A STATIC EVALUATION OF SOIL PLUG BEHAVIOR WITH APPLICATION TO THE PILE PLUGGING PROBLEM

by

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Submitted to the Department of Civil Engineering in partial fulfillment of the requirements for the degree of Doctor of Science in Civil Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Dept. of Civil Engineering, February 21, 1989

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Prof. Robert V. Whitman, Thesis Supervisor

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Departmental Committee on Graduate Students
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ABSTRACT

During the initial stage of installation of open pipe piles, soil enters the pile at a rate equal to the pile penetration. As penetration continues, the inner soil cylinder may develop sufficient frictional resistance to prevent further soil intrusion, causing the pile to become "plugged". The open ended pile then assumes the penetration characteristics of a closed ended pile. The mode of pile penetration significantly alters the soil–pile interaction during and after installation. This affects the ultimate static bearing capacity (mainly in granular materials), the time–dependent pile capacity (in clays) and the dynamic behavior and analysis of the piles.

Following analyses that demonstrate the effects of pile plugging, a review and interpretation of existing data is undertaken, utilizing data from soil samplers, model pipe piles and full scale piles.

A simplified 'silo approach' is then used as a first tool in the static evaluation of the soil plug behavior. Interpretation of the analysis in light of experimental data provides an explanation for some aspects of the plugging phenomenon. However, the 'silo approach' method does not account for the complexity of the soil plug–pile interaction, resulting in inconsistent predictions when compared to experimental data.

A better understanding of the phenomenon is gained by examining the micro behavior of granular material. Based on the micro mechanics of an assembly of particles, two original models are suggested:

1. The S.G. model – describing the mechanics of the granular material/interface shear resistance, which can be applied to the general shear mechanism of granular media.

2. The soil arching model – utilizing the arching phenomenon to describe the resistance of granular media to loading, explaining the build–up of the plug/wall friction.

The proposed method of analysis provides a more comprehensive elucidation of the plug resistance mechanism, enabling better predictions of driving resistance and pile capacity. Because of their fundamental nature, the solutions are applicable to other relevant fields of study (e.g. powder technology).

Thesis Supervisor: Professor Robert V. Whitman
Title: Professor of Civil Engineering
"To see what everyone has seen and think what no one has thought"

Albert Szent-Gyorgyi
to my dear son Oren
ACKNOWLEDGEMENTS

The period of my doctoral studies at M.I.T. was an educational and social experience of great impact on my life. I wish to acknowledge the following people for their contributions:

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Special thanks to my dear parents for enabling me to have that which was deprived from them by Nazi–Germany: education and a loving, supporting family.

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REFERENCES 493

APPENDIX I. 507
An evaluation of the effect of a soil plug
upon the static capacity of the pile

APPENDIX II. 513
Analysis of tests on model piles

APPENDIX III. 517
Solutions for Equations 7.11 and 7.18
TABLE OF CONTENTS (in detail)

ABSTRACT 2

ACKNOWLEDGEMENTS 5

TABLE OF CONTENTS (in brief) 6

TABLE OF CONTENTS (in detail) 9

LIST OF SYMBOLS 16

LIST OF TABLES 21

LIST OF FIGURES 23

1. INTRODUCTION
   1.1 General definitions 37
   1.2 Background 38
   1.3 Statement of problem 39
   1.4 Method of solution 40
   1.5 Objectives 43

2. VARIOUS ASPECTS OF PILE PLUGGING
   2.1 Introduction 45
   2.2 The influence of a soil plug on the ultimate static capacity of piles 45
   2.3 The influence of a soil plug on time dependent pile capacity
      2.3.1 Introduction 48
      2.3.2 Radial consolidation 48
      2.3.3 Soil displacements and pore pressure dissipation around open vs. closed-ended piles 50
      2.3.4 Prediction of pore pressure dissipation time around open vs. closed-ended piles 52
      2.3.5 Prediction of gain in capacity with time for open vs. closed-ended piles 53
   2.4 The influence of a soil plug on the dynamic behavior and analysis of piles
      2.4.1 Introduction 56
      2.4.2 The 1-D W.E. and its underlying assumptions 56
      2.4.3 The 1-D W.E. in light of pile plugging 58
   2.5 Summary and conclusions 62

3. REVIEW AND INTERPRETATION OF EXISTING MEASUREMENTS
   3.1 Introduction 71
   3.2 Plug development during penetration
      3.2.1 Foreword 71
3.2.2 Analogy between soil samplers and open-ended piles
(a) Geometry
(b) Penetration
(c) Conclusions

3.2.3 Soil samplers

3.2.4 Small-scale models

3.2.5 Full-scale piles
(a) Introduction
(b) Large model piles
(c) Offshore piles
(d) Conclusions

3.3 The effect of penetration on the inner soil plug
3.3.1 Introduction
3.3.2 Deformation and stress changes in sampled soil
3.3.3 Deformations around plugged/unplugged piles
3.3.4 The effect of plugging on the inner soil (plug) in model piles
(a) Introduction
(b) Analysis of data
(c) Conclusions and discussion
3.3.5 Changes in the soil plug following full-scale pile penetration

3.4 The behavior of open-ended piles under static loads
3.4.1 Introduction
3.4.2 Small-scale models
3.4.3 Full-scale soil plugs
3.4.4 Full-scale load tests

3.5 Summary and conclusions

4. BEHAVIOR OF SOIL PLUGS UNDER STATIC LOADS — THE SILO APPROACH
4.1 Principle of analysis
4.2 Development of governing equations
4.3 The underlying assumptions and inherent simplifications
4.4 Investigation of governing equations
4.5 The meaning of the developed relations
4.6 Assessment of the inner soil deformations
4.7 The silo approach in light of experimental data

4.7.1 Purpose
4.7.2 Distribution of forces along the soil plug
4.7.3 Deformations of the soil plug
4.7.4 Plug forces and stresses for a constant soil height over diameter ratio
4.7.5 The meaning of the plugging and the developed relations as interpreted in Section 4.5
5. THE 'MICRO' APPROACH TO THE BEHAVIOR OF GRANULAR MATERIAL

5.1 Introduction
5.2 Packing of spheres
5.3 The strength and shear mechanism of an assembly of individual particles in contact
   5.3.1 Analysis using friction of inclined planes model
      (a) Underlying concept
      (b) Uniform rods in a parallel stack
      (c) Uniform spheres in F.C.C. packing
      (d) Uniform spheres in H.C.P. packing
      (e) Volume change and energy consideration
      (f) Experimental results
   5.3.2 Detailed 3-D analysis of a compression test on an F.C.C. model of a granular medium
5.4 Silo analysis using uniform discs
   5.4.1 The 'systematic arching theory'
   5.4.2 The 'no arching case'
   5.4.3 The 'full arching case'
   5.4.4 Investigation of the obtained relations
      (a) The maximum stresses
      (b) The stress ratio
      (c) Comparison to the standard silo analysis
   5.4.5 The 'intermediate arching case' and experimental data
5.4.6 Summary and conclusions
5.5 Soil behavior in light of soil structure
   5.5.1 Introduction
   5.5.2 Anisotropy and initial fabric
   5.5.3 The mechanism of fabric changes during deformation of granular material
   5.5.4 The shear mechanism of granular material
5.6 Summary and conclusions of the 'micro' approach

6. GRANULAR SOIL – INTERFACE SHEAR RESISTANCE

6.1 Introduction
6.2 The friction mechanism along an interface
   6.2.1 The micro approach
   6.2.2 The Sphere in the Groove (S.G.) model
      (a) Underlying concept
      (b) A sphere between two planes ($\beta = 90^\circ$)
         limiting equilibrium of interparticle friction
      (c) A sphere in a groove ($\beta \neq 90^\circ$),
         limiting equilibrium of interparticle friction
      (d) A sphere between two planes ($\beta = 90^\circ$)
limiting equilibrium of interface friction
(e) A sphere in a groove ($\beta \neq 90^o$),
(limiting equilibrium of interface friction
(f) Discussion
(g) Limiting equilibrium of interface friction for non-spherical particles between two planes ($\beta = 90^o$)

6.3 The controlling parameters of the interface shear resistance for the S.G. model
6.3.1 Introduction
6.3.2 Expansion of the S.G. model to account for the interface roughness
(a) Underlying concept
(b) The modified S.G. model for spherical particles
(c) The modified S.G. model for non-spherical particles
(d) The general equations of the S.G. model
(e) Discussion

6.3.3 Parameters which are determined by soil characteristics
(a) Relative density
(b) Appearance and composition
(c) Moisture content
(d) Stress–strain behavior

6.3.4 Parameters which are determined by the interface conditions
(a) Stresses
(b) Rate of shear

6.4 Summary and conclusions of the S.G. model

7. THE STRESS STATE OF THE INNER SOIL PLUG
7.1 Introduction
7.2 Elastic solution
7.3 Plastic solution
7.4 The soil arching approach
7.4.1 Underlying concept
7.4.2 The possible trajectories
7.4.3 The stress state along the interface
7.4.4 Possible stress conditions in the soil plug
(a) $45^o < \psi < 90^o ; K_1 < 1$
  1. $\delta = 0$
  2. $\delta = \phi$
  3. $\delta \leq \phi$
(b) $\psi = 45^o ; K_1 = 1$
(c) $0^o < \psi < 45^o , K_1 > 1$
  1. $\delta_p(\phi) \text{ Limiting interface friction}$
  2. $\delta < \delta_p(\phi)$
(d) Summary
7.5 The shape of the arch and its mechanism
7.6 Summary and conclusions

8. CRITICAL REVIEW OF EXPERIMENTAL DATA IN LIGHT OF THE PROPOSED INTERFACE SHEAR RESISTANCE MECHANISM

8.1 Introduction

8.2 The parameter $\theta$ of the S.G. model
   8.2.1 The principal stress trajectory and the S,G, model
   8.2.2 Minimum energy solution for the preferred intergranular contact
   8.2.3 Sphere and rod packings

8.3 Failure criteria for granular material according to the S.G. model
   8.3.1 Introduction
   8.3.2 The interparticle failure criterion
   8.3.3 The shear strength and the principal stress rotation
   8.3.4 Discussion
      (a) Conclusions from Figs. 8.4 and 8.5
      (b) The frictional component of the shear resistance
      (c) Conclusions
      (d) Critical review of Skinner's data
      (e) The controlling factors of the failure criterion

8.4 Simple shear and principal stress rotation
   8.4.1 Introduction
   8.4.2 Principal stress rotation under small strains
   8.4.3 Additional relevant observations Concerning the simple shear test device

8.5 Predictions and measurements of the interface shear resistance of sliding smooth surfaces
   8.5.1 Introduction
   8.5.2 The predictions of the S.G. model
   8.5.3 Experimental results from the ring torsion apparatus
      (a) The apparatus
      (b) The tests
      (c) Test results from rounded sand
      (d) Test results from angular sand
      (e) The sliding and shearing mechanism of interfaces
      (f) Test results from subrounded sand
      (g) Maximum and residual interface friction coefficients
   8.5.4 Discussion and conclusions

8.6 Predictions and measurements of the interface shear resistance of rough surfaces
   8.6.1 Introduction
   8.6.2 The predictions of the S.G. model
   8.6.3 Experimental Results from the simple shear apparatus
8.6.4 Consideration of principal stress rotation
   (a) Test results from rounded sand  
   (b) Test results from angular sand

8.6.5 Simplified analysis for shear resistance of rough surfaces
   (a) The controlling factors 
   (b) The required roughness range 
   (c) The requirements of the simplified analysis 
   (d) Possible simplified relations 
   (e) The suggested simplified relations

8.7 The effect of various factors on the interface shear resistance as determined through predictions and measurements
   8.7.1 Introduction 
   8.7.2 Stresses normal to the interface 
   8.7.3 Grain size and surface roughness 
   8.7.4 Test type

9. SOIL PLUG RESISTANCE TO STATIC LOADS – EXPANSION OF THE SILO APPROACH TO ACCOUNT FOR THE PRINCIPAL STRESS ORIENTATION AND INTERFACE SHEAR MECHANISM

9.1 Introduction
9.2 Development of governing equations
   9.2.1 The convex arch 
   9.2.2 The concave arch 
   9.2.3 The comprehensive analysis of the soil plug resistance

9.3 The soil plug resistance analysis in light of experimental data
   9.3.1 Introduction 
   9.3.2 The convex arch analysis 
   9.3.3 The comprehensive analysis

9.4 Summary and conclusions

10. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

10.1 Summary and conclusions
10.2 Recommendations
10.3 A final note

REFERENCES

APPENDIX I. An evaluation of the effect of a soil plug upon the static capacity of the pile
   I.1 Scope 
   I.2 Assumptions
APPENDIX II.
Analysis of tests of model piles
II.1 Remarks
II.2 Reservations
II.3 The table lines
  Table II.1 Summary of results
  Figure AII.1
  Figure AII.2

APPENDIX III.
Solution for Equations 7.11 and 7.18
Table A: $45 + \phi/2 \leq \psi < 90^\circ$
Table B: $0 \leq \psi \leq 45 - \phi/2$
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Arching factor</td>
</tr>
<tr>
<td>Ai, A₀</td>
<td>Shaft area of pile, inside and outside</td>
</tr>
<tr>
<td>Aₚ</td>
<td>total cross-sectional area of pile</td>
</tr>
<tr>
<td>Aₜ</td>
<td>cross-sectional area of steel tip</td>
</tr>
<tr>
<td>B</td>
<td>outside diameter of pile</td>
</tr>
<tr>
<td>Bₑ</td>
<td>inside diameter of cutting edge</td>
</tr>
<tr>
<td>Bₛ</td>
<td>inside diameter of tubing</td>
</tr>
<tr>
<td>Bₜ</td>
<td>outside diameter of sampling tube</td>
</tr>
<tr>
<td>Cₘ</td>
<td>area ratio</td>
</tr>
<tr>
<td>Cₕ</td>
<td>coefficient of radial consolidation</td>
</tr>
<tr>
<td>Cᵢ</td>
<td>inside clearance</td>
</tr>
<tr>
<td>Cₒ</td>
<td>outside clearance</td>
</tr>
<tr>
<td>Cᵤ(0)</td>
<td>initial unconfined compressive strength</td>
</tr>
<tr>
<td>d</td>
<td>particle size</td>
</tr>
<tr>
<td>D</td>
<td>penetration depth</td>
</tr>
<tr>
<td>Dₘₐₓ</td>
<td>maximum penetration depth</td>
</tr>
<tr>
<td>Dᵣ</td>
<td>relative density</td>
</tr>
<tr>
<td>D₅₀</td>
<td>median grain size by weight</td>
</tr>
<tr>
<td>e</td>
<td>void ratio</td>
</tr>
<tr>
<td>Eₚ</td>
<td>modulus of elasticity of the pile material</td>
</tr>
<tr>
<td>F</td>
<td>interparticle friction force</td>
</tr>
<tr>
<td>fₛ</td>
<td>friction stresses, pile skin friction</td>
</tr>
<tr>
<td>fₛ₁, fₛ₀</td>
<td>unit shaft friction, inside and outside the pile</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>h</td>
<td>roughness, height of asperity needed to create a contact point</td>
</tr>
<tr>
<td>ID</td>
<td>inside diameter of pile = diameter of soil plug</td>
</tr>
<tr>
<td>K</td>
<td>average coefficient of lateral stress</td>
</tr>
<tr>
<td>K_i</td>
<td>coefficient of earth pressure along the interface</td>
</tr>
<tr>
<td>K_o</td>
<td>coefficient of earth pressure at rest</td>
</tr>
<tr>
<td>L</td>
<td>length (height) of soil plug</td>
</tr>
<tr>
<td>N</td>
<td>SPT blow count</td>
</tr>
<tr>
<td>N</td>
<td>normal force</td>
</tr>
<tr>
<td>N_s</td>
<td>non-dimensional friction coefficient</td>
</tr>
<tr>
<td>P</td>
<td>applied shear force</td>
</tr>
<tr>
<td>P_max</td>
<td>total resistance</td>
</tr>
<tr>
<td>P_z</td>
<td>force of soil plug at depth z</td>
</tr>
<tr>
<td>q_p</td>
<td>unit end-bearing capacity</td>
</tr>
<tr>
<td>Q</td>
<td>force normal to the direction of P</td>
</tr>
<tr>
<td>Q_closed</td>
<td>total capacity of a closed-ended pile</td>
</tr>
<tr>
<td>Q_final</td>
<td>pile capacity after complete dissipation of pore pressure</td>
</tr>
<tr>
<td>Q_open</td>
<td>total capacity of an open-ended pile</td>
</tr>
<tr>
<td>Q_p</td>
<td>point capacity</td>
</tr>
<tr>
<td>Q_plugged</td>
<td>total capacity of plugged pile</td>
</tr>
<tr>
<td>Q_unplugged</td>
<td>total capacity of unplugged pile</td>
</tr>
<tr>
<td>Q_t</td>
<td>pile capacity at time t after driving</td>
</tr>
<tr>
<td>r</td>
<td>radial distance</td>
</tr>
<tr>
<td>R</td>
<td>radius of pile</td>
</tr>
<tr>
<td>R_a</td>
<td>ratio between the axes of an ellipse</td>
</tr>
<tr>
<td>R_i</td>
<td>inner radius of open-ended pile</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$R_o$</td>
<td>outer radius of pile</td>
</tr>
<tr>
<td>$R_n$</td>
<td>normalized roughness</td>
</tr>
<tr>
<td>$S_p$</td>
<td>pile circumference</td>
</tr>
<tr>
<td>$S_x$</td>
<td>area of contacts normal to the $x$ or $y$ axis</td>
</tr>
<tr>
<td>$S_z$</td>
<td>area of contacts normal to the $z$ axis</td>
</tr>
<tr>
<td>$T$</td>
<td>resistance of interface sliding</td>
</tr>
<tr>
<td>$t$</td>
<td>wall thickness</td>
</tr>
<tr>
<td>$t_1$</td>
<td>wall thickness at pile tip</td>
</tr>
<tr>
<td>$t_2$</td>
<td>wall thickness at a segment above the pile tip</td>
</tr>
<tr>
<td>$T_h$</td>
<td>time factor</td>
</tr>
<tr>
<td>$U$</td>
<td>infinitesimal circumferential area of soil plug cross-section</td>
</tr>
<tr>
<td>$u(x,t)$</td>
<td>longitudinal displacement of infinitesimal segment</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>degree of consolidation</td>
</tr>
<tr>
<td>$u_{max}$</td>
<td>maximum pore pressure</td>
</tr>
<tr>
<td>$u_r$</td>
<td>radial soil displacement</td>
</tr>
<tr>
<td>$\Delta V_r$</td>
<td>volume change in soil up to a given distance of $r/R$ away from the pile</td>
</tr>
<tr>
<td>$z$</td>
<td>distance from top of soil plug</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>roughness angle</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle between the average direction of particle plane of movement and plane of principal stress, according to Rowe</td>
</tr>
<tr>
<td>$\beta$</td>
<td>groove angle</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>specific recovery ratio $(dL/dD)$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>unit weight of soil</td>
</tr>
<tr>
<td>$\gamma_{max}$</td>
<td>maximum shear strain</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$\delta$</td>
<td>interface friction angle</td>
</tr>
<tr>
<td>$\operatorname{tg} \delta$</td>
<td>interface friction coefficient</td>
</tr>
<tr>
<td>$\operatorname{tg} \delta_b$</td>
<td>interface friction coefficient calculated by the S.G. model</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>major principal strain</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>minor principal strain</td>
</tr>
<tr>
<td>$\epsilon_v$</td>
<td>volumetric strain</td>
</tr>
<tr>
<td>.</td>
<td>major principal strain rate</td>
</tr>
<tr>
<td>.</td>
<td>minor principal strain rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>angle between wall of groove and z axis</td>
</tr>
<tr>
<td>$\theta$</td>
<td>groove inclination</td>
</tr>
<tr>
<td>$\mu$</td>
<td>friction coefficient $= \operatorname{tg} \delta$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>non-dimensionalized time factor</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>unit density of the pile material</td>
</tr>
<tr>
<td>$\sigma_{h}', \sigma_{v}'$</td>
<td>horizontal and vertical effective stresses *</td>
</tr>
<tr>
<td>$\bar{\sigma}_h$</td>
<td>average lateral stress *</td>
</tr>
<tr>
<td>$\sigma_{n}'$</td>
<td>effective normal stress *</td>
</tr>
<tr>
<td>$\sigma_{r}'$</td>
<td>effective radial stress *</td>
</tr>
<tr>
<td>$\sigma_x, \sigma_y$</td>
<td>wall and vertical stresses on silo under load of disc assembly</td>
</tr>
<tr>
<td>$\sigma_x(II)$</td>
<td>wall pressure at point $x$ above which exists fill to a depth $H$</td>
</tr>
<tr>
<td>$\sigma_y(II)$</td>
<td>base pressure at point $x$ above which exists fill to a depth $H$</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>major principal stress *</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>minor principal stress *</td>
</tr>
<tr>
<td>$\bar{\sigma}_z$</td>
<td>average vertical stresses in a soil plug cross-section *</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
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<td>--------</td>
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</tr>
<tr>
<td>$\tau$</td>
<td>friction stresses along interface</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>interface shear resistance</td>
</tr>
<tr>
<td>$\phi, \phi'$</td>
<td>internal friction angle $^*$</td>
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<tr>
<td>$\tan \phi'$</td>
<td>internal shear resistance $^*$</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>frictional component of shear angle $^*$</td>
</tr>
<tr>
<td>$\phi_\mu$</td>
<td>interparticle friction angle $^*$</td>
</tr>
<tr>
<td>$\tan \phi_\mu$</td>
<td>interparticle friction coefficient $^*$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>orientation of major principal stress</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>direction in which interparticle sliding occurs</td>
</tr>
</tbody>
</table>

* Except for Section 2.3, all relevant analyses in this work refer to drained or dry sand; therefore, effective and total stresses are the same, and the prime notation is not always carefully noted.
<table>
<thead>
<tr>
<th>Table No.</th>
<th>Title</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1A</td>
<td>Typical Dimensions of Pipes Frequently Used as Piles</td>
<td>120</td>
</tr>
<tr>
<td>3.1B</td>
<td>Typical Dimensions of Offshore Piles</td>
<td>121</td>
</tr>
<tr>
<td>3.2</td>
<td>Methods of Driving Samplers into the Soil</td>
<td>122</td>
</tr>
<tr>
<td>3.3</td>
<td>Plug Formation for Different Soil Volumes Entering Pile Tip vs. Penetration (assuming no volume change)</td>
<td>123</td>
</tr>
<tr>
<td>3.4</td>
<td>Geometry of Piles Presented in the Case Histories</td>
<td>124</td>
</tr>
<tr>
<td>3.5</td>
<td>Pile Plugging Statistics in the Gulf of Mexico (Data provided by Heerma on 60 piles at 10 sites)</td>
<td>125</td>
</tr>
<tr>
<td>4.1</td>
<td>Force Distribution Along the Soil Plug and Plug Deformation</td>
<td>188</td>
</tr>
<tr>
<td>4.2</td>
<td>Stresses and Forces for a Constant Ratio of Plug Length over Diameter</td>
<td>189</td>
</tr>
<tr>
<td>6.1</td>
<td>The Granular Soil/Interface Shear Resistance and its Relations to the Micro Approach, the S.G. Model, and the Stress State of the Inner Soil Plug</td>
<td>304</td>
</tr>
<tr>
<td>6.2</td>
<td>Parameters for Sphere Packings and Granular Soil</td>
<td>305</td>
</tr>
<tr>
<td>7.1</td>
<td>The Interrelations between Lateral Earth Pressure Coefficient Along Soil/Pile Interface and $\phi$, $\delta$ and $\psi$.</td>
<td>343</td>
</tr>
<tr>
<td>7.2</td>
<td>Values of the Limiting Interface Friction Angle $\delta_p$ and the Associated Lateral Earth Pressure Coefficient $K_i$ for Different Values of the Soil Shear Angle $\phi$</td>
<td>344</td>
</tr>
<tr>
<td>Table No.</td>
<td>Title</td>
<td>Page No.</td>
</tr>
<tr>
<td>----------</td>
<td>----------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>8.1</td>
<td>Friction Coefficient of Smooth Surfaces ($\alpha = 0^\circ$) and Quartzic Sand ($\theta = 32^\circ$) of Different Densities and Particle Shape, According to the S.G. Model</td>
<td>421</td>
</tr>
<tr>
<td>8.2</td>
<td>Measured Friction Coefficient of Smooth Surfaces and Different Sands (Yoshimi and Kishida -1981a,b)</td>
<td>421</td>
</tr>
<tr>
<td>8.3</td>
<td>Friction Coefficient ($tg\delta_b$) of Surfaces of Different Roughnesses ($\alpha$) and Quartzic Sand ($\theta = 32^\circ$) of Different Densities ($\beta$) and Particle Shapes ($R_a$) According to the S.G. Model</td>
<td>422</td>
</tr>
<tr>
<td>8.4a</td>
<td>Measured Friction Coefficient of Toyoura Rounded Sand for Surfaces of Different Roughnesses (Uesugi and Kishida -1986b) Compared to the Simplified Predictions of the S.G. Model, According to Eq.6.38</td>
<td>423</td>
</tr>
<tr>
<td>8.4b</td>
<td>Measured Friction Coefficient of Fujigawa Angular Sand for Surfaces of Different Roughnesses (Uesugi and Kishida -1986b) Compared to the Simplified Predictions of the S.G. Model, According to Eq.6.38</td>
<td>424</td>
</tr>
<tr>
<td>8.5</td>
<td>Friction Coefficient ($tg\delta_b$) of Surfaces of Different Roughnesses ($\alpha$) and Quartzic Sands of Different Grain Shapes ($R_a$) According to the Simplified Relations of the S.G. Model ($\beta = 90^\circ$, $\theta = 25^\circ$)</td>
<td>425</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Title</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The Three Possible Penetration States of the Inner Soil:</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>(a) Unplugged — Free Soil Intrusion</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) Partially Plugged — Limited Soil Intrusion</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Fully Plugged — No Soil Intrusion</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Stresses Acting on Piles Under (a) Unplugged and (b) Plugged Conditions</td>
<td>63</td>
</tr>
<tr>
<td>2.2</td>
<td>The Effect of a Plug Upon the Static Capacity of a Pile in (a) Clay (b) Sand</td>
<td>64</td>
</tr>
<tr>
<td>2.3</td>
<td>(a) Radial Soil Movements around Closed and Open Ended Piles (Randolph et al. –1979)</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>(b) Normal Radial Displacement for a 48-inch Pile</td>
<td></td>
</tr>
<tr>
<td>2.4</td>
<td>Decay of Excess Pore Pressure around Piles Driven in Boston Blue Clay (initial O.C.R =2), (after Carter et al. –1979)</td>
<td>66</td>
</tr>
<tr>
<td>2.5</td>
<td>Predicted Pore Pressure Dissipation for Plugged and Unplugged Piles (Based on PLS measurements)</td>
<td>67</td>
</tr>
<tr>
<td>2.6</td>
<td>Comparison between Predicted Set-Up Time for a 48-inch Pile with Field Data Collected by Vesic (1977)</td>
<td>68</td>
</tr>
<tr>
<td>2.7</td>
<td>(a) Measurements of Force and Velocity vs. Time for an Unplugged Pile.</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>(b) Plot of Measured Force Compared to Force Obtained from TEPWAP Analysis.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Comparison Between Measured Force near Tip of Pile and Calculated Force Obtained from TEPWAP Analysis (Referring to Measurements at Pile Top) (Paikowsky –1982)</td>
<td></td>
</tr>
<tr>
<td>2.8</td>
<td>(a) Measurements of Force and Velocity vs. Time for a Plugged Pile</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>(b) Plot of Measured Force Compared to Force Obtained from TEPWAP Analysis (Paikowsky –1982)</td>
<td></td>
</tr>
<tr>
<td>3.1</td>
<td>Geometrical Characteristics of Soil Samplers (Hvorslev –1949)</td>
<td>126</td>
</tr>
<tr>
<td>3.2</td>
<td>Typical Pile for a Drilling Platform — Offshore Louisiana (McClelland –1974).</td>
<td>127</td>
</tr>
</tbody>
</table>
3.3 Geometrical Characteristics of Samplers and Open-Ended Piles

3.4 Displacement Rate of Samplers and Piles During Penetration:
(a) Displacement of Pile Top and Tip Under a Single Blow. The top displacement was calculated from acceleration records; the tip displacement obtained from analysis (Paikowsky -1982).
(b) Typical Displacement vs. Time Curves for Quasi-Static and Average Dynamic Penetration
(c) Penetration of a Split-Spoon Barrel During SPT (Schmertmann and Palacios -1979).

3.5 Plugging Measurements on a Sampler with a Uniform Diameter, Under Different Driving Methods (Hvorslev -1949).

3.6 Plugging Measurements on a Sampler with Enlarged Tip Wall Thickness, Under Different Driving Methods (Hvorslev -1949).

3.7 Typical Results of Tests on Model Pipe Piles Driven in Sand (Kishida -1967b).
(a) State of Plug During Pile Penetration
(b) Sand Compaction Inside the Shaft

3.8 Analysis of Plugging Measurements on Model Pipe Piles Obtained by Different Researchers

3.9 Analysis of Plugging Measurements on an Open-Ended Pile Using Data From NGI -1981

3.10 The Ratio Between the Resistance Force of Open to Closed-Ended Piles vs. Normalized Depth as Obtained Experimentally (data from Kaarlsrud and Haugen -1981), and Theoretical Calculations.

3.11 Measurements of Pore Pressure Due to Installation of a Closed-Ended Pile (NGI -1981)

3.12 Measurements of Pore Pressure Due to Installation of an Open-Ended Pile (NGI -1981)

3.13 Plugging Measurements for Piles in the Gulf of Mexico (after Kindel -1977)

3.14 Adjustments for Plugging Measurements:
(a) Average and Adjusted Average Plug Movements, Sites A, B, & C.
(b) Plug Length Ratio and Adjusted Plug Length Ratio vs. Penetration, Sites A, B, & C (after Kindel -1977)
3.15 Statistics of Pile Plugging in the Gulf of Mexico
   (a) Kindel -1977
   (b) Using data provided by Heerma Engineering Services -1980

3.16 Typical Soil Profiles in the Gulf of Mexico

3.17 Soil Deformation Possibilities Ahead of Open-Ended Piles

3.18 Analysis of Plugging Measurements for Diameter $B = 60$ in. Piles in the Gulf of Mexico

3.19 Analysis of Plugging Measurements for Diameter $B = 42$ in. Piles in the Gulf of Mexico

3.20 Analysis of Plugging Measurements for Diameter $B = 48$ in. Piles in the Gulf of Mexico

3.21 Driving Record of Diameter $B = 48$ in. Pile in the Gulf of Mexico

3.22 Deformation of Sampled Soil Due to Stress Changes at Sampler Tip (after Hvorslev -1949)

3.23 Distortion of Soil in Sampler Due to Entrance of Excess Soil. Thick-Walled Piggot Sampler $C_A = 79\%$, $C_i = 2.7\%$, $C_o = 0\%$ (Hvorslev -1949)

3.24 Distortion of Soil in Sampler Due to Inside Friction. Thin-Walled Shelby Tube $C_A = 10\%$, $C_i = 1.2\%$, $C_o = 0\%$ (Hvorslev -1949)

3.25 Distortion of Soil Due to Sampler Plugging
   (a) Start of Downward Deflection of Soil Layers Ahead of Sampler
   (b) Deformation of Sampled Soil Caused by Overdriving (Hvorslev -1949)

3.26 Results from Model Simulating Pile Penetration in Clay
   (a) Deformation Pattern Around a Flat Open-Ended Pile with $B/t = 20$
   (b) Contours of Maximum Shear Strain, $\gamma_{max}$, $\frac{1}{2}(\varepsilon_1 - \varepsilon_3)$, for a Flat Open-Ended Pile with $B/t = 20$ (after Azzouz and Baligh -1984)

3.27 Soil Formations Ahead of Sampler
   (a) Cone
   (b) Bulb (Hvorslev -1949)

3.28 Formation of a Soil Cone Ahead of:
   (a) Overdriven Sampler (Hvorslev -1949)
(b) Pile Tip at a Depth of 12.0 m. (dimensions are in cm.), (BCP Committee—1969)

3.29 Soil State Near the Pile Tip (Fig. 3.28-b):
(a) Dry Density Contours (BCP Committee—1969) with Volumetric Strain Values using Eq. 3.2
(b) Dry Density Values vs. Distance from Pile Center for Different Depths under the Pile Tip (Z') (BCP Committee—1969)
The relations of Eq. 3.3 for a = 0.23, b = 1.6 are transformed into density using Eq. 3.2 and presented as a solid line, and as a dashed line for the ranges of b = 1.0 and 2.0

3.30 Effect of Pile Penetration on the State of the Sand Around the Pile:
(a) Volumetric Strain vs. Normalized Distance from the Pile Shaft
(b) Volume Change in the Soil as a Function of the Pile Volume vs. Normalized Distance from the Pile Shaft

3.31 Changes in Porosity Due to Penetration of Open and Closed-Ended Model Piles in Sand (data from Szechy—1961)

3.32 Porosity Measurements at the Top and Bottom of Soil Plugs vs. Model Pile Diameter (data from Szechy—1959)

3.33 Effect of Pile Penetration on the State of the Soil Inside the Pile. Volumetric change of plug length in respect to penetration depth and volumetric strain of the plug soil.

3.34 Values of Internal Friction Angle at Various Locations Along the Soil Plug vs. Model Pile Diameter (Based on data from experiments of Kishida—1967b)

3.35 Effect of Pile Penetration on the Soil Plug and Load Test Results of Open and Closed-Ended Piles (Kishida—1967a)

3.36 Installation and Load Tests on Open and Closed-Ended Model Pipe Piles
(a) Pile Resistance vs. Penetration Depth
(b) Load Settlement Relations at the Final Installation Depth (Prepared using data obtained from Kishida 1967b)

3.37 Analysis of Typical Test Results on the Behavior
of Sand Plugs in Open–Ended Steel Pipe Piles
(a) Relationship Between Ultimate Stress and Pile Diameter
(b) Normalized Sum of Calculated Frictional Stress vs. Normalized Height of Sand
(Kishida and Isemoto –1977)

3.38 (a) Soil Profile and SPT Results Inside and Outside Open–Ended Piles
(b) Comparison of Load–Settlement Relations Between Open and Closed–Ended Piles
(Data from Auki –1982, attributed to Soo –1980)

4.1 The ‘Silo Approach’ Analysis
4.2 The Force at the Bottom of a Soil Plug vs. the Plug length for Different K·tgδ Values
4.3 Representation of the ‘Silo Approach’ equations
4.4 Determination of Minimum Plug Force Location
4.5 Plug Force vs. Inside Diameter for Various Plug Lengths
4.6 The ‘Silo Approach’ in conjunction with the Tip Bearing Capacity
4.7 Determination of Plugging Conditions
4.8 Distribution of Normalized Friction Forces along the Plug Length
4.9 Deformation of the Plug
4.10 Measured and Calculated Stresses for Various Plug Diameters and the Appropriate Calculated K Values
4.11 Determination of Plugging Condition for Experimental Data
4.12 Relationship between $\sigma_s/\gamma ID$ and Downward Displacement; ID= 300mm, L/ID = 1.0 to 8.0 (after Kishida et al. –1985)
4.13 Relationship between $\sigma_s/\gamma ID$ and Downward Displacement; ID= 300 to 800mm L/ID = 2.5 (after Kishida et al. –1985)

5.1 Possible Sphere Packings. The B Spheres (shaded) rest on the A Spheres. A third close-packed layer
may then rest on the B spheres, either directly above the A spheres or in C positions. 
(after Illston et al. —1979).

5.2  (a) Expanded unit cell of Hexagonal Close-Packed Structure 241
      (b) Expanded unit cell of Face-Centered Cubic Structure (after Illston et al. —1979)

5.3  Basic Equilibrium Diagram for Friction of Inclined Planes Representing two Particles 242

5.4  Direction of Particle Movement ($\phi$) and Average Indination of Plane ($\alpha$) 242

5.5  Uniform Rods in a Parallel Stack 243

5.6  Uniform Spheres in a Face — Centered Cubic Packing 243

5.7  Direction of Particle Movements (Rowe —1962) 244

5.8  The Elementary F.C.C Cube and the Axes Notations (Scott —1963) 244

5.9  Unit Hexagonal Area and the Projection of Area Normal to $(\sigma_1 - \sigma_2)$ Direction (Scott —1963) 245

5.10 Forces Acting on a Typical Sphere B and the Path of B during Motion (Thurston and Dersiewicz —1959) 245

5.11 The Interparticle Force Distribution. (a) Full Contact (b) No Arching (c) Full Arching to the left 246

5.12 Load Transfer and Relative Movement for (a) No Arching and (b) Full Arching Conditions 247

5.13 Results of 2-D Silo Analysis based on a Discrete Model and the Standard Silo Analysis 248

5.14 Radiograph of Dense Sand after Surface Displacement of 4mm. Light arch zone is the area of arching in which the sand has expanded as a result of shear deformation. The black dots are lead spheres (Bransby et al. —1975). 249

5.15 Effect of Direction of Loading on the Friction Angle of Three Sands (Ladd et al. —1977) 249

5.16 Distribution of Interparticle Contact Normals as a function of Axial Strain for Sand Samples Prepared by (a) Tapping (b) Tamping (Oda 1972b) 250
5.17 (a) Stress Strain and Volumetric Strain Curves for Two Samples of Sand at the Same Initial Void Ratio, but Prepared by Different Methods (Oda 1972b)
(b) Representation of Interparticle Relations.

5.18 Stress Ratio and Internal Friction angle at Failure vs. Bedding Angle (Oda et al. –1982)

5.19 Relationship between the Fabric Characteristic $S_x/S_y$ of Anisotropic Sand and (a) The Mobilized Stress Ratio $\sigma_1/\sigma_3$ (b) The Dilatancy Rate $-dv/de$ (after Oda –1972b)

5.20 Reorientation of Particles during Loading
(a) Process where the Long Axes of the Particles Rotate so as to be Normal to the Major Principal Stress, causing Appearance of Contacts Normal to the Major Principal Stress, and
(b) Causing Disappearance of Contacts Parallel to it (Oda et al. –1982)

5.21 Frequency Distribution of Mobilized Angle $\delta$ in a Two-Dimensional Granular Assembly of Loose Model (Oda and Konishi –1974)

5.22 Shear Load and Vertical Displacement of Loading Platen plotted against Horizontal Displacement for Shear Box Tests on 1mm dia. Ballotini, all at the same Initial Porosity of 34.9% under a Normal Load of 20 lb. (Skinner –1969)

5.23 Shear Load vs. Horizontal Displacement and Dilation vs. Horizontal Displacement Curves Obtained from a Special Test with 3mm diameter Glass Ballotini in the Top Half of the Shear Box, the bottom half being replaced by plate glass, the test being performed partly dry and partly flooded. Normal load 20 lb. and initial porosity 37.5% (Skinner –1969)


6.1 The Sphere in the Groove (S.G.) Model: (a) An Isometric Presentation with Angles Detailed (b) A Sphere Between Two Planes ($\beta = 90^\circ$) (c) A Sphere in a Groove ($\beta \neq 90^\circ$).

6.2 Results of the Sphere in the Groove Model for Limiting Equilibria Conditions of:
(a) The Interparticle Friction
(b) The Interface Friction
(c) The Interface and the Interparticle Friction for $\beta = 90^\circ$, Considering Elongated Particles.

6.3 Geometrical Relations for the Sphere in the Groove Model (Smooth Surface).

6.4 Extreme Conditions for the Sphere in the Groove
(a) The Groove Normal to the Interface where No Rolling can take place
(b) The Groove Parallel to the Interface where No Sliding can take place.

6.5 The Sphere in the Groove Model: Details of Force and Geometrical Relations for the Analysis of Moment Equilibrium around the Contact Points C.

6.6 Details of Forces and Geometrical Relations for an Elliptical Particle Between Two Planes ($\beta = 90^\circ$).

6.7 (a) Profile of 'Smooth' Quartz Surface Traces.
   Scale: Vertical, 1 division = $2 \times 10^{-6}$ inch;
   Horizontal, 1 division = $2 \times 10^{-3}$ inch
   (from Dickey -1966).
(b) The Profile Described in (a) for Vertical = Horizontal Scale.
(c) Example of Profile of 'Rough' Surface in which the Vertical and the Horizontal Scales are Equal.

6.8 The S.G. Model for Rough Surfaces
(a) Geometrical Relations for Moment Equilibrium of a Sphere
(b) Evaluation of the Asperity Height
(c) Geometrical Relations for Moment Equilibrium of an Ellipse

6.9 Results of the Sphere in the Groove Model for Limiting Equilibria Conditions of the Interface Friction, Considering the Surface Roughness of the Contacting Body along the Interface.

