NOISE ANALYSIS OF A SUSPENDED HIGH POWER MICHELSON INTERFEROMETER

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Abstract

The Laser Interferometer Gravitational Wave Observatory (LIGO Project) will search for
gravitational waves by observing shifts in the interference of a Michelson interferometer. To
start detecting gravitational waves with any measure of confidence, current estimates require
the interferometer to be sensitive to differences of at least $10^{-9}$ radians in the phase of light.
Ground-based LIGO will offer this sensitivity in a band around 100 Hz. The sensitivity of
LIGO is limited at frequencies below 200 Hz by random (mainly seismic and thermal) forces
acting on its optical elements. Around and above 200 Hz — where this "displacement noise" is
no longer significant — the sensitivity is determined by how well the interference shift can be
determined at the detector. The quantum nature of coherent laser light in the interferometer
imply a power fluctuation at the detector that scales as $\sqrt{I}$, where $I$ is the light intensity at
the Michelson beam-splitter. With the signal power scaling as $I$, a fundamental sensitivity
limit is thus set for detection. To obtain the desired sensitivity given this limit, LIGO will
have close to 100 watts of laser light incident on the beam-splitter. No laboratory in the
world, to the best of our knowledge, has had experience with interferometry at these high
power levels prior to this thesis. This thesis experimentally tests an important assumption
used in the noise estimates — that the noise above 200 Hz will be quantum limited.

To investigate the noise in interference detection at high power levels, a team at MIT (to
which the author belongs)$^{1}$ has constructed a suspended Michelson interferometer. The noise

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$^{1}$Prof. R. Weiss and Dr. D. Shoemaker gave birth to the Phase Noise Interferometer (PNI) project after they
in the detection of the differential phase of this interferometer was investigated at two stages. At the first stage, several hundreds of milliwatts from a frequency stabilized Ar⁺ gas laser was incident directly on the beam-splitter. At the second stage, the input light was constructively built (recycled) to above 30 watts at the beam-splitter using an optical cavity — this cavity was formed by placing a partially transmitting mirror in the input light as the front (power recycling) mirror and the Michelson interferometer as the back mirror. Our experience showed that above 1 kHz, the noise indeed was quantum limited consistent with the incident power. This led to the measurement of a phase noise sensitivity of about $3 \times 10^{-10}$ radian/$\sqrt{\text{Hz}}$ in the recycled interferometer, better than any known measurement to date. Below 1 kHz, we examined the “technical” noise sources that caused the noise to be above the quantum limit. We concluded that back scattering of light, input beam jitter, and residual frequency noise need to be controlled to get down to the fundamental limit required by LIGO at these frequencies.

This thesis discusses the construction of the interferometer, the noise models and experiments used to analyze the measured phase noise at the two stages, and the implications of the experimental results to LIGO.

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wrote the proposal in 1992; the actual work started in full swing around mid 1994. The team at MIT consisted of Brian Lantz and Partha Saha as graduate students; Drs. Peter Fritschel (charge), Gabriela Gonzalez (from April 1995), and Mike Zucker (actively from September 1994 to early 1995, in an advisory role thereafter) as research scientists. Mr. Tom Evans, Mr. Ed Kruzel, and Mr. Ralph Burgess offered technical help, while Prof. H. Weiss and Dr. D. Shoemaker offered advice and guidance. The project drew support and technical help from the entire LIGO team in many ways — especially Dr. Sigg and Ms. Mavalvals, who gave the project its first working wavefront sensor.
Contents

Abstract ........................................................................................................ iii

1 Gravitational Wave Detection Through Interferometry 1
   1.1 Gravitational waves and their detection ........................................... 1
   1.2 Sources of gravitational waves ....................................................... 4
   1.3 History of gravitational wave detection and LIGO ......................... 5
   1.4 The LIGO noise budget ................................................................. 8

2 A Suspended High Power Michelson Interferometer 13
   2.1 Interferometry techniques used in the PNI ................................. 14
       2.1.1 A simple Michelson interferometer ...................................... 14
       2.1.2 A phase modulated Michelson interferometer with asymmetry 15
       2.1.3 A recycled Michelson interferometer ................................... 19
   2.2 Sources of phase noise in the PNI ................................................. 22
       2.2.1 Noise in the input light ....................................................... 22
       2.2.2 Noise sources in the interferometer .................................... 23
   2.3 Construction of the PNI .............................................................. 24
       2.3.1 PNI in the first stage ......................................................... 24
       2.3.2 Laser stabilization subsystems ......................................... 24
       2.3.3 Suspensions ................................................................. 27
       2.3.4 Seismic isolation ........................................................... 30
       2.3.5 Optics ....................................................................... 31
       2.3.6 Michelson length control ............................................... 33
2.3.7 PNI in the second stage ........................................... 34
2.3.8 The recycled Michelson length control ......................... 35
2.3.9 The common mode servo ............................................ 36
2.3.10 The active differential alignment system .................... 39
2.4 Performance of the PNI ................................................. 41

3 Noise Sources in the Light ............................................ 43
3.1 Fundamental or quantum Noise in light ............................. 43
3.2 Frequency noise .......................................................... 49
3.3 Amplitude Noise .......................................................... 52
3.4 Beam jitter noise .......................................................... 55

4 Noise Sources in the Interferometer ................................ 63
4.1 Displacement noise ....................................................... 63
  4.1.1 Seismic noise ......................................................... 63
  4.1.2 Thermal noise ......................................................... 64
  4.1.3 Radiation pressure .................................................. 66
4.2 Interferometer misalignment .......................................... 67
4.3 Parasitic Interferometry ............................................... 69
4.4 Instrumentation noise .................................................. 75
  4.4.1 Amplitude modulation by Pockels cell ............................ 75
  4.4.2 Noise in the differential length sensing and control loop .... 76

5 Final Remarks ............................................................... 79
5.1 The PNI spectra ......................................................... 81
5.2 Implications for LIGO .................................................. 83

Appendices ................................................................. 85

A The Michelson Interferometer as an Optical Element .......... 85

B The Fabry Perot Cavity: Optical Parameters and Resonance .. 89
C Feedback Control Systems  95

Bibliography  99
Chapter 1

Gravitational Wave Detection Through Interferometry

This chapter will offer a brief and simple introduction to the physics of gravitational waves and their detection. The aim will be to motivate the focus of this thesis, that the phase difference measured by a Michelson interferometer can achieve a sensitivity of $10^{-10}$ radian/$\sqrt{\text{Hz}}$ in a bandwidth around several hundreds of Hertz.

1.1 Gravitational waves and their detection

The theory of special relativity relates events across inertial frames of reference through Lorentz transformations. It gives us the metric $g_{\mu\nu}$ which, acting on a space-time interval, gives us an invariant across all inertial frames of reference.

General theory of relativity introduces non-inertial effects to $g_{\mu\nu}$. The motion of massive objects causes changes in $g_{\mu\nu}$ to propagate away at the speed of light. In the coordinate frame of an observation point far away from a massive object in motion, small perturbations $h_{\mu\nu}$ to the special relativistic metric $\eta_{\mu\nu}$ can be sensed:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (1.1)$$

It is this $h_{\mu\nu}$ that gets referred to as gravitational waves.

Einstein’s equation, assuming small variations in $g_{\mu\nu}$ and excluding the source, reduces to a wave equation for $h_{\mu\nu}$ — a transverse variation of $\eta_{\mu\nu}$ with respect to the direction of
propagation can then be assumed. The freedom to choose a coordinate frame allows us to select one where \( h_{\mu \nu} \) is traceless (the transverse traceless or TT gauge). If the propagation direction be \( z \), the non-zero elements of \( h_{\mu \nu} \) correspond to those that multiply the \( x \) and \( y \) coordinates, and get symmetrically located within a matrix representation (metric property). Thus the most general TT \( h_{ij} \) can be written as

\[
h_+ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + h_\times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{1.2}
\]

where the first piece is referred to as the \('+' (h_+)\) polarization, and the second one the \('\times' (h_\times)\) polarization.

The effect of a gravitational wave can be demonstrated if a measurement of time interval with light is attempted in some coordinate frame of reference. Arbitrarily, we choose an \( h_+ \) wave; the \( h_\times \) wave has exactly the same effect in a coordinate frame rotated 45° with the one we will use for our calculations. We send a light pulse from the origin along each of the transverse orthogonal axes of the coordinate frame given to us by the \( h_+ \) wave. These light pulses are retro-reflected by mirrors back to the origin after traveling the same coordinate length \( l \) in the two orthogonal directions, and we compare the two round trip times in the presence of an \( h_+ \) wave. For the light ray sent along the \( x \) axis,

\[
-(c dt)^2 + (1 + h_+)(dx)^2 = 0 \tag{1.3}
\]

\[
\Rightarrow \int_{\text{round trip}} dt = \int_{\text{round trip}} \sqrt{1 + h_+ \frac{dx}{c}}
\]

\[
\Rightarrow \int_{\text{round trip}} dt = \int_{\text{round trip}} \frac{dx}{c} + \int_{\text{round trip}} \frac{1}{2} h_+ dt + O(|h_+|^2).
\]

For the other pulse launched at the same time in the other orthogonal direction \( y \), its round-trip time differs by \( \int_{\text{round trip}} h_+ dt \) from the one along \( x \). Thus the phase difference between two phase-fronts starting at the same time via a beam splitter at the origin and then combined after retro-reflection at the two mirrors is

\[
\delta \phi(t) = \omega_0 \int_{t-2l/c}^{t} h_+ dt' \tag{1.4}
\]

where \( \omega_0 \) is the frequency of light. Let us now assume that \( h_+ = h_0 \cos(\omega_g t) \) in our coordinate frame, where \( \omega_g \) is the frequency of the gravitational wave reaching us. Carrying out the
integration in Equation 1.4, we finally obtain,

\[ \delta \phi(t) \approx h_0 \cdot \frac{2l}{c} \cdot \frac{\omega_0 \sin(\omega_g l/c)}{\omega_g (l/c)} \cdot \cos[\omega_g (t - l/c)]. \]  

(1.5)

If \( \omega_g l/c \) is much less than 1, we get

\[ |\delta \phi(t)| \approx h_0 \omega_0 (2l/c). \]  

(1.6)

Thus we are naturally led to the concept of interferometry for detection of gravitational waves; the phase difference measured by a Michelson interferometer can be attributed to passing gravitational waves in absence of other sources of such phase change. We also notice from Equation 1.6 that the longer the arms of this Michelson interferometer, the larger is the measured phase difference owing to gravitational waves.

In the example we just described, the two masses were aligned along the principal axes of \( h_{ij} \); to understand what happens when the two masses lie along two arbitrarily chosen orthogonal axes, we refer to Figure 1.1. We show masses distributed in a ring and their relative separations, as a gravitational wave propagates normally through the ring. Any two masses at the end of lines intersecting at 90° can be used as mirrors to define a Michelson interferometer with a beam-splitter at the origin; as the arm lengths from the origin to these two masses show any relative change, so will the phase difference measured at the beam-splitter. We thus observe that even if the wave may not be properly aligned with the two arms of the interferometer, we will measure some phase difference owing to the passing gravitational wave — this phase difference, however, may not be as large as that in the aligned case. Let \( \delta \phi_{+,x}(t) \) refer to the maximum possible phase difference that can be measured by properly aligning the interferometer for the corresponding polarization. The phase difference, measured by an interferometer with its arms defining the \( x \) and \( y \) axes of an arbitrary coordinate frame in which the gravitational wave is incident at a zenith angle of \( \theta_{gw} \), and an azimuth angle of \( \phi_{gw} \), is then given by

\[ \delta \phi(t) = \frac{1}{2} (1 + \cos^2 \theta_{gw}) \cos(2\phi_{gw}) \phi_+(t) + \cos \theta_{gw} \sin(2\phi_{gw}) \phi_x(t). \]  

(1.7)
Figure 1.1: A ring of free masses in some coordinate frame as the gravitational wave propagates normally through it; the relative separation of the masses change as indicated for the two different polarizations.

1.2 Sources of gravitational waves

We can try to estimate the magnitude of $h_0$ by drawing analogs with electro-magnetic radiation. Mass and momentum conservation rule out radiation from the monopole and dipole parts of a system of moving masses; the quadrupole is then the first term that radiates, and this happens to be the $\dot{Q}$ term. To have the energy propagating out, we must have this term associated with an $1/r$ spatial dependence. We form a dimensionless $h_0$ with the fundamental constants $G$ and $c$ and the radiating $\dot{Q}/r$ (the radiation, being in the classical limit, cannot involve $h$) as

$$h_0 \sim \frac{G}{c^4} \cdot \frac{\dot{Q}}{r}. \quad (1.8)$$

Defining $E^Q$ to be the kinetic energy in quadrupolar form, and putting the relevant numbers in Equation 1.8 above, we find:

$$h_0 \sim 10^{-20} \cdot \frac{E^Q}{M_\odot c^2} \cdot \frac{10 \text{ Mpc}}{r} \text{ where} \quad (1.9)$$
1 \ M_\odot \ = \ 1.99 \times 10^{30} \ \text{kg} = 1 \ \text{Solar Mass}; \ \text{and} \n10 \ \text{Mpc} \ = \ 3.09 \times 10^{20} \ \text{kilometers} = \ \text{distance to Virgo Cluster.}

The numbers quickly establish that gravitational wave sources cannot be terrestrial; they must be generated by objects that are of interest in astrophysics.

Some estimates of gravitational wave strength are offered in [1] for different kinds of astrophysical sources. Bursts of gravitational waves immediately precede a collision of compact binaries or a supernova collapse; these bursts produce radiation at frequencies that are very low initially, but increase to about 10 kHz as the impending catastrophe draws near. Thorne [2, 1] calculates the strength for gravitational waves from a supernova collapse to be

\[ h_{rms} \approx 3 \times 10^{-20} \left( \frac{\Delta E}{M_\odot c^2} \right)^{1/2} \left( \frac{1 \ \text{kHz}}{f} \right)^{1/2} \left( \frac{10 \ \text{Mpc}}{r} \right), \]

(1.10)

where \( \Delta E \) represents the energy lost to gravitational waves. In fact, energy lost to gravitational waves (extremely slowly over years of observation) have been shown to be consistent with the decay of the orbital period of the binary system PSR 1913+16 (this is the famous binary pulsar system that obtained a Nobel prize for Hulse and Taylor in 1993).

Gravitational waves of well defined periodicity are supposed to be generated by rotating stars; Thorne [1] calculates the gravitational wave strength generated by a rotating neutron star of ellipticity \( \epsilon \) to be (\( \epsilon \leq 10^{-6} \))

\[ h_{rms} \approx 8 \times 10^{-20} \cdot \epsilon \cdot \left( \frac{f}{1 \ \text{kHz}} \right)^2 \left( \frac{10 \ \text{kpc}}{r} \right), \]

(1.11)

where \( f \) is the frequency of rotation.

Another source of gravitational radiation may well be a stochastic background which, because of its lack of any fixed direction of propagation, allows for easier data analysis — however, its strength cannot be predicted very well.

1.3 History of gravitational wave detection and LIGO

Attempts to detect gravitational waves started with the work of Joe Weber in the sixties, when he searched for coincident excitations of normal modes in two massive aluminum cylinders separated by a large distance. In 1970, Weber claimed that two such cylinders indeed were
registering vibrations owing to passing gravitational waves. This attracted a lot of experimenters; Weiss [3] conceived of an interferometric detection of the waves, and performed a detailed study of an instrument that could achieve this goal. Most of Weber's claims were later disputed, but the work on gravitational wave detection that had been initiated still continues unabated. The original idea of Weber has been developed by many groups, and the resonant "bar" technology — operated at low temperatures with better sensors — now claims detections at the level of $h \approx 10^{-18}$ in a bandwidth of few Hz around a resonance of approximately 1 kHz [4].

The idea of Weiss has undergone many modifications, and groups around the world have seriously pursued the idea of gravitational wave detection through interferometry. In the first section, we showed how a passing gravitational wave could be demonstrated by a change in the differential phase of a Michelson interferometer,

$$\delta \phi \approx h_0 \omega_0 (2l/c). \tag{1.12}$$

We notice that we can amplify the phase difference caused by a gravitational wave at low frequencies, if we can make the light traverse a long distance $l$. The original instrument constructed by Weiss [5], and a German group at Garching [6], involved a delay line in which the light bounced back and forth many times between two mirrors to build up the phase-shift before making its way out for interference. A French group [7], a Scottish group [8], and a group at Caltech [9] pursued the idea of using Fabry-Perot cavities as the arms of the Michelson interferometer — in this scheme, a partially transmitting mirror allowed the incident light to enter an optical cavity while letting a part of the light inside the cavity to escape out. Figure 1.2 illustrates the difference between a delay line and a Fabry-Perot interferometer.

The work with proto-types [6, 5, 9] quickly established that the gravitational waves reaching us could not have amplitudes larger than $h_{rms} \approx 10^{-16} \text{--} 10^{-17}$, and that the dominant ones most probably have frequencies below 1 kHz. Given this uncertain description of the waves, interferometric methods offer more advantages than the resonant "bar" technology as they allow a wider bandwidth search around low frequencies. A number of groups therefore have pooled their resources to develop the interferometer technology for direct detection and monitoring of gravitational waves. The Fabry Perot scheme of Figure 1.2 has become the preferred
overall design, as optical cavities allow high and spatially pure light intensities which then lead to better signal to noise ratios. The LIGO project of United States, a joint collaboration between MIT and Caltech, aims at building two interferometers with 4 km long arms — one in Hanford, Washington and another in Livingston Parish, Louisiana. The VIRGO project, a joint collaboration between a group in Italy and another in France, plans to build one interferometer with arm lengths of 3 km. GEO-600, a project pursued by a British and a German group, wishes to build one interferometer with arms 600 meters long, while TAMA-300, a Japanese effort, wants to construct its interferometer with 300 meter arms.

The work that is documented in this thesis was undertaken under the auspices of the LIGO project. This project [10] builds on the experience of Caltech and MIT with laser interferometers, and has two primary tasks: to build two 4 km long interferometers as stated before, and to understand and reduce the noise in these interferometers so that their sensitivity can be enhanced to where continual monitoring of gravitational waves is possible. While construction of the two interferometers are vigorously under way, scientists at the two institutions are also trying to understand the noise in laser interferometers by constructing proto-types. The 40 meter proto-type at Caltech recently achieved a (potential) sensitivity to gravitational waves of $h \approx 3 \times 10^{-19}$ around 450 Hz [11]. To understand what is needed to make this sensitivity even better and motivate the work documented in this thesis, we need to present the LIGO
noise estimates. We do this in the next section.

