Predicting Minimum Savings in Thai Villages

by

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Abstract: Amador Werning Angeletos (2006) characterize the conditions under which optimal saving/consumption decision are determined by a minimum savings policy. I test this model empirically against data from the Townsend Thai Monthly Survey. Each household’s income distribution, measure of risk aversion, and hyperbolic discount rate are estimated and then inputted into the model. In the overall sample, the minimum saving value as predicted by the model does register a statistically significant relationship with the actual amount saved by the household. This is expected, since minimum saving policy is not optimal for all households. Limiting the sample to the appropriate subgroup produces a strong positive correlation.
0. Introduction

At the Macroeconomics level, growth economists attribute the lack of convergence on poor country's low savings rate. At the Microeconomics level, development economists are puzzled by poor's inability to escape the poverty trap: over-consuming what little they have, despite the fact that investing will yield a high rate of return. Even in the developed world, overconsumption contributed towards the recent financial crisis, despite savings being promoted through social security and retirement saving tax benefit.

My undergraduate thesis was heavily influenced by the work of Demirguc-Kunt, Levine, and Min (1998), with the idea that banking penetration is the key to economic growth. My early years in graduate school was accompanied by the microfinance hype, with success stories on how a small loan is able to finance a business profitable enough to repay the high interest rate, and enable the borrower to escape poverty. My collaboration with the Bank of Thailand confirms the view that policy makers view financial access as a benchmark of success. But if financial access is so important, why is there still undersaving in the United States, where opening a savings account online is as easy as a mouse click.

Financial access in itself might not be very useful, if the wrong financial products are being offered. In a related project investigating government subsidized microfinance loans. I find that households take out loan for consumption, instead of investing in a high return project. With the principal consumed, the household is then stuck indefinitely repaying interest. When interest rate is low, it might be in the household's interest to evergreen the loan: but I observe households that are stuck in the viscous debt cycle, paying the majority of their income towards lining the loanshark's pocket. For anecdotal evidence, look no further than the 30% APR credit card in your pocket. While the reader might simply use credit card for convenience, never having to pay a dollar of fee; the credit card companies must surely be making money off someone.

To consume alot is not overconsumption. The agent can rationally consume his entire wealth if he does not care about the future. But most people do care about the future. Conventional discount rate cannot explain a 30% APR loan. As such, the agent must not be rational. Ex ante, the agent knows that it is optimal to spread out consumption over time. But when the opportunity to spend arises, future consumption becomes unimportant compared to present consumption, and he cannot overcome his desire to splurge. This time-inconsitency is due to hyperbolic discounting, first formalized in Ainslie (1975). In this context, financial access which results in greater choice might be detrimental. Loans can exacerbate poverty through consumption. The standard savings account might be inferior to the pledge savings account where deposits cannot be withdrawn at will.

Amador Werning Angeletos (2006) shows that when an agent suffers from hyperbolic discounting, it is optimal. under certain conditions, for him/her to commit to a minimum saving rule. Their model builds upon quasi-hyperbolic discounting of Laibson (1997), incorporated into the self-control framework of Gul and Pesendorfer (2001). Put simply, hyperbolic discounting implies that agent will choose the ex-ante inferior choice to overconsume. As such, it is optimal to commit to minimum saving, even though this results in less flexibility. Practically, this is done by pledging to save a certain amount with the bank, with the commitment made credible through bank fines. The policy implication is that for certain groups of people, pledge savings account is the optimal financial product to combat overconsumption.

This paper attempt to test the model of Amador Werning Angeletos (2006) against data from the Townsend Thai Monthly Survey. The model requires an input of three microeconomic variables: income distribution, coefficient of absolute risk aversion \( \alpha \) and measure of hyperbolic discounting \( \beta \). From these inputs, the model can predict i) whether the minimum saving rule is optimal and ii) what is the optimal minimum saving amount. The model's output provide a testable implication, as it can be compared against the household's actual minimum saving amount as observed in the data.

To my knowledge, this paper is the first to empirically test the model of Amador Werning Angeletos (2006). This is not to say that there is no interest in the model. However most datasets do not measure all three required input variables. Economic panel dataset with income time series do not typically measure hyperbolic discounting. On the other hand, psychology dataset are often cross sectional. I am fortunate enough to have access to the Townsend Thai Monthly Data, which is a monthly panel over a decade old. with a supplement survey measuring household's physchological attributes.

The rest of the paper will proceed as follow. Section 1 introduces the Townsend Thai Monthly Data. From this data, I estimate the actual amount household commit to minimum save in Section 2. Section 3 introduces the beta-delta model, which provides the theoretical backbone. Section 4 shows that minimum savings can
be predicted by three variables: coefficient of absolute risk aversion $\alpha$, measure of hyperbolic discounting $\beta$, and income distribution income distribution. These three components are estimated in Sections 5, 6, 7 respectively. Section 8 combines the three variables into predicted minimum saving. Section 9 performs regression analysis which compares actual to predicted minimum saving. Section 10 concludes.

1. Townsend Thai Data

The Townsend Thai project covers a wide range of surveys conducted over the past two decades in Thailand. The project is described in detail on its website. Part of the project involves household surveys. The impressive feature of these household surveys is that they continue, until this date, to survey the same households as it did a decade ago. Thus, they are among the longest socioeconomic panel data. This paper uses the monthly version of the household survey from the project, commonly known as the Townsend Thai Monthly Household Survey.

The Townsend Thai project was initially designed as a cross sectional survey in 1997. Two of Thailand’s regions were chosen: the relatively rich central region, and the relatively poor northeastern region; and within each region, two provinces. These were chosen deliberately (instead of randomized) to capture the variation in wealth level across the country.

A stratified random sample of households was chosen within each province. Each province is first divided into forested and non-forested area using GIS data. From both strata, a total of 12 sub-districts were selected per province: and 16 villages were selected from each sub-district. This totals to 192 villages. Initially 15 households were surveyed in each village.

The 1997 economic crisis provided a motivation for the survey to be continued annually. The unexpected crisis put the survey in a unique position to collect panel data, with the first year serving as the pre-crisis baseline. The project also realized the need for increased detail, at a higher frequency. As such, in each province, one of the twelve sub-districts was chosen into the monthly version, while others remained in the existing annual format.

In each of the four subdistrict (one per province) of the Townsend Thai Monthly Household Survey, four of the original twelve villages were selected. This reduction in village number is compensated by the tripling of household per village from 15 to 45: with the goal of capturing networks within village. The increase in frequency from annual to monthly obviously reduces measurement error associated with recall. However, the forte of the monthly version lies in its great detail: thousands of variables across twenty modules are collected monthly, from the same households.

Recall that the goal is to compare two variables estimated from the data: i) actual minimum saving, versus ii) predicted minimum saving. The Townsend Thai Survey measures exactly which types of financial product each household uses in each time period, and from this I can estimate the actual amount of minimum saving that household commit/pledge to save in each time period. From the Townsend Thai Monthly survey, I can also estimate the three components used to predict minimum savings: income distribution, coefficient of absolute risk aversion $\alpha$, and measure of hyperbolic discounting $\beta$.

