Analysis of Ice-Induced Acoustic Events in the Central Arctic

by

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Submitted to the Department of Civil and Environmental Engineering
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 1997

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Doctor of Philosophy in the field of Acoustics

Abstract

The ambient noise in the Arctic ocean is an aggregate of individual acoustic events, the
majority of which are caused by ice-motion related processes. Ice-induced acoustic events
in the Central Arctic are analyzed in this thesis, with the purpose to estimate parameters
that characterize the motion of their generating mechanisms and ultimately to identify and
distinguish the latter.

The analysis is carried out at two levels. First, individual events, detected in ambi-
tent noise data from the SIMI experiment are analyzed in the frequency range 10-350 Hz.
Parameters that pertain both to particle motion and source propagation are estimated. In
particular, slip, rise time, particle velocity and the slip time function are first determined
from corresponding time-domain event parameters. Due to the lack of independent mea-
surements of ice-borne seismic waves (compressional and/or shear), for each event an
appropriate fault model is assumed, based on the ever:t radiation characteristics. The im-
portant physical conclusion from this analysis is that particle slip for ice processes is in the
range $O(10^{-4}) - O(10^{-2})$ m, at least 3 orders of magnitude lower than the corresponding
values for earthquakes. Also, particle velocity is in the range $\approx 1 - 67$ cm/s, and on
average about 50% lower than particle velocity of a displacement discontinuity during an
earthquake. Slip functions indicate that the displacement offset of a discontinuity in the ice
is highly non-linear in time, and in some cases there is also evidence of stick-slip motion.
The wide range of variation of particle motion-related parameters for the detected events
indicates that multiple ice processes and/or individual fault processes, at different stages in
their formation, radiate sound into the water.

Source parameters that pertain to source propagation, namely source speed, orientation
(strike-angle) and dimensions are then estimated. On average, fracture speed estimates
are in the range 200-1100 m/s, significantly lower than the previously assumed Rayleigh
wave speed (1700 m/s). This result implies that the characteristic parameters for fracture
propagation in Arctic ice vary significantly and deviate from theoretical limits, as is the
case with rock. The wide range of speed estimates supports the hypothesis that either a
multitude of event mechanisms or propagation features of a particular mechanism in the
ice, are associated with the detected acoustic events. Source dimensions have also been
determined from the displacement spectrum of the source-wave parameter, estimated from
the acoustic event signal. Source length or range of deformation is in the range 0.7-100 m, and depth is 0.4 - 4 m. Both length-controlled and depth-controlled mechanisms are identified, based on the agreement of the data with predictions of existing fault models.

Finally, the event radiation characteristics are estimated and modeled, with the purpose to identify the dominant types of ice-motion processes. Unloading motion and fracture, both shear and tensile are the most plausible mechanisms. Unloading motion, which is preceded by complete fracture of an ice floe, is shown to be a plausible event mechanism, both in the low- and mid-frequency ranges, i.e., below 100 Hz and 100-350 Hz, respectively. Events that are attributed to this process are consistently detected in ambient noise, following intervals of high ice fracture activity. This is an important observation, given that unloading motion is a post-fracture phenomenon.

Fracture dominates the mid-frequency range, although it is shown that some low-frequency events may be associated with secondary cracks, probably generated at an early stage in the fault formation. Source dimensions, particle slip and velocity and propagation speed are highest for events associated with shear fracture. Source speed estimates for events attributed to tensile fractures are on average lower than the corresponding estimates for shear fracture-induced events. This result agrees with measurements of propagation speed of tensile and shear fractures in rock. Finally, secondary fracture features are shown to contribute to the field of sound radiation, particularly from a shear fracture. Arrays of tensile cracks which are formed at the tips and edges of shear cracks, to enable them to propagate, affect the event radiation pattern induced by a shear fracture. The resulting event horizontal directivity is best represented by a hybrid acoustic model, developed in the thesis, that takes into account the two simultaneous modes of fracture propagation. The contribution of sound radiation from secondary cracks to the acoustic pressure field due to a shear fault is an important physical conclusion, and shows that fractures in sea ice behave in a similar manner as fractures in rock.

The variation of estimated source parameters, propagation speed and frequency content in particular, indicates that ice mechanisms radiate sound possibly at different stages in their formation and temporal evolution, which can be detected in ambient noise. Thus, the analysis of events is also carried out at the level of clusters or event sequences. Ice processes of different scales, including coalescence of en echelon arrays of secondary cracks and the formation of crack process zones, which macroscopically appears as a slow kinematic growth of a large fracture, are shown to be plausible event generating mechanisms.

Thesis Supervisor: Ira Dyer
Title: Professor of Ocean Engineering
Dedicated to my mother
Acknowledgments

This thesis marks the end of my studies, ten unforgettable years here at MIT, during which many people have played an important role in my education and my life and I thank them for that.

Four years ago, I was fortunate to meet Prof. Ira Dyer, in my quest for a PhD topic. I thank him for giving me the opportunity to work with him, for his tremendous guidance during the progress of this work and also for giving me the freedom to develop my own ideas and apply them to my research. This PhD and a career in acoustics would not have been possible for me without his help and guidance; I will always be grateful to him.

I would like to thank Prof. Daniele Veneziano, my academic advisor since my undergraduate years, Master's thesis advisor and chairman of my doctoral committee. His support, guidance, kindness and patience have helped me tremendously. I will always be very grateful to him.

I would like to thank the members of my thesis committee, Prof. Henrik Schmidt for always being willing to answer my questions on the experiment, for his comments on the thesis, and for his support and most appreciated advice when things got rough. I also thank Prof. Jerome Connor, for being a great teacher and for his support over the years.


In the Ocean Engineering Acoustics Group I have made several friends, who helped me and supported me during the four years of my PhD studies. I thank them all. Four people deserve special thanks: Sabina Rataj, Dr. Joe Bondaryk, Dr. Tarun Kapoor and Peter Daly. Sabina has always been there for me. She has helped me and supported me tremendously. Her kindness and cheerfulness have helped me deal with some very rough situations and I will always be very grateful to her. I thank Joe for so many things over these past four years. Whenever I have had a problem Joe has always helped. I have learned a lot from
our discussions on my work, from his suggestions, and I admire his dedication to teaching and to helping his colleagues and students. I will always be indebted to him for taking the time to read my thesis, for his most helpful comments and for the support he has given me during the writing and revisions of the thesis. I wish him the very best; he truly deserves it. Tarun is the first friend I made in the Acoustics group. I thank him for his help, particularly the long hours he spent helping and guiding me during the first stages of my work, and for his support and friendship. Peter has always been there to answer my questions, to help in any way he could. I greatly appreciate his help over the years.

I also thank Vincent Lupien for his friendship, his support, for listening to my troubles when I needed a friend and for helping me, through our discussions, cope with some very difficult situations. Brian Sperry, Matt Conti, Pierre Elisseef have made the time I spent in the acoustics lab a pleasant experience.

I also thank Peter Kofinas, for being a good friend since the first day I came to MIT and for sharing with me so many things, and Liana Alvarez for her support over the years and for her friendship.

Finally, most of all I thank my family; I would not have made it without them. My mother to whom I owe everything, for her sacrifices, the strength that she has always given me, for always believing in me and for trying so hard to give us the best she could. The rough years during which we have been apart are finally over. I thank my sister Christiana, with whom I have shared many years at MIT, for the laughs and the all-nighters we pulled together, and for the strength that she has given me.
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Chapter 1

Introduction

1.1 Motivation

Ice activity, which includes deformation, fracture, collision and ridge building, is the predominant contributor to underwater ambient noise in the Arctic ocean. In the past, military interest in the strategic role of the under-ice environment in the Arctic motivated much of the research on ambient noise. In regard to the ice itself, multiappropriate ice in particular which is of great concern in navigation and off-shore structural engineering, research was and still is driven by the need to understand the response of the ice cover to different loads. Finally, the research on the behavior of Arctic ice under different environmental conditions is of significant interest for climatological purposes. For example, monitoring of Arctic ice thickness variation may provide useful information on atmospheric phenomena, such as global warming. In order to understand both the characteristics of the noise and the generating ice mechanisms, individual transients in the noise time series, termed events, need to be studied. Depending on the particular Arctic region of interest and seasonal environmental conditions, different ice processes and possibly their combinations are responsible for these events.

One of the ultimate goals of a detailed study of event generating ice mechanisms is better understanding of the behavior of the Arctic sea ice under different environmental stress conditions, and its rheological properties which still remain only partially known. Mechanical studies confirm that sea ice is a highly anisotropic and inhomogeneous material
with non-uniform behavior. It displays a wide range of characteristics, from elastic behavior and brittle fracture to ductility and plastic yielding. However, information from in-situ experiments on the mechanical properties of sea ice in the Arctic is very limited. In addition, there is significant uncertainty associated with the nature and magnitude of environmental loads applied to the ice cover. Although such loads have been identified and estimated, it may be difficult to determine which combination produces sufficient forcing to exceed the material strength of sea ice and cause it to deform and fracture. Therefore, the study of ambient noise may provide valuable insight into the mechanisms of ice motion that would otherwise be hard to achieve through more direct techniques.

1.2 Objectives

The objective of the research is to understand the characteristics of ice processes, deformation and fracture in particular, through analysis of experimental acoustic data at two levels. First, individual events are studied. Source parameters are estimated and radiation characteristics are described using appropriate acoustic models with the purpose to identify and distinguish the generating mechanisms. Therefore, the analysis is carried out at the individual fracture or deformation level.

In order to gain a better and broader insight into the behavior of Arctic sea ice under environmental loading, acoustic events need to be studied also at a larger scale, or even in the aggregate. Therefore, the analysis is also carried out at the level of faulting. This requires the study of event sequences or clusters, both spatial and temporal. In the case of earthquakes, a fault is formed in several stages. The seismic cycle is composed of the pre-seismic stage in which a precursor to the main shock occurs, the co-seismic stage in which the principal fracture propagates, and the post-seismic stage in which the transition to a new equilibrium stress state in the medium occurs. Therefore, temporal and spatial sequences or clusters of events are studied in order to identify the possible occurrence of a seismic cycle during ice motion. Source parameters of events generated at the different stages of faulting, such as frequency content and propagation speed, are believed to be distinct. In contrast, it is not clear how the radiation characteristics of the induced acoustic
events vary in these stages. Other plausible phenomena during any seismic stage include the formation of crack process zones, fault-locking, coalescence of crack arrays and variations in the fracture mode along a fault. Therefore, the goal of the analysis at the level of event clusters is to identify and understand the formation and evolution of faulting in the ice. On the other hand, the analysis at the level of individual events aims at the understanding of the characteristics of the process at each stage and the identification of fracture details and their relation to the mechanical properties of sea ice.

The investigation of ice mechanisms through the study of the physics of ice-induced acoustic events is a collective effort. The problem of sound radiation from ice motion processes itself offers far more challenges and raises more scientific questions that can be addressed in a single thesis. The Arctic region of interest, ice conditions, season and frequency range are among the key factors that determine which aspects of the problem need to be studied. There are a number of combinations of ice type, environmental conditions and frequency range on which the characteristics of ice mechanisms depend. Typically, the frequency range of interest in the analysis of ocean ambient noise is between 1 Hz and 100 kHz. It can be divided into low-, mid- and high-frequency range, taken to be 1-100 Hz, 100-3000 Hz and 3-100 kHz, respectively [11]. In this thesis, acoustic events in the low- and mid-frequency ranges, i.e., below 100 Hz and between 100 and 350 Hz, respectively, are analyzed. The upper limit is set by the filter in the data acquisition system. The data have been collected during the Sea Ice Mechanics Initiative (SIMI) experiment, in the central Arctic, in pack ice, during the spring of 1994.

In summary, the objective of the thesis is to analyze and interpret acoustic events induced by physical processes in the ice, in order to identify and distinguish the characteristics of these processes, both at the scale of a single mechanism and at the scale of an aggregate of mechanisms.

1.3 Approach

I have selected only a one-hour ambient noise data segment to analyze, for two reasons. First, for the purpose of my study, this segment in which 196 acoustic events have been detected,
is sufficiently large. Second, environmental conditions such as temperature variation which affect the ice strength are not expected to change within one hour. Therefore, the spatial and temporal characteristics of the event population, such as clustering are expected to be directly related to ice motion processes. The organization of the analysis of acoustic events and the estimation of fault parameters is summarized in the following diagram:

Figure 1-1: Organization of Study.

In general, the parameters that characterize a fault mechanism are:
• Length or range of deformation

• Depth

• Strike angle (horizontal)

• Dip angle (vertical)

• Propagation velocity

• Average dislocation (slip)

• Seismic moment

• Rise time

• Particle velocity

• Stress-drop

All are estimated in this study, as shown in the diagram in Figure 1-1, except dip angle, since in the far-field it is difficult to extract information pertinent to this parameter, from the data; ice thickness is much smaller than the wavelength of the source. The parameters that characterize particle motion of a fault mechanism, including slip, rise time, particle velocity and the slip time function are estimated from the event acoustic signals in the time domain. Techniques that are commonly used to analyze seismic signals have been adapted for this purpose. Source parameters that characterize propagation, including fracture speed, orientation and fault dimensions, are estimated from event parameters in the frequency domain. The type of motion, seismic moment and stress drop are determined from the event radiation patterns and source strength.

1.4 Thesis Outline

In this chapter, previous research on the physics of ice-induced acoustic events is presented. Individual efforts and issues pertinent to the problem are summarized. My goal is not only
to discuss the complexity of the problem and its multiple aspects, but also to emphasize the
barriers which have prevented some of the critical issues from being completely resolved.

Chapter 2 discusses the event time-domain analysis. The methodology for event de-
tection in the acoustic data and location in the ice, and the classification of signature types
are described. There are two major categories of events in the data, namely simple and
compound, distinguished by the number of their signal components. Within each category,
four signature types have been identified. The predominant types are a damped sinusoid
and a series of randomly separated peaks. Compound events have two signal components,
usually of the same type.

Particle motion of the event generating mechanisms is also estimated. Due to lack of
measurements of the ice-borne seismic waves (compressional and/or shear), from which
particle slip, velocity and rise time are typically estimated, these parameters are determined
here directly from the acoustic signals. For this purpose, the event radiation characteristics
are first investigated. Based on the directivity of the event source, an appropriate acoustic
and subsequently fault model is selected. There are five event sub-populations, based
on the acoustic model that best describes the radiation patterns of the constituent events.
In the first, radiation is modeled by a stationary vertical dipole and the most plausible
generating physical mechanism is floe unloading. In the other four sub-populations, the
constituent events are attributed to fracture and they are distinguished by the type of
fracture that has caused them, and thus by their radiation characteristics. Source motion
is included in the model to account for fracture propagation. In the first of these four
sub-populations, radiation is modeled either by a moving vertical dipole or possibly by
a longitudinal octapole. The adequacy of each model is not clear. Finally, in the last
three event sub-populations radiation is modeled either by a single octapole, lateral or
longitudinal according to the generating fracture process, shear and tensile, respectively,
or a combination of octapoles. This hybrid model adequately describes radiation from
shear fractures which propagate in the medium by the formation of arrays of tensile cracks
at their tips and edges and contribute to the acoustic pressure field. Once an appropriate
fault model has been selected, the event acoustic signals are integrated thrice in the time
domain to obtain the corresponding slip source function. From the latter, mean slip, in
the range $O(10^{-4}) - O(10^{-2})$ m, particle velocity between 1 and 67 cm/s and rise time, which depends on the event signature, are determined. I show that in comparison to particle motion during an earthquake, although slip and rise time are respectively, 3-4 and 2-3 orders of magnitude lower, particle motion in the ice is fast; particle velocity reaches a maximum of 70% of the typical particle velocity of an earthquake (100 cm/s). Finally, the source displacement signals (equivalent to the shear-wave displacement pulses in the case of earthquakes) and corresponding slip time functions, estimated through repeated integration of the acoustic signals, are modeled by the Gaussian and Rayleigh functions and their respective integrals. Slip functions of some compound events indicate the possible occurrence of stick-slip motion.

Chapter 3 discusses the event frequency-domain analysis. Event parameters, including bandwidth, peak frequency for narrowband events, Doppler shift associated with source motion are first estimated. Both narrowband or narrow-width (defined as events with bandwidth of one octave or less) and broadband events have been detected, though mostly compound events are broadband. In regard to their peak frequencies, approximately 40% of the total number of events are in the low-frequency, and 60% in the mid-frequency ranges, respectively. It has been possible to measure Doppler shifts from the event spectra, ranging between 1-90 Hz. Consequently, source speed and orientation have also been estimated. In regard to source speed, it is on average 200-1100 m/s, lower than the Rayleigh wave speed (1700 m/s for sea ice), assumed as the fracture propagation speed in previous studies [6]. There are 47 speed estimates outside the above-mentioned range. These results indicate that more than one mechanism is associated with the detected events. In addition, source speed is correlated with event signature. In the last stage of the frequency-domain analysis, the dimensions of the event generating mechanisms are estimated from the characteristic corner frequencies of the source displacement spectra, and the corresponding propagation speeds. Length, or range of deformation for mechanisms that are not directly related to fracture propagation, varies between 0.7 and 100 m, and depth is on average in the range of the ice thickness. Based on the agreement of the data with existing fault models, which are used to determine the scaling dimension of a seismic source, there are both depth-controlled and length-controlled event mechanisms, although for approximately 40% of the analyzed
events the results are inconclusive.

In Chapter 4, the radiation characteristics of events, used in Chapter 2 to select an appropriate fault model, are more fully described. As previously mentioned, appropriate acoustic models for non-directional and directional sources are developed or assumed. In particular, a vertical dipole in the absence of source motion, a dipole modified by a Doppler factor, longitudinal and lateral octopoles and higher order multipoles are adequate models for different groups of events. Based on the results of this analysis and the previously estimated source parameters, the identified ice mechanisms include floe unloading motion, and fracture, both shear and tensile. The former mechanism is preceded by fracture. It is shown to occur both in the low- and mid-frequency ranges, a surprising result, given the conclusions of previous studies [6]. Although the majority of events attributed to fracture have peak frequencies in the range 80-313 Hz, there are also about 60 low-frequency events with source speed in the range 400-850 m/s that may have been induced by some fracture process. This result is also surprising, given that mid- and high-frequency events have been associated with fracture in previous studies [30] [38] [6]. 50% of these events precede high-speed, mid-frequency events, the radiation characteristics of which are modeled by lateral octopoles, and are thus believed to have been induced by shear fractures. It is, therefore, possible that pre-cursory motion of the ice is responsible for these events. The frequency content of such motion may be different from that of the main fracture and of post-cursory motion. Not only peak event frequency varies considerably for events attributed to fracture, but so does source speed. Secondary cracks, particularly ones that occur in process zones, in a macroscopic description appear as a slow kinematic growth and it is possible that there are acoustic events associated with such cracks. Finally, the complexity of the fracture process and the possible change of crack mode during the propagation of a fault is suggested by the radiation characteristics of about 45 events, which are modeled by a higher order hybrid acoustic multipole.

Event sequences and clusters are analyzed in Chapter 5. In particular, temporal, spatial and statistical characteristics of the clusters are identified, with the purpose to associate them with the possible occurrence of fracture cycles. The event radiation characteristics, source speed estimates and orientations within individual clusters indicate that arrays of
small tensile cracks may be formed off the primary propagation paths of shear fractures, a phenomenon that has also been observed during earthquakes. The formation of crack process zones is also shown to be a plausible event generating mechanism. Finally, the occurrence of fracture cycles is suggested by the characteristics of some event clusters, but it is not clearly identified.

1.5 Previous Work

Two major areas of research on the ambient noise in the Arctic ocean may be distinguished. One focuses on the correlation of the variation of noise with composite measures of environmental stresses. The other involves the study of event physics. Here, I review previous work on both areas, for completeness, but emphasize the study of event physics as it pertains to my research.

1.5.1 Correlation of Environmental Forces with Ocean Ambient Noise in the Arctic

In each frequency range, the variation of the ambient noise under the ice cover correlates highly with different environmental forces. Makris and Dyer [27] showed that the variation of low-frequency ambient noise under pack ice in the central Arctic, is highly correlated with composite measures of stress, horizontal stress and ice moment in particular, caused by wind, current and drift. In his study of the Marginal Ice Zone, Makris [7] [28] suggested that low-frequency noise originates from sound sources in drifting ice floes, while wind stress is poorly correlated with noise. He showed that surface gravity wave forcing is the primary correlate of long-term noise and that other environmental forces are weakly correlated to noise in the Marginal Ice Zone, in contrast to the corresponding results for the central Arctic.

Milne and Ganton [30] investigated seasonal changes of environmental conditions and their respective correlation to noise for shorefast ice. They suggested that thermal tensile cracks are the source of the ambient noise for both spring and winter, and that floe motion
is the predominant sound generating mechanism in the summer. They also distinguished variation of the noise in two frequency ranges. From 200 Hz to 800 Hz, the noise varied with changes of air temperature and was impulsive. From 1000 Hz to 10 kHz, the noise was Gaussian and appeared to depend on wind speed. They suggested that thermal cracking and wind-related forcing caused the noise in the first and second frequency ranges, respectively.

1.5.2 Study of Event Physics - Ice Mechanisms

Townsend-Manning [40] was the first to consistently examine a large data set from the FRAM IV experiment in the central Arctic. Her work focused mainly on the detection, location and analysis of the statistical characteristics of a large event population. In particular, she investigated the temporal distribution of events, and suggested the dependence of her results on environmental conditions.

Several researchers have studied ice fracture as a source which radiates acoustic energy in the water. Different models have been proposed to describe the radiation from ice motion processes. Stein [39] modeled tensile cracks using an acoustic monopole in the ice and derived the underwater acoustic pressure field. He also investigated individual events [38], identified the contributing waves (flexural, longitudinal and acoustic) to the event signatures, and suggested that the acoustic wave directivity and power could be one of the most important parameters in the problem of ambient noise. He emphasized that his proposed model is an ad hoc radiation model in the sense that it may be meaningful only when discussing events whose openings are small compared to ice thickness. Therefore, before the monopole radiation is averaged over source depth, it must be understood how the fracture itself changes the radiation characteristics. However, the nature of his data was such that further analysis of the fracture process was not possible. This is a common barrier for most researchers in this area. Experiments in the Arctic are not controlled and thus, changes in the physical characteristics of sound-generating ice processes or details of these processes, cannot be closely monitored, as in a laboratory. For example, propagation of a fracture cannot be followed from its initiation to its arrest. Therefore, changes in the propagation speed, from low values when the fracture is shorter than the critical length, to
high limit values (of the order of the shear or compressional-wave speed) as the fracture accelerates after it has reached its critical length, cannot be observed. Consequently, parameters that are useful in understanding the material characteristics of sea ice, such as its resistance or strength parameter, discussed in Chapter 3, cannot be estimated. Also, it is not possible to associate recorded acoustic events with the particular physical processes that have generated them, in situ. Due to the multitude of fractures in the ice, visual inspection of individual cracks is almost impossible, unless they occur at a very large scale, e.g., leads opening at particular locations. Therefore, details of the fracture process, such as variation of the slip mode, bifurcation and arrest, which are likely to change the source radiation characteristics, may be difficult to extract from the data. On the other hand, a controlled experiment in the laboratory, during which such features could be studied, may be inaccurate. As previously mentioned, the rheological properties of sea ice and its mechanical behavior are not sufficiently well known to adequately reproduce the material. To address this issue of the variation of the radiation characteristics of a crack, Langley [26] suggested that the monopole model used to represent tensile cracks, should be combined with an acoustic quadrupole, as a more realistic representation of a crack whose field may not be axisymmetric.

Xie and Farmer [41], studied events from tensile cracks in shorefast ice. Based on the event characteristics, they proposed the following hypothesis for the fracture process. The initial ice breakup emits individual sound pulses, the density of which increases with further fracture, particularly if the stress due to wind and current continues. The process persists until large-scale faults are found, causing the complete breakup of ice floes into smaller ones. Then, under the influence of wind and current, the individual floes gain momentum and slide horizontally, therefore rubbing against each other. The rubbing excites a pure-tone signal which they observed in the noise time series. They subsequently developed a model to represent this process, assuming that friction between adjacent ice floes excites horizontal shear waves in the ice. The signal and radiation characteristics of the acoustic events analyzed in this thesis do not indicate the occurrence of ice floe rubbing.

Farmer and Xie, also investigated crack propagation in landfast ice [12]. They measured the sound generated by cracks up to 20 kHz. The high-frequency components of these
signals showed similarities to seismic observations and they, therefore, adapted a theory of
earthquake mechanics to the case of a floating ice sheet. The theory incorporated the effects
of crack propagation which was believed to be responsible for certain characteristics of the
observed signals. They examined only four events in the noise time series, and assumed
that they were caused by tensile cracks. From the frequency components of the detected
events they deduced that more than one scale is associated with the fracture process and that
the event source moves with the developing crack. They also suggested that although the
horizontal and vertical scales are the two primary scales associated with a crack, finer scale
features may also contribute to noise signals. Assuming stick-slip motion, i.e that the crack
advances in small jumps determined by the thickness of the ice sheet, they modeled the
source slip function as a sinusoidally roughened ramp function. This model accounts for
crack length increase with propagation from both sides of the epicenter of the crack, but not
for crack orientation. It is consistent with their observations that for near-field events, there
is significant variability in the signal between hydrophones, whereas for far-field sources,
the signal is independent of hydrophone location. Their work serves as a guidance in this
thesis.

The work most pertinent to my research is that of Chen [6] and Dyer [11]. Dyer
investigated several possible event mechanisms in the low- and mid-frequency ranges and
presented a broader picture of the event physics. Based on the results of Makris and
Dyer [27], for pack ice, he suggested that moments at ridge hummocks and stresses at
overthrusting sites may account for the noise. He emphasized that the relief of local
deformation of the ice sheet is likely to be the responsible mechanism for sound radiation
at low frequencies. Dyer proposed two models, one to represent the fracture process driven
by the environmental stress moment and a horizontal stress model. In a different study [10],
hel also proposed a slip model for ice fracture. Chen used both this and the ridge unloading
models (modified by Dyer to include the creep effect of ice) in her study of fracture
mechanisms in the Marginal Ice Zone. She investigated four ice mechanisms, namely ridge
unloading, wave-induced floe breakup, floe collision and shear, and shear fracture, and used
equivalent dipoles and octopoles to represent the event sources. She assumed compactness
of the source, a far-field approximation for the pressure field, and assumed that the ice
properties below 300 Hz (where the wavelength is large compared to the ice thickness), and apart from the immediate fracture zone, to be identical to those of water. An interesting observation from her results is that when she calculated the dipole force of the detected events, at low frequencies, the spread of the distribution of force with peak frequency indicated a more complicated source. The requirement for a composite acoustic model, also emphasized by Langley, is discussed in detail in this thesis. Chen also used a vertical dipole to represent the volume change associated with the ice unloading motion. Using phase-tapered array ideas, she modeled fracture propagation and estimated the fault plane dimensions from the peak event frequencies, fault length in particular. However, she was unable to extract the fracture propagation speed and orientation. A barrier associated with the estimation of these parameters is the location and spatial concentration of the receivers. For distant sources, if receivers are close to each other, the Doppler shift due to source motion may not be measurable or may be negligible. Consequently, both source speed and orientation cannot be estimated accurately. The most important conclusion of Chen’s work is that shear fracture is the most plausible event mechanism in the mid-frequency range (above 100 Hz) and ridge-unloading is a likely source at frequencies below 100 Hz. Her results serve as an initial guide to the work in this thesis, though they are applicable to the Marginal Ice Zone where the ice stress field is different from that in the Central Arctic.

1.6 Conclusions and contributions of the thesis

My ultimate goal in studying ice-induced acoustic events in the Arctic has been to present as complete a description as possible, based on the nature of the acoustic data, of the characteristics of the identified event generating ice mechanisms. As previously discussed, a complete description of these mechanisms involves measurements of parameters that characterize particle or local motion of the ice processes and parameters that describe propagation of faults, identification of the type of motion associated with these faults and finally an identification of either secondary fracture processes or large-scale fracture features that radiate sound and thus contribute to the ambient noise in the Arctic ocean. The nature of the acoustic data that I have analyzed is such that the above-mentioned parameters have
been estimated with sufficient accuracy.

The following are the most important contributions of this thesis:

- Estimation of particle motion of fault processes in Arctic ice.

To the best of my knowledge, particle velocity, slip and rise time have not been consistently estimated from acoustic signals, for the entire event population, in previous studies. Evidently, particle slip has been implicitly estimated from the event pressure signals. For example, Chen [6] estimated fracture moment from the octopole source strength, which is in turn a function of particle slip and fault area. However, she did not have accurate estimates, neither of source dimensions to determine the fault area, nor of the source speed and orientation to include in the calculation of octopole strength. Thus, her estimates of particle slip are several orders of magnitude lower than both mine and Kim's [21]. In regard to the modeling of slip functions, although both Dyer [10] and Farmer [12] have suggested models for these functions, for particular types of event signals, no consistent study of a large number of such functions has been done, in previous studies. Slip functions are important for understanding the local motion of a discontinuity or dislocation in the medium. Also, measurements of particle slip are important for estimating stress or strength parameters. It is proportional to the friction drop, from the static to the dynamic value, at sliding of the discontinuity. The corresponding stress drop, which depends both on slip and the friction drop, is also a function of normal stress. Consequently, the latter can also be estimated and provide information on the stress field in the medium. Due to the lack of previous measurements of slip parameters in any Arctic region, the estimated values are compared to corresponding values for earthquakes. The most important conclusion is that particle displacement (mean slip) of ice processes is measurable, in the range $O(10^{-4}) - O(10^{-2})$ m, though at least 3-4 orders of magnitude lower than slip of earthquake faults. Particle velocity is roughly between 1-67 cm/s, the maximum estimate being only 30% lower than the typical particle velocity of a fault in the earth crust (100 cm/s). In addition to the different material characteristics of the two media, the ice cover is a thin plate, the thickness of which controls the depth and consequently the slip of fault processes. Thus, the difference in particle displacement values between ice faults and earthquakes is expected. Seismic
moment, stress drop and consequently strength measures of the medium are functions of particle slip, among other variables. Thus, independent measurements of slip are important in calculating these fault parameters.

- Estimation of the characteristic parameters of propagation for fault processes in Arctic ice.

The estimated fracture speed is on average, in the range 200-1100 m/s, significantly lower than the previously assumed values (of the order of the Rayleigh wave speed). The wide range of speed estimates provides additional insight into the multitude of processes or features of particular processes that induce acoustic events. The variation of source speed is attributed to physical phenomena. For example, low-speed events are either associated with mechanisms that do not involve propagation, such as floe unloading, or may be associated with secondary fracture features, such as crack zones, which in the aggregate appear as a large, slowly-moving fracture. On the other hand, higher speed events are probably associated with fracture processes. Therefore, source parameters serve as additional information in the process of identifying the event generating mechanisms. Estimates of source orientation, also obtained here, are important in modeling the event radiation characteristics, particularly for events with a characteristic horizontal radiation pattern. In previous studies, horizontal directivity values averaged over azimuth have been used [7].

- Identification of ice mechanisms based on the analysis of the acoustic event radiation characteristics

Although previously proposed mechanisms such as floe unloading and fracture (both shear and tensile) are plausible generating mechanisms of the events analyzed here, they have no predominant frequency range in which they occur. This result supports the hypothesis that ice mechanisms in the ice occur at different stages, the frequency content of which may span the entire frequency range of interest, i.e. 3 Hz - 100 kHz. Note that the plausibility of floe unloading as one of the event inducing processes is enhanced by the fact that events attributed to this process are consistently detected following sequences of high-speed events, the radiation characteristics of which indicate that they are associated
with fracture phenomena. Another important result, which is supported by the physics of the shear fracture formation and propagation, is that for some events, their radiation patterns indicate that a possibly secondary fracture process affects radiation from the shear fracture. Shear fractures propagate through the formation of arrays of tensile cracks which also radiate sound. A complicated radiation pattern results which is best described by a hybrid acoustic model, proposed in the thesis, that takes into account both processes.

- Identification of large-scale fracture features

Large-scale analysis of acoustic events in the aggregate and particularly in terms of temporal and spatial clusters has been performed. Event clusters have been analyzed in previous studies but no fracture details have been identified. Although in this thesis individual stages in the fault formation have not been identified, since their frequency content and inter-arrival time are unknown, I show that large-scale fracture features, such as process or crack zones, occur in the ice and contribute to the ambient noise. Events that are believed to be associated with such zones have low source speeds. Echelon arrays of tensile fractures, with orientations perpendicular to large shear fractures have also been identified. To the best of my knowledge, although such cracks have been observed in rock, during earthquakes, their occurrence in Arctic ice has not been suggested from the results of previous studies of either seismic signals or acoustic events.
Chapter 2

Time Domain Analysis

2.1 Overview

Analysis of acoustic events in the time domain is achieved in five stages, summarized in the following diagram:

![Diagram of time domain analysis stages]

Figure 2-1: Stages of the analysis of acoustic events in the time-domain.

First, data are selected according to the purpose of the study. For example, when environmental correlates to the ocean ambient noise are sought, a large data set is required, spanning over several hours or even days, to ensure variation in environmental conditions during data collection. The purpose of this thesis is to study individual acoustic events and understand the characteristics of their generating mechanisms in the ice. Therefore, a smaller ambient noise data set is sufficient, as long as a large event population can be
detected in it to support the results of the study. Therefore, a one-hour data segment from the Sea Ice Mechanics Initiative (SIMI) experiment was selected, in which 196 acoustic events have been detected. Ice and environmental conditions during the experiment and data collection are discussed in Section 2.2.

The remaining four stages pertain to the processing and analysis of the data. The detailed methodology followed at each stage is described in Sections 2.3 - 2.7. First, events are detected in the ambient noise time series and their arrivals at the hydrophones of the receiving array are calculated. The results from this stage are used to localize the detected events in the ice. All events are located in an area approximately 1.5 km$^2$, within which regions of high ice activity are identified. In addition, measurements of ice thickness next to the hydrophones of the receiving array indicate variation of ice concentration in this area. Events cluster both in time and space. It is believed [6] [11] [28] that temporal clustering is directly associated with variations in environmental forces applied to the ice cover. Within one hour these forces are unlikely to change significantly. Therefore, the relation between event clustering in time and the temporal evolution of ice motion, e.g., stick-slip fracture propagation, is investigated in this study. Spatial event clustering is probably associated with the propagation of faults in the ice and the formation of crack zones. The analysis of event clusters is presented in Chapter 5.

In the fourth stage of the time domain analysis, events are classified according to their signature types. Two major categories are distinguished, namely single- and multiple-component events. 61% of the total number of events are in the first category and 39% are in the second. In addition, four basic signature types are identified, shown in Figure 2-2. Simple (single-component) and compound (multiple-component) event signals are of any of these types. The nomenclature previously introduced by Chen [6] is used in their classification. The association of individual signature types with corresponding distinct mechanisms has been the goal of several researchers [6] [38]. In this study it is shown that although some source parameters, such as particle velocity and slip function, are correlated with signature type, there is no one-to-one mapping between ice mechanisms and event types.
Figure 2-2: Basic Event Signature Types

The statistics of events as a function of signature type, irrespective of the number of signal components are: 12% have signatures of the first type, 13% of the second, 34% of the third and 41% of the fourth. The latter is thus the predominant type, a signature characterized by a series of pulses of random separation. Statistics and characteristics of event types are discussed in detail in Section 2.6.

Correlation of the distribution of event signature types with ice thickness variations is
then investigated, but the results are inconclusive. The highest event concentration is in regions where ice is 2.4-3 m thick, in which signatures of the third and fourth types are predominant. However, this is a general characteristic of the event population, irrespective of location. There is no area of particular ice thickness where clustering of events of a single type occurs. Therefore, the two variables in question appear uncorrelated.

Signatures are also examined in order to detect the possible arrival of the longitudinal wave from the ice at the receivers, in addition to the predominant arrival of the acoustic wave. The longitudinal wave is the fastest of the two and is thus expected to reach the receivers first. Indeed two arrivals have been detected in roughly 20% of the events, though not at all hydrophones. Using the distances between source and receivers and corresponding arrival times, I estimated the propagation speed for the first arrival. 60% of these estimates were in the range of the speed of the longitudinal wave, namely between 2900 and 3200 m/s. The remaining estimates, though, were of the order of the sound speed in water, on average 1440 m/s. This result indicates that for some events, the detected first arrival is unrelated to the longitudinal wave arrival.

The event signals are finally integrated with respect to time with the purpose to estimate ice fault parameters at the level of particle motion. This process requires the assumption of a fault (or source radiation) model, which is selected based on the event radiation characteristics. Appropriate acoustic models include a vertical dipole, a dipole modified by the Doppler factor to account for source motion, longitudinal and lateral octopoles and hybrid multipoles, consisting of a superposition of a dipole and a lateral octopole or a combination of octopoles. They are described in detail in Chapter 4, where the procedure for selecting an adequate acoustic model, the detailed analysis of the event radiation characteristics and the proposed ice mechanisms are presented. The type of ice motion, such a shearing due to a propagating fracture or volume change in the medium due to unloading of an ice floe determine the characteristics of the source parameters, the source-wave displacement type in particular. Acoustic pressure is proportional to the first derivative of traction in the medium, to the second derivative of the source-wave displacement parameter (equivalent to the compression (P)- or shear (S)-wave displacement pulse, in the case of earthquakes) and to the third derivative of slip displacement. Notice that
no independent measurements of the P- and/or S-wave displacement signals are available, during the selected one-hour data collection time interval. Therefore, the source (or slip) time function of each type of event is estimated based on the assumed source model, and consequently the characteristic parameters of particle motion in the ice, namely the time to complete slip offset at any point in the fault (rise time), particle velocity and maximum slip.

Parameters that characterize particle motion of the ice mechanisms and particularly their slip functions are clearly dependent on event signature. Although events of different signatures are attributed to the same ice mechanism, such as shear fracture or floe unloading, there is a unique slip function associated with each event type. Therefore, slip parameters are discussed here as a function of signature and examples are presented for each identified type, both simple and compound. The statistics of these parameters for the event population show that compound events have the longest rise times, in the range 0.055-0.14 s, and events of the first type have the shortest. In regard to particle slip, minimum values have been estimated for events of the second and fourth types, both simple and compound, and maximum values for the third type. The fastest slip process(es) is that associated with events of the first and third types; the maximum particle velocity is 67 cm/s. The slip function of compound events of long duration (0.14-0.2 s), indicates that stick-slip motion may be associated with their generating ice mechanism. Existing slip function models, such as the Haskell and Brune models which assume that the source mechanism is a moving dislocation, are finally discussed and it is shown that the Gaussian pulse, the Rayleigh function and their first integrals, best describe the estimated source-wave displacement and slip time functions, respectively.

2.2 SIMI Experiment: Ice and Environmental Conditions and Data Collection

Conducted in the spring of 1994 in the central Arctic, the SIMI experiment had several objectives, among which was the detection and localization of active ice zones and the recording of acoustic emissions from natural ice processes, such as fractures. Two hy-
drophone arrays, one horizontal crossed array and one vertical line array, of 32 elements each, were deployed for ambient noise data acquisition. A smaller geophone array was also used, at selected times during the experiment. Only data recorded by the horizontal hydrophone array have been examined in this thesis. The hydrophones were suspended 60 m below the ice and the maximum aperture of the array was approximately 700 m. The sampling rate for the data was 1000 Hz. The data were filtered with a bandpass filter between 1 and 350 Hz, in the acquisition system. The array geometry and its position relative to areas of potential high ice activity, such as old cracks, ridges and leads, is shown in Figure 2-3:

Figure 2-3: Hydrophone locations. The horizontal array is surrounded by areas of existing cracks, leads, and ridges. Data from day J98 have been analyzed. During that day, no significant ice activity was observed in these areas, except north of Mount Odyssey.

A one-hour segment from day J98 (Julian day 98), between local times 2300:03 and 2359:54 has been analyzed in this thesis. During that day, approximately at time 2340:00, strong ice motion was observed, north of Mount Odyssey; it was an old crack that had slipped, creating a new fracture zone. No such significant ice activity in other regions with
existing faults was recorded. Therefore, the majority of the detected events, near or at that area are probably associated with this large crack. Also, during that day only the first 24 elements of the horizontal array were functioning.