1.4 The LIGO noise budget

The approximate form of Equation 1.5, given in Equation 1.6, holds true for a Fabry-Perot interferometer at very low (gravitational wave) frequencies, if the transit time $2l/c$ is replaced by the equivalent "storage time", $\tau_s$, of an optical cavity. The storage time of an optical cavity represents approximately the time spent by light inside the cavity before it escapes back out again; for a cavity with low loss (of light power in a round trip), we can approximate $\tau_s \approx (2l/c) \cdot (1/T)$ where $T$ is the transmission of the input mirror and $l$ is the length of the cavity. The measured phase difference spectrum gets related to a gravitational wave amplitude in a Fabry-Perot cavity as [2]:

$$\delta \phi(\omega) = 4\omega_0 \tau_s \cdot h(\omega) \cdot \frac{1}{\sqrt{1 + 4\omega^2 \tau_s^2}},$$

where $\omega_0$ corresponds to the frequency of the laser used. If we use the parameters for the optical cavities used in LIGO [12], we obtain a $\tau_s$ equal to 0.87 milli-seconds; assuming a wavelength of 0.5 microns, we can thus write

$$\delta \phi(f) = 2 \times 10^{12} \cdot h(f) \cdot \frac{1}{\sqrt{1 + (f/91.57)^2}}.$$  

(1.14)

LIGO has recently decided to switch to the Nd:YAG laser with a wavelength of 1.06 microns; if we used this laser, the factor on the right hand side of Equation 1.14 $(2 \times 10^{12})$ will be halved.

How small a $\delta \phi$ can LIGO hope to measure? Light used for interferometry introduces quantum noise in detection of phase by an interferometer, and this noise scales up as $\sqrt{P}$ as $P$, the power inside the interferometer, is increased; the differential phase signal, on the other hand, is also amplified as it gets multiplied by the power $P$. The equivalent phase noise thus scales as $1/\sqrt{P}$ — in Chapter 2 we show how a power level of about 80 watts at a wavelength of $514.5 \times 10^{-9}$ meter inside the interferometer can get the equivalent phase noise down to $1 \times 10^{-10}$ radian/$\sqrt{\text{Hz}}$.

However, quantum noise is not the only noise present in the detection of phase — numerous other noise sources, described extensively elsewhere [3, 10] and reviewed recently [12], can
Figure 1.3: Noise in the LIGO interferometer that limits its sensitivity; the vertical axis is in terms of $h$, the equivalent gravitational wave amplitude.

appear as $\delta \phi$ at the output of the interferometer. We present the most dominant ones in the different frequency bands in the LIGO interferometer in Figure 1.3. Below about 60 Hz, seismic noise causes the interferometer mirrors to shake, thus compromising the sensitivity of the interferometer to phase shifts from gravitational waves. The ground seismic noise can be approximated to be about $10^{-7}/f^2$ m/\sqrt{Hz}$ above 10 Hz; the passive vibration isolation system of LIGO limits the transfer of the ground noise to the interferometer table to about $10^{-6}$ around 100 Hz. There is additional vibration isolation of the mirrors from the interferometer table as the mirrors are hung as pendulums with a resonant frequency of 0.74 Hz — the seismic noise contribution therefore falls off sharply as shown in Figure 1.3. Above 70 Hz, the off-resonance thermal excitations of the pendulum motion and the transverse violin modes of the suspension wire dominate the spectrum. The exact models used to estimate the off-resonance contributions from thermal excitation are presented in Chapter 4. The Q of the pendulum designed for LIGO is assumed to be about $3.33 \times 10^5$; the Q for the violin modes is taken to be $1.7 \times 10^5$ with the fundamental frequency at 376 Hz. Where the seismic and thermal noise
estimates intersect, we achieve a sensitivity to gravitational waves of strengths $h \simeq 6 \times 10^{-23}$; if we try to ensure that the quantum noise in light does not undermine this sensitivity, we see from Equation 1.14 that the quantum noise should be no more than $1 \times 10^{-10}$ radian/$\sqrt{\text{Hz}}$ as used in Figure 1.3. We also see why the choice of an optical storage time of about 1 millisecond, as shown in Figure 1.3, was made for LIGO — any larger does not give us any more sensitivity, and a smaller storage time compromises the sensitivity that the seismic and thermal noise estimates allow.

Given the overall noise envelope shown in Figure 1.3, we may wonder what kind of gravitational waves LIGO might detect. Theoretical work predicts that the most promising source of gravitational waves are binaries in coalescence, and that these occur at the rate of three per year if one can look up to 100 Mpc [2]. These coalescences produce a gravitational wave chirp that sweeps up from tens of Hertz to about 1 kHz in a few minutes, and have strengths of about $h_0 \approx 10^{-21}$. LIGO will be able to detect these waves with a signal to noise ratio of about 5 if the sources are located within 30 Mpc, optimally oriented, and produce a signal that lies in a bandwidth of about 100 Hz.

We observed before that we need a light level of about 80 watts inside the interferometer, to get the quantum noise down to $1 \times 10^{-10}$ radian/$\sqrt{\text{Hz}}$ with a Argon gas laser at 0.5 microns. At the time LIGO was conceived, there was no experience with interferometry at these high power levels — the best phase noise sensitivity known at that time was about $1.5 \times 10^{-9}$ radian/$\sqrt{\text{Hz}}$ [6] with a power level of 730 milliwatts. Though no new fundamental source of noise was expected at these high power levels, a sense of uncertainty existed in the project over this lack of experience. A proposal to build an interferometer to study the problems of phase detection at these high power levels was therefore presented to the LIGO project by Weiss and Shoemaker [13]. The idea was to build a Michelson interferometer with several tens of watts of light incident on the beam-splitter, and compare the measured phase noise to that predicted by quantum noise. The Fabry-Perot cavities were avoided in the arms of the Michelson interferometer to make the interferometer as insensitive to “displacement” noise as possible. While there were research efforts undertaken to understand and reduce displacement noise in the 40 meter interferometer caused by seismic and thermal excitations [11], the task at MIT was to build a simple high power Michelson interferometer with 50 cm arms and
investigate the "readout" problems with light. This thesis documents the construction of this interferometer at MIT, the measured phase noise, and its analysis into contributions from the different noise sources.
Chapter 2

A Suspended High Power Michelson Interferometer

The last chapter showed why it was considered necessary to construct an interferometer and study the noise in optical phase detection at the level of $10^{-10}$ radian/$\sqrt{\text{Hz}}$ in a bandwidth around 200 Hz. In this chapter, we develop the background required to understand the noise spectral density of such an interferometer. The interferometer, given its purpose, is henceforth referred to as the Phase Noise Interferometer, or PNI.

An outline of the contents of this rather long chapter is appropriate before we begin. The fundamental noise in phase detection is given by the quantum noise in light, and this determines the smallest phase difference that can be resolved in interferometers using nonsqueezed light. In the first section, we address signal to noise issues given this fundamental noise. In the next section, we itemize the other noise sources in interferometry, and present estimates that influenced the design of the PNI. These noise sources are revisited in chapters 3 and 4, where we develop models to understand their coupling to optical phase, and describe experiments that carefully determine their contributions to the PNI noise spectra. The actual construction and layout of the PNI through its two stages form the subsequent section. Finally, we present the performance of the PNI through noise spectra measured in its two stages of construction, as a prelude to detailed noise analyses undertaken in chapters 3 and 4.
2.1 Interferometry techniques used in the PNI

2.1.1 A simple Michelson interferometer

To motivate the signal extraction technique used, we start with a simple Michelson interferometer whose optical layout appears in Figure 2.1. A laser beam with incident power, $P_1$, is split by a 50-50 beam-splitter and sent along two orthogonal paths to be retro-reflected back to the splitter. The returned beams are combined; a portion then makes its way to the laser, and the remainder to a photo-detector as shown in the figure. The side of the beam-splitter from where light is incident is often labeled the symmetric port if most of the light is returned that way, while the (dark) side the photo-detector then sees gets referred to as the anti-symmetric port. The light at the anti-symmetric port, $P_{out}$, is termed the output of the interferometer because it carries information (signal) about the phase difference between the two orthogonal optical paths, $\phi = 2k(l_1 - l_2)$, as measured by light:

$$P_{out} = \frac{P_1}{2}(1 - \cos \phi). \quad (2.1)$$

Equation 2.1 is non-linear; however, if the Michelson interferometer is held at a fixed $\phi_d$ away from where $P_{out}$ attains an extremum, and our signal $\Delta \phi$ causes small and therefore linear changes in $P_{out}$, then a measurable quantity proportional to the signal is obtained. Usually, a
feedback control system is required to "lock" \( \phi \) because of the noise in the environment, and this is usually chosen — to maximize the signal — where \( \frac{\partial P_{\text{out}}}{\partial \phi} \) is maximum, \( \phi_d = \frac{\pi}{2} \). The quantity \( P_i \) is not constant, and if we assume its variation to be given by \( \Delta P_i \), then — at our optimal operating point — we measure:

\[
P_{\text{out}} = \frac{P_i}{2} + \frac{\Delta P_i}{2} + \frac{P_i}{2} \cdot \Delta \phi. \tag{2.2}
\]

Since the noise owing to \( \Delta P_i \) competes with the signal \( \Delta \phi \), it is desirable to measure the signal where \( \Delta P_i \) is smallest — this happens at radio-frequencies where the laser intensity noise reaches its quantum (lower) limit. Thus the phase difference at the output of the interferometer is modulated at a fixed radio-frequency, and the signal is upconverted to sideband frequencies using the non-linearity of Equation 2.1. The next sub-section discusses this method.

2.1.2 A phase modulated Michelson interferometer with asymmetry

If \( \phi \) in Equation 2.1 is modulated by \( \Gamma \cos(\omega_m t) \), then we can write the equation as,

\[
P_{\text{out}} = \frac{P_i}{2} (1 - \cos \phi \cdot \cos(\Gamma \cos(\omega_m t)) + \sin \phi \cdot \sin(\Gamma \cos(\omega_m t))). \tag{2.3}
\]

With \( \Gamma \ll 1 \), the signal that multiplies \( \Gamma \cos(\omega_m t) \) is maximized if our operating \( \phi_d = 0 \) or a multiple of \( \pi \). The output of the interferometer then reads:

\[
P_{\text{out}} = \frac{P_i}{4} \cdot (\Delta \phi)^2 + \frac{P_i}{2} \cdot \Delta \phi \cdot \Gamma \cos(\omega_m t) + \frac{P_i}{4} \cdot (\Gamma \cos(\omega_m t))^2. \tag{2.4}
\]

\( \Delta P_i \) disappears in first order — however, since it still comes multiplied by the slow components of \( (\Delta \phi)^2 \), or \( \frac{\Gamma^2}{2} \) from the third term, modulation at radio-frequencies remains necessary for rejection of intensity noise. As the anti-symmetric port is held at a dark fringe — i.e., since \( l_1 \approx l_2 \), the properties of light that vary (like power, frequency, etc.) exactly cancel out in first order. Most of the incident light is now reflected (at the symmetric port), and this naturally leads to "recycling of light" as will be shown in the next sub-section. We may wonder why \( (\Delta \phi)^2 \) is shown in Equation 2.4 — this is because the two mirrors' surfaces may not be perfectly alike, causing the two wavefronts reaching the splitter to have an average phase difference of \( \Delta \phi = 0 \), but the rms \( (\Delta \phi)^2 \) to be non-zero. We will return to this shortly.
Figure 2.2: A phase modulated Michelson interferometer with asymmetry

Though a modulation of $\phi$ is desirable, it requires the use of phase modulators in the two arms of the Michelson interferometer. This leads to problems at high laser powers, because phase modulators tend to corrupt the spatial quality of the beams and thus degrade the cancellation that is important for good recycling. More discussion about the problems with this "in-line modulation" appears in [2].

A different technique uses a phase modulator in the input light. We still wish to hold the anti-symmetric port at a dark fringe for recycling of light; however, if we do so, there is no signal derived at that port synchronous with the phase modulation. Therefore, we decide to put an intentional asymmetry between the two arms of the interferometer, i.e., we have $l_1 - l_2 = \delta l$ of macroscopic size. Referring to Figure 2.2, we notice that this implies that Equation 2.1 has to be rewritten — with the intentional phase difference caused by the modulation separated from that caused by noise or signal — as

$$P_{out} = \frac{P_i}{2}(1 - \cos(\phi + \Gamma \cos(\omega_m(t - t_c - t_\Delta)) - \Gamma \cos(\omega_m(t - t_c + t_\Delta)))).$$  

(2.5)

Here we have introduced two time intervals for light that are important: $t_c$ refers to a time interval "common" to both paths and $t_\Delta = \frac{\delta l}{c}$ captures the asymmetry in the intervals. Notice how the asymmetry prevents the phase modulation from canceling out. A trigonometric
identity simplifies Equation 2.5:

\[ P_{out} = \frac{P_i}{2} (1 - \cos(\phi + 2\Gamma \sin[\omega_m t_{\Delta}] \sin[\omega_m (t - t_c)])) \]  

(2.6)

If we define \( \Gamma_e = 2\Gamma \sin[\omega_m t_{\Delta}] \), then we can write Equation 2.6 as,

\[ P_{out} = \frac{P_i}{2} (1 - \cos(\phi + \Gamma_e \sin[\omega_m (t - t_c)])) \]  

(2.7)

Expanding the above equation now gives us a form similar to Equation 2.3. Though we have a phase modulator in the path before the beam-splitter and thus common to both beams, we still get differential phase modulation through the asymmetry.

The first part of the PNI work involved the construction of an asymmetric Michelson interferometer with phase modulation at 25 MHz. The asymmetry was set at \( \delta l = 20.8 \text{ cm} \). With \( \Gamma \) chosen to be 1, the asymmetry fixed \( \Gamma_e \) to be 0.22. When we expand Equation 2.7 to second order in both \( \phi \) and \( \Gamma_e \), we obtain (similar to Equation 2.4):

\[ P_{out} = \frac{1}{4} P_i (\Delta \phi)^2 + \frac{1}{8} P_i \Gamma_e^2 + \frac{1}{2} P_i \Gamma_e \sin(\omega_m (t - t_c)) \Delta \phi - \frac{1}{8} P_i \Gamma_e^2 \cos(2\omega_m (t - t_c)) \]  

(2.8)

The above equation captures all the elements that we need for our first signal to noise calculation. The first term, \( \frac{1}{4} P_i (\Delta \phi)^2 \), is a measure of the defect in contrast of the interferometer, a term which we will now define. We must understand that laser light in our experiment is not a plane wave with a fixed phase across its (infinite) planar front — it is better represented by a superposition of Hermite-Gaussian functions [14] whose variation in field amplitude and phase in a transverse cross-section is given by:

\[ E(x) = \Sigma_n C_n \cdot H_n(\frac{\sqrt{2}x}{w}) \cdot \exp[-\frac{x^2}{w^2} - i \frac{kx^2}{2R}] = \Sigma_n U_n \]  

(2.9)

where \( C_n \) is a normalization constant, and \( H_n \) is a Hermite polynomial of order \( n \). Two parameters in the above equation, the spot size, \( w \), and the radius of curvature, \( R \), determine how the amplitude and phase of the field change across the transverse axis. As the beam propagates in space, these parameters also change owing to diffraction [15]; the wavefront, in addition, picks up an overall phase — a longitudinal part \( (kz) \), and another part (the Cuoy phase) that comes from the finite transverse extent of the beam. Thus, when we write the phase difference as \( \Delta \phi \), we imply \( \iint \Delta \phi(x,y) dA \) and, in a similar manner, \( (\Delta \phi)^2 \equiv \iint(\Delta \phi(x,y))^2 dA \).
Clearly, $\Delta \phi$ averages over the transverse variation but keeps the overall phase, while $(\Delta \phi)^2$ adds the square of the transverse variation to that of the overall phase. If the two end mirrors of the Michelson interferometer have surfaces that are not exactly alike or are misaligned with respect to each other, we can force $\Delta \phi$ to zero, but not $(\Delta \phi)^2$ necessarily. Referring to Equation 2.1, we hence notice that the minimum $P_{\text{out}}$ possible is $\frac{1}{4} P_i(\Delta \phi)^2$. One figure of merit of a Michelson interferometer is how close the contrast is to unity where

$$\text{contrast} = \frac{P_{\text{out}}^\text{max} - P_{\text{out}}^\text{min}}{P_{\text{out}}^\text{max} + P_{\text{out}}^\text{min}}. \quad (2.10)$$

or, equivalently, how small the defect in contrast, (c.d.) $\equiv 1 - \text{contrast}$, is measured to be. Some amount of algebra shows that the c.d. of a simple Michelson interferometer is $(\Delta \phi)^2/2$, an expression that also shows up in the first term of Equation 2.8 multiplied by $P_i/2$.

The second term in Equation 2.8 shows that phase modulation causes additional light to leak out at the dark port. The first two terms have sizeable dc components and some low frequency variation as the alignment of the interferometer or the level of incident light changes. Apart from the direct amplitude noise that they then create, they also constitute a source of broadband quantum noise, the power spectral density of which, in watts/$\sqrt{\text{Hz}}$ [2], is

$$\sqrt{2h\nu(\frac{1}{4} P_i(\Delta \phi)^2 + \frac{1}{8} P_i \Gamma_e^2)} . \quad (2.11)$$

The signal $\Delta \phi$ occurs at frequencies much lower than the radio-frequency $\omega_m$ — thus the third term, oscillating at $\omega_m$, shows the effect of $\Delta \phi$ through a modulation of its amplitude. If a mixer multiplied $P_{\text{out}}$ with $\sin[\omega_m(t - t_c)]$, and a low pass filter removed from the product all frequencies above some fraction of the modulation frequency, $\omega_m$,

$$\frac{1}{2} P_i \Gamma_e (\Delta \phi) \quad (2.12)$$

is obtained. This quantity — in watts of power — is what is measured and gives our signal $\Delta \phi$ in radians when divided by the calibration, $\frac{1}{2} P_i \Gamma_e$, in watts per unit radian change in phase. The broadband quantum noise also enters through the process just described — at any non-zero frequency of $\Delta \phi$ there are 2 sidebands at a level given by (2.11) that add in quadrature. Thus, given the calibration just derived, the fundamental noise power spectral
The Michelson Interferometer

density in radian/\sqrt{Hz} is
\[ \sqrt{\frac{2h\nu}{P_i}} \cdot (1 + \frac{2(\Delta \phi)^2}{\Gamma_e^2})^{1/2}, \] (2.13)
where \( h \) is the Planck's constant, and \( \nu \) is the frequency of light. This is also our fundamental noise limited sensitivity. If the contrast were perfect (i.e., 1), we can calculate what \( P_i \) implies a quantum noise limited sensitivity of \( 10^{-10} \) radian/\sqrt{Hz} ; we thus find that we need \( P_i = 77.4 \) watts with a wavelength of 514.5 nm. This is beyond a conventional single mode laser — hence, a new technique of "Recycling", which we introduce in the next sub-section, becomes imperative.

