to the two-stage game exists and is characterized by a distribution function over strategies (interest rates and credit denial probability) for each bank, \(F_i(S), i=1,2\). The equilibrium has the following properties:

1. The bank with the smaller market share (bank 2) makes zero profits. The bank with the larger market share (bank 1) makes strictly positive profits.

2. Conditional on offering credit, both banks play completely mixed strategies over the interval \([\underline{r}, R]\).

3. There exist no atoms in the mixing probabilities of either bank over the interval \([\underline{r}, R]\).
4. The bank with the smaller market share (bank 2) “stays out” with positive probability (i.e. does not bid on the free market with some probability, so that \(1 - F_2(R) > 0\), and the probability that bank 2 plays the strategy \(D\) is positive).

5. Bank 1 bids \(S_i = R\) with positive probability (\(\mu_1(R) > 0\)).

**Proof:** By claim 1 above, bank \(j\) will charge its old customers \(S_i\) if \(\theta \geq \frac{1}{S_j}\) and will charge \(r > S_i\) (or deny credit) if \(\theta < \frac{1}{S_j}\). This implies that bank \(i\) gets all \(\theta\) belonging to bank \(j\) for whom \(\theta < \frac{1}{S_j}\). For a bid by bank \(i\) of \(S_i\), we obtain the payoffs:

\[
\pi_i(S_i) = \begin{cases} 
\lambda(S_i \theta - 1) + \alpha_j(1 - \lambda)G\left(\frac{1}{S_j}\right)\left(S_i m\left(\frac{1}{S_i}\right) - 1\right) & \text{if } S_i < S_j \\
\frac{1}{2} \lambda(S_i \theta - 1) + \alpha_j(1 - \lambda)G\left(\frac{1}{S_i}\right)\left(S_i m\left(\frac{1}{S_i}\right) - 1\right) & \text{if } S_i = S_j \\
\alpha_j(1 - \lambda)G\left(\frac{1}{S_i}\right)\left(S_i m\left(\frac{1}{S_i}\right) - 1\right) & \text{if } S_i > S_j 
\end{cases}
\]

Since the action space is a real interval, given by \([r, R]\) (to be defined), we can observe that this payoff function (and consequent game) satisfies the conditions in Dasgupta and Maskin (1986) for the existence of a mixed strategy equilibrium. Using the definition of \(F_i(s) = \text{prob}(S_i \leq s)\), we define \(\text{prob}(S_i < s) = F_i(s) - \mu_i(s)\), where \(\mu_i(s)\) represents the mass \(F_i\) puts on \(s\), if any. We then have the following important lemma, the proof of which can be found in the appendix.

**Lemma 2** \(F_i(s)\) and \(F_j(s)\) will be continuous and strictly monotone increasing on an interval \((r, R)\) (i.e. \(f_i(s) > 0 \forall s\), so that \(\beta\) an interval \((s_1, s_2) \subseteq (r, R)\) where \(f_i(s) \equiv 0 \forall s \in (s_1, s_2)\)).

We can now write

\[
\pi_i(S_i) = \lambda(1 - F_j(S_i))(S_i \theta - 1) + \alpha_j(1 - \lambda)G\left(\frac{1}{S_i}\right)\left(S_i m\left(\frac{1}{S_i}\right) - 1\right) \quad \forall s \in [r, R) \quad (1.4)
\]

Since \(m(\frac{1}{S_i}) < \frac{1}{S_i}\), the second term is negative, so that it will be optimal for bank \(i\) to enter only if \(\lambda(1 - F_j(S_i))(S_i \theta - 1) \geq -(1 - \lambda)\alpha_j G(\frac{1}{S_i})(S_i m(\frac{1}{S_i}) - 1)\). In order for this to continue to hold as \(s \to R\), we require that

\[
\lim_{s \to R} (1 - F_j(s)) \geq \frac{(1 - \lambda)}{\lambda} \frac{\alpha_j G(\frac{1}{R})(R m(\frac{1}{R}) - 1)}{R \theta - 1}
\]
But the only way for \( \lim_{s \to R} (1 - F_j(s)) > 0 \) is if either there is an atom at \( R \) in \( F_j \) \( (\mu_j(R) > 0) \), or if bank \( j \) does not always bid on the free market \( (1 - F_j(R) > 0) \). There cannot be an atom in both \( F_1 \) and \( F_2 \) since then neither \( S_2 = R \) nor \( S_1 = R \) would ever be optimal. Therefore at least one bank must be not entering the free market with positive probability. Note that this establishes that at least one bank is making zero expected profit. Let us call this bank 2 (the smaller bank), and we will later show that it must indeed be the smaller of the two banks.

We can now use the zero profit condition for bank 2 to solve for \( \tau \), since \( \tau \) must satisfy:

\[
\lambda (\tau \bar{\theta} - 1) + \alpha_1 (1 - \lambda) G \left( \frac{1}{\tau} \right) \left( \tau m \left( \frac{1}{\tau} \right) - 1 \right) = 0 \quad (1.5)
\]

because at \( \tau \), bank 2 wins the free market with probability one.\(^{14}\) That bank 2 must indeed be the zero profit bank can now be verified by comparing equation 1.5 with equation 1.6 below, the profits for bank 1, and noting that the profits for bank 2 are unambiguously lower at \( \tau \) given that \( \alpha_2 < \alpha_1 \).

We then have that, for bank 1,

\[
\pi_1(\tau) = \lambda (\tau \bar{\theta} - 1) + \alpha_2 (1 - \lambda) G \left( \frac{1}{\tau} \right) \left( \tau m \left( \frac{1}{\tau} \right) - 1 \right) \quad (1.6)
\]

and using condition above for bank 2, we can rewrite this as:

\[
\pi_1(\tau) = - (\alpha_1 - \alpha_2)(1 - \lambda) G \left( \frac{1}{\tau} \right) \left( \tau m \left( \frac{1}{\tau} \right) - 1 \right) = \left( 1 - \frac{\alpha_2}{\alpha_1} \right) \lambda (\tau \bar{\theta} - 1) \equiv \bar{\pi},
\]

which is greater than zero, and specifies the positive profits that bank 1 makes. Bank 1 must make identical profits \( \forall s \in (\tau, R) \), since bank 1 is mixing in equilibrium. We can now

\(^{14}\)Note that for \( \lambda \) very small, \( \tau > R \), so that in effect there is no interest rate at which bank 2 can make non-negative expected profits, even assuming it wins the market with probability 1. Call such a cutoff value \( \Lambda(\alpha_1) \). That it is a function of \( \alpha_1 \) (given a fixed \( R \)) is clear from equation (1.5). As a matter of fact, the exact condition that we need for this result is that \( \pi_2(R|w) > 0 \) (that the profits of bank 2 when it charges the highest interest rate \( R \) and wins the free market be positive,) which we can solve to obtain:

\[
\Lambda(\alpha_1) = \frac{\alpha_1 G \left( \frac{1}{R} \right) (1 - R m \left( \frac{1}{R} \right))}{R \bar{\theta} - 1 + \alpha_1 G \left( \frac{1}{R} \right) (1 - R m \left( \frac{1}{R} \right))} > 0
\]

If this condition, that \( \lambda > \Lambda(\alpha_1) \), is not satisfied, then a monopoly will be the only viable market structure. Note that this is always less than 1 if \( G \left( \frac{1}{R} \right) > 0 \), so for any distribution of types such that \( G \left( \frac{1}{R} \right) > 0 \) there is a \( \lambda \) which results in a duopoly. \( \Lambda(\alpha_1) \) is an increasing function so there are values of \( \lambda \) such that if \( \alpha_1 \) is below a critical value the second bank enters, but if \( \alpha_1 \) is higher, then the second bank never enters, suggesting the possibility of path dependence in the market structure.
use $\bar{\pi}$ to solve for $F_2(s)$ directly as:

$$
\pi_1(s) = \bar{\pi} = \lambda(1 - F_2(s))(s\bar{\theta} - 1) + \alpha_2(1 - \lambda)G\left(\frac{1}{s}\right)\left(sm\left(\frac{1}{s}\right) - 1\right)
$$

$$\Rightarrow F_2(s) = 1 + \frac{\alpha_2(1 - \lambda)G\left(\frac{1}{s}\right)(sm\left(\frac{1}{s}\right) - 1)}{\lambda(s\bar{\theta} - 1)} - \frac{(1 - \frac{\alpha_2}{\alpha_1})(r\bar{\theta} - 1)}{s\bar{\theta} - 1}
$$

(1.7)

We can similarly solve for the other mixing probability, using the zero profit condition for bank 2.

$$0 = \lambda(1 - F_1(s))(s\bar{\theta} - 1) + \alpha_1(1 - \lambda)G\left(\frac{1}{s}\right)\left(sm\left(\frac{1}{s}\right) - 1\right)
$$

$$\Rightarrow F_1(s) = 1 + \frac{\alpha_1(1 - \lambda)G\left(\frac{1}{s}\right)(sm\left(\frac{1}{s}\right) - 1)}{\lambda(s\bar{\theta} - 1)},
$$

(1.8)

which concludes the proof with the equilibrium characterized by $F_1, F_2$. □.

This equilibrium fits well with our intuition and with other known results for Bertrand competition. We can illustrate this by doing some comparative statics on $F_i$, and observing how we converge to standard results as we vary the proportion of new borrowers in the market. As $\lambda$ increases, we observe that $F_i(s)$ increases for any $s$. This implies that an increase in $\lambda$ leads to a decrease in the expected free market rate. This is comforting, since it implies that a drop in the importance of the adverse selection term (as proxied by $\lambda$) leads to an increase in competition and a fall in the expected interest rate. The decrease in the market rate coincides with a decrease in the rents being extracted from high quality old borrowers. Notice further that in the limit, as $\lambda \to 1$, $F_i(s) \to 1 \forall s$ in the interior of the support of the distribution. This essentially says that we are concentrating all mass on the bottom of the support, or in other words that we are coming closer to playing a pure strategy. To see what this strategy is, we look at what happens to $r$. From equation (1.5), we see that as $\lambda \to 1$, $r$ has to satisfy $\lambda(r\bar{\theta} - 1) = 0$, or that $r = 1/\bar{\theta}$. This is what we should expect, that we obtain a pure strategy equilibrium where everyone bids $S_i = S_j = 1/\bar{\theta}$ and breaks even: as the proportion of new borrowers in the market goes to 1, the proportion of old borrowers goes to zero, and there is no longer any adverse selection effect from other bank's rejectees.
It is worth noticing that the total profit of the industry increases with $\alpha_1$, the large bank’s previous market share, or size. In other words, the banking system becomes less competitive as the two banks become more asymmetric. This is primarily because the smaller bank’s costs rise since it faces a higher adverse selection cost stemming from the larger bank’s rejected customers. Thus the smaller bank’s disciplining effect on bank 1’s pricing decreases. An example of this equilibrium is provided in section 5, where we explicitly compute the equilibrium strategies for the case where $\theta$ is uniformly distributed between 0 and 1.

We should also point out that while the assumption that a bank is unable to ascertain whether an applicant borrower has had a previous banking relationship or not is important to our analysis, it is not crucial that it take anywhere near so extreme a form. We could just as well allow banks to review the credit histories of applicant borrowers. Then, as long as this review process comes at a positive per borrower cost, we immediately obtain that entrant banks face higher costs of reviewing these records than incumbent banks, since there are more borrowers that are unknown to the entrant. With this setup, our main result concerning blockaded entry (demonstrated in the next section) continues to hold as stated.\textsuperscript{15}

1.4 More than two potential banks: Blockaded entry

In the previous section we characterized the equilibrium for the case when there are two banks in the market. However, we did not consider explicitly the possibility of entry by new competitors. In this section we show that our equilibrium is still valid when we assume the existence of a third potential entrant with zero market share. Moreover, we show that the equilibrium described in section 3 is in fact unique. In other words, using Bain’s terminology, we show that we are in a situation of blockaded entry.

The proof of proposition 1 from the last section provides us with an intuition as to why,

\textsuperscript{15}Suppose you have to pay a screening cost $c$ per borrower, and this allows you to observe whether they have newly left a competitor bank, or are new to the market. Then the cost for each bank will be:

$$
C_1 = \lambda c + \alpha_2 c(1 - \lambda)G\left(\frac{1}{S_1}\right)
$$

$$
C_2 = \lambda c + \alpha_1 c(1 - \lambda)G\left(\frac{1}{S_2}\right)
$$

and as long as $\alpha_1 > \alpha_2$, we will have that $C_1 < C_2$, and the result of blockaded entry will continue to hold. We can then interpret our approach as saying that the cost of getting information about a borrower is to make them a loan.
given the equilibrium of section 3, a third potential bank finds entry blockaded and is unable to penetrate the market without incurring losses. The reason is that a third potential bank considering entry faces a significantly different distribution of borrowers than the two current incumbents, and given that competition à la Bertrand by the incumbents already forces one bank’s profit down to zero, a potential entrant can only expect to do (strictly) worse than the next smallest bank.\(^{16}\) Loosely speaking, a potential entrant faces two “adverse selection terms”, one from each of the incumbent banks, instead of the one term that is faced by each incumbent. Thus the entrant’s payoff function is unambiguously worse than that for the smaller incumbent bank, since it differs by a term that must be strictly negative. The entrant faces losses from both bank 2 and bank 1’s rejects, while bank 2 only need worry about bank 1’s rejects.

The reasoning above leads us to the main result of this paper, that of blockaded entry by a third potential bank with no previous market presence. Because of the qualitatively larger cost associated with entry for the third bank, we are able to obtain the result that given the equilibrium we obtained in proposition 1 above for 2 banks, a third bank never finds it optimal to enter the market with positive probability when the two incumbents are competing according to this equilibrium. That is, fixing the competition among the 2 banks (who may be ignoring the possibility of entry), a third bank nevertheless finds entry blockaded when deciding whether to submit a market bid or not. However, this is in some sense a weak result, since it doesn’t address the issue of the timing of entry and of competition, and may not be robust to changes in these, or in a bank’s ability to pre-commit to enter. This pre-commitment issue is particularly important when we consider entry, since a sequential entry / competition decision might lead banks to behave differently than if they are simultaneously making their entry and pricing decisions. If we accept that entry may be a decision that takes place over a longer period of time and is necessarily a decision made prior to competition, then this kind of result needs to be strengthened to

\(^{16}\)Note that what we name the “adverse selection” effect can be loosely interpreted as a fixed cost of entry, \((1 - \lambda)\sigma_i G \left( \frac{1}{2} \right) \left( s_m \left( \frac{1}{2} \right) - 1 \right)\), and hence the proof of proposition 1 is similar to the analysis of the mixed strategy equilibrium in Varian (1980). (Although this “fixed cost” is dependent on \(S_i\), we can imagine the fixed cost as the portion that a bank pays for a bid of \(R\), and a variable part that is the incremental cost when bidding \(S_i < R\), and that the borrower’s reservation interest rate, \(R\), still leads to negative profits for that bank whenever it obtains its opponents worst borrowers.) However, this cost is dependent on the opponent’s market share, \(\sigma_j\), so that we cannot look for symmetric equilibria when market shares differ. This leads us to obtain zero profits for the smaller bank (the one with the highest fixed costs), but positive expected profits for the larger bank.
rule out any kind of entry by a third bank. In fact, the result we do obtain is that there
does not exist any equilibrium where a third bank enters with positive probability. In other
words, the only equilibrium of this game is for bank 3 never to enter, and for banks 1 and
2 to compete as in a duopoly.

To obtain these results, consider a slightly modified version of the model from section
2, with the only modification being the introduction of a third competitor bank. This bank
(called the “entrant”) has a market share of old borrowers \( \alpha_3 = 0 \). The rest of the structure
of the game remains the same as before, including the fact that all banks bid simultaneously
on the free market. We first find it convenient to establish some notation and preliminary
results. Define \( L(s) \equiv (1 - \lambda)G(\frac{1}{2})(s\theta(\frac{1}{2}) - 1) \), and note that \( L(s) < 0 \ \forall \ s \) by definition of
\( m(\cdot) \). We can then write the payoffs in the three bank game as

\[
\pi_i(s) = \lambda(1 - F_j(s))(1 - F_k(s))(s\theta - 1) + (1 - F_j(s))\alpha_k L(s) \\
+ (1 - F_k(s))\alpha_j L(s)
\]

(1.9)

Now consider the specific case where \( \alpha_3 = 0 \), and \( \alpha_1 > \alpha_2 > 0 \), which reflects the fact that
the third bank is a potential entrant with zero market share. Then profits for the three
banks are given explicitly by: \(^{17}\)

\[
\pi_1(s) = \lambda(1 - F_2(s))(1 - F_3(s))(s\theta - 1) + (1 - F_3(s))\alpha_2 L(s) \\ (1.10)
\]

\[
\pi_2(s) = \lambda(1 - F_1(s))(1 - F_3(s))(s\theta - 1) + (1 - F_3(s))\alpha_1 L(s) \\ (1.11)
\]

\[
\pi_3(s) = \lambda(1 - F_1(s))(1 - F_2(s))(s\theta - 1) + (1 - F_1(s))\alpha_2 L(s) \\ + (1 - F_2(s))\alpha_1 L(s) \\ (1.12)
\]

We know from proposition 1 what the equilibrium for the 2-bank game is. Let this equilib-
rium be denoted by \( \{F^*_1, F^*_2\} \). We then have the following result.

**Proposition 2** An equilibrium to the three-bank game is given by: \( \{F^*_1, F^*_2, F_3(R) = 0\} \),
where \( F_3(R) = 0 \) implies that bank 3 never submits a bid on the free market.

**Proof:** Given \( F_3(R) = 0 \), clearly \( \{F^*_1, F^*_2\} \) is an equilibrium for both banks that stay in,
since this case is identical to two-bank case. What we need to show is that given \( \{F^*_1, F^*_2\} \),

\(^{17}\)Here we are assuming that the distribution function are continuous and strictly increasing, so that there
are no atoms. That this is true is demonstrated in the appendix.
$F_3(R) = 0$ is optimal for bank 3.

We write bank 3's payoffs as:

$$
\pi_3(s) = \lambda (1 - F_1^*(s))(1 - F_2^*(s))(s\bar{\theta} - 1) + (1 - F_1^*(s))\alpha_2 L(s) + (1 - F_2^*(s))\alpha_1 L(s)
$$

$$
= (1 - F_2^*(s)) [\lambda (1 - F_1^*(s))(s\bar{\theta} - 1) + \alpha_1 L(s)] + (1 - F_1^*(s))\alpha_2 L(s)
$$

$$
= (1 - F_2^*(s))\pi_2^*(s) + (1 - F_1^*(s))\alpha_2 L(s)
$$

But $\pi_2^*(s) = 0$, by construction of $F_1^*, F_2^*$, implying that $\pi_3(s) < 0 \forall s$ as long as $F_1^*(s) < 1 \forall s$. Therefore, $F_3(R) = 0$ is an optimal response. \(\Box\)

We can provide intuition as to why bank 3's profit is negative. Bank 3 is competing on an equal basis with bank 2 for new borrowers and for bank 1's old borrowers, but bank 3 also faces bank 2's rejected customers. Since bank 2 is being held to zero profits, bank 3 gets zero profit on this segment of the market as well but negative profit on bank 2's rejected borrowers. Hence bank 3 must get negative profit overall, if it ever enters. Put simply, entrant banks effectively face higher costs of operation during their period of entry than incumbent banks, which leads to the deterred entry result.

Now we proceed to address the issue of uniqueness of this equilibrium. As argued above, we show in the following that there does not exist any equilibrium to this game for which $F_3(R) > 0$, or, in other words, that there is no equilibrium in which bank 3 ever bids on the free market. The intuition of this result is worth restating, and it is simply that a third bank faces a worse distribution than the other two incumbent banks, and hence never finds it profitable to enter. Note that it is also true that the second bank (the smaller of the two incumbents) also faces a worse distribution than the larger bank, bank 1, because it receives a larger share of bad borrowers than bank 1 does. However, we have already constructed an equilibrium for this case, and noted that it necessarily implies one bank must be making zero expected profits, and we noted that this bank is indeed the smaller bank. It is exactly this fact, that competition must force some banks' profits to zero, that gives us the impossibility of entry by a third bank, who must compete in the already "tough" competition in the market.

As a result of the arguments above, we offer the following proposition. The mechanics of the proof, which involve checking a number of special cases and noting that a number of conditions that are implied by the existence of an equilibrium where bank 3 enters with
positive probability cannot be satisfied, are rather lengthy and are relegated to the appendix.

**Proposition 3** There does not exist an equilibrium where bank 3 enters the free market with positive probability (i.e. where $F_3(R) > 0$).

**Proof:** See appendix.

This proposition fully demonstrates our result of blockaded entry in this banking market. Note importantly that the only equilibrium with which we are left is the one obtained in proposition 2, that the two incumbent banks are able to safely compete ignoring the threat of potential entry.

Moreover, this result is robust to some generalizations. In particular, it does not rely on the single-period nature of our model. In the appendix we present a two-period version of the model, and show that even with the prospect of future profits from old customers, the result of blockaded entry continues to hold. The intuition for this is that the possibility of profits in the future leads to fiercer competition in the initial period, so that all the future rents are bid away in the first period. Since our emphasis was on showing that information can create a barrier to entry rather than on how increased competition can lead to lower introductory rates for borrowers\(^\text{18}\), we have for simplicity focused on the single-period model in the main text. Similarly, the result does not rely on assuming that $\alpha_3 = 0$, so that the entrant bank has no previous market share. In the proof to the proposition in the appendix, we prove a more general result, that with three banks, the one with the smallest market share will never bid for the free market of borrowers. We state the result as above since the goal of the paper was to analyze a potential entrant's decision to try to obtain new customers, and as such would not be expected to have any pre-existing market share.

### 1.5 An example

In this section we try to provide some more intuition about the results of our model solving it for the case where $\theta$ is uniformly distributed between $[0, 1]$. Therefore, let $G(\theta)$ denote a uniform distribution function over $[0, 1]$, with density $g(\theta) = 1$. Then we can explicitly

\(^{18}\)See Sharpe (1990) for an analysis of this case.
solve for \( F_1, F_2, \) and \( r, \) and they will be given as follows.

\[
r = 1 + \frac{\sqrt{(1 - \alpha_1)\lambda^2 + \alpha_1\lambda}}{\lambda} > 2 \tag{1.13}
\]

\[
F_1(s) = 1 - \frac{\frac{1}{2}(1 - \lambda)\alpha_1 \frac{1}{2}}{\frac{1}{2}s - 1} \tag{1.14}
\]

\[
F_2(s) = 1 - \frac{\frac{1}{2}(1 - \lambda)\alpha_2 \frac{1}{2}}{\frac{1}{2}s - 1} - \frac{(1 - \frac{\alpha_2}{\alpha_1}) \left[ \frac{1}{\lambda} \sqrt{(1 - \alpha_1)\lambda^2 + \alpha_1\lambda} \right]}{(\frac{1}{2}s - 1)} \tag{1.15}
\]

Remember that: \( \alpha_2 = 1 - \alpha_1 \)

Thus we have:

\[
F_2(s) = 1 - \frac{\frac{1}{2}(1 - \lambda)(1 - \alpha_1) \frac{1}{2}}{\frac{1}{2}s - 1} - \frac{(\frac{2\alpha_2 - 1}{\alpha_1}) \left[ \frac{1}{\lambda} \sqrt{(1 - \alpha_1)\lambda^2 + \alpha_1\lambda} \right]}{(\frac{1}{2}s - 1)} \tag{1.16}
\]

Notice that the lower bound for the free market interest rate is increasing in the market share of the largest bank (in this case \( \alpha_1 \)). The larger bank 1’s share, the larger the adverse selection problem for bank 2. In other words, when bank 2 gets smaller the distribution that it faces on the free market gets worse. Hence in order to make zero profit bank 2 has to charge a higher interest rate.\(^{19}\) It is interesting to notice that the effect of \( \alpha_1 \) on the interest rate lower bound is stronger when \( \lambda \) is smaller. \( \lambda \) is in some way an index of the importance of the adverse selection (or cream-skimming) problem. If \( \lambda = 1 \) all borrowers are new borrowers and the incumbent’s advantage disappears, so that previous market share do not matter. If \( \lambda \) is small most borrowers are old borrowers, so that market shares become relevant because they represent valuable knowledge for the banks.

Figure 1 shows plots of \( F_1(s) \) and \( F_2(s) \) for three different values of \( \lambda, \) the arrival rate of new borrowers. It illustrates that as \( \lambda \) increases, the importance of the adverse selection effect decreases, leading to increasing probability of lower interest rates. Furthermore as \( \lambda \)

\(^{19}\) The derivative of (1.13) with respect to \( \alpha_1 \) is:

\[
\frac{1}{2}(1 - \lambda) \left[(1 - \alpha_1)\lambda^2 + \alpha_1\lambda \right]^{-\frac{1}{2}} > 0.
\]
approaches 1 the two banks’ strategies become closer to each other, and the curves become steeper, indicating less variance in the market interest rate charged. As \( \lambda \) approaches 1 the banks’ strategies begin to approximate those predicted by a perfectly competitive market, with each borrower putting all their probability weight on the break-even interest rate (i.e. they play a pure strategy).\(^{20}\)

The effects of adverse selection on the bidding strategies of the two banks are clear from (1.14) and (1.15). Bank 1 will play less aggressively when \( \alpha_1 \) is large. This result becomes clearer if we think about \( \alpha_1 \) as a measure of bank 2’s costs: the larger \( \alpha_1 \), the larger the proportion of bad borrowers bank 2 faces in the free market. Hence when \( \alpha_1 \) increases bank 1 is facing a less dangerous opponent (since \( \alpha_2 \) clearly decreases) and can afford to increase the interest rate it charges. More technically, if we look at the equilibrium as a whole, we know that bank 1 has to bid less aggressively in order to keep a less efficient bank 2 at zero profits. In other words, since bank 2 is more “at risk” now because of banks 1’s larger size, bank 1 needs to be less aggressive in its bidding strategy given the extra risk burden faced by bank 2. The interpretation for (1.15) is similar. The first term is symmetric to (1.14). The second term goes in the opposite direction. We can think of it as a “cost term”: the larger \( \alpha_1 \), the higher the “informational cost” for bank 2, so that even if it has an incentive to lower its interest rate in order to win more often, it also has to raise it in order to cover costs.

Figure 2 shows \( F_1(s) \) for two different distributions of borrowers over \( \theta \), both with the same mean. Under the truncated uniform distribution the borrowers are uniformly distributed over the interval \([.864, 1]\). The two-step distribution represents a mean preserving spread of the truncated uniform, with 3.6% of the borrowers distributed uniformly over \([0, 1]\) and the remaining 96.4% distributed uniformly over \([.9, 1]\). It can be seen that \( F_1 \) for the truncated uniform lies strictly above \( F_1 \) for the two-step. This implies that banks bid more aggressively under the truncated uniform than under the two-step. In fact, average interest rates faced by new banks are more than 1% lower, even though the average credit worthiness of the whole market is the same. The difference between the two distributions is that borrowers are more heterogeneous under the two-step, so the adverse selection problem

\(^{20}\)Note that the interest rate charged, \( r \), is extremely high in this example. This is simply because we allow borrowers to be of an unboundedly poor quality, and there are a larger number of these low quality borrowers. In figure 2 we allow for a more reasonable distribution of borrower qualities, and obtain more realistic interest rates.
is greater. In other words, the incumbent's advantage is larger and so is its market power.

