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A Teaching Note on “Real Options”

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Introduction

In Spring 2010 I taught—for the first time—an undergraduate required subject in *project evaluation* (1.011). In the course of that teaching, I developed a teaching note on “real options” as part of a unit about how one deals with uncertainty.

This teaching note tries to illustrate how one values *flexibility* in engineering system design. The concept, while conceptually straight forward, turns out to be anything but when one tries to implement it, even for relatively simple cases. The case here deals with a bridge builder making a decision at $t=0$. He/she thinks at $t=0$ about providing additional but currently unneeded strength to a bridge to allow subsequent capacity expansions of that bridge, if needed. The idea is that actually building that additional capacity at $t=25$ would be at lower cost than would have been the case if the additional strength hadn't been in place at $t=0$. This example should give students some sense of what we mean by flexibility in engineering system design and how one would value it, depending on the environment in which decisions were made and the uncertainties about future needs.

This teaching note would be of interest to the reader who is concerned with teaching these kinds of concepts to undergraduates, and may be of value to those who teach introductory graduate classes as well. Any comments on the substance, content, clarity, and value of the approach would be greatly appreciated.

Real Options—A Simple Illustration

J. Sussman

April 28, 2010

The engineering life is uncertain. We often need to make decisions now that we hope will be effective some years from now. One approach to this conundrum is to design “flexibility” into our projects, hedging against future uncertainties. This sounds reasonable and it is, but it is not “free”. Including flexibility in our design can be costly and if it turns out not to be needed in the future, it is money down the drain. So how do we know when to design in flexibility and when not to? Intuitively, the more uncertain we are about the future, the more the chances are that we will need flexibility. “Real Options Analysis” (ROA) is a method we can use to put a value on flexibility and decide whether or not we should spend the resources now to hedge against future uncertainties.

Let’s use a simple engineering example to illustrate these ideas.

In 2010, we have three choices for building our bridge.

A) We can build a bridge with one deck that costs CA.

B) We can build a bridge with two decks that costs CB

C) Or we can build a bridge with one deck but add structural strength such that if we decide to add a second deck, we can do so more economically than we could in case A. The cost here is CC.

So $CB > CC > CA$

To make our lives simple, let’s suppose we can decide in 2030 and **only then**—20 years hence--- to add a second deck if we need it.

If in 2030, we decide to add a second deck, here are the costs we would incur at that time.

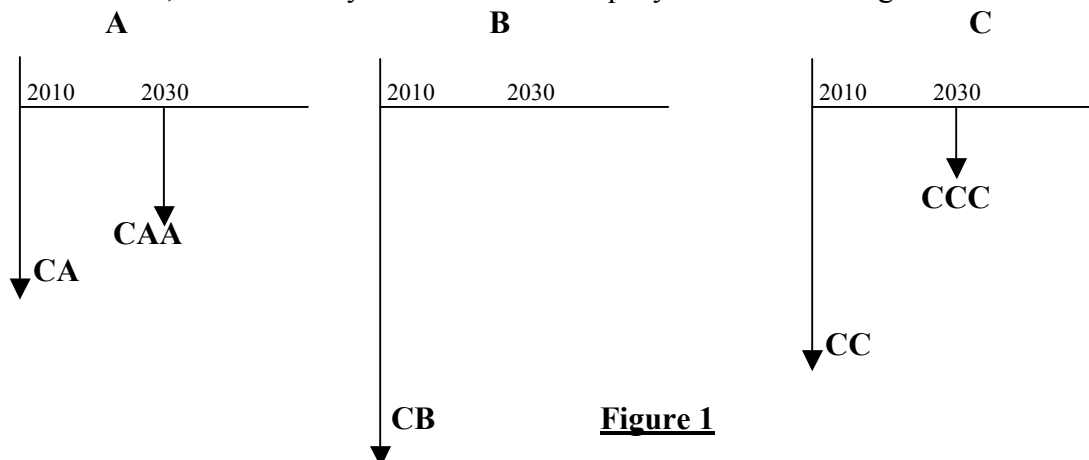
A) CAA

B) Zero (since we already have two decks in this case)

C) CCC

We would expect that $CAA > CCC$

Here are the cash flow diagrams for the three possibilities assuming we decide in 2030 to build the second deck, either initially or in 2030. To simplify our lives we’ll ignore maintenance for now.