Income is the product of Initial Asset $w_t$ and Return on Asset (ROA) $R^p_t$. Coefficient of Absolute Risk Aversion $\alpha$ is calculated from ROA and consumption data $c_t$. The variables $w_t$, $R^p_t$, and $c_t$ comes directly from the Townsend Thai Monthly Survey. However, they are not readily available, but require aggregating answers from over a hundred questions across 10 survey modules. Fortunately, this has already been completed in the accounting work of Srisivat et al. (2011).

Hyperbolic Discount Factor $\beta$ is estimated by using data from a supplemental survey conducted in

\footnote{http://ciec.uchicago.edu/data/}
January 2010 by Christopher Woodruff. This survey asks questions from the psychology literature, including risk aversion, discounting, cognitive abilities, and personality. The survey is conducted at the individual level, drawing from almost all households in the Townsend Thai Monthly survey. The variable of interest, $\beta$, is backed out by comparing reward choice in the present to various points in the future.

Although the survey started out with 720 households, there is attrition, as well as temporary absences. I limit analysis to 484 households that were present throughout the 13 years (1999-2011) which I have data. This can be matched to 580 individuals from 482 households who answered the relevant questions from the Woodruff survey. I exclude household with unreasonable $\alpha < 0$ values (11 households), and $\beta > 1$ values (45 households), leaving 426 households with predicted minimum saving values. The following table summarizes the data sources. The link between the variables will be elaborated upon in Sections 5, 6, 7.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Level</th>
<th>Source</th>
<th># Household</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROA ($R_t$)</td>
<td>Household-Month</td>
<td>Srisivil</td>
<td>484</td>
</tr>
<tr>
<td>Initial Asset ($w_t$)</td>
<td>Household</td>
<td>Srisivil</td>
<td>484</td>
</tr>
<tr>
<td>Consumption ($c_t$)</td>
<td>Household-Month</td>
<td>Srisivil</td>
<td>484</td>
</tr>
<tr>
<td>CARA ($\alpha$)</td>
<td>Household</td>
<td>ROA, Consumption</td>
<td>473</td>
</tr>
<tr>
<td>Hyperbolic Discount ($\beta$)</td>
<td>Individual</td>
<td>Woodruff</td>
<td>437 (580 Individuals)</td>
</tr>
<tr>
<td>Predicted Minimum Saving</td>
<td>Household-Month</td>
<td>$\beta, \alpha, ROA, Asset$</td>
<td>426</td>
</tr>
<tr>
<td>Actual Minimum Saving</td>
<td>Household-Month</td>
<td>Saving Module 14, Insurance Module 21</td>
<td>484</td>
</tr>
</tbody>
</table>

2. Estimating Actual Minimum Saving

As mentioned in the data section, I know exactly which types of financial product each household uses in at each point in time. This is possible because the list of product is enumerated at the baseline, and for each subsequent month, households are asked to identify which existing product they stop using, as well as list any new products. For each financial product, a form is filled to collect the product’s attribute. The survey has a separate form for each type of financial product. Of relevance here are the Savings form and the Insurance form.

There are two main financial products that commits household to save in the future. The first is the typical pledge saving product, whereby household pledge to save a certain amount every time period (typically a month). The second is life insurance products, whereby household commit to pay a fee in each future time period. The pledge saving product can be easily distinguish from the typical saving product. The Savings form has a variable which specifically ask whether product is pledged saving or not. If the product is pledge saving, the required saving amount per time period (typically a month) is recorded.

Pledge saving account within the formal sector (i.e. banks) is the same as what is typically offered in a U.S. bank. There is a fix term, typically several years: and during each month during the term, the household must save an agreed upon amount. Missing payment results in monetary loss, either a fine or reduction in interest: and thus serve as a commitment device.

On the other hand, pledge saving account with the informal sector (e.g. village fund) usually have an open-ended term. Anecdotal evidence from field visits suggests that withdrawal can only occur upon an adverse event, such as crop failure: or when household moves away from the village. Failure to deposit without a good reason can lead to expulsion from village fund membership. This is a strong commitment device, because membership is also required to access cheap village fund credit.

Indeed, maintaining a pledge saving account is a prerequisite for village fund membership. This however poses a problem, because even for household that does not find it optimal to save a minimum amount, data will show that they do. To address this issue, these mandatory account, which can be identified by having the least possible available commitment of 500 baht/month, will not count towards the measure of actual minimum saving.

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2Amount from these accounts will only count if household has at least one other non-mandatory account, because this shows that the mandatory account constraint is not binding, and household prefers to commit to a larger amount.
There are a variety of life insurance products, each exhibiting a different degree of saving. In one extreme are pure insurance products whereby households only receive a return if death occurs during the insured period. The insurance payment is entirely forgone otherwise. On the other extreme are policies where the face value of the cumulative payments is recouped upon policy expiration, even without death. The payments into these policies could be interpreted as deposits, and the interest from these deposits used to pay for the insurance premium. The commitment mechanism here is especially strong, because the fees paid are legally not deposits. Many households tell us that multiple failure to pay fees will lead to policy cancellation with zero payout.

Let \( Y_D \) be the payout in the case of death, and \( Y_{ND} \) be the payout otherwise. In this context, \( Y_{ND} \) could be interpreted as the amount of saving into the insurance policy. \( Y_{ND} = 0 \) correspond to pure insurance, while \( Y_{ND} = Y_D \) is pure saving. Unfortunately, the survey does not record \( Y_{ND} \). As such, I will have to measure it indirectly. Let \( P_D \) be yearly probability of death, and \( C \) be the yearly insurance fee. Assuming risk neutrality and competitive market\(^3\), I get the following zero profit condition:

\[
Y_{ND} = \frac{C - Y_D P_D}{1 - P_D}
\]

With \( P_D \) and \( C \) measured per year, then \( Y_{ND} \) should be interpreted as yearly amount of minimum saving. Note that households don't actually receive \( Y_{ND} \) at year end; instead it goes towards the total payment when policy expires. The variables \( C \) and \( Y_D \) are measured in the survey. For \( P_D \), I use Thailand's death rate as estimated by the World Health Organization\(^1\). The rate is age and gender specific which I match\(^5\) to the particular policy holder within the household.

For both insurance and saving, I sum across all policies. For insurance, I take the average over the 13 years period. For savings, I only take average after village fund was introduced in 2001. There was supply side limitation before 2001: and household would have saved at post 2001 level during pre 2001 years if the products were available. 199 households used pledge insurance, while 232 households used pledge savings. Correlation between the two usage indicator variables is surprisingly low at 0.0724. The histogram of the per year average insurance fee (\( Y_{ND} \)) and minimum saving are graphed below. Zero value observations are excluded. The sum of pledge insurance and pledge saving gives the estimated value for actual minimum saving, which will be compared to predicted minimum saving.