Consider the equilibrium that would obtain if there were no heterogeneity concerning borrowers' credit worthiness. In that case, all banks would break even by offering an interest equal to $\frac{1}{\beta}$, the inverse of each borrower's success probability, and there would be no incumbency advantage. Moving away from this extreme, but preserving the mean success probability, asymmetric information begins to play a part and the advantage for incumbent banks increases. This emphasizes the fact that our results are driven by the presence of asymmetric information and not just by the riskiness of the market.

1.6 Conclusion

We have argued above that asymmetric information can *per se* constitute a barrier to entry into a banking market, and that this barrier arises endogenously out of the nature of competition. While the industrial organization literature has often focused on the notion of large fixed costs imposing a barrier to entry, and this is particularly true of models of Bertrand competition, the "fixed cost" we have in mind is one that arises out of a bank's decision to enter a credit market populated by heterogeneous borrowers, and is not a direct fixed cost that must be paid up front to enter, or a production cost as in much of the industrial organization literature.

The incentive for a bank to enter at all into the credit market is provided by the fact that with a new pool of borrowers seeking financing each period, banks would always like to enter as long as the average payback rate for the new borrowers is sufficiently high to cover the bank's investment. However, a bank attempting to enter a loan market is always faced with the prospect of receiving some of a competitor's worst risks, and losing money on them to the extent that it is unable to distinguish between the good and bad risks. If an incumbent bank is able to distinguish between its good and bad borrowers, then an entrant bank is virtually guaranteed to receive all of these bad borrowers. This form of adverse selection is in some sense an unavoidable part of the entry decision, and is what leads banks to eschew entry to avoid the expected losses from this pool of "bad" borrowers.

Our result that the equilibrium number of banks in the market is limited to 2 is clearly an extreme result and is not meant to be a prediction for an actual banking market. It stems from our use of a very extreme form of competition, and under a less extreme form of
competition such as one that included differentiation between banks, or capacity constraints, we would expect to see a larger, but still finite, number of banks. What we wish to emphasize is the fact that this informational asymmetry can make entry extremely difficult rather than the specific predictions of our model regarding the equilibrium number of banks. This limit on entry due to adverse selection continues to play an important role in models that soften the competition among banks.

We have been able to characterize the equilibrium with two banks, and also to show that with three banks, we never obtain an equilibrium where the smallest (in this case, the entrant) bank actually does enter the market with positive probability. But entry is primarily a dynamic issue, and while we obtain these results in a static setting, we think it warrants attention to consider these decisions in a multi-period setting. With more than one period, a potential entrant bank may have an incentive to enter even if it expects to make first period losses, since it knows that in the future it will be able to reap some benefits from the customers it already knows, and deny credit to the borrowers that are bad risks. For these reasons we provided a simple example of a two-period game. There we show that, with two banks in the market, the prospect of future profits increases competition in the early period to such an extent that all future informational rents are competed away. Hence, our main result of blockaded entry continues to hold.\textsuperscript{21} However, further research along these lines is necessary to adequately address these issues.

Finally, it should be clear that the basic structure of our model could easily be extended to areas other than banking markets. An example of how it could be applied to labor markets is that it might not be possible to start a law firm by raiding other law firms, because the old firms will fight to hang on to their good lawyers. This same framework could also be applied to analyze entry in the insurance market or in the managed care (HMO) market, since both of these are characterized by large amounts of informational asymmetries which would put entrant firms at risk of receiving "low quality" (high risk, high cost) customers.

\textsuperscript{21}Moreover, Dell’Ariccia (1997) shows that if banks are differentiated by transportation costs, adverse selection still leads to a finite number of banks in equilibrium, even in the absence of any exogenous fixed costs. He shows that in a dynamic OLG model in which banks live forever that our main result continues to hold.
1.7 Appendix

1.7.1 Proofs

Proof of lemma 2: Suppose $F_j$ is discontinuous at $s^*$ (i.e. $\exists$ an atom in $F_j$), then bank
$i$'s action of playing $s^* - \epsilon$ strictly dominates playing $s^* + \epsilon$, $\epsilon > 0$. Therefore bank $i$ will
not bid a free-market interest rate $S_i \in [s^*, s^* + \epsilon)$. But then bank $j$ can raise its interest
rate without losing customers and so $s^*$ cannot be an optimal action for bank $j$. Hence $F_j$
must be continuous.

To prove the second part, suppose $F_j$ is non-increasing over some interval, or in other
words that $\exists$ some interval $(s_1, s_2) \subseteq (r, R)$ for which $f_i(s) =: 0 \forall s \in (s_1, s_2)$. But then
$\text{prob}(S_i < S_j|S_i = s_1) = \text{prob}(S_i < S_j|S_i \in (s_1, s_2))$, but profits are strictly higher for
$S_i > s_1$ (conditional on winning,) so that bank $i$ maximizes its payoff by playing $S_i = s_2$, and hence would never offer an interest rate in the interval $(s_1, s_2))$. But then bank $j$ can
increase its profits by playing $S_j = s_2 - \epsilon$ with positive probability, where $\epsilon < s_2 - s_1$, since
this will lead to strictly higher profits than any interest rate offer in a neighborhood of $s_1$.
However, this contradicts the assumption that $f_j(s) \equiv 0 \forall s \in (s_1, s_2)$. □

Here we offer a more general restatement of proposition 3, under the assumption that
$\alpha_3$ is not restricted to being zero. All that we require for the result of blockaded entry is
that bank 3 have a smaller market share than either of the two incumbent banks.

Proposition 3a Let $\alpha_1 > \alpha_2 > \alpha_3$, $\alpha_3 \geq 0$. Then there does not exist an equilibrium where
bank 3 enters the market with positive probability (i.e. where $F(R) > 0$).

Proof: We prove the proposition using a number of preliminary results.

Claim 2 In any equilibrium, no two banks may have an atom at the same interest rate,
i.e. $\exists s'$ such that $\mu_i(s') > 0$, $\mu_j(s') > 0$, with $i \neq j$.

Proof: The proof of this result is relatively standard. WLOG suppose $\mu_i(s') > 0$. Then
$\pi_j(s) > \pi_j(s')$ for $s \in (s^*, s')$, which implies that $\mu_j(s') = 0$. □

Claim 3 Assume $\alpha_1 > \alpha_2 > \alpha_3$. Then $\pi_1 > \pi_2 > \pi_3$, where $\pi_i$ represents the equilibrium
profits for bank $i$ conditional on making a bid.

Proof: WLOG assume $\alpha_j > \alpha_k$. Since we have established that for $s \in (r, R)$ bank $k$ will
never charge a rate at which bank $j$ has an atom, we can write the profit as
\[
\begin{align*}
\pi_j(s) &= (1 - F_i(s))(1 - F_k(s))\lambda(s\theta - 1) + \alpha_i(1 - F_k(s))L(s) + \alpha_k(1 - F_i(s))L(s) \\
\pi_k(s) &= (1 - F_i(s))(1 - F_j(s))\lambda(s\theta - 1) + \alpha_i(1 - F_j(s))L(s) + \alpha_j(1 - F_i(s))L(s)
\end{align*}
\]

Define \( A(s) = (1 - F_i(s))(1 - F_j(s))\lambda(s\theta - 1) + \alpha_i(1 - F_j(s))L(s) \) and \( B(s) = \alpha_j(1 - F_i(s))L(s) \). Note that \( B < 0 \), so that in order for \( \pi_k(s) \geq 0 \), it must be that \( A > 0 \). Note also that, for all \( s \),

\[
\pi_j(s) = \frac{(1 - F_k(s))}{(1 - F_j(s))}A(s) + \frac{\alpha_k}{\alpha_j}B(s)
\]

Let \( \underline{g}_k \) be the lower bound of bank \( k \)'s support. Thus \( \pi_k = \pi_k(\underline{g}_k) \). Consider:

\[
\pi_j(\underline{g}_k) = \frac{(1 - F_k(\underline{g}_k))}{(1 - F_j(\underline{g}_k))}A(\underline{g}_k) + \frac{\alpha_k}{\alpha_j}B(\underline{g}_k)
\]

Since \( F_k(\underline{g}_k) = 0 \) and \( \frac{\alpha_k}{\alpha_j} < 1 \), \( \pi_j(\underline{g}_k) > \pi_k = A(\underline{g}_k) + B(\underline{g}_k) \). Now, since \( \pi_j \geq \pi_j(\underline{g}_k) \),

\[ \pi_j > \pi_k. \]

**Claim 4** \( \pi_2 = 0. \)

**Proof:** Suppose \( \pi_2 > 0 \). Then \( F_2(R) = 1 \), and bank 2 always bids on the free market. Let \( r^* \leq R \) be the upper bound of bank 2's support.

Either \( \mu_2(r^*) > 0 \) or \( \lim_{s \to r^*} F_2(s) = 1 \). Suppose the latter holds. Then, for any bank \( j \neq 2 \), \( \lim_{s \to r^*} \pi_j(s) < 0 \). Therefore it must be that \( f_j(s) = 0 \) for any \( s \in (r', r^* \] \) and \( \pi_2(r^*) > \pi_2(s) \) for any \( s \in (r', r^* \] \). Since \( r^* \) is the upper bound of bank 2's support, it must be that \( \mu_2(r^*) > 0 \), contradicting the assumption that \( \lim_{s \to r^*} F_2(s) = 1 \). Suppose instead that \( \mu_2(r^*) > 0 \). Then \( \lim_{s \to r^*} F_1(s) < 1 \). But for any \( s > r^* \), \( \pi_1(s) < 0 \). Therefore \( \lim_{s \to r^*} F_1(s) = 1 \), which implies that \( \mu_1(r^*) > 0 \). But this violates Claim 2 since \( \mu_2(r^*) > 0 \). This leads us to a contradiction that \( \pi_2 > 0 \). \( \square \)

Combining Claims 2 and 4 we see that \( \pi_3(s) < 0 \) for any \( s \neq \mathcal{D} \). Thus \( F_3(R) = 0 \) in any equilibrium, and bank 3 never bids on the free market. \( \square \)
1.7.2 An Example of a Two-period Game

In this section, we demonstrate that the result of blockaded entry (proposition 3) is not an artifact of the single-period nature of our model. Here we provide an example in which, even with the prospect of future profits, having two incumbent banks still creates enough competition so that a third bank would never enter. To do so, we need only add one more period to our model, changing it from a one-period to a two-period model.

Specifically, we assume that borrowers live for only 2 periods, and that half of the borrowers exit in each period, to be replaced by an equal mass of new borrowers with the same distribution over success probabilities. We use an overlapping generations (OLG) structure for tractability, since it simplifies the calculation of each bank's equilibrium strategies. The rest of the model remains the same as in the main text, so that, in period 1, there are two incumbent banks, bank 1 and bank 2, each with a share of the market equal to $\alpha_1$ and $\alpha_2$, respectively, and that $\alpha_1 > \alpha_2$. Banks compete in each of two periods, period 1 and period 2. The overlapping generations assumption implies that any borrowers carried over into period 1 (from a previous unmodelled period) will not be part of the market in period 2.

We solve the game by backward induction. Consider first period 2. Given the OLG structure of the game, the only bank to have a positive market share going into period 2 will be the bank with the lowest interest rate in period 1. Therefore, the outcome in this subgame is exactly that of the model in the main text, when one bank has a market share equal to one ($\alpha_i = 1$, $\alpha_j = 0$). As in the main model, this bank obtains positive expected profits equal to the sum of the profits on new borrowers and on old customers (those obtained in period 1). Any other bank earns zero profits. Another simplification due to the OLG assumption is that, since only one bank has all the market share, the expected profits in period 2 do not depend on the market shares in period 1. We can therefore write these as a constant, which we will denote by $\Pi^W$. Note that there still exists the possibility of a tie in two banks' interest rates. We will denote the expected profits in period 2 in this instance as $\Pi^T$. To simplify exposition, we assume that the tie-breaking rule is that any tying bank has an equal probability of either winning the entire market or losing the market, so that clearly $\Pi^T < \Pi^W$.

Consider the effect that bank $i$'s bid on the free market will have on $i$'s profits. As before, bank $i$'s bid will not affect the profits from old customers in period 1. We therefore
write the total payoff to a bid of $S_i$ as:

$$
\pi_i(S_i) = \begin{cases} 
.5(S_i\bar{\theta} - 1) + \Pi^W + \alpha_j L(S_i) & \text{if } S_i < S_j \\
.25(S_i\bar{\theta} - 1) + \Pi^T + \alpha_j L(S_i) & \text{if } S_i = S_j \\
\alpha_j L(S_i) & \text{if } S_i > S_j 
\end{cases}
$$

We note that our argument about the impossibility of two concurrent atoms still applies in this setting. Therefore, profits in the 3 bank game are given by:

$$
\pi_i(S_i) = (1 - F_j(S_i))(1 - F_k(S_i))(.5(S_i\bar{\theta} - 1) + \Pi^W) \\
+ \alpha_j (1 - F_k(S_i))L(S_i) + \alpha_k (1 - F_j(S_i))L(S_i)
$$

Again let us define $A(S_i) = (1 - F_j(S_i))(1 - F_k(S_i))(.5(S_i\bar{\theta} - 1) + \Pi^T) + \alpha_j (1 - F_k(S_i))L(S_i)$ and $B(S_i) = \alpha_k (1 - F_j(S_i))L(S_i)$. We note that $B$ must still be negative whenever $F_j(S_i) < 1$. Therefore our proof that $\pi_1 > \pi_2 > \pi_3$ applies here as well. The remainder of the proof is identical to the proof of proposition 3 (the one-period case), and therefore need not be repeated.

In essence what occurs in the two period game is that the net increment to profit of making a bid for the second bank is driven down to zero. Since the third bank is still in a worse position relative to the second bank, its increment to profit of making a bid is negative. Therefore, even with the expectation of future profits from trapped customers, competition in the first period is sufficiently intense that all future expected profits are bid away. In essence, banks are willing to sustain losses in the first period in order to learn about some customers and make profits in the future.22 Since our concern at this point is not with characterizing the equilibrium in this game, but with establishing the result of blockaded entry for a third bank, we also need not repeat the construction of the equilibrium strategies found in proposition 1.

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22We see this by looking at a bank's profit at the lowest interest rate, $r$. A bank bidding this rate wins the free market with probability one, and makes profits of $\Pi^W$ in period 2. However, this rate $r$ must be sufficiently low that in fact first period profits are negative, and equal to $-\Pi^W$. 

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Chapter 2

Lending Capacity and Adverse Selection in the Banking Industry

2.1 Introduction

One of the primary functions of banks is to gather information about borrowers, and screen creditworthy borrowers from non-creditworthy ones. Much of this information is obtained in the process of lending and in the subsequent "monitoring" role that is often seen as a defining characteristic of bank financing.\(^1\) Moreover, most of this information is proprietary to the lending bank. Creditworthy borrowers may thus find themselves locked in to their banks if they are unable to credibly signal their high quality to other financiers, which in turn allows banks to extract rents from these borrowers. At the same time, if credit markets are composed of borrowers of differing qualities, banks will be faced with an adverse selection problem to the extent that borrowers are better informed about their prospects. Moreover, entrant banks may face a greater adverse selection problem than incumbent banks. Incumbent banks may possess an informational advantage by virtue of the information acquired through their previous lending relationships. This advantage may constitute an economic barrier to entry as entrant banks face a worse pool of borrowers than incumbent banks. This paper explores the implications of this information problem for competition and market structure in the banking industry.

\(^1\)James (1987) provides evidence on the special nature of bank loans over other kinds of finance. Lummer and McConnell (1989) argue that banks may have access to private information during the life of a loan, so that loan renewals (but not new loans) provide a signal of the debtor firm's creditworthiness.
To address these issues, we present a model of competition in banking where potential entrant banks find themselves at an informational disadvantage relative to incumbent banks, and where this disadvantage serves to limit entry into the credit market. In our model, banks compete in interest rates for loan customers. However, each bank is constrained in the number of loans it can grant (they have “lending capacities”). Moreover, we assume that there is some heterogeneity across borrowers in that some borrowers (“good borrowers”) have profitable investment opportunities, while others (“bad borrowers”) do not. While this characteristic of borrowers is ex-ante unobservable, we assume that banks, by virtue of the relationships they establish with borrowers upon the granting of a loan, may observe whether borrowers have good projects or not. As a final element, we assume that there is some turnover among borrowers, so that at any given moment there are some borrowers whose characteristics are unknown to all banks. This creates an incentive for banks to compete over these “new” borrowers.

Using this simple model, we can derive a number of interesting results regarding the nature of competition among banks. We find that banks always have an incentive to undercut each other’s interest rate offers, and that this result, common in models of Bertrand competition, arises here for a different reason and even when lending capacities are binding. Since banks learn their borrowers’ “types” after granting a loan, any profit-maximizing bank would refuse to continue financing borrowers revealed to be bad. Since information is proprietary and not transferable, these borrowers become part of the pool of customers that are unknown to all other banks. A bank that is able to lower its rate below its competitors’ rate suffers less from adverse selection since it is able to grant loans to a larger fraction of the “new” borrowers, and not just to other banks’ rejected borrowers.

Another result, alluded to above, is that incumbent banks’ informational advantage can create serious difficulties for banks attempting to enter this market. Banks have an incentive to enter a market in order to lend to “new” borrowers, but they risk refinancing incumbent banks’ poor credit risks. This “adverse selection effect” makes entry difficult and provides an incumbency advantage. This implies that new banks may only choose to enter if they expect to be able to compete on a more or less equal footing with incumbent banks, which occurs when there is a high degree of turnover among borrowers. High turnover erodes an incumbent’s informational advantage, thus reducing the adverse selection effect and permitting entry.
We also argue that focusing exclusively on the number of banks may not provide a very good indicator of the "competitiveness" of the market. Instead, we need to consider both the composition of borrowers as well as the constraints on banks' lending capacities. In particular, we show that markets composed of many small banks may actually have higher expected interest rates in equilibrium than markets composed of a few large banks. In our model, increased competition among banks leads to an inefficiency: since each small bank has less information about the market than a large bank would, it is less effective in its screening, so that more bad borrowers are able to obtain financing. Under some circumstances, this effect can be sufficiently strong as to overwhelm the competitive effect of increasing the number of banks, and may lead to an increase in interest rates.

We can use this framework to offer some predictions for the degree of competition and the patterns of entry likely to take place after a financial liberalization or deregulation in the banking industry. For instance, if incumbent banks face constraints on lending capacity, an entrant may be able to gain a foothold in a market either by entering and competing directly or by buying out an incumbent bank. However, because of the adverse selection it faces upon entry, banks will find entry difficult unless they can quickly acquire information about the market. Therefore, we argue that most entry should be in the form of a merger with or an acquisition of an incumbent bank, since this provides an entrant bank with a customer base and hence reduces its informational disadvantage.

A specific application of this analysis is to the financial integration taking place in the European Union through the formation of the common market, or to the deregulation on limits on interstate branching that is under way in the U.S. There has been much debate concerning the likely effects of these deregulations on the competitiveness of the banking industry. Some recent applied work on the issue of cross border banking finds that foreign banks have found it rather difficult to enter domestic markets even after the drop of most regulatory barriers (see Hoschka (1993) and Vesala (1995)). The pattern that seems to be emerging is that the few foreign banks that have taken the plunge into domestic financial markets have done so mainly through acquisitions of or mergers with domestic banks, and have entered predominantly into the commercial or wholesale banking market rather than the retail market. This seems consistent with the implications we obtain regarding entry, as we might expect retail markets to be characterized by larger informational asymmetries.

Much of the existing literature has analyzed the monitoring and information gathering
role of bank finance (Diamond (1984)). More recently, the informational advantage of inside banks over alternative sources of finance has been emphasized (see Rajan (1992), Sharpe (1990), or von Thadden (1994)). Yet, the effect of these information asymmetries on the structure of the industry, and its consequent implications for the issue of surplus extraction, has been ignored. Concerning the possible welfare effects following a financial liberalization, Riordan (1993) finds that, due to the “Winner's Curse”, having more banks may lower the average quality of firms obtaining financing and hence lead to higher interest rates. This result is similar to ours, that increased bank competition leads to a lower quality pool of borrowers obtaining financing. However, Riordan assumes entry does indeed take place, which is one of the premises that this paper challenges.

Entry under information asymmetries has been analyzed by Dell’Ariccia, Friedman, and Marquez (1996), who argue that entrant banks are faced with the prospect of having to make loans to incumbent banks' most risky firms. This leads to new banks finding entry blockaded as long as there are already two incumbent banks. Their model is extended here by introducing constraints on lending capacity so as to analyze the implications of different characteristics of credit markets on the extent of competition in the banking sector.

The importance of constraints on lending capacity has been the subject of some of the recent empirical literature on banking and intermediation. Kashyap and Stein (1995) find that the lending behavior of banks seems to be quite sensitive to exogenous changes in monetary policy. They conclude that banks effectively face some limits on their loanable funds. Thakor (1996) provides both theory and evidence that increases in capital requirements for banks decrease aggregate lending. From a theoretical point of view, many explanations have been offered as to why banks may face lending constraints. For example, a bank's supply of loanable funds may be limited by the total pool of customer deposits in its market. Alternatively, informational asymmetries between potential outside investors and bank managers regarding the value of existing bank assets may limit a bank's ability to raise funds (Stein (1995)).

The rest of the paper proceeds as follows. Section 2 lays out the basic model and assumptions. Section 3 begins the analysis of the model by deriving some useful preliminary results. The bulk of the analysis is contained in section 4. We characterize the equilibrium of the model for both the case with 2 banks and for \( N \) banks, and derive some comparative

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\(^2\)See section 2 for further reasons why banks may have a limited ability to grant loans.
statics. Section 5 analyzes the issue of entry by outside banks and determines the conditions under which entry will be more or less difficult, and what form of entry should take place. Section 6 concludes with a discussion and some extensions for future research.

2.2 Model

Our basic setup is a form of two-period Bertrand competition among banks for loans to borrowers. In period 1, there is a continuum of borrowers, which we normalize to have measure 1. Each potential borrower has an investment project that requires a capital inflow of $1 and generates a cashflow $R$ with probability $\theta$, and 0 with probability $1 - \theta$. This outcome is observable and contractible, but $\theta$ is unknown to either borrower or lender.\(^3\)

For simplicity, we assume that $\theta \in \{\theta_l, \theta_h\}$, $\theta_l < \theta_h$, that $\text{prob}(\theta = \theta_h) = q$, and that this distribution of types of borrowers is common knowledge. Borrowers have no private resources, so they must look to banks to obtain financing. We assume that $\theta_l R < 1$ and $\theta_h R > 1$, so that it is efficient to finance good borrowers but not bad borrowers. Moreover, letting $\bar{\theta} = q\theta_h + (1 - q)\theta_l$ denote the average success probability, we assume that $\bar{\theta} R > 1$, so that it is ex-ante efficient to grant a loan.

There are $N$ banks, each with a limited amount of loanable funds, so that no single bank can offer financing to every borrower in the market. We will denote the constraint on lending capacity for bank $i$ (i.e. the pool of funds available to bank $i$ for loans) as $\overline{K}_i$. For most of the analysis, we will assume that all banks are identical, and that $\overline{K}_i = \frac{1}{N}$, so that the aggregate lending capacity of the banking sector is 1, and therefore sufficient to meet demand. We will, however, provide a partial characterization for the asymmetric case when $N = 2$. We assume that, having granted a loan, a bank learns that borrower's type ($\theta$).

The nature of competition among banks in period 1 is such that each bank lends to a fraction $\alpha_i$ of borrowers. If all banks are symmetric, we simply assume that $\alpha_i = \frac{1}{N}$, $\forall i = 1, \ldots, N$.\(^4\) In period 2, a fraction $\lambda$ of borrowers dies (exits the market), and is replaced by

\(^3\)We will use the term "borrower" and "firm" interchangeably, since the model applies equally well for household borrowers as for small private firms. All the results in the model carry through if borrowers know their own type ex-ante, but creditors learn it only after granting a loan.

\(^4\)We leave unmodelled the first period competition for borrowers, when no bank possesses any private information regarding firms' success probabilities. Instead, we focus on competition after all firms have already obtained financing from some bank. Since there is no question of project choice (as in Rajan (1992) or von Thadden (1994)), adding a previous period of competition among banks would only complicate the analysis and notation without contributing to the insight of the problem, which is the effect of a bank's
an equal mass of new borrowers that have the same distribution over types as the original
borrowers. This exit from the market is independent of borrower type, so that we maintain
both the population size and the distribution of borrowers the same.\footnote{inside information on the industry structure.}

In period 2, banks compete à la Bertrand for the pool of borrowers. We assume that
banks are unable to distinguish between new borrowers and borrowers that have been
rejected by a competitor bank or who are simply switching banks to take advantage of
lower rates. These firms comprise what we call the “free market”. The timing at in period
2 is as follows. First, all banks simultaneously choose an interest rate for the free market.
Then, having observed the other banks’ rates, they simultaneously choose interest rates for
their old customers. This is equivalent to allowing old borrowers to observe the “market
rate”, and use that to bargain for lower rates from their banks. If an old borrower can get
the same interest rate from its old bank and from a competitor, we assume that it stays
with its current bank, while if there is a tie in the free market, all tying banks split the
market equally.\footnote{We do this so as to not confound the effects due to adverse selection, and those due to the fact that the
market may simply be growing. Also, this simplifies the interpretation of \( \lambda \) if we think of \( \lambda \) as an exogenous
separation rate between borrowers and banks. See the discussion below.}
Finally, banks make loans to all borrowers that have applied to them for
credit, up to their capacity to supply loans. Borrowers not served by the banks with the
lowest interest rate are able to apply for loans at the bank with the next lowest interest
rate, and so on until either the industry supply of funds is exhausted or until every borrower
obtains a loan. We assume that banks give credit to unknown borrowers in proportion to
their relative frequency in the population. Therefore, the quality distribution of borrowers
that any given bank obtains will be representative of the pool of borrowers that still have
not obtained financing.

A few comments are in order. The assumption that banks learn by lending is meant to
capture the stylized notion that a borrower’s old bank may know more than what is available
on a credit record, either from monitoring or having access to books or by observing the
kind of projects being undertaken. Therefore, banks are able to charge differential rates
to their old customers. Borrowers are able to switch banks in order to obtain the “market
rate” that is offered to potential new firms by a competitor bank.

The capacity limitation we have in mind may be due to a number of reasons, such
as there being a fixed (at least in the short run) amount of deposits for which banks are competing. Banks may face increasing costs of raising loanable funds, such as by having to offer higher rates on deposits or by needing to put up a certain amount of their own capital in order to have the proper incentives to monitor. Banks may also have a limited ability to process information, or even a particular limited ability or expertise to monitor loans they make, and unmonitored loans may not be profitable.\(^7\) Also, in a more general model, the lending capacity of each bank could be obtained endogenously by assuming that banks compete for external funds,\(^8\) so that total loanable funds will be the sum of bank capital, deposits, and other external sources such as external equity or debt (see Stein (1995)). Yannelle (1995) provides a model of double-sided strategic interaction where banks compete over both deposits and loans by making interest rate offers. While in her model banks are ex-post monopolists (since one bank obtains all deposits), an extension to introduce some sort of product differentiation would lead to a distribution of deposits across the incumbent banks.\(^9\) Here, our focus is on the ex post competition for loans after funds have been obtained by banks, so we simply fix this capacity exogenously.

\section{2.3 Preliminaries}

Before proceeding with the main part of the analysis, we first characterize the equilibrium in the subgame after free market bids have been submitted. Let \(r_i\) be the rate that bank \(i\) has offered on the free market, and let \(r_{i\theta}\) be the rate that it offers to a known borrower of type \(\theta\).

