Financial Market Failures and Systemic Crises

by

Diego Feijer

M.S., Electrical Engineering and Computer Science, Massachusetts Institute of Technology (2011)

Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2015

© Massachusetts Institute of Technology 2015. All rights reserved.

Signature redacted

Author ........................................................... ∙ ∙ ∙ ∙
Department of Electrical Engineering and Computer Science
August 21, 2015

Signature redacted

Certified by.....

Munther A. Dahleh
William A. Coolidge Professor of EECS
Thesis Supervisor

Signature redacted

Accepted by .................

Leslie A. Kolodziejski
Chairman, Department Committee on Graduate Theses
Financial Market Failures and Systemic Crises
by
Diego Feijer

Submitted to the Department of Electrical Engineering and Computer Science on August 21, 2015, in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Abstract
This thesis contributes to the theoretical literature that studies the macroeconomic implications of financial frictions. It develops frameworks to address different financial market failures, and evaluate preventive policies to mitigate the vulnerability of the economy to costly systemic crises.

First, it identifies a credit risk (fire sale) externality that justifies the macroprudential regulation of short-term debt to mitigate the probability of systemic bank runs. Without regulation, banks do not internalize how their funding decisions affect the terms at which other market participants can obtain credit. The formal welfare study conducted, provides a general equilibrium notion of systemic risk that captures both fundamental insolvency and illiquidity risk. It also connects this measure with the optimal Pigouvian (corrective) tax.

Second, it shows that liquidity crises may arise as the result of endogenous information panics. It finds that collective ignorance is welfare maximizing but it is fragile, susceptible to self-fulfilling fears about asymmetric information. Adverse selection may thus obtain in equilibrium, sustained by negative aggregate expectations. The mechanism that gives rise to multiple equilibria is robust to the introduction of noisy private signals, and warrants the regulation of information acquisition for rent-seeking (speculative) motives.

Finally, it demonstrates the limitations of unconventional credit easing policies to stimulate lending during market freezes. With inter-temporal investment complementarities, credit to non-financial firms may be curtailed as the result of dynamic coordination failures. Interest rate cuts mitigate coordination risk, but increase the average duration of credit market freezes when the productivity of capital is high. Capital injections in the banking sector, or direct lending to non-financial firms, are completely ineffective, because reductions in deposits from households crowd out government spending. In contrast, government guarantees improve welfare by reducing strategic uncertainty.

Thesis Supervisor: Munther A. Dahleh
Title: William A. Coolidge Professor of EECS
Acknowledgments

First and foremost, I would like to express my gratitude to my advisor Munther Dahleh. I am extremely grateful for the freedom he gave me to develop my own research ideas while providing me with constant guidance, support, and feedback along the way. It has been a joy to work under his supervision; I do not recall ever leaving his office without feeling enthusiastic.

I am deeply indebted to Ricardo Caballero for sharing his economic insights, and for providing me with invaluable advice; and Andrew Lo for his generosity, and for motivating me to do research on financial crises. It has been a true privilege to have both of them on my thesis committee. I am also grateful to Fernando Paganini, who taught me how to approach research questions, and always inspired me to keep on learning. His mentorship during my time as an undergraduate student in Uruguay proved invaluable throughout my doctoral studies at MIT. In addition, I would like to acknowledge Iván Werning for giving me the chance to present my work at the Macro-International Lunch in the Economics Department.

I sincerely thank my closest friends Roberto, Pascual, Nico, David, Eugene, Paula, Mariana, Laura, Enrique and Hoeskuldur for their companionship and all the unforgettable moments we have shared together. This journey would not have been nearly as enjoyable without their presence. I specially thank Estefanía for being so wonderful, supportive, and caring during the last years of the program. Fellow students at LIDS also played an important role, in particular Munzer’s research group and my officemates: Yola, Mitra, and Spyros.

Last but not least, my deepest appreciation goes to my family for their unconditional love and unwavering support. I wish I could have also shared this achievement with my paternal grandparents, Muti and Abu. Undeniably, the dedication and strength of my mother, Madelón, which allowed my sister Mariuc and I to grow up in a loving and stimulating environment, are the bedrock upon which this thesis has been built. From my father, Victor, I received a little of his intellectual curiosity; his long-lasting memory is present in every equation.

Boston, Massachusetts, USA
August, 2015
# Table of Contents

1 Introduction 1

2 Fire Sales and the Credit Risk Channel 5
   2.1 The Environment ........................................ 9
   2.2 Crises and Systemic Risk .............................. 14
   2.3 Debt and Fire Sales .................................... 16
   2.4 Welfare .................................................. 20
   2.5 Policy .................................................... 27

3 Information Panics and Liquidity Crises 37
   3.1 The Model ................................................ 42
   3.2 Equilibrium .............................................. 47
   3.3 Endogenous Asymmetric Information ................. 48
   3.4 Normative Analysis .................................... 56

4 Coordinating Inefficient Credit Flows 65
   4.1 The Baseline Model .................................... 70
   4.2 Strategic Uncertainty and Multiple Equilibria .... 73
   4.3 Model with Incomplete Information .................... 76
   4.4 Coordination Failures and Policy Interventions .... 84

5 Concluding Remarks 93

Bibliography 95
CHAPTER 1

Introduction

This thesis studies why financial markets sometimes malfunction; where their instability is rooted; how their fragility can trigger adverse feedback mechanisms, which spill over to the real economy; and what the role for macroprudential policy and regulation is in preventing costly systemic crises.

This work contributes to the theoretical literature that lies at the intersection of macroeconomics and finance. It develops models that isolate the destabilizing effects of fire sales, asymmetric information, and investment complementarities. Historically, these factors have played a relevant role during financial crises.

The crisis of 2007–2009 was a "classic financial panic" (Bernanke, 2013). The bursting of the housing bubble led to a sharp increase in subprime mortgage defaults. Amid widespread concern about the size and incidence of credit losses, a wide range of financial institutions experienced withdrawals of short-term funding. These funding pressures forced fire sales and deleveraging, which exerted downward pressure on asset prices. The sharp decline in asset prices, in turn, further eroded banks’ net worth, which prompted more fire sales, and thus amplified initial capital losses.

Chapter 2 delves into the workings of such downward liquidity spirals. By formally studying the welfare properties of competitive equilibria in a model of debt markets and investment, it identifies a credit risk externality that justifies the macroprudential regulation of short-term debt issuance to mitigate the probability of panic bank runs. Without regulation banks do not internalize that an incremental unit of short-term debt, depresses fire sale prices, and thus increases the funding
costs of other agents. This, in turn, heightens their credit risk, and leaves the economy extremely vulnerable to a systemic run. The model provides a measure of systemic risk that captures both fundamental insolvency and illiquidity risk, and connects its sensitivity to changes in short-term funding with the optimal Pigouvian (corrective) tax.

Another element that contributed to the severity of the recent financial crisis was pervasive uncertainty about the location of losses and the size of risks of securities related to subprime mortgages, which clogged the balance sheets of financial institutions. Liquidity dried up when market participants began to question the value of many structured products, as they realized how seriously deficient they were in their underwriting and disclosures. Historical accounts suggest that asymmetric information problems, created in the particular institutional context of the modern financial system via the complexity of the securitization process, play a critical role during financial crises (Mishkin, 1991).

Chapter 3 explores how endogenous liquidity crises can be ignited by information panics. It finds that collective ignorance is welfare maximizing but it is fragile, susceptible to self-fulfilling fears about asymmetric information. When investors become worried about the potential of adverse selection, they raise interest rates. In turn, fearing unfairly high rates, borrowers have incentives to acquire information about their probability of repayment. More information worsens the average credit risk of borrowers in the market, and thus justifies investors' initial concerns. Adverse selection may obtain in equilibrium even though it is not justified by the fundamentals in the economy. Importantly, information panics amplify small aggregate shocks to asset qualities, and cause large detrimental effects to real economic activity.

Starting in the summer of 2007, the Federal Reserve responded aggressively to contain the crisis. It began by easing monetary policy, and then by deploying a host of credit easing tools with the goal of reducing financial strains. Such unconventional interventions were justified by a generalized agreement about the significance of macroeconomic externalities (Bernanke, 2009). They included unprecedented amounts of liquidity injections, purchases of credit instruments and toxic assets, guaranteeing bank liabilities, and infusions of capital into the financial system. But even as credit spreads across markets widely dissipated as the result of massive liquidity provisions, intermediaries remain reluctant to extend loans due
to a lack of confidence about capital, asset quality, and credit risks. The crisis highlighted the difficulty in reviving private lending once the acute phase of the turmoil had ended.

Chapter 4 precisely focuses on dynamic credit market freezes sustained by self-fulfilling expectations, and analyzes the limitations of unconventional policy interventions in stimulating credit. In a macroeconomic model with inter-temporal investment complementarities, the normal credit flow to non-financial firms may be impaired when financial intermediaries expect others not to lend. These coordination failures have persistent real effects through the accumulation of aggregate wealth in the financial sector. They are also inefficient, and thus justify interventions, because otherwise profitable projects cannot be undertaken due to a lack of working capital. Interest rate cuts mitigate coordination risk, but increase the average duration of credit market freezes when the productivity of capital is high. Once financial intermediaries cease to be balance-sheet constrained, capital injections in the banking sector or direct lending to non-financial firms are completely ineffective in encouraging lending because reductions in deposits from households crowd out government spending. In contrast, government guarantees improve welfare by reducing strategic uncertainty.
Chapter 2

Fire Sales and the Credit Risk Channel

Fire sales and their negative systemic spillovers are the main theoretical rationale to prevent the excessive accumulation of short-term funding. Distressed sales by financial intermediaries to meet immediate liquidity demands often cause deep price dislocations, which transmit to other market participants through common exposures of their balance sheets, forcing them to also liquidate assets. The result is a self-reinforcing downward spiral in asset prices, which impinges on the entire financial system.¹

In this chapter, I develop a general equilibrium model of investment with financial frictions, identifying a channel that connects the severity of fires sales with the likelihood of banking runs, thus linking systemic risk together with macro-prudential regulatory policies. This connection is based on a different kind of fire sale externality, which I label the credit risk externality. This externality operates through endogenous credit risk constraints, and captures how runs on individual financial institutions can disrupt overall financial stability. It arises because banks do not internalize that, by increasing interest rates, an additional unit of fire sold assets worsens the terms at which others can raise funding, and thus increases their probability of default.

By assuming that bad states of nature (crises) are driven by exogenous stochastic processes, the existing literature on fire sales shuts down this mechanism. Indeed, as explained by Dávila (2015), the ensuing welfare losses are commonly associated

¹Shleifer and Vishny (2011) provide a survey of the literature on fire sales in finance and macroeconomics. Stein (2013) discusses the role of welfare-improving macroprudential policies.
with two distinct fire sale externalities: a **collateral externality** and a **terms of trade externality**. The former arises when agents do not internalize the impact that their individual funding decisions have on the value of assets that other agents use as collateral, affecting their borrowing capacity. The latter appears when the marginal rates of substitution across periods/states differ across agents, in which case a fire sale in a particular period/state worsens the terms of trade of other sellers with relatively higher marginal utility in that period/state.\(^2\) The credit risk externality is different from the collateral externality in the sense that changes in the fire sale price have a direct impact not on the borrowing **capacity** of banks, but on their borrowing **terms**; and it is also different in nature from the terms of trade externality because it does not require heterogeneity across agents in the economy.

The key feature of the model is the fact that the incidence of financial crises depends on aggregate market variables determined in equilibrium. Systemic runs are the result of a coordination failure among short-term creditors. I model short-term creditors’ rollover decision problem as a coordination game with incomplete information, and rely on global games techniques to solve for the unique optimal (symmetric) threshold strategy. Runs are based on panics yet they occur when fundamentals fall below this threshold. In equilibrium, a change in the threshold alters the probability of repayment and thus affects interest rates, which in turn, feedback to the threshold. I show that this fixed-point problem admits a unique solution, which determines the overall credit risk of banks and therefore the probability of crises.\(^3\) This probability is decomposed into a fundamental insolvency component and an illiquidity component, and it is increasing in both the fragility and illiquidity risk of banks’ balance sheets.

In the model, banks maximize profits but are subject to two financing frictions. First, banks are credit constrained by the possibility of runs as just described. Second, short-term debt needs to be completely safe if it is to command a lower interest

---

\(^2\)See e.g. Bianchi, 2011; Oliver and Korinek, 2012; Stein, 2012; and Gersbach and Rochet, 2013 for models with collateral externalities; and Geanakoplos and Polemarchakis (1986); Gromb and Vayanos, 2002; Lorenzoni, 2008; Korinek, 2012; and He and Kondor, 2014 for models featuring terms-of-trade externalities.

\(^3\)In the baseline model I assume “full recovery” after default. With “partial recovery” I show at the end of the chapter, that this feedback loop between interest rates and credit risks may result in multiple equilibria driven by self-fulfilling insolvency concerns, a phenomenon reminiscent of Calvo (1988).
rate, which imposes an upper bound on the amount of short-term financing given by the collateral value. I study constrained efficiency by consider a social planner that faces the same constraints as the private economy, but internalizes the effect that funding decisions have on market prices. I show that both the credit risk and the collateral externality are negative, causing (short-term) over-borrowing ex-ante and over-selling ex-post. When the regulator reduces the aggregate amount of short-term debt, it reduce fire sales and thereby redistributes resources away from underpriced financial assets towards real investment. In addition, it decreases the probability of a systemic run by directly reducing the fragility of balance sheets and indirectly increasing their liquidity.

The main normative implication is that constrained efficiency can be restored with a Pigouvian (corrective) tax levied on each unit of short-term debt raised by banks. The optimal tax is decomposed in two terms: one that is proportional to the social shadow value of borrowing short-term against collateral, and another one that corresponds to the marginal credit risk externality at the social planner’s solution. The marginal credit risk externality captures private agents’ overvaluation of short-term debt, and is equal to the difference between the marginal change in the probability of a crisis weighted by the real economic costs of such an event, and the marginal change in the credit risk of a representative bank weighted by the cost of a run on its short-term liabilities. In this respect, the model illustrates how measures of aggregate systemic risk should discount individual contributions to set the tax at the appropriate level.

Finally, the model offers a distinctive account of how better economic times may lead to higher inefficiencies, thus calling for tighter macroprudential regulation. As investment prospects improve in a second order stochastic dominance sense, the probability of financial crises decreases. However, because the distribution of returns becomes more “concentrated” around its mean, banks grow less concerned about tail risks and the possibility of damaging runs, hence increase their reliance on short-term funding. Interestingly then, even though the incidence of runs is lower, their severity can actually be greater because of more significant market-wide liquidations.
Chapter 2. Fire Sales and the Credit Risk Channel

Related Literature

My model builds on Stein (2012). The main ingredient I borrow is the fact that banks can manufacture safe assets in the form of short-term debt backed by long-term investments. Short-term financing is cheaper (albeit riskier) because households directly derive utility from holding safe assets (private money). This is meant to capture the spirit of Gorton and Pennacchi (1990) who articulate the idea that banks create liquid securities that are extremely valuable for transaction purposes; given that these securities are riskless, agents do not fear the loss of value in transactions with better informed counterparties.\(^4\) In this respect, the chapter is related to the nascent literature on the supply of safe assets and its impact on the real economy (Gorton and Ordoñez, 2013; Krishnamurthy and Vissing-Jorgensen, 2013; and Caballero and Farhi, 2014).

Unlike Stein (2012), I explicitly model bank runs and defaults as the result of a coordination failure among short-term creditors. In my setting, balance sheet compositions and market conditions interact with each other and impact the probability of crises, ultimately determining equilibrium outcomes. Gorton and Metrick (2012) argue that the crisis of 2007–2009 was a systemic run on the sale and repurchase market. In other words, the run was not on the traditional banking sector, rather it took place on the shadow banking system. To capture the essence of this argument, I consider banks as profit-seeking financial institutions and model runs similarly to Rochet and Vives (2004) and Morris and Shin (2010). This conception differs from the seminal work of Diamond and Dybvig (1983), where banks use long-term investments to back demand deposits, and households need for deposits stems from a desire to insure against liquidity shocks and thereby smooth consumption. From a methodological standpoint, my analysis of runs relies on global games techniques similar to those in Morris and Shin (2003) and Goldstein and Pauzner (2005).

This chapter contributes to the literature that analyzes the positive effects of fire sales, which dates back to Shleifer and Vishny (1992, 1997) and Kiyotaki and Moore (1997). In particular, Brunnermeier and Pedersen (2009) and Acharya and

\(^4\)Other theories of short-term debt include Diamond and Rajan (2001), who demonstrate its role as a disciplining device for bank managers; or Brunnermeier and Oehmke (2013), who argue that extreme reliance on short-term financing may be the outcome of a "maturity rat race" among creditors.
Viswanathan (2011) stress the relationship between market liquidity and funding liquidity; Allen and Gale (1994, 1998) and Geanakoplos (2009) study limited market participation by potential buyers of assets due to financial constraints result in "cash-in-the-market" pricing; and Shleifer and Vishny (2010) and Diamond and Rajan (2011) focus on how scarce capital for arbitrage opportunities becomes the hurdle for new investments. Crucially, in all these models, when a large number of agents engage in liquidation and deleveraging activities, prices for assets do not reflect fundamental values but are rather determined by the spare capacity in the economy.

In addition, the model is related to the literature on amplification mechanisms in credit markets through financial frictions, whereby a small negative shock to agents' balance sheets can have a big impact on the economy, as it leads to the liquidation of assets, which lowers prices and further deteriorates balance sheets. This is surveyed by Krishnamurthy (2010) and Brunnermeier and Oehmke (2012).

Outline of the Chapter. The rest of the chapter is organized as follows. Section 2.1 introduces the basic environment. Section 2.2 determines the probability of bank runs as the solution to a fixed-point problem, which captures the feedback loop between credit risks and interest rates. Section 2.3 studies general equilibria by endogenizing bank's choice of short-term debt and connecting fire sale prices with the fragility of balance sheets. Section 2.4 analyzes the welfare properties of the unique competitive equilibrium of the model. Policy interventions are discussed in Section 2.5.

### 2.1 The Environment

There are three periods, and the economy is populated by two groups of agents of equal unitary mass: households and banks. There is a unique consumption good.

#### A. Households

Households have an initial endowment of one unit of the consumption good. At date 0 they have a choice: either consume this endowment, or invest a part of it in financial assets and consume the proceeds at date 2. There are two types of assets
they can invest in: risky “bonds” and riskless “money,” with gross real returns \( R_B \) and \( R_M \), respectively. Rates of interest are endogenous in the model, determined in equilibrium.

Preferences are linear over early and late consumption. In addition, along the lines of Stein (2012), households derive utility from the monetary services that safe assets provide. Specifically, the expected utility function of a representative household is given by (Sidrauski, 1967)

\[
U(\{C_t\}) = C_0 + \beta \mathbb{E}[C_2] + \gamma M, \quad \text{with} \quad \beta + \gamma < 1, 
\]

where \( M \) is the guaranteed command over late consumption goods represented by the total initial holdings of money. This is a simple reduced-form way of capturing households demand for extremely safe assets, and in line with standard banking theories (Diamond and Dybvig, 1983; Gorton and Pennacchi, 1990), it purposefully brings to the forefront the special role of banks as intermediaries that transform risky and illiquid assets into safe and liquid ones.\(^5\)

B. Banks

Banks are identical, and they all have the same risky investment opportunity: one unit of the consumption good invested at date 0 has a real return of \( \theta \) at date 2, with mean \( \bar{\theta} \). Let \( F \) denote the continuous cumulative probability distribution of \( \theta \), with support over \( \Theta = [\theta_{\min}, \theta_{\max}] \), with \( \theta_{\min} > 0 \) which allows banks to manufacture safe assets in the economy; \( f \) is the density function.

Banks receive no initial endowment. In order to invest they need to raise funds from households, which can be achieved by issuing two types of financial claims: \( m \in (0,1) \) units of money (collaterized short-term debt) and \( 1-m \) bonds (long-term debt). I will refer to households holding short-term (long-term) debt claims as short-term (long-term) creditors. For now, the balance sheet composition \((m, 1-m)\) is fixed. Short-term debt claims need to be rolled over at date 1. At date 1, to meet potential redemption demands, banks can sell any fraction of their assets at a fire-

---

sale discount in a secondary market. Specifically, an asset that pays off $\theta$ at date 2 can be sold for $k\theta$ at date 1, with $k \in [0,1]$. Note that $k$ is a measure of the market liquidity for banks' assets: the smaller $k$, the smaller is the cash pool that banks can draw on in the interim.

Short-term debt claims have to be completely safe if they are to carry a return $R_M$. Hence, if at date 1 short-term creditors decide to withdraw their funds (run), banks have to be able to raise $M \equiv mR_M$ regardless of the future return on investment. Therefore, banks face an upper bound on money creation:

$$m \leq \frac{k\theta_{\min}}{R_M}. \tag{2.2}$$

In the absence of fire sales, banks would not be financially constrained by (2.2); however, when future cash flows cannot be pledged to raise additional funds due to long-term debt overhang problems (Myers, 1977; Hart and Moore, 1995), fire sales are very difficult to avoid.

To make borrowing and investment attractive, I assume that projects have positive NPV by imposing the following condition which will become apparent later.

**Assumption 1 (NPV).** The mean return on investment satisfies: $\bar{\theta} > \max \left\{ \frac{1}{B'}, \frac{1}{(\beta+\gamma)k} \right\}$.  

### C. Panic Runs

At date 1, the value of the future payoff $\theta$ is revealed. But before this happens and before the secondary market opens, short-term creditors have to decide whether to rollover or withdraw their funds.\footnote{When the price of assets in the secondary market is determined in equilibrium, it acts as an informative public signal that can potentially reveal the value of $\theta$. The timing of events described rules out the possibility of using the market as a coordination device.} At this point, each creditor $j$ only observes a private noisy signal $\theta_i = \theta + \epsilon_j$, where $\{\epsilon_j\}$ are very small error terms, independently and uniformly distributed over $[-\epsilon, \epsilon]$. An agent's signal can be thought of as his own opinion about the return on the risky investment undertaken by banks. Information is heterogeneous among creditors, but no one has an advantage over the others in terms of the precision of their signal.

Now imagine that a fraction $\lambda$ of the short-term creditors to a given bank, each based on his private signal, decide to withdraw their funds at date 1. The bank is
thus forced to liquidate a fraction \( q \) of its assets to raise \( \lambda m R_M \) to pay its departing creditors. Can a run result in the failure of the bank at date 2? This will be the case if,

\[
(1 - q) \theta < (1 - \lambda)m R_M + (1 - m)R_B, \quad \text{where} \quad (q \lambda) = \lambda m R_M. \tag{2.3}
\]

Equivalently, for a given return on investment \( \theta \), a bank fails at date 2 if

\[
\theta < \theta_{\text{run}}(\lambda) \equiv m R_M + (1 - m)R_B + \lambda m R_M \left( \frac{1}{k} - 1 \right). \tag{2.4}
\]

In case of failure, I make the following simplifying assumption:

**Assumption 2 (Zero Recovery Value).** Creditors holding short-term debt claims to a bankrupt bank at date 2 receive a payoff of zero.