7.1 Elastic Solution for Pushing an Isotropic Circular Cylinder in all-around Confinement
(a) The Analyzed Problem
(b) Lines of Principal Stress
(c) Major and Minor Principal Stresses
   (after Milne-Thomson -1962)

7.2 Plastic Solution for the Slip Lines of Massless Media ($\phi = 20^\circ$, $\delta = 15^\circ$), Pushed Between Two Walls

7.3 The 'Arching Approach' to the State of Stress in the Inner Soil Plug
(a) Continuous Possible Arches Defined by the Principal Stress Trajectory
(b) Free Body Diagram
(c) Mohr Circle for the
State of Stress Along the Interface

7.4 Force Equilibrium on an Interface Boundary Element

7.5 The Range of Possible Major and Minor Principal Stress Trajectories (0 ≤ ψ ≤ 90°) Presented as Segments of Circular Arches

7.6 Possible Stress States in the Inner Soil Plug for 45° ≤ ψ ≤ 90°

7.7 Rotation of Principal Stresses and Failure Planes for Major Principal Stresses Along the Center Line, and δ = φ

7.8 Relations Between Horizontal and Vertical Stresses along the Interface as a Function of the Internal Friction Angle (φ) and the Interface Friction Angle (δ) for the Range of 45° + φ/2 ≤ ψ ≤ 90°

7.9 Possible Stress States in the Inner Soil Plug for 0 ≤ ψ ≤ 45°

7.10 Rotation of Principal Stresses and Failure Planes for Minor Principal Stress along the Center Line, and δ = δp·(φ)

7.11 Relations Between φ and δp·(φ)

7.12 Relations Between φ and tgδp·(φ)

7.13 Relations Between φ and δp·(φ)/φ. The +++ line shows the exact relations and the continuous line shows the approximated relations of Eq. 6.28

7.14 Relations Between δp·(φ)/φ and K1 Along the Interface

7.15 Relations Between Horizontal and Vertical Stresses along the Interface as a Function of the Internal Friction Angle (φ) and the Interface Friction Angle (δ) for the Range of 0 ≤ ψ ≤ 90°

7.16 Catenaries and Circular Arches for Different ψ Angles

8.1 Comparison Between the Principal Stress Orientation of (a) the S.G. Model (b) the Trajectory of the Arching Approach (c) Mohr–Coulomb Failure Criterion under T.C State of Stress (d) Rowe's Friction of Inclined Planes Principle.
8.2 The Infinitesimal Displacement of the Sphere in the Groove. (a) The Movement (b) Analogy to Friction of Inclined Planes.

8.3 Deformation of 2-D Rods or Spheres in Orthorhombic Packing. (a) The ‘Standard’ Arrangement (b) The Deformed Configuration (c) Geometrical Relations (d) Relation to the S.G. Model.

8.4 Orientation of Principal Stresses During Shear of Spherical Particles:
(a) Using the Groove Orientation of the S.G. Model for Interparticle Limiting Equilibrium
(b) Using the Principal Stress Trajectory Based on the Mohr–Coulomb Failure Criterion
(c) Consideration of the Grain Shape by the S.G. Model

8.5 Relations of the S.G. Model Between the Groove Orientation $\theta$ and the Interface Friction Coefficient $\tan \delta_s$ for Elliptical Particles and Different $\beta$ Angles, Compared with Experimental Results of Principal Stress Orientation $\psi$ and Soil Friction Coefficient of Different Sands $\tan \phi'$.

8.6 Experimental Results from Drained Triaxial Compression Tests ($\sigma_3 = 30$ psi) on Saturated Quartz Grains of Different Shapes and Identical Size, as a Function of the Relative Density:
(a) Drained Shear Angle
(b) Dilatational Component of the Shear Angle
(c) Frictional Component of the Shear Angle (after Koerner –1970)
(d) Frictional Resistance for $\beta = 90^\circ$ and Different Grain Shapes as Calculated Using the S.G. Model, Compared to Experimental Data

8.7 Theoretical and Experimental Relations Between $\phi_\mu$ and $\phi_{cv}$:
(a) According to Skinner (1969) (See Bishop and Skinner –1976)
(b) The Above Relations Plotted with the Theoretical Relations of the S.G. Model for $\beta = 90^\circ$, $R_a = 1$, and the Limiting Friction Interface Angle (Eq. 7.15), with Data from Rowe (1962), and with Additional Data of Skinner (1969).

8.8 The Influence of the Grain Size on the Frictional Resistance of Saturated Quartz Soils (Koerner –1970)

8.9 Deformation in Simple Shear of:
(a) Axially Loaded Pile
(b) Simple Shear Apparatus
Mohr Circle of the Simple Shear Describes:
(c) The Effective Stress
(d) The Effective Stress Increment
(e) Strain Increment for the Whole Sample

8.10 Results of Simple Shear Tests:
(a) Stress Ratio and Average Void Ratio Change
    for Whole Sample, Against Shear Strain
    (Roscoe et al. —1967)
(b) Inclination to Horizontal of Major Principal
    Planes of Stress $\psi$, Stress Increment $\chi$, and
    Strain Increment $\xi$, as well as of Plane of
    Maximum Shear Stress $\beta$ and Maximum
    Obliquity $\omega$ (Roscoe et al. —1967)
(c) Relations Between Shear Force $S$, Shear
    Distortion $\gamma$, Rate of Thickness Change $\Delta H/H$,
    Inclination Angle of Maximum Principal Stress
    Axis $\psi$, and Inclination Angle of Maximum
    Principal Strain Increment Axis $\xi$, for
    Experimental Series of Dense Model
    (Oda and Konishi —1974b)

8.11 Results of Simple Shear Tests:
(a) Stress Ratio and Average Void Ratio Change for
    Whole Sample, Against Shear Strain
    (Roscoe et al. —1967)
(b) Inclination to Horizontal of Major Principal
    Planes of Stress $\psi$, Stress Increment $\chi$, and
    Strain Increment $\xi$, as well as of Plane of
    Maximum Shear Stress $\beta$ and Maximum
    Obliquity $\omega$ (Roscoe et al. —1967)
(c) Relations Between Shear Force $S$, Shear
    Distortion $\gamma$, Rate of Thickness Change $\Delta H/H$,
    Inclination Angle of Maximum Principal Stress
    Axis $\psi$, and Inclination Angle of Maximum
    Principal Strain Increment Axis $\xi$, for
    Experimental Series of Loose Model
    (Oda and Konishi —1974b)
(d) Direction of Maximum Principal Stress Axis and
    its Relation to Preferred Direction of $N_i$;
    Rosette Diagram and Schmidt’s Equal Area
    Projection Show 2- and 3-Dimensional
    Distribution of $N_i$ (Oda and Konishi —1974b)

8.12 Rotation of Principal Axes of Stress and Strain
    (Budhu —1988)

8.13 Rupture Zone in Direct Simple Shear Tests
    (Sketches of Radiograph):
    (a) NGI DSS, Leighton—Buzzard Sand
    (b) Cambridge DSS, Leighton—Buzzard Sand
    (c) Cambridge DSS, Fine Sand
        (Wood and Budhu —1980)
8.14 Sequence of Structures Indicating the Rupture Zone in a Direct Shear Test on Kaolin (Morgenstern and Tchalenko —1967)

8.15 Ring Torsion Apparatus for Measuring Friction between Soil and Metal Surfaces (Yoshimi and Kishida —1981b)

8.16 Comparison Between Measurements of the Effect of Sand Density and Surface Roughness on the Coefficient of Friction (Yoshimi and Kishida —1981a,b) and Friction Coefficient for Smooth Surfaces and Different Densities Calculated According to the S.G. Model.
(a) Rounded to Subrounded Quartzic Sand
(b) Angular Sand of Different Minerals (23% quartz)

8.17 Test Results from Tonegawa Sand and Steel Surfaces (Yoshimi and Kishida —1981b):
(a) Typical Test Results
(b) Sand Deformations Visualized by Radiograph
(c) X—Ray Photographs

8.18 Comparison Between Measurements of the Effect of Sand Type and Surface Roughness on the Coefficient of Friction (Yoshimi and Kishida —1981a,b) and Friction Coefficient for Smooth Surfaces and Different Densities According to the S.G. Model.
(a) Subrounded Smooth—Surfaced Sand of Different Minerals (60% quartz)
(b) Different Sand Types

8.19 Comparison between Measurements of Maximum and Residual Friction Coefficient between Steel and Dry Sand (Yoshimi and Kishida —1981b) and Friction Coefficient for Smooth Surfaces and Different Densities According to the S.G. Model

8.20 Rowe's Shear Strength Analysis:
(a) Test Results (Rowe —1962)
(b) Representation of the Shear Strength Components

8.21A The Simple Shear Apparatus for Measuring Friction between Sand and Metal Surfaces (Uesugi and Kishida —1986a, and Kishida and Uesugi —1987)
(a) The Test Apparatus and its Details
(b) The Measured and the Calculated Displacements
(c) Schematic Diagram of the Friction process

8.21B (a) Simple Shear/Shear Box Apparatus for Measuring Friction between Sand and Metal Surfaces
(b) The Displacement Measurements for the 2 Tests (Uesugi and Kishida —1986b)

8.22 The Surface Roughness Evaluation:
(a) $R_{\text{max}}(L = 2.5\text{mm})$ (Yoshimi and Kishida -1981a,b)
(b) $R_{\text{max}}(L = 0.2\text{mm})$ (Uesugi and Kishida -1986a,b)

8.23 Definition of Coefficient of Friction at Yield by Uesugi and Kishida (1986a,b), Designated as $\text{tg}\delta_s$
When Compared with the S.G. Model Analyses

8.24 Comparison of Measurements of the Effect of Surface Roughness on the Coefficient of Friction to Predicted Friction Coefficient of Rough Surfaces and Rounded Sand According to the S.G. Model:
(a) Considering Rotation of Principal Stress/Inter-
particle Contact Orientation with Stress Ratio
(b) Relations for Simplified Predictions

8.25 Comparison of Measurements of the Effect of Surface Roughness on the Coefficient of Friction to Predicted Friction Coefficient of Rough Surfaces and Angular Sand According to the S.G. Model:
(a) Considering Rotation of Principal Stress/Inter-
particle Contact Orientation with Stress Ratio
(b) Relations for Simplified Predictions

8.26 Comparison of Measurements of the Effect of Surface Roughness on the Coefficient of Friction to Predicted Friction Coefficient of Rough Surfaces and Rounded Sand According to the S.G. Model:
(a) Considering Rotation of Principal Stress/Inter-
particle Contact Orientation with Stress Ratio
(b) Relations for Simplified Predictions

8.27 Comparison of Measurements of the Effect of Surface Roughness on the Coefficient of Friction to Predicted Friction Coefficient of Rough Surfaces and Different Sand Types Using Simplified S.G. Model Relations ($\theta = 25^\circ$, $\beta = 90^\circ$) and Considering the Grain Shape (a) Toyoura Sand, Round Particles
(b) Seto Sand, Angular Particles (c) Fujigawa Sand, Angular Particles

8.28 The Influence of the Normal Stress on the Interface Friction Coefficient. Measurements by Uesugi and Kishida (1986a) and Predictions Using the Simplified Relations of the S.G. Model

8.29 Comparison of Measurements to Analyses of the Coefficient of Friction of Different Median Grain Size Fujigawa Angular Sands and Surface Roughness Measurements along $L = 0.2\text{mm}$.
(a) Linear Regression Relationship for Data of Each $D_{50}$ (Uesugi and Kishida -1986b)
(b) S.G. Model:
(1) Considering Principal Stress Rotation
(2) Simplified Relations for $\theta = 32^\circ$, $\beta = 60^\circ$
(3) Simplified Relations for $\theta = 32^\circ$, $\beta = 45^\circ$

8.30 Comparison of Measurements (Uesugi and Kishida -1986b) to S.G. Model Predictions of the Friction Coefficients of Different Median Grain Sizes of Fujigawa Angular Sands and Suitable Surface Roughness Measurements
(a) $D_{50} = 0.16\text{mm}, R_{\text{max}}(L = 0.2\text{mm})$
(b) $D_{50} = 0.54\text{mm}, R_{\text{max}}(L = 0.5\text{mm})$
(c) $D_{50} = 1.82\text{mm}, R_{\text{max}}(L = 2.00\text{mm})$

8.31 The Effect of Test Type on the Measurement of the Interface Friction Coefficient (Kishida and (Uesugi -1987) Shear Interface Tests of:
(a) Toyoura Sand in Ring and Simple Shear
(b) Fujigawa (?) sand in the Large Simple and Direct Shear Apparatus
(c) Fujigawa sand in the Small Simple and Direct Shear Apparatus
(d) Interface Displacement in Simple and Direct Shear Interface Test.

9.1 The Convex Arch Analysis

9.2 The Relations between the Minor and Major Principal Stresses for $\delta = \phi$ and Failure along the Interface

9.3 Analysis for the Total Vertical Force

9.4 The Relations between the Convex 'Active' and the Concave 'Passive' Arches and the Concave Arch Analysis

9.5 The Transition from Active to Passive Arching

9.6 Determination of Plugging Condition for Experimental Data Using the Silo Approach (see Fig. 4.11) and the Convex Arch Analysis

9.7 Determination of Plugging Condition Using the Comprehensive Analysis of the Soil Plug Resistance
CHAPTER 1
INTRODUCTION

1.1 GENERAL DEFINITIONS

Piles are the most commonly used foundation tool for transmitting or transferring loads. This function is required for construction in water and/or where upper soil strata are compressible, weak, expansive or collapsible. In addition, piles provide appropriate solutions for engineering problems such as horizontal loads, uplift forces, location of future excavation or possible scour.

Open pipe piles are widely used for land and offshore construction, as they constitute a strong and light-weight structure, easy to handle and splice.

During the initial stage of installation, the soil elevation inside the pile does not change; i.e. the length of the inner soil cylinder is approximately equal to the depth of penetration, and the pile is considered unplugged (see Fig. 1.1a). As penetration continues, the inner soil cylinder develops frictional resistance, which may prevent some of the soil ahead of the opening from entering the pile; i.e. the length of the inner soil cylinder is less than the penetration depth, and the pile is considered partially plugged (see Fig. 1.1b). With further penetration, the inner soil cylinder may develop sufficient frictional resistance to prevent any soil intrusion, causing the pile to become "plugged". The open-ended pile then assumes the penetration characteristics of a closed-ended pile.

Although technically the inner soil can be referred to as a ‘plug’ only when it prevents entry of additional soil during penetration, the term ‘soil plug’ is commonly used in reference to any soil mass inside the pile, regardless of its state during installation.
1.2 BACKGROUND

My personal interest in the subject of pile plugging arose during the research of "Use of Dynamic Measurements to Predict Pile Capacity Under Local Conditions" (Paikowsky –1982). This research was undertaken for the purpose of designing and constructing an offshore unloading terminal, where investment in reliable tools for predictions prior to and during construction was justified by the potential saving in construction costs. In spite of the fact that this research produced excellent predictions of the pile capacities when compared with static load tests, two main difficulties related to the plugging phenomenon were encountered:

1. The piles penetrated in an unplugged mode, resulting in a lower ultimate capacity and a greater settlement than expected. This was due to the fact that no quantitative methods are available to predict the plugging potential or to take into consideration the contribution of the inner soil to the pile’s performance under static loading.

2. When the piles were artificially plugged (with a concrete plug or internal annulus), the increase in pile capacity was indicated by blow count observed in the field during driving. However, the dynamic analysis, which showed excellent agreement with the measurements from unplugged piles, failed to correctly model the behavior of the plugged piles.
1.3 STATEMENT OF PROBLEM

The general problem of Static Evaluation of Soil Plug Behavior can be broken down into the following elements:

1. Identification of the effects of pile plugging

   Due to the complexity of the plugging phenomenon and the difficulties in identifying when and whether it occurs, only a limited amount of research has been devoted to the subject. Analysis of the effects of the change in mode of penetration, from open to closed-ended, is required as a first step in order to establish the significance of the phenomenon.

2. Identification of the plugging mechanism.

   To this end, a review and interpretation of existing data is required. Since only limited data directly related to the plugging phenomenon exist, the possibility of utilizing other relevant data (e.g. soil sampling, silo research) must also be examined.

   The available data can then be used to extract any relevant information, such as: identifying the frequency with which plugging occurs, its significance, and the possible mechanism behind it.

3. Identification of the controlling parameters of the plugging mechanism.

   Once the major parameters that control the plugging mechanism are identified, further investigation can focus on their evaluation.

4. Evaluation of the inner soil resistance.

   Once the controlling parameters are identified and assessed, they can be utilized to evaluate the inner soil resistance for two purposes:

   (a) Determination of conditions in which plugging will take place.

   (b) Assessment of the resistance of a soil mass inside the pile following penetration in either a plugged or unplugged mode.
1.4 METHOD OF SOLUTION

The following steps are taken in order to address the aforementioned problem:

1. Analysis of Various Aspects of Pile Plugging.

   Assuming that plugged piles behave like closed-ended piles, the effect of the penetration mode transition from unplugged to plugged is examined in Chapter 2 for the following aspects: ultimate static capacity, time-dependent pile capacity, and dynamic behavior and analysis.

2. Review and Interpretation of Existing Measurements.

   Once the analogy between open-ended piles and soil samplers is established, data related to penetration of soil samplers, small scale model piles and full-scale piles are utilized in Chapter 3 to examine:
   (a) The plug development during penetration.
   (b) The effect of penetration on the inner soil plug.
   (c) The behavior of open-ended piles under static loads.

3. The Silo Approach.

   In order to identify the controlling parameters of the plugging mechanism in sand, a simplified analysis is performed in Chapter 4 using the 'Silo Approach'. Investigation of the governing equations of that analysis and comparison with experimental results in light of its underlying assumptions indicates that two major parameters control the plugging mechanism. These are the interface friction coefficient (tgδ) and the coefficient of earth pressure (Kᵢ), the ratio between vertical to horizontal stresses along the inside pile wall. Both are believed to be determined by the particulate behavior of the cohesionless soil.


   As both controlling parameters seem to be functions of the micro behavior of the soil, a review of relevant information is presented in Chapter 5. This is done in order to (1) obtain the knowledge needed to develop and support solutions for the
controlling parameters, and (2) examine the possibilities of explaining the silo approach by the particulate arching mechanism.

5. The Interface Friction Coefficient.

Based on the mechanism that controls the granular material behavior under loading and shear, as obtained through the above review, a model for the analysis of friction of soil along an interface is developed in Chapter 6.

6. The Coefficient of Earth Pressure.

Considering the nature of the granular material, the previous explanation for the silo analysis by arching (Chapter 5), and possible stress trajectories within the inner soil mass, a solution for the stress state within the soil plug is suggested using the arching approach in Chapter 7. This solution enables the calculation of the coefficient of earth pressure along the interface (K_i).

7. The Limiting Interface Friction Coefficient.

According to the arching approach, the interface friction coefficient (tgδ) and the coefficient of earth pressure (K_i) are interdependent. For the special condition in which the expected arch exerts normal stresses on the pile wall which are greater than the tangential stresses (K_i > 1), a limiting friction coefficient is suggested in Chapter 7 as the result of a proposed shear mechanism.


Chapter 8 examines the proposed interface shear resistance mechanism consisting of the interface friction coefficient model of Chapter 6 and the limiting friction coefficient suggested in Chapter 7. The interface shear resistance and the principles behind it are compared to experimental data in two stages: (a) comparison to soil shear strength measurements in order to examine the principles of the model under soil shear conditions and (b) comparison to interface shear resistance tests considering the different parameters of the suggested model.

9. The Inner Soil Resistance
The previously proposed steps for modeling of the soil and the plugging mechanisms are assembled in Chapter 9 to produce a comprehensive analysis, which consists of the following stages:

(a) A suggested mechanism of plug formation, based on:
- the mechanism identified in Step 2
- the silo approach (Step 3)
- examination of the silo approach in light of the arching theory (Step 4)
- the micro behavior of the granular material (Step 4).

(b) Development of the governing equations and their solutions, based on:
- the silo approach (Step 3)
- the stress state of the inner soil plug, based on the arching approach (Step 6)
- the coefficient of earth pressure (Step 6)
- the interface friction coefficient (Step 5) and its limiting value (Step 7)

(c) Comparison of the proposed mechanism and solutions to measurements of:
1. Plugging states of piles of different diameters installed to the same penetration depth
2. Plug lengths for different pile diameters
3. Plug resistance force for different plug lengths and diameters
1.5 OBJECTIVES

Being the first attempt to systematically investigate the pile plugging phenomenon, the first objective of this study was to identify the effects of pile plugging. This was done in Chapter 2, leading to the conclusions that:

1. Plugging of piles driven in clay has a negative effect on the pile performance. Further research concerning plugging in clays should be directed towards identifying its occurrence.

2. Plugging of piles driven in sand has a major contribution to the pile resistance. Further research in this area should be focused on methods of analysis.

The addressing of these two conclusions through the interpretation of existing measurements is the objective of Chapter 3. The plugging of piles in clay is investigated and the extent of the phenomenon in offshore piling is determined, leading to practical conclusions. The rest of the study concentrates on plugging of piles in sand.

Aside from the ultimate objective of evaluating the soil plug behavior with application to the pile plugging problem, as detailed in Section 1.3 (statement of problem), several intermediate objectives must be met along the path to solution followed by the study. These intermediate objectives are:

1. Identification of the micro mechanism of the discrete soil grains.
2. Use of this mechanism to develop a physical model which takes into account the parameters that control the discrete grain behavior.
3. Relation of the single grain model to the entire soil mass
4. Application of the model for the solution of the ultimate problem.
The Three Possible Penetration States of the Inner Soil:
(a) Unplugged — Free Soil Intrusion
(b) Partially Plugged — Limited Soil Intrusion
(c) Fully Plugged — No Soil Intrusion
CHAPTER 2
VARIOUS ASPECTS OF PILE PLUGGING

2.1 INTRODUCTION
The mode of pile penetration significantly controls the soil–pile interaction during and after driving. Deformations and stresses in the surrounding soil during open-ended penetration differ substantially from those created by plugged (closed-ended) or partially plugged (partial soil penetration) piles. Investigation of the various aspects of pile plugging focuses on three major issues which are discussed below.

2.2 THE INFLUENCE OF A SOIL PLUG ON THE ULTIMATE STATIC CAPACITY OF PILES
The ultimate static capacity\(^1\) of an open–ended pipe pile can be estimated in the following manner (see Figure 2.1):

\[
Q_{\text{unplugged}} = \sum f_{s0} \cdot A_0 + \sum f_{s1} \cdot A_i + q_p \cdot A_t
\]
\[
Q_{\text{plugged}} = \sum f_{s0} \cdot A_0 + q_p \cdot A_p = Q_{\text{closed-ended}}
\]

The magnitude of \(q_p\) acting on the steel area may differ in general from that acting on the total cross–section. For undrained penetration in clay, deep penetration in sand, and for the purpose of the present discussion, they are assumed to be the same.

The plug of an open–ended pile is mobilized when the accumulated inside skin friction (reduced by the plug weight) exceeds the ultimate static bearing capacity of

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\(^1\)This section does not deal with load vs. settlement for open–unplugged, open–plugged, or closed–ended piles, an issue of importance which is discussed in Chapter 3.
the soil below the toe of the pile. The pile then behaves as though it is closed-ended, and Eq. 2.2 defines its maximum static capacity.

Equations 2.1 and 2.2 are used in Appendix (I) to present a rough evaluation of the effect of a plug upon the static capacity of a pile. The results of this evaluation are presented in Figure 2.2 in the following ways:

1. The ratio between the total capacity of an open-ended unplugged pile \(Q_{\text{open}}\) to that of a closed-ended pile \(Q_{\text{closed}}\) vs. depth of penetration \(D\), normalized with respect to the pile diameter \(B\).

2. The contribution of the point capacity \(Q_p\) to the total capacity of a closed-ended pile \(Q_{\text{closed}}\) vs. normalized depth of penetration.

Both relations are presented for two ‘typical’ soils, a normally to lightly overconsolidated clay and a medium dense sand. The analysis assumes that the shear resistance along the outside of the pile is the same whether installed open or closed-ended. The influence of an enlarged wall thickness at the pile’s toe (‘shoe’) is treated in Appendix (I) and included in Figure 2.2a. This analysis does not consider dynamic effects during driving; therefore, the relations in Fig. 2.2 are applicable to quasi-static penetration. The normalized fashion in which the results are presented implies that size effect is assumed not to influence the mobilization of the plug and/or the pile capacity (see for example, Meyerhof –1983; Trofimenkov et al. –1985).

The relations in Figure 2.2 indicate that:

1. The point contribution to the total capacity of a pile in clay is reduced rapidly with depth. At a depth ratio of \(D/B = 30\) the point carries about 25% of the total ultimate load, and practically nothing under working loads. (The exact amount of the load that will reach the tip in this case is controlled by the relative stiffness of the pile to that of the soil.)

2. Uniform pipe piles in clay are expected to plug at a depth ratio of \(D/B \approx 10\) (i.e. when \(Q_{\text{open}}\) reaches \(Q_{\text{closed}}\)), which increases to about \(D/B \approx 20\) when the pile
3. Plugging of open piles in clay, therefore, does not contribute significantly to the capacity of the pile.

4. The point contribution to the total resistance of a pile in sand decreases gradually with depth, but cannot be ignored even at deep penetrations.

5. Full plugging of piles in sand is expected to take place at a depth ratio of about $D/B = 25$ to $30$.

6. Plugging of open piles in sand contributes significantly to the capacity of the pile.
2.3 THE INFLUENCE OF A SOIL PLUG ON TIME–DEPENDENT PILE CAPACITY

2.3.1 Introduction

The allowable load, and the stage and manner in which the load will be applied, is determined by the rate of gain in pile capacity with time after driving. An estimate of the effect of the mode of penetration (open vs. closed) on the rate of capacity gain follows.

2.3.2 Radial consolidation

It is assumed that the piles are driven into insensitive clay in which thixotropy (Skempton and Northey –1952) is not a factor. In such clay, the increase of skin resistance with time is associated only with migration of pore water. This migration is caused by excess pore pressure initiated during pile penetration.

The material presented in Section 2.2 has shown that the capacity of a pile in clay is controlled primarily by the skin friction. The increase in bearing capacity of such piles is therefore determined by radial consolidation of the clay (Soderberg –1962). For this reason, the gain in resistance can be analyzed using radial diffusion theory. The time associated with a specified degree of consolidation is given by:

\[ t = \frac{T_h \cdot R^2}{C_h} \]  

(2.3)

in which:

- \( t \) — elapsed time since pile driving
- \( C_h \) — coefficient of radial consolidation
- \( T_h \) — time factor
- \( R \) — radius of pile (closed–ended, \( R = B/2 \))

Thus the time needed to develop pile capacity is proportional to the square of
the pile size. By normalizing Eq. 2.3, the dissipation time around a closed-ended pile of one size \((R_1,t_1)\) can be used to estimate that of another size \((R_2,t_2)\), such that:

\[
\frac{t_1}{t_2} = \left(\frac{R_1}{R_2}\right)^2
\]  

(2.4)

The use of Equation 2.4 assumes no effect of pile diameter on the magnitude of soil stresses or pore pressure. These assumptions were confirmed by Baligh and Levadoux (1980), who compiled measurements of excess pore pressure due to installation of closed-ended piles from various case histories. The data were presented in the form of normalized excess pore pressure (with respect to the original vertical effective stress \(-\sigma_{vo}'\)) versus normalized radial distance (with respect to the pile radius). The different data points fall within a narrow range (e.g. the excess pore pressure near the pile shaft was about \(2\sigma_{vo}'\)). This shows that the magnitude of the soil stresses and the pore pressures is not a function of the pile diameter.

Grosch and Reese (1980) reinterpreted data gathered by Vesic (1977), which describe the increase in pile capacity with the time after installation. They plotted the time required for the piles to reach 50% of their maximum capacity vs. the pile diameter. The obtained relations were those of Eq. 2.4, despite the fact that the data were gathered at various sites having different clays (mostly soft).

The above findings can be summarized as follows:

1. Equation 2.4 can be used to estimate the time required for dissipation of excess pore pressure around one closed-ended pile when given the dissipation measurements of another pile of a different size.

2. Since the magnitude of the stresses and excess pore pressure around the pile is not determined by the size of the pile, the dissipation time must therefore be controlled by the volume of soil displaced during penetration. For closed-ended
piles, the volume of the displaced soil is proportional to the square of the pile size, which is expressed by Eq. 2.4.

3. The bearing capacity of a pile in clay is determined by the skin friction. The skin friction is in turn controlled by the effective stresses near the pile shaft. These stresses are themselves controlled by the pore pressure, which is determined by the volume of soil displaced by the pile penetration. Logic dictates that Eq. 2.4, which relates pile size to dissipation time of excess pore pressure, can also be used to predict a gain of pile bearing capacity with time. These assumptions were proven correct by the work of Grosch and Reese, as mentioned above.

2.3.3 Soil displacements and pore pressure dissipation around open vs. closed-ended piles

Study of the soil displacement around open vs. closed-ended piles may be utilized, therefore, to determine the ratio of the gain in bearing capacity between the two types.

The volume of the displaced soil during open-ended unplugged pile penetration depends on the pile diameter and wall thickness. Simplified expressions which describe the radial soil displacements for open and closed-ended piles were developed by Randolph et al. (1979). These expressions (see Figure 2.3a) are based on radial volume conservation and assume the following: (1) no change in soil volume during penetration; (2) soil displacement is radial; and (3) only outward displacements occur during the penetration of open-ended piles. The equations, along with the calculated soil displacements due to penetration of a 48-inch diameter pile in plugged (closed-ended) and unplugged modes are presented in Figure 2.3. Figure 2.3b demonstrates the large soil displacements caused by penetration of plugged piles as opposed to the much smaller soil displacements of open-ended pile penetration.
Bogard and Matlock (1985) presented data describing pore pressure dissipation following the penetration of open unplugged and closed-ended model piles. Both cases exhibit the same normalized pattern of pore pressure dissipation. Since the dissipation patterns were shown to be similar for both types of piles, Eq. 2.3, which describes radial consolidation, can be applied in both cases. Equation 2.4 can therefore be used to assess the dissipation time around an open-ended unplugged pile based on the known dissipation time around a closed-ended pile. However, a value which represents the displaced soil volumes and corresponding soil behavior for the two types of piles must be substituted for the ratio of $R_1$ over $R_2$ in Eq. 2.4.

One way to derive a value for this ratio is to follow the logic of Eq. 2.4. Calculation of the ratio of the displaced soil volumes of closed vs. open-ended piles of the same outer diameter leads to:

$$\frac{t_1}{t_2} = \frac{R_o^2}{R_o^2 - R_i^2}$$  \hspace{1cm} (2.4a)

for which:

$R_o =$ outer radius of the piles ($= B/2$)

$R_i =$ $R_o - t =$ inner radius of the open-ended pile.

A more comprehensive solution is presented by Carter et al. (1979). In their study, a soil model based on modified cam clay was used, in which undrained conditions with subsequent consolidation could be simulated. Figure 2.4 shows the results obtained, presented as a set of curves which describe the pore pressure (normalized w.r.t. the maximum pore pressure) as a function of a non-dimensionalized time factor for piles of different wall thicknesses penetrating into BBC (see Fig. 2.3a for the definition of $\rho$). It should be noted that when predicting the maximum pore pressure [$U_{\text{max}}$ normalized by the initial unconfined compressive strength $C_u(o)$] for the
open-ended piles, the pressure is dependent on the dimensions expressed by $\rho$. 

e.g: These relations (presented in the upper right corner of Fig. 2.4) predict a constant ratio of $U_{\text{max}}/C_{u}(0) = 4.05$ for the pore pressure around any closed-ended piles ($\rho = 1.0$).

### 2.3.4 Prediction of pore pressure dissipation time around open vs. closed-ended piles

The approach outlined above was used to prepare Figure 2.5, which presents predictions of pore pressure dissipation for a 48-inch diameter pipe pile with a wall thickness of 1.5 inches. The curves were constructed in the following manner:

1. Curve No. 1 shows the pore pressure dissipation data measured by a PLS² cell during penetration in BBC (from Morrison —1984).
2. Curve No. 2 was constructed using Eq. 2.4 to adapt Curve No. 1 to the diameter of the closed-ended 48-inch pile.
3. Curve No. 3 represents a rough estimation of the pore pressure dissipation around an open-ended pile, with a diameter of 48 inches, which penetrates in an unplugged mode. The ratio between the decay of excess pore pressure around the open-ended pile (Curve No. 3) to that of the closed-ended pile (Curve No. 2) is based on: (1) the relations described by Figure 2.4 (Carter et al. —1979) at 50% consolidation for relative displacement of $\rho = 1$ (for the 48-inch closed-ended pile) and $\rho = 0.12$ (for the 48-inch open-ended pile); and (2) the relations of Eq. 2.4a.

The soil displacements associated with Curve No. 3 (open-ended, unplugged) and Curve No. 2 (closed-ended, plugged) have been presented in Figure 2.3. Using

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²The Piezo—Lateral Stress cell (PLS), is a device developed at MIT to measure simultaneously the total horizontal stress and pore pressure on the shaft of a model pile. The cell diameter is 1.51 inches, and it is pushed at a constant rate of penetration of 2 cm/sec. (Morrison —1984).
Figure 2.4, this difference is estimated to generate a pore pressure next to the plugged pile about twice that of the unplugged pile. As a result of the larger displaced soil volume and the higher pore pressure, the time which is required for the pore pressure to dissipate around the 48-inch closed-ended pile (of Fig. 2.5) is about an order of magnitude greater than the time required for pore pressure dissipation around the 48-inch open-ended pile.

2.3.5 Prediction of gain in capacity with time for open vs. closed ended piles

The relations of Figure 2.5 can be further analyzed in order to evaluate the gain in capacity with time. If no other measurements exist, a simplified assumption of a direct relation between gain in pile capacity to pore pressure dissipation can be used. This assumption (supported by the previous discussion) leads to the following relations:

\[
\frac{Q(t)}{Q_{\text{final}}} = 1 - \bar{u}
\]

(2.5)

where:
\[
Q(t) = \text{pile capacity at time } t \text{ after driving}
\]
\[
Q_{\text{final}} = \text{pile capacity after full dissipation}
\]
\[
\bar{u} = \text{degree of consolidation}
\]

The simplified relations of Eq. 2.5 imply that the pile capacity at the penetration time (\(\bar{u} = 1\)) is zero. Measured values of the horizontal effective stresses (\(\sigma'_n\)) during penetration (Morrison –1984) while assuming that the skin friction, and therefore the pile capacity, are proportional to \(\sigma'_n\) at any time during consolidation, allow adjustment of Eq. 2.5 in the following manner:

\[
\frac{Q(t)}{Q_{\text{final}}} = K_i + (1-K_i)(1-\bar{u})
\]

(2.6)
54

where:

\[ K_i = \text{Ratio of the horizontal (radial) effective stress at time of penetration to the original vertical effective stress (} \sigma'_h / \sigma'_v). \]

Use of Eq. 2.6 and data provided by Morrison (1984) enable estimation of the gain in capacity with time for the 48-inch pile. The estimations for the two modes of penetration (plugged/unplugged) are presented in Figure 2.6 along with measurements compiled by Vesic (1977). Fig. 2.6 shows that:

1. The calculated gain in capacity with time fits in with the general trend of actual measured data.
2. For the considered 48-inch friction piles driven in BBC, the time required for the plugged pile to achieve its maximal bearing capacity is an order of magnitude greater than that required for the unplugged pile.
3. An unplugged pile with a diameter of 48 inches in BBC requires 2.5 days to reach 50% and 25 days to reach 90% of its maximum capacity. A plugged pile of the same diameter will require a much longer time, such that 50% of its maximum capacity is reached after 50 days and 90% after 500 days.

The following should be emphasized in regard to the relations suggested in Sections 2.3.4 and 2.3.5:

1. The data compiled by Vesic (1977) were obtained by direct pile load testing at different time stages (e.g. Seed and Reese –1955). The calculated curves are based on measurements of pore water dissipation around a model pile, and are subject to the assumptions made for Eq. 2.6. Morrison (1984) measured values of \( \sigma'_h \) and \( u \) with time, but relations such as those of Eq. 2.6 need additional confirmation of the direct influence of those values on the skin friction.
2. The accuracy of this process when applied to a sensitive \( (s_t = 6) \) clay, such as the BBC, is questionable, taking its thixotropic behavior into consideration (O’Neill
For these reasons, the calculated curves in Figure 2.6 should be regarded as an approximation that serves the purpose of the present illustration.

3. All analyses assumed that the plugged pile is analogous to a closed-ended pile. While this assumption is entirely correct for the end-bearing consideration, it is conditional for the time-dependent pile capacity. If the pile penetrates in a plugged mode along a length which is significant in relation to its penetration depth in an unplugged mode, then these analyses are correct. If, however, the pile penetrated in a plugged mode for only a short distance at the end of its driving, then the effect on the gain in capacity with time will be smaller than that described in Sections 2.3.4 and 2.3.5, despite the fact that the friction along the lower portion of the pile is more significant than in the upper portions.
2.4 THE INFLUENCE OF A SOIL PLUG ON THE DYNAMIC BEHAVIOR AND ANALYSIS OF PILES

2.4.1 Introduction

Dynamic analysis of pile driving is performed for two major reasons:

(1) Prediction of the dynamic behavior of piles prior to construction. Such analysis enables the relation between observed dynamic resistance and anticipated static capacity to be established.

(2) Analysis of dynamic measurements obtained during driving in order to predict the pile behavior under static loads.

Both analyses are typically based on a numerical solution of the one dimensional wave equation. The solution utilizes mathematical models for the pile, the pile/soil interaction and, in the case of the predriving analysis, for the driving system also.

In the case of open-ended pile penetration, the validity of the theory and the models used must be reviewed in light of the various possible inner soil/pile interactions.

2.4.2 The 1–D wave equation and its underlying assumptions

Stress propagation in a pile during driving can be described by the following equation of movement (Paikowsky –1984):

\[ \frac{E_p}{A_p} \frac{\partial^2 u}{\partial x^2} - \frac{S_p}{A_p} \frac{1}{E_p} \tilde{f}_s = \rho_p \frac{\partial^2 u}{\partial t^2} \]  \hspace{1cm} (2.7)

in which:

- \( u(x, t) \) = longitudinal displacement of infinitesimal segment
- \( A_p, S_p \) = pile area and circumference, respectively
- \( E_p, \rho_p \) = modulus of elasticity and unit density of the pile material
- The friction stresses (\( \tilde{f}_s \)) are activated by the pile movement and under free
wave motion ($\ddot{s} = 0$) Eq. 2.7 becomes the familiar 1-D W.E.

Several assumptions, such as prismatic shape and homogeneity, are implicit in the development of the 1-D W.E. It is also assumed that, under loading, plane parallel cross-sections remain plane and parallel and that a uniform distribution of stress exists across each plane. It should be noted that the assumption of uniaxial stress does not include uniaxial strain. Thus, owing to Poisson's effect, there are lateral expansions and contractions arising from the axial stress which are associated with lateral inertia (Graff—1975). The additional friction term was included under the assumption that the soil is stationary (has no inertia effects) and the action of the friction forces does not violate any of the previous assumptions.

The different assumptions are generally satisfied. Piles are symmetric and usually homogeneous. In the case of combined material (e.g. reinforced concrete), as long as full compatibility exists and plane cross-sections remain plane, the material can be regarded as one of equivalent qualities. The external friction forces violate the assumption of uniform stress distribution. However, the ultimate friction stresses are very small in relation to the propagating stress (less than 1%) and their effect on the distribution can therefore be neglected. The basic frequency of typical stress pulses induced by driving hammers is less than a few hundred Hertz. The frequencies in which radial-inertia effects have to be included are on the order of several thousand Hertz (e.g. about 7KHz for a 60-inch pile). Therefore, the lateral inertia effect can be neglected as well.

Pile penetration is accompanied by a displacement of soil equal in volume to the penetrating object (neglecting the volume change of the soil). An open-ended pile induces a small displacement, and therefore the inertia effect of the soil can be ignored. A closed-ended pile causes large soil displacements, particularly in the vicinity of the pile tip.

The evaluation of the dynamic soil resistance is customarily done by assuming
damping to be viscous and a soil property. However, the damping values, which are used for the analysis of large displacement piles (particularly at the tip), differ from those used for small displacement piles driven in the same soil conditions. It is therefore assumed that the inertia effect of the soil displacement is taken indirectly into account through an empirical damping coefficient. Theoretical improvements of the driving analysis, which account for the effect of the soil inertia, are believed to be of small practical value. Such analysis is complicated and becomes meaningless in light of the uncertainty associated with the dynamic behavior of the soil. Moreover, the present analysis seems to lead to satisfactory results, especially when the soil parameters are backfigured to fit dynamic measurements.

From the aforementioned discussion, it can be concluded that the underlying assumptions used for the theoretical description of the pile condition during driving are adequate for most practical purposes.

2.4.3 The 1-D W.E. in light of pile plugging

In order to ensure the validity of the above analysis for open-ended pile penetration, three possible inner-soil pile interactions must be examined:

(a) Intrusion of soil into the pipe pile during its penetration in an unplugged mode. During unplugged penetration, the friction inside and outside can be cumulatively represented by one friction force. Analysis of the dynamic measurements obtained during driving of unplugged, open-ended, large diameter piles, using the above approach, was presented by Paikowsky (1982). Four static load tests of up to 1200 tons (about four times the load ever known to be used in proving this method) were found to be in 90% agreement with the predicted static capacity. Figure 2.7 presents a comparison between calculated and measured force at the top of a 60-inch unplugged pile. The calculated force was obtained by applying the measured velocity at the top of the pile to Eq. 2.7
and changing the dynamic soil resistance until the calculated force matched the measured force, as shown in Figure 2.7. From this resistance, the static force acting cumulatively on the inside and outside walls and tip of the pile was predicted to be 1090 tons, compared to 1200 tons measured in a static load test. It can therefore be concluded that when an open ended pile penetrates in an unplugged mode, a dynamic analysis can be conducted by considering the inner and outer wall friction as one.

(b) Partial plugging for which the soil inside the pipe is displaced a distance different than the steel pipe itself. This condition represents an intermediate stage between no plugging and full plugging (see Fig. 1.1b). A dynamic analysis under such conditions would have to be adapted to one state or another (unplugged or fully plugged) according to the stage of plugging.

(c) Fully plugged mode in which the inside soil is displaced the same distance as the steel pipe. When a pipe pile becomes fully plugged during penetration, the dynamic conditions differ from those previously described for closed and open-ended unplugged piles.