Though all our quantities have been in laser power so far, the more convenient unit is the photo-current \( I \) (in Amperes) in the photo-detector — as that is what gets measured. The conversion is given by \( I_{out} = (\eta e)/(h\nu) \cdot P_{out} \) where \( e \) is the electronic charge, and \( \eta \) the quantum efficiency (of the conversion of photon to electron). For the wavelength of our choice (5.14 \times 10^{-7} \text{ m}), the conversion factor \((\eta e)/(h\nu)\) is about 0.3 Ampere/Watt for the Silicon photo-detectors used in our experiment with light incident near Brewster's angle. In units of current, the expression 2.13 can be written as:
\[ \sqrt{\frac{2e}{I_i}} \cdot (1 + \frac{4 \text{ c.d.}}{\Gamma_e^2})^{1/2}, \] (2.14)
where \( I_i \) is the "bright fringe" current (or the power incident on the beam-splitter converted to a current).

2.1.3 A recycled Michelson interferometer

Figure 2.3 shows the optical layout of the PNI in its second stage, where the light lost at the symmetric port is reflected (or "recycled") back by a mirror placed in the path of the input light. This mirror forms an optical cavity with the Michelson interferometer as its other (back) mirror.

Phase modulation produces sideband frequencies on the light,
\[ E e^{i(\omega t + \Gamma \cos(\omega_m t) + \phi)} = E e^{i(\omega t + \phi)} [J_0(\Gamma) + iJ_1(\Gamma) (e^{i\omega_m t} + e^{-i\omega_m t}) + \ldots]. \] (2.15)

Using the field transmission and reflectivity derived for the Michelson interferometer in Appendix A, we can rewrite Equation 2.8 in terms of \( J_0(\Gamma) \equiv J_0 \) and \( J_1(\Gamma) \equiv J_1 \) (ignoring the
term at twice the modulation frequency):

\[ P_{\text{out}} = \frac{1}{4}P_1 J_0^2 (\Delta \phi)^2 + 2P_1 J_1 (\Gamma)^2 \sin^2(\omega_m t_\Delta) + \frac{(\Delta \phi)^2}{4} + 2P_1 J_0 J_1 \sin(\omega_m t_\Delta) \sin[\omega_m(t - t_c)] \Delta \phi, \]

(2.16)

where all the parameters, \( P_1, J_0, \) and \( J_1, \) are for light at the beam-splitter. With a simple Michelson interferometer these parameters are the same as that for the input light. With recycling, these parameters get modified — as discussed in Appendix B, the recycling optical cavity provides internal field gain (an increased \( P_1 \)) that then gives us the high phase sensitivity we desire. However, an asymmetric Michelson interferometer causes the “back” mirror of the recycling optical cavity to have different “transmissions” at different frequencies — hence the field gain as seen at the carrier frequency is not necessarily the same as that seen at the sidebands caused by phase modulation. The proportion of carrier to sideband fields at the beam splitter is then different from that in the input light. There are thus two effects of recycling on the light incident at the beam-splitter (which decides the fundamental noise limited sensitivity) — an increased light level, and a modified carrier to sideband field ratio.

Before we calculate the revised parameters, we should note that our recycling method assumes that both the carrier and the sideband fields are resonant in the cavity. In Appendix B, we notice that if the frequency of the light is changed by a full spectral range (fsr) \( (c/2L \) where \( L \) is the length of the cavity), the light is resonant again — thus to get the sideband field reso-
nant we should have the modulation frequency at least at the first fsr: this fixes the length of the cavity to be \( c/(2 \cdot (f_m = 25 \text{ MHz})) = 6 \text{ meters} \). In the actual experiment, the resonant frequency was found to be 25.33 MHz, so the length of the cavity was actually 5.92 meters. From Appendix A, we find that the field reflection coefficient at the beam-splitter of the Michelson interferometer is \( e^{-i\alpha_k \cos \delta_k} \) where \( \alpha_k \) involves the sum of the lengths of the two Michelson arms — hence, the recycling mirror was placed about 5.5 meters from the beam-splitter (thus, \( 2 \times \text{beam-splitter to recycling mirror distance} + \text{sum of two Michelson arm lengths} = 2 \times \text{length of common mode path} = 12 \text{ meters} \)).

Since the Michelson interferometer is held at a dark fringe for the carrier frequency, the loss (or transmission) of the field at this frequency through the "back mirror" can be assumed to be much smaller than the transmission of the input mirror. The loss comes from an imperfect contrast; from Appendix A, the power transmission through a Michelson interferometer was derived as \( \delta^2 = (1/2) \cdot \text{(c.d.)} \). Given the typical c.d.s for the PNI of around 1000 ppm, the power transmission could be estimated to be 500 ppm. The transmission of the input mirror, on the other hand, was about 8200 ppm. The field gain at the carrier frequency could thus be approximated as \( 2/\sqrt{T_1} \) for the PNI (Appendix B), or the recycling power gain as \( R_G = 4/T_1 \).

For the sidebands, the transmission through the Michelson interferometer to the dark port is \( \sin^2(\delta_1 = \omega_m t_\Delta) \), equal to about 12,000 ppm for the PNI. Since this transmission through the "back mirror" is larger than the transmission of the input mirror that we used (8200 ppm), the recycling cavity was slightly undercoupled for the sidebands. The field gain is \( (2\sqrt{T_1})/(\sin^2(\delta_1) + T_1 + L) \) where \( L \) lumps all the other losses seen by light inside the cavity distinct from the transmissions at the two end mirrors. If we divide the sideband field gain by the carrier field gain, we obtain \( g_r = L_{\text{common}}/(\sin^2(\delta_1) + L_{\text{common}}) \) where \( L_{\text{common}} = T_1 + (1/2) \cdot \text{(c.d.)} + L \). Since we are working where the approximation \( J_1 \approx \Gamma/2 \) holds true, we thus notice that if we revise \( \Gamma \rightarrow g_r \Gamma \), and \( P_i \rightarrow R_G P_i \) for all our unrecycled formulas, we get the corresponding relationships that hold for the recycled case. For example, the fundamental noise power spectral density, Equation 2.13, for the recycled interferometer is

\[
\sqrt{\frac{2h
u}{R_G P_i}} \cdot (1 + \frac{2(\Delta \phi)^2}{g_r^2 \Gamma_e^2})^{1/2}, \tag{2.17}
\]

where \( P_i \) is the input power before the recycling mirror. Given that \( R_G \) is about 500, we thus
see that recycling requires only 200 milli-watts of input power to get the required 80 watts at the beam-splitter for a quantum noise spectral density of $10^{-10} \text{ rad}/\sqrt{\text{Hz}}$.

### 2.2 Sources of phase noise in the PNI

In the last section, we presented techniques that help in reducing quantum noise to the level of the required detection sensitivity of $10^{-10} \text{ rad}/\sqrt{\text{Hz}}$. However, quantum noise is not the only noise that shows up in the detected phase — and the goal of the construction of PNI, as indicated in Chapter 1, was to investigate which of the other sources (if any at all) showed up as limiting "sensing" noise in the bandwidth of interest around several hundreds of Hertz. At the time the construction of PNI was proposed [13], several of these noise sources influenced the construction and design of the PNI. In this section, we will briefly introduce these sources — with an eye to explaining the construction of the PNI that occupies the next section — and then return to them again for a detailed look in chapters 3 and 4.

#### 2.2.1 Noise in the input light

**Frequency noise:** The measurement of $\phi = 2k(l_1 - l_2)$ is affected by both a change of $\Delta k$ as much as $\Delta(l_1 - l_2)$ — thus a fixed asymmetry, i.e. a fixed $(l_1 - l_2)$, causes sensitivity to $\Delta k$. Given our $\delta l = 20.8$ cm, the PNI thus had a sensitivity of $(4\pi \delta l)/c = 8.71 \times 10^{-9} \text{ radian/Hz}$. To achieve a phase noise of $10^{-10} \text{ radian}/\sqrt{\text{Hz}}$, we thus required a frequency stability of about $10^{-2} \text{ Hz}/\sqrt{\text{Hz}}$ in our bandwidth of interest.

Scattered light can interfere at the photo-detector after traveling through a path different than the main beam. If this path differs in length by a large extent from the main one, and the element reflecting the scattered light back to the photo-detector is more or less stationary, the spurious interference is another way of adding frequency noise coherently to the detected phase. If the element shakes, it causes additional spurious interference to appear at the dark port — we discuss this kind of "parasitic interferometry" in detail in chapter 4.

**Amplitude noise:** There is no first order dependence of the phase noise in Equation (2.16) on amplitude noise — except that imperfections in the construction of the interferometer, like an offset from the minimum possible null at the dark port, can give rise to such a dependence. If the phase modulation process gives rise to some amplitude modulation (AM), then we again
get a contribution from amplitude noise in the phase noise spectrum through the AM in the light leaking out at the dark port. Otherwise, the amplitude noise contribution is through its bilinear coupling with the most dominant features in the noise in $\phi$. An amplitude stabilization system was kept ready, but the main strategy planned was to get rid of the imperfections that cause the amplitude noise to influence the measured phase noise.

**Beam pointing noise:** Light varies in direction at the input of the interferometer, and this gets converted to a varying phase-difference through a differential misalignment of the Michelson mirrors. Once again, the emphasis was to align the Michelson mirrors as best as possible and keep them aligned through active feedback, rather than trying to fix the input beam direction.

### 2.2.2 Noise sources in the interferometer

Seismic noise causes the optical elements in the interferometer to shake — leading to a differential displacement of the Michelson mirrors that gets converted to an optical phase noise, and then also to misalignment of the elements with respect to each other which disturbs the operating point of the interferometer. Great pains were therefore taken to isolate the optical elements from seismic noise — the elements were suspended as pendulums from structures that in turn rested on passive and active isolation stages. These isolation systems will be discussed in the section on construction of the PNI. Thermal excitation of the mirror surface and the suspending wires can also be a source of phase noise — this influenced the dimensions of the mirrors used, while modeling of the thermal noise in the suspension wires was carried out to see the effect on the phase noise sensitivity. We will return to these issues later.

**Instrumentation noise** in the electro-optic and electronic components (e.g., the dark noise in the photo-detector, or the amplitude modulation accompanying the phase modulation in the modulators) was carefully measured, and care was taken to ensure that it did not affect our noise sensitivity. At the time the PNI was constructed, we did not anticipate the extent of parasitic or unintended interferometry caused by scattered light sent back to the dark port photo-detector by a reflector in motion — so no special precautions, apart from the inclusion of a Faraday isolator along with the input optics, were taken in the design.
2.3 Construction of the PNI

The PNI was constructed in two stages — first, a phase-modulated asymmetric Michelson interferometer was assembled to undertake a preliminary study of noise in phase detection consistent with low operating power levels. Steps to exclude the noise sources discovered were then implemented, and the interferometer recycled in power. At this second (recycled) stage, sources that could cause deviation from the fundamental noise limited sensitivity were again investigated.

Given the construction process described, we organize this section accordingly in two parts; the first part presents the construction of the unrecycled Michelson interferometer, while the second part points out the changes carried out before recycling of light.

2.3.1 PNI in the first stage

Figure 2.4 shows an overview of the PNI in its first stage of construction. Laser light was stabilized in frequency and amplitude in one room; it was then steered into a clean vacuum envelope in the other room. The vacuum envelope housed a near tank, a bigger central tank, and a 5 meter long tube that connected the two tanks. The main purpose of the vacuum (about a few µtorrs) was to keep the mirror and beam-splitter surfaces free from hydro-carbon contaminants (that can introduce significant optical loss). There were other benefits — e.g., the elimination of air drifts that can excite the suspended optical elements, and phase noise via refractive index fluctuations of air. The Michelson interferometer with 50 cm nominal arm lengths was located on a table inside the bigger tank. The near tank had steering mirrors on the table inside it in the first stage; the second stage added a recycling mirror as shown outlined in Figure 2.4.

We now discuss each subsystem separately.

2.3.2 Laser stabilization subsystems

An Ar⁺ laser from Spectra Physics (model 2080) was used for the experiment. The frequency and amplitude of the laser were stabilized before the light was used for interferometry — Figure 2.5 shows the optical layout for the stabilization subsystems.
Figure 2.4: Overview of the PNI subsystems in the first stage
Figure 2.5: Optical layout of the laser stabilization subsystems (first stage). Error signal obtained from the photo-diode in the figure was used to stabilize the frequency of the laser by electronic feedback to PZTs on the laser cavity mirrors.

Although not obvious in Figure 2.5, the laser plasma tube had to be put on a stack made of alternating layers of lead and rubber elements to isolate the vibration of (pumped) cooling water from the optical table. The resonator mirrors were put on the optical table, and not in physical contact with the plasma tube so that the laser resonator cavity length and alignment did not fluctuate with the cooling water flow.

The Ar$^+$ laser (wavelength = $5.14 \times 10^{-7}$ m) had significant frequency noise, about 10 kHz/$\sqrt{\text{Hz}}$ at about 100 Hz and falling roughly as $1/f^2$. Thus stabilization of its frequency was essential for reaching a level of $10^{-10}$ radian/$\sqrt{\text{Hz}}$. The required noise suppression of $10^6$ at 100 Hz was carried out in two stages: first by stabilizing the frequency with respect to a passive quartz cavity (the reference cavity in Figure 2.5), and then by using the recycling cavity as an additional reference in the second part of the experiment. Stabilization with respect to a passive cavity has been shown to achieve about 1 Hz/$\sqrt{\text{Hz}}$ at several hundreds of Hertz and to keep roughly that level until 10 kHz [16]. Thus the frequency noise at the first stage of our experiment provided a lower limit to the best phase noise sensitivity.

To get the requisite frequency noise suppression with the reference cavity, the first stage stabilization system had a unity gain frequency of close to 1 MHz. A number of different
The Michelson Interferometer

actuators were used to get this bandwidth. The mirrors that were used to define the laser resonator cavity (of length 2.16 meters) were put on PZT actuators — the high reflector (flat) mirror assembly reached its first resonance at about 230 kHz with a maximum dynamic range of 80 nanometers, while the output coupler (with a mirror of radius of curvature 8 meters) end resonated at about 8 kHz but had a maximum dynamic range of 8 μmeters. A Pockels cell phase corrector extended the bandwidth of the servo to 1 MHz. The laser was observed to be multi-mode above 2.5 watts of output power, thus establishing an upper-bound to the power that we could use for our experiment.

About 25 milliwatts of phase modulated (frequency of modulation = 12.33 MHz) light (with 40% of the power in the sideband fields, i.e. \( J_2^0 / J_2^1 \approx 3 \)) was incident on the reference cavity, and the Pound Drever technique described in Appendix B was used to obtain the error signal for frequency stabilization. The control signal was applied to the actuators described in the last paragraph. The reference cavity was made of two mirrors separated by a quartz spacer of length 35 cm (fsr = 428.6 MHz); the input mirror was flat while the other mirror had a radius of curvature of 1 meter. Both the mirrors had the same nominal transmission of 700 ppm. A ring down experiment (explained in Chapter 3) with the passive cavity yielded a total loss of 1600 ppm — thus the finesse of the cavity was about 2000.

An Acousto-optic modulator appears in Figure 2.4 which deflected about 20% of the incident power into the first diffraction order. It was intended to keep the amplitude of light constant by adding or taking out light in the first diffracted order. A photo-detector was included in the Input Optics chain for monitoring the amplitude fluctuations.

### 2.3.3 Suspensions

A mass suspended as a pendulum from a tower is free in an inertial space at frequencies above the pendulum resonance — its displacement, \( d \), in the inertial space at frequency \( \omega \) much above the pendulum resonance \( \omega_0 \) can be approximated by: \( d = (\omega_0^2 / \omega^2) \cdot d_t \), where \( d_t \) is the displacement of the tower in the same space. This is the idea behind suspending a mirror to isolate it from ambient vibration; the pendulum frequency in our experiment was about 1 Hz, thus there was vibration isolation of about \( 10^{-4} \) starting at around 100 Hz, our bandwidth of interest. Figure 2.6 shows the structure from which we suspended our mirror
as a pendulum. The tower had a height of 15.7 inches, and it was about 5.4 inches wide and 2.5 inches deep; the mirror was suspended 9.8 inches below the top plate to get the pendulum resonance at about 1 Hz. The tower had its first resonance at about 157 Hz. The mirror also had resonances in its yaw (a $\phi$ or side to side motion) and pitch (a $\theta$ or up and down motion) motions at about 0.5 Hz. Steel wire, $2 \times 10^{-3}$ inches thick, was used to suspend the mirror with a single loop as shown in Figure 2.6 in dotted lines. The wire took off tangentially at about 0.017 inches above the center of mass of the mirror. The mass of the mirror was about 0.25 kg, this caused a vertical "bounce" resonance at about 18.7 Hz. The wire could also be excited transversely like a violin string — the first such "violin" resonance was found to be at 550 Hz.

The $Q$ of the pendulum was high ($\approx 10^6$), hence the mirror needed damping at its low frequency resonances to hold it steady for interferometry. With this purpose in mind, four magnets (2.2 mm in length and 1.5 mm in diameter) were glued on top of Aluminum stand-offs (1/16 inches in diameter and 1/8 inches long) at four points on the mirror as shown in Figure 2.6. A fin with a slit in it was slipped on top of each magnet and then brought inside.
an OSEM (Optical Sensor Electric motor) unit. Each OSEM unit had an LED that was made to shine through the slit on to a split photo-detector — when the two photo-detectors saw equal light there was no signal; however, when the fin moved along with the mirror, one photo-detector saw more light than the other and a signal was produced by subtraction. The four signals from the four OSEM units were brought inside an OSEM processor box where they were linearly combined and then corrected for the corresponding resonance frequencies to form the longitudinal shift, and the pitch and yaw displacements of the mirror. Next, a derivative was taken electronically to get the velocity and a proportionate force applied, via coils around the OSEM units, on the magnets glued to the mirror to counter the mirror motion. Velocity damping was obtained in this manner and used to hold the mirror stationary at low frequencies.

Since electronic noise making its way onto the coils is a source of displacement noise, the velocity damping feedback was filtered by a 7 pole filter at 22 Hz. The final displacement noise owing to electronics was due to the thermal noise in the resistor used to convert a voltage to current in the coils. There was concern about thermal noise in the mechanical structures causing displacement in the measurement bandwidth — this happens through off resonance excitation of the pendulum, the violin modes of the suspending wire, or the mirror structure modes [17]. Great care was therefore taken to keep the Q of these resonances high, so that the off resonance contribution was small. For example, the suspension tower was made of stainless steel to prevent the reduction of the pendulum Q through eddy current damping of the magnets glued to the mirror (as well as being compatible with a clean vacuum system). The residual contributions to phase noise from the sources just described are calculated in Chapter 4.