So if we select a discount rate r we can calculate the NPV in the three cases

$$NPVA = -CA - CAA / (1+r)^{20}$$

$$NPVB = -CB$$

$$NPVC = -CC - CCC / (1+r)^{20}$$

So are we done? Can we simply calculate the three NPVs, choose the best one and call it a day? I said we were going to simplify but not quite this simple! Remember for these NPVs, we have assumed we *were* going to build a second deck and perhaps in reality we will decide in 2030 we don't need it. *Also, we haven't accounted for the different level of service provided by the bridge to travelers in the various cases.* To deal with this, we have to add some further ideas.

Suppose we assume LOS in any given year is inversely proportional to the travel time (tt).

$$LOS = 1/tt$$

Also, LOS is a function of the volume (VOL) of traffic. So as volume grows, congestion begins to develop and LOS deteriorates. Here is the characteristic relationship between LOS and VOL.

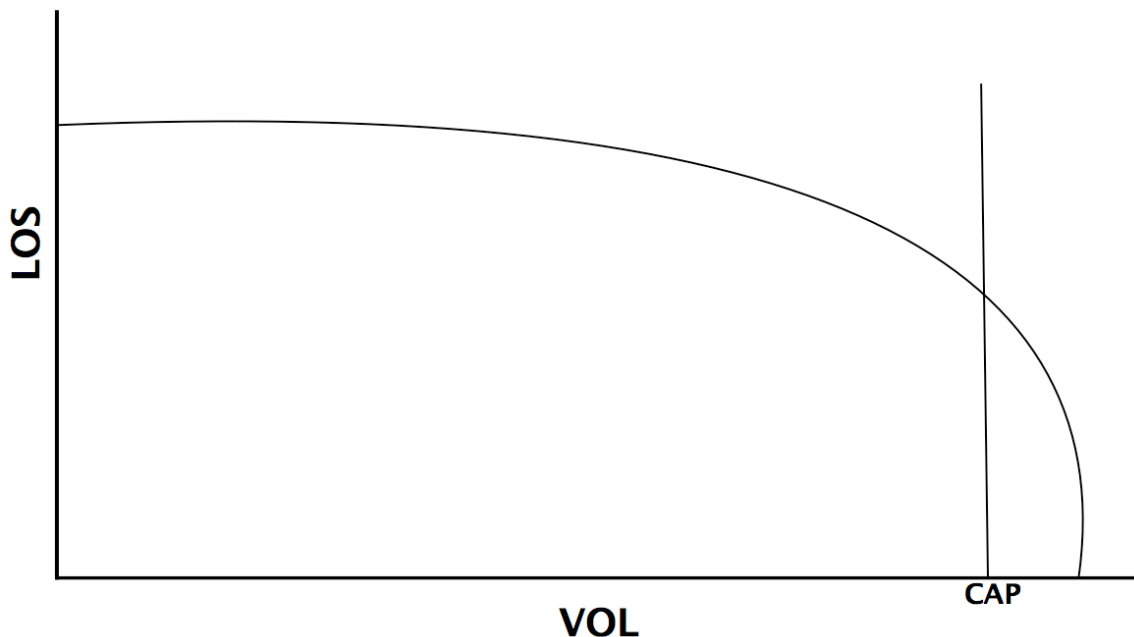


Figure 2

This is the characteristic hockey stick shape well known to transportation specialists. As VOL approaches the capacity (CAP), LOS deteriorates sharply. Going from a VOL of 80% of CAP to 90% of CAP can have a large detrimental effect on LOS.

Now let's consider uncertainty. We know that VOL will grow over time from 2010 through 2030 – when we have a 1-time chance to add a second deck – and then through 2050 when (let's assume for

simplicity) we decide to decommission the bridge. Let's assume we have two possible traffic growth curves as shown, low growth with probability p and high growth with probability $(1-p)$ as shown.