Although stark, this assumption is motivated by the fear and uncertainty that surround bankruptcy proceedings among counter parties to debt claims. The fact that overnight repurchase agreements do not perfectly protect lenders in case of default is well documented by Duffie (2010a). In addition, short-term creditors may not be legally eligible to hold collateral, or they may face funding strains and may need to sell this collateral in a distressed secondary market. In any case, the immediate price of such assets is likely to be well below its value in best use; for simplicity, I treat this value as zero.

In the interval \([\theta_{\text{min}}, \theta_{\text{run}}(0)]\) the bank fails even in the absence of a run due to fundamental insolvency. When his signal assures him that this is indeed the case, an individual short-term creditor is better off withdrawing his funds regardless of the actions taken by others. I thus refer to \([\theta_{\text{min}}, \theta_{\text{run}}(0)]\) as the lower dominance region. On the other hand, in the interval \((\theta_{\text{run}}(1), \theta_{\text{max}}]\) fundamentals are so strong that the bank cannot fail even when at date 1 most of its short-term creditors refuse to roll over. In this case, the short-term creditor is indifferent between withdrawing and rolling over whatever his belief about the behavior of other creditors. This is because money does not carry an interest from date 1 to date 2.

**Assumption 3 (Indifference).** Whenever indifferent between rolling over and withdrawing, short-term creditors always choose the former.

Within the interval \((\theta_{\text{run}}(1), \theta_{\text{max}}]\), the assumption implies that rolling over is the dominant strategy. I thus refer to this interval as the upper dominance region. As-
sumption 3 is analog to Goldstein and Pauzner (2005), in which the existence of an upper dominance region of fundamentals is also imposed.

Note that a larger $\lambda$ (size of the run) implies a larger $\theta_{\text{run}}$, thus a higher probability of the bank failing at date 2. Therefore, when $\theta$ is believed to have fallen in neither of the dominance regions, the key variable driving an individual short-term creditor’s decision to rollover is his belief about the proportion of others rolling over at date 1. Hence, creditors play a coordination game with incomplete information and strategic complementarities: an agent’s incentive to take a particular action (withdraw or roll over) is higher when other agents take that action. As the next result shows, similar to Rochet and Vives (2004), this game has a unique equilibrium:

**Proposition 1 (Runs).** Under Assumptions 2 and 3, and in the limit as $\epsilon \to 0$, the coordination game among the short-term creditors of a bank has a unique (symmetric) perfect Bayesian equilibrium in which agents rollover when their signal is above a threshold $x$, and run otherwise. The threshold is given by

$$x = mR_M + (1 - m)R_B + mR_M \left(\frac{1}{k} - 1\right). \quad (2.5)$$

The proof is based on standard global games arguments; however, the intuition behind this result is very simple. Rolling over pays no dividends. Given that creditors are not able to coordinate their actions due to the presence of noise in their private signals (Carlsson and van Damme, 1993; Morris and Shin, 1998), it is optimal for them to rollover only when they know for certain that the bank will not fail at date 2. This is only the case when $\theta > \theta_{\text{run}}(1) = x$. Because of the fixed-value of collateralized short-term debt claims, the optimal rollover decision taken by creditors in the interim dilutes other creditors.

The decomposition of the run threshold into an insolvency component and an illiquidity component is reminiscent of Morris and Shin (2010). The reason short-term creditors run is their fear about others doing the same. Such illiquidity panics may drive an otherwise solvent bank to bankruptcy and result in a coordination failure: the smaller $k$, the more distressed the fire sale and the more significant the failure.
2.2 Crises and Systemic Risk

In order to complete the equilibrium characterization I determine interest rates at date 0 and then employ them to solve equation (2.5).

A. Interest Rates

Interest rates on money and bonds are pinned down by households break-even conditions at date 0. First, a household is indifferent between having $\beta + \gamma$ units of date 0 consumption; or a completely safe claim that promises one unit of consumption at date 2 since such claim provides $\beta$ of utility from future consumption plus $\gamma$ from monetary services. Therefore, the return on money must be given by

$$R_M = \frac{1}{\beta + \gamma},$$

which is constant due to households’ linear preferences. This is for starkness and tractability, not realism.

Now suppose that a household holds a bond issued by a representative bank. The bond promises a payment of $R_B$ at date 2 if the bank does not fail. If the bank does fail, the remaining assets in its balance sheet are evenly divided among its long-term creditors. The repayment function at date 2 is then,

$$\rho(\theta) \equiv \min \left\{ R_B, \frac{1}{1 - m} \left( \theta - \frac{mR_M}{k} \right) \right\}.$$

Hence, the break-even condition at date 0 is $\mathbb{E}[\rho(\theta)|x] = \frac{1}{\beta}$, which defines

$$R_B(x) = \frac{1}{\beta(1 - F(x))} - \frac{1}{1 - m} \left( \mathbb{E}[\theta|\theta < x] - \frac{mR_M}{k} \right) \frac{F(x)}{1 - F(x)}.$$

Unlike $R_M$, $R_B$ is not constant and depends on the probability of failure $F(x)$. Also, naturally $R_B$ is smaller than the interest rate that households would demand if they did not receive anything in case of default, as represented by the first term in (2.7). Finally, notice that in light of (2.5), Assumption 1 yields $\frac{\bar{\theta}}{\beta} > \frac{mR_M}{k} + \frac{1-m}{\beta}$ for all $m \in [0, 1]$, which implies that investment projects have positive NPV at date 0 for any mix of short-term and long-term debt financing.
B. Equilibrium

Employ (2.6) and (2.7) in (2.5) to define

$$\zeta(x; m, k) \equiv \frac{mR_M}{k} + (1 - m)R_B(x).$$

(2.8)

Then, for a given $m$ and $k$, equilibrium run thresholds are solutions to the following fixed-point equation:

$$x = P_\Theta[\zeta(x; m, k)],$$

(2.9)

which captures the feedback loop between interest rates and credit risks. The operator $P_\Theta[\cdot]$ denotes the projection onto the compact set $\Theta$.

**Theorem 1 (Equilibrium).** There exists a unique equilibrium run threshold $\theta^*(m, k)$, with the following properties:

$$R = R_M - \frac{\partial \theta^*}{\partial m} \frac{R_M}{k} \frac{1 - \beta}{1 - F(\theta^*)} \quad \text{and} \quad \frac{\partial \theta^*}{\partial k} = \frac{-mR_M}{1 - F(\theta^*)} \leq 0.$$

Theorem 1 has several implications which I now discuss in turn.\(^7\)

**Illiquidity and Insolvency.** An increase in the amount of short-term financing can either increase or decrease the run threshold depending on the predominant issue behind a bank’s credit risk: illiquidity or insolvency. When the fire sale discount $k$ is small such that $\frac{R_M}{k} > \frac{1}{\beta}$, illiquidity risk is high and an increase in $m$ always increases the run threshold $\theta^*$. In contrast, when $k$ is large enough that $\frac{R_M}{k} < \frac{1}{\beta}$, an increase in $m$ decreases insolvency risk by lowering financing costs thus decreasing the run threshold.

**Systemic Risk.** Based on Proposition 1, the probability of a systemic run is $F(\theta^*)$. Notice that this probability depends on the aggregate amount of private money in the economy, and is decreasing in the market liquidity for banks’ assets as measured by the aggregate variable $k$.

\(^7\)As a technical remark, notice that a rigorous computation of the equilibrium would entail solving the fixed-point problem (2.9) for an arbitrary $\epsilon > 0$ and then take the limit as $\epsilon \to 0$. Implicitly, I have interchanged the order. However, this simpler approach is justified by the fact that the convergence of agents’ posterior beliefs about the fraction of other agents running to the uniform prior is uniform (see Morris and Shin, 2003), and the fact that the operator $P_\Theta[\cdot]$ is continuous.
Cheap Money Financing. In equilibrium, it follows from (2.8) that

\[ E[\theta | \theta < \theta^*] - \frac{mR_M}{k} \leq \theta^* - \frac{mR_M}{k} = (1 - m)R_B(\theta^*). \]

Then, equation (2.7) implies that the interest rate on bonds is \( R_B(\theta^*) \geq \frac{1}{\beta} \). Hence, given that the interest rate on short-term debt is \( R_M = \frac{1}{\beta + \gamma} < \frac{1}{\beta} \), private money financing constitutes a cheaper funding source (albeit riskier). This is because money offers a convenience yield that bonds do not, thus in equilibrium households are willing to pay a higher price for the former.

2.3 Debt and Fire Sales

In this section I characterize general equilibria of the model, considering banks as profit maximizing agents and linking fire sale prices to the fragility of balance sheets.

A. Patient Investors

Following Stein (2012), I introduce another type of intermediaries which I refer to as “patient investors.” Collectively they receive a fixed endowment of \( I > \frac{1}{\beta + \gamma} \) at date 1. The crucial assumption is that even if they were allowed to raise resources at date 0 to set aside for future trading opportunities, \( I \) must be fixed at date 1, independent on any information that may be revealed. This assumption is a straightforward way of modeling institutional impediments to capital formation, as documented and studied by Duffie (2010b), which is the cause of price distortions in the secondary market for bank assets.\(^8\)

Patient investors have access to a productive technology \( g(\cdot) \), which is increasing and strictly concave. In case of a systemic run they spend a total of \( \int mR_M = M \) purchasing banks assets, and invest the remaining \( I - M \) in their productive technology which yields a total output of \( g(I - M) \). For patient investors to be willing to allocate funds in this manner, marginal returns from both investment

---

activities have to be equalized in equilibrium:

\[ \frac{1}{k} = g'(I - M). \]  

(2.10)

Condition (2.10) is key to the model. First, it relates the funding liquidity available in the economy with the market liquidity for banks' assets. Moreover, it also explains how decisions made by banks at the individual level propagate through the market and affect the whole economy. The larger the amount of short-term debt in the system, the more assets patient investors need to absorb in a crisis, and the more distressed the fire sale:

\[ \frac{dk}{dM} = g''(I - M)k^2 < 0. \]  

(2.11)

Equation (2.11) follows from the fact that \( g \) has decreasing marginal returns. This also implies that in a crisis patient investors earn a profit of

\[ Q(m) = \frac{mR_M}{k} - (g(I) - g(I - mR_M)). \]  

(2.12)

which is increasing in \( M \), and positive in equilibrium because \( g \) is upper bounded by its first order Taylor approximation.

For expositional purposes I will assume that \( \theta_{\text{min}} \leq \frac{1}{\beta} \), and impose the following conditions:

\[ g'(I) = \frac{1}{\beta R_M} \quad \text{and} \quad g'(I - R_M) = \frac{\theta}{R_M}. \]

These imply that, in equilibrium, \( k \in [R_M/\theta, \beta R_M] \). Hence, \( \frac{R_M}{k} \geq \frac{1}{\beta} \geq \theta_{\text{min}} \) which renders illiquidity risk dominant over fundamental insolvency risk and restricts private money creation through (2.2). The lower bound is simply to satisfy Assumption 1.

**B. Private Money Creation**

Banks objective at date 0 is to choose a capital structure that maximizes net expected profits at date 2. In choosing their capital structure, banks must then balance between lower financing costs and greater liquidations in case of a run. Notice that the Modigliani and Miller (1958) theorem is rendered inapplicable by the convenience yield offered by safe short-term debt. The optimization problem of a representative
bank is then to pick \( m \) to solve

\[
\max_{m \in [0, 1]} \Pi(m; k) \equiv (1 - F(\theta^*)) \left\{ \mathbb{E} \left[ \theta | \theta \geq \theta^* \right] - m R_M - (1 - m) R_B(\theta^*) \right\}
\]

subject to the collateral constraint (2.2), with \( R_M \) and \( R_B \) given by (2.6) and (2.7) respectively, and where \( \theta^* \) is characterized by Theorem 1.

The profit function \( \Pi \) can be decomposed into three terms:

\[
\left( \bar{\theta} - \frac{1}{\beta} \right) + m \left( \frac{1}{\beta} - R_M \right) - F(\theta^*) m R_M \left( \frac{1}{k} - 1 \right).
\]

The first term is the net expected profit from investment financed fully financed by long-term debt; the second term shows the expected savings from issuing \( m \) units of short-term debt; the final term represents the fire sale costs associated with this riskier short-term capital structure. Notice that banks understand how their choice of capital structure affects their own individual credit risk (default probability) defined by the run threshold \( \theta^* \); but as a single atom in a continuum they disregard their contribution to the severity of fire sales, considering the aggregate variable \( k \) as a constant.

C. Competitive Equilibrium

The Lagrangian associated with (2.13) is

\[
\mathcal{L}(m, \mu; k) \equiv \Pi(m; k) - \mu \left( m - \frac{k \theta_{\min}}{R_M} \right),
\]

where \( \mu \) is the shadow value of the financial constraint (2.2). Then, competitive equilibria are characterized by (2.10) and the Kuhn-Tucker first-order conditions for optimality:

\[
\begin{align*}
\text{either,} & \quad m = \frac{k \theta_{\min}}{R_M} \quad \text{and} \quad \mu = \Pi'(m; k) > 0; \\
\text{or,} & \quad 0 \leq m < \frac{k \theta_{\min}}{R_M} \quad \text{and} \quad \mu = \Pi'(m; k) = 0,
\end{align*}
\]
with the marginal expected profit given by

$$
\Pi'(m,k) = \left( \frac{1}{\beta} - R_M \right) - R_M \left( \frac{1}{k} - 1 \right) \left\{ F(\theta^*) + m \frac{\partial F(\theta^*)}{\partial m} \right\}.
$$

(2.16)

When \( \frac{1}{\beta} - R_M > R_M \left( \frac{1}{k} - 1 \right) \left\{ F(\theta^*) + m \frac{\partial F(\theta^*)}{\partial m} \right\} \), money financing is so cheap that banks find it optimal to set it to the maximum allowed by their collateral constraint, determined in equilibrium by:

$$
m_{\text{max}} g'(I - m_{\text{max}} R_M) = \frac{\theta_{\text{min}}}{R_M}.
$$

(2.17)

Alternatively, banks choose an interior \( 0 < m < m_{\text{max}} \) solution to

$$
\left( \frac{1}{\beta} - R_M \right) = R_M \left( \frac{1}{k} - 1 \right) \left\{ F(\theta^*) + m \frac{f(\theta^*)}{1 - F(\theta^*)} \left( \frac{R_M}{k} - \frac{1}{\beta} \right) \right\},
$$

(2.18)

at which the collateral constraint is not binding. The second term in (2.18) captures the private marginal cost of issuing short-term debt, which, making use of the comparative statics in Theorem 1, is increasing in \( m \) provided the hazard rate \( h(x) \equiv \frac{f(x)}{1 - F(x)} \) is increasing. Then,

**Theorem 2 (Competitive Equilibrium).** If \( h(x) \) is increasing, the economy admits a unique competitive equilibrium \((m^*, k^*, \theta^*)\). When the spread \( \frac{1}{\beta} - R_M \) is relatively large, private money creation is at \( m_{\text{max}} \); alternatively, \( 0 < m^* < m_{\text{max}} \). Moreover, \( \frac{\partial m^*}{\partial R_M} < 0 \).

Figure 2.1 shows the effects of a decrease in the short-term interest rate \( R_M \). First, it increases the savings from issuing an incremental unit of private money. Second, it decreases the incidence of runs, the hazard rate, and the severity of fire sales, thereby decreasing the marginal cost associated with this riskier form of financing.

Finally, notice that in the baseline model banks were deemed to invest all the funds raised at the initial date in risky projects. But, would banks prefer to save resources as a buffer against losses in the interim? There are two ways banks can reduce fire sales by one unit: they can issue one more bond and one less money; or, they can issue one more bond and store the proceeds. The former increases

---

9The quantity \( m_{\text{max}} \in (0,1) \) is well defined because the left-hand side in (2.17) is increasing in \( m \) and, by assumption, \( g'(I - R_M) = \frac{\beta}{R_M} > \frac{\theta_{\text{min}}}{R_M} \).
Chapter 2. Fire Sales and the Credit Risk Channel

Figure 2.1. Private money creation. A decrease in the short-term interest rate $R_M$, shifts the marginal gains from issuing money upwards and the cost to the right, increasing private money creation in equilibrium.

the run threshold by $R_B - R_M$, and has a net cost of $R'_B - R_M$ in equilibrium. The latter increases the run threshold by $R_B - 1$, and has a net cost of $R''_B - R_M$ in equilibrium. Given that both alternatives reduce fire sales by one unit, the fact that $R_B - 1 > R_B - R_M$ implies that $R''_B - R_M > R'_B - R_M$. The former strategy is hence strictly preferred, and in equilibrium banks decide not to storage.

2.4 Welfare

I now study the welfare properties of the competitive equilibrium in the economy.

A. Planning Problem

Consider a social planner who, at date 0, can choose the aggregate amount of short-term debt in the system, but whose ability to create money is subject to the same financial constraints as the private economy, namely, (2.2) and (2.9). Given that proceeds from all investment activities carried by banks and patient investors are ultimately rebated back to households in lump-sum fashion, the objective of the social planner is to maximize the expected utility of a representative household, as defined by (2.1).
Notice that consumption at date 2 is given by: \( \mathbb{E}[\theta|\theta > \theta^*] + g(I) \), with probability \( 1 - F(\theta^*) \); and \( \mathbb{E}[\theta|\theta < \theta^*] + g(I - mR_M) + mR_M \), with probability \( F(\theta^*) \). Hence, the problem for the planner can be written as

\[
\max_{m \in [0,1]} W(m) = \left( \frac{\bar{\theta} - 1}{\bar{\beta}} + m \left( \frac{1}{\bar{\beta}} - R_M \right) \right) + \left\{ (1 - F(\theta^*))g(I) + F(\theta^*) \left( g(I - mR_M) + mR_M \right) - \frac{1}{\bar{\beta}} \right\},
\]

subject to constraints (2.2), (2.10) and (2.6), with \( \theta^* \) given by Theorem 1. The welfare function \( W \) comprises three terms: the first one is the net expected return to investment by banks; the second one represents monetary services in the economy; and the last one is the net expected return to investment by patient investors.

The difference between the planner’s problem and that of an individual bank can be clearly seen by comparing (2.19) with the expected profit of an individual bank (2.13). The benefit of issuing short-term debt is the same for both, but the cost is different. For the planner, fire sales are a transfer of resources from banks to patient investors. In case of a run there is a reallocation of \( M \) units of resources away from real investment towards underpriced financial assets. Given that \( g'(I) \geq \frac{\bar{\beta} + \gamma}{\bar{\beta}} > 1 \) this reallocation undermines the real economy by

\[
F(\theta^*) \left( g(I) - g(I - M) - M \right) \geq 0,
\]

obtained by rearranging the expression inside brackets in (2.19) and ignoring constants. Let \( C(m) \equiv g(I) - g(I - mR_M) - mR_M \) denote the real cost of a systemic run, increasing in \( m \).

**Constrained Efficiency.** A constrained efficient equilibrium is an allocation \( m^P \) that solves the planner’s optimization problem.

The existence of constrained efficient equilibria follows from the continuity of \( W \) over the compact choice set

\[
\left\{ m \in [0,1] : mg'(I - mR_M) \leq \frac{\theta_{\min}}{R_M} \right\} = [0, m_{\max}].
\]

\[10\] Ivashina and Scharfstein (2010) document how banks with spare capacity during the financial crisis decided to buy fire-sold securities rather than lend to firms.
Chapter 2. Fire Sales and the Credit Risk Channel

The Lagrangian associated with (2.19) is

\[ L^P(m, \zeta) = W(m) - \zeta \left( m - \frac{k\theta_{\min}}{R_M} \right) \text{ with } k = \frac{1}{g'(1 - mR_M)}, \]

where \( \zeta \) is the shadow value of the constraint (2.2). Constrained efficient equilibria are then characterized by the following Kuhn-Tucker first-order optimality conditions:

\[
\begin{align*}
\text{either, } m &= m_{\max} \quad \text{and} \quad \zeta = \frac{W'(m)}{1 - \theta_{\min} \frac{dk}{dm}} > 0; \\
or, \quad 0 &\leq m < m_{\max} \quad \text{and} \quad \zeta = W'(m) = 0.
\end{align*}
\]

(2.21) (2.22)

Because the planner recognizes how different capital structures affect the aggregate variable \( k \), marginal welfare is given by

\[ W'(m) = \left( \frac{1}{\beta} - R_M \right) - F(\theta^*) R_M \left( \frac{1}{k} - 1 \right) - C(m) \frac{dF(\theta^*)}{dm}. \]

(2.23)

**Proposition 2 (Uniqueness).** If \( \frac{dk}{dm} \) is decreasing in \( m \), then the constrained efficient equilibrium is unique. In addition, \( \frac{\partial m^p}{\partial R_M} \leq 0. \)

When \( \frac{dk}{dm} \leq 0 \), which is equivalent to \( g'' > \frac{[g''']^2}{g''} > 0 \), it follows that \( \frac{dF(\theta^*)}{dm} \) is increasing in \( m \) and the equation \( W'(m) = 0 \) admits a unique solution. On the other hand, the reason why a decrease in the short-term rate of interest \( R_M \) promotes the issuance of short-term debt is the same as in the competitive economy.

**B. Inefficiency**

To understand whether the process of private money creation described in the preceding section involves externalities not internalized by banks, compare the optimality conditions (2.21) and (2.22) with (2.14) and (2.15). There is a wedge between the bank’s solution and the planner’s solution given by,

\[ \tau^* = -\frac{\theta_{\min} \frac{dk}{dm}}{R_M} + \left[ C(m^p) \frac{dF(\theta^*)}{dm} - m^p R_M \left( \frac{1}{k^p} - 1 \right) \frac{dF(\theta^*)}{dm} \right], \]

(2.24)

\[ \text{collateral} \quad \text{credit risk} \]
with all the expressions evaluated at the constrained efficient allocation.

In other words, the size of short-term financing picked individually by banks diverges from the one chosen by a social planner. The fact that banks ignore the general equilibrium effects that an incremental change in \( m \) have on the fire sale discount \( k \), result in two distinct fire sale externalities captured by the expression in (2.24): a collateral externality and a credit risk externality. These externalities arise because banks do not internalize that changing \( m \) alters the fire sale discount by \( \frac{dk}{dm} \) and has two consequences:

**Collateral Externality.** First, it affects the value of assets that all banks can pledge as collateral which, in turn, loosens or tightens the constraint (2.2) by \( \frac{dk}{dm} \), and has a social shadow value of \( \xi \). Given that \( \frac{dk}{dm} < 0 \) and \( \xi \geq 0 \), the collateral externality is always negative and generates short-term over-borrowing ex-ante and over-selling ex-post.