3. The Beta-Delta Hyperbolic Discounting Model

In this section, I introduce the notation for the hyperbolic discounting model, which will be applied when applicable to the remaining sections of the paper. To understand time-inconsistency generated by hyperbolic...
discounting. Let \( f(t, t_0) \) be the discount rate faced by consumption at time \( t \) when evaluated at \( t_0 \). Hyperbolic discounting requires that discount rate only depend on time from the present \( f(t, t_0) = f(t - t_0) \). and for discount factor between \( t \) and \( t + 1 \) to decrease as the present \( t_0 \) moves closer towards \( t \).

\[
\frac{f(t+1, t_0)}{f(t, t_0)} = \frac{f(t+1 - t_0)}{f(t - t_0)} < \frac{f(t-t_0)}{f(t-t_0-1)} = \frac{f(t+1, t_0+1)}{f(t, t_0+1)}
\]

(1)

See that this violate time-consistency of standard discounting which requires \( \frac{f(t+1, t_0)}{f(t, t_0)} = \frac{f(t+1, t_0+1)}{f(t, t_0+1)} \). One should also not confuse a model which allows variable discount rate over time with hyperbolic discounting. The particular model I follow the beta-delta model introduced by Laibson (1997).

\[
f(t,t_0) = \begin{cases} 
1 & t = t_0, \\
\beta \delta & t = t_0 + 1, \\
\beta \delta^t & t > t_0 + 1.
\end{cases}
\]

This model is actually considered quasi-hyperbolic because the inequality of (1) is no longer strict except at except at \( t = t_0 + 1 \). The main reason behind using this model is that it is tractable and its parameters are comparatively easy to estimate. Here, \( \delta \) is the standard discount factor, with \( \beta \) reserved for hyperbolic discounting. The reader of the standard economic literature should notice the notational faux pas. To study hyperbolic discounting, I incorporate it into standard intertemporal consumption/saving model with uncertainty given by the following sequence problem.

\[
\max_{c_1(s'),a_1^*(s')} E_0 [\sum_{t=0}^{T} f(t,0) u(c_t(s'))] \\
\text{s.t. } c_1(s') + \sum_{k=0}^{K} a_k^*(s') \leq u_1(s') = \sum_{k=0}^{K} R_k^b(s') a_{k-1}^*(s'-1)
\]

(2)

Throughout, contemporaneous utility \( u(c_t(s')) \) will be a constant absolute risk aversion (CARA) utility function. The only fancy feature here can be the allocation of multiple \( (K) \) assets, which is important for estimating the CARA coefficient. Uncertainty is introduced by the fact that rate of return \( R_k^b(s') \) depends on the history \( s' \). Adding up the income from all asset gives total wealth \( w_t(s') \), which can either be consumed \( c_t(s') \), or reinvested \( a_t^*(s') \). \( T \) could be finite or infinite. and \( t \) could vary from a month to a year in the future.

4. Predicted Minimum Saving

The model below closely mimics that of Amador, Werning, and Angeletos (2006), henceforth AWA. Here, I will make the following simplifications. \( T \) is finite and limited to 2 (so that \( t = 0, 1, 2 \)). Household only have access to one asset \( a_1 \) (which can be thought off as a unit of the portfolio). Furthermore assume that at \( t = 1 \) household can only save \( a_1 \) with zero interest rate \( (R_2 = 1) \). Finally, for simplicity assume that \( c_o \) has already been decided (which will be normalized to zero), and \( \delta = 1 \). This simplifies the problem to

\[
\max_{a_1(s'),c_1(s')} E_0 [V(s^1)] \\
V(s^1) = -\beta (e^{-\alpha (R_1(s') a_1(s') - a_1(s'))} + e^{-\alpha c_1(s^1)}) \\
RC : c_1(s^1) \leq a_1(s^1)
\]

(3)

Note that \( t = 1 \) savings \( a_1(s^1) \) is purposely written as a choice variable instead of consumption \( c_1(s^1) \). This is because because predicted saving is the variable of interest. There is a distinction between \( a_1(s^1) \) and \( c_1(s^1) \) because the resource constraint does not necessarily bind. The obvious solution here is \( c_1^*(s^1) = c_2^*(s^1) = B \delta^a_1 \), which is optimal from the perspective of self-0 (the household at \( t = 0 \)). Of course, the issue with the hyperbolic discounter is that when \( t = 1 \) actually arrives. \( c_1 \) is treated as the present. and thus no longer discounted by \( \beta \). The household instead solves
\[
\begin{align*}
\max &_{a_1(s^1), c_2(s^1)} - (e^{-\alpha(R_1(s^1)a_0-a_1(s^1))} + \beta e^{-\alpha c_2(s^1)}) \\
RC : & c_2(s^1) \leq a_1(s^1)
\end{align*}
\]

The solution is \( c_1^*(s^1) = \frac{R_1 a_0}{2} - \frac{\ln(3)}{2a} > \frac{R_1 a_0}{2} + \frac{\ln(3)}{2a} = c_2^*(s^1) = a_1^*(s^1) \), which is inferior from the point of view of self-0. To alleviate this problem, assume that at \( t = 0 \) household has a mechanism that allows it to commit at \( t = 0 \) to both \( a_1(s^1) \) and \( c_2(s^1) \), contingent on return \( R_1(s^1) \). Whether this commitment can be implemented in practice will be addressed later on. One should view this as a game played between two selves, where self-0 is the principle, and self-1 is the agent with hidden type \( R_1(s^1) \). Note that since \( R_1 \) is sufficient statistic for consumption, \( s^1 \) will be omitted for brevity. By revelation principle, the problem becomes

\[
\begin{align*}
\max &_{a_1(s^1), c_2(s^1)} E_0[V(R_1)] \\
RC : & c_2(R_1) \leq a_1(R_1) \\
IC : & U(R_1, R_1) \geq U(R_1, R_1') \\
U(X, Y) = & -(e^{-\alpha(Xa_0-a_1(Y))} + \beta e^{-\alpha c_2(Y)}) \\
(X\text{-actual type}: Y\text{-reported type})
\end{align*}
\]

It is useful to rewrite self-0 utility \( V(R_1) \) in term of self-1 utility \( U(R_1, R_1) \). The approach here is to simplify the problem by first characterizing \( U(R_1, R_1) \) with the IC:

\[
\begin{align*}
V(R_1) & = U(R_1, R_1) + (1-\beta) e^{-\alpha(R_1 a_0-a_1(R_1))} \\
-\beta(e^{-\alpha(R_1(s^1)a_0-a_1(s^1))}) & = -(e^{-\alpha(Xa_0-a_1(Y))} + (1-\beta) e^{-\alpha(R_1 a_0-a_1(R_1))})
\end{align*}
\]

Following Mirlees' (1971), (5) can be replaced with LIC and Monotone.

\[
\begin{align*}
U_2(R_1, R_1) = & -\alpha a_1(R_1)e^{-\alpha(R_1 a_0-a_1(R_1))} + \alpha c_2(R_1)e^{-\alpha c_2(R_1)} = 0 \\
\alpha_1'(R_1) & \geq 0
\end{align*}
\]

\( \text{LIC} \)

\( \text{Monotone} \)

The LIC implies that the total derivative of \( U(R_1, R_1) \) is just the derivative with respect to the first argument (actual type), due to the application of the envelope theorem. We can thus characterize \( U(R_1, R_1) \) as (6) by integrating its first derivative

\[
\begin{align*}
\frac{dU(R_1, R_1)}{dR_1} & = U_1(R_1, R_1) = \alpha a_0 e^{-\alpha(R_1 a_0-a_1(R_1))} \\
U(R_1, R_1) & = U(R_1^{\text{max}}, R_1^{\text{max}}) - \int_{R_1} \alpha a_0 e^{-\alpha(R_1 a_0-a_1(R_1))} dx 
\end{align*}
\]