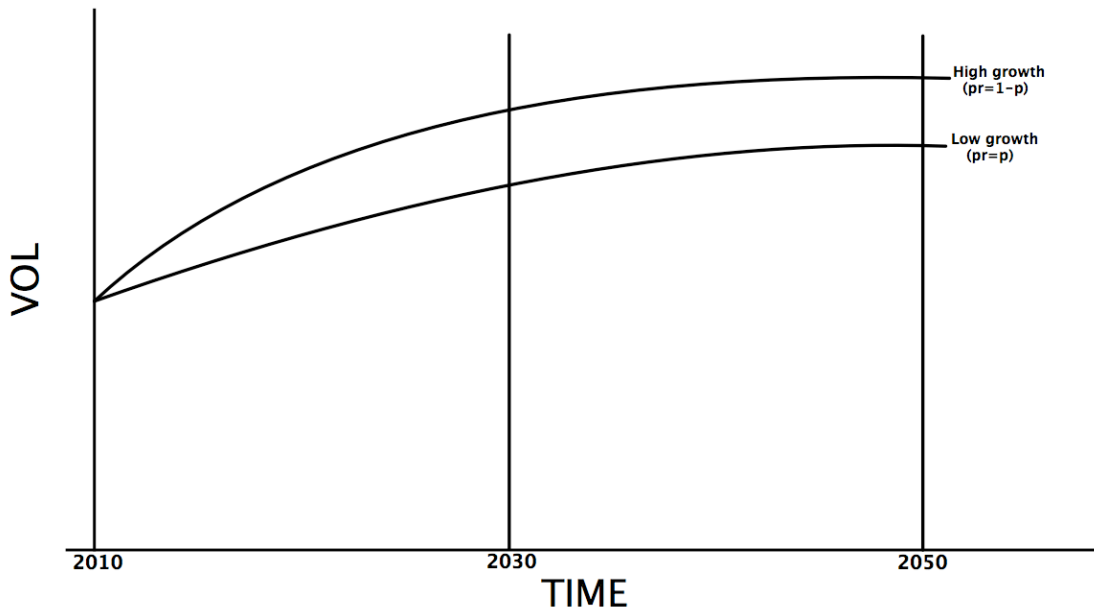
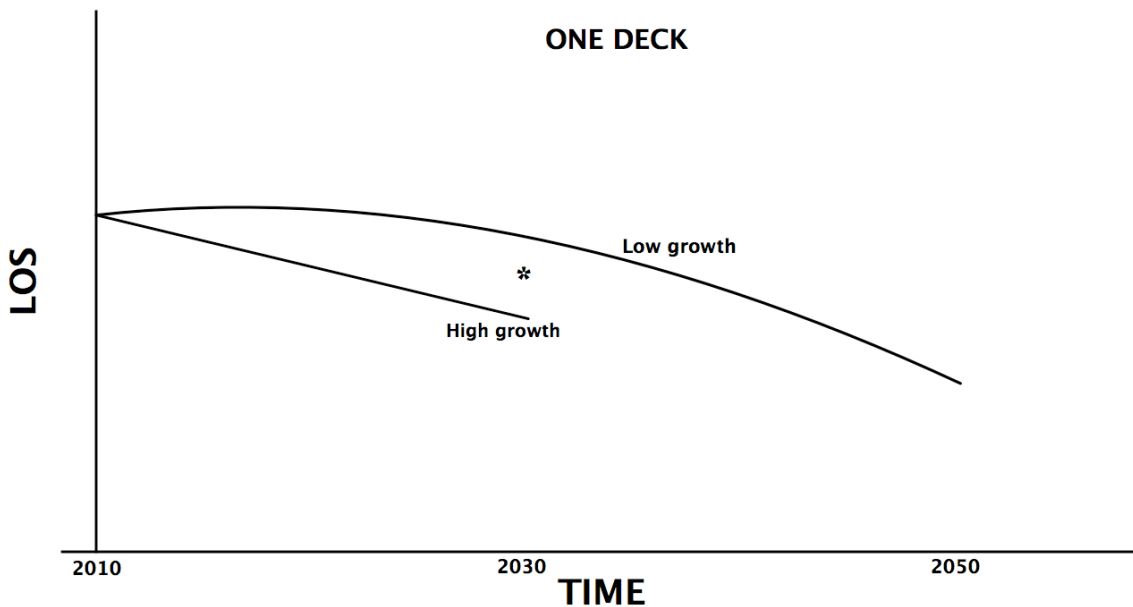


Figure 3

Now how would we make the decision as to whether to add the second deck in 2030. Presumably we would look at the LOS in 2030 –the only time we are allowed to add a second deck -- and decide whether we need more capacity. So we could have a trigger point LOS that leads us to our decision. Let's suppose that under the low growth scenario, we decide we don't need the second deck and under the high growth scenario, we do. This is illustrated as shown in Figure 4.