**Credit Risk Externality.** Second, it affects the market liquidity for banks assets and therefore their credit risk through (2.9). Such a change heightens or abates systemic risk by \( \frac{dF(\theta^*)}{dm} = \frac{\partial F(\theta^*)}{\partial m} + \frac{\partial F(\theta^*)}{\partial k} \frac{dk}{dm} \), which costs the real economy \( C(m) \); out of this change, however, banks only take into account \( mR_M(\frac{1}{k} - 1)\frac{\partial F(\theta^*)}{\partial m} \). To determine the sign of the credit risk externality, I decompose the expression inside brackets in (2.24) as

\[
\phi(m) = C(m)\frac{\partial F(\theta^*)}{\partial k} \frac{dk}{dm} - Q(m)\frac{\partial F(\theta^*)}{\partial m}.
\]

Given that both terms are non-negative, banks are penalized for not considering how their individual choice of \( m \) indirectly affects the credit risk of all the other market participants through the variable \( k \); and they are compensated for the fire sale transfers to patient investors in case of a run. Notice that the function \( Q(m) \), defined by (2.12), represents the financial profits from purchasing underpriced fire-sold assets and coincides with the difference between the private and social cost of a run is \( mR_M(\frac{1}{k} - 1) - C(m) \). I prove at the end of the chapter, that of this two opposing forces, the former is always stronger and thus renders the credit risk externality negative, \( \phi(m^P) \geq 0 \).

The next theorem is one of the main results of the chapter.
Theorem 3 (Constrained Inefficiency). Both the collateral and the credit risk externality are negative, rendering private money creation excessive from a social perspective.

It is instructive to understand the nature of the fire sales externalities. Theorem 3 is a particular case of a generic inefficiency result in economies with incomplete markets, which can be traced back to the work of Greenwald and Stiglitz (1986). When prices affect investments through channels other than budget constraints, the First Welfare Theorem no longer applies. In this setting, the two fire sale externalities not internalized by banks when choosing their capital structures are respectively associated to the two channels that tie financial constraints to credit market prices; namely, the collateral channel (2.2) and the credit risk channel (2.9).

Given that $\phi(m) = 1'(m; k) - W'(m)$, the fact that the credit risk externality is negative implies that banks overvalue the issuance of private money relative to the social planner. The associated welfare loss is depicted in Figure 2.2. However, notice that both social and private equilibrium allocations coincide at the corner $m_{\text{max}}$ when $\mu^p > 0$. The reason is that in the model the investment scale is fixed. If banks were allowed to optimally choose the size of their investment, they would be incentivized to increase investment in order to loosen the collateral constraint whenever money financing is very attractive. In this case, investment and financing decisions would be coupled together, and as a result, the high-spread region where the collateral constraint is binding would feature both over-borrowing and over-investment.

C. Uncertainty and the Severity of Crises

Suppose that the payoff of the risky asset is $\theta = \bar{\theta} + \sigma v$, where the noise $v$ is uniformly distributed over the interval $[-\frac{1}{2}, \frac{1}{2}]$, and $\sigma > 0$ measures the size of the uncertainty. Then, (2.9) is a quadratic equation, and

$$\theta^*(m, k) = \theta_{\text{max}} - \sqrt{2\sigma \left( \bar{\theta} - \frac{mR_M}{k} - \frac{1 - m}{\beta} \right)}.$$  

First, Figure 2.3 considers the effects of a change in $R_M$, with $\sigma = \frac{\bar{\theta}}{\beta}$ and $\bar{\theta} = \frac{\bar{\theta}}{\beta}$. Also, $\frac{1}{\beta} = 1.04$, and the spread $\frac{1}{\beta} - R_M$ moves between 10 and 400 basis points. Private money creation is excessive. For example, at $R_M = 1.01$ banks set the
Figure 2.2. Welfare loss. The shaded region represents the welfare losses associated with the credit risk externality. The solution \( m^* \) is not constrained efficient because the planner's marginal cost of issuing short-term debt exceeds its marginal benefit.

Figure 2.3. Variations in the short-term interest rate. The black (grey) curve corresponds to private (social) solutions. Assume that patient investors' investment technology is \( \varphi(\cdot) = a \log(\cdot) \) with \( a = (\beta - \frac{1}{\beta})^{-1} \). Furthermore, \( I = a\beta R_M \), which implies that \( k(m) = R_M[\beta - m(\beta - \frac{1}{\beta})] \).
Figure 2.4. Effect of uncertainty in investment return. The black (grey) curve corresponds to private (social) solutions. Parametric and functional assumptions are the same as in Figure 2.3.

issuance of short-term debt at approximately 40%, which is 4 percentage points above the choice of the social planner. The result is an increase in the probability of a systemic crisis from 4% to 5%. Notice also that, as the spread compresses, issuing money becomes more expensive thus less attractive, and private and social incentives become more align.

Figure 2.4 illustrates how, as banks’ investment prospects improve in a second order stochastic dominance sense, the probability of financial crises decreases (bottom panel). When the distribution of returns $F$ becomes more “concentrated” around the mean $\bar{\theta}$ (smaller $\sigma$), banks become less concerned about tail risks and the possibility of damaging runs, increasing their reliance on short-term funding. Interestingly then, even though the incidence of runs is lower, their severity, measured by the fire sale discount, can actually be greater because of more significant market-wide liquidations.\(^\text{11}\) As the example shows (top pannel), better economic

\(^{11}\text{Acharya and Viswanathan (2011) describe a similar result in a model where good economic}
times may lead to higher fire sale inefficiencies.

### 2.5 Policy

The presence of a collateral and a credit risk externality provides a rationale for welfare improving policy interventions by a central authority. Assuming a perfectly informed planner, there is no advantage of quantity regulation over price regulation, as shown in the seminal work of Weitzman (1974).

#### A. Macroprudential Regulation and Financial Stability

The simplest approach to rein in private money creation and restore constrained efficiency is to set a maximum cap at its constrained efficient level, $m^P$. Alternatively, a Pigouvian (corrective) tax $\tau^*$ can be levied on each unit of short-term debt raised by banks. Banks’ optimization problem is the same as before, except that expected profits from investment are now

$$
\left( \bar{\theta} - \frac{1}{\bar{\rho}} \right) + m \left( \frac{1}{\bar{\rho}} - R_M \right) - F(\theta^*) m R_M \left( \frac{1}{k} - 1 \right) - m \tau^*. \tag{2.26}
$$

Faced with a tax $\tau^*$, banks are then led to carry out an individual benefit-cost analysis that internalizes the fire sale externalities that they impose on the rest of the financial system. Indeed, the new competitive equilibrium $(m^*_k, k^*_e)$ is characterized by conditions analog to (2.14), (2.15). Employing (2.24) and introducing (2.23) in the new shadow value $y^*$ yields,

$$
\mu^*_f = \zeta \frac{\theta_{\text{min}}}{R_M} \frac{dk}{dm} + W'(m^*_f) + \left\{ C(m^*_f) \frac{dF(\theta^*_t)}{dm} - C(m^P) \frac{dF(\theta^P)}{dm} \right\}
\right.
$$

funds yield cheaper short-term debt, and induce the entry of higher-leverage firms. Adverse asset shocks in good times then lead to greater de-leveraging ex-post, resulting in deeper fire-sale discounts and more severe crises. This phenomenon resembles the “volatility paradox” described by Brunnermeier and Sannikov (2014). Times of low volatility tend to be associated with a buildup of leverage, hence lower exogenous risk can lead to more extreme financial crises.
where, for notational simplicity, I have defined \( \theta^P \equiv \theta^*(m^P, k^P) \). Then, substituting for \((m^P, k^P, \xi)\) gives

\[
\xi \left(1 - \frac{\theta_{\min}}{R_M \frac{dk}{dm}}\right) = W'(m^P),
\]

which reduces to (2.21) and (2.22), and shows that \((m^*_T, k^*_T)\) is constrained efficient.

Notice that when the regulator reduces the aggregate amount of short-term debt, it reduce fire sales and thereby redistributes resources away from underpriced financial assets towards real investment. In addition, based on the comparative statics of Theorem 1, it improves the systems' resilience to panic runs on the banking sector by directly reducing the fragility of balance sheets and indirectly increasing market liquidity. Formally, \( m^* \geq m^P \) results in \( k^* \leq k^P \), and implies that \( \theta^*(m^P, k^P) \leq \theta^*(m^*, k^*) \). Hence,

**Corollary (Money and Crises).** Macroprudential regulations that limit the aggregate amount of short-term debt ex-ante reduce the probability of a systemic crisis.

### B. Lender of Last Resort and Debt Guarantees

According to Bagehot (1873) doctrine, a central authority acting as a Lender of Last Resort can inject liquidity into the financial system and prevent destabilizing fire sales of assets should short-term investors begin to lose confidence and demand their credit back. In the model, suppose the government has limited taxing powers and can only raise an aggregate amount of resources \( G \) which is not enough to satisfy total liquidity needs by banks. Then, a LOLR policy amounts to the central bank investing \( G \) along side patient investors' \( I \), reducing the fire sale discount to

\[
k = \frac{1}{g'(I - M + G)}.
\]  

(2.27)

Alternatively, suppose that the government employs those resources to insures a number \( m^G \) of short-term debt claims. Now, \( M = (m - m^G)R_M + G \) where \( m - m^G \) represent the amount of uninsured claims and \( G = m^G R_M \). To avoid insurance fraud, assume that the government prohibits banks from liquidating more than the relative fraction of uninsured claims. Then, \( \frac{(m-m^G)R_M}{k^*_{\min}} \leq \frac{(m-m^G)R_M}{M} \), implying that private money creation is subject to the same collateral constraint (2.2). Given that only uninsured claims may be subject to runs, the fire sale discount is also
determined by (2.27). However, compared to a LOLR, deposit insurance is more effective as it has the added benefit of decreasing the run threshold:

\[ x = \mathcal{P} \left[ \frac{mR_M}{k} + (1 - m)R_B(x) - \frac{G}{k} \right] \]

thus the probability of a systemic crisis.

In other words, interventions as a LOLR or government (short-term) debt guarantees can make fire sales less severe, allowing for more private money to be created in equilibrium. However, as long as \( \frac{dK}{dm} < 0 \), the competitive equilibrium is still constrained inefficient and there is scope for macroprudential policies to control short-term debt financing and improve welfare in the economy.

**Proofs and Extensions**

**A. Coordination Failures**

The proof of Proposition 1 follows standard global games arguments, and reduces to checking some simple properties on the payoff and information structures.

**Proof of Proposition 1.** Given a realized investment payoff \( \theta \) and a fraction \( \lambda \) of creditors withdrawing, the differential return from withdrawing over rolling over for an individual short-term creditor is given by,

\[ \Delta(\lambda, \theta) \equiv 1 - \mathbb{I}\{\theta > \theta_{\text{run}}(\lambda)\}, \]

where \( \mathbb{I} \) denotes the indicator function of the set. Clearly, \( \Delta \) is monotone in \( \theta \) and \( \lambda \). Let \([\theta_{\text{run}}]^{-1}(\theta)\) be the unique value of \( \lambda \in [0, 1] \) that solves \( \theta_{\text{run}}(\lambda) = \theta \). Based on (2.4), this is simply

\[ [\theta_{\text{run}}]^{-1}(\theta) = \frac{\theta - \theta_{\text{run}}(0)}{mR_M \left( \frac{1}{k} - 1 \right)}. \]

Then, the Laplacian indifference condition:

\[ \int_0^1 \Delta(\lambda, \theta^*)d\lambda = 0 \quad (2.28) \]
has a unique solution $\theta^*$, determined by $[\theta_{\text{run}}]^{-1}(\theta^*) = 1$. It is straightforward to see that $\theta^*$ is equivalently given by (2.5). As Morris and Shin (2003) and Goldstein and Pauzner (2005) show, the above properties combined with the existence of lower and upper dominance regions, and the fact that the noise terms $\{\epsilon_j\}$ are independently uniformly distributed, are sufficient to establish the desired result.

Public Information and Strategic Uncertainty. The introduction of a noisy public signal has no effect on the outcome of the coordination game. With public information—or a common informative prior for that matter—Laplacian beliefs no longer determine the optimal switching threshold; this is because the proportion of other creditors withdrawing may not be uniform. Hence, condition (2.28) needs to be replaced by

$$
\int_0^1 \Delta(\lambda, \theta^*) \varphi(\lambda) d\lambda = 0,
$$

where $\varphi$ is a probability density that factorizes the posterior belief about the size of the run. Nevertheless, given that $\varphi > 0$ for every $\lambda \in (0,1)$, the optimal run threshold is still defined by $[\theta_{\text{run}}]^{-1}(\theta^*) = 1$ as before.

B. Equilibrium Run Thresholds

The proof of Theorem 1 is based on the observation that fixed points (2.9) are local maxima of the function $\zeta(\cdot; m, k)$, as illustrated in Figure 2.5.

Proof of Theorem 1. By definition,

$$
\zeta(x; m, k) = \frac{1}{1 - F(x)} \left( \frac{mR_M}{k} + \frac{1-m}{\beta} - F(x)E[\theta|\theta < x] \right).
$$

Integrating by parts, yields

$$
F(x)E[\theta|\theta < x] = \int_{\theta_{\min}}^{x} \theta f(\theta) d\theta = xF(x) - \int_{\theta_{\min}}^{x} F(\theta)d\theta,
$$

which implies that $\frac{d}{dx}F(x)E[\theta|\theta < x] = xf(x)$.

Then, after some simple algebraic manipulations, it can be shown that

$$
\frac{\partial \zeta}{\partial x}(x; m, k) = h(x)\left(\zeta(x; m, k) - x\right),
$$

(2.29)
Figure 2.5. Equilibrium run threshold. The equation $x = \mathcal{P}[\zeta(x; m, k)|_\Theta$ admits a unique (fixed-point) solution $\theta^*(m, k)$. The figure illustrates the intersection between the graphs of $\zeta(\cdot; m, k)$ and the identity map.

where $h(x) = \frac{f(x)}{1-F(x)}$ is the hazard rate associated with $F$, which is positive for all $x \in \Theta$. Then, $\frac{\partial \zeta}{\partial x} \geq 0$ if and only if $\zeta(x) \geq x$. Moreover,

$$\frac{\partial^2 \zeta}{\partial x^2}(x; m, k) = h'(x) \left( \zeta(x; m, k) - x \right) + h(x) \left( \frac{\partial \zeta}{\partial x}(x; m, k) - 1 \right). \tag{2.30}$$

Consequently, from (2.29), all interior fixed points $x^*$ satisfy:

$$\frac{\partial \zeta}{\partial x}(x^*; m, k) = 0.$$

In addition, in light of (2.30), it follows that

$$\frac{\partial^2 \zeta}{\partial x^2}(x^*; m, k) = -h(x^*) < 0,$$

which demonstrates that all interior fixed points are local maxima.

As a result, if $\zeta(\theta_{\min}; m, k) \leq \theta_{\min}$ it follows that $\zeta(x; m, k) \leq x$ for all $x \in \Theta$, and the unique solution to (2.9) is $\theta^*(m, k) = \theta_{\min}$, the product of the projection operator $\mathcal{P}[\cdot]_\Theta$. Alternatively, if $\zeta(\theta_{\min}; m, k) > \theta_{\min}$, then (2.9) admits a unique solution $\theta^*(m, k) \in \text{int} \Theta$; this is because, by Assumption 1, $\bar{\theta} > \frac{mR_M}{k} + \frac{1-m}{\bar{\theta}}$ for all $m \in [0, 1]$ thus $\lim_{x \to \theta_{\max}} \zeta(x; m, k) = -\infty$. 
Comparative statics follow immediately by implicit differentiation
\[
\frac{\partial \theta^*}{\partial m} = \frac{\partial \xi}{\partial m} \bigg|_{\theta^*} = \frac{\partial \xi}{\partial m} \bigg|_{\theta^*} \quad \text{and} \quad \frac{\partial \theta^*}{\partial k} = \frac{\partial \xi}{\partial k} \bigg|_{\theta^*} = \frac{\partial \xi}{\partial k} \bigg|_{\theta^*}.
\]

Self-Fulfilling Solvency Crises

When banks default in the baseline model, the remaining assets in their balance sheets after fire sale liquidations are equally divided among long-term creditors. Alternatively, with "partial" recovery after default the interest rate on bonds becomes,
\[
R_B(x) = \frac{1}{\beta(1-F(x))} - \frac{b}{1-m} \left( \mathbb{E}[\theta|\theta < x] - \frac{mR_M}{k} \right) \frac{F(x)}{1-F(x)},
\]
where the parameter \( b \in [0,1] \) measures the size of the recovery.

In particular, consider the scenario without recovery after default \((b = 0)\) where creditors holding debt claims to a bankrupt bank receive a payout of zero; in this case, \( R_B(x) = \frac{1}{\beta(1-F(x))} \) which is concave in \( x \) because the hazard rate \( h(x) \) is increasing. To understand equation (2.9) in this case, suppose initially that banks are only financed by bonds, so that \( m = 0 \). For starkness, assume that \( \theta_{\text{min}} = \frac{1}{\beta} \) and \( \bar{\theta} = \frac{2}{\beta} \).

Given that \( \theta_{\text{min}} = \frac{1}{\beta} \), it follows that \( \theta_{\text{min}} \) is a fixed point with \( R_B = \frac{1}{\beta} \). However, \( \bar{\theta} \) and \( \theta_{\text{max}} \) are also fixed points: the former because \( \bar{\theta} = \frac{2}{\beta} \) and \( F(\bar{\theta}) = \frac{1}{2} \); the latter as a consequence of \( \lim_{x \to \theta_{\text{max}}} \zeta(x;0,k) = +\infty \). In the first equilibrium, banks are always solvent and never fail hence the interest rate on bonds equals the inverse of households utility discount factor. In contrast, in the other equilibrium solutions creditors fear default so they demand higher interest rates on bonds; in turn, elevated interest rates raise the run threshold, which increases the probability of default and thus justifies the hike in interest rates in the first place. Given that such high interest rates are unwarranted, I label these equilibria (self-fulfilling) "solvency crises."
The following proposition characterizes the solutions to (2.9) in general.

**Proposition 3 (Multiple Equilibria).** For all \( m \in [0, 1) \) and \( k \in [R_{M}, \beta R_{M}] \), the economy admits three equilibria: \( \theta^{*}, \theta^{*}_{u} \) and \( \theta^{*}_{sc} \). In addition: \( \theta^{*} \) is increasing in \( m \) and decreasing in \( k \), and \( \theta^{*}_{sc} \equiv \theta^{*}_{max} \).

Proposition 3 is best understood graphically from Figure 2.6. The proof is based on the observation that the equation \( \frac{\partial \xi}{\partial m}(x; m, k) = 0 \) admits a unique solution \( x_{T} \) which is independent of \( m \), hence \( \xi(x_{T}; m, k) = \xi(x_{T}; 0, k) \) for all \( m \in [0, 1] \). Consequently, the graph of \( \xi(x; m, k) \) must lie between \( \xi(x; 0, k) \) and \( \xi(x; 1, k) \equiv \frac{R_{M}}{k} \).

The source of equilibrium multiplicity is not the coordination problem of Diamond and Dybvig (1983). In the model bank runs are based on panics but the realization of \( \theta \) allows creditors beliefs to coordinate and thus determines whether they occur or not. Here multiplicity arises as the result of the two-way feedback between higher interest rates and higher probability of default, as originally formalized by Calvo (1988). The crucial assumption is that of “price taking,” whereby banks may face different financing costs depending on aggregate expectations in the credit market. Risky investments then sow the seed of indeterminacy.

The equilibrium solution \( \theta^{*}_{u} \) is locally unstable in the sense that a perturbation in the fundamentals of the model will drive the economy towards either \( \theta^{*} \) or \( \theta^{*}_{sc} \). This
equilibrium is also pathological in other ways. First, Frankel, Morris, and Pauzner (2003) show that unstable equilibria of this kind are not selected by players in global games. Second, this equilibrium exhibits “paradoxical” comparative statics. To illustrate this point, consider the scenario sketched in Figure 2.6. Intuitively, \( \theta_u^* \) should be decreasing in both \( m \) and \( k \); the former because an increase in \( m \) alleviates insolvency concerns, given that \( \theta_u^* > x_T \), and the latter because illiquidity risk subsides as \( k \) increases. However, notice that at this equilibrium \( \frac{\partial c}{\partial m} > 0 \) and \( \frac{\partial c}{\partial x} > 1 \). Therefore, by implicit differentiation:

\[
\frac{\partial \theta_u^*}{\partial m} = \frac{\frac{\partial c}{\partial m}}{1 - \frac{\partial c}{\partial x} |_{\theta_u^*}} \geq 0 \quad \text{and} \quad \frac{\partial \theta_u^*}{\partial k} = \frac{\frac{\partial c}{\partial k}}{1 - \frac{\partial c}{\partial x} |_{\theta_u^*}} \geq 0.
\]

C. (Short-Term) Over-Borrowing

In the main text, I argued that the credit risk externality is negative, thus it led to excessive private money creation. In order to prove this result, it suffices to show that (2.25) is non-negative.

First, from Theorem 1 and (2.11), it follows that

\[
\frac{\partial F}{\partial k} \frac{dk}{dm} / \frac{\partial F}{\partial m} = \frac{-mR_M}{k} \frac{k}{k-1} \frac{g''(I-mR_M)R_Mk^2}{g'(I-M) + M^2 g''(I-M)} \geq 0.
\]

Interestingly, notice that this expression is independent of the probability distribution \( F \). Now recall that \( C(m) \equiv g(I) - g(I - mR_M) - mR_M \) and \( Q(m) \equiv \frac{mR_M}{k} - (g(I) - g(I - mR_M)) \). Given that the second-order Taylor expansion of the function \( g(I) \) is given by

\[
g(I) \approx g(I - M) + Mg'(I-M) + \frac{M^2}{2} g''(I-M),
\]

it then follows that

\[
\frac{Q(m)}{C(m)} = -\frac{(mR_M)^2}{2C(m)} g''(I-mR_M),
\]

where I have discarded all terms higher than third-order in the numerator.
The objective is now to demonstrate that (2.25) is non-negative at the constrained efficient equilibrium \((m^P, k^P)\). Based on the expressions above, this is equivalent to showing that

\[
\frac{m^p}{2} \left( \frac{R_M}{k^P} - \frac{1}{\beta} \right) \leq C(m^P).
\]

Notice that the above inequality is equivalent to

\[
R_M - \frac{1}{2\beta} \leq \frac{g(I) - g(I - m^P R_M)}{m^p} - \frac{1}{2} R_M g' \left( I - m^P R_M \right).
\]

Because \(g\) is concave, the right-hand side is larger than \(\frac{1}{2} R_M g' \left( I - m^P R_M \right)\) as implied by (2.31). The result then obtains by observing that

\[
R_M - \frac{1}{2\beta} < \frac{1}{2} \frac{R_M}{k^P}.
\]

Indeed, notice that because \(k^P \leq 1\),

\[
\frac{1}{2} \left( \frac{R_M}{k^P} + \frac{1}{\beta} \right) \geq \frac{1}{2} \left( R_M + \frac{1}{\beta} \right) > R_M.
\]

The last inequality is due to the fact that, in equilibrium, \(R_M = \frac{1}{\beta + \gamma} < \frac{1}{\beta}\).  \(\blacksquare\)


CHAPTER 3

Information Panics and Liquidity Crises

Asymmetric information and uncertainty about asset quality can create adverse selection and disrupt the efficient functioning of markets, even when the gains from trade are common knowledge (Akerlof, 1970). A “lemons problem” occurs in credit markets when lenders cannot distinguish between good and bad-risk borrowers. The result may be that the former drop out of the market and thus forgo profitable investment opportunities. Gorton (2008, 2009) describe the beginning of the financial crisis of 2007-09 as a panic, documenting how the information asymmetry that originated by the complexity of securitized assets, combined with a negative shock to the fundamentals in the economy, caused a sudden evaporation of liquidity and the breakdown of credit markets.