The expression then goes into \( V(R_1) \) as planned, which simplifies the problem. To get to (7), I use
integration by parts\(^6\) and factor out \(e^{-\alpha R_1 a_o}\).

\[
E_0[V(R_1)] \quad = \quad E_0 \left[ U(R_1, R_1) + (1 - \beta) e^{-\alpha(R_1 a_o - a_1(R_1))} \right] \\
= \quad E_0 \left[ U(R_1^{\text{max}}, R_1^{\text{max}}) + \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} -a_o \alpha e^{-\alpha(R_1 a_o - a_1(R_1))} dx + (1 - \beta) e^{-\alpha(R_1 a_o - a_1(R_1))} \right] \\
= \quad U(R_1^{\text{max}}, R_1^{\text{max}}) - \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} a_o \alpha e^{-\alpha(R_1 a_o - a_1(R_1))} dx f(R_1) dR_1 + \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} (1 - \beta) e^{-\alpha(R_1 a_o - a_1(R_1))} f(R_1) dR_1 \\
= \quad U(R_1^{\text{max}}, R_1^{\text{max}}) + \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} G(R_1) \left( e^{\alpha_1(R_1)} \right) dR_1 \quad \text{with} \quad G(R_1) = (f(R_1)(1 - \beta) - a_o \alpha F(R_1)) \left( e^{-\alpha_2(R_1)} \right)(7)
\]

The problem then becomes maximizing (7) with respect to \(a_1(R_1)\) subjected to the resource constraint \(RC\). It should be noted at this point that if we let \(\theta = e^{-\alpha_2(R_1)}\), then the problem above is isomorphic\(^7\) to the model in AWA where \(\theta\) is interpreted as a taste shock. As such, for the rest of this section, I will draw upon the results from AWA, without replicating the rigorous proofs, as the focus of this paper will be to empirically test AWA’s theory. To gain some intuition regarding the solution, look at the factor \(G(R_1)\).

Roughly speaking, the term \(f(R_1)(1 - \beta)\) represents the benefit of saving. This is because \(1 - \beta\) represents the degree of under-saving, scaled by the density of type \(R_1\). On the other hand, \(a_o \alpha F(R_1)\) represents the cost of saving. The term \(a_o \alpha\) is the scale between utility and marginal utility. \(F(R_1)\) represents the fact that increasing saving for type \(R_1\) entail increasing saving for all types types below \(R_1\), in order for type \(R_1\) to not have incentive to underreport income. See that at \(R_1^{\text{min}}\), the cost is zero because there is no type below \(R_1^{\text{min}}\). This imply that the second-best solution (subjected to LIC) would have saving high at \(R_1^{\text{min}}\) then decreasing as the \(a_o \alpha F(R_1)\) term starts to dominate.

The solution as described is however not feasible because a decreasing \(a_1(R_1)\) violates the monotonicity. As such, the third-best solution (subjected to both LIC and monotonicity) features bunching at the bottom of \(R_1\). The critical point \(R_1^*\) is given by (8). To justify the choice of \(R_1^*\), consider the contribution towards

\[
E_0 \left[ - \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} \alpha e^{-\alpha(c_1(R_1)))} dR_1 \right] \\
= \quad \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} -\alpha e^{-\alpha(c_1(x)))} dx f(R_1) dR_1 \\
= \quad \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} m dR_1 \\
= \quad \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} \alpha e^{-\alpha(c_1(R_1)))} dR_1 \\
= \quad [m]_{R_1^{\text{min}}}^{R_1^{\text{max}}} - \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} \alpha e^{-\alpha(c_1(R_1)))} F(R_1) dR_1 \\
= \quad - \int_{R_1^{\text{min}}}^{R_1^{\text{max}}} \alpha e^{-\alpha(c_1(R_1)))} F(R_1) dR_1
\]

\(^6\)The middle term can be simplified through integration by parts

\(^7\)The solution is not exactly the same. In the original paper, as \(\theta\) increases, \(c_1\) increases while \(c_2\) decreases. Increasing \(\theta\) is equivalent to decreasing \(R_1\). This equivalency implies that \(c_2\) should be increasing with \(R_1\), which is exactly the case here. The discrepancy lies in \(c_1\), which instead maps to \(c_1 - R_1\) here. However, in the end, it is \(c_2\), the policy variable that matters.
self-0's expected utility from types \( R_1 \leq R_i \). Integrate by parts\(^8\) to get (9)

\[
R^p_i = \max \left\{ R_1 \mid \int_{R_i}^{R_i^p} G(R_1')dR_1' \geq 0 \right\} 
\]

\[
\int_{R_i^{\min}}^{R_i^p} G(R_1) \left( e^{\alpha a_i'(R_1)} \right) dR_1 = \left( e^{\alpha a_i'(R_i^p)} \right) \int_{R_i^{\min}}^{R_i^p} G(R_1')dR_1' - \int_{R_i^{\min}}^{R_i^p} \left( \int_{R_i^{\min}}^{R_i^p} G(R_1')dR_1' \right) dR_1' (\alpha a_i'(R_1)e^{\alpha a_i(R_1)}) dR_1 
\]

See that the way \( R^p_i \) is defined exactly identifies the region of \( R_1 \) where \( a'(R_1) < 0 \) is second best. Of course, monotonicity requires \( a'(R_1) \geq 0 \), thus resulting in bunching \( a'(R_1) = 0 \). Consider the following solution:

\[
c^p_i(R_1) = a^p_i(R_1) = \begin{cases} a_i'(R_1^p) & \text{for } R_1 < R_i^p \\ a_i'(R_1) & \text{for } R_1 \geq R_i^p \end{cases} 
\]

where \( a^p_i(R_1) = \frac{R_i}{2a} + \frac{\ln(2)}{2a} \) is self-1's preferred solution.

The interesting feature of (10) is that it is possible to implement in practice. In the setup, I assumed that self-0 can commit to \( a^p_i(R_1) \). However, it is no easy task to enforce this commitment. In particular, an outside party (i.e. bank) must be able to punish the household if it does not follow through: and all this done without observing income! In (10), this is feasible because for \( R_1 \geq R_i^p \), \( a^p_i(R_1) \) is exactly self-1 preferred solution \( a_i'(R_1) \), so no enforcement is needed. For \( R_1 < R_i^p \), self-1 would prefer to save a lower amount. However \( a^p_i(R_1) \) is constant in that region, so the bank can determine whether household followed its commitment without observing the household's income. If the household does not save (at least) \( a_i'(R_1^p) \), the bank can punish with a fee. Notice, that the arrangement I just described is exactly the pledge saving product.

AWA's propositions 3 and 4 establish that (10) is the optimal solution to the mechanism design problem (5) if and only if \( \alpha a_o F(R_1) - (1 - \beta) f(R_1) \) is nondecreasing in the range \( R_1 \geq R_i^p \). One can easily check that the nonincreasing condition here is exactly the same as AWA's second assumption\(^9\). The condition can be rewritten in terms of the first derivative as (11). The intuition behind this condition is that it ensures that the monotonicity condition is not further violated (outside \( R_1 < R_i^p \)), so that there is no additional bunching, apart from that at the bottom of \( R_1 \).

\[
\frac{f'(R_1)}{f(R_1)} \leq \frac{\alpha a_o}{(1 - \beta)} \text{ for all } R_1 \geq R_i^p
\]

To recap, the three variables coefficient of risk aversion \( \alpha \), measure of hyperbolic discounting \( \beta \), and income distribution income distribution (\( R_1 \) and \( a_o \)) determine whether the minimum saving policy is optimal through (11). When minimum saving policy is optimal, these three variables also predicts minimum saving per (10). The following three sections will estimate each of these three variables.