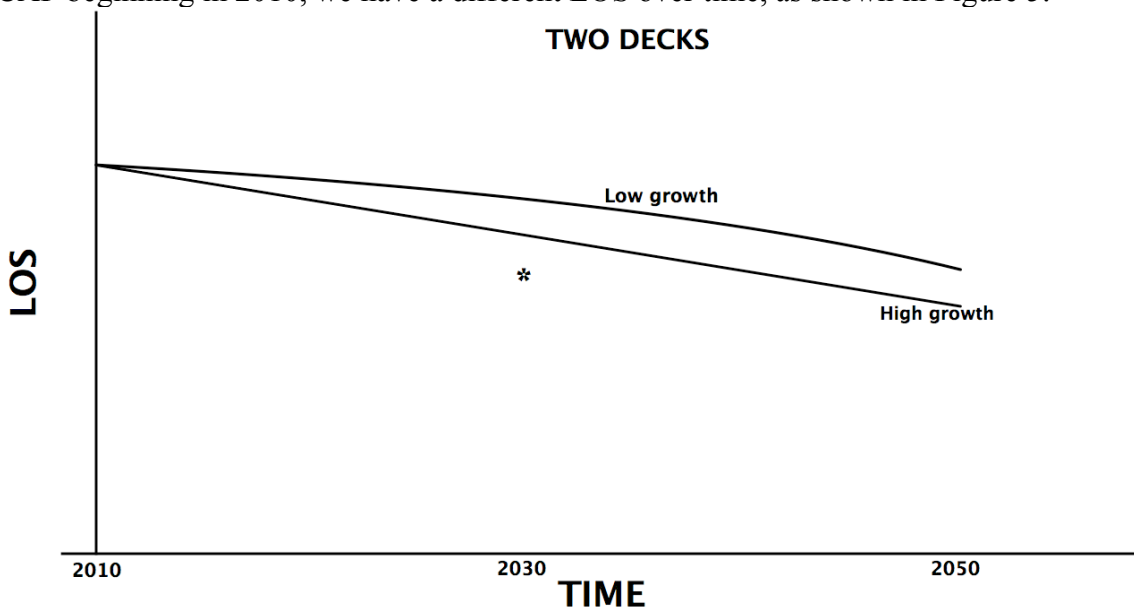


* Trigger LOS (permitted to build second deck *only* in 2030)

Figure 4

We don't know *yet* what the LOS in the high growth scenario is after 2030.

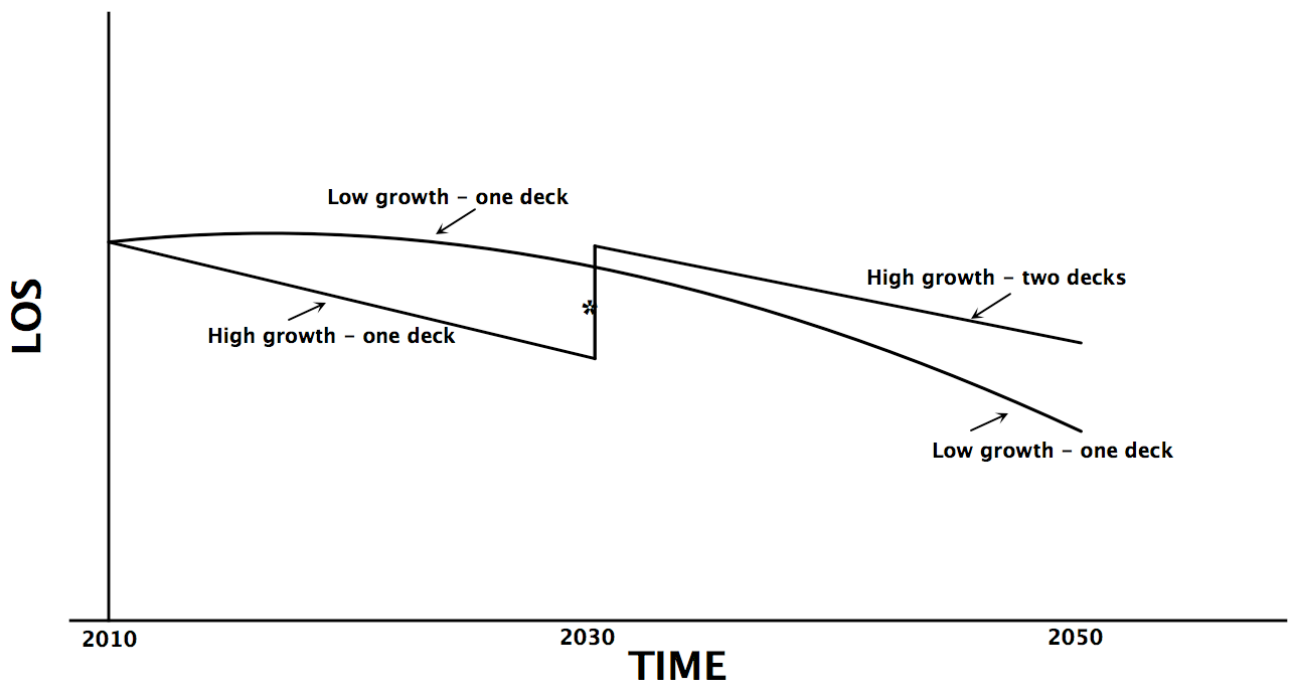
So we need to show the LOS for B when we simply built two decks in 2010. Since we have more CAP beginning in 2010, we have a different LOS over time, as shown in Figure 5.



