

INSTRUMENT FOR THE MEASUREMENT
OF THE DROPLET SIZE DISTRIBUTION IN CLOUDS

by

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ABSTRACT

An instrument has been developed for the measurement of cloud drop-size distributions. Measurements can be made of distributions comprising droplets whose range of diameters is from 2 to 50 microns at an average counting rate of 2000 per second.

The operating principle is based upon the concept that the amount of electrostatic charge a droplet can acquire is limited by the breakdown potential gradient of the medium surrounding of the droplet. It is shown that this charge is proportional to the surface area of the droplet.

The limiting charge is obtained by passing the droplets through a coronal discharge chamber. The droplets are collected by means of a jet impinger in which the collector plate is an electrical probe. The distribution of signal pulses generated by the collector probe is amplified and then directed into a special evaluator which automatically computes the droplet size distribution.

Equations describing the performance characteristics of the instrument are reported.

An experimental technique for the determination of the collection efficiency of the instrument as a function of a droplet size is included.

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INTRODUCTION

The determination of droplet size distribution in clouds has been a problem of great interest to meteorologists since the inception of the science. Undoubtedly natural inquisitiveness was the motivating factor in the initial measurements. However, when fundamental investigations of clouds were undertaken in relation to the origin and release of precipitation (1), the natural limitations upon visibility (2), and the mechanism of icing (3) it was realized that the size distribution of the droplets constituting the cloud is one of its most important physical characteristics. Presumably these distributions are functions of both time and space. In order to increase the fund of knowledge it would be desirable to measure these partial variations separately. This end can be accomplished most economically by taking the observations from aircraft.

CHAPTER I

OBSERVATIONS OF DROPLETS IN NATURAL CLOUDS

The literature contains many references to measured drop-size distributions (4) (5) (6) (7) (8). The most extensive and possibly the most accurate observations are those obtained (with the aid of glass slides) from aircraft by Diem (5). These data have been summarized by Houghton (9).

Cloud Type	Peak Diameter (μ)	Range of Curve (μ)
Dense Cumulus	14.5	3-40
Fair-Weather Cumulus	8.5	2-20
Stratocumulus	7.9	2-24
Nimbostratus	13.2	2-42

Stratus	12.9	2-42
Altostratus	10.6	2-30

Diem notes that many distributions have secondary maxima.

Howell (10) has summarized data taken on Mount Washington using the Rotating Cylinder technique which seems to be in substantial agreement with Diem as to the bounds on droplet sizes existing in clouds. In addition Howell reports that 95% of the clouds have droplet concentrations of less than 2000 droplets per cubic centimeter, while Diem's work indicates an upper limit of 400 droplet per cubic centimeter.

PRESENT METHODS USED FOR DROP SIZE MEASUREMENTS

Although the techniques of drop-size measurement which have been developed for use in aircraft have definite failings, a short discussion of the physical processes involved will be beneficial as background material. Greater detail is given in reference (11).

Multicylinder Method (12)

The operation of this instrument is based upon the principle that a cylinder moving through a cloud will intercept an amount of the cloud which is a function of both the drop-size distribution and the diameter of the cylinder. The collection efficiency which is defined as the ratio of the amount of water collected by the cylinder to the amount of water contained in the volume of atmosphere swept out by the cylinder has been calculated by Langmuir and Blodgett (13) but has not been verified experimentally.

In order to use the multicylinder technique to obtain a unique drop-size distribution it is necessary to bound the distribution

by some means and then divide the droplets within these bounds into a number of class intervals equal to the number of cylinders used in the measurement. In this manner a closed set of equations may be obtained and an n-point distribution drawn. This procedure is not followed in practice because of the uncertainty in the values of the collection efficiency for cylinders. The drop-size distribution is actually determined by a comparison of the shape of the curve obtained by plotting the logarithm of the rate of water interception per unit area against the logarithm of the cylinder diameter with a set of theoretical curves calculated for five different single maximum drop-size distributions. The specified distributions supposedly embrace the great majority of real distributions. Since at best it is only possible to obtain by these means artificially specified distributions with single maxima, this approach will be considered unsatisfactory.

Slide Technique (14)

If a glass slide covered with oil or soot is momentarily exposed perpendicular to the air stream flowing past the aircraft it will collect a sample of the cloud equal to the number of droplets in the volume swept out and modified by the collection efficiency of the slide. If the collection efficiency and also the relation between the trace left on the slide and the drop diameter is known, it is possible to obtain the drop-size distribution of the cloud. The disadvantages of this method are that droplets larger than 20μ shatter upon impact leaving a poorly defined trace and that the evaluation of the data is extremely tedious.

Drop Photography(15)

It is possible to obtain drop-size distributions by

photographing a sample of the cloud. The difficulty in this method is the magnification required to resolve the smaller droplets limits the focal volume to include only 1 to 10 droplets. Another point which must be considered is that droplets outside the focal volume will contribute an error count since they will appear larger than normal. This last factor may be obviated by restricting either the flow or the illumination to the focal volume only. The single feature which makes this system undesirable is that as many as 100 photographs must be evaluated in order to obtain one distribution.

Cascade Impactors (16)

The operating principle of the cascade impactor is similar to that of the Multicylinder Method in that they both rely upon the variation of collection of object in the air stream as a function of the droplet size. In the impactor the droplets are collected upon several glass slides oriented at right angles to stream flow. The collection efficiencies of the slides are adjusted such that only the larger droplets impinge of the first glass slide. As the name implies, this process is repeated so that in principle each slide counts droplets in a narrow class interval. If it were possible to adjust the collection efficiencies of the slides such that sharp discontinuities existed with respect to drop size, this would be an excellent instrument. This is not the case and each slide collects a rather wide distribution requiring a modified Sooted Slide evaluation. A further weakness of the impactor is that the resulting distribution has been averaged over space rather than yielding the distribution at a point.

CHAPTER II

POSSIBLE METHODS TO BE USED IN OBTAINING DROP-SIZE DISTRIBUTIONS

In considering new approaches to the problem of drop-size determination it was felt that any proposed design must satisfy three criteria. The chosen method must incorporate a means for a direct, overall calibration of the system. It must yield a continuous record of drop-size distribution as a function of time, and the labor involved in the evaluation procedures should be kept to a minimum.

It is implicit in the second criterion that techniques which result in the time averaging of distributions be excluded. An example of the disallowed technique is the previously discussed Slide Method. As opposed to the Slide Method which requires a finite measuring time, the proposed instrument should obtain a drop-size distribution in an interval of time ideally approaching zero length. Techniques based upon the measurement of electrical properties associated with droplets appear attractive because the measuring time constant of such systems can be made as smaller than those of any others, subject to the limitations imposed by statistical fluctuations of the sample. Another factor which contributes to the desirability of the electrical technique is that the output response is electrical in nature and can be fed directly into electronic equipment for storage or immediate evaluation.

Because of the arguments stated above, an investigation of the feasibility of an all electronic drop-size determination equipment was conducted. A summary of the results is reported in the following section.

CAPACITANCE METHOD

When a stream of air containing water droplets is passed through the plates of an electrical capacitor, the capacitance of the unit will vary in accordance with the amount of water and water vapor in the stream. If the physical dimensions of the capacitor are small it may be possible to observe the passage of a single droplet of water between the plates with consequent determination of droplet size. It remains to determine if the effect is measurable with present or probable instrumentation.

Suppose that the air stream has a particle concentration of 100 droplets per cubic centimeter (cc.). The volume of a parallel plate capacitor required to resolve a single droplet should be approximately 1/10 millimeter (mm.). The capacity of such a capacitor with an air dielectric is 0.044 microfarads (mmfd.)

The change of capacitance of this unit due to the presence of a cloud droplet may be approximately determined by computing the change of stored energy in the system due to the presence of a dielectric sphere. This has been done in reference (17) and the resulting equation is:

$$\Delta C = \frac{C \pi (e_1 - e_0) D^3}{2 (2 e_0 + e_1) V}$$

where C = value of capacitor
 e_0 = dielectric constant of air
 e_1 = dielectric constant of water
 V = volume of capacitor
 D = diameter of droplet

Substituting numerical values for the capacitor under consideration the following expression is obtained:

$$\Delta C = 6.62 \times D^3 \times 10^{-10} \text{ } \mu\text{fd}$$

where D is in microns

Thus for a cloud droplet whose diameter is 2 microns

$$\Delta C = 5.3 \times 10^{-9} \text{ } \mu\text{fd}$$

and for a cloud droplet whose diameter is 50 microns

$$\Delta C = 8.3 \times 10^{-5} \text{ } \mu\text{fd}$$

These changes would occur at an initial capacitance level of 4.4×10^{-2} mmfd.

A survey of commercial laboratory grade capacitance meters indicates that the lowest available inaccuracy is $.1\% \pm .5$ mmfd. at capacitance levels from 1 to 1000 mmfd. The lower level of 1 mmfd. is 10^2 above the ambient capacity of the proposed capacitor.

There is described in the literature (18) a capacitance meter which has a full scale range of .005 mmfd. Assuming an indicating meter of 1/2% accuracy, this instrument could possibly detect the 50 micron droplet.

Conclusions

It hardly seems likely that this method could be refined to detect single droplets. The argument given above has neglected several important factors, listed below, which introduce further errors in measurement.

1. Parallel capacitance of the supports and the attached

electrical circuits.

2. The effects of the variation of the ambient temperature upon the spacing of the condenser plates.
3. The changes in the air dielectric constant due to the variation of the vapor pressure of water.

THE ELECTROSTATIC CHARGE METHOD

The work of Ladenburg (19) and others (20) (21) indicate that the time required for electrostatic charging of spherical particles passing through a coronal discharge is independent of particle size, and that the maximum charge on a particle varies directly with the electrical field strength and the square of the particle diameter.

The equation relating the variables which determine the amount of charge collected by a particle is given by Brun (21) to be the following:

$$Q = \left(1 + 2 \frac{\epsilon - 1}{\epsilon + 2}\right) \frac{\pi a^2 E_0 j t}{E_0 + j \pi t} \quad (1)$$

where

Q = electrostatic charge

ϵ = dielectric constant of particle

E_0 = electrostatic field strength

j = current density

t = time

This equation has been verified by Fuchs et al (22) and Pauthenier and Moreau-Hanot (20) for droplet particles larger than 1 micron. For particles smaller than 1 micron, the influences of diffusion ions and of mirror-forces which were neglected in the derivations given in the literature become extremely important. The experimental work of

Fuchs (22) indicates that equation (1) can only be considered to give the lower limit to the charging rate of particles below 1 micron in diameter.

It would appear from these expressions that the charge on the particle may be made as large as desired by merely increasing the electrostatic field strength. Actually there is a theoretical limit to the amount of charge which can exist on a particle determined by the ionization potential of the atmosphere. Physically, the limiting process may be described in this manner. As the charge on the particle is increased, a point is reached where the potential gradient at the surface of the droplets becomes sufficiently great so as to produce a coronal discharge. The coronal discharge decreases the charge on the droplet until a point is reached where the surface potential gradient will no longer support the discharge. The particle then begins to collect more charge until the gradient initiates corona action again and the cycle then repeats. Experiments by Fuchs (22) indicate that the magnitude of these charge oscillations is less than 2%.

The physical explanation may be interpreted mathematically by considering the water droplet to be a dielectric sphere. The electrostatic field strength at the surface of an isolated sphere is given by (23)

$$E_0 = \frac{dV}{dr} = - \frac{q}{r^2}$$

where

r = radius of sphere

q = surface charge

If this field strength is set equal to the atmospheric breakdown

potential, an equation relating limiting charge to droplet diameter is obtained.

$$q = 100 r^2 \quad (2)$$

The time required to obtain the limiting charge may be found by setting equation (2) equal to equation (1).

$$t = \frac{E_b E_0}{\pi j \left[\left(1 + 2 \frac{\epsilon - 1}{\epsilon + 2} \right) E_0 - E_b \right]} \quad (3)$$

It is important that the droplets be exposed to the coronal discharge for such a period of time if the results of this technique are to be valid. It is well known that cloud droplets carry varying amounts of surface charge in their normal state. Exposure to the corona will simply add an increment of charge to the existing charge unless the limiting exposure is used.