When a non-homogeneous pile is subjected to an impact stress, the assumption that a plane parallel cross-section remains plane implies equal strain throughout the cross-section. Therefore, a combined modulus of elasticity can be used. However, in a plugged pile only the steel pipe is subjected to the impact. When the propagating stress wave encounters the plug, it is subjected to an abrupt change in the cross-section. If full compatibility between the pile and the soil plug is assumed, the problem can be overcome by considering the different impedances of the two sections.

A simple dynamic evaluation of the plug behavior, assuming a rigid soil plug and full compatibility, can be developed as follows. When the soil mass accelerates with the pile wall, the inside shaft friction is given by:
The acceleration during driving is on the order of a few hundred g’s (say \( a_{cc} = 200g \)) and therefore the friction is about 100B ton/m² (B = pile diameter in meters). Calculations of friction stresses using Eq. 2.8 for large diameter pipe piles would indicate that such piles can never plug, especially when considering soil softening during pile driving. However, since such piles do plug (see Chapter 3), the assumption of full compatibility is unlikely. The soil plug, being incompatible, is subjected to radial shear stress propagation in addition to longitudinal compressive stress. An analysis of this system requires separate consideration of the inertia of the soil plug itself, even though the pile and the plug undergo equal displacement. This complex pile–plug behavior is not consistent with the simplified underlying assumptions of Eq. 2.7, so that fully plugged piles cannot be analyzed utilizing the 1–D W.E.

Analysis of the dynamic measurements obtained during driving of an artificially plugged pile was performed by Paikowsky (1982). Figure 2.8 presents a comparison between calculated and measured force at the top of a plugged pile. Even though a reasonable match is observed in Figure 2.8, the predicted capacity resulting from this analysis was unacceptable.

Several attempts were made to resolve this problem of the dynamic analysis of plugged piles. Heerma and DeJong (1980) modeled the inner soil plug as an independent impact mass–spring system subjected to limited friction induced by the inner pile wall. This system does not correctly model the actual mechanism as described above, and is therefore adequate only as long as the pile remains unplugged.

Other analyses utilizing the F.E. method (e.g. Simons –1985; Smith and Chow
—1982) failed to address the problem by using models which could not simulate the complex mechanism of the system.

It can therefore be concluded that when an open ended pile penetrates in a plugged mode, a complex mechanism of pile—plug interaction and independent plug behavior renders the prevailing dynamic pile analysis method invalid.
2.5 SUMMARY AND CONCLUSIONS

The plugging of a pile alters its mode of penetration, thereby affecting its static capacity and dynamic behavior.

1. A fully plugged open ended pile acquires the tip resistance of a closed-ended pile, resulting in a substantial increase in its bearing capacity.

2. The plugged pile penetration mode is accompanied by large soil displacements typical of closed-ended piles. The increased volume of displaced soil results in a more extensive region of pore pressure increase around a pile driven in clay. Therefore, the time required for the pile capacity gain increases in accordance with the longer time needed for the pore pressure dissipation. This time factor is of great significance and must be taken into account in pile design.

3. The inner soil in a fully plugged pile exhibits a complex behavior under the dynamic loads of driving. Analyses using the 1-D W.E. fail to model correctly the actual plug–pile interaction and so do not accurately predict field behavior or explain dynamic measurements.

This chapter has reviewed the importance of pile plugging in affecting the behavior, design and analysis of piles. As plugging was found to have a great influence, it leads to the conclusion that an investigation of the extent of this phenomenon is required. This must be followed by an understanding and an in-depth analysis of the mechanism of pile plugging.
Unit shaft friction outside and inside the pile
Shaft area of pile, outside and inside
Cross section area of steel tip
Total cross sectional area of the pile
Unit end bearing capacity

Fig. 2.1: Stresses Acting on Piles Under (a) Unplugged and (b) Plugged Conditions
The Effect of a Plug Upon the Static Capacity of a Pile in (a) Clay and in (b) Sand
CLOSED ENDED PILE

\[ \rho = 1 - \left( \frac{R_0 - t}{R_0} \right)^2 \]

\[ u_r/R_0 = \sqrt{(r/R_0)^2 + 1} - r/R_0 \]

OPEN ENDED PILE

\[ u_r/R_0 = \sqrt{(r/R_0)^2 + \rho} - r/R_0 \approx \frac{\rho R_0}{r} \]

Fig. 2.3: (a) Radial Soil Movements around Closed and Open – Ended Piles, (Randolph et al. –1979)
(b) Normal Radial Displacement for a 48 inch Pile.
Fig. 2.4: Decay of Excess Pore Pressure around Piles Driven in Boston Blue Clay (initial O.C.R = 2), after Carter et al. -1979.
Fig. 2.5: Predicted Pore Pressure Dissipation for Plugged and Unplugged Piles (Based on PLS measurements)
<table>
<thead>
<tr>
<th>TYPE</th>
<th>DIA.</th>
<th>LENGTH ft.</th>
<th>SOIL TYPE</th>
<th>LOCATION</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>steel H</td>
<td>14&quot;</td>
<td>191</td>
<td>Silt</td>
<td>Tappan Zee, N.Y.</td>
<td>Yang, 1956</td>
</tr>
<tr>
<td>closed ended steel pipe</td>
<td>6&quot;</td>
<td>22</td>
<td>Soft Clay</td>
<td>San Francisco</td>
<td>Seed &amp; Reese, 1957</td>
</tr>
<tr>
<td>steel pipe</td>
<td>12&quot;</td>
<td>60</td>
<td>Soft Clay</td>
<td>Michigan</td>
<td>Housel, 1958</td>
</tr>
<tr>
<td>steel pipe</td>
<td>24&quot;</td>
<td>316</td>
<td>Soft to Stiff Clay</td>
<td>Eugene Island</td>
<td>McCleland, 1969</td>
</tr>
</tbody>
</table>

**Fig. 2.6:** Comparison between Predicted Set Up Time for a 48 inch Pile with Field Data Collected by Vesic' - 1977
Fig. 2.7:

(a) Measurements of Force and Velocity vs. Time for an Unplugged Pile.
(b) Plot of Measured Force Compared to Force Obtained from TEPWAP Analysis.
(c) Comparison Between Measured Force near Tip of Pile and Calculated Force obtained from TEPWAP Analysis (Referring to Measurements at Pile Top), Paikowsky -1982
Fig. 2.8:
(a) Measurements of Force and Velocity vs. Time for a Plugged Pile.
(b) Plot of Measured Force compared to Force obtained from TEPWAP Analysis, Paikowsky—1982
CHAPTER 3

REVIEW AND INTERPRETATION OF EXISTING MEASUREMENTS

3.1 INTRODUCTION

Plugging has been shown to be of major importance in the behavior of pipe piles during and after installation. To forward understanding of the plugging phenomenon, existing data are interpreted and presented herein. The data were sorted into three categories. The first deals with the plug formation during penetration, in order to establish the existence of the phenomenon. The second and the third deal with the behavior of the soil plug under static and dynamic loads.

3.2 PLUG DEVELOPMENT DURING PENETRATION

3.2.1 Foreword

The extent of the plugging phenomenon can be assessed by using measurements of soil intrusion, relative to pile penetration. Three types of data sources are used for this purpose: (1) soil samplers; (2) small-scale models and (3) full scale piles.

The research dealing with soil samplers is naturally focused on the factors which control soil penetration into the sampler. Once the relations between a sampler and a pile are established, these data can be used to evaluate the development of soil plugs in piles.

As measurements on actual piles are costly and complicated, research on plugging was mainly carried out using model piles. The data obtained from these experiments describe the development of a soil plug and its influence on the pile behavior.

Lastly, data of soil penetration into full-scale piles are presented and interpreted in light of the conclusions derived from the first two sources.
3.2.2 **Analogy between soil samplers and open-ended piles**

Considerable experience exists concerning the different factors which control soil penetration during sampling operations. In order to identify the appropriate data for the purpose of plug development, an analogy between samplers and open-ended piles has to be established.

(a) **Geometry**

Figures 3.1 and 3.2 show typical sampler and pile geometries, respectively. Important geometrical characteristics used for samplers and their equivalents for piles are:

(a) The area ratio:

\[ C_A = \frac{B_w^2 - B_E^2}{B_E^2} = \frac{4t_1(B - t_1)}{(B - 2t_1)^2} = \frac{4(B_t^2 - t_1^2)}{(B - 2t_1)^2} \]

(b) The inside clearance:

\[ C_i = \frac{B_S - B_E}{B_E} = \frac{2(t_1 - t_2)}{B - 2t_1} \] \hspace{1cm} (3.1)

(c) The outside clearance:

\[ C_o = \frac{B_w - B_t}{B_t} \]

where:

- \( B \) = outside diameter of a pile
- \( B_t \) = outside diameter of the sampling tube
- \( B_w \) = outside diameter of cutting edge
- \( B_E \) = inside diameter of the cutting edge
- \( B_S \) = inside diameter of the tubing
- \( t_1 \) = wall thickness at pile tip
- \( t_2 \) = wall thickness at a segment above the pile tip
Thin-walled samplers have an outside clearance \( C_O = 0 \), and a requirement for an inside clearance ratio of \( C_I = 0.5 \) to 3\% (ASTM-74). Standard thin-walled sample tubes range in diameter from 2 to 5 inches, and have an area ratio of \( C_A = 11.4 \) to 12.8\% (ASTM), which is within the maximum recommended ratio of \( C_A = 15\% \) (Hvorslev-1949).

Tables 3.1A,B summarize typical dimensions and characteristics of pipe piles. Offshore piles are commonly constructed with a uniform outside diameter (B), and hence have an outside clearance \( C_O = 0 \). The tables also show that piles usually have a ratio of outside diameter to tip wall thickness (B/t\(_I\)) ranging from 26 to 96, where offshore piles have an average ratio of 34. This means that samplers with area ratios of \( C_A = 4.3\% \) to \( C_A = 17.4\% \) would be of major interest in the study of pile plugging.

A graphical representation of the relations of Eq. 3.1 between the ratio B/t and the area ratio is given in Figure 3.3, along with a typical ratio for a thick-walled sampler (SPT split barrel).

Figure 3.3, Table 3.1 and the ASTM recommendations show that geometrical similarities exist between pipe piles and thin walled samplers.

(b) **Penetration**

Piles are driven by hammers which operate at a rate of about 40 blows per minute (diesel or single acting steam/air hammers). The pile penetration per blow depends on the soil conditions, penetration depth, pile geometry and the driving system. During easy driving (e.g. offshore piles in the Gulf of Mexico), a driving rate of 2 to 3 cm/blow (10-15 blows/foot) is typical. The actual penetration takes place in pulses according to the stress propagation in the pile. The penetration time depends on several factors, but can be roughly estimated to last about 15 to 30 milliseconds under easy driving conditions. Based on these average data, typical rates of penetration (offshore piles/easy driving) can be calculated. Using these rates with
additional supporting information, the following can be established:

(1) Actual pile penetration is very rapid. Figure 3.4a presents a record of pile displacement in which the primary penetration takes place at an average rate of about 135 cm/sec (top) and 200 cm/sec (tip) (Paikowsky –1982).

(2) The penetration of a split spoon barrel during SPT was found to take place at rates similar to those of the piles, as presented in Figure 3.4b (Schmertmann and Palacios –1979).

(3) The average penetration speed of the pile, taking into consideration the time intervals between blows, is roughly 1.7 cm/sec. Figure 3.4c compares this rate of penetration to that recommended for deep, quasi–static penetration tests of soil (1 to 2 cm/sec, ASTM –1979).

(4) The rate of penetration for thin–walled sample tubes is not specified by ASTM standards, which direct one to "push the tube into the soil by a continuous and rapid motion, without impact or twisting". A penetration speed of 15 cm/sec was recommended by Hvorslev (1949). Small diameter samplers are pushed continuously by mechanical devices.

(c) Conclusions

Typical pipe piles share geometrical characteristics with thin–walled samplers, but not with thick–walled samplers (e.g. split barrel used in SPT).

The actual advance of the pile in the soil is similar to the dynamic penetration taking place during SPT. The average penetration speed of the pile, however, is similar to that of quasi–static penetration or pushed sampling. It seems, therefore, reasonable to use the known thin–walled sampling results in order to study the disturbance of soil and the plugging of open pipe piles.
3.2.3 Soil Samplers

Hvorslev (1949) presents measurements for plugging of thin-walled samplers for different driving methods: slow jacking, fast pushing, hammering and single blow, as defined in Table 3.2. These measurements basically consist of records of the sample length, \( L \), at various sampler penetrations, \( D \) (see Figure 3.1). If perfect sampling is achieved with perfect "cookie-cutter" penetration, a value of \( D = L \) would be expected. On the other hand, when \( L \) is less than \( D \), some plugging has taken place during penetration.

A better indication of plugging is provided by the specific (incremental) recovery ratio, \( \gamma \). This ratio is defined as the increment in sample length corresponding to a unit increment of sampler penetration, i.e. \( \gamma \) is the first derivative of \( L \) with respect to \( D \) (see Fig. 1.1). For perfect sampling conditions (unplugged sampler), \( \gamma = 100\% \). When \( \gamma \) decreases, more plugging takes place as the sampler penetrates. A constant value of \( \gamma \) between 0 and 1 means that a steady state partial plugging occurs. A value of \( \gamma \) in excess of unity was observed during some thick-walled sampler penetrations. For samplers with inside clearances greater than or equal to zero, \( \gamma \) in excess of unity means that part of the soil ahead of the cutting edge enters the sampler. Since thick-walled samplers have a large area ratio (e.g. \( C_A = 75\% \) for SPT sampler), the displaced volume of soil ahead of the cutting edge is significant in relation to the volume of soil ahead of the opening.

Figure 3.5 shows plugging measurements as relationships of \( L \) vs. \( D \) and \( \gamma \) vs. \( D \). The data of Fig. 3.5 correspond to a brass thin-walled sampler with a uniform wall thickness (no cutting shoe; i.e. \( C_o = C_i = 0 \)) penetrating into soft varved clay.

In examining Figure 3.5 the following is noted:

1. The effects of the driving method on plugging can be observed by comparing curves A, B and C to the ideal sampling line R, corresponding to \( L = D \). Clearly, fast
pushing (A) leads to a penetration mode which is closest to unplugged penetration, followed by single blow penetration (curve B), and finally hammering (C). However, viewed globally on the basis of final sample length (35, 32 and 29 in. for A, B and C, respectively) compared to final sampler penetration (about 47 in.), differences due to the method of driving are not as significant as the deviation from the expected perfect sampling. This means that significant plugging took place with all three methods of driving.

2. A clearer indication of when and how plugging takes place can be obtained from the relationship between $\gamma$ and D for various driving methods (A', B' and C'). This comparison indicates that:

(a) During early penetration of the sampler ($D < 5$ in. or $D/B < 2.5$) all three driving methods lead to $\gamma = 100\%$, or unplugged penetration (line R').

(b) The effects of the driving method are clear at intermediate penetration depths ($5 < D < 30$ in. or $2.5 < D/B < 15$). Partial plugging of the hammered sample (C') starts at an early stage ($D = 7$ in.), followed by the single blow sample (B', $D = 10$ in.) and the "fast—pushed" sample (A', $D = 22$ in.).

(c) At later penetration stages ($D > 30$ in., or $D/B > 15$), all three driving methods lead to roughly the same mode of penetration with partial plugging of approximately $\gamma = 40\%$.

3. The important results derived from the $\gamma$ vs. D plots are not easily seen from a plot of L vs. D.

4. Reasonably reliable $\gamma$ vs. D plots can only be obtained if a sufficient number of measurements of plugging are recorded during penetration.

The results in Figure 3.6 correspond to a steel sampler with an inside clearance $C_i = 1.2\%$ and an area ratio $C_A = 10\%$. In examining these results it can be noted that:

1. Based on the L vs. D plot, the effect of the driving method on plugging is
negligible. (Compare curves D and E to the ideal sampling line, R).

2. Based on the $\gamma$ vs. D plot, the effect of various driving methods on plugging is clear and significant (see curves D' and E'). During the first 35 in. ($D/B < 17.5$) both driving methods lead to $\gamma = 100$ to 93%, i.e., nearly unplugged penetration (line R'). After 35 in. penetration, plugging of the slow-jacked sampler starts to increase almost linearly from 7% ($\gamma = 93\%$) to 53% plugging ($\gamma = 47\%$) whereas the fast-pushed sample continues to penetrate in a perfect unplugged mode ($\gamma = 100\%$).

The data in Figure 3.6 were obtained under controlled conditions (soil type, penetration depth, sampler diameter, area ratio and driving method), and hence the effect of sampler geometry (as expressed by the inside clearance $C_i$) can be determined. By comparing curves A and A' in Fig. 3.5 to E and E' in Fig. 3.6, respectively, it is clear that the sampler with $C_i = 0$ has a greater tendency to plug compared to the sampler with $C_i = 1.2\%$, which penetrates in a perfect unplugged mode. This means that a small inside clearance is sufficient to significantly decrease the probability of pile plugging during penetration in clay.

3.2.4 Small-scale models

Plugging tests using small-scale model piles enable creation of a controlled environment at an affordable cost, in comparison to full-scale testing.

A series of tests on closed and open-ended model pipe piles was carried out by Kishida (1967a, 1967b). Piles of 50 cm in length, ranging in diameter from 1 to 15 cm, were pushed slowly (0.2 cm/sec) into a very loose dry sand ($D_r = 15.3\%$). The following measurements were provided: (1) resistance force vs. depth of penetration; (2) height of inner soil at penetration intervals of 5.0 cm; (3) dynamic small size (6 mm dia.) penetration test inside the shaft measuring the blows per each 5.0 cm penetration

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1See Appendix II, Section II-2b for remarks concerning the sand’s low relative density.
(A correlation was established between the relative density of the sand and the number of blows; see Fig. AII-2 in Appendix II); (4) load vs. settlement relations at the final penetration depth of 45.0 cm.

A detailed analysis of Kishida’s data is presented in Appendix II. The presented results are further used in subsequent chapters. A summary of Kishida’s results with regard to plug formation is presented in Figure 3.7. The penetration depth, $D$, vs. the inner soil height for different pile diameters, $L$, is shown in Figure 3.7a. Each of the curves in Figure 3.7a is an averaged result of several tests. This information indicates that the smallest diameter pile (1 cm) plugged immediately and developed a plug of 1.5 cm in its final penetration. This is 1.6 times its inside diameter (ID). The piles ranging in diameter from 2.5 to 7.5 cm plugged partially or fully at a penetration depth of about 15 cm with final $L/ID$ ratios varying from 2.88 to 5.58. The two larger diameter piles of 10.0 and 15.0 cm did not plug. Indications of partial plugging, however, were observed in the last penetration stages of the 10.0 cm diameter pile (when $L/ID \geq 4.5$).

The state of the sand inside the shaft of two piles, as reflected by the miniature dynamic penetration test results, is shown in Figure 3.7b. A short and stiff plug was formed in the smaller diameter pile. The larger diameter pile, on the other hand, developed a looser inner soil plug of a height approximately equal to the final penetration depth. In both piles, a denser soil layer (in relation to other areas of the soil plug) exists in the lower portion of the pile, some distance (from about 0.5 ID) above the opening.

A series of tests on model pipe piles were also carried out by Klos and Tejchman (1977) and Szechy (1959). Both researchers drove the piles using gravity hammers and measured the length of the the soil plug during penetration.

The plug length and the specific recovery ratio ($\gamma$) vs. depth of penetration for similar pile diameters (46.5 to 60.0 mm) used by the different researchers, are
presented in Figure 3.8. The data of Fig. 3.8 indicate that the soil plug development is affected by the pile geometry and installation method. The effect of the soil type and condition is unclear from Fig. 3.8.

Some strong similarities exist between the results obtained by Szechy and Klos and Tejchman, to that of Kishida, presented in Fig. 3.7. In each of the cases for a certain combination of pile geometry, soil, and driving conditions there is a particular pile diameter for which a firm plug will not be created and the plug length increases almost linearly with the penetration depth. Fig. 3.7a indicates that piles having diameters smaller than 7.5 cm developed a steady plug length once they reached a certain penetration depth, e.g. Fig. 3.8 shows that the 5.0 cm pile created a firm plug at a penetration depth of about 25 cm where the recovery ratio dropped to about 20%. In piles larger than or equal to 10 cm, the plug length continued to increase constantly as penetration progressed.

The data of Fig. 3.7b are similar to those reported by Szechy. The inner soil mass of the plugged piles was stiffer and denser than in the unplugged. This effect was enhanced within a distance of about 2 to 3 diameters from the pile tip.

3.2.5 Full-Scale Piles

(a) Introduction

Limited information is available about plugging of actual piles, of which only a small portion includes continuous plugging records. The lack of data can be attributed to:

1) Unawareness of the consequences of pile plugging.
2) Interpretation of plugging by referring to the average plug length rather than to the incremental changes.
3) Complications that plug measurements cause in pile driving operations.
(b) Large model piles

Tests on large scale model piles were carried out by the Norwegian Geotechnical Institute (Karlsrud and Haugen -1985a; NGI -1981). Two piles of diameter \( B = 15.5 \) cm, wall thickness \( t = 4.5 \) mm and length 5.15 m, one closed-ended and the other open-ended, were pushed into overconsolidated clay. An analysis of the plugging measurements is presented in Figure 3.9 in the form of plug length \( L \) and specific recovery ratio \( \gamma \) vs. pile penetration, \( D \). To a depth of 16.5 diameters, the penetrating soil had a surface elevation higher than the ground elevation at the site. From the specific recovery ratio, it is evident that partial plugging begins at a depth of 11 diameters and full plugging is achieved at a depth of 21 diameters. With further penetration, additional soil (\( \gamma < 15\% \)) enters the pile; however, towards the end of the penetration, a tendency for full plugging is again observed.

The resistance to penetration [pushing at a rate of 0.25 cm/sec (open) and 0.075 cm/sec (closed)] of the open and the closed-ended piles was used to prepare Figure 3.10. The ratio of the static load test results (performed about 1.5 months after installation) at the final penetration depth was added. The load test results indicate an 11% increase in the capacity ratio in relation to the ratio of the resistances to penetration at the final depth. The range of resistance ratio from the plugging analysis of Chapter 2 (see Section 2.2, Figure 2.2 and Appendix I) was predicted before these data became available, and was added to Figure 3.10. The data of Fig. 3.10 show:

1. The ratio of open to closed-ended pile resistances and the plugging depth during quasi-static penetration fits the predicted behavior excellently.

2. When plugged, the open-ended pile developed a resistance equal to that of the closed-ended pile.

3. Figures 3.11 and 3.12 present the pore pressure measurements with time after installation of the closed and open-ended piles. In reference to the data of Fig. 3.10, where the tested capacity of the open-ended pile was greater than
that of the closed-ended pile, the following can be observed:

* The open-ended pile was tested 56 days after installation. A piezometer one diameter away from the pile shaft recorded an increase of about 40KN/m² in the pore water pressure at the end of the installation, which reduced to about 7.5 KN/m² in about 3 days.

* The closed-ended pile was tested 33 days after installation. A piezometer one diameter away from the pile shaft recorded an increase of pore water pressure of about 65KN/m² at the end of the installation, which decreased to about 7.5KN/m² in 4 days.

* The transition in the penetration mode of the open-ended pile, from that of unplugged to fully plugged, brought about changes similar to those created by the closed-ended pile. This subject is further considered in Section 3.4.

(c) Offshore piles

The common view of offshore pile plugging

Kindel (1977) presented plugging measurements obtained during driving of large piles (30 ≤ B ≤ 48 inch) in the Gulf of Mexico. The pile penetration depths were in the range of 120 to 400 ft., most likely in soft clays (the common subsurface soil condition in the Gulf of Mexico). Detailed measurements of soil elevations inside 8 piles driven at 2 sites are presented in Figure 3.13. The relations in Fig. 3.13 were obtained by plotting the penetration depth vs. the distance from the mudline to the top of the soil inside the pile. The measurements were taken usually at the end of each pile add-on section, using a steel tape and a ballast. The complete plugging line in Fig. 3.13 indicates the conditions for which the inner soil moves the same distance as the pile penetration. In reference to the data of Fig. 3.13, Kindel stated:
"the plug movement pattern is at times erratic"; and "some measurements indicate an upward plug movement with respect to the former plug location".

In order to reduce some of the irregularities in the individual plug measurements, Kindel processed the data as shown in Figure 3.14, performing the following steps:

1. Averaged the plug movement for all piles at each site, e.g. the 3 piles of site ‘A’ and the 5 piles of site ‘C’ in Fig. 3.13 are shown as a single continuous line in Figure 3.14 a–1 and a–2, respectively.

2. Calculated for each measurement depth the corresponding Plug Length Ratio (PLR), which is defined as the length of the soil column inside the pile over the total penetration, e.g. the 6 dots of Fig. 3.14 b–1 were obtained from the 6 measurements for the average curve of Fig. 3.14 a–1. The PLR for the 100 foot penetration is \((100-14.5)/100 = 0.855\), and so on.

3. Extrapolated the curve of PLR vs. penetration in order to estimate the PLR at the end of the pile run (free run)

Pile run is defined as the pile penetration due to the dead weight (pile plus hammer) prior to driving, e.g. the piles of site ‘A’ had an average free run of 56 feet. The associated PLR, extrapolating the curve of Fig. 3.14 b–1, is 0.81.

4. The PLR at the end of the pile run was then used to calculate the length of the plug. This stage was referred to as the initial condition (where pile driving started), and all subsequent plug measurements were adjusted accordingly, e.g. for the piles of site ‘A’ the average PLR of 0.81 [see Step (3)] means that the average plug length at the end of the free run is \(56 \cdot 0.81 = 45.4\) ft. The average PLR for the first measurement [see Step (2)] is 0.855, which means that the plug length at this stage was \(100 \cdot 0.855 = 85.5\) ft. The plug movements at the end of the free run and the first stage were therefore 10.6
and 14.5 ft., respectively. If the end of the free run is referred to as initial stage, then the 'adjusted' curve of Fig. 3.14 a–1 starts at pile penetration of 56 feet and 0 plug movement, and progresses to the first measurement point for which the penetration was 100 ft. and the adjusted plug movement $14.5 - 10.4 = 3.9$ feet.

5. The adjusted plug lengths were used to correct 'new' PLR's which were then plotted against the penetration depth, e.g. the 'adjusted' PLR for the 100 ft. penetration is $(100-3.9)/100 = 0.96$.

Correction of the data obtained from 3 sites according to the above procedures (Fig. 3.13 shows data of only 2 sites out of the 3) leads to 'corrected average' PLR's of 0.97, 0.96 and 0.93 as shown in Fig. 3.14 b–1, b–2 and b–3.

From the relations of Figure 3.14 Kindel concluded the following:

1) The plug movements tend to stabilize at greater penetration depths as indicated by the increase of the actual PLR with depth, shown in Figure 3.14b. This may be due to a true stabilizing of the plug. However, it is most likely due to a decrease in inside volume resulting from an increase in the pile wall thickness. Correction for this effect could not be made by Kindel, as the exact pile section sizes were not available.

2) Most of the plugging occurs during the free run of the pile prior to driving, e.g. at site 'A' the average pile free run was 56 ft. and the difference in soil elevations was 11 ft. The final penetration was about 400 ft. with an additional difference in soil elevation of only 9 ft.
The PLR values at final penetration of 268² piles from 62 different sites in the Gulf of Mexico were sorted by Kindel in a statistical histogram, as shown in Figure 3.15a. A normal distribution was fitted to Kindel's block diagram.

Figure 3.15 indicates that:

1) The mean PLR for all piles is 0.92.

2) About 10% of the piles have a PLR < 0.8 and 2% of the piles have a PLR < 0.7. Note that a PLR = 0.8 for a 300 ft. pile means that the difference between outer and inner soil elevations is 60 ft.

3) A significant percentage of the piles (12%) have a measured PLR in excess of unity. This means that after driving, the soil inside the pile is at a higher elevation than the mudline. As mentioned earlier, this might be caused by changes in the wall thickness of the pile, and hence suggests that the degree of plugging might actually be higher than that indicated in Fig. 3.15a.

4) About 47% of the cases fall in the range of PLR values above 0.95. Based on simplified analysis, Kindel refers to this range as one for which no significant plugging action takes place.

Kindel's data present valuable information for plugging of large pipe piles in soft clay. He concluded that:

"It seems reasonable to assume that the majority of the smaller PLR values are due primarily to plug movement occurring during pile run. Further evidence is required to shed light on those situations where significant movements occurred during driving. The preceding analysis of soil plug data indicates that for the vast majority of piles significant plugging action does not occur".

²The total number of piles obtained by using the vertical scale in Fig. 3.15 does not add up to 268 piles. It is assumed that the error is the vertical scale or the total number of piles, but not the distribution.
Kindel's conclusions represent the common view of the profession regarding plugging of piles in clay and of integrating the available data.

The analysis of the plugging measurements in the form presented for the large model piles in Fig. 3.9 clearly explains the limitations of the aforementioned plugging analysis by Kindel. Assume that the pile of Fig. 3.9 would have been driven to a depth of 3.2 m only. According to Kindel's analysis, at this penetration depth the PLR is 94% and the pile is unplugged; however, at the same time a recovery ratio of $\gamma = 0$ indicates that the pile is fully plugged.

While this example clearly demonstrates the possible error of Kindel's conclusions, representing the common view of the profession, only actual data of offshore piles enables clarification of the subject.

Analysis of case histories of plugging during pile driving

Data on pile plugging during driving were provided by Heerma Engineering Services, Leiden, the Netherlands, at ten different sites in the Gulf of Mexico where 60 piles were driven. The pile diameters varied from 36 to 60 in., total pile lengths from 300 to 750 ft., and the penetration depths from 200 to 500 ft., approximately. Table 3.1B provides more detailed information on the piles. This section presents results of a data analysis conducted in order to:

1. Observe the nature of actual pile plugging, e.g. partial vs. total plugging.
2. Identify conditions leading to pile plugging.
3. Check the validity of Kindel’s conclusions.

Typical Soil Conditions

Soil conditions at the ten sites considered herein exhibit strong similarities. In order to investigate the plugging phenomenon and to achieve simplicity and clarity, the deposits at these sites are classified into two major groups depending on whether
sand layers of sufficiently meaningful thickness (more than 5 ft.) exist in the upper 100 ft. below mudline:

Group A: Deposit involving sand layers in the upper 100 ft. This group includes 5 of the 10 sites, and is divided into two categories:
Category A-1 is characterized by various soft to stiff layers separated by a relatively large number of thin layers of sands and silts (less than 30 ft.), Figure 3.16a.
Category A-2 is characterized by thick sand layers near the mudline, Figure 3.16b.

Group B: Deposits involving no sand layers in the upper 100 ft. Figure 3.16c presents a typical group B profile.

The Plugging Measurements

The plugging measurements considered herein consist basically of the difference in soil elevations inside and outside the pile. The soil level inside the pile is determined by means of a wire attached to a ballast weight which is dropped into the pile at depth intervals determined by the need for welding (add-on) of pile segments. Therefore, available measurements have a limited accuracy, since they do not have the sufficient resolution to make a careful pile plugging analysis, as was done in the case of the large model pile of Fig. 3.9, and hence conclusions here involve some uncertainties.

Analyzing plugging measurements using the inner pile volume

Offshore piles have a variable wall thickness (see Figure 3.2 and Table 3.1B) that tends to complicate and obscure the interpretation of plugging measurements. To

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3Due to wire elongation or slack, uncertainties in reference elevation and mudline, etc.
reduce the difficulties caused by these variations, the given wall thickness distribution along the pile was used to compute the difference between inner and outer soil elevations, assuming no change in volume of the entering soil. Results are obtained for three separate conditions regarding the volume of soil that enters into the pile at the elevation of the cutting shoe, Figure 3.17.

a. The soil inside the pile at the cutting shoe is not deformed by pile penetration, i.e. all soil ahead of the cutting shoe wall is pushed outside the pile.

b. Half the soil ahead of the pile wall (at the cutting shoe) enters into the pile.

c. The outer soil suffers no distortions due to pile installation. This means that all the soil ahead of the pile wall eventually enters into the pile.

Table 3.3 presents estimates of plug elevations based on conditions, a, b and c for a 60" diameter pile with typical geometry, as specified in Table 3.4.

Typical Plugging Observations and Interpretations

Figure 3.18 presents typical plug elevation measurements for four piles driven to a penetration depth of 300 ft. at a site where the soil deposit profile is given by A–1 in Figure 3.16a. The piles had a diameter B = 60 in. and segment dimensions as given in Table 3.4.

Predictions based on mode of soil penetration into the pile (a, b and c given in Table 3.3) are all plotted in Figure 3.18, where it is clear that:

1. Piles at this site driven to a penetration depth of 300 ft. (D/B = 60) do not exhibit any tendency to plug because differences between inner and outer soil elevations are very small (± 5 ft.). In other words, results are far from the complete plugging line determined by the condition that plug elevation equals the penetration depth.
2. Most data points indicate that the soil elevation inside the piles is lower than the mudline elevation after a penetration depth of 200 to 250 ft., with the exception of two piles.

3. Experimental data fall within the band of uncertainty given by modes of soil penetration conditions 'a' and 'b' (discussed above, see Figure 3.17) with the exception of one doubtful measurement.

Figure 3.19 presents typical plugging data during driving of four 42 in. diameter piles to a penetration depth of 240 ft. at a site where the soil deposit profile is given by A–1 in Fig. 3.16a.

The pile geometry is that given in Table 3.4 for B = 42 in. Results in Fig. 3.19 are very similar to Fig. 3.18, and again show no indication of plugging. Most of the measurements, however, are near the estimated mode of soil penetration condition 'a' (see Figure 3.17).

Figure 3.20 presents plugging data during driving of eight 48 in. diameter piles to penetration depths of 365 to 375 ft. The soil deposit at the site is given by profile B in Figure 3.16c, which shows that it contains a deep clay layer to a depth of 328 ft.

The piles can be classified into two groups (4 piles each), differing by their geometry starting at 260 ft. above the tip. The detailed geometry of the piles in groups A and B is given in Table 3.4 for B = 48 in.

We note in Figure 3.20 that:

1. Measurements of group A piles are slightly different from group B piles, indicating that slight changes in wall thickness (e.g. from \( t_5 = 1.250 \text{ in.} \) to \( t_6 = 1.375 \text{ in.} \) for group A vs. \( t_6 = 1.500 \text{ in.} \) for group B) 260 ft. above the tip can make some difference in plugging behavior.

2. In spite of the scarcity of measurements, evidence of some pile plugging is observed from the first depth of penetration readings. Measurements
on three piles show a difference in plug elevation of 20 to 25 ft. at a penetration depth of 146 ft.

3. All piles had a typical free run of about 100 ft., and then were driven about 46 ft. to a depth of 146 ft. below mudline (see Figure 3.21). Since:
   (a) subsequent plugging measurements indicate no plugging tendency during dynamic pile driving (before a penetration depth of 300 ft. is reached);
   (b) Kindel's data, as previously discussed, suggests substantial plugging during the free run;
   (c) The quasi-static analysis of section 2 (which was confirmed in Figure 3.10), suggests pile plugging under quasi-static penetration at a depth of about 20 D/B;

it is safe to assume that some plugging took place during the first 100 ft. of free run or quasi-static penetration under the action of the pile and hammer weight. Therefore, line q.s. (quasi static) presents the best estimate of the plugging history of the piles up to 146 ft. penetration: (I) during the initial penetration (0 to 75 ft.) piles penetrate in the mode of soil penetration 'a'; (II) plugging probably took place at a depth of 75 to 80 ft., and line q.s. thus becomes roughly parallel to the complete plugging line; (III) Further dynamic pile penetration (from 100 to 145 ft.) probably took place with the mode of soil penetration 'a'.

Although the above estimated plug behavior was reconstructed from the known measurements at the depth of 145 ft. working backwards, it provides an excellent fit to the quasi-static plugging predictions of Section 2.2. (see Figure 2.2a). The estimated behavior led to pile plugging at a depth of 75 ft, equal to 19 D/B, as predicted (see also Figure 3.10 and Appendix I).

4. A very clear change in plugging tendency takes place for all piles at a
penetration depth in excess of 300 ft. \((D/B \geq 75)\). Lines joining data points at a depth of 300 ft. to the final penetration depths \((365 - 375 \text{ ft.})\) have a slope that is close to the complete plugging line (average plugging of 67% and 74%, which represent the difference between the completely unplugged mode of \(\gamma = 100\%\) and the recorded recovery ratios of 33% and 26% for groups A and B, respectively). The actual penetration mode of the piles at a depth greater than 300 ft. was therefore either in (a) a steady partial plugging mode, as illustrated by the lines in Figure 3.20 (and suggested by sampler penetration data in Figure 3.6), or: (b) a cookie-cutter mode (mode of soil penetration ‘a’) to a depth of 325 ft., followed by penetration in a completely plugged mode for about 45 ft.

5. Figure 3.21 shows the driving record of a typical pile at this site, where we note that:

(a) The penetration resistance expressed by the number of blows per foot of penetration, \(N\), exhibits a slight change due to plug formation. However, in view of other factors contributing to changes in \(N\), it is believed that, at least in clay, the detection of plugging by means of changes in \(N\) is not practically feasible.

(b) For six of the eight piles, the driving resistance in the granular material \((D = 365 \text{ ft.})\) was 3.4 times the driving resistance in the overlying clay. A separate dynamic analysis (not presented here) shows that this increase in resistance can only be explained by the fact that the pile was already plugged as it encountered the sand.

Statistics of pile plugging

An evaluation of the frequency of occurrence of pile plugging is important in determining the significance of the problem in offshore piles. It was previously
demonstrated that Kindel's statistical analysis of Figure 3.15a can be misleading in estimating the importance of plugging on the behavior of piles.

In view of the present scarcity of information and poor understanding of plugging mechanisms, it is believed that more pertinent questions regarding the importance of plugging are:

1) What is the percentage of piles where plugging should be of concern?
2) What are the pile and soil characteristics (depth, diameter, soil types, etc.) that have a significant effect on pile plugging?

In an attempt to answer these questions, based on the treatment of plugging presented earlier, it can be noted that:

1) The rational utilization of plugging data requires that locations along the pile (i.e. elevations) where penetration took place in a plugged mode be known.
2) Since penetration can take place in a partially plugged mode, the degree of plugging at various penetration depths needs to be known, for completeness.
3) In view of the incomplete nature of existing data on plugging, a more practical classification is suggested in order to evaluate the importance of plugging:

   (a) A pile is considered to be 'plugged' if sufficient data exists to show that along a significant penetration distance\(^4\), the incremental soil column entering the pile is less than 50% of the increment in penetration depth (i.e. if the pile penetrates with an incremental recovery ratio \(\gamma \leq 50\%\));

   (b) A pile is considered to be 'unplugged' (or to penetrate in a 'cookie-cutter' mode throughout its length) if sufficient measurements exist to show that the plugging criterion (\(\gamma \leq 50\%\) at any depth) is very unlikely to have taken place; and

   (c) A pile is considered to be 'most likely plugged' if measurements strongly

\(^4\)Say 10% of the total penetration length of the pile.
suggest that the plugging condition (γ ≤ 50% at any depth) must have taken place during a significant distance of pile penetration.

For example, if only one plugging measurement is recorded after driving a pile to its final penetration, and it indicates a significant difference in elevation between the inner and the outer soil (say more than 5–10%), this pile would be classified as 'most likely plugged'. This is because the pile penetrated at least 10% of its final penetration depth in a 'plugged' mode with a recovery ratio of 50% or less.

Table 3.5 and Figure 3.15b present results of a statistical analysis conducted on the plugging data provided by Heerma Engineering Services for 60 piles at 10 sites in the Gulf of Mexico. The diameter, B, and the maximum penetration depth, Dmax, of the piles are in the range 36 < B < 60 in. and 200 < Dmax < 500 ft. The average final PLR value (at final penetration) for the 60 piles in Figure 3.15b is the same as obtained by Kindel (Fig. 3.15a) for another set of 268 piles (30 < B < 48 in, 120 < Dmax < 400 ft) at 68 sites, also in the Gulf of Mexico. This suggests that other conclusions derived from the presented analyses are probably applicable to a wider range of piles and sites in the Gulf of Mexico than those considered. However, a comparison of Figs. 3.15a and b indicates that the variability of Kindel's data is greater than that of the presented data.

Table 3.5 also shows that:

(1) Based on the classification described above, and when the whole set of 60 piles is considered as one entity, results indicate that 15% of the piles are plugged and an additional 25% of the piles are most likely plugged. This means that concern over plugging should extend to 40% of the piles, whereas 60% penetrated in a 'cookie-cutter' mode.

(2) Plugging statistics change drastically if the relative penetration depth of the
piles, as expressed by the ratio $D_{\text{max}}/B$, is taken into consideration. For relatively long piles ($D_{\text{max}}/B > 75$) concern over plugging should extend to 73% of the piles, whereas for relatively short piles ($D_{\text{max}}/B < 75$) this percentage is only 21%.

The degree of concern over pile plugging depends on the location below mudline where plugging takes place, because the contribution of a given pile length to the total capacity of friction piles increases with depth below mudline. For example, the contribution of a 30 ft. pile segment to the total capacity of a 300 ft. friction pile is about 1% when the segment is right underneath the mudline, whereas it is closer to 20% if the pile segment is located right above the tip.

(d) Conclusions

Based on the presented analysis of plugging data on 60 piles at 10 sites in the Gulf of Mexico, the following conclusions may be advanced:

1. Pile plugging evaluation from existing measurements is complicated and involves uncertainties. However, it is clear that the pile plugging phenomenon is of frequent occurrence and is of greater significance than that presently accorded it by the profession.

2. More reliable measurements of pile plugging are definitely needed. In particular, complete (continuous) records of the elevation of the soil inside the pile should be obtained vs. penetration depth. Without such measurements, which can be obtained at minor additional efforts and costs, no definite conclusions regarding pile plugging can be reached.