**Lemma 1** On the equilibrium path, in the subgame after free market bids are submitted, bank \(i\) (1) retains all its old high quality borrowers by charging \(r_{i\theta_h} = \min_{j \neq i} \{r_j\}\), and; (2) denies credit to all its old low quality (\(\theta_l\)) borrowers.

**Proof:** See appendix.

The intuition for this result is straightforward. A bank will always want to deny funding to all its low quality borrowers and to retain its high quality borrowers by offering them

\(^7\)For a moral hazard model that incorporates the interaction between monitoring expertise and bank capital, see Almazan (1996).

\(^8\)Matutes and Vives (1996) consider the competition for deposits followed by competition for loans.

\(^9\)For a spatial model of this form, where each bank obtains the deposits of those customers "close" to it, see Chiappori et al. (1995).
a rate that is competitive with its opponents' market rate, as long as this rate is not too low. A competitor bank that offers a rate low enough to attract high quality borrowers will also attract all low quality borrowers. Therefore, such a low rate cannot be profitable as it would lead to losses for the competitor bank. Hence each bank indeed retains all its old high quality borrowers.

From lemma 1, we know that every bank will offer financing to all its old high quality borrowers. Since a fraction \( q \) of firms are of high quality, and a fraction \( (1 - \lambda) \) of these firms survived from period 1, the mass of loans made to old borrowers by bank \( i \) is \( \alpha_i q (1 - \lambda) \), so that \( K_i = \bar{K}_i - \alpha_i q (1 - \lambda) \) is left over for financing new borrowers. If banks are symmetric, so that \( \bar{K}_i = \alpha_i = \frac{1}{N} \), we obtain \( K_i = \frac{1}{N} (1 - q + \lambda q) \) as the amount available for new loans by each bank. It is this remaining capacity, \( K_i \), that will be most relevant to our analysis.

We first compute the overall distribution of borrowers in the population. To calculate explicit expressions for the distribution of borrowers each bank obtains and to illustrate the form of each bank’s payoff function, assume for now that we have only two banks in the market, with market shares \( \alpha_i \) and \( \alpha_j \), \( \alpha_i + \alpha_j = 1 \). In period 2, there is a mass \( \lambda \) of new borrowers with average success probability \( \bar{\theta} \). There are also a number of bad borrowers that have been rejected by competitor banks. The mass of bad borrowers rejected by bank \( i \) in period 2 is \( \alpha_i (1 - \lambda) (1 - q) \), where \( (1 - \lambda) \) represents the probability that these borrowers survived from period 1 and \( (1 - q) \) represents the mass of borrowers that were of low quality.

In most analysis of Bertrand-Edgeworth equilibria, the form of market rationing plays a key role.\(^{10}\) The heterogeneity of borrowers applying for credit makes it particularly important here as well. The rationing rule we use has the implication that the distribution of borrowers to whom any given bank offers credit is representative of the remaining pool of borrowers. Any other rationing scheme presupposes that there are some kinds of borrowers that are more likely to obtain funding. In contrast, our rationing rule makes no presumptions about any borrower’s ability to obtain funding as a function of its probability of success on its investment project.

Suppose that \( r_i < r_j \). If \( K_i \geq \lambda + \alpha_j (1 - \lambda) (1 - q) \), bank \( i \) lends to all the new borrowers and all of bank \( j \)'s rejected borrowers. If, however, \( K_i < \lambda + \alpha_j (1 - \lambda) (1 - q) \), then bank \( i \)

\(^{10}\)See Davidson and Deneckere (1986) for a discussion of this point.
obtains:

\[ K_i \left( \frac{\lambda}{\lambda + \alpha_j(1 - \lambda)(1 - q)} \right) \text{ new firms} + K_i \left( \frac{\alpha_j(1 - \lambda)(1 - q)}{\lambda + \alpha_j(1 - \lambda)(1 - q)} \right) \text{ rejects} = K_i \text{ total.} \]

Let \( \pi_i(r_i|W) \) denote the profits to bank \( i \) conditional on having the lowest interest rate (the “winning” bank: \( r_i < r_j \)). For ease of notation, define \( A_i \equiv \alpha_i(1 - \lambda)(1 - q) \). We then have:

\[
\pi_i(r_i|W) = \min \left( \lambda, K_i \left[ \frac{\lambda}{\lambda + A_j} \right] \right) (r_i - \bar{\theta} - 1) + \min \left( A_i, K_i \left[ \frac{A_j}{\lambda + A_j} \right] \right) (r_i - \theta_i - 1) \tag{2.1}
\]

The profit for bank \( i \) when it has the higher interest rate (\( r_i > r_j \), so it “loses”) depends on the proportion of new borrowers left over, which is a function of the competitor’s capacity. For the two-bank case, we obtain:

\[
\pi_i(r_i|L) = \max \left\{ 0, \min \left( \lambda \left( 1 - \frac{K_j}{\lambda + A_i} \right), K_i \left[ \frac{\lambda \left( 1 - \frac{K_j}{\lambda + A_i} \right)}{\lambda \left( 1 - \frac{K_j}{\lambda + A_i} \right) + A_j} \right] \right), \right. \\
+ \min \left\{ A_i, \min \left( K_i, K_i \left[ \frac{A_j}{\lambda \left( 1 - \frac{K_j}{\lambda + A_i} \right) + A_j} \right] \right) \right\} (r_i - \bar{\theta} - 1) \tag{2.2}
\]

As is usual in capacity constrained games, each bank’s payoff depends on the other bank’s capacity as well as its own. However, we concentrate our analysis on the case where each bank’s capacity is “small” relative to the total demand for loans. This smoothes out our payoff functions and allows us to ignore the kinks caused by the “mins” and “maxes”.

With more than two banks, we will for simplicity assume that (1) each bank has an equal share of period 1’s market: \( \alpha_i = \alpha = \frac{1}{N}, i = 1, 2, \ldots, N; \) (2) all banks have the same capacity, \( K_i = \frac{1}{N} \), so that \( K_i = \alpha_i(1 - q + \lambda q) = \frac{1}{N}(1 - q + \lambda q) \); and (3) their total capacity equals the demand for loans.\(^{12}\)

\(^{11}\)If \( K_i = \frac{1}{2} \), then \( K_i = \frac{1}{2}(1 - q + \lambda q) \). Observe that \( \lambda + \frac{1}{2}(1 - \lambda)(1 - q) = \frac{1}{2} \lambda + \frac{1}{2}(1 - q + \lambda q) = \frac{1}{2} \lambda + K_i \), so that \( K_i < \lambda + \alpha_j(1 - \lambda)(1 - q) \).

\(^{12}\)The demand for loans from the entire market is just \( D^M = \lambda + \sum_{j=1}^{N} \alpha_j(1 - \lambda)(1 - q) = \lambda + \left( 1 - \lambda \right)(1 - q) = (1 - q + \lambda q) = \sum_{j=1}^{N} \alpha_j(1 - q + \lambda q) = \sum_{j=1}^{N} K_j \), so that entire demand for new loans can be served. That all old good customers are served is also implicit in our definition of \( K_i \) as the remaining funds from \( K_i \) after these firms have been financed.
Recall that we assumed that in case of a tie in interest rate offers each tying bank receives an equal fraction \( \frac{1}{M} \), where \( M \) is the number of tying banks) of the remaining pool of free market borrowers. Letting \( \pi_i(r|T,M) \) represent the payoff in case of a tie, we derive the following.

**Proposition 1** The average quality of borrowers financed by bank \( i \) is higher if it has a lower interest rate than if it ties with other banks.

**Proof:** If \( r_i = r_j \) for some \( j \neq i \), then bank \( i \) lends to a proportion \( \frac{A'}{2} \) of new borrowers, where \( A' \) represents the fraction of new borrowers who have not yet obtained financing, and to a proportion \( \frac{B'}{2} + b_j \) of bad borrowers, where \( B' \) and \( b_j \) represents the mass of rejected borrowers that have not yet obtained financing from the other \( N - 2 \) banks and from bank \( j \), respectively. By undercutting slightly \( (r_i = r_j - \epsilon) \), bank \( i \) lends to a proportion \( A' \) of new borrowers and \( B' + b_j \) old borrowers. Therefore, the relative proportion of new to bad borrowers for bank \( i \), \( \frac{A'}{B' + b_j} \), has improved. \( \Box \)

**Corollary 1** Each bank’s profit is strictly greater when it has the lowest interest rate than if it ties with \( M \) other banks: \( \pi_i(r|T,M) < \pi_i(r|W) \).

**Proof:** This follows directly from proposition 1 and a generalization of equation (2.1) to include \( N \) banks. In case of a tie, a bank’s payoff is as in (2.1), but where the first term is divided by the number of tying banks, \( M \). Since \( \pi_i(r_i|W) \) is increasing in \( r_i \), we have that \( \lim_{r_i \to r} \pi_i(r_i|W) > \pi_i(r|T,M) \). \( \Box \)

In other words, when a bank has the lowest rate, it obtains a qualitatively better distribution of borrowers than in the case of a tie since it does not share the new borrowers with other banks (up to its capacity, that is). This is because, when it undercutts slightly, it increases the proportion of new borrowers to whom it offers credit relative to bad, rejected borrowers, thus increasing the average quality of the loans it grants. Finally, we assume that \( \pi_i(R|W) > 0 \), so that it is always profitable for at least one bank to serve the market.

### 2.4 Equilibrium analysis and the effects of competition

We begin with a simple result that will be important for the subsequent analysis.
Lemma 2 There does not exist an equilibrium with banks playing pure strategies on the free market.

Proof: Suppose that \( \{r_i\}_1^N \) defines a pure strategy equilibrium. Since \( \pi_i(r_i|T) < \pi_i(r_i|W) \), this implies that \( r_1 \neq r_2, \forall i, j \), since if \( r_i = r_j \) for some \( i \) and \( j \), either bank would always have the incentive to undercut slightly. WLOG, suppose that interest rate bids are ordered as \( r_1 < r_2 < \ldots < r_N \). Since \( \pi_i(r_i|W) \) is strictly increasing in \( r_i \), bank \( i \) would want to increase its bid to \( r_{i+1} \), implying that \( r_i = r_{i+1} \). This leads to a contradiction. Therefore, no equilibrium in pure strategies exists. \( \square \)

This result contrasts with the usual results of Bertrand-Edgeworth games, where for very small or very large capacities the game admits a pure strategy equilibrium. The reason for this result is exactly the adverse selection effect. In the usual cases of capacity constrained competition, an already constrained firm has no incentive to lower its price in order to increase its market share.\(^{13}\) However, as argued above, here a bank is able to improve its distribution of applicants by undercutting its opponent, and hence obtain a higher expected payoff, independent of its capacity.

2.4.1 The 2-bank case

While no pure-strategy equilibrium exists, an equilibrium in mixed strategies does exist. As a first step, we prove the following result concerning each bank's equilibrium profits. We note at this point that this result also applies to any two-bank equilibrium, independent of whether capacities are symmetric or not. Moreover, it should be clear that a similar result generalizes to any number of banks.

Lemma 3 In any equilibrium, at least one bank must make expected profits on the free market equal to what it would obtain by charging the highest possible interest rate (\( R \)), but having the highest rate with probability one: \( \pi(R|L) \).

Proof: See appendix.

We can effectively think of \( R \) as each bank's reservation interest rate, as \( \pi_i(R|L) \) is a lower bound on its profits. Therefore, \( \pi_i(R|L) \) establishes a bank's "outside option" as long

\(^{13}\)See Kreps and Scheinkman (1983) or Osborne and Pitchik (1986).
as $\pi_i(R|L) \geq 0$. Otherwise, a bank can opt to not participate in the free market at all, so that its profits are zero. In any mixed strategy equilibrium, it must be that each possible interest rate charged yields the same profit level, and that at the highest interest rate the bank offers, it know that it will be undercut by competitors with probability one. Therefore, in equilibrium, at least one bank's expected profit is in fact equal to this outside option.

While most of the analysis will focus on the symmetric case where $\overline{K}_1 = \overline{K}_2 = \alpha_1 = \alpha_2 = \frac{1}{2}$, and we will restrict the analysis to symmetric equilibria, we also provide a partial characterization of the equilibrium in the two bank case when banks have different capacities. This is necessary to perform comparative statics on each bank's strategies and profits when we vary lending capacities. With a slight abuse of notation, we make profits for each bank an explicit function of capacity, $\pi_i(R|L, K_1, K_2)$. In the symmetric case, by lemma 3, equilibrium profits for both banks are given by $\pi_1(R|L) = \pi_2(R|L) = \pi(R|L)$.\footnote{In the two-bank case, the restriction to symmetric equilibria is without loss of generality, since the equilibrium obtained is unique. With more than 2 banks, the symmetric equilibrium may not be unique, so that it is unclear how serious a restriction this is.} We now characterize the equilibrium when capacities are such that the entire market can be served.\footnote{A similar result holds whenever both banks together do not have sufficient lending capacity to serve the entire market: $K_1 + K_2 < \lambda + 2\alpha(1 - \lambda)(1 - q)$. We offer the result as stated since it highlights the role played by information without having to resort to imposing binding capacity constraints in order to attain positive profits.}

**Proposition 2** Let $\overline{K}_1 = \overline{K}_2 = \frac{1}{2}$. A symmetric mixed-strategy equilibrium exists, given by $\{(\sigma_1, F_1), (\sigma_2, F_2)\}$, where $\sigma_i$ represents the probability that bank $i$ bids on the free market, and $F_i$ is bank $i$'s cumulative distribution function over interest rate.

1. $F_1$ and $F_2$ are continuous and increasing.

2. These mixing probabilities contain no mass points.

3. If $\pi_i(R|L, K, K) > 0$, profits for each bank are given by $\overline{\pi} = \overline{\pi}_i = \pi_i(R|L, K, K)$, $i = 1, 2$, and both banks always bid on the free market ($\sigma_1 = \sigma_2 = \sigma = 1$).

4. If $\pi_i(R|L, K, K) < 0$, profits are $\overline{\pi}_i = 0$, and $\sigma_i < 1$, $i = 1, 2$, so that there is a positive probability that either bank does not bid on the free market.

5. (Asymmetry) If $K_1 > K_2$ and $K_1 + K_2 \geq \lambda + 2\alpha(1 - \lambda)(1 - q)$, there is an asymmetric equilibrium where bank 1 offers interest rate $R$ with positive probability and obtains
profits $\pi_1(R|L, K_1, K_2)$.

**Proof:** See appendix.

The interpretation of mixed strategy equilibria is not always straightforward. If a bank's strategy were predictable, it is a standard result in models of Bertrand competition with fixed costs that competitors would have an incentive to undercut that bank's offer, so that it would face losses with probability one. Therefore, banks must randomize their interest rate offers in order to prevent competitors from always cornering the market.\(^{16}\) One possible interpretation is that the randomization is the result of a process in which banks bargain over interest rate offers separately with each borrower. If clients have heterogeneous bargaining skills that are uncorrelated with their creditworthiness, the outcome of this bargaining process will be a distribution over interest rates, as in the proposition. The only difference is that this distribution would be an observed distribution over interest rates across the population of borrowers, instead of one from which a single interest rate offer is drawn.

Note that if $\pi_i(R|L, K_1, K_2) > 0$, both banks make positive expected profits on the free market in equilibrium, while if $\pi_i(R|L, K_1, K_2) \leq 0$, the banks make zero profits. Whether $\pi_i(R|\cdot)$ is greater than or less than zero will depend largely on $K_1$ and $K_2$, each bank's capacity. For $K_i, K_j$ large enough, we are essentially in the unlimited capacity case, and $\pi_i = 0$. For levels of aggregate capacity less than or equal to 1, profits for any given bank increase in its own capacity and decrease in its competitor's capacity. To see this, we focus on the asymmetric game and allow the capacity of each bank to vary. The following result holds as long as capacities are not too large (as long as $\pi_i(R|L, K_1, K_2) > 0$, $i = 1, 2$).

**Corollary 2** Suppose that $\pi_i(R|L, K_1, K_2) > 0$, $i = 1, 2$. Then each bank's equilibrium free market profit is (weakly) increasing in its own capacity: $\frac{\partial \pi_i}{\partial K_i} \geq 0$ and decreasing in its opponent's capacity: $\frac{\partial \pi_i}{\partial K_j} < 0$.

**Proof:** See appendix.

As long as profits are expected to be positive in equilibrium, any bank would be better off if it could finance a larger pool of applicants. However, when a competitor's capacity

\(^{16}\)Rajan (1992) analyzes a similar situation where he allows firms to seek competitive bids from many banks. He argues that it is not uncommon to see firms seeking sealed bids from a number of different banks, and that this kind of behavior leads to mixed strategy equilibria.
increases, two effects operate: less applicant firms are left for the high rate bank, and moreover the pool of applicants worsens. Therefore, profits must decrease.

A comment about one of the assumptions may be useful at this point. The assumption that banks are unable to distinguish between new and old borrowers may seem somewhat extreme given that a firm's credit history is often available.\textsuperscript{17} We claim that allowing banks to review the credit records of applicant firms before making any loans would not change the results in any qualitative way as long as this review comes at some per firm cost. Entrant banks face higher costs of reviewing all these records since they face a larger mass of unknown borrowers: all the new, incoming borrowers, and every incumbent bank's poor quality borrowers. Alternatively, we obtain the same results (and all those that follow) if we assume that some bank-borrower relationships end for exogenous reasons (such as dissatisfaction with services). Banks will then be unable to distinguish borrowers that have been rejected from those that are changing banks for "taste" considerations. This provides an alternative interpretation for \( \lambda \) as the rate at which borrowers dissolve their banking relationships.\textsuperscript{18}

**Implications for surplus extraction**

We have up to now been focusing exclusively on the incremental profits from the decision to compete (only on "free market" profits) and ignoring the profit that each bank makes from its old customers. This is the "surplus extraction" issue, that banks obtain a form of informational monopoly through their lending activity. What rents a bank is able to extract from its old locked-in good customers depends on how capacities and information affect competition. For bank \( i \), its profit from its old customers is, in expectation: \( \alpha_i(1 - \lambda)q(E[r_j]\theta_h - 1) \), since bank \( i \) charges a borrower of type \( \theta_h \) an interest rate \( r_{i\theta_h} = r_j \) (see lemma 1), but only \( (1 - \lambda) \) of the \( q \) good borrowers from period 1 survived to period 2. Therefore, total expected profits for bank \( i \), including the free market and its old customers,

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\textsuperscript{17} The availability of information concerning borrowers' previous credit arrangements is itself an equilibrium phenomenon (see Padilla and Pagano (1997)). While banks never have an incentive to share information about their old borrowers in our static model, this incentive may arise in a dynamic framework. We can, however, analyze the effects of assuming that banks operate in an environment that requires information sharing.

\textsuperscript{18} This idea of exogenous separations has also been used by Greenwald (1986) in his analysis of the labor market. A further reason for venturing elsewhere for financing might be the constraints on lending capacity of banks, which may bind when a firm wants to obtain a loan in order to expand output.
are:

\[
\Pi_i = E[\pi_i(r)] + \alpha_i(1 - \lambda)q(E[r_j]\theta_h - 1) \\
= \pi_i(R|L, K_1, K_2) + \alpha_i(1 - \lambda)q(E[r_j]\theta_h - 1)
\]  \hspace{1cm} (2.3)

We then have the following result:

**Corollary 3** Total profit (free market plus old customers) for bank $i$ is, (1) increasing in its own capacity: $\frac{\partial}{\partial K_i}\Pi_i > 0$; but (2) may increase or decrease in its opponent's capacity: $\frac{\partial}{\partial K_j}\Pi_i$ may be greater or less than zero.

**Proof:** See appendix.

Changes in $K_i$ have no direct effect on bank $i$'s competitor's interest rate, $E[r_j]$, so that $\frac{\partial}{\partial K_i}E[r_j] = 0$, and therefore $\frac{\partial}{\partial K_i}\Pi_i = \frac{\partial}{\partial K_i}\pi_i(R|L) > 0$. This is because increasing bank $i$'s capacity does not affect how bank $j$ is willing to mix among interest rates, since bank $j$ mixes so as to keep bank $i$ indifferent between its actions. Increasing $K_i$ acts as a constant change in bank $i$'s payoff, and so does not require a corresponding change in bank $j$'s mixing probability. An increase in competitor's capacity ($K_j$), on the other hand, has a direct negative effect on bank $i$'s profit, since it lowers its free market profits (corollary 2). However, increasing bank $j$'s capacity has another effect as well: it softens the competition by bank $j$, leading to higher expected interest rates ($\frac{\partial E[r_j]}{\partial K_j} > 0$). Again, this stems from the usual equilibrium logic that since the bank whose capacity increase mixes so as to keep the other bank indifferent, it must shift its distribution up in order to lower its probability of winning.\textsuperscript{19} This allows bank $i$ to obtain higher profits from its old customers. In other words, the surplus that each bank is able to extract from its old borrowers thanks to its informational advantage depends indirectly on its competitor's capacity through its effect on competition.

**Corollary 4** Total expected profit is increasing in opponent's capacity ($\frac{\partial}{\partial K_j}\Pi_i > 0$) if $q$ is greater than some cutoff value $q$.

\textsuperscript{19}Kepr and Scheinkman (1983) obtain a similar result on the distribution $F_j$, and offer the following intuition: since bank $j$ now has a relatively larger capacity, it must be less aggressive in its bidding in order to lower the otherwise increased “risk” of losing that bank $i$ faces.
The proof of this result is straightforward, and therefore omitted. From the definition of $\Pi_i$ (equation (2.3)), we see that bank $i$'s profit will be increasing in its opponent's capacity when the rents coming from its old customers are large, so that the surplus extraction effect dominates the loss in profits on the free market. When $q$ is large, the majority of old borrowers are of high quality, so that the pool of old borrowers refinanced by bank $i$ is large and the profit stemming from these borrowers is large. Moreover, $E[r_i]$ is increasing in $q$ for value of $q$ near 1, which further increases the surplus extraction effect. In other words, when most borrowers are good, the value of bidding a high interest rate is very large, even if it comes with an increased probability of being the highest rate.

While it would seems that low values of $\lambda$ lead to more surplus extraction (the last term in equation (2.3) is large), as we will see in a later section, the expected interest rate also depends on $\lambda$: as $\lambda$ converges to 1, $E[r_i]$ converges to $R$. Therefore, as information asymmetries decrease, more surplus is extracted from each old creditworthy borrower (although there are less of them). The explanation for this result relies on focusing on the limits to lending capacity. With no adverse selection, banks with limited capacity retain a degree of market power. Introducing an element of adverse selection into the model actually prompts banks to compete more aggressively, since then only when they have low rates can they expect to make profits from lending on the free market.20

2.4.2 The case with $N > 2$ banks

We now turn to the analysis of equilibria with more than two banks. We will concentrate exclusively on the case of symmetric banks from now on. With constraints on lending capacity, any single bank's inability to serve all new borrowers implies that other banks with higher interest rates need not make zero profits. However, whether all $N$ banks are able to obtain positive profits depends on the rate of turnover among borrowers ($\lambda$) and the proportion of high quality borrowers ($q$).

We first offer the following result concerning the equilibrium distribution of firms obtaining financing. All results from now on apply to the symmetric mixed strategy equilibrium of this game.

**Proposition 3** Let $\alpha_i = \overline{K}_i = \frac{1}{N}$, so that $\sum_{i=1}^{N} \overline{K}_i = 1$. The average quality of borrowers

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20 As capacity increases, the amount of surplus that can be extracted diminishes, since the competition for borrowers increases, lowering the interest rate.
obtaining credit decreases with \( N \), the number of banks.

**Proof:** See appendix.

In effect, even though we postulate that capacity should be sufficient to finance everyone, in equilibrium some borrowers are denied credit since banks need not refinance their own old customers that are known to be poor credit risks. The lemma demonstrates that this rationing is reduced as we increase the number of banks. The intuition comes from noticing that as the number of banks increases, each bank has knowledge of a relatively smaller fraction of the customers. The total pool of free market borrowers remains unchanged, but the fraction that each bank can "screen" based on its previous knowledge is reduced, so that in effect it faces a worse distribution of borrower. In equilibrium, this leads to more low quality borrowers obtaining financing.\(^{21}\) This points to an inefficiency of increased competition that does not rely on simply increasing industry lending capacity but rather works directly through the increased competition among banks. This allows us to state the following result concerning equilibrium profits:

**Proposition 4** Consider \( N \) identical banks with \( \sum_{i=1}^{N} K_i = 1 \). There exists some \( \lambda_{N}^{+} < 1 \) such that for \( \lambda > \lambda_{N}^{+} \), free market profits for each bank are positive \( (\pi_i > 0) \), and are equal to zero for \( \lambda \leq \lambda_{N}^{+}, \forall \ i = 1, \ldots, N \). Moreover, \( \lambda_{N}^{+} \) is increasing in \( N \).

**Proof:** See appendix.

This proposition establishes that if the proportion of new borrowers is large enough, in equilibrium each bank obtains positive profits on the free market. This seems in accordance with our intuition, since if there are "enough" new borrowers each period \( (\lambda \approx 1) \), so that information asymmetries are low, one possible strategy by a bank is simply to offer the interest rate \( r_i = R \) on all loans, and make positive profits. However, if the proportion of new borrowers is small, then the bank with the highest interest rate will be left with almost all bad borrowers, and hence cannot expect to make positive profits. The only symmetric equilibrium then is for each bank to make zero expected profits, and to not enter with some probability.\(^{22}\) The reason why \( \lambda_{N}^{+} \) is increasing in \( N \) even though aggregate lending

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\(^{21}\)One possible interpretation for this result is that as \( N \) increases, the borrower-specific information becomes more dispersed or fragmented, as each bank acquires information about a smaller number of borrowers.

\(^{22}\)Strictly speaking, at the cutoff value \( \lambda_{N}^{+} \), each bank bids with probability 1, and makes zero expected profits. For \( \lambda < \lambda_{N}^{+} \), every bank will have some positive probability of not bidding \( (\sigma < 1) \).
capacity is fixed is that as the number of banks increases, the pool of borrowers obtaining credit worsens (proposition 3). The following limit result highlights the role of adverse selection in determining bank profits and interest rates.

**Corollary 5 (Comparative statics results on the market equilibrium)**

1. Fix aggregate capacity $\bar{K}$ equal to 1. For large values of $\lambda$, each bank offers an interest rate close to $R$: $\lim_{\lambda \to 1} E[r_i] = R$.

2. Fix $\lambda = 1$. As aggregate capacity increases, each bank's interest rate offer decreases: $\lim_{K \to \infty} E[r_i] = 1/\bar{\theta}$.

As adverse selection becomes less important, each bank maximizes profits by charging the highest possible interest rate. The limit on aggregate lending capacity prevents banks from competing with each other for customers. However, increasing capacity leads to the usual Bertrand result of zero profits. Using similar logic, we can obtain a related result concerning the proportion of good to bad quality borrowers. If $q$ is large enough, each bank also makes positive profits. If $q$ is close to 1, each incumbent bank's informational advantage is effectively diminished since "most" borrowers are good, i.e. the proportion of bad borrowers rejected by each bank is very small compared to the fraction of new borrowers seeking financing. Positive profits are therefore possible even for the highest interest rate bank, when the number of banks is fixed exogenously.