THE EFFECT OF CHARGE UPON DROPLET SIZE

It is interesting to consider the effect of the surface charge acquired by the droplet upon the equilibrium water vapor pressure existing at the droplet surface. In the absence of free charge, the surface vapor pressure is governed by the surface tension of the water.

$$L_v \frac{e_r}{E} = \frac{2 \gamma}{R T r} \quad (4)$$

where

e_r = equilibrium vapor pressure at droplet surface

E = equilibrium vapor pressure at a plane surface

r = droplet radius

R = gas constant

ρ = density of water

T = absolute temperature

The equation may be written in the following form:

$$\rho RT \ln \frac{e_r}{E} = \frac{2\gamma}{r}$$

The term on the right of the equality sign represents the pressure differential across the droplet surface. When a surface charge is collected there is an additional pressure differential term of opposite sense due to the Coulomb forces. The equation combining these pressures is

$$\rho RT \ln \frac{e_r}{E} = \frac{2\gamma}{r} - \frac{q^2}{8\pi r^4}$$

where q = total surface charge

The amount of surface charge required to counter balance the surface tension effect is found by setting the two pressure terms equal to each other.

$$q = \sqrt{16\pi r \gamma^3}$$

If numerical values are substituted in this expression it will be found that the computed charge is at least an order of magnitude greater than the limiting charge for cloud droplets, consequently it is felt that charge acquisition will have little effect upon cloud droplet distributions.

Measurement of Drop-Size Distribution by the Multicylinder
Collection of Charged Particles

In order to obtain the drop-size distribution of the charged droplets it is necessary to measure either the current flow due to a stream of the droplets impinging upon a conductor or to measure the charge on individual particles.

The first mode of attack is based upon the previously discussed variation of the collection efficiency of cylinders as functions of drop size. Since the collection efficiency of a cylinder is a unique function for a particular droplet, the flow of current from each cylinder will also be unique. However, it can be shown that at the present time the knowledge of collection efficiencies is inadequate for a precise determination of droplet distribution.

Consider the method of evaluation for the multicylinder technique. The distribution is divided into N classes. Under the assumption that all the droplets give up their charge upon hitting the cylinders, the following equations may be written:

$$\begin{aligned} i_1 &= N_1 Q_1 U_{11} V A_1 + \dots + N_N Q_N U_{1N} V A_N \\ &\vdots \\ i_N &= N_1 Q_1 U_{N1} V A_1 + \dots + N_N Q_N U_{NN} V A_N \end{aligned}$$

where

- i_n = current flow from the N^{th} cylinder
- N_n = number of droplets/cc. in class n
- Q_n = mean charge on class n droplets
- U_{1n} = collection efficiency of cylinder for droplets of class n

V = velocity of air stream

A_N = projected area of cylinder N

If there are N different cylinders and N classes chosen, the system of equations form a closed system and a solution is obtainable. The major difficulty in this approach is that the collection efficiencies must be known very accurately. Consider the case where it is desired to measure a distribution whose droplets range from 2 to 60 microns. This implies that the ratio of Q_1 to Q_N is 10^{-3} . Now if it is necessary to determine N_1 to one significant place it must be possible to add the term $Q_1 U_1 V A_1$ to the term $Q_N U_N V A_N$ and still retain the one place significance of N_1 in the sum. This means that every term in the product must be known to three places. The collection efficiencies as calculated by Langmuir and Blodgett (13) are given to three places but with the third place doubtful, consequently it is not felt that this scheme is suitable for measuring drop-size distributions in clouds.

Such a system as the one described above might be feasible if the width of interest in the distribution were reduced. If the desired lower limit of the distribution were raised to 8 microns, a 50:1 current ratio between the minimum and maximum terms would exist and an evaluation of the 8 micron class would involve only a 20% uncertainty.

Conclusions

From theoretical considerations it appears that an instrument of this type is capable of measuring droplets whose distribution lies between 8 and 60 microns with approximately 20% uncertainty at the

low end. It is necessary that collection efficiencies be known to three places in this instrument if the uncertainty is to be kept to the indicated amount. Most cloud droplet distributions extend to 2 microns or smaller, so this instrument would be of little utility in obtaining cloud distributions.

The Measurement of the Charge on Individual Droplets

The most direct method for the determination of the drop-size distribution of a charged cloud is to measure the charge of the individual droplet. This can be done most conveniently by constraining the droplets so that they impinge one at a time upon a conducting probe. The potential of the probe will in turn respond in a unique manner depending upon the electrical time constant of the probe circuit and the approach velocity of the droplet. The equation relating probe potential and the approach of the charged droplet has been derived (see appendix) and is given below

$$\frac{\phi rc}{a Q} = e^{r/Rcv} \left[e^{-r/Rcv} + \frac{r}{Rcv} \int_0^{r/Rcv} \frac{e^{-u}}{u} du \right] \quad (5)$$

where

ϕ = probe potential

a = radius of probe

C = capacity of probe

Q = charge of the droplet

r = distance of droplet from probe

R = resistance in the probe circuit

v = approach velocity of the droplet

This equation is best described by a plot of the dimensionless parameter

$C\phi/aQ$ versus the dimensionless parameter r/\sqrt{RC} . This plot is given in figure 1 .

The smallest droplet which can be measured with this technique is determined by the ratio of the random noise voltage introduced by the electrical resistance in the probe circuit to the signal voltage developed across the probe resistor. When computing the noise voltage generated by an electrical resistor it is important to include the modifying effects of the shunt capacitance across the resistor. The noise voltage may be computed using an equation due to Schotky (24)

$$E = \left(4KT \int_0^{\infty} R(f) df \right)^{1/2} \quad (6)$$

where

k = Boltzman's constant

T = absolute temperature

f = frequency

R = resistance

R in this case is equal to

$$\frac{R}{1 + (2\pi R C f)^2}$$

where

C = capacity

Integrating over all frequency, the interesting result is obtained that noise voltage is independent of resistance.

$$E = \sqrt{\frac{KT}{C}}$$

Random fluctuations of noise voltage can equal three times the effective

value of the noise voltage computed above.

The magnitude of the signal voltage is obtained from equation (11) and is of the order Q/C . The ratio of signal to noise is then

$$\frac{S}{N} = \frac{Q}{3\sqrt{KTC}}$$

The minimum detectable signal can be obtained by setting equation equal to unity. The signal obtained corresponds to a droplet diameter of approximately 2 microns.

Impinger Considerations

If a simple probe is moved through a charged cloud, counts will be indicated due to droplets which impinge upon the probe and also due to droplets which pass in the vicinity of the probe because of displacement current. It is important that the influence of these droplets passing in the vicinity of the probe be removed as they introduce an error count in the resulting measured distribution. The most direct manner of obviating this error is to make use of the jet impinger principle so that the probe would have a very high collection efficiency for all droplets above two microns. This can be done easily by placing a cap with an orifice drilled in it over the probe and then attaching the probe to a vacuum system. The geometry of the probe is then arranged in accordance with jet impinger theory. May (25) and Sell (26) have shown that in jet impingers the collection efficiency is directly proportional to the dimensionless parameter

$$\frac{\rho v D^2}{\eta l}$$

where ρ = particle density
V = particle velocity
D = particle diameter
l = associated with the length of the collector
 η = viscosity of the air

Ferry (27) has experimentally verified this relation for bacteria of 1 micron in diameter. He obtained collection efficiencies greater than 50% using relatively high jet velocities (150 m/sec.) and large collecting plates (2 cm.). Ferry states that the actual collection efficiency was considerably higher as there was a loss of counts due to bacteria being blown off the collector by the jet action. In this application the jet velocity would be sonic and the collecting probe of the order of .1 cm., consequently it is felt that the requisite high efficiency is obtainable.

Coincidence Errors

The last factor which must be investigated concerns the probability that two or more droplets will impinge upon the collector probe simultaneously, or within the time required for the probe to return to equilibrium potential after the impingement of a droplet.

Assuming the cloud droplets are randomly distributed in space, the probe counts will occur randomly in time and the probability of obtaining exactly m counts during the recovery time T_r is given by the Poisson relation. (See appendix 2)

$$P_m(t_r) = \frac{(NT_r)^m}{m!} e^{-NT_r}$$

where m = coincidence counts
 T_r = recovery time
 n = average number of events occurring per unit time

is obtained by summing the Poisson relation from m equal to 2 to m equal to infinity.

$$\sum_2^{\infty} P_m(t_r) = 1 - e^{-nT_r} - nT_r e^{-nT_r} \quad (7)$$

The probability of coincidence can be made less than any assigned value by reducing the counting rate suitably. Using a recovery time of 50 microseconds as suggested by Guyton's work (28) a probability of coincidence of 1% is determined for a counting rate of 2000 droplets per second. Professor Houghton has determined experimentally (37) that a count of 400 droplets is sufficient to resolve a single cloud distribution.

Conclusions

It is seen from this analysis that an instrument can be designed which will measure cloud droplets greater than 2 microns in diameter at a average rate of 2000 droplets per second.

The design of this instrument is given in the following chapter.

CHAPTER III

DESIGN OF THE ELECTROSTATIC DROP-SIZE DISTRIBUTION DETERMINER LABORATORY MODEL

Properties of the Input Signal

For convenience, the significant properties of the input signal will be restated.

Maximum droplet diameter	50 microns
Minimum droplet diameter	2 microns
Maximum droplet concentration	2000 droplets per cc.
Dielectric constant of water	81

Basic Components of the Instrument

There are four basic sequential processes which take place in the resolution of a drop-size distribution by the electrostatic technique. The droplets are first charged by passing through the Coronal Charging Chamber and then detected by the Collecting Probe. The resulting signals are fed into a Pulse Amplifier to raise their signal level so that they can actuate the Evaluator. It is this Evaluator which ultimately computes the drop-size distribution. Each of the above processes is integrally associated with a basic component of this instrument. A block diagram indicating this operational sequence is given in figure 2 . The design of the individual components will be discussed in the succeeding sections.

Coronal Charging Chamber

The design parameters for this chamber are obtained from equation (1) which relates the electric field strength and the current density in the chamber to the time required for droplets passing through to obtain limiting charge.

$$t = \frac{E_b E_0}{\pi j [3E_0 \cdot E_b]}$$

It can be readily seen that the charging time is minimized by maximizing the current density, subject to the condition that E_0 be less than E_b and greater than $E_b/3$.

A survey of the literature indicates that the design of coronal discharge plates is conducted in an empirical manner. The general design used in the Cottrel Precipitrons employs a needle studded plate and a smooth anode plate. The purpose of the needles on the cathode is to provide local points of high field strength so that ionization will occur in this region. Electron flow, which corresponds to ionization at the cathode, is more desirable than positive ion flow because the collision cross section for electrons is smaller than for positive ions and probability of secondary ionization occurring in the air stream is reduced according to this ratio. It is important that secondary ionization be held to a minimum particularly in the region of the droplet stream since this process sets free ions of both signs thereby making the magnitude and sign of the charge collected by the droplets problematical.

In order to function properly, the needles must be separated from each other by approximately 1/4 of an inch. It is important in this application that the needles be arranged such that the magnitude of the current density within the chamber is independent of position. This is most easily accomplished by locating the needles along rows which make a small angle with the center line of the cathode plate. With this arrangement, the droplets see a continuous coronal sheet and a constant

average current density. Another practical consideration is that the droplets must flow above the needles, otherwise they will not receive the proper charge. This difficulty can be removed by placing an insulated shield, suitably drilled, over the needles such that the tips are flush with the upper surface.

A chamber was constructed with these factors in mind. Design particulars are given in figure 3. A plot of coronal current as a function of the potential applied to the plates is given for this chamber in figure 13. It was found that with a potential gradient of 30,000 volts per inch, the coronal current density was 4×10^{-6} amperes. Converting to electrostatic units and substituting into equation (3), a limiting time T_1 of .06 second was obtained. It should be emphasized that much smaller limiting times are possible if larger potential gradients are permitted. Thus if E_0 is made equal to 1/2 of E_p (37,000 volts/inch) and j is taken at the extrapolated value at 5×10^{-6} , a limiting time of .002 seconds is obtained. The limiting times should be doubled to take into account those droplets which already have a limiting positive charge and must be discharged to zero before being charged negatively.