This chapter offers a theory of endogenous liquidity crises based on “information panics,” equilibrium scenarios where fears of asymmetric information become self-fulfilling. It traces the fragile nature of liquidity to agents’ individual rent-seeking motives to gain informational advantages. It highlights how, when aggregate shocks generate enough uncertainty about the quality of the assets that back debt securities, high rates of interest and the acquisition of financial expertise to extract rents from uninformed investors in the market can reinforce each other, and lead to an inefficient outcome in which adverse selection problems undermine real economic activity.

The firms featured in the model have the ability to acquire costly financial expertise, which allows them to better interpret future private information regarding the quality of preexisting legacy assets in their balance sheets. Legacy assets are
used to back debt securities which firms sell in a competitive market in order to finance new projects. Investment opportunities are identical across firms and have positive net present value. The only heterogeneity is on the types of legacy assets, which outside investors cannot distinguish. This is precisely the source of adverse selection in the credit market. Firms with safer legacy assets subsidize borrowing by firms with riskier ones. By acquiring financial expertise, a firm can more accurately evaluate the type of its legacy asset, and thus its probability of repayment. More precise information therefore minimizes the expected losses accrued by borrowing with safe legacy assets, and maximizes the expected gains extracted from uninformed investors by borrowing with riskier ones.

When the marginal cost of information is relatively low firms do not invest in expertise, hence the unique equilibrium in the economy features symmetric ignorance and low interest rates. On the other hand, when this cost is relatively high, the unique equilibrium is characterized by full asymmetric information and high interest rates. In the former, all firms borrow and invest in projects; in the latter, adverse selection causes firms with safe legacy assets to withdraw from the credit market and thus forgo their investment opportunities. When the marginal cost of information is neither too low nor too high, both equilibria may coexist. In this case, I label the equilibrium with asymmetric information an “information panic” to emphasize the fact that its features are not justified by the fundamentals in the economy.

Information panics can be understood as follows. When investors in the market become worried about the potential of asymmetric information about the quality of legacy assets, they raise interest rates to make up for their feared disadvantage. In turn, anticipating an increase in the cost of borrowing, firms have incentives to acquire financial expertise that helps them better evaluate their probability of repayment. More information then worsens the average credit risk of the pool of borrowers, and thus justifies investors’ initial concerns. Asymmetric information materializes as a self-fulfilling equilibrium phenomenon.

The mechanism that gives rise to such adverse dynamics is a feedback loop between the extent of asymmetric information and the cost of borrowing from the market. The possibility of multiple equilibria is due to the fact that a firm’s optimal strategy to acquire financial expertise is non-monotonic in the interest rate. To see this, notice that the incentives to acquire financial expertise are entirely driven
by the expected losses from borrowing at an unfairly high interest rate in case of owning a safe legacy asset. For low interest rates, firms might borrow regardless of their private information because of the potential gains in case their legacy assets are of the risky type. As interest rates increase these gains decrease, hence the incentives to acquire financial expertise increase. But once interest rates reach a certain threshold, the cost of borrowing becomes excessively high that it is optimal for firms to limit losses by deciding not to borrow if their private information signals the ownership of safe legacy assets. From this point forward, costly expertise only reduces the expected informational rents.

The multiplicity of equilibria is robust to the introduction of noisy private signals about fundamentals in the economy. The reason is that information panics are not a “coordination failure.” Given that firms take interest rates as given, there is no explicit strategic motive for them to coordinate their actions regarding the acquisition of financial expertise. As a result, global games arguments based on incomplete information do not affect the findings of the model regarding the fragility of liquidity in financial markets as a consequence of information panics.

In the model, information has no social value for it only serves “rent-seeking” purposes instead of “value-creation” (Hirshleifer, 1971). This is for starkness; in reality, information about productive technologies can be socially valuable by allowing, for example, a more efficient allocation of resources. However, the model highlights how information acquisition that is purely based on individual speculative motives can be detrimental to the economy as a whole: first, it can destroy value by causing adverse selection which hinders good firms’ ability to obtain financing; and second, it can leave financial markets susceptible to self-fulfilling fears of asymmetric information. Acquiring financial expertise is like a prisoners’ dilemma: it benefits firms individually, but collectively they would be better off if they remained uninformed. Symmetric ignorance maximizes welfare because it allows full investment in the economy.

Symmetric ignorance can be preserved by requiring firms to disclose to lenders what they know about the quality of their legacy assets. Mandatory disclosures have a deterrent effect: they impede the extraction of informational rents from the credit market, and thus discourage the acquisition of financial expertise for speculative purposes. Credit rating agencies can also play an important role in regulating the production of rent-seeking information. Although the model abstracts away
from the intricacies of the industry, it shows that ratings made publicly available regarding the quality of legacy assets do not have to be perfectly accurate to be effective. Assessments with small errors can still reduce the extent of asymmetric information to the point that gaining informational advantages is no longer profitable.

Related Literature

I build on the classic framework of Myers and Majluf (1984), in which a firm must raise cash to invest in a positive net present value project. Debt is the optimal security to raise capital because, as demonstrated by Nachman and Noe (1994), it minimizes the mispricing of securities: the undervaluation of safe (high quality) firms, and the overvaluation of risky (low quality). Other theories about the optimality of debt include DeMarzo and Duffie (1999), who study the problem of designing a security that maximizes the payoff of a seller that will receive private information prior to the selling. Given that the demand curve for uninformed buyers is downward sloping due to the potential of adverse selection, debt is optimal because it reduces the price responsiveness to the quantity sold. Closer to my model is the work of Dang, Gorton, and Holmstrom (2012), who argue that money markets function well as long as agents feel no need to question the value of the securities being traded. Debt is optimal in liquidity provision because it minimizes parties incentives to produce private information. In their model, in stark contrast with mine, adverse selection never materializes in equilibrium since in case of bad public news about fundamentals, information acquisition is avoided by reducing the trading volume below the expected value of debt.

The conclusion that over-investment in costly financial expertise by firms may destabilize markets by causing adverse selection is also shared by other papers. Glode, Green, and Lowery (2012) develop a bargaining model where expertise acquired by firms intermediating trade improves their ability to value securities, and thus protects them from opportunistic bargaining by counter parties. In their setting, different from my model, information acquisition acts as a threat that improves the terms of trade and it is therefore never used in equilibrium. Market collapses occur when there is a jump in volatility. In a similar bargaining model of over-the-counter trading with common knowledge about the gains from trade, Dang (2008)
shows that the mere possibility of information acquisition is a source of inefficiency. Fishman and Parker (2015) focus on the externalities involved in the process of information acquisition. Their model features strategic complementarities in firms’ valuation capacity, which results in multiple equilibria. However, in their setting credit crunches would never occur if there were always gains from trade, as it is the case in mine; in addition, strategic complementarities in my model are not global.

Previous work has linked fluctuations in market liquidity to changes in the severity of adverse selection. For example, in Eisfeldt (2004) liquidity is endogenously determined by the amount of trade for reasons other than information, such as consumption, investment, or portfolio rebalancing; in Bolton, Santos, and Scheinkman (2011) liquidity crises worsen adverse selection problems as the extent of asymmetric information grows over time; and in Morris and Shin (2012), adverse selection can become contagious and trigger a confidence crisis in markets through uninformed traders’ participation decisions. I offer an alternative explanation as to why adverse selection may arise endogenously as an equilibrium outcome; namely, the possibility of speculation in markets granted by some parties’ ability to acquire private foreknowledge. Moreover, adverse selection may be the result of information panics, which highlights the fragile nature of the notion of liquidity. In the same vein, Plantin (2009) and Malherbe (2014) also argue that liquidity dry-ups may be the result of self-fulfilling prophecies; the former driven by a “learning by holding” mechanism, and the latter by firms’ strategic decisions to hold cash.

Finally, the model developed is related to the work of Philippon and Skreta (2012) and Tirole (2012). Their focus is the study of cost-minimizing policy interventions by the government to improve lending and investment in markets impaired by adverse selection. From a normative perspective, I complement this line of research by studying ex-ante policies to limit the acquisition of speculative information and thus preserve symmetric ignorance in credit markets.

Outline of the Chapter. The rest of the chapter proceeds as follows. Section 3.1 introduces the basic environment. Section 3.2 studies the effect that different degrees of asymmetric information on (partial) equilibrium market rates. Section 3.3 endogenizes firms’ decisions to acquire financial expertise. Finally, Section 3.4 conducts a welfare analysis, which warrants the regulation of information for speculative purposes.
3.1 The Model

The economy is populated by a continuum of firms with unitary mass that can borrow from a competitive credit market. There are two dates: investment and contracts take place at \( t = 1 \); payoff returns happen at \( t = 2 \). All parties are risk neutral, and the risk-free rate is zero.

A. Financial Assets

All firms start without cash or preexisting liabilities; the only thing in their balance sheet is a "legacy asset" which vary in quality. All legacy assets have the same expected return \( \bar{a} \) at date 2, but differ in their riskiness which I label by \( \theta \). Specifically, an asset of type \( \theta \) pays \( a_\theta \) with probability \( p_\theta \), and 0 with probability \( 1 - p_\theta \). For simplicity I assume that there are only two types \( \{L, H\} \): \( \bar{a} = p_L a_L = p_H a_H \), \( p_L > p_H \), and \( a_L < a_H \). Notice that subscripts denote the "riskiness" of the asset. Types \( \theta \in \{L, H\} \) are in proportion \((\lambda, 1 - \lambda)\), respectively. For notational convenience, let \( \lambda_L = \lambda \) and \( \lambda_H = 1 - \lambda \).

Information Structure. Each firm \( i \) knows the prior distribution of types, and also receives a private signal \( s_i \in \{L, H\} \) indicating the particular type of its legacy asset. The probability that the type indicated by a signal \( s \) is indeed the true type, is given by

\[
\pi_s(\mu) \equiv \lambda_s + (1 - \lambda_s)\mu, \tag{3.1}
\]

where the parameter \( \mu \in [0, 1] \) represents the accuracy of the signal, and depends on financial expertise acquired at a prior date. Notice that if \( \mu = 0 \), then signals are completely uninformative. On the other hand, signals are completely revealing when \( \mu = 1 \). Therefore, \( \mu \) determines the degree of asymmetric information in the economy. For now, \( \mu \) is fixed.

B. Real Projects

At date 1 all firms have access to the same investment opportunity. Investment requires a fixed amount of funding \( l \) and delivers a random payoff \( v \in \{V, 0\} \) at date 2, distributed according to \((p_v, 1 - p_v)\). Let \( \overline{v} \equiv \mathbb{E}[v] \) denote the expected value.
of $v$. I assume that investment projects have positive NPV: $\overline{v} > l$.

Conditional on investment, a firm's total income at date 2 is $y = a + v$ with mean $\overline{y} = \overline{a} + \overline{v}$. For a legacy asset of type $\theta$, $F(y|\theta)$ and $f(y|\theta)$ denote the probability distribution and density of $y$, respectively. Notice that, by construction, $F(y|H)$ is a mean preserving spread of $F(y|L)$.

C. Contracts

Because firms have no cash at date 1, they have to raise $l$ from the market if they wish to invest in their projects. Loans need to be repaid at date 2 after the investment payoff is realized. I make two assumptions regarding the contracting game.

Assumption 4 (Fungibility). The only observable outcome is a firm's total income $y$.

This renders private information about the quality of legacy assets relevant as it prevents contracts from being written directly on $v$, and it is the same restriction imposed by Myers and Majluf (1984) and Philippon and Skreta (2012).

Assumption 5 (Schedules). Repayment schedules are nondecreasing in total income $y$.

This condition is standard in the literature of financial contracting and dates back to Innes (1990) and Nachman and Noe (1994), and can be justified by the possibility of hidden trades. If schedules were decreasing in $y$, the borrower could receive a higher return by borrowing from a third party and secretly adding this cash to its balance, obtaining a lower repayment and then paying back the third party immediately.

Notice that raising external capital is plagued by adverse selection in that the securities issued by firms with high quality assets may be imitated by firms with low quality assets, which results in the undervaluation of the former and the overvaluation of the latter. Under Assumptions 4 and 5, Nachman and Noe (1994) show that in this context the contracting game has a unique equilibrium where all the firms that invest pool on the same security, and this security is a simple debt contract. Debt is optimal because it minimizes the aforementioned mispricing of securities. Let $r$ denote the gross interest rate at which firms can borrow; $R = rl$ is the promised repayment at date 2. I will work with both variables interchangeably.
Discussion. Debt is the optimal borrowing security provided the distribution of payoffs can be ranked according to hazard rate dominance:

\[
\frac{f(y|L)}{1 - F(y|L)} < \frac{f(y|H)}{1 - F(y|H)} \quad \text{for all} \quad y \in \bigcap_\theta \{y : f(y|\theta) > 0\}.
\]

In the current setting, the above is equivalent to \(p_L > p_H\). However, with a more general distribution of types such ordering would need to be imposed, allowing legacy assets to be ranked by their quality. It is worth emphasizing that the conclusions drawn in the sequel do not hinge on the existence of only two types and the discrete nature of probability distributions; this is only for expositional purposes.

D. Investment Decisions

The net expected payoff from investment for a firm with a legacy asset of type \(\theta\) is given by

\[
U_\theta(R) \equiv \mathbb{E}_y[\max\{y - R, 0\}|\theta] \geq 0. \tag{3.2}
\]

Because \(\max\{y - R, 0\}\) is convex in \(y\) due to limited liability on the side of the firm, by definition of mean preserving spread it follows that firms with riskier legacy assets expect higher returns, \(U_H \geq U_L\). Integrating by parts, (3.2) can be rewritten as

\[
U_\theta(R) = \bar{y} - \int_0^R \{1 - F(y|\theta)\}dy. \tag{3.3}
\]

Notice that \(U'_\theta(R) = -(1 - F(R|\theta))\) which shows that \(U_\theta(R)\) is decreasing and convex. The second term in (3.3) is the firm’s expected repayment function. Indeed,

\[
D(\theta, R) \equiv \int_0^R \{1 - F(y|\theta)\}dy = \mathbb{E}_y[\min\{y, R\}|\theta] = \bar{y} - U_\theta(R). \tag{3.4}
\]

If investors in the market could invest directly in projects they would receive, on average, \(\bar{a}\). Because they can only act as intermediaries, they give up \(U_\theta(R) - \bar{a}\) to firms, which represent the “gains from trade.”

Let \(r_\theta\) denote the interest rate at which a firm with a legacy asset of type \(\theta\) is indifferent between consuming the asset, and investing and then consuming the proceeds at date 2:

\[
U_\theta(R_\theta) \equiv \bar{a} \quad \text{with} \quad R_\theta = r_\theta l. \tag{3.5}
\]
Intuitively, based on (3.4), \( r_\theta \) equates the expected repayment of a loan with the expected return on the investment project; that is, \( D(\theta, R_\theta) = \overline{v} \). For simplicity I will assume that \( a_\theta > V \), as illustrated in Figure 3.1; then,

\[
R_\theta = \frac{\overline{v}}{1 - F(0|\theta)} = \frac{\overline{v}}{p_v + p_\theta(1 - p_v)} \in (I, V).
\]  

This parametric assumption is not crucial; what is important for the problem to be interesting is that \( R_L < R_H \).\(^1\)

A natural way of measuring the dispersion in the quality of firms is by comparing their expected repayments at \( R_L \). Define,

\[
\Delta = \frac{\overline{v}}{D(H, R_L)} = \frac{1 - F(0|L)}{1 - F(0|H)} = \frac{p_v + p_L(1 - p_v)}{p_v + p_H(1 - p_v)},
\]  

and notice that \( \Delta \geq 1 \), and \( \Delta = 1 \) if and only if \( p_L = p_H \). Keeping \( p_L \) constant, a bigger \( \Delta \) is equivalent to a smaller \( p_H \), hence a larger difference in the quality of legacy assets. The next two assumptions will be useful in the analysis.

---

\(^1\)The fact that \( U_L \leq U_H \) implies that \( R_L \leq R_H \). However, notice that if \( a_\theta < V \) then \( U_L(R) = U_H(R) = \overline{v} - p_v(R - \overline{v}) \) for all \( R \in [a_H, V] \), which leads to \( R_L = R_H \) in some circumstances.
Assumption 6 (Risky Assets). The parameters $\lambda$ and $\Delta$ satisfy:

$$\Delta \geq \frac{\delta}{I}, \quad \text{and} \quad \lambda \leq \frac{\Delta I - 1}{\Delta - 1}.$$  

These conditions state that the two types of legacy assets are sufficiently different in quality; and that there are enough legacy assets of type $H$. They are necessary and sufficient to guarantee that when $\mu = 1$, firms with legacy assets of type $L$ withdraw from the credit market as a consequence of an adverse selection problem.

Now, in general, for a firm that receives a signal $s \in \{L, H\}$ with precision $\mu$, the net expected payoff from investment is

$$W_s(R, \mu) \equiv \pi_s(\mu)U_s(R) + (1 - \pi_s(\mu))U_{-s}(R);$$  

hence its expected repayment is $E[p(\theta, R)|s] = \bar{y} - W_s(R, \mu)$. Notice that $W_s$ is decreasing in $R$, and

$$\frac{\partial W_s}{\partial \mu} = (1 - \lambda_s)[U_s(R) - U_{-s}(R)],$$  

implying that $W_H$ is increasing in $\mu$, while $W_L$ is decreasing. Intuitively, a more precise signal $s = H$ provides higher certainty that the legacy asset is of type $H$, which commands a lower probability of repayment thus a higher expected payoff. The opposite holds for signal $s = L$.

A firm that receives a signal $s$ decides to borrow $l$ from the credit market to undertake its investment opportunity, if and only if,

$$W_s \geq \bar{a}.$$  

Based on (3.10), the following result identifies investment thresholds for interest rates.

Lemma 1 (Investment Thresholds). Define $\bar{r}_\theta(\mu)$ as: $W_\theta(\bar{r}_\theta(\mu)l, \mu) \equiv \bar{a}$. Then, $\bar{r}_\theta$ is a continuous function, decreasing and increasing if $\theta = \{L, H\}$, respectively. In addition, $\bar{r}_H(1) = r_\theta$; and $\bar{r}_L(0) = \bar{r}_H(0) = \bar{r}$, where $\bar{r}$ satisfies:

$$\lambda U_L(\bar{r}l) + (1 - \lambda)U_H(\bar{r}l) = \bar{a}.$$

(3.11)
When the market rate $r$ exceeds $\bar{r}_L(\mu)$, firms that received the signal $s = \theta$ do not invest at date 1 and simply sit on their legacy assets until date 2 when its payoff is realized. Firms whose signals indicate that the legacy asset is of type $H$ expect higher returns from investment, and thus tolerate higher credit market rates. Figure 3.2 summarizes these conclusions.

### 3.2 Equilibrium

From the previous argument, the market perception about the quality of the firms that borrow depends on the borrowing rate and the degree of asymmetric information. For an arbitrary firm that borrows, let $\varphi(s)$ denote the market belief that the firm received the signal $s$. In accordance with Bayes’ law, it follows that: $\varphi(s) = \lambda_s$ when $r < \bar{r}_L(\mu)$; whereas $\varphi(H) = 1$ when $\bar{r}_L(\mu) \leq r < \bar{r}_H(\mu)$.

Equilibrium rates, $r^*$, are then determined by investors’ break-even condition:

$$l = \sum_{s \in \{L,H\}} \mathbb{E}[p(\theta, r^* l) | s | \varphi(s)]$$

$$= \bar{y} - \begin{cases} 
& \lambda W_L(r^* l, \mu) + (1 - \lambda) W_H(r^* l, \mu) \quad \text{if} \quad r^* < \bar{r}_L(\mu); \\
& W_H(r^* l, \mu) \quad \text{if} \quad \bar{r}_L(\mu) \leq r^* \leq \bar{r}_H(\mu).
\end{cases}$$

As the interest rate slightly exceeds $\bar{r}_L(\mu)$ the mix of firms that apply for funds changes drastically, for those who believe to have a greater chance of owning a legacy asset of type $L$ withdraw from the market. Hence, the expected return on loans abruptly decreases from $\bar{y} - \lambda \bar{y} - (1 - \lambda) W_H(\bar{r}_L(\mu) l, \mu)$ to $\bar{y} - W_H(\bar{r}_L(\mu) l, \mu)$. This discontinuity may result in multiple equilibrium interest rates. For starkness, I will assume that in such cases the market selects the lowest one. This is to highlight the fact that the mechanism that gives rise to equilibrium multiplicity in the model is different from that in Stiglitz and Weiss (1981) and signaling games in general.
Chapter 3. Information Panics and Liquidity Crises

The next proposition characterizes the equilibrium interest rate \( r^*(\mu) \).

**Proposition 4 (Interest Rates).** Under Assumption 6, the equilibrium market rate satisfies: \( r^*(\mu) \equiv r^*(0) \in (r_L, r_H) \) for all \( \mu < \mu_c \); and \( r^*(\mu) \in (r_L(\mu), r_H(\mu)) \) increasing for all \( \mu \geq \mu_c \), where \( \mu_c \) is the unique solution to

\[
- \alpha_i - (1 - \alpha) f(T(\mu), \theta) = 0.
\]

When \( \mu < \mu_c \), the proportion of firms with legacy assets of type \((L, H)\) that borrow is constant at \((\lambda, 1 - \lambda)\). Consequently, investors can always break even regardless of firms’ private knowledge by charging an interest rate \( r^*(0) \), which is too high for safe firms but too low for risky ones. This *cross-subsidization* is precisely the source of adverse selection in the credit market. Once \( \mu > \mu_c \), out of the \( 1 - \lambda \) firms that now borrow, \( \pi_H(\mu) = 1 - \lambda + \lambda \mu \) have legacy assets of type \( H \). Because this fraction increases with \( \mu \), the equilibrium interest rate \( r^*(\mu) \) must increase to maintain the zero-profit condition.

Before moving on, it is instructive to analyze two extreme equilibrium scenarios.

**Symmetric Ignorance.** When \( \mu = 0 \), private signals are completely uninformative; hence, neither firms nor the market can discern the quality of legacy assets. The interest rate \( \bar{r} \) defines the investment cutoff, as illustrated in Figure 3.3a, and \( r^*(0) \) is the lowest interest rate that satisfies investors’ zero-profit condition (3.12). All firms borrow and invest in projects.

**Full Asymmetric Information.** When \( \mu = 1 \), superior financial expertise allows firms to perfectly infer from their private signals the types of their legacy assets. With the parametric restrictions in Assumption 6, the equilibrium \( r^*(1) \) is unique and lies in \((r_L, r_H)\) as shown in Figure 3.3b. In this scenario, the process of raising external financing is marred with adverse selection; facing unfairly high interest rates, firms with legacy assets of type \( L \) drop out of the credit market and forgo valuable investment opportunities.