In regard to environmental conditions, winds ranged between 10-25 knots during the experiment, temperatures were between 0°C and −30°C, and the ocean current was approximately 1 knot. No significant variations in these conditions were observed during the time at which the data segment of interest was recorded. Ice thickness varied between 0.3 and 4.2 m, in areas where no ice features, such as ridges or 'mountains' were located, whereas the latter were roughly 4-5 m and 5-8 m high, respectively, as shown in Figure 2-4:

![Image of ice thickness variation](image)

Figure 2-4: Spatial ice thickness variation, between 0.3 and 7.8 m, estimated by extrapolating between the measured values next to each hydrophone: contour heights are in m; increments are 0.5 m.

Direct measurements of ice thickness were taken only next to each hydrophone. Therefore, no accurate information is available on ice thickness at other locations. Estimates of the height of ridges and 'mountains' are very rough [33]. Contour heights for ice thickness, shown in Figure 2-4, are extrapolated estimates. If the correlation between event char-
acteristics, such as signature type, clustering and later peak frequency, source speed and directivity with ice thickness is sought, only those events close to the hydrophones should be examined for this purpose.

The sound speed profile used in all calculations is that measured during the experiment for day J98, shown in Figure 2-5. It is characterized by the typical deep ocean gradient below 300 m. Segmentation of the profile to simplify refraction calculations, required for event localization, is discussed in Appendix C.

![Central Arctic Ocean Sound Speed Profile: SiMI Experiment, day J98](image)

Figure 2-5: Measured sound speed profile for day J98. Data are available only up to a depth of 462 m. Sound speed is between 1432 m/s at the ocean surface ($z = 0$) and 1462 m/s at depth $z = 462$ m.

### 2.3 Event Detection

Detection of acoustic events in the data is achieved through the use of a simple square law detector. The following assumptions are made: the a-priori probability of the event signal occurrence in the noise is unknown. Therefore, the basic likelihood ratio test cannot be
used. Also, the waveform of the signal is not known \textit{a-priori}, and thus a matched filter, often used in detection algorithms, is not of use. The square law detector allows both these parameters to be unknown. The noise is assumed Gaussian, statistically independent of the event signal, with variance \( \sigma^2 \) and covariance matrix \( K = \text{diag}(\sigma^2) \). In cases where its mean is non-zero, it is subtracted from the processed time series to obtain a zero-mean noise sequence.

The hypothesis testing problem is:

\[
H_0 : z_i(t) = n_i(t)
\]
\[
H_1 : z_i(t) = s_i(t) + n_i(t)
\]

where \( z \) is the data vector of length \( L, i = 0, ..., L - 1 \), and \( n_i \) and \( s_i \) the noise and signal values, respectively. Thus, hypothesis \( H_0 \) is that the data vector is pure noise and hypothesis \( H_1 \) is that a signal is present in the data in addition to noise.

The generalized likelihood ratio test, or square-law detector in this case, is

\[
z_i(t)K_{ii}^{-1}z_i(t) \gtrless_{H_0}^{H_1} \gamma
\]

where \( \gamma \) is the threshold. For unit-variance noise, the left-side of Equation (2.2) is the sum of the squares of the data values:

\[
\Upsilon = \sum_{i=0}^{L-1} z_i^2(t) \gtrless_{H_0}^{H_1} \gamma
\]

\( \Upsilon \) is the so-called \textit{sufficient statistic}, i.e the quantity that best describes the data. The variance of the noise in the data of interest may not be unity and thus the following equation is used instead:

\[
\sum_{i=0}^{L-1} \frac{|z_i(t)|^2}{\sigma_i^2} \gtrless_{H_0}^{H_1} \gamma
\]

with \( \sigma_i^2 = \sigma^2 \). In order to compute the threshold \( \gamma \), an estimate of the probability of false-alarm must be known. In the detection process, I used different estimates of this probability, according to the signal-to-noise ratio in the event time series. Under the assumption of a \( \chi^2 \)
distribution of the sufficient statistic, $\gamma$ is given by

$$\gamma = \sigma^2 \sqrt{2L} G^{-1}(P_F) + \sigma^2 L$$ (2.5)

where $G$ is the Gaussian distribution function, and $P_F$ the probability of false-alarm. The derivation of this expression is described in Appendix A. The performance of the detector is affected by the lack of a-priori knowledge of the signal waveform and the signal-to-noise ratio (SNR). There are different ways of expressing this quantity. Here, the following equation is used:

$$SNR = 10\log_{10}(\frac{<\tilde{s}^T \tilde{s}>_{rms}}{<\tilde{n}^T \tilde{n}>_{rms}})$$ (2.6)

where $\tilde{s}$ and $\tilde{n}$ are the noise and signal vectors, and $< >_{rms}$ denotes the root-mean-squared values. As the signal-to-noise ratio decreases, the probability of false-alarm increases. Consequently, the performance of the detector deteriorates.

The detection procedure is summarized in the following diagram:

---

Figure 2-6: Schematic representation of the methodology for event detection in ambient noise.

First, processing and detection sliding data windows are selected. The size of the detection window is equal to the time required to travel a distance equal to the maximum aperture of the array (700 m) at an average over depth speed $c = 1437$ m/s, i.e., $dt_{det} = \frac{700}{1437} \approx 0.5$ s. The size of the processing window is ten times larger. This choice is based on the size of the data set. For data within each processing window, detection is performed separately for each channel (hydrophone), following the procedure previously described.
In cases of low detection threshold values, the probability of false-alarm is adjusted. For an event detection to be recorded, the event signal must be present in at least five channels. This process is repeated until the entire one-hour data segment has been examined.

Finally, visual inspection confirms the actual occurrence of the detected events and eliminates false-alarms and redundant detections. Since the duration of each event is not known a-priori, it is impossible to adjust the size of the detection window. In some cases only part of an event may be present in a particular window; the remaining part will be in the next one. Therefore, two detections are recorded. Visual examination of the time series is necessary to eliminate one of the detections. Waterfall plots of the data are used for this purpose. 196 events have been detected in the data of interest. Two events, under high and low signal-to-noise ratio, respectively, are shown in Figures 2-7 and 2-8:

Figure 2-7: Detected event in the ambient noise time series (unfiltered data); $SNR \simeq 12$ dB. This is a single-component (simple) event of the first type, of duration approximately 0.030 s.
Figure 2-8: Detected event in the ambient noise time series (unfiltered data); $SNR \simeq 5$ dB. This is a multiple-component (compound) event, of duration approximately 0.4 s.

2.4 Time-delay estimation

For each event, time delays of the event arrival at different channels are calculated using a standard cross-correlation procedure. A reference channel is first selected, usually the one where the event arrives first. The signal in that channel is then cross-correlated with the corresponding signals in all the other channels in which the event has been detected. The time delay is the time at which the cross-correlation coefficient attains an absolute maximum. Consider time series $x_i(t)$ and $x_j(t)$, such that

\begin{align}
    x_i(t) &= s(t) + n_i(t) \\
    x_j(t) &= \alpha s(t + \tau_j) + n_j(t)
\end{align}  \tag{2.7}
where $s$ is the event signal, $n_{i,j}$ the noise components at channels $i$ and $j$, respectively, $\alpha$ the difference in transmission loss between the channels and $\tau_j$ the time delay at channel $j$, relatively to channel $i$. $s(t)$, $n_i(t)$ and $n_j(t)$ are assumed to be uncorrelated. The time-delay estimator used is that proposed by Knapp and Carter [22]. The cross-correlation function is defined as

$$\mathcal{R}_{x_i,x_j}(\tau) = \mathcal{E}[x_i(t)x_j(t - \tau)] \quad (2.8)$$

where $\mathcal{E}$ denotes expectation. Due to the finite duration of the time series, only an estimate of $\mathcal{R}_{x_i,x_j}$ can be obtained. The value of $\tau$ that maximizes the expression in Equation (2.8) is the time-delay estimate. To improve its accuracy, $x_i(t)$ and $x_j(t)$ can be pre-filtered using a weighing function, in the frequency domain. Commonly used pre-filters are the Phase Transform and the Hannan-Thomson processor. Both are appropriate for estimating time-delays for bandlimited signals in the presence of white noise.

After pre-filtering, the cross-correlation function is simply given by

$$\mathcal{R}_{x_i,x_j}(\tau) = \int_{-\infty}^{\infty} G_{y_i,y_j}(f)e^{j2\pi f \tau} df \quad (2.9)$$

Given the filtered series $y_i$ and $y_j$

$$G_{y_i,y_j}(f) = H_i(f)H_j^*(f)G_{x_i,x_j}(f) \quad (2.10)$$

where $H_i$ and $H_j$ are the frequency responses of the filters for $x_i$ and $x_j$, respectively, $(\ast)$ denotes complex conjugate, and $G_{x_i,x_j}(f)$ is the cross-spectral density of the two original sequences. The product $H_iH_j^*$ is the general frequency weighing $W(f)$. For the basic cross-correlation processor (no pre-filtering) $W(f)$ is given by

$$W(f) = H_iH_j^* = \begin{cases} 1 & f_{\text{min}} < f < f_{\text{max}} \\ 0 & \text{otherwise} \end{cases} \quad (2.11)$$

where $f_{\text{min}}$ and $f_{\text{max}}$ are the low- and high-frequency limits, respectively.

To pre-filter the event time series, I used the Hannan-Thomson processor. This weighs the cross-spectral phase according to the magnitude of the coherence. Maximum weight is
placed on the cross-spectral phase with the minimum variance [13]. The weighing function is given by

$$ W(f) = \frac{\gamma_{x_i,x_j}(f)^2}{|G_{x_i,x_j}(f)|(1 - \gamma_{x_i,x_j}(f)^2)} \quad (2.12) $$

where

$$ |\gamma_{x_i,x_j}(f)|^2 = \frac{|G_{x_i,x_j}(f)|^2}{G_{x_i,x_i}G_{x_j,x_j}} \quad (2.13) $$

is the mean-squared coherence. $G_{x_i,x_i}$ and $G_{x_j,x_j}$ are the auto-spectral density functions of $x_i(t)$ and $x_j(t)$.

The higher the cross-correlation coefficient, the more accurate the time-delay estimate. For the majority of the events, the time-delay error ranged between 0.001 and 0.005 s [37]. In cases of poor cross-correlation, due to the presence of a 'weak' event signal in some channels, a low signal-to-noise ratio, or the complexity of the event signature, particularly for multiple-component events, the time-delay error was as high as 0.050 s. Note that implicit in this estimation procedure is the assumption that both the source and the receivers are not moving. It will be shown in Chapter 3 that the event sources are actually in motion. When this motion is neglected in the time-delay estimation, event series $x_i$ and $x_j$ may appear uncorrelated. The component of the source propagation velocity in the direction of each receiver is different. It is maximum when a receiver is in the direction of propagation of the source. To account for this effect, the time series $x_i(t)$ and $x_j(t)$ in Equation 2.7 can be expressed as

$$ x_i(t) = s(\beta_i t) + n_i(t) $$

$$ x_j(t) = s(\alpha s(\beta_j (t + \tau_j)) + n_j(t) \quad (2.14) $$

where $\beta_i$ and $\beta_j$ are the time compressions resulting from source motion [23], and are related to the propagation velocity $\bar{v}$ by

$$ \beta_i = 1 + \frac{|\bar{v}|cos(\phi_i - \phi_s)cos(\theta_i)}{c} \quad (2.15) $$

where $\phi_i$ is the horizontal angle of receiver $i$ (with the origin of the coordinate system at the moving source), $\phi_s$ the horizontal angle of the source, $\theta_i$ the vertical angle from the source to receiver $i$ and $c$ the sound speed in water. The expression $|\bar{v}|cos(\phi_i - \phi_s)cos(\theta_i)$ is the
component of velocity in the direction of the receiver. It is assumed that all receivers are at rest. Prior to cross-correlation, \( x_i(t) \) must be passed through an 'expander' or 'compressor' to eliminate the relative time compression \( \frac{\beta_i}{\beta_j} \). The maximum-likelihood (ML) function used to obtain the best time-delay estimates in the presence of source motion [23] is described in Appendix B. The difficulty in this type of estimation is that \( x_i(t) \) and \( x_j(t) \) are jointly non-stationary, unless \( \beta_i = \beta_j \). Since the source speed is not \textit{a-priori} known, I first calculated time delays and event locations assuming no source motion. Once I had estimated source speed, I corrected both time delays and event locations. The correction resulted in higher cross-correlation coefficients between event time series at different hydrophones.

### 2.5 Event location

Time delays, hydrophone locations and knowledge of the sound speed profile are needed to locate events. Localization is achieved in two stages. In the first stage, the straight path (slant range) between source and receivers is used, whereas in the second stage, the refractive path is used for ray-tracing, as shown in Figure 2-9:

![Sound Speed Profile](image1)

![Straight Path of Sound Ray](image2)

![Refractive Path of Sound Ray](image3)

**Figure 2-9:** Straight and refractive paths of a sound ray in the water. Refraction is taken into account only in the second stage of the event location process. The sound speed profile up to the depth of the hydrophones (60 m) is also shown.
The procedure for event localization in the ice is summarized in the following diagram:

Figure 2-10: Schematic representation of the methodology for event localization in the ice.

First, time delays are re-calculated to ensure that they are all positive. Then, a polar grid is constructed, composed of concentric circles, whose radii differ progressively by 10 m. The cell size is 10 m by 1 degree. It is assumed that the sound speed in water is $c = 1437$ m/s, an average value over a depth of 60 m. The source is placed at each grid point. The slant distance from the source location to each hydrophone is then computed, as well as the corresponding time delays. Their standard deviation from the measured delay values is then calculated:

$$\sigma_\tau = \sqrt{\frac{\sum_{i=1}^{N} (\tau_i - \tau_{i,est})^2}{N}}$$

(2.16)

where $N = 24$ is the number of hydrophones and $\tau_i$ and $\tau_{i,est}$, the measured and estimated time delays, respectively. The propagation speed is also estimated as the slope of the best fitted regression line through the points $(\tau_{i,est}, R_i)$, with $R_i$ the slant range for each hydrophone. The error between this estimate and the assumed sound speed is then computed as $e = (c - c_{est})^2$. The location that minimizes both this error and the standard deviation of the time delay error $\sigma_\tau$ is the best event location. The procedure followed at this stage is similar to the one used by Townsend-Manning [40] and Stein [38], although the criteria and thresholds used are different. At the second stage of the location process, a finer and smaller grid is constructed around the best location from stage 1 processing. The cell size is now 1 m by 1 degree. Again the source is placed at each grid point and the optimization process is repeated. However, the refractive range between the source and each hydrophone is now used. The required refraction calculations are described in Appendix C.

For each event, its location, time of occurrence and launch angle for every hydrophone
are recorded. The locations of all detected events are shown in Figure 2-11:

![Event population plot](image)

Figure 2-11: Event population of 196 events detected in an one-hour data segment. The locations of the hydrophones are superimposed in the plot.

Reflection of sound rays off the ice cover are not taken into account in the estimation of event locations. The critical horizontal distance $X_{crit}$ (cycle distance) is approximately 1500 m. There are only 5 events whose distance from some hydrophones is greater than $X_{crit}$. The number of reflections in these cases is calculated by simply dividing the horizontal distance of each hydrophone from the source by $X_{crit}$. An example is presented
in Appendix C. Each reflection results in a sign change in the peak event amplitude at a particular hydrophone. Sign variation is important particularly in modeling the event radiation characteristics. The loss that sound rays experience as they are reflected off the ice is less than 1 dB per bounce [40] and is therefore neglected in the transmission loss calculation. Although roughness of the ice cover is important, only specular reflections are considered in this study. The problem of sound reflection from inhomogeneities is a research topic on its own and is beyond the scope of this thesis.

There are two sources of error in the location estimates. One is due to the error in hydrophone location. I have no information on this error for the SIMI experiment; in other experiments it was found to be of the order of 1 m, based on results from a sensor tracking system. The other error is due to cable motion induced by currents in the ocean. Chen estimated this error as a function of current speed and found it to be of the order of 10 m. In my location procedure, for the best location of each event I only calculated the error between the measured times delays and the ones estimated based on the event location. They were rarely above 0.005 s (on average) with a standard deviation of less than 0.001 s, corresponding to a range error of about 7 m. This is a very satisfactory result given the assumptions made and the uncertainties associated with the various parameters involved in the estimation.

2.6 Identification and Classification of Event Signatures

Events of different durations and signature types have been detected in the data. Their classification according to signal characteristics is a useful source of information, in reaching physical conclusions on the generating ice mechanisms. First, two categories of events are distinguished, namely single- and multiple-signal component events, which in the thesis I term simple and compound, respectively. Four basic signature types are subsequently identified and both simple events and individual components of compound events are classified accordingly. The following diagram summarizes this procedure:
Figure 2-12: Schematic representation of the two-level event signature classification.

The statistics of occurrence of simple and compound events are:

<table>
<thead>
<tr>
<th>Event type</th>
<th>No. of events</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>simple</td>
<td>120</td>
<td>61</td>
</tr>
<tr>
<td>compound</td>
<td>76</td>
<td>39</td>
</tr>
</tbody>
</table>

Table 2.1: Number of detected simple and compound events.

Although the majority are simple events, there is a significant number of compound events. Their occurrence implicitly introduces an additional parameter to the analysis, namely the inter-arrival time between signal components and its relation to the temporal evolution of the ice processes that have caused this type of events.

There are different ways in which event signatures can be classified. Since there is no commonly accepted nomenclature, for consistency I use that introduced by Chen [6]; my taxonomy is, however, different. As previously mentioned, four basic event signatures are identified, for simple events and individual components of compound events. They are shown in Figure 2-2. Type I is called a pop or burst, types II and III are damped sinusoids, the former a complex pop usually consisting of two type I pops, and the latter a longer sinusoid. The fourth signature type is a series of pulses, usually of random separation. Type IV events are different from compound events in that the separation between pulses is much smaller, i.e., about an order of magnitude lower, than the separation between signal
components in compound events. The latter are classified according to the signature type of their individual components, which for most events is of the same type. However, there are some exceptions; in such cases events are classified according to the type of their first component. The following are typical compound event signatures; a compound III-I signature is also shown in Figure 2-13:

![Compound Event Signatures](image)

Figure 2-13: Signatures of Compound Events
The main difference between my taxinomy and that of Chen is that I distinguish between single- and multiple-component events. Types V and VI in her classification, both series of pulses with constant and random pulse separation, respectively, are identified either as type IV or as compound events here.

The statistics of occurrence of different signatures, first irrespective of the number of components and then separately for simple and compound events are shown in Tables 2.2 - 2.4:

<table>
<thead>
<tr>
<th>Event Population</th>
<th>Signature Type</th>
<th>No. of events</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type I</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Type II</td>
<td>25</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Type III</td>
<td>66</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>Type IV</td>
<td>81</td>
<td>41</td>
</tr>
</tbody>
</table>

Table 2.2: Distribution of event population with signature type.

<table>
<thead>
<tr>
<th>Simple Events</th>
<th>Signature Type</th>
<th>Duration (s)</th>
<th>No. of events</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type I</td>
<td>0.015-0.040</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Type II</td>
<td>0.020-0.045</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Type III</td>
<td>0.050-0.065</td>
<td>49</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Type IV</td>
<td>0.08-0.12</td>
<td>43</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 2.3: Statistics of simple events’ signatures.

<table>
<thead>
<tr>
<th>Compound Events</th>
<th>Signature</th>
<th>Dur. (s)</th>
<th>Separation of components (s)</th>
<th>No. of events</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type I</td>
<td>0.065-0.12</td>
<td>0.015-0.025</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Type II</td>
<td>0.060-0.12</td>
<td>0.015-0.030</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Type III</td>
<td>0.080-0.12</td>
<td>0.020-0.040</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Type IV</td>
<td>0.12-0.2</td>
<td>0.020-0.030</td>
<td>38</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 2.4: Statistics of compound events’ signatures.

Only 12% and 13%, respectively, of the total number of events have type I and type
II signatures. The predominant types are III and IV. Note also that for compound events, inter-arrival times between their individual components are of the same order of magnitude, namely between 0.015 and 0.040 s. I will not attempt to explain these statistics at this point, since I have not yet presented the analysis of other event characteristics. If the hypothesis that different signatures are associated with distinct ice motion mechanisms or particular stages in a fault formation process is valid, the above results indicate that one or two mechanisms dominated the ice activity during the time at which these data were collected. The validity of the hypothesis needs to be assessed through further analysis. In Section 2.7 and in Chapter 4, the relationship between event signature types and ice mechanisms is discussed. It is shown that there are no unique signatures associated with particular mechanisms. For example, events which are believed to have been induced by fracture processes in the ice are both simple and compound and have signatures of any of the four identified types. It is, therefore, possible that the multitude of signatures is associated with other physical parameters, such as the formation stage of the ice fault radiates sound or the structure of the medium in the vicinity of the fault or in the region of its propagation. Although there is no one-to-one relationship between ice mechanisms and event types, source parameters are shown to be correlated with signature, particularly propagation speed, slip function and particle velocity.

2.6.1 Detection of the arrival of the longitudinal wave in event signals

At the beginning of this chapter, I mentioned that in some cases of events, the arrivals of both the longitudinal and the acoustic waves can be detected in an event signature. Assuming that the thickness of the ice cover is on average of the order of 3 m, at frequencies below approximately 100 Hz three waves can theoretically propagate away from a source in the ice and into the water: the flexural, longitudinal and acoustic waves. The flexural wave is subsonic and can, therefore, be recorded only by receivers very close to the ice cover. At frequencies above 100 Hz, only the acoustic and longitudinal waves can propagate into the water [38]. In both cases, all other waves are highly attenuated. Also, since propagation of the flexural wave decays with depth, at the depth of the hydrophones and in both frequency
ranges, only the longitudinal and acoustic waves are expected to propagate. The former is the fastest, with speed \( c_l \approx 3100 \text{ m/s} \), and thus arrives first at the hydrophones. The paths of the two waves are shown in Figure 2-14:

![Diagram of acoustic wave in water and longitudinal wave in ice](image)

Figure 2-14: Paths of acoustic wave in water and longitudinal wave in ice. For sketching purposes the slant path between source and receiver is shown. In all calculations the refractive path is considered.

Langley [26] showed that vertical radiation from the longitudinal wave is concentrated in a narrow beam at a vertical angle \( \theta_l = \cos^{-1}\left(\frac{1}{M_a}\right) \approx 62^\circ \), where \( M_a \) is the Mach number for this wave. Therefore, the presence of the wave in an event signal recorded by hydrophones at different locations depends also on the observation or launch angle of the sound ray that connects the source and each hydrophone, in other words on their respective distance. Also, due to attenuation it may be undetectable in the ambient noise. The presence of the longitudinal wave is unlikely to affect significantly the event radiation characteristics and consequently the choice of acoustic model; however, when appropriate, the vertical radiation pattern needs to be included in the model. Figure 2-15 is an example where the arrival of the longitudinal wave can be seen in the event time series, although it is 'weak' compared to that of the acoustic wave.
Figure 2-15: Event signal in which the arrival of the longitudinal wave (marked with the upward arrows) is detected in the first 6 hydrophones. The strongest arrival is that of the acoustic wave. The distance of the source to the first hydrophone is 466 m.

The noise level is about 94 dB re 1μPa, the pressure level \(20 \log_{10}(\frac{p_{\text{peak}}}{p_{\text{ref}}=1\mu \text{Pa}})\), where \(p_{\text{peak}}\) is the peak event pressure) of the acoustic wave signal is 111 dB re 1μPa, and that of the longitudinal wave signal is 99 dB re 1μPa.

To ensure that this is indeed the arrival of the longitudinal wave, its speed can be estimated by plotting the event's first arrival times at different hydrophones against horizontal range. Having estimated the event location using the procedure described in Section 2.5, I can determine whether the first arrivals are those of the longitudinal wave by determining its phase speed. Thus, I assume that the path of propagation is that shown in Figure 2-14, with propagation in the water at sound speed \(c = 1437 \text{ m/s}\) and propagation in the ice at speed \(c_i = 3100 \text{ m/s}\). The launch angle \(\theta_0\) of the acoustic wave, obtained from the refraction calculations (see Appendix C) and radiation of the longitudinal wave from the ice down
to the hydrophones at the critical angle $\theta_l = \cos^{-1}(\frac{c_i}{c_l})$ are assumed. The equation for the longitudinal arrival time at hydrophone $i$ is given by:

$$\tau_i = \frac{r_i}{c_i} + \frac{R_{l,i}}{c} + t_e \quad (2.17)$$

where $r_i$ is the horizontal distance that the longitudinal wave travels in the ice, $R_{l,i}$ the distance between the point at which the wave leaves the ice and the receiver, i.e., if refraction is neglected this would correspond to the distance $R_{l,i} = \sqrt{(X - r_i)^2 + z^2}$, with $X$ the horizontal distance shown in Figure 2-14 and $z$ the depth of the hydrophones. Because the refracted path is at 62°, refraction can be neglected [26]. $t_e$ is the event occurrence time, estimated from the arrivals of the acoustic wave. Measured time delays are plotted versus range, for each hydrophone. The slope of the least squares best fitted line is the speed estimate sought [38], as shown in Figure 2-16.

Figure 2-16: Estimation of the speed of the longitudinal wave from the slope of the best fitted regression line. The estimated speed is 3000 m/s, very close to the nominal value. Channel 1 has been used arbitrarily as the reference channel; therefore, some arrival times are negative.
I followed this procedure for 39 events, in the time series of which I suspected the presence of the longitudinal wave. For 23 of these events the estimated longitudinal speed in ice was in the range 3000-3200 m/s; for the remaining 16 it was of the order of the sound speed in water. This result indicates that the first arrival was incorrectly assumed to be that of the longitudinal wave. It may instead be associated with the characteristics of the event generating mechanism.

2.6.2 Distribution of event signature types with ice thickness

A study of the relationship between event signature types and ice thickness variation is meaningful only for events close to the hydrophones, next to which ice thickness was directly measured. Thus, only a sub-population of events is studied, shown in Figure 2-17:

![Image of Figure 2-17: Locations of events of different signature types in the area of interest and ice thickness variation. Contour heights are in m; increments are 0.5 m. The symbols used for signature types are: (*) for type I, (o) for type II, (+) for type III and (x) for type IV.]

Figure 2-17: Locations of events of different signature types in the area of interest and ice thickness variation. Contour heights are in m; increments are 0.5 m. The symbols used for signature types are: (*) for type I, (o) for type II, (+) for type III and (x) for type IV.
109 events are located in this area. There are 6 type I, 12 type II, 57 type III and 34 type IV events, both simple and compound. The statistics of their spatial distribution as a function of signature type, irrespective of the number of signal components, are summarized in Table 2.5. A 0.3 m (≈ 1 ft) increment has been used for ice thickness variation.

<table>
<thead>
<tr>
<th>Ice Thickness (m)</th>
<th>Type I # of Events</th>
<th>Type II # of Events</th>
<th>Type III # of Events</th>
<th>Type IV # of Events</th>
<th>Total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3 - 0.6</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>0.6 - 0.9</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0.9 - 1.2</td>
<td>-</td>
<td>-</td>
<td>6</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>1.2 - 1.5</td>
<td>-</td>
<td>2</td>
<td>6</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td>1.5 - 1.8</td>
<td>1</td>
<td>-</td>
<td>3</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>1.8 - 2.1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>2.1 - 2.4</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>2.4 - 2.7</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>17</td>
<td>31</td>
</tr>
<tr>
<td>2.7 - 3.0</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3.0 - 3.3</td>
<td>-</td>
<td>1</td>
<td>6</td>
<td>-</td>
<td>7</td>
</tr>
<tr>
<td>3.3 - 3.6</td>
<td>-</td>
<td>-</td>
<td>6</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>3.6 - 3.9</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3.9 - 4.2</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 2.5: Distribution of event signature types with ice thickness.

62 of the 109 events located in the area shown in Figure 2-17 are located in regions where ice is about 2-3 m thick. Although knowledge of the event generating physical mechanisms is useful in interpreting this observation, it is known that multi-year ice in the Arctic (≈ ≥ 1 m thick) contains a large number of cracks, many of which are old and thus inactive. Therefore, application of sufficient forcing at a particular area, as well as the formation and propagation of new fractures are likely to activate old cracks, too, resulting in high activity, and consequently high density of induced acoustic events.

Based on the above statistics, no correlation between the occurrence of type I events and ice thickness can be deduced, due to the lack of a sufficiently large sample of such events and the fact that they are not clustered in a particular region. This is also the case for type II events, though they appear to be concentrated in areas where ice thickness is between 1.8 and 3.0 m. Although 92% of the total number of type III events are located in the area
shown in Figure 2-17, and thus the sample of these events is sufficiently large, there is no clear evidence of correlation between their occurrence and ice thickness variations. The highest concentration of type III events is in two regions, of ice thickness 0.9-1.5 ft and 2.4-3.6 m, respectively. Note that in the former region there is no other dominant event type, whereas in the latter events of all types are located. These results, are more likely to be related to the characteristics of the event generating ice mechanisms in these areas than to variations in ice thickness and consequently the material structure. In Chapter 1, I mentioned that the micro-mechanical structure of first-year ice is different from that of multi-year ice. In this area, ice is mainly multi-year, with the exception of areas where thickness is 0.3-0.9 m, in which only 4 events are located. Finally, in regard to type IV events, their highest concentration (22 out of 34 events) is in an area where ice is 2.1-2.7 m thick. Based on these results it is difficult to reach a confident conclusion on the correlation of signature type to ice thickness. For this particular event subpopulation, signature type and ice thickness appear uncorrelated.

2.7 Estimation of the characteristic displacement rise time, maximum slip and particle velocity

In addition to the distinction between event types, signatures may provide information on the generating ice mechanisms, their characteristic times in particular. In seismology, a fault process can be described by two characteristic times. The first is related to the rupture propagation (duration of the fault) and the second is the time required for a propagating displacement discontinuity in the medium to complete a slip offset at any point in the fault, commonly termed rise time. It is often considered as the time required for the discontinuity to reach approximately 90% of the permanent slip [20], although its precise definition depends on the individual fault model used. Consequently, two characteristic velocities are defined, fracture propagation velocity and particle velocity on the fault surface. Rise time and permanent (or maximum) slip are two of the parameters that describe a fault, discussed in Chapter 1.
In the case of earthquakes, particle motion is commonly estimated from the compressional (P)- and/or shear (S)-wave displacement signals which can be measured directly. During the SIMI experiment, geophones were deployed only at particular crack zones (visually selected) and at particular times. Thus, independent measurements of the above-mentioned signals are limited; to the best of my knowledge, they are not available for the entire area in which the detected acoustic events are located and at the time in which the data that I have selected were collected. In order to estimate parameters that pertain to particle motion of ice processes from the acoustic event signals, knowledge of the latter’s generating physical mechanisms is necessary, in order to have an appropriate fault model. This implies that the event radiation characteristics must be analyzed and an acoustic model which describes their horizontal and vertical radiation patterns must be selected. An estimate of source speed is also necessary to determine if propagation is associated with the mechanism, e.g., a propagating fracture. The detailed analysis of event radiation characteristics, the procedure for selecting an adequate acoustic model and the proposed ice mechanisms are presented in Chapter 4. Here, the acoustic models are only briefly discussed, to support the choice of the assumed ice mechanism in the estimation of the source-wave parameter (equivalent to the P- and/or S-waves).

It is possible that a single acoustic model does not describe the event radiation adequately. Faulting in the ice is complicated and involves sub-processes. However, initially two basic models can be distinguished, namely the dipole (a monopole and its image) and the octopole (a quadrupole and its image). There is a fundamental physical difference implied by these models: the mechanism which induces events, the radiation patterns of which are best described by a dipole, is associated with a volume change in the medium and force is the pertinent source strength parameter. In contrast, the mechanisms which induce events, the radiation patterns of which are modeled by either a longitudinal or a lateral octopole, or a combination of the two, are associated with fracture processes and thus a fault area and a moment are the pertinent source strength parameters. In Chapter 4, it is shown that unloading motion of an ice floe induces acoustic events that are best represented by dipoles, and that longitudinal and lateral octopoles describe the radiation characteristics of events induced by tensile and shear fractures, respectively.
The estimation of parameters that describe particle motion is meaningful for events associated with faulting processes, which involve motion of a discontinuity and particularly slip offsets. Unloading motion is a post-fracture phenomenon; it is the motion of an ice floe which has completely fractured. For events attributed to this process, estimation of slip parameters is meaningless. Therefore, the event radiation characteristics need to be investigated in order to determine if the unloading is the generating ice mechanism, and consequently exclude the induced events from the estimation.

The acoustic pressure is proportional to the third derivative of slip displacement, since the seismic (or double couple) moment \( M \) (to be stated in Equation 2.20) is proportional to slip, i.e.,

\[
\begin{align*}
p(R, \theta, \phi; t) \propto \frac{g(\theta)B(\phi)\mathcal{F}(R)}{2\pi c} \frac{\partial^3 u}{\partial t^3} \bigg|_{t' - \frac{t}{c}} \cdot \frac{1}{|1 - M \cos(\phi - \phi_s)|}
\end{align*}
\]  

(2.18)

Specific equations for pressure are presented in the discussion of each selected acoustic model. \( u \) is the slip sought, at the time of sound emission from the source, \( t' \) the vertical (launch angle), \( \phi \) the horizontal angle from the source to a receiver, and \( R \) the corresponding distance. Thus, in order to calculate the third time derivative of slip as a function of the acoustic pressure, prior to integration in the time-domain to obtain the slip function, the following source parameters must be known:

- source speed \( V \) and thus the Mach number \( \mathcal{M} = \frac{V}{c} \), with \( c \) the sound speed in water
- source orientation \( \phi_s \)
- vertical directivity \( g(\theta) \)
- horizontal directivity \( B(\phi) \)
- spreading function \( \mathcal{F}(R) \)

The estimation of source speed, orientation and spreading function is discussed in detail in Chapter 3. For a non-refractive medium, \( \mathcal{F}(R) = \frac{1}{R} \), but in general a more complicated function of \( R \). For each event, source speed and orientation are calculated from the Doppler shift, measured from the event spectra at different hydrophones. The vertical and horizontal
directivity functions are determined by examining the event radiation patterns and assuming an appropriate acoustic model.

The following is a brief discussion of the acoustic models that best describe radiation induced by faulting processes in the ice. Although the pertinent equations are given assuming a non-refractive medium, corrections have been made to account for refraction, when necessary. Notice, however, that events located at large distances from the hydrophone array, at which refraction becomes important, are not analyzed since the azimuth range of observation is small and the horizontal event radiation pattern cannot be resolved. There are four pertinent event groups, classified according to their radiation characteristics. Source motion is taken into account in all models by the inclusion of the Doppler factor \( \frac{1}{|1 - M \cos(\phi - \phi_s)|} \).

Table 2.6 summarizes the statistics of the four event groups, as a function of the selected acoustic model.

<table>
<thead>
<tr>
<th>Acoustic model</th>
<th>No. of Events</th>
<th>% of total # of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole modified by Doppler effect</td>
<td>52</td>
<td>29</td>
</tr>
<tr>
<td>Octopole (lateral)</td>
<td>46</td>
<td>25</td>
</tr>
<tr>
<td>Octopole (longitudinal)</td>
<td>28</td>
<td>15</td>
</tr>
<tr>
<td>Hybrid multipoles</td>
<td>35</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2.6: Statistics of events as a function of the appropriate acoustic model.

Notice that events of different signature types are included in each group. Signature and source directivity appear uncorrelated.

**Vertical, moving dipole model** The sound radiation resulting from a compact point force in a medium with zero shear modulus is described by a vertical dipole acoustic model. The pressure field due to a dipole source in the presence of source motion is given by Equation 2.19:

\[
p = \frac{\sin \theta \mathcal{F}(R)}{4\pi} \left( F(t - \frac{R}{c}) \mathcal{F}(R) + \frac{1}{c} \frac{\partial F(t - \frac{R}{c})}{\partial t} \right) \cdot \frac{1}{|1 - M \cos(\phi - \phi_s)|}
\]  (2.19)

where \( F(t - \frac{R}{c}) \) is the dipole force function delayed by time \( \frac{R}{c} \). There are 52 events that are best described by this model, with \( R \) the refractive distance (path) between source and
receiver. Due to source motion, they are believed to be associated with a fracture process in the ice. In Chapter 4, I show that possibly another model, such as a vertical longitudinal octopole, may also be appropriate. In the estimation of slip parameters, for each of the 52 events the moving dipole is assumed.

**Moving octopole model** The sound radiation resulting from a double force couple system in the medium is described by an octopole, resulting from the transformation of a quadrupole by the free surface. The pressure field due to an octopole in the presence of source motion is given by

\[
p(\vec{R}, t) = \frac{1}{\left| 1 - M \cos(\phi - \phi_s) \right|} \frac{g(\theta) B(\phi) F(R)}{2\pi} \cdot \left\{ \frac{1}{c^2} \frac{\partial^3}{\partial t^3}(Mh) + \frac{3F(R)}{c^2} \frac{\partial^2}{\partial t^2}(Mh) + 6F(R^2) \frac{\partial}{\partial t}(Mh) + 6F(R^3)(Mh) \right\}
\]

(2.20)

where \( g(\theta) \) is the vertical directivity, \( B(\phi) \) the horizontal directivity, with \( \phi \) the source orientation and \( \phi_s \) the source orientation, \( M \) the force couple moment, \( h \) the ice thickness and \( M \) the Mach number. \( B(\phi) \) and \( g(\theta) \) depend on the type, i.e., *lateral* and *longitudinal*, and the orientation of the octopole. Appendix G describes the two types and the derivation of their respective horizontal and vertical radiation patterns. Table 2.7 summarizes the different directivity functions.

<table>
<thead>
<tr>
<th>Type</th>
<th>( g(\theta) )</th>
<th>( B(\phi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>lateral (x-y)</td>
<td>( \sin \theta \cos^2 \theta )</td>
<td>( \sin \phi \cos \phi )</td>
</tr>
<tr>
<td>lateral (x-z)</td>
<td>( \sin^2 \theta \cos \theta )</td>
<td>( \cos \phi )</td>
</tr>
<tr>
<td>lateral (y-z)</td>
<td>( \sin^2 \theta \cos \theta )</td>
<td>( \sin \phi )</td>
</tr>
<tr>
<td>longitudinal (x-x)</td>
<td>( \cos^2 \theta \sin \theta )</td>
<td>( \cos \phi )</td>
</tr>
<tr>
<td>longitudinal (y-y)</td>
<td>( \cos^2 \theta \sin \theta )</td>
<td>( \sin \phi )</td>
</tr>
<tr>
<td>longitudinal (z-z)</td>
<td>( \sin^3 \theta )</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2.7: Vertical and horizontal directivity patterns of different types of octopole and corresponding double couples.

There are 46 events, the radiation of which is best described by a lateral octopole. Shear fracture is the most plausible generating mechanism. Therefore, in the estimation
of slip parameters, for each of these events it is assumed that the source is a moving shear dislocation. There are 28 events modeled by a longitudinal octopole. Tensile fracture is the most plausible mechanism. This fault model is thus assumed for each of these events. Finally, there are 35 events, the radiation of which is best described by a combination of lateral and longitudinal octopoles. The resulting radiation pattern is not included in Table 2.7, but is discussed in detail in Chapter 4. These events are attributed to shear fracture, the radiation pattern of which is affected by the formation of tensile cracks which coalesce and thus enable the propagation of the primary fracture for large distances. For each of the events in this group, the source is assumed to be a shear-tensile moving dislocation. Notice that the above-mentioned sub-populations are composed of both simple and compound events, of any of the identified signature types, although type III and IV events are predominantly associated with fracture processes.

Particle velocity and rise time are clearly associated with signature. The shape of the slip functions depends on the waveform of the acoustic signal from which it is obtained. Therefore, slip parameters discussed as a function of this parameter. Examples are presented for each of the identified event types. As previously mentioned, the displacement rise time can thus be estimated directly by integrating the event signal thrice in the time domain. The source-wave displacement parameter can also be estimated from the event signal; it is proportional to the second derivative of acoustic pressure. The procedure for first approximating the event signal by a polynomial and then repeatedly integrating to obtain the source-wave pulse and slip displacement functions corresponding to different signature types is described in Appendix D.