The precautions which must be followed if this unit is to be carried in an airplane are enumerated by Brun et al (22). The length of the chamber assuming a potential gradient of $E_p/2$ and an air velocity of 100 meters is .5 meters.

Collecting Probe

The design criteria for the collecting probe are that the overall collection efficiency should be a maximum, the recovery time a

minimum, and that the geometry of the orifice and collector be arranged such that a maximum number of droplets is counted.

The requirement that the collection efficiency of the probe be a maximum is approached by making the probe as small as physically possible and by using a high orifice velocity. An approximate value which neglects the effect of the orifice velocity can be calculated by using the Langmuir and Blodgett data. A much more precise value can be obtained by passing a known number of droplets of fixed diameter through the instrument and comparing the indicated counts to the actual number. This will be described more completely in the section on calibration.

The recovery time of the probe is determined by the time required for the droplet to clear the influence field of the collector. It is therefore a direct function of the ratio of the collector diameter to droplet velocity. Both of these factors are defined by the design of the jet impinger.

The orifice and the collector which together form the jet impinger are designed to accordance with the work of May (25). The equation for efficiency of impaction is

$$I = \frac{\rho v D^2}{\eta l} \quad (18)$$

where

- ρ = droplet density
- D = droplet diameter
- V = jet velocity
- η = viscosity of air
- L = orifice diameter

The ratio of the aperture diameter to the distance the collector is away

from the aperture should be nearly unity. The ratio of collector diameter to orifice diameter should be 5:1. For values of I (impaction) greater than 10, the efficiency of collection is 100%. With an orifice opening of .5 mm in diameter operating under critical flow, the minimum diameter for 100% collection efficiency is .4 microns. This equation has been verified in the region of .8 microns so it is assumed that with this aperture will result 100% collection of 2 micron droplets. Using the criterion that the ratio of collector to orifice diameters should be five to one, the collector diameter is computed to be 2.5 mm. See Fig. 4

The value of T_r , found by the method described in the preceding section, is approximately 50 microseconds. This corresponds to a counting rate of 2000 random droplets per second.

There is the possibility that large droplets may impinge upon the edge of the orifice and in doing so discharge themselves. The probability of this occurrence is found by taking the ratio of the projected area of the droplet to the area of the orifice. This probability for a 50 micron droplet is calculated to be 1% and is negligible.

Pulse Amplifier

The Pulse Amplifier is divided into two sections, the pre-amplifier and the main amplifier. The pre-amplifier is mounted in the same housing as the Probe so that its capacitance to ground would be as small as possible. The main amplifier is connected to the pre-amplifier by a co-axial cable and may be located in any convenient position.

The frequency response required for this amplifier is determined by the pulse at the high frequency end of the spectrum and by the

pulse length at the low frequency end. The rise time of the pulse is approximately 10 microseconds and the length of the pulse is approximately 60 microseconds. Reference to Elmore and Sand (29) indicates that these limits specify a low frequency half power point of 785 cycles per second and a high frequency half power point of 100,000 cycles. It is important that this amplifier be designed such that it has only a single over shoot because the oscillations which follow the pulse decay in standard amplifiers result in spurious droplet measurements. The over shoot may be kept to less than one percent if one of the coupling time constants in the circuit is made 100 times smaller than the other coupling time constants. The location of this short time constant is determined by the amount of low frequency noise affecting the amplifier. Its position must be a compromise between location at the high gain end of the amplifier where it acts as a filter and discriminates against microphonics and hum and an intermediate point where it prevents overloading of the amplifier and only partially filters the low frequency noise.

The dynamic range of the amplifier is determined by the ratio of the probe potential produced by a 2 micron droplet to that produced by a 50 micron droplet. This ratio is determined by the ratio of the squares of the droplet diameters and is 625:1. Standard amplifiers cannot handle such large excursions and it will be necessary to incorporate either a square rooting circuit^s in one of the low-level stages or to divide the range and use separate amplifiers to handle each division. In the laboratory model this latter approach was used.

The noise voltage developed in the grid resistor is obtained from equation (12) and is 12 microvolts. The noise voltage

developed in the pre-amplifier tube and in the first tube of the main amplifier amounts to another 8 micro volts. The maximum value of the noise voltage is three times the sum of these noise voltages or 60 microvolts. The overall gain of the amplifier should be sufficient to raise a signal of 60 microvolts to a value sufficient to operate conventional oscillographic equipment, i.e. 0.1 volts. This specifies an overall gain of 1700.

The pre-amplifier is a standard 6AK5 cathode follower circuit. This circuit was chosen because of the convenient low output impedance offered in such a design which makes the transmission of signals from the pre-amplifier to the main amplifier less susceptible to interference and pick up from extraneous sources. The major design problem was involved in the choice of the grid resistor. This resistor determines to some extent the sensitibility and recovery time of the amplifier and to a greater extent the random noise which is mixed with the signal. Reference to figure 1 shows that with critical jet velocity and an input capacitance of 10^{-11} farads, a grid of 10^5 ohms provides .4 of the maximum Q/C signal. This is only an apparent loss as the actual signal to noise ratio is greater with this value of resistance than with a higher value. The grid resistor and the input probe capacity determine the time constant of the probe circuit. With the components chosen above, the time constant of this circuit is 1 microsecond. Any change which is transferred to the probe will be discharged in 6 microseconds to a value which will not create an uncertainty in drop-size determination.

The main amplifier was built in two sections, each consisting of a three stage feedback loop. The individual amplifier gains are respectively 30 and 200. Variable coupling time constants are incorporated in both amplifier units so that each may be used separately. The design of feedback loops is similar to that used by Elmore and Sands (29). Initially it was thought that delay line pulse shaping would be necessary, but it was found possible to reduce the time constant of the probe circuit as discussed previously and eliminate the separate shaping circuits. Circuit diagrams for the complete amplifier are given in figure 5.

The Evaluator

The general technique used in the evaluation of the drop-size distribution involves portraying the pulses from the amplifier upon the Y axis of an oscilloscope. Film is drawn slowly across the face of the tube so that the pulses overlap one another. The film is then developed and a densiometer trace taken across the width of the film. The density of the trace at any particular distance from the zero point represents the number of pulses which are greater than the pulse height. Thus a plot of density as a function of the distance from the zero can be interpreted as a plot of the cumulative size distribution as a function of drop diameter. The actual number of counts in any particular size interval can be obtained by subtracting the number of counts occurring at the upper bound from the number of counts occurring at the lower bound. In order for this method to be used successfully it is necessary that the film be calibrated by applying a known number of calibrating pulses to the film and using their density as a reference.

The method described above is indirect and somewhat tedious.

Attention was then directed towards obtaining a simpler evaluation scheme. It was felt that it was desirable to retain the photographic recording method and the densiometer trace evaluation. After some thought, the conclusion was reached that a scheme in which only the peaks of the pulses were recorded would satisfy the objections to the first method. At the time this method was under consideration there were no references to such a method in the literature. The explanation of the electronic circuitry which performs this function is best understood by referring to figure 6. The signal pulse may be represented by a symmetrical trapezoidal wave. This wave is first differentiated by means of a short time constant coupling circuit, and results in the wave form shown in line B. The differentiated signal is used to trigger a voltage sensing circuit which has been set to fire when the applied voltage falls below 0 volts. The sensing circuit used here is one of the Schmitt type (30) and has extremely good stability, maintaining a reference trigger voltage to 0.1 volts over many hours of operation. The output of the Schmitt circuit shown in line C is then differentiated by another short time constant coupling circuit, line D, and the negative pulses used to trigger an univibrator trigger circuit. The output of this trigger circuit is a pulse 6 microseconds long which fires at the maximum value of each signal pulse, and is used as the intensifying pulse. In practice, the signal pulse is applied both to the Y axis of the oscilloscope and to the "Pulse Peak Intensifier" circuit. The output of the intensifier circuit is applied to the Z axis of the oscilloscope. The oscilloscope is then blank until a signal pulse occurs

which places a dot of fixed duration at the appropriate height.

The precision possible with these techniques is determined by the ratio of spot diameter on the oscilloscope tube face to the diameter of the face. According to measurements (31), the spot diameter on a 7 inch cathode ray tube is 0.02 inches. Therefore, this ratio is 350:1 and permits height determinations with a precision of 0.3%. The amount of illumination produced by cathode ray screen due to the electron stream excitation is directly proportional to the number of times the screen is excited until saturation occurs. Standard tubes can be excited 500 times per second without becoming saturated. The probability of any spot on the screen being illuminated is 0.003 and the probability that this spot will be illuminated by the next following pulse is 10^{-5} . This may be interpreted as a 1% chance that a particular spot will be illuminated twice in one second. The errors involved in the densiometer trace evaluation are less than 1%. Perhaps the largest uncertainty will result from variations in the sensitivity of the film. The calibration procedure described above must be followed for each reel of film.

CHAPTER IV

CALIBRATION OF THE ELECTROSTATIC DROP-SIZE DETERMINER

In order for this instrument to be of value as a measuring tool it is necessary to obtain its performance characteristics. That is, it is necessary to determine experimentally the relationship between the droplet diameter and the resulting signal pulse. Also, the relationship is desired between the probe collection efficiency and droplet diameter. The problems and techniques involved in these determinations will be discussed separately.

It would be desirable to perform this calibration by passing a stream of uniform water droplets through the instrument at a fixed velocity. However, there are no convenient methods available for the generation of homogeneous droplets in the quantities necessary for this work. An investigation was then undertaken to determine substitutes for the water droplets.

Droplet Substitutes

The significant properties of cloud water droplets, in order of importance, are shape (spherical), size range (2-50 microns), density (1), and the dielectric constant (81). These factors were used to test the suitability of the particles under consideration.

Metal powders are available in the desired size range. The particles are non-spherical in shape and frequently have jagged edges. This was considered cause for rejection.

Glass beads are available in distributions which encompass the 2-50 micron range. The problem of sizing these is formidable (32), and was considered cause for rejection.

Attention was then directed towards particles produced by natural processes. It would seem logical that in keeping with the

consistency of nature, spores and pollens should be fairly uniform in size. A survey of the literature yielded this information.

Pollens

Paper Mulberry - *Broussonetia papyrifera*. Grains irregular, but tending to be spherical 13.4 microns in diameter.

Ragweed Short - *Ambrisia elatior*; grains spherical; 18.3 microns in diameter.

Privet - *Ligustrum* sp. Grains spherical; 28.5 to 31 microns in diameter.

Sweet Vernal Grass - *Anthoxanthum odoratum*. Grains nearly spherical averaging 42 microns in diameter.

Spores

Yeast - Ellipsoidal in shape (9-11) by (6-8) microns.

Mold

A *Chrysogenum* - Ellipsoidal in shape (3-4) by (2.8-3.5) microns.

A *Pseridoglaucus* - Globular in shape; 5.5 to 7.5 microns in diameter depending upon the particular strain.

The pollens were easily obtained from standard pharmaceutical houses. The spores and molds were grown in a culture medium, but difficulty in obtaining uniform air suspensions precluded their use.

The density of the pollens is of the order of $1/3$.

The dielectric constant of the pollen is of the order of 2.

Experimental Determination of Drop-Size Versus Signal Pulse

This determination is made by passing the pollen grains through the instrument and recording the signal pulse heights with the oscilloscope. The procedure is repeated for each of pollen sizes.

The pulse height heights obtained in this manner will not correspond to the pulse heights obtained from water droplets of the same diameter unless a higher potential gradient is used in the

coronal chamber. The reason for this lies in the difference between the dielectric constants of the two materials. Reference to equation (1) shows that for equal pulse heights from the water droplets and the pollen grains, a 9/5 greater potential gradient must be used in the calibrating run.

Experimental Determination of the Collection Efficiency

The collection efficiency of the instrument was determined with the aid of pollen grains. The procedure followed is most easily understood by reference to figure 5.