3. A distinction between the initial penetration stages of a pile during its quasi-static penetration under its own weight (pile free run) vs.

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5Assuming a linearly increasing skin friction with depth (in clay).
subsequent penetration due to dynamic driving is required. Available data suggest that the likelihood of plugging during dynamic driving is less than during quasi-static penetration.

4. The plugging during quasi-static penetration in clay can be predicted with a reasonable reliability. The suggested analysis of Section 2.2 (see Appendix I and Figure 2.2) was found to fit the experimental data of Figures 3.10 and 3.20.

5. The plugging during dynamic penetration was found to differ substantially from that during quasi-static penetration, proving correct the conclusions of Chapter 2 regarding the complications involved with the plugging mechanism during dynamic driving.

6. For situations where free run penetrations are small (e.g. at sites involving sufficiently thick sand layers in the upper 75 to 100 ft.) no plugging takes place during pile free run penetration.

7. For typical offshore pile geometries utilized in practice, there appears to be a critical depth for pile plugging under conditions of dynamic driving. For the sites analyzed in the Gulf of Mexico this critical depth appears to be in the neighborhood of 75 times the pile diameter.

8. The limited existing data on plugging suggests that plugging tendency is more pronounced after the driving delays required for add-on segments to be welded.

9. The wall thickness variation along the pile is believed to represent an important parameter controlling plugging. Piles with a cutting shoe wall thickness in excess of the wall thickness of the pile at higher elevations have less tendency to plug in clays than piles with a uniform (constant) wall thickness.

10. Pile plugging during quasi-static and dynamic driving appears to be a
consistent phenomenon for given soil and pile conditions and hence rational, systematic, and useful prediction methods of plugging are conceivably possible to achieve.

11. Cookie-cutter penetration and complete plugging represent reasonable limiting penetration modes for analysis purposes. Actual penetration of open ended piles (in either quasi-static or dynamic regimes) is believed to be more complicated to analyze because it can also take place under unsteady pile plugging conditions.
3.3 THE EFFECT OF PENETRATION ON THE INNER SOIL PLUG

3.3.1 Introduction

The available data regarding the influence of the plug development on the state of the soil are presented. Elucidation of the effects of penetration on the inner soil comprising the plug leads to a better understanding of the mechanism of plug development and behavior. This knowledge is integrated into the proposed model for the plugging mechanism (Chapter 9).

3.3.2 Deformation and stress changes in sampled soil

Figure 3.22, which was prepared from data presented by Hvorslev (1949), shows three different patterns of soil deformations inside a sampler and their causes. The upper zone of soil in the sampler exhibits a convex upward curvature. This curvature is attributed to a bulb of reduced stresses, due to removal of the soil from inside the borehole. The decrease in vertical stresses in respect to the surrounding medium brings about swelling and possible soil failure under the opening.

The central zone of soil results from ‘perfect sampling’, where the sampler underwent continuous penetration into an undisturbed soil with a recovery ratio \( \gamma = 100\% \). The soil in the lower zone exhibits a concave downward curvature, attributed to a bulb of increased vertical stresses due to penetration in a closed/plugged mode. This leads to formation of a soil cone in advance of the plugged sampler or pile (BCP Committee –1969, 1971), which creates a deflection of the soil layers ahead of the penetrating column.

The two cases where the sampler penetrates soil layers previously subjected to either a decrease (due to removal) or increase (due to oversampling) in vertical stresses have limited practical relevance to piles. Similar deformations, however, are expected to take place during plug formation due to similar stress fields. These deformations
may be due to:

1. Entrance of excess soil — The penetrating pile displaces a volume of soil equal to its own volume (neglecting volume changes). This displaced soil can either penetrate to the inside or flow outside the pipe (different approaches to this subject were presented in Section 3.2.5). During the initial penetration, it can be assumed that a smaller internal resistance increases the tendency of the soil to enter the pile. This penetrating soil causes an upward convex distortion in the upper zone of the soil in the sampler, as presented in Figure 3.23a,b (Hvorslev -1949). These figures refer to soil penetration in a thick–walled sampler. However, the specific recovery ratios they present can be routinely found during initial stages of plug formation (e.g. see Figs. 3.9, 3.13).

2. Influence of inside wall friction — The inside wall friction can cause large convex distortions in the soil, mostly confined to a drag zone near the interface with the sampler wall, as presented in Figure 3.24a. The parabolic distortions of Figure 3.24b were obtained by hammering a tube with a small area ratio, producing conditions similar to those of pile driving.

3. Deflection and failure of the soil below — The inside wall friction also governs the pressure on the soil below the sampler. When the inside friction increases, it reduces the soil penetration (partial plugging). When the cumulative inside friction exceeds the soil bearing capacity at the tip, the layers below the sampler deflect downward, stretching and thinning out before entering the sampler (Hvorslev -1949). Figure 3.25a presents the beginning of the downward deflection of the soil layers. Figure 3.25b shows the distortions caused by overdriving (complete plugging; H refers to the penetration of the sampler below the bottom of
the borehole during the actual sampling). This subject is further discussed in Chapters 7 and 9, where it is suggested that the plugging which is required to produce the conditions of Figure 3.25b (according to Hvorslev) would not allow further soil penetration, and therefore cannot explain the concave deflection of soil along the sampler away from the tip. It seems that the increased friction is conducive to the development of 'arching' where the stresses and deformations produce the maximum resistance to the penetration.

The disturbance of soil samples entering a sampler was investigated theoretically by Azzouz and Baligh (1984). Considering the clay to behave as an inviscid incompressible fluid under an axially symmetric steady state penetration, Azzouz and Baligh could reduce the process of penetration to a flow problem where the soil 'particles' move along streamlines around the pile. Estimating the velocity field (satisfying conservation of volume and boundary conditions) enabled them to estimate soil deformations, strain rates along stream lines, and strain path of different soil elements. Figures 3.26a,b present the predicted deformation pattern and the contours of maximum shear strain (respectively) for an open-ended pile with a diameter to wall thickness ratio of $B/t = 20$. From their research on soil deformations due to open-ended pile penetration, Azzouz and Baligh concluded the following: (1) soil distortion is evident only in the immediate vicinity of the pile wall (within a region of about 3t width); (2) a steady state of soil deformation is reached after a distance of approximately 20t (about one diameter) behind the tip for piles with $B/t = 20$; (3) As the $B/t$ ratio increases, soil deformation decreases; (4) For a pile with a $B/t$ ratio of 20, it was found that 92% of the displaced soil would move outwards when the tip is rounded and 76% if the tip was flat-ended; (5) Figure 3.26b shows that the maximum shear strain levels $\gamma_{\text{max}} = \frac{1}{2} (\epsilon_1 - \epsilon_3)$, where $\epsilon_1$, $\epsilon_3$, are the major and minor principal strains, respectively] increase sharply near the pile wall.
**In summary:** The displacement patterns which are seen during sampling depend on a combination of various factors, such as the geometry of the sampler, method of driving, type of soil, and penetration stage. The observed behavior is used to support a plugging mechanism and model in Chapters 7 and 9.

### 3.3.3 Deformations around plugged/unplugged piles

When full plugging ultimately takes place, further entrance of soil into the sampler is prevented. A permanent cone or bulb of soil is then formed below, and forced into the soil ahead of the sampler (Figure 3.27), which now acts as a solid body. A similar cone is also formed when the sampler is full (overdriven), or under closed-ended piles, shown in Figures 3.28a and 3.28b, respectively.

The results of dry density measurements ($\gamma_f$) below the pile tip ($B = 20\text{cm}$, $D = 12.0\text{m}$) in the area shown in Figure 3.28b are presented in Figures 3.29a and 3.29b, where $z'$ denotes the depth below the bottom of the pile (BCP Committee 1969, 1971). Knowing the initial void ratio and dry unit weight, $e_i = 0.89$, $\gamma_i = 1.47 \text{ t/m}^3$, allows evaluation of the volumetric strain and the penetration effect in the following way:

$$\epsilon_v = \frac{\Delta v}{v_o} = \frac{\Delta e}{1 + e_0} = \frac{e_f - e_i}{1 + e_i} = \frac{\gamma_i - \gamma_f}{\gamma_f} = \frac{\Delta \gamma}{\gamma_f}$$

(3.2)

Values of the volumetric strain using Equation 3.2 were added to the equi-density (equi-volumetric strain) contours of Fig. 3.29a. Equation 3.2 can be used to transform the relations of Fig. 3.29b to those of volumetric strain as a function of the distance $r$ from the center of the pile. The relations of Fig. 3.29b can then be represented using:

$$\epsilon_v = a \cdot (r/R)^{-b}$$

(3.3)
where
\[ a = -0.23 \quad \text{(based on the evaluation of the average final dry density at the pile wall } \gamma = 1.91 \text{t/m}^3), \text{ and} \]
\[ b = 1.6, \text{ for the estimation of the average change.} \]

The relations of Equation 3.3 for \( a = -0.23 \) and \( b = 1.6 \) were used with Equation 3.2 to calculate the solid–line curve shown in Fig. 3.29b for the average dry density under the pile tip vs. distance from the pile center. Possible upper and lower ranges were added in dashed lines using \( b \) values of 1.0 and 2.0, respectively.

The formation of a permanent cone ahead of plugged/closed–ended piles leads to a penetration mechanism of the soil cone similar to that of the pile itself. The compressed conical wedge pushes a radial shear zone sideways into a plastic zone (where the densities were measured). Thus, the advancement of the pile into dense sand is made possible by lateral expansion of the soil along a circular ring existing approximately as a continuation of the pile shaft. This mechanism was supported by displacement measurements of the sand below the tip of the pile shown in Figure 3.28 (BCP Committee -1969). It is logical then to assume that the displacements formed by the cone ahead of the pile are similar to those of the pile shaft.

The relations of Equation 3.3 can therefore serve as an estimation of the volumetric strain around the shaft of a plugged/closed–ended pile driven in dense sand. These relations are presented in Figure 3.30a in the form of volumetric strain vs. normalized distance from the pile center (note: the soil is compacted and the volumetric strain is therefore negative). The trend of the obtained relations in Figure 3.30a compares well to the trend of the relations in Figure 2.3b for the normalized radial soil movement (in relation to the displacement at the pile wall) around a closed–ended pile in an incompressible material. The absolute strain values of the incompressible material are about twice those of Fig. 3.30a for \( b = 1.0 \).

The relations of Figure 3.30a indicate that within a distance of about 3.0 radii
from the pile shaft the volumetric strain reduces to be less than 5%.

The relations of Equation 3.3 can be used to evaluate the 'influence zone' around plugged piles driven in sand. The volume change in soil up to a given distance of $r/R$ away from the pile is obtained by:

$$\Delta V_r = \int_{R}^{(r/R)} 2\pi r \cdot \epsilon_v \cdot dr$$

substituting Equation 3.3 into the above and integrating leads to:

$$\Delta V_r = \frac{2\pi a R^2}{2 - b} \left[ \left( \frac{r}{R} \right)^{2-b} - 1 \right] \quad \text{for } b \neq 2$$

normalizing the volume change in respect to $V_{pile} = \pi R^2$ gives:

$$\frac{\Delta V_r}{V_{pile}} = \frac{2a}{2 - b} \left[ \left( \frac{r}{R} \right)^{2-b} - 1 \right] \quad \text{for } b \neq 2$$

$$(3.4)$$

$$\frac{\Delta V_r}{V_{pile}} = 2a \cdot \ln \left( \frac{r}{R} \right) \quad \text{for } b = 2$$

The relations of Equation 3.4 are presented in Figure 3.30b for the average relations of $b = 1.6$ and for the upper and lower ranges, using $b = 1.0$ and $b = 2.0$, respectively. Note that the relations of Figures 3.30a,b are extrapolated to $r/R = 4$, using the data of Figure 3.29, limited to the distance of $r/R = 2.5$. The relations of Figure 3.30b indicate that about 50% of the soil volume change due to closed-ended/plugged pile penetration in sand takes place within a distance of 1.5 radii around the pile. The average relations indicate that within a distance of 3 radii around the pile, about 90% of the volume change takes place.
The effects of model pile penetration by vibration and driving were studied by Szychy (1961). Figure 3.31 presents some of the data concerning porosity measurements around open and closed-ended piles driven in sand. Measurements taken in eight different locations (see Fig. 3.31a) are shown for 2 different pile sizes driven closed (Fig. 3.31b) and open-ended (Fig. 3.31c). For the given soil conditions, the volumetric strain is about equal to the measured changes in the porosity ($\epsilon_v \approx \Delta n$).

The following observations can be made regarding the data of Figure 3.31:

1. The measurements around the shaft were taken at distances equal to or greater than 2.7 radii (for the $B = 70$ mm) from the pile center. In reference to Figure 3.30a, at that distance the volumetric strain drops to less than 5% (compared to 25% next to the pile shaft) and, therefore, the presented measurements should be reviewed qualitatively.

2. Noticeable compaction took place just below the ground surface and along the upper part of the pile shaft (for all but the $B = 25$ mm closed-ended pile). Greater compaction occurred around the closed-ended piles as compared to the open-ended pile. The single digit volumetric strains fall within the range of compaction of Figure 3.30a.

3. Compaction took place under the toes of all closed-ended piles. This compaction reduces with movement away from the pile and changes into dilation (loosening). The measurement point No. 7 is within the expected cone under the 70 mm closed-ended pile. The compaction found is lower than expected, referring to Figure 3.29.

4. Loosening of soil took place under the toes and around the lower part of the open-ended piles. Since the piles were unplugged at the reported stage, the information of Figure 3.30c implies that the loosening is due to the soil flow inside the pile. These results can explain lower shaft friction and higher settlements of unplugged open-ended piles when compared to plugged ones.
5. The compaction of the inner soil was reported by Szechy to be from 6 to 14%, which is in reasonable agreement with the data of Figure 3.33.

3.3.4 The effect of plugging on the inner soil (plug) in model piles

(a) Introduction

In the controlled environment of the laboratory, detailed data regarding the changes which take place during penetration may be obtained (see Section 3.2.4).

Evaluation of the soil state inside the pile allows one to estimate the influence of plugging on this soil, and therefore helps to elucidate the possible plugging mechanism.

In order to evaluate the sand state inside the shaft, direct and indirect measurements are analyzed.

(b) Analysis of data

Porosity measurements at the bottoms and tops of soil plugs were reported by Szechy (1959). The processed data are presented in Figure 3.32 as the void ratio at the top ($e_t$) and the bottom ($e_b$) of the soil plug vs. the model pile diameter. A trend of decrease in void ratio (compaction) with the decrease in pile diameter is observed in all measurements. The soil at the top is always looser than the soil at the bottom. For the 'large' diameter pile ($B = 7$ cm) the void ratio at the top is equal to the initial void ratio prior to penetration ($e_0$).

The data of Figure 3.32 were further assessed by calculating the volumetric strain undergone in the soil plug. The void ratio at the bottom of the plug ($e_b$) and the average void ratio for the entire plug ($e_{avg}$) were calculated using Equation 3.2. The results are shown in Figure 3.33 as ratios of 'bottom' and 'average' volumetric strain vs. model pile diameter.

A measure of volumetric change due to the pile plugging was added to Figure
3.33 in the form of the change in plug length in relation to the penetration depth. It was calculated in the following way:

\[
\frac{\Delta V}{V} = \frac{D - L}{D} \quad (3.5)
\]

where \( D = \) depth of penetration

\( L = \) plug length.

The sand compaction inside the shaft was also evaluated by Kishida (1967a,b) using small scale SPT-type tests \((B = 6 \text{ mm})\). These test results can be interpreted through the use of two correlations (Kishida -1967a): (1) between the number of blows \((N)\) and the relative density \((D_r)\), and (2) between \(D_r\) and the internal friction angle \((\phi')\). A summary of Kishida's results and their analysis are presented in Appendix II.

The average void ratio for the entire plug \((e_{avg})\) from Kishida's experiments was used with Equation 3.2 to evaluate the average volumetric strain of the plug for different pile diameters. These data are also presented in Figure 3.33, along with the volumetric change due to the plugging, using Equation 3.5.

Kishida's miniature SPT results were analyzed in Appendix II according to segments of similar resistance. The blow count resistance was then translated into internal friction angle (using Kishida's 1967b correlation), presented in Figure 3.34. The data are presented as the soil plug internal friction angle for the plug top, bottom, segment above the bottom, segment of one diameter ahead of the tip, and average for the entire plug length.

(c) Conclusions and discussion

The relations of Figures 3.33 and 3.34 provide interesting information
concerning the plugging mechanism and its influence on the formation of the inner soil plug.

1. A table within Figure 3.33 contains some basic information for both sets of tests (Szechy -1959; Kishida -1967b) concerning the different factors which were found to control pile plugging. Although the experiments differ completely in the model pile geometry (thin vs. thick-walled), the initial state of the soil (very loose vs. dense), and the installation method (pushing vs. driving), all soil plugs (plugged and unplugged) experienced similar volumetric strains, not exceeding 10%.

2. The volumetric strain of the unplugged piles ($B > 4.5$ cm for Szechy and $B > 7.5$ cm for Kishida's experiments) is similar to the volumetric change obtained from Equation 3.5. It seems, therefore, that the volume changes inside the shaft due to densification (as a result of the circumferential friction stresses) can explain the difference between the plug length and the penetration depth when piles penetrate in sand in an unplugged mode. These volumetric changes took place under motion and are about one order of magnitude larger than those calculated and tested under 1-D static plug loading (see Section 4.6 for the assessment of the inner soil deformations and Section 4.7.3 for comparison with experimental data).

3. An additional conclusion to the above is that the ‘shorter’ soil plugs of the plugged piles are not a result of compaction. When penetration takes place in a plugged mode, the soil ahead of the tip must flow away from the opening in a penetration mode similar to that of a closed-ended pile.

4. Figure 3.32 shows that the sand in the lower plug zone of the piles used by Szechy was consistently denser than the soil in the upper plug zone. The void ratio at the plug top of the unplugged pile ($B = 7$ cm) was equal to the initial void ratio of the sand, before penetration.
5. The volumetric strains calculated from the results of both tests are comparable (conclusion No. 1). The distribution of the internal friction angle along the plug of Kishida's piles (shown in Figure 3.34) suggests, however, different conclusions than the aforementioned denoted as No. 4:

(a) The lower part of the soil plug of all piles has a similar friction angle regardless of its plugging state. Note that the blow count is related to density and is reflected therefore, by the friction angles.

(b) The soil density measurements under the pile tip\(^6\) are in good agreement (difference < 5%) with the soil density measurements at the lower part of the soil plug. The measurements inside the pile may be influenced by a scale effect due to the instrument's own penetrating diameter of B = 6 mm, and can vary from one pile diameter to another. However, the uniformity of these measurements supports the reliability of the test results from the lower zone of the pile shaft, as no scale effect takes place under the tip. These results are troublesome; it is expected that the zone under the plugged piles will be denser than that under the open-ended piles due to the cone formation mentioned earlier. The visible slip surfaces and lateral expansion produced by the compressed cone of Figure 3.28 are indications of general failure in very dense sand. Kishida's experiments were conducted in a very loose material and although a compressed wedge is expected to be formed under the tip of the plugged pile, its motion through the mass would not produce the same effect. This explains the fact that the area under the piles, as well as the soil plug, is much denser than the surrounding soil, but it leaves questions as to the reason for similar densities under the plugged and

---

\(^6\)Within one diameter ahead of the tip except for piles with B \(\geq\) 7.5 cm, where the given data refer to a shorter distance.
unplugged piles.

(c) In spite of the comparable densities at the lower part of the plug, the density distributions within the soil plugs can be clearly classified into two groups according to the plugging state. The 4 piles with $B \leq 5$ cm are all plugged and exhibit one distribution. The two piles with $B \geq 10$ cm are unplugged and exhibit another type of distribution. The data related to the pile diameter $B = 7.5$ cm are unclear and may be attributed either to error in measurements or to the fact that this pile is in transition between an unplugged to plugged state. This grouping is further supported by the analysis of Chapter 4, and can be reviewed in Figure 4.11.

(d) All plugged piles contain a very dense layer some distance above the pile tip. Using a rough approximation, this layer starts about a quarter of a diameter ($B/4$) away from the tip, and has the most dense consistency of the entire soil plug. The density at the top of the plug, although greater than that at the tip, is less than that of the very dense zone.

(e) The unplugged piles have their most dense zone in the bottom part of the soil plug. The density then lessens from the bottom upwards, where the top of the plug consists of soil in its most loose state.

6. The above observations clearly indicate that a certain mechanism transforms the pile from unplugged to plugged. This mechanism, which takes place a short distance above the tip, compresses a zone of soil inside the shaft to form a very dense layer which acts as a ‘plug’, preventing any further movement of the inner soil, and thus transforms the pile’s mode of penetration from that of an open-ended to that of a closed-ended pile. The existence of this mechanism and its effect can be further supported by re-examination of the plugging state for the pile of diameter $B = 7.5$ cm. Figure 4.11 and the
analysis of Chapter 4 indicate that this pile is in the final stages of a transition state from unplugged to plugged. Its specific recovery ratio is $\gamma = 13\%$ in comparison to $\gamma \leq 5\%$ for the plugged piles and $\gamma \geq 91\%$ for the unplugged piles.

A speculation can be made as to the changes in the density distribution of this soil plug. With further penetration, additional compaction of the lower zone and its movement upwards is expected to transform the segment above the bottom into the most dense zone, resulting in a density distribution similar to that of the plugged piles. It should be remembered that the penetration test inside the upper zone of the pile (on which the data of Fig. 3.34 are based) may be affected by the shaft diameter; therefore, the absolute values of the different friction angles may differ somewhat from those presented. The 6 mm penetrating probe occupies about 8% of the inner volume of the $B = 2.25$ cm diameter pile, and only about 0.7% of the inner volume of the $B = 7.5$ cm diameter pile. The above speculation regarding the change in density distribution with further penetration would apply to any of the unplugged piles. The dense zone at the bottom is expected to move upwards until full plugging conditions are created. The density distribution will then be reversed, with the soil at the top becoming denser, and the most dense sand being found in the lower segment above the tip.

7. The described soil plug behavior supports the plugging mechanism proposed in subsequent chapters of this work. An increase in local resistance in the lower zone of the plug due to arching enables the mobilization of friction which exceeds the tip bearing capacity, thus plugging the pile, and preventing further soil penetration. This arching is possible due to reorientation of the soil particles in an arch formation (having the spatial shape of a spherical cap) along the major principal stress trajectory. Such an arrangement, when
subjected to loading, leads to soil compression, straining, and dilation along the arch, identified as a denser zone in the data of Figure 3.34.

3.3.5 Changes in the soil plug following full-scale pile penetration

Soil profiles and SPT results (inside the shaft), prior to and following installation of open-ended piles at three different sites, are presented in Figure 3.35 (Kishida -1967a). Additional data gathered by Auki (1982) did not contain any new information. These data were, however, reviewed carefully and contributed to the following observations:

1. In most cases, the thickness of the clay layer inside the shaft did not exhibit visible differences when compared to the soil profile prior to the installation. This observation is in agreement with the analysis and data of unplugged offshore piles presented in Section 3.2.5c. The analyses indicated that the predictions based on mode ‘a’ of soil entrance into the pile (assuming that the soil inside the pile at the cutting shoe is not deformed by pile penetration and the entering volume is equal to that ahead of the opening, see Figure 3.17a) matched the measured data, therefore complying with the data of Figure 3.35 as well.

2. The clay inside the shaft was strained and probably remolded during penetration. According to Figure 3.35, unconfined compression tests indicated higher strain values at failure for the clay inside the shaft, compared with the undisturbed clay prior to installation. These results are in agreement with those obtained by Karlsrud and Haugen (1985), who found the strength of reconsolidated remolded clay due to pile penetration to be greater than the strength of the original normally consolidated clay.

3. Marked changes are observed in the thickness of the sand layers. However, in all three cases shown in Figure 3.35 the volumetric change of the sand did not
exceed 10%. This matches nicely the presented material of Figure 3.33, where it was concluded that regardless of the pile–sand installation differences, the volumetric strain of the soil plug did not exceed 10% (see Section 3.3.4c, No.1). This agreement leads to the conclusion that the compaction of the sand layers was due to the circumferential friction stresses, and was not a consequence of plugging.

4. Comparison of the SPT results prior to driving and inside the shaft shows the following:
   (a) No significant changes in the clay layers
   (b) Some small increases in the SPT results of the silt inside the shaft.
   (c) Marked increases in the SPT results of the sand layers. These results seem to indicate two trends: (I) the SPT results are significantly higher for the cases where the sand was compacted. No or moderate increases were observed when the sand was subjected to little or no compaction; (II) The repetitive transition between high and low blow counts (as observed in Figure 3.35 for the case of the Miayi Bridge) in the densified sand layer may be an indication of the densification process in which the sand was alternately subjected to pulse loading. This may be a result of the driving operation, or the arching mechanism which is formed and destroyed during the pile penetration, or a combination of the two.

5. Soils which are mixtures of cohesive and granular materials (i.e. sandy clay), are also subjected to some volume changes due to compaction. Such compaction increases the amount of granular material in a certain volume and therefore alters the parameters of the soil (e.g. decrease of liquid limit).
3.4 THE BEHAVIOR OF OPEN–ENDED PILES UNDER STATIC LOADS

3.4.1 Introduction

The ultimate goal to be kept in mind while designing piles is the prediction of the pile behavior under static loads. The influence of the plug state on the ultimate static capacity of the pile was demonstrated in Section 2.2. The analyses there were based on simplified pile–plug relations and did not consider the load–settlement behavior of the piles.

This section summarizes the available information concerning the behavior of open–ended piles and soil plugs under static loads. The data were divided into three categories, presented in the following sections.

3.4.2 Small–scale models

Section 3.2.4 described experiments on small–size pipes which were carried out by Kishida (1967a, 1967b), Klos and Tejchman (1977), and Szechy (1959). Capacity of piles versus depth of penetration, and/or a load–settlement curve at a particular depth were reported by all three researchers. Kishida (1967b) presents such comparisons for a variety of open and closed ended piles under the same conditions. Figure 3.36 was prepared based on Kishida's data. The resistance to penetration vs. penetration depth for closed (B = 5.0 cm) and open–ended (B = 5.0 cm, 10 cm) piles pushed into sand is shown in Figure 3.36a. The load–settlement behavior of these piles at their final penetration depth (D = 45 cm) is shown in Figure 3.36b. Each data point in Figure 3.36 represents the average value obtained from 3 tests for the B = 5.0 cm diameter piles and from 2 tests for the B = 10 cm diameter piles. The range of values for each measurement is represented by a line across the data points.

Examination of Figure 3.36 reveals that:

1. Comparison between the open and closed–ended 5.0 cm piles shows that in the initial stages of penetration, the resistance of the closed–ended piles is greater
than that of the open-ended piles. For example, at a penetration depth of 10 cm, the force required to push the closed-ended piles is twice that required for the open-ended piles. As penetration progresses, the pushing force required for the open-ended pile increases at a greater rate and approaches that of the closed-ended pile, so that at a depth of 25.0 cm both piles exhibit the same resistance to penetration. With further penetration the pushing force required for the open-ended pile exceeds that of the closed-ended pile. At the final penetration depth of 45 cm the average resistances are 50 and 47 Kg for the open and closed-ended piles, respectively.

2. Examination of the plugging process of the open-ended pile in Figure 3.8 reveals that when penetrating from a depth of 10 cm to a depth of 25 cm the specific recovery ratio of the open pile decreased from $\gamma = 97\%$ to $\gamma = 20\%$ as a result of significant plugging. At its final penetration depth the recovery ratio of the open ended-pile was about $\gamma = 4\%$, which is indicative of complete plugging.

The parallel between the pile resistance and its plugging state leads to the conclusion that monitoring of the specific recovery ratio will explain the different pile resistances of Figure 3.36a.

3. The 10 cm diameter open ended pile did not plug (see Figure 3.7), penetrating in an unplugged mode. Figure 3.36a shows that the curve describing the pile resistance at different penetration depths follows the same pattern as that of the 5.0 cm open-ended pile prior to plugging. At the final penetration depth the 5 cm plugged pile exhibited a resistance approximately equal to that of the 10 cm unplugged pile, which has twice its diameter.

4. Comparison of the load-settlement behavior of the three piles in Figure 3.36b reveals that:
   a) The open and the closed-ended 5 cm diameter piles exhibit the same
ultimate bearing capacity of about 40 kg.

b) Following ultimate load, the open ended pile exhibited larger settlements than the closed ended pile. However, their entire load–settlement curves are very similar.

c) The above leads to the conclusions that under static loading, plugged piles behave almost identically to closed–ended piles and under working loads both will have the same response.

d) The 10 cm diameter unplugged pile exhibited a load–settlement behavior similar to that of the 5 cm diameter closed–ended and plugged piles. Its ultimate bearing capacity of about 50 kg (analyzed by Kishida to be 40 kg) is much smaller however, than its potential when plugged (about 175 kg, see Figure 4.11).

3.4.3 Full–scale soil plugs

Isemoto (1976) and Kishida and Isemoto (1977) describe a series of tests carried out on full–scale soil plugs in order to investigate the behavior of sand inside open–ended pipe piles. Dense dry sand ($D_r = 90\%$) was placed on steel discs installed at the bottom of steel pipes ranging in diameter from 30 to 100 cm. While pushing the disc upwards, measurements of displacements and loads were recorded. The behavior of the sand in the pipe was also investigated using Finite Element analysis, considering the increase of the sand modulus of elasticity with the increase of the load and the frictional resistance between the sand and the pile. These tests are further discussed in Chapters 4 and 9. Typical results of the tests and their analyses are presented in Figure 3.37, where it can be noted that:

1. For a constant ratio of plug height to pile diameter ($H/B \text{ const}$) there is an abrupt decrease in the ultimate stresses (pushing load over disc area) as the pile diameter increases (see Figure 3.37a). The analysis in Chapter 4 (see
Figure 4.10) suggests that this is a result of a lower ratio between horizontal to vertical stresses ($K$) depending on the pile diameter.

2. About 85% of the total plug load is carried by the frictional stresses along the shaft, within a distance of about two diameters from the tip of the pile (Figure 3.37b). This result is in agreement with the data of Section 3.3.4 regarding model piles and the densification of the lower plug zone, following pile plugging.

3. The absolute height of the soil plug has a substantial influence on the capacity of the smaller diameter piles ($B \leq 700$ mm). For a certain pile diameter the capacity increases with increase in plug height (following a pile diameter in Figure 3.37a on vertical line through increasing $H/B$ ratios).

3.4.4 Full-scale load tests

Data of load tests performed on closed and/or open-ended piles are reported by Szechy (1959), Klos and Tejchman (1977), and Kishida (1967a). Auki (1982) summarized results of 50 load tests from different sources. In 36 cases the data was related to load tests of open-ended piles only; in 7 other cases, comparisons were made between closed and open-ended piles. Unfortunately, for most of the cases there are insufficient data regarding the plug development inside the pile. Three of the cases attributed to Kishida were presented in Figure 3.35. An additional case attributed to Soo (1980) is presented in Figure 3.38.

The piles of the N.K.K.building (Figure 3.35b) are of diameters $B = 50.8$ cm, and have an unknown wall thickness, $t$. The piles were driven 24.0 m, mainly through silt and clay deposits, and finally penetrated into a sand bearing layer ($L_b = 4.0$ m). The piles in Figure 3.38 are of diameter $B = 50.8$ cm as well, wall thickness $t = 0.9$ cm, and penetration depth of 32.0 m, of which $L_b = 1.5$ m. Figures 3.35b, 3.38 and the additional information provided by Auki (1982), suggest the following:
1. The open-ended pile of Figure 3.35b most likely did not plug at all. Its load settlement behavior is similar to that of the closed-ended pile, although showing larger settlement for the same loads beyond the ultimate bearing capacity. For a total settlement of about 35 mm the capacities are 445 and 495 tons for the open and closed-ended piles, respectively. Comparing the settlements of the piles under a load of 450 tons shows that the open-ended pile settled about 35 mm, while the closed-ended pile settled only 13 mm. For practical purposes, under a design load less than or equal to 300 tons (considering a minimal factor of safety of 1.5), both piles would exhibit the same behavior.

2. Substantial plugging apparently developed in the open-ended pile of Figure 3.38. The load settlement curves of the piles however, indicate that major differences exist. Under a settlement of about 35 mm the closed and open-ended piles carried about 375 and 215 tons respectively. Their ultimate capacities were about 360 and 205 tons.

3. Detailed information concerning the plug development, load-test schedule (time after installation), soil quality (especially in the bearing stratum) and other factors is essential to identify the confusing results of the piles in Figure 3.35b vs. those of Figure 3.38. It may well be that the incomplete data and the fact that the piles were driven in a layered medium, where their behavior will be time dependent, can explain some of the results. The two sets of tests demonstrate the need for complete information when plugged piles are analyzed.

4. Auki concluded the following from his entire data set:
   a) The ratio between the maximum bearing capacity of the open-ended piles to that of the corresponding closed-ended piles varied between 50% to 95%.
b) Certain correlations were evident between the increase of the ratio $L_b/B$ and the increase in capacity of the open-ended piles when compared to the closed-ended.

c) No particular attention was paid to the plug length, most likely due to lack of information.
3.5 SUMMARY AND CONCLUSIONS

1. Typical pipe piles share geometrical characteristics with thin-walled samplers.

2. The dynamic penetration of piles is similar to that which takes place during SPT. The average speed of pile penetration is similar to that of quasi-static penetration.

3. The known sampling results can therefore be used to study the disturbance of the soil and the plugging of open pipe piles.

4. Examination of plugging of thin-walled samplers shows that a good indication of plugging is provided by the specific (incremental) recovery ratio $\gamma$, defined as the increment in sample length corresponding to unit increment of sampler penetration ($\gamma = \frac{dL}{dD}$).

5. Sampler geometry and driving method affect the plugging process. Thin-walled samplers with constant wall thickness have a greater tendency to plug compared to thick-walled samplers, or to samplers with inside clearance.

6. The mechanism of pile plugging during quasi-static penetration differs from that during dynamic penetration. Piles have a greater tendency to plug during quasi-static penetration, and this plugging can be predicted.

7. The pile plugging phenomenon is of frequent occurrence, and has much greater significance than that presently accorded it by the profession. This lack of recognition is mainly due to misinterpretation of data.

8. Pile plugging during quasi-static and dynamic driving appears to be a consistent phenomenon for given soil and pile conditions. Hence rational, systematic, and useful plugging prediction methods are conceivably possible to achieve.

9. Theoretical and experimental results show that clay most likely enters the pile in a penetration mode where only that clay ahead of the opening enters.
10. No volume change appears to take place during clay penetration into piles.
11. A maximum volumetric strain of 10% takes place in the pile during sand compaction due to penetration and plugging.
12. Small-scale model piles and full-scale model soil plugs indicate that a certain mechanism transforms the pile from unplugged to plugged. The point at which this occurs depends on pile geometry, soil consistency, and method of penetration.
13. In sand plugs, this mechanism creates a zone of compressed soil a short distance above the opening, which prevents any further movement of the inner soil.
14. When full plugging occurs, any further entrance of soil is prevented. A permanent soil cone or bulb is created ahead of the pile, identical to that formed ahead of closed-ended piles. In dense sands, this compressed conical wedge pushes a radial shear zone sideways into a plastic zone, resulting in a lateral expansion of the soil along a circular ring approximating a continuation of the pile shaft.
15. The plugged mode of penetration in sands densifies the soil around the pile, resulting in greater friction. This effect is less profound in loose sands. When penetration occurs in an unplugged mode, the flow inside seems to loosen the soil under and around the pile.
16. Plugged piles exhibit a load settlement behavior similar to that of closed-ended piles. However, the end bearing capacity of a plugged pile is mobilized through friction along the soil plug. This explains the fact that under ultimate capacity load, the settlement required for a plugged pile to mobilize its full capacity may be greater than that for a closed-ended pile.
17. Unplugged piles develop a bearing capacity smaller than that of plugged/closed-ended piles. Due to differences in the penetration mode, the
friction and end bearing capacity of unplugged piles are significantly smaller, resulting in greater settlements. Load-settlement behavior seems to vary in proportion to the plugging state. The friction and end bearing capacity of partially plugged piles are expected to increase to an intermediate level between plugged and unplugged piles, resulting in lower settlements and greater bearing capacity as plugging progresses.
Table 3.1A

Typical dimensions of pipes frequently used as piles

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<th>B (inch)</th>
<th>t (inch)</th>
<th>B/t</th>
<th>Average B/t</th>
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<th>t (inch)</th>
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Remarks:

a. Data obtained from Hal and Hunt (1979); Chellis (1961)
b. Average B/t refers to the average of the available range of pipe thicknesses.
c. In common practice; open ended piles with diameters less than 10 inches and minimum wall thickness of 1/4" for B < 14"; 3/16" for 18" > B > 14" and 3/8" for B > 18", are not driven.
d. Inside and outside driving shoes may be used to protect the pile tip. For pipes ranging in diameter from 10 3/4 to 60 inch, insid shoes provide outside clearance of C₀ = 0.6 to 1.2% and inside clearance ranging from Cᵢ = 2.8 to 11.4%.