I must emphasize that in general, rise time estimated from far-field teleseismic signals during earthquakes is very sensitive to errors in these recordings [20]. It is a near-field parameter, much dependent on the recorded displacement signal, its duration, amplitude and waveform in particular. In the case of earthquakes, the recording stations may be thousands of km from the source. The distortion of the seismic signals, as they travel through the earth, a highly inhomogeneous medium, may affect significantly the accuracy of the calculated rise time. In this study, both the fact that it must be estimated by integrating the acoustic signal and that the receivers are in the far-field of the source, are likely to affect
the accuracy of the estimates. Integration is sensitive to the selection of the appropriate time interval. For a particular event, a 10% longer than the event duration time interval results in particle velocity being under-estimated by 10%, since rise time depends on the source-wave pulse width. Under ideal data collection conditions, rise time could be used as a source of information on the initial stress level at the origin of a fault. However, its estimation must be sufficiently accurate for this purpose. I do not claim that from these data the exact rise times for different ice mechanisms can be obtained. Assuming that the errors in recordings are of the same order of magnitude for all events, I want to estimate the relative difference in rise time and particle velocity for event generating mechanisms and the approximate shapes of the corresponding slip functions. Although only the events in Figures 2-2 and 2-13 are discussed in detail, the statistics of the above-mentioned parameters for the entire event population are also summarized. Using this information, appropriate slip function models are selected. The exponential function proposed by Brune [3], the ramp function proposed by Haskell [16], the Gaussian pulse proposed by Dyer [10] to model source-wave displacement corresponding to an acoustic burst (type I event), and the Rayleigh function which I show to be a more appropriate source slip model for compound events are discussed.

In Figures 2-18 to 2-25, each event signature, source-wave displacement parameter and slip function are shown. I have omitted the type IV compound signature due to inaccurate results from the integration process. Prior to integration, the event pressure signals have been normalized by \( \frac{2\pi c}{\rho F(R)B(\phi)g(\theta)} \), where \( B(\phi) \) and \( g(\theta) \) are selected according to the appropriate acoustic radiation model (including the Doppler factor), previously discussed. \( \rho = 1000 \ kg \ m^{-3} \) is the density of water. The time interval of integration has been chosen according to the duration of each event. Note that for the simple type II signal in Figure 2-2, I estimated its duration to be about 0.025 s. However, from its first integral I observed that stress in the medium fluctuates for approximately 0.050 s. A longer interval is thus selected.
Figure 2-18: Simple type I event signature and corresponding source-wave displacement pulse and slip function. The displacement pulse width $\tau$ is approximately 0.015 s, maximum slip $u_{max}$ is 0.4 cm, rise time $T$ is 0.030 s and particle velocity $v$ is approximately 13 cm/s.
Figure 2-19: Simple type II event signature and corresponding source-wave displacement pulse and slip function. The displacement pulse width \( \tau \) is approximately 0.040 s, maximum slip \( u_{\text{max}} \) is 0.12 cm, rise time \( T \) is 0.060 s and particle velocity \( v \) is approximately 2 cm/s.
Figure 2-20: Simple type III event signature and corresponding source-wave displacement pulse and slip function. The displacement pulse width $\tau$ is approximately 0.040 s, maximum slip $u_{max}$ is 1.4 cm, rise time $T$ is 0.050 s and particle velocity $v$ is approximately 28 cm/s.
Figure 2-21: Simple type IV event signature and corresponding source-wave displacement pulse and slip function. The displacement pulse width $\tau$ is approximately 0.070 s, maximum slip $u_{\text{max}}$ is 0.9 cm, rise time $T$ is 0.080 s and particle velocity $v$ is approximately 11 cm/s.
Figure 2-22: Compound type I event signature and corresponding source-wave displacement pulse and slip function. The displacement pulse width $\tau$ is approximately 0.055 s, maximum slip $u_{\text{max}}$ is 0.8 cm, rise time $T$ is 0.070 s and particle velocity $v$ is approximately 11.4 cm/s.
Figure 2-23: Compound type II event signature and corresponding source-wave displacement pulse and slip function. The displacement pulse width $\tau$ is approximately 0.060 s, maximum slip $u_{max}$ is 0.73 cm, rise time $T$ is 0.075 s and particle velocity $v$ is approximately 10 cm/s.
Figure 2-24: Compound type III event signature and corresponding source-wave displacement pulse and slip function. The displacement pulse width $\tau$ is approximately 0.090 s, maximum slip $u_{\text{max}}$ is 1.4 cm, rise time $T$ is 0.12 s and particle velocity $v$ is approximately 11.7 cm/s.
Figure 2-25: Compound type III-I event signature and corresponding source-wave displacement pulse and slip function. The displacement pulse width $\tau$ is approximately 0.06 s, maximum slip $u_{\text{max}}$ is 1.62 cm, rise time $T$ is 0.080 s and particle velocity $v$ is approximately 20 cm/s.
In the case of compound events, when the separation between signal components is small compared to the duration of the latter, the source-wave parameter and slip functions are as shown in Figures 2-22 - 2-25. However, as the separation increases, the slip function changes. An example is shown in Figure 2-26:

![Slip function of compound event](image)

Figure 2-26: Slip function of compound event of duration 200 ms. Maximum slip is 2.6 cm. Rise time must be estimated for individual signal components of the event.

The above slip function is probably associated with stick-slip motion. When the frictional resistance of the medium varies during a displacement offset, a dynamic instability may occur, resulting in abrupt slip followed by a period of no motion, as the stress in the medium builds up again. In this case, initial particle motion is followed by a 25 ms interval in which no increase in slip occurs. This is in turn followed by a new offset of the motion. When the separation between signal components of compound events is small this phenomenon cannot be observed. The integration procedure itself is partially responsible for this problem. Thus, it has been possible to estimate this type of slip function only
for compound III and IV events. In Chapter 5 where event sequences are discussed, it is shown that stick slip motion may occur at different scales, i.e., both compound events and sequences of events, the inter-arrival times of which are larger than the separation of signal components, may be induced by this type of motion.

A shear wave from a perfectly rectangular fault, appears in the far-field as a rectangular pulse. For more realistic fault geometries, the displacement signal for this wave must be approximately a uni-directional pulse with no marked oscillations. The integrated signals are in accord with this requirement. The pulse width is proportional to the fault duration and consequently the fault length. The latter can be estimated from the spectrum of the displacement signal, as it will be discussed in Chapter 3.

The distribution of particle velocity with signature type, for simple and compound events is shown in Figures 2-27 and 2-28, respectively:

![Distribution of particle velocity with signature type (simple)](image)

Figure 2-27: Distribution of particle velocity with signature type, for simple events.

The highest concentration of events is in the particle velocity range 2-30 cm/s. On
average, type IV events have the lowest particle velocities and types I and III are the predominant event types in the particle velocity range 30-67 cm/s. The samples of type I and type II events are too few to make any meaningful conclusions. Particle velocity for type III events is distributed in the entire range of this parameter. These results indicate that there is no clear correlation between signature type and particle velocity.

Figure 2-28: Distribution of particle velocity with signature type, for compound events.

The decrease in number of events with increasing particle velocity is significantly more abrupt than that of simple events. The highest concentration of events is in the particle velocity range 1-10 cm/s and no compound events have velocities larger than 50 cm/s. Beyond 30 cm/s there are only 5 compound type I and 4 compound type III events. Again, there is no clear correlation between signature type and particle velocity.

Figures 2-29 and 2-30 show the distribution of slip with simple and compound signatures, respectively. For compound events with slip functions as that shown in Figure 2-26, the maximum slip reached at the end of the entire displacement offset is recorded.
Figure 2-29: Distribution of slip with signature type, for simple events.

Figure 2-30: Distribution of slip with signature type, for compound events.
The highest concentration of simple events is in the slip range 0.1-1 cm. There is no single event type that characterizes this range. Although the largest number of events is of type IV signature, this is due to the fact that the latter is the predominant type in the entire event population. Also, there are only 5 events for which slip is above 3 cm. In the case of compound events, their highest concentration is also in the range 0.1-1 cm and there is an abrupt decrease in the number of events with increasing slip, as is the case for simple events.

The detailed statistics of source-wave pulse width, maximum slip, rise time and particle velocity for the analyzed event population are summarized in Table 2.6:

<table>
<thead>
<tr>
<th>Signature</th>
<th>Source-wave pulse width (s)</th>
<th>Max. slip (cm)</th>
<th>Rise time (s)</th>
<th>Particle vel. (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type I</td>
<td>0.015-0.020</td>
<td>0.1-1.3</td>
<td>0.010-0.020</td>
<td>10-65</td>
</tr>
<tr>
<td>Type II</td>
<td>0.030-0.040</td>
<td>0.06-1.5</td>
<td>0.020-0.04</td>
<td>3-37</td>
</tr>
<tr>
<td>Type III</td>
<td>0.040-0.060</td>
<td>0.2-4</td>
<td>0.035-0.06</td>
<td>6-67</td>
</tr>
<tr>
<td>Type IV</td>
<td>0.060-0.12</td>
<td>0.07-6</td>
<td>0.060-0.12</td>
<td>1.2-50</td>
</tr>
<tr>
<td>Compound</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type I</td>
<td>0.055-0.070</td>
<td>0.3-3.5</td>
<td>0.050-0.070</td>
<td>6-50</td>
</tr>
<tr>
<td>Type II</td>
<td>0.060-0.080</td>
<td>0.02-1.5</td>
<td>0.04-0.07</td>
<td>0.5-22</td>
</tr>
<tr>
<td>Type III</td>
<td>0.070-0.1</td>
<td>0.6-4</td>
<td>0.050-0.1</td>
<td>12-50</td>
</tr>
<tr>
<td>Type III-I/IV</td>
<td>0.060-0.14</td>
<td>0.01-5</td>
<td>0.050-0.13</td>
<td>0.2-36</td>
</tr>
</tbody>
</table>

Table 2.8: Variation of source-wave and particle slip parameters for the entire event population.

For earthquakes, maximum slip is between 30 and 400 cm, for corresponding fault lengths between 18 and 75 km and fault depths between 15 and 60 km. Particle velocity is on average 100 cm/s; this is a typical value for most earthquakes. Fractures in the ice that cause the acoustic events analyzed here have lengths in the range 2-100 m, as will be discussed in Chapter 3. Their maximum possible depth is equal to the ice thickness, here 4.2 m, three orders of magnitude lower than the shallowest earthquake. The maximum estimated slip is 6 cm and the maximum particle velocity is 67 cm/s. With the exception of type I and III events, both simple and compound, all others have particle velocities in the range 0.2-45 cm/s. In regard to slip, it appears uncorrelated with signature type. Note, however, the minimum slip values have been obtained for both simple and compound type
II and IV events.

Based on the above results, the most interesting observation is that although ice mechanisms have characteristic dimensions about three orders of magnitude lower than earthquakes, their particle velocity is on average less than one order of magnitude lower. In fact, the ice processes that induce some of the analyzed type I and III events, have particle velocities only 30% lower. It will be shown in Chapter 3 that there is a similar analogy between fracture propagation velocity in ice and in rock, during an earthquake.

In regard to rise time \( T \), for ordinary earthquakes it is in the range of 1-10 s, for tsunami earthquakes it is 100-1000 s and there are also extremely slow focal process for which \( T > 1000 \) s. Ice processes that generate type IV and compound events have the longest rise times, in the range 0.055-1.14 s. The mechanism that generates a type I event has the shortest rise time, between 0.015 and 0.020 s. Rise times for ice mechanisms are, therefore, at least two orders of magnitude lower than those of ordinary earthquakes. Estimates of particle slip for fracture-induced events are important since they are used to calculate seismic moment and consequently stress drop in the medium and some measure of the latter’s strength, e.g., the dimensionless strength parameter \( S \) (see Chapter 3). Rise time is a function of the elastic properties of the medium, including granularity, in which the slip offset occurs. Ice and rock have different material properties and this is one explanation for the difference in slip values. Also, particle slip is proportional to the friction drop, from an initial static value to a dynamic value, upon the displacement offset [36]. Friction in a material is a function of the area of inhomogeneities or asperities, which interact during particle motion of a discontinuity. The size of these asperities in rock are probably significantly larger than that of asperities in sea ice. Thus, the difference in friction drop in rock and ice may also account for the corresponding difference in slip.

### 2.7.1 Source time function models

Different source function models have been proposed to describe slip or stress change on a fault surface. Often a step function is assumed for this purpose which represents the stress drop or displacement offset of a fracture by an abrupt discontinuity at time \( t = 0 \). From a
physical point of view, this may not be an appropriate model since the abrupt change, i.e., 'he step in slip and the resulting infinite gradient, implies that singularities occur in other source parameters. Also, in regard to the acoustic pressure signal, since it is proportional to the third derivative of slip a perfect step function would lead to a white spectrum for pressure. Therefore, the slip time function must be modified so that the initial gradient is finite. Brune [3] and Haskell [15], among others, have proposed two alternative source time function models to address this issue. Several modified models have also been developed [32]. Here I will discuss only the Haskell and Brune models and will state modifications that have been made [32]. The fault is represented as a moving dislocation in both models.

It has been shown [10] that the Gaussian pulse is a better model for source-wave displacement corresponding to a type I event. I will, therefore, discuss this model and also show that for more complicated types of events the Rayleigh function represents more accurately the estimated displacement signals.

Haskell model

A rectangular fault, propagating with uniform velocity is assumed, as shown in Figure 2-31.

![Figure 2-31: Rectangular fault moving with velocity $v_r$. $L$ and $D$ are the length and depth of the fault, respectively. The fault propagates in the plane $z = 0$.](image-url)
It has been previously mentioned that the roughest approximation to a slip time function is the step function, of the form

\[ u(t) = u_\infty H(t - \frac{x}{v}) \]  (2.21)

where \( u_\infty \) denotes the final displacement (or permanent slip) of the propagating discontinuity and \( v \) is the rupture speed of the fault. \( H(t) \) is the Heaviside unit step function defined as

\[ H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \]  (2.22)

The Haskell model assumes that particle displacement can be described by a ramp function. Thus, \( u(t) \) is given by

\[ u(t) = u_\infty G(t - \frac{x}{v}) \]  (2.23)

where \( G(\cdot) \) is of the form shown in Figure 2-32:

![Ramp function](image)

Figure 2-32: Ramp function used in the Haskell model to represent slip at the rupture front. \( T \) is the rise time and \( u_{\text{max}} \) the permanent slip.

Slip motion is represented by this model as a uniform process that occurs progressively along the fault. The model assumes that slip occurs simultaneously across the fault depth
D. The far-field displacement of the shear wave is therefore given by

\[ u(t) = [G(t - \frac{r_0}{c}) + G(t - \frac{r_0}{c} - T_0)] \cdot R(\phi, \theta, r) \cdot (\mu LDu_\infty T) \] (2.24)

where \( R(\phi, \theta, r) \) is the radiation pattern (in the spherical coordinate system), \( \mu \) the shear rigidity of the medium (approximately \( 3 \times 10^9 \) Pa for ice), \( L, D \) the fault dimensions and \( T_0 \) the shear wave pulse width, given by

\[ T_0 = L \frac{\xi - \cos(\Phi)}{c} \] (2.25)

where \( c \) is the sound speed and \( \Phi = \phi_i - \phi_s \), the angle between the direction of fracture propagation and the direction of receiver \( i \). The term \( \frac{L}{u} \) is the travel time to the receiver from the point of initiation of the fracture and \( \frac{L}{c} \cos(\Phi) \) the travel time from the point of termination. Note that Equation 2.24 assumes a unilateral fault, which propagates only in one direction. It can readily be generalized for a bilateral fault, i.e., one that propagates in two opposite directions [32]. The Fourier transform of the displacement, from which the fault dimensions can be estimated is consequently given by

\[ U(\omega) = R(\phi, \theta, r) \mu Au_\infty \omega \hat{G} F_1(\omega, T_0) \] (2.26)

where \( A \) is the fault area, i.e., \( A = LD \), \( \hat{G} \) is the Fourier transform of the ramp function, \( r \) is the distance between source and receiver, and \( F_1 \) is given by

\[ F_1 = \frac{\sin(\omega T/2)}{\omega T/2} \] (2.27)

The advantage of the Haskell model is that it takes into account the effects of fracture propagation and is thus appropriate for problems where these effects are of concern. One of its disadvantages is that the ramp source function is chosen in an arbitrary manner. A second disadvantage which is, however, common to several source time function models, is that it assumes that the initial far-field displacement is characterized by a linear increase in time. This may not be an accurate representation of the physics of fault initiation. Savage
proposed an alternative model [32] which assumes initial quadratic time dependence of particle slip.

**Brune model**

To describe the motion of a propagating dislocation, Brune used an exponential source time function (assuming a circular fault):

\[ u(t) = \frac{\sigma}{\mu} \beta \tau (1 - e^{-\frac{t}{\tau}}) \]  \hspace{1cm} (2.28)

where \( \sigma \) is the effective stress, \( \beta \) the shear-wave speed (\( \approx 1800 \) m/s for ice), \( \mu \) the shear rigidity and \( \tau \) a time constant, \( \mathcal{O}(\frac{R}{\beta}) \), i.e the travel time of the displacement signal. The model does not take into account the effects of fracture propagation, explicitly. The source time function is of the form shown in Figure 2-33:

![Figure 2-33: Source time function used in the Brune model to represent slip at the rupture front. \( T \) is the rise time and \( u_{\text{max}} \) the permanent slip.](image)

The derivation of Equation 2.28 can be understood by considering a tangential stress step applied to the interior of the fault surface [3]. This causes the fracture to move in both directions. A shear stress wave is generated, which propagates at a direction perpendicular to the fault surface. The time function for this wave is, therefore, given by
\[ \sigma(r, t) = \sigma H(t - \frac{r}{\beta}) \]  

(2.29)

where \( H \) is the Heaviside step function, defined in Equation 2.22, \( \beta \) the shear wave speed, \( \sigma \) the effective shear stress and \( r = (x, 0, 0) \), for a fault propagating in the x-direction. The shear stress \( \sigma \) is related to the tangential displacement by \( \sigma = \frac{\partial u}{\partial x} \). By integrating Equation 2.29, with respect to \( x \), the following expression is obtained for \( u \):

\[ u(t) = \begin{cases} 
0 & t < 0 \\
\left(\frac{\sigma}{\mu}\right)\beta t & t \geq 0 
\end{cases} \]  

(2.30)

As the effects of the edges of the fault surface become felt at the observation point, the slip particle velocity will drop and approach zero as slip reaches its maximum value. This effect can be included in the source time function by introducing the term \( \tau (1 - e^{-\frac{t}{\tau}}) \) in Equation 2.30. The displacement is now given by Equation 2.28, and particle velocity \( \dot{u} \) by

\[ \dot{u}(x = 0, t) = \left(\frac{\sigma}{\mu}\right)\beta e^{-\frac{t}{\tau}} \]  

(2.31)

As \( t \to \infty, \dot{u} \to 0 \), as required. The corresponding spectrum for this model is given by

\[ U(\omega) = \left(\frac{\sigma\beta}{\mu}\right) \frac{1}{\omega(\omega^2 + \tau^{-2})^{\frac{1}{2}}} \]  

(2.32)

The advantage of the Brune model is that it uses an exponential source time function which appropriately describes particle velocity and in general the temporal behavior of slip in the vicinity of the rupture front. The main disadvantage of the model is that it does not explicitly account for fracture propagation. Therefore, it is appropriate for near-field problems, where the mechanisms of rupture initiation or slip offset are important, whereas the Haskell model is appropriate for far-field problems where rupture propagation is important. Note that in both models, the predicted displacement spectrum has a distinct trend proportional to \( \omega^{-1} \), in the mid-frequency regime, i.e., between the two characteristic corner frequencies, \( \omega_1 \) and \( \omega_2 \), which are related to the dimensions of the fault (length \( L \) and depth \( D \)). The cause of this trend is different in the two models. In the Haskell model, it occurs only when \( \omega_1 \)
and $\omega_2$ differ significantly, i.e., for a long and narrow fault (typically strike slip), for which $\frac{D}{L} \ll 1$. In the Brune model, the $\omega^{-1}$ trend is due to the premature arrest of the fault [3]; only part of the stress is released as the fault slips.

**Gaussian and Rayleigh functions**

To describe the characteristics of the type I signal, Dyer [10] modeled the source-wave pulse $u_s(t)$ by a Gaussian pulse, of the form

$$u_s(t) = Ae^{\frac{(t-\eta)^2}{2\sigma^2}}$$

(2.33)

where $A$ is a function of the maximum slip and the standard deviation $\sigma$ of the duration of the displacement signal, and $\eta$ is the latter's mean. The resulting function for slip displacement, i.e., the Gaussian cumulative distribution, is in good agreement with the estimated slip function for the type I event, as shown in Figure 2-34:

![Slip function for type I simple event](image)

Figure 2-34: Comparison between the estimated slip function and those predicted by the Haskell, Brune and Gaussian function models. The latter best fits the type I slip function.
In the above plot, the estimated function and the Haskeli, Brune and Gaussian functions are superimposed. Both the Haskell and the Brune models fail to describe the non-linear initial increase in particle velocity for this type of event. This is a characteristic trend of the slip functions of all simple event types. As previously mentioned, the assumption that slip linearly increases from its offset, made by both these models, is indeed one of their disadvantages. In contrast, the Gaussian function adequately describes this characteristic of particle motion. Stein [38] also assessed the validity of this model and computed synthetic event traces and their integral functions by superimposing two Gaussian volume velocities. He showed that this may be a plausible model for events more complicated than type I. To the best of my knowledge, no other such analysis has been done in the context of the study of ice-induced acoustic events, for different signature types.

Slip functions for the four types of simple events are best modeled by a Gaussian function. However, for compound and type IV events, the corresponding source-wave pulses are not characterized by the symmetry of the Gaussian pulse. A different model may thus be more appropriate. I have chosen the Rayleigh function, which is of the form

$$f(t) = 2\alpha te^{-\alpha t} \quad 0 \leq t, \quad 0 < \alpha$$

(2.34)

where $\alpha$ is a constant. The first integral of this function can be computed analytically:

$$\int f(t)dt = 1 - e^{-\alpha t^2}$$

(2.35)

Note that a superposition of twice differentiated Rayleigh pulses is needed to reconstruct the corresponding model pressure signal. The choice of the Rayleigh function is not unique. For example, the Maxwell function may also be appropriate, or even a superposition of other well-known functions. In choosing a model source-wave pulse in a meaningful manner, one must examine not only the shape of the corresponding slip function (first integral), but also the agreement between corresponding pressure signal (second derivative) and the recorded signal.

In Figure 2-35, the estimated slip functions, the Gaussian function and the integrated Rayleigh function are superimposed, for a compound III-I event:
Figure 2-35: Comparison between the estimated slip function for a type III-I compound event, and those predicted by the Gaussian and Rayleigh models.

Typically, for compound events particle velocity increases more slowly after the slip offset than for simple events. Thus, the slope of the slip function is less steep than that of the Gaussian function. The Haskell and Brune models are also inadequate since they both assume rapid offset of particle motion. The Rayleigh function best describes particle motion of the mechanisms that generate compound events.

2.8 Summary and Observations

In this chapter, I have presented the analysis of acoustic events in the time domain. A population of 196 events has been detected in ambient noise, during one hour of data collection. Events are located in an area approximately 1.5 km$^2$, around the hydrophone array and they are clustered both in time and space. Individual clusters of events are analyzed in Chapter 5, so that both their temporal and frequency characteristics can be discussed simultaneously.
Simple and compound events have been distinguished, based on the number of their signal components. In addition, four basic signature types have been identified: a pop or burst (type I), a complex pop (type II), a damped sinusoid (type III) and a multi-pulse signal (type IV). 41\% of the total number of events in either of the two major categories have signatures of the latter type. Also, in the signature of 20\% of the events, the arrivals of both the acoustic and the longitudinal waves have been detected. The longitudinal wave signal is 'weak' in comparison to the acoustic wave signal, usually 7-10 dB lower and 3-6 dB higher than the noise level. The duration of simple events of the first three types is in the same range, namely 0.020-0.065 s, and that of type IV is longer, between 0.08 and 0.12 s, and thus comparable to the duration of compound events. For the latter, the separation between signal components is in the same range, between 0.015 and 0.040 s. In Chapter 5, I discuss how this parameter may be related to the temporal evolution of the mechanisms in the ice that cause these types of events. The relation between signature type and ice thickness has also been investigated but no correlation between the two variables has been found. Based on these results, the following issues need to be addressed: first, if events are induced by processes such as fractures, one would expect that they would be predominantly compound, given that faults are formed at different stages, and their propagation is characterized by stick-slip motion. However, the time scales associated with the latter phenomenon and with the different fault stages are unknown. It is possible that the inter-arrival times between stages may be longer than the separation of individual components of compound events. Therefore, temporal sequences of events may instead be associated with the progressive formation of faults. This issue is discussed in Chapter 5. Also, the occurrence, of simple events, possibly indicates the occurrence of mechanisms, such as cracks, which are of smaller scale compared to large faults. The pre-cursors and post-cursors of these mechanisms may be undetectable in ambient noise.

In order to gain insight on the relation between different types of events and their generating mechanisms, I have also estimated source-wave and slip displacement functions corresponding to the four basic event signatures and four of the five compound signatures. The characteristics of the event generating mechanisms have been briefly described, in order to support the choice of fault model assumed in the estimation. From the slip functions,
two of the five characteristic parameters that describe a fault, namely maximum particle
slip and rise time, have been estimated, as well as particle velocity. It is found that the ice
process associated with simple type I and III events, simple or compound, has the highest
particle velocity up to 67 cm/s. In comparison to earthquakes, although maximum slip of
ice processes is on average about 2-3 orders of magnitude lower, particle velocity is less
than one order of magnitude lower and its maximum estimated value is only 30% lower
than the average value of a typical earthquake. The lower bound of particle velocity is on
average 5 cm/s. In regard to permanent slip, the maximum estimated value is 6 cm and the
minimum is 0.01 cm.

Mechanisms that generate type IV and compound events have the longest rise times,
in the range 0.060-0.14 s. Note that these are average times, over the total duration of the
corresponding acoustic events. Perhaps there is a more appropriate procedure for estimating
rise time for compound events, particularly if their individual signal components are
associated with some type of stick-slip motion. For example, rise time could be calculated
by integrating the individual signal components separately. However, for compound events
of long duration (0.1-0.2 sec) it has been possible to estimate slip functions, the shape of
which indicate the occurrence of stick slip motion.

Finally, particle slip models and their adequacy in representing the estimated slip func-
tions have been discussed. It has been shown that neither the Brune nor the Haskell models,
commonly used in seismology to describe propagating discontinuities, are suitable. Both
assume that particle velocity increases rapidly at the slip offset. In contrast, all measured
curves have a characteristic trend, namely a slow, non-linear initial increase in slip. The
Gaussian function, proposed by Dyer as a better model to describe slip of the ice mecha-
nism that generates as type I event, has this particular initial trend. By superimposing the
measured and theoretical curves I have concluded that this is an appropriate model of slip
for at least the first three types of simple events. For compound events, the source-wave
displacement pulse is asymmetric and the initial slip increase is also non-linear. I have se-
lected the Rayleigh function and its integral to model the source-wave displacement pulses
and corresponding slip functions for compound events and have shown that it describes the
data more accurately than the Gaussian model.
Having completed the time domain analysis of acoustic events and the estimation of source parameters at the level of particle motion, I proceed with the event frequency domain analysis. The latter also includes the estimation of source parameters at the level of fault motion, namely propagation speed and orientation, intrinsic frequency and characteristic dimensions.
Chapter 3

Frequency domain analysis and source parameter estimation

3.1 Overview

The parameters that characterize particle motion of the event generating mechanisms have been estimated in Chapter 2, through the analysis of acoustic events in the time domain. In this chapter, parameters that characterize fault propagation in the ice are estimated. For this purpose, the analysis of acoustic events in the frequency domain is convenient. In particular, the peak event frequency, bandwidth and Doppler shift, estimated from the event spectrum, are needed in the calculation of source speed, direction of propagation and intrinsic source frequency. The characteristic dimensions (length and depth) of the event-inducing faults, or the range of deformation in the case of ice mechanisms which are unrelated to fracture, are determined from the spectrum of the estimated source-wave displacement pulse. Typically, two corner frequencies are identified in this spectrum which are directly related to the fault geometry. The measure of dimension does not solely refer to length and depth. In the case of fracture, the latter are indeed the characteristic dimensions of the process. In Arctic ice, however, there are also other event generating mechanisms, such as unloading motion of ice floes. For these mechanisms, dimension refers to a range of deformation. Consequently, the above-mentioned estimation procedure may not be appropriate. This issue is further discussed in Chapter 4.

Once the source parameters have been estimated, the event radiation
patterns are finally required in order to identify the type of ice mechanism, e.g., fracture or other motion-related process.

The procedure for analyzing acoustic events in the frequency domain is summarized in the following diagram:

![Diagram](image)

Figure 3-1: Stages of the procedure for estimating acoustic event and fault parameters.

The analysis is performed channel by channel. The frequency range of analysis is between 10 and 350 Hz. The low cutoff frequency has been established based on the spectral characteristics of cable strum. As previously mentioned, the receivers are suspended 60 m below the ice cover and are moving due to hydrodynamic forces. For each receiver, a peak in the ambient noise spectrum is present in the frequency range between 1 and 10 Hz, though not necessarily at all times, and is attributed to strum. In the frequency domain analysis of events, this portion of the spectrum is discarded. The upper cutoff frequency is
that of the filter of the data acquisition system. In general, the frequency domain of ocean ambient noise in the Arctic is between 3 Hz and 100 kHz; it can be divided into low-, mid- and high-frequency regimes. The high-frequency regime is between 3 and 100 kHz and thus beyond the range of this analysis. The frequency content of the detected events is in the low- and part of the mid-frequency regimes, which are 3-100 Hz and 100-3000 Hz, respectively [11]. According to the intrinsic source frequency estimated from Doppler shift and source speed, or the observed peak event frequency, 41% and 59% of the event population, are low-frequency and mid-frequency events, respectively, i.e., in the ranges 10-100 Hz and 100-350 Hz. In regard to the events’ frequency range, two parameters are important to this study, namely bandwidth and peak frequency. They are estimated in the first stage of the analysis, from the source level spectrum of each event.

Once the event spectrum has been estimated at each hydrophone, cross-correlation of the spectra is performed in the second stage of the analysis. Ice mechanisms, fractures in particular, can be thought of as moving sources. Depending on receiver and source locations, source motion is reflected on the event spectrum at different hydrophones. A Doppler shift is observed, from which source speed, orientation and the actual frequency of the source can be estimated. When the distance between hydrophones is small in comparison to their distance from the source, this shift may not be measurable. For this event population though, it has been possible to measure Doppler shifts, ranging from 1 to 86 Hz. In previous studies of event physics, the Rayleigh wave speed, approximately 1700 m/s for sea ice, has been assumed as the source speed, particularly in cases when fracture is the most plausible event generating mechanism. This is definitely not a unique value; even as an upper limit of fracture speed it is not unique, as argued by several seismologists [36] [2]. The actual fracture propagating speed may vary significantly. Indeed, it is shown in this chapter that fracture speed for 76% of the total number of events is significantly lower, in the range 100-1000 m/s. There are 33 events, predominantly type I and III, for which the estimated fracture speed is between 1000 and 1698 m/s. Also, lower than 100 m/s speed estimates have been obtained for 14 events, the majority of which have type IV signatures. There are ice processes, such as floe unloading, which have been previously suggested as plausible event generating mechanisms [6], that do not involve source motion. Therefore, the low
source speeds may be due to the occurrence of such a physical mechanism. Alternatively, they may be attributed to inaccurate measurements of Doppler shift, in cases where the distance between the event location and the receivers is large, or to the inability to capture the motion of the source through this particular type of estimation procedure.

In the last stage of the frequency domain analysis, the dimensions of the event generating mechanisms are estimated from the characteristic corner frequencies of the source-wave displacement spectra, and the corresponding propagation speeds. The $L$- and $W$- models are subsequently discussed in an attempt to distinguish between length-controlled and depth-controlled event mechanisms. Although events described by either model have been identified, for approximately 40% of the total number of events neither of the two models appears adequate.

### 3.2 Event spectrum estimation

Standard Fast Fourier Transformation (FFT) of the event signal is used to obtain its spectrum. Corrections are made to account for transmission loss, sensitivity of the hydrophones (155 dB re 1 Volt/1 $\mu$Pa) and for the pre-gain setting of the data acquisition (20 dB). The analysis time window used is in the range 0.050-0.2 s, according to the event duration, and is centered at the time of occurrence of the peak event amplitude.

Spreading of sound pressure in a refractive medium is assumed in all calculations, instead of spherical spreading. Under the assumption of spherical spreading, the transmission loss is $H_i = 20 \log_{10} R_i$ in dB re 1 m, where $R_i$ is the distance between the source and receiver $i$. The spreading function for a refractive medium is calculated as follows. Consider two sound rays traveling in the medium, as shown in Figure 3-2. According to ray theory, acoustic energy does not cross the rays and is thus conserved between them. Transmission loss depends on the distance between source and receiver, the separation between two adjacent (closely-spaced) rays and the change in sound speed with depth. Values for sound speed at the source and receiver depths are available from the measured sound speed profile, discussed in Chapter 2 and Appendix C.
The respective transmission equations (in terms of sound pressure) for non-refractive and refractive media are:

\[ p^2 = \frac{p_{ref}^2 R_{ref}^2}{R^2} \]  \hspace{1cm} (3.1)

and

\[ p^2 = \frac{p_{ref}^2 R_{ref}^2 \Delta \theta \rho_0 c_0 |\cos \theta_0|}{X^2 h \rho_1 c_1 |\cos \theta_1|} \]  \hspace{1cm} (3.2)

where \( R_{ref} \) is the reference distance, here 1 m, \( p_{ref} \) is the reference pressure, here 1 \( \mu \)Pa, \( c_0, \rho_0 \) are the sound speed and density at the source depth, \( c_1, \rho_1 \) are the corresponding values at the receiver depth, and \( h = \Delta X \tan \theta_1 \) is the vertical separation between rays, with \( X \) the horizontal distance between source and receiver.

The following expression is used to calculate the transmission loss \( H_{i, ref} \) in dB re 1 m
\[ H_{i,refr} = 10 \log_{10}(R_i) + 10 \log_{10}\left(\frac{h_i}{\Delta \theta}\right) - 10 \log_{10}\left(\frac{\rho_0 c_0}{\rho_1 c_1, i}\right) - 10 \log_{10}\left(\frac{\cos \theta_{0,i}}{\cos \theta_{1,i}}\right) \quad (3.3) \]

Note that the above expression assumes the angles \( \theta_0 \) and \( \theta_1 \) to be from the horizontal axis. In sea water, \( \rho_0 \simeq \rho_1 \) is a good approximation. From the measured sound speed profile, \( c_0 = 1432 \text{ m/s} \) and \( c_1 = 1440 \text{ m/s} \). Therefore, the term \( 10 \log_{10}\left(\frac{\rho_0 c_0}{\rho_1 c_1, i}\right) \) in Equation 3.3 is negligible (\( \simeq 0.02 \text{ dB} \)).

Figure 3-3 shows the difference in transmission loss under the assumption of spherical spreading and refractive spreading, respectively:

![Graph showing transmission loss versus range](image)

Figure 3-3: Transmission loss versus range. Solid line: refractive spreading, dashed line: spherical spreading. The maximum difference of the two transmission loss values is \( \simeq 7 \) dB at a distance 1.0 km. The events for which their horizontal radiation pattern can be estimated, i.e., for which the azimuth range of observation is greater than 20 – 30°, are at distances at most 700 m from the hydrophone array. At this range, the difference in transmission loss, under the two spreading assumptions is 1.3 dB.
The bandwidth of an event is defined as the frequency range within the half-power (or -3 dB) points, at which the peak pressure is reduced by a factor $\frac{1}{\sqrt{2}}$. A second parameter that needs to be estimated from the event spectrum is the frequency range at which the ratio of signal-to-noise spectral levels is above a particular threshold (I use 5 dB in this study as a conservative threshold). An estimate of this frequency range is necessary to determine which part(s) of the event spectrum will be used in the cross-correlation procedure used to estimate Doppler shift. It is very important, particularly in the case of compound events, the spectrum of which is characterized by multiple peaks, separated by segments of the spectrum in which the ratio of the signal-to-noise levels is less than 5 dB (typically 1-3 dB). This ratio can be estimated by superimposing the event spectrum and the ambient noise spectrum prior to the event.

Both narrowband and broadband events have been detected, though only very few are broadband. According to my definition, narrowband events have bandwidths of about one octave or less and their peak frequency, which is approximately equal to or larger than their bandwidth, is clearly seen in their spectrum. Perhaps a more appropriate term for such events is 'narrow-width', to distinguish them from the narrowband events with bandwidth approximately $\frac{1}{10}$ to $\frac{1}{4}$ of an octave, detected in data from previous experiments in pack ice [40] [11]. The minimum bandwidth of events in the selected data is approximately $\frac{1}{3}$ of an octave. The characteristics of narrow-width events can be studied as a function of peak frequency. The latter is defined as the frequency component of the major energy lobe of the event.

The peak frequency of broadband events is smaller than their bandwidth, which may span over several octaves. Simple events may fall in either category, although only type VI events were found to be broadband. In contrast, 70% of compound events are broadband. For the latter, the frequency range of highest spectral energy is considered in the calculations of source parameters, such as Doppler shift and consequently source speed and orientation.

The spectrum of a typical narrow-width event in shown in Figure 3-4. The event and noise spectra are superimposed.
Figure 3-4: Spectrum of narrow-width event in channel 4. The peak frequency of the event is 32 Hz and its bandwidth is approximately 10 Hz. The maximum ratio of signal-to-noise levels is about 17 dB.

The occurrence of several distinct spectral peaks of comparable amplitude level in an event spectrum, indicates that there is more than one dominant energy lobe for the event, and consequently more than one scale associated with the generating process in the ice. One of the goals of this study is to understand this phenomenon and possibly relate its occurrence to the characteristics of the event generating physical processes. In particular, two hypotheses are considered. The first is related to the formation of a fault at different stages. It is well known that in the case of earthquakes, the main shock is accompanied
by a pre-cursor and a post-cursor. The frequency content and propagation speed of these processes may differ significantly from those of the main shock. The occurrence of such a phenomenon is possible in the case of faults in the ice but its detection depends on a number of parameters. The first is the inter-arrival time between the occurrence of individual stages in the fault formation. If it is larger than the temporal separation between signal components of compound events, which is probably the case, then the multiple peaks in corresponding event spectra must be attributed to some other phenomenon.

I do not know what are typical values for the inter-arrival time between individual seismic stages for faults in the ice, and I believe it is incorrect to use earthquakes as a guide. The duration of seismic signals and that of acoustic events are incomparable. For example, in the case of the Macquarie Ridge earthquake, where a slow pre-cursor was detected, the main shock was approximately 20 s long and its separation from the pre-cursor was about 100 s [17]. However, there is no scaling law for the inter-arrival time that may be applicable in this study. The maximum duration of a compound acoustic event detected in these data is approximately 0.2 s and the separation between signal components is about 0.030 s. Therefore, the separation between a pre-cursor to an earthquake and a pre-cursor to an ice fracture are probably incomparable, too. The second parameter of concern is the strength of secondary processes and the ability to detect them in ambient noise. It may also be possible that the occurrence of compound events is associated with the motion characteristics of the main physical process, e.g., stick slip, as indicated by some of the estimated slip functions. This is the second hypothesis considered in the study and is further discussed in Chapter 4, in the context of the proposed ice mechanisms. To address the issue of fault formation at different stages, event sequences and their corresponding frequency characteristics are also analyzed, in Chapter 5. The separation between individual stages may be of the order of event inter-arrival times in a sequence or temporal cluster.

The spectrum of a compound event, is shown in Figure 3-5:
Figure 3-5: Compound event spectrum in channel 7. In the frequency ranges 12-32 Hz and 90-350 Hz the ratio of the signal-to-noise levels is above 5 dB.