### 5. Estimating CARA coefficient

\[
m' = G(R_1) \\
n = (-e^{\alpha a_i(R_1)})
\]

\[
m = \int_{R_i}^{R_i^p} G(R_1')dR_1' \\
n' = (-\alpha a_i'(R_1)e^{\alpha a_i(R_1)})
\]

\[
(1 - \beta) \frac{dh(\theta)}{d\theta} + H(\theta) \text{ non-decreasing} \\
\frac{d[(1 - \beta) \frac{dh(\theta)}{d\theta} + H(\theta)]}{d\theta} \geq 0 \\
\frac{d[(1 - \beta) \frac{dh(\theta)}{d\theta} + H(\theta)]}{d\theta} \geq 0 \\
\frac{d[(1 - \beta) \frac{dh(\theta)}{d\theta} + H(\theta)]}{d\theta} \geq 0 \\
\alpha a_o F(R_1) - (1 - \beta) f(R_1) \text{ non-decreasing}
\]

9
To estimate the CARA coefficient, I follow the portfolio choice method of Mehra and Prescott (1985). My strategy mirrors that of Chiappori, Samphantarak, Shulhofer-Wohl (2013), which uses the same dataset. There are two main differences: i) they use CRRA and ii) they do not feature hyperbolic discounting. Note that while I allow for hyperbolic discounting, I do not allow here the commitment mechanism to alleviate it. Again, start with the general sequence problem of the beta-delta model (2), which is reproduced below:

\[
\max_{c_t(s'), a^*_t(s') \in \mathbb{A}} \mathbb{E}_t \left[ \sum_{t=0}^{T} f(t, 0) u(c_t(s')) \right]
\]

subject to

\[
c_t(s') + \sum_{k=0}^{K} a_k^*(s') \leq w_t(s') = \sum_{k=0}^{K} R_k^t(s') a_{k-1}^*(s'^{-1})
\]

In this section, \(T\) is infinite with \(t\) corresponds to a period of one month. This is so that the monthly nature of the data can be fully exploit. Correspondingly, utility \(u(c_t(s'))\) must allow for seasonal shock to consumption, so that a spike in consumption due to an event such as the Thai new year in April will not be interpreted as the household being risk tolerant. This is done through the variable \(\xi_m(t)\) with \(m \in \{Jan, Feb, ..., Dec\}\).

\[
u(c_t(s')) = e^{-\alpha(c_t(s') - \xi_m(t))}
\]

Given this configuration, I can translate sequence problem (2) into the corresponding functional equation. As expected, hyperbolic discounting leads to time inconsistency: the consumption path that household expects at \(t = 0\) does not correspond to the consumption path that actually materializes. It is illustrative to look at the functional equation governing these two consumption paths. To distinguish the two FEs, I call the former the naive FE \(V(w_t|s')\) defined in (12), and the latter the actual FE \(W(w_t|s')\) defined in (14).

\[
V(w_t|s') = \max_{a_t^*} e^{-\alpha(w_t - \sum_{k=0}^{K} a^*_t - \xi_m(t))} + \delta \mathbb{E}_t \left[ V \left( \sum_{k=0}^{K} R_k^t(s'^{+1}) a_k^*|s'^{+1} \right) \right]
\]

Let \(\{c^*_t(s')\}_{t=0}^{\infty}\) be the solution to the problem (12), and is thus characterized by the following Euler equation for each asset \(k\) that the household invests in at time \(t\).

\[
e^{\alpha \xi_m(t)} e^{-\alpha c_t^*} = \begin{cases} 
\beta \delta e^{\alpha \xi_m(t+1)} E_t[R_k^{t+1} e^{-\alpha c_{t+1}^*}] & \text{for } t = 0 \\
\delta e^{\alpha \xi_m(t+1)} E_t[R_k^{t+1} e^{-\alpha c_{t+1}^*}] & \text{for } t > 0
\end{cases}
\]

The naive FE problem reflects how the naive household incorrectly values consumptions between \(t\) and \(t + 1\). When time \(t\) actually arrives, the household will treat time \(t\) as the present and no longer discount \(c_t\) with hyperbolic discount factor \(\beta\) as it does for \(t + 1\), which is still in the future. As such the actual FE household faces is (14) with corresponding Euler equation (15). Let \(\{c^*_t(s')\}_{t=0}^{\infty}\) be the solution, and thus the actual realized consumption path. It is easy to see that \(c^*_t(s')\) features overconsumption earlier on, when compared to \(\{c^*_t(s')\}_{t=0}^{\infty}\), which is first best from the perspective of agent at \(t = 0\). From (15), rearrange and apply the unconditional expectation\(^{10}\) to get (16). For brevity I also substitute in the seasonally adjusted consumption \(x_t^* = c_t^* - \xi_m(t)\).

\[
W(w_t|s') = \max_{a_t^*} e^{-\alpha(w_t - \sum_{k=0}^{K} a^*_t - \xi_m(t))} + \delta \mathbb{E}_t \left[ W \left( \sum_{k=0}^{K} R_k^t(s'^{+1}) a_k^*|s'^{+1} \right) \right] 
\]

\[
e^{\alpha \xi_m(t)} e^{-\alpha c_t^*} = \beta \delta e^{\alpha \xi_m(t+1)} E_t[R_k^{t+1} e^{-\alpha c_{t+1}^*}] \text{ for } t \geq 0
\]

\[
1 = \beta \delta E_t[e^{\alpha(c_t^* - \xi_m(t)) + \alpha x_t^*}] E_t[R_k^{t+1} e^{-\alpha c_{t+1}^*}]
\]

\[
1 = \beta \delta E_t[e^{\alpha(c_t^* - \xi_m(t))}] E_t[R_k^{t+1} e^{-\alpha c_{t+1}^*}]
\]

\[
1 = \beta \delta E_t[e^{\alpha(c_t^* - \xi_m(t))}] + \text{Cov}[e^{\alpha(c_t^* - \xi_m(t))}, R_k^{t+1}]
\]
\[ 1 = 36E[e^{\alpha(x_t^* - x_{t-1}^*)}]E[R^k_{t+1}] + \beta\delta \sqrt{Var[e^{\alpha(x_t^* - x_{t-1}^*)}]}Var[R^k_{t+1}]Corr[e^{\alpha(x_t^* - x_{t-1}^*)}, R^k_{t+1}] \]  

(16)

As documented by Alvarez, Pawasutipaisit, and Townsend (2012), households hold large amount of cash at home, implying that the riskfree saving asset \( k = s \) has zero interest. This implies \( E[R^k_{t+1}] = R^k_{t+1} = 1 \). The Euler equation for this particular asset pins down discounted expected marginal utility in (17).

\[ \frac{1}{E[R^k_{t+1}] - 1} = \frac{36\delta E[e^{\alpha(x_t^* - x_{t-1}^*)}]}{\delta\beta \sqrt{Var[e^{\alpha(x_t^* - x_{t-1}^*)}]} E[e^{\alpha(x_t^* - x_{t-1}^*)}]} \]  