When we set up the suspensions in our experiment, we found that the mirrors were disturbed by tens of μradian in alignment over time scales of tens of seconds. The cause of these alignment fluctuations was not completely understood. To counter the alignment changes, position feedback was introduced in the damping loops with a single pole filter at 0.06 Hz (there was negligible gain at 2 Hz to avoid coupling the mirror to the stack resonance).
2.3.4 Seismic isolation

The suspension towers rested on tables that were seismically isolated by active and passive methods. Passive isolation was provided by a cascade (or stack) of steel mass and viton spring elements as laid out in Figure 2.7. This stack, which has been extensively studied elsewhere [18], has been shown to transmit less than $10^{-4}$ of ground noise at 100 Hz from the base to the top. However, it does amplify motion by a roughly a factor of 5 at its resonances between 2 and 10 Hz in the horizontal and vertical directions.

The base of the stack rested on three legs that provided active isolation — these legs were commercial units available from Barry controls and their performance has been described in a recent technical report [19]. Each of these units has a central mass whose velocity is sensed in the three orthogonal directions by geophones and damped using PZTs that can withstand a payload of about 2500 lbs per foot. As shown in Figure 2.8, the system eliminated amplification of ground noise from the stack resonances at the interferometer table, and offered about 20 dB of isolation from 2 to 100 Hz in the horizontal direction. In the vertical direction, it gave 20 dB of isolation at about 1 Hz, rose to about 40 dB of isolation at 10 Hz, and then started to fall again from 20 Hz upwards.

Of more relevance to interferometry were impulsive seismic events that reached upto 80 microns in amplitude, and had characteristic frequencies between 5 and 10 Hz. Events upto
Figure 2.8: Performance of the Barry isolators, from [19]

40 microns (the dynamic range limit of the Barry Control PZTs) were suppressed by factors between 20 to 30 at the top of the stack by the active isolation units.

2.3.5 Optics

The interferometer mirrors, as described before, were suspended as pendulums and placed within a vacuum envelope. There were additional optical elements, referred to collectively as the “input optics” which, as shown in Figure 2.9, prepared the light for interferometry and steered it into the vacuum envelope. The optical fiber stabilized the beam pointing. The
Pockels cell phase-modulated the light at 25 MHz with a modulation index of $\Gamma \approx 1$; care was taken to orient the Brewster windows and the crystal so that the associated AM (peak) depth at 25 MHz was less than $5 \times 10^{-5}$. The $\lambda/2$ was oriented along with the Faraday rotator to get p-polarized light incident on the beam-splitter. The Faraday isolator provided 40 dB of isolation (in power) from light reflected back. The lenses were used to mode-match the light from the output characteristics of the fiber to a waist radius of about 0.9 mm at the end of the common mode path; however, without the recycling mirror which acts as a diverging lens with its radius of curvature of 10 meters, the spot radius was about 1.2 mm. The entire optical train, except the steering mirrors at the end, was mounted on rails that were damped from resonances. The first steering mirror transmitted some light so that the amplitude noise in the input light could be monitored, thus allowing for necessary corrective signals to be sent to the AOM on the laser stabilization table.

The Michelson interferometer was made of optical elements that were 3 inches in diameter and 1 inch thick; the two end mirrors were coated for high reflectivity (with transmission at about 10 ppm), and the beam splitter had a (power) transmission of 49.75% and a reflectivity
of 50.25% for p-polarized light (with AR reflectivity of 130 ppm). The Michelson asymmetry \((l_1 - l_2)\) was measured to be 20.8 cm.

2.3.6 Michelson length control

Figure 2.10 shows the layout of the feedback system that was used to hold the Michelson interferometer at the dark fringe (a description of this system in the traditional language of feedback control systems appears in Appendix C). We present here the details of the different settings as used in the first stage of the experiment. The oscillator output was at 2.00 volts pp and set at a frequency of 25 MHz. This output was then split 50-50 in power, half of which was amplified 40 dB before coupling it to a Gsanger PM-25 Pockels cell through a tuned transformer. The other half was phase-shifted and boosted to 23 dBm before being fed as the LO to a Mini-Circuits ZAY-1B mixer. The photo-detector at the dark port used the capacity
of the photo-diode and an external inductor to amplify its signal at 25 MHz through a low
Q resonance. The signal was further amplified by a ZFI-500 amplifier from Mini-Circuits,
before being low-pass filtered at 30 MHz to remove all harmonics of the modulation frequency.
Next, the signal was down-converted by the mixer, and the IF signal low-pass filtered at 5
MHz. Filter 3 in Figure 2.10 received the signal through a buffer and set the servo gain at
low frequencies. At its low gain setting, Filter 3 had unity gain until 1 Hz, a zero at 1 Hz,
and finally a pole at 1 kHz. In the high gain setting, there was gain of about 36 dB upto
63 Hz, and the first zero occurred there (the pole frequency was left at its previous value).
A variable resistor (usually set at 2.5 kΩ) offered gain control by varying the current given
a fixed voltage (each coil presented 500 Ω looking into the Fast Z input as shown in Figure
2.10). The overall loop gain varied with input light power — the u.g.f of the servo varied from
50 to 250 Hz in the different data runs (with a phase modulation index, Γ', equal to 1).

2.3.7 PNI in the second stage

Based on the experience in the first stage, the PNI underwent some changes in the second
recycled stage. As the phase noise spectrum in the first stage suffered from beam pointing
fluctuations, the entire input optics chain was moved to a separate table, closer to the vacuum
envelope, and covered with acoustic shielding. The mode-matching of light to the recycling
cavity had to be redone. The recycling mirror was suspended inside the near vacuum tank; to
reduce the seismic excitations of the mirror, active Barry isolation feet were now added under
the passive stack inside the tank. The light retro-reflected off the recycling mirror was steered
out onto the input optics table by the Faraday isolator, and received on a photo-detector.
This photo-detector, exactly like the one at the dark port, was tuned to the phase-modulation
frequency with a resonant circuit. Signals at the phase modulation frequency were thus
obtained for frequency and common mode length correction, and used in a servo loop to keep
the recycling cavity resonant. This "common mode" servo, as pointed out before, was also
designed to obtain the necessary frequency noise suppression. Changes were also made in the
differential length control servo, and a wavefront sensor was used for active control of the
differential misalignment. On the laser stabilization table, the light going to the reference
cavity was sampled before the AOM — the laser frequency stabilization was now thus isolated
from the effects of the amplitude stabilization system. An overview of the different subsystems in the second stage appears in Figure 2.11.

We discuss below the changes in the differential length control servo, and the two new additions in the second stage: the common mode servo and the wavefront sensing system.

2.3.8 The recycled Michelson length control

Recycling built up light power at the beam splitter on resonance from 1% (recycling mirror power transmission) of the input light intensity to a factor of 500 — this required careful tailoring of the differential length control servo so that the increased gain from the recycling cavity power build-up did not lead to saturation. Given the earlier (Michelson) length control servo design as discussed in section 2.3.6 and shown in Figure 2.10, we cut the signal going into Filter 3 by a factor of 10. Filter 3 was changed drastically — it could now provide gain increments of 2 dB from 2 dB upto 22 dB (with its highest setting gain comparable to that obtained with the earlier Michelson servo); also, the filter now had a zero at around 2 Hz but the pole was shifted to around 12 kHz. Thus a wide-band servo was now possible. Mirror
structure resonance at around 23 kHz forced us to put a 4 pole Chebyshev filter at around 14 kHz in Filter 2 along with the 5 MHz low pass filter. Instead of the attenuator after Filter 3, the impedance seen in the Fast Z path varied from 10 kΩ to 2kΩ from 3 kHz to 15 kHz respectively — this helped in extending the bandwidth of the servo without high voltage gain. At the operating parameters of the recycled Michelson, the lowest gain setting of Filter 3 set the u.g.f to be around 400 Hz. The phase modulation index was reduced to 1/2 — this was done to prevent the dark port photo-detector from seeing too much light.

Filter 3 also had a bypass stage that increased the gain upto 2 Hz by about 100, and then in decreasing amounts upto 120 Hz — this helped in reducing the measured phase noise at these frequencies and thus the contribution from the bilinear coupling with amplitude noise. We will discuss this more in the next chapter.

2.3.9 The common mode servo

The common mode servo, shown schematically in Figure 2.12 served two purposes: it had to keep the recycling cavity on resonance, and it had to provide an additional frequency noise suppression of 10² over that obtained from frequency comparison with a reference quartz cavity. The recycling cavity deviates from resonance for two reasons; at low frequencies upto about 100 Hz, the length changes of the cavity dominate, while — at higher frequencies — the
frequency noise of the laser becomes important. Thus the error signal had to be sent to one of the two actuators shown in Figure 2.12 depending on its frequency. If the deviation from resonance be $u$ (in radians), then the application of a servo as shown in Figure 2.12 reduces it to $u/(1 + H_R + H_L)$. At the point where $H_R$ equals $H_L$ in gain (the cross-over point), the phase difference between $H_R$ and $H_L$ is important — a $\pi$ phase-shift can reduce the servo gain to unity.

Given the constraints described, the two paths, $H_R$ and $H_L$, were designed as shown in Figure 2.13. The feedback path ($H_L$ in Figures 2.12 and 2.13) to the laser cavity actuators was made to see a 6 dB/octave high pass filter with a corner frequency of 100 Hz, a flat portion from 100 Hz until 1 kHz whence it dropped 12 dB/octave to 10 kHz, and then, finally, a 6 dB/octave fall after 10 kHz. The feedback path to the recycling cavity ($H_R$) mirror followed the $1/f^2$ response of the mirror after its resonance at around 1 Hz until 80 Hz when an electronic "zero" caused a slower $1/f$ fall to 1 kHz — from that point onwards a 5 pole Butterworth filter caused a precipitous $1/f^6$ drop. Configured in this manner, the cross-over between the two paths happened around 150 Hz depending on operating conditions, and the $\text{u.g.f}$ of the servo was around 30 kHz. This implied that though the $H_L$ path had a gain of 300 from 100 Hz to 1 kHz, it only corrected for 50% of the deviation from resonance at 150 Hz and about 90% at 1 kHz. Thus, if frequency noise of 1 Hz/$\sqrt{\text{Hz}}$ caused the deviation from resonance, we could expect the common mode servo to suppress the frequency noise to only 0.5 Hz/$\sqrt{\text{Hz}}$ at 150 Hz, and to 0.1 Hz/$\sqrt{\text{Hz}}$ at 1 kHz. However, the Butterworth helped the $H_L$ path to account for about 99% of the resonance deviation at about 1.7 kHz where the gain in that path dropped to about 100 — hence, we could expect to obtain our best phase noise around that frequency.

A bypass stage was turned on after the recycling cavity was resonant that increased the servo gain as $f^3$ from 13 Hz until 1 Hz and as $f$ from 1 to 0.5 Hz. The increased low frequency gain decreased the fluctuations from resonance owing to the large excitations at these frequencies.

Appendix B describes the Pound-Drever scheme of holding an optical cavity resonant at a carrier frequency with non-resonant side band frequencies. We used a variant of this technique, where the carrier and sidebands were both resonant with different couplings; while
Figure 2.13: Relative gain and phase of the two actuator paths used in the common mode servo (bypass stage turned off).
the carrier was over-coupled, the sidebands were slightly undercoupled. We can write the real ($R$) and imaginary ($Im$) reflectivities for the carrier field as $(r_R^c + i \cdot r_{im}^c \cdot \psi)$, and for the sideband field as $(r_R^s + i \cdot r_{im}^s \cdot \psi)$, where $\psi$ is the deviation in the phase of the carrier field from resonance (Appendix B discusses why the reflectivities may be written this way). We then notice that the term at radio-frequency $\omega_m$ in Equation B.17 can be written as $4J_1 J_0 (r_{im}^s - r_{im}^c \delta) \cos(\omega_m t) \psi$. Thus, if the carrier and sidebands are both resonant, their reflectivities have to be different (through different couplings as in our experiment) for an error signal to be obtained to hold the optical cavity resonant.

2.3.10 The active differential alignment system

If we put the signal wavefront at the modulation radio-frequency, as shown in Equation 2.12, on a split photo-diode face and subtract the two signals from the two photo-detectors, we reject the phase information common to both parts but pick up that information which is anti-symmetric in the two. Differential misalignments create such anti-symmetric phase patterns (refer Figure 2.14) — e.g., a small differential mirror tilt $\phi$ about the vertical axis $z$ creates a phase pattern $(2k \cdot \phi \cdot y)$ if $x$ be the path along which light travels — this phase pattern is anti-symmetric about $y = 0$. In this way, a quad photo-detector can be used to extract information about the differential pitch and yaw displacements of the Michelson mirrors. Once the information is obtained, active feedback can be used to stabilize the differential alignment of the mirrors (after the necessary down-conversion using a mixer and appropriate compensation filters) through the OSEM actuators. Extensive literature [20, 21] already exists in the LIGO project about the details of this technique.

A quad wavefront sensor was installed at the dark port. There were substantial differential pitch and yaw displacements between 2 and 10 Hz because of stack resonances, and some at sub Hertz frequencies as discussed before. A feedback loop was designed with high gain around 2 Hz — this, with position feedback discussed before, was instrumental in stabilizing the power inside the recycling cavity to within a few percent.
Figure 2.14: A simplified schematic to illustrate the principle behind the wavefront sensor.
Figure 2.15: P.s.d in radian/√Hz of the PNI in its first stage

2.4 Performance of the PNI

The performance of the PNI was gauged by the phase noise spectrum that was measured from the signal at the dark port. The measured spectrum was corrected by the feedback gain and then converted to a power spectral density in radian/√Hz. It was then compared to what would be obtained from the fundamental noise alone. In this section, we present two spectra that were obtained with an unrecycled Michelson in the first stage of PNI, and then the spectrum obtained with the recycled Michelson during the second stage of PNI. In the next two chapters we undertake a study of these spectra.
Figure 2.16: P.s.d in radian/$\sqrt{\text{Hz}}$ of the PNI in its first and second stage

Figure 2.15 shows the power spectral density of the unrecycled Michelson interferometer; the lower phase noise was obtained with three times more input light than the upper one, and with acoustic shielding around the input optics after it was moved to a new table.

Figure 2.16 shows the p.s.d of the PNI in its second recycled stage. We notice that the high frequency end reached $3.5 \times 10^{-10}$ radian/$\sqrt{\text{Hz}}$, about 15% above what is expected for the quantum noise level. This plot shows a phase sensitivity in interferometers that seems not to have been achieved before.

We turn now to a detailed study of phase noise in the next two chapters.
Chapter 3

Noise Sources in the Light

In this chapter, we look at the noise sources present in light used for interferometry and estimate their contributions to the dark port phase noise of the PNI (Phase Noise Interferometer). These noise sources are inherent in light, independent of the interferometer — we investigated which parameters of the interferometer were most influential in determining their contributions. There are other sources of noise that exist only when an interferometer interacts with light, and cease to have meaning without an interferometer — we reserve these sources for Chapter 4.

To analyze the dark port noise into elements from the different (classical) noise sources in light, we usually started by measuring the noise source directly. Then we simulated the noise source we were investigating with a calibrated stimulus, and measured the corresponding change in the phase noise spectrum. The observed change in the spectrum highlighted the particular mechanisms that were at work in the interferometer, to form a contribution from the noise source to the dark port detected phase. Once the mechanisms were identified, we could predict, given the measured level of the noise source, how much of the detected phase noise was due to that particular source.

3.1 Fundamental or quantum Noise in light

In section 2.1, we obtained the quantum noise (referred to usually as "shot noise") p.s.d in the detection of phase for the unrecycled and recycled Michelson interferometers. For
the Michelson interferometer, the parameters required for this estimate were the input light power at the beam-splitter (the bright fringe photo-current), the c.d., and the additional photo-current that is measured owing to phase modulation. Recasting expression (2.14) in the quantities measured with the dark port photo-detector, we obtain

\[ \sqrt{\frac{2e}{I_{\max}}} \cdot (1 + \frac{I_{\min}}{I_{\Gamma}})^{1/2}, \]  

(3.1)

where \( I_{\max} \) is the current measured when the unlocked interferometer swings through a bright fringe, \( I_{\min} \) is the photo-current when the interferometer is locked with a very small modulation index, and \( I_{\Gamma} \) is the increase in this current after the modulation index is brought to its desired level. The expression (3.1) assumes that the modulation index is sufficiently small so that the \( J_2 \) and higher order Bessel expansion terms (Appendix A) may be neglected.

The quantum noise in light is added in quadrature with the dark noise of the photodetector (this includes the thermal noise from the photo-diode and some contribution from the transimpedance pre-amplifier) — to express the dark noise in units of phase power spectral density, we need to multiply it by the calibration of the measured quantity in radians of phase difference. This calibration constant is obtained experimentally by injecting a known sinusoidal current (at a frequency where the driven mirror behaves as a free mass) into the OSEM coils of one of the Michelson mirrors. With the known force constant of the OSEM coils (in Newtons per ampere) and the mass of the mirror, we can calculate the phase difference in radians produced by this motion. The phase noise spectrum would show this signal in volts/√Hz and thus give us the calibration constant. Figures 2.15 and 2.16 show the calibration signal as peaks at 2 kHz.

The November 95 spectrum in Figure 3.1 was taken for an \( I_{\max} \) of 21.2 ma, an \( I_{\min} \) of 60 μa, and an \( I_{\Gamma} \) of 0.2 ma — with these values, \( 4.43 \times 10^{-9} \) radian/√Hz is obtained from expression 3.1. The dark noise floor was at \( 6.9 \times 10^{-9} \) radian/√Hz; together with the quantum noise in quadrature, we get \( 8.2 \times 10^{-9} \) radian/√Hz, the level that we see from 5 kHz onwards.

The January 96 spectrum in Figure 3.1 was taken for an \( I_{\max} \) of 64 ma, an \( I_{\min} \) of 124 μa, and an \( I_{\Gamma} \) of 0.6 ma — these values establish a quantum noise limited sensitivity of \( 2.46 \times 10^{-9} \) radian/√Hz. The dark noise floor now was at \( 2.3 \times 10^{-9} \) radian/√Hz — thus we expect \( 3.37 \times 10^{-9} \) radian/√Hz if the phase noise is quantum noise limited. However, the
Figure 3.1: Shot noise level comparisons with measured unrecycled PNI spectra
measured spectrum noise asymptotes to between $4.5 \times 10^{-9}$ and $5 \times 10^{-9}$ radian/$\sqrt{\text{Hz}}$ above 2 kHz — obviously, some other source of noise was more dominant.

The measurements of the recycled interferometer parameters needed for the quantum noise estimate requires a somewhat different approach. The value of $\Gamma$ cannot be reduced to determine the contrast defect of the interferometer, as the modulation index influences the gain in two length control loops and one alignment control loop — also, a change in $\Gamma$ causes a slight change in the beam direction which then affects the recycling cavity alignment [22]. With recycling, it is not possible to directly measure the bright fringe current at the dark port as well.