A sample volume of pollen grains was weighed and its density determined. The number of droplets in the sample was then found by using the equation

$$N = \frac{6VF}{\pi D^3}$$

where

V = volume of the sample

F = packing factor of the particles

D = particle diameter

The packing factor represents the amount of void space in the sample volume. Graton and Fraser (33) cite a value for F of 26% for closely packed spheres.

This sample is placed in the loading chute of the mixing nozzle and needle valve on the nozzle opened so that the pollen is mixed with the air stream at a known rate. The suspension is further agitated in the mixing chamber and then directed into the "Determiner". A densiometer trace is made along the length of the film which has been exposed by the signal pulses and the number of droplets which have been

intercepted by the measuring area of the probe evaluated. The number of particles which a probe having unit collection efficiency would intercept is found by taking the ratio of the probe measuring area to the tube area and multiplying it by the number of particles in the pollen sample. The ratio of these two numbers is the probe collection efficiency.

The collection efficiency obtained above is valid only for the particular particles used in the experiment. In order to correct these results so that they are valid for water droplets, it is necessary to make use of the dimensionless ratio which governs impaction

$$\frac{\rho V D^2}{4\eta}$$

where

- ρ = particle density
- D = particle diameter
- V = particle velocity
- η = air viscosity

It follows that if the collection efficiency for a pollen grain of diameter D_p is I_p , then this collection efficiency is valid for a water droplet of diameter.

$$D_w = \sqrt{\rho_p} D_p$$

CHAPTER V

DISCUSSION OF RESULTS

The results of this investigation are summarized in three figures describing the operational characteristics of the instrument.

Calibration of the Probe Potential as a Function of Drop-Size

Figure 7 represents the calibration of the potential probe as a function of drop-size. This calibration was made with the aid of pollen grains and has been corrected so that it applies to water droplets. The data were plotted with a square root Y axis scale. This was done so that a comparison between the measured probe potential and the theoretically calculated probe potential might more easily be made. The agreement between the calculated line and the measured points appears to be quite good. However there is some question as to the correct value of droplet velocity (in the probe) to be used in the argument of equation 4. It is certainly true that the charge is proportional to the surface area of the droplet.

A further point concerns the lower measurable droplet size. Extrapolation of the measured curve until an intersection occurs with the noise level abscissa yields a minimum detectable drop size of 3.4 microns. The lower limit can be extended by modifications of the amplifier circuit which are discussed in Chapter VI.

Collection Efficiency as a Function of Drop Size

The measured values of collection efficiency, corrected so that they apply to water droplets are given in figure 8. A calculated curve of collection efficiency based upon the data of Langmuir and Blodgett (13) is plotted on the same graph. There is considerable discrepancy between the two curves in the region below 15 microns. The

sense of the disagreement may be understood when it is recalled that the probe is operating at a pressure which is less than one half atmospheric pressure. The high sonic jet velocity essentially bends the stream lines from their normal course into the probe so that the measuring area of the probe is increased. This entrainment effect obviously is more pronounced for the smaller droplets and results in an increase of the collection efficiency for these droplets. The reduction in the curvature of the collection efficiency curve is very desirable.

The Measured Drop-Size Distribution from a Spray Nozzle

A cloud chamber was constructed from a galvanized iron cylinder and fitted with a spray nozzle (Spraying Systems Co. j/4-2850) and a ventilating system design to control the concentration of droplets. Provisions were made so that samples could be drawn from various locations on the chamber. The drop-size distributions presumably vary at differing locations due to the sedimentation process occurring in the chamber. A measurement was taken at a sampling point directly below the nozzle. The purpose of this was to obtain a distribution which discriminated against the larger spray droplets and more closely resembled a natural cloud. The film record was evaluated by the method described in Chapter III.

It should be noted that the distribution is considerably skewed and that the droplet number maximum occurs in a band whose mean value is less than 4 microns.

CHAPTER VI

RECOMMENDATIONS FOR MODIFICATION OF THE "DETERMINER" CALIBRATION

The instrument as it exists is capable of measuring droplets in range from 2 to 50 microns. However there is some uncertainty as to the validity of the correction terms used in the reduction of the calibrating curves obtained from the pollen grains to calibrating curves for water droplets. It is therefore desirable that a technique be developed for the production of droplets of uniform sizes, to be used in a direct calibration. A spray nozzle of the spinning disk type such as the one designed by Dimmock (35) might prove suitable for this application.

Coronal Charging Chamber

Difficulty is experienced in obtaining sufficiently high potential gradients in the charging chamber. The radio frequency power supply used in the investigation (Spellman 2040) was rated at 30,000 volts at 1 milliamperere load current. It actually delivered 20,000 volts at 1 milliamperere. This point should be rectified.

An investigation of the effect of water vapor upon coronal action should be undertaken. There is some evidence that the vapor has a quenching effect upon the corona which reduced the current density in the chamber.

Pulse Amplifier

The pulse amplifier should be modified to include non over-loading features.

The pre-amplifier circuit should be changed so that it

includes a stage of amplification (10 x) in addition to the cathode follower output. The reason for this is that such a design would limit the noise contribution of the first tube in the main amplifier to 1% instead of the present 30%.

Evaluator

The evaluation procedure using pulse peak intensification should be improved. At the present stage of development difficulty is experienced in obtaining proper manual circuit adjustment. It is felt that the basic design approach is correct.

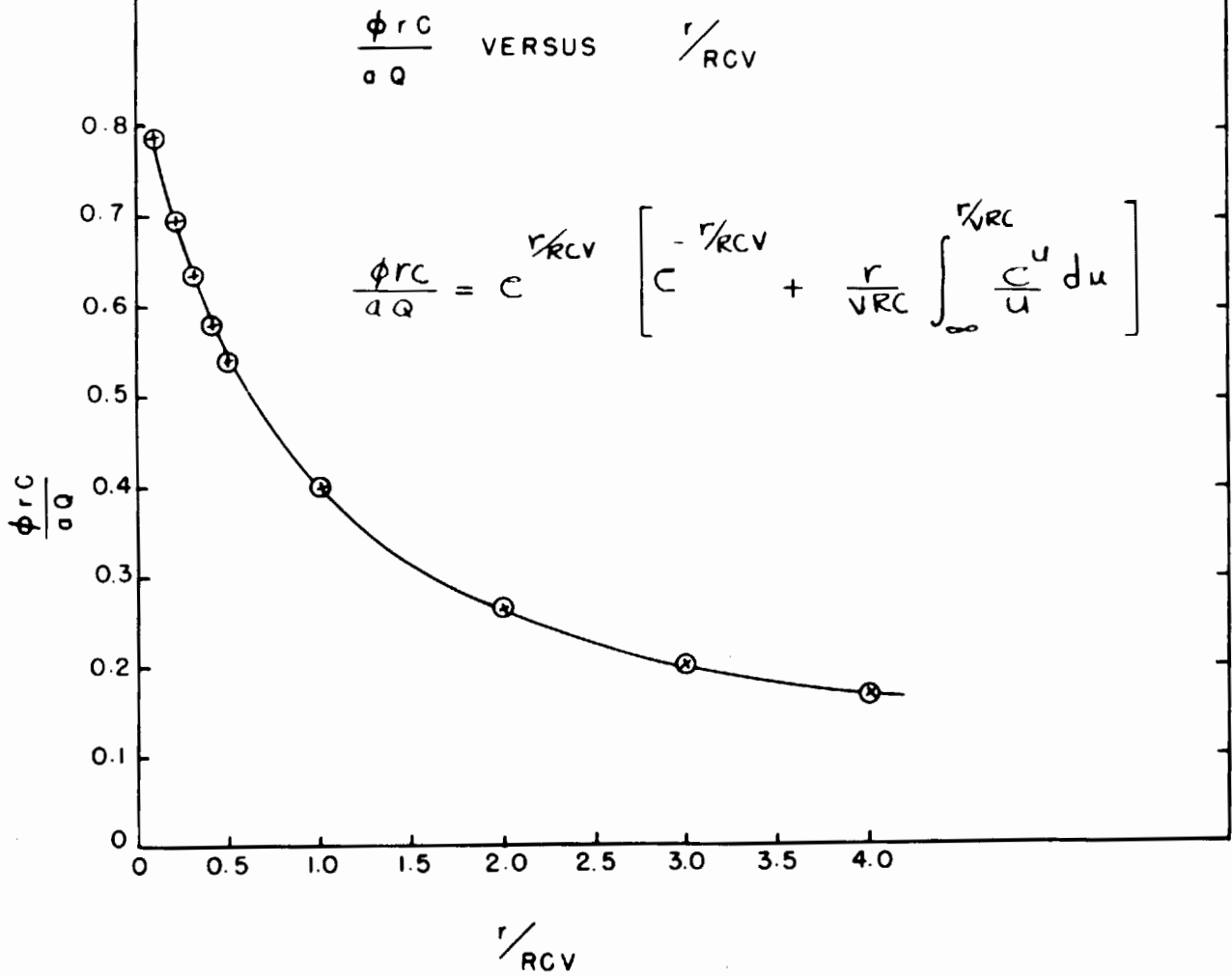
Since drop size is proportional to the square root of the limiting charge, it would be desirable to incorporate a square rooting circuit in the evaluator.

Figure 1 .

Plot of Probe Potential Function

$\frac{\phi r C}{a Q}$ versus $r/R C V$

PLOT OF PROBE POTENTIAL FUNCTIONS



- ϕ = PROBE POTENTIAL
- C = PROBE NETWORK CAPACITOR
- R = PROBE NETWORK RESISTOR
- a = PROBE RADIUS
- V = DROPLET VELOCITY
- r = DROPLET DISTANCE FROM PROBE
- Q = DROPLET CHARGE