The analysis of this section suggests that shocks to the banking industry can have permanent (or long-lasting) effects on its composition. For example, recent studies have argued that shocks that reduce bank capital affect banks' ability to grant loans (Kashyap and Stein (1995)). Banks that reduce their lending activity also lose information about the borrowers in the market, making competition for future market share more difficult. Banks that are better able to weather these shocks should find themselves in a significantly improved position thanks to their relatively larger information base. However, to the extent that macroeconomic or monetary policy shocks hit banks across the board, this may open

---

23 This is similar to a result in Riordan (1993), that increased competition among banks leads to a higher probability of a bad firm obtaining credit from at least one bank.

24 For $\lambda = 1$, there is no information asymmetry. The equilibrium in this case is in pure strategies, since the undercutting incentive disappears when capacity is limited.
the door for other competitors (other non-bank intermediaries) to enter these markets, and permanently establish a presence.\textsuperscript{25}

\subsection*{2.4.3 A comparison across markets}

Consider two different banking markets, one a highly concentrated market with two large banks, and another characterized by competition among a large number of small banks. We address the question of which market is the more "competitive" one, where by competitive we mean having lower interest rates. Most models of competition have the property that a larger number of potential competitors leads to more surplus for consumers, which in this model translates to lower average interest rates for borrowers.\textsuperscript{26} However, in capacity-constrained models, competition is generally softened as capacity is decreased, since the incentives to compete aggressively are reduced when there is a limit to how much of the market any given competitor can capture. Therefore, it is the interaction between banks' lending capacity and the degree of informational asymmetries that determine the expected interest rates, so that the number of banks by itself may not be a sufficiently good proxy for the degree of competition.

We wish to analyze the effect on expected interest rates of increasing the number of banks, keeping the aggregate lending capacity fixed.\textsuperscript{27} As we showed in the previous section (proposition 3), the average quality of borrowers who obtain credit is decreasing in the number of competing banks. This has the negative effect of pushing interest rates up to compensate for the reduced average quality of borrowers. Overall, whether interest rates are lowered because of increased competition or increased because of the worsening of the pool of borrowers will depend on the particulars of the market.

\textsuperscript{25}Berger, Kashyap, and Scalise (1995) find that in the period 1989-1992, the so-called "credit crunch", banks curtailed their small business lending, without a significant rebound post 1992, even though overall growth in commercial and industrial lending seems to have recovered. Bernanke, Gertler, and Gilchrist (1994) have suggested that this might be consistent with the "flight to quality" phenomenon, although they do not explain the lack of a rebound in this sector. Our analysis suggests that the rebound failed to occur precisely because of the information loss concerning small business loans, which theory would predict are the most subject to information problems. Whether this pattern will continue remains to be seen.

\textsuperscript{26}For an example where increased competition leads to higher prices, see Sharkey and Sibley (1993). An interesting example of a model where competition in banking leads to higher interest rates can be found in Hoff and Stiglitz (1997).

\textsuperscript{27}In the context of a standard Bertrand-Edgeworth game, Allen and Hellwig (1986) show that the equilibrium of the capacity constrained game converges in distribution to the competitive outcome as the number of firms is increased at the same time as their capacity is decreased. However, in our model there is a countervailing factor that puts a limit on the beneficial effects of increased competition.
<table>
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<tr>
<th>Expected Interest Rates Across Markets</th>
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<tr>
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Table 2.1: Expected interest rates for 3 and 4 bank equilibria. $\overline{K}_i = \frac{1}{3}$ or $\frac{1}{4}$, $\alpha_i = \frac{1}{3}$ or $\frac{1}{4}$, respectively. $\theta_h = .75$, $\theta_l = .25$, $R = 3$. Values reported only for equilibria that yielded positive profits. Bold values represent those for which interest rates are lower with 4 banks.

Some simple numerical calculations shed some light on when a particular effect can be expected to dominate. Table 3.1 presents some results concerning the expected interest rate for different values of $\lambda$ and $q$ for a market with either 3 or 4 banks. We assume that $\theta_h = .75$, $\theta_l = .25$, $R = 3$, and that $\overline{K}_i = \frac{1}{3}$ or $\frac{1}{4}$, $\alpha_i = \frac{1}{3}$ or $\frac{1}{4}$, respectively. For the most part, we find that expected interest rates are higher with 4 banks than with 3, suggesting that the negative effect of the worsening pool dominates any gain from increased competition. Only for very low values of $q$, the fraction of high quality borrowers in the market, do interest rates drop when we increase the number of banks. This occurs because when there are very few high quality borrowers, the only way to make positive profits is by offering credit to some of these borrowers, and only the bank with the lowest rate will be able to do so. Increasing $N$, while keeping the aggregate capacity fixed, increases the competition for these few borrowers.

This last discussion raises another important point, which is that aggregate capacity clearly matters. While increasing the number of banks may cause interest rates to rise or fall, increasing capacity should push interest rates down.

**Proposition 5** There exists a set of parameters $(\lambda, q)$ for which: with $N$ active banks with
aggregate lending capacity of $\bar{K}_N = \sum_{i=1}^{N} \bar{K}_i = 1$ such that each bank makes positive profits in equilibrium, there exists an aggregate lending capacity $\bar{K}^2 = \bar{K}_1 + \bar{K}_2 > \bar{K}^N$ for a market with only two active banks such that $E[r|N, \bar{K}^N] > E[r|2, \bar{K}^2]$.

Proof: See appendix.

In other words, two banks with a large capacity of loanable funds may compete more aggressively for customers than a large number of very small banks. Effectively, the degree of competition and the aggressiveness of banks in obtaining customers depends on the financial constraints that each bank faces as well as on how disperse is the knowledge they have about the industry. If aggregate lending resources vary across different industries or markets, then simply using the total number of banks as a proxy for the degree of competition might be missing one of the main determinants of competition and its effects on consumer surplus.

An equivalent interpretation of these results is to consolidation in the banking industry. To the extent that mergers lead to a consolidation of information as well as assets, banks should be able to reduce the degree of inefficient lending, which may in fact push in the direction of lower interest rates. There has been much debate as to whether the wave of recent mergers and acquisitions in the U.S. has had a negative impact on the volume of small business loans (Berger et al. (1995), Peek and Rosengren (1997)). Our analysis suggests that perhaps at least a part of this decreased volume may be attributed to more efficient screening of borrowers when information is more consolidated.\(^{28}\)

2.5 Entry: Patterns and Conditions

A pattern seems to be emerging, particularly in the European market, for foreign banks who enter a domestic market. They tend to do so either via acquisitions or via mergers, with very little domestic bank branching. We argue that, because of informational asymmetries that create difficulties for entry, the most effective way to enter a market may be to buy out an incumbent bank. This provides an entrant not only with an established network,

\(^{28}\)There are reasons to think, however, that larger institutions may be worse at monitoring small, regional loans. These loans often require either specialized or local knowledge, and as such may be better monitored by small local banks with strong relationships to the community. The implication is that post merger, the new, larger bank may fail to renew many of these small loans. The analysis of this aspect of consolidation is outside the scope of this paper. See Peek and Rosengren (1997) and Berger, Saunders, Scalise, and Udell (1997) for an analysis of this issue.
but more importantly with information about the market.\textsuperscript{29}

2.5.1 Conditions for entry

As alluded to in the introduction, we expect banks to find it easier to enter a market where they are able to compete on a more equal footing with incumbent banks. Incumbent banks' constraints on lending capacity may open the door for potential entry by a bank with available loanable funds. With two incumbent banks, a third bank may be able to enter even if it does not expect to have the lowest interest rate as long as the incumbents are sufficiently constrained, and information asymmetries are limited.

However, the results can be quite different when banks' lending capacity increases. Dell'Ariccia et al. (1996) focus on the extreme case where banks face no limits on lending, and argue that if banks compete à la Bertrand in the credit market, then whenever there are already two active banks in any given market, a third bank will always find entry blocked. This entrant faces a worse distribution of firms than the incumbent banks, and so cannot expect to obtain non-negative profits. Therefore, the free-entry equilibrium number of banks will depend on the capacity constraints each bank faces.

Proposition 6 Entry is more difficult when more banks participate in a market. Specifically,

1. Suppose there are $N$ symmetric incumbent banks, each with capacity $K_i = \frac{1}{N}$. There exists some cutoff $\lambda_N < 1$ such that an equilibrium where an $N + 1$\textsuperscript{st} bank enters the free market with positive probability exists only if $\lambda > \lambda_N$.

2. This cutoff value $\lambda_N$ is increasing in $N$.

Proof: See appendix.

The proposition implies that banks may have an easier time entering a market with a large number of new borrowers seeking financing, and that, most importantly, are unknown to local banks. As in proposition 4, the result is again driven by the changes in the quality

\textsuperscript{29}"Moreover, by acquiring a domestic bank, a foreign bank gets direct access to a customer base and local information." Vesala (1995), p. 37, and "Actual foreign rivalry is still fairly limited in many European countries as the market shares of foreign owned banks are quite small in most countries. ... the foreign owned banks have mostly positioned themselves in the wholesale and corporate banking markets ... " , p. 38.
of the pool of borrowers obtaining financing as \( N \) increases. When \( \lambda \) is sufficiently large, entrant banks are less disadvantaged relative to incumbent banks and so may find entry profitable. As \( N \) increases, incumbents' expected profit decreases, making entry more difficult for fixed values of \( \lambda \).\(^{30}\) While this lends itself to the interpretation of a "growing market", we should emphasize that for the purposes of the proposition, we keep the total mass of borrowers in the economy fixed. If the economy is truly growing, so that each period there are \( \lambda \) more borrowers, then either entry or capacity expansion will be necessary in order for these borrowers to be able to obtain financing. Instead, proposition 6 states that even if the mass of borrowers is not growing, there is still an incentive for a new bank to enter if there are a sufficient number of new, unknown borrowers each period. What we have is a story of high turnover rather than market growth.

In the limit, if every period all borrowers exit the market and are replaced by new borrowers (\( \lambda = 1 \)), entry by an equal-sized \( N + 1 \)st bank would always occur (independently of \( N \)), and no further entry beyond that would be profitable.\(^{31}\) In this situation, any informational advantage of incumbents would be completely diluted, and potential entrant banks may have an easier time getting a foothold in this industry.

### 2.5.2 Bank acquisitions

We know, from corollary 3, that \( \frac{\partial}{\partial K_i} \Pi_i \geq 0 \). This implies that a new bank with large capacity can add value to an existing bank with a smaller capacity to make loans. That this is preferable to de novo entry that increases the number of banks from \( N \) to \( N + 1 \) is clear from proposition 4. For any value of \( \lambda \), the equilibrium with \( N + 1 \) banks will lead to lower expected profits than the equilibrium with \( N \) banks. Moreover, by increasing the aggregate capacity with de novo entry, post-entry equilibrium profits are reduced even further relative to the buyout case. Since profits are increasing in capacity, a small incumbent will be worth more to an entrant with a larger lending capacity than as a stand alone, and will be bought out as a form of entry. We have therefore established the following result.

---

\(^{30}\)Thanks to the incumbent banks' inside information, not all borrowers are served in equilibrium, so that even the \( N \)th incumbent bank retains a measure of market power, and can make positive profits. The increased competition and the worsening of the borrowers' quality after entry leads to zero profits.

\(^{31}\)Notice however than when \( \lambda = 1 \) the market will support any number of identical banks independent of their capacities. This is similar to other models of symmetric Bertrand competition where there is really no limit to the number of banks that can participate (See Sharkey and Sibley (1993)). Introducing a small fixed cost of entry or a positive cost of bank funds would be sufficient to pin down the equilibrium number of banks.
Proposition 7 Banks with large lending capacity, $\bar{K}$, should find it more profitable to enter through an acquisition than through de novo entry.

By buying out an incumbent, the entrant bank acquires knowledge of local borrowers, enabling it to avoid some of the adverse selection that a de novo entrant would face.\footnote{There is a caveat here, however. In the aggregate, banks' portfolios are not risky in this model, even if individual loans are. Banks' inability to raise external funds becomes an important issue when considering buyout decisions, since without some measure of asymmetric information at the bank level, banks should be able to raise external capital. Nevertheless, it is still true that potential competitors would prefer to enter via acquisition. Also, banks limited capacity may exist for other reasons as well, such as competition for deposits (Matutes and Vives (1996)) or limits to their information processing ability (Gale (1993)).} Banks with small capacities will be unable to buy out larger banks, since unless they also acquire that bank's funds, they will be unable to expand activities sufficiently to cover the cost they paid. Therefore, this provides a theory of large banks' expansion through acquisitions, leading to a possible consolidation in the industry.\footnote{Hoskca (1993) provides some evidence that only the biggest European banks are involved in cross-border operations. Smaller banks operate almost exclusively in domestic markets.}

2.5.3 Role of information sharing and expertise

In the context of this simple model, banks never have an incentive to share information concerning their old borrowers. However, this incentive may exist in a dynamic version of this model, where borrowers have to make some decision over project choice.\footnote{Padilla and Pagano (1997) provide a model where the sharing of information among banks serves as a commitment against extracting too much surplus from borrowers in later periods, after information about them has been acquired. This in turn raises borrowers' returns from exerting effort, and leads to more efficient investment decisions.} Also, there may be institutional or legal reasons why banks have to share information about customers with competitors. In this section we briefly analyze the implications of information sharing for entry.

Information sharing among incumbent banks decreases the adverse selection each bank faces, since it allows them to sort out other banks' high and low quality borrowers. This has a positive effect on each bank’s profit, as it no longer risk financing other banks' bad customers. But it also allows banks to compete more aggressively for both old and new borrowers, exerting a negative pressure on profits. This occurs because each bank now has some unused lending capacity to the extent that old low quality borrowers are denied credit. A third possibility is that sharing information may facilitate collusion among incumbents, since it allows them to maintain high profits and to punish transgressors by cutting off the
flow of information.

However, these same factors that affect incumbents' profits have implications for the possibility of entry by outside banks. If incumbent banks are required to share information with entrant banks, this reduces the adverse selection cost that entrant banks face, and hence facilitates entry. Even if incumbents remain better informed, as long as entrants are able to screen out some low quality borrowers, their position will be improved. In the context of proposition 6, the minimum \( \lambda \) that will be required in order for entry to occur will be reduced.

A similar analysis applies to entrant banks who may have some expertise in a particular industry. A bank specializing in granting loans to a specific kind of industry or type of borrower should also find it easier to enter to the extent that it is able to distinguish among good and bad borrowers before granting a loan. Therefore, it suffers less from adverse selection than an entrant with no particular market or local knowledge, and should have less difficulty entering.\(^{35}\)

We summarize the discussion of this section in the following proposition.

**Proposition 8** Entry by a new bank should be easier if:

1. **Incumbent banks are required to share information concerning borrowers with entrant banks.**

2. **Entrant banks have market-specific expertise in evaluating borrowers' creditworthiness.**

### 2.6 Conclusion and extensions

We have argued that the presence of information asymmetries among banks regarding the quality of borrowers can pose a significant barrier to entry. Even when banks face constraints on lending capacity, this adverse selection can have two opposing effects on competition. It may induce tougher competition as banks try to minimize the adverse effects of these information asymmetries by competing aggressively for new customers. However, it also lowers the incentives to compete since the presence of low quality borrowers lowers each bank's return. We argue that these information asymmetries can have large effects on

\(^{35}\)A model that incorporates the idea of specialization in monitoring and its effects for competition is Almazan (1996).
banks' profits and ability to enter new markets. In particular, we find that as information asymmetries are reduced, both the possibility of realizing positive profits and of entering an existing market are increased. Moreover, even if entry does take place, its impact on interest rates is not clear. If entrants do not add significant aggregate capacity to the market, increasing the number of competing banks may actually push interest rates up and lead to more low quality borrowers obtaining financing.

There is a good deal of casual evidence that lends support to our theory. Vives (1991) has argued that most trade in banking services in Europe remains limited, even after deregulation. Most of the trade is intra-industry, and varies substantially across different countries, suggesting hidden and persistent economic barriers to entry. This is consistent with our theory that banks find it easier to enter a market when they are either specialized in an industry or when they are on a more equal footing with incumbent banks. Similarly, Hoschka (1993) presents evidence that the small amount of entry that has been observed has been done by the largest banks, and that moreover it has mostly taken place in the commercial banking sector, rather than the retail sector where we expect information problems to be greater.

However, there remain a number of unresolved issues. In a dynamic framework, we might be interested in analyzing exactly what determines these lending capacities, and what role is played by the competition for funds. While we have taken the lending capacity of banks to be exogenous, competition for deposits imposes another cost that needs to be borne under increased competition. Also, a bank’s ability to raise funds should be correlated with depositors’/investors’ expectations concerning profitability, so that banks that are perceived as being successful should have an easier time raising funds (Matutes and Vives (1996)). A bank’s ability to enter a market may be further reduced to the extent that depositors anticipate (correctly) that it will have difficulty entering successfully. Thus, they refrain from depositing there, which rationalizes the entrant bank’s low profits.

Another interesting extension is to focus on the ex-ante competition for customers. Knowing that in future periods each bank will have some “trapped” customers from which it can extract surplus, each bank has the incentive to raise a lot of funds in order to control a large share of the market. However, raising capacity also lowers future expected interest rates, leading to less surplus extraction from old customers, and hence lowering the incentive to raise lending capacity. The optimum should represent a tradeoff between these
two effects.

While these extensions must await further research, we believe that the main point remains that the adverse consequences of information asymmetries can have a large impact on the degree of competition, and particularly on the ease of entry.
2.7 Appendix

2.7.1 Proofs

Proof of Lemma 1: The payoff from type $\theta_h$ borrowers will be given by $\Gamma(r_i, \theta_h - 1)$, where $\Gamma$ is some function of $\alpha_i$, $\lambda$, and $q$, each bank's previous market share, the fraction of new firms, and the ex ante probability that a firm is good, respectively. Profits on these firms are maximized at the highest possible $r_i$ such that a firm of type $\theta_h$ doesn't switch banks, i.e. $r_i \theta_h = r_j$. Profits will be non-negative as long as $r_j \geq \frac{1}{\theta_h}$. Suppose bank $j$ charges $r_j = \frac{1}{\theta_h}$ and obtains all of bank $i$'s good customers. Then, ignoring any capacity constraints, profits to bank $j$ will be:

$$
\pi_j(r_j) = \lambda(r_j \bar{\theta} - 1) + \alpha_j(1 - \lambda)(1 - q)(r_j \theta_i - 1) + \alpha_j(1 - \lambda)q(r_j \theta_h - 1) \\
= \lambda(r_j \bar{\theta} - 1) + \alpha_j(1 - \lambda)(r_j \bar{\theta} - 1) = (r_j \bar{\theta} - 1)(\lambda + \alpha_j(1 - \lambda)) \\
= \left(q + (1 - q) \frac{\theta_i}{\theta_h} - 1\right)(\lambda + \alpha_j(1 - \lambda)) < 0,
$$

since $\theta_i < \theta_h$, so that the first term is negative. Therefore bank $j$ never offers $r_j \leq \frac{1}{\theta_h}$, and so never obtains bank $i$'s good firms. That firms of type $\theta_i$ are denied credit should be clear given that we assumed that $R \theta_i < 1$, which establishes our claim. □

Proof of Lemma 3: Payoffs are:

$$
\pi_i(r_i) = \begin{cases} 
\pi_i(r_i | W) & \text{if } r_i < r_j \\
\pi_i(r_i | T) & \text{if } r_i = r_j \\
\pi_i(r_i | L) & \text{if } r_i > r_j
\end{cases}
$$

(2.4)

and note that $R = \arg\max_{r_i} \pi_i(r_i | L)$. Let $(F_1, F_2)$ define an equilibrium, where $F_i$ represents the mixing distribution over interest rate offers (cdf) by bank $i$. Let $\mu_i(r)$ represent the mass $F_i$ puts on interest rate $r$, if any. With a slight abuse of notation, let $\pi_i(F_1, F_2)$ represent the profits of bank $i$ under the proposed equilibrium strategies $F_i$, $F_j$. We then know that $\pi_i(F_1, F_2) \geq \pi_i(R | L)$, since we require that, $\forall r \in \text{supp}(F_i)$, $\pi_i(r) \geq \pi_i(R | L) \Rightarrow \pi_i(F_1, F_2) \geq \pi_i(R | L)$. Let $\bar{r}_1 = \max \text{supp}(F_1)$ and $\bar{r}_2 = \max \text{supp}(F_2)$. Suppose that $\bar{r}_1 > \bar{r}_2$. Then $\text{prob}(r_2 > \bar{r}_1) = 0$, so that $\pi_1(\bar{r}_1) = \pi_1(\bar{r}_1 | L)$. From equation (2.4) above, we must have that $\bar{r}_1 = R$. Suppose then that $\bar{r}_1 = \bar{r}_2 = \bar{r}$. If $\mu_2(\bar{r}) = 0$
(so that bank 2 does not have an atom at $\bar{r}$,) then again $\text{prob}(r_2 > \bar{r}_1) = 0$, and then

$$\pi_1(\bar{r}) = \pi_1(\bar{r}|L) = \pi_1(R|L).$$

If $\mu_2(\bar{r}) > 0$, so that bank 2 puts positive mass on an interest of $\bar{r}$, we can show that $\mu_1(\bar{r}) = 0$, which by a symmetric argument gives us that

$$\pi_2(\bar{r}) = \pi_2(\bar{r}|L) = \pi_2(R|L),$$

as desired. Therefore at least one bank has payoff $\pi_i(R|L)$ in equilibrium. $\square$

**Proof of Proposition 2:** First we need to show that $\not\exists$ mass points in the mixing distributions of both banks. We look for a symmetric equilibrium. Suppose that bank $i$ is mixing over $[\bar{r}, R)$, and that there is an atom at some $r = r'$. Then the payoff for bank $j$ is $\pi_j(r'|T)$ for playing $r'$, but $\lim_{r \to r'} \pi_j(r) = \pi_j(r'|W) > \pi_j(r'|T)$, so that bank $j$ would put no mass at $r'$, contradicting the assumption that $F$ contains an atom there.\textsuperscript{36}

We can now state bank $i$'s expected payoffs as:

$$\begin{align*}
\pi_i(r_i|K_i, K_j) &= (1 - F_j(r_i))K_i \left\{ \frac{\lambda}{\lambda + A_j} (r_i \bar{\theta} - 1) + \frac{A_j}{\lambda + A_j} (r_i \theta_t - 1) \right\} \\
+ &F_j(r_i)K_i \left\{ \frac{\lambda}{\lambda (1 - K_j/\lambda + A_i)} (r_i \bar{\theta} - 1) + \frac{A_j}{\lambda (1 - K_j/\lambda + A_i) + A_j} (r_i \theta_t - 1) \right\}
\end{align*}$$

where again we use the notation that $A_i \equiv \alpha_i(1 - \lambda)(1 - q)$. We can now rewrite this as:

$$\begin{align*}
\pi_i(r_i|K_i, K_j) &= K_i \left\{ \frac{\lambda}{\lambda + A_j} (r_i \bar{\theta} - 1) + \frac{A_j}{\lambda + A_j} (r_i \theta_t - 1) \right\} \\
+ &F_j(r_i)K_i \left\{ \frac{\lambda}{\lambda (1 - K_j/\lambda + A_i)} (r_i \bar{\theta} - 1) - \frac{\lambda}{\lambda + A_j} (r_i \theta_t - 1) \right\} \\
+ &F_j(r_i)K_i \left\{ \frac{A_j}{\lambda (1 - K_j/\lambda + A_i) + A_j} - \frac{A_j}{\lambda + A_j} (r_i \theta_t - 1) \right\}
\end{align*}$$

Let $C = \lambda \left(1 - \frac{K_j}{\lambda + A_i}\right)$. We can solve for the distribution function $F_j$ as:

$$F_j(r) = \frac{\frac{r}{K_i} - \left[ \frac{\lambda}{\lambda + A_i} (r \bar{\theta} - 1) + \frac{A_j}{\lambda + A_i} (r \theta_t - 1) \right]}{\left[ \frac{C}{C + A_i} - \frac{\lambda}{\lambda + A_i} \right] (r \bar{\theta} - 1) + \left[ \frac{A_i}{C + A_i} - \frac{A_j}{C + A_i} \right] (r \theta_t - 1)}$$

(2.5)

\textsuperscript{36}A similar (though slightly more complicated) argument can be used when strategies are not symmetric. The mechanics of the proof in this case are omitted.
Also, using the lemma 3, allows us to pin down profits:

\[
\pi_i(R|L,K) = K_i \left[ \frac{C}{C + A_i} (R\bar{\theta} - 1) + \frac{A_i}{C + A_i} (R\theta_l - 1) \right] \tag{2.6}
\]

Note that if \( K_1 + K_2 \geq \lambda + 2\alpha(1 - \lambda)(1 - q) \), then these two equations simplify to:

\[
F_j(r) = \frac{\bar{\pi}_i}{K_i} - \frac{\lambda}{\lambda + A_i} (r\bar{\theta} - 1) + \frac{A_j}{\lambda + A_i} (r\theta_l - 1)
\]

\[
\frac{C}{C - \lambda + A_i} (r\bar{\theta} - 1) + \frac{A_i}{C - \lambda + A_i} (r\theta_l - 1) \tag{2.7}
\]

\[
\pi_i(R|L,K) = K_i \left[ C(R\bar{\theta} - 1) + A_j(R\theta_l - 1) \right]
\]

\[
= K_i \left[ \lambda \left( 1 - \frac{K_j}{\lambda + A_i} \right) (R\bar{\theta} - 1) + \alpha_j(1 - \lambda)(1 - q)(R\theta_l - 1) \right] \tag{2.8}
\]

which we can now substitute into equation (2.7) above, as \( \bar{\pi}_i = \pi_i(R|L,K) \) in equilibrium.

Note finally that if \( K_j < \lambda + \alpha_i(1 - \lambda)(1 - q) \), then \( \exists \lambda < 1 \) such that \( \pi_i(R|L,K) > 0 \). This gives us that for \( \lambda > \lambda \), \( \bar{\pi}_i = \pi_i(R|L,K) > 0 \), which necessarily implies that it is always optimal for both banks to bid. Conversely, for \( \lambda \leq \lambda \), \( \pi_i(R|L,K) \leq 0 \), and it will be strictly negative for \( \lambda < \lambda \), so that it cannot be optimal for both banks to always bid. This gives us that both banks must abstain from bidding with positive probability. Therefore \( \sigma_i < 1 \) and both banks earn zero profits in equilibrium. \( \square \)

**Proof of Corollary 2:** From proposition 2, we have that for capacities sufficiently small so that profits are positive, profits will be given by \( \pi_i(R|L,K_1,K_2) > 0 \). From equation (2.8), this is strictly increasing in \( K_i \), but decreasing in opponent's capacity, \( K_j \). \( \square \)

**Proof of Corollary 3:** \( F_i, F_j \) were computed in proposition 2. From equation (2.5), we see, after substituting for \( \bar{\pi}_i \) (given in equation (2.8)), that \( \frac{\partial F_i}{\partial K_i} = 0 \). If we express \( \pi_i(R|L,K_1,K_2) \) as \( E[\pi_i|K_1,K_2] \), this gives us that:

\[
\frac{\partial}{\partial K_i} \Pi_i = \frac{\partial}{\partial K_i} E[\pi_i|K_1,K_2] + \alpha_i(1 - \lambda)q \left( \frac{\partial}{\partial K_i} E[r_j|\theta_l - 1] \right) > 0
\]

\[
> 0, \text{ from corollary 2}
\]

\[
= 0, \text{ since } \frac{\partial F_j}{\partial K_i} = 0
\]

\[\text{Note that this analysis also holds whenever } K_1 + K_2 \leq \lambda + 2\alpha(1 - \lambda)(1 - q), \text{ as claimed, with the distribution function given by equation (2.5) instead of (2.7).}\]

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Notice however that $\frac{\partial F_i}{\partial K_j} < 0$, from equations (2.7) and (2.8), so that increasing one’s own capacity has the effect of a first order stochastic dominance (FOSD) shift on $F_j$ (we see this by taking the limit as $K_j \to 0$ and $K_j \to \lambda + \alpha(1 - \lambda)(1 - q)$). We now have:

$$
\frac{\partial}{\partial K_j} \Pi_i = \frac{\partial}{\partial K_j} E[\pi_i|K_1, K_2] + \alpha_i(1 - \lambda)q \left( \frac{\partial}{\partial K_j} E[r_j|\theta_n] - 1 \right)
$$

$<$0, from corollary 2 $>$0, since $\frac{\partial F_i}{\partial K_j} < 0$

so that the result is ambiguous and will depend on the specific parameters of the model. □

For the proofs of propositions 3, 4, and 6, we need the following preliminary result:

**Lemma A1** Let $\gamma_i$ denote the ratio of new to bad firms that bank $i$ faces, where we order banks by their interest rate offer. Then $\gamma_i > \gamma_{i+1}$. In other words, the distribution of firms each bank faces gets worse in $i$.