The round trip power loss of the light inside the recycling cavity can be measured by modulating the input light in amplitude with a square wave, and monitoring the light intensity variation transmitted through one of the Michelson end mirrors (which approximately had 10 ppm of power transmission). The recycling cavity of the PNI contained light at different frequencies (the carrier and its side-bands), each of which saw a different (transmissive) loss through the Michelson interferometer. However, as the sideband field was only about 1/8 of the carrier field in amplitude, and the (transmissive power) loss for each sideband was more than twice that for the carrier — after the first half-life interval\textsuperscript{1}, the monitored time decay in the light intensity could be expected to be entirely due to the carrier field. Since our modulation did not exceed more than 10% of the existing light level, we measured half-lives that captured field decay times [23]. These measured half-lives were averaged and converted to a power decay time ($t_e$) by dividing by 2 ln(2); the measured round trip power loss for the carrier could then be calculated as $t_{\text{roundtrip}}/t_e$ given the high finesse. We thus obtained about 9730 ppm for $L_{\text{common}}$, defined as in Section 2.1.3, for our recycling cavity.

An optical spectrum analyzer was used to look at the carrier to sideband power ratios in the light at several places — at the input to the interferometer ($J_1^2|_{\text{inpt}}$), at the dark port of the interferometer ($J_1^2|_{\text{dark}}$), and in the light inside the recycling cavity seen transmitted through one of the Michelson mirrors ($J_1^2|_{\text{inside}}$). From Section 2.1.3,

$$g_r^2 = (J_1^2|_{\text{inpt}}) \div (J_1^2|_{\text{inside}}).$$

\textsuperscript{1}This is the time in which the monitored light intensity decays to half its value.
We measured $g^2$ to be $12.8/65 = 0.197$, and given the $L_{\text{common}}$ measured before, we calculated $\sin^2(\delta_1)$ to be about 12192 ppm. With 20.8 cm of asymmetry as measured, and our actual modulation frequency of 25.33 MHz, we get $\sin^2(\delta_1) = 12176$ ppm — thus the two numbers agreed within 5%.

The carrier to sideband power ratio at the beam-splitter is altered at the anti-symmetric port as the two frequencies see different transmissions through the Michelson interferometer — the carrier transmission is through the contrast defect only, while the sideband transmission gets dominated by a term that involves the asymmetry as well. Writing $c.d. = 2 \times \delta_0^2$, the change in ratio can be expressed as

$$
\left( \frac{J^2_0}{J^2_1} \right)_{\text{inside}} \cdot \frac{\delta_0^2}{\delta_0^2 + \sin^2(\delta_1)} = \left( \frac{J^2_0}{J^2_1} \right)_{\text{dark}}. \tag{3.3}
$$

We thus could solve for $\delta_0^2$ given our measured carrier-sideband power ratio of 3.7 at the dark port, and found this to be 736 ppm (the contrast defect was thus about $1.4 \times 10^{-3}$).

The relationship (3.3) also allows us to deduce what the photo-current (as measured by the anti-symmetric photo-detector) at the beam-splitter must be, given the measured dark port current. If the light power (measured in photo-current) in the carrier at the beam-splitter be $pJ_0^2$ and that of one of the sidebands be $pJ_1^2$ — we can write,

$$
pJ_1^2 \left[ \left( \frac{J^2_0}{J^2_1} \right)_{\text{inside}} \cdot \delta_0^2 + 2 \cdot (\delta_0^2 + \sin^2 \delta_1) \right] = \text{dark port current}. \tag{3.4}
$$

The measured dark port current for Figure 3.2 was 11 ma; given this value, $pJ_1^2$ is about 149 ma and $pJ_0^2 = 0.149 \times 65 = 9.7$ amperes. The photo-current at the beam-splitter, $I_{\text{max}}$, is then equivalent to 10 amperes, and we can recast Equation 2.14 for the parameters of the recycled Michelson interferometer as

$$
\sqrt{\frac{2e}{I_{\text{max}}} \cdot [1 + \frac{1}{2} \cdot \left( \frac{J^2_0}{J^2_1} \right)_{\text{dark}}]}^{1/2}. \tag{3.5}
$$

If we use the numbers as measured when the plot of Figure 3.2 was recorded, we get a quantum noise limited sensitivity of $3 \times 10^{-10}$ radian/$\sqrt{\text{Hz}}$ — we can see that the measured high frequency noise in the figure asymptotes to this level within less than 25%.
Figure 3.2: Comparison of recycled PNI phase noise with quantum noise.
3.2 Frequency noise

As discussed before, frequency noise shows up in the phase noise of the PNI primarily because of the asymmetry in the two arms of the Michelson interferometer — given the asymmetry of $\delta l = 20.8 \text{ cm}$ in the PNI, we could thus expect a coupling of $8.7 \times 10^{-6} \text{ radian/Hz}$. With the unrecycled Michelson interferometer, the spectrum that we show for January 96 in Figure 3.1 asymptotes to about $4.5 \times 10^{-9} \text{ radian/}\sqrt{\text{Hz}}$, at a level above that expected for the quantum and dark noise; if we assume that this excess is all due to the frequency noise added in quadrature, we find that our frequency noise was no greater than $0.34 \text{ Hz/}\sqrt{\text{Hz}}$ from 2 kHz onwards (this frequency noise sets a phase noise of $2.98 \times 10^{-9} \text{ radian/}\sqrt{\text{Hz}}$ which was overwhelmed by the dark noise in the November 95 spectrum). The level of stability is slightly better than the expected $1 \text{ Hz/}\sqrt{\text{Hz}}$ as reported before from the use of a quartz cavity longitudinal mode as a reference [16]. We did not have another analyzer cavity in the first stage of PNI, so we did not have an independent measure of the frequency noise — however, a spectrum of the error signal in the frequency stabilization feedback loop, as shown in Figure 3.3, provided a lower limit of about $0.06 \text{ Hz/}\sqrt{\text{Hz}}$ to the frequency noise above 2 kHz. To calibrate the measured spectrum in frequency units, we wiggled one of the laser cavity mirrors with a known displacement and observed the peak in the spectrum — a modulation $\delta L$ of the laser cavity length $L$ produces a frequency modulation of $f \cdot (\delta L/L)$ where $f$ is the frequency of light.

Where the recycling cavity does not deviate from resonance from its length fluctuations, the error signal obtained for holding it resonant is proportional to the residual frequency noise after the light has been stabilized in frequency with the quartz reference cavity. The length changes of the recycling cavity are mainly seismically driven and seismic excitation drops off fast above some 10s of Hz — however, the cross-over between actuators must happen gradually to avoid phase-shifts between the two paths that can then compromise the servo gain. If we change the laser resonator cavity length to correct for the measured residual frequency noise, we cannot allow for any large corrections that would cause a shift from the frequency at which the reference quartz cavity is resonant. All these considerations had to be taken into account at the second stage of the PNI as we tried to achieve the required frequency noise suppression.
Figure 3.3: Error signal in the reference cavity stabilization loop

to reach a phase noise level of $10^{-10}$ radian/√Hz.

The design of the common mode servo has been discussed in Chapter 2. The common mode error signal shown in Figure 3.4 was first converted to frequency units using the calibration method described before and then converted to phase noise using the asymmetry. The error signal captures the lower limit to frequency noise; the upper limit is set by the measured phase noise of the PNI. Thus, where the two traces meet — as around 10 kHz — the phase noise could be attributed to the residual frequency noise coupled through the asymmetry. We measured the phase coherence between the measured dark port phase noise and the common mode error signal in the bandwidth shown in Figure 3.4. Since seismic noise drops off within few tens of Hertz, the deviation of the recycling cavity from resonance should be dominated by frequency noise above 100 Hz. However, as discussed in Chapter 2, frequency noise correction did not take full effect until beyond 1 kHz (the deviation from resonance was corrected by a corresponding change in the length of the recycling cavity) — hence we could expect phase noise owing to the residual frequency noise, phase coherent with the common mode error signal, from 100 Hz to around 1 kHz. At first no such phase coherence was detected; later
Figure 3.4: Common mode error signal converted to phase noise in the recycled PNI

experiments (by Dr. Gonzalez and Mr. Lantz) which put an extra Faraday isolator in the input optics chain brought down the phase noise in these frequencies by a factor between 3 and 5 and showed phase coherence with the common mode error signal from 400 Hz downwards. We will return to back scattered light in the section on parasitic interferometry.

Light scattered from the main beam and then recombined back at the dark port photodetector after reflecting off a surface can be a source of frequency noise. We noticed a strong dependence of the dark port noise at frequencies above 2 kHz to the setting of the phase-shifter used with the local oscillator — the noise increased if the setting was disturbed slightly from the optimal, and became coherent with the common mode error signal. Since the noise came roughly at quadrature, this scattered path must have had a reflector about 1.5 meters away from the recycling mirror, most possibly at the vacuum entry port (the reflector could also have been half way in the vacuum tube connecting the tanks though this was unlikely). Efforts to locate this scatterer did not succeed — however, the scatterer was also not important as its noise came at quadrature to the phase signal.
3.3 Amplitude Noise

Amplitude noise in light and its coupling to the phase noise was extensively investigated during both stages of the PNI work; the noise was found to not have any significant influence at either of the two stages. As shown in Chapter 2, radio-frequency phase modulation does isolate the phase noise from amplitude noise in the bandwidth of interest; however, phase modulation with a Pockels cell leads to a slight amplitude modulation that can re-introduce the amplitude noise. We will look at this amplitude modulation under the section on instrumentation noise in the next chapter and show how it can be avoided.

The Relative Intensity Noise (RIN) of the free running laser — defined as $(\Delta I)/I$ where $I$ is the dc power level — had a measured spectral density of $1 \times 10^{-3}/\sqrt{\text{Hz}}$ at around 10 Hz, falling off roughly as $1/f$ up to 10 kHz. We had an active stabilization system which achieved a suppression of the amplitude noise by a factor of 100 at 100 Hz, dropping to around 30 at 1 kHz and unity around 16 kHz. With the stabilization system in operation, we thus had an RIN of about $1 \times 10^{-6}$ around 1 kHz.

Expression 2.12, which captures the signal from which the phase-difference is inferred, shows how the RIN can contribute to the dark port phase noise. Recasting this expression in terms of the Bessel functions used in Equation 2.16, we obtain

$$2P_i \cdot (1 + \text{RIN}) \cdot [J_0 J_1 \sin(\omega_m t \Delta)] \cdot \sin[\omega_m (t - t_c)] \cdot \Delta \phi. \quad (3.6)$$

We notice that if the Michelson interferometer has a fixed offset from its null, there is phase noise coherent with the RIN; if there are larger residual $\Delta \phi$ changes at say frequency $f_\phi$, the RIN at frequencies $f \pm f_\phi$ would be the main sources of phase noise at frequency $f$. Thus — to ascertain how much phase noise is contributed by the RIN — we need to identify the largest features in $\Delta \phi$ and find their frequencies. This is easily obtained by injecting a sizeable RIN at some frequency where the phase noise is at its minimum and identifying the resulting (most dominant) peaks that then appear as sidebands of this frequency. For example, Figure 3.5 shows the phase noise spectrum of the recycled PNI when an RIN of $7.7 \times 10^{-3}$ was introduced in the input light — a number of sidebands which can be identified with mechanical resonances were obtained. From the measured phase noise at the peaks and the known RIN, the $\Delta \phi_{\text{peak}}$ at the resonant frequencies can be determined. We can also determine the most dominant
Figure 3.5: Phase noise peaks produced by RIN injection at 3 kHz

Figure 3.6: Phase noise owing to RIN injection at 3 kHz with high bypass gain at low frequencies
peak, the one that would cause phase noise coupled with the RIN. Figure 3.6 shows what happened when a high gain (at low frequencies) bypass stage was turned on in the Michelson length control servo — the sideband peaks disappeared and the one that was left was due to the offset of the Michelson interferometer from the minimum possible null. Again, from the known RIN and phase noise at the RIN peak, we determined this offset to be $3.7 \times 10^{-5}$ radian in the recycled PNI. From Equation 3.6, it is clear that the relevant RIN for the recycled PNI is that of the light at the beam-splitter; this was obtained by measuring the RIN of the light transmitted through one of the Michelson end mirrors. As shown in Figure 3.7, we noticed

![Figure 3.7: RIN of the light transmitted through the recycling cavity](image)

that this RIN, with the active differential alignment system installed, was not much different from that of the input light (shown with an $1/f$ line in Figure 3.7). Thus, given the recycled phase noise spectrum of October 10 (shown in Figure 2.16), the unstabilized RIN multiplied by $3.7 \times 10^{-5}$ radian, the dc offset, gave rise to a phase noise that was below the measured p.s.d.

During the first stage of the PNI, the unrecycled interferometer suffered from large $\Delta \phi$ variations at around 19 Hz, the vertical bounce resonance of the suspensions. Our recorded
spectrum rejected any (peak) excursions beyond 1 milli-radian — with the amplitude stabilization in operation, an RIN of $1 \times 10^{-6}/\sqrt{Hz}$ in the input light resulted in a phase noise of $1 \times 10^{-9}$ radian$/\sqrt{Hz}$, below the phase noise shown in Figure 2.15.

From Equation 3.6, we notice that variations in the term $J_0 J_1 \sin(\omega_m t \Delta)$ will have the same effect on the phase noise as RIN — this noise can be measured by injecting a $\Delta \phi$ at some frequency where the phase noise has reached a minimum and measuring the increased phase noise that result at side band frequencies of the excitation. However, this requires that the the excitation be more stable than the RIN or the fractional noise in the calibration terms, $J_0 J_1 \sin(\omega_m t \Delta)$; when we did this experiment we measured the RIN until our oscillator (excitation) noise dominated at a level of $10^{-5}/\sqrt{Hz}$ from 400 Hz onwards. The oscillator noise level of $10^{-5}$ converted to a phase reached the noise level of our measured spectrum from 2.5 kHz onwards — this implied that the noise in the elements that set the phase calibration, and the RIN, did not dominate in our phase noise spectrum.

### 3.4 Beam jitter noise

In this section we look at how fluctuations of the beam pointing at the interferometer input results in phase noise. We first present the formulation that describes this noise in an unrecycled Michelson interferometer, and then an experiment that was carried out to verify our result. We next show which parts of our Michelson spectra in the first stage were dominated by beam pointing jitter. Finally, we show how we need to revise our formulation to address the recycled interferometer, and present experiments that established the contribution of beam jitter to the recycled PNI phase noise spectrum.

Referring to Figure 3.8, we set up a coordinate frame on the beam-splitter, in which the beam-splitter itself lies diagonally at $45^0$ to the positive X axis. In this coordinate frame, the input light beam comes and strikes the splitter at a point whose $y$ coordinate is given by $y_i$; the beam also makes a counter-clockwise angle $\theta_i$ with the negative X axis. The aligned situation thus assumes that $\theta_i$ and $y_i$ are 0.

After the beam hits the splitter, half the power is transmitted while the other half is reflected. The reflected beam makes its way to one of the mirrors — which we henceforth refer as mirror 1 — and reaches the point A to be reflected back again. Mirror 1 is misaligned
Figure 3.8: Definition of variables in the geometry of a beam varying in direction on a misaligned mirror.

with (clockwise) angle \( \theta_1 \) to the negative X axis; it also intersects the Y axis at the point \((0, y_1)\). The beam, after reflection from the mirror and transmission through the splitter, meets the X axis at B.

The distance \( y_1 \) is given by the corresponding arm length of the Michelson interferometer. To determine the interference detected at the anti-symmetric port, we need to follow the path of the other beam transmitted through the splitter to the X axis also. We simplify the calculation by assuming that the second beam follows the path of the first (reflected) beam — i.e., it reflects off the beam-splitter like the first and then off the second mirror which is now placed to intersect the Y axis at \((0, y_2)\). The beam reflected off this second mirror is then transmitted through the splitter to the X axis, exactly like the first one, for interference. The asymmetry in the two arms is captured in \( y_2 - y_1 \). When the two phase-fronts are brought to the X axis for interference, they differ in their

(1) gaussian parameters;
(2) positions of centers on the X axis;
(3) tilts, given the angle between each beam and the vertical where it crosses the X axis; 
(4) extra distances they had to travel over their nominal ones (2y₁ or 2y₂).

The gaussian parameters are set by how the beams are shaped by lenses and other optical 
elements. Expressions for the terms in (2),(3), and (4) for the beam reflected off mirror 1 can 
be formulated with the parameters θᵢ, yᵢ, θ₁, and y₁; the subscript 2 substituted for 1 gives 
us corresponding expressions for the beam reflected off mirror 2. We need to first define AP, 
the length of the perpendicular on the Y axis in Figure 3.8,

\[ AP = \frac{\tan(\thetaᵢ) \cdot (y₁ - yᵢ) + yᵢ}{1 + \tan(\thetaᵢ) \cdot \tan(\theta₁)}. \]  

(3.7)

The point B then has an x coordinate of

\[ AP - (y₁ - AP \cdot \tan(\theta₁)) \cdot \tan(2\theta₁ - \thetaᵢ), \]

(3.8)

while the angle the reflected beam makes with the vertical at B is 2 \cdot θ₁ - θᵢ.

The extra distance that each of the beams has to travel given a misaligned mirror and a 
mis-pointed beam contributes most significantly to the phase noise. This is

\[ \frac{y₁ - yᵢ - AP \cdot \tan(\theta₁)}{\cos(\thetaᵢ)} + \frac{y₁ - AP \cdot \tan(\theta₁)}{\cos(2\theta₁ - \thetaᵢ)} - 2 \cdot y₁ \]

(3.9)

for mirror 1. The extra distances, converted to an extra optical phase for each of the beams, 
need to be subtracted to form the phase difference between the two beams at the anti-
symmetric port. Accordingly, we define

\[ y₁ + y₂ = L \]

\[ y₁ - y₂ = \Delta, \]

(3.10)

and enumerate terms upto second order (in misalignment and beam pointing angle) in the 
phase difference:

\[ k \cdot (\theta₂ - \theta₁) \cdot (2yᵢ + 2L\theta₁ - 2yᵢ\thetaᵢ) \]

\[ + k \cdot (\theta₂ + \theta₁) \cdot (2\Delta \thetaᵢ) \]

\[ + k \cdot L \cdot (\theta₂ + \theta₁)(\theta₁ - \theta₂) \]

\[ + k \cdot \Delta \cdot (\theta₁² + \theta₂² + 2\thetaᵢ²), \]

(3.11)
where $k = \frac{2\pi}{\lambda}$.