### 3.3 Endogenous Asymmetric Information

I now study equilibrium outcomes when firms have the capability of acquiring knowledge about their legacy assets.
Figure 3.3. Equilibrium interest rates. Interest rates $r^*(\mu)$ are determined by intersecting the supply of loans (black) with the demand curve (grey).
A. Information Choice

Suppose that firms can invest in financial expertise \( \mu \in [0, 1] \) at an initial date 0. Expertise directly determines the precision of the future private signals they receive at date 1, and thus affects their knowledge about the quality of their legacy assets, and ultimately their borrowing decisions as discussed in the previous section.

Acquiring a level of expertise \( \mu \) costs \( c(\mu) = \gamma \mu \) units of resources at date 2, with \( \gamma > 0 \). Then, at date 0, taking the borrowing rate \( r \) as given, firms face the following optimization problem:

\[
\max_{\mu \in [0, 1]} \mathbb{E}_s[W_s(rl, \mu)] - c(\mu).
\]  

In order to solve (4.1), it is important to understand the incentives to gain an informational advantage. I thus begin by analyzing a firm’s expected wealth \( \mathbb{E}_s[W_s(rl, \mu)] \). Because all firms are ex-ante identical, the probability of each receiving a particular signal \( s \in \{L, H\} \) at date 1 is \( \lambda_s \).

First, suppose that \( r < r_L \). Given that \( r_H(\mu) \geq \bar{r} \) for all \( \mu \in [0, 1] \), the firm will decide to borrow from the market if the signal received is \( s = H \). Alternatively, imagine that the firm receives a signal \( s = L \), and define \( \mu_L(r) \equiv \bar{r}_L^{-1}(r) \) over \( [r_L, \bar{r}] \), as the maximum level of knowledge that the firm may have in order not to leave the market. Let \( \mu_L(r) \equiv 1 \) for \( r < r_L \). Because debt contracts backed by legacy assets of type \( L \) are underpriced for all \( r > r_L \), credit is perceived as more expensive the more precise the signal becomes. Indeed, notice that based on Lemma 1, \( \mu < \mu_L(r) \) if and only if \( \bar{r}_L(\mu) > r \).

Therefore, for \( r \leq \bar{r} \), the total expected utility of the firm is given by

\[
\mathbb{E}_s[W_s(rl, \mu)] = \begin{cases} 
\lambda W_L(rl, \mu) + (1 - \lambda) W_H(rl, \mu) & \text{if } \mu \leq \mu_L(r); \\
\lambda \bar{a} + (1 - \lambda) W_H(rl, \mu) & \text{if } \mu \geq \mu_L(r).
\end{cases}
\]  

If \( \mu < \mu_L(r) \), there is enough uncertainty about the quality of the legacy asset that the firm will decide to borrow regardless of the signal received at date 1. Interestingly, (3.8) implies that

\[
\lambda W_L(rl, \mu) + (1 - \lambda) W_H(rl, \mu) = \lambda U_L(rl) + (1 - \lambda) U_H(rl),
\]  

Chapter 3. Information Panics and Liquidity Crises
which is independent of $\mu$. This fact reveals that the gains from acquiring expertise and investing after receiving a signal $s = H$, are exactly offset by the losses accrued when $s = L$. On the other hand, when $\mu > \mu_L(r)$ the aforementioned losses are too big that the firm decides not to borrow when $s = L$.

Now, consider the case $r \geq \bar{r}$. Because $r_L(\mu) \leq \bar{r}$ for all $\mu \in [0,1]$, the firm will decide to drop out of the credit market if the signal received is $s = L$. In analogous fashion let $\mu_H(r) \equiv \bar{r}_H^{-1}(r)$ defined over $[\bar{r}, r_H]$, measure the minimum level of knowledge that the firm needs to attain in order to borrow from the market. Notice that, again based on Lemma 1, $\mu < \mu_H(r)$ if and only if $\bar{r}_H(\mu) < r$. Consequently,

$$E_s[W_s(rl, \mu)] = \begin{cases} \bar{a} & \text{if } \mu \leq \mu_H(r); \\ \lambda \bar{a} + (1 - \lambda)W_H(rl, \mu) & \text{if } \mu \geq \mu_H(r). \end{cases} \quad (3.16)$$

Figure 3.4 summarizes the previous analysis. The fact that $E_s[W_s(rl, \mu)]$ is convex in $\mu$ implies that (4.1) only admits corner solutions. Specifically,

**Proposition 5 (Optimal Expertise).** If $\gamma > \Gamma \equiv \bar{a} \lambda \left( \frac{1}{\lambda + 1} - 1 \right)$, then $\mu^*(r) \equiv 0$. Otherwise, there exists two cutoffs $\rho_L \in (r_L, \bar{r})$ and $\rho_H \in (\bar{r}, r_H)$ defined by $\lambda(\bar{a} - U_L(\rho_L)) = \gamma = (1 - \lambda)(U_H(\rho_H) - \bar{a})$, such that, $\mu^*(r) = 1$ for all $r \in [\rho_L, \rho_H]$. In addition, $\rho_\theta \to r_\theta$ as $\gamma \to 0$.

Notice that firms' incentives to acquire financial expertise are entirely driven by the potential losses from borrowing at an unfairly high interest rate in case of owning a legacy asset of type $L$. Proposition 5 intuitively states that when the marginal cost of information exceeds the minimum expected losses—equivalently, the maximum expected gains—from borrowing ignoring the type of the legacy asset, it is not optimal for firms to learn about the quality of their legacy assets. Indeed, I show in the proof that $\lambda(\bar{a} - U_L(\bar{r}l)) = \Gamma = (1 - \lambda)(U_H(\bar{r}l) - \bar{a})$.

On the other hand, when $\gamma < \Gamma$, the incentives to learn are non-monotonic in $r$. They are increasing when interest rates are below $\bar{r}$, because the expected losses from borrowing mount up as credit becomes more expensive; but once interest rates exceed $\bar{r}$, they turn decreasing because the expected losses are now controlled by deciding not to borrow when the private signal indicates that the legacy asset is of type $L$, and the potential informational gains of having a legacy asset of type $H$ decrease as interest rates increase.
Chapter 3. Information Panics and Liquidity Crises

52

Figure 3.4. Total expected wealth. The figure plots $E_x[W_x(rl, \mu)]$ as a function of $\mu$, for a fixed $r$. The flat portion of the curve is equal to $\lambda U_L(r) + (1 - \lambda)U_H(r)$ when $r = \bar{r}$; and $\bar{a}$ when $r \geq \bar{r}$. Notice that, by definition, $\bar{r}_\theta(\mu_\theta(r)) = r$.

B. Multiple Equilibria

With endogenous asymmetric information, equilibrium interest rates are solutions to the following fixed-point equation:

$$r = r^*(\mu^*(r)),$$

where the mappings $r^*(\mu)$ and $\mu^*(r)$ are given by Propositions 4 and 5, respectively.

For the characterization of equilibria, it is useful to define $\Gamma_0 \equiv \bar{v}\lambda \left( \frac{1/\bar{v}}{\lambda + 1/\bar{v}} - 1 \right)$ and $\Gamma_1 \equiv (1 - \lambda)(\bar{v} - l)$. Notice that $\Gamma_0 \leq \Gamma$, and in addition, Assumption 6 implies that $\Gamma_0 \geq 0$. In order to guarantee the existence of solutions to (3.17) for every $\gamma > 0$, I will assume that $\Gamma_0 \leq \Gamma_1$.\footnote{By definition, it can be shown that $\Gamma_1 - \Gamma_0 = \bar{v} - l \left[ 1 + (1 - \lambda) \frac{\lambda(\Delta - 1)}{\lambda(\Delta - 1) + 1} \right] \geq \bar{v} - l(2 - \lambda)$; hence, $\bar{v} \geq 2l$ is a sufficient condition for $\Gamma_1 \geq \Gamma_0$.}

The next theorem is one of the main results of the chapter.

Theorem 4 (Equilibria). The following holds:

(i) If $\gamma \leq \Gamma_0$, the unique equilibrium is $r^*(1)$;

(ii) If $\gamma \in [\Gamma_0, \min\{\Gamma_1, \Gamma\}]$, both $r^*(0)$ and $r^*(1)$ are equilibrium solutions;
(iii) If $\gamma \geq \min\{\Gamma_0, \Gamma\}$, the unique equilibrium is $r^*(0)$.

The proof is immediate from Proposition 5 once it is understood that the inequalities $\gamma > \Gamma_0$ and $\gamma < \Gamma_1$ are equivalent to $\rho_L > r^*(0)$ and $\rho_H > r^*(1)$, respectively. Figure 3.5 provides a graphical illustration of how equilibria are determined. When $\gamma$ is relatively small, the benefits of learning the types of legacy assets outweigh the marginal cost of information, hence the unique equilibrium features full asymmetric information. At the other extreme, the opposite holds and the equilibrium is characterized by collective ignorance in the economy.

In the intermediate region $[\Gamma_0, \min\{\Gamma_1, \Gamma\}]$, both equilibria may coexist. The reason is that at $r^*(0)$, the cost of borrowing is low enough that it is optimal for firms not to acquire financial expertise; on the other hand, at $r^*(1)$ the market rate is high enough to incentivize firms to learn in order to minimize their expected losses in case they have legacy asset of type $L$. Consequently, given that interest rates are set based on the market’s belief about the quality of the pool of firms that borrow, and thus depend on the degree of asymmetric information, equilibria in the economy are ultimately driven by aggregate expectations.

Disruptions in the credit market due to adverse selection can then be the result of a self-fulfilling prophecy. When investors become worried about the potential of adverse selection they raise interest rates to break even, which, in turn, gives
firms more incentives to acquire expertise that helps them better evaluate their probability of repayment. More information worsens the average credit risk of the pool of borrowers, and thus justifies investors' initial concerns. As a result, the economy settles at an equilibrium with high interest rates \( r^*(1) \) and full asymmetric information, even though it could have settled at one with a low interest rate \( r^*(0) \) and collective ignorance. I label this scenario an "information panic" to emphasize the fact that such features are not granted by fundamentals, as opposed to the equilibrium solution (i) in Theorem 4.

C. Aggregate Shocks and Amplification

Theorem 4 can be interpreted in terms of the economy's response to aggregate shocks to the quality of legacy assets. Imagine that, initially, all legacy assets are of type \( L \). All firms invest in projects by borrowing at the fair rate \( r^*_L(1) \), determined in equilibrium by: 

\[
I = Y - U_L(r^*_L(1)).
\]

Now, suppose that the economy is hit by a shock that turns a fraction \( 1 - \lambda \) of the legacy assets riskier. If the shock is small \( (\lambda \approx 1) \), then the lemons problem is virtually unimportant. Firms have no incentives to acquire financial expertise to distinguish between the two types, and the equilibrium adjusts by slightly increasing interest rates. If instead the shock is big \( (\lambda \approx 0) \), firms also choose to remained uninformed. The reason is that owning a legacy asset of type \( L \) is extremely unlikely, which implies that both the expected losses and gains from adverse selection are very limited —the latter is explained by the fact that, despite outside investors' inability to identify legacy assets of different types, they expect most firms to own type \( H \). Hence, even tough interest rates may increase significantly in equilibrium, all firms still borrow in the credit market.

Full asymmetric information only obtains in equilibrium when aggregate shocks generate enough uncertainty about the quality of legacy assets backing debt securities. This conclusion is in contrast with the common intuition that a larger fraction of lemons causes more severe disruptions in financial markets. This is true when asymmetric information is exogenously installed, but not the case when it endogenously responds to quality shocks as in the model.

The effect of relatively small shocks may be only an increase in interest rates from \( r^*_L(1) \) to \( r^*(0) \). However, information panics can make them have dispro-
portionately large effects, amplifying the increase in interest rates from \( r_L^f(1) \) to \( r^*(1) > r^*(0) \) and, more importantly, causing firms with legacy assets of type \( L \) to withdraw from the credit market. The next result formally specifies the most favorable conditions for information panics to develop.

**Proposition 6 (Information Panics).** The likelihood of information panics is maximized when

\[
1 - \lambda = \frac{\bar{\mu} - 1}{\bar{\mu} - 1} \quad \text{and} \quad \Delta = \frac{\bar{\mu}}{1} \left( \frac{\bar{\mu} - 1}{1 - \bar{\mu}} \right),
\]

where the former corresponds to the minimum fraction of legacy assets of type \( H \) that has the power to disrupt trade.

**D. The Nature of Information Panics**

Information panics arise as the result of the feedback between market interest rates and firms’ incentives to learn about their probability of repayment. As made clear by Figure 3.5, the possibility of multiple equilibria is due to the non-monotonicity of the optimal strategies to acquire information, described by Proposition 5. In this regard, notice the following:

**Strategic Complementarity and Substitutability.** In the low region \( \{ r \leq \bar{r} \} \), firms’ actions exhibit strategic complementarities. While \( \mu < \mu_c \), market rates do not respond to increasing information asymmetry; but once the level of firms’ expertise reaches \( \mu_c \), more information results in higher interest rates, which makes borrowing more expensive and thus increases the benefits of acquiring information.

On the other hand, in the high region \( \{ r \geq \bar{r} \} \) there is strategic substitutability in firms’ initial decisions to learn. In this case, because a firm only decides to borrow when its signal indicates that the legacy asset is of the risky type, the incentives to acquire information are driven by the profits extracted from limited liability. Then, more information leads to higher borrowing rates, which in turn reduces firms’ expected private benefits of having a more accurate understanding of their own credit risk.

**Robustness to Heterogeneous Information.** Equilibrium multiplicity in the model survives the introduction of heterogeneous information. First, based on the previous argument, standard *global games* results (e.g. Carlsson and van Damme,
1993; Morris and Shin, 2003) are not directly applicable. But most importantly, information panics are not the product of a "coordination failure" among firms. Once the interest rate is fixed, a firm's individual decision to invest in expertise is not at all influenced by the actions taken by the other firms; hence, there is no explicit strategic coordination motive at date 0. Thus, even if firms had access to a noisy —yet extremely precise— private signal about some of the fundamentals in the economy, say $\bar{y}$, the optimal information acquisition rule would remain non-monotonic and would still render multiple equilibrium interest rates.

## 3.4 Normative Analysis

In this section I examine the welfare properties of competitive equilibria, and the policy implications of information panics.

### A. Welfare and the Social Value of Information

Given that outside investors make zero profits in equilibrium, welfare in the economy is determined by firms' consumption (output) at date 2.

**Theorem 5 (Welfare).** Collective ignorance is welfare maximizing, and strictly Pareto-dominates full asymmetric information.

With collective ignorance, welfare is given by

$$W^*_0 \equiv \lambda U_L(r^*(0)l) + (1 - \lambda) U_H(r^*(0)l) = \bar{y} - l,$$

where I have made use of (3.15). In contrast, under full asymmetric information, welfare is reduced to

$$W^*_1 \equiv \lambda \bar{a} + (1 - \lambda) \underbrace{U_H(r^*(1)l)}_{\bar{y} - l} - \gamma = \bar{y} - l - \lambda (\bar{b} - l) - \gamma < W^*_0.$$

Notice that the decrease in aggregate consumption is explained by two factors: first, the cost of information acquisition; and second, firms with legacy assets of type $L$, which constitute a fraction $\lambda$, drop out of the credit market and thus forgo their investment opportunities, which have positive NPV $\bar{v} - l$. 

In the model, information has no social value because creditors and firms bargain over the fixed surplus \( \bar{y} \). Information only serves “rent-seeking” purposes, allowing firms to limit losses due to adverse selection, and extract gains at the expense of uninformed investors. In reality, information about productive opportunities can be socially valuable as it permits financial markets better distinguish good projects from bad projects and hence allocate capital in a more efficient way. Theorem 5 highlights the fact that information acquisition that is purely based on individual speculative motives can be detrimental to the economy as a whole. In this sense, acquiring information is like a prisoners’ dilemma: it benefits firms individually, but collectively they would be better off if they remained uninformed.

How can a state of collective ignorance be preserved in the economy?

B. Mandatory Disclosures as Deterrents

A first approach is to levy a tax \( \tau = \min\{\Gamma_1, \Gamma\} - \gamma \) on the acquisition of financial expertise. Then, the marginal cost of information becomes \( \gamma + \tau \), and from Theorem 4 it follows that the equilibrium in the economy is \( r^*(0) \). The problem with this solution is that, because it may be difficult to distinguish between speculative information and other types of socially valuable information, it may have the unintended consequence of impairing the economy’s ability to select good investment projects.

To remedy this problem, the alternative I discuss next specifically targets the source of the inefficiency. Imagine that firms were required to disclose their private signals at date 1. Then, under symmetric information, the interest rate at which a firm could borrow would depend on the signal \( s \in \{L, H\} \) received, and its precision \( \mu \). Let \( r^F_s(\mu) \) denote the equilibrium interest rates, defined by investors’ break-even conditions:

\[
I = \bar{y} - W_s(r^F_s(\mu), \mu).
\]  

(3.18)

Because \( W_s(r^F_s(\mu), \mu) > \bar{a} \), it follows that \( r^F_s < \bar{s} \). Moreover, notice that \( r^F_s(0) = r^*(0) \). The reason is that, for all \( \mu \in [0, 1] \),

\[
W_s(rl, 0) = \lambda U_L(rl) + (1 - \lambda)U_H(rl) = \lambda W_L(rl, \mu) + (1 - \lambda)W_H(rl, \mu),
\]
where I have made use of (3.15) for the last equality. By definition, the right-hand side is equal to \( \bar{y} - l \) at \( r^*(0) \); hence, the fact that \( W_s \) is strictly decreasing in \( r \) implies that \( r^*_0(0) = r^*(0) \).

Now, using (3.18), firms' expected profits at date 0 from borrowing and investing are constant, \( \mathbb{E}_s[W_s(r_1, \mu)] = \bar{y} - l \). Consequently, given that acquiring information is costly, it is optimal for firms to choose \( \mu^*(r) = 0 \) and remain uninformed. Intuitively, faced with mandatory disclosure requirements firms can no longer extract informational rents from the credit market, and as a result, have no incentives to acquire costly expertise.

Mandatory information disclosures thus have a deterrent effect, and improve welfare by discouraging the acquisition of financial expertise for rent-seeking purposes. Without expertise, the information that firms disclose at date 1 is already common knowledge. This is in contrast with Kurlat and Veldkamp (2015), who argue that by making payoffs less uncertain, information disclosures reduce risk and therefore reduce returns, harming investors and reducing welfare in some situations.

C. Credit Rating Agencies

Rating agencies can also play an important role in the regulation of speculative information acquisition by publicly announcing the probability distribution of the legacy assets backing debt securities issued by individual firms.\(^3\) Such public announcements would have the same effect as the mandatory disclosures just described, if the information provided allows investors to perfectly elicit firms private signals.

Of course, in reality, the opinion of rating agencies about the credit quality of a firm is also vulnerable to a lemons problem because the firm might still know more about its legacy asset. In addition, investors' inability to assess the efficacy of rating agency models leaves them susceptible to the agencies' errors.\(^4\) However, the certifications issued by agencies do not need to be perfectly accurate to be effec-

---

\(^3\)I abstract away from potential conflict of interests that may arise between credit rating agencies and the firms whose securities they are rating, as analyzed by Bolton, Freixas, and Shapiro (2012).

\(^4\)Ashcraft and Schuermann (2008) discuss the securitization of subprime mortgage credit, placing special emphasis on the informational frictions that arise in the process and how rating agencies assign credit grades to mortgage-backed securities. They argue that credit ratings were assigned to subprime mortgage-backed securities with significant error prior to the financial crisis of 2007–2009.
tive. Indeed, suppose that agencies can correctly predict the signal received by an individual firm with probability \( q \in (0, 1) \). Then,

**Proposition 7 (Credit Ratings).** There exists a credit rating error \( \bar{q} \in (0, 1) \), such that, \( r^*(0) \) is the unique equilibrium in the economy for all \( q \geq \bar{q} \).

The intuition behind this result is that for \( q \) close enough to 1, the informational rents that firms can extract from the market become very limited to the point that gaining informational advantages by acquiring costly financial expertise is no longer profitable.

**D. Interest Rate Policy and Business Cycles**

Suppose that firms have access to an alternative investment opportunity, in addition to \( v \). This new project requires the same amount of external funding \( l \), and delivers a payoff \( \bar{\delta} \in \{\bar{V}, 0\} \), distributed according to \((\bar{p}_v, 1 - \bar{p}_v)\), such that, \( E[\bar{\delta}] = \bar{v} \) and \( \bar{p}_v < p_v \). The decision to invest in either project is made before firms raise credit.

For starkness, I assume that market participants can detect this choice of projects, but the government cannot. Hence, the introduction of this risk-shifting technology does not alter the decentralized equilibrium previously described, and Theorem 4 still holds. In other words, because creditors are able to adjust interest rates to break even, firms have no incentives to choose the riskier project.

Now, consider the case \( \gamma \in [\Gamma_0, \min\{\Gamma_1, \Gamma\}] \), and imagine a scenario where credit interventions by a central authority prevent market rates from exceeding \( \rho_L \); for example, this could be the product of credit easing policies aiming at stimulating the recovery of the economy from a severe crisis. Then, from Theorem 4, it follows that the unique equilibrium in the economy is \( r^*(0) \), which features collective ignorance and full investment. Such "interest-rate interventions" shut down the feedback mechanism that gives rise to information panics; however, should they be used to prevent information panics and regulate the acquisition of information for speculative purposes?

The problem with this type of market regulation is that it might induce risk shifting by firms, as depicted in Figure 3.6. Since, by assumption, market participants can detect risk-shifting, credit is rationed and the government ends up paying
Figure 3.6. Risk shifting. The equilibrium with government interventions is \( r^*(0) \), but the government pays information costs due to its inability to detect firms' choices of projects. This cost is equal to the difference between the grey lines at \( r^*(0) \), which represent the expected equilibrium repayment function for both projects when \( y = 0 \) (see eq. (3.12)).

The bottom line is that, even though credit easing policies may be warranted by economic forces outside of the model, they are not ideal to maintain symmetric ignorance in markets.

Proofs

Proof or Lemma 1. Because \( R_L < R_H \) it follows that \( U_L(R_H) < \bar{a} < U_H(R_L) \), hence \( W_s(R_H, \mu) < \bar{a} < W_s(R_L, \mu) \). Then, given the fact that the function \( W_s(\cdot, \mu) \) is continuous and decreasing over \([R_L, R_H]\), an application of the intermediate value theorem shows that \( \bar{R}_\theta(\mu) = \bar{r}_\theta(\mu) I \) is well defined. Now, notice that \( W_L(R, 0) = W_H(R, 0) = \lambda U_L(R) + (1 - \lambda) U_H(R) \), which implies that \( \bar{R}_L(0) = \bar{R}_H(0) = \bar{R} \); moreover, \( W_\theta(R, 1) = U_\theta(R) \), thus \( \bar{R}_\theta(1) = R_\theta \). Finally, implicit differentiation yields

\[
\frac{d\bar{R}_\theta}{d\mu} = -\frac{\partial W_\theta}{\partial \mu} \frac{\partial W_\theta}{\partial R}.
\]
and because the denominator is negative, this shows that $R_\theta(x)$ inherits the same monotonicity that $W_\theta$ features in $x$.