Figure 2

Components of the Determiner

BLOCK DIAGRAM OF ELECTROSTATIC DROPLET DISTRIBUTION DETERMINER

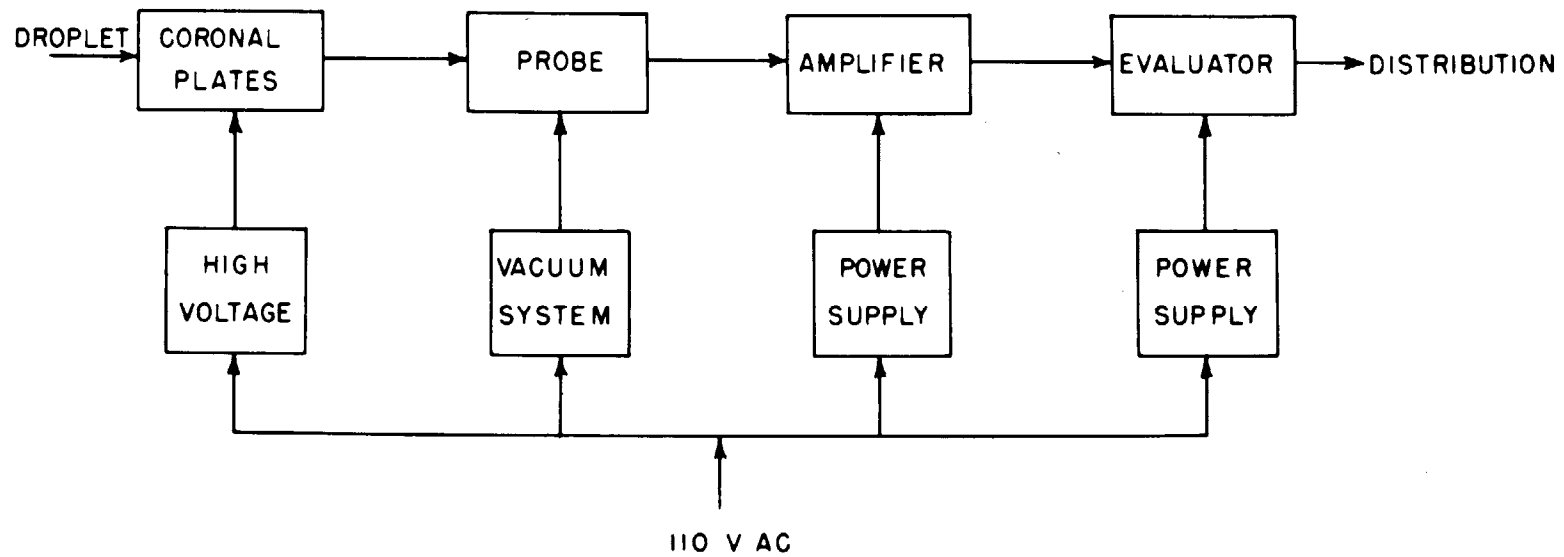
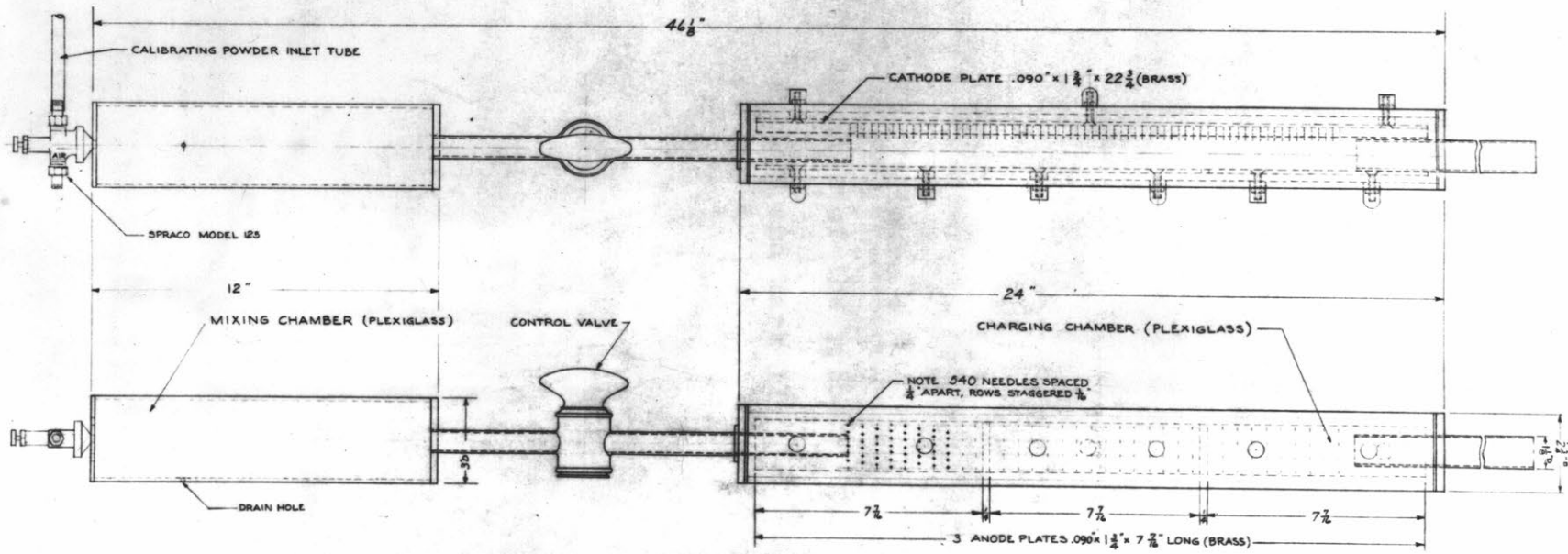


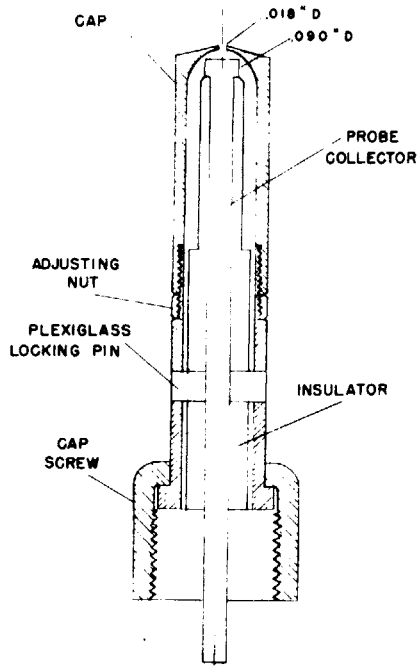
Figure 3
Charging and Mixing Chambers



CALIBRATION UNIT

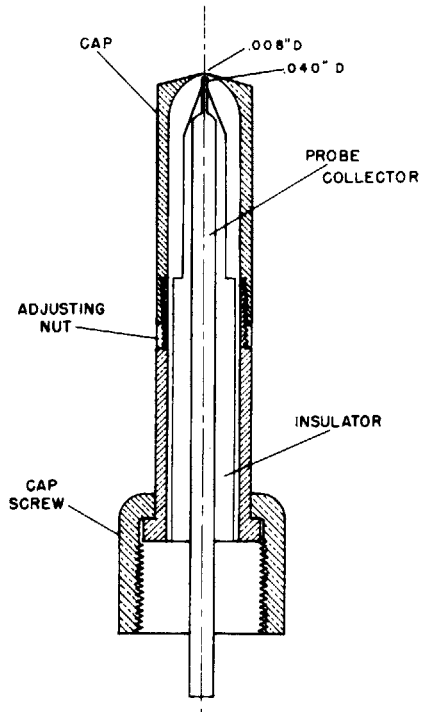
SCALE: 0 1 2 3 INCH

Figure 4
Details of the Probe
Mark I and Mark II



SCALE : _____ 1 inch.

PROBE Mark II

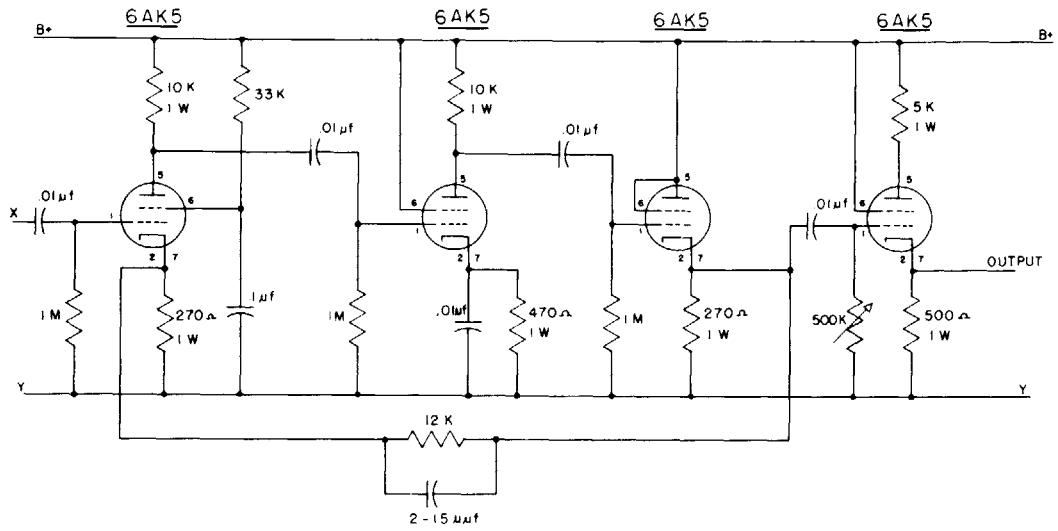
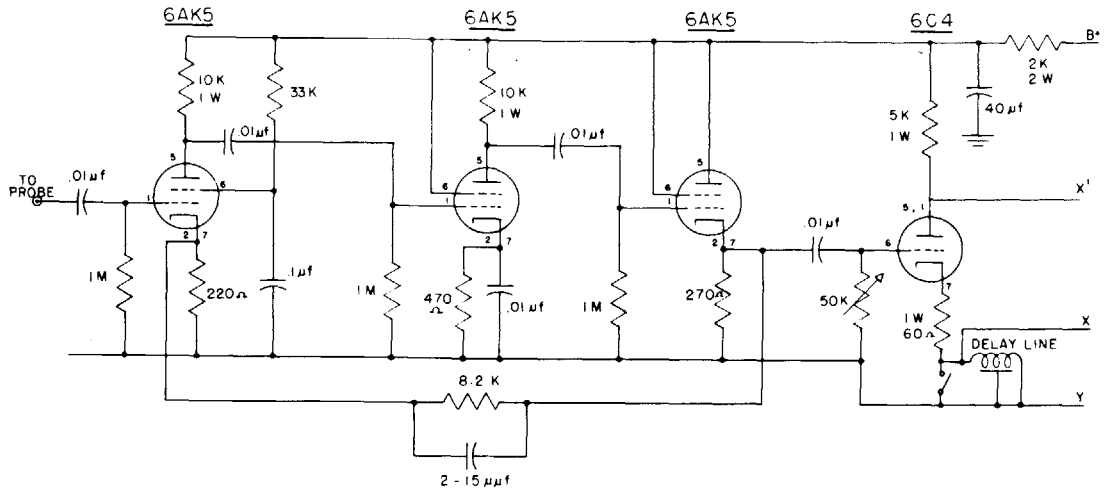


SCALE : _____ 1 in.