The low-frequency range of the event, between 12 and 32 Hz, is about one octave wide. The high-frequency range between 90 and 350 Hz, in this particular spectrum, and on average between 100 and 300 Hz for all hydrophones, is about 1.5 octaves wide. In general, I observed that these well-separated event frequency bands are each about one octave wide. It may be difficult to distinguish the frequency ranges in which the ratio of signal-to-noise levels is 5 dB or higher and thus, the spectrum can be averaged in equivalent bands, e.g., 1/3-octaves to facilitate the process, as shown in Figure 3-6:
Figure 3-6: Compound event and noise spectra in channel 7, averaged in 1/3-octave bands. Spectral peaks at 22, 100 and 330 Hz are clearly seen. The difference at the two frequency ranges in pressure level is about 9 dB; adding $10 \log_{10}(\text{bandwidth})$, a measure of their respective energies, corresponds to 19 and 32 dB, respectively.

The frequency range of analysis is divided in 5 octave bands, namely 11-22 Hz, 22-44 Hz, 44-88 Hz, 88-175 Hz, 175-350 Hz. The distribution of events with bandwidth, measured in octave bands, is shown in Table 3.1, irrespective of signature type and number of signal components. All simple events are narrow-width (or narrowband by my definition).
<table>
<thead>
<tr>
<th>Bandwidth (octaves)</th>
<th># of Events</th>
<th>% of total # of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 or &lt; 1</td>
<td>187</td>
<td>95</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.1: Distribution of event population with bandwidth.

The statistics of low- and mid-frequency events, i.e., with peak frequencies below and above 100 Hz, respectively, are summarized in Table 3.2. For simple events, peak frequency is directly measured from their spectrum, at each hydrophone and then averaged over all hydrophones. For compound events, it is assumed to be the frequency of the highest spectral energy (proportional to one-half the product of the squared peak pressure and the bandwidth). As previously mentioned, the characteristics of these events cannot be studied as a function of this peak frequency, only.

<table>
<thead>
<tr>
<th>Signature Type</th>
<th>No. of Events</th>
<th>No. of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_{\text{peak}} &lt; 100 \text{ Hz} )</td>
<td>( f_{\text{peak}} &gt; 100 \text{ Hz} )</td>
</tr>
<tr>
<td>I</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>II</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>III</td>
<td>13</td>
<td>36</td>
</tr>
<tr>
<td>IV</td>
<td>18</td>
<td>25</td>
</tr>
<tr>
<td>Compound I</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Compound II</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Compound III</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>Compound IV</td>
<td>15</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 3.2: Statistics of low- and mid-frequency events.

There are 81 low-frequency and 115 mid-frequency events, respectively. Although type I and II events, both simple and compound, are almost equally distributed in the two ranges, simple and compound type III and IV events occur predominantly in the mid-frequency range. For compound events, signature type and peak frequency appear uncorrelated. The detailed distribution of events with peak frequency is shown in Figure 3-7, for the entire population and in Figures 3-8 and 3-9 for individual signature types.
Figure 3-7: Distribution of the entire event population with peak frequency, irrespective of signature type. The data are best fitted by the lognormal distribution function (superimposed function).

The highest concentration of low- and mid-frequency events occurs between 30 and 90 Hz and 100 to 200 Hz, respectively. As frequency increases, the number of events decreases. The distribution function that approximately fits the data is the lognormal distribution of the form

$$F(f) = \frac{1}{f} e^{-\ln^2(f)}$$

(3.4)

where \( f \) is the peak event frequency. However, as frequency increases, the decrease in the number of events is not as rapid as that predicted by this distribution function.

It may be more meaningful to examine the distribution of peak frequency with individual event signature types, instead of the entire population. The variation of peak event frequency for simple signatures is shown in Figure 3-8; the statistics are summarized in Table 3.3.
Figure 3-8: Distribution of peak event frequency for simple event types.

<table>
<thead>
<tr>
<th>Signature</th>
<th>Low-Frequency Events</th>
<th>Mid-Frequency Events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$ (Hz)</td>
<td>$\sigma$ (Hz)</td>
</tr>
<tr>
<td>Type I</td>
<td>52</td>
<td>25</td>
</tr>
<tr>
<td>Type II</td>
<td>50</td>
<td>16</td>
</tr>
<tr>
<td>Type III</td>
<td>63</td>
<td>13</td>
</tr>
<tr>
<td>Type IV</td>
<td>73</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 3.3: Statistics of simple event low- and mid-frequencies. $\mu$ is the mean frequency and $\sigma$ is the standard deviation.

Notice that there are no type I and II events at frequencies above 200 Hz. Similarly,
there are no type III and IV events at frequencies below 50 Hz. These are the only clear observations in regard to the correlation between signature type and peak event frequency. The distribution of type III and IV events can be fitted approximately by the lognormal distribution function in Equation 3.4. In contrast, the corresponding distribution of type I and II events cannot be fitted by any of the known statistical functions.

The variation of peak event frequency for compound event signature types is shown in Figure 3-9 and the statistics are summarized in Table 3.4.

Figure 3-9: Distribution of peak event frequency for compound event types.
<table>
<thead>
<tr>
<th>Signature Type</th>
<th>Low-Frequency Events</th>
<th>Mid-Frequency Events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$ (Hz)</td>
<td>$\sigma$ (Hz)</td>
</tr>
<tr>
<td>I</td>
<td>56</td>
<td>14</td>
</tr>
<tr>
<td>II</td>
<td>55</td>
<td>19</td>
</tr>
<tr>
<td>III</td>
<td>61</td>
<td>29</td>
</tr>
<tr>
<td>IV</td>
<td>63</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 3.4: Statistics of compound event low- and mid-frequencies. $\mu$ is the mean frequency and $\sigma$ is the standard deviation.

There are no type I and II events at frequencies above 200 Hz and no compound events in general, below 30 Hz. The distribution of compound events with peak event frequency cannot be fitted by any of the known distribution functions.

3.2.1 Distribution of particle slip with peak event frequency

In Chapter 2, the maximum particle slip for each event has been estimated. A previously mentioned, slip is a meaningful source parameter for events associated with fracture processes. Since it is the permanent slip corresponding to the rise time of the event generating mechanism, I refer to it as the mean slip $\bar{\mu}$ over the duration of the event. The variation of this parameter as a function of peak event frequency is investigated.

For events attributed to fracture, Chen, [6] estimated mean slip $\bar{\mu}$ from the static seismic moment $M_0$, using the relation $M_0 = \mu \bar{\mu} A$, where $\mu$ is the rigidity of the medium ($3 \times 10^9$ for sea ice) and $A$ the fault area. She calculated $M_0$ from the event source strength and obtained an empirical formula for the frequency dependence of mean slip:

$$\bar{\mu} \propto f^{-1.5}$$

(3.5)

The constant of proportionality that she used is $3.4 \times 10^{-4}$. Mean slip versus peak frequency for narrow-width events is shown in Figure 3-10:
The empirical function does not fit the data. The spread of the data is such that no relation between mean slip and frequency can be determined. This result is not surprising. In the case of earthquakes, it has been observed [20] that integration of both short period \( T = 1-10 \) s and moderate period \( T = 20 \) s seismic waves results in rise time and slip estimates of roughly the same magnitude. The amplitudes of these waves are, however, different. Chen's estimates of mean slip are of the order of \( 10^{-8} \) m, significantly lower than mine, which are \( O(10^{-4}) - O(10^{-1}) \) m. My estimates agree better with those obtained by Kim [21].

3.3 Estimation of Doppler shift

Event generating processes in the ice, such as fractures, can be treated as moving sources. When an event source is moving relative to a hydrophone which is assumed to be at rest,
the received frequency is not equal to the intrinsic source frequency; it is shifted according to the velocity and orientation of the source.

3.3.1 Galilean Transformation

Consider two coordinate systems, one fixed and the other moving at constant speed relative to the first. A transformation from the first to the second system is called a Galilean transformation. A Doppler shift results from this change in coordinates, since frequency is not Galilean invariant [31]. Let the moving source and the receivers be in coordinate systems 1 and 2, respectively. Then, the position of receiver $i$ in coordinate system 2 is

$$x_{i2}(x_{i1}, t) = x_{i2} + (t - t_0)\vec{V} \tag{3.6}$$

where $\vec{V}$ is the velocity of coordinate system 1 with respect to 2, and $t_0$ is the time of acoustic emission. The received frequency $\omega_i$ is related to the intrinsic frequency of the source $\omega_0$ by

$$\omega_i = \omega_0 + \vec{V} \cdot \frac{\vec{R}_i}{c} \quad \iff \quad \omega_i = \frac{\omega_0 c}{c - \vec{V} \cdot \vec{R}} \quad i = 1, 2, \ldots, 24 \tag{3.7}$$

where $c$ is the sound speed and $\vec{R}_i = \frac{\vec{R}}{|\vec{R}|}$ the unit vector in the direction of the receiver. $\vec{R}$ is the vector from the point at which the sound ray was emitted into the water (here the source location) to the reception point $(x_i, y_i, z = 60)$ and $|\vec{R}|$ is the distance between source and receiver. The difference $\Delta \omega_i = \omega_i - \omega_0$ is the so-called Doppler shift. The source frequency $\omega_0$ is not known and thus the true Doppler shift $\Delta \omega_i$ can only be estimated. The peak event frequency at each hydrophone is known and so is the relative Doppler shift $\Delta \omega_{i,j} = \omega_i - \omega_j$, between any two received peak frequencies $i$ and $j$. It is reflected on the event spectrum at each hydrophone and can be determined through spectrum cross-correlation. The Doppler effect is shown in Figure 3-11, where the event spectra at two channels are superimposed:
Figure 3-11: Superimposed event spectra at hydrophones 2 and 21. Their refractive distance from the event is 530 and 672 m, respectively. The distance between hydrophones is 244 m.

Note that for the superimposed spectra, the difference in pressure levels at the two hydrophones (about 12 dB at the peak level) is due to Doppler effects and the horizontal directivity of the source. The frequency resolution is 1 Hz and thus, this is also the minimum measurable Doppler shift. During the estimation of the latter for different events, I observed that when the distance between the source and the hydrophones was greater than the cycle distance $X_{\text{crit}} = 1514$ m, no Doppler shift was detected between the received peak frequencies, particularly at hydrophones in the circular part of the array. As previously
mentioned, only 5 events are at greater distances than $X_{\text{crit}}$ from the hydrophone array.

### 3.3.2 Spectrum cross-correlation

Although Doppler shift can be roughly estimated by comparing the peak event frequencies, cross-correlation of the spectra is a more appropriate procedure, since it yields an estimate of the shift of the event spectrum instead of the shift in peak frequency, only.

The distribution of Doppler shift with peak event frequency is shown in Figure 3-12:

![Figure 3-12](image)

Figure 3-12: Distribution of Doppler shift with peak event frequency, for the entire event population. The three lines are least-squares best fits through the data. Line 1 (-.-): $\Delta f = \frac{f}{2}$, line 2 (.-.): $\Delta f = \frac{f}{5}$, line 3 (-.-): $\Delta f = \frac{f}{25}$;

Low Doppler shift values have been obtained for both low- and mid-frequency events. The above plot shows that there is no unique relation between Doppler shift and peak event frequency. There is more than one possible best fit line through the data.

The error in estimating Doppler shift through comparison of peak event frequencies at different hydrophones instead of spectrum cross-correlation, is discussed in Appendix D. It

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becomes appreciably large in the case of compound events. In the cross-correlation, only part of the spectrum is used. The portion of the spectrum outside the event's frequency content does not provide any useful information in the Doppler shift estimation. Its inclusion reduces the spectra cross-correlation coefficient and consequently the accuracy of the estimated frequency shift. An example to demonstrate this effect is given in Appendix D.

For compound events, two or more values of Doppler shift are obtained, corresponding to the two or more frequency ranges in the event spectrum where the ratio of the signal-to-noise levels is 5 dB or higher. The Doppler shift value used in the estimation of source speed is that for the range of higher spectral energy.

### 3.4 Estimation of source orientation, speed and frequency

Source motion affects the radiated sound field, i.e., the peak event pressures recorded at each hydrophone. Thus, prior to the estimation of the event radiation characteristics, source speed and orientation must be determined in order to appropriately correct the recorded peak event pressures for the Doppler effect.

Consider a fracture, propagating at a vertical angle $\theta_s$ in the ice and cutting through the entire ice sheet, as shown in Figure 3-13:

![Figure 3-13: Propagating fracture in the ice.](image-url)

where
1. \( \phi_i \) = horizontal angle of receiver \( i \), measured from the \( x \)-axis

2. \( \phi_s \) = horizontal angle between the propagation direction of the source and the \( x \)-axis

3. \( \vec{V} \) = velocity of the source

4. \( \vec{V} \cos(\theta_s) \) = component of velocity in the \( x - y \) plane

The ice thickness is on average 3 m and thus much smaller than the wavelength of the event source. Also, the dip angle \( \theta_s \) is neglected, as discussed in Chapter 1. I assume that the fracture propagates in the \( x - y \) plane. The unknown variables are, therefore, the propagation speed \( |\vec{V}| \) and the source orientation \( \phi_s \). I estimate this angle first, then the source speed and finally the intrinsic source frequency, using Equation 3.7.

In the overview to this chapter, I mentioned that in previous studies of event physics [7] [12], the Rayleigh wave speed (1700 m/s for sea ice) has been assumed as the event source speed, when fracture is the most plausible event generating mechanism. This choice is based on the fact that if finite fracture energy is absorbed per unit area of crack surface as the crack propagates, its velocity must be less than or equal to the Rayleigh wave speed [24]. This is one of several existing hypotheses on the upper bound of fracture speed.

It has been shown [5] that for shear cracks in a non-ideally brittle solid, if slip begins when shear stress reaches a certain static friction level, fracture speed cannot be less than the compressional wave speed. The transition between propagation below and above the Rayleigh wave speed depends on the shear stress the medium can support and the fracture surface energy and length [2]. The two bounds on fracture speed can be more clearly understood by considering the three types of cracks. It may seem premature to introduce fracture mechanisms at this point, since their occurrence has not yet been deduced from the analysis of the characteristics of acoustic events. However, a large number of cracks have been visually detected during the experiment in the area in which the events are located, although no individual cracks have been directly associated with induced acoustic events. Therefore, fracture in the ice is a highly plausible event generating mechanism. Also, I will not discuss the radiation characteristics of fracture, since they are unrelated to the issue of the upper limit of propagation speed. In Chapter 4, a detailed discussion of these characteristics is presented.
3.4.1 Crack modes

A crack in a solid medium can be stressed in three different modes, shown in Figure 3-14:

![Diagram of crack propagation modes: Mode I is the tensile mode, Mode II the in-plane shear and Mode III is the anti-plane shear.]

Figure 3-14: Crack propagation modes: Mode I is the tensile mode, Mode II the in-plane shear and Mode III is the anti-plane shear.

Mode I is the tensile or opening mode. The displacements of the crack surfaces are perpendicular to the plane of the crack. Modes II and III involve shearing. In-plane shear results in Mode II cracking, also called sliding mode. The displacements are in the plane of the crack and normal to the crack edge. Mode III, also called tearing mode, results from anti-plane (or out-of-plane) shearing. Displacements are in the plane of the crack and parallel to its leading edge. The three fracture modes are not necessarily physically dependent.

The limit of the propagation velocity of a Mode III crack is the shear wave velocity, since only shear stresses exist at the crack-tip. These stresses are limited to travel at the shear wave velocity, and thus the same must be true for the fracture. In the crack-tip field of a Mode I crack only normal stresses exist. Therefore, the limit of the propagation velocity of a crack of this type is the compressional or P-wave velocity. It is argued, however, [24] that depending on the brittleness of the medium, the velocity may instead be limited by the Rayleigh wave speed. Mode II rupture propagation is more complicated since both shear and normal stresses exist in the crack-tip field. The shear components travel at the Rayleigh wave velocity (due to in-plane shear) and the normal components at the P-wave velocity. The shear and Rayleigh wave velocities are roughly of the same magnitude, for sea ice 1800 m/s and 1700 m/s, respectively. At the limit, a Mode II crack propagates at
the Rayleigh wave velocity due to out-of-plane shear displacements whereas the Mode III crack propagates at the shear velocity due to the in-plane shear displacements. The P-wave velocity is higher; for sea ice it is about 3500 m/s. For any of the three types of fracture, constant propagation velocity is physically almost impossible. The rupture propagation characteristics depend on a dimensionless strength (or resistance) parameter $S$, defined as the ratio of the stress increase needed to initiate slip, and the dynamic stress drop [36]:

$$S = \frac{\sigma_y - \sigma_f}{\sigma_1 - \sigma_f} \quad (3.8)$$

where $\sigma_y$ is the yield stress of the medium, $\sigma_1$ is the shear stress, which is constant at large distances from the crack, and $\sigma_f$ is the shear stress at the crack plane, equal to the sliding friction stress. If, for example, $S$ is chosen to be much smaller than 1, which implies low rupture resistance, the propagation velocity of a Mode II crack can very quickly approach the P-wave velocity. The transition between sub-Rayleigh and super-Rayleigh propagation is a function of critical fracture length $L_c$, defined as

$$L_c = \left(\frac{\mu}{\pi}\right) \frac{\sigma_y - \sigma_f}{\sigma_1 - \sigma_f} d_0 \quad (3.9)$$

where $\mu$ is the coefficient of internal friction, and $d_0$ is the critical slip distance over which the yield stress decreases to the friction stress. When the crack length reaches approximately twice the value of $L_c$, fracture velocity increases and rapidly reaches one of the two limiting values. In the case of a Mode II crack, rupture starts slowly first and approaches the Rayleigh wave speed as $L \to L_c$. Then, as the crack accelerates, the rupture front becomes narrower. Its velocity remains in the range of the Rayleigh wave velocity for some distance and then bifurcates and accelerates to the P-wave velocity. Again, this is a theoretical situation. In reality the ultimate speed is lower.

The difference between the two types of propagation, can be also explained in terms of the stress drop as slip begins at the rupture front. For a Mode III fracture, stress drop is abrupt and the propagation speed is independent of the rupture distribution function. In contrast, for a Mode II fracture, the rupture front cannot be singular and propagation depends on the spatial and temporal rupture distribution i.e., crack breakdown occurs over
some finite distance. Consequently, stress drop and the associated stress concentration are smoothed out [2].

### 3.4.2 Estimation of propagation direction

Equation 3.7 can be rewritten as

$$\omega_i = \frac{\omega_0 c}{c - |\vec{V}| \cos(\phi_i - \phi_s) \cos(\theta_i)} \quad (3.10)$$

The dot product $\vec{V} \cdot \hat{R}_i$ in Equation 3.7 represents the magnitude of the component of velocity $\vec{V}$ in the direction $\hat{R}_i$ of receiver $i$. Assuming propagation of the source in the horizontal plane of ice, this velocity component is $\vec{V} \cos(\phi_i - \phi_s)$ and its projection in the slant direction of the receiver is $\vec{V} \cos(\phi_i - \phi_s) \cos\theta_i$, where $\theta_i$ is the launch angle. In Equation 3.7, the three unknowns are the source orientation $\phi_s$, speed $|\vec{V}|$ and frequency $\omega_0$. Mathematically, given the observed frequencies and horizontal angles for three hydrophones, source orientation $\phi_s$, speed $V$ and intrinsic frequency $\omega_0$ can be calculated uniquely.

Since it is not \textit{a-priori} known which combination of hydrophones will give the most accurate estimate of the parameters sought, all combinations of hydrophones $(i, j, k)$, $i, j, k, = 1, ..., 24$ are used in the procedure. A least-squares fit is used to obtain the best estimates, first of the direction of propagation, then of source speed and finally of the source frequency. Given hydrophones $i, j, k$, the source orientation is given by

$$\phi_s = \tan^{-1} \left[ \frac{\omega_j \Delta \omega_{k,i} \cos \phi_j \cos \theta_j - \omega_i \Delta \omega_{k,j} \cos \phi_i \cos \theta_i - \omega_k \Delta \omega_{i,j} \cos \phi_k \cos \theta_k}{-\omega_j \Delta \omega_{k,i} \sin \phi_j \cos \theta_j + \omega_i \Delta \omega_{k,j} \sin \phi_i \cos \theta_i + \omega_k \Delta \omega_{i,j} \sin \phi_k \cos \theta_k} \right] \quad (3.11)$$

Instead of the difference in peak frequencies $(\omega_i - \omega_j)$, for any hydrophones $i$ and $j$, the Doppler shift $\Delta \omega_{ij}$, estimated from spectrum cross-correlation is used. Equation 3.11 is an exact expression for $\phi_s$. Theoretically, all combinations should yield the same value, assuming Doppler shifts have been estimated exactly. In reality, this is not the case. As discussed in Appendix D, there are several errors associated with the Doppler shift estimates.
3.4.3 Estimation of propagation speed

Once the source orientation $\phi_s$ has been estimated, for any two hydrophones, the propagation speed $V$ can be obtained, using the equation

$$|\vec{V}| = \frac{(\omega_j - \omega_i)c}{\omega_j \cos(\phi_j - \phi_s) \cos \theta_j - \omega_i \cos(\phi_i - \phi_s) \cos \theta_i}$$

(3.12)

Again, all combinations $i, j$ are used in the estimation and a least-squares fit is then performed to obtain the best speed estimate. Once both $|\vec{V}|$ and $\phi_s$ are known, the intrinsic source frequency $\omega_0$ is obtained from Equation 3.7. To determine the accuracy of the estimated source parameters, the expected peak event frequencies are calculated, using the parameter values in Equation 3.7, and are compared to the measured values. The difference between estimates and measurements is shown in Figure 3-15:

![Comparison between Measured and Estimated Peak Event Frequencies, Event d12_35](image)

**Figure 3-15:** Comparison between estimated and measured peak event frequencies. Error bar indicates their difference for each hydrophone. The mean difference of the two sets of values is -0.182 Hz and the standard deviation is 1.32 Hz.
The distribution of estimated source speed for the entire population is shown in Figure 3-16:

![Distribution of source speed](image)

Figure 3-16: Distribution of source speed, irrespective of signature type.

69% of the total number of events have speeds in the range 200-1100 m/s. Only 28 estimates are below this range and 23 are above. The upper bound of this range (1100 m/s) is $0.65v_R$, where $v_R$ is the Rayleigh wave speed. This result is not surprising; even in the case of earthquakes, typical propagation speeds are approximately $0.7 - 0.8\beta, v_R$, where $\beta$ is the shear wave speed, for shear fractures and about $0.6\beta, v_R$, for tensile fractures. Values as low as $0.2\beta$ have also been recorded [20], particularly during precursory fault motion and in shear process zones. At this point I conclude that the wide range of speed estimates indicates that there is more than one event generating ice mechanism and that fracture is indeed one of these mechanisms. The study of event radiation characteristics is very important in supporting this conclusion.

The distribution of source speed for individual simple event types is shown in Figure
3-17, and variation of this parameter as a function of peak event frequency is shown in Figure 3-18.

![Graphs showing distribution of source speed for different event types](image)

Figure 3-17: Distribution of source speed, for simple events.

The wide range of speed estimates for each event type is clearly seen in Figure 3-17. There are two interesting observations. First, for type III events, the minimum estimated source speed is of the order of 400 m/s and the maximum speed for type IV events is of the order of 1000 m/s. Notice also that there are only 3 events with source speeds of the order of the Rayleigh wave speed. These results are discussed in more detail, later.
Figure 3-18: Source speed versus peak frequency, for simple events. The best fit least-squares lines are superimposed to the data.

For type I events, source speed is in the range 500-1500 m/s. In regard to the relation between this parameter and peak event frequency, on average there is some agreement of the data with the best fitted regression line, i.e., there is a linear increase in source speed with increasing peak frequency. However, there are also a few marked deviations, possibly due to the implicit assumption of the estimation procedure that source propagation is unilateral. Only one source orientation is estimated and it is impossible to determine if a crack propagates in opposite directions from its tips (bilateral). This may explain lower
than expected speed estimates. Higher estimates may be due to erroneous Doppler shifts, obtained when the event spectra at different hydrophones are poorly correlated.

For type II events, there is a wide range of speed estimates; the highest event density occurs between 300 and 400 m/s. The event sample is, however, too small to reach any useful conclusions. Also, there is no clear relation between source speed and peak event frequency, as indicated by the scatter of the data, particularly in the low-frequency range. The deviations from the least-squares fit to the data are more significant than those of type I events.

For Type III events, source speed is in the range 400-1440 m/s and appears uncorrelated with peak frequency. Notice that in the source speed estimation procedure described earlier, two upper bounds have been used, to prevent divergence of the speed estimates, namely the Rayleigh wave and compressional wave speeds. As previously mentioned, it is theoretically possible for Mode I and II fractures to propagate at the compressional wave speed, depending on their length (above or below the critical crack length). Only for type I and III events, I obtained two sets of source speed estimates, under the two theoretical upper bound assumptions. The second set of estimates, not shown here, are in the range 1726 - 3182 m/s.

Finally, in regard to type IV events, as previously mentioned there are no estimated speed values above 1000 m/s. The highest event concentration is in the range 8-400 m/s. The results of all four simple event types indicate that there is a correlation between source speed and signature. The statistics of these parameters, summarized in Table 3.4, show this result more clearly. Another important observation is that there is no clear relation between source speed and peak event frequency. Source motion is typically associated with fracture propagation. The variation in speed and its lack of frequency dependence indicates that this process possibly radiates sound at different stages in its formation, of distinct frequency content.

The distribution of source speed for individual compound event types is shown in Figure 3-19, and variation of this parameter as a function of peak event frequency is shown in Figure 3-20.
Figure 3-19: Distribution of source speed, for compound events.

Notice that there is only one compound event with source speed of the order of the Rayleigh wave speed. The sample of type I and II events is too small to reach any meaningful conclusions. For type III events, there is a wide spread of speed estimates and no concentration in a particular range. Finally, for type IV events source speeds are concentrated in the range 5-800 m/s. The low-speed estimates discussed in the overview to this chapter are mostly for compound and type IV events. However, there is no clear correlation between individual signature types and source speed for compound events.
Figure 3-20: Source speed versus peak frequency, for compound events. The best fit least-squares lines are superimposed to the data.

The spread of the source speed data in each of the plots in Figure 3-20 indicates that there is no apparent relationship between compound event signatures and peak frequency. For type III and IV events this spread is more pronounced. This is expected given that the more complicated the signature is the more difficult it is to estimate the peak frequency from the event spectrum. In particular, for type IV events, the ratio of the signal-to-noise levels was rarely above 5 dB, within the frequency range of interest. Consequently, Doppler shifts and in turn speed estimates for these events may also be inaccurate.
The statistics for source speed for different types of event signatures are summarized in Table 3.5:

<table>
<thead>
<tr>
<th>Signature Type</th>
<th>Mean Speed (m/s)</th>
<th>$\sigma$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1030</td>
<td>309</td>
</tr>
<tr>
<td>II</td>
<td>620</td>
<td>366</td>
</tr>
<tr>
<td>III</td>
<td>920</td>
<td>318</td>
</tr>
<tr>
<td>IV</td>
<td>355</td>
<td>235</td>
</tr>
<tr>
<td>Comp. I</td>
<td>838</td>
<td>263</td>
</tr>
<tr>
<td>Comp. II</td>
<td>357</td>
<td>158</td>
</tr>
<tr>
<td>Comp. III</td>
<td>465</td>
<td>432</td>
</tr>
<tr>
<td>Comp. IV</td>
<td>390</td>
<td>265</td>
</tr>
</tbody>
</table>

Table 3.5: Statistics of estimated source speed as a function of signature type.

Type I and III events are associated with the fastest ice mechanisms, whereas type IV events with the slowest. With the exception of type I, the mean speed of all other compound events is in the same range. Recall that 73% of type III events have peak frequencies in the mid-frequency range, in which the dominant ice mechanism has been found to be fracture, in previous studies of event physics [6] [30] [38]. In Chapter 4, it will be shown that mid-frequency type III events are indeed attributed to fracture processes, although there is no unique signature type characteristic of these processes.

### 3.5 Displacement spectrum of source-wave parameter

In Chapter 2, the source-wave parameter has been obtained by integrating the acoustic event signal, twice in the time-domain. This parameter is equivalent to the measured P- and S-wave pulses for earthquakes. Although based on the event radiation patterns the type of ice motion to which the events are attributed has been deduced, I use the term 'source-wave parameter' instead of P- and/or S-wave(s), since there are no independent measurements of these waves. However, I apply the technique of estimating the source dimensions from the displacement spectrum of the source-wave parameter, which is commonly used for earthquakes.

Consider a propagating rectangular fault. Figure 3-21 shows its geometry:
Figure 3-21: Diagram of a rectangular propagating fault. $L$ is its length, $D$ its depth, $u$ the slip and $v$, the propagation velocity.

From the source-wave displacement spectrum, the *corner frequency*, i.e., the frequency at the intersection of the low- and high-frequency trends in the spectrum can be estimated. It is expected that the corner frequency is well-defined [3] [15] [32]. As previously mentioned, the displacement signal of the source wave is approximately a uni-directional pulse with no marked oscillations. This is clearly seen in Figures 2-18 - 2-25. Therefore, its spectrum must have a maximum value at zero-frequency and the flat asymptote (zero gradient) for lower frequencies is a line roughly parallel to the frequency axis. This is a common spectral trend, irrespective of the fault geometry. The high-frequency part of the spectrum is defined by the highest order discontinuity in the displacement signal and its proportional to an inverse power of frequency; the slope of this part usually shows a $\omega^{-2}$ dependence. Corner frequency is an approximate measure of the effective width of the spectrum. If the fault length and width are of significantly different magnitude, two corner frequencies instead of one can be observed in the spectrum. The second corner frequency results from an intermediate slope with a $\omega^{-1}$ gradient.

There is an inverse relationship between pulse width of the displacement parameter and spectrum width. The pulse width is proportional to the duration of the rupture, approximately equal to the ratio of fault length $L$ to propagation speed $v$. The spectrum width is
proportional to the corner frequency. Consequently, fault length is inversely proportional to the corner frequency $\omega_1$: $\omega_1 \propto \frac{v}{L}$. The second corner frequency is used to estimate the fault depth $D$: $\omega_2 \propto \frac{v}{D}$. Thus, $\omega_1$ and $\omega_2$ are directly related to the reciprocal of rupture propagation time and of rise time, respectively. Separation of the two frequencies depends on the ratio of these two characteristic times of the fault, and is consequently sensitive to the ratio $\frac{L}{D}$.

Different types of displacement spectra have been identified for the analyzed events. Two corner frequencies are clearly detected in the source-wave displacement spectra of simple type I and III events. The following is an example of a typical source-wave displacement spectrum of a type I event:

![Displacement spectrum of source-wave parameter – type I event](image)

Figure 3-22: Source-wave displacement spectrum, for a type I event. Frequency (in rad/s) has been normalized by a factor $\frac{1}{2v_p}$ where $v_p$ is the propagation velocity; $v_p = 524$ m/s. The factor of 2 by which frequency is divided is required in order to compute the total fault length and width instead of their half-values. The corner frequencies are $\omega_1 = 0.2$ (normalized rad/s) and $\omega_2 = 0.4$ (normalized rad/s).
For type I events, in addition to the $\omega^{-1}$ dependence, as frequency increases the decrease in the spectral amplitude is proportional to $\omega^{-3}$, instead of $\omega^{-2}$. The three curves $\propto \omega^{-1}, \omega^{-2}, \omega^{-3}$ are superimposed on the spectrum. The spectral fall in the high-frequency range is clearly not proportional to $\omega^{-2}$. This trend characterizes the source-wave displacement spectrum only of type I events. In the case of earthquakes, it has been reported [1] that fault nucleation is responsible for this type of source-wave spectrum. It is impossible to verify such occurrence for ice faults, given the available acoustic data. Since frequency has already been normalized, the reciprocals of the estimated corner frequencies give directly the fault length and width. For this example, $L = 5$ m and $D = 1.25$ m. The two characteristic frequencies are well-separated ($\frac{L}{D} = 4$).

Figure 3-23 shows a second type of displacement spectrum:

![Displacement spectrum of source-wave parameter – type IV event](image)

Figure 3-23: Source-wave displacement spectrum, of a type IV event. Frequency (in rad/s) has been normalized as in the previous figure.

In this example, the normalized characteristic frequencies are 0.1 and 0.2 respectively,
corresponding to $L = 10$ m and $D = 5$ m. The separation between the two frequencies is smaller than that in the previous example. Notice the $\omega^{-2}$ spectral fall in the high-frequency range for this event, versus the $\omega^{-3}$ relation for the previous event. This type of displacement spectrum is neither unique to type IV events nor characteristic of them. For approximately 50% of type IV events, the spectral shape is similar to that of compound events. In Chapter 2, it has been discussed that it may be difficult to clearly distinguish type IV from compound events. Thus, this result is justified.

The following example is that of an event possibly induced by a circular fault, i.e., $L \approx D$.

![Displacement spectrum of source-wave parameter – type III event](image)

**Figure 3-24**: Source-wave displacement spectrum of type III event.

The fault dimensions are approximately $L = 4$ m and $W = 3$ m. In cases when the two corner frequencies are very close to each other, the small fault dimension is determined from the frequency $\omega_3 = (\omega_1\omega_2)^{\frac{1}{4}}$. Finally, there is another type of source wave-displacement spectrum, in which no corner frequency $\omega_2$ can be distinguished. The spectral fall at high
frequencies shows a $\omega^{-1}$ dependence. In the case of earthquakes, spectral fall-offs that show an $\omega^{-1.3}$ dependence have been reported, too. The scaling with rupture propagation velocity is an important factor in these spectra, in addition to the actual shape of the source-wave pulse, i.e., low propagation speed may be responsible for this type of displacement spectrum.

![Displacement spectrum of source-wave parameter - compound type III event](image)

Figure 3-25: Source-wave displacement spectrum of a compound type III event.

### 3.5.1 Source dimensions of the entire event population

Following the above-described procedure, fault dimensions have been estimated for the entire event population. Fracture, however, is not the only event generating mechanism. The above procedure is typically used for fracture-induced events. It may thus be inaccurate for estimating the range of deformation associated with other motion-related processes in the ice, such as floe unloading. These results will be examined further, once the event
mechanisms have been identified, in Chapter 4.

The distribution of source length with signature type, for simple and compound events is shown in Figures 3-26 and 3-27, respectively:

![Graphs showing distribution of source length with signature type](image)

Figure 3-26: Distribution of estimated source length with signature type, for simple events.

Type III events are associated with processes of the longest length. Above 60 m, only events of this type exist. Their highest concentration, though, is in the range 20-50 m. In contrast, type IV events are predominantly associated with processes of lengths in the range 1-20 m. Recall that type IV events also have source speeds predominantly in the range 200-400 m/s, whereas the highest source speed estimates have been obtained for type I and
Figure 3-27: Distribution of estimated source length with signature type, for compound events.

Irrespective of signature, compound events have source lengths predominantly in the range 0.7-40 m, and the number of events rapidly decreases with increasing length. There is no clear correlation between this source parameter and signature type for compound events.

The depth of the event generating fault mechanisms is evidently limited by the ice thickness. The distribution of this parameter for the entire event population is shown in Figure 3-28:
Figure 3-28: Distribution of estimated source depth for the entire event population.

64% of the total number of events have source depths in the ranges 3-3.5 m and 3.75-4 m, i.e., of the order of the ice thickness. This implies that in the case of fracture, the entire ice sheet is cut through as the fracture propagates. Depth for the remaining events is distributed in the range 0.5-3 m. In regard to individual signature types, for type III events depth is predominantly in the range 3-4 m (the maximum measured ice thickness is 4.2 m). There is no clear relation between source depth and signature type for other types of simple events and for compound events. The fact that source dimensions appear un-correlated with signature type is not surprising. The results of the analysis so far have shown that there is no one-to-one relationship between source parameters and signature. This in turn indicates that the hypothesis that distinct ice mechanisms are associated with particular signatures may be incorrect. In fact, it is possible that events of different types may have been induced by the same process. This issue is further addressed in Chapters 4 and 5.

The statistics of source dimensions are summarized in Table 3.6:
<table>
<thead>
<tr>
<th>Signature</th>
<th>Mean length</th>
<th>$\sigma_L$ (m)</th>
<th>Mean depth</th>
<th>$\sigma_D$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>31.4</td>
<td>10</td>
<td>3.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Type II</td>
<td>25.8</td>
<td>18.8</td>
<td>3.3</td>
<td>0.65</td>
</tr>
<tr>
<td>Type III</td>
<td>41.5</td>
<td>12</td>
<td>3.3</td>
<td>0.64</td>
</tr>
<tr>
<td>Type IV</td>
<td>17</td>
<td>11</td>
<td>2.4</td>
<td>1.1</td>
</tr>
<tr>
<td>Comp. Type I</td>
<td>39.7</td>
<td>23.2</td>
<td>3.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Comp. Type II</td>
<td>17.4</td>
<td>11.2</td>
<td>2.6</td>
<td>1.2</td>
</tr>
<tr>
<td>Comp. Type III</td>
<td>24.2</td>
<td>14</td>
<td>2.9</td>
<td>0.7</td>
</tr>
<tr>
<td>Comp. Type IV</td>
<td>22</td>
<td>17.6</td>
<td>2.26</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 3.6: Statistics of source dimensions as a function of signature type.

Simple type III and compound type I events have the highest mean source length. With the exception of type IV, all other simple events have mean depths 3.3 m. The mean values of compound events are lower, with the exception of compound type I. The above estimates show that motion associated with at least 50% of event types penetrates the ice sheet completely.

Once the fault dimensions have been estimated and since the maximum particle slip for each event is already known, the seismic moment can be directly calculated from the equation:

$$M_0 = \mu \Delta \bar{u} A$$  \hspace{1cm} (3.13)

where $M_0$ is the static seismic moment, $\mu$ the shear rigidity of the medium, $\Delta \bar{u}$ the maximum slip (an average over the entire fault length) and $A$ the fault area. I will not discuss the seismic moment estimates yet for two reasons. First, seismic moment is a quantity that describes a faulting source associated with fracture. Thus, in Chapter 4, once the event mechanisms have been identified and associated with appropriate acoustic models, in the case of fracture-induced events seismic moment can be estimated from the event source strength. The estimates calculated directly from Equation 3.13 will be compared to those obtained from the event source strength. For events that are associated with mechanisms other than fracture, e.g., ones involving a volume change, force instead of moment is the pertinent source strength parameter.
3.5.2 L- and W- models

In terms of the controlling dimension of a fault, two models are commonly used, namely the \( L \)-model and the \( W \)-model. Scaling relations for faults are often discussed in terms of these models. The former assumes that the appropriate scale of the fault is length, and is typically used only in cases of large earthquakes, for which mean slip increases linearly with length, and consequently the static moment \( M_0 \propto L^2W \). The \( W \)-model assumes that the fault is controlled by the smaller dimension, i.e., its depth and that \( M_0 \propto LW^2 \). Thus, faults of different lengths have the same width. Other observed scalings for seismic moments include \( M_0 \propto L^3 \). Rise time is assumed to be as follows:

\[
t_r \propto \frac{D}{u_r} \quad W\text{-model} \\
t_r \propto \frac{L}{u_r} \quad L\text{-model}
\]  

(3.14)

where \( u_r \) is the propagation velocity. In all calculations of source dimensions, the assumption of the \( W \)-model is considered. The two models are shown in Figure 3-29:

Figure 3-29: Schematic diagram to illustrate the difference between \( L \)- and \( W \)- models.

The predicted seismic moment by the \( W \)- and \( L \)-models, given the estimated source dimensions, is shown in Figure 3-30. The model values are compared to the data, in Chapter 4, in order to gain additional insight on the characteristics of the event generating fracture mechanisms. In the case of earthquakes, the \( W \)- and \( L \)-models are used for small and large seismic events, respectively.
Figure 3-30: Predicted seismic moment variation with source length, by the $W$- and $L$- models.

Event self-similarity in terms of the controlling dimension is evident even from the statistics of source dimensions. Events of different source lengths have been found to have the same depth. This issue is discussed in more detail in Chapter 4, in terms of the estimated stress drop and its dependence on seismic moment.

### 3.6 Summary and Observations

The frequency domain analysis of acoustic events, in the range 10-350 Hz, and the estimation of source parameters that characterize propagation of the event generating mechanisms have been presented in this chapter. Both low- and mid-frequency events have been detected, with peak event frequencies in the ranges $< 100$ Hz and 100-350 Hz, respectively. In regard to the correlation of peak event frequency and signature type, it has been observed that type
III and IV events dominate the mid-frequency range, whereas type I and II events have peak frequencies no larger than 200 Hz. In terms of bandwidth, 95% of the total number of events are narrowband and compound events are on average broadband. Their spectrum is characterized by several peaks, often separated by more than one octave.