(17)

\[ \frac{1}{\sqrt{Var[R^k_{t+1}]} - \beta\delta \sqrt{Var[e^{\alpha(x_t^* - x_{t-1}^*)}]}} \approx \alpha \sigma \]  

(18)

Now consider the household’s actual portfolio, which can be considered as the aggregate asset \( k = P \) with rate of return \( R^P_{t+1} \). Assume that \( P \) attains the theoretically risk premium bound given by Hansen-Jagannathan (1991), so that \( Corr[e^{\alpha(x_t^* - x_{t-1}^*)}, R^P_{t+1}] = -1 \). Then divide (16) by (17) to and rearrange to obtain (18), which is free of discount variables \( \beta \) and \( \delta \). This is crucial in estimating \( \alpha \), because \( \beta \) and \( \delta \) are also unknowns, and thus this avoids the complication of joint estimation. The LHS is the Sharpe ratio, while the RHS resembles a coefficient of variation. Assuming that \( R^P_{t+1} \) follows an i.i.d. process, the Sharpe ratio can be easily estimated from the monthly rate of return data as calculated in Srivisal’s accounting work. To simplify the RHS, assume that the change in seasonally adjusted consumption follows an i.i.d. normal distribution:

\[ \Delta x_t^* = (x_t^* - x_{t-1}^*) \sim N(\mu_{\Delta x}, \sigma^2_{\Delta x}) \]

It follows that \( e^{\alpha(x_t^* - x_{t-1}^*)} \) will follow an an i.i.d. log normal distribution, which has a nice expression for the coefficient of variation (19). Further simplify by taking a linear approximation around \( \sigma_{\Delta x}^2 = 0 \) to get (20).

\[ \frac{\sqrt{Var[e^{-\alpha \Delta x_{t+1}^*}]} - 1}{E[e^{-\alpha \Delta x_{t+1}^*}]} \approx \alpha \sigma \]  

(19)

\[ \frac{\sqrt{Var[e^{-\alpha \Delta x_{t+1}^*}]} - 1}{E[e^{-\alpha \Delta x_{t+1}^*}]} \approx \alpha \sigma \]  

(20)

If I observe \( \Delta x_t^* \), then \( \sigma_{\Delta x} \) could naturally be estimated with the root mean squared error off the regression on a constant. However, what is actually observed is \( \Delta c_t \), which leads instead to the regression (21).

\[ \Delta c_t = \Delta \xi_{m(t)} + \mu_{\Delta x} + \epsilon_t \]  

(21)

An implicit assumption here is that \( \Delta \xi_{m(t)} \) is not endogenous, which is reasonable if we think of \( \Delta \xi_{m(t)} \) representing some predetermined cultural factor. I can now rearrange (20) to get the estimator for \( \alpha \) (22). I acknowledge that both \( \epsilon_t \) and \( R^k_{t+1} \) is likely measured with error, and thus \( \alpha \) is likely underestimated, in a fashion similar to the standard attenuation bias.

\[ \hat{\alpha} = \frac{\hat{E}[R^P_{t+1}] - 1}{\hat{\sigma}_{\Delta x} \sqrt{Var[R^k_{t+1}]} } \]  

(22)

\[ ^{11} \text{ These factor likely depend on past consumption pattern. It is no coincidence that the Thai new year event described earlier happen at time of rice harvest. However, what is important is that these seasonal adjustment were already predetermined before \( t = 0 \) when household starts making consumption decision.} \]

\[ ^{12} \text{ As CARA is unit dependent, I will be consistently use the consumption unit of 1 USD (33 THB). This is for comparison purposes with studies using U.S. data.} \]
Each component on the RHS can be estimated for each household in the sample, ultimately producing the CARA estimate for each household. The histogram, over households, of the four estimates are graphed below. See that $E[R'_{1}]$ is below one for a few households, translating to risk seeking preference. This is due to finite sample: with each household observed for 156 months (13 years). Households with $\alpha < 0$ will be excluded from further analysis. This leaves a truncated mean of $0.0058$ and median $0.0025$, which is relatively low when compared to a range of $0.124$ to $0.158$ estimated by Gregory, Lamarche and Smith (2002) using yearly United States macro data; but is more comparable a mean of $0.037$ and median of $0.004$ from Cohen and Elav (2007) using yearly Israeli micro data.

6. Estimating Hyperbolic Discount Factor

The hyperbolic discount factor is estimated using data from Christopher Woodruff’s 2010 survey measuring ability, talent and risk. Of relevance are a series of questions in the survey, each which ask for a reward amount household must receive $o$ months from present, so that the household is indifferent to receiving 5000 baht in $p$ months (with $o > p \geq 0$). Note that this is purely a theoretical exercise for the household, and no reward were actually given. Denote the household’s answer by $m_{o,p}$.

The four values measured in the survey are:

- Receiving $m_{3,0}$ in 3 months is equivalent to receiving 5000 baht in the present
- Receiving $m_{1,0}$ in 1 month is equivalent to receiving 5000 baht in the present
- Receiving $m_{13,12}$ in 13 months is equivalent to receiving 5000 baht in 12 months
- Receiving $m_{6,3}$ in 6 months is equivalent to receiving 5000 baht in 3 months

$^{13}$from 103 to 132 when using $10000$/year. The adjustment is therefore to divide by 10,000 and multiply by 12. The detail of adjusting CARA will be addressed later on.

$^{14}$They use yearly data, so I multiplied their estimates of $0.0031$ and $0.000034$ by 12.
The measurement of hyperbolic discount \( \beta \), since the pioneering work of Thaler (1981), requires two assumptions: (i) the reward \( m_{o,p} \) is added directly into the anticipated consumption stream; and (ii) the (expected) marginal utility of consumption is constant over time.

The first assumption is to avoid the complication of smoothing the reward over time. To illustrate this point, suppose that household only has access to the riskless asset. Then \( m_{o,p} \) would only reflect the riskfree interest rate between time \( o \) and \( p \). This assumption is more plausible if the reward is small relative to monthly consumption: as a large amount would prompt household to think about smoothing. 5,000 baht is hardly negligible for some households, being slightly less than the consumption sample mean (across households and months) of 6.500 baht.

Recall the naive FE (12) where at \( t = 0 \) the household naively thinks that it will follow consumption path \( \{c^*_{t}(s^f)\}_{t=0}^\infty \) which will provide lifetime utility:

\[
E_0[- \sum_{t=0}^{\infty} \beta^{t(t>0)} \delta^t e^{-\alpha(s^*_t)}]
\]

Given the first assumption, the reward \( m_{o,p} \) at time \( o \) will increase lifetime utility by the amount given on the LHS of (23). On the other hand, reward 5000 at time \( p \) will increase lifetime utility by amount given on the RHS of (23). Because the household is indifferent between the two rewards, the two utility gains are equalized. Assuming that \( \Delta x^*_t \) follows an i.i.d. distribution, (23) can be rearranged into (24)

\[
E_0[\beta^{t(p>0)} \delta^t (e^{-\alpha(s^*_t+m_{o,p})} - e^{-\alpha(s^*_t)})] = E_0[\beta^{t(p>0)} \delta^t (e^{-\alpha(s^*_t+5000)} - e^{-\alpha(s^*_t)})]
\]

What this implies is that the degree to which reward data \( m_{o,p} \) differs from 5000 will reflect the difference in discounted marginal utility over time. As Noor (2009) points out, reward data simply cannot distinguish whether household require higher reward in the future \( (m_{o,p} > 5000) \) because of a discounting due to either \( \beta \) or \( \delta \); or b) smaller expected marginal utility \( (\alpha E_0[ae^{-\alpha(s^*_t)}] < \alpha E_0[ae^{-\alpha(s^*_t)}]) \). This motivates the second assumption. Constant (expected) marginal utility over time implies \( E_0[ae^{-\alpha(s^*_t)}] = E_0[ae^{-\alpha(s^*_t)}] \), so that the reward data can be attributed solely to discounting.