**Proof:** (We prove this by defining the distribution each bank faces recursively.) Suppose there are $N$ banks operating, each with a capacity $K_i$ and market share $\alpha_i = \frac{1}{N}$. Fix $i$, so that bank $i$, as long as it is capacity constrained, finances $A_i$ new firms and $K_i - A_i$ old firms. Of the $K_i - A_i$ bad firms financed by bank $i$, a fraction $\frac{1}{N-1}$ were bank $i+1$’s rejected firms. Now, let $B_i$ represent the mass of bad firms bank $i$ faces, of which $b_{i+1}$ of them are the rejects from bank $i+1$, and suppose there are $b_i$ bad firms of its own that it doesn’t risk having to refinance. Since all the market shares are the same, we have that $b_i = b_{i+1}$. Therefore, the mass of bad firms that bank $i+1$ faces is: $B_{i+1} = B_i + b_i - (b_{i+1} - \frac{1}{N-1}(K_i - A_i)) - (K_i - A_i) = B_i - \frac{N-2}{N-1}(K_i - A_i)$.

Suppose that bank $i$ faces $G_i$ new firms. Then bank $i+1$ faces $G_{i+1} = G_i - A_i$ new firms. Finally, we have that bank $i+1$ offers credit to $A_{i+1}$ new firms and $K_{i+1} - A_{i+1}$ old firms, where

$$A_{i+1} = \left( \frac{G_{i+1}}{G_{i+1} + B_{i+1}} \right) K_{i+1} = \left( \frac{G_i - A_i}{G_i - A_i + B_i - \frac{N-2}{N-1}(K_i - A_i)} \right) K_{i+1} \quad (2.9)$$

Now the ratio of new to old firms obtained by bank $i+1$, $\gamma_{i+1}$, is simply given by

$$\gamma_{i+1} = \frac{A_{i+1}}{K_{i+1} - A_{i+1}} = \frac{G_{i+1}}{B_{i+1}} = \frac{G_i - A_i}{B_i - \frac{N-1}{N-2}(K_i - A_i)}.$$

We want to show that $\gamma_{i+1}$ can never be greater than $\gamma_i = \frac{A_i}{K_i - A_i}$. Suppose that $\gamma_{i+1} > \gamma_i$. 

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This implies:
\[ G_i > A_i \left( \frac{B_i}{K_i - A_i} + \frac{1}{N - 1} \right) \]

Notice that it must be true that \( K_i - A_i = \left( \frac{B_i}{G_i + B_i} \right) K_i \). If we substitute this in, we obtain:
\[ G_i > K_i \left( \frac{G_i}{G_i + B_i} \right) \left[ \frac{B_i}{G_i + B_i} + \frac{1}{N - 1} \right] = G_i + \frac{A_i}{N - 1} > G_i \forall N, \]

a contradiction. Therefore, the ratio of new firms to old firms must be strictly decreasing over \( i = 1, \ldots, N \). \( \square \)

**Proof of Proposition 3:** (By induction) As in lemma A1, let \( \gamma_i^N \) be the ratio of new to bad firms faced by bank \( i \), when we have \( N \) banks. We claim that if \( \gamma_i^N > \gamma_i^{N+1} \), then \( \gamma_i^{N+1} > \gamma_i^{N+1} \). From lemma A1, we know that \( \gamma_i^N = \frac{G_i}{B_i} \) and \( \gamma_i^{N+1} = \frac{G_i'}{B_i'} \), where \( G_i, B_i, G'_i, B'_i \) represent the mass of new and bad borrowers faced by bank \( i \) in the \( N \) and \( N + 1 \) bank case, respectively (primes represent variables for the \( N + 1 \)-bank case). Also,
\[ \gamma_i^{N+1} = \frac{G_i - A_i}{B_i - \frac{N-1}{N-2}(K_i - A_i)} \quad \gamma_i^{N+1} = \frac{G_i' - A'_i}{B_i' - \frac{N}{N-1}(K'_i - A'_i)} \]
where \( A_i, A'_i \) is the number of new firms to whom bank \( i \) offers credit. By definition, this is \( A_i = \frac{G_i}{G_i + B_i} K_i \), \( A'_i = \frac{G_i'}{G_i' + B_i'} K'_i \). This implies that
\[ \gamma_i^{N+1} = \frac{G_i - \frac{G_i}{G_i + B_i} K_i}{B_i - \frac{N-1}{N-2}(K_i - K_i \frac{G_i}{G_i + B_i})} \quad \gamma_i^{N+1} = \frac{G_i' - \frac{G_i'}{G_i' + B_i'} K'_i}{B_i' - \frac{N}{N-1}(K'_i - K'_i \frac{G_i'}{G_i' + B_i'})} \]

Since we assume that \( \gamma_i^N > \gamma_i^{N+1} \), this implies that \( G_i B'_i > G'_i B_i \). Using this, and substituting for \( K_i \) and \( K'_i \), we obtain that \( \gamma_i^N > \gamma_i^{N+1} \), as desired. Finally, note that \( \gamma_i^N = \frac{\lambda}{N-2(1-\lambda)(1-\theta)} \) and \( \gamma_i^{N+1} = \frac{\lambda}{N-1(1-\lambda)(1-\theta)} \), so that \( \gamma_i^N > \gamma_i^{N+1} \). An application of mathematical induction establishes our result, that the overall distribution of firms obtaining financing worsens with \( N \). \( \square \)

**Proof of Proposition 4:** To obtain the result, we calculate the proportion of left over firms for bank \( i \) given that \( i - 1 \) banks have a lower interest rate. While this proportion depends on the capacities of the previous \( i - 1 \) banks, all that is required is that the distribution of firms that bank \( i \) faces is worse than that bank \( i - 1 \) faces, for all \( i = 2, \ldots, N \). For that, we use lemma A1. In order for bank \( N \) to make positive profits, it must be that \( \pi_N(R|L, \{K_i\}_{i=1}^N) > 0 \). By a similar construction as was used in proposition 2 we can write
expected profits for each bank as a function of the number of new and bad customers to whom they lend. Let \( A_i \) be the number of new firms that a bank obtains if it has the \( i^{th} \) lowest interest rate, and let \( L_i \) be number of low quality firms. The expected payoff for any bank is:

\[
\pi_i^N(r) = (1 - F(r))^{N-1} \left\{ A_1(r \bar{\theta} - 1) + L_1(r \theta_l - 1) \right\} \\
+ \binom{N-1}{1} F(r)(1 - F(r))^{N-2} \left\{ A_2(r \bar{\theta} - 1) + L_2(r \theta_l - 1) \right\} \\
\vdots \\
+ \binom{N-1}{N-2} F(r)^{N-2}(1 - F(r)) \left\{ A_{N-1}(r \bar{\theta} - 1) + L_{N-1}(r \theta_l - 1) \right\} \\
+ F(r)^{N-1} \left\{ A_N(r \bar{\theta} - 1) + L_N(r \theta_l - 1) \right\} \\
= \sum_{i=1}^{N} \binom{N-1}{i-1} F(r)^{i-1}(1 - F(r))^{N-1} \left\{ A_i(r \bar{\theta} - 1) + L_i(r \theta_l - 1) \right\}
\]

Evaluated at \( r = R \), by lemma 3, we need that

\[
\pi_i^N(R) = \{ A_N(R \bar{\theta} - 1) + L_N(R \theta_l - 1) \} > 0, 
\] (2.10)

which will be true for \( \gamma_N = \frac{A_N}{L_N} \) large enough. Note that \( \gamma_i \) is an increasing function of \( \lambda \), and that as \( \lambda \to 1, \gamma_i \to \infty \) for all \( i \), since there are essentially no old firms. This shows that there must be some \( \lambda_N^+ \) such that for \( \lambda > \lambda_N^+ \), \( \pi_N(R|L, \{K_i\}_{i=1}^N) > 0 \), and an \( N^{th} \) bank makes positive profits. Since, by lemma A1, \( \gamma_i \) is decreasing in \( i \), all other banks must also be making positive profits. That \( \lambda_N^+ < \lambda_{N+1}^+ \) follows from proposition 3, since the pool of borrowers the \( N + 1^{st} \) bank faces is worse. Therefore, a higher \( \lambda \) is required in order for equation 2.10 to be positive with \( N + 1 \) banks. \( \Box \)

**Proof of Proposition 5:** Fix the aggregate lending capacity of the \( N \) active banks, \( \bar{K}^N \). Now consider the 2 bank equilibrium, and simply take the limit as each bank's capacity goes to infinity. Specifically,

\[
\lim_{K_i \to \infty} \pi_i(r_i|K_i) = (1 - F_j(r_i))\lambda(r_i \bar{\theta} - 1) + \alpha_j(1 - \lambda)(1 - q)(r_i \theta_l - 1),
\]

which must equal zero in equilibrium. Since in the \( N \) bank equilibrium with fixed aggregate
capacity $K^N$ each bank is making positive profits, there must be some set $(\lambda, q)$ for which $E[r|N, K^N] > E[r|2, K^2]$. Since equilibrium profits in the 2 banks case go smoothly to zero as lending capacity increases, we see that there is some value $K^2$ such that if $K = K_1 + K_2 > K^2$, the expected interest rate will be lower than in the $N$ bank case. □

**Proof of Proposition 6:** With $N$ banks, we have $K_i = \frac{1}{N}$ and $K_i = \frac{1}{N}(1 - q + \lambda q)$, so that $\sum_{i=1}^{N} K_i = 1$. Let the equilibrium strategies be given by $\{F_i\}_1^N$, and profits by $\pi_i(R|L, \{K_i\})$. Note that $\pi_{N+1}(R|L, \{K_i\}) = 0$, since $\text{prob}(R < \min_j\{r_j\}) = 0$. Therefore, we focus on $r$, the minimum of the support of $F_i$. We have:

$$\pi_{N+1}(r) = \pi_{N+1}(r|W, \{K_i\})$$

$$= \frac{\lambda}{\lambda + \sum A_i} K_{N+1}(r\bar{\theta} - 1) + \frac{\sum A_i}{\lambda + \sum A_i} K_{N+1}(r\theta_i - 1)$$

$$= K_{N+1} \left\{ \frac{\lambda}{\lambda + (1 - \lambda)(1 - q)} (r\bar{\theta} - 1) + \frac{(1 - \lambda)(1 - q)}{\lambda + (1 - \lambda)(1 - q)} (r\theta_i - 1) \right\}$$

since $\sum_{i=1}^{N} \alpha_i = 1$. For any other bank $i$, with a market share $\alpha_i = \frac{1}{N_i}$, we have that:

$$\pi_i(r|W, \{K_i\}) = K_i \left\{ \frac{\lambda}{\lambda + \frac{N-1}{N}(1 - \lambda)(1 - q)} (r\bar{\theta} - 1) + \frac{\frac{N-1}{N}(1 - \lambda)(1 - q)}{\lambda + \frac{N-1}{N}(1 - \lambda)(1 - q)} (r\theta_i - 1) \right\}$$

For simplicity, let $K_{N+1} = \lambda + (1 - \lambda)(1 - q)$. Comparing these last two equations, we see that we can express profits for the $N + 1^{st}$ bank as:

$$\pi_{N+1}(r) = \left[ \frac{\lambda + \frac{N-1}{N}(1 - \lambda)(1 - q)}{K_i} \right] \pi_i(r|W, \{K_i\}) + \frac{1}{N} (1 - \lambda)(1 - q)(r\theta_i - 1)$$

As $\lambda \to 1$, $\pi_{N+1} \to \frac{1}{K_i} \lim_{\lambda \to 1} \pi_i(r|W, \{K_i\})$. However, from corollary 5, we have that $r \to R$ as $\lambda \to 1$, and so $\lim_{\lambda \to 1} \pi_i(r|W, \{K_i\}) = K_i(R\bar{\theta} - 1)$. This implies that $\lim_{\lambda \to 1} \pi_{N+1} = \frac{1}{K_i} K_i(R\bar{\theta} - 1) = (R\bar{\theta} - 1)$, which is greater than zero by assumption. Therefore, $\exists \lambda_{N+1}$ such that for $\lambda > \lambda_{N+1}, \pi_{N+1}(r) > 0$, so that entry is profitable, and $(\{F_i\}, \sigma_{N+1})$ cannot be an equilibrium.

Conversely, suppose that $\bar{\pi}_i = 0$. Since for any interest rate offer, $\pi_{N+1}(r) < \pi_i(r)$, no entry can take place. Therefore, for $\lambda < \lambda_{N+1}$, entry is not profitable.
That $\lambda_N$ is increasing in $N$ follows directly from the fact that the profits of the entrant bank are some constant fraction lower than for incumbent banks, and that $\lambda_N^+$ is increasing in $N$. □

References


Chapter 3

Competition and the Choice
Between Informed and Uninformed
Finance

3.1 Introduction

This paper attempts to shed some light on how a firm's choice between contracting with either an informed or an arm's length lender affects (and is affected by) the competition it faces in its product market. We argue that a firm's product market struggles can have an impact on its choice between different sources of debt financing, so that we cannot look at the firm in isolation in trying to explain this choice. In doing so, we draw a distinction between two different types of "monitoring" activities that are usually ascribed to informed or inside lenders. The first of these is control. To the extent that inside lenders are often in a position to exert direct influence and control over firm managers' behavior, this will imply that by resorting to an informed lender, a firm will be able to resolve any (or most) managerial moral hazard or asset substitution problems that may exist. The other activity is simply information gathering, that inside lenders are able to obtain information about firms (or managers) that is not available to outsiders. This has the predicted effect of enabling more efficient continuation decisions, without necessarily allowing insiders to directly control the
actions of the firm’s managers. While a host of papers have appealed to both of these characteristics to explain the benefits (and costs) of bank financing, our contention is that these two different “benefits” of inside financing have very different implications for a firm’s product market strategy.

In particular, we argue that while an inside lender’s information enables efficient continuation decisions, it also allows managers to engage in the pursuit of privately beneficial activities, such as “empire building” (see Jensen (1986), or more recently, Hart and Moore (1995)). This occurs because informed lenders will only pull the plug when a manager is indentified as being of low quality, and not necessarily when cashflows are low. In essence, an inside lender is able to disentangle a firm’s expected profitability from its realized profits, which may be subject to random fluctuations due to market disturbances, or may be low because of predatory actions by rivals. This destroys the credibility of a threat to liquidate a firm if cashflows are low if the manager is found out to be of high quality. This is a form of the “soft budget constraint,” that informed principals may not have the incentive ex-post to enforce aspects of the contract to which they agreed ex-ante.

Arm’s length finance, on the other hand, may be a way to provide discipline for empire building managers, to the extent that uninformed creditors rely only on publicly observable variables (such as profits or debt repayment) to make their continuation decisions. This forces managers to concentrate on maximizing firm value in order to convince creditors that they have the potential for high profits. Unfortunately, to the extent that public signals are not perfectly informative even ex-post, choosing to obtain financing from an uninformed source also leads to inefficient continuation decisions being taken, as some good firms are sometimes shut down while some bad firms are allowed to continue. This is the case, for example, if each firm’s demand is subject to random and unobservable shocks, so that the observation of profits does not allow creditors to directly infer expected profitability.

1It may also have inefficient effects ex-ante, to the extent that it affects managerial incentives. See Rajan (1992).
2See Dewatripont and Maskia (1995) for a formalization of this idea in the context of a model where centralized principals have the ability to take actions in order to boost profits. Their inability to commit not to do so leads to the soft budget constraint and its consequent inefficiencies.
3If uninformed creditors condition their continuation decisions on publicly observable but endogenous variables, managers may have an incentive to manipulate these signals in an attempt to influence creditors’ beliefs concerning the viability of the enterprise, or the quality of the manager. The manipulation of profits is a way of jamming creditors’ inference problem. As in much of the signal jamming literature (e.g. Fudenberg and Tirole (1986), Holmström (1982)), this attempt at manipulating beliefs occurs whether or not it is likely to be successful, and as such can have a real impact on firms’ profits.
This leads us to look at the role of competition in influencing this choice. To the extent that managers have private objectives that may not exactly coincide with profit maximization, it is reasonable to believe that the degree of competition in the firm's product market will have some effect on managerial actions. But increased competition, by making profits more sensitive to managerial actions or to the firm's underlying cost structure, also makes profits a more informative signal about expected profitability. This increases a manager's incentive to attempt to influence creditors' expectations about managerial quality, since it increases the return to doing so. Moreover, by making the publicly observable signal more precise, competition decreases the probability that an inefficient continuation decision is taken. Therefore, arm's length finance and product market competition have a complementary role in the provision of implicit incentives for empire-building managers.

There are a number of further implications offered by this model. Competition may also affect a firm's incentive to drive out competitors, as the gain to doing so may depend on the degree of competition. This "predatory" incentive, if sufficiently large, can lead firms to prefer informed sources of financing despite the added discipline provided by uninformed sources for two reasons: 1) it allows them to be less concerned about their own current profits and hence to take actions to lower competitors' profits in order to drive them out of the market, and 2) it insulates them from rivals' predatory actions, since rivals have no incentive to prey on a firm that is backed by an informed creditor. We can also draw some implications relating the informativeness of public signals to the firm's financing choice. We find that very uninformative public signals have little impact on managerial incentives to behave, so that arm's length financing provides little market discipline, but leads to large ex-post inefficiencies. Therefore, the optimal solution in this case is to obtain financing from an informed lender. One implication of this is that young firms, or firms in R&D intensive industries, for whom current profit does not provide an accurate signal of future profitability, should be more likely to seek financing from informed sources.

There has been a good deal of literature attempting to explore the link between firms' financial structure and the subsequent competition in the product market. In particular, a number of papers have focused on the strategic benefits of leverage, emphasizing the commitment value that leverage provides when it comes to making output or pricing decisions (e.g. Brander and Lewis (1986), Maksimovic (1988, 1990)). Our approach attempts to abstract from these strategic elements that might motivate the choice of financing, and focuses
on how reactions to the competitive environment help determine firms' choice of one form of financing over the other. Related to this is the literature on the effects of product market competition on managerial incentives (Scharfstein (1988), Hermalin (1992), and Schmidt (1997)). In a similar vein, we explore the link between competition and incentives, but in a framework where competition changes the nature of the incentive problem through its effect on the information content of observable signals.

The notion that bank financing can be thought of as a form of delegated monitoring which serves to control the firm can be found in Diamond (1984). This view of "monitoring" has been further developed in Besanko and Kanatas (1993), who argue that managerial moral hazard or asset substitution problems that may exist within the firm can be resolved by resorting to an informed lender. Our work is closer to Dewatripont and Maskin (1995), who analyze how a creditor's inability to commit to certain actions ex-post leads to ex-ante inefficiency. One key difference between our paper and theirs is in the assumption on the benefits of "monitoring". While Dewatripont and Maskin (1995) assume that insiders have the ability to take actions to boost firms' profitability, we assume that the only difference between insiders and outsiders is the information each has.

There has also been considerable research on the choice between bank debt and public debt, both theoretical (Berlin and Loeys (1988)\(^4\), Diamond (1991), Rajan (1992), Von Thadden (1994)\(^5\)) and empirical (Hoshi, Kashyap, and Scharfstein (1993), Houston and James (1996)). The idea of maintaining an arm's length relationships as a commitment to being tough has been formalised by Crémer (1995) in the context of a principal-agent model. In that model, as in ours, it is precisely the paucity of information acquired by principals who remain at arm's length that makes credible their threat to terminate the relationship.\(^6\) However, neither Crémer's paper, nor to our knowledge anyone else, has considered the effects of this role of information acquisition by inside lenders on firms' behavior in the

\(^4\)Berlin and Loeys (1988) present a related model that argues that debt contracts based on noisy indicators of profitability may lead to large continuation inefficiencies. Firms may instead choose to hire a delegated monitor who must be provided with the proper (costly) incentives to monitor.

\(^5\)A complementary approach to the one here is developed by Von Thadden (1994), who demonstrates that bank (informed) finance can help alleviate a firm's bias towards short term projects with quick returns, since informed lenders can make decisions that are contingent on the "quality" of the firm directly. However, he does not address the issue of providing incentives for effort, but rather only for project choice.

\(^6\)Burkart, Gromb, and Panunzi (1997) employ a model closely related to Crémer's to analyze the effects on managerial incentives of having a large shareholder with an incentive to monitor. In their model, monitoring increases the probability of directly assessing the manager's quality, which has a negative impact on the manager's incentive to work hard.
product market.

3.2 A Simple Model of Financing: Monopoly

To illustrate the disciplinary aspect of arm's length financing, we first analyze a firm's choice of financing when it faces no competition in its product market. Suppose a firm operates for two periods, faces a linear inverse demand curve given by $D(p) = A - p$ and has constant marginal costs of $c$ in each period. In order to produce it needs to obtain financing in each period. The investment requires a capital inflow of $I$. In addition, there is a random fixed cost component $F$, which is distributed according to the distribution function $H(F)$. Neither the manager nor the lender knows the value of $F$, but its distribution is common knowledge. Finally, there is a random shock component of profits, $\tilde{\epsilon}$, which can reflect uncertain market conditions or shocks to costs, and which is identically and independently distributed across both periods.\(^7\) Profits, gross of the cost of investment, are therefore given by:

$$\pi = D(p)(p - c) + \epsilon - F$$  \hspace{1cm} (3.1)

Assuming that $\epsilon$ and $F$ are both normally distributed, $\epsilon \sim N(0, \sigma_{\epsilon}^2)$, $F \sim N(F^e, \sigma_F^2)$, expected profits are:

$$E[\pi] = (A - p)(p - c) - F^e.$$  \hspace{1cm} (3.2)

We assume that there are two possible sources of finance available to a firm: (1) an informed lender (we will use a bank as an example), and (2) a public (arm's length) debt market. The main difference between these two is that we assume that a creditor from the public debt market has no "inside" knowledge of the firm's quality, and is only able to observe what is public information to all parties, which includes the realization of profits each period. In contrast, a bank, being an insider, can at a cost observe not only realized profits, but also the firm's type, $F^i$.\(^8\) We assume that after granting a loan once, an inside

\(^7\)A signal jamming model where the observed signal incorporates an element of random noise can be found in Mirman, Samuelson, and Urbano (1993).

\(^8\)We use fixed costs as the relevant variable to differentiate among firms' qualities so that we can ignore the effects on second period strategies of learning the firm's type. We could just as well suppose that firms
lender learns the value of $F$. We also assume that the market for credit is competitive, both in the arm’s length market and in the banking market.

We restrict ourselves to the analysis of debt contracts but assume that profits, $\pi$, are verifiable, so that creditors can always get cash out of the firm. In other words, profits act like a liquidation value, guaranteeing that creditors always get paid whenever possible. However, we assume that the price level, $p$, is not observable by creditors. This can be justified by arguing that creditors may not be able to observe secret price cuts that managers might make, or that there is also some uncertainty in the level of demand that is uncorrelated over time, so that managers cannot be made to target the realized price directly.

To incorporate the idea that managers are not solely interested in maximizing profits but rather like to build large empires, we assume that a manager’s objective is to maximize a weighted average of profits and sales in the first period: $\pi + \beta D(p)$. Since we are dealing with only a two-period model, managerial empire building tendencies in the second (the last) period are not affected by the source of financing in the first period. Therefore, we can immediately solve for the equilibrium in the last period. In the second period, managers are unconstrained in their empire building ability, and simply maximize:

$$\max_p \pi + \beta D(p) = \max_p E[(A - p)(p - c + \beta) + \epsilon - F]$$  \hspace{1cm} (3.3)

The equilibrium second period price level is $p^E_2 = \left(\frac{A + c - \beta}{2}\right)$, which yields expected profits of $\pi^E_2 = \left(\frac{A - c}{2}\right)^2 - \frac{\beta^2}{4} - F^e$. The second period private benefit is $B = \beta D(p^E_2) = \beta \left(\frac{A - c + \beta}{2}\right)$.

Finally, we assume that everyone, including the manager, is risk neutral, and that firms consume any net profits each period, so that they need to finance the full amount in period 2 externally.\footnote{Lummer and McConnell (1989) provide evidence that bank loan renewals (as opposed to loan originations) provide a positive signal about the firm’s investment prospects and profitability, and hence lead to a positive share price response. They argue that this is consistent with the view that banks obtain information about their debtors during the course of their relationships.}

In order to motivate the problem faced by the firm and its creditors, we make the following two key assumptions. First, define $R^E \equiv D(p^E_2)(p^E_2 - c)$, the level of expected differences in the demand functions they face, so that high quality firms are more profitable. However, behavior in the second period would then be affected by what is learned in the first period, making it more difficult to isolate our effect. See Fudenberg and Tirole (1986) for a discussion of this point.

\footnote{Dewatripont and Maskin (1995) employ the same assumption in their comparison of centralized versus decentralized economies. Relaxing this assumption does not seem to have any qualitative effects on the outcome of the model.}
profits gross of the fixed cost.

**Assumption 1** There exists some value of $F$, $\tilde{F}$, such that, for $F < \tilde{F}$, $R^E - F > I$, but $R^E - F < I$ for $F > \tilde{F}$.

**Assumption 2** $R^E - F^e > I$.

Assumption 1 tells us that firms with sufficiently high fixed costs should be closed down, since they will be unable to pay back their creditors. Assumption 2 is merely that, even is the manager were to be totally unconstrained in its empire-building, at least ex-ante firms have the ability to pay back their loans, so that creditors are willing to lend.

### 3.2.1 Bank Finance

As is usual, we solve the model starting from the end, and work our way backwards. In fact, we have already found the solution to the manager’s problem in period 2, since the manager simply maximizes $\pi + \beta D(p)$.$^{11}$ This yields $p^E_2$ as the equilibrium price and $B$ as the manager’s private benefit derived from empire building.