The event spectra at different hydrophone have been used in a cross-correlation procedure, to estimate the observed Doppler shift, resulting from source motion. It has been possible to measure Doppler shifts ranging between 1 and 90 Hz. Estimated frequency shifts and peak event frequencies have been used to calculate source speed and orientation (strike angle), two of the parameters that characterize propagation of faults in a medium. In previous studies of event physics, the Rayleigh wave speed, approximately 1700 m/s for sea ice, has been assumed as the source speed, particularly in cases when fracture is the most plausible event generating mechanism. Since Doppler shift is measurable in the analyzed data, due to both the event locations and the geometry of the array, it has been possible to estimate source speed more accurately. For 76% of the total number of events, it is significantly lower than the Rayleigh wave speed, in the range 100-1000 m/s. There are 33 events, predominantly type I and III, for which the estimated fracture speed is between 1000 and 1698 m/s. Also, lower than 100 m/s speed estimates have been obtained for 14 events, the majority of which have type IV signatures. A possible explanation for the low velocity estimates is that the medium may undergo a rapid transition from velocity weakening to velocity strengthening. The source speed results indicate that there is more than one event generating mechanism. Also, peak event frequency appears uncorrelated to source speed. One of the weaknesses of the employed estimation procedure is that bilateral and uni-lateral propagation cannot be distinguished; only one source orientation is determined. Lower than expected speed values may be attributed to this problem. Also, there are known event generating mechanisms, such as floe unloading, which do not involve source motion. Their occurrence in the ice and consequent generation of acoustic events also explain low source speed estimates. The most important conclusion from this analysis is that fracture, possibly at different scales, is indeed a plausible event inducing mechanism.

Finally, source dimensions are estimated from the spectrum of the source-wave displacement signal. In Chapter 2, the latter has been obtained by integrating the acoustic
pressure signal twice in the time domain. The spectrum is typically characterized by a flat asymptote at low frequencies and a negative slope in the high-frequency range. Based on the dependence of the spectral fall-off on frequency, four types of source-wave displacement spectra have been identified for the analyzed events. The spectrum corresponding to type I events is characterized by a $\omega^{-3}$-dependent slope. This phenomenon cannot be explained with the available data. Such a trend has, nevertheless, been observed in the case of earthquakes. The second type of displacement spectrum is that in which two corner frequencies are distinguished and are well-separated. The corresponding geometry of the wave-inducing source is a long and narrow, approximately rectangular fault. For the third type of spectrum the corner frequencies are very close to each other, thus suggesting a circular fault. Finally, for several compound events the high-frequency spectral trend is $\omega^{-1}$-dependent and only one corner frequency can be estimated. This phenomenon may be attributed to a low propagation speed, which is used to normalize the spectrum. The statistics of source dimensions suggest that events of different lengths have the same width. The estimated length range is 0.7-100 m and the corresponding depth range 0.4-4 m. According to the W-model, for such events the controlling dimension of their source is depth. Event self-similarity and relation between controlling dimension, seismic moment and stress drop is further investigated in Chapter 4. 64% of the total number of events have source depth of the order of the ice thickness. In the case of fracture, this result indicates that the entire ice sheet is cut as the fracture propagates.

In regard to the relation between the occurrence of broadband compound events and a fracture cycle, no sufficient information has yet been obtained to draw any conclusions. The results of the analysis, however, suggest that it is likely that these events are associated with a single physical process.

Having estimated source parameters that pertain both to particle motion and propagation of the event generating mechanisms, I now proceed to the estimation of the event radiation patterns and the identification of the ice mechanisms. The study of event radiation characteristics is very important in supporting the conclusion on the multitude of mechanisms that have caused the acoustic events, drawn from the variation of source speed estimates.
Chapter 4

Estimation of event radiation patterns, acoustic models and proposed ice mechanisms

4.1 Overview

Parameters that characterize particle motion and propagation of acoustic event-generating mechanisms have been estimated in Chapters 2 and 3. The type of ice motion associated with these mechanisms is determined from the event radiation patterns, the analysis of which is presented here. This chapter is organized in three parts. First, the representation of seismic sources by force equivalents is briefly described. In particular, the single point force, force couples and their use in modeling displacement discontinuities in a medium, such as those introduced along a fracture plane, are discussed. Force couples and their combinations are typically used to represent different fracture modes.

The analysis of the event radiation characteristics is subsequently presented. Examples of events are used to demonstrate their distinct radiation patterns. Acoustic models corresponding to body force equivalents are then selected. In particular, the dipole (a monopole and its image) associated with a point force, and the octopole (a quadrupole and its image) equivalent to a double force couple are the two basic models that are initially discussed.
The methodology for estimating event radiation patterns and appropriate acoustic models is summarized in the following diagram:

```
Peak
Event Amplitudes

Check Sign

Amplitudes & Source Polarity

Acoustic Model

Adjustment of Model

Corrections of Data:
1. Doppler Factor
2. Vertical Directivity
3. Large aperture effects

Event Horizontal Radiation Pattern

Comparison of Data with Model

Best Acoustic Model for Event Radiation
```

Figure 4-1: Schematic representation of the procedure followed for selecting an appropriate acoustic model for an event radiation pattern.

The peak event amplitudes are first examined in order to detect changes in their sign. Different factors cause such changes but the one that is of interest to this analysis is the horizontal directivity pattern of the source. Then, according to the variation of source level with azimuth, an acoustic model is selected. As previously mentioned, initially either the dipole or the octopole are chosen. Both models have been previously proposed [38] [6] to describe the radiation characteristics of ice-induced acoustic events in the Central Arctic.
and the Marginal Ice Zone. The data are corrected by a Doppler factor in order to take into account source propagation and then, according to the selected model, they are also corrected for the corresponding vertical directivity. The corrected data are examined as a function of azimuth and compared to the predicted by the model radiation pattern. Only events with azimuth observation ranges greater than 30° are analyzed. For events that are located at large distances from the array, no useful information can be obtained on their horizontal directivity. Modifications to the selected model are subsequently made. They are necessary when the mechanics of the event generating process appear to disagree with the implicit physical assumptions made by the model. For example, the models must be modified to account for source motion. Alternatively, if a small-aperture source is initially assumed and the ratio of source length to wavelength is larger than 1, large aperture of the source must be accounted for.

It is possible that a single acoustic model does not describe the event radiation adequately. The faulting process in a medium is complicated and involves sub-processes. This issue is addressed here and hybrid multipole acoustic models, composed of a combination of a dipole and an octopole or a combination of lateral and longitudinal octopoles are developed. Table 4.1 summarizes the statistics of analyzed events (181 of the 196 detected events) as a function of the selected acoustic model.

<table>
<thead>
<tr>
<th>Acoustic model</th>
<th>No. of Events</th>
<th>% of total # of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole (no source motion)</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>Dipole modified by Doppler effect</td>
<td>52</td>
<td>29</td>
</tr>
<tr>
<td>Octopole (lateral)</td>
<td>46</td>
<td>25</td>
</tr>
<tr>
<td>Octopole (longitudinal)</td>
<td>28</td>
<td>15</td>
</tr>
<tr>
<td>Hybrid multipoles</td>
<td>35</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 4.1: Statistics of events as a function of radiation characteristics.

The results from the analysis and modeling of the event radiation patterns are then used, together with previously estimated source parameters, to identify the event generating ice mechanisms. The following mechanisms are discussed:

- Floe unloading motion
Fracture

- Tensile (Mode I)
- In-plane (Mode II) and out-of-plane (Mode III) shear

Process-zone formation

Stick-slip motion

The last two processes are only briefly described. Although the occurrence of stick-slip motion is suggested by the slip curves of some compound events, limited information can be obtained from the event radiation patterns to confirm it as a characteristic of ice motion fracture processes. Process zone formation is further investigated in Chapter 5. Finally, event self-similarity and scaling relations are discussed.

4.2 Representation of ice motion sources in terms of body force equivalents

In general, two types of sources are distinguished, faulting sources associated with an internal surface across which discontinuities in displacement occur, and volume sources, associated with an internal volume across which discontinuities in strain occur. It has been shown [1] that some volume sources, e.g., spherical, are equivalent to superimposed perpendicular force couples, which I briefly discuss in the context of faulting sources.

4.2.1 Faulting sources

The following discussion is based on [1]. Body force equivalents can be used to describe a faulting source, such as slip across a fracture plane. For a discontinuity in traction, the body force equivalent $f_T$ is given by

$$f_T(\eta, \tau) = - \int_{\Sigma} T(u(\bar{x}, \tau), \nu) \delta(\eta - \bar{x}) d\Sigma(\bar{x})$$

(4.1)
where $\Sigma$ is the internal fault surface, $\eta$ is an arbitrary position within the volume of concern $V$, $\bar{x}$ is an arbitrary position on $\Sigma$, $T$ is the traction and $\nu$ is the outward normal to the surface, vector. Analogous to the above force is the body force equivalent to a discontinuity in displacement:

$$ f_u(\bar{\eta}, \tau) = - \int \int_{\Sigma} u_i(\bar{x}, \tau, \nu) c_{ijkl} \nu_j \frac{\partial}{\partial \eta_l} \delta(\bar{\eta} - \bar{x}) d\Sigma(\bar{x}) $$

(4.2)

where $c_{ijkl}$ is the constitutive tensor of the medium. This expression is derived in [1].

Faulting motion within volume $V$ is an internal mechanism and thus the total momentum and angular momentum must be conserved. Invoking equilibrium, the total force due to $f_u$ and the total moment of $f_u$ about any fixed point within the volume must be zero, i.e.,

$$ \int \int \int_{V} f_u(\bar{\eta}, \tau) dV(\bar{\eta}) = 0 \quad \forall \tau $$

(4.3)

and

$$ \int \int \int_{V} (\bar{\eta} - \bar{\eta}_0) \times f_u(\bar{\eta}, \tau) dV(\bar{\eta}) = 0 \quad \forall \tau, \bar{\eta}_0 \text{ fixed} $$

(4.4)

The following example of fault slip in the $x_1 - x_2$ plane, shows the representation of fracture by a system of force couples [1]. The surface $\Sigma$ lies in the plane $x_3 = 0$ and thus $\bar{u}$ has no component in the $x_3$-direction. Let $x_1$ be the direction of slip; $u_2 = u_3 = 0$ and $\nu_1 = \nu_2 = 0$. The body-force equivalent for a displacement discontinuity is given by

$$ f_u(\bar{\eta}, \tau) = - \int \int_{\Sigma} u_1(\bar{x}, \tau) c_{13kl} \nu_j \frac{\partial}{\partial \eta_l} \delta(\bar{\eta} - \bar{x}) dx_1 dx_2 $$

(4.5)

For an isotropic inhomogeneous medium, the elements of $c_{ijkl}$ are zero except $c_{1313} = c_{1331} = \mu$. Thus, Equation 4.5 becomes

$$ f_{1u}(\bar{\eta}, \tau) = - \int \int_{\Sigma} \mu(x) u_1(\bar{x}, \tau) \delta(\eta_1 - x_1) \delta(\eta_2 - x_2) \frac{\partial}{\partial \eta_l} \delta(\eta_3) dx_1 dx_2 $$

$$ f_{2u}(\bar{x}, \tau) = 0 $$

$$ f_{3u}(\bar{\eta}, \tau) = - \int \int_{\Sigma} \mu(x) u_1(\bar{x}, \tau) \frac{\partial}{\partial \eta_l} \delta(\eta_1 - x_1) \delta(\eta_2 - x_2) \delta(\eta_3) dx_1 dx_2 $$

(4.6)
\( f_1(\tilde{\eta}, \tau) \) represents a system of single couples distributed over the fault surface \( \Sigma \). Upon evaluation of the integral in Equation 4.6, the resulting force \( f_1 \) is of the form

\[
f_1(\tilde{\eta}, \tau) = -\mu(\tilde{\eta}) u_1(\tilde{\eta}, \tau) \frac{\partial}{\partial \eta_3} \delta(\eta_3)
\]

(4.7)

Since the above is a system of single couples acting in opposite directions, both parallel to the \( x_1 \)-axis, the total force due to \( f_1 \) is zero. However, the moment of the system about the \( \eta_2 \)-direction does not vanish and is given by

\[
\int \int \int_V \eta_3 f_1 u' dV = \int \int_\Sigma \mu u_1(\tilde{x}, \tau) d\Sigma
\]

(4.8)

If \( \mu \) is constant, i.e., the medium is isotropic, the total moment is \( M = \mu \tilde{u} A \), where \( \tilde{u} \) is the mean slip over the fault surface \( \Sigma \) and \( A \) is the fault area.

\( f_3(\tilde{\eta}, \tau) \) represents a system of single forces, as opposed to \( f_1 \) which is a system of single couples. Upon evaluation of the second integral in Equation 4.6, \( f_3 \) is of the form

\[
\int \int \int_V \epsilon_{213} f_3 dV = -\int \int_\Sigma \mu u_1(\tilde{x}, \tau) d\Sigma
\]

(4.9)

where \( \epsilon \) is the permutation tensor. The total moment due to the system \( f_3 \) is \(-\mu \tilde{u} A\).

The important conclusion from this example is that slip is equivalent to a distribution of single couples and single forces. It may appear confusing, given that slip is typically represented by a double-couple distributed over the fault surface. The fact is that there is always a single couple made up of forces in the same direction as the fault displacements (\( f_1 \) in this example), but a complete equivalent to fault slip has two terms, the second being a single force, as is the case in this example, or another single couple or a linear combination of the two [1].

In the far-field, \( \Sigma \) can be treated as a point source region. Then \( f_1 \) and \( f_3 \) are both single couples. Thus, a double-couple equivalent to fault slip is given by
\[ f_1(\tilde{\eta}, \tau) = -M_0 \delta(\eta_1) \delta(\eta_2) \frac{\partial}{\partial \eta_3} \delta(\eta_3) F(\tau) \]
\[ f_2(\tilde{\eta}, \tau) = 0 \]
\[ f_3(\tilde{\eta}, \tau) = -M_0 \frac{\partial}{\partial \eta_1} \delta(\eta_1) \delta(\eta_2) \delta(\eta_3) F(\tau) \]

where \( F \) is the source (or slip function) discussed in detail in Chapter 2, and \( M_0 \) is the seismic moment, given by

\[ M_0 = \mu \bar{u} A \] (4.11)

\( M_0 \) is used to measure the strength of the motion process, such as an earthquake or an ice fracture, in this study. The above expression implies that \( M_0 \) is a static quantity, but in reality \( M_0 \) is time-dependent, since slip is time-dependent. The static quantity is an average over the duration of the seismic signal and/or over the fault surface.

The seismic moment tensor \( M \) depends on source strength and fault orientation (strike and dip angles) and characterizes a compact source completely. For sources of finite length, the seismic moment density tensor \( m \) is used, where \( m = \frac{dM}{d\Sigma} \) for a faulting source and \( m = \frac{dM}{dV} \) for a volume source, with units of moment per unit area and moment per unit volume, respectively. Using Equation 4.11, but considering a general case of a medium (not isotropic for which \( c_{ijkl} = \mu \)), we obtain the expression for the moment density tensor for a faulting source:

\[ m_{kl} = u_i \nu_j c_{ijkl} \] (4.12)

where \( u_i \) is the displacement, \( \nu_j \) the outward normal unit vector and \( c_{ijkl} \) the constitutive tensor of the medium. For an isotropic medium, in which the displacement discontinuity is parallel to the fault surface \( \Sigma \) at some point \( \bar{x} \), the dot product \( \bar{u} \cdot \bar{\nu} \) is zero and thus

\[ m_{kl} = \mu (\nu_k u_k + \nu_l u_l) \] (4.13)

For a volume source in an inhomogeneous anisotropic medium, the seismic moment is given by

\[ M_{ij} = \int \int \int_V c_{ijkl} \Delta e_{kl} dV \] (4.14)

where \( \Delta e_{kl} \) is the strain the material undergoes, which characterizes the seismic source. For
the special case of an isotropic medium, the moment is given by

\[ M_0 = 2\mu \Delta e_{ij} V \]  

(4.15)

where Equation 4.15 is the equivalent to Equation 4.11 for a volume source. Thus, the moment density tensor is given by

\[ m_{ij} = c_{ijkl} \Delta e_{kl} \]  

(4.16)

Consider again a faulting source. If \( \Sigma \) lies in the \( x_1 - x_2 \) plane, with slip in the \( x_1 \) direction, as in the example previously described, the moment density tensor is given by

\[
m = \begin{pmatrix}
0 & 0 & \mu u_1(\vec{x}, \tau) \\
0 & 0 & 0 \\
\mu u_1(\vec{x}, \tau) & 0 & 0
\end{pmatrix}
\]  

(4.17)

This is for the double couple, and in this particular case it is used to represent pure shear in the medium (Mode II fracture).

For a tensile fracture (Mode I), in the \( x_3 = 0 \) plane, only the slip component \( u_3 \) is non-zero. In this case,

\[
m = \begin{pmatrix}
\lambda u_3(\vec{x}, \tau) & 0 & 0 \\
0 & \lambda u_3(\vec{x}, \tau) & 0 \\
0 & 0 & (\lambda + 2\mu) u_3(\vec{x}, \tau)
\end{pmatrix}
\]  

(4.18)

For the three components of force there are nine possible couples, shown in Figure 4-2:
(1,1)  (1,2)  (1,3)
(2,1)  (2,2)  (2,3)
(3,1)  (3,2)  (3,3)

Figure 4-2: Possible force couples.

In Chapter 3, the three modes of fracture have been discussed. Their body force equivalents are combinations of force couples. For example, the tensile crack for which displacements are perpendicular to the plane of the crack, can be represented by a superposition of three double couples. These must be such that the force and moment arm are in the same direction, i.e., as in (1, 1), (2, 2), (3, 3). For example, a tensile crack in the $x_3 = 0$ plane is represented by the following system of couples:
Figure 4-3: Body-force equivalents of a tensile fracture in the $x_3 = 0$ plane.

From Equation 4.18, the magnitudes of the three couples are in the ratio $1:1: \frac{(\lambda + 2\mu)}{\lambda}$, where $\lambda$ and $\mu$ are the Lamé constants of the medium [1]. Body-force equivalents and corresponding acoustic models are discussed in detail in the next section.

4.3 Acoustic models and proposed ice mechanisms

In this section, the event radiation characteristics, pertinent acoustic models and proposed ice mechanisms, based on the results of the entire analysis, are discussed. In regard to the acoustic models, the following assumptions are made:

- **Far-field approximation:** it is assumed that the distance between source and receivers is much larger than the source wavelength, i.e., $kR \gg 1$, where $R$ is the distance between source and receiver and $k$ the wavenumber.

- **The presence of ice is neglected:** it is assumed that the source and its image are located directly above and below in the ice cover and that their distance is small in comparison to the wavelength, as shown in Figure 4-4:
Figure 4-4: Geometry of the source, its image and their respective distances from a hydrophone. Although straight paths from the source and its image to the receiver are drawn, for the purpose of the sketch, refractive paths are assumed in all calculations.

- **Compactness**: it is assumed that the separation of the source and its image is much smaller than the wavelength, i.e., $2kh\sin\theta \ll 1$, where $2h$ is the maximum ice thickness, and $\theta$ the launch angle at the source from the horizontal axis.

- **Refraction is taken into account**: In all calculations, the refractive path between source and receivers is assumed. In regard to transmission loss, refraction is accounted for as described in Chapter 3.

- **The peak event pressures (absolute values, since pressure sign is separately recorded for each channel) are used in all calculations.**

Note that the analysis is performed channel by channel, using the recorded peak event pressure at each channel to calculate dipole force or octopole moment. To obtain a single value for source strength for each event, an arithmetic average of the source strength values at all channels is computed. In regard to the size of the source, the ratio of source length
$L$ to wavelength $\lambda$ determines if the source must be treated as a large-aperture source and appropriate corrections must be introduced.

For each event, the fracture or deformation length from the corresponding spectrum of the source displacement signal has been estimated in Chapter 3. As it has been discussed, the term 'length' in the case of events that have been induced by mechanisms other than fracture, e.g., unloading of the ice refers to a range of deformation. The ratio of source length to wavelength, for the entire event population is shown in Figure 4-5:

Figure 4-5: Ratio of source length to wavelength. Solid line is $L = \lambda$.

This ratio determines the number of minor lobes of the horizontal directivity pattern (it is assumed that in the vertical direction the source is compact) and the width of the major lobe. The statistics of these parameters for the event population are summarized in Table 4.2. As discussed in Chapter 3, the estimation of length from the source displacement spectrum, implicitly assumes that the source is associated with some type of fracture or faulting. Therefore, it may be incorrect to use this procedure for estimating the range of deformation.
for a source that is associated with some other type of ice motion. Consequently, some of the above estimates of number of minor lobes may also be inaccurate. For the moment, I will assume that they are correct. The data will show if the assumption of a large-aperture source is appropriate.

<table>
<thead>
<tr>
<th>$\frac{L}{\lambda}$</th>
<th>No. of events</th>
<th>No. of minor lobes</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>1−&lt;2</td>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td>2−&lt;3</td>
<td>36</td>
<td>2</td>
</tr>
<tr>
<td>3−&lt;4</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>4−&lt;5</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>5−&lt;6</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>6−&lt;7</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>7−&lt;8</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>8−&lt;9</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 4.2: Statistics of the ratio of source length to wavelength and the number of minor lobes of the directivity pattern.

Section 4.3.1 describes the corrections that must be introduced in the event horizontal radiation pattern in the case of a large-aperture source.

4.3.1 Large-aperture sources

When the source length is no longer small compared to the wavelength, the assumption of a small-aperture source is violated. In regard to the source depth which is at most equal to the ice thickness, small vertical aperture is assured. The highest peak event frequency is $f = 313$ Hz; the wavelength is $\lambda = 4.6$ m. I assume that ice thickness is on average 3 m.

In the horizontal, the source can be modeled as a uniform line array of length $L$, composed of tightly packed, individually compact sources. I assume that each source element radiates at the same amplitude and phase. Notice that phase changes due to propagation have been accounted for by the inclusion of the Doppler factor in the directivity, assuming unilateral propagation. For bilateral propagation, an additional phase term must be introduced to account for propagation in two opposite directions from the crack tips, i.e., the direction of motion of half of the source array elements is opposite to that of
the other half. I do not consider this case in the thesis. Given the data and particularly the event locations with respect to the array location, it is very difficult to account for bilateral propagation in the estimation of Doppler shift and source speed. Also, in terms of the physics of a fracture or fault, often bilateral propagation becomes unilateral once a particular fracture length has been reached [20].

Figure 4-6: Sketch of a large-aperture source. Line array elements are not necessarily spherical sources. According to the model used they may be dipoles, octopoles, etc. The distance between source elements is assumed to be $\frac{\lambda}{2L}$.

Suppose the distance between source elements in the array is in general $\Delta L$, and $R$ the distance of the first element from the receiver; the corresponding horizontal distance is $R \cos \theta$. Using the law of cosines, the horizontal distance of the element $i$ of the array,
\( i \neq 1 \), is given by

\[
d^2 = (\Delta L)^2 + (R \cos \theta)^2 - 2\Delta LR \cos \theta \cos(\phi - \phi_s) \tag{4.19}
\]

Thus, the distance \( R_i \) of source element \( i \) from the receiver is

\[
R_i = [(\Delta L)^2 + (R \cos \theta)^2 - 2\Delta LR \cos \theta \cos(\phi - \phi_s) + z^2]^{\frac{1}{2}} = \\
R[(\frac{\Delta L}{R})^2 + 1 - 2(\frac{\Delta L}{R}) \cos \theta \cos(\phi - \phi_s)]^{\frac{1}{2}} \approx \\
R[1 - 2(\frac{\Delta L}{R}) \cos \theta \cos(\phi - \phi_s)]^{\frac{1}{2}} \tag{4.20}
\]

given that \( z = R \sin \theta \), and using the far-field approximation.

Let \( A \) be the source strength. The equivalent strength of an array element is \( \frac{A}{L} dL \). Suppose the source is represented by an array of dipoles. Under the far-field approximation and assuming source motion, the total pressure field is given by [9]

\[
p(R, t) = \frac{A}{L} \int_{-R_i}^{R_i} \frac{\sin \theta}{|1 - M \cos(\phi - \phi_s)|} \frac{1}{4\pi R_i} \left( \frac{1}{c} \frac{\partial}{\partial t} F(t - \frac{R_i}{c}) \right) dL \tag{4.21}
\]

Since the separation between the elements of the array is small compared to the distance of the source from the receivers, \( \frac{1}{R_i} \) can be expressed in powers of \( \frac{\Delta L}{R} \). Upon evaluation of the integral in Equation 4.21, the correction term in the directivity pattern due to large aperture of the source is given by

\[
b(\phi, \theta) = \frac{\sin(kL/2 \cos \theta \cos(\phi - \phi_s))}{kL/2 \cos \theta \cos(\phi - \phi_s)} \tag{4.22}
\]

The spatial phase interference due to the inclusion of this term in the directivity pattern causes the latter to vary rapidly in azimuth, depending on the number of minor lobes.
4.3.2 Correction in the vertical directivity pattern due to the arrival of the longitudinal wave

In Chapter 2, I have discussed that in some event time series two arrivals are detected, the first being that of the longitudinal wave and the second that of the acoustic wave. The energy from the longitudinal wave remains trapped in the ice for long distances as it is radiated into the water. The vertical radiation pattern of this wave is a narrow beam around the corresponding Mach angle, i.e., \( \theta_l = \cos^{-1}(1/M_l) \approx 62^\circ \), where \( M_l \) is the ratio of the propagation speed of this wave to the sound speed in water [26]. Refraction and scattering of the wave from discontinuities in the ice cover significantly affect its radiation pattern. Finally, events that are very distant from the hydrophone array are not analyzed here, for the reasons previously stated. Among the events analyzed here, there are only 8 events, in the time series of which the arrival of the longitudinal wave is detected. Therefore, although for other problems it may be important to include the Mach beam due to this wave in the vertical directivity pattern of the assumed model, I have neglected it in this analysis. I believe that the uncertainty associated with other event parameters, as well as estimation errors in the event location, peak frequency and source speed are more important than the effect of the omitted longitudinal Mach beam.

Prior to the estimation of the horizontal and vertical radiation patterns of each event, the recorded peak amplitudes are examined for changes in their sign. Polarity is important for sources with a characteristic horizontal directivity pattern, e.g., double couples. The results are summarized in Table 4.3.

<table>
<thead>
<tr>
<th>Sign change</th>
<th>No. of events</th>
<th>% of total # of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>1 or 2 channels</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td>3 or more channels</td>
<td>142</td>
<td>79</td>
</tr>
</tbody>
</table>

Table 4.3: Event statistics as a function of peak amplitude sign change.

In Chapter 2, it has been discussed that reflection of a sound ray off the ice cover causes a change in the sign of the peak event pressure. However, there are only 5 events the distance
of which from some hydrophones is greater than the cycle distance, computed in Appendix C. These events are not analyzed here since their large distance from the array results in a small azimuth range of observation, $10^\circ - 20^\circ$. Consequently, no useful information on their horizontal radiation pattern can be extracted. In general, therefore, a sign change in the observed peak amplitude at some hydrophones must be attributed to the event horizontal radiation pattern. The vertical directivity, which is a function of the cosine and/or sine of the launch angle $\theta$ is restricted to a range $0^\circ - 90^\circ$, within which both trigonometric functions and consequently their combinations are non-negative. Also, the correction term due to source motion is $\frac{1}{|1-M\cos(\phi - \phi_s)|}$ and thus always positive. A sign change in peak event amplitude only in 1 or 2 channels is probably not associated with the event radiation pattern, but may be due to the hydrophone itself (although I have no information on any hydrophone being incorrectly cabled). I observe that these hydrophones are usually the outermost ones of the legs of the array, i.e., any two of channels 15,18, 21 and 24, shown in Figure 2.1. I, therefore, assume that for 39 events there is no change in the sign of the peak event amplitude. For the remaining events, I observed that the change in the sign of the peak event amplitude is consistent in groups of hydrophones, e.g., in all hydrophones at any particular leg of the array, such as 13-15, 16-18, etc.

4.3.3 Dipole model

In the previous section, the concepts of a point force and a system of point forces couples have been introduced. The sound radiation resulting from a compact point force in the medium is described by a dipole acoustic model. The pressure field due to a dipole source, first assuming source motion and then the special case of a motionless source is given by Equations 4.23 and 4.24, respectively:

$$p = \frac{\mathcal{F}(R)}{4\pi} \left( \frac{\sin \theta}{|1-M\cos(\phi - \phi_s)|} \right) \left( F(t - \frac{R}{c}) \mathcal{F}(R) + \frac{1}{c} \frac{\partial F(t - \frac{R}{c})}{\partial t} \right)$$ (4.23)

$$p = \frac{\sin \theta \mathcal{F}(R)}{4\pi} \left( F(t - \frac{R}{c}) \mathcal{F}(R) + \frac{1}{c} \frac{\partial F(t - \frac{R}{c})}{\partial t} \right)$$ (4.24)
where \( F(t - \frac{R}{c}) \) is the dipole force function delayed by time \( \frac{R}{c} \), \( \mathcal{F}(R) \) the spreading function, \( \phi_s \) the source orientation, \( \theta \) the launch angle and \( M \) the Mach number, i.e., \( \frac{V}{c} \), the ratio of source speed \( V \) to the sound speed in water \( c \). In Equation 4.24 the Doppler factor is 1, since \( M = 0 \). In the absence of source motion, radiation from a dipole source is independent of azimuth. The Doppler factor \( \frac{1}{|1 - M \cos(\phi - \phi_s)|} \) introduces a horizontal directivity, though, when \( M \neq 0 \). The horizontal and vertical radiation patterns of a dipole source, corrected by the Doppler factor are shown in Figure 4-7. It is assumed that source orientation is 60\(^\circ\) and the Mach number \( M = 0.5 \).

![Horizontal radiation pattern of a modified dipole: Doppler factor (1/1 - Mcos(\phi))](image1)

![Radiation pattern of a vertical dipole: g(\theta) = sin(\theta)](image2)

Figure 4-7: Horizontal and vertical radiation patterns of a dipole modified by the Doppler factor. Radial increments correspond to values of \(|B(\phi)|\), versus \( \phi \) the horizontal angle (\(^{\circ}\)), in (a) and \(|g(\theta)|\) in (b), versus \( \theta \) the vertical angle (\(^{\circ}\)), respectively.

Since both the Doppler factor and the vertical directivity of the compact dipole source do not introduce any sign change in the peak event amplitude as a function of the horizontal angle \( \phi \), no sign change is expected for events the radiation of which is best described by
this model. In regard to the analyzed events, dipole radiation under no source motion is assumed when the difference between the maximum and minimum source levels is 3-4 dB or less and the estimated propagation speed is negligible.

Example 1: The radiation characteristics of a detected simple type I event are discussed in this example. The peak event frequency is 54 Hz and the source speed is 11 m/s. There is no sign change in peak event amplitude at any hydrophone. The data are corrected assuming a vertical dipole. The horizontal event radiation pattern is shown in Figure 4-8:

![Graph showing the horizontal event radiation pattern](image)

Figure 4-8: Horizontal event radiation pattern. Source level (in dB re 1 μPa, 1 Hz and 1 m) as a function of azimuth $\phi$. The estimated horizontal dipole pattern, modified by the Doppler factor, is superimposed to the data. The difference between maximum and minimum source levels is 4.5 dB and the azimuth range of observation is $148^\circ$. The source speed is negligible ($M = 0.008$), and thus event radiation is approximately independent of azimuth : $|B(\phi)| \simeq 1$.

The mean error between the model and the data is 1.3 dB, and the standard deviation
is 1.2 dB. This type of radiation pattern is not unique to type I events. There are 20 narrow-width events of different signature types, and source speeds 200 m/sec or lower, the radiation characteristics of which are best described by this model. Their peak frequencies are both below and above 100 Hz (50% low-frequency and 50% mid-frequency events).

Under the far-field approximation, the first term in Equations 4.23 and 4.24 can be neglected. Then, the peak dipole force \( F_{\text{peak}} \) for narrow-width events is given by

\[
F_{\text{peak}} = \frac{4\pi(1 - M\cos(\phi - \phi_s))}{\mathcal{F}(R)ksin\theta} p_{\text{peak}} \xrightarrow{\lim_{\lambda \to 0}} F_{\text{peak}} = \frac{4\pi}{\mathcal{F}(R)ksin\theta} p_{\text{peak}} \tag{4.25}
\]

where \( p_{\text{peak}} \) is the peak event amplitude. The source strength is estimated as the mean peak dipole force among the channels that have recorded the event. Dipole force as a function of peak frequency is shown in Figure 4-9, for the 20 events that are represented by dipoles. The data have been corrected for large aperture of the source, when necessary.

![Graph showing dipole force versus peak event frequency](image)

Figure 4-9: Dipole force versus peak event frequency. The decrease in force with increasing frequency is best described by a function \( F_{\text{dipole}}(f) \propto f^{-3} \) (solid line).
For these data, dipole force is inversely proportional to $f^3$. The scatter of the data as a function of frequency is less than one order of magnitude. Dyer [11] used an empirical formula for the relationship between dipole force and peak frequency, assuming no source motion and for low-frequency events, i.e., $f < 80$ Hz. The formula has the form

$$F_{\text{peak}} = \frac{2.5 \times 10^7}{1 + \left(\frac{f}{f_0}\right)^2}[N]$$  \hspace{1cm} (4.26)

where $f_0 = 7.5$ Hz. The latter value has been assumed, based on Dyer's results; no independent evaluation of $f_0$ can be achieved with these data. However, the data on which this formula is based showed significant spread; Dyer thus suggested that a function of $f^{-3}$ could be used instead. The empirical relation between dipole force and peak event frequency that best describes the data in this thesis, assuming negligible source motion, is

$$F_{\text{peak}} = \frac{3 \times 10^{8}}{1 + \left(\frac{f}{f_0}\right)^3}[N]$$  \hspace{1cm} (4.27)

The above function fits the data both in the low- and mid-frequency ranges ($20 \leq f \leq 350$).

4.3.4 Floe unloading

Dyer [11] and later Chen [6] proposed the ridge or floe unloading mechanism, as a plausible event generating ice mechanism at low frequencies. Under accumulated moment due to the wind and current drag, a deformed ice floe breaks and returns to its buoyant position. This unloading motion involves a volume change and can thus be modeled by an acoustic vertical dipole. It is a mechanism that involves no source motion. A detailed description of the model can be found in [11] and [6]. Dyer considered a beam attached to a ridge cone. The beam deforms under a continuously applied load, such as the environmental loads applied to the ice cover, until it breaks. The deformed beam returns to its buoyant position and this motion introduces a volume change in the water. The vertical dipole force induced by this type of motion is given by [11][6]

$$F = \left(\frac{\sigma}{E}\right)\left(\frac{1 - \nu^2}{2}\right)\frac{32\rho_w l_s a_2}{\pi f} \frac{l_0^2}{H b_1 c_g^3} \left[1 + \frac{4c_g}{2\pi fl_0}\right]$$  \hspace{1cm} (4.28)
where $\sigma$ is the maximum flexural stress in ice, $E$ the Young's modulus, $\nu$ the Poisson ratio of ice, assumed here to be 0.33, $l_s$ the separation of the volume change and its image, i.e., $l_s = 2h$, $a_2$ the ratio of the ice beam width to the characteristic length of the ice beam, $\rho_w$ the density of water, $H = 2h$ the ice thickness, $l_0$ is the diameter of the ridge cone, $b_1$ the parallel scale of the applied moment and $c_g$ the group speed of the flexural wave in ice. Assuming that ice thickness is on average 3 m, $c_g$ (in m/sec) is given by: $c_g = 200 \times f^{0.6}$ for $f < 30$ Hz, $c_g = 1414$ for $f = 100$ Hz, and $c_g = 1200$ for $f > 300$ Hz [6][38]. To compare the predicted dipole force with that estimated for each of the 20 events, I used the following values for the various parameters in Equation 4.28: $\frac{\sigma}{E} = 3 \times 10^{-4}$, $l_s = H = 3.0$ m, $a_2 = 0.1 \left( \frac{2}{L} \right)$ and $l_0 = b_1 = L$, where $L$ is the length that I have estimated for each event in Chapter 3. Superimposed data and predicted dipole force values are shown in Figure 4-10. The solid and dashed lines are the functions in Equations 4.26 and 4.27, respectively.

![Predicted dipole force by unloading model, superimposed to data](image)

**Figure 4-10**: Superimposed estimated dipole force for the analyzed events and predicted dipole force by the floe unloading model.
At low frequencies, the predicted and estimated dipole force values differ by roughly an order of magnitude (as \( f \to 100 \) Hz), or less (for \( f < 40 \) Hz). Also, since dipole force is inversely proportional to \( f^3 \) for these data, and thus decreases faster at high frequencies than the model dipole force, which is inversely proportional to \( f^2 \), the error between the two sets of values increases with increasing frequency. For example at frequencies above 300 Hz it is about 1.5 orders of magnitude. The frequency dependence of dipole force in Equation 4.28 must be modified to include the \( f^{-3} \) dependence of the data. Dyer modified the unloading model to account for creep of the ice. Creep is a time-dependent phenomenon; thus the introduction of a time-dependent parameter (linear in time) in Equation 4.28 would correspond to a change from the \( \simeq f^{-2} \) relationship between dipole force and peak event frequency to the desired \( f^{-3} \). Dipole force, as derived by Dyer for unloading, is a function of several parameters, among which are the dimensions of the deformed floe. Figure 4-11 shows both the predicted and estimated event dipole force as a function of length:

Figure 4-11: Dipole force versus source length. Solid line: best fitted function \( F_{dipole} \propto L^3 \).
Notice that since length is proportional to source speed, the estimated lengths for the low-speed events are also low, in the range 1-25 m. The agreement between data and theory is fair. The error is much less than one order of magnitude for the entire length range. Source length, as estimated in Chapter 3, is inversely proportional to the corner frequency \( \omega_1 \), not the peak event frequency. It is actually a function of speed \( V \) which, in turn, is a function of peak frequency. Therefore, the exact relationship between source length and peak frequency is not known. Figure 4-11 shows that the data are proportional to \( L^3 \). Deviations from the best fitted curve occur at lengths \( L \approx 2 \) m, and \( L \approx 7 \) m. Since the data are also inversely proportional to \( f^3 \), the relationship between the two variables is \( L \propto \frac{1}{f} \), for this event sub-population. The theoretical values are proportional to \( L^2 \).

There are two issues that must be addressed at this point. First, the estimated source speeds for these events, though low, are nevertheless non-zero. Their mean speed is 116 m/sec and the standard deviation is 74 m/sec. It is possible that they may have been over-estimated by the procedure described in Chapter 3, which is sensitive to several parameters including the accuracy of the estimated Doppler shift and that of peak event frequency. The second issue of concern is that both Dyer and Chen have deduced that unloading motion is a plausible mechanism at low frequencies, whereas the above-discussed events have peak frequencies both in the low- and mid-frequency ranges. Chen's conclusion is based on the fact that her data spread at frequencies above 100 Hz was 3 to 4 orders of magnitude and that the calculated dipole force values deviated appreciably from those predicted by the model. For these data, the spread in the entire frequency range of interest is smaller than 1 order of magnitude. The deviation of the calculated dipole force values from those predicted by the unloading model is due to the difference between the frequency dependence of the data and that of the model. I expect that the modified creep effect model fits that data in a more satisfactory manner. An important observation that also leads me to believe that the analyzed events are associated with some type of unloading motion is their occurrence in the ambient noise data relative to that of other types of events. Acoustic events attributed to floe unloading are consistently detected following time intervals of high ice fracture activity. In particular, mid-frequency events the radiation characteristics of which are best described by more complicated acoustic models, and for which the estimated propagation
speeds are in the range 600-1400 m/sec, typically precede the discussed low-speed events. An example is shown in Figure 4-12:

![Figure 4-12: Event sequence. Only 10 channels are shown. At approximately time 3.5 sec, a low frequency type I event occurs, which is modeled by a dipole. The estimated source speed of this events is 32 m/sec. Both this event and the ones preceding it are located in the same area.](image)

The event that occurs at time $t \simeq 3.5$ sec is low-frequency ($f_{peak} = 54$ Hz), low-speed and its radiation pattern is best represented by a vertical dipole. In contrast, the two low-frequency events at times $t \simeq 1.6$ sec and $t \simeq 2.8$ sec, respectively, i.e., both prior and after the strong activity in the interval $t \simeq 2 - 2.7$ sec have both higher source speeds and their radiation patterns are described by more complicated acoustic models. The analysis of event sequences, such as that shown in 4-12, may provide insight on ice motion at a large scale and on the mechanisms associated with different stages in the fault formation, as discussed
in Chapter 3. At this point I conclude that unloading motion of the ice is a plausible event generating mechanism both in the low- and mid-frequency ranges. The scaling dimension is the range of deformation (L^3-dependence) and dipole force shows a f^{-3} dependence on peak event frequency. The two variables are, therefore, inversely proportional to each other. The physical mechanism appears to be uncorrelated with the event signature type although the event sample is not sufficiently large to assume such independence with confidence.