* Trigger LOS (permitted to build second deck *only* in 2030- although in this case, we have the second deck already)

Figure 5

So now we can complete Figure 4. Under the high growth scenario from 2030 on, we have two decks. So LOS from 2030 on reflects that as shown in Figure 6.



* Trigger LOS (permitted to build second deck *only* in 2030)

Figure 6

Now we are ready to calculate the NPVs of the three cases, under *both* the low growth and high growth scenarios. We incorporate the LOSs by using the NPV provided by each of the cases by considering case B in which we have two decks from 2010 forward as the “base case”. So two decks from 2010 may be “overbuilding” but we will still get some value through improved LOS anyway. And costs due to a poorer LOS because you built only one deck initially must be considered.

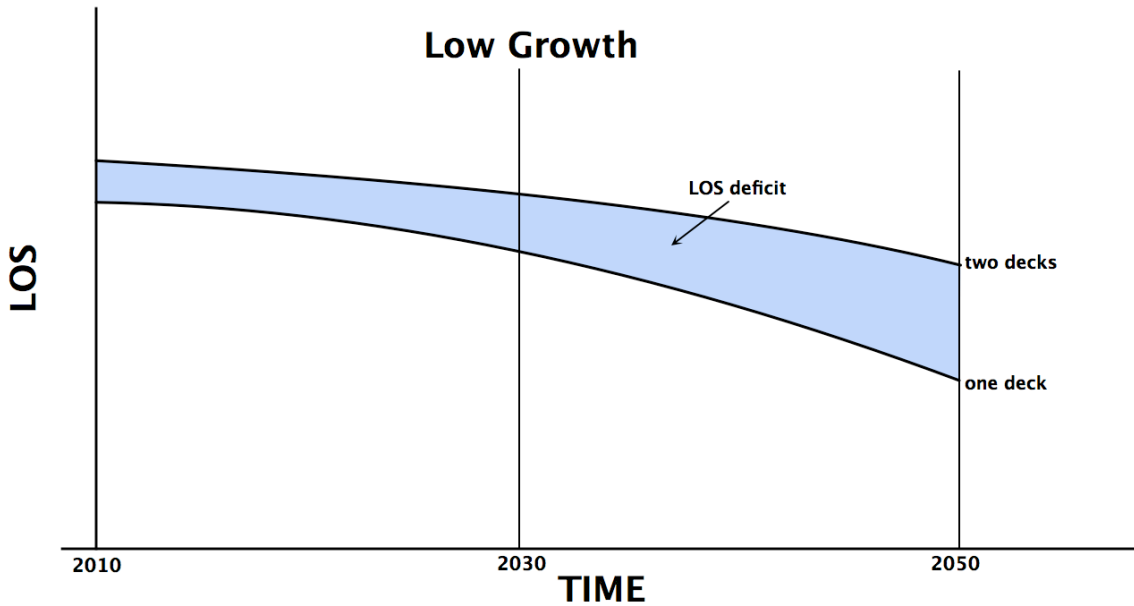
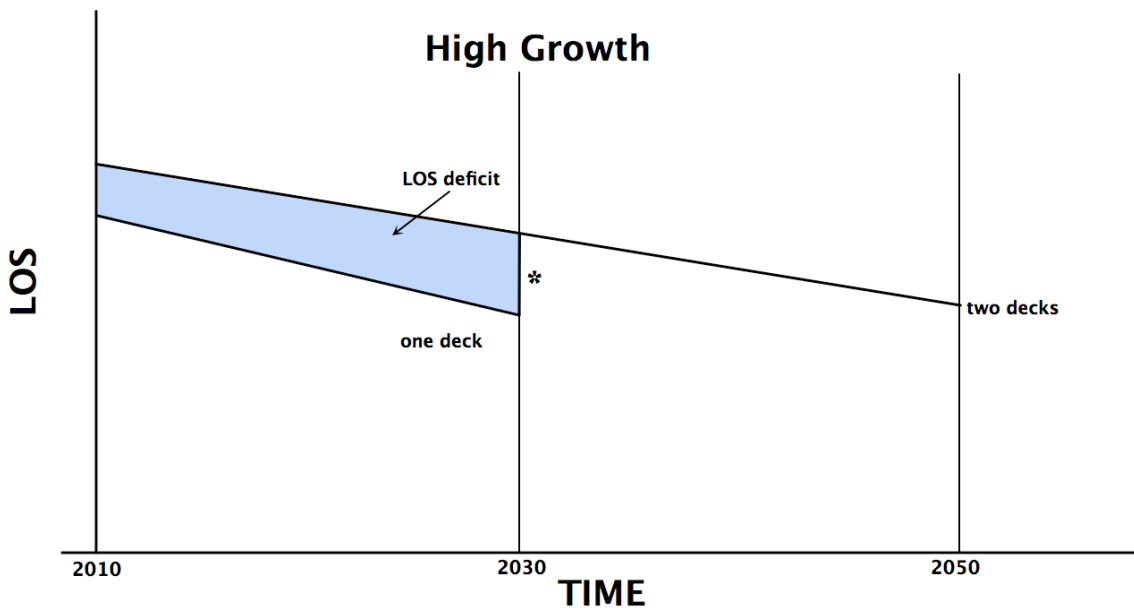