Unfortunately, the second assumption is outright violated in this framework because \( \{c^*_t(s^f)\}_{t=0}^\infty \) is a solution to the naive Euler equation (13), and thus endogenous. Assuming \( \varphi \) that household’s naive consumption plan involves investing in the riskfree asset \( s \) (with return \( R_{o+1}^s \)). I can simplify the euler equation to (25): then iterate from time \( p \) to time \( o - 1 \) to get (26). See that this expression can be compared to (24) by taking expectation at time 0 and using law of iterated expectation.

\[
e^{-\alpha s^*_t} = \begin{cases} \beta \delta R_{o+1}^s E_t[e^{-\alpha s^*_t+1}] & \text{for } t = 0 \\ \delta R_{t+1}^s E_t[e^{-\alpha s^*_t+1}] & \text{for } t > 0 \end{cases}
\]

\[
E_p[e^{-\alpha(s^*_t)}] \delta^{o-p} \beta^{t(p=0)} = (R_{o,p}^s)^{-1}
\]

This result is hardly surprising: the optimal consumption path must be such that the difference in discounted expected marginal utility reflect the rate of return. For example, if \( R_{o,p}^s = 1 \) (as is the case for actual consumption path), then expected marginal utility should increase over time. It is important that household does not necessarily expect consumption to fall over time; but rather \( \text{Var}_p[x^*_o] \) is increasing.

---

15 The amount \( m_{o,p} \) at time \( o \) would thus be chosen such that it increases the lifetime budget constraint by the same amount as 5000 at time \( p \). Let \( R_{o,p}^s \) be the return household receive at time \( o \) after investing 1 unit at time \( p \). Then \( m_{o,p} = 5000R_{o,p}^s \).

16 Although I observe that household use safe asset \( s \) to achieved the actual plan \( \{c^*_t(s^f)\}_{t=0}^\infty \), it might not be the case for the naive plan \( \{c^*_t(s^f)\}_{t=0}^\infty \).
with $o$. Comparing (26) to (24) leads to a disappointing result: in this framework $m_{o,p}$ does not reflect discounting parameters $\beta$ and $\delta$, but only $R^o_{o,p}$, the interest rate of the riskfree asset.

One must ask whether household, in practice, take into account the endogenous expected marginal utility when answering $m_{o,p}$. That is to say, assuming for now that household does not discount ($\beta = \delta = 1$), and consumption is expected to stay constant: will a household demand lower reward in the future solely because consumption is variable in the future. At the risk of sounding self-serving, I must profess that I would not have taken this factor into account if I were in the household’s shoes. This motivates the assumption that when answering reward values $m_{o,p}$, household follow a special set of belief $E'$ such that $E'_0[ae^{-\alpha(x^*)}] = E'_0[ae^{-\alpha(x^*)}]$, which leads to the desired result (27) after the linear approximation of $e^{-\alpha(x)} - 1$ around $x = 0$.

$$\beta(p=0)\delta_{o-p} = \frac{5000}{m_{o,p}}$$

For each household, $\log \beta$ and correspondingly $\beta$ can be estimated by running the regression (28) of $\log \left( \frac{m_{o,p}}{5000} \right)$ on $-I(p = 0)$ while controlling for $(p - o)$. The data is pooled for households with more than one individual surveyed. In the histogram, see that $\hat{\beta}$ for a few households exceeds one. I attribute this to measurement error and will exclude these households from further analysis.

$$\log \left( \frac{m_{o,p}}{5000} \right) = \log \beta(-I(p = 0)) + \log(p - o)$$

7. Estimating Income Distribution

An issue I have not addressed thus far is timing. In the theory Section 4, the model allows for three periods ($t = 0, 1, 2$). Realistically, this would correspond to young, middle and old age. Of importance is $t = 1$, where the model predicts a relationship between income distribution and saving. The Townsend Thai data has 13 years of data. My approach will be to treat this entire 13 years as a single period. The theoretical prediction would of course apply to middle aged households.

Note that this imply that household view the entire period of 13 years as the present. It is hardly realistic, as household realistically makes decision at greater frequency. Since our data is collected monthly: this imply that at the very least one period should correspond to a month. Unfortunately, I was unable to extend AWA model beyond 3 periods in way that retain its predictive ability. As such, I will proceed with the aforementioned timing.

17For example, if $\Delta x^*_{t+1}$ were i.i.d normal then $x^*_{t} - x^*_{t+1}$ follows Brownian motion, with $\text{Var}_p(x^*_{t})$ increasing linearly with $\alpha - \rho$ and $E_p[e^{-\alpha(x^*_{t})}] \propto e^{-\alpha \text{Var}_p(x^*_{t})/2}$
Recall that Income distribution has two components: the initial wealth $a_0$. I take this variable directly from Srivisal's accounting work. Recall that Srivisal's accounting also provides monthly rate of return. However, a period spanning 13 years will leave me with only one observation per household, thus making it difficult to estimate the distribution.

To proceed, I first take the yearly average and then assume that each year correspond to a subperiod with return $R_{1,y}$. Investment $a_0$ is invested in the first subperiod, and its return fully reinvested in subsequent subperiods. $R_1$ is the geometric sum of the rate of return $R_{1,y}$ of the subperiods.

$$\ln(R_1) = \sum_{t=1}^{14} \ln(R_{1,y})$$

Further assume that $R_{1,y}$ follow an i.i.d log-normal distribution $\ln N(\mu, \sigma)$, so that $\ln(R_1)$ has log-normal distribution $\ln N(\mu, \sigma)$. I estimate $\mu$ and $\sigma$ for each household using the arithmetic moments of yearly data. This allows the distributions $f(R_1)$ and $f'(R_1)$ to be parametrized.

$$\frac{\sigma}{\sqrt{13}} = \ln(1 + \frac{\text{Var}[R_{1,y}]}{E[R_{1,y}]^2})$$

$$\frac{\mu}{13} = \ln E[R_{1,y}] - \frac{\sigma^2}{2}$$

### 8. Estimating the Prediction from the Components

Having estimated the three components, I am now ready to combine them to estimate the predicted minimum saving. First, I correct for timing issues regarding $\alpha$ and $\beta$, which were estimated in the context where one time period is a month. I adjust $\alpha$ by dividing by 13 and then 12 (13 years and 12 months/year). This is such that consumption $c_1$ at time period 1 is equivalent to spreading $c_1$ over the 156 months. The parameter $\beta$ on the other hand is not adjusted: with the interpretation that concept of present now spanning a longer period. Recall that I exclude household with implausible $a_\alpha$ and $\beta$ household, leaving 426 households. Combined with $R$ distribution and $a_\alpha$, I can calculate $R'_{1,y}$ computationally and graph it in the histogram below. There are 23 households with $\beta = 1$, and for them it is clear that $R'_{1,y} = 0$.

$$R'_1 = \max\{R_1\int_{R_1}^{R_1}\left(f(R_1) (1- \beta) - a_\alpha F(R_1) \right) \left(e^{-\alpha R'_1 a_\alpha} \right) \, dR'_1 \geq 0\}$$

---

$^1$It is also possible to do this monthly. But I feel that doing so will simply introduce autocorrelation and seasonality issues (which results in some observation with negative rate of return).