Outside shoes provide C₀ = 4.6 to 13% and Cᵢ = 1 to 4% (APF-1982).
### Table 3.1B

Typical dimensions of offshore piles

<table>
<thead>
<tr>
<th>B (inch)</th>
<th>t₁ (inch)</th>
<th>t₂ (inch)</th>
<th>tavg. (inch)</th>
<th>B/t₁</th>
<th>B/t₂</th>
<th>B/t avg.</th>
<th>CA %</th>
<th>C1 %</th>
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<th>Data Source</th>
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<td>1.875</td>
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<td>1.000</td>
<td>1.129</td>
<td>44</td>
<td>60</td>
<td>53</td>
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<td>1.31</td>
<td>G.O.M.</td>
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<td>1.968</td>
<td>1.968</td>
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<td>1.125</td>
<td>27</td>
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<td>54</td>
<td>16.6</td>
<td>4.05</td>
<td>I.L.</td>
<td>2</td>
</tr>
<tr>
<td>56</td>
<td>1.750</td>
<td>0.875</td>
<td>0.875</td>
<td>32</td>
<td>64</td>
<td>64</td>
<td>14.7</td>
<td>3.33</td>
<td>I.L.</td>
<td>2</td>
</tr>
<tr>
<td>54</td>
<td>1.250</td>
<td>0.875</td>
<td>1.131</td>
<td>43</td>
<td>62</td>
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<td>9.9</td>
<td>1.46</td>
<td>G.O.M.</td>
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</tr>
<tr>
<td>54</td>
<td>1.750</td>
<td>2.250</td>
<td>1.900</td>
<td>31</td>
<td>24</td>
<td>28</td>
<td>14.3</td>
<td>0.00</td>
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<tr>
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<td>1.500</td>
<td>1.648</td>
<td>27</td>
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<td>16.3</td>
<td>1.12</td>
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</tr>
<tr>
<td>48</td>
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<td>1.250</td>
<td>1.338</td>
<td>32</td>
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<td>36</td>
<td>13.8</td>
<td>1.11</td>
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<td>48</td>
<td>1.500</td>
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<td>1.010</td>
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<td>1.000</td>
<td>1.540</td>
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<td>2.22</td>
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<td>1.250</td>
<td>0.750</td>
<td>0.997</td>
<td>38</td>
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<td>0.929</td>
<td>42</td>
<td>56</td>
<td>45</td>
<td>10.3</td>
<td>1.25</td>
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<tr>
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<td>0.750</td>
<td>0.935</td>
<td>34</td>
<td>56</td>
<td>45</td>
<td>12.9</td>
<td>2.53</td>
<td>G.O.M.</td>
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</tr>
<tr>
<td>42</td>
<td>1.250</td>
<td>0.625</td>
<td>0.625</td>
<td>34</td>
<td>68</td>
<td>68</td>
<td>12.9</td>
<td>3.16</td>
<td>I.L.</td>
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</tr>
<tr>
<td>42</td>
<td>1.250</td>
<td>0.875</td>
<td>1.223</td>
<td>34</td>
<td>48</td>
<td>34</td>
<td>12.9</td>
<td>1.90</td>
<td>G.O.M.</td>
<td>4</td>
</tr>
<tr>
<td>42</td>
<td>1.250</td>
<td>0.750</td>
<td>1.312</td>
<td>34</td>
<td>56</td>
<td>32</td>
<td>12.9</td>
<td>2.53</td>
<td>G.O.M.</td>
<td>4</td>
</tr>
<tr>
<td>36</td>
<td>1.375</td>
<td>0.875</td>
<td>0.940</td>
<td>26</td>
<td>52</td>
<td>38</td>
<td>17.2</td>
<td>3.00</td>
<td>G.O.M.</td>
<td>1</td>
</tr>
</tbody>
</table>

Remarks:

a. For definitions see Equation 3.1 and Figures 3.1 and 3.2.
b. Location: G.O.M. = Gulf of Mexico; N.S. = North Sea; I.L. = Israel.
c. Average B/t refers to average wall thickness along the embedded part of the pile.
d. Data source:
   1) Heerema Eng. Service, Leiden, The Netherlands
   2) Paikowsky (1982)
   3) Kindel (1977)
   4) Continental Oil Company
TABLE 3.2: Methods of driving samplers into the soil  
(Hovorslev, 1949)

<table>
<thead>
<tr>
<th>Method</th>
<th>Description of Operation</th>
<th>Type of Penetration</th>
<th>Average Rate of Penetration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow jacking</td>
<td>Levers or short commercial jacks</td>
<td>Intermittent slow motion</td>
<td>.025-.050(a) cm/sec</td>
</tr>
<tr>
<td>Fast jacking</td>
<td>Steady force, no interruptions</td>
<td>Continuous uniform</td>
<td>15-30 cm/sec</td>
</tr>
<tr>
<td>Hammering</td>
<td>Repeated blows of a drop hammer 140 lb. - 12&quot; drop</td>
<td>Intermittent fast motion</td>
<td>0.25-0.50(a)</td>
</tr>
<tr>
<td>Single blow</td>
<td>Blow of a heavy drop hammer 500 lb. - 4 to 9 ft.</td>
<td>Continuous fast motion</td>
<td></td>
</tr>
<tr>
<td>Shooting</td>
<td>Force supplied by explosives</td>
<td>Continuous very fast motion</td>
<td></td>
</tr>
</tbody>
</table>

(a) The average penetration rate is significantly decreased by the delays required for measurements of recovery ratio and resetting of drive heads and jacks.
TABLE 3.3: Plug Formation for Different Soil Volume Entering Pile Tip vs. Penetration (assuming no volume change)

GULF OF MEXICO

SITE No. 1

PLUG FORMATION FOR DIFFERENT SOIL VOLUME ENTERING PILE TIP VS. PENETRATION (assuming no volume change)

Pile outside diameter = 60 [inch]
Pile total length = 455 [ft]
Pile final penetration = 300 [ft]

Pile Segements, Length [ft.], Wall Thickness [inch], from bottom to top.
1. L = 10 T = 1.375
2. L = 200 T = 1.000
3. L = 10 T = 1.250
4. L = 90 T = 1.375
5. L = 145 T = 1.250

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>24.61 -0.39</td>
<td>25.80 0.80</td>
<td>27.01 2.01</td>
</tr>
<tr>
<td>50</td>
<td>48.97 -1.03</td>
<td>51.34 1.34</td>
<td>53.76 3.76</td>
</tr>
<tr>
<td>75</td>
<td>73.33 -1.67</td>
<td>76.88 1.88</td>
<td>80.32 5.32</td>
</tr>
<tr>
<td>100</td>
<td>97.69 -2.31</td>
<td>102.42 2.42</td>
<td>107.27 7.27</td>
</tr>
<tr>
<td>125</td>
<td>122.05 -2.95</td>
<td>127.97 2.97</td>
<td>134.03 9.03</td>
</tr>
<tr>
<td>150</td>
<td>146.40 -3.60</td>
<td>153.51 3.51</td>
<td>160.78 10.78</td>
</tr>
<tr>
<td>175</td>
<td>170.76 -4.24</td>
<td>179.05 4.05</td>
<td>187.53 12.53</td>
</tr>
<tr>
<td>200</td>
<td>195.12 -4.88</td>
<td>204.59 4.59</td>
<td>214.36 14.36</td>
</tr>
<tr>
<td>225</td>
<td>219.64 -5.36</td>
<td>230.58 5.58</td>
<td>241.77 16.77</td>
</tr>
<tr>
<td>250</td>
<td>244.64 -5.36</td>
<td>256.79 6.79</td>
<td>269.23 19.23</td>
</tr>
<tr>
<td>275</td>
<td>269.64 -5.36</td>
<td>283.01 8.01</td>
<td>296.69 21.69</td>
</tr>
<tr>
<td>300</td>
<td>294.64 -5.36</td>
<td>309.22 9.22</td>
<td>324.03 24.03</td>
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</tbody>
</table>
TABLE 3.4: Geometry of Piles Presented in the Case Histories

<table>
<thead>
<tr>
<th>PILE DIAMETER</th>
<th>B=60in</th>
<th>B=42in</th>
<th>B=48in (A)</th>
<th>B=48in (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEGMENT NUMBER</td>
<td>LENGTH WALL THICK. L(ft)</td>
<td>WALL THICK. t(in)</td>
<td>LENGTH WALL THICK. L(ft)</td>
<td>WALL THICK. t(in)</td>
</tr>
<tr>
<td>1(tip)</td>
<td>10</td>
<td>1.375</td>
<td>10</td>
<td>1.250</td>
</tr>
<tr>
<td>2</td>
<td>190</td>
<td>1.000</td>
<td>145</td>
<td>0.750</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>1.000</td>
<td>10</td>
<td>0.875</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>1.250</td>
<td>35</td>
<td>1.125</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>1.375</td>
<td>15</td>
<td>1.375</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>1.375</td>
<td>50</td>
<td>1.375</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
<td>1.250</td>
<td>30</td>
<td>1.000</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
<td>1.250</td>
<td>10</td>
<td>1.000</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td>65</td>
<td>0.875</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$L = \text{Length segment along the pile}$

$t = \text{wall thickness}$

Note: Lines in table indicate locations where add on pile segments were welded.
Table 3.5: Pile plugging statistics in the Gulf of Mexico
(Data provided by Heerema on 60 piles at 10 sites)

<table>
<thead>
<tr>
<th>D&lt;sub&gt;max&lt;/sub&gt;/B</th>
<th>Total</th>
<th>Plugged</th>
<th>Most Likely Plugged</th>
<th>Unplugged</th>
<th>PLR (final)</th>
</tr>
</thead>
<tbody>
<tr>
<td>more than 75</td>
<td>22</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>0.90</td>
</tr>
<tr>
<td>less than 75</td>
<td>38</td>
<td>1</td>
<td>7</td>
<td>30</td>
<td>0.94</td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td>9</td>
<td>15</td>
<td>36</td>
<td>0.92</td>
</tr>
</tbody>
</table>

D<sub>max</sub> = Maximum penetration depth for final plug measurement.
B = pile diameter
PLR(final) = Final Penetration length Ratio
\[ PLR(final) = \frac{L(soil)}{D_{max}} \text{ at final location of measurement.} \]
L(s(soil)) = Length of soil column inside pile.
INSIDE CLEARANCE  
\[ C_i = \frac{B_s - B_E}{B_E} \]

OUTSIDE CLEARANCE  
\[ C_o = \frac{B_w - B_t}{B_t} \]

AREA RATIO  
\[ C_A = \frac{B_w^2 - B_E^2}{B_E^2} \]

D = Depth of Penetration  
L = Original Length of Sample  
\[ \gamma = \frac{\Delta L}{\Delta D} = \text{Specific Recovery Ratio} \]

**Fig. 3.1:** Geometrical Characteristics of Soil Samplers, (Hvorslev –1949)
Fig. 3.2: Typical Pile for a Drilling Platform – Offshore Louisiana, (McClelland –1974).
Fig. 3.3: Geometrical Characteristics of Samplers and Open-Ended Piles.
Displacement Rate of Samplers and Piles During Penetration.

(a) Displacement of Pile Top and Tip Under a Single Blow. The top displacement was calculated from acceleration records; the tip displacement obtained from analysis (Paikowsky -1982).

(b) Typical Displacement vs. Time Curves for Quasi-Static and Average Dynamic Penetration

(c) Penetration of a Split-Spoon Barrel During SPT (Schmertmann and Palacios -1979).
Fig. 3.5: Plugging Measurements on a Sampler with a Uniform Diameter, Under Different Driving Methods (Hvorslev -1949).
$\gamma = \frac{dL}{dD} = \text{SPECIFIC RECOVERY RATIO IN } \%$

- SOIL
- SOFT VARVED CLAY

STEEL TUBING $B=2''$

- $C_A = 10 \%$
- $C_I = 1.2 \%$
- $C_D = 0 \%$

D.D = SLOW JACKING
E.E' = FAST PUSHING

Fig.3.6: Plugging Measurements on a Sampler with Enlarged Tip Wall Thickness, Under Different Driving Methods (Hvorslev -1949).
Fig. 3.7: Typical Results of Tests on Model Pipe Piles Driven in Sand (Kishida—1967—b).
(a) State of Plug During Pile Penetration
(b) Sand Compaction Inside the Shaft
$\gamma = \frac{dL}{dD}$ - SPECIFIC RECOVERY RATIO IN %

![Graph showing analysis of plugging measurements on model pipe piles obtained by different researchers.](image)

**Table:**

<table>
<thead>
<tr>
<th>NOTATION</th>
<th>METHOD OF DRIVING</th>
<th>B (mm)</th>
<th>t (mm)</th>
<th>B/t</th>
<th>Dr%</th>
<th>REFERENCE</th>
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<tbody>
<tr>
<td></td>
<td>Slow Pushing</td>
<td>50.0</td>
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<td>5.00</td>
<td>12.0</td>
<td>41.0</td>
<td>Klos &amp; Tejchman -1977</td>
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<tr>
<td></td>
<td>Driving</td>
<td>46.5</td>
<td>3.00</td>
<td>15.5</td>
<td>70**</td>
<td>Szechy -1959</td>
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</tbody>
</table>

* Average of 3 tests
** Estimated

Fig. 3.8: Analysis of Plugging Measurements on Model Pipe Piles Obtained by Different Researchers.
\[ \gamma = \frac{dL}{dD} \text{ - SPECIFIC RECOVERY RATIO IN } \% \]

**MODEL PILE**

\[ B = 15.24 \text{ cm} \]
\[ t = 0.45 \text{ cm} \]

slow pushing, average rate 0.25 cm/sec

---

**Fig. 3.9:** Analysis of Plugging Measurements on an Open - Ended Pile. Using Data From NGI -1981.
Fig. 3.10: The Ratio Between the Resistance Force of Open to Closed-Ended Piles vs. Normalized Depth as Obtained Experimentally (data from Kaarsrud and Haugen -1981), and Theoretical Calculations.
Fig. 3.11: Measurements of Pore Pressure Due to Installation of a Closed-Ended Pile (NGI–1981)

piezometer no. | distance r/r₀
--- | ---
P₂ | 19.4
P₃ | 13.1
P₄ | 6.4
P₅ | 3.0
P₆ | 3.5
P₇ | 3.5

B = 15.24 cm
t = 0.45 cm
L = 5.15 m

completion of Installation

Start of Installation

CYCLIC LOADING OF PILES, HAGA

INSTALLATION OF INSTRUMENTED PILE A₂
ABSOLUTE PORE PRESSURE FOR THE PIEZOMETERS IN THEGROUND BETWEEN PILE A₁ AND A₂
PLOTTED AGAINST TIME FOR THE PERIOD 000000 2114 - 000000 1621

NORWEGIAN GEOTECHNICAL INSTITUTE
Fig. 3.12: Measurements of Pore Pressure Due to Installation of an Open-Ended Pile (NGI-1981)
Plugging Measurements for Piles in the Gulf of Mexico
(after Kindel - 1977)

Fig. 3.13:
Adjustments for Plugging Measurements (after Kindel –1977):
(a) Average and Adjusted Average Plug Movements, Sites A, B, & C.
(b) Plug Length Ratio and Adjusted Plug Length Ratio vs. Penetration, Sites A, B, & C
Fig. 3.15: Statistics of Pile Plugging in the Gulf of Mexico
(a) Kindel - 1977
(b) Using data provided by Heerma Engineering Services - 1980
Fig. 3.16: Typical Soil Profiles in the Gulf of Mexico
Fig. 3.17: Soil Deformation Possibilities Ahead of Open-Ended Piles
Fig. 3.18: Analysis of Plugging Measurements for Diameter $B = 60$ in. Piles in the Gulf of Mexico
Fig. 3.19: Analysis of Plugging Measurements for Diameter B = 42 in. Piles in the Gulf of Mexico.
Fig. 3.20: Analysis of Plugging Measurements for Diameter B = 48 in. Piles in the Gulf of Mexico
Fig. 3.21: Driving Record of Diameter B = 48 in. Pile in the Gulf of Mexico
Deformation of Sampled Soil Due to Stress Changes at Sampler Tip (after Hvorslev -1949)
Fig. 3.23: Distortion of Soil in Sampler Due to Entrance of Excess Soil. Thick-Walled Piggot Sampler $C_A = 79\%, C_i = 2.7\%, C_o = 0\%$ (Hvorslev -1949)

Fig. 3.24: Distortion of Soil in Sampler Due to Inside Friction. Thin-Walled Shelby Tube $C_A = 10\%, C_i = 1.2\%, C_o = 0\%$ (Hvorslev -1949)
Fig. 3.25: Distortion of Soil Due to Sampler Plugging
(a) Start of Downward Deflection of Soil Layers Ahead of Sampler
(b) Deformation of Sampled Soil Caused by Overdriving
(Hvorslev –1949)
Fig. 3.26: Results from Model Simulating Pile Penetration in Clay
(a) Deformation Pattern Around a Flat Open-Ended Pile with B/t = 20
(b) Contours of Maximum Shear Strain, $\gamma_{\text{max}}$, $1/2(\varepsilon_1-\varepsilon_3)$, for a Flat Open-Ended Pile with B/t = 20 (after Azzouz and Baligh –1984)
Fig. 3.27: Soil Formations Ahead of Sampler (a) Cone (b) Bulb (Hvorslev -1949)
Formation of a Soil Cone Ahead of:
(a) Overdriven Sampler, (Hvorslev -1949)
(b) Pile Tip at a Depth of 12.0 m. (dimensions are in cm.),
(BCP Committee -1969)
Fig. 3.28-b:
State Near the Pile Tip (Fig. 3.28-b):
(a) Dry Density Contours (BCP Committee 1969) with Volumetric Strain Values using Eq. 3.2
(b) Dry Density Values vs. Distance from Pile Center for Different Depths under the Pile Tip (Z') (BCP Committee 1969)
The relations of Eq. 3.3 for $a = 0.23b = 1.6$ are transformed into density using Eq. 3.2 and presented as a solid line, and as a dashed line for the ranges of $b = 1.0$ and $2.0$. 

Test 5C
Effect of Pile Penetration on the State of the Sand Around the Pile:
(a) Volumetric Strain vs. Normalized Distance from the Pile Shaft
(b) Volume Change in the Soil as a Function of the Pile Volume vs.
Normalized Distance from the Pile Shaft
Fig. 3.31: Changes in Porosity Due to Penetration of Open and Closed-Ended Model Piles in Sand (data from Szechy – 1961)
Porosity Measurements at the Top and Bottom of Soil Plugs vs. Model Pile Diameter (data from Szechy -1959)
Fig. 3.33: Effect of Pile Penetration on the State of the Soil Inside the Pile.

Volumetric strain of the plug soil.

Volumetric change of plug length in respect to penetration depth and

Effect of Pile Penetration on the State of the Soil Inside the Pile.
Values of Internal Friction Angle at Various Locations Along the Soil Plug vs. Model Pile Diameter (Based on data from experiments of Kishida —1967—b)

Fig. 3.34: Values of Internal Friction Angle along the Soil Plug vs. Model Pile Diameter (Based on data from experiments of Kishida —1967—b)

SOIL PLUG INTERNAL FRICTION ANGLE

Values of Internal Friction Angle at Various Locations Along the Soil Plug vs. Model Pile Diameter (Based on data from experiments of Kishida —1967—b)

Fig. 3.34: Values of Internal Friction Angle along the Soil Plug vs. Model Pile Diameter (Based on data from experiments of Kishida —1967—b)
Fig. 3.35: Effect of Pile Penetration on the Soil Plug and Load Test Results of Open and Closed-Ended Piles (Kishida—1967—a)
Fig. 3.36: Installation and Load Tests on Open and Closed - Ended Model Pipe Piles
(a) Pile Resistance vs. Penetration Depth
(b) Load Settlement Relations at the Final Installation Depth
(Prepared using data obtained from Kishida 1967-b)
Fig. 3.37: Analysis of Typical Test Results on the Behavior of Sand Plugs in Open-Ended Steel Pipe Piles
(a) Relationship Between Ultimate Stress and Pile Diameter
(b) Normalized Sum of Calculated Frictional Stress vs. Normalized Height of Sand
(Kishida and Isemoto -1977)
Fig. 3.38: (a) Soil Profile and SPT Results Inside and Outside Open-Ended Piles
(b) Comparison of Load-Settlement Relations Between Open and Closed-Ended Piles
(Data from Auki -1982, attributed to Soo -1980)
CHAPTER 4

BEHAVIOR OF SOIL PLUGS UNDER STATIC LOADS – THE SILO APPROACH

4.1 PRINCIPLE OF ANALYSIS

The stresses along the soil plug can be analyzed using the same approach as that utilized by the Silo Theory. H.A. Jansen in 1895 (Jakobson –1958) developed the commonly used formula for determining the force exerted on the base of a silo containing granular material. The only difference between the two cases lies in the direction of the shear stresses. In a silo, there is a downward movement of the grains with respect to the walls. In a pile, there is a downward displacement of the pipe with respect to the soil, and therefore downward shear forces on the soil.

4.2 DEVELOPMENT OF GOVERNING EQUATIONS

Figure 4.1 presents a soil plug of inside diameter ID and height L. Consider the forces acting at a depth z on an element made by a horizontal differential cross-section of height dz and diameter ID. The average lateral stress (\(\hat{\sigma}_h = \sigma_{radial}\)) is symmetric about the axis of the center line, producing zero net force on the element. The force equilibrium in the vertical direction (using the notations and relations of Fig. 4.1) is:

\[
A\tilde{\sigma}_z + A \cdot \gamma \cdot dz + U \cdot \tau = A \cdot \tilde{\sigma}_z + A \cdot \frac{\partial \tilde{\sigma}_z}{\partial z} \quad (4.1)
\]

or

\[
\gamma \cdot dz + \frac{U}{A} \cdot \tau = \frac{\partial \tilde{\sigma}_z}{\partial z} \quad (4.2)
\]

or

\[
\gamma \cdot dz + \left[\frac{4K \cdot t g \delta}{ID}\right] \tilde{\sigma}_z dz = \frac{\partial \tilde{\sigma}_z}{\partial z} dz
\]

Assuming pure frictional relations such that \(\tau = \tilde{\sigma}_h \cdot \mu = \tilde{\sigma}_h \cdot t g \delta\) produces the following relations:

\[
\gamma \cdot dz + \left[\frac{4K \cdot t g \delta}{ID}\right] \tilde{\sigma}_z dz = \frac{\partial \tilde{\sigma}_z}{\partial z} dz
\]

or

\[
\frac{\partial \tilde{\sigma}_z}{\partial z} - C \cdot \tilde{\sigma}_z = \gamma
\]

where \(C = \frac{4K \cdot t g \delta}{ID}\)
The obtained linear first order equation has the following solution:

\[ -\sigma_z e^{\int -\gamma \, dz} = \int \gamma \cdot e^{\int -\gamma \, dz} \, dz + \text{Const.} \]

\[ \tilde{\sigma}_z \cdot e^{-\gamma z} = -\frac{\gamma}{C} \cdot e^{-\gamma z} + \text{Const.} \]

The known boundary conditions are \( \sigma_z = 0 \) at \( z = 0 \)

\[ \tilde{\sigma}_z = -\frac{\gamma}{C} + \text{Const.} \rightarrow \text{Const.} = \frac{\gamma}{C} \]

The obtained equation therefore is:

\[ \tilde{\sigma}_z = \frac{\gamma}{C} (e^{\gamma z} - 1) \]

or

\[ \tilde{\sigma}_z = \frac{\gamma I D}{4 K t g} \cdot \left[ e^{\frac{4K t g \delta}{1D} z} - 1 \right] \]

(4.3a)

The average force over a horizontal cross-section along the soil plug is \( \bar{P}_z = \tilde{\sigma}_z \cdot A \), which is:

\[ \bar{P}_z = \frac{\pi \gamma I D^3}{16 K t g} \cdot \left[ e^{\frac{4K t g \delta}{1D} z} - 1 \right] \]

(4.3b)

In a more general case, the friction along the interface is assumed to consist of adhesion and normal stress–dependent components: \( \tau = C_A + \tilde{\sigma}_h t g \delta \). Substituting this relation into Eq. (4.2) leads to:
\[ \gamma dz + \frac{4}{ID} \left[ C_A + K \cdot \sigma_z \cdot t g \delta \right] dz = \frac{\partial \sigma_z}{\partial z} dz \]

or

\[ \frac{\partial \sigma_z}{\partial z} - C \sigma_z = \gamma + \frac{4CA}{ID} \] (4.4)

The following relations can be obtained from the solution of Eq. (4.4) in a way similar to that presented previously:

\[ \sigma_z = \frac{(\gamma + \frac{4CA}{ID})}{C} \left[ e^{cz} - 1 \right] \] (4.5a)

or

\[ \sigma_z = \frac{\gamma ID + 4CA}{4K tg \delta} \left[ e^{\frac{4K tg \delta}{ID} z} - 1 \right] \] (4.5b)

\[ P_z = \frac{\pi ID^2 (\gamma ID + 4CA)}{16K tg \delta} \left[ e^{\frac{4K tg \delta}{ID} z} - 1 \right] \] (4.6a)

Similar relations can be obtained from assuming the existence of adhesion only, where \( \tau = CA \). In this case, \( \tau \) is not \( \sigma_z \) dependent and direct relations can be obtained from the summation of \( CA \) over the circumferential area:

\[ \gamma dz + \frac{4}{ID} C_A \cdot dz = \frac{\partial \sigma_z}{\partial z} dz \]

\[ \frac{\partial \sigma_z}{\partial z} = \gamma + \frac{4CA}{ID} \]

\[ \sigma_z = \left[ \gamma + \frac{4CA}{ID} \right] z \] (4.6a)

\[ P_z = 0.25 \cdot \pi \cdot ID (4CA + \gamma ID) \cdot z \] (4.6b)
4.3 THE UNDERLYING ASSUMPTIONS AND INHERENT SIMPLIFICATIONS

Equations 4.3, 4.5 and 4.6 describe the average vertical stresses and forces at a distance $z$ from the top of a soil plug. Before any further use is made of these equations, the underlying assumptions which lead to them should be emphasized.

1. Friction stresses are being mobilized all along the soil–pile interface, i.e., sufficient relative displacement takes place (soil being pushed upwards or pipe being pushed downwards) in order to activate this friction.

2. The friction coefficient ($\tan \delta$) is considered as constant ($z$ independent).

3. The coefficient of lateral stress ($K = \sigma_h/\sigma_z$) is considered as constant ($z$ and $r$ independent), i.e., the horizontal stresses are constant across any given depth. For that reason $\tilde{\sigma}_z$ and $\tilde{P}_z$ are treated as the average stress and force acting on the cross-section.

4. No assumptions are made concerning the material qualities other than those which are inherent in assumptions 2 and 3. It is therefore notable that:
   a) Regarding the material as a continuum, the Poisson effect of elastic behavior or dilation of soils is ignored. Such phenomena are expected to increase lateral pressure and, therefore friction, due to the tendency of the material to expand laterally under the confined environment.
   b) Regarding the material as a particulate media (in the cohesionless case), the so called 'arching effect' is ignored. This phenomenon is expected to increase lateral pressure, and therefore friction, due to the ability of the particulate media to redistribute shear stresses through transformation from one zone to the other.
4.4 INVESTIGATION OF GOVERNING EQUATIONS

The relations which are shown as a set of curves in Fig. 4.2 were developed in order to examine the possible range of parameters to be used with Eq. 4.3b (for the cohesionless material). The chosen values were:

1. Soil unit weight of $\gamma = 1 \text{t/m}^3$, which approximates the effective unit weight of a submerged soil.
2. Inside diameter $ID = 1.0 \text{m}$ which agrees with the typical dimensions of offshore piles as well as allows use of the axis $z$ for absolute dimensions (in meters) or relative length ($z/ID$).

The relations of Fig. 4.2 show enormous forces for relatively shallow depths and small unit weight and friction. In order to demonstrate the friction contribution, it can be integrated along the plug.

The friction $dF$ on the circumference of the differential cross-section of Fig. 4.1 was presented in Eq. 4.1 as $U\tau$, using the notations of Fig. 4.1:

$$dF = U \cdot \tau = \pi \cdot ID \cdot K \cdot \tan \delta \cdot \sigma_z \cdot dz$$

Substituting for $\sigma_z$ of Eq. 4.3a and integrating along the plug:

$$F = \frac{\gamma \pi ID^2}{4} \int_0^L (e^{cz} - 1) \, dz$$

$$C = \frac{4K \tan \delta}{ID}$$

$$F = \frac{\gamma \pi ID^3}{16K \tan \delta} \left[ e^{\frac{4K \tan \delta}{ID} z} - 1 - L \cdot C \right]$$

Comparing Eq. 4.7 to 4.3b we find that:

$$P_z - F = L \cdot \gamma \cdot \frac{\pi ID^2}{4}$$

The obtained value is the weight of the soil inside the pile, represented by the dashed...
The friction, therefore, is the difference between the weight of the soil and the given curve; e.g., for $K \cdot \tan \delta = 0.04$ and $z = 20\text{m}$ the total resisting force at the bottom of the plug is approximately $P_z = 115\text{ tons}$, the weight of the soil is $W_s = 15\text{ tons}$, and the frictional resistance is therefore $F = 100\text{ tons}$. The relations which are presented in Fig. 4.2 demonstrate the significant resistance forces which accumulate in the inner soil compared to the soil's weight.

The obtained relations of Fig. 4.2 indicate the very high sensitivity of the solution to small increases in $K \cdot \tan \delta$ values. Moreover, even at short plug lengths ($z$ values), a rapid increase in $P_z$ takes place for relatively small values of $K \cdot \tan \delta$. Therefore, a lower value of $K \cdot \tan \delta$ is used in order to enable the study of the 'silo approach' equations within a reasonable range of depth and forces, as presented in the following analyses.

Equations 4.3a, 4.3b, 4.5b and 4.6b are presented in Figs. 4.3a, b as relations of the diameter $ID$ in meters vs. the vertical force $P_z$ in tons. The relations of Fig. 4.3 were obtained for specific soil parameters ($\gamma, K, \tan \delta, C_A$) and a plug length of 15 meters. While the force $P_z$ increases with the diameter for the pure cohesive soil, it presents interesting relations for the frictional stress-dependent materials. It seems that for given conditions, there is a certain plug diameter for which the force $P_z$ will be minimal. Such relations may imply the existence of so-called 'arching'. In arching, the displacement between the soil and the structure mobilizes shear stresses which are transferred from or to the structure, depending on their relative stiffnesses and geometry.

The behavior of the vertical stresses and forces of cohesionless material, as indicated by Eqs. 4.3a, b are compared in Fig. 4.3b. It is interesting to note that while the vertical stresses increase as the diameter decreases (for $z = \text{const.}$), the force behaves in the manner described above.

In order to investigate the behavior of the force, the location of the minimum $P_z$
of Eq. 4.3b is identified. Differentiating Eq. 4.3b with respect to \( ID \) and equating it to zero leads to the following relations:

\[
\frac{P_{z\text{min}}}{ID} = \frac{3}{4K\tan\delta} \left[ 1 - e^{-4K\tan\delta \frac{Z_m}{ID}} \right]
\]  

(4.8)

where \( Z_m \) is the plug length for the minimum force \( P_{z\text{min}} \).

Equation 4.8 can be simplified with reasonable accuracy to:

\[
\frac{Z_m}{ID} = \frac{3}{4} \cdot K\tan\delta
\]  

(4.9)

For a given soil plug length \( (z) \) and \( K \cdot \tan\delta \) value, there is a certain diameter at which the minimal \( P_z \) force is obtained. Plugs with smaller or larger diameters would lead to forces greater than the minimal value.

The same procedure when used for Eq. 4.3a leads to the following relations:

\[
\frac{\sigma_{z\text{min}}}{ID} = \frac{1}{4K\tan\delta} \left[ 1 - e^{-4K\tan\delta \frac{Z_m}{ID}} \right]
\]

The stress increases or decreases with an increase or a decrease in the ratio of \( z/ID \), respectively. In the case of a constant plug length \( (z = \text{const.}) \), an increase in diameter would lead to a decrease in the stress, as observed in Fig. 4.3.b. For the condition where \( z/ID = \text{const.} \), the relations in Eq. 4.3a indicate that a linear increase in the stress is expected with an increase in \( ID \), as there is no influence on the exponential term as long as \( z/ID \) remains constant.

The relations given by Eqs. 4.8 and 4.9 of \( K \cdot \tan\delta \) and the plug length normalized by its diameter, are presented in Fig. 4.4. Using Fig. 4.4 it can be observed that for the soil parameters used in Fig. 4.3 \( (K \cdot \tan\delta = 0.0375) \) the minimal average force \( P_z \)
would occur at a depth to diameter ratio ($z/ID$) of 20. Figure 4.5 presents the relations of Eq. 4.3b for three different plug lengths. The minimal force $P_z$ for all three depths is located along the relations described in Eq. 4.9. For the particular chosen $Kt\tan\delta$ value of 0.0375, all minimum forces will be achieved when at the depths of 10, 15 and 20m the pile diameter will be 0.5, 0.75 and 1.0m, respectively (for all $z/ID = 20$).
4.5 THE MEANING OF THE DEVELOPED RELATIONS

The practical meaning of the developed relations can be explained through the curves of Figs. 4.6 and 4.7. The curves of Fig. 4.6 describe the relations between the force $P_z$ and the diameter $ID$: (1) according to Eq. 4.3b for a particular $Ktgd$ and an inner soil length of $z = 15m$; (2) assuming a range of point bearing stresses ($q_p$) for a depth of 15m; the total point force is plotted for different pile diameters, (ID).

For illustration purposes, Fig. 4.7 was constructed out of Fig. 4.6 using only one representative curve describing the $P_z = q_p \cdot A$ relations. The curves of Fig. 4.7 imply that for given soil–pile conditions and referring to $z$ as a penetration depth, there is a certain inner pile diameter which would fulfill the assumed plugging conditions. For the case of Fig. 4.7, a pile with ID = 0.75m will reach the state where the tip force resistance will be equal to the inner soil plug resistance (calculated from Eq. 4.3b) at a depth of 15m. From that point on, the soil plug length will remain constant, assuming no major increase of the point bearing capacity with further penetration. In the presented example, the pile plugs at the depth to diameter ratio of 20 which is approximately the critical depth for the point bearing stresses. Referring to the the two curves of Fig. 4.7 we can conclude that the force at the tip of a certain pile diameter (under the given conditions) will be the smaller of the two possible values (marked by shaded areas in Fig. 4.7). For piles with a diameter smaller than the particular diameter, the force at the tip is determined by the soil bearing capacity. For piles with a diameter greater than the particular one, the force at the tip is approximated by the resistance of the plug. Under the assumed parameters of Fig. 4.7, piles with diameters smaller than 0.75m which have reached the depth of 15.0m are already plugged, as their soil–plug resistance exceeded the tip bearing capacity prior to that depth. Piles with diameters larger than 0.75m are not yet plugged at the depth of 15.0m, and their tip resistance can be approximated to be that of the inner soil resistance.
4.6 ASSESSMENT OF THE INNER SOIL DEFORMATIONS

The relations of Eq. 4.3 can be further developed in order to assess the soil deformations which are associated with the calculated forces. In order to do so, the soil modulus of elasticity has to be estimated. The relations suggested by Hardin and Richart (1963) for angular sand in conjunction with small strains ($\gamma \approx 10^{-5}$) are:

\[ G = 320 \cdot \frac{(3-e)^2}{1+e} \sqrt{\sigma_0'} \]  

(4.10a)

where $\sigma_0'$ is the average confining stress in kg/cm$^2$ and G is in the same units.

For $\sigma_r = K\cdot \sigma_z$ and $\sigma_\theta = \nu\cdot \sigma_z$, $\sigma_0'$ can be estimated by $\sigma_0' = \frac{1}{3}(1+K+\nu)\sigma_z$.

Seed and Idriss (1970) have rewritten Eq. 4.10a as:

\[ G = 100 \cdot C_1 \cdot \sqrt{\sigma_0'} \]  

(4.10b)

and have suggested the following values for the parameter $C_1$:

<table>
<thead>
<tr>
<th>State of Sand</th>
<th>Loose</th>
<th>Medium</th>
<th>Dense</th>
<th>Very Dense</th>
<th>Very Dense Sand &amp; Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dense</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>7.5</td>
<td>11</td>
<td>15</td>
<td>20</td>
<td>28–41</td>
</tr>
</tbody>
</table>

Since the deformations occur in a mode of confined compression, we are interested in the constrained modulus, which is expressed via modulus of elasticity and Poisson's ratio as:

\[ D = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \]

Substituting for $E = 2G(1+\nu)$, the above expression for $\sigma_0'$ and Eqs. 4.10 for G, leads to:

\[ D = \frac{(1-\nu) \cdot 2C}{(1-2\nu)} \left[ \frac{(1+K+\nu)\sigma_z}{3} \right]^{1/2} \]

where $C = 320 \cdot \frac{(3-e)^2}{1+e}$ for the relations of Eq. 4.10a.
or $C = 100 \cdot C_1$ for the relations of Eq. 4.10b.

Simplifying the above, we get:

$$D = K_1 \cdot \sigma_z^{1/2}$$

where $K_1 = \left(\frac{(1-\nu) \cdot 2C}{(1-2\nu)}\right) \left[\frac{1}{3} (1+K+\nu)\right]^{1/2}$

The deformation $dw$ of an element with a thickness of $dz$ is:

$$dw = \frac{\sigma'_z}{D} dz = \frac{1}{K_1} \sigma'_z^{1/2} dz$$

Integrating and substituting $\sigma'_z$ for the calculated vertical stress in the soil plug, Eq. 4.3a leads to:

$$w = \frac{(K_2 \cdot \gamma)^{1/2}}{K_1} \int_0^L \left(\frac{e^z}{K_2} - 1\right)^{1/2} dz$$

where $K_2 = \frac{L \cdot D}{4K \cdot \delta}$

For the substitution of $y = \left(\frac{e^z}{K_2} - 1\right)^{1/2}$

$z = \ln(y^2+1) \cdot K_2$, and differentiation with respect to $y$ gives:

$$dz = 2K_2 \frac{y}{y^2+1} dy.$$ The above Equation can then be transformed into:

$$w = \frac{(K_2 \cdot \gamma)^{1/2}}{K_1} \int_0^L y \ dy = \frac{2\gamma^{1/2} \cdot K_2^{3/2}}{K_1} \int_0^e \frac{y^2}{y^2+1} \ dy$$

From which the deformation is obtained as:

$$w = \frac{2\gamma^{1/2} \cdot K_2^{3/2}}{K_1} \left(F - \tan^{-1} F\right)$$

(4.11)

where $F = \left(\frac{L}{K_2} - 1\right)^{1/2}$
4.7 THE 'SILO APPROACH' IN LIGHT OF EXPERIMENTAL DATA

4.7.1 Purpose

The 'Silo Approach' was developed as an initial simplified analysis in order to study soil plug behavior under static loads. For this reason, careful observations of the obtained relations in comparison with experimental data are needed. Experimental data enable the assessment of the approach through its assumptions, which may lead to conclusions regarding its adequacy, points of weakness, and necessity of further development.

4.7.2 Distribution of Forces along the Soil Plug

Isemoto (1976) and Kishida and Isemoto (1977) described a series of tests where 'soil plugs' were pushed. Pipes ranging in diameter from 0.3 to 1.0m were filled with sand to different heights. The sand rested on a rigid plate which was then pushed upwards. The applied forces as well as the bottom and top displacements were monitored (see Fig. 3.37).

A finite element analysis, which was modified to account for the wall friction, was used by Isemoto to analyze the experimental data. Figure 4.8 describes the cumulative friction force (from the plug bottom upwards) in relation to the total plug force vs. the distance normalized by the diameter of the plug. The relations which are attributed to the F.E. analysis indicate:

1. A build-up of friction forces occurs at the lower part of the soil plug. About 85% of the total force is attributed to friction along the lower portion of about 2 diameters in length.

2. Regardless of the soil height inside the pile, the same friction distribution is observed. Analyses for soil height to diameter ratio ranging from 2.6 to 4.1 show exactly the same distribution in which the major part of the force is attributed to the lower part of the soil plug, as mentioned above.

No direct measurements of friction distribution along the soil were performed by Isemoto. The following analysis was therefore used in order to assess the missing information:

1. Data of plug resistance, in which for the same pile diameter different soil heights
were tested, were utilized; e.g., for I.D. = 0.581m, four soil heights ranging from 1.5 to 2.4m (corresponding to height over inside diameter ratio ranging from 2.6 to 4.1, respectively) were checked. (See illustration within Fig. 4.8 and Table 4.1.)

2. Taking the maximum soil height as a 100% condition, the consequences of soil removal were checked, assuming that the differences in resistance can be attributed to the removed part; e.g., considering the case where I.D. = 0.581m, the maximum soil height was 2.4m (z*/ID = 4.13) with a resistance force of 496 tons. When the soil height was 2.1m, the measured resistance was 312 tons. Based on the stated assumption, it was then calculated that the lower 0.3m corresponding to z*/ID = 0.52 had carried 184 tons, or 37% of the total friction force. The weight of the soil was neglected in relation to the observed high resistance forces.

Although our assumption about the role of each additional layer is a given fact, as every case was tested to its full capacity, the force distribution (and the friction stresses to a lesser degree) is not necessarily unique and therefore not proven by this method.

3. This method was used in order to obtain the data points and construct the curve for the two plug diameters which are denoted on Fig. 4.8 as "Analysis of Isemoto's (1976) Experimental Results." (For details refer to Table 4.1).

4. Utilizing the 100% condition (maximum height), the K·tgδ value was calculated through Eq. 4.3b such that the calculated Pz will be in close agreement to the measured one.

5. Using the back–calculated K·tgδ value for the maximum soil height condition, Pz was calculated for the other heights according to the tests. The force distribution was then established in the same manner as was presented above for the measured values (for details refer to Table 4.1).

6. The obtained data points and curve are presented in Fig. 4.8, denoted as 'Silo Approach Analysis.'
Both analyses resulted in distributions which are in fairly good agreement with each other, as can be observed in Fig. 4.8. The results emphasize the role of the lower part of the soil plug, even more than the results of the F.E. analysis.

About 85% of the load was found to be carried by friction, within a distance which is approximately one diameter from the tip upwards.

Another observation which emerges from that analysis is that back-calculation of $K \cdot \tan \delta$ values using the test results leads to [see columns (8) and (9) of Table 4.1]:

1. Different values for different soil heights, such that the lower the soil height the higher the $K \cdot \tan \delta$ value.
2. Different values for different pile diameters, such that the larger the diameter the lower the $K \cdot \tan \delta$ value.
3. Estimation of the forces using one backfigured $K \cdot \tan \delta$ leads to an error of up to 50%.

### 4.7.3 Deformations of the Soil Plug

For most cases, the total displacement of the plug is of main concern (e.g. for load deformation predictions). Even though, it is interesting to compare the calculated deformations of the soil as developed in Sec. 4.6 with those observed in experiments.

Isemoto (1976) and Kishida and Isemoto (1977) tested the sand which they used to fill the pipes. As average representing relations they suggested the use of

$$ E = 1000 \sigma'_0^{1/2} $$

where $E$ is the soil's modulus of elasticity and $\sigma'_0$ is in kg/cm².

Inspection of Isemoto's test results reveals, however, that the suggested relations are adequate only for the stress range of $1 \leq \sigma'_0 \leq 20$ Kg/cm². Under ultimate plug capacity, the stresses usually exceed the above range. Based on Isemoto’s data, the relations of

$$ E = 1340 \cdot \sigma'_0^{1/2} $$

would seem to fit better for the range of stresses where $\sigma'_0 \geq 20$ Kg/cm². Using the above
relations enables one to obtain the value of $C$, as expressed in Section 4.6, which was calculated to be $C = 515$ (for $\nu = 0.3$). Using this value with Eq. 4.10 enables estimation of the plug's deformation under the ultimate loads.

The comparison between the measured and the calculated values was performed in the following way:

1. The measured displacements at the top and the bottom of the plug were obtained (under ultimate loads).
2. The difference between the displacements would account for the deformation of the soil plug itself (see Table 4.1).
3. Using the best match values of $P_2$ and $K_tg\delta$ [see column (8) of Table 4.1] and the above calculated $C$ value, enables utilization of Eq. 4.11 for the assessment of the plug deformations.
4. The data of both approaches are presented in Fig. 4.9.

The obtained results are in excellent agreement with the calculated ones. It should be noted, however, that if the suggested relations of Eqs. 4.10 were used, it would lead to $C$ values about 3 times higher than those obtained from Isemoto's tests.

The relations of Eq. 4.10 are to be used with small strains as noted previously. To use those suggested values, a correction factor should be applied in order to account for higher strain values (e.g. Seed and Idriss—1970).

4.7.4 Plug Forces and Stresses for a Constant Soil Height Over Diameter Ratio

No assumptions were made concerning the behavior of the plug material in the 'Silo Approach' analysis. Therefore, based on Section 4.4, an increase in the diameter under the conditions where a given $z/ID$ remains constant would not change the exponential term, and would lead to:

1. Linear increase in the stress $\sigma_z$
2. Rapid increase in the force.

Different cases out of Isemoto’s (1976) experiments were compiled in Table 4.2. Six cases refer to plug length over diameter ratios of about 3 and four cases refer to ratios of about 4.

Comparison of the measured forces of the different plug diameters to the above anticipated behavior shows no clear trend.

The relations of the measured plug stress vs. the inside plug diameter (for the two length over diameter values) are presented in Fig. 4.10. The obtained relations indicated that:

1. The smaller the plug diameter, the higher the stresses measured.
2. For the smaller diameters (less than 0.75m) a significant influence of the soil height is clearly seen. Stresses are doubled and tripled when changing the plug height over diameter ratio from 3 to 4.
3. For the larger diameters, no significant influence of the soil height is observed and the stresses are substantially smaller than those found in the smaller diameters.

This unexpected behavior becomes clearer upon examination of the results through the analysis of the ‘Silo Approach’.

Column (7) of Table 4.2 presents the results of the analysis in which the K values were calculated in order to comply with the measured plug forces, assuming a constant interface friction of $\tan \delta = 0.5$, as recommended by Isemoto (1976) and Kishida and Isemoto (1977).