PROBE Mark I

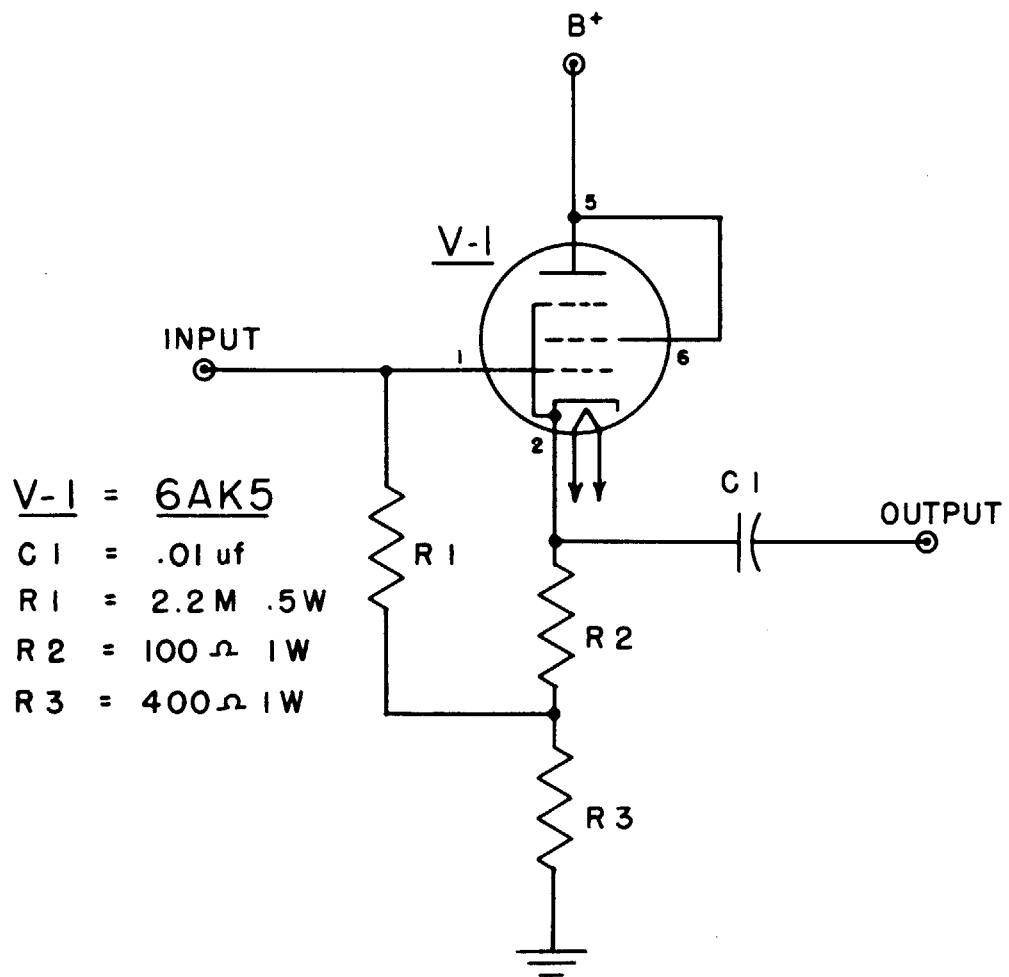
Figure 5

Schematic Drawing of the Pulse Amplifier



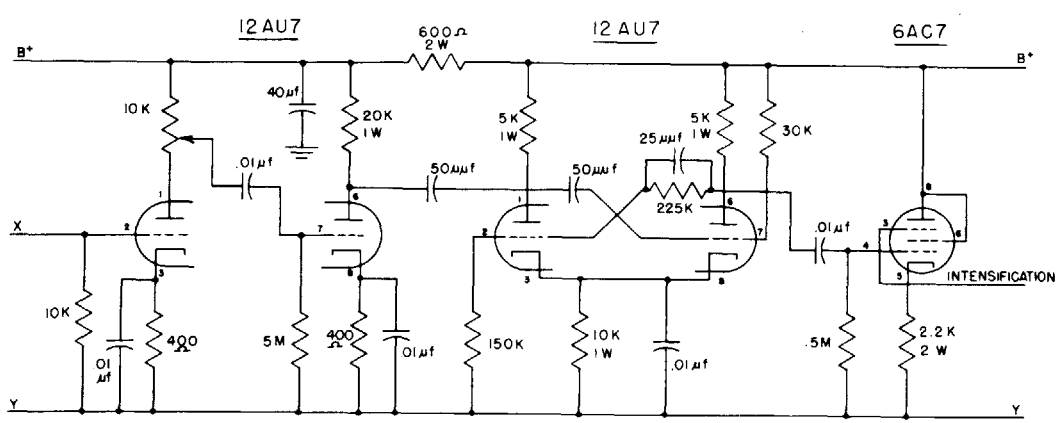
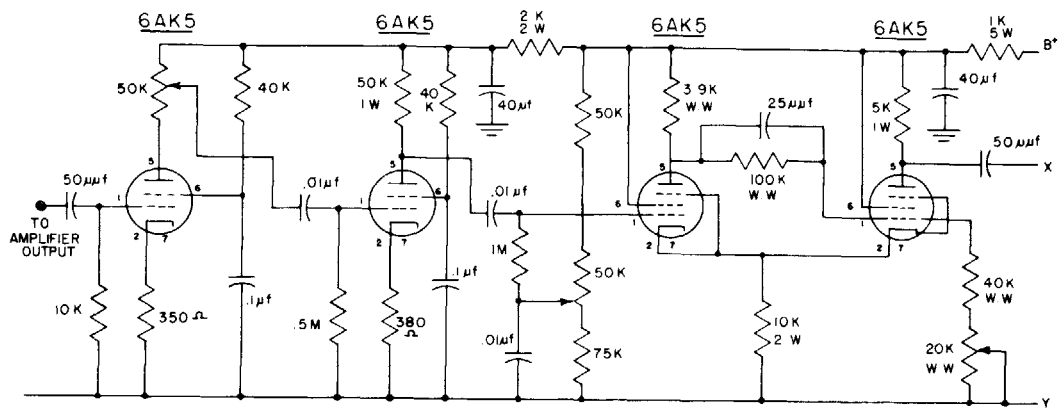
PULSE AMPLIFIER AND
CLIPPING CIRCUIT

CATHODE FOLLOWER USED AS
PRE - AMP



PROBE PRE-AMPLIFIER

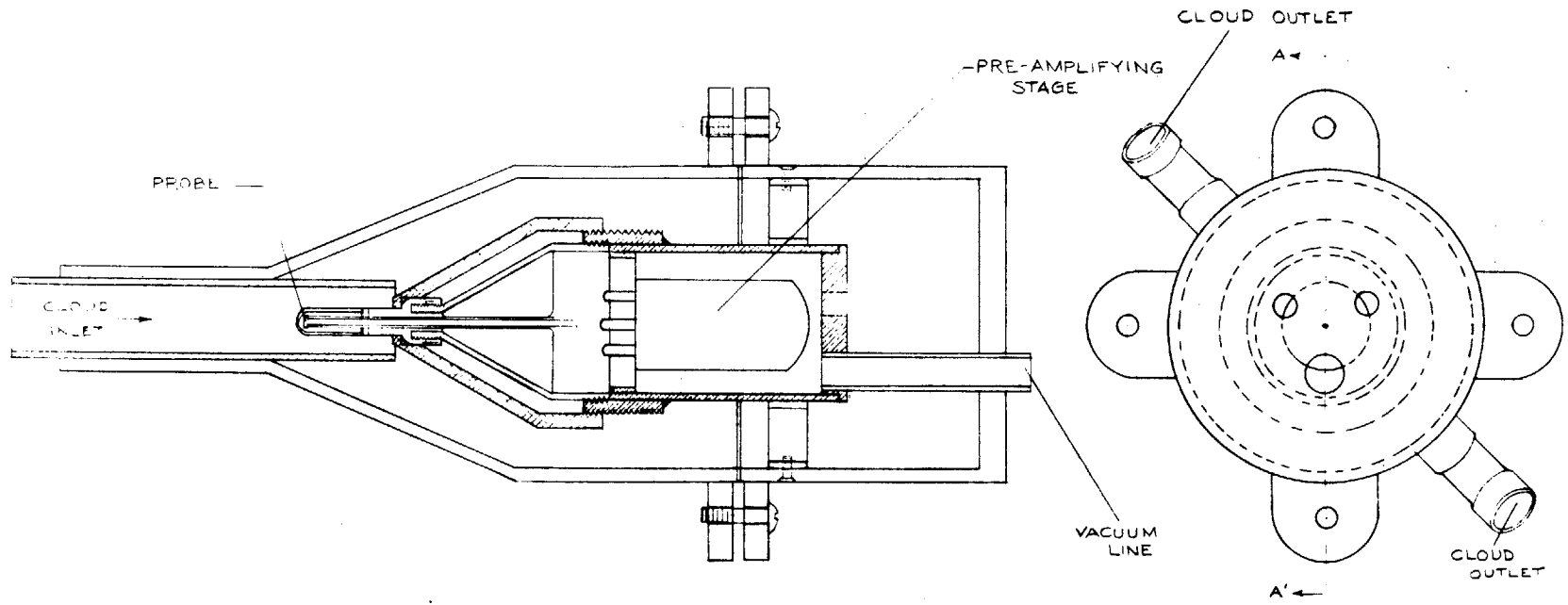
Figure 6
Schematic Drawing of the Pulse
Peak Intensifier



PULSE PEAK INTENSIFICATION
CIRCUIT

Figure 11

Probe Collection Unit
and Pre-amplifying Stage



PROBE COLLECTOR UNIT AND PRE-AMPLIFYING STAGE
 SCALE: 0 ——— 1 INCH

Figure 7

Probe Signal Pulse Voltage
as a Function of Droplet Diameter

SIGNAL PULSE VOLTAGE AS A FUNCTION OF DROPLET DIAMETER

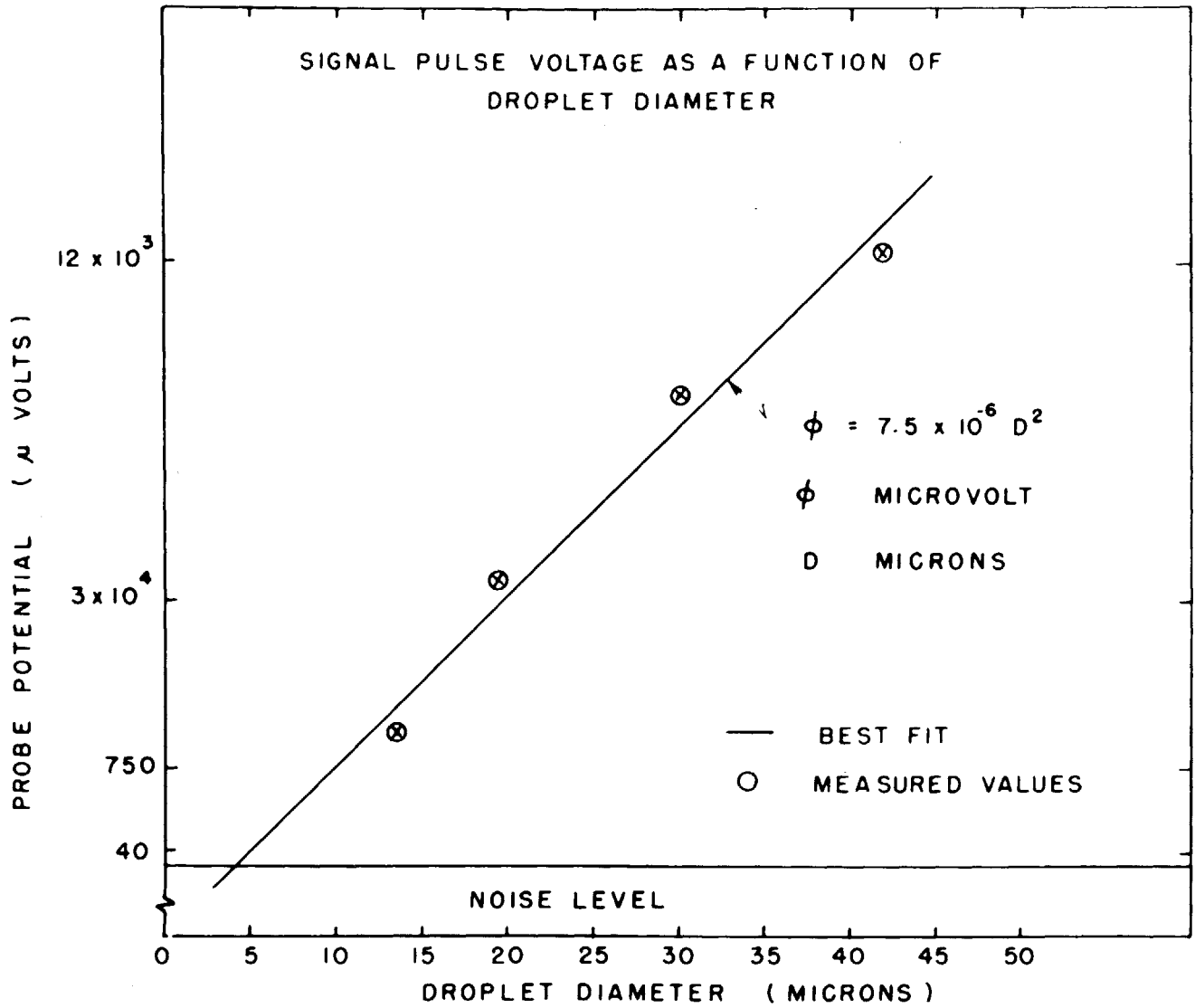


Figure 8

Collection Efficiency of the Probe
as a Function of Drop Size

100
90
80
70
60
50
40
30
20
10
0

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

M. 1000

COLLECTION EFFICIENCY OF PROBE AS A FUNCTION OF DROP SIZE.

VELOCITY 10 M / SEC.