**Example 2:** The radiation characteristics of a type III event are discussed in this example. \( f_{\text{peak}} = 76 \text{ Hz} \) and source speed \( V = 420 \text{ m/sec} \). There is no sign change in peak event amplitude at any hydrophone. The data are corrected assuming a modified vertical dipole. Its horizontal radiation pattern is shown in Figure 4-13:

![Figure 4-13: Event horizontal radiation (source level (dB re 1\mu Pa, 1 Hz, 1 m) versus horizontal angle (°)) and superimposed Doppler factor. The difference between maximum and minimum source levels is about 7 dB. The data spread in azimuth is 126°.](image-url)
Source motion is clearly associated with this event generating mechanism. The mean error between the data and the radiation pattern of the model is 1.5 dB and the standard deviation is 0.6 dB. For this particular event, the agreement between the model and the data is good. A preliminary analysis of the radiation characteristics of the detected event population, excluding events that are believed to be associated with unloading motion, showed that there are 52 events with peak frequencies below 100 Hz and speeds 400-850 m/sec, the radiation characteristics of which are described by a modified dipole. Dipole force as a function of peak event frequency is shown in Figure 4-14. The data have been corrected for large aperture source effects when necessary.

![Modified dipole force vs peak event frequency](image1)

![Modified dipole force vs peak event frequency](image2)

Figure 4-14: Dipole force versus peak event frequency for low-frequency events (log-log plot). Superimposed are the curves in Equations 4.26 (lower plot) and 4.27 (upper plot), for $F_{dip}$ proportional to $f^{-2}$ and $f^{-3}$, respectively.
The relationship between dipole force and peak event frequency is not as clear as in the previously discussed event sub-population, despite the fact that the correction for large aperture source effects has reduced the scatter of the data, particularly as \( f \to 100 \text{ Hz} \). This suggests that an alternative acoustic model may be more appropriate. The data spread as a function of frequency is on average 1-1.5 orders of magnitude. The functions in Equations 4.26 and 4.27 are superimposed on the \( \lambda \). In the former equation, the constant \( 2.5 \times 10^7 \) in the numerator is replaced by \( 7 \times 10^7 \). At frequencies below 30 Hz, the data are sparse and it is, therefore, difficult to determine which function fits the data better. At higher frequencies, two trends are observed due to the spread of the data. The function which assumes \( F_{dp} \propto f^{-3} \) fits part of the data, which show a rapid decrease with increasing frequency. The function which assumes \( F_{dp} \propto f^{-2} \) fits another part of the data, as shown in Figure 4-14. Thus, there is no unique integer power-law relationship between dipole force and peak frequency.

There are two possible explanations for the above results. First, given that 60% are either compound or type IV events and since their peak event frequency cannot be clearly distinguished in the spectrum of such events, some of the estimated frequency values may be inaccurate. Consequently, the representation of dipole force as a function of this parameter may not be meaningful for all analyzed events. Alternatively, the model itself may be inadequate or may require modification. Although the fit between the data and the assumed modified dipole is satisfactory within the azimuthal range of observation, when the latter is restricted to a sector of about 90° or less, the adequacy of the model cannot be deduced with confidence. Given the estimated source speeds, it is possible that fracture is the event generating mechanism. Although mid- and high-frequency events have been associated with fracture processes in past studies [38] [6], these events may be associated with some type of precursory, low-frequency motion. Indeed, upon examination of the ambient noise data, I observed that approximately 50% of the analyzed events precede mid-frequency events which are attributed to fracture. An example is the event that occurs at time \( t \simeq 1.6 \) sec, in the data segment shown in Figure 4-12.

The adequacy of the model must be investigated further. If fracture is the event generating mechanism and given that event horizontal radiation patterns are characterized only by
the Doppler factor, other possible models include an axial (longitudinal) octopole along the z-axis, and a superposition of three longitudinal octopoles that represents tensile fracture in the $z = 0$ plane. The directivity pattern of both models is a function of the launch angle $\theta$ (excluding the Doppler factor). The models are discussed in the next section and the events are re-examined. Another issue of concern is that only for 10 of the 52 events there is no sign change in their peak amplitudes at any hydrophones. This is explained, if violation of the small aperture source effect is taken into account. Next, I examine the relationship between dipole force and source length. Figure 4-15 shows the distribution of dipole force with this parameter:

![Dipole force versus source length](image)

Figure 4-15: Dipole force versus source length for low-frequency events. Superimposed is the best fitted curve, $F_{\text{dipole}} \propto L^2$. The data have been corrected for large aperture of the source.

Despite the fact that the spread of the data has been reduced by taking into account large aperture of the source, scaling of dipole force with source length is inadequate. In Chapters 2 and 3, mean slip, source dimensions and consequently seismic moment have
been estimated for the analyzed events, using Equation 4.11. Although this parameter is also estimated from the octopole source strength, I use the estimates from Chapter 3, initially to assess the relationship between the moment and source geometry and determine if a different acoustic model is more appropriate to describe the radiation pattern of the above-discussed events. Figure 4-16 shows the estimated seismic moment as a function of source length and superimposed are predicted values by the $W$-model. The latter assumes that depth is the controlling dimension of the source. i.e., $M_n \propto LW^2$.

![Seismic Moment versus length](image)

Figure 4-16: Seismic moment versus source length for low-frequency events. Superimposed are the predicted by the $W$-model moment values.

The only significant deviations of the data from the predicted values occur at $L \approx 35$ m and are approximately 0.5 of an order of magnitude. On average, the agreement of the data with the $W$-model is good and suggests first, that the analyzed events are probably best represented by a different acoustic model and second, that the controlling dimension of the source is depth. Notice that the $W$-model is typically used for scaling small earthquakes.
The above observations lead me to the presentation of the octopole model, equivalent to the double-couple body force system, used to represent fracture modes. The 52 events analyzed in Section 4.3 are further examined under the assumption of an octopole acoustic model, as well as the remaining events which have not yet been discussed.

4.3.5 Octopole model

The assumptions made for this model have been stated at the beginning of Section 4.3. In all calculations, appropriate corrections for a large aperture source have been made, when necessary. The octopole model results from the transformation of a quadrupole (or double force couple) by the free surface. There are two types of quadrupoles, and consequently two types of octopoles, the lateral and longitudinal. Appendix G describes the two types and the derivation of their respective horizontal and vertical radiation patterns. The pressure field due to an octopole in the presence of source motion is given by

\[
p(\vec{R}, t) = \frac{1}{|1 - \mathcal{M}\cos(\phi - \phi_s)|} \frac{g(\theta)B(\phi)\mathcal{F}(R)}{2\pi} \cdot \left\{ \frac{1}{c^3} \frac{\partial^3}{\partial t^3} (Mh) + \frac{3\mathcal{F}(R)}{c^2} \frac{\partial^2}{\partial t^2} (Mh) + 6\mathcal{F}(R^2) \frac{\partial}{\partial t} (Mh) + 6\mathcal{F}(R^3)(Mh) \right\}
\]

(4.29)

where \(g(\theta)\) is the vertical directivity, \(B(\phi)\) the horizontal directivity, \(M\) the force couple moment and \(\mathcal{F}(R)\) the spreading function. Horizontal and vertical radiation patterns of different types of octopoles and corresponding force couples in Figure 4-2 are summarized in Table 4.4:

<table>
<thead>
<tr>
<th>Type</th>
<th>(g(\theta))</th>
<th>(B(\phi))</th>
<th>Double Couple</th>
</tr>
</thead>
<tbody>
<tr>
<td>lateral (x-y)</td>
<td>(\sin\theta\cos^2\theta)</td>
<td>(\sin\phi\cos\phi)</td>
<td>(1,2), (2,1)</td>
</tr>
<tr>
<td>lateral (x-z)</td>
<td>(\sin^2\theta\cos\theta)</td>
<td>(\cos\phi)</td>
<td>(1,3), (3,1)</td>
</tr>
<tr>
<td>lateral (y-z)</td>
<td>(\sin^2\theta\cos\theta)</td>
<td>(\sin\phi)</td>
<td>(2,3), (3,2)</td>
</tr>
<tr>
<td>longitudinal (x-x)</td>
<td>(\cos^2\theta\sin\theta)</td>
<td>(\cos\phi)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>longitudinal (y-y)</td>
<td>(\cos^2\theta\sin\theta)</td>
<td>(\sin\phi)</td>
<td>(2,2)</td>
</tr>
<tr>
<td>longitudinal (z-z)</td>
<td>(\sin^3\theta)</td>
<td>1</td>
<td>(3,3)</td>
</tr>
</tbody>
</table>

Table 4.4: Vertical and horizontal directivity patterns of different types of octopole and corresponding double couples.
Individual octopoles or their combinations are used to describe the radiation field induced by fracture processes. The following are examples of a lateral octopole in the x-y plane, and a longitudinal octopole along the x-axis.

The radiation pattern of the lateral octopole, with horizontal and vertical directivity $B(\phi) = \cos\phi \sin\phi$ and $g(\theta) = \sin\theta \cos^2\theta$, respectively, is shown in Figure 4-17:

![Horizontal and vertical radiation patterns of a lateral octopole.](image)

Figure 4-17: Horizontal and vertical radiation patterns of a lateral octopole in the $x - y$ plane. Plot (a) on the left, shows the horizontal directivity $|B(\phi)|$ versus azimuth (°). Plot (b) shows the vertical directivity function $|g(\theta)|$ versus vertical angle $\theta$ (°).

The polarity of the source is important; adjacent segments of the octopole horizontal directivity pattern are of opposite sign, as shown in the above plot. Thus, it is expected that for events, the radiation characteristics of which are described by this model, there is variation in the sign of their peak amplitudes at different hydrophones, according to the latter's location relative to the source.

The radiation pattern of a longitudinal octopole, with horizontal and vertical directivity
\[ B(\phi) = \cos^2 \phi \text{ and } g(\theta) = \sin \theta \cos^2 \theta, \text{ respectively, is shown in Figure 4-18:} \]

Figure 4-18: Horizontal and vertical radiation patterns of a longitudinal octopole, along the x-axis. Plot (a) on the left, shows the horizontal directivity \( |B(\phi)| \) versus azimuth (°). Plot (b) shows the vertical directivity function \( |g(\theta)| \) versus vertical angle \( \theta \) (°).

Under the far-field approximation, only the first term in Equation 4.29 is retained:

\[
p(\vec{R}, t) \simeq \frac{g(\theta)B(\phi)\mathcal{F}(R)}{2\pi} \frac{1}{c^3} \frac{1}{\partial t^3} (Mh) \cdot \frac{1}{|1 - M \cos(\phi - \phi_s)|} \quad (4.30)
\]

In the case of narrowband events, the peak pressure due to an octopole can be approximated by

\[
p(\vec{R}) \simeq \frac{1}{|1 - M \cos(\phi - \phi_s)|} \frac{g(\theta)B(\phi)\mathcal{F}(R)}{2\pi} k^3 Mh \quad (4.31)
\]
where $p(\vec{R})$ is the peak event amplitude. Therefore the source strength is given by

$$S = k^3 Mh = p(\vec{R}) \frac{2\pi(1 - \mathcal{M} \cos(\phi - \phi_s))}{g(\theta) B(\phi) \mathcal{F}(R)}$$  \hspace{1cm} (4.32)$$

and consequently the moment $M$ is

$$M = p(\vec{R}) \frac{2\pi(1 - \mathcal{M} \cos(\phi - \phi_s))}{k^3 h g(\theta) B(\phi) \mathcal{F}(R)}$$  \hspace{1cm} (4.33)$$

A large-aperture source can be represented by an array of octopoles. Under the far-field approximation, the total pressure field is given by [9]

$$p(\vec{R}, t) = \frac{A}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{|1 - \mathcal{M} \cos(\phi - \phi_s)|} \frac{g(\theta) B(\phi) \mathcal{F}(R_s)}{2\pi} \frac{1}{c^3 \partial^3} (Mh) dL$$  \hspace{1cm} (4.34)$$

where $A$ is the source strength of each array element and $R_s$ is given by Equation 4.20. The above equation is analogous to Equation 4.21 for the large-aperture dipole. The correction to the directivity pattern is the expression $b(\phi, \theta)$, given by Equation 4.22. Corrections for large aperture of the source are introduced for all events for which $\frac{A}{L} \ll 1$.

### 4.3.6 Fracture

Sound radiation from a shear fracture is modeled by a lateral octopole. A superposition of longitudinal octopoles is used to model sound radiation from tensile fractures. Although the two types of fracture processes are not necessarily physically dependent, it has been observed [36] that the propagation of shear cracks is enabled by the formation and coalescence of arrays of tensile cracks.

#### Tensile fracture

Large faults typically occur due to the propagation of shear cracks, not tensile cracks. However, the latter are part of the fault formation and acoustic emissions from such cracks are detectable in the ocean ambient noise. Indeed, according to previous studies of event physics [30] [38], tensile fracture is a plausible event generating mechanism in the mid-
and high-frequency ranges. In Section 4.1 it has been shown that tensile fracture may be represented by three superimposed double couples, or longitudinal octopoles and their ratio of magnitudes depends on the plane at which fracture occurs. I assume that the Lamé constants of ice are such that $\lambda = \mu$. First, consider a tensile fracture in the $x - y$ plane, i.e., the tensile forces are in this plane, as shown in Figure 4-19:

![Diagram of tensile fracture](image)

Figure 4-19: Tensile fracture in the $z = 0$ plane and its body-force equivalent. Displacements are normal to the crack tip.

In the ice, such a crack would occur under changes in temperature. The ratio of the magnitudes of the octopoles is 1:1:3. Therefore, the directivity pattern $g(\theta)B(\phi)$ is given by

$$g(\theta)B(\phi) = \cos^2 \phi \cos^2 \theta \sin \theta + \sin^2 \phi \cos^2 \theta \sin \theta + 3 \sin^3 \theta = \sin \theta(1 + 2 \sin^2 \theta)$$

(4.35)

The above equation has been obtained simply by superimposing the three longitudinal octopoles with the desired ratio of magnitudes and using elementary trigonometric identities. The directivity pattern of this crack is independent of azimuth, as expected, since crack propagation is in the $z$-plane. In this case, the direction of propagation is the dip-angle equal to $90^\circ$. The vertical directivity pattern of the crack is shown in Figure 4-20:
Figure 4-20: Vertical directivity pattern of tensile crack propagating in the z-direction. Directivity $|g(\theta)|$ versus vertical angle $\theta (^o)$.

This pattern is dominated by that of a longitudinal octopole in the z-direction ($sin^3 \theta$ term in Equation 4.35) but there are contributions from the longitudinal octopoles in the other two directions, of which the pattern of a vertical tensile crack is composed.

In Section 4.3.4 it has been shown that there are 52 events, with peak frequencies in the low-frequency range, and propagation speeds 400-850 m/sec, the horizontal directivity of which is best described solely by the Doppler factor. The modified dipole model may not be the most appropriate model, given the spread of the estimated dipole force with peak frequency and source length. The horizontal directivity pattern of a vertical tensile crack is also characterized solely by the Doppler factor, i.e. $B(\phi) = 1$. I, therefore, examine these events here, assuming their directivity pattern is given by Equation 4.35. Corrections are made for large aperture of the source, when necessary.

First, I compare the octopole (seismic) moment, calculated from the event source strength, i.e., using Equation 4.33, with that calculated directly from the fault area and
mean slip, i.e., using Equation 3.13. The two sets of estimates are shown in 4-21, as a function of source length:

![Comparison of estimated seismic moment](image)

Figure 4-21: Comparison between estimated seismic moment from the source dimensions and mean slip (marked with +) and estimated moment from the event octopole source strength (marked with o). The mean and standard deviation of the difference of the two estimates is 6%.

The agreement of the results from the two estimation procedures is good. The maximum difference between the two sets of values is about 23%. Given that slip has been estimated as accurately as possible, the same corrections, i.e., horizontal and vertical directivity, Doppler factor and transmission loss, have been made in both estimations of seismic moment from the event acoustic signals. The agreement of the two sets of values indicates that the fault dimensions and consequently the fault area has been estimated accurately, for each event.

Next, I examine the variation of octopole moment as a function of peak event frequency, shown in Figure 4-22. I use the moment estimates obtained from octopole source strength, including corrections for large aperture of the source:
Figure 4-22: Octopole moment as a function of peak event frequency. The data have been corrected for large aperture source effects, when necessary. Superimposed is the curve $M_{oct} \propto f^{-2}$.

The spread of data as a function of peak event frequency is significant (2-3 orders of magnitude at frequencies between 40 and 80 Hz), and much more pronounced than the corresponding spread of dipole force. Thus, in terms of this comparison, the modified dipole is a more appropriate model. But, peak event frequency may not be the most appropriate scaling parameter for seismic moment. Thus, I also examine the relationship between octopole moment and source length, shown in Figure 4-23:
Figure 4-23: Octopole moment as a function of source length. Superimposed are the predicted moment values, by the $W$-model.

A result that appears suspicious is that source length reaches 100 m for these events, the highest estimate of source length for the entire event population. If tensile fracture is the event generating mechanism, this is surprising. Tensile fractures are not usually associated with large faults, at least in rock. Also, the analyzed events are in the low-frequency range, and tensile fracture has been shown to induce mid- and high-frequency acoustic events. On the other hand, there is a clear relationship between octopole moment and source length, as well as an agreement between the data and the predicted seismic moment by the $W$-model which is typically used for scaling fractures. The results are, therefore, inconclusive in regard to the event generating mechanism of the above-discussed events.

Now, consider a tensile crack in the $x - z$ plane, shown in Figure 4-24:
Figure 4-24: Tensile fracture in the $y = 0$ plane and its body-force equivalent.

Such a crack could occur in an ice floe if, for example, adjacent floes exert force on it. The ratio of the magnitudes of the octopoles is 1:1:3. Assuming that the dip angle is zero and the propagation direction is along the $y$-axis, the directivity pattern for this type of fracture is given by

$$B(\phi)g(\theta) = \cos^2\phi\cos^2\theta\sin\theta + 3\sin^2\phi\cos^2\theta\sin\theta + \sin^3\theta = \sin\theta(1 + 2\sin^2\phi\cos^2\theta) \quad (4.36)$$

Similarly, for a tensile crack in the $y - z$ plane,

$$B(\phi)g(\theta) = 3\cos^2\phi\cos^2\theta\sin\theta + \sin^2\phi\cos^2\theta\sin\theta + \sin^3\theta = \sin\theta(1 + 2\cos^2\phi\cos^2\theta) \quad (4.37)$$

Both patterns are dominated by the pattern of the longitudinal octopole in the respective directions of propagation. However, there are contributions by the 2 other octopoles. Tensile fracture is associated with a volume opening in the medium, in the sense that the radiation pattern is affected by three-dimensional motion.

There are 28 events, the radiation characteristics of which are modeled by the above combinations of longitudinal octopoles and are thus believed to have been induced by tensile
fracture. All events are simple, but there is no predominant signature type that characterizes this group. The source speed of the events is in the range 600-1098 m/sec. Their peak frequencies are between 80 and 262 Hz. Only two events are in the low-frequency range. First, I compare the octopole (seismic) moment, calculated from the event source strength, i.e., using Equation 4.33, with that calculated directly from the fault area and mean slip, i.e., using Equation 3.13. The error between the two estimates is shown in Figure 4-25:

![Comparison of estimated seismic moment](image)

Figure 4-25: Comparison between estimated seismic moment from the source dimensions and mean slip (marked with +) and estimated moment from the event octopole source strength (marked with o). The mean and standard deviation of the difference of the two estimates are 0.4 and 0.35 of one order of magnitude, respectively.

On average, the agreement between the two sets of values is good. Their maximum difference is about 1 order of magnitude; the values obtained from the event octopole source strength are consistently lower. Moment as a function of peak event frequency is shown in Figure 4-26:
Figure 4-26: Octopole moment as a function of peak event frequency. Superimposed are the curves $M_{oct} \propto f^{-3}$ (solid line), which fits the data at frequencies below 150 Hz, and $M_{oct} \propto f^{-5}$ (dotted line), at frequencies above 150 Hz.

There is no consistent relationship between octopole moment and peak event frequency. At frequencies approximately above 150 Hz, the estimated moment is significantly lower than that predicted by the $M_{oct} \propto f^{-3}$ dependence at lower frequencies. Also, the decrease of moment with increasing frequency is faster, i.e., $M_{oct} \propto f^{-5}$. A possible explanation is that peak event frequency is averaged over all hydrophones. Source motion is significant, and thus the Doppler shift between peak event frequencies at different hydrophones are also large. Consequently, this averaging results in higher or lower than expected mean frequency values. Thus, it may be more appropriate to examine the data as a function of source length, which is inversely proportional to frequency or as a function of the estimated intrinsic source frequency. Figure 4-27 shows the variation of octopole moment as a function of source length:
Figure 4-27: Octopole moment as a function of source length. Superimposed is the curve $M_{oct} \propto L^3$.

Source length varies between 4 and 25 m. The spread of the data is on average less than one order of magnitude. The deviations of the data from the best fitted curve $M_{oct} \propto I^3$ are even smaller. The observed dependence of fracture moment on length is surprising. It implies that the $L$-model is the most appropriate scaling model and thus the reference dimension for strain is the fault length. This is a characteristic of large fault processes and tensile fractures do not belong in this category. However, the agreement of the selected acoustic model with the data, the range of peak frequencies of the events and the source speed estimates, indicate that the tensile fracture is the most plausible generating mechanism of the above-discussed events and occurs predominantly at frequencies above 100 Hz.
4.3.7 Shear fracture

Systems of double couples (lateral octopoles) are used to model shear fractures. So far in the discussion, no events of a particular type have been discussed. The three event sub-populations already analyzed, are not characterized by a predominant signature. 60% of the events modeled by lateral octopoles are type III. The remaining 40% includes both type I and type IV events. Type III events have the highest particle and propagation speeds, as well as the highest mean slip and source length.

Example 3: A type III event is considered, with peak frequency 183 Hz and source speed 1064 m/s. A lateral compact octopole is assumed and the peak event amplitudes are corrected by the Doppler factor and vertical directivity, assuming $g(\theta) = \sin\theta\cos^2\theta$. The event source level is examined as a function of azimuth. The range of observation is $240^\circ$.

![Source Level (dB) - Event d1_2](image)

Figure 4-28: Event radiation pattern as a function of azimuth. The horizontal directivity pattern of the assumed lateral octopole, $B(\phi) = \cos(\phi - \phi_s)\sin(\phi - \phi_s)$ is superimposed to the data. The difference between minimum and maximum source level is 21 dB.
Although there are no data in the range $-20^\circ$ to $60^\circ$, the lateral octopole best describes the azimuthal dependence of source level. The sign of the peak event amplitudes has also been examined. In the octopole quadrant where the majority of the data are located, the peak event amplitudes are negative. In the other two quadrants they are positive.

There are 46 events, 35 of which are type III, with peak frequencies above 100 Hz, the radiation characteristics of which are best described by a lateral octopole. Source speed varies between approximately 700-1400 m/s. The relation between peak event frequency and octopole moment, both assuming a lateral octopole, corrected for large aperture of the source when necessary, and a higher order multipole, described later, is not clear. Thus, I examine the distribution of octopole moment with source length.

![Seismic moment versus source length](image)

Figure 4-29: Octopole moment as a function of source length. Superimposed is the curve $M_{\text{oct}} \propto L^2 W$, i.e., the predicted variation of seismic moment, under the assumption of the $L$-model. Type III events are marked with (+) and all other types with (*).
The spread of the data is on average one order of magnitude. The deviation of the data from that predicted by the $L$-model is roughly of this order, too. Notice that the estimated fracture lengths are predominantly in the range 12-70 m. The estimates of octopole moment agree with those obtained directly from mean slip and fault area. They are, however, about 2-3 orders of magnitude higher than those estimated by Chen, for shear-induced acoustic events in the Marginal Ice Zone. Kim [21] estimated the seismic moment required to produce a pressure of 1 Pa, at a range 300 m from the source and at depth 60 m, as $M \approx 8 \times 10^6$ Nm. For the same range, and pressure, I estimate the moment to be $M \approx 1 \times 10^8$ Nm. The seismic moment of an ordinary earthquake is on average $1 \times 10^{20}$ Nm, although there is significant variation of this parameter for earthquakes.

At this point I conclude that, *shear fracture is a plausible event generating mechanism in the mid-frequency range*. The relationship between fracture moment and peak event frequency is, however, not well-defined.

There are 35 events, the radiation characteristics of which are not adequately represented by any of the above-discussed models, even when large aperture of the source is taken into account. Variation of the crack mode as a fracture propagates, or the formation of secondary arrays of tensile cracks off the edges and tips of a shear fracture may be responsible for the complexity of the event radiation pattern.

A shear crack in an elastic medium cannot grow in its own plane for large distances. This may appear confusing since large faults are associated with shear fractures. The propagation of the latter, is actually accompanied by the formation of tensile cracks, at the tips of the shear cracks, in the direction of the applied force and perpendicular to the plane of shearing. Arrays of axial tensile cracks coalesce to form a new shear crack. The repetitive generation of the latter result in the formation of a process zone which is the one of the stages in the formation of a fault. This phenomenon has been observed both in the laboratory and in field studies of rock. It is shown in Figure 4-30:
Figure 4-30: Propagation of tensile cracks from the tips and edges of shear crack in an elastic medium (after Scholz [37]).

4.3.8 High-order multipole

The total moment of a propagating dislocation in a medium is given by the weighted sum of elementary moment tensors, i.e.,

\[ M = f_1(\delta, \phi_s)M^{(1)} + f_2(\delta, \phi_s)M^{(2)} + f_3(\delta, \phi_s)M^{(3)} + f_4(\delta, \phi_s)M^{(4)} \]  
(4.38)

where \( f_1, \ldots, f_4 \) are functions of the dip angle of the dislocation \( \delta \) and the strike angle \( \phi_s \), and the four moment tensors correspond to: \( M^{(1)} \) is the moment for a horizontal fault plane, with slip direction defining the direction of propagation, \( M^{(2)} \) is for a pure strike-slip fault,
i.e., $\delta = \frac{\pi}{2}$, $M^{(3)}$ is for a pure dip-slip fault, i.e., $\delta$ gives the direction of propagation and $M^{(4)}$ is for up-dip slip, i.e., $\delta = \frac{\pi}{4}$. Thus, the radiation field implied by Equation 4.38 can be written as the sum of four fields, induced by different elementary types of dislocation [1].

Now, suppose that in the direction perpendicular to the plane of shear, tensile cracks are developed, i.e., one of the types of dislocation is tensile fracture. Since the dip angle is not known, as a first approximation I assume that equal weighting is placed on the elementary moment components. Thus, the directivity pattern of this composite model is given as a superposition of a lateral octopole in the plane of shear and an appropriate combination of longitudinal octopoles for a perpendicular to this plane tensile fracture.

For example, consider shear in the $y - z$ plane and tension along the $x$-axis.

$$B(\phi)g(\theta) = \sin^2 \theta \cos \phi \sin \phi + \sin \theta (1 + 2 \cos^2 \phi \cos^2 \theta) =$$

$$\sin \theta [1 + \sin \phi \cos \theta + 2 \cos^2 \theta \cos^2 \phi]$$

(4.39)

Table 4.5 summarizes the possible directivity patterns $B(\phi)g(\theta)$ of this model, according to the direction of propagation:

<table>
<thead>
<tr>
<th>Plane of shear</th>
<th>Dir. of tension</th>
<th>$B(\phi)g(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x-y$</td>
<td>$z$</td>
<td>$\sin \theta [1 + 2 \sin^2 \theta + \cos^2 \theta \sin \phi \cos \phi]$</td>
</tr>
<tr>
<td>$x-z$</td>
<td>$y$</td>
<td>$\sin \theta [1 + \cos \phi \cos \theta + 2 \cos^2 \theta \sin^2 \phi]$</td>
</tr>
<tr>
<td>$y-z$</td>
<td>$x$</td>
<td>$\sin \theta [1 + \sin \phi \cos \theta + 2 \cos^2 \theta \cos^2 \phi]$</td>
</tr>
</tbody>
</table>

Table 4.5: Directivity patterns of higher-order multipoles.

**Example 4:** A type II event is considered, with peak frequency 215 Hz and source speed 517 m/s. The radiation pattern is described by the pattern given in Equation 4.39. The variation of source level with azimuth is shown in Figure 4-31, for a given launch angle $\theta = 38^\circ$. 
Figure 4-31: Event horizontal radiation pattern (source level in dB re 1 mPa, 1 Hz, 1m versus horizontal angle (°)). Superimposed multipole model pattern. The range of observation is 255°.

It is possible that the model which describes the radiation pattern of this event is not unique. I have chosen a possible model, and have assumed equal magnitude of the longitudinal and lateral octopoles since there is fracture propagation in two planes. Although the two fracture modes are independent the radiation field of one affects the other. This is clear from the superimposed model pattern in Figure 4-31. The horizontal pattern of the lateral octopole is distorted due to the effect of the longitudinal octopole pattern on it. This problem requires further study in order to determine the appropriate ratio of magnitudes of the two octopoles, from physical consideration.

The radiation patterns of 35 mid-frequency events are represented by one of the models in Table 4.5. Type IV is the predominant signature (29 events); the remaining ones are type I. The source speed of these events is in the range 517-1100 m/s. Seismic moment as a function of peak event frequency is shown in Figure 4-32. Compactness has also been
accounted for, in the estimation of the events' directivity:

![Seismic moment versus peak event frequency](image)

Figure 4-32: Seismic moment as a function of peak event frequency. The best fitted slope to the log-log data is proportional to $f^{-5}$. Type I events are marked with (+) and type IV with (*).

There is an $f^{-5}$ dependence of seismic moment to peak frequency, although both the deviation of the data from the best-fitted curve and the spread of the data are on average larger than one order of magnitude. This indicates that either the relationship between the two parameters cannot be clearly defined, or more probably the model may require modifications. Notice that the seismic moment for these events is on average higher than that of tensile and pure shear fractures, previously discussed.

Seismic moment is also examined as a function of source length, and is shown in Figure 4-33:
Figure 4-33: Seismic moment as a function of source length. The best fitted line to the log-log data is proportional to $L^3$. Type I events are marked with (+) and type IV with (*).

The data show approximately an $L^3$-dependence, but both the spread of the data as a function of source length and its deviation from the best-fitted curve indicate that modifications must be introduced to the model. As previously mentioned, although the variation of source level in azimuth is well represented by the proposed model, the latter is not unique. An alternative model which, however, includes the effect of radiation from a tensile fracture on the field of the shear fracture, must be fitted to the data and the corresponding results can be compared to determine the adequacy of either of the two. The strength of secondary cracks or fracture details determines their effect on the radiation from the primary fracture.

4.3.9 Stick-slip motion

If the frictional resistance of the medium varies during fault sliding, a dynamic instability may occur, resulting in abrupt slip and stress drop. It may be followed by a period of
no motion and then re-occur, as the stresses in the medium build-up again. This type of motion is called regular stick-slip. It is typically modeled using a frictional slider loaded with a spring, to which an external force is applied. The stiffness of the spring represents the elastic properties of the medium surrounding the fault. Regular stick-slip is often observed in materials in which friction has a negative dependence on sliding (particle) velocity, which leads to a dynamic friction that is lower than static friction. This is the so-called velocity-weakening. Earthquakes are recurring slip instabilities on pre-existing faults and thus they are by definition stick-slip phenomena. This may also be the case of ice fractures. The estimated slip functions of compound events, the individual signal components of which are separated by roughly 20-25 msec are characteristic of stick-slip motion. Individual components of these events are not, however, separately analyzed. Thus, I can only speculate that this type of motion is plausible in the ice, and generates compound events.

There are 12 events, with peak-frequencies predominantly in the mid-frequency range, which are attributed to fracture and which occur in sequences of two events of the same signature type. The inter-arrival time between events is less than 0.5 sec and their distance is less than 50 m. The estimated seismic moment and slip parameters of the second event are consistently lower, particle velocity in particular, though of the same order of magnitude. An example is presented in Chapter 5. A possible generating mechanism of these sequences is stick-slip motion.

4.3.10 Formation of crack process zones

Seismic moment, estimated for fracture-induced events varies in the range $O(10^6) - O(10^{10})$ Nm. Evidently, this result is associated with the variation of the fracture type. Yet, even for events that are believed to have been generated by a particular type of fracture process, there is still a 2-3 orders of magnitude variation of seismic moment. In Chapter 3, it was shown that source speed also varies significantly for the analyzed event sub-populations. Therefore, both primary and secondary processes, of the same type may be responsible for the observed variation in source parameters. As a fracture propagates, secondary cracks,
which at a later stage coalesce to form primary fractures, are generated in the vicinity of a propagating fault, in the so-called process zones. The propagation speed, fault dimensions and seismic moment of these cracks are lower than the corresponding values for the primary fracture. In particular, seismic moment is typically 3-4 orders of magnitude lower [20]. In order to assess the occurrence of secondary cracks in the ice, the locations of events need to be examined. Spatial event clustering is expected, as a result of the formation of crack zones. This phenomenon is further investigated in Chapter 5.

4.4 Stress drop, scaling relations and event self-similarity

Geometric similarity of event sources has been assumed throughout the thesis and particularly in the calculation of source dimensions; this type of similarity is implied when assuming that corner frequencies are inversely proportional to source length and depth, respectively. Other proposed similarities [20] include: fixed strain drop, i.e., slip is directly proportional to source length, and dynamic similarity, i.e., the product of rise time and fracture propagation velocity is directly proportional to source length. Studies of a large number of earthquakes have shown that there are cases for which these similarity relationships between fault parameters hold [1]. To the best of my knowledge, the validity of these relationships for faults in Arctic ice has not been assessed. I, therefore, investigate similarity of fractures, using the estimated parameters for events attributed to fracture processes.

Figure 4-34 shows particle slip as a function of source length. Figure 4-35 shows the product of rise time and fracture speed as a function of source length. Recall that in Section 3.2.1, the distribution of slip with peak event frequency has been examined. Figure 3-10 shows that there is no single frequency power-law that fits the data. Thus, peak event frequency is not an appropriate scaling parameter.
Figure 4-34: Particle slip $\bar{u}$ versus source length $L$. Superimposed to the data is the best fitted curve $\bar{u} \propto L$.

There is a significant spread of the data, in Figure 4-34 over several orders of magnitudes with increasing length, particularly in the range 5-60 m, which indicates that there is probably no fixed strain drop for fractures in sea ice. Note, however, that in comparison to peak event frequency, source length is a better scaling parameter for slip. Figure 4-34 includes all events attributed to fracture, irrespective of fracture mode. Events induced by tensile and shear cracks were separately examined, but no clear relationship between slip and source length was identified.
Figure 4-35: Product of rise time $T$ and fracture speed $V$ versus source length $L$. Superimposed to the data is the best fitted curve $T \times V \propto L$.

The dynamic similarity of ice fractures is evident from Figure 4-35; the product of rise time and fracture speed is indeed directly proportional to source length. The spread of the data is less than one order of magnitude. In this study, static fault parameters have been investigated. In particular seismic moment and slip, which are both dynamic quantities, have been averaged over the event duration. The latter probably is much shorter than the fault duration, which cannot be accurately estimated since it is not known how many acoustic events have been induced by a single fault. Therefore, kinematic similarity, implied by Figures 4-34 and Figure 4-36 (to be shown on p. 203) may not be clear for ice fractures. In contrast, no averaging over the fault duration was needed in the estimation of rise time and fracture speed. Therefore, the expected dynamic similarity of fractures is more clearly seen. In the case of earthquakes, the line $T \times V = constant \times L$ defines a four-quadrant plane and data rarely fall on this line. Usually, according to the quadrant in which the data are, the relationship between magnitude and seismic moment and that between source
length and magnitude are determined.

From the discussion on dynamic crack models one important conclusion is that the dynamic characteristics of fracture slip and velocity all scale linearly with stress drop. The latter is, therefore, the fundamental scaling parameter. In principle, measurements of any of the observed parameters can be inverted for stress drop. However, in practice, such inversion is usually difficult. Measurements of the slip parameters are averages (temporal in the case of dynamic, and spatial in the case of static measurements). In addition, a model must be used to relate radiation to source parameters and simplifying assumptions must be made in regard to the geometry of the fault, e.g., the models of a rectangular fault and a circular fault, discussed in Chapters 2 and 3 and the radiation models discussed in this chapter. In studies of seismic sources, due to different choices of estimation methodologies and models used, stress drop estimates usually do not agree [36]. Due to the uncertainty associated with the underlying physics, no unique model or methodology is widely accepted.

For static cracks, the relation between average stress drop and slip is given by

\[ \Delta \sigma = C \mu \left( \frac{\Delta \bar{u}}{\Lambda} \right) \]  

(4.40)

where \( \Delta \bar{u} \) is the mean slip, i.e., the maximum slip estimated in Chapter 2, averaged over the duration of the fault, \( \Lambda \) is the characteristics rupture dimension, \( C \) is a constant that depends on the geometry of the rupture and \( \mu \) the shear rigidity of the medium. The term in parentheses is a co-seismic strain change, averaged over the scale length \( \Lambda \), and the ratio \( \frac{\mu}{\Lambda} \) is a stiffness, relating \( \Delta \sigma \) and \( \Delta \bar{u} \), as in the spring-slider model. For a circular crack \( \Lambda = a \), where \( a \) is the crack radius, and \( C = \frac{7\pi}{16} \). For infinite length fractures, \( \Lambda = W \), where \( W \) is \( \frac{D}{2} \), \( D \) the depth of the fracture, \( C = \frac{2}{\pi} \), for a strike-slip fracture and \( C = \frac{4(\lambda+\mu)}{\pi(\mu+2\mu)} \), for a dip-slip fracture. In order to estimate stress drop from the data, a time-history of slip must also be prescribed from a model, or directly estimated, as shown in Chapter 2.

As an example, consider a circular fracture. The relation between seismic moment and stress drop is given by:

\[ M_0 = \frac{16}{7} \Delta \sigma a^3 \]  

(4.41)

where \( \Delta \sigma \) is often referred to as the Brune stress drop. There is a general \( a^3 \)-dependence
on moment over a wide length range, indicating that stress drop is approximately constant and independent of source strength. This observation is a fundamental argument for the self-similarity of earthquakes. The relation is examined here for ice fractures. The fit of the $L$- and $W$-models to the data, discussed both in Chapter 3 and in this chapter, gives an indication of the scaling dimension $\Lambda$ of the event mechanisms. It has been shown that there are both length- and depth-controlled ice processes.

Stress drop has been calculated from Equation 4.40 using the appropriate scaling dimension for each event sub-population, as suggested by the agreement of the seismic moment data with that predicted by the fault models. Stress drop as a function of fault area ($A = LD$) is shown in Figure 4-36:

Figure 4-36: Stress drop as a function of fault area and best fitted least-squares line ($\Delta\sigma \propto (LD)^{-0.5}$).

Stress drop ranges between 110 Pa, at $f = 300$ Hz to $2.25 \times 10^4$ Pa at $f = 40$ Hz. The ice fracture strength is in the range $2 \times 10^5$ to $1 \times 10^6$ Pa. So, the estimated stress drop is
1-3 orders of magnitude lower. In the case of earthquakes, there is usually a difference of 2-4 orders of magnitude between measured stress drop and the shear strength of rock. The spread of the stress drop data with fault area is significant, with a maximum of about two orders of magnitude. There is no clear relationship between stress drop and fault area; the best fitted curve is $\Delta \sigma \propto (LD)^{-0.5}$, but the deviations of the data from this curve are about one order of magnitude.