**Proof of Proposition 4.** Based on (3.8),

$$\bar{y} - \lambda W_L(rL, \mu) - (1 - \lambda) W_H(rL, \mu) = \bar{y} - \lambda U_L(rL) - (1 - \lambda) U_H(rL),$$

which is independent of $\mu$. Consequently, for all $\mu$ for which there exists an interest rate $r^*(\mu) < \bar{r}_L(\mu)$ that equates (3.19) to $l$, it must be that $r^*(\mu) = r^*(0)$. To see this clearly, start with $\mu = 0$ as depicted in Figure 3.3a and recall from Lemma 1 that $\bar{r}_L(0) = \bar{r}$. Then, the only effect of increasing $\mu$ is to shift $\bar{r}_L(\mu)$ to the left towards $r^*(0)$. The cutoff $\mu_c > 0$ is precisely defined as: $\bar{r}_L(\mu_c) = r^*(0)$.

Now, notice that $U_L(rL) = \bar{a}$ by definition, and $U_H(rL) = \bar{y} - rL(1 - F(0|H))$ because $rL < V$. Then, employing (3.19) and (3.7) it follows that

$$\bar{y} - \lambda W_L(rL) - (1 - \lambda) W_H(rL) = \bar{y} \left( \lambda + \frac{1 - \lambda}{\Delta} \right).$$

The expression on the right-hand side is upper bounded by $l$, if and only if, the parametric conditions in Assumption 6 are satisfied. Hence, $\bar{r}_L(\mu_c) > rL = \bar{r}_L(1)$; equivalently, $\mu_c < 1$. Therefore, for all $\mu > \mu_c$, the zero-profit equilibrium condition (3.12) admits a unique solution $r^*(\mu) \in (\bar{r}_L(\mu), \bar{r}_H(\mu))$, determined by

$$\bar{y} - W_H(r^*(\mu), \mu) = l.$$ 

Because $W_H$ is increasing in $\mu$, it follows that $\frac{dr^*}{d\mu} > 0$ as sketched in Figure 3.7.

**Proof of Proposition 5.** The convexity of the objective function in (4.1) implies that $\mu^*(r) = 0$ if and only if

$$\mathbb{E}_s[W_s(rL, 0)] - c(0) > \mathbb{E}_s[W_s(rL, 1)] - c(1).$$

(3.20)

First, suppose that $r \leq \bar{r}$. Based on (3.14), condition (3.20) is equivalent to $\gamma > \lambda(\bar{a} - U_L(rL))$. Notice that the right-hand side is an increasing function of $r$, and it is 0 at $rL$. Then, if $\gamma > \lambda(\bar{a} - U_L(\bar{r}))$ it follows that $\mu^*(r) \equiv 0$ for all $r \leq \bar{r}$. 


Figure 3.7. Comparative statics. An increase in $\mu$ shifts the second portion of the demand curve to the rights, increasing the equilibrium interest rate $r^*$ as a result.

Alternatively, there exists $\rho_L \in (r_L, \bar{r})$, such that, $\gamma = \lambda(\bar{a} - U_L(\rho_L))$ and

$$\mu^*(r) = \begin{cases} 0 & \text{if } r \leq \rho_L; \\ 1 & \text{if } \rho_L \leq r \leq \bar{r}. \end{cases}$$

On the other hand, according to (3.16), when $r \geq \bar{r}$ condition (3.20) takes the form: $\gamma > (1 - \lambda)(U_H(r) - \bar{a})$. The right-hand side is now decreasing in $r$, and it is 0 at $r_H$. Hence, if $\gamma > (1 - \lambda)(U_H(\bar{r}) - \bar{a})$ it follows that $\mu^*(r) \equiv 0$ for all $r \geq \bar{r}$. Otherwise, there exists $\rho_H \in (\bar{r}, r_H)$, such that, $\gamma = \lambda(U_H(\rho_H) - \bar{a})$ and

$$\mu^*(r) = \begin{cases} 1 & \text{if } r \in [\bar{r}, \rho_H]; \\ 0 & \text{if } r \geq \rho_H. \end{cases}$$

The last step is to show that $\Gamma = \lambda(\bar{a} - U_L(\bar{r})) = (1 - \lambda)(U_H(\bar{r}) - \bar{a})$. Indeed, the second equality is implied by definition (3.11); now, notice that because $\bar{r} < r_H < V$, it follows that $U_\theta(\bar{r}) = \bar{y} - \bar{r}(1 - F(0|\theta))$. Therefore, employing (3.7) yields $U_L(\bar{r}) = \bar{y} - \frac{\bar{v}}{\lambda + 1/\lambda}$, which completes the proof.

**Proof of Theorem 4.** First, I determine $r^*(0)$, $r^*(1)$, $\rho_L$, and $\rho_H$. Because $\rho_\theta <
$r_H I < V$, it follows from (3.2) that $U_\theta(\rho_\theta l) = \bar{y} - \rho_\theta l(1 - F(0|\theta))$, which implies that

$$
\rho_L l = \frac{\bar{y} + \frac{\gamma}{\lambda}}{1 - F(0|L)} \quad \text{and} \quad \rho_H l = \frac{\bar{y} - \frac{\gamma}{1-\lambda}}{1 - F(0|H)}.
$$

Moreover, Proposition 4 established that $r^*(0) < \tilde{r} < r^*(1) < r_H$, thus $U_\theta(r^*(0) l) = \bar{y} - r^*(0) l (1 - F(0|\theta))$ and $U_H(r^*(1) l) = \bar{y} - r^*(1) l (1 - F(0|H))$. Finally, recall that by definition, $\bar{y} - \lambda U_L(r^*(0) l) - (1 - \lambda) U_H(r^*(0) l) = I = \bar{y} - U_H(r^*(1) l)$; hence

$$
r^*(0) = \frac{1}{\sum_\theta \lambda_\theta (1 - F(0|\theta))} \quad \text{and} \quad r^*(1) = \frac{1}{1 - F(0|H)}.
$$

Using the above, it is now straightforward to show that $\rho_L > r^*(0) \iff \gamma > \Gamma_0$ and $\rho_H > r^*(1) \iff \gamma < \Gamma_1$, as I claimed in the main text.

Proof of Proposition 6. Let $I = \min\{\Gamma_1, \Gamma\} - \Gamma_0$ denote the measure of the interval for marginal costs $\gamma$ that renders multiple equilibria. For the moment, fix $\lambda \in (0, 1)$. Observe that $\Gamma(\Delta)$ is increasing, with horizontal asymptote $(1 - \lambda)\bar{y}$. In addition, $\Gamma(\frac{\bar{y}}{\lambda}) - \Gamma_1 = (1 - \lambda)^2 (\frac{1}{\bar{y}} - 1) < 0$. Therefore, there exists a unique $\Delta_c > \frac{\bar{y}}{\lambda}$ such that $\Gamma(\Delta_c) = \Gamma_1$; in fact, it is easy to find that $\Delta_c(\lambda) = \frac{\bar{y} - (1 - \lambda)I}{\lambda}$.

As a result,

$$
I = \begin{cases} 
\Gamma - \Gamma_0 & \text{if} \quad \Delta \leq \Delta_c;
\Gamma_1 - \Gamma_0 & \text{if} \quad \Delta \geq \Delta_c.
\end{cases}
$$

Notice that $\Gamma_0(\Delta)$ is increasing, and so is $\Gamma - \Gamma_0 = \frac{\bar{y}^2}{\lambda + \frac{\lambda}{\Delta} - 1} (1 - \frac{I}{\bar{y}})$. Consequently,

$$
\max_\Delta I(\Delta) = I(\Delta_c) = \left(1 - \frac{I}{\bar{y}}\right) (\bar{y} - (1 - \lambda)I),
$$

which is increasing in $\lambda$. Now, under Assumption 6, for any given $\Delta \geq \frac{\bar{y}}{\lambda}$, the fraction $\lambda$ is upper bounded by $\frac{\Delta - 1}{\Delta - 1}$. Then, it follows that the pair $(\Delta_c(\lambda_c), \lambda_c)$ maximizes $I$, where $\lambda_c$ is implicitly defined by:

$$
\lambda = \frac{\Delta_c(\lambda_c) l}{\Delta_c(\lambda_c) - 1}.
$$

Finally, after substituting for $\Delta_c$ and performing some simple algebraic manipula-
tions, it can be shown that \( \lambda_c = \frac{1-a}{\frac{a}{1-a}} \) which gives \( \Delta_c(\lambda_c) = \frac{\gamma}{1 - \frac{\gamma}{1-a}} \).

**Proof of Proposition 7.** At date 1, for fixed \( \mu \), the interest rate at which a firm that receives a signal \( s \in \{L, H\} \) can borrow, \( r^q(\mu) \), is determined by investors zero-profit conditions:

\[
l = \bar{y} - \left( q W_s(r^q_s(\mu)l, \mu) + (1 - q) W_s(r^q_s(\mu)l, \mu) \right).
\]

Then, at date 2, a firm with a legacy asset of type \( \theta \) receives a net payoff of

\[
W_\theta(r^q_\theta(\mu)l, \mu) = \frac{\bar{y} - l}{q} - \frac{1 - q}{q} W_\theta(r^q_\theta(\mu)l, \mu).
\]

Hence, the expected payoff at date 0 for fixed \( r^q_L < r^q_H \) is given by,

\[
\mathbb{E}_s[W_s(r^q_s l, \mu)] = \frac{\bar{y} - l}{q} - \frac{1 - q}{q} \left( W_L(r^q_H l, \mu) + (1 - \lambda) W_H(r^q_L l, \mu) \right),
\]

which converges to \( \bar{y} - l \) as \( q \to 1 \).

As I argued in the main text, \( \lim_{q \to 1} r^q_\theta(\mu) = r^f_\theta(\mu) < \bar{r}_\theta(\mu) \) for all \( \mu \in [0,1] \). Let \( \epsilon_\theta = \max_{\mu \in [0,1]} \{ \bar{r}_\theta(\mu) - r^f_\theta(\mu) \} \), and assume that \( q \) is close enough to 1, such that, \( |r^q_\theta(\mu) - r^f_\theta(\mu)| < \epsilon_\theta \) for all \( \mu \in [0,1] \) and \( \theta \in \{L, H\} \). Then, \( r^q_\theta < \bar{r}_\theta \) and using (3.9) it follows that,

\[
\frac{d}{d\mu} \mathbb{E}_s[W_s(r^q_\theta l, \mu)] = \frac{1 - q}{q} \lambda (1 - \lambda) (F(0|L) - F(0|H)) (r^q_L - r^q_H) l \geq 0.
\]

Consequently, the optimization problem \( \max_{\mu} \{ \mathbb{E}_s[W_s(r^q_s l, \mu)] - c(\mu) \} \) only admits corner solutions. Now, because \( W_\theta(\cdot, \mu) \) is continuous for \( r \in [0, r_H] \), it follows that for every \( \epsilon > 0 \), there exists \( \bar{q} \) with the property that, for all \( q \geq \bar{q} \), \( |\mathbb{E}_s[W_s(r^q_s l, \mu)] - (\bar{y} - l)| < \epsilon \). This implies that \( \mu^* \equiv 0 \) is the unique maximizer for every \( r^q_L \) and \( r^q_H \). Finally, notice from (3.21) that: \( r^q_L(0) = r^q_H(0) = r^*(0) \).
CHAPTER 4

Coordinating Inefficient Credit Flows

MACROECONOMIC COMPLEMENTARITIES can lead to coordination failures. The reason is that, when interactions in the economy are characterized by coordination motives, multiple equilibria can often be sustained by rational self-fulfilling expectations. As described in the seminal paper by Cooper and John (1988), complementarities may stem from externalities in technologies, demand spillovers, and trading in incomplete financial markets. But even when the equilibrium is unique, coordination risk can critically affect aggregate behavior and its response to exogenous disturbances.

In a dynamic macroeconomic model with investment complementarities I study the limited, and in cases potentially detrimental, effects of different government responses to stimulate bank lending during self-fulfilling credit market freezes, equilibrium scenarios where banks abstain from extending credit to non-financial firms because they expect other banks not to lend. I show that traditional interest rate cuts reduce the incidence of market freezes, but can make them harder to escape when they occur. On the other hand, unconventional policy interventions in the form of equity injections or liquidity provisions, which aim at recapitalizing the financial sector, become completely ineffective once intermediaries cease to be liquidity constrained.

The starting point is that banks have better skills in evaluating and monitoring projects, which implies that it is efficient for credit in the economy to flow from households to firms through financial intermediaries (Diamond, 1984). There is no friction that constraints the ability of intermediaries to raise deposits from house-
holds. Individual banks have access to a real investment technology whose return depends on a stochastic macroeconomic fundamental, but most importantly, it is non-decreasing in the aggregate level of investment. Wishing to maximize equity value, banks thus have a motive to coordinate their portfolio choices. When fundamentals are very strong, banks supply credit regardless of the actions of others. Similarly, when fundamentals are extremely bleak, it is not profitable for banks to invest with non-financial firms even if all the other banks decided to invest. But in the intermediate region, the choice of an individual bank becomes a function of its beliefs about the choice of others: when banks fear that other will not lend they might choose not to lend, thus driving the economy into a self-fulfilling credit market freeze. Such coordination failures undermine welfare because otherwise profitable projects are not undertaken due to a lack of operating capital.

In order to determine the probability of inefficient credit market freezes, and thus study the effects of government policies on the transitional dynamics, I introduce heterogenous information about the fundamentals in the economy and solve the coordination problem faced by banks using global games techniques. With incomplete information, the equilibrium path is now unique and features credit market freezes whenever the fundamentals fall below a certain threshold. This investment threshold is decreasing in the aggregate wealth of financial intermediaries, which highlights the role of the latter as a key aggregate economic indicator. It shows that a better capitalized financial sector is more resilient to exogenous shocks, and less susceptible to credit market freezes. Moreover, it links strategic complementarities across time periods, and therefore provides the inter-temporal channel by which freezes can have persistent real effects.

I find that low interest rates decrease the incidence of credit market freezes, but increases their duration when the productivity of capital is high. The intuition is as follows. A reduction today in the safe rate of interest decreases the likelihood of a credit market freeze because it renders investment with non-financial firms relatively more profitable. As a result, strategic uncertainty decreases and coordination risk subsides. However, if the freeze does occur, the impact on the likelihood that it extends from tomorrow to the next period is twofold. First, there is the same effect as before operating through the risk perception of banks, and facilitating coordination. But there is a second influence working on the opposite direction: when banks engage in storage, lower interest rates result in a more significant shrinkage in the
growth rate of funds, which renders coordination harder to achieve. This force prevails when the profitability of firms’ projects is high, because in that case, coordination risk is already low; without exogenous innovations that improve macroeconomic prospects, this feedback mechanism makes the coordination of investment in the future even more difficult.

I also analyze a host of unconventional policy measures intended to encourage bank lending. Among them, I consider: equity injections, whereby the government acquires ownership stakes in banks; the use of discount window operations to lend funds to banks; public-private partnership programs; and direct lending to non-financial firms. Given that the source of inefficiency in the model is the coordination failure, I assume that these credit policies are financed with lump-sum (non-distortionary) taxes. But precisely because there is no financial friction affecting banks' ability to raise deposits, Ricardian Equivalence (Barro, 1974) holds and renders all these interventions fruitless in their attempt to unfreeze credit markets and restore bank lending. Reductions in the inflow of deposits from households reduce the aggregate wealth of financial intermediaries, and thus completely crowd out government spending. Government guarantees, on the other hand, are welfare improving. By limiting the downside risk faced by banks, they reduce strategic uncertainty and hence promote coordination. This non-Ricardian effect hinges on the fact that, in equilibrium, there are no losses to be covered by the government; what it is required, however, is credibility in their implementation.

The results in this chapter can shed light on the effectiveness of the policies taken by the Federal Reserve in response to the financial crisis of 2007–2009. During that period, Ivashina and Scharfstein (2010) and Acharya and Mora (2015) document an alarming fall in bank lending for real investment. As pressures in the financial system mounted, and once the federal funds rate reached its zero lower bound in December 2008, a multiplicity of credit easing tools were deployed to contain the crisis (Bernanke, 2009). These included unprecedented amounts of liquidity injections, purchases of commercial paper and corporate bonds, guaranteeing bank liabilities, and infusing capital into financial institutions. Although many observers acknowledge these unconventional interventions for the wide reductions in credit spreads across financial markets that followed their implementation, there is anecdotal evidence suggesting that these policies did little to encourage banks to resume their role as liquidity providers. For example, in April 2009 under the headline
"Bank Lending Keeps Dropping," the Wall Street Journal reported that "lending at the biggest U.S. banks has fallen more sharply than realized, despite government efforts to pump billions of dollars into the financial sector." Indeed, the model suggests that, provided the cause behind the credit freeze is a coordination failure, once financial stress subsides and intermediaries cease to be balance-sheet constrained, government spending will be unsuccessful in increasing lending to pre-crisis levels; instead, policies aimed directly at restoring confidence, such as credible (costless) announcements, can be much more effective. It seems, however, that during the financial crisis guarantees were only implemented to limit losses on existing liabilities, not to improve the extension of credit.

**Related Literature**

The chapter contributes to the theoretical literature on financial crises, their transmission to the real economy, and their implications for policy. I build on the work of Bebchuk and Goldstein (2011), who in a static model analyze the ability of various government interventions to get the economy out of a self-fulfilling credit market freeze. I embed their coordination problem in a dynamic general equilibrium macroeconomic setting where the aggregate wealth of financial intermediaries evolves endogenously. In stark contrast with their finding that capital infusions to the banking sector cannot fully eliminate the coordination failure, I show that they have zero effect in the absence of frictions in the market for deposits.

Benmelech and Bergman (2012) also study the limitations of liquidity injections in inducing banks to increase their supply credit to the corporate sector beyond a certain point. The describe how the economy can get stuck in a "credit trap" equilibrium where aggregate lending is constrained by low collateral values. Their model focuses on collateral constraints in the real sector; my model emphasizes intertemporal investment complementarities. In Philippon and Schnabl (2013), complementarities among banks stem from a debt overhang problem, causing banks to rationally forgo profitable lending as they expect other banks not to lend. In their framework, it is a feedback mechanism through the repayment of household debt to the financial sector that gives rise to negative externalities, and justifies the recapitalization of banks. Similarly, Wilson (2009) studies government purchases of preferred stock and common stock in an attempt to encourage efficient lending.
Both papers analyze the social costs of bailout programs and the participation decision of banks. I argue for government guarantees that limit losses on new loans as an effective way to improve welfare in scenarios where funding stresses have subsided, but liquidity remains stuck in the financial sector.

Some of the policies I consider in the chapter are also analyzed by Gertler and Kiyotaki (2010), who develop a calibrated model to study credit easing policies in a macroeconomic model where financial intermediation plays a crucial role. Along the same lines, Gertler and Karadi (2011) include nominal rigidities, and evaluate the effects of both conventional and unconventional monetary policy. While these important papers focus on agency problems that constrain the ability of intermediaries to obtain funding from depositors, in the spirit of Holmstrom and Tirole (1997), I analyze policy responses when banks refuse to lend because of self-fulfilling fears.

A key feature of the model is that coordination failures can propagate from one period to the next, and thus exhibit path dependence and persistence. The dynamics of coordination and the implications for equilibrium determinacy is the focus of the large literature on dynamic global games. Frankel and Pauzner (2000) and Burdzy et al. (2001) consider an environment where fundamentals evolves according to a stochastic process and players experience frictions in changing their action; they show that aggregate shocks and idiosyncratic inertia deliver a unique equilibrium. Angeletos and Pavan (2007) study how endogenous learning affects the level of strategic uncertainty; in their setting, knowledge of the result of previous actions combined with the arrival of new private information over time can lead to multiple equilibrium outcomes. Steiner (2008) and Giannitsarou and Toxvaerd (2012) analyze dynamic strategic complementarities whereby players' present actions influence their future payoffs; existence of a unique equilibrium is thus established based on a recursive approach. In my setting, banks are only operate for one period which effectively reduces the analysis to a sequence of static coordination games with incomplete information featuring a unique equilibrium (Carlsson and van Damme, 1993; Morris and Shin, 1998); the inter-temporal link between stages is provided by the accumulation of aggregate wealth in the financial sector.

The fact that coordination risk yields multiple equilibrium steady states is also shared by other papers. For example, Azariadis and Drazen (1990) introduce threshold externalities in a neoclassical growth model, and obtain multiple locally stable
balanced growth paths. Their model features technological externalities that allows returns to scale to rise rapidly once economic state variables reach a “critical mass.” Recently, Schaal and Taschereau-Dumouchel (2014) develop an otherwise standard neoclassical growth model with monopolistic competition, but where an aggregate demand externality links production decisions across firms. Although the transitional dynamics in my model are similar to the ones described by these papers, their focus is on economic development and business cycles, respectively; mine is on the role played by financial intermediaries in channeling credit from households to the real economy.

Outline of the Chapter. The rest of the chapter is organized as follows. Section 4.1 introduces the baseline model. Section 4.2 provides an informal description of equilibria under perfect information. Section 4.3 formally solves for the competitive equilibrium of the economy under incomplete information. Section 4.4 studies welfare and the effect of different policies.

4.1 The Baseline Model

The core framework is a canonical real business cycle model. Shocks to TFP are endogenous and depend on the aggregate amount of credit supplied by financial intermediaries (banks) to non-financial firms in the traditional sector. There is a unique consumption good that serves as the numeraire, with its price is normalize to one. Moreover, the economy is populated by three types of agents which I describe in detail below.

A. Households

There is a continuum of identical households of measure unity. Within each household, half of the members are workers and the remaining half are bankers. Over time, even though this fraction is unchanged, an individual can switch between occupations. Workers supply labor inelastically, and each banker manages a financial intermediary for one period. They return their wages and transfer dividends back to the household family, respectively.

Within the family there is perfect consumption insurance, hence all of its mem-
bers enjoy the same level of consumption. This simple form of heterogeneity follows Gertler and Karadi (2011), and as it will become apparent later, provides tractability by allowing a representative agent approach.

Instead of holding capital directly, households finance future consumption by holding deposits with financial intermediaries that it does not own. Bank deposits are one-period securities that pay a net real return \( r_t \) from date \( t - 1 \) to \( t \), determined endogenously. Let \( d_t \) denote the quantity of debt held, \( w_t \) wage earnings, and \( \pi_t \) net payouts from ownership of both banks and non-financial firms. All variables are expressed in per capita household terms, thus the proportion of workers within a household does not appear in the analysis.

Then, the household chooses non-negative consumption and savings plans, \( \{c_t\} \) and \( \{d_{t+1}\} \), to maximize its discounted expected lifetime utility

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),
\]

with \( \beta \in (0, 1) \), and subject to the budget constraint:

\[
c_t + d_{t+1} = (1 + r_t)d_t + w_t + \pi_t,
\]

for a given initial level of deposits \( d_0 > 0 \).

For simplicity, I will assume that the instantaneous utility function is logarithmic, \( u(\cdot) = \log(\cdot) \), which allows to solve problem (4.1) in closed form. Let \( u' \) denote the household’s marginal utility, and define

\[
\Lambda_{t,t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)},
\]

as the stochastic discount factor measuring the inter-temporal marginal rate of substitution between consumption in periods \( t \) and \( t + 1 \).