COLLECTION EFFICIENCY

— COMPUTED COLLECTION EFFICIENCY
- - MEASURED COLLECTION EFFICIENCY

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

DROPLET DIAMETER (MICRONS)

0 5 10 15 20 25 30 35 40 45 50

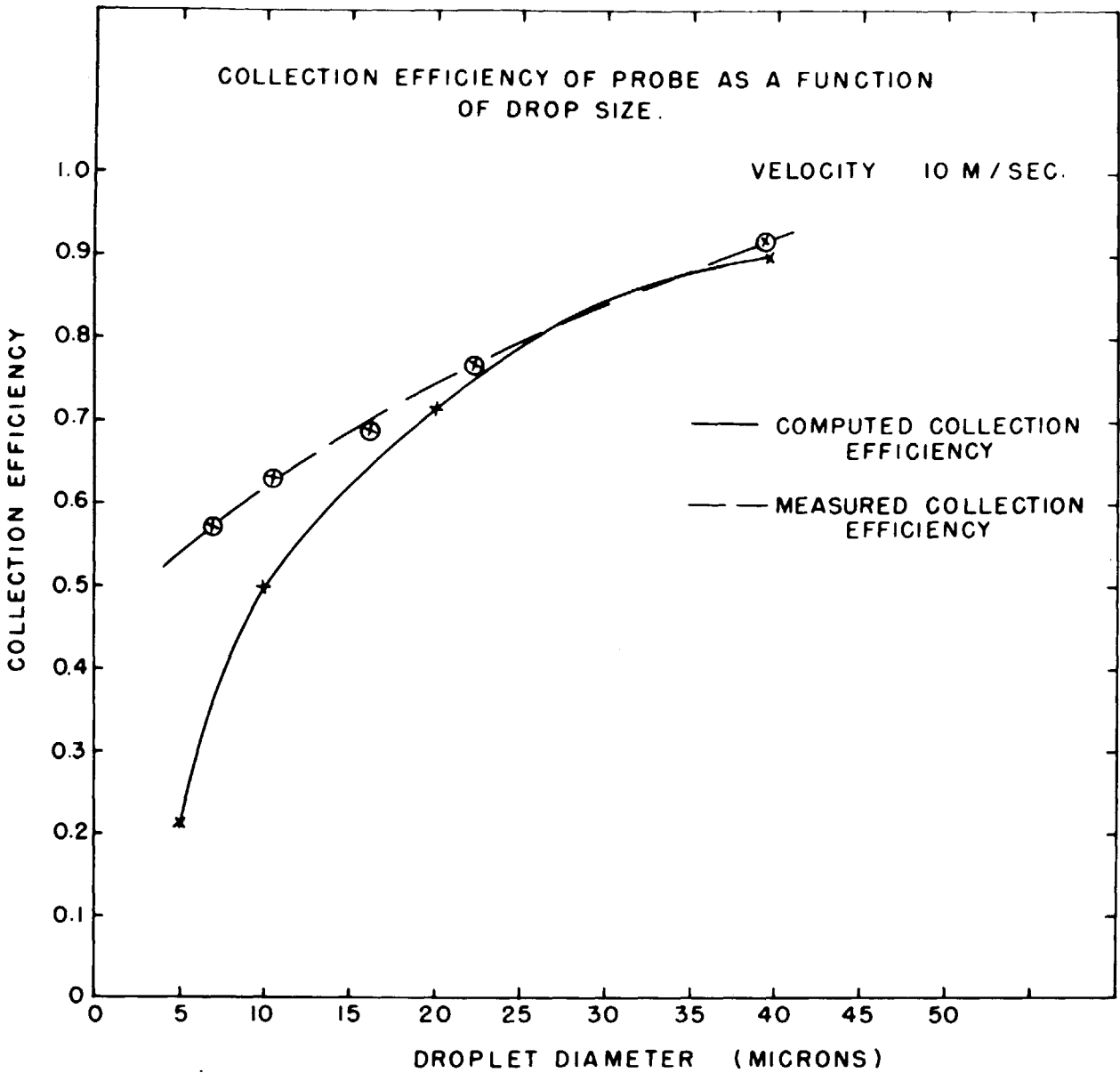


Figure 9

Measured Droplet Distribution
from a Spray Nozzle

MEASURED DROPLET DISTRIBUTION FROM
SPRAY NOZZLE

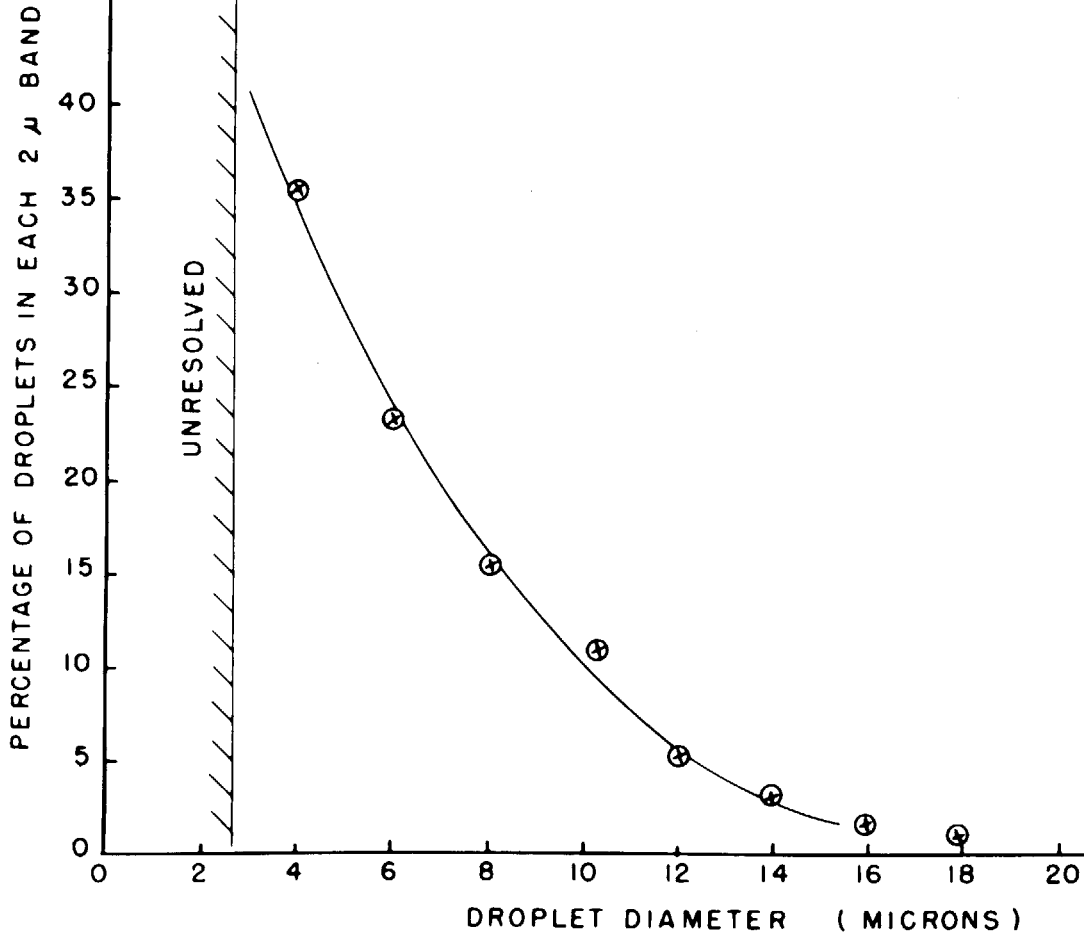


Figure 10
Signal Flow Diagram through
Pulse Peak Intensifier

SIGNAL DIAGRAM
FOR PULSE PEAK INTENSIFIER

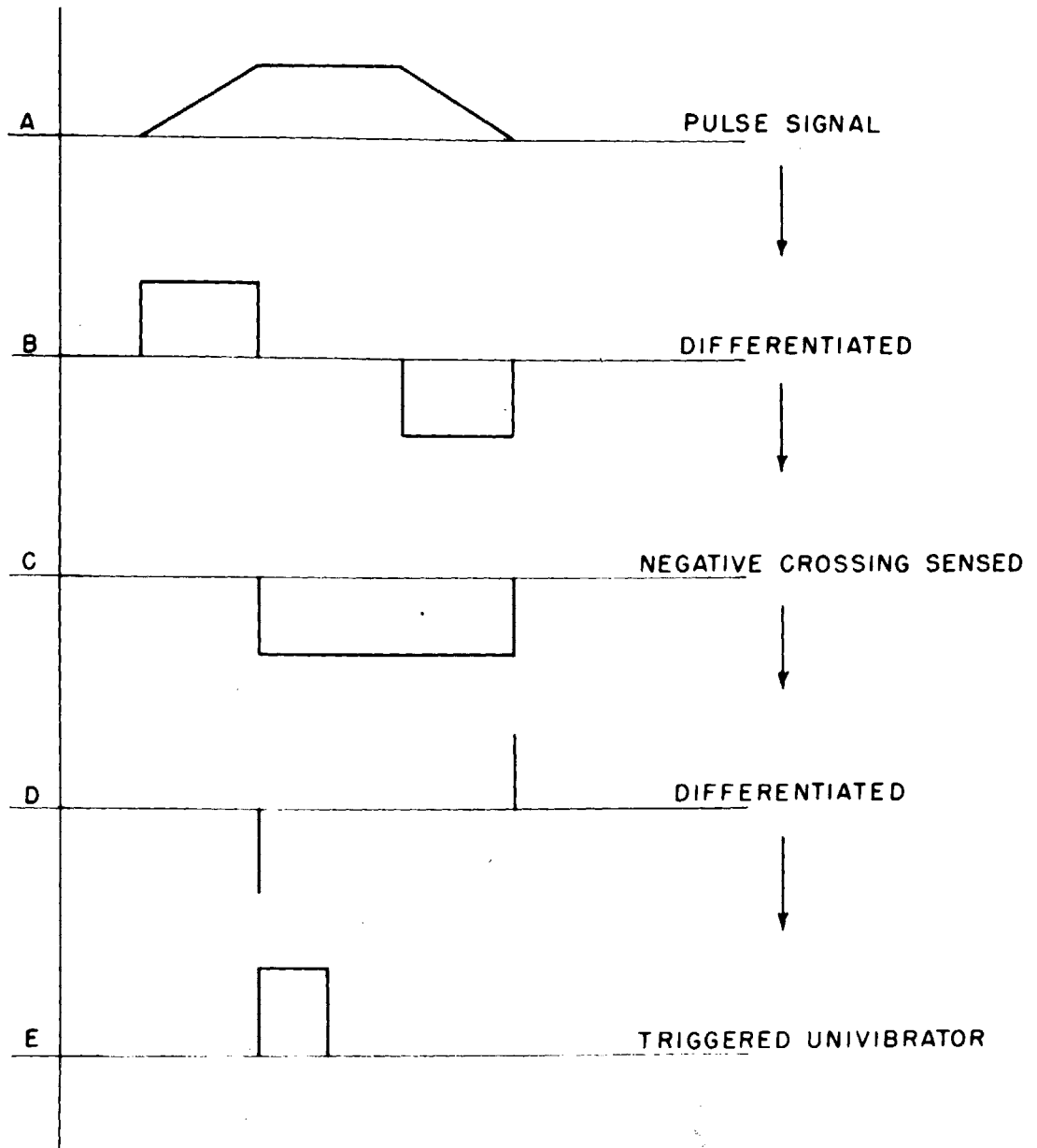


Figure 12

Variation of Breakdown Potential Gradient
as a Function of Temperature and Pressure

BREAKDOWN POTENTIAL GRADIENT AS A FUNCTION OF PRESSURE AND TEMPERATURE.