Not all terms in the expression above contribute equally to the longitudinal phase difference; given our asymmetry ($\Delta$) of 20.8 cm, the term that dominates is

$$k \cdot (\theta_2 - \theta_1) \cdot (2y_i + 2L\theta_i)$$

(3.12)

To verify that this term dominates, we wiggled one of the steering mirrors that sent the light over a distance of 10 meters to the beam-splitter of the unrecycled Michelson interferometer. Figure 3.9 shows two time traces — the error signal of the Michelson length control loop and the simultaneous low frequency variation of the Michelson dark port photo-current. As can be seen clearly, the modulation of the error signal at our wiggle frequency followed the variation in the low frequency photo-current quite closely. In the last chapter, we discussed how the low frequency changes were dominated by differential misalignment in the unrecycled PNI, i.e., a variation in $(\theta_2 - \theta_1)$. A comparison is thus possible between the modulation of the error signal and the low frequency change in photo-current. Working out the values for the dip in the middle of the trace, $\theta_i = (\text{Calibration of } 0.4 \mu \text{rad/Volt}) \times (50 \text{ mV peak input}) = 2 \times 10^{-8}$ rad pk; phase modulation amplitude $= 2 \times 10^{-4}$ rad/Volt calibration $\times 0.75$ volt amplitude on trace) $= 1.5 \times 10^{-4}$ rad — we thus find that expression 3.12 establishes $(\theta_2 - \theta_1)$ to be about 30 $\mu$rad.

The fluctuation in the low frequency trace corresponded to roughly 3% in contrast variation. Integrating the interference pattern that result at the dark port of a Michelson interferometer (with equal arms) when the two end mirrors are misaligned, we get a contrast variation of

$$\frac{1}{4} \cdot k^2(\theta_1 - \theta_2)^2w^2,$$

(3.13)

where $w$ is the waist radius of the two beams. Using $w = 1$ mm at the beam-splitter for the unrecycled PNI, we find $(\theta_1 - \theta_2) \approx 30 \mu \text{rad}$ as obtained in the last paragraph.

Figure 3.10 shows the influence of beam jitter on the phase noise of the unrecycled PNI. As shown in the figure, the November 95 spectrum was dominated by mechanical resonances of the elements in the input optics chain; care was taken to therefore damp these resonances and an acoustic shield was used to isolate the input optics from ambient excitation. This resulted in elimination of the phase noise owing to input beam jitter above 1 kHz as the
Figure 3.9: Time traces of the length error signal and the low frequency light variation at the dark port of the unrecycled PNI, as the input beam is wiggled in direction.

spectrum from Jan 96 clearly shows; the beam jitter peaks between 400 and 500 Hz were also substantially reduced. The signal from a quad photo-detector looking at the light reflected from the unrecycled PNI showed phase-coherence with the phase noise between 400 Hz and 2 kHz — thus the phase noise in this region could be attributed to variation in input beam pointing.

The recycled PNI requires some modification of the above formulation. We rewrite the expression in 3.12 for the unrecycled PNI slightly differently,

\[ k \cdot (\theta_2 - \theta_1) \cdot (2d_i). \]  \hspace{1cm} (3.14)

where \( d_i \) is the displacement of the beam from its axis (the path it would follow if it were properly aligned). If the source of the beam jitter is a steering mirror that shakes (with angle \( \theta_i \)) at a distance \( s \) from where the common mode path terminates, then \( d_i = s \cdot \theta_i \). When we introduce the recycling mirror (but not recycling of light, e.g., by misaligning the recycling mirror), we introduce the effects of a diverging lens via the 10 meter radius of curvature of the
mirror — the source of the wiggle appears to move closer to the beam splitter, and thus the effective \( d_i \) is less than before. For example, if the source is at a distance \( p \) from the recycling mirror, \( s = 6 + (5p)/(5 + p) \), instead of \( 6 + p \). With the recycling mirror in place, the beam forms a waist at the end of its common mode path — at this location, given the wiggle \( \theta_i \) and the displacement \( d_i \), we can write a modal (field) description of beam as [24]:

\[
\psi(x) = A[U_0(x) + \frac{d_i}{w_0}U_1(x)],
\]

(3.15)

where \( w_0 \) is the waist radius, and \( U_{0,1} \) represent the Hermite Gaussian functions of order 0 and 1. When we begin to recycle light, the field \( U_1 \) does not build up as much as \( U_0 \) — if light sees a round trip power loss of \( L \), we can write the following (approximate) expression for the recycled light:

\[
\psi(x) = A[U_0(x) + \frac{L}{4} \cdot \frac{d_i}{w_0}U_1(x)].
\]

(3.16)

The carrier and the sidebands thus see different "effective" displacements at the end of the common mode path inside the recycling cavity, as their losses are different. These fields next see the differentially misaligned Michelson interferometer which give rise to further \( U_1 \) fields;
the sideband and the carrier $U_1$ fields next interfere to form a signal at our phase modulation frequency which add noise to the detected phase. Using the model discussed in [20] and the parameters of the recycled PNI, we obtain about 2.2 radian of optical phase for 1 $\mu$radian of $(\theta_2 - \theta_1)$, 1 radian of $\theta_i$, and $s = 8$ meters. An experiment performed with a steering mirror located at this value of $s$ ($\approx 8$ meters) yielded about 7 radian of optical phase per radian of $\theta_i$ — this seemed to indicate that the static differential misalignment was about 3 $\mu$radian.

Looking at the light reflected off the PNI with a quad photo-detector, and calibrating the signal by dithering the recycling mirror in angle, we can obtain the power spectral density of $\theta_i$. Using the coupling measured by a calibrated input beam jitter as described in the last paragraph, the contribution of beam jitter to optical phase noise can be determined. Figure 3.11 shows exactly such a comparison. The phase noise predicted from input beam pointing comes close to explaining the measured phase noise on October 10 from 400 Hz to about 2 kHz — however, at the time the beam pointing noise was measured, the phase noise in this bandwidth was higher by roughly a factor of 3 than the noise measured on Oct 10. The plot serves to illustrate that the noise in beam pointing, unless actively controlled, can become the limiting noise from several hundreds of Hertz to a few kHz. The PNI spectrum in this bandwidth clearly had many sources of noise — the residual frequency noise in laser, some back scattering, and definitely beam pointing noise. A lot of effort thus needs to be concentrated on controlling these sources of noise to stretch the level of $10^{-10}$ radian phase noise into this bandwidth.

We now turn to the sources of noise that come from the interaction of light with the elements that make up the interferometer.
Figure 3.11: Phase noise from input beam jitter compared with the Oct 10 spectrum of the PNI
Chapter 4

Noise Sources in the Interferometer

The elements that comprise the interferometer introduce noise in the detected optical phase — e.g., the mirrors are disturbed from their positions by random forces which can have seismic or thermal origin; the sensing and active control systems are limited by noise from the electronics used to implement them; and, finally, light scattered off the intended path of travel can find its way to the photo-detector used at the dark port and interfere with the light carrying the phase signal. The purpose of this chapter is to look at these sources of noise and estimate their levels in the detected phase noise.

4.1 Displacement noise

The goal of the PNI (Phase Noise Interferometer) was to minimize the conversion of differential displacement in two orthogonal directions (required for the detection of gravitational waves) to optical phase; hence, our interferometer constructed a Michelson interferometer with 50 cm arms without optical cavities in them. However, displacement noise still played a role in the detected phase noise and we discuss the sources of this noise in this section.

4.1.1 Seismic noise

The ground shakes at low frequencies to produce differential displacements between the two Michelson arms; this seismic noise has a stationary part that usually rises from 0.1 Hz to 1 Hz, reaches a flat level between 1 and 10 Hz, and, finally, falls off as $1/f^{-2}$ from 10 Hz
upwards. The stationary ground noise at MIT (where we built the PNI) was approximately $3 \times 10^{-10}$ m/√Hz at around 80 Hz and isotropic — given the attenuation (of around $10^{-4}$) of the stack, and (of around $3 \times 10^{-4}$) of the pendulum — we expected the differential motion of the Michelson mirrors to not exceed $3 \times 10^{-18}$ m/√Hz. This level of ground noise produces less than $1 \times 10^{-10}$ radian/√Hz of optical phase noise.

The site of the PNI was affected by impulsive seismic events which lasted several seconds at frequencies between 5 and 10 Hz, and reached amplitudes as large as 80 microns. Most of this ground noise excited the vertical bounce resonance of the pendulums in the first stage of the PNI; in the second stage, a servo with high gain at around 20 Hz (the bounce resonance) was implemented to avoid the bilinear coupling of this low frequency phase noise to the phase noise above 100 Hz.

### 4.1.2 Thermal noise

Apart from the longitudinal resonance at about 1 Hz of the pendulum, the mirror position can also be disturbed by transverse excitations of the wire used for suspension, or the excitations of the "internal mirror" modes that are defined by the mirror material and geometry. These resonances are damped out by mechanisms internal to the excited element — and, by the Fluctuation Dissipation Theorem (see [17] for example), are also therefore excited thermally. Internal damping is modeled by adding an imaginary part to the usual spring constant $k$, i.e., the force $F$ in Hooke’s law is written as:

$$F = -k[1 + i\phi(\omega)]x,$$

where $x$ is the corresponding change in the quantity of interest with the application of force.

The Fluctuation Dissipation Theorem then establishes the thermal power spectral density for $x$ [17]:

$$x^2(\omega) = \frac{4k_BTk\phi(\omega)}{\omega[(k - m\omega^2)^2 + k^2\phi^2]},$$

where $k_B$ is the Boltzmann constant, $T$ the absolute temperature, and $m$ the mass of the object in motion. It is clear that knowledge of the frequency dependence of $\phi$ is important in predicting how the excitation would affect the optical phase noise — given the resonances we are usually concerned with and our frequencies of interest (several hundreds of Hz to a kHz), it
seems that \( \phi \) can be assumed to be a constant (the "structure damping hypothesis" \([17, 25, 26]\)). It is clear from Equation 4.2 that the Q of the resonance (defined to be \( \omega_0/\delta\omega \), where \( \omega_0 \) is the resonant frequency and \( \delta\omega \) the full width of the resonance at the half power points) is then \( 1/\phi(\omega_0) = 1/\phi \). We present here some thermal noise estimates given the parameters of the PNI.

The longitudinal pendulum mode is the first \((n = 0)\) of a series of modes called the violin modes (the term is usually reserved for the \(n > 0\) modes) — the displacement of the mirror owing to these modes may be written as \([17, 25]\):

\[
x^2(\omega) = \sum_n \frac{4k_BT}{\omega\mu_n} \cdot \frac{\omega_n^2\phi_n}{(\omega_n^2 - \omega^2)^2 + \omega_n^4\phi_n^2}.
\] (4.3)

where \( \mu_n = m \), the mass of the pendulum for the longitudinal mode (with frequency \( \omega_p \)), and \( \mu_n = (m\omega_n^2)/(2\omega_p^2) \) for the violin modes with a resonant frequency of \( \omega_n = 2\pi f_n \) for the \(n\)th mode. The first violin frequency was measured to be at 550 Hz with a Q of \( 1.7 \times 10^5 \) — at this resonant frequency, we find,

\[
x^2(550\ \text{Hz}) = \frac{8k_BTQ\omega_p^2}{m\omega_n^5} = 1.81 \times 10^{-30} \cdot (\frac{550}{f_n})^5 \cdot (\frac{Q}{1.7 \times 10^5}) \text{ m}^2/\sqrt{\text{Hz}}. \tag{4.4}
\]

The above displacement noise corresponds to about \( 3 \times 10^{-8} \) radian/\( \sqrt{\text{Hz}} \) in the optical phase which we should have been able to detect in the second stage (the first stage noise was approximately at this level); however, no peaked structure at this level was apparent in the spectrum around 550 Hz. In the bandwidth where we show the optical phase noise power spectral density in Figures 2.15 and 2.16, the longitudinal mode of the pendulum introduces its off resonant thermal noise; from Equation 4.3 we find this to be

\[
x^2(f) = \frac{4k_BT(2\pi f_p)^2}{m(2\pi f)^5Q} \text{ m}^2/\sqrt{\text{Hz}}. \tag{4.5}
\]

The Q of the pendulum is hard to measure; however, the work of Gillespie and Raab \([25]\) shows how this can be approximated to roughly two times the Q of the violin resonance. We thus approximate Q to be of the order of \( 3 \times 10^5 \), and can approximate Equation 4.5 to be

\[
x^2(f) = 1.34 \times 10^{-38} \cdot (\frac{100}{f})^5 \text{ m}^2/\sqrt{\text{Hz}}. \tag{4.6}
\]
The expected phase noise is then about $4 \times 10^{-12}$ radian/√Hz at 100 Hz, and below our measured phase noise.

Gillespie and Raab [26] have studied the physical deformation modes of the mirror modeled as a right solid cylinder. They have looked at the change in overall phase of a Gaussian beam reflected off a mirror whose modes have been excited thermally, and have added the (off resonance) contributions from these mechanical modes in the bandwidth of our interest until the sum converged to a specific value. Their modeling has shown that given the dimensions of our mirrors (3 inches in diameter and 1 inch in thickness), and summing the first 20 modes, the thermal displacement power spectral density can be given by

$$x_{th} = 2.4 \times 10^{-18} \cdot \left(\frac{100\text{Hz}}{f}\right)^{1/2} \cdot \left(\frac{\phi}{10^{-4}}\right)^{1/2} \text{m/√Hz}.$$  \hspace{1cm} (4.7)

A differential displacement of $4 \times 10^{-18}$ m/√Hz corresponds to a phase difference of $1 \times 10^{-10}$ radian/√Hz. The first mirror structure resonance seems to be between 23 and 29 kHz in the mirrors, where an attempt at the measurement of $Q$ gave a low value of around 1000 (fused silica is expected to have $Q$ values of around $10^6$); this may be because the first resonance sees more loss owing to the attachments to the mirror. If we assume this low value of $Q$ for all the modes, we get a phase noise spectral density of around $2 \times 10^{-10}$ radian/√Hz which is still below the measured phase noise.

### 4.1.3 Radiation pressure

The force exerted on an object by light is given by $P/c$ where $P$ is the incident power, and $c$ the speed of light [27]. The incident power has quantum fluctuations whose p.s.d is given by $\sqrt{2h\nu P}$, where $h$ is the Planck's constant and $\nu$ the frequency of light. These fluctuations then correspond to fluctuating forces on the mirrors which result in their displacement. Adding the displacements in the two arms in quadrature, we obtain a measured phase noise density at frequency $\omega$ (much larger than the pendulum resonant frequency) of

$$\phi_{ns} = \frac{4k}{m\omega^2c} \cdot \sqrt{\eta} \cdot \left(\frac{h\nu}{\eta e}\right) \cdot \sqrt{\frac{eI_{max}}{2}},$$  \hspace{1cm} (4.8)

where $k$ is the wave number, $m$ the mass of the mirrors, $\eta$ the quantum efficiency of the photo-detector used, $e$ the electronic charge, and $I_{max}$ the photo-current at the beam-splitter.
Noise Sources in the Interferometer

Given 10 amperes of current at the beam splitter in the recycled stage (Chapter 3), we thus obtain,

\[ \phi_{ns} = 3 \times 10^{-15} \cdot \left( \frac{I_{\text{max}}}{10 \text{ amp}} \right)^{1/2} \cdot \left( \frac{100}{f} \right)^2 \cdot \left( \frac{0.25 \text{ kg}}{m} \right) \text{ radian}/\sqrt{\text{Hz}}. \quad (4.9) \]

This shows us that the radiation pressure from the quantum fluctuations in light did not play a major role in the detected phase noise — however, if the beam-splitter is not exactly 50-50, or, if the masses of the two mirrors are not exactly equal, we can obtain a differential displacement owing to the intensity noise in light (RIN). If \( r \) represents the fractional difference in the two masses (i.e., \( 2(m_1 - m_2)/(m_1 + m_2) \)), or the difference in the power reflected off the beam-splitter from that transmitted, then

\[ \phi_{ns} = \frac{kP}{m \omega^2 c} \cdot (r \cdot \text{RIN}) = 1.23 \times 10^{-8} \cdot \left( \frac{I_{\text{max}}}{10 \text{ amp}} \right) \cdot \left( \frac{100}{f} \right)^2 \cdot (r \cdot \text{RIN}) \cdot \left( \frac{0.25 \text{ kg}}{m} \right) \text{ radian}/\sqrt{\text{Hz}}. \quad (4.10) \]

Given that the RIN was roughly at \( 1 \times 10^{-6} \), and \( r \) definitely not greater than 10%, we thus find that the displacement noise owing to the radiation pressure in light could not have contributed to the phase noise of the recycled PNI.

The feedback system that damped the pendulums could also convert its electronic noise to a displacement signal — we will look at this in the section on instrumentation noise.

### 4.2 Interferometer misalignment

As the suspended mirrors get disturbed from their equilibrium positions, they produce a misalignment of the optical elements used for interferometry, in addition to a change in the distances between mirrors (longitudinal displacement). We can distinguish between two kinds of misalignments in the PNI — a common mode misalignment, where the input beam in a round trip to its starting point sees optical elements that are not in their intended alignments; a differential misalignment, where the two beams interfered at the anti-symmetric port see their wavefronts rotated or shifted with respect to each other.

As discussed in the section on beam jitter in Chapter 2, the primary cause of differential misalignment is the difference, \( (\theta_2 - \theta_1) \), of the two Michelson mirrors. We have already seen how this difference in angle, coupled with beam jitter, gives rise to a longitudinal phase. This difference also can cause light to leak out of the dark port and cause a poor contrast. In
the first stage of the PNI, the c.d. owing to differential misalignment was sometimes as high as 3 % (see Figure 3.9 for example) — this corresponded to a “transmissive” loss of about 15,000 ppm through the Michelson interferometer. During the unrecycled stage, apart from the phase noise through the bilinear coupling with beam jitter, this differential misalignment was of not much consequence — causing only about 1.5 % change in the quantum noise level at its worst. However, this was a serious impediment to recycling; given that the input transmission to the recycling cavity was only 8200 ppm, this level of loss could cause the carrier to be undercoupled and hence not build up the light power to the desired extent at the beam-splitter. Some position feedback in the damping control of the mirrors, along with the wavefront sensor described in Chapter 2, had to be therefore implemented to stabilize the differential misalignment of the Michelson mirrors at the second stage of the PNI.

The recycling cavity can be thought to be made of the recycling mirror as the front mirror, and the Michelson interferometer as the “back” mirror. While the differential misalignment influences the reflectivity and transmission of this back mirror, the alignment of the back mirror is determined by the angle $\theta = (\theta_1 + \theta_2)$ of the two Michelson mirrors with respect to a perpendicular on the path of the input laser beam. When the two mirrors of the recycling cavity are disturbed from their correctly aligned positions, the input beam projects onto the first order Hermite-Gaussian modes at the expense of the zeroth order mode — this causes a reduction in power at the beam-splitter inside the recycling cavity. If the angle of the recycling mirror from its correctly aligned orientation be given by $\alpha$, then the power lost can be expressed as a reduction in transmission of the input light power of [2]

$$\frac{R}{w} \cdot \alpha - \left(\frac{R - L}{w}\right) \cdot \theta)^2 + \left(\frac{kw\theta_2}{2}\right)^2, \quad (4.11)$$

where $R$ is the radius of curvature of the recycling mirror (10 meters), and $w$ is the waist radius at the end of the common mode path (0.9 mm). Thus while a 10 $\mu$radian change in the recycling mirror orientation produces about an 1 % drop in power at the beam-splitter, the same change in $\theta$ results in only a 0.4 % reduction in power.