B. Firms

In the traditional sector, competitive (non-financial) firms produce output using physical capital. Let \( K_t \) denote the aggregate amount of capital in the economy. Firms have access to a technology that builds capital from goods one-for-one. Fol-
Following Bebchuk and Goldstein (2011), an individual firm holding $k_t$ units of capital in period $t$ produces output according to:

$$y_t = \begin{cases} a k_t & \text{if } \eta K_t + \theta_t \geq \tau, \\ 0 & \text{otherwise}, \end{cases}$$

(4.4)

where $a > \frac{1}{\beta}$ is a parameter. The specified technology (4.4) is meant to capture in reduced form the positive complementarities that stem from the linkages across different sectors in the production side of the economy. The parameter $\eta > 0$ measures the strength of this interdependence among firms; $\tau > 0$ is a fixed cost. In addition, the profitability of the production technology depends on the stochastic macroeconomic fundamental $\theta_t \in \Theta$, which follows a Markov process characterized by the state transition function $Q(z, \cdot)$ with the Feller property.\(^1\) The set $\Theta$ is a closed interval.

Because firms receive no endowments from households, they finance operation through financial intermediaries. In order to obtain the funds needed to acquire capital for production, firms issue claims against output next period equal to the amount of working capital $k_t$. Like in Bernanke and Gertler (1988), for simplicity, capital fully depreciates after one period.

C. Financial Intermediaries

Financial intermediaries act as specialist and invest on behalf of households. In the current period $t$, bank $h$ raises $d^h_t$ deposits from other households. It then invests a fraction $l^h_t \in [0, 1]$ of its wealth with non-financial firms by purchasing securities that promise a risky gross real return $R_{t+1}$ per unit in period $t+1$; and saves the remaining fraction $1 - l^h_t$ in a linear (risk-free) storage technology with a low net rate of return $0 \leq r^f < \frac{1}{\beta} - 1$. In next period $t+1$, the bank pays back its creditors and transfers revenues from investments and savings back to its household $h$.

I assume that $\theta_t$ is realized at the beginning of period $t$ before decisions (consumption, savings, and investments) are made, and its value is common knowledge.

\(^{1}\)This means that for any continuous and bounded function $\phi : \Theta \to \mathbb{R}$, it follows that $E[\phi(z')|z] = \int \phi(z')Q(z, dz')$ is a bounded and continuous function of $z$ on $\Theta$. 

The banker's portfolio choice problem is then to maximize its equity value:

$$\max_{d_t^h \geq 0, I_t^h \in [0, 1]} \Lambda_t t+1 \pi_{t+1} (d_t^h, I_t^h), \quad \text{with} \quad \pi_{t+1} = \left\{ R_t^h (I_t^h) - (1 + r_{t+1}) \right\} d_t^h \quad (4.5)$$

and where the gross real return on bank's $h$ assets is given by

$$R_t^h (I_t^h) = 1 + r_f + [R_t + 1 - (1 + r_f)] I_t^h.$$ 

Notice that bankers discount expected return according to (4.3); however, given that there is no consumption risk between contiguous periods, the only uncertainty faced by financial intermediaries is of strategic nature. In addition, because bankers only live for one period, their optimization problem (4.5) is static.

With the above market structure, unlike in a Walrasian setting, non-financial firms may not be able to raise any amount of capital they choose at a given price. This is because banks' investment decisions are not solely based on prices, but also depend on quantities. Hence, financial intermediaries have a strategic motive to coordinate their decisions, which is precisely the source of the financial friction affecting the supply of credit to non-financial firms in the model. As I describe next, the solution of this game is greatly simplified by assuming that bankers are essentially myopic; allowing them to operate for a longer time horizon will substantially complicate the analysis without qualitatively changing any of the results.

### 4.2 Strategic Uncertainty and Multiple Equilibria

I now informally describe competitive equilibria of the economy. I focus on interior equilibria where both consumption and savings are positive. Let $\theta_t = (\cdots, \theta_{t-1}, \theta_t)$ denote the history of macroeconomic shocks up to time $t$.

**Definition (Competitive Equilibrium).** A competitive equilibrium in this economy consists of contingent plans of consumption and savings $\{c_t(\theta_t), d_{t+1}(\theta_t)\}_{t \geq 0}$ for households, investment strategies and deposit holdings $\{I_t^h(\theta_t), d_{t+1}(\theta_t)\}_{t \geq 0}$ for each bank $h$, capital $\{k_t(\theta_t)\}_{t \geq 0}$ for firms, and prices $\{r_t(\theta_{t-1}), R_t(\theta_{t-1}), w_t(\theta_t)\}_{t \geq 0}$ such that: (i) the representative household $h$ maximizes (4.1) subject to (4.2), given initial savings $d_0$; (ii) banks solve (4.5); and (iii) prices clear markets for deposits, labor, and capital.
A. Consumption and Deposits

I simplify notation by making the dependence in history \( \theta^t \) implicit. To characterize equilibria, first note that because the production function (4.4) does not depend on labor, wages \( w_t \) are equal to zero. Moreover, perfect competition implies that firms earn no rents, thus the gross return on capital \( R_{t+1} \) is equal to \( a \) if \( \eta K_t + \theta_t \geq \tau \), and 0 otherwise.

On the other side of the economy, demand for deposits from financial intermediaries must equal the supply from households

\[
\begin{align*}
\frac{d^h_t}{d_t} = d_{t+1} > 0.
\end{align*}
\]

Hence, it follows from (4.5) that regardless of the portfolio choice \( I^h_t \), at any interior equilibrium:

\[
\Lambda_{t,t+1} \left\{ R^h_{t+1} - (1 + r_{t+1}) \right\} = 0;
\]

that is, the risk adjusted premium (excess return) must be zero. But given that \( u'(\cdot) > 0 \), condition (4.7) is equivalent to: \( R^h_{t+1} = 1 + r_{t+1} \). To see this clearly, imagine for the sake of contradiction that \( R^h_{t+1} < 1 + r_{t+1} \); then, banks would choose not to raise deposits, \( d^h_t = 0 \), which violates the market clearing condition (4.6). Alternatively, if \( R^h_{t+1} > 1 + r_{t+1} \) banks would want to expand deposits indefinitely; however, their ability to borrow from households is constrained by \( d_t \leq a^t d_0 \). Therefore, banks earn no rents from holding deposits, \( \pi_t = 0 \).

From (4.2) and (4.6), the evolution of the aggregate amount of total wealth of financial intermediaries, \( D_t \), is thus governed by the law:

\[
\begin{align*}
D_{t+1} = (1 + r_t)D_t - C_t,
\end{align*}
\]

where \( C_t \) represents aggregate consumption in the economy. Individual consumption by households is determined by the usual Euler equation

\[
\begin{align*}
\Lambda_{t,t+1}(1 + r_{t+1}) = 1,
\end{align*}
\]

which captures the tradeoff between the loss in utility in the current period, \( u'(c_t) \), if the household deposits one more unit of resources with banks and the increase in (discounted) utility, \( \Lambda_{t,t+1} u'(c_{t+1})(1 + r_{t+1}) \), gained from the extra payoff next
In light of (4.7), fluctuations in the level of consumption are caused by changes in the productivity of the real sector, which in turn are entirely induced by the volatility in the aggregate supply of credit to firms via financial intermediaries.

B. Investment with Complementarities

The remaining variable that needs to be determined in order to fully characterize equilibria is the gross return $R_{t+1}^h$. Notice from the above discussion that, based on conditions (4.7) and (4.9), the investment/saving decision of bank $h$ at time $t$ reduces to maximize its equity value:

$$\max_{I_t^h \in [0,1]} \left[ R_{t+1} - (1 + r_f) \right] I_t^h.$$ 

Because the realization $\theta^t$ is known at time $t$, whether $R_{t+1}$ is a or 0 only depends on the aggregate wealth of financial intermediaries, $D_{t+1}$. Bank $h$ considers $D_{t+1}$ as given, and thus faces a coordination problem which I now proceed to analyze.

**Upper Dominance Region.** Imagine that $\theta_t \geq \tau$. In this case, the macroeconomic fundamental is so strong that the gross return $R_{t+1} = a$ regardless of aggregate capital $K_t$. Because $a > \frac{1}{\beta} > 1 + r_f$, it follows that the bank's optimal decision is to choose $I_t^h = 1$; that is, invest all of its wealth $d_{t+1}$ in securities issued by firms independently of the amount of credit supplied by other intermediaries. By symmetry, aggregating quantities yields $K_t = D_{t+1}$, and in light of (4.9), consumption next period $t+1$ grows by $\beta a$.

**Lower Dominance Region.** Alternatively, suppose that $\theta_t \leq \tau - \eta D_{t+1}$. With this bleak macroeconomic picture, even if the aggregate amount of total wealth of financial intermediaries was lent to non-financial firms the return on capital would still be zero. Hence, in this scenario, claims to output are worthless and the optimal strategy for the bank is to transfer all of its wealth to the next period using its safe storage technology. Consequently: $I_t^h = 0$, $K_t = 0$, $R_{t+1}^h = 1 + r_f$, and consumption at $t + 1$ shrinks by $\beta(1 + r_f)$.

What happens when $\tau - \eta D_{t+1} < \theta_t < \tau$? In this intermediate region, the decision of the bank is driven by its belief about the actions of other banks in the
Chapter 4. Coordinating Inefficient Credit Flows

4.3 Model with Incomplete Information

I now remove the assumption of perfect knowledge about \( \theta_t \), and formally study the existence and uniqueness of the competitive equilibrium in recursive form.

A. Timing and Information Structure

Recall that \( \theta_t \) is drawn from the distribution \( Q(\cdot, \theta_{t-1}) \) at the beginning of time \( t \); however, in contrast to the previous section, I assume that its value is not revealed to
agents. Instead, each banker receives a private signal $x^h_t = \theta_t + \epsilon^h_t$ where $\epsilon^h_t$ are vanishingly small noise terms that are independently and uniformly distributed across agents and time, with support over $[-\epsilon, \epsilon]$. Although information is heterogenous, no banker has an informational advantage over the others.

The timing of events within period $t$ is as follows: first, based on their private signals $x^h_t$ and the history of macroeconomic shocks $\theta^{t-1}$ banks choose the investment portfolio $l^h_t$; after that, $\theta_t$ is revealed, and households consume and save resources according to $c_t$ and $d_{t+1}$, respectively.

B. Recursive Formulation

It is easier to consider the optimization problem of a representative household $h$ in recursive form. I follow the usual convention of denoting current variables without subscript and next period variables with the prime superscript. Let $\theta_{-1}$ represent the previous macroeconomic fundamental, and define $\theta = (\theta_{-1}, \theta)$. Then, the aggregate state of the economy is determined by the variable $(D, \theta)$. Denote the law of motion for aggregate deposits by $G : D' = G(D, \theta)$, which households use to forecast returns. The problem of the household can then be written as,

\[
V(d; D, \theta) = \max_{c \geq 0, d' \geq 0} \{ u(c) + \beta \mathbb{E}[V(d'; G(D, \theta), \theta')|\theta]\}, \tag{4.10}
\]

\[
s.t. \quad c + d' = (1 + r(D, \theta_{-1}))d. \tag{4.11}
\]

Definition (Recursive Competitive Equilibrium). A recursive competitive equilibrium is defined by a law of motion $G(D, \theta)$, pricing functions $R(G(D, \theta), \theta)$ and $r(G(D, \theta), \theta)$, a value function $V(d, D, \theta)$, and decision rules $\{c(d, D, \theta), d'(d, D, \theta), l^h(D, \theta)\}$, that satisfy the conditions below:

(i) Households optimization: $\{c(d, D, \theta), d'(d, D, \theta)\}$ solve (4.10) subject to (4.11), taking $G(D, \theta)$ and $r(G(D, \theta), \theta)$ as given;

(ii) Portfolio choice: bank $h$ selects $l^h(D, \theta)$ to solve

\[
\max_{l^h \in \{0,1\}} \lim_{\epsilon \to 0} \mathbb{E}\left[\left(R(G(D, \theta), \theta) - (1 + r_f)\right) l^h | \theta_{-1}, x^h\right] \tag{4.12}
\]

taking $G(D, \theta)$ and $R(G(D, \theta), \theta)$ as given;
(iii) Rational expectations: the perceived law of motion for aggregate deposits is consistent with individual decisions, \( d'(D, D, \theta) = G(D, \theta) \);

(iv) Markets for capital and deposits clear: defining \( \mu \equiv \int \mathbb{1}\{l^h(D, \theta) = 1\} \, dh \), then

\[
R(G(D, \theta), \theta) = \begin{cases} 
\eta \mu G(D, \theta) + \theta & \text{if } \theta \geq \tau, \\
0 & \text{otherwise},
\end{cases}
\]

and \( r(G(D, \theta), \theta) = r^f + [R(G(D, \theta), \theta) - (1 + r^f)] \mu \).

A few remarks about this definition are in order. First, notice that in the limit as private signals become extremely precise, any interior equilibrium must satisfy condition (4.7). Consequently, following the arguments in the previous section verbatim justifies reducing bank’s portfolio choice problem to (4.12). Second, I have already incorporated into the household’s budget constraint (4.11) the results that both financial intermediaries and non-financial firms earn zero profits in equilibrium, and workers receive zero wages.

C. Equilibrium Characterization

In order to solve for the equilibrium, I proceed in two steps. I begin by studying household’s consumption and saving decisions as solutions to (4.10)-(4.11), which yields the law of motion \( G(D, \theta) \). Then, I solve (4.12) using global games techniques, finding the optimal strategy \( l^h(D, \theta) \) for banks. Finally, I determine the price functions.

Policy Function

Define the non-empty and compact valued correspondence

\[
\Gamma(d; D, \theta_{-1}) = [0, (1 + r(D, \theta_{-1}))d],
\]  

\( (4.13) \)

\footnote{This is because, \( \lim_{\tau \to 0} E_t[\Lambda_{t, t+1} \{ R^h_{t+1} - (1 + r_{t+1}) \} | \theta_{t-1}, y_{t+1}^h] = \Lambda_{t, t+1} \{ R^h_{t+1} - (1 + r_{t+1}) \} \), which follows from an application of the dominated convergence theorem using the fact that the conditional density has compact support over.}
which represent the feasible set for future deposits, and rewrite the household problem described by (4.10)-(4.11) as:

\[ V(d; D, \theta) = \max_{d' \in \Gamma} \left\{ u \left( (1 + r(D, \theta_{-1}))d - d' \right) + \beta \mathbb{E} \left[ V(d'; G(D, \theta), \theta') | \theta \right] \right\}. \]  

(4.14)

I solve this functional equation at the end of the chapter, and show that due to log preferences the optimal policy is linear and involves saving resources for next period according to: \( d'(d, D, \theta) = \beta (1 + r(D, \theta_{-1}))d \). In words, every period the household deposits a fraction \( \beta \) of its current income \((1 + r(D, \theta_{-1}))d \) with banks, and consumes the remaining fraction \( 1 - \beta \). Notice that because this decision only depends on the previous macroeconomic fundamental \( \theta_{-1} \), I can write \( d'(d, D, \theta_{-1}) \) without loss of generality. Thus, in equilibrium

\[ D' = G(D, \theta_{-1}) = \beta (1 + r(D, \theta_{-1}))D. \]  

(4.15)

Optimal Investment Threshold

Suppose that the proportion of financial intermediaries that purchase securities issued by non-financial firms is \( \mu \in [0, 1] \). Hence, the amount of working capital that firms have access to is \( \mu D' \). If the realized value of the macroeconomic fundamental is \( \theta \), then the differential payoff for bank \( h \) obtained from supplying credit to non-financial firms rather than engaging in storage is given by

\[ \Delta(\mu, \theta) \equiv \begin{cases} 
  a - (1 + r^f) & \text{if } \theta \geq \tau - \eta \mu D', \\
  -(1 + r^f) & \text{otherwise}.
\end{cases} \]

The function \( \Delta \) is non-decreasing in both \( \mu \) and \( \theta \). The former property shows the strategic complementarities among bankers investment decisions. Moreover, for all \( \mu \in [0, 1] \), it follows that \( \Delta(\mu, \theta) = -(1 + r^f) < 0 \) for all \( \theta \leq \tau - \eta D' \); and \( \Delta(\mu, \theta) = a - (1 + r^f) > 0 \) for all \( \theta \geq \tau \).

Outside of the dominance regions \( \{ \theta \leq \tau - \eta D' \} \) and \( \{ \theta \geq \tau \} \), the decision of bank \( h \) is driven by his belief regarding the actions of the other banks. As I argued in Section 4.2, complete information about \( \theta \) leads to indeterminacy; however, this is not the case with incomplete information. The next proposition formally characterizes the unique (symmetric) solution of the coordination game.
Proposition 8 (Investment Threshold). The coordination game is solvable by iterated deletion of strictly dominated strategies. At the unique equilibrium, all banks use a switching strategy around a threshold $\theta^*$, and supply credit to non-financial firms if and only if $\theta \geq \theta^*$. In the limit as $\varepsilon \to 0$, the threshold is given by $\theta^* = \tau - \eta D' \left( 1 - \frac{1+r^f}{a} \right)$.

The symmetric nature of the solution implies that banks either save or invest with non-financial firms en masse; hence, in equilibrium $\mu \in \{0,1\}$. I refer to the former scenario as a "credit market freeze," whereby firms cannot operate due to lack of working capital. Therefore, the probability of such an event is $Q(\theta < \theta^* | \theta_{-1})$.

The proof of Proposition 8 is based on the theory of global games, and follows from a direct application of the results in Morris and Shin (2003). They show that for extremely precise private signals, the probability distribution of the fraction of banks that invest, conditional on using the above switching strategy, is uniform over the unit interval $[0,1]$. These Laplacian beliefs entail the maximum degree of strategic uncertainty for an individual bank, and pin down $\theta^*$ as the unique solution to the indifference condition between investing and saving:

$$\int_0^1 \Delta(\mu, \theta^*) d\mu \equiv 0.$$ 

In fact, notice that the threshold can be rewritten as

$$1 + r^f = \left( 1 - \frac{\tau - \theta^*}{\eta D'} \right) a,$$

where $\left( 1 - \frac{\tau - \theta^*}{\eta D'} \right)$ represents the probability that the marginal bank who receives the signal $\theta^*$ assigns to the fraction of banks investing in firms.

Theorem 6 (Equilibrium Dynamics). In the competitive equilibrium of the economy, the aggregate quantity of deposits—and thus total financial intermediaries' wealth—obeys the law of motion

$$D_{t+1} = \beta(1 + r_t(D_t, \theta_{t-1})) D_t,$$  \hspace{1cm} (4.16)

where the gross return from time $t-1$ to $t$ is given by

$$1 + r_t(D_t, \theta_{t-1}) = \begin{cases} a & \text{if } \theta_{t-1} \geq \theta^*(D_t), \\ 1 + r^f & \text{otherwise}, \end{cases}$$  \hspace{1cm} (4.17)
with the investment threshold defined as

$$\theta^*(D_t) = \tau - \eta D_t \left(1 - \frac{1+r_f}{a}\right). \quad (4.18)$$

Figure 4.2 depicts the transitional dynamics along the equilibrium path. A credit market freeze is more likely to occur when both the total wealth of financial intermediaries and the normalized relative return on investment $\rho \equiv 1 - \frac{1+r_f}{a}$ are low.

I further explore the properties of these transitions in the next two sections.

D. Aggregate Shocks and Persistence

To illustrate the importance of $D_t$ as an aggregate macroeconomic variable, consider the following setting. Start with $D_0 \geq \frac{\tau}{\eta \rho}$ and $\theta_{-1} = 0$. Then, because $\theta^*(D_0) = \tau - \eta D_0 \rho \leq 0$, banks lend credit to non-financial firms and $D_1 = \beta a D_0 > D_0$. Suppose that at $t = 0$ there is a negative aggregate shock and $\theta_0 < \theta^*(D_1) = \tau - \eta D_1 \rho$. The shock thus drives the economy into a credit market freeze. Therefore, the amount of resources that financial intermediaries have available next period is equal to $D_2 = \beta(1+r_f)D_1 = AD_0$, where $A = \beta^2(1+r_f)a < 1$. 
Chapter 4. Coordinating Inefficient Credit Flows

Figure 4.3. Persistence. Effects of an aggregate one-time shock to fundamentals at \( t = 0 \) for different initial bank capitalizations: \( D_0 \geq \frac{T}{\eta\rho A} \) (solid) and \( D_0 < \frac{T}{\eta\rho A} \) (dotted). In the latter, the shock switches the accumulation dynamics of the economy.

Now imagine that at \( t = 1 \) the fundamental is back to its original level, \( \theta_1 = 0 \). Does credit to non-financial firms start flowing again once the macroeconomic outlook is back to what it was? The answer depends on the wealth \( D_0 \) that financial intermediaries had before the crisis. If \( D_0 \geq \frac{T}{\eta\rho A} \), then \( \theta^*(D_2) = \tau - \eta AD_0 \rho \leq 0 \), and the supply of credit is back to normal; however, if \( D_0 < \frac{T}{\eta\rho A} \) instead, the investment threshold \( \theta^*(D_2) \in (0, \tau(1 - A)] \) and the credit market freeze persists. In fact, as shown in Figure 4.3 for starkness, without any future innovations in the macroeconomic fundamental, the economy remains stuck in this adverse scenario forever even though the trigger was a one-time negative shock.

The point to take away from this exercise is that the amount of liquid resources of financial intermediaries is a key factor of equilibrium outcomes and the level of real activity, as it determines how resilient or fragile the economy is to aggregate shocks.
E. Interest Rates and the Risk-Taking Channel

To simplify notation, let $\theta^*_t = \theta^*(D_{t+1})$. Recall that the probability of a credit market freeze at time $t$ is given by

$$Q(\theta_t < \theta^*_t | \theta_{t-1}).$$

Hence, in light of the fact that $\frac{\partial \theta^*_t}{\partial r^f} > 0$, a reduction in the safe rate of interest $r^f$ at time $t$ decreases the probability of a market freeze from $t$ to $t + 1$. This result is reminiscent of the "risk-taking channel" in the transmission mechanism of monetary policy envisioned by Borio and Zhu (2012) and documented by Adrian and Shin (2010), which refers to the real effects that rate policies have through changes on risk perception and risk tolerance. In this concrete environment, low interest rates reduce strategic uncertainty, and thus coordination risk, by rendering investment with non-financial firms relatively more profitable.

However, what is the effect tomorrow of a reduction in $r^f$ today, given that the market freeze could not be avoided? In such a scenario, $D_{t+2} = \beta(1 + r^f)D_{t+1}$; hence,

$$d\theta^*_{t+1} - \theta^*_{t+1} = \rho^r \frac{\partial D_{t+2}}{\partial r^f} = \eta \beta D_{t+1} \left( \frac{1 + r^f}{a} - \rho \right).$$

Notice that this expression is positive whenever the relative return on investment is smaller than the gross risk-free return: $a - (1 + r^f) > 1 + r^f$. In this case, the probability that the credit market freeze persists from $t + 1$ to $t + 2$, has increased.