The obtained results are presented in the upper section of Fig. 4.10, where the backfigured K values were plotted against the inside diameter. Additional data were added for the case where the ratio of plug height to diameter was equal to one, based on Kishida, et al. (1985).

In all three cases a clear trend is observed. With decrease of plug diameter,
higher K values are required in order to fit the ‘Silo Approach’ analysis to the observed data. When changing the plug diameter from 1.0 to 0.3 m the K values double from 0.7 to 1.4.

Two additional curves based on calculated values [see column (8) of Table 4.2] were added to Fig. 4.10 (denoted as ‘Silo Approach’ analysis). These curves enable a better understanding of the observed experiment in comparison to the analysis, for which a constant Ktg6 was assumed for all diameters.

The immediate conclusion that can be made, based on Fig. 4.10, is that the pile diameter has a great influence on the manner in which the soil behaves. A further assessment of the meaning of these results will be presented in Section 4.8 which concludes this chapter.

4.7.5 The Meaning of the Plugging and the Developed Relations As Interpreted in Section 4.5

Kishida (1967a, b) described a series of tests carried out on model brass piles ranging in diameter from 1.0 to 15.0 cm. The piles were pushed, open and closed-ended, to a depth of 45.0 cm. Details about the tests and results of data reduction are presented in Appendix II.

In order to examine the presented ‘Silo Approach’ analysis, Kishida’s data were utilized in Fig. 4.11 in the following way:

1. An average point bearing capacity stress q_p = 2.3 kg/cm³ was calculated from the results of load tests conducted on the closed-ended piles. Considering the existing average unit weight and friction angle of γ = 1.38 g/cm³ and φ = 33°, and 45 cm depth of penetration, the bearing capacity stress leads to a bearing capacity factor of N_q = 37 which is in agreement with the values suggested by Zeitlen and Paikowsky (1982).

2. The curve of P_z = Q_p = 2.3 A was then constructed. A is calculated as the area of the pile opening using the inside diameter denoted as ID (Q_p in kg for A in
3. Based on the uplift tests, an average skin friction angle of $\delta = 9.5^\circ$ was calculated. Admittedly low, this average value does not agree with the $\delta = 15.6^\circ$ quoted by Kishida (1967a) as a result of a pulling test on a single pile. Both numbers imply a very smoothly finished brass surface, as will be explained and analyzed in an upcoming chapter discussing the subject of interface friction. No data were furnished concerning the inner part of the brass tubes; it was therefore assumed that the internal brass finished condition is equal to the external one, and the same friction angles can be utilized for the inner soil–brass interface.

4. Due to the assumptions and simplifications of the ‘Silo Approach’ analysis, the $K$ value is not inherited as part of the overall considered factors. In this preliminary stage, the value of $K$ is assessed crudely using the average plug internal friction angle ($\phi_{avg} = 33.8^\circ$) and values obtained by an approach which will be presented later (see Fig. 7.8).

   a) Minimum value of $K$:

   $$K_{min} = K_a = \tan^2(45 - \phi/2) = 0.285$$

   b) Maximum value of $K$ assuming ‘Active arching’ conditions:

   $$K_{max} = \frac{1}{1 + 2\tan^2 \phi} = 0.527$$

   c) Using $K_{min}$ with the higher $\delta$ value and $K_{max}$ with the lower $\delta$ value as an approximation to obtain an average estimation leads to the values of $K \cdot \tan \delta = 0.08$ and $K \cdot \tan \delta = 0.09$, respectively.

5. Using Eq. 4.3b with the value of $K \cdot \tan \delta = 0.09$ and $z = 45$cm leads to the relations of $P_z$ vs. ID, as presented in Fig. 4.11.

6. The two curves then predict that piles with inside diameters smaller than approximately 3cm will be plugged at a depth of 45cm, while piles with larger diameters will not.
7. The data of the different plug capacities as calculated in Appendix II, based on Kishida's experiments, were then added to Fig. 4.11. For each test, the average specific plugging ratio for the last 15cm of penetration was denoted. Recalling the specific recovery ratio for samples as described in Chapter 3, $\gamma$ approaching zero indicates a completely plugged condition while $\gamma$ approaching 100% and over indicates an unplugged state, where free flow of soil into the opening takes place along with the penetration of the pile.

8. The obtained results showed remarkable agreement with the "Silo Approach" principle. However, it was indicated that the 'certain diameter' (in relation to Fig. 4.11) is about 7cm for Kishida's experiments. Piles with inside diameters smaller than or equal to 7cm were found plugged at a depth of 45cm and their tip resistance was in excellent agreement with the expected one, based on the tip bearing capacity of the closed-ended piles. Piles with inside diameters exceeding 7cm were found to be completely unplugged. However, the $K_{tg\delta}$ value of 0.09, which was first crudely estimated, was found to be smaller than the actual number describing the conditions according to Eq. 4.3b (as could be expected).

9. $K \cdot tg\delta = 0.2$ was then used in Eq. 4.3b in order to reconstruct the relations between $P_z$ and ID according to the 'Silo Approach'.

The purpose of this section was to judge the value of the presented 'Silo Approach' in light of experimental data. It is therefore suggested not to criticize the crudeness of the process, as it is by definition a by-product of its simplistic assumptions. This process indicates the procedure's points of weakness (as referred to in the summary), but nevertheless allows for assessment of the value of the method.
4.8 SUMMARY DISCUSSION AND CONCLUSIONS

1. Simple equations were developed for determining the stresses, forces and deformations of a soil plug. As the development was inspired by the approach which is used for silo analysis, it is termed 'The Silo Approach'.

2. The predominant factor which controls the plug resistance is $K \cdot \tan \delta$, consisting of the ratio between horizontal to vertical stresses along the pile interface ($K$), and the interface friction coefficient ($\tan \delta$) (see Fig. 4.2).

3. The behavior of plugs consisting of cohesionless soil was analyzed utilizing the 'Silo Approach' and compared to experimental data.

3.1 $\sigma_z$ - The vertical stress

a) The vertical stress, as well as the vertical force, increases or decreases with the soil plug length: i.e., the plug resistance increases along with soil penetration to the inside of the pile during driving. This was confirmed experimentally as shown in Column (5) of Table 4.1. For the two tested plugs, the force increased with the increase in soil height.

b) The vertical stress increases or decreases with an increase or decrease of the ratio $z/ID$: i.e., for a constant plug length (say, same penetration depth), an increase in the pile diameter would lead to a decrease in the vertical stresses and vice versa. This is confirmed in Fig. 4.10, where the stresses are much higher for the case where $z/ID = 4.0$ compared to $z/ID = 3.0$. The influence of the diameter under the same plug length can be seen through the data presented in Table 4.2: e.g., for identical plug lengths of 2.4m, the pile with a diameter of 0.581m had developed stresses of 1871t/m$^2$ compared to 175t/m$^2$ for the pile with a diameter of 0.800m.

c) For a constant plug length over diameter ratio ($z/ID = \text{const.}$), an increase in pile diameter is expected to cause a linear increase in the
vertical stresses. The experimental data presented in Fig. 4.10 show the opposite. The stresses were found to decrease substantially when the diameter increases under constant z/ID conditions.

3.2 $P_z$ – The vertical force

a) The force approaches a minimum value at a particular $z/ID$ ratio. Values of $z/ID$ smaller or larger than this ratio would lead to an increase of the anticipated force. The practical meaning of these relations is explained by combining the tip force capacity with the plug capacity. Such an explanation enables one to examine the mechanism of plugging as first presented in Chapter 2. Analysis and presentation of experimental data, as shown in Sections 4.5, 4.7.5 and Fig. 4.11, verify the assumed concept.

b) The cumulative friction force at each plug level can be calculated using Eq. 4.3b (neglecting the soil’s weight). Using this analysis in comparison with processed experimental data and F.E. analysis has indicated (see Fig. 4.8) that about 85% of the total force is attributed to friction along the lower portion of the soil plug, about one to two diameters in length.

3.3 The calculated deformation of the plug, as expressed in Eq. 4.11, was in good agreement with the experimental data. The sand modulus of elasticity was obtained from test results.

4. The importance of the $K \cdot \tan \delta$ factor has been described. Additional information was obtained from comparison of the ‘Silo Approach’ to the experimental data.

4.1 The stresses and forces increased and decreased with the appropriate change in the soil plug length, as described. However, back–calculation of $K \cdot \tan \delta$ values indicated that larger values were needed for the analysis to describe the experimental results of the smaller soil columns. When these results are combined with the observed force distribution, it leads
to the conclusion that the $K \cdot \tan \delta$ factor cannot be assumed to be constant along a certain soil plug.

4.2 The analytical predictions from the ‘Silo Approach’ were opposite to the experimental data, which clearly showed a substantial increase in the stresses when decreasing the diameter (under constant length over diameter ratio). The gathered calculated $K$ values of three different cases, shown in the upper part of Fig. 4.10, clearly indicated that if similar friction conditions are to be assumed for all plugs, then the ratio between the horizontal to the vertical stress, $K$, is a function of the plug diameter.

5. Relations were found to exist between the stresses and the forces of the plug to the plug diameter and the soil height. No assumptions were made in the ‘Silo Approach’ regarding the material properties. These relations are therefore reflected through the calculated $K$ values which indicate the manner in which the soil behaves. The increase of $K$ with the decrease in plug diameter is dominant; however, an increase of $K$ with a decrease in plug height could also be identified. The increase in $K$ value means an increase in horizontal stresses, which may be the result of mainly two mechanisms: soil dilatancy and/or arching.

6. Kishida et al. (1985) concluded that dilatancy is the reason for increasing horizontal stresses on the inner soil–pile interface. Dilatancy can be expected to be of major importance in the case of dense sand under low confining stresses. Therefore, for a certain I.D., the decrease of $K$ with the increase in plug height (due to decreased dilatancy) can be accepted as a reasonable explanation. Those low confining stress conditions refer to very small plug heights ($z/ID \approx 1$). These are not of any significance for the practical aspect of pile plugging. Moreover, Kishida et al. (1985) report that the sand in pipes following its
placement is at 'rest' condition \( K \approx K_0 \approx 1 - \sin \phi \). The higher K values for higher soil plugs, which are much in excess of \( K_0 \), cannot therefore be explained by dilatancy only.

7. The strong correlation between the higher K values and the smaller diameter can theoretically be explained by both dilatancy and arching. The arching phenomenon seems, however, to provide a better and more consistent explanation due to several reasons:

a) The piles with the smaller diameters are engaged with large plug resistances and very high confining stresses which would prevent dilatancy from developing.

b) In all diameters under all heights the K values exceed any 'reasonable' values (see Fig. 4.10 where all K values are over 1.0). This includes all cases where dilatancy can or cannot exist.

c) Kishida et al. (1985) described an additional set of tests. In these tests, the disc (on which the sand was resting) was first displaced downwards rather than pushed upwards against the soil. The tests clearly indicate two results:

i) Regardless of the soil height \( z/ID \) ratio ranged from 2.0 to 8.0 with \( ID = 0.3m \) an abrupt decrease in the stresses took place with the bottom downward displacement. With the first displacement from 0 to 3mm, the stress on the bottom disc decreased by 50 to 80% of its original value. With further displacement of up to 15mm the stresses increased gradually, but remained significantly lower than those at rest (see Fig. 4.12).

ii) For a constant ratio of \( z/ID = 2.5 \), the decrease in stresses of the smaller diameters was of greater magnitude than that of the larger diameters. The stresses on the disc under the soil plug
with ID = 0.3m were reduced by 70%, while in the case of the plug with ID = 0.8m the stresses were reduced by 35% only (see Fig. 4.13).

If it is assumed that the entire soil plug moved downwards when the disc was displaced by 3mm, this displacement corresponds to a longitudinal strain of $\varepsilon_a = 0.125\%$ to 1% (for 2.4m and 0.3m plug length, respectively). This 'extension' cannot cause dilatancy in the soil medium, considering the reduction of the vertical stresses.

The aforementioned test results can only be explained through an arching mechanism. Due to the downward displacement of the disc, the soil loses its 'support' and the load is transferred to the rigid pile walls, where equilibrium is maintained by the shear forces along the soil–pile interface. The vertical stress reduction results in a convex upwards arch of grain contacts which follows the major principal stress trajectory. This arch in axisymmetric conditions assumes the shape of a spherical cap. Being along the principal stress trajectory, the arch has zero shear stress along its boundaries, and therefore is under triaxial compression conditions. During the increase in loading the arch dilates, enabling the transfer of large horizontal stresses to the pile wall. This results in large frictional stresses which allow the support of the soil above the arch. In the case of large diameters, this arch is destroyed by the soil above it.

The described concept is further examined in reference to the orientation of soil grains and arching in silos in Chapter 5, and in reference to pile plugging in Chapter 7.

8. In summary, $K \cdot \tan \delta$ is the dominant factor in controlling the soil plug mechanism. The $K$ value, which represents the ratio between the horizontal to the vertical stresses, is influenced mainly by the arching mechanism.
9. In order to obtain a better understanding and analysis of the soil plug behavior under static loads, it is essential to investigate both these areas of importance:

i) What is the mechanism which controls the shear of the soil along the interface: i.e. what is the appropriate $\tan \delta$ to be used in conjunction with the soil plug displacement?

ii) What are the expected relations between the normal and tangential stresses along the interface: i.e. what is the appropriate $K$ value to be used in conjunction with the arching mechanism which takes place in the soil plug?
Table 4.1 - Force Distribution Along the Soil Plug and Plug Deformation

<table>
<thead>
<tr>
<th>I.D. (1)</th>
<th>Z* (2)</th>
<th>ΔZ* (3)</th>
<th>γ (4)</th>
<th>Measured</th>
<th>Calculated</th>
<th>Measured</th>
<th>Cal.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.581</td>
<td>2.40</td>
<td>-</td>
<td>1.71</td>
<td>496</td>
<td>-</td>
<td>-</td>
<td>510-0.50</td>
</tr>
<tr>
<td></td>
<td>2.10</td>
<td>0.30</td>
<td>1.72</td>
<td>312</td>
<td>0.52</td>
<td>.37</td>
<td>303-0.54</td>
</tr>
<tr>
<td></td>
<td>1.80</td>
<td>0.60</td>
<td>1.73</td>
<td>64</td>
<td>1.03</td>
<td>.87</td>
<td>62-0.50</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>0.90</td>
<td>1.75</td>
<td>48</td>
<td>1.55</td>
<td>.90</td>
<td>47-0.58</td>
</tr>
<tr>
<td>0.6872</td>
<td>2.80</td>
<td>-</td>
<td>1.70</td>
<td>312</td>
<td>-</td>
<td>-</td>
<td>320-0.44</td>
</tr>
<tr>
<td></td>
<td>2.45</td>
<td>0.35</td>
<td>1.72</td>
<td>192</td>
<td>0.51</td>
<td>.39</td>
<td>186-0.44</td>
</tr>
<tr>
<td></td>
<td>2.10</td>
<td>0.70</td>
<td>1.67</td>
<td>56</td>
<td>1.02</td>
<td>.82</td>
<td>55-0.45</td>
</tr>
<tr>
<td></td>
<td>1.75</td>
<td>1.05</td>
<td>1.73</td>
<td>48</td>
<td>1.53</td>
<td>.85</td>
<td>45-0.53</td>
</tr>
</tbody>
</table>

(1) Inside diameter (meters)  
(2) Z* - Length of soil plugs (meters)  
(3) ΔZ* - Additional soil layer (e.g., 2.4-Z* for I.D. = 0.581)  
(4) γ - Unit of weight of soil (t/m³)  
(5) Pmax - Ultimate capacity of plug (tons)  
(6) ΔZ*/ID - Normalized segment length  
(7) ΔP/P100% - where ΔP = P100% - Pmax (P100% = 496, 312 for I.P. = 0.581, 0.6872m respectively)  
(8) Calculated values using Equation 4.3B  
(9) Calculated Pz for ktgδ = 0.50, 0.44 for ID = 0.581, 0.6872 respectively  
(10) ΔP/P100% - where ΔP = P100% - Pz (P100% = 510, 320 for I.P. = 0.581, 0.6872 respectively)  
(11, 12, 13) Measured displacements at top and bottom of plug and their difference in mm.  
(14) Calculated deformation using values of Column 8 and Equation 4.11.
Table 4.2 - Stresses and Forces for a Constant Ratio of Plug Length over Diameter

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>(m)</td>
<td>-</td>
<td>t/m³</td>
<td>ton</td>
<td>t/m²</td>
<td>tgδ = 0.5</td>
<td>Pz - Kcal</td>
</tr>
<tr>
<td>I.D.</td>
<td>L</td>
<td>L/I.D.</td>
<td>γ</td>
<td>Pmax</td>
<td>σz</td>
<td>Pz</td>
<td>σz</td>
</tr>
<tr>
<td>0.1562</td>
<td>0.469</td>
<td>3.00</td>
<td>1.65</td>
<td>9</td>
<td>470</td>
<td>9 - 1.420</td>
<td>1</td>
</tr>
<tr>
<td>0.2979</td>
<td>0.907</td>
<td>3.04</td>
<td>1.71</td>
<td>72</td>
<td>1033</td>
<td>71 - 1.420</td>
<td>8</td>
</tr>
<tr>
<td>0.5810</td>
<td>1.800</td>
<td>3.10</td>
<td>1.73</td>
<td>64</td>
<td>241</td>
<td>64 - 0.995</td>
<td>65</td>
</tr>
<tr>
<td>0.6872</td>
<td>2.100</td>
<td>3.06</td>
<td>1.67</td>
<td>56</td>
<td>151</td>
<td>55 - 0.890</td>
<td>96</td>
</tr>
<tr>
<td>0.8000</td>
<td>2.405</td>
<td>3.01</td>
<td>1.70</td>
<td>88</td>
<td>175</td>
<td>87 - 0.905</td>
<td>139</td>
</tr>
<tr>
<td>1.0002</td>
<td>3.080</td>
<td>3.08</td>
<td>1.71</td>
<td>88</td>
<td>112</td>
<td>86 - 0.740</td>
<td>317</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5810</td>
<td>2.400</td>
<td>4.13</td>
<td>1.71</td>
<td>496</td>
<td>1871</td>
<td>492 - 0.995</td>
<td>509</td>
</tr>
<tr>
<td>0.6872</td>
<td>2.800</td>
<td>4.07</td>
<td>1.70</td>
<td>312</td>
<td>841</td>
<td>309 - 0.875</td>
<td>749</td>
</tr>
<tr>
<td>0.8000</td>
<td>3.200</td>
<td>4.00</td>
<td>1.66</td>
<td>271</td>
<td>271</td>
<td>137 - 0.710</td>
<td>995</td>
</tr>
<tr>
<td>1.0002</td>
<td>3.970</td>
<td>3.97</td>
<td>1.68</td>
<td>152</td>
<td>194</td>
<td>155 - 0.630</td>
<td>1851</td>
</tr>
</tbody>
</table>

K = 1

Kgδ = 0.5
I.D = Inside Diameter of the Pipe  (ID = B - 2t)
L = Length of Soil Inside the Pile  (Plug Length)
γ = Unit Weight of Soil

σ_z, σ_h = Average Vertical and Horizontal (=radial) Stresses in the Cross-Section

K = Average Coefficient of Lateral Stress  (σ_h = K · σ_v)
τ = Friction Stress Along Interface  (τ = C_A + σ_h · μ)

C_A = Adhesion between Soil and Pipe
μ = Friction Coefficient  (μ = tg δ)
δ = Interface Friction Angle  (Soil/Pile)

A = Cross Section Area  [A = π(ID)^2/4]
U = Circumferential Area  [U = π · ID · dz]

Fig. 4.1: The ‘Silo Approach’ Analysis.
The Force at the Bottom of a Soil Plug vs. the Plug length for Different $K \cdot \tan \delta$ Values.

**Fig. 4.2:**

**Eq. 4.3B**

$\gamma = 1\text{t/m}^3$

$ID = 1.0\text{m}$

$K \cdot \tan \delta = 0.01$

$W_s = \text{Weight of Soil}$
Fig. 4.3: Representation of the 'Silo Approach' equations.
Determination of Minimum Plug Force Location.

Approximate Relations $Z/ID \approx 0.75 \frac{K}{tg \delta}$ (Eq. 4.9)

Fig 4.4: Determination of Minimum Plug Force Location.
Plug Force vs. Inside Diameter for Various Plug Lengths.

Fig. 4.5
Fig. 4.6: The 'Silo Approach' in conjunction with the Tip Bearing Capacity.

\[ A = \pi ID^2 / 4 \]
\[ \gamma = 1t/m^3 \]
\[ K \cdot \tan \delta = 0.375 \]
Determination of Plugging Conditions.

Fig. 4.7:

\[ A = \pi \cdot ID^2 / 4 \]
\[ K \cdot tg\delta = 0.0375 \]
\[ \gamma = 1.0 \text{ t/m}^3 \]
\[ z = 15.0 \text{ m} \]
Fig. 4.8: Distribution of Normalized Friction Forces along the Plug Length.
Fig. 4.9: Deformation of the Plug.
Experimental Results
Isemoto (1976)

'Silo Approach' Analysis
K\cdot\tan \delta = 0.5

Measured and Calculated Stresses for various Plug Diameters and the Appropriate Calculated K Values.

Fig. 4.10:
Determination of Plugging Condition for Experimental Data

Fig. 4.11:

Equation 4.3B

\[
A = \pi \cdot \frac{ID^2}{4}
\]

\[
\gamma = 1.49 \text{ gr/cm}^3
\]

\[
z = 45.0 \text{ cm}
\]
Fig. 4.12: Relationship between $\sigma_z/\gamma ID$ and Downward Displacement; ID = 300mm L/ID = 1.0 to 8.0 (after Kishida et al. –1985)

Fig. 4.13: Relationship between $\sigma_z/\gamma ID$ and Downward Displacement; ID = 300 to 800mm L/ID = 2.5 (after Kishida et al. –1985)
CHAPTER 5
THE ‘MICRO’ APPROACH TO THE BEHAVIOR OF GRANULAR MATERIAL

5.1 INTRODUCTION

Soil is commonly classified according to grain size. Even though division into several soil types based on grain size is arbitrary and artificial, two main types of soils can be considered: (1) soils in which body forces predominate, i.e., cohesionless soils; and (2) soils in which surface forces are important, i.e., cohesive soils.

Most of the properties peculiar to cohesive soils result from the electrochemical properties of clay minerals. As the clay particles exhibit colloidal action, their study and description through a mechanical model is not likely to be feasible. However, the cohesionless—granular material can be analyzed using a simplified model consisting of a mass of dry spheres, discs, or ovals.

Earlier applications of models composed of packings of discs and spheres to problems related to the mechanics of granular materials have been made by Jenkin (1931). Later studies investigated and explained different aspects of the granular material behavior.

This chapter reviews three areas of the micro approach. Following are brief descriptions of the subjects and reasons for their review.

*Subject (1)*

The mechanical models approach, and some of the obtained results. Introduction of the simplified models and their analyses, performed in order to permit their criticism, improvement, and utilization at a later stage.

*Subject (2)*

The analysis of loads in a silo using a model of uniform discs. This allows comparison of the results of a discrete model analysis utilizing an arching presentation with the results of an analysis based on a continuum. The subject was discussed previously, and
will be incorporated into the following chapters.

*Subject (3)* Elucidation of the micro behavior of soil particles enables one to better understand the structure of the soil and the real mechanism that controls its behavior. This knowledge is obtained by reviewing tests performed on soil and soil models with the purpose of studying the individual particles, their fabric, and behavior.

Review of the above subjects in the present work aids in understanding the soil behavior in conjunction with different aspects of pile plugging in granular material. Such an understanding allows the development of different models and concepts for the analysis of the stated problem, as discussed in Chapters 6 through 9.
5.2 PACKING OF SPHERES

Modeling of the cohesionless soil as a mass of dry uniform spheres is considered. Such a model may be represented by pure metal crystals in which all the ions are of equal size (e.g. Illston et al. —1979).

In general, the packing of the spheres (as well as the metal ion) will deform through a slip of the closely-packed rows in any one closely-packed plane. Six spheres is the maximum number which can be in contact with one particular sphere in a single plane (see Fig. 5.1—hexagonal shape). To extend this structure to 3-D, these closely-packed planes must be stacked on top of each other as closely as possible. Placing a second plane of spheres on top of the first (labeled B in Fig. 5.1) allows two possibilities for a third layer. The first possibility, labeled C, is not vertically above either the first or the second layer. The second is an A layer again. These arrangements lead to sequences of ABC ABC, known as F.C.C. (Face Centered Cubic), and AB AB, known as H.C.P. (Hexagonal Close-Packed). The two structures which show the closest packing and their slip planes are presented in Fig. 5.2. Calculating the void ratio for each repetitive unit of these structures leads to $e = 0.35$ (equal to $e_{\text{min}}$ for uniform spheres) and $e = 0.43$ for the F.C.C. and H.C.P., respectively. Both ratios are for dense material when compared to granular soils (see, for example, Table 6—D.M. 7.1, 1982 and Table 6.2 of Chapter 6).
5.3 THE STRENGTH AND SHEAR MECHANISM OF AN ASSEMBLY OF INDIVIDUAL PARTICLES IN CONTACT

5.3.1 Analyses using friction of inclined planes model

(a) Underlying concept

The stress dilatancy relation for static equilibrium of an assembly of particles in contact was investigated theoretically and experimentally by Rowe (1962), and was later implemented for earth pressure and slopes (Rowe –1963). Rowe used the following observations as a ground for his interparticle relations investigation:

1. It is generally assumed that the Mohr–Coulomb failure criterion, which is applicable to the shear of continuous materials, may also be applied to discontinuous assemblies of particles such as sand.

2. The Mohr–Coulomb criterion predicts a straight slip failure plane and shear parallel to a slip plane. Its application to a discontinuous assembly suggests that the assembly slides at failure, with no volume change.

3. The fact that contraction and dilation take place during shear suggests that particle movements during deformation and failure are not necessarily in the direction of the applied shear stress.

4. Study of the intergranular relations permits, therefore, an understanding of the mechanism which leads to failure and of soil behavior during shear.

Based on these observations, Rowe assumed that the basic concept of friction of inclined planes may be applied to describe the interparticle sliding. In general, the interparticle sliding will occur in a direction \( \varphi \) from the applied shear force \( P \) when \( Q \) is the force normal to the direction of \( P \) (see Fig. 5.3). Assuming cohesionless particles, the forces presented in Fig. 5.3 can be resolved in the following way:

\[
P = Q \cdot \tan (\phi \mu + \varphi)
\]
where $\phi \mu$ is the intergranular friction angle.

Figure 5.4 describes the relations between the direction of particle movement and the average inclination of the shear plane. Experiments performed on steel rods and steel spheres (Rowe -1962) confirmed that $\varphi$ is the inclination to the minor principal plane at which the particles tend instantaneously to move, while the final "slip plane" is in $\alpha$ inclination to the major principal plane at which the particles are interlocked. With reference to Fig. 5.4, $P = \sigma'_1 \cdot b$ and $Q = \sigma'_3 \cdot b \cdot \tan \alpha$. When substituted into the above equation, the following stress ratio is obtained:

$$\frac{\sigma'_1}{\sigma'_3} = \tan \alpha \cdot \tan (\phi \mu + \varphi)$$  \hspace{1cm} (5.1)

Equation 5.1 is applicable to any packing in which an imaginary plane of interlocked particles at angle $\alpha$ (to the direction of the minor principal stress) is drawn together with particles moving in contact in a direction $\varphi$ (to the major principal stress). In a closure to discussions regarding his approach, Rowe (1964) explained that the values of $\alpha$ and $\varphi$ are such that the lowest ratio of $\sigma'_1 / \sigma'_3$ is obtained. The following analyses are based on this approach; therefore, any stress rotation in respect to the packing would alter the angles, resulting in an increase of the ratio calculated by Eq. 5.1.

(b) **Uniform rods in a parallel stack**

A simple arrangement of rods or discs in a parallel stack subjected to a 2-D stress system is presented in Fig. 5.5a. From the geometrical relations it can be calculated that $\alpha = 60^\circ$ and $\varphi = 60^\circ$. Rowe (1962) describes the observed mode of failure in the following words:

"Slip took the form of a catastrophic movement...The rods on the $\alpha$ plane moved right past their supporting rods, to form a new contact on the row below while the rods in the zones on either side of this plane returned to their original packing."
Figure 5.5 describes this movement of the two rows, one in relation to the other (b), and the movement of an individual rod from its original position to its final state (c). It is obvious that the rod is sliding and/or rolling over its adjacent supporting rod from one position to the other, while the direction of the plane of contact varies with this movement.

(c) Uniform spheres in F.C.C. packing

Rowe's presentation of the F.C.C. packing is shown in Fig. 5.6a, b. A vertical cut (marked by A–A in Fig. 5.6) reflects the conventional F.C.C. face as shown in Fig. 5.2 According to Rowe (1964), the angles $\alpha_1 = 63.5^\circ$ and $\varphi = 45^\circ$ (shown in Fig. 5.6b) are those leading to the lowest ratio in Eq. 5.1; they were also confirmed by his experiments. The angle $\alpha_2 = 54.7^\circ$ of Fig. 5.6c differs from that presented by Rowe and is related to an analysis of a single sphere in the F.C.C. packing, to be presented in Section 5.3.2.

(d) Uniform spheres in H.C.P. packing

The packing is identical to that presented in Fig. 5.2 where $\sigma_3$ is the all-around stress in the plane parallel to the hexagon, and $\sigma_1$ is normal to the hexagon. For this arrangement $\alpha = 70.6^\circ$ and $\varphi = 54.7^\circ$.

(e) Volume change and energy consideration

The motion of the rods in relation to each other in the 2-D system explains the dilation followed by contraction, which takes place during shear. The rods move apart in a horizontal direction and then move vertically into the space left, as shown in Fig. 5.5d.

Along with a small increase in the effective stress ratio, there is a small change in the major and minor principal strains ($\delta_1, \delta_2$), strain rates ($\dot{\varepsilon}_1, \dot{\varepsilon}_2$), and in the
corresponding volume change $d\dot{v}$. From geometrical relations, Rowe (1962) showed that:

\[
\dot{\delta}_2 = \tan \varphi \quad \text{and} \quad \dot{\varepsilon}_2 = \tan \alpha \cdot \tan \varphi \left[ 1 + \frac{d\dot{v}}{\nu \dot{\varepsilon}_1} \right]
\]

where longitudinal compression and lateral expansion are positive.

For the 3–D case where $\dot{\delta}_2 = \dot{\delta}_3$:

\[
\frac{\dot{\varepsilon}_3}{\dot{\varepsilon}_1} = \frac{1}{2} \tan \alpha \cdot \tan \varphi = \frac{1}{2} \left[ 1 + \frac{d\dot{v}}{\nu \dot{\varepsilon}_1} \right]
\]  \hspace{1cm} (5.2)

These relations together with those of Eq. 5.1 can be used to calculate the energy ratio, which is the instantaneous rate of work done on the sample by $\sigma_i'$ to that done by the sample on $\sigma_3'$.

\[
\frac{\dot{E}}{2\sigma_3'} = \frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_3} = \frac{\tan (\phi \mu + \varphi)}{\tan \varphi}
\]  \hspace{1cm} (5.3)

If the particles are frictionless, $\phi \mu = 0$ and the energy ratio is unity. The energy ratio increases with $\phi \mu$ owing to internal work converted to frictional heat. The supplied energy increment ($\sigma_i' \cdot \dot{\varepsilon}_1$) is shared, therefore, between absorbed energy and external work.

It should be noted that Rowe eliminated the angle $\alpha$ from the energy equation. He explains it by the fact that the packing orientation adds a further degree of freedom, and $\alpha$ increases with $\sigma_i'/\sigma_3'$, where the ratio may obtain a minimum value (Rowe –1964).

The equation of internal energy (5.3) may be minimized in relation to $\varphi$. 
Differentiating and minimizing \( \frac{dE}{d\varphi} = 0 \) gives:

\[
\varphi = 45 - \phi\mu/2
\]

Substituting \( \varphi \) into Eq. 5.1 leads to:

\[
\frac{\sigma_1'}{\sigma_3} = \tan\alpha \cdot \tan(45 + \phi\mu/2)
\]  \hspace{1cm} (5.4)

Substituting the relations of Eqs. 5.2 and 5.3 into Eq. 5.4 leads to:

\[
\frac{\sigma_1'}{\sigma_3 \cdot \left[ 1 + \frac{d\dot{v}}{v\epsilon} \right]} = \tan^2(45 + \phi\mu/2)
\]  \hspace{1cm} (5.5)

When there is no volume change during shear, Eq. 5.5 becomes the familiar Mohr–Coulomb equation for ideal frictional material, for which \( \phi = \phi\mu \).

\( \textit{ff) Experimental results} \)

Experiments on packings of metal rods \( (\phi\mu = 10^\circ) \) and metal spheres \( (\phi\mu = 7^\circ) \) were conducted by Rowe (1962). The direction of the particle movements as found experimentally is presented in Figure 5.7. The essential findings for the packings were:

1. The stress ratio at the peak strength and during subsequent states of deformation followed Equation 5.1 for all particle packings.

2. The energy ratio for a fixed orientation of particle movement is given by Eq. 5.3.

3. The slip plane was not the cause of failure at the peak, but rather the result of particle movement in their contact directions. The slip occurred well past the peak, when the stress ratio reached unity.
A series of triaxial compression tests and direct measurements of $\phi \mu$ on quartzic sands, glass, and steel were carried out so that all the terms of Eq. 5.5 were measured independently. The essential findings for these tests were as follows:

1. Eq. 5.5 was valid for dense overconsolidated or reloaded soils throughout deformations, where the strains to the peak were of the order of a few percentage points.

2. Loose, normally consolidated soils on first loading suffered an additional loss of energy and it was necessary to introduce a value of $\phi_f$ in place of $\phi \mu$ in order to satisfy Eq. 5.5 (Rowe -1963). The loss of energy is explained to be due to additional friction as a result of deviation from the preferred direction, associated with a process of rearrangement of the particle assembly during deformation.

Rowe’s stress-dilatancy theory will be discussed further in Chapter 8, where it will be compared to a new model of granular material behavior (developed in Chapter 6), and will be examined in light of additional data to be presented in subsequent sections.

5.3.2 Detailed 3-D analysis of a compression test on an F.C.C model of a granular medium.

Rowe (1962) developed a general concept regarding the failure mechanism of an assembly of particles, as previously presented. The principle of friction of inclined planes was applied for the analysis of various rod and sphere arrangements under a certain state of stress. This method of analysis made it possible to avoid of two major difficulties:

1. Each particle assembly can be oriented in any direction, and for a certain applied stress system there will be a preferred orientation of what may be called failure planes.
2. A real 3-D analysis of a certain sphere under a general stress system is statically indeterminate. Each sphere makes contact with other spheres at up to twelve points, and at each point there are generally one normal and two tangential forces.

A comprehensive 3-D analysis of sphere assembly was developed by Thurston and Dersiewicz (1959), and further explained and used by Scott (1963). The abovementioned difficulties were overcome by choosing the most dense packing (F.C.C) under dry conditions, subjected to a known stress history in a certain direction. Analysis of an individual sphere under hydrostatic stress ($\sigma_3$) and under deviatoric stress ($\sigma_1 - \sigma_3$) enables one to follow the motion of the displaced sphere.

Figure 5.8 describes the elementary F.C.C cube in the conventional axes notation. When the cube is subjected to a hydrostatic compression stress $\sigma_3$, the force on each face of the cube is $8 \cdot R^2 \cdot \sigma_3$. The force on each individual sphere is assumed to be proportional to the exposed cross-sectional area on the cube's face, and therefore is $4 \cdot R^2 \cdot \sigma_3$. Under hydrostatic compression, no tangential forces exist and the above forces will be balanced by the contact forces between the spheres, each of $\sqrt{2} \cdot R^2 \cdot \sigma_3$.

Considering a single sphere in the closely packed hexagonal layer, reference is made to Figure 5.2b, where the F.C.C structure and the four slip planes are presented. Each of the planes is octahedral, and for the convenience of the analysis, a new system of axes is defined and presented in Figure 5.8. The $\bar{z}$ axis is perpendicular to the octahedral plane, and therefore to the plane of the hexagonal layer. Figure 5.9 presents the hexagonal layer in reference to the newly defined coordinate system. Sphere B is a typical sphere in the considered plane. It is confined by six surrounding spheres in its own layer and three spheres in each of the overlying and underlying layers. The fact that each sphere is confined by six others in its own plane and only three in the underlying plane justifies referring to the movement of one plane over the underlying
one, and not to movements within the plane itself. Assuming that the spheres are of
equal roughness, it can be concluded that movement is more likely to occur when each
sphere rises over the ‘valley’ between two spheres in the underlying layer and not by
one sphere climbing over the surface of another. This motion was also experimentally
inspected and failure was found to occur when a sphere such as B moves over the
valley between E and F (see Figure 5.10)

The six forces due to contacts with adjacent spheres have no component
perpendicular to the plane of the layer. The resultant of the three forces from the
overlying spheres is \( Q = 2\sqrt{3}R^2\sigma_3 \), which acts in the negative \( \bar{z} \) direction.

In addition to the hydrostatic state, the force \( L \) is imposed such that it is
inclined at angles \( \beta \) and \( \gamma \) to the positive \( \bar{y} \) and \( \bar{z} \) axes respectively. The force is
considered to be in the \( \bar{y} - \bar{z} \) plane only, as we are interested in the minimal force
sufficient to cause movement of B (in the \( \bar{y} - \bar{z} \) plane). Consequently, it is assumed
that there is no force between G and B. The forces which are acting on B are therefore
\( Q, L, \) two equal normal forces \( N \) at the points of contact with E and F, and two
tangential forces at these points of contact, which may be taken to be \( T = N \cdot \tan \phi \mu \), as
\( \tan \phi \mu \) is the coefficient of friction between two spheres (see Figure 5.10). The two equal
tangential forces \( T \) tend to rotate the sphere B. Thurston and Dersiewicz assumed in
their analysis that no rotation occurs, as the torque will be opposed by tangential
forces exerted by the other nine spheres in contact. It should be noted that
experimental data, including that of Thurston and Dersiewicz, suggest that the spheres
actually move by rotation.

The resolution of the forces acting on sphere B indicates that the force \( L \)
required at equilibrium when slipping is about to take place is given by the expression:

\[
\frac{L}{2R^2\sigma_3} = \frac{3 + 4\sqrt{3} \cdot \tan \phi \mu}{2(\sqrt{6} - \tan \phi \mu) \cos \beta - (\sqrt{3} + 4\sqrt{2} \cdot \tan \phi \mu) \cos \gamma} 
\]  

(5.6)
When considering the ratio of shearing to normal forces we get:

\[
\frac{L \cdot \cos \beta}{2\sqrt{3} \cdot R^2 \cdot \sigma_3 + L \cdot \cos \gamma} = \frac{\sqrt{3} + 4\sqrt{2} \cdot \tan \phi \mu}{2(\sqrt{6} - \tan \phi \mu)} \tag{5.7}
\]

which is the obliquity.

Considering the force L as arising from a principal stress difference (in failure), \((\sigma_1 - \sigma_3)_f\), as shown in Figure 5.9, it can be substituted by:

\[
L = 2\sqrt{3} \cdot R^2 (\sigma_1 - \sigma_3) \sin \beta
\]

and when substituting into Equation 5.7 we obtain:

\[
\frac{\left[ (\sigma_1 - \sigma_3)_f / 2 \right] \cdot \sin 2\beta}{(\sigma_1 + \sigma_3)_f / 2 - \left[ (\sigma_1 - \sigma_3)_f / 2 \right] \cdot \cos 2\beta} = \tan \phi = \frac{\sqrt{3} + 4\sqrt{2} \cdot \tan \phi \mu}{2(\sqrt{6} - \tan \phi \mu)} \tag{5.8}
\]

Considering a constant principal stress difference acting at a varying angle \(\beta\), the angle at which the maximum obliquity is obtained can be found by differentiating the L.H.S of Equation 5.8 with respect to \(\beta\) and setting it to zero:

\[
\cos 2\beta = (\sigma_1 - \sigma_3) / (\sigma_1 + \sigma_3)
\]

Substituting the above into Equation 5.8 we obtain:

\[
\tan \phi = \frac{(\sigma_1 - \sigma_3)_f / 2}{\sqrt{\sigma_{1f} \cdot \sigma_{3f}}} = \frac{\sqrt{3} + 4\sqrt{2} \cdot \tan \phi \mu}{2(\sqrt{6} - \tan \phi \mu)} \tag{5.9}
\]

The expression of Equation 5.9 corresponds to a maximum obliquity, describing the failure mechanism of an idealized granular material in the form of spheres in an F.C.C packing under certain stress conditions. It states that failure by slipping will
occur on a particular surface when the ratio (obliquity) of tangential (shearing) stress to normal effective stress on the surface reaches a certain maximal value. The above analysis provides, therefore, a theoretical basis for Mohr's theory. Variation of $\phi \mu$ between 0 to $20^\circ$ in Equation 5.9 leads to friction angles ranging from $19.5^\circ$ to $42.3^\circ$. 
5.4 SILO ANALYSIS USING UNIFORM DISCS

5.4.1 The 'systematic arching theory'

Trollope (1957) measured the distribution of pressure across the base of model embankments composed of granular material. The results led to a theoretical study of the body—force distribution within a system of uniform, smooth rigid discs.

The disc arrangement is presented in Fig. 5.11a, and is identical to the "uniform rods in a parallel stack" (Fig. 5.5), used by Rowe. The hexagonal arrangement around a typical disc forms six contact forces on each disc. Three of the forces are determined from the known boundary conditions and two from equations of statics, so the system is singly indeterminate.