It may seem that during the first stage of PNI, common mode alignment — i.e., the adjustment of $\theta_i$, the input beam angle as defined in the section on beam jitter in Chapter 3, or $\theta = (\theta_1 + \theta_2)$ of the two Michelson mirrors — was not as important as differential alignment
Figure 4.1: Figure showing where we put a photo-detector to monitor back-scattered light; an extra beam scattered off the main beam to interfere with the phase signal is also shown.

for a good contrast at the output of the interferometer. This is not true — the asymmetry in the Michelson interferometer makes the common mode alignment just as important. From Equation 3.8 expanded to second order in the terms $\theta_{1,2,i}$ and $y_i$, we obtain the separation of the centers of the two Gaussian beams interfering at the dark port to be

$$\Delta \cdot [2\theta_i - \theta] + L \cdot (\theta_2 - \theta_1),$$  

(4.12)

where $\Delta$ and $L$ are as defined in Chapter 3. The angle between the two Gaussian beam cross-sections is $2(\theta_2 - \theta_1)$; thus given any arbitrary $\theta_i$ and $\theta$, it is hard to have the centers coincident and the profiles aligned, as are necessary for a good contrast. Only when the input beam is retro-reflected are we able to satisfy both these criteria.

4.3 Parasitic Interferometry

The light incident on the interferometer is primarily made of fields at three frequencies — the carrier frequency, $\omega$, and the two sideband frequencies, $\omega \pm \omega_m$. We can combine the two
fields at the two sideband frequencies, and obtain an expression for the (time dependent) field amplitude at the input of the interferometer as,

\[ J_0 E_{in} + 2i J_1 E_{in} \cos(\omega_m t), \]  

(4.13)

where \( J_0 \) and \( J_1 \) are the zero and first order Bessel functions with the modulation index \( \Gamma \) as their arguments. A photo-detector looking at this light (as shown in Figure 4.1) does not see any signal at the modulation radio-frequency — thus, if we down-converted the photo-detector output from a bandwidth around the modulation frequency, we expect to see shot noise consistent with the light level. In practice, we may see some intensity noise owing to the slight amplitude modulation that accompanies the phase modulation process. When we blocked the light going to the interferometer at the point shown in Figure 4.1, we found that the input light was in fact shot noise limited as shown in Figure 4.2. When the light was allowed

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig42.png}
\caption{Input light intensity noise around the phase modulation frequency — as shown, it is quantum noise limited to about 100 Hz}
\end{figure}

to be incident on the interferometer, the signal on the photo-detector would periodically rise sharply above the shot noise level, as shown in Figure 4.3, usually around the time of increased
seismic activity. This is because a small portion of the light reflected off the interferometer is sent back along with the input light by a reflector in the input optics chain. The amplitude of this additional field can be written as,

$$ [r' J_0' E_{in} + 2ir' J_1' E_{in} \cos(\omega_m t + \phi_s)] \cdot e^{i\phi_c}, $$  \hspace{1cm} (4.14)

where $r'$ equals the products of the reflectivities of the interferometer and the back-scatterer, $J_{0,1}'$ represents the changed modulation index in the reflected light, and $\phi_{c,s}$ are the extra carrier and sideband phases respectively that this light accrues with respect to the input light owing to its extra path of travel. The modulation index is unchanged in the case of the unrecycled PNI unless the light travels through the phase modulator; however, in the case of the recycled PNI, the modulation index is affected by the differing reflectivities of the carrier and the sidebands. This light then interferes with the input light to produce a signal at the modulation frequency of

$$ 4r' |E_{in}|^2 \cdot [J_0' J_1 \cos(\omega_m t) - J_1' J_0 \cos(\omega_m t + \phi_s)] \cdot \sin \phi_c, $$  \hspace{1cm} (4.15)
The quantity $\phi_c$ contains the effect of the changes in the extra travel path — this path is modulated as the reflecting surface of the interferometer or the reflector in the input optics chain shakes. If either of these is excited as $A \cos(\omega_r t)$, then we obtain $\sin \phi_c = \sin(\phi_0 + 2kA \cos(\omega_r t))$; hence, the frequency of the signal in expression (4.15) chirps to a maximum of $(2A/\lambda)$ where $\lambda$ is the wavelength of light. Thus the spectrum shows a sudden drop at a maximum frequency as evident in Figure 4.3. The level of the spectrum is set by the expression multiplying $\sin \phi_c$ — as can be seen from expression (4.15), if the modulation index is unchanged as in the unrecycled PNI, a modulation frequency can be chosen so that $\phi_s$ is a multiple of $2\pi$ and the signal forced to disappear. If there are more than one scatterer, we will obtain a sum of expressions of the form of 4.15 for each scatterer. It is clear that this "parasitic" interference can be diminished by reducing the reflectivity $r'$, or, alternatively, restricted to low frequencies by lowering the amplitude of vibration of the reflectors with respect to each other.

The input light amplitude given by 4.13 converts to that of

$$i J_0 E_{bs} \delta_0 + 2i J_1 E_{bs} \sin(\delta_1) \sin(\omega_m t + \phi_D) + 2J_1 \delta_0 E_{bs} \cos(\omega_m t + \phi_D)$$  \hspace{1cm} (4.16)

at the anti-symmetric port, where $\delta_0$ is determined by the Michelson differential phase ($\delta_0 = (1/2) \cdot \Delta \phi$) and the contrast defect ($2\delta_0^2 = c.d.$), and $\delta_1 = (\omega_m/c \cdot (l_2 - l_1)$, while $\phi_D$ contains the time delay owing to travel to the dark port from the input to the interferometer. As discussed before, $E_{bs}$ is the same as the input field $E_{in}$ in the unrecycled PNI, but amplified by the recycling field gain at the second stage of the PNI. The modulation index also needs to be multiplied by $g_r$ at the second stage. The extra field owing to back scattering shows up with amplitude

$$r'E_{bs}[i J'_0 \delta_0 + 2iJ'_1 \sin(\delta_1) \sin(\omega_m t + \phi_s + \phi_D) + 2J'_1 \delta_0 \cos(\omega_m t + \phi_s + \phi_D)] \cdot e^{i\phi_c}.$$  \hspace{1cm} (4.17)

Thus a spurious signal is obtained at the modulation frequency, equal to

$$2r'(\Delta \phi) \sin(\delta_1)|E_{bs}|^2[J'_0 J_1 \sin(\omega_m t + \phi_D) + J_0 J'_1 \sin(\omega_m t + \phi_D + \phi_s)] \cos(\phi_c)$$

$$+ 4r'\delta_0^2 |E_{bs}|^2 [J'_0 J_1 \cos(\phi_c) \cos(\omega_m t + \phi_D) + J_0 J'_1 \sin(\phi_c) \cos(\omega_m t + \phi_D + \phi_s)].$$  \hspace{1cm} (4.18)
With a big excitation of the path making up $\phi_c$, we obtain a spectrum similar to the one discussed before with respect to the input light monitoring photo-detector; Figure 4.4 shows such a spectrum obtained for the increased phase noise at the anti-symmetric port for the recycled PNI. The lower figure shows the phase coherence of the anti-symmetric signal with the signal obtained at the input light photo-detector (this quantity is the projection of one signal on the other — if one is entirely due to the other, then we obtain a value of 1, otherwise, a value less than 1 is obtained); we observe that the additional light making its way to the
anti-symmetric port can account for the increased noise up to 2 kHz. This kind of coherence was non-stationary, however, and required big excitations which would cause the scatterer to vibrate through many wavelengths. We can see some influence of this noise in the spectra of the unrecycled PNI also, as indicated in Figure 4.5.

![Michelson phase noise graph](image.png)

Figure 4.5: Influence of back scattering on the phase noise of the unrecycled PNI.

The path $\phi_c$, when disturbed by big excitations, led to signals that were coherent in both the input light and the anti-symmetric output as shown in Figure 4.4. When the excitations were less and probably via stationary sources like ambient acoustics, the modulation of $\phi_c$ could still cause phase noise, but coherence with the input light signal would not necessarily be obtained (since one is proportional to $\sin\phi_c$, and the other to a complicated superposition of $\sin\phi_c$ and $\cos\phi_c$). Thus, when $\phi_c$ excitations were small and no coherence was observed with the signal at the input light photo-detector, back scattering as a way of adding stationary phase noise to the detected signal at the anti-symmetric port could not be ruled out — in fact, addition of an extra Faraday isolator (reported by Mr. Lantz and Dr. Gonzalez) in later experiments seemed to have reduced the phase noise in the stationary spectrum shown in Figure 2.16 by a factor of about 3 from 400 Hz to a kHz.
In Figure 4.1 we show a beam that is scattered off the main path at an angle to recombine back at the dark port photo-detector. The noise owing to this beam is added to the phase noise exactly the way we have just described, with necessary adjustments of \( J_0, J_1, \tau', \text{ and } E \). If the length of the path taken by the beam is quite different from the main one, \( \phi_c \) can be dominated by frequency noise — especially, if the surface scattering back the light onto the photo-detector is not subject to large vibrations. Also, from Equation 4.18, we notice that this noise can come at the modulation frequency with a different phase than the signal we are trying to measure; hence, we can reject this noise by properly adjusting the phase of the local oscillator. We indeed observed such increased phase noise above 2 kHz, coherent with the common mode error signal, if the local oscillator was not optimally set for the anti-symmetric output.

4.4 Instrumentation noise

4.4.1 Amplitude modulation by Pockels cell

The polarization of the laser beam needs to be aligned with the birefringent axis of the electro-optic crystal for pure phase modulation — misalignment leads to amplitude modulation at that frequency. The Pockels cell that we used for the PNI had brewster entry and exit windows (a PM-25 from Gsänger) — these windows also needed alignment with the crystal. For a phase modulation index of \( \Gamma = 1 \), we measured an AM depth of \( 4 \times 10^{-6} \) at the modulation frequency.

Referring to Equation 2.8, we notice that the amplitude modulation at radio-frequency comes multiplied with \( P_i \) in every term; however, the amplitude modulation comes as RIN \( \cdot \cos(\omega_m(t - t_c)) \) which is in quadrature to the term carrying the phase signal \( (\Delta \phi) \). This helps us reject the the low frequency terms in Equation 2.8 that would have otherwise come at the modulation radio-frequency multiplied by the AM, and competed with the measured phase.

4.4.2 Noise in the differential length sensing and control loop

The photo-detector used at the dark port of the PNI produced current noise (primarily owing to the thermal noise of its real impedance, and pre-amplifier input noise) with a flat power
spectral density of $1.06 \times 10^{-11}$ amp/$\sqrt{\text{Hz}}$ — this is equivalent to the quantum noise produced by 0.35 ma of photo-current. The photo-current at the dark port during the unrecycled stage of the PNI was measured to be 0.26 ma for the November 95 spectrum and 0.66 ma for the January 96 spectrum (Figure 3.1); thus, during the unrecycled stage, the quantum noise in the photo-current measured was quite close to the dark noise in the photo-detector. In the second recycled stage of PNI, the dark port photo-current was 11 ma, producing quantum noise significantly above that owing to the photo-detector.

The coils of each of the OSEM units produced a force of 0.05 N/amp — if we used a resistor $R$ to convert a voltage to current in these coils, we could expect a Johnson noise of $\sqrt{(4kT)/R}$ amp/$\sqrt{\text{Hz}}$ which then would produce a displacement noise in the mirror of

$$x(f) = \frac{1}{(2\pi f)^2 m} \cdot \sqrt{\frac{4kT}{R}} \cdot (0.05 \text{ N/amp}) \text{ m/}\sqrt{\text{Hz}}.$$

(4.19)

Efforts were taken to make sure that the feedback control loop did not produce noise above that produced by the Johnson noise of a 2k resistor; with the displacement noise in the two Michelson mirrors added in quadrature, we obtained a phase noise power spectral density of

$$5 \times 10^{-11} : \left( \frac{100 \text{ Hz}}{f} \right)^2 \text{ radian/}\sqrt{\text{Hz}}.$$

(4.20)

which then was below the sensitivity we attempted to measure.

Though attempts were made to make the coil and magnet actuator have a constant current to force conversion at all frequencies of interest, the actuator did end up having resonances that caused displacement noise in the measured bandwidth. In Figure 4.6, we show a feature in the measured phase noise spectrum that could be attributed to this kind of resonance. The resonant system was defined by the magnet pieces and the glue with which they were attached to the mirror. As this system was thermally excited, the measured phase noise was consistent with the recoil motion of the mirror.
Figure 4.6: A feature in the phase noise measured with the recycled PNI that could be attributed to resonances in the actuators.
Chapter 5

Final Remarks

This chapter will attempt to summarize the results of this thesis and their implications. The aim of the work was to investigate the noise sources that come along with the use of light to read the phase difference of a Michelson interferometer. The phase difference is caused by the change of one arm length of the interferometer with respect to the other. The change in length is converted to an optical phase, which then gets measured as a light level at the output of the interferometer. Chapter 1 showed that the change attributed to gravitational waves can be monitored if the changes caused by other sources are kept small — thermal and seismic excitations cause the arm lengths to vary, and constitute “displacement” noise that directly competes with the signal from gravitational waves. The use of light for “readout” causes additional noise — as the measured phase may not be owing to a change in arm lengths but because of change in frequency, or the varying light level at the interferometer output may be due to the fluctuations of light intensity or because light adds its own quantum noise in its detection. Scattered light recombined with the interferometer output at the dark port will also cause spurious signals that compete with the phase to be detected. We will look at the PNI spectrum in its two stages in this chapter and discuss what we learnt about this “readout” noise. Next, we will revisit the LIGO noise estimate discussed in Chapter 1, and show what our experimental results imply.
Figure 5.1: Unrecycled Michelson interferometer spectra showing the dominant noise sources at different frequencies.
5.1 The PNI spectra

Figure 5.1 shows the different noise sources at different frequencies in the unrecycled Michelson interferometer. Above 3 kHz, the noise asymptotes to a level that is given by three noise sources added in quadrature: the quantum noise in light, the residual frequency noise, and the dark noise in the photo-detector (which refers to the thermal noise of the photo-diode and the pre-amplifier noise added in quadrature). These three noise sources, converted to an equivalent phase noise power spectral density, show different scalings with the input light power $P$ — while the frequency noise contribution is a constant, the photo-detector dark noise gets reduced as $1/P$, and the quantum noise as $1/\sqrt{P}$. The two spectra show the low and high power limits; at low powers ($I_{\text{max}}$ of about 20 ma), the photo-detector dark noise dominates while at high powers ($I_{\text{max}}$ of about 65 ma), the frequency noise contribution becomes the limiting noise.

Between 300 Hz and 1.5 kHz in the Jan 96 spectrum, and upto 3 kHz in the Nov 95 spectrum, input beam jitter, through a bilinear coupling with the differential misalignment of the Michelson mirrors, was the main contributor of phase noise. Acoustic shielding and damping of the resonances of the optical elements at the input to the interferometer helped in reducing the beam jitter above 1.5 kHz and between 300 and 500 Hz — this caused the improvement in the Jan 96 spectrum over the one of Nov 95. Parasitic interferometry, as discussed in Chapter 4, caused by back scattering of the light returned to the symmetric port, brought the phase noise up sharply below 200 Hz.

Analysis of the noise below 2 kHz detected at the dark port of the recycled PNI was more complicated because a number of noise sources were involved at the same time. The spectrum that was analyzed for this thesis, taken on Oct 10 96, did not clearly show any dominant noise source as discussed; however, subsequent work by Lantz and Gonzales improved the spectrum and elucidated which noise sources must have been at work. We show two recent and improved spectra in Figure 5.2 along with the one analyzed for the thesis. The improvement shown in the spectrum of 12/07/96 (a factor of 3 reduction in the phase noise between 200 Hz and 1 kHz) followed from a lower contrast defect (most probably from better alignment of the interferometer), and an extra Faraday isolator included in the input optics chain. At this
Figure 5.2: Two recent and improved recycled PNI spectra compared with the one analyzed for this thesis.
Figure 5.3: The PNI phase noise compared to the LIGO requirement.

stage, the spectrum showed coherence with the common mode error signal below 400 Hz. This spectrum also shows quantum limited noise from around 1.1 kHz at a level slightly lower than the spectrum of Oct 10 because of a better contrast. Thus back scattering of light must have contributed significantly to the phase noise of Oct 10 below 2 kHz. No detailed analysis of the spectrum of 12/07 has been carried out — however, apart from the resonant feature from the actuators around 1 kHz (as discussed in Chapter 4), the excess noise from 400 Hz to 1.1 kHz can be suspected to be mainly from input beam jitter. The limiting noise, ultimately, will be the contribution from residual frequency noise. The improvement in the spectrum of 12/09/96 followed from an extra 10 dB of frequency noise suppression (implemented by Lantz) from 100 Hz to 400 Hz.

5.2 Implications for LIGO

Figure 5.3 shows the sensitivity of the LIGO interferometer if its dark port noise was limited to the noise measured with the recycled PNI, in the bandwidth where "readout" noise is expected
to dominate (the spectrum of 12/09 has been used). At frequencies above 1.1 kHz, the phase noise is about $3 \times 10^{-10}$ radian$/\sqrt{\text{Hz}}$ instead of the $1 \times 10^{-10}$ required by LIGO; below 1.1 kHz, the deviation is explained by technical noise sources as described in the last section. Though not exactly according to the LIGO requirement, the phase noise measured — consistent with above 30 watts of light incident on the beam-splitter — is better than any known measurement known until now. The Garching group [6] showed a phase noise consistent with 720 milliwatts at the beam-splitter, and the 40 meter prototype — at its best sensitivity [11] — only had 150 milliwatts at the splitter. A better contrast (through the use of mirrors with better surface figure than the ones used in the PNI), or a higher modulation index will allow the high frequency spectrum to approach the LIGO goal. If higher modulation index is preferred, a photo-detector capable of handling high light powers (without saturating or becoming nonlinear) must be used — also, more power at the input should be available. LIGO is currently developing photo-detectors capable of handling high light powers (the recycled PNI dark port photo-detector had to handle somewhere between 7 to 11 ma of photo-current, close to the limits of linearity in the detector used).

LIGO has recently decided to use Nd:YAG lasers instead of the green Ar* laser used for the PNI. To achieve the required phase noise sensitivity above 1 kHz, LIGO then needs twice the input power. It seems that such high powers are indeed possible with the solid state lasers — but photo-detectors capable of handling higher light levels become imperative. Below 1 kHz, the excess noise, as we discovered, was mostly due to back scattering, residual frequency noise, and input beam jitter. The problem of back scattering will require high quality Faraday isolators with wedged surfaces. The frequency noise of Nd:YAG lasers seems to be a factor of 100 better than the Ar* gas lasers [28], hence the required frequency noise suppression will be easily obtained. The input beam direction will also be actively controlled with the wavefront sensors described before, hence the input beam jitter should not be a problem for LIGO.