The intuition behind this result can be understood as follows. A marginal reduction in $r^f$ at time $t$ has two opposing inter-temporal effects at $t + 1$. On the one hand, it has the same positive effect as before: it increases the normalized relative return by $\frac{\partial \rho^r}{\partial r^f} = \frac{1}{a}$, which decreases the probability of a credit market freeze from $t + 1$ to $t + 2$ by $\frac{\eta}{a} D_{t+2}$, and thus makes investment with non-financial firms more attractive. On the other hand, if a credit market freeze does occur at time $t$, it shrinks financial intermediaries’ aggregate wealth next period by $\frac{\partial D_{t+2}}{\partial r^f} = \beta D_{t+1}$, which renders coordination more difficult as it increases the probability that the freeze extends in time from $t + 1$ to $t + 2$ by $\eta \rho \beta D_{t+1}$.

The profitability of the productive technology, $a$, determines which of these two forces prevails. Specifically, when $a > 2(1 + r^f)$ the latter is stronger, and policies
of reducing interest rates can then have the unintended consequence of making potential credit market freezes harder to escape. The reason is that, in this case, coordination risk is already low (\( \rho \) is small), and a reduction in the safe interest rate does little to further encourage investment.

**Proposition 9 (Interest Rate Cuts).** Low interest rates decrease the incidence of credit market freezes, but increases their average duration when the productivity of capital is high.

### 4.4 Coordination Failures and Policy Interventions

I now focus on the welfare properties of the competitive equilibrium, and study the effectiveness of different policy interventions in restoring efficiency.

#### A. Welfare

It is important to begin by understanding the source of the inefficiency in the model. Consider a social planner that makes consumption and deposit decisions on behalf of households, and also chooses the investment technology. The planner receives a stream of noisy signals about the macroeconomic fundamental \( \{ \theta_t \} \), and therefore does not have any informational advantage over individual agents. Suppose that, like in the decentralized economy, the signals have infinite precision. This assumption renders the issue of whether the planner is allowed to aggregate private information inconsequential.\(^3\)

Consumption and deposit decisions, \( \{ C_t \} \) and \( \{ D_{t+1} \} \), maximize the expected discounted lifetime utility of households, subject to the aggregate resource constraint of the economy (4.8). As before, these decisions are made under complete information. Then, any optimal consumption plan has to satisfy the condition:

\[
\beta \frac{u'(C_{t+1})}{u'(C_t)} = \frac{1}{1 + \rho_{t+1}(D_{t+1}, \theta_t)}
\]

which coincides with the Euler equation (4.9). Consequently, given an investment strategy selected at time \( t \), the gross return \( 1 + \rho_{t+1} \in \{ a, 1 + \rho' \} \) is determined,

\(^3\)See Angeletos and Pavan, 2007 for a general welfare analysis in economies with strategic interactions and heterogenous information.
and individual households would choose the same contingent plans as the social planner: make deposits equal to a fraction \( \beta \) of their current wealth, and consume the remaining \( 1 - \beta \).

The inefficiency of the decentralized equilibrium lies on the noncooperative solution of the global game played by banks. The efficient outcome would feature banks engaging in storage only when \( \Delta(1, \theta) < 0 \); that is, when the macroeconomic fundamental is so low that the profitability of the productive technology is zero even under full investment of the resources available in the economy. Rather, as showed by Proposition 8 and its subsequent explanation, banks choose not to invest in non-financial firms whenever \( \theta \) is such that \( \int_{0}^{1} \Delta(\mu, \theta) d\mu < 0 \). The magnitude if this coordination failure is thus measured by the difference between the threshold \( \theta^*(D_t) \) and the upper limit of the lower dominance region, \( \tau - \eta D_t \).

**Proposition 10 (Efficiency).** The solution to the social planner's problem involves choosing aggregate deposits according to (4.16)-(4.17), but using a lower investment threshold:

\[
\theta^*_{SP}(D_t) \equiv \tau - \eta D_t < \theta^*(D_t).
\]

Notice that the planner does not eliminate the possibility of credit market freezes; doing so would entail a waste of economic resources when macroeconomic prospects are bleak. The key difference is that the occurrence of such events is completely justified by the fundamentals in the economy, as opposed to being the result of self-fulfilling fears, like in the decentralized setting.

Figure 4.4 contrasts the evolutions of aggregate deposits for a given sample path of shocks \( \{\theta_t\} \). Periods of credit market freezes are indicated by the shaded regions. The fact that freezes happen less often for the social planner is reinforced by a higher average growth rate of wealth of financial intermediaries. In addition, this inter-temporal effect in the accumulation of \( D_t \) enables the economy to exit credit market freezes much faster than in the competitive equilibrium.

In light of Proposition 10, I continue the normative analysis by considering different unconventional credit interventions. To assess their effectiveness in mitigating the coordination failure, it suffices to look at the resulting gap \( \theta^* - \theta^*_{SP} \). Justified by the fact that the consumption/saving decision of households is efficient, I focus on policies financed by lump-sum (non-distortionary) taxes.
Chapter 4. Coordinating Inefficient Credit Flows

Competitive Equilibrium

Figure 4.4. Evolution of aggregate wealth in the financial sector. Shaded regions represent credit market freezes. The ones in the bottom figure are efficient.

B. Equity Injections

Suppose that, at time $t$, the government levies a lump-sum tax $T_t$ on each household, and with the revenue it purchases equity from banks. I assume that the government cannot choose a particular one bank. Then, at time $t + 1$, the government rebates back any profits from its shared ownership of financial intermediaries. For simplicity, I assume that to acquire equity, the government pays no premium over the market price.

As before, banks earn no rents from holding deposits; but now they make profits from their net equity capital $T_t$. Therefore, at time $t + 1$ households receive net payouts from ownership of banks—both private and through the government—in the amount of $\pi_{t+1} = (1 + r_{t+1})T_t$. The sequential budget constraint (4.2) becomes,

$$c_t + d_{t+1} = (1 + r_t)(d_t + T_{t-1}) - T_t.$$  \hfill (4.19)
Notice that the exact share of public vs. private ownership becomes irrelevant.

In order to solve for household's optimization problem in recursive form, I augment the state space to include taxation in the previous period, \( T_{t-1} \), and re-write (4.13)-(4.14) with \( \bar{D} = (D, T_{t-1}) \). It is then straightforward to see that the optimal deposit policy is given by \( d_{t+1} = \beta(1 + r_t(\bar{D}_t, \theta_{t-1}))(d_t + T_{t-1}) - T_t \). Consequently, the aggregate wealth of financial intermediaries at time \( t \) amounts to

\[
D_{t+1} + T_t = \beta(1 + r_t(D_t + T_{t-1}, \theta_{t-1}))(D_t + T_{t-1}),
\]

which is the same as in the case without taxation at time \( t \), as it can be seen by comparing this expression with (4.16).

This result shows that, in this setting, any attempt to infuse capital into the banking sector imposing lump-sum taxes on households is entirely offset by a fall in aggregate deposits in the same amount.

C. Liquidity Provisions and Lending Facilities

Alternatively, imagine that the government expands discount window operations, whereby at time \( t \), it lends the revenue from taxes to banks at a prescribed interest rate \( \zeta(1 + r_{t+1}) \); in this context, the factor \( \zeta \geq 1 \) represents the penalty rate. At time \( t + 1 \), the household receives payouts in the amount of

\[
\pi_{t+1} + \zeta(1 + r_{t+1})T_t = (1 + r_{t+1})[(1 - \zeta)T_t + \zeta T_t] = (1 + r_{t+1})T_t.
\]

Hence, with this policy intervention, household's modified budget constraint in period \( t \) is the same as in (4.19). As a result, (4.20) also holds.

On the other hand, consider public–private partnerships by which the government invests in the productive sector alongside banks wishing to invest as well; otherwise resources are saved in the storage technology. This program is preferred over direct lending to the traditional sector, as it benefits from using the intermediation expertise of banks in monitoring and screening operating non-financial firms.\footnote{See Bebchuk and Goldstein, 2011 for a thorough description of such ventures in a partial equilibrium setting.} Notice then that the gross return on each unit of government spending is also equal to the gross return on bank deposits, \( 1 + r_{t+1} \). Hence, (4.19) again holds, and consequently,
this policy is as futile as the previous ones.

To summarize, the idea behind these credit easing policies is that an increase in the aggregate wealth of financial intermediaries, either directly or indirectly, would provide additional capital that could be supplied to non-financial firms and would also facilitate the coordination of investment. However, these interventions are totally ineffective in encouraging aggregate lending because, in a perfect market for bank deposits, forward looking households anticipate the increase in tax commitments and reduce the amount of deposits accordingly, keeping their consumption/saving pattern unchanged. Ricardian equivalence thus obtains. Formally,

**Proposition 11 (Neutrality).** Reductions in aggregate deposits from households completely crowd out government spending attempts to stimulate bank lending.

### D. Government Guarantees

Finally, consider government guarantees that limit the potential losses faced by financial intermediaries at the time they make their portfolio choices. Specifically, suppose that banks are guaranteed a gross return $R_g \in (0, 1 + r_f)$ on loans made to non-financial firms whose projects fail. Hence, for an individual bank, the differential payoff between its investment and storage technology is now given by

$$\Delta_g(\mu, \theta) \equiv \begin{cases} a - (1 + r_f) \quad \text{if} \quad \theta \geq \tau - \eta \mu D', \\ R_g - (1 + r_f) \quad \text{otherwise}. \end{cases}$$

Like before, the investment threshold characterizing the outcome of the global game is defined as the unique solution to the Laplacian condition: $\int_0^1 \Delta_g(\mu, \theta^*_g) d\mu \equiv 0$. Then, a straightforward calculation yields,

$$\theta^*_g(D') = \tau - \eta D' + \eta D' \frac{(1 + r_f) - R_g}{a - R_g} < \theta^*. \quad (4.21)$$

Crucially, because in equilibrium banks only supply credit when the macroeconomic fundamental is at least $\theta^*_g$, they experience no losses; hence, as long as $R_g < 1 + r_f$, the intervention does not actually require public spending. As a result, the neutrality result that left the previous credit policies ineffective does not
apply, and the aggregate wealth of financial intermediaries is the same as without
government guarantees. Therefore, the competitive equilibrium is characterized by
(4.16) and (4.17), with $\theta^*_g(D_t)$ given by (4.21). This leads to the next proposition.

**Proposition 12 (Government Guarantees).** In the limit as $R_g \uparrow 1 + r^f$, it follows that
$\theta^*_g \downarrow \theta^*_s$, which imply that government guarantees completely eliminate inefficient credit
market freezes.

The fact that guarantees can encourage the flow of credit to non-financial firms,
and thus improve welfare, hinges crucially on the assumption that the only source
of downside risk in the model is coordination. However, this is for starkness, and
serves to highlights how the mere possibility of such interventions works by di-
rectly reducing strategic uncertainty, restoring confidence in the financial sector.
Critically, what is needed for them to be effective are credible announcements by
the government.

**Dynamic Programming Proofs**

**A. Solving the Bellman Equation**

Take the functional equation (4.14), and consider a solution of the form:

$$V(d; D, \theta) = \alpha \log(d) + B(D, \theta),$$

(4.22)

where $\alpha > 0$ is a constant, and $B$ is a function to be determined. Based on this
guess, the first order optimality condition for $d'$ implies that

$$d'(d, D, \theta) = \frac{\beta \alpha}{1 + \beta \alpha} (1 + r(D, \theta_{-1})) d.$$  

(4.23)

After some simple algebraic manipulations it is easy to see that (4.22) is verified for
$\alpha = \frac{1}{1 - \beta}$ and $B$ implicitly given by

$$B(D, \theta) = \gamma + (1 + \beta \alpha) \log(1 + r(D, \theta_{-1})) + \beta \mathbb{E} \left[ B(G(D, \theta), \theta') | \theta \right],$$

(4.24)
where \( \gamma = \log \frac{1}{1+\beta\alpha} + \beta\alpha \log \frac{\beta\alpha}{1+\beta\alpha} \). Notice from (4.24) that, for any bounded non-negative return function \( r(D, \theta_{-1}) \), \( B \) is defined as the fixed point of an operator that maps the space of continuous and bounded functions \( \mathbb{R}_+ \times \Theta \mapsto \mathbb{R} \), endowed with the sup norm, into itself. This operator inherits the monotonicity of \( \mathbb{E}[-] \), and it also satisfies discounting; hence, it verifies Blackwell's sufficient conditions for a contraction (with modulus \( \beta \)). Consequently, there exists a unique function \( B \) that solves (4.24) and has the aforementioned characteristics.

### B. Optimality of the Value Function

For a feasible plan for deposits \( \{d_t\} \) starting from some \( d_0 > 0 \) and \( \theta_{-1} \in \Theta \), define the expected discounted lifetime utility as

\[
\nu(\{d_t\}) \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log \left( (1 + r_t)d_t - d_{t+1} \right).
\]

The household's optimization problem in sequential form is:

\[
\nu_0^* = \max_{\{d_t\} \geq 0} \nu(\{d_t\}) \quad \text{with} \quad d_0 > 0, \theta_{-1} \in \Theta. \quad (4.25)
\]

What remains to be established is that the value function \( V \) defined by (4.22), and the associated policy \( \{d_t^*\} \) in (4.23), actually solve (4.25):

\[
\nu_0^* = \nu(\{d_t^*\}). \quad (4.26)
\]

In order to probe this, I resort to Theorem 9.12 in Stokey and Lucas (1989).

I begin by checking some regularity conditions. First, notice that the correspondence \( \Gamma \) define in (4.13) admits many measurable selections; for example, all constant savings rate policies. Second,

**Lemma 2 (Regularity).** Let \( U(d_t, d_{t+1}) = \log((1 + r_t)d_t - d_{t+1}) \). Then, for any initial condition \( (d_0, \theta_{-1}) \in \mathbb{R}_+ \times \Theta \), \( \lim_{n \to \infty} \mathbb{E}_0 \sum_{t=0}^{n} \beta^t U(d_t, d_{t+1}) \) exists but it may be infinite.
Proof. Define \( U^+ = \max\{U, 0\} \). It suffices to prove that
\[
\lim_{n \to \infty} \mathbb{E}_0 \sum_{t=0}^{n} \beta^t U^+(d_t, d_{t+1}) < \infty.
\]
This limit exists because the sequence in question is non-negative and non-decreasing. Notice that \( d_{t+1} \geq 0 \) implies that,
\[
\log((1 + r_t)d_t - d_{t+1}) \leq \log(1 + r_t) + \log d_t.
\]
On the other hand, \( d_{t+1} \leq (1 + r_t)d_t \). Hence, \( \log d_{t+1} \leq \log(1 + r_t) + \log d_t \), which gives
\[
\log d_t \leq \sum_{n=0}^{t-1} \log(1 + r_t) + \log d_0 \leq t \log a + \log d_0,
\]
where I have made use of the fact that the interest rate on deposits is upper bounded by \( a > 1 \) in equilibrium. Then,
\[
\mathbb{E}_0 \sum_{t=0}^{n} \beta^t U^+(d_t, d_{t+1}) \leq \mathbb{E}_0 \sum_{t=0}^{n} \beta^t |\log((1 + r_t)d_t - d_{t+1})|
\leq \sum_{t=0}^{n} \beta^t (t + 1) \log a + \sum_{t=0}^{n} \beta^t |\log d_0|.
\]
Taking limits on both sides of the above inequality yields,
\[
\lim_{n \to \infty} \mathbb{E}_0 \sum_{t=0}^{n} \beta^t U^+(d_t, d_{t+1}) \leq \frac{\log a}{(1 - \beta)^2} + \frac{|\log d_0|}{1 - \beta} < \infty,
\]
which renders the desired result.

Now, consider the solution function \( V \) in (4.22) and the associated policy function described by (4.23): \( d_{t+1}^* = \beta(1 + r_t)d_t^* \). To simplify notation I write \( r_t = r(D_t, \theta_{t-1}) \). Under this policy, the deposit rule \( \{d_t^*\} \) generated is given (in logs) by:
\[
\log d_t^* = \sum_{n=0}^{t-1} \{\log \beta + \log(1 + r_n)\} + \log d_0,
\]
This plan satisfies two important properties. First, it verifies the "transversality
Chapter 4. Coordinating Inefficient Credit Flows

condition:"

\[ \lim_{t \to \infty} \beta^t \mathbb{E}_0 V(d_t^*; D_t^*, \theta_t) = 0, \quad \text{for all } d_0 > 0, \theta_{-1} \in \Theta. \] \tag{4.29}

Indeed, the fact that the function \( B \) is bounded renders

\[ \lim_{t \to \infty} \beta^t \mathbb{E}_0 V(d_t^*; D_t^*, \theta_t) = \lim_{t \to \infty} \frac{\beta^t}{1 - \beta} \left\{ t \log \beta + \log d_0 + \sum_{n=0}^{t-1} \mathbb{E}_0 \log(1 + r_n) \right\} ; \]

and because \( 0 \leq \mathbb{E}_0 \log(1 + r_n) \leq a \), the limit must be zero. Second, it achieves a finite expected discounted lifetime utility, which is seen by making use of (4.28) as follows,

\[ v(d_t^*) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log ((1 - \beta)(1 + r_t)d_t^*) \]

\[ = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(1 - \beta) + t \log \beta + \log d_0 + \sum_{n=0}^{t} \log(1 + r_n) \right] < \infty. \tag{4.30} \]

Finally, the last condition needed to invoke the result in Stokey and Lucas (1989) is gathered in the next lemma; its subsequent proof thus establishes (4.26). Let \( \hat{G} \) denote the set of all feasible policies for which (4.29) holds, and notice from (4.29) that \( \{d_t^*\} \in \hat{G} \). Then,

**Lemma 3 (Dominance).** For any initial condition \((d_0, \theta_{-1}) \in \mathbb{R}_+ \times \Theta\) and any feasible plan \(\{d_t\}\), there exists a policy \(\{d_t^*\} \in \hat{G}\) such that \(v(\{d_t^*\}) \geq v(\{d_t\})\).

**Proof.** Consider a feasible policy \(\{d_t\} \notin \hat{G}\). From the previous analysis, this is equivalent to \(\lim_{t \to \infty} \frac{\beta^t}{1 - \beta} \mathbb{E}_0 \log d_t \neq 0\). Hence, the series \(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log d_t\) diverges; in addition, the bound found in (4.27) implies that the series must diverge to \(-\infty\). But then, given that

\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(d_t, d_{t+1}) \leq \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log((1 + r_t)d_t), \]

it follows that \(v(\{d_t\}) = -\infty\). Choosing \(\{d_t^*\} = \{d_t^*\}\) and recalling from (4.30) that \(v(\{d_t^*\})\) is finite, completes the proof.

\(\square\)
CHAPTER 5

Concluding Remarks

THIS THESIS was concerned with the channels by which fire sale externalities, asymmetric information, and investment complementarities leave the economy excessively vulnerable to costly financial crises. All the models presented, emphasized the prominent role played by financial frictions as drivers of economic fluctuations. Below, I summarize the key contributions of each chapter, and offer some final thoughts on potential directions for future research.

Thesis Summary

Chapter 2 developed a theoretical foundation for macroprudential policy to reduce the likelihood of baking crises. Without regulation, banks do not internalize that an additional unit of short-term debt funding impairs the borrowing terms of other financial institutions, and heightens the vulnerability of the whole economy to a systemic panic run. The model provided an endogenous metric for systemic risk, which encompasses both fundamental insolvency and illiquidity risk. More importantly, it explained how this measure should be used to optimally tax the issuance of short-term debt in order to restore constrained efficiency.

Chapter 3 analyzed the detrimental effects of financial expertise for rent-seeking purposes. Liquidity may evaporate as the result of information panics, equilibrium scenarios where fears of asymmetric information become self-fulfilling and cause adverse selection problems. Regulations that prevent firms from gaining informational advantages that allow them to extract rents at the expense of uninformed investors, can discourage the acquisition of financial expertise for such specula-
tive purposes and thus maximize welfare. Mandatory information disclosures and credit rating agencies may play an important role in preserving symmetric ignorance in credit markets.

Finally, Chapter 4 considered a dynamic macroeconomic setting where financial intermediaries are not constrained in their ability to raise funding, but nonetheless refuse to expand the issuance of loans to non-financial firms because of self-fulfilling fears of others not lending. The model highlighted the inability of capital injections, liquidity provisions, and lending facilities in mitigating the inefficient coordination failure. In contrast, although proportional reductions in household deposits completely crowd out public spending, credible announcements of government guarantees limiting potential losses on investment can actually facilitate the flow of credit by reducing the level of coordination risk. Traditional interest rate cuts increase the resilience of the economy to credit market freezes, but it may impair its ability to escape them when they occur.

Future Research Directions

Systemic Risk Measures and Macroprudential Policy

The financial crisis of 2007–2009 prompted a comprehensive reform of the financial regulatory framework, one that underscored the importance of systemic risk and financial stability. Numerous measures of systemic risk had been proposed by academics, each emphasizing different aspects of the problem (see e.g. Acharya et al., 2010; Brunnermeier and Adrian, 2014). The idea shared by all these proposals is that failures and losses of individual financial institutions can impose externalities on other market participants; the goal of financial regulation is to force institutions to internalize such neglected costs. It is important that the principles behind macroprudential policies be guided by general equilibrium theories that allow for rigorous welfare analysis; yet, there seems to be a gap between theory and practice. The model in Chapter 2 is very stylized, thus its recommendations may lack practical relevance. A natural next step is therefore to translate its insights into a more general macroeconomic setting, one in which the magnitude of the optimal Pigouvian tax could be quantified. This effort may help guide policymakers in the design and implementation of optimal financial regulation.
Dynamics of Information Production and Acquisition

Information panics may explain the fragility of financial markets in terms of their vulnerability to sudden breakdowns. Recent research has embedded financial markets into dynamic macroeconomic environments to study how asymmetric information about the quality of assets can amplify aggregate shocks. For example, Gorton and Ordoñez (2014) describe how credit booms may develop over time as agents disregard the quality of collateral; as this trend continues, small shocks can have large effects by triggering a switch in the “information regime.” Kurlat (2013) shows that when entrepreneurs need to sell assets to undertake investment opportunities, if some of these assets are lemons and buyers cannot identify them, a wedge between the return on savings and the cost of funding appears which encourages some entrepreneurs to stay out of the market; this informational friction makes exogenous aggregate shocks more severe. The model in Chapter 3 identifies an alternative mechanism, based on self-fulfilling fears of asymmetric information, by which shocks may also be amplified. Understanding how these panics affect the production and evolution of information in those dynamic settings could be a direction to pursue in the future.

Liquidity Constraints and Macroeconomic Complementarities

Recent work has augmented DSGE models to include financial sectors, in an effort to quantitatively evaluate the impact of financial frictions in business cycle fluctuations (see e.g. Gilchrist, Yankov, and Zakrajsek, 2009; Christiano, Motto, and Rostagno, 2010; Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; and references therein). The main focus has been on understanding how feedback loops linking liquidity constraints and the value of collateral can amplify disruptions in financial markets and spill over to the real economy, with large and persistent effects. Incorporating macroeconomic complementarities into these studies, in the spirit of Chapter 4, could capture the interplay between funding stresses triggered by exogenous shocks and endogenous aggregate market expectations, and would allow to quantitatively assess the effectiveness of different government interventions in normalizing market conditions in the short term, and spurring growth in the long term. This topic is left for future research.
Bibliography