4.5 Summary and Observations

In this chapter, the event radiation characteristics have been analyzed and plausible generating ice mechanisms have been identified. According to the acoustic model that best describes their radiation patterns, events fall into 5 categories.

A vertical dipole has been used to describe event radiation characteristics that are independent of azimuth. Unloading motion of the ice cover is the most plausible corresponding mechanism. 20 such events, of a total of 196 events measured, with peak frequencies both below and above 100 Hz, and mean source speed of the order of 100 m/sec have been associated with this process. In regard to their occurrence in the ambient noise data, these events are consistently detected following intervals of multiple arrivals of mid-frequency events, which are attributed to fracture. This result supports the hypothesis that unloading motion is the event generating mechanism. In contrast to the results of a previous study by Chen [6], unloading motion is shown to occur both in the low- and mid-frequency ranges and appears uncorrelated with event signature type. It has been observed that dipole force is inversely proportional to $f^3$, the peak event frequency. The dipole force data has been compared to the predicted force values, by Dyer's unloading model [11]. The model predicts a $f^{-2}$ dependence of force; I expect that inclusion of the creep effect of ice in the model will account for the discrepancy between the data and the predicted values.

A vertical dipole, modified by the Doppler factor to account for source motion has been used to describe event radiation characteristics which have no other azimuthal variation except that due to source motion. Low-frequency events, with source speeds in the range 400-850 m/sec have initially been modeled by this dipole. However, the spread of the data
as a function of both peak frequency and source length suggests that an alternative model, also with no other characteristic horizontal directivity except that introduced by the Doppler factor, may be more appropriate. I have, therefore, used a superposition of longitudinal octopoles, which represents vertical tensile fracture, to model the event radiation pattern. Although the relationship between octopole moment and peak event frequency is worse than the corresponding relationship of dipole force, the small spread of the moment data with source length, the agreement of the data with the predicted values by the \( W \) fault model, which assumes that the fracture is depth controlled, and the range of source speed indicate that a fracture process is the most plausible event generating mechanism. Tensile cracks are commonly observed at the initial stage of fault formation and thus, the detected events may have been generated by fractures during precursory motion of the ice, the frequency content of which may be different than that of the coseismic or primary fault motion.

In general, a superposition of longitudinal octopoles has been used to model the radiation characteristics of events that are attributed to tensile fracture. There are 28 such events, with peak frequencies predominantly in the mid-frequency range and source speeds 600-1098 m/sec. No particular signature type characterizes this event sub-population. The relationship of octopole moment with peak frequency is not clear; below 150 Hz it is \( f^{-3} \)-dependent and above 150 Hz it is \( f^{-5} \)-dependent. There is a clear relationship, though, between seismic moment and source length; the moment is proportional to \( L^3 \), in the entire range of estimated source length. The source mechanism is shown to be depth-controlled.

Lateral octopoles have been used to model the radiation characteristics of events that are attributed to shear fractures. There are 46 such events, of a total of 196 events measured, with peak frequencies above 100 Hz, and source speeds 700-1400 m/sec which are associated with this mechanism. Type III is the predominant signature type of this event sub-population. There is no clear relationship between octopole moment and peak event frequency. However, the moment data are proportional to \( L^2 \) and agrees well with the predicted values by the \( L \)-model. This implies that the source mechanism is length-controlled.

Finally, the complexity of the mechanics of a propagating fracture suggests that a different acoustic model may be more appropriate to represent the radiation pattern of shear-induced acoustic events. The model must account for the characteristics of propagation
and possible variation of fracture mode, particularly the generation of tensile cracks off the 
edges and tips of shear fractures. Since the radiation field of a dislocation can be expressed 
as the sum of elementary fields of dislocations of particular types, I superimposed the 
radiation patterns of a lateral and a combination of longitudinal octopoles, in order to 
represent the effect of tensile fracture propagation, perpendicular to the plane to shear, on 
the radiation field of the shear fracture. Although the event horizontal directivity and that 
of the model agree well, the relation between peak event frequency or source length and 
seismic moment is not clear. Further investigation of the model is required. There are 35 
such mid-frequency events, of a total of 196 events measured, the radiation characteristics 
of which are described by this model.

Seismic moment, for all events that are attributed to fracture processes is in the range 
$O(10^6) - O(10^{10})$ Nm. The wide range of this parameter indicates that fracture processes 
occurs at different scales in the ice. Sound emission from secondary processes is detectable 
in ambient noise. Stress drop has also been estimated; it is in the range $O(10^2) - O(10^5)$ 
Pa, at least one order of magnitude lower than the ice fracture strength.

In summary, unloading motion, tensile and shear fracture, are the most plausible event 
generating mechanisms and occur at various scales. Unloading motion may occur both 
below and above 100 Hz, whereas fracture is predominant in the mid-frequency range. The 
important physical conclusions from this chapter are: sound radiation from shear fractures 
is affected by secondary crack features, such as arrays of tensile cracks which enable 
propagation of the shear fracture, and which in the aggregate affect the radiated sound field. 
I believe that the contribution of these processes to the event radiation pattern is a function 
of several parameters, among which is the processes' strength, relative to that of the main 
fracture. Also, generated events may not always be detectable in ambient noise. This may 
partially explain the fact that not all event radiation patterns resulting from shear fracture, 
appear affected by this phenomenon. In regard to unloading motion, a very interesting 
observation is that events attributed to this process are consistently detected in the ambient 
oice time series, following periods of high ice fracture activity. Unloading motion is a 
post-fracture process.

Having estimated all pertinent source and event parameters, including plausible event
generating mechanisms, I proceed to a preliminary analysis of events at the level of temporal and spatial clusters, in an attempt to understand large-scale characteristics of ice motion.
Chapter 5

Analysis of Temporal and Spatial Event Clusters

5.1 Overview

Individual acoustic events have been analyzed both in the time and frequency domains and their radiation characteristics have been estimated. In the introduction to the thesis, I discussed the importance of analyzing events at a larger scale, namely sequences or clusters of events, temporal and spatial. Although difficult, it is necessary to identify and understand large scale features of ice motion through the analysis of acoustic events. For example, the formation of a fault is preceded by the occurrence of several small fractures. As previously discussed, dynamic propagation of small cracks in a macroscopic description appears as a slow kinematic growth of a large fracture. There are events with propagation speeds in the range 10-200 m/s and source lengths less than 1 m. It is possible that they have been induced by small cracks. Such cracks not only precede a fault but are also formed due to its propagation in the medium. They are located in process zones in the vicinity of the fault. The induced acoustic events may be detected in ambient noise, depending on their corresponding source strengths. If detected, they are expected to be clustered, at least in space.

In Chapter 3, I discussed the concept of fault cycle. The results from the analysis of compound events, particularly the event spectra and the estimates of source speed, indicate
that the multiple spectral peaks detected both in the low- and mid-frequency regimes are probably associated with the mechanics of a single ice process, at a particular stage in its formation. Therefore, individual signal components of compound events appear to be unrelated to the different stages involved in the formation of the fault, particularly due to the small separation between signal components. Stick-slip motion is a more plausible phenomenon to which the occurrence of compound events can be attributed. The slip functions of some compound events are characteristic of such motion. To address the possible occurrence of fault cycles in ice, in this chapter I investigate temporal event sequences and clusters. For faults in Arctic pack ice, typical inter-arrival times, or pertinent time scales, associated with the occurrence of seismic stages during a fault cycle, are unknown. They have been studied for the Marginal Ice Zone [6]. Even for earthquakes, little is known on pre-cursory and post-cursory fault motion, particularly in regard to the characteristic parameters of this motion, including its frequency content, propagation speed and strength, and arrival time relative to that of the main shock. In this chapter, my goal is to identify such time-scales, based on temporal patterns of event arrivals, and consequently to determine the possible occurrence of a fault cycle or more realistically a sub-cycle. I believe that an entire fault cycle cannot be identified in this study. First, the frequency range of the analysis (10-350 Hz) does span over the entire range of interest (1 Hz - 100 kHz). Fracture mechanisms have also been identified in the high-frequency range (3-100 kHz) [6] [38] and may be part of a fault cycle in the ice. Second, the time interval of analysis may be too small. The inter-arrival time between individual seismic stages may be longer than one hour.

Events cluster in time and space due to ice motion, induced by environmental forces, such as winds, applied to the ice cover. Temporal event clusters are believed [6] to be directly associated with changes in environmental conditions. Since it is difficult to associate event clustering with significant changes in these conditions within one hour, the most meaningful clusters to this analysis are those that occur simultaneously in time and space, as they are likely to be directly related to ice motion processes. The parameters required for the analysis of event clusters are: the event detections in the data and their inter-arrival times, the event locations, signal characteristics, peak frequency and bandwidth and the estimated source
speed, orientation, dimensions, acoustic model and corresponding dipole force or seismic moment. Variation in stress drop with time is also examined to determine the occurrence of fracture deceleration and arrest.

5.2 Analysis of event sequences and temporal clusters

I distinguish between sequences and clusters of events, and define a temporal sequence as a series of at most 3 events, whose inter-arrival times are at least twice as long as the inter-arrival times between signal components of compound events. This threshold has been set in order to ensure that individual components of compound events are not identified as part of a temporal sequence. In the ideal physical case where sound emissions from a fault at the three stages of its formation, i.e., the pre-cursor, the main shock and the post-cursor, reach the hydrophones, three distinct events would be detected in ambient noise. This number has, therefore, been set as the upper bound of the number of events in a sequence. Only one sequence of 3 events has been found; all others are composed of two events, only. I define a temporal cluster as a series of at least 4 events, whose inter-arrival times are limited by the same bound as those for events sequences.

Characteristics that are common to all sequences include: constituent events have the same type of signature and approximate duration, their frequency content is in the same range, i.e., typically, the difference in their peak frequencies is no larger than 20-50 Hz and both are in the mid-frequency range. The source speed and strength of the first event is higher than the corresponding values of the second. Also, the radiation patterns of both events, modeled either by a lateral octopole or by a higher-order multipole, suggest that they have been induced by the same physical process. Events in all sequences are believed to have been generated by shear fracture. There are 6 sequence and 14 temporal clusters of events in the data, of which 11 are composed of events that also cluster in space. Characteristic spatial patterns, such as zones of events and uni-directional paths, are distinguished from the event locations and the estimated directions of propagation.

Example of an event sequence: The time series of the sequence is shown in Figure 5-1:
Figure 5-1: Temporal event sequence. The event inter-arrival time is about 0.26 s.

Tables 5.1 and 5.2 summarize the event and source parameters, respectively:

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Signature</th>
<th>Duration (s)</th>
<th>$f_{peak}$ (Hz)</th>
<th>Bandwidth (octaves)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Type II</td>
<td>0.025</td>
<td>224</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>2</td>
<td>Type II</td>
<td>0.025</td>
<td>196</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>

Table 5.1: Event parameters of sequence.

<table>
<thead>
<tr>
<th>Event #</th>
<th>$v_p$ (cm/s)</th>
<th>$\bar{u}$ (cm)</th>
<th>$V$ (m/s)</th>
<th>$\phi_s$ (°)</th>
<th>$L$ (m)</th>
<th>$D$ (m)</th>
<th>$M_0$ (Nm)</th>
<th>Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0.2</td>
<td>446</td>
<td>217</td>
<td>8</td>
<td>3.3</td>
<td>3.17e+08</td>
<td>Shear</td>
</tr>
<tr>
<td>2</td>
<td>2.4</td>
<td>0.12</td>
<td>380</td>
<td>196</td>
<td>5.5</td>
<td>3.3</td>
<td>1.62e+08</td>
<td>Shear</td>
</tr>
</tbody>
</table>

Table 5.2: Slip and propagation parameters. $v_p$ is the particle velocity, $\bar{u}$ the mean slip, $V$ the source speed, $\phi_s$ the source orientation, $L$ the length, $D$ the depth and $M_0$ the seismic moment.
The peak pressure amplitude of the first event is approximately twice as large as the amplitude of the second. Although for both events the 'primary' signal is type II, each is followed by a less well-defined signal. An interesting observation from the event parameters is that both slip and particle velocity are lower for the second event. Stick slip motion is characterized by velocity weakening, i.e., a decrease in particle velocity as the fracture advances in a discontinuous manner. Although slip functions of some compound events indicate such motion, it may also occur at different scales, i.e., the time interval between advancements of the fracture may be of the order of the separation between signal components in compound events or the inter-arrival time between events in a sequence. Figure 5-2 shows the spectrogram of the sequence:

Figure 5-2: Spectrogram of event sequence. Contours are in dB re 1 μPa, 1 Hz. The difference in peak pressure level between the two events is 9 dB.

The peak event frequency, propagation speed, and seismic moment of the second event are lower, although still in the same range. The source orientations differ by 20°, so propagation is at least approximately in a single direction. The distance between the two
events is 7 m and the length of each event is in this range, too. Thus, the characteristics of the event sequence suggest that both events are probably induced by the same physical process, i.e., they have been generated at the same stage of fracture formation. There is no evidence of pre-cursory or post-cursory motion. Unstable slip is a plausible generation mechanism of this sequence. Similar results have been obtained for all detected event sequences.

5.2.1 Event clusters

In order to identify time intervals in the one-hour data segment where clusters of events occur, the inter-arrival times for the entire event population are determined as shown in Figure 5-3. Time has been counted, from the arrival of the first event to the arrival of the last event. There are approximately 10 min in the data (6 prior to the first event and 4 after the last event) in which no events have been detected.

Figure 5-3: Event inter-arrival time function.
The time intervals of event clustering are at the minima of the inter-arrival function in Figure 5-3. The last 60 events (30% of the total population) have been detected between times 2350-2355, i.e., within 5 min. As previously mentioned, at that time a large fracture North of the recording array began to slip.

Assuming that environmental forces do not vary significantly in one hour, I have no pertinent information to choose the duration of clusters or time intervals for the analysis in a physically meaningful manner. For example, in the Marginal Ice Zone, Chen [6] used the period of ocean swells as the threshold for the duration of event clusters. I, therefore, first roughly divide the data into one-minute intervals. Notice that the mean event inter-arrival time serves as a first indication of where in the data temporal event clusters occur. The distribution of the number of events per minute is shown in Figure 5-4:

![Figure 5-4: Number of events per one-minute interval. Times $t = 0$ and $t = 49$ min correspond to the times of arrival of the first last detected events, respectively.](image)

In the first 20 min, the mean is 2 events per minute; in the last 30 min, it is 6 events per minute. Consequently, the mean event inter-arrival time decreases fast; in the last 10 min
it is of the order of 5 s whereas at the beginning of the data it as high as 68 s. There is a strong correlation between event location and event inter-arrival time as it will be shown through the analysis of simultaneous temporal and spatial event clusters.

5.2.2 Temporal and spatial clusters

The following assumptions are made, based on the actual events in the data: the minimum number of events per cluster is 4, its maximum duration is 60 s and the maximum event inter-arrival time in a cluster is 5 s.

Example of temporal cluster: The times series of the cluster is shown in Figure 5-5:

![Temporal event sequence # 12](image)

Figure 5-5: Temporal cluster, composed of 6 events. Event inter-arrival times are between 0.1 s and 0.48 s.

Notice that the event amplitudes increase with time. The event with the largest amplitude is believed to have been induced by unloading motion of the ice. The events preceding it, are attributed to shear fracture (their radiation patterns are best described by lateral octopoles).
Their source parameters vary significantly. Tables 5.3 and 5.4 summarize the event and source parameters, respectively:

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Signature</th>
<th>Duration (s)</th>
<th>$f_{\text{peak}}$ (Hz)</th>
<th>Bandwidth (octaves)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Type III</td>
<td>0.06</td>
<td>59</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>2</td>
<td>Type III</td>
<td>0.06</td>
<td>67</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>3</td>
<td>Type IV</td>
<td>0.08</td>
<td>72</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>4</td>
<td>Type IV</td>
<td>0.1</td>
<td>73</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>5</td>
<td>Type III</td>
<td>0.045</td>
<td>62</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>6</td>
<td>Type I</td>
<td>0.03</td>
<td>56</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>

Table 5.3: Event parameters of sequence.

<table>
<thead>
<tr>
<th>Event #</th>
<th>$v_p$ (cm/s)</th>
<th>$\bar{u}$ (cm)</th>
<th>$V$ (m/s)</th>
<th>$\phi_s$ (°)</th>
<th>$L$ (m)</th>
<th>$D$ (m)</th>
<th>$M_0$ (Nm)</th>
<th>Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>1.2</td>
<td>402</td>
<td>204</td>
<td>25.5</td>
<td>2.3</td>
<td>2.1e+09</td>
<td>Shear</td>
</tr>
<tr>
<td>2</td>
<td>7.5</td>
<td>0.8</td>
<td>547</td>
<td>232</td>
<td>23.3</td>
<td>1.4</td>
<td>8.17e+08</td>
<td>Shear</td>
</tr>
<tr>
<td>3</td>
<td>7.5</td>
<td>0.25</td>
<td>960</td>
<td>236</td>
<td>33</td>
<td>1.4</td>
<td>4.46e+08</td>
<td>Shear</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.7</td>
<td>1041</td>
<td>202</td>
<td>10</td>
<td>1.0</td>
<td>2.16e+08</td>
<td>Shear</td>
</tr>
<tr>
<td>5</td>
<td>36.7</td>
<td>2</td>
<td>622</td>
<td>109</td>
<td>2.0</td>
<td>3.1</td>
<td>3.96e+08</td>
<td>Tensile</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>-</td>
<td>11</td>
<td>-</td>
<td>40</td>
<td>3.3</td>
<td>-</td>
<td>Unloading</td>
</tr>
</tbody>
</table>

Table 5.4: Slip and propagation parameters. $v_p$ is the particle velocity, $\bar{u}$ the mean slip, $V$ the source speed, $\phi_s$ the source orientation, $L$ the length, $D$ the depth and $M_0$ the seismic moment.

Notice that slip and particle velocity for Event #1 are higher than the corresponding values of the following 3 events. The decrease in particle velocity may be due to velocity weakening in the medium and the events may have been generated by the stick-slip motion of a single fracture. For the Event #6, which is attributed to unloading motion, seismic moment and slip parameters are not meaningful source variables. Instead, dipole force ($3 \times 10^6 N$) is the pertinent parameter. Notice also that all events are in the low-frequency range, with peak frequencies which vary at most by 17 Hz.

The analysis of event clusters is organized as follows: the statistics of detected temporal event clusters are first summarized in Table 5.5. Events that cluster both in time and space are then discussed, since they directly pertain to the goal of the study of large-scale features
of ice motion. There are 11 simultaneous temporal and spatial clusters. According to identified spatial patterns, such as zones of events and uni-directional paths, pertinent event and source parameters are described. Figure 5-6 shows the arrival times of the clusters versus the number of events per cluster. Time $t = 0$ corresponds to the arrival time of the first cluster.

Figure 5-6: Arrival times of temporal clusters.
<table>
<thead>
<tr>
<th>Cluster #</th>
<th># of Events</th>
<th>Duration (s)</th>
<th>Event Mean Int. Time (s)</th>
<th>Signature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2.94</td>
<td>0.9</td>
<td>Type: IV (4)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>3.96</td>
<td>1.27</td>
<td>Type: III (4)</td>
</tr>
</tbody>
</table>
| 3        | 5           | 1.83         | 0.4                      | Type: II (2)  
IV (3) |
| 4        | 4           | 3.54         | 1.13                     | Type: II (1)  
III (2)  
IV (1) |
| 5        | 9           | 15.34        | 1.84                     | Type: III (2)  
IV (7) |
| 6        | 8           | 12.26        | 1.68                     | Type: I (2)  
II (1)  
IV (5) |
| 7        | 6           | 7.88         | 1.48                     | Type: IV (6) |
| 8        | 5           | 3.56         | 0.82                     | Type: II (1)  
III (2)  
IV (2) |
| 9        | 9           | 18.33        | 2.25                     | Type: I (1)  
II (3)  
III (4)  
IV (1) |
| 10       | 7           | 12.36        | 1.98                     | Type: II (1)  
III (1)  
IV (5) |
| 11       | 14          | 16.01        | 1.18                     | Type: I (3)  
II (2)  
III (3)  
IV (6) |
| 12       | 14          | 26.35        | 1.96                     | Type: I (2)  
II (2)  
III (1)  
IV (9) |
| 13       | 14          | 17.94        | 1.32                     | Type: I (2)  
II (1)  
III (5)  
IV (6) |
| 14       | 4           | 3.92         | 1.2                      | Type: IV (4) |

Table 5.5: Statistics of temporal event clusters. The number of events per signature type is shown in parentheses.
Figure 5-7 shows the locations of the simultaneous temporal and spatial clusters, relative to the hydrophone array.

Figure 5-7: Locations of simultaneous temporal and spatial clusters. Events are marked by (*) , hydrophones by (o). The approximate areas of the clusters are also shown. Clusters 9,10,11 and 12 have two sub-clusters, (a) and (b), composed of events which are located at two different areas.

With the exception of cluster 13, all others are located closely in space. Notice that cluster 13 is North of the hydrophone array and its arrival time is around the time that a large fracture was observed to slip during the experiment. There are three clusters of exclusively type IV events and one of compound type III only. All others are composed of events of different signatures, although type IV is predominant in six more clusters. The
inter-arrival time of clusters decreases rapidly with time with a minimum of 10 s. Although approximately 20.5 min have elapsed between the occurrence of clusters 1 and 2, all others are at most 5 min apart. Clusters 4, 13 and 14 are not further discussed since their constituent events are at large distances from each other. Clusters 2 to 5, 6 and 7, 9 and 10, 11 and 12, are shown together in four different plots because in these groups, events are located in the same area. Also, 9 through 12 occur within less than 2 min of each other. Clusters 1 and 8 are shown separately. The temporal sequence of events is also marked in the plots. Time $t = 0.0$ corresponds to the arrival time of the first event.

![Cluster # 1](image)

Figure 5-8: Event locations for cluster 1. The area of the cluster is about 250 $m^2$. The maximum distance between events is 18 m. Arrows indicate the estimated source orientations. The dashed line shows a possible propagation path.

Event and source parameters for cluster 1 are summarized in Tables 5.6 and 5.7, respectively.
<table>
<thead>
<tr>
<th>Event No.</th>
<th>Signature</th>
<th>Arrival time (s)</th>
<th>$f_{\text{peak}}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Type IV</td>
<td>0.0</td>
<td>139</td>
</tr>
<tr>
<td>2</td>
<td>Type IV</td>
<td>0.4</td>
<td>231</td>
</tr>
<tr>
<td>3</td>
<td>Type IV</td>
<td>2.4</td>
<td>139</td>
</tr>
<tr>
<td>4</td>
<td>Type IV</td>
<td>2.7</td>
<td>259</td>
</tr>
</tbody>
</table>

Table 5.6: Event parameters for cluster # 1.

<table>
<thead>
<tr>
<th>#</th>
<th>$v_p$ (cm/s)</th>
<th>$\bar{u}$ (cm)</th>
<th>$V$ (m/s)</th>
<th>$\phi_s$ (°)</th>
<th>$A$ ($m^2$)</th>
<th>$M_0$ (Nm)</th>
<th>Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7</td>
<td>0.05</td>
<td>117</td>
<td>47</td>
<td>4</td>
<td>5.5e+07</td>
<td>Shear</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>0.06</td>
<td>98</td>
<td>27</td>
<td>7</td>
<td>1.62e+08</td>
<td>Shear</td>
</tr>
<tr>
<td>3</td>
<td>1.15</td>
<td>0.08</td>
<td>68</td>
<td>188</td>
<td>5</td>
<td>1.30e+08</td>
<td>Shear</td>
</tr>
<tr>
<td>4</td>
<td>1.4</td>
<td>0.15</td>
<td>49</td>
<td>218</td>
<td>6</td>
<td>2.1e+08</td>
<td>Shear</td>
</tr>
</tbody>
</table>

Table 5.7: Source parameters for cluster # 1.

Events in this cluster have type IV signatures and are located uni-directionally in space. Their peak frequencies are in the mid-frequency range. Although the estimated source speeds are low, the event radiation characteristics suggest that shear fracture is their generating mechanism. The seismic moment of these events is lower than the average estimated seismic moment for the entire sub-population of events attributed to fracture, about $4 \times 10^8$ Nm. Notice that for events induced by shear fractures, the highest moment observed is $O(10^{10})$ Nm, again considering all fracture-induced events. Small, or secondary shear cracks, which possibly coalesce to form a larger fracture along the identified path, may have induced these events.

Figure 5-9 shows the change in seismic moment and stress drop from this cluster, as a function of time. For such a cluster, in which a uni-directional path can be traced, which may correspond to a fracture propagation path, examination of the variation of stress drop is meaningful, to detect the occurrence of fracture arrest. In the discussion of the next cluster I show that for tensile cracks in a process zone, this parameter varies in a random manner.
The high stress drop for the last event indicates deceleration and eventual arrest of the fracture. However, the increase in stress drop is not progressive within the cluster. There is a decrease in stress drop between the first and third events and then an increase. This indicates that the fracture accelerates first, which is contradicted by the estimated source speeds. Either the latter are too low due to bilateral fracture propagation which cannot be measured by the employed techniques, or the stress drop values are incorrect. Stress drop is a function of the characteristic dimension of the source (length or depth) which may have been under-estimated, too. Notice also the proportionality of stress drop and seismic moment, predicted by fault models, such as the Brune dislocation model. Seismic moment, though, remains almost constant between the second and third events.
Figure 5-10: Event locations for clusters 2-5. Cluster #4 has been omitted. The maximum distance between events in cluster #2 is 48 m; in #3 it is 247 m and in #5 it is 342 m. Arrows indicate the estimated source orientations. The dashed line represents a possible path of fracture propagation.

There are two areas in which events cluster, marked by the squares in Figure 5-10. In area A, 4 of the 6 events have type III signatures and belong to temporal cluster #2. Cluster #3 occurs 58 s later; 2 of its constituent events are in this area, too. The second area of ice activity is B, composed solely of type IV events, which belong to temporal cluster #5.

Based on the locations and orientations of events in cluster #3, a uni-directional path is traced. Shear fracture is the source mechanism of these events. Their peak frequencies are in the range 157-225 Hz, mean source speed is 872 m/s, and mean seismic moment is
$\bar{M}_0 = 1.6 \times 10^9$ Nm. In area A, the events of cluster #2 are attributed to tensile fracture. Their peak frequencies are in the range 222-258 Hz, mean source speed is 517 m/s and mean seismic moment is $\bar{M}_0 = 3.4 \times 10^8$ Nm. An interesting observation is that their orientation is almost perpendicular to the uni-directional path, along which the shear fracture is believed to propagate. An echelon of tensile crack arrays, perpendicular to a propagating shear fracture has been observed in several instances, during earthquakes. Coalescence of such crack arrays is part of the faulting process. Neither the locations nor the orientations of events in area B suggest a particular spatial path. The mean source speed of these events is 342 m/s and the mean seismic moment is $\bar{M}_0 = 8.1 \times 10^7$ Nm. The random orientations, low source speeds and mean seismic moments suggest that these events may have been generated by secondary cracks, and thus area B may correspond to a process zone. Figure 5-11 shows a typical temporal variation of seismic moment and stress drop in an area of secondary cracks (area B in this case).

![Diagram 1](image1)

![Diagram 2](image2)

Figure 5-11: Variation of seismic moment and stress drop with time.
Since no path can be traced from the location of events in this area, based on the source orientations, the events have probably been induced by distinct cracks. It is evident from the above plots that the relationship between stress drop and seismic moment is not clear. On average it appears that the two variables are independent. Tensile fractures are not extensively examined during seismic studies, since they are not responsible for large faults. There is a gradual increase in stress drop for the first 5 events and then a decrease. This indicates that there may be deceleration of the cracks. As in the previous cluster, the variations of stress drop and source speed disagree; the events have speeds in the same range.

Figure 5-12: Event locations for cluster 6-7. A path, about 330 m long, can be traced through the locations of the first three events in cluster #7, indicated by the dashed line. The two dot lines represent plausible boundaries of a fracture zone.
The area of highest event spatial concentration is marked as zone A in the above plot. Paths can be traced only between a few events in the two clusters, as shown. Zone A is 200 m long and 100 m wide; the majority of its constituent events have type IV signatures. Their orientations suggest uni-directional propagation. With the exception of one event in cluster #6, which occurs at time $t = 9.8$ s, all other events in zone A are mid-frequency and are attributed to both shear and tensile fractures. The only low-frequency event is believed to have been induced by unloading motion of the ice. Its source speed is 71 m/s. The mean source speed of the other events is 724 m/s and the mean seismic moment is $\bar{M}_0 = 5.7 \times 10^9$ Nm. Therefore, zone A is an area of high ice activity. It is possible that events located in this area belong to a single fracture. The uni-directional path traced from the locations and orientations of three of the events of cluster #7 are possibly associated with shear fracture (the azimuth range of observation of two of the three events was less than 90° and thus the event directivity pattern could not be resolved). The mean speed of these events is 480 m/s. The change in seismic moment and stress drop along this path is shown in Figure 5-13:

![Graph showing seismic moment and stress drop](image)

Figure 5-13: Variation of seismic moment and stress drop with time.
The proportionality between seismic moment and stress drop is clear in the above plot, as well the gradual increase of both. Notice that the source speed for the three events also decreases with increasing time. This supports the possibility of fracture deceleration indicated by the increasing stress drop.

Figure 5-14: Event locations for cluster 8. The maximum distance and inter-arrival time between events are 210 m and 1.4 s, respectively. The two dashed lines represent plausible boundaries of a fracture zone.

Although events in this cluster are located uni-directionally in space, as suggested by their orientations, there is no clear relation between their successive temporal and spatial occurrences. Therefore, it is difficult to trace a particular path. Instead, a spatial zone of ice activity is identified. In this zone, possibly a single primary fracture propagates and the detected events may have been induced by this fracture. Its length and maximum width are approximately 180 m and 50 m, respectively. There is no predominant event signature type in this cluster.
Event and source parameters for this cluster are summarized in Tables 5.6 and 5.7, respectively.

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Signature</th>
<th>Arrival time (s)</th>
<th>$f_{peak}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Comp. III</td>
<td>0.0</td>
<td>85</td>
</tr>
<tr>
<td>2</td>
<td>Comp. III</td>
<td>1.2</td>
<td>98</td>
</tr>
<tr>
<td>3</td>
<td>Type IV</td>
<td>1.6</td>
<td>96</td>
</tr>
<tr>
<td>4</td>
<td>Type IV</td>
<td>1.9</td>
<td>89</td>
</tr>
<tr>
<td>4</td>
<td>Type IV</td>
<td>3.3</td>
<td>95</td>
</tr>
</tbody>
</table>

Table 5.8: Event parameters for cluster # 8.

<table>
<thead>
<tr>
<th>#</th>
<th>$v_p$ (cm/s)</th>
<th>$\bar{u}$ (cm)</th>
<th>$V$ (m/s)</th>
<th>$\phi_s$ (°)</th>
<th>$A$ (m$^2$)</th>
<th>$M_0$ (Nm)</th>
<th>Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.6</td>
<td>0.6</td>
<td>374</td>
<td>106</td>
<td>2.7</td>
<td>8.5e+07</td>
<td>Tensile crack</td>
</tr>
<tr>
<td>2</td>
<td>15.0</td>
<td>0.9</td>
<td>586</td>
<td>119</td>
<td>26.8</td>
<td>7.1e+08</td>
<td>Shear</td>
</tr>
<tr>
<td>3</td>
<td>14.0</td>
<td>1</td>
<td>224</td>
<td>142</td>
<td>7.4</td>
<td>2.6e+08</td>
<td>Tensile crack</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>1.2</td>
<td>346</td>
<td>145</td>
<td>16.7</td>
<td>2.2e+08</td>
<td>Tensile crack</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.8</td>
<td>340</td>
<td>180</td>
<td>16.7</td>
<td>2.3e+08</td>
<td>Tensile crack</td>
</tr>
</tbody>
</table>

Table 5.9: Source parameters for cluster # 8.

Notice that the propagation speed of the shear crack is higher than that of the tensile cracks. Peak event frequencies are in the same range and below 100 Hz. Slip parameters are also in the same range, with small variations. The fault area and seismic moment of the shear crack are slightly higher.
Figure 5-15: Event locations for clusters 9-10. The maximum distance between events in areas A and B is 320 m and 100 m, respectively. A propagation path of a fracture can be traced through the locations of some events, as indicated by the dashed line.

Two areas of ice activity can again be distinguished, marked as A and B in Figure 5-15, but no particular pattern can be identified from the event locations in clusters #9 and #10 in these areas. The inter-arrival time between the two clusters is 10 s and their constituent events are located in both areas. Only one path can be traced, as shown, between four events in cluster #10. Note that the boundaries of the areas shown in all the above plots have been selected using a threshold for the distance between events. Within area A, a smaller spatial event cluster can be identified. Its constituent events are low-speed, low-frequency and
are believed to have been induced by unloading motion. All the events in area B, are mid-frequency, with mean speed 432 m/s and mean seismic moment $M_0 = 3.4 \times 10^8$ Nm. Possibly these events belong to a process zone, although the seismic moment is not as low as in the previously detected process zone. The events along the traced uni-directional path have been induced by a shear fracture. Their mean seismic moment is $M_0 = 4.5 \times 10^9$ Nm.

Figure 5-16: Event locations for clusters 11-12. The maximum distance between events in areas A, B and C, are 300 m, 125 and 130 m, respectively. The dashed line indicates a possible path of fracture propagation and the dotted lines mark hypothetical boundaries of a fracture zone.

A long zone of ice activity is identified in the above plot, within which events cluster
in two areas, marked A and B. There is a third area of high event concentration, marked as area C. The inter-arrival time between clusters #11 and #12 is 28 s. From all spatial event clusters examined so far, the one within area A is the most densely populated. The majority of events have type IV signatures although a few of types II and III are also present. Only two paths can be distinguished, based on event locations and temporal occurrence, also shown above. The random orientation of events in areas A and C indicate that these may be cracks zones, formed in the vicinity of a primary fracture, possibly propagating along the identified path. The mean seismic moment in area A is \( \tilde{M}_0 = 1.8 \times 10^8 \), in area B it is \( \tilde{M}_0 = 2.55 \times 10^8 \) and in area C it is \( \tilde{M}_0 = 4.8 \times 10^7 \). Similarly, the mean event source speed is lowest in area C. For events along the uni-directional path, the mean seismic moment is \( \tilde{M}_0 = 5.6 \times 10^9 \). These events have probably been induced by shear. The type of mechanism that has induced both these events and ones in area C is not known. They are located approximately 1 km from the hydrophone array and thus their directivity patterns cannot be determined.

So far I have discussed event clusters that occur simultaneously in time and space. There are also spatial clusters in which the event inter-arrival time is large, i.e., about 5 min or more. The question that arises in regard to these clusters is if the event inter-arrival time corresponds to the separation between the stages in the formation of a fault. From a physical point of view, it is difficult to associate the location and successive occurrence of these events with the temporal evolution of an ice process, e.g. the propagation of a fracture, at least within a single cycle of motion. The limits of such a cycle are defined by the point of offset of stress release in the medium and the time at which a new stress equilibrium is reached. Offset followed by stress recharge and a new stress equilibrium state in the medium occurs at several scales. The fault cycle is probably one of them but it cannot be identified, at least from these data. Long-term observations of the temporal occurrence of events are more appropriate for this purpose.
5.3 Summary and Observations

A preliminary analysis of the event population in the aggregate shows that events cluster both in time and space, either concurrently or separately. First, event sequences have been briefly described. The most plausible sequence generating mechanism is stick-slip motion, due to the observed particle velocity weakening. The data and the results of the entire analysis do not offer additional information to support this hypothesis.

Temporal event clusters have been subsequently identified and their statistics have been computed. Although clustering of events in time is believed to be directly associated with changes in environmental conditions, due to the lack of variation in these conditions within the one-hour of data collection, temporal clusters are probably associated with characteristics of ice motion. The analysis of clusters focuses on ones that occur simultaneously in time and space.

In regard to the event locations, several areas or zones have been identified, in which events occasionally of the same signature type, are concentrated. Their boundaries are usually well-defined. In some cases of simultaneous temporal and spatial clustering of events, a uni-directional path can be traced, indicating a possible propagation pattern of an ice fracture. Secondary features of ice motion, such as the formation of process zones and the coalescence of arrays of tensile cracks have been shown to be plausible mechanisms, based on the strength of the events, their source speeds and radiation characteristics.

Stress drop has also been examined for events in each cluster, and particularly along possible fracture propagation paths. A progressively large stress drop may indicate deceleration of the fracture and eventual crack arrest. In cluster 6-7, for events events along the identified propagation path, stress drop increased progressively with increasing time, from about 980 Pa to 6700 Pa. Variation in stress drop for events in other clusters does not suggest any particular characteristics of crack motion. Another interesting observation is that events within an identified fracture zone have a random variation in stress drop.

Evidently, the results from this analysis must be further investigated, possibly through the examination of a different type of data. The important points of this study is first that events cluster in space and time due to the characteristics of their generating mechanisms
and second, that ice motion processes at different scales radiate sound that is detectable in ambient noise.
Chapter 6

Conclusion: Summary of Results and Contributions

Ice-induced acoustic events, detected in ambient noise data from the SIMI experiment, have been analyzed with the purpose to estimate the characteristics of their physical generating mechanisms. There are three sets of results regarding the ice mechanisms and they are separately discussed in this chapter. The first pertains to particle motion and includes estimates of permanent slip, particle velocity and rise time. The second pertains to the propagation of the mechanisms and includes estimates of propagation speed, orientation and type of source motion, determined from the event radiation characteristics. The last set of results pertains to large-scale features of ice motion, estimated from the analysis of event spatial and temporal clusters.

6.1 Time-domain event parameters and particle motion of ice mechanisms

A total of 196 events have been detected, in an one-hour ambient noise data segment collected in the Central Arctic during the SIMI experiment. The events are located in an area of about $1.5 \text{ km}^2$. Simple and compound events have been distinguished, based on the number of their signal components. In addition, four basic signature types have been
identified: a pop or burst (type I), a complex pop (type II), a damped sinusoid (type III) and a multi-pulse signal (type IV). 45% of the total number of events in either of the two major categories have signatures of the latter type.

In order to gain insight on the relation between different types of events and their generating mechanisms, shear-wave and slip displacement functions have been estimated for the entire event population, through integration of the acoustic signals. In regard to particle motion of ice mechanisms, the important result is:

- Particle slip in the ice is $O(10^{-4}) - O(10^{-2})$ m and particle velocity is in the range $< 1 - 67$ cm/s

In comparison to particle motion of faults in rock, slip in ice is at least 3-4 orders lower, whereas particle velocity is on average only 50% lower. Type III events have the highest particle velocity and mean slip. No other correlation between the variation of source parameters that pertain to particle motion and signature type has been found.

### 6.2 Frequency-domain event parameters and propagation of ice mechanisms

Both low- and mid-frequency events have been detected, with peak event frequencies in the ranges $< 100$ Hz and within 100-350 Hz, respectively. In regard to the correlation of peak event frequency and signature type, it has been observed that type III and IV events dominate the mid-frequency range, whereas type I and II events have peak frequencies no larger than 200 Hz. In terms of bandwidth, 95% of the total number of events are narrow-width, i.e., with bandwidths of about one octave or less, and compound events are on average broadband, i.e., with bandwidths which span over more than one octave. Their spectrum is characterized by several peaks, often separated by more than one octave.

Doppler shifts between 1 and 90 Hz, associated with source motion, have been estimated through spectrum cross correlation. Consequently both the source speed and the source orientation have been estimated. In regard to propagation of ice mechanisms, the important result is:
Propagation speed is predominantly in the range 200-1100 m/s, significantly lower than the previously assumed Rayleigh wave speed.

Mid-frequency type I and type III events have the highest source speeds and type IV events have speeds no higher than 1000 m/s. There is no clear correlation between source speed variation and compound signature types.