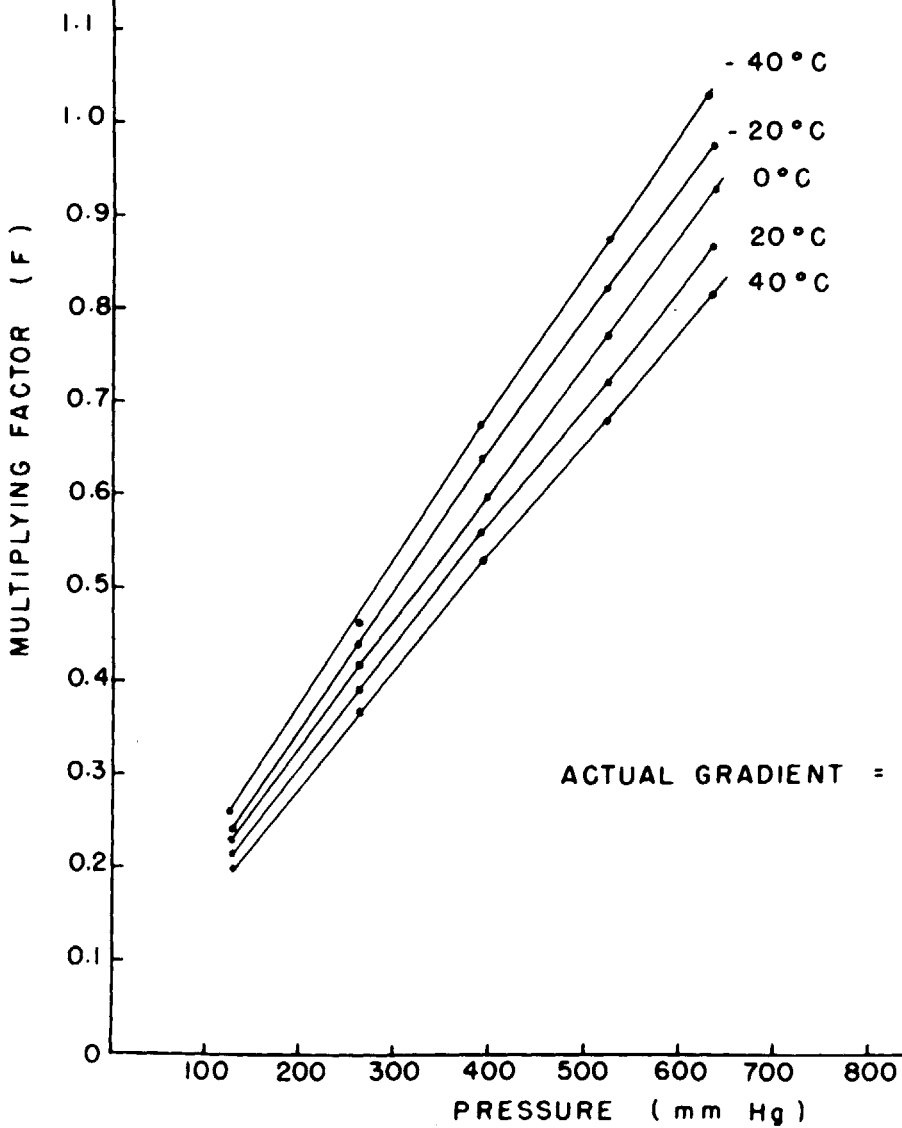


Figure 13

Coronal Discharge Current
as a Function of Applied Voltage

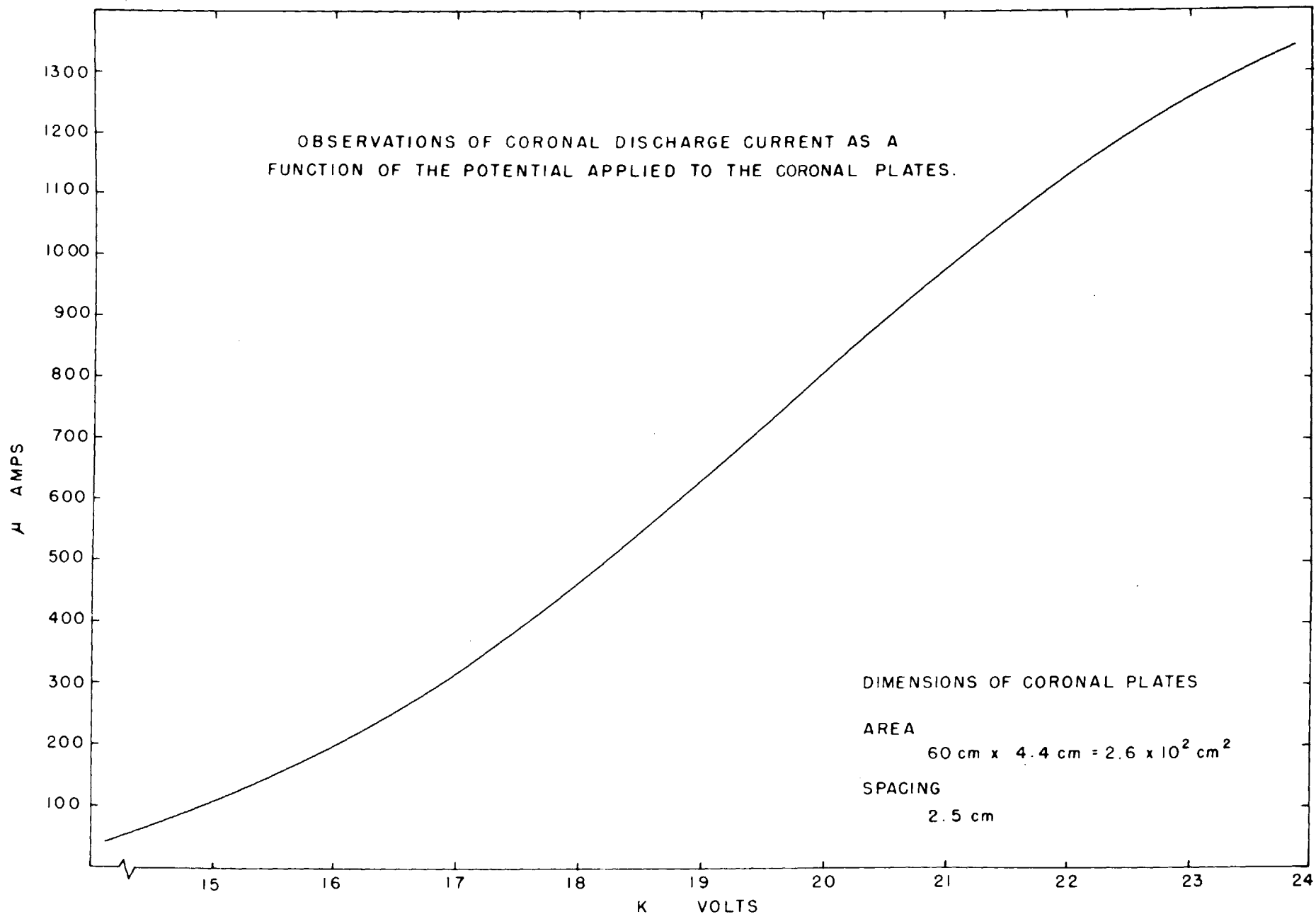
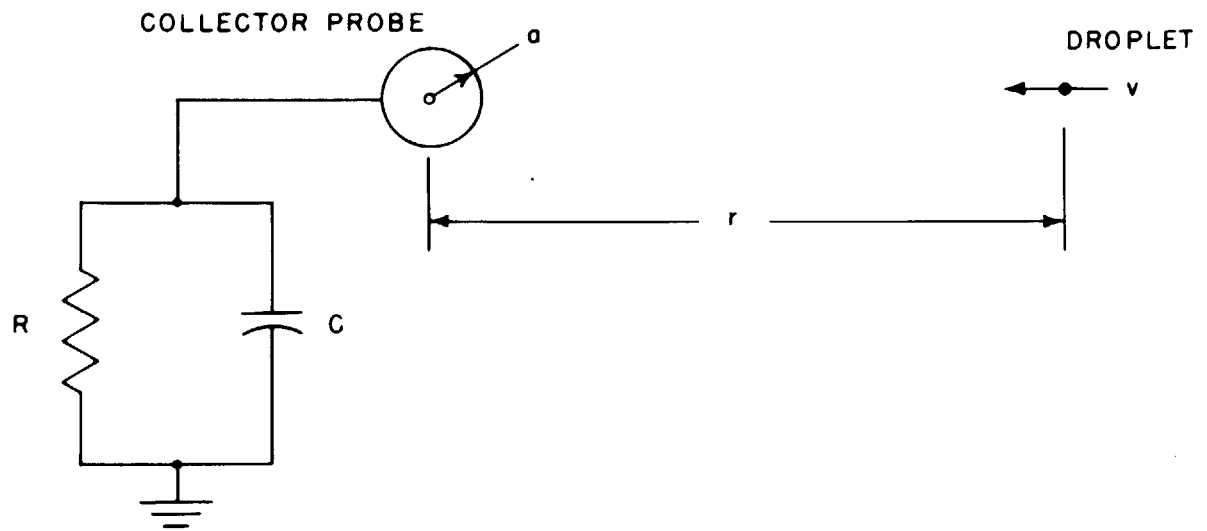


Figure 14

Idealized Schematic Diagram
of Probe and Droplet

ELECTRICAL EQUIVALENT CIRCUIT OF THE PROBE AND DROPLET



APPENDIX I. DERIVATION OF THE EQUATION DESCRIBING PROBE POTENTIAL AS A
FUNCTION OF THE APPROACH OF A CHARGED DROPLET

This problem has been idealized in the following manner:

1. The probe collector is assumed to be spherical in shape.
2. The droplet is assumed to be a point charge with respect to the probe collector.

The equivalent circuit upon which this derivation is based is given in figure 1 .

Let ϕ be the potential of the probe. The current which flows from ground through the resistor-capacitor network is given by

$$i_c = \phi G + \dot{\phi} C$$

where G = conductance of probe resistor
 C = capacity of the probe network to ground
 i_c = conduction current

The displacement current which is caused by the motion of the charged particle is found by taking the time rate of change of the electrostatic field due to the point charge and multiplying it by the physical capacitance of the probe collector sphere.

$$i_d = - \frac{a \phi}{r^2} \dot{r}$$

where i_d = displacement current
 a = probe collector radius
 r = distance the droplet is away from probe

From continuity, the displacement current must equal the conduction current

$$\phi G + \dot{\phi} C = - \frac{aQ}{r^2} \dot{r} \quad (3)$$

This equation is a standard linear first order differential equation whose solution is of the form

$$\phi e^{t/RC} = \frac{a}{C} \int_0^t e^{t/RC} \frac{dv}{(r_0 - vt)^2} + C$$

Integrating between the limits of r and r_0

$$\phi = - \frac{aQ}{RC} e^{r/RCV} \left[\frac{e^{-r_0/RCV}}{r_0} - \frac{e^{-r/RCV}}{r} - \frac{1}{RCV} \int_{r_0/VC}^{r/VC} \frac{e^u}{u} du \right]$$

where r_0 represents the position of the droplet at t equal to zero.

Let r_0 equal minus infinity at t equal to zero.

The integral

$$\int \frac{e^u}{u} du$$

is tabulated in tables (34)

A plot of $\phi C/aQ$ is given in figure 1 .

$$\frac{\phi RC}{aQ} = e^{r/RCV} \left[e^{-r/RCV} + \frac{r}{RCV} \int_{-\infty}^{r/VC} \frac{e^u}{u} du \right]$$

APPENDIX 2. DERIVATION OF THE COINCIDENCE ERROR FOUND IN A MEASURED DISTRIBUTION

This problem has been idealized in the following manner.

1. The droplets are assumed to be randomly distributed in space.
2. The droplets are assumed to be points. This implies that a droplet either impinges upon the collector or misses the collector completely.

If a stream of droplets which are randomly distributed in space is passed through the "Determiner", the pulses generated by the collecting probe will occur randomly in time. Since the sample is large, the probability of a droplet impinging upon the collector is independent of the number of droplets already collected. The statistical relationship which describes the probability that exactly M pulses will occur in a time t when the average droplet count is N per second is given by the Poisson distribution (35).

$$P_M(t) = \frac{(Nt)^M}{M!} e^{-Nt}$$

The probability that two or more droplets will impinge upon the collector during a time interval t is given by the summation of the separate probabilities that 2, 3, . . . ∞ events will occur during the time t .

$$E = \sum_{M=2}^{\infty} P_M(t) = 1 - e^{-Nt} - Nt e^{-Nt}$$

This equation may be evaluated by expressing the exponential terms in a power series

$$E \cong \frac{(Nt)^2}{2}$$

The equation may be solved for N

$$N = \sqrt{\frac{2E}{T^2}}$$

The effect of neglecting the higher order terms is nearly the effect of the neglect of co-incidences greater than 2.

It is possible by means of statistical methods to reduce the co-incidence error to as small a value as desired (37). However, with a counting rate of 1000 per second and a recovery time of 50 microseconds the error amounts to .12% or 1.2 drops in error per thousand droplets sized.

BIOGRAPHICAL SKETCH

Born February 11, 1927 at Irvington, New Jersey.

Educated in the Public Schools of Bloomfield, New Jersey.

Attended Massachusetts Institute of Technology, 1944.

Served in the U. S. Navy, 1944-1946.

Attended Columbia University, 1945.

Reentered Massachusetts Institute of Technology, 1946.

Awarded S. B. Degree in Electrical Engineering, 1948.

Attended Massachusetts Institute of Technology, 1948.

Awarded S. M. Degree in Meteorology, 1950.

Awarded Massachusetts Institute of Technology Overseas Fellowship, 1950.

Awarded Research Fellowship from University of Uppsala, 1950.

Attended Massachusetts Institute of Technology to present date.

Member of Sigma XI.

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