With bank finance, the firm gets refinanced if the bank finds that $F \leq \tilde{F}$, and shut down otherwise. Therefore, in period 1, the manager realizes that his actions will have no effect on the probability of continuation, so he maximizes:

$$\max_p E[D(p)(p - c + \beta) + \epsilon - F],$$

which, just as in period 2, yields $p^B_1 = (\frac{A + \epsilon}{2}) - \frac{\beta}{2}$ and profits of $\pi^*_1 = (\frac{A - \epsilon}{2})^2 - \frac{\beta^2}{4} - F^e$. In other words, the manager is free to engage in empire building activities (offers a low price, which is equivalent to overproducing), but only gets refinanced when he is known to have low costs, i.e. we have an efficient ex-post continuation decision. Note that it is exactly

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$^{11}$Note that for now we ignore the debt payment, $D$, that would normally have to made to creditors for the financing of $I$. The main reason for doing this is that we abstract from the limited liability effect of debt emphasized by Brander and Lewis (1986). They argue that if profits are given by a reduced form function $R(q, z)$, where $z$ is a random term affecting profits, then a necessary and sufficient condition for output to be affected by the existence of a debt payment is that $\frac{\partial R(q, z)}{\partial q_i \partial z} \neq 0$. However, in the present model, profits are $\pi(q, z) = R(q) + z$, so that $\frac{\partial \pi(q, z)}{\partial q_i \partial z} = 0$, and there is no limited liability effect of debt. In other words, maximizing profits with or without consideration of the relevant debt payment yields the same result. Of course, when adding continuation payoffs, the value of $D$ does matter, since it reduces the benefit to the manager.
the imposition of ex-post efficiency that leads to an ex-ante inefficiency on the part of the manager.

3.2.2 Arm’s Length Finance

Again, since any information learned about $F$ does not affect the second period output level, $p_2^E$ solves the second period problem for the manager, and $\pi_2^E = (\frac{A-e}{2})^2 - \frac{\beta^2}{4} - E[F|\pi_1]$, where this expectation is taken over the posterior distribution of $F$, having observed profits in the first period. However, now the firm’s ability to obtain financing for period 2 can no longer depend on $F$, as this is not observed by arm’s length creditors. Instead, creditors offer to refinance only if they infer that $F \leq \hat{F}$ after observing first period profits. Let the probability that the firm is allowed to continue be given by $\varphi(p; F)$ for each value of $F$. We then have the manager’s optimization problem:

$$\max_p R(p) + \beta D(p) - F^c + \int_{-\infty}^{\infty} \varphi(p; F)(R_2^E + B - F)dH(F)$$

(3.5)

where $R(p) = D(p)(p - c)$. The second term in the maximization problem reflects the fact that the benefit in period 2, $R_2^E + B - F$, is only enjoyed by the manager if the firm is able to obtain continuation finance, which occurs with probability $\varphi(p; F)$ for each value of $F$. This yields a first order condition of:

$$D'(p)(p - c + \beta) + D(p) + \int_{-\infty}^{\infty} \frac{\partial}{\partial p}\varphi(p; F)(R_2^E + B - F)dH(F) = 0$$

(3.6)

What remains is to define the function $\varphi(p; F)$. From assumption 1, we know that as long as $F$ is believed to be below $\hat{F}$, the firm will be allowed to continue. In other words, if $\pi_1$ represents the firm’s profits in the first period, then as long as $E[R_2^E + \epsilon - F|\pi_1] \geq I$, the firm will be able to obtain continuation financing. Since $\epsilon$ is independent across periods, $E[R_2^E + \epsilon|\pi_1] = R_2^E$, so that the cutoff rule is that the firm is refinanced as long as $E[F|\pi_1] \leq R_2^E - I = \hat{F}$.

Since we have assumed that both $\epsilon$ and $F$ are normally distributed, high profits represent “good news” about firm 1’s probability of having low costs.\textsuperscript{12} We first compute the posterior

\textsuperscript{12}This is in the sense of Milgrom (1981). It is easy to show that profits, $\pi(p, \epsilon|F)$, satisfy the monotone
expectation of $F$ after observing first period profit $\pi_1$ as:

$$E[F|\pi_1] = \frac{h_F F^c}{h_F + h_\epsilon} + \frac{h_\epsilon}{h_F + h_\epsilon} (R(p) - \pi_1),$$

where $h_F$ is the (prior) precision of $F$ ($h_F = \frac{1}{\sigma_F^2}$) and $h_\epsilon$ is the precision of $\epsilon$ ($h_\epsilon = \frac{1}{\sigma_\epsilon^2}$). Therefore, since $\varphi(p; \hat{F})$ represents the probability of continuation, it must be that $\varphi(p; \hat{F}) = \text{prob}(E[F|\pi_1] \leq \hat{F})$. In the appendix, we demonstrate that

$$\frac{\partial}{\partial p} \varphi(p; \hat{F}) = \phi_\epsilon \left( F + F^c \frac{h_F}{h_\epsilon} - \hat{F} \frac{(h_F + h_\epsilon)}{h_\epsilon} \right) \frac{h_\epsilon}{h_F + h_\epsilon} R'(p),$$

(3.7)

where $\phi_\epsilon$ is the density function of $\epsilon$, the density of a normally distributed random variable with mean zero and precision $h_\epsilon$. Substituting this into the FOC above, and letting $\bar{F} = F + F^c \frac{h_F}{h_\epsilon} - \hat{F} \frac{(h_F + h_\epsilon)}{h_\epsilon}$, we obtain that the equilibrium price level, $p_1^4$, needs to satisfy:

$$R'(p) + \beta D'(p) + \frac{h_\epsilon}{h_F + h_\epsilon} R'(p) \int_{-\infty}^{\infty} \phi_\epsilon (\bar{F}) (R_2^E + B - F) dH(F) = 0$$

(3.8)

Notice that, as is usual in signal jamming models, creditors here are not fooled by the manager’s attempt to manipulate the signal, but rather correctly anticipate the price level that satisfies equation (3.8). In fact, it can easily be shown that the probability of continued operation, $\varphi(p; \hat{F})$, defines a cutoff rule on profits, so that the firm is allowed to continue if profits are above some threshold $\pi^t$, and shut down otherwise. The cutoff level of profit is given by that profit level such that the expected profit in the second period, net of the expected fixed cost, is greater than the amount that is lent ($I$). In other words, since the firm is allowed to continue as long as $E[F|\pi_1] \leq \hat{F}$, the cutoff level of profits, $\pi^t$, must satisfy: $E[F|\pi^t] = \hat{F}$. For a conjectured price level of $p_1^t$, creditors expect profits $\pi(p_1^t) = R(p_1^t) + \epsilon - F$, and draw their inferences accordingly from the realized value $\pi_1$. Of course, in equilibrium creditors’ conjectures must be correct, so that $p_1^t = p_1^4$, and the equilibrium cutoff level of profit is

likelihood ratio property (MLRP) in $\pi$, in the sense that:

$$\frac{f(\pi|F^L)}{f(\pi|F^H)}$$

is increasing in $\pi$ for $F^L < F^H$,

where $f(\pi|F)$ is the conditional density of $\pi$ given $F$. This is true since $\pi = R(p) + \epsilon - F$, so that $f(\pi|F) = \phi(\pi - R(p) + F)$. Therefore, $\frac{f(\pi|F^L)}{f(\pi|F^H)} = \frac{\phi(\pi - R(p) + F^L)}{\phi(\pi - R(p) + F^H)}$, which is increasing in $\pi$, and hence satisfies the MLRP.

This is a well-known formula from statistical decision theory. See DeGroot(1970).
\[ \pi' = R(p_1^A) - \hat{P}_{he} + \hat{h}_e + F_{he}. \]

A comparison of the price level \( p_1^B \) that satisfies equation (3.4) and that which solves (3.8) (given by \( p_1^A \)) shows that clearly \( p_1^A > p_1^B \), i.e. in equilibrium a manager who is financed by the arm’s length market prices higher (or produces less) than one who is bank financed. Given investors’ decision rule, managers try to influence investors’ beliefs concerning expected profitability. This is similar to other results in much of the signal jamming literature (see in particular Fudenberg and Tirole (1986)), where firms attempt to influence the inference process of other parties even when they know that in equilibrium their attempt to muddy the signal will not be successful. In this case, however, the signal jamming is efficient from the point of view of the owner of the firm, since it serves to boost first period profits at the expense of managerial empire building.\(^{14}\) In contrast to other strands of the literature, such as Stein’s (1989) model of managerial myopia, capital market pressure can serve to provide discipline to managers through their attempt to influence the market’s beliefs, and as such can be good for owners (or equity holders, in the context of Stein’s model).\(^{15}\) Since bank financed managers engage in empire building to their hearts’ content and overproduce relative to the profit maximizing strategy, we see that arm’s length financing provides a measure of discipline for managers to the extent that it forces them to be concerned about current profitability.\(^ {16}\)

### 3.2.3 Choice of Financing

Notice that, at least at first sight, the firm’s owner has an incentive to try to boost profits by obtaining financing from the arm’s length market, since she knows that the manager will

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\(^{14}\) Note, however, that it may not be socially efficient. Not only does the manager lose a portion of his private, non-transferable benefit of empire building, but consumers pay a higher price and are supplied less output. In short, we are closer to the static monopoly solution.

\(^{15}\) Stein (1989) assumes that managers are concerned only about profits and the current stock price, and that the only way they have of boosting current profits is by inefficiently shifting profits forward. In the context of our model, introducing the possibility that managers can also decrease investment in activities that promote future growth (such as R&D or selling useful assets) introduces an inefficiency to arm’s length finance. Managers may now allocate fewer resources to these activities in order to concentrate on current profits. However, the general result that managers invest less in empire building continues to hold.

\(^{16}\) Diamond (1991) proposes a model where bank monitoring provides an incentive to choose a safe project if banks can commit to cut off funding if they find out the borrower has chosen a bad project. This relies on the assumption that banks can withdraw their loan after monitoring takes place and information is learned, but before the money is spent. Firms therefore have an incentive to build up their reputation by selecting good projects, since monitoring is not informative about firm type but rather about project selection. In contrast, in our model firms that have bank financing are not concerned about their reputation since there is nothing they can do to affect it. It is only when financed by an uninformed lender that they can take actions that have an impact on their credit rating.

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otherwise have free rein to engage in empire-building activities. However, this ignores that uninformed finance is subject to inefficient continuation, which leads debt to be overpriced in the second period relative to the terms that would apply were the firm identified as being good. This lowers the value of continuing for a second period to the owner of the firm. Therefore, if arm's length finance is to be preferred, it must be that first period profits are raised sufficiently to compensate the owner for the increased debt payments that must be made in the second period.

**Lemma 1** The equilibrium price under arm's length financing ($p^A$) is increasing in $h_e$.

**Proof:** See appendix.

In essence, as profits become a more informative signal about fixed costs, a firm financed by an arm's length creditor will have a larger incentive to signal jam by boosting profits, since the perceived return to doing so increases with the precision of the signal. As the signal becomes more informative, managerial actions have a larger impact on creditors' posterior beliefs, so that in the limit as the variance of profits goes to zero, managers obtain a one for one return to their signal jamming reflected in creditors' beliefs. On the other hand, if current observed profit is an extremely noisy signal of true firm profitability (i.e. if $\sigma^2$ is large), managers know that their action will have little impact on realized profits, and little influence on their creditors' inference process, since profits are subject to large random shocks with high probability. Therefore, managers have little incentive to be disciplined (signal jam) when arm's length financed.

**Proposition 1** There exists a value of $h_e$, $\bar{h}_e$, such that for all $h_e > \bar{h}_e$, the ex-ante value of the firm is maximized by obtaining financing from the arm's length market.

**Proof:** See appendix.

In essence, as the precision of profits as a signal regarding fixed costs increases, not only does the incentive to signal jam increase, but also the inefficiency associated with arm's length financing decreases. This inefficiency is precisely creditors' inability to observe fixed costs directly and their consequent need to base their continuation decisions on observable variables (profits). As the variance of $\epsilon$ decreases, the probability of making an erroneous decision decreases, since the inference on costs is more precise. Therefore, for some value
of $h_e$, the gain in profitability dominates the loss due to the inefficiency in continuation, leading firms to choose the arm's length market as their source of financing.

A simple re-interpretation of the noise component of profit gives us a theory for the choice of bank versus public debt financing consistent with ones that have been much in the literature lately (e.g. Diamond (1991)). If we think of young, inexperienced firms as ones for which current profits may not be a good indicator of future profitability ($\sigma^2_e$ is large), then we have a theory that predicts that young firms choose to appeal to an informed lender in order to obtain financing. In this model, however, bank (informed) lending does not serve to control the agency problems within the firm, but merely enables an efficient continuation decision for the firm. It is precisely only because arm's length financing affords no further discipline that informed financing is preferred.

### 3.2.4 Caveat: The role of incentive contracts

As a final point, we note that so far we have ignored the fact that firm owners (or investors, for that matter, if they are not restricted to debt contracts) might like to write incentive contracts with their managers, and thus prevent them from overproducing. We have done so in order to establish the exact nature of the tradeoff, and the impact that arm’s length finance has on behavior independent of any other incentives that may be layered on top. However, we argue that allowing for incentive contracts would not change our results in any qualitative way, but would merely muddy the picture of the role of information in providing (or destroying) incentives. In fact, using a similar setup, Crémer (1995) examines the role of incentive contracts in his analysis of an employment relationship. He finds that in many instances, providing incentives when the principal is informed is significantly more costly than if she is uninformed, so that committing to remain ignorant can be ex-ante optimal. Remaining ignorant effectively provides the principal with one more instrument: the ability to commit to terminate the relationship when performance is low, independent of the reason.\(^{17}\) In our paper, we make a similar point but through the channel of the implicit incentives provided by managers’ career concerns. Schmidt (1997) argues that even with

\(^{17}\)Moreover, Hart (1995, p.128) has argued that “... while an incentive scheme may work well in motivating managers to exert effort, it is likely to be less effective in getting managers to cut back on empire building or relinquish control.” He argues that a large bribe may be required to get managers to rein in their interests in power, so that it may be better to force them to do so by placing debt in the capital structure. This is similar to the idea proposed here of forcing managers through a commitment to remain uninformed.
incentive contracts, a firm may be able to benefit from being able to commit to liquidate a firm, but will usually be unable to do so if it knows that in equilibrium bad outputs are simply due to bad shocks.

3.3 Competition and Financing Choice

While we have explicitly modelled the product market in our analysis of monopoly and highlighted the disciplinary role of arm's length finance, the introduction of competition brings with it a number of new elements. Now we must consider the incentives to obtain one form of finance versus the other as a function of competitors' sources of funding. For example, to the extent that a firm's ability to obtain second period financing relies on its creditor's inference regarding its costs, competitors may have an incentive to tamper with this inference process. This may lead firms competing against bank-financed firms to obtain bank financing as well, as a way of deterring overly aggressive behavior by rivals.

Perhaps most importantly, we will explore what we believe to be a close link between information provision and competition. In particular, we will argue that increased competition serves to increase the precision of some noisy signal about costs, such as profits. As such, this will have a direct effect on firms' choice of financing, since we have already developed a link between the informativeness of the signal, and the incentive to curtail empire-building activities in order to boost profits. This will create a close link between the degree of competition and the disciplinary role of arm's length finance that will be a key element in the subsequent analysis.

3.3.1 Differentiated Products

We consider a model of price competition with differentiated products where each firm has a choice between obtaining an informed or an arm's length lender. We adapt Hotelling's model of competition on the line and assume that firms have a two period horizon. Specifically, we assume that consumers are distributed uniformly along a line segment of length 1, with endpoints at 0 and 1. However, since we concentrate on shocks to demand and their consequent implications for firm profitability, we introduce shocks to a firm's demand as shocks to the mass of consumers distributed along this line. Specifically, we assume that the density of consumers at point $x$ is $(1 + \epsilon)$, where $\epsilon \sim N(0, \sigma^2)$. Firm 1 is located at
point 0 and firm 2 at point 1 (maximal differentiation), and each firm has constant unit costs of \( c \). As in the last section, each firm is characterized by its fixed cost, \( F_i \), which is distributed according to \( H_i(F) \). Consumers have unit demands, with a reservation value of \( V \). Consumers also face a transportation cost of \( t \) per unit of distance traveled to the firm of their choice, so that a consumer located at point \( x \) would have to pay a cost of \( p + xt \) were he to buy at price \( p \) from a firm located at point 0. The transportation cost therefore provides us with a measure of the degree of differentiation between the products.

Given the above, a consumer located at point \( x \) will consume from firm 1 if

\[
p_1 + xt < p_2 + (1 - x)t,
\]

where \( p_1, p_2 \) are the prices quoted by firms 1 and 2, respectively. Using this, we can solve for the demand for firm 1 as: \( D^1(p_1, p_2) = \left( \frac{p_2 - p_1 + t}{2t} \right) (1 + \epsilon) \). It is convenient for what follows to define \( d^i(p_i, p_j) = E[D^i(p_i, p_j)] = \left( \frac{p_2 - p_1 + t}{2t} \right) \). Now let \( p^E_i = \max_{p_i} d^i(p_i, p_j)(p_i - c + \beta) \), and define \( R^D = d^i(p^E_1, p^E_2)(p^E_i - c) \), so that \( R^D \) defines expected profits for each firm for the duopoly equilibrium when both firms are unconstrained in their empire-building activities. As in section 2, \( R^D \) will represent the expected profits in the second period, when neither firm has any incentives to refrain from empire building. Similarly, we define \( B = \beta d^i(p^E_1, p^E_2)(p^E_i - c) \) to be the private benefit of empire building in the second period.

We modify assumptions 1 and 2 as follows.

**Assumption 3** There exists some value of \( F, \hat{F} \), such that, for \( F < \hat{F} \), \( R^D - F > I \), but \( R^D - F < I \) for \( F > \hat{F} \).

**Assumption 4** \( R^D - F^e > I \).

Contrary to the analysis of the previous section, the constraints imposed by assumptions 3 and 4 are now important for the viability and optimality of one form of finance over the other. \( R^D \) is in reality an increasing function of \( t \), the degree of competition. A more precise interpretation of the assumptions is that we will only focus on values of \( t \) for which \( R^D(t) \) satisfies the two assumptions, since otherwise the problem becomes trivial.

\[18\] This assumes that the reservation value, \( V \), is sufficiently high that consumers are actually willing to purchase.
As before, we assume that managers care not only about profits, but also about sales (or market share), since they care about having a "large empire". We therefore argue that, in a one period problem, managers would seek to maximize:

$$\max_{p_i} E[d^i(p_i, p_j)(p_i - c + \beta) + d^i(p_i, p_j)(p_i - c + \beta)\epsilon - F] \quad (3.9)$$

As we will see, a consequence of this setup is that the variability of profits will depend on the intensity of competition. Observe that $\text{var}(\pi^i) = \text{var}(d^i(p_i, p_j)(p_i - c)\epsilon) = d^i(p_i, p_j)^2(p_i - c)^2\sigma^2_\epsilon$. Therefore, as $p_i \to c$, $\text{var}(\pi^i) \to 0$: profits become a very informative signal of the firm's fixed cost. In fact, since this is precisely the effect in which we are interested, it is useful to re-express the model as one where profits are given by:

$$\pi^i = d^i(p_i, p_j)(p_i - c) + \epsilon t,$$

so that as $t \to 0$ (the market becomes more competitive), $\text{var}(\pi^i) \to 0$. The equivalence of these two approaches for our purposes is demonstrated in the appendix, by showing that $p_i \to c$ in the first model as well, so that the informativeness of the signal increases with the intensity of competition.\footnote{The problem with the original setup is that, since firms choose the price, they can also affect the informativeness of the signal, thus turning the model from one of signal jamming to one of signal manipulation. This may affect managers' incentives, to the extent that they may find it more or less beneficial to hide information from creditors. In order to abstract from this effect, we show in the appendix that the two models are equivalent for limiting values of $t$.}

In order to obtain an equilibrium, we need to consider a number of cases.

**Both firms bank financed**

Since we assume that banks observe the firm's type after granting them a loan, managers financed by a bank (an informed lender) know that whether they are granted a second period loan or not depends only on their inherent profitability and not on their realized profits in the first period. They also know that this is true as well for their competitor, who is also bank-financed. Therefore, the maximization problem for each firm in period 1 is equivalent to the one period problem:

$$\max_{p_i} E[d^i(p)(p_i - c + \beta) + \epsilon t] = \max_{p_i} R^i(p) + \beta d^i(p) \quad (3.10)$$
where \( p = (p_1, p_2) \) and \( R^i(p) = d^i(p)(p_i - c) \).

This problem is identical to the maximization problem in period 2, where both firms are unconstrained in their empire-building activities. Therefore, the solution is:

\[
\begin{align*}
p_1 &= p_1^F = c + t - \beta \\
p_2 &= p_2^F = c + t - \beta
\end{align*}
\]

which implies that first period profits are \( \pi_1^{BB} = \frac{1}{2}(t - \beta) - F^e = R^D - F^e, \ i = 1, 2. \)

**Firm 1 bank financed, firm 2 arm’s length financed**

Now each firm faces a very different maximization problem. Since firm 1 only obtains financing for a second period if its costs are found to be sufficiently low, the objective for firm 1 is to maximize:

\[
\max_{p_1} R^1(p) + \beta d^1(p) + \int_{-\infty}^{\hat{F}} \int_{-\infty}^{\infty} \left[ \varphi^2(p; F)(R^D + B - \hat{F}) \\
+ (1 - \varphi^2(p; F))(R^M + B - \hat{F}) \right] dH_2(F)dH_1(\hat{F})
\]

Equation (3.11) merits some interpretation. The first two terms are simply the terms for first period profit and empire-building. The first integral represents that probability that firm 1 has a low enough fixed cost \( \text{prob}(F \leq \hat{F}) \), so that it is allowed to continue by its informed creditor. \( R^M \) represents firm 1’s monopoly profits in period 2, which it obtains if its competitor is denied financing, and the term \( \varphi^2(p; F) \) represents the probability that firm 2 obtains refinancing, which now depends on both \( p_1 \) and \( p_2 \). Notice that firm 1 may have an incentive to try to influence this probability since if firm 2 leaves the market, firm 1 obtains monopoly profits (gross of the fixed cost) of \( R^M \).

For firm 2, its survival probability depends on its realized level of profits. Its objective function is:

\[
\max_{p_2} R^2(p) + \beta d^2(p) + \int_{-\infty}^{\hat{F}} \int_{-\infty}^{\infty} \varphi^2(p; F)(R^D + B - F)dH_2(F)dH_1(\hat{F}) \\
+ \int_{\hat{F}}^{\infty} \int_{-\infty}^{\infty} \varphi^2(p; F)(R^M + B - F)dH_2(F)dH_1(\hat{F}),
\]
since whenever firm 1 turns out to have high fixed costs (if $F > \hat{F}$) it fails to obtain a second period loan, so that firm 2 obtains monopoly profits. These two maximization problems yield the FOCs

$$R_1^1(p) + \beta d_1^1(p) + \int_{-\infty}^{\hat{F}} \int_{-\infty}^{\infty} \frac{\partial}{\partial p_1} \varphi^2(p; F)(R^D - R^M - B)dH_2(F)dH_1(\hat{F}) = 0 \quad (3.13)$$

$$R_2^2(p) + \beta d_2^2(p) + \int_{-\infty}^{\hat{F}} \int_{-\infty}^{\infty} \frac{\partial}{\partial p_2} \varphi^2(p; F)(R^D + B - F)dH_2(F)dH_1(\hat{F}) + \int_{\hat{F}}^{\infty} \int_{-\infty}^{\infty} \frac{\partial}{\partial p_2} \varphi^2(p; F)(R^M + B - F)dH_2(F)dH_1(\hat{F}) = 0, \quad (3.14)$$

where subscript $i$ refers to the partial derivative with respect to the $i^{th}$ argument. Let $p_1^{BA}$, $p_2^{AB}$ be the equilibrium prices that satisfy these two FOC's.

As before, we need to specify the probability of continuation, $\varphi(p; F)$. For refinancing to take place, we require that $E[R^2(p) + \epsilon|\pi^2] \geq I$, where $\pi^2$ represents firm 2's profits in the first period. Since $R^i(p) = d^i(p)(p_i - c)$, and $\epsilon$ is independently distributed across periods, $E[R^2(p) + \epsilon|\pi^2] = R^2(p)$, so that the firm gets refinanced as long as $E[\pi^2] \leq R^2(p) - I$, for the equilibrium values of $p_1$, $p_2$ in the second period. Much as in the previous section, we find that:

$$E[\pi^2] = \frac{h_F}{h_F + h_{tc}} F^e + \frac{h_{tc}}{h_F + h_{tc}} (R(p) - \pi^2) \quad (3.15)$$

where $h_{tc} = \frac{1}{\text{var}(\epsilon)} = \frac{1}{\epsilon} h_{tc}$. From this, it can be established that:

$$\frac{\partial}{\partial p_i} \varphi^2(p; F) = \phi_e \left( F + F^e \frac{h_F}{h_{tc}} - \hat{F} \frac{h_F + h_{tc}}{h_{tc}} \right) \frac{h_{tc}}{h_F + h_{tc}} \frac{\partial}{\partial p_i} R^2(p)$$

Substituting this into the FOC's gives us:

$$R_1^1(p) + \beta d_1^1(p) + \frac{h_{tc}}{h_F + h_{tc}} R_2^2(p) \int_{-\infty}^{\hat{F}} \int_{-\infty}^{\infty} \phi_e(\cdot)(R^D - R^M)dH_2(F)dH_1(\hat{F}) = 0 \quad (3.16)$$
\[ R_2^2(p) + \beta d_2^2(p) + \frac{ht}{h_F + h_t} R_2^2(p) \int_{-\infty}^{\infty} \phi_\epsilon(\cdot)(R^D + B - F) dH_2(F) \]
\[ + \frac{ht}{h_F + h_t} R_2^2(p) (1 - \Phi_F(\hat{F})) \int_{-\infty}^{\infty} \phi_\epsilon(\cdot)(R^M - R^D) dH_2(F) = 0 \] (3.17)

We can point out a few things from equations (3.16) and (3.17). The first is that even if \( R^M - R^D \approx 0 \), firm 1 overproduces (prices lower than under strict profit maximization) since \( \beta > 0 \). Second, if \( R^M - R^D > 0 \), firm 1 overproduces relative to the single period empire-building equilibrium, and that for a fixed \( \varphi^2(\cdot) \), this overproduction is increasing in \( \Delta R \equiv R^M - R^D \). From equation (3.17), we also see two effects. Focusing on the last term, we see that the the prospect of future monopoly profits induces a further positive (disciplinary) effect on price (for prices below the profit maximizing price \( p_2^* \), the price which would prevail under strict managerial discipline, \( \varphi^2_2(p; F) \) will be positive). This disciplinary effect is also increasing in \( \Delta R \). The penultimate term is the equivalent for the duopoly case of the disciplinary effect that we saw for a monopoly.

Note that, for all values of \( p_2 \), we have that \( p_1^{BA}(p_2) < p_1^{BB}(p_2) < p_1^*(p_2) \), where the second value is the reaction function of firm 1 when both firms are bank financed, and the last term is the profit maximizing price. Also, from equation (3.17), we see that \( p_2^{BB} < p_2^{AB}(p_1) \leq p_2^*(p_1) \), since profits (and hence the probability of being refinanced) are maximized at \( p_2^*(p_1) \).