Figure 7

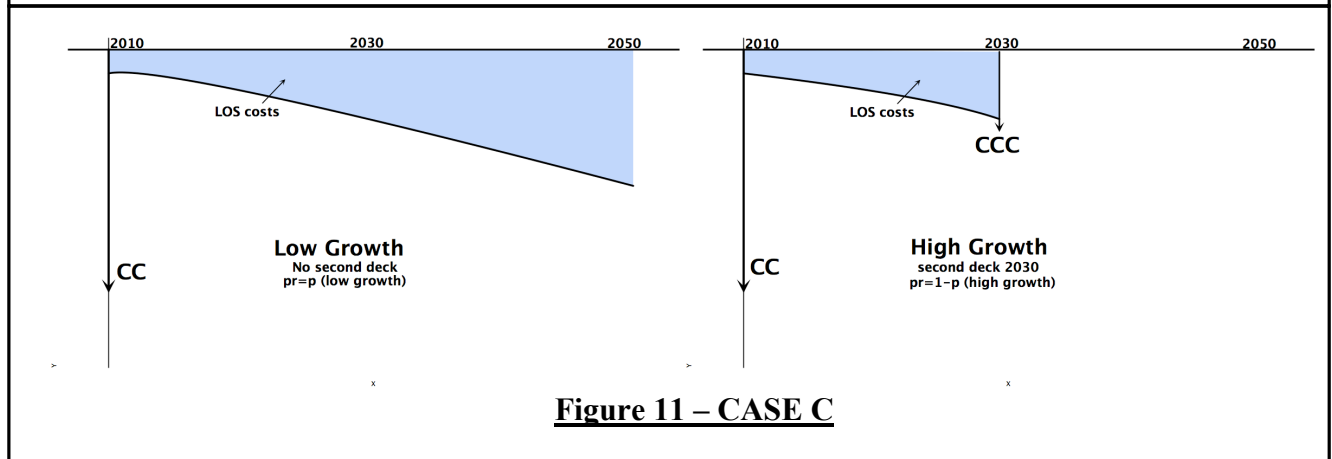
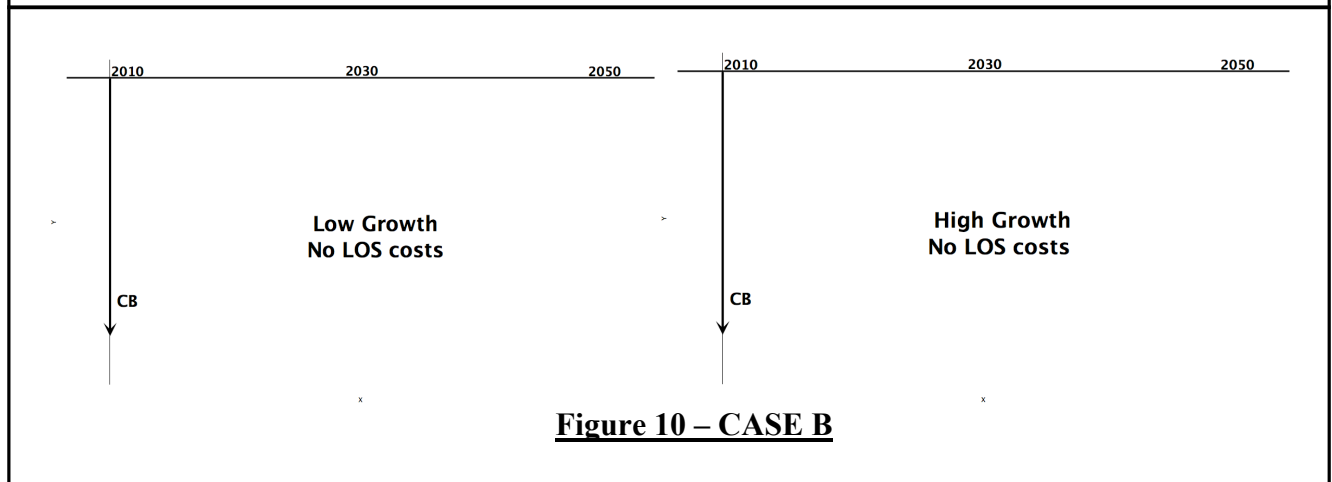
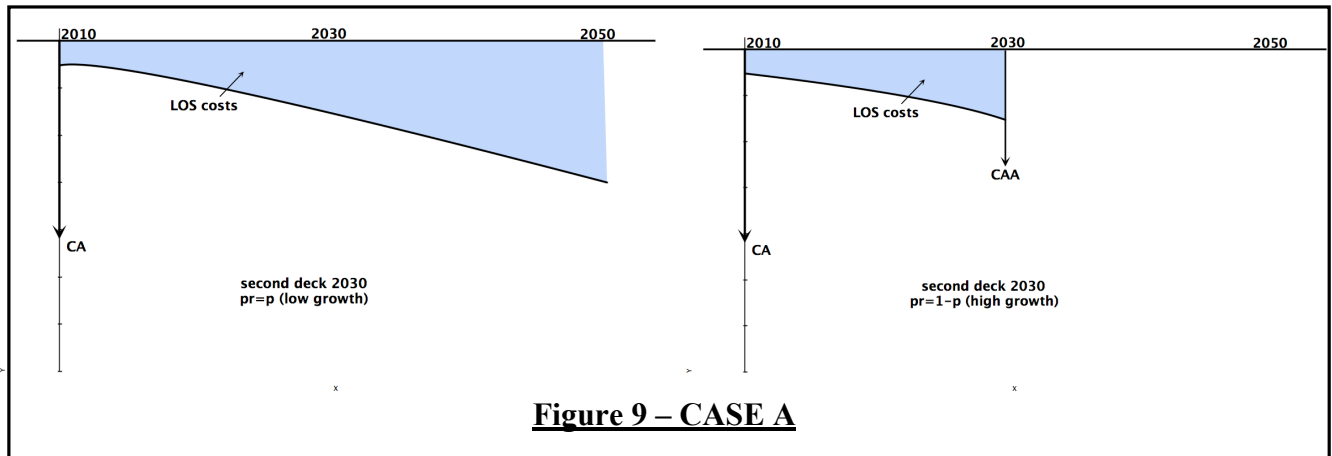


N.B. In the high growth scenario, when we start in 2010 with one deck, in 2030 we build a second deck

Figure 8

So we have an “LOS deficit” relative to the two deck case for both low growth and high growth. In the case of high growth, the deficit goes only until 2030, because we then build a second deck, eliminating the deficit. In the low growth case, the deficit extends until 2050 because we never build the second deck.

Now we re-plot Figure 1 but this time including LOS costs which are calculated from the LOS deficits by choosing a value of time and weighing it by the volumes in each year for which there is a deficit.



Now *finally* the picture emerges.