A solution can be obtained by assuming certain mechanisms of load transfer. Trollope relates these conditions to certain zero forces due to contacts that do not take place. In Figure 5.11, the same effect is obtained by demonstrating a slight rearrangement of the particles / discs, which may result from a preferred load transfer and will be discussed later in this chapter. The two mechanisms are:

1. No arching, where no contacts and therefore no horizontal forces exist, as presented in Fig. 5.11b. The obtained arrangement is similar to that of the F.C.C face.

2. Full arching, where the major forces are the horizontal forces, as shown in Fig. 5.11c. This arrangement has a mirror image in which $L_\text{q}$ and $l_\text{q}$ would be equal to zero and the contacts will be through $M_\text{q}$ and $m_\text{q}$ ($\alpha > 60^\circ$, measured from the opposite direction).

The two limiting conditions involve arrangements which are statically indeterminate, and for which solutions can be developed. Trollope's arching idea was used by Butterfield (1969) to conduct a theoretical study of the pressures developed in a 2-D silo containing the aforementioned frictionless discs.

The method of analysis consisted of a progressive summation of the interparticle
forces. Starting from the top horizontal fill surface layer of particles, \( l = m = n = 0 \). In the second layer, \( L = l+1 \), \( M = m+1 \), and so on, as shown in Fig. 5.11a. The interparticle forces result from the weight of the discs, as shown in Fig. 5.11d.

Introduction of an 'arching factor', \( A \), enables intermediate arching conditions to be described. The expression used by Butterfield (differing somewhat from that of Trollope) is:

\[
\frac{M}{L} = \frac{(1-A)}{(1+A)} \cdot \frac{(m+1)}{(1+1)} \quad (5.10)
\]

in which \( A \) varies between \( A = +1 \) describing full arching to the left, \( A = -1 \) describing full arching to the right (for which \( M = 0 \) as shown by dotted lines of Fig. 5.11c), and \( A = 0 \) in between, for no arching.

The results of the 'silo analysis' for the smooth uniform discs will be presented, using the following notations:

\((\sigma_x)_H, (\sigma_y)_H\) — The wall and the base pressure at point \( x \) above which exists fill to a depth \( H \).

\( \gamma \) — unit weight of modeled soil

\( D \) — width of silo

\( w, w' \) — positive whole number such that depth of fill \( H \) is:

\[
H = w \cdot D \cdot \tan \alpha \quad \text{and} \quad w' = (H-H')/2H, \quad H' = (D\tan \alpha)/2
\]

\( t \) — force transmission factor which enables derivation of the forces arising at the silo walls, \( M = t \cdot l \). \( t \) can be calculated by resolution of forces:

\[
t = \frac{\tan \alpha - \tan \delta}{\tan \alpha + \tan \delta} \quad 0 \leq t \leq 1
\]
for which $\alpha$ is the angle shown in Figure 5.11 and $\delta$ is the friction angle between the discs and the wall.

5.4.2 The 'no arching case' ($A = 0, \alpha < 60^\circ$)

Analysis of the case where no horizontal forces are transferred through the mass leads to the following solution:

(a) For $H < 2H'$

\[
(\sigma_x)_H = \frac{(1+t)}{\tan^2 \alpha} \cdot \frac{\gamma H}{2}
\]

(b) For $H = \text{multiple of } 2H'$

\[
(\sigma_x)_H = \frac{\gamma D}{2 \tan \delta} \cdot (1 - tw)
\]

\[
(\sigma_y)_H = \frac{\gamma D \tan \alpha}{2} \cdot \frac{(1 + t - 2tw'^*t)}{(1 - t)}
\]

$(\sigma_x)_H$ and $(\sigma_y)_H$ are linear functions of $H$ for each interval of $(n-1)2H' < H < n2H'$, $n = 1,2,3,...$. As $H' = (D \tan \alpha)/2$, the relations between the stresses and the height of the fill normalized by $D(H/D)$ will be linear between the points of $H/D = \tan \alpha \cdot w$, $w = 1,2,3,...$ as presented in Figure 5.13.

Equation 5.12b was developed independently, as the equation suggested by Butterfield was found to be erroneous. It should be noted, however that equation 5.12b differs slightly from equation 5.6b at the boundary.
5.4.3 The ‘full arching case’ \((A = \pm 1, \alpha \geq 60^\circ)\)

Analysis of the case where horizontal forces are transferred through the mass leads to the following solution:

For \(H \leq H'\)

\[
\begin{align*}
(\sigma_x)_H &= \frac{\gamma H}{\tan \frac{\alpha}{2}} \cdot \frac{(1 + t)}{(1 - t)} \\
(\sigma_y)_H &= \frac{\gamma H}{\tan \frac{\alpha}{2}}
\end{align*}
\]

Both functions increase linearly to their maximum value at \(H'\) and then remain constant for any \(H \geq H'\) (see Figure 5.13).

5.4.4 Investigation of the obtained relations

The presented equations are analyzed so that their behavior and implications can be studied.

(a) The maximum stresses

Equations 5.12 (a) and (b) can be checked for the condition of a very high fill. In such a case, \(w\) and \(w'\) approach infinity and the horizontal stresses on the wall \((\sigma_x)\) and the vertical stresses at the bottom become (for the no arching case):

\[
\begin{align*}
(\sigma_x)_{H \to \infty} &= \frac{\gamma D}{2 \tan \delta} \\
(\sigma_y)_{H \to \infty} &= \frac{\gamma D \tan \alpha}{2} \cdot \frac{(1 + t)}{(1 - t)} = \frac{\gamma D \tan^2 \alpha}{2 \tan \delta}
\end{align*}
\]

Equations 5.13 already indicate that, for the full arching case, \(H'\) will be the height for which the maximum stresses will be developed:
\[(\sigma_x)_H' = \frac{\gamma H'}{t \tan^2 \alpha} \cdot \frac{(1 + t)}{(1 - t)} = \frac{\gamma D}{2 t \tan \delta} \quad (a)\]

\[(\sigma_y)_H' = \gamma H' = \frac{\gamma D}{2 t \tan \delta} \quad (b)\]

(b) **The stress ratio**

Equations 5.12 through 5.15 enable one to assess the ratio between horizontal to vertical stresses obtained by the discrete analysis.

For the ‘no arching case’, use of equations 5.11 and 5.14 leads to:

\[K = \frac{(\sigma_x)_H}{(\sigma_y)_H} = \frac{(\sigma_x)_w}{(\sigma_y)_w} = \frac{1}{t \tan^2 \alpha} \quad (5.16)\]

For the ‘full arching case’, use of equations 5.13 or 5.15 leads to:

\[K = \frac{(\sigma_x)}{(\sigma_y)} = \frac{(\sigma_x)_w}{(\sigma_y)_w} = \frac{t \tan \alpha}{t \tan \delta} \quad (5.17)\]

Noting that \(\alpha = 60^\circ\) and the only friction angle which exists in the system is \(\delta\), the obtained typical values for \(\delta = 30^\circ\) are the following:

For no arching \((\alpha = 60^\circ)\) \(K = \sigma_x/\sigma_y = 1/3 = K_a\)

Active ratio for friction angle of 30°

For full arching \((\alpha = 60^\circ)\) \(K = \sigma_x/\sigma_y = 3 = K_p\)

Passive ratio for friction angle of 30°

These stress ratios indicate the relations between the relative movements and their associated load transfer mechanisms, as shown in figure 5.12. For the ‘no arching’ case the load transfer is that of figure 5.11b, associated with the active state of
stress. When the walls are flexible, allowing radial displacements, the stresses on the walls are reduced to the minimum possible stresses, while the stresses on the relatively fixed and rigid bottom are maximal. These conditions are described in figure 5.12a. For the ‘full arching’ (to the left) case the load transfer is that of figure 5.11c, associated with the passive state of stress. When the bottom is flexible, allowing downwards displacement, the stresses on the bottom are reduced to the minimum, while the stresses on the fixed and rigid walls are maximal. These conditions are described in figure 5.12b, where full arching is described by combining the load transfer of full arching to the right and to the left.

(c) **Comparison to the standard silo analysis**

The principles of the silo analysis were reviewed in section 4.1. The established Janssen’s formula (1895, see Jakobson –1958) is:

\[
\sigma_x = \frac{\gamma}{t \sin \theta} \cdot \frac{U}{A} \left(1 - e^{-\frac{ht \sin \theta}{U}}\right)
\] (5.18)

The variables are presented and defined in figure 4.1.

In order to make a comparison with the results obtained by the uniform disc model, equation 5.18 must be modified to fit a 2-D case. When considering the problem of a soil between two parallel vertical walls, only the terms \(U\) and \(A\) must be modified. The internal area for a unit length is \(A = D \times 1\), and the circumferential area \(U\) (which is actually the surface area on which friction acts) is 2 units. The ratio \(U/A\) for the 2-D case is then \(D/2\), compared with \(D/4\) for the circular case. The comparison will be based, therefore, on the results obtained by the disc approach to the silo analysis using equation 5.18 in the following form:
\[ \sigma_x = \frac{\gamma D}{2t g \theta} \cdot (1 - e^{-2t g \theta D \delta}) \quad (5.18 \text{a}) \]

Figure 5.13 compares the results of the discrete disc model with those of the standard silo analysis modified for the 2-D case. Equations 5.11a and 5.12a, with \( \alpha = 60^\circ \) and \( \delta = 30^\circ \) for the full and no arching cases, are plotted using dashed lines for the horizontal stress at the bottom of the silo. The solid line curves are the results of equation 5.18a for \( \phi = \delta = 30^\circ \). \( \delta = 30^\circ \) was also used in the frictionless disc analysis. The vertical stresses at the bottom of the silo for the full and no arching cases are plotted as well.

5.4.5 The ‘intermediate arching’ case and experimental data

General expressions for the stresses acting on the bottom and walls of a silo were developed by Butterfield (1969) for the use of intermediate values of the arching factor \(-1 \leq A \leq +1\). Subsequently, the results of different stress measurements in silos were compared by Butterfield to the theoretical best fit solution. The obtained results indicated the following:

1. The stress distribution for a slight arching state (e.g. \( A = 0.1 \)) is similar to that of the full arching state.

2. Comparison of the experimental data from circular silos with the results from the 2-D discrete disc model was made by referring to a square silo (for which \( \sigma_x \) is expected to be half that of the 2-D case). In reference to the silo equations, it should be noted that \( R = D/4 \) for a circular silo, as well as for a square silo.

3. Comparison of experimental data to the calculated stresses was made on a ‘best fit’ basis. The results did not exceed the range between the upper and lower bound solutions, suggesting that the arching concept provides a rational guide to the mechanism behind the pressure in silos. It was also found that full
arching was likely to develop in the case of an emptying silo (comparable to lowering of the bottom, or to the existence of a flexible base), and no arching in the case of a filling silo.

Analyses and experiments aimed at studying the flow of granular materials from silos have calculated and detected the location and the shape of the actual arch which coincides with the arching phenomenon (Bransby and Blair–Fish -1973, 1975 a&b, Perry and Handley -1967). Figure 5.14 presents a radiograph of sand in a 2–D silo after a movement of 4 mm. The same arch shape was obtained through a computer analysis of a discrete element simulation (Williams -1988). In both cases an actual arch consisting of a zone of particles in a preferred orientation is observed. Lighter zones in a radiograph indicate areas of lesser density or cavities. The arch in figure 5.14 delineates between the upper denser sand, which it supports, and the flowing sand under it. It can therefore be concluded that the sand in the arch dilates under shear deformation while supporting the soil above it. The experimental results (Blair–Fish and Bransby -1973) and the numerical simulation (Williams -1988) both indicate that the arch is actually a ‘dynamic’ one. It is destroyed and recreated under the loading which is associated with the flow of the granular mass.

The stress state which supports the theory of the arch shape and mechanisms is examined in Chapters 7 and 9.

5.4.6 Summary and Conclusions

1. Analysis of an idealized system of uniform smooth discs between two friction walls (Butterfield -1969) was based upon assumption of certain mechanisms in conjunction with the ‘systematic arching theory’ (Trolleye -1957).

2. In the state of ‘no arching’, horizontal forces cannot transfer directly through the disc assembly due to lack of contacts in that direction. This leads to
minimum normal stresses on the wall.

3. In the state of 'full arching', major horizontal forces transfer directly through the disc assembly due to lack of some of the diagonal contacts. This leads to maximum normal stresses on the wall.

4. The no arching case produces maximum stress conditions on the bottom of the silo, while the full arching case produces minimum stress conditions.

5. Comparison of the results obtained from the disc assembly analysis to the standard silo analysis, modified for 2-D, shows:
   a. The no arching mechanism is comparable to the stress state associated with active stress conditions, and represents a lower bound solution.
   b. The full arching mechanism is comparable to the stress state associated with passive stress conditions, and represents an upper bound solution.
   c. Numerically, for the condition of $\phi = \delta = 30^\circ$ in the no arching case, $K = \sigma_x/\sigma_y = 1/3 = K_a$, and therefore $\sigma_y = 3\sigma_x$. In the full arching case, $K = \sigma_x/\sigma_y = 3 = K_p$, and therefore $\sigma_y = \sigma_x/3$. As a result, the average stresses on the bottom of the silo can vary under the two extreme conditions by almost a full order of magnitude.

6. Studies of partial arching have shown that small amounts of arching ($A = 0.1$ for the range from 0 to 1) lead to results similar to the full arching state.

7. Comparisons between experimental data and the silo analysis modified for the circular case have shown that the measurements did not exceed the upper and the lower range as determined by full arching and no arching states.

8. A full arching case was developed under a state of silo emptying. This was demonstrated by the actual arch shape observed on a radiograph (Figure 5.14). The lighter arch shape zone of Fig. 5.14 shows that the sand of the arch is of lesser density, probably due to dilation. The arch delineates between the upper denser sand and the flowing sand under it. In motion, there is actually a
‘dynamic arch’ which is built up and destroyed under the loading.

9. The aforementioned summary shows that the disc model provides a very good fit to the silo theory. The study of an idealized model under the simplified arching approach explains the physical phenomena and the behavior of the granular material in a silo.
5.5 SOIL BEHAVIOR IN LIGHT OF SOIL STRUCTURE

5.5.1 Introduction

Conventionally, the friction angle of granular material is recognized to depend primarily on its relative density. Empirical correlations are used between static and dynamic penetration tests, directly or indirectly (through the relative density) with the friction angle.

This attitude discouraged investigations of granular material where the micro approach is used to analyze its structure, mechanism, and behavior under loading. Simplified mechanical models, previously reviewed in this chapter, were aimed at illustrating granular behavior, as were the model tests (e.g. Thurston and Dersiewicz—1959, Rowe—1962). The test results usually confirmed the relations between the loading and the general shear resistance. However, they did not follow the individual particles, and therefore could not provide the knowledge to support the models regarding the particle configuration and mechanism. Anisotropy in the sand behavior was observed (e.g. Ladd et al.—1977, see Figure 5.15), and brought about recognition of the soil structure role in the mechanisms of deformation and strength of granular materials (Konishi—1978).

Soil structure is conventionally defined by two components: (1) the soil fabric, which is the orientation and distribution of particles in a soil mass, and (2) the nature and magnitude of interparticle forces (Lambe—1958).

In granular material the two are closely related, as interparticle forces are directly related by the contacts which are governed by the fabric.

Close observations of the granular fabrics of sands and particle models in initial and deformed specimens will be used for the following purposes:

(1) to validate the use of particle models for the study of granular soil

(2) to describe the mechanism that controls the soil behavior under loading.

Both points are made in order to (1) examine the use of granular models as
previously described, and as will be developed later; (2) support the mechanism and idea of arching by checking preferred contact orientations as they coincide with interparticle forces and stress orientations; (3) look into the importance of the relative displacement and the interparticle friction during shear.

5.5.2 Anisotropy and initial fabric

Figure 5.15 summarizes data of triaxial tests run on samples formed in a tilting mold in order to vary the angle $\beta$ between the direction of sand deposition and the major principal stress ($\sigma_1$) during shear (Ladd et al. -1977). The greatest friction angle is obtained when the direction of $\sigma_1$ coincides with the direction of sand deposition.

Oda (1972 –a,b) studied the initial fabric and its relations to the mechanical properties of granular materials. The fabric was examined in thin sections which were prepared from sand specimens fixed with polyester resin. These sections were studied under an optical microscope in order to determine the preferred orientation and spatial relations of constituting grain particles.

The distribution of interparticle contact normals is shown in Figure 5.16. The angle $\beta$ is the angle between the normal of the interparticle contact and the direction of the major principal stress, as shown in Figure 5.17b. For $\beta = 0$, the contacts are normal to the major principal stress direction. The same sand (rounded to sub rounded) at the same initial void ratio ($e_i = 0.64$) was used in tests (a) and (b) of Fig. 5.16. The contact distribution of the sample which was prepared by tapping (Fig. 5.16a–1) showed a marked orientation of particles. The contacts of the sample which was prepared by plunging (plunging of the hand tamper directly into the sand as opposed to tapping on the side wall of the mold) showed no preferred orientation (Fig. 5.16b–1). Figure 5.17a presents the marked difference in the stress–strain behavior of the two samples.
A set of tests on sand composed of varied amounts of grains (mainly quartz and feldspar) was conducted by Oda (1972a). It was found that the samples composed mainly of non-spherical particles had a more pronounced initial orientation than the samples of rounded ones. However, even glass balls which were allowed to fall freely under the action of gravity formed an anisotropic assembly, in which the balls tend to be arranged in chains (Kallstenius and Bergau –1961).

The effect of the particle shape was demonstrated by a set of tests on oval cross-sectional rods subjected to a biaxial compression (Oda et al. –1982). Figure 5.18 presents the results of those tests, in which the less rounded rods (oval II) demonstrated a profound anisotropy and higher shear resistance in comparison to the more rounded rods (oval I).

The presented data suggest that the initial fabric of granular material controls its strength. For a sand with a given density, the compaction (deposition) method and the particle shape are the main factors which determine the fabric.

5.5.3 The mechanism of fabric changes during deformation of granular material

The stress-strain relations of Figure 5.17 showed the remarkable effect of the initial fabric on the mechanical behavior. The mobilized strength of the specimen compacted by the tapping method is greater than that of the specimen compacted by the plunging method, although both specimens had the same initial void ratio. Some differences were observed in Figures 5.16a–1 and 5.16b–1. The tapping compaction resulted in a fabric of preferred intergranular contact orientation, normal to the loading and therefore normal to the major principal stress. It is reasonable to assume that the plunging action on the sand in the mold (during the sample preparation) is similar to a dynamic 1-D loading. The loading led to the preferred orientation of the soil grains, enabling them to react better to the unidirectional load. The tapping, on the other hand, provided equally distributed shaking, lacking a distinct load action,
and therefore led to a random grain orientation.

If the above explanation is correct, then a detailed observation of the fabric changes during loading should result in similar consequences. Figures 5.16a,b present the distribution of the interparticle contact normals as a function of the axial strain during loading. These distributions should be observed in relation to the stress–strain curves of Fig. 5.17a. Fig. 5.16 indicates that the grain fabric continuously changes, and the normals to the intergranular contact planes gradually concentrate within the range of angle \(0 \leq \beta \leq 50^\circ\) (or \(130^\circ \leq \beta \leq 180^\circ\)). With the increase in axial strain, the fabric becomes more anisotropic. The dotted lines in Fig. 5.16 represent the theoretical lines for the case of anisotropic assembly. The function \(E(\beta)\) (mean probability density of points of contacts in ranges of \(10^\circ\)) changes mostly during the course of increasing stress ratio. After peak stress ratio, the function does not change markedly.

In addition to determining the direction of the normals, Oda (1972b) measured the angles between the long axes of grain particles and a reference axis. Knowing the number and area of the contacts allows calculation of the projected area of the contact in the chosen axes directions. The ratio \(S_z/S_x\) is the area of contacts normal to the \(z\) axis over the area of contacts normal to the \(y\) or \(x\) axes. The areas over the \(y\) or \(x\) axes are identical due to symmetry. Figures 5.19a,b present the ratio of \(S_z/S_x\) to the mobilized stress ratio \(\sigma_1/\sigma_3\) and the dilatancy rate \(-dv/d\varepsilon_1\).

The conclusion which can be derived from Figure 5.19 is that the contacts normal to the direction of axial load (\(\sigma_1\) in direction of \(-z\)) are most effective in supporting the load, while the contacts which are perpendicular to \(\sigma_1\) may have no actual role in supporting it. Fig. 5.19 indicates that as the stress ratio increases, the ratio between the contact area normal to the major principal stress and that normal to the minor principal stress increases in order to support the load increase. Oda (1972b) also showed that when movement of the particles occurs it is the long axis that
becomes generally perpendicular to the direction of the maximum principal stress. The increase of the ratio $S_z/S_x$ is best explained from the results shown in Figure 5.20 (Oda et al. –1982). Fig. 5.20 indicates that the ratio $S_z/S_x$ increases due to the combination of a decrease in $S_x$ and an increase in $S_z$, which results from particle reorientation as the long axis rotates to become normal to the major principal stress.

The observed particle movement can be explained by the need for a preferred contact orientation that will be normal to the encountered increasing stress. By following the soil grains or the model particles, it was possible to identify this phenomenon. It remains, however, to explain how the load is transferred from one contact to another.

Kallstenius and Bergau (1961) tested the texture of granular masses by using glass balls. They found that following the free fall of the balls,

"Both theory and practical tests prove that there is a tendency for the balls to arrange themselves in 'chains'...".

In other words, the preferred orientation of the particles is expressed through the orientation of the intergranular contact, and the formation of a continuous chain of contacts that allows stress to 'flow'. Konishi (1978) conducted 2-D model experiments on photoelastic cylindrical rods of three different diameters. Konishi followed the fabric structure under simple shear, $K_0$, and compression and biaxial tests. He noted that not only do the contacts align normal to the principal stress direction, but they also reorient themselves if a principal stress rotation takes place during loading. In parallel, he photographed and noted the following:

"The ability of supporting the major principal stress $\sigma_1$, should be maximum at the contact of $\beta = 0^\circ$ and minimum at $\beta = 90^\circ$ from observation of the interparticle forces. On the other hand, the applied stress is transmitted through the granular medium through many contact points between particles as shown [in a photograph] where the paths of forces can be observed as the 'chains' of heavily stressed particles. It seems that the main direction of the 'chains' coincides with the major principal stress direction".
Konishi’s observations showed that more effective contacts to support \( \sigma_1 \) will develop in the ‘chains’, and in return they will increase the ability of the chains to support \( \sigma_1 \).

Oda et al. (1982) followed the creation and the disappearance of the ‘chains’ during loading. They concluded that:

"during the course of deformation and up to the peak stress, new contacts are continually formed in such a manner that the contact unit normals tend to concentrate more in a direction parallel to the maximum principal compression. This concentration of unit normals seems to be closely related to the formation of new column–like load paths which carry the increasing axial stress under constant lateral force. After the peak stress, such a column–like microstructure disappears and considerable rearrangement of the load paths takes place, leading to more diffused (homogeneous) microstructure in the critical state."

### 5.5.4 The shear mechanism of granular material

It was commonly assumed that frictional sliding was the major component of granular shear deformation. Sliding, and therefore the intergranular friction angle, was thought to control the microscopic deformation mechanism (see Section 5.3.1—Rowe’s analysis). When the stress–strain behavior was explained on the basis of models using spheres in conjunction with sand testing, it was noted (Lambe and Whitman—1969) that the main cause of strain is a movement consisting of sliding and rolling between particles (see Section 5.3.1e). The relative importance of the two mechanisms was not established. Although a detailed analysis of the F.C.C. model implied that the displaced sphere rolls over the groove between the adjacent spheres (Section 5.3.2), it was still assumed by Thurston and Dersiewicz that no rotation takes place.

The questions of interest regarding the micro behavior are the following: (1) What is the importance of particle rolling in the microscopic deformation mechanism? (2) What is the influence of the intergranular friction angle on the local and general shear mechanism? and (3) What type of shear zone exists in granular materials?

Roscoe and Schofield (1964) noted in their discussion (regarding Rowe—1963),
that they observed equal and opposite rotations of neighboring particles of dense sand which were against the glass walls of their plane strain earth pressure test apparatus. Oda (1972b) followed the movement of the long axis of the sand grains during reorientation and found that the particles moved and rotated to the most stable position. He concluded that rolling of particles must be an important factor controlling the mechanism of fabric reconstruction.

More detailed information concerning particle movements was obtained from model tests in which round and oval rods, as well as spheres, were used to simulate the granular particles. Konishi (1978) and others have established the validity of the use of such models for granular material representation. It was demonstrated that the models' stress-strain behavior, as well as their mechanism of fabric changes during shear, are similar to those observed in a real sand under the same type of testing (usually biaxial compression, simple shear and direct shear tests, which will be discussed further in Chapter 8.).

Oda and Konishi (1974) used an assembly of randomly packed photoelastic cylinders in a 2-D simple shear apparatus. They followed the angles between the loading axis and (a) the normal to the contact and (b) the the contact force (see Figure 5.17b for β and θ respectively). The mobilized friction angle is obtained from the difference of the two: δ_1 = θ₁ - β₁. When the absolute value of δ₁ is equal to the interparticle friction angle φ_μ, interparticle sliding takes place at the contact. Figure 5.21 shows the frequency distribution of the mobilized angle δ₁ during a simple shear test. The obtained results indicate that sliding on a microscopic scale in the assembly was confined to some preferred contacts (φ_μ = δ₁ = ±22°) and did not occur at a majority of the contacts, even when the granular assembly was sheared (L-9 and L-12). Similar results were obtained by Konishi (1978) during biaxial compression tests. These findings do not yet contradict Rowe's model, for which he suggested that deformation occurs as relative motion between instantaneously rigid groups of
particles, as described in Section 5.3.1.

Oda et al. (1982) facilitated a better understanding of the shear mechanism by using two types of oval rods under two intergranular friction conditions ($\phi_\mu = 26^\circ, 52^\circ$). One set of ovals was rounded (I) and the other less rounded (II), having an axis length ratio of 1.1 and 1.4 respectively. The following results were obtained (see Fig. 5.18):

1. When the major principal stress was normal to the long axis ($\theta = 0^\circ$), the greater the interparticle friction and the greater the angularity, the greater the stress ratio in failure: i.e. referring to the measured internal friction angles, for $\phi_\mu = 52^\circ$ and $26^\circ$, $\varphi_{\text{max}} = 60^\circ$ and $58^\circ$ for oval II and $\varphi_{\text{max}} = 49^\circ$ and $42^\circ$ for oval I, respectively.

2. Referring to the entire orientation range ($0^\circ \leq \theta \leq 90^\circ$), no distinct difference in the strength of the rounded ovals was observed under the two different intergranular friction conditions (see Figure 5.18). Such independence of the overall effective friction coefficient may suggest that particle rolling is dominant during shear deformation. This is in agreement with Skinner (1969), whose results were obtained by shearing glass ballotini in a shear box as presented in Figure 5.22. Although the wet glass interparticle friction angle was about $\phi_\mu = 33^\circ$ and the dry one was about $3^\circ$, both exhibited essentially the same overall shear resistance. The stress–strain relations of Fig. 5.22 also show that rolling of particles is accompanied by a volume change. If the packing is dense this volume change is dilation. This dilation is independent of the interparticle friction coefficient.

3. The less rounded particles showed a gain of about 40% in strength with increase of the intergranular friction angle. The ratio between the number of contacts where sliding was found to be dominant to the number of contacts where rolling was dominant was denoted by R. For the smaller intergranular friction case, R
was found to range between 0.20 and 0.51. For the greater intergranular friction case, R was found to range between 0.12 and 0.23. Although in both cases rolling is dominant, it is clear that rolling is more likely to occur in the unrounded ovals with a greater friction angle, where sliding encounters greater interlocking resistance. The same conclusion can be derived from the experimental results which are presented in Figure 5.23 (Skinner -1969). By testing the glass ballotini against a smooth glass plate, the change from dry to wet conditions resulted in an increase in shearing resistance. The shearing angle increased from $\phi' = 11^\circ$ to $\phi' = 20^\circ$, with an increase in the dilation (expressed by the vertical displacement) from $0.5 \times 10^{-3}$ inch to over $8 \times 10^{-3}$ inch. This indicates that once interlocking is forced, either due to the angularity of the particle shape (Oda et al. -1982) or to a high friction interface (Skinner -1969), sliding decreases and rolling, accompanied by volume change, increases.

The effective shear strength of the mass is therefore a result of the particle mechanism, which depends on the particle shape and the interparticle friction coefficient. Skinner's results suggest that the round particles roll easily; interlocking is negligible, and therefore the intergranular friction angle is of no influence. The angular particles or a contact with a plane would introduce 'locking' between the particles or between the plane and the particles only under greater interparticle friction. This would increase the incidence of rolling over sliding, changing the mechanism of the shear from one (sliding) to the other (rolling).

If this is correct, then Rowe's assumptions of the stress dilatancy theory as presented in Section 5.3.1 are invalid, as no correlation exists between $\phi\mu$ and the shear strength. This subject of the fundamentals of soil mechanics is of vital importance, and a considerable debate (with no clear conclusions) took place regarding it (e.g.
Bishop —1971, Bishop and Skinner —1977). A new mechanical model which considers both sliding and rolling is presented in Chapter 6, which sheds some light on this subject. The model predictions are compared to experimental results in Chapter 8. This comparison helps to elucidate the possible shear mechanism of granular media, and explains the discrepancy between Rowe's theory and Skinner's test results.

Particle rolling was found to be important even when the assembly reached the overall failure state. Therefore, it is also expected to influence the shear zone. Oda (1972b) followed the fabric changes during the deformation of triaxial compression tests on sand. He concluded that rolling of particles must be an important factor controlling the mechanism of fabric reconstruction. This process of fabric reconstruction was found to be continuous, and no gap was observed between granular fabrics formed at pre-peak stress and those formed at peak stress. Neither failure plane nor failure zone could be seen microscopically in any specimen throughout the process of progressive deformation up to peak stress state ($\varepsilon_a < 15\%$). Oda could not find any evidence that the long axes of particles tend to be arranged parallel to potential failure planes ($\phi/2$), traditionally adopted from Mohr's failure criterion.

Lupini et al. (1981) studied the drained residual strength of cohesive soils. The mechanisms which control the residual shearing were studied through three series of tests on different soil mixtures in which the gradings of the soils could be varied artificially. The structure of the shear surfaces was examined visually and in thin sections. A summary of the results is presented in Figure 5.24. Three modes of residual shear behavior are recognized in this figure: a turbulent mode, a transitional mode, and a sliding mode. The mode of shear was found to depend on dominant particle shape, and on the coefficient of interparticle friction.

Focusing on granular material, it is of interest to examine the turbulent mode typical for materials with a Plasticity Index lower than 25% (see also Kenney —1977). The turbulent mode occurs when behavior is dominated by rotund particles, or
possibly in soils dominated by platey particles, when the coefficient of interparticle friction between these particles is high. When residual strength is high, no preferred particle orientation occurs, and brittleness is due to dilatant behavior only. The residual friction angle depends primarily on the shape and packing of the rotund particles and not on the coefficient of interparticle friction. A shear zone, once formed, is a zone of different porosity only, and it is considerably modified by subsequent stress history. The work by Lupini et al. was found to be in close agreement to that of Oda, Konishi and others (as described above), and completes the overall picture presented in this chapter of the micro approach to the behavior of granular material.
5.6 SUMMARY AND CONCLUSIONS OF THE 'MICRO APPROACH'

The Mohr–Coulomb failure criterion of continuous materials is commonly applied to discontinuous assemblies of particles such as granular soil. It anticipates a slip failure plane with parallel shear. The observations of volume changes during shear suggest that the intergranular mechanism differs from that of continuous media.

Models consisting of assemblies of spheres and rods were analyzed and tested. Even the most simplified models provide an acceptable explanation of the volume change mechanism (e.g. Rowe –1962). The shear was essentially perceived as a sliding mechanism, and therefore was analyzed as friction of inclined planes. The inclination of the plane was taken to be equal to the angle of the direction of the interparticle sliding, measured from the applied shear stress. The friction angle between the planes was taken to be equal to the intergranular friction angle. Although this approach was aimed at analyzing granular assemblies, it was not based on any close observations of individual grain behavior under loading.

The model tests indicated that the shear had the form of a sudden movement of two distinct sphere structures, moving one in relation to the other. The obtained slip plane was found not to be the cause of the failure, but rather the result of the particle movement.

A detailed 3–D analysis of an assembly of spheres under a compression test was presented. Resolution of the forces acting on an individual sphere enables one to follow its most probable path. The displacement of the sphere over the groove between adjacent spheres was assumed to occur by sliding, even though rolling was more feasible.

Modeling of granular material as an assembly of rigid smooth discs was presented. The model was utilized to study the stresses acting on the sides and bottom of a 2–D friction wall silo. The solutions were obtained by introducing a preferred orientation for stress transfer. The state of stress in which the disc assembly has a lack
of contacts in the horizontal direction is referred to as the 'no arching state'. In this state, the normal stresses on the walls are minimal, and are comparable to those obtained from the continuum approach of the silo analysis with active stress conditions. The state of stress in which the disc assembly transfers mainly horizontal forces is referred to as the 'full arching state'. In this state, the normal stresses on the walls are maximal and are comparable to those obtained from the continuum approach of the silo analysis with passive stress conditions. When compared with experimental data, the above states were confirmed as lower and upper bound solutions.

Successful modeling of granular material as an assembly of individual particles needs to be supported by observations at the micro level. In order to examine the aforementioned models and analyses and to develop new methods of modeling, a review of the microstructure behavior was presented.

The experimental data revealed the changes which take place in the granular fabric during deformation under unequal principal stresses. Sand grains, as well as spherical and oval rods, move and rotate to the most stable position in order to react to the applied major principal stress. Continuous reorientation was observed in the cases where principal stresses rotation took place during loading. While the long axes of the particles become generally perpendicular to the direction of the maximum principal stress, the contact area normal to the major stress increases as the contact area normal to the minor stress decreases. This concentration of contacts is closely related to the formation of new column-like load paths which carry the increasing axial stress. These contacts/load paths could be observed and traced (Oda –1974, Oda et al. –1982). After the peak stress, such a column-like microstructure disappears and considerable rearrangement of the load paths takes place, leading to a more homogeneous microstructure at the post-peak (critical) state.

In contradiction to the common assumption that particle sliding is the major deformation mode, it was found that particle rolling is the major microscopic
deformation mechanism, and is especially enhanced when the interparticle friction is
great. There are relatively few contacts at which sliding is dominant even when the
assembly reaches the overall failure state. The controlling rolling mechanism develops
to a turbulent mode of shear, in which a clear failure plane or failure zone cannot be
seen microscopically, and there is no separation or preferred orientation of a shear
zone. The initiation of a slip plane is therefore a result of failure (and not a cause),
that does not occur at the peak stress ratio.

The following conclusions can be derived from the presented models and the
observed microstructure behavior:

1. An assembly of spheres or round rods can explain the volume change
mechanism of granular materials during shear. An assembly of oval rods
exhibits stress–strain behavior very similar to that obtained from a sand sample
under the same loading conditions.

2. The individual grains move and rotate to the most stable condition as a
reaction to an applied load. The preferred arrangement is achieved by
orientation of the long axes of the particles perpendicular to the direction of the
maximum principal stress.

3. As a result of this orientation, the contact area normal to the major principal
stress increases. The concentration of contacts in the preferred direction forms
a column–like load path which transfers the increasing axial stress along its
trajectory.

4. Behavior of the granular material under loading is in agreement with the
presented rigid disc model for the silo analysis. The preferred stress
transference, as expressed by the arching hypothesis, was fully supported by
observations of the microstructure behavior. Analysis of granular soil under the
examined plugging conditions must therefore take into consideration the
particle reorientation and the preferred stress path in the direction of the major
principal stress trajectory.

5. The effective shear strength of a granular mass is a result of the particle displacement mechanism, which is either rolling or sliding. Rolling was identified as the major displacement mechanism, and it is controlled by the particle shape and intergranular friction in the following way:

   a. In granular material consisting mostly of round particles, rolling takes place with greater ease, and therefore the interparticle friction angle has a very small influence on the overall shear resistance (according to Skinner).

   b. Rolling encounters more opposition in granular material consisting mostly of elongated particles. The intergranular friction here, therefore, has a greater influence on the overall shear resistance. However, this influence is limited by a trade-off in the sliding/rolling mechanisms. Greater intergranular friction impedes sliding, thereby facilitating rolling; reduced intergranular friction has the opposite effect.

   c. The interface friction between the granular material and a plane has a similar effect. Lower interface friction allows sliding of the interface on the grains. Greater interface friction impedes sliding, and thereby enables rolling to take place between the particles. Figure 5.23 (Skinner -1969) was obtained for the same plane/granular material (glass) in order to demonstrate the sliding/rolling mechanism during shear of granular material.

6. Models which simulate granular material as an assembly of individual particles must allow for rolling to occur. Moreover, analysis of friction of granular material along an interface must include both rolling and sliding mechanisms and their interactions.

7. The granular materials shear in what can be best described as a turbulent mode
of shear. No preferred particle orientation occurs, and the formed shear zone is of an increased porosity due to dilatancy, reflected as brittleness in the stress-strain behavior.

8. The actual shear of the granular material differs substantially from the sudden shear observed in the early models. This may be a result of the use of an assembly of uniform round particles in an exact repetitive structure. Such a structure prevents the independent movement of the individual particles, resulting in a situation where shear could take place only as a general movement of two distinct whole bodies.

Some of the above ideas reflect only the material which was reviewed in the present chapter but not necessarily the results of this entire study. A model which utilizes the above information is presented in Chapter 6 and discussed in Chapter 8. Its results, however, do not entirely agree with some of the above conclusions. This may be due to the inability of the researchers to weigh and consider the contribution of each of the controlling factors.
Fig. 5.1: Possible Sphere Packings. The B Spheres (shaded) rest on the A Spheres. A third close-packed layer may then rest on the B spheres, either directly above the A spheres or in C positions. (after Illston et al. —1979).

Fig. 5.2: (a) Expanded unit cell of Hexagonal Close-Packed Structure. (b) Expanded unit cell of Face-Centered Cubic Structure.
Fig. 5.3: Basic Equilibrium Diagram for Friction of Inclined Planes Representing two Particles.

Fig. 5.4: Direction of Particle Movement ($\phi$) and Average Inclination of Plane ($\alpha$).
Fig. 5.5: Uniform Rods in a Parallel Stack.

(a) Plan View
(b) Vertical Cut Through A-A

Vertical Cut Through X-X
View of Plane Perpendicular to $\alpha_2$

Fig. 5.6: Uniform Spheres in a Face Center Cubic Packing.
\( \psi_o \)-Direction of Initial Contact

\( \psi_e \)-Direction of Final Contact

Fig. 5.7: Direction of Particle Movements (Rowe—1962)

Fig. 5.8: The Elementary F.C.C cube and the Axes Notations (Scott—1963)
**Fig. 5.9:** Unit Hexagonal area and the Projection of area Normal to \((σ_1-σ_3)\)-direction (Scott -1963)

**Fig. 5.10:** Forces Acting on a Typical Sphere B and the Path of B during Motion (Thurston and Dersiewicz -1959)
Fig. 5.11: The Interparticle Force Distribution (a) Full Contact (b) No Arching (c) Full Arching to the left.
Fig. 5.12: Load Transfer and Relative Movement for (a) No Arching and (b) Full Arching Conditions.
Results of 2-D Silo Analysis based on a Discrete Model and the Standard Silo Analysis.

Fig. 5.13
Radiograph of Dense Sand after Surface Displacement of 4mm. Light arch zone is the area of arching in which the sand has expanded as a result of shear deformation. The black dots are lead spheres (Bransby et al. -1975).

Fig. 5.14:

Effect of Direction of Loading on the Friction Angle of Three Sands (Ladd et al. -1977)

![Fig. 5.14 Diagram](image)

![Fig. 5.15 Diagram](image)
Fig. 5.16: Distribution of Interparticle Contact Normals as a function of Axial Strain for Sand Samples Prepared by (a) Tapping (b) Tamping (Oda 1972–b).