Both firms arm's length financed

When each firm is arm's length financed, they are faced with two conflicting incentives. First, each firm has an incentive to prey on the other, since predatory behavior may be rewarded by an increased probability of exit by the competitor firm. However, predatory behavior also decreases the preying firm's profits, which serves to increase its own probability of exit, and hence causes managers to lose their private benefit \( B \). In essence, each firm has only one instrument (price) with which to try to affect two different probabilities, and so faces a tradeoff between the two. Skipping most of the details, the FOC obtained for firm
1 is:

\[
R_1^1(p) + \beta d_1^1(p) + \int_{-\infty}^{\infty} \frac{\partial}{\partial p_1} \varphi^1(p; \tilde{F}) \left[ \varphi^2(p; F)(R^D + B - \tilde{F}) + (1 - \varphi^2(p; F))(R^M + B - \tilde{F}) \right] dH_2(F) dH_1(\tilde{F}) \\
+ \int_{-\infty}^{\infty} \varphi^1(p; \tilde{F}) \int_{-\infty}^{\infty} \frac{\partial}{\partial p_1} \varphi^2(p; F)(R^D - R^M) dH_2(F) dH_1(\tilde{F}) = 0,
\]

and similarly for firm 2. Let \( p_1^{AA}, p_2^{AA} \) be the prices that satisfy these FOCs. Note from equation (3.18) that, as always, the first two terms give us that \( p_1^{AA} < p_1^* \). The total effect is ambiguous, since lowering price also increases the probability that firm 1’s profit is low and it gets denied refinancing \( (\frac{\partial}{\partial p_1} \varphi^1(p; F) > 0) \), but it increases the probability that firm 2 exits the market in period 2 \( (\frac{\partial}{\partial p_1} \varphi^2(p; F) > 0) \). Which effect dominates depends on the importance ascribed to private benefits \( (B) \) and to the pursuit of empire building activities \( (\beta) \), as well on the relative stringency of the cutoff rule on profits (which depends on the ex-ante profitability of the firm).

### 3.3.2 Choice of financing

As in the section on monopoly, the choice of financing is not dictated simply by which source provides the most discipline, but by which goes furthest towards maximizing firm value. Firms faced with the possibility of predation by a firm with an informed creditor have the incentive to obtain informed financing as well in order to insulate themselves against these predatory actions. However, this permits the manager complete leeway to invest in empire-building and may not be optimal to the firm’s owners, both because it increases the cost of debt, and also because it lowers the level of profits. In order to analyze the choice of financing in more detail, we need to consider the competitiveness of the industry and the benefits of aggressive stances by each firm.

There are two measures along which we can gauge the “competitiveness” of the industry. One is the transportation cost \( t \), which can be interpreted as the degree to which each firm’s product is differentiated from its competitor’s. In the standard model of spatial differentiation, as \( t \) approaches zero, the price each firm charges approaches its marginal cost (firms become Bertrand competitors). The other measure is the difference between
\(R^M\) and \(R^D\), which captures the benefits of driving out a rival by hindering its ability to obtain financing in the future and is determined by consumers' valuation \(V\). While we have implicitly been assuming that \(R^M\) is representative of the profits that would obtain under a monopoly situation (since we have only been considering a duopoly), it may also represent the benefit of preying on rivals in a highly competitive industry with many active firms, or with highly elastic demand, so that the benefit of driving out a rival may be small (\(\Delta R \approx 0\)).\(^{20}\)

We will focus primarily on the choice of financing from the point of view of maximizing overall firm value (over the two periods). When \(\Delta R \approx 0\), the incentive to prey on a rival to induce exit disappears from all our equations above, leaving only the usual strategic effects in addition to each firm’s possible attempt to signal-jam its own creditor's inference. From equations (3.16) and (3.17), we see that the last term in equation (3.16) virtually disappears, leaving only \(R^1(p) + \beta d^1(p)\), so that the bank-financed firm discounts the benefits of “preying” on firm 2. For firm 2, which has arm’s length financing, the last term in equation (3.17) also disappears. Nevertheless, this manager still has an incentive to signal jam in order to assure its ability to continue into the second period.

Notice that, as \(t \to 0\), the benefit from being able to overproduce in period 2 remains strictly positive, even as profits converge to zero. This implies that a firm financed by an arm’s length creditor does not lose its incentive to signal jam as the market becomes more competitive. Conversely, as \(t\) increases (so that the market becomes less competitive), \(h_\ell\) decreases, and profits become a less informative signal about the firm’s fixed cost, decreasing managers’ incentives to signal jam.\(^{21}\) This gives us the following proposition.

**Proposition 2** Assume that \(\Delta R = 0\).

1. There is some value of \(t, t^B\), such that both firms obtaining informed financing is an equilibrium if and only if \(t > t^B\).

\(^{20}\)While these two measures are not really completely independent, it is easy to construct examples where they are nearly so. For a typical model of spatial competition on a circle with transportation cost of \(t\), the equilibrium profits for each firm when there are \(N\) active firms (gross of fixed costs) is \(R^N = \frac{t}{\sqrt{N}}\) Therefore, if \(\Delta R = R^{N-1} - R^N\) (the benefit of inducing the exit of one firm), then \(\Delta R = \frac{t^{2(N-1)}}{R^{(2N-1)}}\), which goes to 0 as \(N \to \infty\), even as the ratio \(\frac{R^N}{\Delta R} \to \infty\). Also, in the context of the Hotelling model being used here, we may assume that consumers' reservation price, \(V\), is low, so that \(R^M\) is not much different from \(R^D\).

\(^{21}\)Notice however that as \(t\) increases, so does \(R^D\), which increases the incentive to signal jam. We will deal with this later.
2. There is some value of \( t, t^A \), such that both firms obtaining uninformed financing is an equilibrium if and only if \( t < t^A \).

**Proof:** See Appendix.

In general, bank financed firms suffer from their inability to commit to "behave". When the market is sufficiently competitive, so that expected profits are low and signalling opportunities are large, they must bond themselves by obtaining financing from a creditor that can commit to be tough when poor outcomes are realized. This prevents managers from overly investing in empire-building activities. In this case, the optimal response to a firm financed by an uninformed creditor is to also obtain that kind of financing, since the informativeness of profits provides managers with a large incentive to signal jam, and moreover the inefficiency in the continuation decision is low. Conversely, in less competitive markets, managers with arm's length financing will have too much freedom to engage in private pursuits, and little incentive to concentrate solely on profit-maximizing. In that case, firms will find it preferable to give managers even more freedom, but be able to make more informed (and hence ex-post efficient) refinancing decisions.

Unfortunately, the relationship between \( t^P \) and \( t^A \) is unclear. It is quite possible for there to be a region where, in equilibrium, both firms obtain either informed or uninformed financing (i.e. that \( t^A > t^P \)). In fact, the possibility that one firm may choose informed financing and the other uninformed is also possible for intermediate values of \( t \). This occurs because of the discrete change in profits when changing from one form of financing to another. For example, suppose firm 1 is financed by an informed creditor. For intermediate values of \( t \), an optimal response by firm 2 may be to obtain uninformed financing, as profits may be seriously hurt under informed financing. Conversely, firm 1 may find it optimal to choose informed financing in this case since it is able to capture a larger share of the market, and uninformed financing may not provide sufficient discipline. In order to rule out equilibria of this sort, we reintroduce a positive return to monopoly over duopoly profits.

**Predatory incentives**

We began our analysis of the previous section under the assumption that \( \Delta R \approx 0 \), so that there was in effect no "strategic" incentive to choose one source of financing versus the other. However, as we remarked earlier, this is the other dimension along which we can measure
the competitiveness of the industry. For example, a highly concentrated industry may be one for which $\Delta R$ is significant, since driving a competitor out may be highly profitable when there are only a few firms in the industry. In this section, we analyze the effect of introducing a predatory motive on a firm’s choice of financing.

Let us first focus on the configuration where firm 1 is financed by a bank and firm 2 by an uninformed creditor. From equation (3.16), the FOC for a bank-financed firm, and equation (3.17), the FOC for an arm’s length-financed firm, it is clear that as we increase the predatory incentive ($\Delta R$), firm 1 reacts by lowering its price in hopes of inducing firm 2’s exit by convincing firm 2’s creditors that it has high costs, while firm 2 reacts by raising its own price to push its creditors’ beliefs in the opposite direction. Therefore, as $\Delta R$ increases, the demand for firm 2 falls so that first period profits drop, even with the added discipline. Moreover, as the informativeness of firm 2’s profit signal increases, not only does firm 2’s incentive to signal jam increase (the usual effect), but firm 1’s incentive to signal jam increases as well, since it perceives itself as more likely to be able to influence firm 2’s creditors. In this case, for a sufficiently large predatory incentive, it becomes optimal for firm 2 to choose informed financing as well, for all values of $t$. We have therefore established the following result.

**Proposition 3** There is a value $\overline{\Delta R}$ such that there exists no equilibrium where one firm chooses bank financing and the other chooses uninformed financing for $\Delta R > \overline{\Delta R}$.

This suggests some symmetry in firms’ financing choices when predatory incentives are large. In essence, a firm’s only possible response to a bank financed competitor is to be bank financed itself, since only when a firm is financed by a “deep pocket” will it be able to avoid being preyed upon. In short, firms choose informed financing as a defense (and deterrent) against predation. However, it may also be that an optimal response to a competitor that is financed by an uninformed creditor is to do likewise. We have already argued that when both firms have an uninformed creditor, they are faced with conflicting objectives. While it is true that conditional on being bank-financed a manager (as would the owner) finds it optimal to attempt to drive his competitor out, this attempt will have little success since in equilibrium his overproduction is anticipated. Moreover, since this overproduction leads to little or no increased exit and has a cost in terms of lower first period profits, an owner may like to commit to remove her manager’s temptation to prey by also choosing an arm’s
length creditor.  

**Within-industry comparison**

While the analysis of the previous section allows us to relate differences in the choice of financing source across industries to variables that describe the competitiveness of the industry, we would also like to explain differences within each industry. There seems to be as much variation in debt ratios within industries as across industries, so that we need a theory to explain this variation across firms in the same or similar industry. For this, we shift our focus to firm-specific characteristics while still applying the insights of the last section.

As a departure from our basic model, we assume that firms are no longer strictly identical but can differ along some dimensions. Specifically, we assume that the shock to demand to which each firm is subjected is not identical across firms ($\epsilon_i$, $i = 1, 2$), but may have different variances $\sigma^2_{\epsilon_i}$. This implies that the inference problem creditors face is different for each firm.

**Proposition 4** Assume that $\Delta R = 0$, so that no predatory incentive exists. There exist values $\overline{\sigma}^2_{\epsilon_1}$ and $\overline{\sigma}^2_{\epsilon_2}$, such that, if

1. $\sigma^2_{\epsilon_1} > \overline{\sigma}^2_{\epsilon_1}$, and
2. $\sigma^2_{\epsilon_2} < \overline{\sigma}^2_{\epsilon_2}$, then

informed finance maximizes the value of firm 1, while arm's length debt maximizes the value of firm 2.

When the informativeness of profits is low, not only does arm's length finance provide little discipline, but also the probability that an inefficient continuation decision is made increases. In this situation, a firm that perceives itself at risk of being liquidated even when the probability that it has a good project is high will opt to obtain financing from

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22This is one instance where the distinction between price and quantity competition may help resolve the ambiguity. Under price competition, having higher prices induces similar responses by competitors, so that arm's length finance may be optimal. In contrast, under quantity competition, committing to overproduce induces rivals to decrease their production, so that all firms may have an incentive to obtain bank finance. This is of course the usual prisoners' dilemma outcome of models of quantity competition, since the commitment to overproduce by both firms lowers overall profits.
an informed source in order to minimize this inefficiency. However, for the case in the proposition, the disciplinary effect of arm’s length debt is high for firm 2, so that it is rational for an owner to choose arm’s length debt in order to maximize the value of the firm.

We can interpret this as suggesting that new entrants (or perhaps firms working to develop a new product) who may have highly variable profits may choose to obtain funds from lenders who will become informed. More established firms, or firms whose profits better reflect future profitability, are able to employ the arm’s length market in order to provide discipline for their managers.

3.4 Conclusion

We have argued that a firm’s choice of source of financing, either borrowing from an informed source of from an uninformed one, cannot be analyzed in isolation since this choice is to some degree dependent on the competition faced by the firm in its product market. This analysis suggests that the degree of competition faced by the firm and the kind of financing obtained interact to have an impact on managerial actions. While the strategic benefits of obtaining a certain form of finance seem worth analyzing, we attempt to turn around the question of whether debt (or rather the kind of debt) influences a firm’s competitive behavior (as has been emphasized by much of the literature) and instead focus on how the competitive nature of the market may dictate a firm’s choice of financing.

We find that to the extent that having too much information available may ruin a lender’s credibility to adhere to certain policies that are ex-ante optimal, choosing to remain uninformed may provide a creditor with the commitment power necessary to impose ex-ante efficiency. The discipline afforded by an uninformed creditor acts as an instrument for the manager to bond himself, since the concern over remaining in control forces the manager to concentrate on maximizing profits. This effect is amplified when the industry is very competitive, so that profits become a more informative signal about the firm’s cost structure. This implies that as the industry becomes more competitive, uninformed finance serves to provide more discipline for managers who would otherwise prefer to dedicate their time to the pursuit of privately beneficial activities.

Conversely, when competition is less intense, managers are afforded more leeway in their
activities, so that uninformed financing provides little discipline. Moreover, since current profits provide a noisier signal of future profitability as competition lessens, obtaining financing from an uninformed source leads to more inefficient continuation decisions. In short, competition and arm's length financing play a complementary role in the provision of incentives through managers' concern for their future employment.

We were also able to suggest when firms may choose one form of financing over the other by looking at firms' differences within an industry, and argue again that information plays a key role here. Of course, in all the analysis, we concentrate on the only one aspect of "informed" or "monitored" lending, and ignore other issues such as control rights that may be attached to some kinds of debt, or the impact on a firm's prospects of its creditor obtaining an information monopoly over that firm. While these issues would provide a more complete story of the choice of financing source, we believe that the basic result on managerial incentives would continue to play a role in any such comprehensive analysis.
3.5 Appendix

3.5.1 Derivation of equation (3.7)

Recall that $\varphi(p; F') = \text{prob}(E[F|\pi] \leq \hat{F}|F')$. Define $\mathcal{F} = E[F|\pi]$. Then:

$$\frac{\partial}{\partial p} \varphi(p; F) = \frac{\partial}{\partial p} \text{prob}(\mathcal{F} \leq \hat{F}|F) = \frac{\partial}{\partial \mathcal{F}} \text{prob}(\mathcal{F} \leq \hat{F}|F) \frac{\partial}{\partial p} E[F|\pi] \quad (3.19)$$

From the definition of $E[F|\pi] = -\frac{h_{\epsilon}}{h_{\epsilon F} + h_{\epsilon}} R'(p)$. However, since $\pi = R(p) + \epsilon - F$, we can rewrite $\text{prob}(\mathcal{F} \leq \hat{F})$ as:

$$\text{prob} \left( \epsilon \geq F + F e^{\frac{h_{\epsilon}}{h_{\epsilon}} h_{F}} - \hat{F} e^{\frac{h_{F} + h_{\epsilon}}{h_{\epsilon}}} \right) = 1 - \Phi_{\epsilon} \left( F + F e^{\frac{h_{F}}{h_{\epsilon}} h_{\epsilon}} - \hat{F} e^{\frac{h_{F} + h_{\epsilon}}{h_{\epsilon}}} \right)$$

Replacing this into (3.19), we obtain:

$$\frac{\partial}{\partial p} \varphi(p; F) = \phi_{\epsilon} \left( F + F e^{\frac{h_{F}}{h_{\epsilon}} h_{\epsilon}} - \hat{F} e^{\frac{h_{F} + h_{\epsilon}}{h_{\epsilon}}} \right) \frac{h_{\epsilon}}{h_{\epsilon F} + h_{\epsilon}} R'(p)$$

as desired.

3.5.2 Proofs

Proof of Lemma 1: To prove the lemma, we focus on equation (3.8). From this, we see that $\frac{h_{\epsilon}}{h_{\epsilon F} + h_{\epsilon}}$ is increasing in $h_{\epsilon}$, and $\frac{h_{\epsilon}}{h_{\epsilon F} + h_{\epsilon}} \rightarrow 1$ as $h_{\epsilon} \rightarrow \infty$. Since $R'(p) > 0$ for $p < p_{1}^*$, the value of $p$ that maximizes profits, this term tells us that the incentive to move closer to profit maximization increases with $h_{\epsilon}$. However, we need to be concerned about the term inside the integral: $\phi_{\epsilon}(\hat{F}) = \phi_{\epsilon} \left( F + F e^{\frac{h_{F}}{h_{\epsilon}} h_{\epsilon}} - \hat{F} e^{\frac{h_{F} + h_{\epsilon}}{h_{\epsilon}}} \right)$. We can rewrite the integral as:

$$\int_{-\infty}^{\infty} \phi_{\epsilon}(\hat{F}) \left( R_{2}^{F} + B - F \right) dH(F)$$

$$= B \int_{-\infty}^{\infty} \phi_{\epsilon}(\hat{F}) dH(F) + \int_{-\infty}^{\hat{F}} \phi_{\epsilon}(\hat{F}) \left( R_{2}^{F} - F \right) dH(F)$$

$$= B \int_{-\infty}^{\infty} \phi_{\epsilon}(\hat{F}) dH(F) + \int_{-\infty}^{\hat{F}} \phi_{\epsilon}(\hat{F}) \left( R_{2}^{F} - F \right) dH(F) + \int_{\hat{F}}^{\infty} \phi_{\epsilon}(\hat{F}) \left( R_{2}^{F} - F \right) dH(F)$$

Notice that the term $(R_{2}^{F} - F)$ in the second integral is greater than $I$ and in the third less than $I$. As $I$ is exactly the expected debt payment that needs to be made to creditors, we can just renormalize this so that $R_{2}^{F} - \hat{F} = 0$. Finally, using the definition of $\hat{F}$
as \( \bar{F} = F + F e^{h_e \frac{1}{h_e}} - F (\frac{h_F + h_e}{h_e}) \), we can stipulate that \( \phi_e \) reaches its peak for \( \epsilon = 0 \), or \( F = \bar{F} + (\bar{F} - F e) \frac{h_e}{h_e} \). Therefore, thinking of \( \phi_e \) as a function of \( F \) (since that is the variable with respect to which we are integrating), we see that as \( h_e \to \infty \), the value of \( F \) for which \( \epsilon = 0 \) converges to \( \bar{F} \). For the first term, this implies that \( \phi_e \) shifts to put more weight on values of \( F \) near \( \bar{F} \). Since the value of \( \phi_F \) is greater at \( \bar{F} \) than at \( \bar{F} + (\bar{F} - F e) \frac{h_e}{h_e} \), we are putting more weight on larger values of \( \phi_F \), so that this term is increasing in \( h_e \). Also, this implies that \( \phi_e \) shifts to put more weight on values of \( F \) for which \( R_2 F - F > 0 \) (the second integral), implying that the whole expression increases. Therefore, the last expression in equation (3.8) increases as \( h_e \) increases, implying that \( p_1^A \) increases with \( h_e \). \( \square \)

**Proof of Proposition 1:** From the previous lemma, we know that as \( h_e \) increases, \( p_1^A \) increases, and that moreover \( p_1^A(h_e) > p_1^B \) for all values of \( h_e \). This implies that first period profits are greater under arm's length financing than under bank financing \( (\pi_1(p_1^A) > \pi_1(p_1^B)) \). However, the ex-ante value of the firm is determined not only by first period profits, but also by the efficiency of the continuation decision. As \( h_e \to \infty \), the probability that a firm is allowed to continue when it should be shut down \( (\text{prob}(\text{continue}|F > \bar{F})) \) and that the firm is shut down when it should be continued \( (\text{prob}(\text{liquidate}|F < \bar{F})) \) both go to zero. Since \( p_1^A \) is continuous and increasing in \( h_e \), there must be some value of \( h_e \) such that the value of the firm is greater under arm's length financing than under bank financing. \( \square \)

**Proof of Proposition 2:** To prove the first part, we need to check whether the optimal response to a firm that is financed by an informed creditor is to obtain informed financing as well. Observe that the degree of empire building does not depend on \( t \) when firms are financed by informed creditors. As \( t \) increases, we know that: 1) the manager of a firm financed by an arm's length creditor will signal jam less, and 2) the variance of profits increases \( (\text{var}(\pi) = t^2 \sigma^2) \), so that the inefficiency in the continuation decision increases. Therefore, there must be some value \( t^B \) such that choosing informed financing is optimal for all \( t > t^B \). As \( t \) decreases, a firm that was financed by uninformed creditors would signal jam more, and would be subject to less inefficiency in continuation, implying that the optimal response would be to obtain uninformed financing. Therefore, both firms obtaining informed financing is an equilibrium iff \( t > t^B \).

To prove the second part, we need to check the opposite result: whether the optimal response to a firm that is financed by an arm's length creditor is to obtain that kind of
financing as well. Suppose that firm 1 chooses uninformed financing. As \( t \) decreases, it must clearly be optimal for firm 2 to choose uninformed financing as well, as the incentive to signal jam increases and the inefficiency of continuation decision decreases. As \( t \) increases, it is clear that at some point the inefficiency in continuation gets so large that it is better to switch to informed financing. Therefore, there is a cutoff value \( t^A \) such that both firms obtaining uninformed financing is an equilibrium iff \( t < t^A \). \( \square \)

3.5.3 Equivalence of approaches

This section demonstrates the equivalence of the two approaches mentioned in section 3.1 for limiting values of \( t \). The key element is that we require that the variance of profits decrease with the level of competition, going to zero as \( t \to 0 \), which is the main characteristic of the second model. In order for this to be true in the original model of competition on the line with a random mass of consumers, we require that \( \lim_{t \to 0} (p_i^* - c) = 0 \), where \( p_i^* \) refers to the equilibrium price under whatever form of financing we are considering.

Let us rewrite the manager's optimization problem:

\[
\max_{p_i} E[d^i(p_i, p_j)(p_i - c + \beta) + \epsilon d^i(p_i, p_j)(p_i - c + \beta) - F + Z^i(p)],
\]

where \( Z^i(p) \) is a proxy for all second period terms. Recall \( d^i(p_i, p_j) = \left( \frac{p_j - p_i + t}{2t} \right) \). We know that for finite value of \( t \), \( \text{var}(\pi^i) = d^i(p_i, p_j)^2(p_i^* - c)^2 \sigma_i^2 \), so that the manager may have an incentive to increase \( p_i \) merely in order to affect \( \text{var}(\pi^i) \). To show that in this is not so for small values of \( t \), we need the following: \( \frac{\partial^2}{\partial t \partial p_i} d^i(p) = \frac{1}{2t^2} > 0 \), so that the marginal impact of lowering price is increasing in \( t \). Therefore, in order for first period profits to be positive, we require, at a minimum, that \( d^i(p_i, p_j) > 0 \). As \( t \to 0 \),

\[
d^i(p_i, p_j) = \begin{cases} 
0 & \text{if } p_i > p_j, \\
\frac{1}{2} & \text{if } p_i = p_j, \\
1 & \text{if } p_i < p_j 
\end{cases}
\]

Therefore, clearly \( \lim_{t \to 0} (p_i^* p_j^*) = 0 \). So suppose that \( \lim_{t \to 0} p_i^* = \bar{p} > c \). Using the expression for \( \frac{\partial^2}{\partial t \partial p_i} d^i(p) \), the return to lowering price slightly for firm \( j \) can be made arbitrarily large by lowering \( t \) sufficiently, so that firm \( j \) would always lower its price. Therefore, it cannot be that the \( \lim_{t \to 0} p_i^* = \bar{p} > c \) and \( \lim_{t \to 0} (p_i^* p_j^*) = 0 \). Therefore, \( \lim_{t \to 0} p_i^* = c \), as
desired. Moreover, since \( p_t^* \) is continuous in \( t \), it must converge smoothly to \( c \), implying that for small values of \( t \), the price charged by a manager will be close to \( c \), implying that the signal that profits provides becomes more informative.

References


Chapter 4

A Note on Bertrand Competition with Asymmetric Fixed Costs

4.1 Introduction

This note considers the possibility of entry into a market characterized by Bertrand competition when entrants face an avoidable fixed (or sunk) cost of entry, which must be paid simultaneous to the decision to enter.\footnote{It is well known that if firms face a sequential entry decision, where they first must pay a sunk cost of entry and then compete à la Bertrand in prices, then the unique equilibrium of the subgame after the entry decision has been made is for all firms to charge at price equal to marginal cost and obtain zero profits. Since all firms cannot recoup their sunk cost, no more than one firm will ever choose to enter in equilibrium.} We use a model similar to that analyzed by Sharkey and Sibley (1993), who focus on the coordination problem between $N$ symmetric firms, each considering entry. They find that, as more firms become potential competitors, expected prices must rise in order to compensate firms for a lower probability of winning the pricing game, since each firm has to incur a fixed entry cost in order to compete. We consider the case where firms are not restricted to being identical and show that asymmetries in the industry structure lead to blockaded entry for higher cost firms. When firms have different fixed costs, competition among low cost firms may force high cost firms to make negative profits. In fact, we show that no high fixed cost firm is able to enter a market with two lower cost firms. This result generalizes to the $N$-firm case, so that a high cost firm is never able to compete in a market with $N$ symmetric lower cost firms. However, if $N = 1$, a second firm is always able to enter, and we obtain a unique equilibrium where the high cost
firm obtains zero profits, and the lower cost one obtains positive profits.

There are many natural settings in which we might expect this kind of competition to arise. For example, Varian (1980), in his model of sales, assumes that firms are characterized by strictly declining average cost curves. He justifies this by arguing that retail stores tend to have fixed costs of rent and employment, and nearly constant variable costs. Similarly, any kind of minimum scale, such as having to hire at least one manager or having to buy one piece of equipment that, at least for some period of time, cannot be released or rented out to an alternative use, will lead to a fixed cost for the period of production.2 This “fixed” cost can usually be avoided by not entering the market at all. The asymmetry in costs can be attributed to different technologies available to the firms, to some sort of incumbency advantage, or to lower costs of capital due to reputation concerns or length of time in the market.3

Most of the previous literature on Bertrand competition has focused on symmetric firms and particularly on duopolies. Other than in the differentiated products (e.g. Salop (1979)) or the Bertrand-Edgeworth literature (see Kreps and Scheinkman (1983), Osborne and Pitchik (1986), or Allen and Hellwig (1986)), with rare exceptions has price competition focused on the interaction of more than a couple of firms (Varian (1980)). The possibility of entry and the interaction among many competitors has been analyzed by Sharkey and Sibley (1993). Here, we extend that analysis to allow firms to have different fixed costs of production, and argue that many of the above models are somewhat fragile to the introduction of heterogeneity across firms, since their results rely on all firms being identical.

4.2 Model: Bertrand price competition

We assume that there are a number of firms that could potentially produce a single good, and we make the simplifying assumption that each unit of the good can be produced at a zero marginal cost, but that there is a positive fixed cost \( C \) of production, which we allow to differ for each firm. This fixed cost is avoidable if the firm chooses not to attempt to

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2Effectively, the distinction is between the relevant period of production and the degree to which costs are “fixed” or “sunk” over that period. See the discussion in Tirole (1988) for further elaboration on this point.

3Dell’Ariccia, Friedman, and Marquez (1998) provide an example where adverse selection considerations force entrant banks to pay an endogenous fixed cost of entry when they decide to offer credit to applicant firms. Incumbent banks suffer less from adverse selection, so that it is the value of incumbency that leads to lower fixed operating costs.
Firms compete for customers in this market à la Bertrand. A decision not to post a price constitutes a decision not to enter. Therefore, each firm’s strategy is composed of a decision to enter and of a posted price. Consumers are perfectly informed about all price realizations, and always choose the lowest offered price.

The function $D(p)$ represents the total quantity demanded of the good at a given price $p$. We make the usual assumption that demand is downward sloping. We define a monopoly price to be a price $p^*$ that maximizes total profit for the industry ($p^* = \arg\max_p pD(p)$), and we let $P^M$ be the set of monopoly prices.

**Assumption 1** Demand, $D(\cdot)$, is a continuous function from $\mathbb{R}_+$ to $\mathbb{R}_+$, and $P^M \neq \emptyset$. Moreover, we assume that $P^M$ contains the single element $p^M$.