We can build two decks in 2010 at a higher capital cost but have no LOS costs throughout the forty years for either the low or high traffic growth case because it is the base case. (Case B)

Or we can build one deck with *no* provision for a second deck. In this case, you have LOS costs throughout the forty-year period in the low growth case ($pr=p$) or LOS costs for the first twenty years only in the high growth case ($pr=1-p$). You incur construction costs of CC in 2030 to build the second deck. (Case A)

Or we can spend more money in 2010 to create structural strength *in case* we want to build a second deck in 2030. In the low growth case that additional cost (CC-CA) is “wasted” since we never build the second deck. But, in the high growth case, we do build the second deck, and our costs in 2030 are CCC which is less than CAA. The LOS costs in A and C are identical. You are trading off spending an additional (CC-CA) in 2010 for spending (CAA-CCC) less in 2030. But again, recognize we have “wasted” (CC-CA) in the low growth scenario. (Case C)

So what should we do? This depends on the probability of low vs. high growth (p vs. $[1-p]$) and the costs associated with cases A, B and C: CA, CB, CC, CAA and CCC); *and* the LOS costs in each of the scenarios; and of course we have to compute the NPV of all these costs, so we need a discount rate r to bring them all back to 2010.

And as if that wasn't enough, for each of A, B, and C we will have two NPVs, one with $pr=p$ (low growth) and one with $pr=[1-p]$ (high growth). So depending on your feelings about risk, you may want to convert these to utilities—but let's assume that away. We have enough here!

So we could do the numbers and decide whether A, B or C is our best strategy. If p is close to 1 -- we are almost sure about low growth—we will likely choose A. On the other hand if p is close to zero -- we are almost sure about high growth -- then we would prefer C over A, and possibly even B could come into play. So you may want to choose some reasonable numbers and see what happens.

The difference [CC-CA] is the price of the **real option**. We are spending money now (in 2010) to have some flexibility in the future, in this case, through lower construction costs [CAA-CCC] in 2030 if we build a second deck. We are buying an option to be able to have lower construction costs in 2030 (CCC, rather than CAA) *if* we need the second deck.

A Partial Solution

Solving part of this problem may be instructive. Let us compare A with C, ignoring B for the moment (so no two decks in 2010).

First, case A:

$$E(NPV)A = p*[CA+LOS40] + (1-p)*[CA+CAA/(1+r)^{20}+LOS20]$$

Where LOS40 is the PV of the LOS costs over the 40 year span) and LOS20 is the PV of the LOS costs for the first 20 years (since we build the second deck and there are no LOS losses for years 21-40.

Next, case C:

$$E(NPV)C = p*[CC+LOS40] + (1-p)*[CC+CCC/(1+r)^20+LOS20]$$

Then calculate the difference in E(NPV)A and E(NPV)C

$$E(NPV)A - E(NPV)C = p*[CA-CC] + (1-p)*[CA-CC+(CAA-CCC)/(1+r)^20] = \\ [CA-CC] + (1-p)*[(CAA-CCC)/(1+r)^20]$$

Note that all the LOS40 and LOS20 terms fall out when we only compare A and C.

Suppose $p=1$, which means we are sure of low growth. So if $CA = -100$ and $CC = -105$, which means it costs us an extra 5 to build in structural strength in case you want to build a second deck. But under low growth, we don't do so, so as we expect, the better case is A. Don't buy the real option.

Suppose $p=0$, which means we are sure of high growth. Then

$$E(NPV)A - E(NPV)C = [CA-CC] + [(CAA-CCC)/(1+r)^20]$$

So, plug in some numbers and see what happens. You decide to buy the real option (or not) based on the relative values of CA, CC, CAA, CCC and r.

The more interesting cases are when p is a value other than 0 or 1.

A Full Solution

Next we consider all three alternatives, A, B and C:

$$E(NPV)A = p*[CA+LOS40] + (1-p)*[CA+CAA/(1+r)^20+LOS20], \text{ as before}$$

$E(NPV)B = p*[CB] + (1-p)[CB]$ which is simply CB. Since two decks is the LOS base case, there are no LOS costs.

$$E(NPV)C = p*[CC+LOS40] + (1-p)*[CC+CCC/(1+r)^20+LOS20], \text{ as before}$$

So again, we can run the numbers for various cases, and can see when the "real option" ($CC-CA$) is worth buying. We leave this as an exercise for the student.

Note that for the full solution, the LOS20 and LOS40 costs do come into play

Concluding Comments

This teaching note dealing with “real options” was inspired by the 25th of April Bridge in Lisbon, Portugal but was substantially simplified for pedagogical reasons—but the analysis is by no means simple!

My goal is that colleagues will find some value in this teaching note and will use them in their own classes.