In summary, the recycled PNI results are encouraging for LIGO. The required phase noise sensitivity will be obtained if — as planned — a photo-detector capable of handling high powers is developed, better Faraday isolators are used to reduce back scattered light, and a switch to the new laser is carried out so that control of frequency noise does not require an complicated servo design.
Appendix A

The Michelson Interferometer as an Optical Element

The purpose of this Appendix is to present the Michelson interferometer as an optical element with frequency dependent field reflectivity and transmission. This approach is useful because phase modulation by a Pockels cell results in a number of fields at (sideband) frequencies spaced by harmonics of the modulation frequency from the main (carrier) frequency of laser light. The Michelson interferometer then reflects and transmits these fields with its corresponding (frequency dependent) field coefficients. The measurement of power at the dark port leads to mixing of the transmitted fields, producing signal at different beat frequencies. A slightly different approach than that taken in the text is thus presented — one that easily ties in with the frequency selective nature of optical cavities.

Referring to Figure A, we assume that the field $E_i$ has been phase-modulated by a Pockels cell; if we have light $E e^{i(\omega t)}$ incident on the Pockels cell, the light that exits the Pockels cell (and is phase modulated by it) may be written as:

$$E e^{i(\omega t + \Gamma \cos(\omega_m t))} = E e^{i(\omega t)} [J_0(\Gamma) + 2 \sum_{k=1}^{\infty} (-)^k J_{2k}(\Gamma) \cos[2k\omega_m t]]$$

$$+ 2i \sum_{k=0}^{\infty} J_{2k+1}(\Gamma) \cos[(2k + 1)\omega_m t]]$$

$$= E e^{i(\omega t)} [J_0(\Gamma) + i J_1(\Gamma)(e^{i\omega_m t} + e^{-i\omega_m t})$$

$$- J_2(\Gamma)(e^{2i\omega_m t} + e^{-2i\omega_m t}) + \ldots],}$$
where we have kept terms up to twice the modulation frequency. The above expression clearly shows how various frequencies are created by phase modulation. We will index the displayed five frequencies in \( E_i \) as \( E_i^k \) where \( k \) is one of \(-2, -1, 0, 1, 2\); e.g., \( E_i^{-2} = -J_2(\Gamma) e^{-2i\omega_m(t-t_{tr})}E_i \), where \( t_{tr} \) refers to the time of transit to the beam-splitter. Note that we have omitted \( e^{i(\omega t)} \) as this is common to all fields, and drops out when the field expressions are converted to an expression for light power, the quantity that is actually measured.

Consider the reflected and transmitted fields of the Michelson interferometer — \( E^k_r \) and \( E^k_t \) respectively, as shown in Figure A. If \( k \) also indexed the frequency of the fields (i.e., \( \omega_0 = \omega \), \( \omega_{\pm} = \omega \pm \omega_m \), etc.), we may write the following relation between the fields:

\[
E^k_r = \frac{1}{2} E^k_t \left( e^{-\frac{2i\omega k l_1}{c}} + e^{-\frac{2i\omega k l_2}{c}} \right), \tag{A.1}
\]

and

\[
E^k_t = \frac{1}{2} E^k_t \left( e^{-\frac{2i\omega k l_1}{c}} - e^{-\frac{2i\omega k l_2}{c}} \right). \tag{A.2}
\]

Let us define \( \alpha_k = \frac{\omega_k l_1 + l_2}{c} \), and \( \delta_k = \frac{\omega_k (l_1 - l_2)}{c} \). Then the above equations can be recast as:

\[
E^k_r = E^k_t e^{-i\alpha_k \cos(\delta_k)}, \tag{A.3}
\]

\[
E^k_t = -i E^k_t e^{-i\alpha_k \sin(\delta_k)}. \tag{A.4}
\]
Thus the Michelson after the beam splitter can be considered to have a field reflection coefficient of \( e^{-i\alpha_k\cos(\delta_k)} \) and a field transmission coefficient of \(-ie^{-i\alpha_k\sin(\delta_k)}\). We see that the \( \phi \) in Equation 2.1 corresponds to \( 2\delta_0 \). To derive Equation 2.1 we have to observe that \( P_{out} = (E^0_i)^* E^0_i \). To obtain Equation 2.8, we have to assume that \( \delta_0 \) is small (and equal to \((\Delta\phi)/2\)) and hence \( \delta_{\pm 1,2} \) are dominated by terms involving \( \omega_m \) and the asymmetry. We next have to add the \( k = 0 \) and \( k = 1 \) fields and multiply by the complex conjugate of the sum; we would then notice that Equation 2.12 is given by

\[
2P_1J_0(\Gamma)J_1(\Gamma)\sin[\omega_m t_{\Delta}](\Delta\phi). \tag{A.5}
\]

For \( \Gamma \ll 1 \), \( J_0(\Gamma) \approx 1 \) and \( J_1(\Gamma) \approx \frac{\Gamma}{2} \); substituting these values, we retrieve the form of Equation 2.12.
Appendix B

The Fabry Perot Cavity: Optical Parameters and Resonance

We will derive expressions for optical cavity parameters in this appendix: the reflectivity, the transmission, and the gain in field strength inside the cavity compared to the incident external field that illuminates it. We will introduce the concept of resonance, and show how these expressions simplify in resonant low loss cavities. Finally, we will discuss the Pound-Drever technique for holding optical cavities on resonance.

We will use the geometry of the optical cavity as shown in Figure B.1 for our discussion. Examining the left ‘input’ mirror and choosing the sign for the field reflectivity of the interface between air and mirror coating to be negative, the incident fields, $E_x$ and $E_b$, and the ones leaving it, $E_r$ and $E_f$, must obey:

\begin{align}
E_r &= t_1 E_b + r_1 E_x, \\
E_f &= t_1 E_x - r_1 E_b. 
\end{align}

(B.1)

Here $t_1$ and $r_1$ are the transmission and reflectivity of the mirror for the electric field. For mirrors with no loss, $|r|^2 + |t|^2 = 1$. The right hand ‘back’ mirror in Figure B.1 similarly stipulates:

\begin{align}
E_r' &= t_2 E_f' + r_2 E_x' \\
E_b' &= t_2 E_x' - r_2 E_f'.
\end{align}

(B.2)
Figure B.1: A Fabry-Perot cavity and the relationship among its fields

For this formulation, let us assume that $E'_x$ is non-existent. Then Equations B.2 simplify to $E'_b = -r_2E'_f$. For the curved back mirror, $r_2$ is a complex function in $x$ and $y$, capturing the change in amplitude and phase of the field reflecting off the mirror "surface.

The field $E'_f$, as shown in Figure B.1, is $E_f$ after propagation in space over the length $L$ of the cavity. This may be expressed, assuming paraxial propagation, as:¹

$$E'_f(x,y) = \text{FT}^{-1}_{[xy]}\left[\exp(-ikL + i\frac{(k_x^2 + k_y^2)L}{2k})\right] \times \text{FT}_{[k_xk_y]}[E_f(x,y)] \quad (B.3)$$

In the above equation, $\lambda$ is the wavelength of light, $\text{FT}_{[k_xk_y]}[\mathcal{E}]$ represents the 2-dimensional Fourier Transform of the field $\mathcal{E}$ resulting in a function in $k_x$ and $k_y$, and $k = 2\pi/\lambda$. We notice that free space propagation mainly involves the change in phase of the Fourier amplitudes at the different spatial frequencies $(k_x, k_y)$.

We will represent free space propagation over a length $L$ by the action of an operator $K(z = L)$ or simply $K$, $E'_f = KE_f$. It should be clear from Figure B.1 that $E_b$ is related to $E'_b$ in exactly the same way — i.e. via a free space propagation over distance $L$: $E_b = KE'_b$. Since $E'_b = -r_2E'_f$, we have $E_b = -Kr_2KE_f$. Now Equations B.1 give us

$$E_f = t_1E_x + r_1Kr_2KE_f. \quad (B.4)$$
This is the steady-state field equation. Now \( U \equiv (r_1/|r_1|)K(r_2/|r_2|)K \) is a unitary operator with eigen-modes and eigen-values which, if the field \( E_f \) is resolved into, simplifies the above problem considerably. If the mirrors are spherical, then the eigen-modes are the Hermite-Gaussian modes which find extensive description in all laser textbooks [29]. In that case, the eigen-value of the operator \( U \) happens to be \( e^{-2ikL+i\psi_G} \), where \( \psi_G \) refers to the Guoy phase that depends on the particular mode chosen. Writing \( \psi \equiv -2kL + \psi_G \) and assuming that \( r_1 \) and \( r_2 \) stand for the amplitudes of the corresponding quantities, we thus can solve for \( E_f \):

\[
E_f = \frac{t_1 E_x}{1 - r_1 r_2 e^{i\psi}}. \tag{B.5}
\]

The reflected and transmitted fields are \( E_r \) and \( E_r' \) respectively and these are — from Equations B.1 and B.2 before —

\[
E_r = \frac{r_1 - (r_1^2 + t_1^2) \cdot r_2 e^{i\psi}}{1 - r_1 r_2 e^{i\psi}} E_x, \tag{B.6}
\]

and

\[
E_r' = \frac{t_1 t_2 e^{i\psi/2}}{1 - r_1 r_2 e^{i\psi}} E_x. \tag{B.7}
\]

By dividing out the expressions (B.6) and (B.7) by \( E_x \), we get the field reflectivity and transmittance for an optical cavity. By dividing out \( E_x \) in Equation (B.5), we obtain the field gain inside the cavity. The denominators of these expressions depend on the values of \( r_1 \) and \( r_2 \) — if these values are close to 1, then for \( \psi \) equal to a multiple of \( 2\pi \), the denominators get very small while the expressions for (B.5), (B.6), and (B.7) become quite large, a condition termed resonance.

In \( \psi \), we capture the source of change in reflectivity, transmittance, and (internal field) gain with the change in the length of the optical cavity, or the change in frequency of the laser. For example, if the optical cavity is resonant, every change of \( \lambda/2 \) in the length of the cavity or, equivalently, every change of \( c/(2L) \) (the full spectral range, or fsr) in the frequency of laser will make it resonant again. Let us write the light intensity transmission of an optical cavity in terms of the maximum transmission \( T_{\text{max}} = (t_1^2 t_2^2)/(1 - r_1 r_2)^2 \) and the detuning of \( \psi \) from resonance:

\[
T = \frac{T_{\text{max}}}{1 + (2F/\pi)^2 \cdot \sin^2(\psi/2)} \tag{B.8}
\]
The expression, $\mathcal{F}$ is thus defined as,

$$\mathcal{F} = \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2}. \quad (B.9)$$

What finesse signifies is easy to understand as the value it takes becomes much larger than 1 — in that case the denominator of Equation B.8 reaches a value of 2 for very small values of $\sin(\psi/2) \approx \psi/2$. If we plotted the intensity transmission as a function of the frequency of light, we would see that it drops from a maximum to half that value when the frequency changes $\Delta f$ — half of the Full Width at Half Maximum (FWHM) value. From the relationship between $\psi$ and the frequency of light, we can easily show that:

$$\mathcal{F} = \frac{\text{far}}{2\Delta f}. \quad (B.10)$$

Thus, given a fixed length of the cavity, the higher the finesse, the narrower the line-width. $\Delta f$ is also referred to as the pole frequency of the cavity in the same approximation (i.e. when $\mathcal{F} \gg 1$), and the storage time of the cavity is defined as $\tau_s = 1/(4\pi \Delta f)$.

When we assume high finesse optical cavities at resonance, a number of simplifications are possible in the formulas given so far. The smaller the fraction of power lost in one round trip of light inside the cavity, the more accurate are these simplified formulas. We assume the delta notation of Siegman [30] in somewhat different way: $T_1 \equiv t_1^2 \equiv \delta_1$, $T_2 \equiv t_2^2 \equiv \delta_2$, and we also assume a fraction of power scattered off the beam inside the cavity, $\delta_0$, in any kind of optical pick-off, etc. It is then easily shown that, our expressions for (B.5), (B.6), and (B.7) become:

$$\frac{E_r}{E_x} = \frac{\delta_2 + \delta_0 - \delta_1}{\delta_2 + \delta_0 + \delta_1} \quad (B.11)$$

$$\left|\frac{E_I}{E_x}\right| = \frac{2\sqrt{\delta_1}}{\delta_2 + \delta_0 + \delta_1} \quad (B.12)$$

$$\left|\frac{E'_r}{E_x}\right| = \frac{2\sqrt{\delta_1 \delta_2}}{\delta_2 + \delta_0 + \delta_1} \quad (B.13)$$

There is distinction made between the input mirror indexed 1 and facing the incident external light, and the output mirror indexed 2 which transmits the light inside the cavity. If $\delta_2 + \delta_0 \gg \delta_1$, the cavity is called undercoupled because the internal field strength is (most probably) smaller than that of the incident field strength, as is obvious from Equation B.12.
If \( \delta_2 + \delta_0 = \delta_1 \), we have a critically coupled optical cavity, since the reflectivity drops to 0 while transmission is maximized. Finally, when \( \delta_2 + \delta_0 \ll \delta_1 \), we have an overcoupled cavity where the field reflectivity becomes negative but reaches unity again and the internal field builds up to high levels (the power gain = \( 4/\delta_1 \)). It is clear that the formulas for finesse and storage time also simplify accordingly.

The Pound Drever Scheme of "locking" optical cavities on resonance will now be briefly introduced. The optical layout for the implementation of this scheme appears in Figure B.2.

A phase modulator is used to modulate light before illuminating the optical cavity. As derived before (Equation A.1), phase modulated light at the input of the optical cavity may be written as:

\[
E e^{i(\omega t + \Gamma \cos(\omega_m t) + \phi)} = E e^{i(\omega t + \phi)} [J_0(\Gamma) + 2iJ_1(\Gamma)\cos(\omega_m t) + \ldots], \tag{B.14}
\]

where we have kept only the terms first order in \( \Gamma \) (small \( \Gamma \) approximation). Using the expression that we derived for the reflected field (Equation B.6), and assuming the delta notation can be used, the (field) reflection coefficient of an optical cavity on resonance varies with small changes of \( \psi \) as

\[
r = \frac{\delta_0 + \delta_2 - \delta_1}{\delta_0 + \delta_2 + \delta_1} - \frac{4\delta_1}{(\delta_0 + \delta_2 + \delta_1)^2} (i\psi), \tag{B.15}
\]
or simply, \( r = r_R + ir_{Im} \psi \). The frequency \( \omega_m \) is selected to be a radio-frequency outside the line FWHM of the optical cavity so that — if the carrier is resonant — the side band frequencies are not. Thus the field reflected off the optical cavity may be written as,

\[
E e^{i(\omega t + \psi)} [r_R J_0(\Gamma) + i(2J_1(\Gamma) \cos(\omega_m t) + r_{Im} J_0 \psi)].
\]  
(B.16)

A photo-detector as shown in Figure B.2 is used to look at the intensity of the reflected light; thus, taking the amplitude square of the field expression (B.16), we obtain

\[
|E|^2 [(r_R J_0)^2 + 2J_1^2 + 2J_1^2 \cos(2\omega_m t) + 4J_1 J_0 r_{Im} \cos(\omega_m t) \psi].
\]  
(B.17)

We notice that the term at radio-frequency \( \omega_m \) is modulated by \( \psi \) that can be extracted by a down conversion process as explained in the text before with respect to a Michelson interferometer. The slowly varying terms in the above terms give rise to shot noise — thus, given the calibration \( 4E^2 J_0 J_1 r_{Im} \), we can find out the fundamental noise limited detection sensitivity of deviation \( \psi \) from resonance.
Appendix C

Feedback Control Systems

It is hard to describe feedback control systems with all their subtleties in a few pages. The purpose of this short description will therefore be to introduce the jargon that goes with these systems since it appears sprinkled through the text of the thesis.

![Block diagram of a Feedback Control System](image)

Figure C.1: A Block diagram of a Feedback Control System

The purpose of the feedback control systems used in our work was primarily to hold a interferometric system at some desired operating point — e.g., the Michelson interferometer at a dark fringe, or a Fabry-Perot cavity on resonance. Let us take an interferometric system therefore — the Michelson interferometer for example — to illustrate the correspondence between what happens in the interferometer, and where it fits in the block diagram in Figure
C.1. The mirrors, and the beam-splitter of the interferometer (refer Figure 2.3) are disturbed from their positions by seismic activity and the resulting displacement noise, labeled “External Disturbance”, then enters a summing junction indicated by $\Sigma$ on the top left corner of the block diagram. The “Plant” is our Michelson interferometer which converts the length disturbance into a phase difference, and then into a power fluctuation at the dark port of the interferometer (details of this process have already been presented in Chapter 2). The photo-detector at the dark port is our “Sensor” which picks-up the power fluctuation and converts it into a measured voltage. We can decide to hold the Michelson interferometer at a dark fringe or any fraction of the bright fringe; this is set by an external input at the summing junction, $\Sigma$, in the bottom right corner of Figure C.1. Finally, the difference between our set input and the actual state of interference (the error signal) is amplified and filtered by additional electronics forming the “Feedback” path to an actuator — where actuator refers to a device that converts an electric signal to physical motion, like the coils that exert force on the little magnets glued on the mirrors in our actual experiment. The actuator thus pushes one of the mirrors to counter the phase difference caused by the “External Disturbance”. We now arrive at the summing junction where we started. The signal that feeds the actuator is known as the control signal.

The different boxes, $P(s)$, $S(s)$, $F(s)$, and $A(s)$, represent the filtering and change of units that happen at each stage described in the paragraph above. Let us define $G(s) = P(s) \cdot S(s) \cdot F(s) \cdot A(s)$. Assume that the seismic disturbance correspond to a length change of $l_d$, which, through the plant and sensor, appears as voltage, $v_d$. If our set voltage is $v_s$, we can show easily that the error signal monitor will show:

$$\text{Error monitor} = \frac{v_d - v_s}{1 + G}, \quad \text{(C.1)}$$

while the output of the Actuator $A(s)$ will be $G \cdot l_d/(1 + G)$ and the control monitor will read:

$$\text{Control monitor} = \frac{G(l_d/A)}{1 + G}, \quad \text{(C.2)}$$

were $l_d/A$ is the measure of the displacement in terms of the actuator's electrical input. From the above two equations we notice that higher the Gain $G$, the less is the error and more faithfully does the actuator follow the disturbance. However, it is impossible to have high gain at all frequencies — mechanical and electronic constraints roll off the gain at high
frequencies — thus $G$ equals unity at some frequency, the so called unity gain frequency (u.g.f). This frequency then gives us a measure of frequencies (bandwidth) over which there is active feedback (servo). It is desirable that $G$ not have a phase of $180^\circ$ at this frequency — the vanishing denominator of Equation C.1 leads to undesirable instability which often manifests itself in (unity gain) oscillation.
Bibliography