Profits for any firm, conditional on having the lowest bid, will then be a continuous function of price, with a maximum at a positive and finite $p^M$.  

### 4.3 Analysis

The demand faced by each firm is given by:

$$d_i(p_i) = \begin{cases} 
D(p_i) & \text{if } p_i < p_j \ \forall \ j, \\
\frac{D(p_i)}{k} & \text{if } p_i \leq p_j \ \forall \ j, \text{ with equality for } k \text{ firms,} \\
0 & \text{if } p_i > p_j \ \text{for some } j \neq i
\end{cases} \quad (4.1)$$

Profits for firm $i$ are then $\pi_i(p_i) = p_id_i(p_i) - C_i$, where $C_i$ represents its fixed cost. It is known from previous work that no pure strategy equilibrium exists for this game if the number of potential firms is greater than or equal to 2, and if each firm’s fixed cost is greater than zero, since firms always have the incentive to undercut each other in order to increase their sales, but would prefer to exit rather than incur the fixed cost. However, a mixed

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4 As the analysis below will make clear, we do not require that these fixed costs be large, only that the fixed cost be incurred along with the decision to offer the good for sale (i.e. that no cost be incurred if the firm decides not to post a price).

5 This assumption could be relaxed to yield the same conclusions. See Allen and Hellwig (1986). Note that the assumption of global concavity of the revenue function, which is often assumed in the literature, would be sufficient for the results.

6 See Varian (1980) or Sharkey and Sibley (1993). While these models are ones with identical firms, the logic of their results applies here as well.
strategy equilibrium does exist. Sharkey and Sibley (1993) characterize this equilibrium when firms are symmetric, and find that as $N$ increases, the equilibrium distribution of prices shifts to put more weight on higher prices in order to compensate each firm for its lower probability of having the lowest price. With symmetric firms, an equilibrium with any number of active firms always exists. We argue that this is an artifact of the symmetric nature of the game, and that when fixed costs differ across firms, their result no longer holds.

4.3.1 The case with 2 potential firms

Consider an industry with two firms, and assume that firm 1 has to pay a fixed cost of $C_1$ and firm 2 a cost of $C_2$, with $C_1 < C_2$. As argued above, no pure strategy equilibrium exists, but a mixed strategy equilibrium does exist. To simplify notation, define $R(p) = pd(p)$. If we can show that the probability distributions over prices for each firm are strictly increasing and contain no mass points, then we can write each firm's expected payoff as:

$$\pi_i(p) = (1 - \sigma_i F_j(p))R(p) - C_i, \quad i = 1, 2$$

(4.2)

where $\sigma_i$ represents the probability that firm $i$ actually enters the market and $F_i$ is the distribution function over prices charged by firm $i$ if it enters. To fix notation, let $\text{prob}(p_i < p) = F_i(p) - \mu_i(p)$, where $\mu_i(p)$ is the mass of $F_i$ at $p$, if any. We need a couple of preliminary results, the proofs of which are provided in the appendix.

**Lemma 1** The mixing distributions of both firms have the same support, with lower and upper bounds denoted by $\underline{p}$ and $\overline{p}$, respectively. Moreover, $\overline{p} = p^M$, the monopoly price level.

**Lemma 2** There are no mass points in the mixing distributions of either firm over the range $(\underline{p}, \overline{p})$.

We now proceed to define the equilibrium between two asymmetric firms. We offer the result as the following proposition.

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7Dasgupta and Maskin (1986) prove the existence of equilibria in these kinds of games with discontinuous payoffs. It is easy to show that this particular setup satisfies all the necessary conditions for existence.
Proposition 1 In the equilibrium of this pricing game with two asymmetric firms, each firm mixes continuously over an interval \([p, \bar{p}]\). The equilibrium is given by the following strategies: \(\{(\sigma_1, F_1), (\sigma_2, F_2)\}\), where \(\sigma_1 = 1, \mu_1(\bar{p}) > 0, 0 < \sigma_2 < 1, \mu_2(\bar{p}) = 0\), and where \(\bar{p} = p^M\), the monopoly price level.

Proof: Since by lemma 1 both distributions must have the same support, we can write the payoffs for each at the lower bound of the support.

\[
\begin{align*}
\pi_1(p) &= (1 - \sigma_2 F_2(p))R(p) - C_1 = R(p) - C_1 \quad (4.3) \\
\pi_2(p) &= (1 - \sigma_1 F_1(p))R(p) - C_2 = R(p) - C_2 \quad (4.4)
\end{align*}
\]

From these two equations we obtain that \(\pi_1(p) > \pi_2(p)\), and we conjecture that \(p\) is chosen such that \(R(p) = C_2\). Also, since firm 1 makes positive profits in equilibrium, we must have that \(\sigma_1 = 1\). Now look at the upper bound of the support:

\[
\begin{align*}
\pi_1(\bar{p}) &= (1 - \sigma_2)R(\bar{p}) + \sigma_2 \mu_2(\bar{p}) \frac{1}{2} R(\bar{p}) - C_1 \quad (4.5) \\
\pi_2(\bar{p}) &= (1 - \sigma_1)R(\bar{p}) + \sigma_1 \mu_1(\bar{p}) \frac{1}{2} R(\bar{p}) - C_2 \quad (4.6)
\end{align*}
\]

In order for firm 2 to offer prices near \(\bar{p}\), we need that: \(\lim_{p \to \bar{p}} \pi_2(p) = \lim_{p \to \bar{p}} (1 - F_1(p))R(p) - C_2 = 0\). But \(\lim_{p \to \bar{p}} (1 - F_1(p)) = \mu_1(\bar{p})\), which implies that it must be that \(\mu_1(\bar{p}) = \frac{C_2}{R(\bar{p})} > 0\). This also implies that \(\mu_2(\bar{p}) = 0\), since it is not optimal for firm 2 to have an atom there as well. We can now equate the profits for firm 1 at the upper and lower bounds: \(\pi_1(\bar{p}) = (1 - \sigma_2)R(\bar{p}) - C_1 = R(p) - C_1 = \bar{\pi}_1\), which implies that \(\sigma_2 = 1 - \frac{R(p)}{R(\bar{p})}\), verifying our conjecture that \(\bar{\pi}_2 = 0\), and hence that \(R(p) = C_2\). Finally, we obtain \(F_1\) and \(F_2\) by inverting the profit equations:

\[
\begin{align*}
\bar{\pi}_1 &= (1 - \sigma_2 F_2(p))R(p) - C_1 \Rightarrow F_2(p) = \frac{1}{\sigma_2} - \frac{\bar{\pi}_1 + C_1}{R(p)\sigma_2} \quad (4.7) \\
0 &= (1 - \sigma_1 F_1(p))R(p) - C_2 \Rightarrow F_1(p) = 1 - \frac{C_2}{R(p)} \quad (4.8)
\end{align*}
\]

which nearly completes our characterization of the equilibrium, as described above. We only need now to show that \(\bar{p} = p^M\), for which we refer to lemma 1. □

An often overlooked characteristic of equilibria constructed in this fashion is that the supports of the distributions \(F_1\) and \(F_2\) do not coincide exactly, but rather differ by a single
point. Because of the need to guarantee nonnegative profits to firm 2, firm 1 must put positive mass on the single price $\bar{p} = p^M$. But then it would never be optimal for firm 2 to bid $p^M$, since its profits are strictly higher when it bids $p^M - \epsilon$ and avoids the possible tie. Therefore, the support of $F_2$ is $[\bar{p}, p^M)$ instead of the closed interval $[\bar{p}, p^M]$.

In the equilibrium above, firm 2, the high fixed cost firm, enters the market with strictly positive probability, and makes zero profits, whereas the low cost firm always enters the market and makes positive profits. In effect, we need a minimum amount of existing competition in the market before entry can be blockaded. We turn to this next.

4.3.2 The case with $N > 2$ potential firms

Consider now an industry with three potential firms, and assume that each firm has a different fixed cost and that, without loss of generality, $C_1 < C_2 < C_3$. For concreteness, we write the expected payoff to a firm of offering a price $p$ as: $E[\pi_i(p)] = (1 - \sigma_j F_j(p))(1 - \sigma_k F_k(p))R(p) - C_i$.

Consider the equilibrium obtained when only the two low cost firms are potentially in the market, and let the strategies that support that equilibrium be given by $\{(\sigma_1^*, F_1^*), (\sigma_2^*, F_2^*)\}$. As shown in proposition 1, this equilibrium necessarily entails $\sigma_1 = 1$, $0 < \sigma_2 < 1$, so that expected profits for firm 2 are zero. We have the following result:

**Proposition 2** An equilibrium to the game with 3 potential firms is given by the strategies $\{(\sigma_1^*, F_1^*), (\sigma_2^*, F_2^*)\}$ for firms 1 and 2, and $\sigma_3 = 0$ for firm 3.

**Proof**: Given $\sigma_3 = 0$, we know that $\{(\sigma_1^*, F_1^*), (\sigma_2^*, F_2^*)\}$ defines an equilibrium for the two low cost firms. For firm 2, expected profits for any price $p$ are, by proposition 1:

$$\pi_2(p) = (1 - \sigma_1^* F_1^*(p))R(p) - C = 0, \quad (4.9)$$

Given these strategies by firms 1 and 2, if firm 3 enters and offers a price $p$, it obtains:

$$\pi_3(p) = (1 - \sigma_1^* F_1^*(p))(1 - \sigma_2^* F_2^*(p))R(p) - C_3 \quad \text{ (4.10) }$$

$$< (1 - \sigma_1^* F_1^*(p))R(p) - C_3 = 0,$$

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*This observation is also true in models of capacity-constrained competition, where it is known that the firm with the larger capacity assigns positive probability to playing the highest price, and the lower capacity firm does not bid at all with positive probability. See Kreps and Scheinkman (1983).*
where the inequality follows from the fact that \( C_3 > C_2 \), and the final equality from equation (4.9). Hence, \( \sigma_3 = 0 \) is optimal, and we have found an equilibrium. \( \square \)

This result is in contrast to previous results in the literature, although it does not necessarily make a strong case for blockaded entry when costs differ. We would like to demonstrate that there does not exist an equilibrium where the high cost firm is ever able to enter, so that entry is in effect blockaded (in the terminology of Bain). To do this, we first need some preliminary results on the conditions that would need to be satisfied in order for an equilibrium where all 3 firms enter to exist. The proofs may be found in the appendix.

**Lemma 3** In any mixed strategy equilibrium with \( N \geq 2 \), no more than one firm can have an atom (a mass point) at any given price \( p' \).

**Lemma 4** Any equilibrium with \( \sigma_3 > 0 \) necessarily has \( \sigma_1 = \sigma_2 = 1 \), so that both low cost firms enter with probability one.

The logic for this last result is that anytime firm 3 is able to enter and make nonnegative profits, firms 1 or 2 can mimic firm 3's behavior and make strictly positive profits. Therefore, they cannot be indifferent between entering and not entering, since not entering yields zero profits, and hence must enter with probability one if they are to maximize profits.

**Lemma 5** In any equilibrium where \( \sigma_3 > 0 \), \( \bar{p}_3 \leq \bar{p}_1, \bar{p}_2 \).

**Lemma 6** In any equilibrium where \( \sigma_1, \sigma_2, \sigma_3 > 0 \), we must have \( \bar{p}_1 = \bar{p}_2 = \bar{p} \).

We now present our main result.

**Proposition 3** There does not exist an equilibrium to the 3-firm price game with asymmetric costs where \( 0 < \sigma_3 \leq 1 \), i.e. where firm 3 enters the market with positive probability.

**Proof:** Consider the profits for firms 1 and 2 at values near \( \bar{p} \), which is the same for firms 1 and 2 by lemma 6.

\[
\lim_{\epsilon \to 0} \pi_1(\bar{p} - \epsilon) = \lim_{\epsilon \to 0} (1 - \sigma_2 F_2(\bar{p} - \epsilon))(1 - \sigma_3 F_3(\bar{p} - \epsilon))R(\bar{p} - \epsilon) - C_1 \quad (4.11)
\]

\[
\lim_{\epsilon \to 0} \pi_2(\bar{p} - \epsilon) = \lim_{\epsilon \to 0} (1 - \sigma_1 F_1(\bar{p} - \epsilon))(1 - \sigma_3 F_3(\bar{p} - \epsilon))R(\bar{p} - \epsilon) - C_2 \quad (4.12)
\]
Using lemma 4, if $\sigma_3 > 0$, we must have that $\sigma_1 = \sigma_2 = 1$. Also, by lemma 5, $\bar{\sigma}_3 \leq \bar{\sigma}$. We can then write:

\[
\begin{align*}
\lim_{\varepsilon \to 0} \pi_1(\bar{p} - \varepsilon) &= \mu_2(\bar{p})(1 - \sigma_3(1 - \mu_3(\bar{p})))R(\bar{p}) - C_1 \\
\lim_{\varepsilon \to 0} \pi_2(\bar{p} - \varepsilon) &= \mu_1(\bar{p})(1 - \sigma_3(1 - \mu_3(\bar{p})))R(\bar{p}) - C_2
\end{align*}
\] (4.13) (4.14)

Note that the value of $\mu_3(\bar{p})$ becomes irrelevant, since in order to satisfy $\pi_1(\bar{p}) \geq 0$ and $\pi_2(\bar{p}) \geq 0$, we require that $\mu_2(\bar{p}) > 0$ and $\mu_1(\bar{p}) > 0$, or that both firms 1 and 2 place positive mass on the highest price, $\bar{p}$. But this is also ruled out by lemma 3. Therefore, we cannot satisfy all the conditions for an equilibrium to exist where $\sigma_3 > 0$. We conclude that any equilibrium must have $\sigma_3 = 0$, so that firm 3 is never able to enter. □

The only equilibrium left is the one established by proposition 2. Therefore we conclude that the three firm game has a unique equilibrium where only the two low cost firms enter with positive probability, only the lowest cost firm obtains positive profits, and the third firm, with the highest fixed cost, finds entry blockaded. It is also not difficult to show that this result generalizes to the case where we consider entry by a high cost firm into a market with $N$ symmetric low cost incumbents.

### 4.4 Discussion

We have argued in the preceding that, contrary to the usual literature that assumes identical firms, competition with asymmetric firms leads to quite different results. To our knowledge no one has analyzed the case where the asymmetry among firms stems from the fixed cost instead of the marginal cost of production, which necessitates a quite different approach to the analysis. The asymmetry in fixed costs, coupled with the intensity of price competition, leads to the result of blockaded entry.

This points to an omission in the existing literature on price competition that usually treats all firms as identical and performs comparative statics on the number of firms to determine the impact of increased competition on equilibrium prices. The preceding has argued that many of these results may be non-robust to the introduction of any amount of heterogeneity across firms when this heterogeneity stems from differences in firms' fixed cost structures.
4.5 Appendix

4.5.1 Proofs of results in Section 3.1 and 3.2

Proof of lemma 1: That the lower bounds of the supports of $F_1$ and $F_2$ are the same should be clear. Suppose not, so that the lower bounds are different. Let $p_i = \inf(supp(F_i)), i = 1, 2$, and assume WLOG that $p_1 > p_2$. If $R(p_1) > R(p_2)$, firm 2 could strictly increase profits by charging $p_1$, since $\text{prob}(p_1 < p_2) = \text{prob}(p_1 < p_1)$. If $R(p_1) \leq R(p_2)$, then firm 1 could strictly increase profits by charging $p_2$, since $\text{prob}(p_2 \leq p_2) < \text{prob}(p_2 \leq p_1)$. Therefore, they must both have the same lower bound. Showing that that the upper bounds of the supports are the same takes a little more work. Let $\bar{p}_i = \sup(supp(F_i))$. We first argue that $\bar{p}_i = p^M$ for at least some firm $i$. Suppose not. Then we have 3 possibilities: (1) $p^M < \bar{p}_i, \bar{p}_j$, (2) $\bar{p}_i < p^M < \bar{p}_j$, and (3) $\bar{p}_i, \bar{p}_j < p^M$. Consider first case (1):

$$
\pi_i(\bar{p}_i) = (1 - \sigma_jF_j(\bar{p}_i))R(\bar{p}_i) - C_i < (1 - \sigma_jF_j(p^M))R(p^M) - C_i = \pi_i(p^M)
$$

(4.15)

so that $(p_i)$ cannot be in the support of $F_i$. Case (2) can be handled similarly by focusing on firm $j$ and observing that profits must be higher at $p^M$, so that $\bar{p}_j$ cannot be in the support of $F_j$. Finally, case (3). WLOG, assume that $\bar{p}_i \geq \bar{p}_j$. Then $\pi_i(\bar{p}_i) = (1 - \sigma_jF_j(\bar{p}_i))R(\bar{p}_i) - C_i$. Note that $\lim_{\epsilon \to 0} F_j(\bar{p}_i - \epsilon) < 1$ only if $\bar{p}_i = \bar{p}_j$. If so, then we can use a simple undercutting argument as in lemma 2 below to argue that $\lim_{\epsilon \to 0} F_i(\bar{p}_i - \epsilon) = 1$, since it would never be optimal for firm $i$ to put positive mass on $\bar{p}_i$ if firm $j$ is doing so. So we reverse the roles and analyze firm $j$'s payoffs:

$$
\pi_j(\bar{p}_i) = (1 - \sigma_i)R(\bar{p}_i) - C_j < (1 - \sigma_i)R(p^M) - C_j = \pi_j(p^M)
$$

(4.16)

Alternatively, if $F_j(\bar{p}_i) = 1$, then equation (4.16), with "i" substituted for "j", gives us that $\pi_i(\bar{p}_i) < \pi_i(p^M)$.

Finally, we now show that $\bar{p}_1 = \bar{p}_2 = p^M$. Suppose not, and that $\bar{p}_1 > \bar{p}_2$. By the above, we have that either $\bar{p}_1 = p^M$, or that $\bar{p}_2 = p^M$. If $\bar{p}_2 = p^M$, then clearly $\pi_1(p^M) > \pi_1(\bar{p}_2)$, so we are done. If $\bar{p}_1 = p^M$, consider the following. For any $p' \in (\bar{p}_2, p^M)$, $\text{prob}(p_2 < p') = \text{prob}(p_2 < p^M)$, so that $\pi_1(p^M) > \pi_1(p')$. Therefore, firm 1 never offers a price $p' \in (\bar{p}_2, p^M)$. Since demand is assumed to be continuous, profits must be increasing in a neighborhood of $p^M$, so that for $p' = p^M - \epsilon$, we must have that $\pi_2(p') > \pi_2(\bar{p}_2)$, with $\pi_2(p')$ reaching its

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maximum as \( p' \to p^M \). Therefore, we conclude that \( \bar{p}_1 = \bar{p}_2 = p^M \). □

**Proof of lemma 2:** Suppose that there is an atom at \( p' \) in firm \( i \)'s distribution \( (\mu_i(p') > 0) \). Then the payoff to firm \( j \) at \( p' \) is: \( \pi_j(p') = (1 - \sigma_iF_i(p'))R(p') + \sigma_i\mu_i(p')\frac{1}{2} R(p') - C_j \), and at \( p' - \epsilon \), \( \pi_j(p' - \epsilon) = (1 - \sigma_iF_i(p' - \epsilon))R(p' - \epsilon) - C_j \). Take the limit, so that

\[
\lim_{\epsilon \to 0} \pi_j(p' - \epsilon) = (1 - \sigma_iF_i(p'))R(p') + \sigma_i\mu_i(p')R(p') - C_j > \pi_j(p'),
\]

so that firm \( j \) never plays \( p' \). Moreover, we can calculate this for values just above \( p' \) as well: \( \pi_j(p' + \epsilon) = (1 - \sigma_iF_i(p' + \epsilon))R(p' + \epsilon) - C_j \). Again, taking the limit, we obtain,

\[
\lim_{\epsilon \to 0} \pi_j(p' + \epsilon) = (1 - \sigma_iF_i(p'))R(p') - C_j < \pi_j(p'),^9
\]

which shows that the distribution \( F_j \) must be zero for some interval \( [p', p' + \epsilon] \). But then firm \( i \) could clearly never find it optimal to have an atom at \( p' \), since \( p' + \epsilon \) yields strictly higher profits with the same probability of being the lowest price. □

**Proof of lemma 3:** (We provide a proof for the case where \( N = 3 \), but for more firms the proof follows similarly.) Suppose not, and that there is a point \( p' \) at which two firms, \( i \) and \( j \), have an atom in their mixing distributions. Write the expected profits for firm \( i \):

\[
\pi_i(p') = (1 - \sigma_jF_j(p'))(1 - \sigma_kF_k(p'))R(p') + (1 - \sigma_kF_k(p'))\sigma_j\mu_j(p')\frac{1}{2} R(p') - C
\]

(4.17)

where \( \mu_j(p') > 0 \), since we assume firm \( j \) puts positive mass on that price. We can now focus on the profits of firm \( i \), and choose \( \epsilon > 0 \) small. Observe that the profit to firm \( i \) satisfies: \( \pi_i(p' - \epsilon) \geq (1 - \sigma_jF_j(p' - \epsilon))(1 - \sigma_kF_k(p' - \epsilon))R(p' - \epsilon) - C \). As usual, we take the limit of this equation as \( \epsilon \to 0 \) to obtain:

\[
\lim_{\epsilon \to 0} \pi_i(p' - \epsilon) \geq \lim_{\epsilon \to 0}(1 - \sigma_jF_j(p' - \epsilon)) \lim_{\epsilon \to 0}(1 - \sigma_kF_k(p' - \epsilon)) \lim_{\epsilon \to 0} R(p' - \epsilon)
\]

\[
= (1 - \sigma_jF_j(p'))(1 - \sigma_kF_k(p'))R(p') + (1 - \sigma_kF_k(p'))\sigma_j\mu_j(p')R(p')
\]

(4.18)

The first term in equation (4.18) equals the same term in equation (4.17), but the second term is strictly greater, so that firm \( i \) would not put any mass at \( p' \), contradicting the

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^9Since \( \lim_{\epsilon \to 0}(1 - F_i(p' - \epsilon))R(p' - \epsilon) = R(p')(1 - \lim_{\epsilon \to 0} F_i(p' - \epsilon)) = R(p')(1 - F_i(p') + \mu_i(p')) \), but \( \lim_{\epsilon \to 0}(1 - F_i(p' + \epsilon))R(p' + \epsilon) = R(p')(1 - \lim_{\epsilon \to 0} F_i(p' + \epsilon)) = R(p')(1 - F_i(p')) \).
assumption that there is an atom in firm i’s distribution at \( p' \). \( \square \)

**Proof of lemma 4:** As before, define \( p_i = \inf(\text{supp}(F_i)) \) and \( \bar{p}_i = \sup(\text{supp}(F_i)), i = 1, 2, 3. \)

Now consider the payoff for firm 3 at \( \bar{p}_3 \):

\[
\pi_3(\bar{p}_3) = (1 - \sigma_1 F_1(\bar{p}_3))(1 - \sigma_2 F_2(\bar{p}_3)) R(\bar{p}_3) + \max \{ (1 - \sigma_1 F_1(\bar{p}_3))\sigma_2 \mu_2(\bar{p}_3), (1 - \sigma_2 F_2(\bar{p}_3))\sigma_1 \mu_1(\bar{p}_3) \} \frac{1}{2} R(\bar{p}_3) - C_3 \quad (4.19)
\]

(The second term is there because we do not place any restrictions on whether firms 1 or 2 can have a mass point at \( \bar{p}_3 \).) In order for this to be an equilibrium action, we require that \( \pi_3(\bar{p}_3) \geq 0 \). Consider the payoffs for firms 1 or 2 at \( \bar{p}_3 \). For at least one of the two firms, \( \mu_i(\bar{p}_3) = 0 \). WLOG, let this be firm 2. Then:

\[
\lim_{\epsilon \to 0} \pi_2(p_3 - \epsilon) = (1 - \sigma_1 \lim_{\epsilon \to 0} F_1(p_3 - \epsilon))(1 - \sigma_3 \lim_{\epsilon \to 0} F_3(p_3 - \epsilon)) \lim_{\epsilon \to 0} R(p_3 - \epsilon) - C_2
\]

\[
= (1 - \sigma_1 F_1(\bar{p}_3)) R(\bar{p}_3) + \sigma_1 \mu_1(\bar{p}_3) - C_2
\]

\[
> (1 - \sigma_1 F_1(\bar{p}_3))(1 - \sigma_2 F_2(\bar{p}_3)) R(\bar{p}_3) + (1 - \sigma_2 F_2(\bar{p}_3))\sigma_1 \mu_1(\bar{p}_3) \frac{1}{2} R(\bar{p}_3) - C_3
\]

\[
= \pi_3(\bar{p}_3) \geq 0
\]

All this gives us that \( \pi_2(\bar{p}_3) > 0 \), which implies that we must have \( \sigma_2 = 1 \). A similar argument gives us that \( \lim_{\epsilon \to 0} \pi_1(p_3 - \epsilon) = (1 - \sigma_2 F_2(\bar{p}_3)) R(\bar{p}_3) + \sigma_2 \mu_2 R(\bar{p}_3) - C_1 > \pi_3(\bar{p}_3) \geq 0 \), so that \( \pi_1(\bar{p}_3) > 0 \), and \( \sigma_1 = 1 \). \( \square \)

**Proof of lemma 5:** Write payoffs for firm 3 as its price approaches \( \bar{p}_3 \):

\[
\lim_{\epsilon \to 0} \pi_3(\bar{p}_3 - \epsilon) = (1 - F_1(\bar{p}_3))(1 - F_2(\bar{p}_3)) R(\bar{p}_3) + \max \{ (1 - F_1(\bar{p}_3))\mu_2(\bar{p}_3), (1 - F_2(\bar{p}_3))\mu_1(\bar{p}_3) \} R(\bar{p}_3) - C_3 \quad (4.20)
\]

where we drop the references to \( \sigma_1 \) and \( \sigma_2 \) by applying lemma 4. In order for \( \pi_3(\bar{p}_3) \geq 0 \), we require either that \( F_1(\bar{p}_3) < 1 \) and \( F_2(\bar{p}_3) < 1 \), or that at least one of \( (1 - F_1(\bar{p}_3))\mu_2(\bar{p}_3) \) or \( (1 - F_2(\bar{p}_3))\mu_1(\bar{p}_3) \) be greater than zero. This implies that \( \bar{p}_1 \geq \bar{p}_3 \) and \( \bar{p}_2 \geq \bar{p}_3 \). \( \square \)

**Proof of lemma 6:** Suppose not, and that \( \bar{p}_1 > \bar{p}_2 \). Consider the payoffs for firm 1 at \( \bar{p}_1 \):

\[
\pi_1(\bar{p}_1) = (1 - \sigma_2 F_2(\bar{p}_1))(1 - \sigma_3 F_3(\bar{p}_1)) R(\bar{p}_1) - C_1 \quad (4.21)
\]
By lemma 4, $\sigma_2 = 1$, and, by assumption, $F_2(\bar{p}_1) = 1$, since $\bar{p}_1 > \bar{p}_2$. This implies that $\pi_1(\bar{p}_1) = -C_1 < 0$, which leads to a contradiction that this is an equilibrium. A similar argument shows that we cannot have $\bar{p}_2 > \bar{p}_1$. Therefore, it must be that $\bar{p}_1 = \bar{p}_2 = \bar{p}$. □

References


Dell’Ariccia, G., Friedman, E., and Marquez, R. (1996). Adverse selection as a barrier to entry in the banking industry. mimeo, MIT.