$^2$The corresponding interpretation on consumption would be that it is either consumed outright at the beginning of the 14 years, or kept on a separate zero account to be consumed over the period.

$^{20}$It is also possible to do this monthly. But I feel that doing so will simply introduce autocorrelation and seasonality issues (which results in some observation with negative rate of return).
I check on the range $R_1 \geq R_1^p$ the condition whether minimum saving policy is optimal. To perform the check, I calculate the maximum over the range. With $R_1$ log-normal, the ratio $\frac{f'(R_1)}{f(R_1)}$ achieves a nice expression. Note that for households with $\beta = 1$, the condition is automatically satisfied because the RHS is infinity. On the other hand, $\beta < 1$ ensures that $R_1^p > 0$, and thus the LHS is bounded and continuous in the relevant range. Of the 403 households with $\beta < 1$, there are 215 household that violate the monotonicity condition. As such the 426 households can be divided into 3 categories:

1) 23 households with $\beta = 1$. Since there is no hyperbolic discounting, we have first best solution, which in this case is also both self-0’s and self-1’s preferred solution. Additional, this also coincide with the minimum saving solution with $R_1^p = R_1^m$.

2) 188 households with $\beta < 1$ satisfying monotonicity condition. Minimum saving for these households is pinned down at $a_1^* (R_1^p) = \frac{R_1^p a_0}{2} + \frac{\ln(\beta)}{2\alpha}$.

3) 215 households with $\beta < 1$ but violate monotonicity condition. The optimal solution for these household feature bunching at multiple (instead of one) income range.

I graph an example optimal saving policy for each of the three categories below. For simplicity I use the parameters $\alpha = 1$, $a_0 = 0$, $R_1^{Max} = 5$, $R_1^{Min} = 0$. The bunching regions would realistically depend on income distribution. Here, I deliberately chose the region as [0.1] for both #2 and #3 and [3.4] for #3. First, see that compared to #2 and #3, #1 has higher saving at $R_1^{Max}$. This is because $\beta$ is higher and thus future consumption is worth more. As mentioned above, #1 can be interpreted as the limit of #2 as $\beta$ approach 1. Category #3 example shows two bunching region. Generally, there can be more bunching regions, depending on how often the monotonicity condition is violated. In section 4, I reasoned how #2 can be implemented in practice. For $R_1 > R_1^p = 1$, the solution coincide with self-1’s preferred solution, and it is constant otherwise. Category #3, on the other hand, cannot be implemented in practice. The solution only coincide with self-1 preferred solution for $R_1 \geq 4$, and thus the bank must ensure household follow through its commitment for $R_1 < 4$. However, the saving policy is not constant in this range. It is therefore impossible for the bank to correctly punish household for deviating without observing the household’s income.

\[
\frac{\mu - x^2 - \ln R_1}{R_1 \sigma^2} = \frac{f'(R_1)}{f(R_1)} \leq \frac{a_0 \alpha}{1 - \beta}
\]

\[f(R_1) \propto \frac{1}{R_1} e^{-(\ln(R_1) - \mu)^2/(\sigma^2)}
\]

\[f'(R_1) = f(R_1) \frac{1}{R_1} + f(R_1) \frac{1}{R_1 \sigma^2} (\ln R_1 - \mu)\]
9. Regression Analysis: Actual versus Predicted Saving

I am now ready to compare actual minimum saving from Section 2 with predicted minimum saving from Section 8. Note that the actual minimum was calculated per year, so in this context, I multiply it by 13. Recall that the prediction applies to the 188 households in group #2 and 23 households from group #1. The correlation is 0.29 (p-value<0.001). As a test for false positive, I repeat this exercise for group #3 to get 0.05 (p-value=0.45); and again pooling the data to get 0.07 (p-value=0.12). This illustrate the importance of checking the condition which ensure minimum saving policy is optimal, as derived by AWA. Group #3 will be excluded from analysis from this point onwards. Below I graph actual against predict minimum saving, with the red identity line.

For most observations (181 out of 211) the actual lies below the predicted value. The robust regression shows a significant relationship (p-value<0.001) but the coefficient is only 0.08, which is below far 1. A possible explanation is the 77 households with zero actual saving. This points towards limited supply of financial services, or that household could be committing through other asset I did not account for such as longterm illiquid investment. Another reason behind the low coefficient value is the 23 $\beta = 1$ households with zero predicted saving due to $R_1^{\min} = 0$ assumption. Excluding these zero observations, however, only results in a slightly higher coefficient of 0.10.

A factor to take into account is whether the 13 years period actually correspond to the middle aged period $t = 1$. If household are either too old or too young, then the prediction should not apply to them. As such, I calculate household's average age at 1999, and include it in the regression as both the control and interaction term. The predicted slope for each age is plotted above. As expected, the slope coefficient is highest when household is middle aged. Nevertheless, at its peak, the slope is still not close to 1.
10. Conclusion

This paper brings the theory of Amador Werning and Angeletos (2006) to the Townsend Thai Monthly Data, and find the two to be consistent. This work has potential policy implication with regards to the availability of pledge savings products through microfinance institution. The next step towards this will be to back out the utility gain from the commitment mechanism such product offer. Since the utility gains depends upon observable characteristics such as risk aversion, hyperbolic discounting, and income distribution, the availability of financial product can be targetted towards communities that benefit most from it. On the flip side, it is clear that patient ($\beta = 1$) households do not benefit from such financial product, so universal access to pledge savings account might not be cost-effective.

The main drawback from the result section is fact that predicted minimum saving, while correlated with, is much larger than the actual value. Apart from the explanation that actual minimum saving is either underestimated or constrained to a low value, it is also possible that the predicted value itself could itself be wrong. The prediction is of course as good as the underlying model. The obvious limitation to the current model is that it only has three periods. Further work will focus towards extending the model to n-period. This is not trivial, as each additional period introduces another self (player) into game. The NBER version of Amador Werning and Angeletos (2006) provides a starting point for the more general model. The n-period model will also help address the inconsistency in the CARA estimation section, which currently does not include commitment mechanism (in order to allow n-period). It will also address timing issues regarding $\alpha$ and $\beta$, which currently has to be adjusted in an ad-hoc fashion.

Another venue for further research is to tackle the problem computationally. This is especially useful for households which violate the monotinicity condition; as the optimal saving policy of these household cannot be characterized analytically. The solution, which involves multiple bunching region, cannot be implemented in practice. Nevertheless, it is still interesting to quantify how much utility has been lost from this limitation. Additionally, since these household are hyperbolic, they should still prefer a minimum saving rule over no commitment. This second-best policy itself can also be solved computationally.
Reference