Source dimensions have also been estimated from the corner frequencies in the spectrum of the shear-wave displacement signals. Source length is in the range 0.7-100 m, and depth is 0.4-4 m. Based on the dependence of the spectral fall-off on frequency, four types of shear-wave displacement spectra have been identified for the analyzed events. The spectrum corresponding to type I events is characterized by a $\omega^{-3}$-dependent slope. Such a trend has been observed in the case of earthquakes, and is attributed to fault nucleation, a hypothesis that cannot be validated in this study, using the available data. The second type of displacement spectrum is that in which two corner frequencies are distinct and are well-separated. The corresponding geometry of the wave-inducing source is a long and narrow, approximately rectangular fault. For the third type of spectrum the corner frequencies are very close to each other, thus suggesting a circular fault. Finally, for several compound events the high-frequency spectral trend is $\omega^{-1}$-dependent and only one corner frequency can be estimated.

6.3 Event radiation characteristics and proposed ice mechanisms

Based on the analyzed event radiation characteristics, the proposed ice mechanisms are:

- Unloading motion, which occurs both in the low- and mid-frequency ranges
- Shear fracture, which occurs in the mid-frequency range
- Tensile fracture, which also predominantly occurs in the mid-frequency range

A vertical dipole has been used to describe radiation characteristics of events which have been induced by unloading motion of the ice, with peak frequencies both below and
above 100 Hz, and mean source speed of the order of 100 m/s have been associated with this process. They are consistently detected in ambient noise, following intervals of multiple arrivals of mid-frequency events, which are attributed to fracture. This result supports the hypothesis that unloading motion is the event generating mechanism. In contrast to the results of a previous study by Chen [6], unloading motion is shown to occur both in the low- and mid-frequency ranges and appears uncorrelated with event signature type. It has been observed that dipole force is proportional to $f^{-3}$, instead of the $f^{-2}$ dependence predicted by Dyer’s unloading model [11]. Inclusion of Dyer’s creep effect of ice in the model may account for the discrepancy between the data and the predicted values.

Low-frequency events, with source speeds in the range 400-850 m/s have initially been modeled by a dipole, modified by the Doppler factor to account for source motion. Due to the spread of the data as a function of both peak frequency and source length, a superposition of longitudinal octopoles, which represents vertical tensile fracture, has also been used to model the event radiation patterns. Although the relationship between octopole moment and peak event frequency is worse than the corresponding relation of dipole force, the small spread of the moment data with source length, the agreement of the data with the predicted values by the $W$ fault model, which assumes that the fracture is depth-controlled, and the range of source speed, indicate that a fracture process is the most plausible event generating mechanism. Tensile cracks are commonly observed at the initial stage of fault formation and thus, the detected events may have been generated by fractures during precursory motion of the ice, the frequency content of which may be different than that of the main fault motion. This may explain the disagreement of my results with those of previous studies on the frequency content of tensile fractures.

A superposition of longitudinal octopoles has also been used to model the radiation characteristics of mid-frequency events that are attributed to tensile fracture. Source speed for these events is in the range 600-1098 m/s. The source mechanism is shown to be depth-controlled.

Lateral octopoles have been used to model the radiation characteristics of events that are attributed to shear fractures. There are 46 events, with peak frequencies above 100 Hz, and source speeds 700-1400 m/s which are associated with this mechanism. Type
III is the predominant signature type of this event sub-population. There is no clear relationship between octopole moment and peak event frequency. However, the moment data are proportional to $L^2$ and agree well with the predicted values by the $L$-model. This implies that the source mechanism is length-controlled. Large seismic events are typically length-controlled.

Finally, the complexity of the mechanics of a propagating fracture suggests that a different acoustic model may be more appropriate to represent the radiation pattern of shear-induced acoustic events. The model must account for possible variation of fracture mode, particularly the generation of tensile cracks off the edges and tips of shear fractures. To address this problem, I superimposed the radiation patterns of a lateral and a combination of longitudinal octopoles, in order to represent the effect of tensile fracture propagation perpendicular to the plane to shear, on the radiation field of the shear fracture. Although the event horizontal directivity and that of the model agree well, the relation between peak event frequency or source length and seismic moment is poor. Further investigation of the model is required. There are 35 mid-frequency events, the radiation characteristics of which are described by this model.

Seismic moment, for all events that are attributed to fracture processes is in the range $O(10^6) - O(10^{11})$ Nm. The wide range of this parameter indicates that fracture processes occur at different scales in the ice. Sound emission from secondary processes is detectable in ambient noise. Stress drop has also been estimated; it is in the range $O(10^2) - O(10^5)$ Pa, at least one order of magnitude lower than the ice fracture strength. This compares well with the ratio of stress drop (typically of the order 10-100 bar) to the strength for rock, during earthquakes.

### 6.4 Large-scale ice motion characteristics

The important results of a preliminary analysis of event clusters are:

- Events cluster both in time and space and clustering is believed to be directly associated with characteristics of ice motion
• Ice fracture processes occur at different scales and radiate sound which is detectable in ambient noise.

In addition to individual ice mechanisms, plausible event generating processes at a larger scale, include the formation of process zones and the coalescence of arrays of tensile fractures in the vicinity of large shear fractures.

6.5 Conclusions on the physics of ice-induced events and contributions of the thesis

The estimation of particle motion of ice mechanisms is the first contribution of this thesis. Individual events have been studied in the past [12] [10] but no consistent study of a large number of events with the purpose to estimate source parameters that pertain to slip motion has been attempted. Evidently, there are limitations to my study. First, due to the lack of direct measurements of seismic wave displacements, and particularly of near-field data, which are commonly processed in such estimation procedures in the case of earthquakes, the acoustic event signals must be used, instead. This implies that the event radiation characteristics must be analyzed in order to determine the type of mechanism that has induced the event and assume an appropriate fault or source model in the estimation of slip parameters. Second, this procedure involves integration of the acoustic signals which may introduce additional inaccuracies. First, it is sensitive to the data window used in the integration, which must coincide with the duration of the event. For example, if the range of integration is 10% longer than the event duration, particle velocity is under-estimated roughly by 10%. Second, the employed integration procedure which invoies a priori approximation of the signal results in errors in the estimated parameters, particularly over-estimation (10% – 20%) of the amplitude of the source-wave parameter and consequently slip. Nevertheless, the value of the study lies in the fact that its results serve as a first indication of the range of slip and particle velocity of faults in the ice and of the characteristics of corresponding slip functions.
Particle slip is measurable, \( O(10^{-4}) \) – \( O(10^{-2}) \) m, and particle velocity, in the upper limit, compares to that of faults in the earth. A similar relationship between faults in ice and in rocks is found for fault length and propagation velocity. The estimated slip functions indicate that particle displacement offset of a discontinuity in the ice is non-linear in time. Some slip functions indicate the stick-slip motion of ice processes. Faults in the earth are by default stick-slip phenomena. The reason that this cannot be deduced also for ice faults is that slip is very small and the duration of the acoustic signals are not sufficiently long to detect this type of motion. Indeed, event signals for which the slip functions indicate stick-slip have the longest duration.

The second contribution of this thesis is the estimation of source speed and orientation. Although the Rayleigh wave speed is often assumed as the source speed in the analysis and interpretation of acoustic events, the estimates obtained in this study indicate that fracture speed of ice processes is significantly lower than the Rayleigh wave speed. There is a wide range of source speed estimates for the detected events, among which very few are of the order of the Rayleigh wave speed. These results indicate that ice processes radiate sound, detectable in ambient noise, at different stages in their formation and propagation. It is almost impossible for propagation speed to remain constant as a fracture evolves spatially. The critical fracture length governs the time at which a slow-propagating crack begins to accelerate and reach a limit value. It remains unknown if several acoustic events have been induced by the same process. The limitation of the estimation procedure used in this thesis is that it cannot account for bilateral propagation. Only one estimate of source orientation if obtained for each event. Thus, fracture phenomena such as back-propagation of a fracture or bilateral propagation cannot be identified and the resulting estimate of source speed may be lower than the actual fracture propagation speed. However, the fact that it has been possible to estimate source motion parameters is very important since appropriate corrections can then be made, e.g., the inclusion of the Doppler factor, in the estimation of the events’ horizontal radiation patterns.

Estimates of source orientation, also obtained here, are important in modeling the event radiation characteristics, particularly for events with a characteristic horizontal radiation pattern. In previous studies, horizontal directivity values averaged over azimuth have been
used [7]. The results of the analysis of event radiation characteristics are used to identify the corresponding ice mechanisms. Previously identified mechanisms such as floe unloading motion and fracture (both shear and tensile) are shown to be plausible processes but not at a particular frequency range. Both low- and mid-frequency events are attributed to the two types of processes. This result supports the hypothesis that ice mechanisms in the ice occur at different stages, the frequency content of which may span the entire frequency range of interest, i.e. 3 Hz - 100 kHz. Another important observation is that events attributed to unloading motion, are consistently detected in the ambient noise series, following high-speed (600-1000 m/s) events associated with fracture. Fracture-induced events have a wide range of source speed. There is no clear correlation between propagation speed and fracture mode, although shear fracture-induced events have, on the average, higher speeds than events attributed to tensile fracture.

Secondary fracture features are shown to affect the event radiation characteristics. *Shear cracks, which cannot grow in their plane for long distances, propagate through the formation of arrays of tensile cracks at their edges and tips. These cracks are important sound radiators, too, perhaps not individually, but in the aggregate, i.e., as arrays. I believe that their effect on the radiation due to shear deteriorates with increasing distance of the source from the receivers. This may explain the result that the radiation patterns of some events attributed to shear fracture appear unaffected by these tensile cracks, i.e., they are represented by lateral octopoles. Coalescence of tensile cracks at the edges of large shear faults have been observed in earthquakes, too, but at a larger scale.*

Finally, large-scale analysis of acoustic events in the aggregate and particularly in terms of temporal and spatial clusters has been performed. Event clusters have been analyzed in previous studies but no fracture details have been identified. Although in this thesis individual stages in the fault formation have not been identified, it has been shown that large-scale fracture features, such as process or crack zones, occur in the ice and contribute to the ambient noise. Events that are believed to be associated with such zones have characteristic low source speeds. These results support the observation, during earthquakes, that secondary fracture phenomena often appear as a slow kinematic growth in a macroscopic description.
6.6 Suggestions for further study

Acoustic radiation from ice-related processes in the Arctic ice is a complicated research problem. Only a few aspects of the problem can be studied in a single thesis and a limited number of issues can be addressed. The following are recommendations for future research, both on individual acoustic events and their aggregate.

Event signature types and their relation to the generating physical processes need to be further analyzed. However, both the results of previous studies and those of my own study suggest to me that it is almost impossible to correlate signature type with the characteristics of distinct physical processes. Possibly more than one signature type is associated with a particular event generating mechanism.

Particle motion of ice processes needs to be further investigated through the analysis of near-field data. Geophones need to be deployed near detected fractures for this purpose. During the SIMI experiment geophone data were collected, though only for a limited number of fractures and for a limited time. The analysis of acoustic data yields only rough estimates of particle motion of fractures. A procedure involving direct measurements of the ice-borne seismic waves, followed by integration in the time-domain (once) and the knowledge of the type of recorded wave types will result in more accurate estimates of particle motion-related parameters.

Source parameters, propagation speed, strike and dip angles, need to be estimated using a different procedure and be compared to the values obtained through the analysis of acoustic data. Again, direct measurements of the compressional and shear-wave signals may be needed, recorded by an array of geophones. Dip angle, which is one of the parameters that characterizes a fault has not been estimated here; it has been assumed to be 0° or 90°, depending on the assumed direction of propagation. Kim [21] has developed numerical models that include the effect of the variation of this angle.

In regard to the acoustic models, at distances where the complexity of fracture propagation, including the generation of arrays of secondary cracks and the variation of crack mode at the tips of shear fractures, become important, the proposed models must account for these effects. Secondary processes affect sound radiation from the primary fracture.
Finally, event clusters must be studied in greater detail and for long periods of time in order to identify large scale ice motion features, such as fracture cycles.
Bibliography


[33] Scheer, E. Personal communication.


Appendix A

Derivation of the mathematical expression for the detection threshold $\gamma$

The probability density of the sum of the squares of $L$ statistically independent, zero-mean, unit variance, Gaussian random variables is well-known as $\chi^2$ (chi-square) with $L$ degrees of freedom. Thus, the probability of false-alarm $P_F$, for the ambient noise data series is given by

$$P_F = Pr[\chi^2_L > \frac{\gamma}{\sigma^2}] = 1 - Pr[\chi^2_L \leq \frac{\gamma}{\sigma^2}] \quad (A.1)$$

where $\sigma^2$ is the variance of the noise and $\gamma$ the detection threshold. Using the fact that

$$\int_{-\infty}^{\infty} p_{\chi^2_L}(x)dx = 1 \quad (A.2)$$

the following expression for $P_F$ is obtained:

$$Pr[\chi^2_L > \frac{\gamma}{\sigma^2}] = \int_{-\infty}^{\infty} p_{\chi^2_L}(x)dx - \int_{-\infty}^{\frac{\gamma}{\sigma^2}} p_{\chi^2_L}(x)dx = \int_{\frac{\gamma}{\sigma^2}}^{\infty} p_{\chi^2_L}(x)dx \quad (A.3)$$

where

$$p_{\chi^2_L}(x) = \frac{1}{2^{\frac{L}{2}}\Gamma\left(\frac{L}{2}\right)}x^{\frac{L}{2}-1}e^{-\frac{x}{2}} \quad (A.4)$$
The *Gamma function* $\Gamma(\cdot)$ is given by

$$\Gamma(\alpha + 1) = \int_0^\infty y^\alpha e^{-y} dy \quad \alpha > -1$$

(A.5)

By the *Central Limit Theorem*, for $L \gg 1$, as is the case here, the $\chi^2$ distribution can be approximated by the normal distribution $\mathcal{N}(L, 2L)$ or $\mathcal{G}(x - \frac{L}{\sqrt{2L}})$, where $x = \frac{3}{\sigma^2}$.

$$\mathcal{G}(\frac{x - L}{\sqrt{2L}}) = \frac{1}{\sqrt{4\pi L}} \int_{-\infty}^x e^{-\frac{(\alpha - L)^2}{4L}} d\alpha$$

(A.6)

Using Equation (A.3) the final expression for $\gamma$ is obtained:

$$\frac{\frac{3}{\sigma^2} - L}{\sqrt{2L}} = \mathcal{G}^{-1}(P_F) \iff \gamma = \sigma^2 \sqrt{2L} \mathcal{G}^{-1}(P_F) + \sigma^2 L$$

(A.7)

Values of $\mathcal{G}^{-1}$ can be found in tables [19].
Appendix B

Maximum-likelihood estimation of time-delays in the presence of source motion

In the presence of source motion, the standard time-delay estimation procedure, described in Chapter 2, must be modified. The method described here has been proposed by Knapp and Carter [23].

Consider the event time series $x_i(t)$ and $x_j(t)$ such that

$$x_i(t) = s(\beta_i t) + n_i(t)$$
$$x_j(t) = s(\beta_j t + D) + n_j(t) \tag{B.1}$$

where $n_i(t)$ and $n_j(t)$ are the noise components of the signals, $D$ the time delay sought and $\beta_i, \beta_j$ the time compressions resulting from source motion, of the form $\beta_i = 1 + \frac{V_i}{c}$, where $V_i$ the component of source velocity in the direction of receiver $i$ and $c$ the sound speed in water. Both are assumed constant over the duration of the event, or the time window used in the detection process. In the presence of source motion, the cross-correlation coefficient of the two signals decreases according to the ratio $\frac{\beta_j}{\beta_i}$. Therefore, prior to cross-correlation, the event signals must be passed through a time compressor or expander to correct them for the effect of $\frac{\beta_j}{\beta_i}$. In cases where the signal-to-noise ratio is low, i.e., less than 5 dB, this
intermediate stage in the estimation of time delays is very important. The cross-correlation coefficient between event signals recorded by hydrophones at a large distance from each other is low, typically 0.4 or less, due to the high noise level. It is further affected by source motion and the resulting time delays are inaccurate.

Although \(x_i\) and \(x_j\) are individually stationary processes, they are jointly non-stationary, thus complicating the development of the Maximum-Likelihood (ML) function that needs to be maximized in the signal cross-correlation, i.e.,

\[
R_{x_i x_j}(t_i, t_j) = \alpha R_s(\beta_i t_i - \beta_j D) \quad (B.2)
\]

where \(\alpha\) is the relative attenuation and \(R_s\) the auto-correlation function of the signal \(s\). Recall that in the absence of source motion, the cross-correlation function \(R_{x_i x_j}\) is given by

\[
R_{x_i x_j}(\tau) = \int_{-\infty}^{\infty} G_{y_i y_j}(f) e^{j2\pi f \tau} df \quad (B.3)
\]

where \(y_i\) and \(y_j\) are the pre-filtered event time-series. An analogous expression must be obtained for the cross-correlation function of the two series in the presence of source motion. \(x_i\) and \(x_j\) can be represented by the Fourier series coefficients \(C_i\) over the detection interval \([0, T]\). These coefficients are given by

\[
C_i = \frac{1}{T} \int_{0}^{T} x_i(t) e^{-2\pi j f \Delta t} dt \quad (B.4)
\]

where \(\Delta f = \frac{1}{T}\). As \(T \to \infty, TC_i \to X_i(f_k)\), where \(X_i(f)\) is the Fourier transform of \(x_i(t)\).

Given \(\beta_i\) and \(\beta_j\), the best time-delay estimate \(D\) is one that maximizes the function

\[
J = \int_{-\infty}^{\infty} X_i(f) X_j^*(\frac{\beta_j}{\beta_i} f) W(f) e^{2\pi j f D \Delta f} df \quad (B.5)
\]

where \((*)\) denotes complex conjugate and \(W(f)\) is given by

\[
W(f) = \frac{\alpha G_s(f)}{G_{n_i}(f) G_{n_j}(f) + G_s(f) [G_{n_i}(f) + G_{n_j}(f)]} \quad (B.6)
\]

where \(G_s\) and \(G_{n_i}, G_{n_j}\), are the auto-spectral density functions of the signal and the noise
components, respectively. The derivation of the function $J$ in Equation B.5 can be found in [23]. The frequency weighing, introduced similarly to the generalized maximum-likelihood estimation procedure is independent of the scaling $\frac{\beta_k}{\beta_j}$.

An issue of concern is when time-scaling is necessary to yield an accurate estimate of the time-delay $D$. If no time-scaling is introduced, $X_j(\frac{\beta_i f}{\beta_i})$ is replaced $X_j(f)$ in Equation B.5.
Appendix C

Linearization of the sound speed profile and refraction calculations

C.1 Refraction calculations

The sound speed profile in Figure (2-5) is linearized through interpolation, to simplify refraction calculations. The resulting profile is shown in Figure C-1:

Figure C-1: Linearized sound speed profile
Assuming vertical stratification of the ocean, the water column is divided into layers; five layers are distinguished. The hydrophones are in the second layer, at depth $z = 60$ m. The sound velocity gradient $g_i$ in layer $i$ is determined by

$$g_i = \frac{dc}{dz} = \frac{c_{i+1} - c_i}{z_{i+1} - z_i}$$  \hspace{1cm} (C.1)$$

It is estimated that there is a change in the gradient at depths $z = 40, 60, 190, 320$ m. The corresponding gradients are approximately $g_0 = 0.05$, $g_1 = 0.3$, $g_2 = 0.023$ $g_3 = 0.061$ and $g_4 = 0.035$. In general, the path of a sound ray, leaving the source at depth $z = 0$ and reaching the receiver at $z > 0$, $0 < z < z_1$, where $z_1$ is the depth of the first layer, is characterized by two angles, the launch angle (with the horizontal) $\theta_0$ and the angle $\theta_1$ at $z$, as shown in Figure C-2:

![Figure C-2: Ray path, from a source at the ocean surface, to an arbitrary depth $z$ within the first layer.](image)

The horizontal range $R$, propagation time and depth $z$ can be calculated using the following general formula, i.e., valid for a sound ray in an arbitrary layer $i$, between depths $z_i$ and $z_j = z_{i+1}$:

$$R_{ij} = \frac{c_{ij}}{g_i} (\sin\theta_i - \sin\theta_j)$$  \hspace{1cm} (C.2)

$$z_{ij} = \frac{c_{ij}}{g_i} (\cos\theta_j - \cos\theta_i)$$  \hspace{1cm} (C.3)
\[ t_{ij} = \frac{1}{2g_i} \ln\left( \frac{(1 + \sin \theta_i)(1 - \sin \theta_j)}{(1 - \sin \theta_i)(1 + \sin \theta_j)} \right) \] (C.4)

where \( \theta_i, \theta_j \) correspond to \( \theta_0, \theta_1 \) in Figure C-2, and \( c_{ij} \) is the sound speed at depth \( z_j \). A special case is that of a vertexing ray, for which \( \theta_j = 0 \). In this case, the distance \( R \) is the so-called critical distance. The minimum launch angle is that of a ray vertexing at the depth of the hydrophones. Here it is approximately, \( \theta_{0,\min} \simeq 5^\circ \). The corresponding critical distance \( (R_{\text{crit}}) \) is 757 m and the cycle distance is 1514 m \((2R_{\text{crit}})\).

A sound ray can reach a receiver at two different times, before or after vertexing, depending on the horizontal distance between source and receiver. The two propagation paths are shown in Figure C-3:

![Figure C-3: A sound ray can reach the hydrophones either before or after vertexing. In the two cases the horizontal distance \( R \) and consequently the propagation time, are different.](image_url)

The refraction calculations are organized as follows: first the maximum and minimum launch angles of the rays which will hit a hydrophone are calculated. Snell's law is then used to determine launch angles and all the subsequent angles of intersection at the layer interfaces, as well as the angle which intercepts the hydrophone at its depth. Once these have been estimated, the propagation time and horizontal range for a ray in each layer are calculated. They are subsequently superimposed to obtain the total horizontal range and propagation time. Two ranges and propagation times are obtained for each launch angle.
An example of a ray path is shown in Figure C-4:

Figure C-4: Sound ray propagating in the layered ocean. Its direction and consequently $\theta_i$ changes according to the variation of sound speed with depth. The total horizontal range is $R_{tot} = R_1 + R_2 + 2R_3 + 2R_4$. Similarly, the total propagation time is $t_{tot} = t_1 + t_2 + 2t_3 + 2t_4$, where $R_i$ and $t_i$ are calculated from Equations C.2-C.4, for a particular launch angle.

At the second stage of event localization, described in Chapter 2, the source is placed at each point of a grid and a standard ray shooting procedure is followed, i.e., rays are launched from the source location to each hydrophone and the refractive propagation time and horizontal distance $R$ are calculated. A limit depth of 300 m is chosen to reduce the number of iterations. Rays propagating at greater depths result in horizontal ranges much larger than those expected. Time delays are also estimated and compared to the measured time delays from the data. The location for which the estimated and measured time delays is minimum is the best event location.

C.2 Reflection calculations

A sound ray can reach the hydrophone after it has bounced once or several times off the ice cover. Only specular reflections of sound rays are considered here, i.e., it is assumed that the ice cover is homogeneous. A reflection of an event sound ray off the ice corresponds to
a sign change in the peak event amplitude, at the hydrophone with is at a distance greater than the cycle distance. In order to ensure that observed sign changes are due to the event radiation characteristics, the possibility of reflection must be investigated. There are only 6 events (3% of the total event population), the distance of which from a few hydrophones is greater than the cycle distance.

In the case of reflection of a ray off the ice, the total horizontal range is a superposition of the cycle distances between bounces, as shown in Figure C-5:

![Figure C-5: Sketch of a sound ray from a source which reaches the receiver after having undergone two reflections off the ice cover.](image)

The number of reflections can be determined by comparing the distance between source and receivers.

**Example:** Consider an event located at $x = 1204$ m, $y = 689$ m. The distance between the event and each of the hydrophones of the horizontal array (Figure 2-3) is calculated and compared to the cycle distance (1514 m). The number of reflections the sound ray undergoes, from the source to a particular hydrophones, is estimated as the integer ratio of the distance between source and receivers and the cycle distance $X$. For this particular events it is found that only five hydrophones are at distances greater than $X$. The number of reflections versus the azimuth angle of the hydrophones is shown in Figure C-5:
Figure C-6: The sound rays reaching seven of the hydrophones have undergone 1 reflection off the ice cover. The concentric circles show the number of reflections and the azimuth range is that of the hydrophones.

The sign change in peak amplitude due to the reflection of the event signal can be observed in the event time series:

Figure C-7: Event time series. A sign change in peak event amplitude is occurs at hydrophones 12-15 and 22-24.
Appendix D

Event signal polynomial approximation and integration

D.1 N-th order Chebychev polynomial

A Chebychev polynomial of degree $n$ is defined as

$$T_n(x) = \cos(n\cos^{-1}(x)) \quad (D.1)$$

In order to approximate an arbitrary function $f(x)$ in the interval $[-1, 1]$ by a Chebychev polynomial of degree $n$, coefficients $c_j$, $j = 0, 1, ..., n - 1$ are first calculated as:

$$c_j = \frac{2}{n} \sum_{k=1}^{n} f(x_k)T_j(x_k) \quad (D.2)$$

Then, the function $f(x)$ is approximately given by

$$f(x) \approx \sum_{k=0}^{n-1} c_kT_k(x) - \frac{1}{2}c_0 \quad (D.3)$$

The approximation formula is exact for $x$ equal to the $n$ zeros of $T_n(x)$. Among polynomial approximations of the same degree, the Chebychev approximation has the smallest deviation from the true function of interest $f(x)$. The computed polynomial is very close to the ideal
minimax polynomial [4].

This method can be used to approximate event signatures by a polynomial and consequently facilitate integration of the functions which best describe these signatures. The following is an example of a type I event signature and its Chebychev polynomial approximation.

![Type I event signature](image1)

![Chebychev polynomial approximation of type I event signature](image2)

Figure D-1: Original and approximated type I event signature, using a Chebychev polynomial of degree equal to the size of the time sequence. The original signal has been smoothed using a Hanning window. The duration of the event $T$ is 0.030 sec.

### D.2 Event signal integration

If $c_i, i = 0, 1, \ldots, n - 1$ are the coefficients that approximate the function $f(x)$, $C_i$ are the coefficients that approximate the indefinite integral of $f$. Although the interval of
integration is known, the shape of the integrated function is sought in this process and not its actual value. The coefficients $C_i$ are given by

$$C_i = \frac{C_{i-1} - C_{i+1}}{2(i-1)} \quad i \geq 1$$  \hfill (D.4)

A value of $C_0$ can be chosen, corresponding to an arbitrary constant of integration. Then, $\int f(x)dx$ is approximated by

$$\int f(x)dx \approx \sum_{k=1}^{n-1} C_k T_k(x) - \frac{1}{2} C_0$$  \hfill (D.5)

The process can be repeated for a more than one integration. Each time, the coefficients of approximation are reduced by 1. In integrating the event sound pressure time series care must be taken in the interval chosen; it must be equal to the duration of the event. The first integral of acoustic pressure with respect to time corresponds physically to stress or traction in the ice. The second integral corresponds to the shear wave displacement; the third is the slip displacement at the fault [10].

For the event signature in Figure D-1 the first two integrals are shown below:

![Graph showing first integral of sound pressure: traction in an unbounded medium.](image)

Figure D-2: First integral (with respect to time) of event sound pressure time series: physically this corresponds to traction in an unbounded medium.
Figure D-3: Second integral (with respect to time) of event sound pressure time series: physically this corresponds to shear wave displacement.

Chapter 2 has a detailed description of different types of slip displacement functions corresponding to distinct event sound pressure signals.
Appendix E

Errors in Doppler shift estimation

E.1 Error due to the use of peak event frequencies in the estimation

In the Doppler shift estimation, the event spectra at different hydrophones are cross-correlated. The frequency shift at the absolute maximum cross-correlation coefficient is the Doppler shift sought. In the case of simple events, their peak event frequency is clearly detected in their spectrum and consequently it can be accurately estimated. The shift in peak event frequency observed at different hydrophones may be used as a measure of the relative Doppler shift, due to source motion. However, this type of estimation may yield very inaccurate results, particularly in the case of compound events, in the spectrum of which the peak frequency is not clearly distinguished. Estimation of Doppler shift by spectrum cross-correlation is a more accurate procedure. The portion of the spectrum within the event frequency range is selected; those parts of the spectrum where the ratio of source-to-noise levels is lower than 5 dB are disregarded.

Figure E-1 shows an example of the error in Doppler shift resulting from the use of peak event frequencies in the estimation, relative to the Doppler shift obtained by spectrum cross-correlation:
Figure E-1: Relative error in Doppler shift, computed as the difference between the estimates obtained through spectrum cross-correlation and comparison of peak event frequencies. Symbols used: (+) for Doppler shift values when spectrum cross-correlation is used; (o) for corresponding values when the estimated peak event frequencies are used.

In the above plot, channel 1 has been used as the reference channel, i.e., its spectrum has been cross-correlated with the event spectrum in all other hydrophones and its peak frequency has been compared to the corresponding peak frequencies, in the two Doppler shift estimation procedures, respectively. The relative error is between 0 and 3 Hz for hydrophones which are close to hydrophone 1 and between 4 and 18 Hz for distant hydrophones.

Even if spectrum cross-correlation is used for estimating the Doppler shift, the frequency portion of the spectrum used in this procedure must be carefully selected. In Sections E.2 and E.3, potential sources of error are discussed.
E.2 Error in Doppler shift due to the use of the entire spectrum in the cross-correlation

In Chapter 3 it has been discussed that in spectrum cross-correlation only the portion of the spectrum within the event bandwidth or frequency range should be included. If the entire spectrum is used instead, errors in the Doppler shift estimates result. Outside the frequency range of the event, the ratio of the source-to-noise levels is 5 dB of less. The noise components of the event time series at different hydrophones are uncorrelated, and this must also be true for their spectra. Therefore, when the entire event spectrum is used, the cross-correlation coefficient will be lower. Consequently the accuracy of the estimated Doppler shift will decrease.

In Figure E-2, the cross-correlation function of an event spectrum in channels 1 and 2 is shown, when only the portion of the spectrum within the event frequency range is used. The distance between the two channels is about 15 m.

![Spectrum Cross-Correlation](image)

**Figure E-2:** Cross-correlation function for an event spectrum, in channels 1 and 2. The event bandwidth is 90 Hz and its peak frequency is 55 Hz. Therefore, the portion of the spectrum used in the cross-correlation is between 10 and 100 Hz.
In Figure E-3 the same cross-correlation function is obtained, but the entire spectrum, i.e., between 10 and 350 Hz, is now used.

Figure E-3: Cross-correlation function for the same event spectrum as that considered in the Figure E-2. The entire spectrum is used in the cross-correlation. The maximum cross-correlation coefficient is 0.72.

The decrease in the maximum cross-correlation coefficient from 0.81 to 0.72 is about 11 % for channels 1 and 2. For the entire array, this change is approximately 12 – 15 %, for hydrophones which are close to each other and 30 – 40 % for distant hydrophones. It is evident that the event spectra are better cross-correlated when only the pertinent segments of these spectra are used. The decrease of the cross-correlation is also reflected in the estimated Doppler shift.

Figure E-4 shows the resulting error in this parameter. Channel 1 has been used as the reference channel.
Figure E.4: Relative error in Doppler shift, computed as the difference between the estimates obtained through spectrum cross-correlation, using the entire spectrum and the portion within the event frequency range, respectively. Symbols used: (o) for Doppler shift values obtained when the entire spectrum is used in the cross-correlation; (+) for corresponding values when only the portion of the spectrum within the event frequency range is used.

The error in Doppler shift is between 0 and 4 Hz for hydrophones which are close to hydrophone 1 and between 1 and 16 Hz for distant hydrophones.

**E.3 Estimation of the lower bound for the cross-correlation coefficient**

In the spectrum cross-correlation procedure, a threshold must be set for the cross-correlation coefficient. Poorly correlated spectra result in large values of Doppler shift and consequently appreciable errors in the estimates of source speed, orientation and intrinsic source frequency. In order to select this threshold, the following procedure has been used: first
two event series are selected, for which it is known that the corresponding event spectra are well correlated. In the example described here, the same spectra used to obtain the cross-correlation function in Figure E-2 are considered. One of the event series is then corrupted with noise. Here, noise prior to the event has been used. The spectrum of the resulting series is computed and cross-correlated with event spectrum at the other channel. The resulting cross-correlation function is shown in Figure E-5:

![Cross-correlation function graph](image)

**Figure E-5**: Cross-correlation function of event spectrum in channels 1 and 2. The original event series in channel 2 has been corrupted with noise.

Originally, the maximum cross-correlation was 0.81. Due to the addition of noise in one of the event series, it has now decreased by 16%. Since all event spectra are cross-correlated with each other, a square correlation matrix is obtained ($n \times n$), where $n$ is the number of hydrophones which participated in the event detection. The threshold for the cross-correlation coefficient has been set at 80% of the maximum entry of the correlation matrix.
Appendix F

Derivation of horizontal and vertical quadrupole radiation patterns

The geometry of the problem is shown in Figure F-1:

Figure F-1: Geometry of the problem.
There are two types of acoustic quadrupoles, the longitudinal and the lateral. They both consist of two superimposed dipoles, with equal but opposite amplitude vectors, as shown in Figure F-2:

![Diagram of Longitudinal and Lateral Quadrupoles](image)

(a) Longitudinal Quadrupole  (b) Lateral Quadrupole

Figure F-2: Longitudinal and lateral quadrupole configurations. $D$ is the dipole amplitude vector, $d$ the distance between the two dipoles and $R$ the distance of the source from the receiver. For the longitudinal quadrupole $D$ and $d$ are parallel and for the lateral they are perpendicular.

Assuming that $d \ll R$ and $kd \ll 1$, the pressure field due to a quadrupole is given by

$$p' = (\vec{V} \cdot \nabla)(\vec{d} \cdot \nabla) = \sum_{i,j=1}^{3} D_i d_j \frac{\partial^2}{\partial x_i \partial x_j} \left( \frac{e^{ikR}}{R} \right)$$  \hspace{1cm} (F.1)

where $p'$ is the pressure perturbation due to the quadrupole [31]. The slant distance between source and receiver is:

$$R = [(x - x_e)^2 + (y - y_e)^2 + (z - z_e)^2]^{\frac{1}{2}}$$  \hspace{1cm} (F.2)

where $(x_e, y_e, z_e)$ are the event coordinates. It is assumed that the source is located at $z = 0$. 

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In spherical coordinates:

\[
(x - x_e) = R \cos \phi \cos \theta \\
(y - y_e) = R \sin \phi \cos \theta \\
z = R \sin \theta
\]

Equation F.1 can be rewritten as:

\[
p' = \sum_{i,j=1}^{3} D_i d_j \left( \frac{\partial}{\partial R} \frac{\partial R}{\partial x_i} \right) \left( \frac{\partial}{\partial R} \frac{\partial R}{\partial x_j} \right) (\frac{e^{ikR}}{R})
\]  

(F.3)

As an example, consider a lateral quadrupole in the \(x \sim y\) plane. From Equations F.3 and F.2:

\[
p' = D_x d_y \left( \frac{x - x_e}{R} \right) \frac{\partial}{\partial R} \left[ \frac{(y - y_e)}{R^3} \left( \frac{e^{ikR}}{R} \right) \right]
\]

(F.4)

\[
D_x d_y \left( \frac{x - x_e}{R^2} \right) \frac{\partial}{\partial R} \left[ \frac{(y - y_e)}{R^3} \left( \frac{ik e^{ikR}}{R^2} \right) \right] =
\]

\[
D_x d_y \left( \frac{(x - x_e)(y - y_e)}{R^2} \right) \left( -k^2 - \frac{3ik}{R} + \frac{3}{R^2} \right) (\frac{e^{ikR}}{R})
\]

Using the expressions for \((x - x_e)\) and \((y - y_e)\) in spherical coordinates, and taking into account the direction of propagation of the source, the following expression is obtained:

\[
p' = D_x d_y \sin(\phi - \phi_s) \cos(\phi - \phi_s) \cos^2 \theta \left( -k^2 - \frac{3ik}{R} + \frac{3}{R^2} \right) (\frac{e^{ikR}}{R})
\]

(F.5)

where \(\phi_s\) is the source orientation. Thus, the directivity pattern of this lateral quadrupole is given by:

\[
B(\phi)g(\theta) = \cos(\phi - \phi_s) \sin(\phi - \phi_s) \cos^2 \theta
\]

(F.6)

Notice that for an octopole, i.e., a quadrupole and its image, a \(\sin \theta\) term must be included in the directivity pattern, due to the Lloyd Mirror effect, described in Appendix G, i.e.,

\[
B(\phi)g(\theta) = \cos(\phi - \phi_s) \sin(\phi - \phi_s) \cos^2 \theta \sin \theta
\]

(F.7)
Similarly, for all possible quadrupoles, lateral and longitudinal, the corresponding directivity patterns are given by:

\[
\begin{align*}
B(\phi)g(\theta) &= \cos(\phi - \phi_s)\sin(\phi - \phi_s)\cos^2\theta & \text{lateral } x - y \\
B(\phi)g(\theta) &= \cos(\phi - \phi_s)\sin\theta\cos\theta & \text{lateral } x - z \\
B(\phi)g(\theta) &= \sin(\phi - \phi_s)\sin\theta\cos\theta & \text{lateral } y - z \\
B(\phi)g(\theta) &= \cos^2(\phi - \phi_s)\cos^2\theta & \text{longitudinal along } x\text{-axis} \\
B(\phi)g(\theta) &= \sin^2(\phi - \phi_s)\cos^2\theta & \text{longitudinal along } y\text{-axis} \\
B(\phi)g(\theta) &= \sin^2\theta & \text{longitudinal along } z\text{-axis}
\end{align*}
\]
Appendix G

Lloyd Mirror Pattern

In Chapter 5 it has been discussed that the presence of the ice is neglected and acoustic field results from the superposition of the source applied directly below the ice cover, i.e., at depth $z = h$ and its image, applied directly above the ice, i.e., at depth $z = -h$, where the ice thickness is $2h$. The Lloyd mirror pattern results from the interference of the source image. The geometry is shown in Figure G-1:

![Figure G-1: Geometry of the source, its image and their distance from the hydrophone.](image)

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Neglecting the time dependence without loss of generality, the total acoustic pressure field is

\[ p(\bar{R}) \propto \frac{e^{ikR_1}}{R_1} - \frac{e^{ikR_2}}{R_2} \]  \hspace{1cm} (G.1)

where \( k \) is the wavenumber, and \( R_1, R_2 \) are given by

\[ R_1 \approx R - h\sin\theta \]  \hspace{1cm} (G.2)
\[ R_2 \approx R + h\sin\theta \]  \hspace{1cm} (G.3)

Thus, Equation G.1 becomes

\[ p(\bar{R}) \propto \frac{1}{R} \left[ e^{ik(R-h\sin\theta)} - e^{ik(R+h\sin\theta)} \right] = \frac{1}{R} \left[ e^{-ikh\sin\theta} - e^{ikh\sin\theta} \right] \]  \hspace{1cm} (G.4)

Using the fact that \( \sin z \) can be expressed as \( \sin z = \frac{e^{iz} - e^{-iz}}{2i} \), Equation G.4 can be written as

\[ p(\bar{R}) \propto -\frac{2i}{R} \sin(kh\sin\theta)e^{ikR} \]  \hspace{1cm} (G.5)

Two cases are distinguished, namely for \( kh\sin\theta \ll 1 \) and \( kh\sin\theta \gg 1 \). Thus, Equation G.5 can be written as

\[ p(\bar{R}) \propto \begin{cases} 
-\frac{2i}{R} kh\sin\theta e^{ikR} & \text{if} \hspace{0.2cm} kh\sin\theta \ll 1 \\
-\frac{2i}{R} \sin(kh\sin\theta)e^{ikR} & \text{if} \hspace{0.2cm} kh\sin\theta \gg 1 
\end{cases} \]  \hspace{1cm} (G.6)

Assuming compactness of the source, i.e., the separation between the source and its image is much smaller than the wavelength of the source, \( k\sin\theta \ll 1 \), only the first of the expressions in Equation G.6 is considered.

For the above described pressure field, the pressure maximum is twice that of a single source (constructive interference) and the pressure minimum is zero (destructive interference) [18]. The number of Lloyd-mirror beams \( M \) of the directive pattern is given by

\[ (2M - 1) \frac{\pi}{2kh} \leq 1 \Leftrightarrow M \leq \frac{c}{4fh} + \frac{1}{2} \]  \hspace{1cm} (G.7)

where \( k = \frac{2\pi f}{c} \), with \( f \) the frequency of the source and \( c \) the sound speed in water.