Fig. 5.17: (a) Stress Strain and Volumetric Strain Curves for two Samples of Sand at the Same Initial Void Ratio, but Prepared by Different Methods (Oda 1972–b) (b) Representation of Interparticle Relations.
### Properties of assemblies in dense packing

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<th>Source</th>
<th>Total Force (N)</th>
<th>Total Force (kN)</th>
<th>Total Force (lbs)</th>
<th>Stress Ratio (σf/σ3)</th>
<th>Internal Friction Angle (θ)</th>
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### Fig. 5.18: Stress Ratio and Internal Friction angle at Failure vs. Bedding Angle (Oda et al. –1982)

### Fig. 5.19: Relationship between the Fabric Characteristic $S_2/S_x$ of Anisotropic Sand and (a) The Mobilized Stress Ratio $σ_f/σ_3$ (b) The Dilatancy Rate $(-dv/dε)$ (after Oda 1972–b)
Fig. 5.20: Reorientation of Particles during Loading (a) A Process in which the Long Axes of the Particles Rotate so to be Normal to the Major Principal Stress, causing Appearance of Contacts Normal to the Major Principal Stress, and (b) Causing Disappearance of Contacts Parallel to it (Oda et al. –1982)

(The dashed lines were added as a rough estimation of the data trend)

Fig. 5.21: Frequency Distribution of Mobilized Angle δ in a 2-D Granular Assembly of Loose Model (Oda and Konishi –1974)
Shear load and Vertical Displacement of Loading Platen plotted against Horizontal Displacement for Shear Box Tests on 1mm dia. Ballotini, all at the same Initial Porosity of 34.9% under a Normal Load of 20 lb. (Skinner –1969)
Shear Load vs. Horizontal Displacement and Dilation vs. Horizontal Displacement Curves Obtained from a Special Test with 3mm. dia. Glass Ballotini in the Top Half of the Shear Box, the bottom half being replaced by plate glass, the test being performed partly dry and partly flooded. Normal load 20 lb. and initial porosity 37.5% (Skinner—1969)
<table>
<thead>
<tr>
<th>P.I. %</th>
<th>Test no.</th>
<th>Clay fraction %</th>
<th>Thin section description</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>2</td>
<td>20</td>
<td>No separation on shear zone. Thin section showed no preferred orientation of the clay matrix (Figs 19, 20)</td>
</tr>
<tr>
<td>22</td>
<td>3</td>
<td>27</td>
<td>No separation on shear zone. Thin section showed no preferred orientation of the clay matrix</td>
</tr>
<tr>
<td>33</td>
<td>4</td>
<td>34</td>
<td>No separation on shear zone. Thin section showed shear zone about 1.5 mm thick containing discontinuous shear surfaces parallel to the direction of shear</td>
</tr>
<tr>
<td>45</td>
<td>5</td>
<td>40</td>
<td>Specimen separated on heavily striated slickenside shear surface. Thin section showed two continuous shear surfaces, undulating in the direction of shear, about 2.5 mm apart. Clay particles between them orientated 0-45° to direction of shear (Fig. 23)</td>
</tr>
<tr>
<td>49</td>
<td>6</td>
<td>48</td>
<td>Specimen separated on shear surface, more polished than test 5, with less well developed striations and no undulations in direction of shear. This surface bounded one side of zone of strongly orientated clay 0.5-2.0 mm thick; other side bounded by less well developed shear surface. Clay on either side of this zone showed partial orientation (Figs 21, 22)</td>
</tr>
</tbody>
</table>

(Particle Sizes: Sand 0.2mm, Clay 0.002 - 0.06 mm)

Fig. 5.24: Happisburgh–London Clay Mixtures: Summary of Post-Failure Structure (Lupini et al. –1981)
CHAPTER 6
GRANULAR SOIL – INTERFACE SHEAR RESISTANCE

6.1 INTRODUCTION

Although the soil/interface shear resistance is of major importance in many soil mechanics problems (e.g. deep foundations, shallow foundations under inclined forces, and retaining structures), surprisingly few attempts have been made to gain an understanding of the underlying mechanism and its controlling factors. All relevant research undertaken thus far has been limited solely to experimental work and its numerical representation. A review of data derived from available interface shear tests of granular material will be presented and analyzed in a Chapter 8. For a general bibliographical review of interface tests see Martins (1983), Lemos (1986), and Kishida and Uesugi (1987).

The friction shear stresses along the inner–soil/pile interface essentially control the soil plug behavior. This was demonstrated through the simplified analysis of the soil plug behavior under static loads (based on the silo approach), presented in Chapter 4. This analysis, as well as that of the pile skin shear resistance in Chapter 2, utilized the common formula for calculation of shear along a vertical granular soil–pile interface:

\[ \tau_i = f_s = \sigma_h' \cdot \tan \delta = K \cdot \sigma_v' \cdot \tan \delta \]  

in which:

- \( \tau_i \) — the interface shear resistance
- \( f_s \) — pile skin friction
- \( \sigma_h', \sigma_v' \) — horizontal and vertical effective stresses along the interface
- \( K \) — coefficient of earth pressure — the ratio between the
horizontal and the vertical effective stresses \( \sigma_h'/\sigma_v' \)

\[ \tan \delta = \text{the interface friction coefficient.} \]

As the analysis in Chapter 4 indicated, the two controlling parameters of the soil plug behavior are \( \tan \delta \) and \( K \) (defined above).

The values of the interface friction angle \( \delta \) are usually given as a function of the internal soil shear angle and the type/surface quality of the body in contact. A standard source for the \( \delta \) values in practice is the Naval Design Manual (NAVFAC D.M. 7.2, 1982). The values given for ultimate friction angles between different granular soils and piles and sheet piles can be compared to the internal shear angle of these soils (\( \phi \)). The obtained \( \delta/\phi \) ratios fall within the range of \( 1/3 \leq \delta/\phi \leq 2/3 \) for steel piles and \( 1/3 \leq \delta/\phi \leq 3/4 \) when considering concrete or timber piles. This range of ratios is standard and is in common use (Nordlund –1963, Bowles –1977, 1988, and others). Based on pile testing, several researchers (e.g. McClelland –1974, Vesic –1977) recommend using \( \tan \delta = \tan \phi \), implying the assumption that shear along the interface actually takes place within the soil itself.

The values of the coefficient of earth pressure \( K \) usually range from \( K_0 \) to about 1.75 depending on the volume displacement, initial soil density, installation method, etc. Bowles (1988) recommends using \( K = K_0 \) (the coefficient of earth pressure at rest), because of long-term soil creep effects. The Naval Design Manual (1982) recommends using \( K = 0.5–1.0 \) for H–piles, \( K = 1.0–1.5 \) for driven single–displacement piles, and \( K = 1.5–2.0 \) for driven single–displacement tapered piles. Based on an extensive pile test program, Mansur and Hunter (1970) found average values of \( K = 1.65 \) for H–piles and \( K = 1.26 \) for pipe piles. The API (1984) recommends using \( K = 0.8 \) for unplugged pipe piles and \( K = 1.0 \) for full displacement piles.

A combined non–dimensional friction coefficient of \( N_s = K \tan \delta \) was suggested
by Vesic (1977). The advantage of using $N_s$ is the fact that $N_s$ values lie in a narrow range. For the above values of $K$ and $\delta$, $N_s$ ranges from about 0.2 to 1.25, considering $20^\circ \leq \delta \leq 40^\circ$ and $0.5 \leq K \leq 1.5$. Vesic (1977) quotes $N_s$ values ranging from 2.0 in very dense sand to 0.4 for very long piles. Based on $K_o$ values, Bowles (1988) uses a range of $N_s$ values from 0.25 to 0.40.

In Chapters 6 and 7, an effort is made to obtain a comprehensive view of the interface shear resistance of granular soils. Although the overt intention is to gain an understanding of the friction mechanism along the inner soil/pile interface, the obtained results are applicable to other soil mechanics problems in which an interface is involved.

The direct interface shear experiments (reviewed in Chapter 8) reveal a series of parameters which control the interface shear resistance. These parameters depend on:

(a) Soil characteristics — density, texture and composition, moisture content, and strength

(b) Characteristics of the contacting body — material type and surface quality

(c) Interface conditions — stresses and rate of shear.

The development of a model which correctly simulates the mechanical process during shear of an interface with granular material allows the tested controlling factors to be examined. An examination of this sort is essential for the proper interpretation and utilization of these various factors, which were identified through the experimental work.

In the case of a soil plug, as well as in other similar situations in which large relative displacements take place between the soil and the body in contact, the $K$ values are closely related to the interface shear resistance. Based on this concept, $K$ values will be developed and presented in Chapter 7.

Table 6.1 summarizes the different topics developed in Chapters 6 and 7, with
their relations. Chapter 8 reviews the available experimental data in light of the proposed mechanism for the interface shear resistance. The following steps outline the logic used to understand and resolve the shear interface problem:

1. The micro behavior of granular soils, which was described in Chapter 5, pointed out the significance of particle rolling as particle reorientation and subsequent shear take place under loading (see summary of main points in Table 6.1). The interface analysis, therefore, must take into consideration two possible mechanisms: (1) sliding of the moving plane on the adjacent grains at the interface; (2) rolling and/or sliding of particles within the soil mass due to the friction between bordering grains at the interface with the moving plane in contact. A physical model is proposed in this Chapter, in which both mechanisms can take place (see summary of S.G. model in Table 6.1). An analysis of this model establishes the conditions under which one mechanism or another will prevail. The model is thus used to examine and analyze experimental data from which the governing factors may be determined and evaluated (see 'The Parameters of the S.G. Model' in Table 6.1).

2. The stress state of the inner soil plug, analyzed in Chapter 7, is related to both controlling factors (K and tγδ) of the soil/interface shear resistance (see Table 6.1) in the following ways:

(a) The average intergranular force is aligned in the direction of the major principal stress. The groove, which represents the intergranular contacts in the S.G. model, is chosen to be normal to the major principal stress trajectory obtained through a continuum approach. This allows us to statistically relate the interface friction acting on a single grain as being representative of the entire soil mass.

(b) In the case where intergranular displacement is the controlling mechanism, the analysis must focus on the conditions which prevail when shear along the
interface takes place in the soil itself. The analysis must consider two major factors: (1) the shear plane is predetermined, resembling soil conditions of an interface shear test in a direct shear apparatus; (2) the stress acting on the soil normal to the interface may be greater than that acting on the soil parallel to the interface. These conditions were obtained by the ‘silo approach’ analysis presented in Chapter 4, where K values in excess of 1.0 matched the experimental data. As reviewed earlier, K values greater than 1.0 were obtained in pile load tests, and are commonly recommended for the analysis of pile skin friction.

The shear mechanism in which a predetermined shear plane is subjected to normal stresses larger than the tangential ones differs from the mechanism usually considered in soil mechanics. This mechanism is therefore reviewed in Chapter 7 with these points in mind: (1) What are the possible stress trajectories under the given load conditions, such that K values will be greater or smaller than 1.0? The possible trajectories must comply with previous observations of: the soil plug behavior (Chapter 3), the plug under static load (Chapter 4), and the intergranular mechanisms in an assembly of particles (Chapter 5) (2) What are the limiting shear stresses along the interface such that the maximal stresses along any plane in the soil mass would not exceed the shear stresses of the Mohr–Coulomb failure criterion?
6.2 THE FRICTION MECHANISM ALONG AN INTERFACE

6.2.1 The micro approach

The mechanism of shear resistance can be examined at different levels concerning the contact details. The basic source of the shear resistance is provided by the attractive forces that act between the surface molecules. The Adhesion Theory of Friction states that only small areas at the asperities are actually in contact, resulting in very high stresses which produce plastic flow at the actual contact points (Bowden and Tabor –1945). The overall shear theory treats the particulate material as a continuum to which the Mohr–Coulomb failure criterion is applied. An interface with particulate material is considered as a plane in a continuum along which the shear to normal stress ratio is limited by the interface friction coefficient, which is equal to or less than the continuum internal friction angle (as stated earlier).

The limitations of treating the granular soil as a continuum were discussed in Chapter 5. It was shown that in order to explain the observed granular soil behavior under shear, a soil model made of an assembly of individual particles must be examined. In the introduction to this chapter, it was stated that no study has related the treatment of shear along an interface as a continuum to the particulate nature of the soil. The interface friction angle is therefore based on experimental data which are directly applied to engineering problems, while ignoring the fundamental mechanism and, often, the limitations of the tests themselves.

In order to understand the friction mechanism along an interface, it is necessary to study a model of particles in contact with a plane in motion. Such a model needs to consider the possible rolling mechanism among the particles, and the interface roughness as expressed by the size of the asperities in relation to the size of the particles. Analysis of a specific 3-D assembly of particles in contact with a plane (e.g. adding a plane to the analysis of Thurston & Dersiewicz –1959, Scott –1963, see
Section 5.3.2) was found to be very complicated and is of limited significance.

A better approach to the study of the granular soil/interface shearing resistances is obtained by development of an equilibrium analysis using the limited principles which were suggested (without provision of any source or details) by Skinner (1969). Skinner used the approach to support the insignificant effect of the interparticle friction coefficient on the internal angle of friction of glass ballotini used in his tests, demonstrating the importance of particle rolling during shear (see Section 5.5.4 and Figs. 5.22 and 5.23).

6.2.2 The Sphere in the Groove (S.G.) Model

(a) Underlying concept

During shear of particulate media consisting of individual spheres, particles are displaced over the valley formed by two adjacent spheres in contact, as presented in Figure 5.10. When conditions of limiting equilibrium are reached in its initial location, the sphere rolls and/or slides, changing its contact orientation such that the angle between the tangent to the contact and a reference axis varies during the displacement (see loci of contacts along the circular trajectory in Figure 5.10). An additional contact point exists where the driving force is applied either by a neighboring sphere or by a contacting plane at the interface.

A general model is used, in which a frictional sphere (modeling a soil particle), resting in an inclined friction groove (modeling the contacts with the supporting particles) is subjected to a moving frictional plane (modeling the body in contact) which induces traction, as shown in Figure 6.1 for a horizontal 'smooth' surface. The groove is aligned in the y–z plane along which the motion is occurring. The limiting case in which the body in contact induces shear in the soil (between the sphere and the groove) is identical to the modeling of shear within the soil mass.

By varying the groove angle ($\beta$), it is possible to vary the location of the
contacts such that all possible particle sizes and packings can be considered. When $\beta = 90^\circ$ there is only one contact point, representing the case when a sphere 'rides' over an adjacent sphere rather than over the valley between spheres. The same case can also be viewed as a transformation of the groove into a plane, providing a 2--D analysis. For the 2--D case the contact point is actually a line normal to the plane in which the displacement takes place, identical to the condition of rods in a parallel stack (Figure 5.5 and Rowe -1962).

By varying the groove inclination ($\theta$), all possible orientations of the particle in relation to the interface and the supporting particle are considered. The particle orientation varies during its movement, as mentioned above (see Figures 5.5 and 5.10). However, since only the limiting equilibrium condition (upon initiation of displacement) is considered, only the initial groove--contact orientation is of interest.

Based on currently available data presented in Chapter 5, it is reasonable to assume that during loading and initial shear the average 'groove' (contact) will be normal to the major principal stress direction (in each stage), coinciding with the direction of the normal to the contact when $\beta = 90^\circ$. Orientation of non--spherical particles will be such that maximum contact area will be provided normal to the major principal stress.

Note that the sphere considered in Figure 6.1 is part of an assembly (e.g. sphere B in Figure 5.10). It is prevented from 'rolling down' the groove by an adjacent sphere which supports it (sphere G in Fig. 5.10), in addition to those which form the groove (spheres E and F in Fig. 5.10). The contact force between spheres G and B diminishes as the driving force increases. When the limiting equilibrium state is attained, upon initiation of motion upwards (positive y direction) this contact force is reduced to zero, and therefore need not be considered in the analysis.

Based on the material presented in Chapter 5, it is expected that the particles will roll rather than slide when moving over the groove. This will be examined in the
model by referring to the equilibrium conditions for rolling/sliding between the sphere and the groove. The obtained conditions can then be compared to those prevailing in actual sand media. The motion mechanism between the moving plane and the soil along the interface is examined in the model by referring to the equilibrium conditions for rolling/sliding between the sphere and the moving plane. If the traction force is not sufficient to mobilize the motion of the sphere, a slip will take place in the interface between the particle and the body in contact. It is the boundary between these two regimes which determines where the shear will occur: along the interface or in the soil.

It is also assumed that, for a typical spherical grain, identical conditions exist at both sphere/groove contact points, leading to symmetry in the groove with respect to the z–y plane of Figure 6.1. It is further believed that this assumption, on the average, would be valid for any grain in a particulate medium and therefore justifies the analysis of the motion along one plane (z–y plane in this case).

(b) A sphere between two planes ($\beta = 90^\circ$); limiting equilibrium of interparticle friction.

A simplified case in which $\beta = 90^\circ$ is presented in Figure 6.1b. As was previously discussed, the sphere in this analysis is located between two planes, having only one intergranular contact point/line (denoted by C in Fig. 6.1). The analysis is therefore applicable to a 2-D case for which the circular cross section of Fig. 6.1b can be viewed as that of a rod or a disc. The contact point between the sphere and the moving plane (at the interface) is denoted by O.

The mechanism of what takes place can be understood by using the free body diagram of Fig. 6.1b. The moving plane in the positive y axis exerts a friction at the interface (between the sphere and the moving plane). The friction force $T$ which acts on the sphere (oriented in the positive y axis) creates a moment that tends to rotate
the sphere clockwise, rolling it up the groove. This moment mobilizes the interparticle friction $F$, which produces a counterclockwise moment. Checking the equilibrium around point $O$ enables determination of the possible motions between the particles (the movement of the sphere along the groove).

The equation for moment equilibrium around point $O$ is:

$$\Sigma M_0 = N \cdot b - F \cdot a = 0$$

From geometry: $a = r \left(1 + \cos \theta\right)$, $b = r \sin \theta$, $r$ being the radius of the sphere.

The maximum possible friction force between the grains is that which develops at the verge of sliding. Rolling is always associated with less friction than sliding. Under limiting equilibrium of interparticle sliding $F = N \cdot \tan \phi_\mu$, where $\phi_\mu$ is the interparticle friction angle (rolling takes place when $F < N \cdot \tan \phi_\mu$). Substitution of $a, b$ and $F$ into the above moment equilibrium equation gives:

$$\tan \phi_\mu = \frac{b}{a} = \frac{\sin \theta}{1 + \cos \theta}$$  \hspace{1cm} (6.2)

The relation of Equation 6.2 is presented as a dashed line in Figure 6.2a. The plotted relations are interpreted in the following way:

(1) If the existing interparticle friction is greater than that required for equilibrium ($\tan \phi_\mu > \frac{b}{a}$), then resistance to sliding remains past the point where rolling begins at point $C$. The area above the line which describes the relation of Equation 6.2 represents, therefore, the domain in which interparticle rolling takes place. In other words, the friction force $N \cdot \frac{b}{a}$ is smaller than that which is developed during sliding: it is a friction which can occur during rolling only. When $\tan \phi_\mu > \frac{b}{a}$ the net moment is clockwise (consistent with the direction of motion of the moving plane), allowing the
sphere to move 'up' the groove. For the S.G. model, this motion is associated with dilation, indicated by an upward movement of the moving plane (in the positive z direction).

(II) If the existing interparticle friction is equal to that required for equilibrium \( \tan \phi_{\mu} = \frac{b}{a} \), then sliding takes place between the sphere and the groove (interparticle sliding). The sphere movement 'up' the groove is associated with dilation as mentioned above.

(III) If the interparticle friction is smaller than that required for equilibrium \( \tan \phi_{\mu} < \frac{b}{a} \), the underlying assumptions of the S.G. model [stated in section (a) above] are violated. The net moment is counterclockwise, rolling the sphere 'down' the groove. [The same conclusion is also derived from examination of the sphere's equilibrium (under the same conditions) along the inclination of the groove.] This situation violates the underlying concept of the S.G. model in two ways: (a) For the model of Figure 6.1a, the sphere loses contact with the moving plane; (b) If the sphere is a part of an assembly, then the force interacting with the supporting sphere (e.g. sphere G of Figure 5.10) must be considered in the analysis. The S.G. analysis, however, was based on the assumption that this force diminishes as the sphere moves 'up' the groove.

It should be noted that the relations of Equation 6.2 describe the upper limit case in which the greatest impediment to interparticle motion exists. When \( \beta = 90^\circ \), the normal force at the contact (N) acts in the z–y plane and is maximal (as can be evaluated from force equilibrium consideration), resulting in maximal interparticle friction (limiting) force (F). This will be demonstrated in the next section, where the interparticle friction angle will be evaluated for the case where \( \beta \neq 90^\circ \), producing smaller values than those of Equation 6.2.
(c) A sphere in a Groove ($\beta \neq 90^\circ$), limiting equilibrium of interparticle friction

For $\theta = 0^\circ$, the angle between the wall of the groove (the plane tangential to the contact) and the $z$ axis is $\beta$, as denoted in Figure 6.1a−1. For the more general case, for which $0 \leq \theta \leq 90^\circ$, the angle between the wall of the groove and the $z$ axis is denoted by $\eta$ (Figure 6.1a−2). The geometrical relation between the two angles is:

$$\eta = \tan^{-1}(\cos \theta \tan \beta)$$

In order to find the normal to the contact, the plane of the wall of the groove can be defined using lines (1) and (2) of Figure 6.1a:

Line (1) = $1i + 0j + \tan \beta^* k$

Line (2) = $0i + 1j + \tan \theta k$

where $\beta^* = 90^\circ - \eta$.

The normal to the contact is obtained by the vector product of lines (1) and (2):

$$N = N_{xyz} = -\tan \beta^* i - \tan \theta j + 1k$$

The normal components in the direction of axes $x$, $y$ and $z$ are:

$$N_x = -\tan \beta^* i / |N|$$

$$N_y = -\tan \theta j / |N|$$

$$N_z = 1k / |N|$$

(6.3)

where $|N| = \sqrt{\tan^2 \beta^* + \tan^2 \theta + 1}$

The above components of the normal are presented in Figure 6.1c. The $z$–$y$
plane is a plane of symmetry, and therefore the normal components \( N_x = \tan \beta \frac{i}{|N|} \) and \( N_y = -\tan \beta \frac{j}{|N|} \) exist at the symmetric contact point.

Referring to Figure 6.1c and Equation 6.3, the analysis of the \( \beta = 90^\circ \) case can be resumed in a more general form:

\[
\Sigma M_o = 2 N_{yz} \cdot b - 2F \cdot a
\]

From geometry (see Figure 6.3): \( a = r(\cos \theta + \sin \beta) \), \( b = rsin\theta \)

At limiting equilibrium of interparticle friction: \( F = N_{xyz} \cdot \tan \phi_\mu \)

Due to the symmetry, the components of the normal forces in the x direction \( (N_x) \) cancel each other. When the moment equilibrium around O is considered, only the resultant of the normal force in the z–y plane contributes to the clockwise moment.

The resultant \( N_{yz} \) of vectors \( N_y \) and \( N_z \) is:

\[
N_{yz} = \sqrt{\tan^2 \theta + 1} / |N|
\]

Substituting \( a, b, N_{yz} \) and \( F \) into the above moment equilibrium equation gives:

\[
tg \phi_\mu = \frac{N_{yz}}{N_{xyz}} \cdot \frac{b}{a} = \frac{\sin \theta}{(\cos \theta + \sin \beta)} \cdot \frac{1}{\sqrt{\tan (90^\circ - \beta) + 1}} \tag{6.4}
\]

When \( \beta = 90^\circ \) Equation 6.4 is equal to Equation 6.2. The relations of Eq. 6.4 are presented in Figure 6.2a. The same interpretation which followed the analysis of the sphere between two planes \( (\beta = 90^\circ) \) holds for these relations as well. Each of the lines represents the interparticle sliding conditions for a different \( \beta \) angle and the range
of $0 \leq \theta \leq 90^\circ$. The domain above the line is that of rolling.

The relations of Figure 6.2a show that:

(I) The 2-D case of $\beta = 90^\circ$ represents the upper boundary between interparticle
sliding and rolling.

(II) Significant variation in the values of $\beta$ leads to a moderate effect on the
boundary between rolling and sliding.

(III) The data regarding $\text{tg}\phi_\mu$ of quartzic sands (Section 6.3.3b), the $\beta$ angle
(Section 6.3.3a), and $\theta$ (Section 8.2) will be discussed subsequently. These
data show that round quartzic sand particles are in the rolling domain of Fig.
6.2a during loading. Progression of loading towards shear seems to shift the
particles closer to the sliding domain. The above analysis verifies analytically
the data of Chapter 5, suggesting that most of the particles will roll during
loading and shear.

Two extreme possible conditions are presented in Figure 6.4:

(a) When $\theta = 90^\circ$, the groove is normal to the interface and $\frac{b}{a} = 1$ for any
$0 \leq \beta \leq 90^\circ$, meaning that only sliding at the interface can take place
(unless $\text{tg}\phi_\mu \geq 1$).

(b) When $\theta = 0^\circ$, $\frac{b}{a} = 0$ for any $\beta$, meaning that when the groove is
parallel to the interface, only rolling can take place (unless $\text{tg}\phi_\mu = 0$).

Both conditions explain the range of relations between $\text{tg}\phi_\mu$ and $\theta$, as
presented in Figure 6.2a.

(d) A sphere between two planes ($\beta = 90^\circ$); limiting equilibrium of interface
friction

Equation 6.2 was developed using the limiting equilibrium of interparticle
friction. Sliding and rolling at the interface are considered by examining the limiting
equilibrium conditions of the interface friction. The resistance to interface sliding is:
Using Figure 6.1b, the following equilibrium equations are applied in the $y$ and $z$ directions respectively:

\[ T + F \cos \theta - N \sin \theta = 0 \]
\[ -P + F \sin \theta + N \cos \theta = 0 \]

Using both equations and substituting for $T$, the interface friction coefficient is obtained:

\[ \tan \delta_s = \frac{N \cdot \sin \theta - F \cdot \cos \theta}{N \cdot \cos \theta + F \cdot \sin \theta} \quad (6.5) \]

For the limiting equilibrium of the interparticle friction $F = N \cdot \tan \phi \mu$, where $\phi \mu$ is described by Equation 6.2. Substituting $F$ and $\phi \mu$ into Equation 6.5 gives:

\[ \tan \delta_s = \frac{\sin \theta}{1 + \cos \theta} \quad (6.6) \]

The obtained relation of Eq. 6.6 is identical to that of Eq. 6.2. This fact suggests that the interface friction coefficient is identical to the interparticle friction coefficient. The interface friction coefficient, under limiting equilibrium of the intergranular contact is equal to the shear strength of the soil itself. These relations indicate, therefore, that for pure frictional relations the shear strength of the soil is equal to the interparticle friction coefficient. This subject will be discussed further in Chapter 8.

As Equations 6.2 and 6.6 have the same relations, the dashed line in Figure 6.2b which describes the boundary between rolling and sliding at the interface ($\tan \delta_s$) as being a function of $\theta$ for the 2-D case ($\beta = 90^\circ$) is identical to the dashed line of Figure 6.2a which describes the boundary between the interparticle rolling and sliding
as $\tan \phi_\mu$, a function of $\theta$.

For the case of $\beta = 90^\circ$ the condition of $\tan \phi_\mu = \tan \delta_s$ also means that $P = N$. This is obtained from moment equilibrium around the center of the spheres (denoted by C.G.), which requires that $T \cdot r = F \cdot r$. For $T = P \cdot \tan \delta_s$, $F = N \cdot \tan \phi_\mu$, and $\tan \delta_s = \tan \phi_\mu$. $P$ must therefore be equal to $N$.

The logic of the two domains (sliding/rolling) concerning the interface will be further examined in the next section.

It should be noted that the relations of Equation 6.6, like those of Equation 6.2, describe the upper limit case ($\beta = 90^\circ$) in which the greatest impediment to motion exists, as was previously discussed.

\[(e)\text{ A sphere in a Groove (}\beta \neq 90^\circ\text{); limiting equilibrium of interface friction}\]

The above equilibrium equations are utilized for the general case. The $N_x$ component of the total normal force at the contact ($N_{xyz}$) contributes to the friction force $F$ but is not considered in the force equilibrium of the yz plane, as shown in Figure 6.1c. Thus, Equation 6.5 in the general case is:

$$
\tan \delta_s = \frac{N_{y\delta} \cdot \sin \theta - F \cdot \cos \theta}{N_{y\delta} \cdot \cos \theta + F \cdot \sin \theta}
$$

(6.5a)

The intergranular frictional force for the limiting equilibrium is $F = N_{xyz} \cdot \tan \phi_\mu$, where $\tan \phi_\mu$ is described by Equation 6.4. Substituting $F$ and $\tan \phi_\mu$ into Equation 6.5a gives:

$$
\tan \delta_s = \frac{\sin \theta \cdot \sin \beta}{1 + \cos \theta \cdot \sin \beta}
$$

(6.7)

A better demonstration of the sphere's rolling and the interface sliding is obtained by referring to the moment equilibrium of the sphere in the groove around its points of rotation (contact points C). Referring to Figure 6.5:
\[ \Sigma M_c = P \cdot d - T \cdot e = 0 \]

The geometrical relations are presented in Figure 6.3:

\[ d = r \cdot \sin \beta \cdot \sin \theta \quad e = r(1 + \sin \beta \cdot \cos \theta). \]

The interface frictional force for the limiting equilibrium of sliding is \( T = P \cdot \tan \delta_s \).

Substituting \( T \), \( e \), and \( d \) into the above moment equilibrium equation leads to:

\[ \tan \delta_s = \frac{d}{e} = \frac{\sin \theta \cdot \sin \beta}{1 + \cos \theta \cdot \sin \beta} \quad (6.7a) \]

which is identical to Equation 6.7.

For \( \beta = 90^\circ \), Equation 6.7 reduces to Equation 6.6. The relations of Eq. 6.7 for different values of the angle \( \beta \) are presented in Figure 6.2b. The same logic which was previously presented in section (b) for the determination of the rolling/sliding domains concerning the interparticle friction is followed here for the interface friction. The maximum possible friction force between the grain and the moving plane at the interface is that which develops at the verge of sliding: \( T = P \cdot \tan \delta_s \). Rolling is associated with less friction than sliding. For given conditions of \( \beta \) and \( \theta \) the relations presented in Fig. 6.2b indicate that:

1) For interface friction of \( \tan \delta_s > \frac{d}{e} \) only rolling can take place between the soil particle and the moving plane at the interface. The friction force \( P \cdot \frac{d}{e} \) is smaller than that which develops during sliding, allowing only rolling to occur. The same conclusion is supported by examination of the particle equilibrium as shown in Figure 6.5. For \( \tan \delta_s > \frac{d}{e} \) a net clockwise moment will be developed. This moment matches the displacement direction of the moving plane, allowing rolling of the sphere ‘up’ the groove. The area above any line
which describes the relation of Equation 6.7 (in Fig. 6.2b) therefore represents the domain in which rolling takes place at the interface.

(II) For interface friction of $\tan \delta_s = \frac{d}{e}$, sliding takes place at the interface.

(III) For interface friction of $\tan \delta_s < \frac{d}{e}$, the underlying assumptions of the model are violated. The counterclockwise moment results in the sphere moving 'down' the groove, losing contact with the moving plane and creating an additional force between the analyzed sphere and an adjacent one, which prevents it from rolling down.

The relations between the conditions for sliding/rolling at the interface and those at the interparticle plane are discussed in the next section.

(f) Discussion

The analyses presented in Sections b, c, d and e used two possible limiting equilibrium conditions:

(1) $F = N \cdot \tan \phi \mu$ for the interparticle friction, and

(2) $T = P \cdot \tan \delta_s$ for the interface friction.

The forces in the model (as, for example, in the case of pile loading) are initiated by the motion of the body in contact along the interface (the 'moving plane'). This motion mobilizes friction which exerts moment on the sphere. The tendency of the sphere to roll due to this moment is countered by a moment in the opposite direction, exerted by the interparticle friction at the contact point/s between the sphere and its supporting sphere/s (the 'groove').

Examination of the limiting equilibrium of the sphere in relation to the interparticle friction led to Equations 6.2 and 6.4. These equations describe the interparticle friction required to maintain the sphere's equilibrium. The obtained relations are based on limiting sliding friction, which is the ultimate possible resistance to motion (and therefore associated with maximal dissipation of energy).
For a set of given conditions \((\beta, \theta)\); if the actual interparticle friction coefficient \(\tan \phi\) is greater than the relations of Equation 6.4, then only rolling is possible between the soil grains. If, on the other hand, it is equal to the calculated value, then the sphere slides up the groove (representing the required conditions for interparticle sliding).

The above concept is significant for soil mechanics. The given interpretation suggests that sliding between particles can take place during deformation and shear if the interparticle friction is small or if the particles have a specific contact orientation. Particles with a greater impedance to motion as a result of a greater interparticle friction or of angular shape (as will be shown later) have a greater tendency to roll during deformation. As a result, a correlation may or may not exist between the interparticle friction coefficient and the shear resistance of granular material. This subject is investigated in Section 8.3, where it is assumed that the driving force of the moving plane is applied by a neighboring grain. The relations of the S.G. model \((\tan \phi \mu, \theta\) representing the \(\sigma_1\) orientation, and \(\tan \delta_s\) representing \(\tan \phi'\)) are then compared with experimental data. Furthermore, this concept of rolling and sliding may explain the phenomenon where materials with a small interparticle friction show a similar shear resistance to those with a high interparticle friction (under the same density-packing conditions), as both dissipate the same energy. If the shear resistance is independent of the interparticle friction, then the total energy dissipated when soil particles slide with a small interparticle friction may be comparable to the total energy dissipated in the case of rolling in a soil with a high interparticle friction (Skinner's tests, Section 5.5.4). This, however, contradicts Rowe's stress dilatancy theory (Section 5.3.1).

The known interparticle friction coefficient of quartz and the possible soil densities (reflected by the factor \(\beta\)) and contact orientations (reflected by the factor \(\theta\)) suggest that the actual interparticle friction of quartzic sand is greater than that calculated in Equation 6.4 for the different loading stages (to be discussed in
subsequent sections). These findings have two implications: (1) Interparticle motion consists mainly of rolling. Particles of round quartzic sands are reoriented and displaced during loading mostly by rolling, as was suggested by the experimental data of Chapter 5; (2) The moment which results from the interparticle friction, and which is required to maintain the equilibrium of the sphere, can be fully mobilized. The ability to mobilize this resistance enables the moving plane to slide on the sphere until the conditions for shear in the soil are met.

Examination of the limiting equilibrium of the sphere in relation to the interface friction led to Equations 6.7 and 6.7a. These equations describe the interface friction coefficient \( \tan \delta_s \) which is required to maintain the equilibrium of the sphere when the moving plane slides on it.

Referring to the case of a sphere between two planes \( (\beta = 90^\circ) \), it was shown that \( P = N \) and \( \frac{b}{a} = \frac{d}{e} = \frac{s \sin \theta}{1 + \cos \theta} \). The moments acting on the sphere are initiated by the interface friction, and the actual interparticle friction of sand is greater than \( \frac{b}{a} \) \( (\tan \phi_\mu > \frac{b}{a} ) \). The mobilized interparticle friction will therefore be equal to the mobilized sliding friction along the interface, maintaining the sphere in equilibrium while the moving plane slides along the interface. The interface friction coefficient \( \tan \delta_s \) is thus equal to the calculated value of Equation 6.7, representing correctly the shear resistance to the contacting body motion along the interface, as long as shear does not take place in the soil itself. Shear within the soil itself can be considered to occur in two instances: (1) According to the S.G. model: when rolling or sliding of the sphere is possible, referring to the above analyzed interparticle motion; and (2) According to the continuum failure criterion: when the sliding resistance (shear) along the interface equals the maximum possible shear stress along any plane in the soil, according to the Mohr–Coulomb failure criterion.

In reference to the aforementioned quartzic sphere between two planes, the sphere will be in the limiting equilibrium state when \( \tan \phi_\mu = \tan \delta_s = 0.5 \). This will
occur when $\theta \approx 53^\circ$. Therefore, for $0 \leq \theta < 53^\circ$, shear will take place along the interface as the moving plane slides on the stable sphere. When $\theta = 53^\circ$ the sphere reaches its limiting equilibrium state, in which theoretically the sphere may slide up the groove. This approach is used as a failure criterion in Section 8.3., where it is analyzed and compared to experimental data.

In the more general case for which $0 < \beta < 90^\circ$, the value of $\frac{b}{a} > \frac{d}{e}$ guarantees the equilibrium of the sphere when $\tan \phi_\mu > \frac{b}{a}$ until the shear conditions are met, as discussed above. For example: for $\beta = 45^\circ$, a quartzic sphere will reach limiting equilibrium when $\tan \phi_\mu = \frac{b}{a} = 0.5$ for $\theta = 60^\circ$. The interface friction coefficient at that stage is: $\tan \delta_s = 0.45$. The maximum possible interface friction at which shear initiates in the soil for the above given conditions is therefore 0.45.

In summary. For a set of given conditions, $[\beta$ — correlated to density, $\theta$ — correlated to stress orientation, particle shape and the roughness of the contacting body (the last two will be included in the analysis)] the S.G. model will predict the interface sliding coefficient limited by a maximum value corresponding to initiation of shear in the soil.

(g) Limiting equilibrium of interface friction for non-spherical particles

between two planes ($\beta = 90^\circ$).

Equations 6.2 and 6.6 were developed using limiting equilibria of interparticle friction for a sphere between two planes. Under these conditions, the sphere is subjected to the greatest impediment to motion, and both equations show an identical upper limit transition from sliding to rolling. From Equation 6.2 it is evident that this transition is determined by the geometrical relation $b/a$. Equation 6.7a was developed for the general case in which interface friction was considered for all possible $\beta$ values. This equation is controlled by the geometrical relation $d/e$. The fact that the analysis
of a spherical grain in a groove is controlled by geometrical relations clearly indicates that the shape of the grain will determine the transition from sliding to rolling under given conditions.

It was previously stated that non-spherical particles will orient themselves such that maximum contact area will be provided normal to the major principal stress, and therefore parallel to the groove (see Section (a) which discusses the underlying concept). For an elongated grain in this orientation, the distances denoted by b and d will increase, while those denoted by a and e will decrease (in relation to a round particle with a radius equal to the average radius of the elongated particle). The ratios b/a and d/e are thus expected to increase, indicating an increase in the impediment to rolling. This results in an upward shift of the transition curve from sliding to rolling.

A general solution for a non-spherical grain in a groove requires a 3-D grain description and vector analysis. Because individual grains express a great degree of variability, such a solution will be very complex and of limited use. Since the 2-D analysis ($\beta = 90^\circ$) of a grain between two planes provides an upper limit solution, using it to analyze an elongated particle of an idealized shape will provide a simple and practical solution which is then modified by assuming a round cross-section.

Figure 6.6 presents the details of the forces, geometrical relations, and distances for an elliptical particle between two planes. The ellipse's long and short axes have the lengths of $2l$ and $2s$ respectively. The ratio between the axes is denoted by $R_a = 1/s$. The dimensions $a_e$ and $b_e$ describe the normal distances from the contact points C and O to the line of action of forces F and P (the arm of the moment), respectively. Figure 6.3 presents the expressions which describe the distances $a_e$ and $b_e$ as a function of the groove inclination $\theta$ and the ellipse axes $l$ and $s$. When $R_a = 1$ (both axes are equal) the expressions for $a_e$ and $b_e$ reduce to those of a and b, previously used for the sphere. Substituting the expressions $a_e$ and $b_e$ into Eqs. 6.2 and 6.6 gives:
\[ \tan \delta_s = \tan \phi = \frac{b_a}{a_c} = \frac{R_a^2 \cdot \tan \theta}{1 + \sqrt{R_a^2 \cdot \tan^2 \theta + 1}} \] 

(6.8)

For \( R_a = 1 \) Equation 6.8 is identical to Equations 6.2 and 6.6 for the range of \( 0 \leq \theta \leq 90^\circ \). Note that although Eq. 6.8 is not valid for \( \theta = 90^\circ \), it has the limit of one for \( \theta \) approaching \( 90^\circ \).

The relations of Equation 6.8 are presented in Figure 6.2c for the cases where \( R_a = 1 \) and \( R_a = 1.25 \). The obtained relations describe the anticipated behavior of elongated particles, for which the transition from sliding to rolling will be shifted upwards in relation to spherical particles. At the same contact orientation and interparticle friction angle, a greater interface friction (rougher surface) is required in order to create shear in a soil of elongated vs. round particles. For the relations of Fig. 6.2c, assuming \( \theta = 30^\circ \), an interface surface with a friction coefficient of 0.27 will cause shear in a soil mass of round particles. The same interface will slide over a soil mass of elongated particles which requires an interface friction coefficient of at least 0.40 to initiate shear in the soil.

Equation 6.8 can be modified for the groove case (\( \beta \neq 90^\circ \)) by assuming an ellipsoidal grain having a longitudinal profile of an ellipse and a round cross-section. The influence of the angle \( \beta \) on the values calculated by Eq. 6.8 is thus identical to the reduction of the ratio due to \( \beta \) as expressed by Equations 6.4 and 6.7 for the interparticle and interface friction, respectively. For the interface case, \( \tan \delta_s \) for an ellipsoid in a groove will be the value of Eq. 6.8 multiplied by the ratio of the values obtained from Eq. 6.7 over that of 6.6. The general equation of the S.G. model for this case and for the contacting plane roughness (Eq. 6.13) is developed in Section 6.3.2c.
6.3 THE CONTROLLING PARAMETERS OF THE INTERFACE SHEAR RESISTANCE FOR THE S.G. MODEL

6.3.1 Introduction

The interface shear resistance is controlled by parameters which can be divided into three categories (Section 6.1): those related to the characteristics of the contacting body, to the characteristics of the soil, and to the interface conditions. These parameters were found to influence the interface shear resistance through various experiments, and will be discussed herein in relation to the S.G. model. A critical review of available experimental data relevant to this discussion will be presented in Chapter 8.

In order to consider a broader range of contacting bodies, the S.G. model is expanded to account for the surface roughness. This expansion of the model, and its conclusions, is followed by a discussion regarding the two additional parameter categories, namely those concerning the soil characteristics and those concerning the interface conditions.

6.3.2 Expansion of the S.G. model to account for the interface roughness

(a) Underlying concept

In the development of the S.G. model, the contacting body was assumed to be a moving frictional plane which was described as a "horizontal ‘smooth’ surface" (Section 6.2.2) with the contact-normal between the sphere and the frictional plane being normal to the direction of the plane movement. The values of the coefficient of friction (tgδₘ) which are calculated by the S.G. model in Section 6.2 refer, therefore, to the predicted transition between sliding and rolling for ‘smooth’ surfaces.

The influence of the surface quality (‘finish’) of the body in contact on the interface shear resistance was recognized long ago (e.g. Potyondy –1961). Kulhawy and
Peterson (1979) proposed quantitative measurements which were based on the average grain size of the soil and the contacting concrete. Yoshimi and Kishida (1981a,b) quantified the surface roughness by measuring the surface profile. They determined the relative height between the highest and the lowest peak over a 2.5 mm length, similar to a method used for evaluation of finished metal surfaces by machinists. Uesugi and Kishida (1986b) and Kishida and Uesugi (1987) modified this roughness evaluation by referring to the relative height over a length equal to the representative grain size diameter of the sand (D_{50}), and then normalized it relative to D_{50}.

Each of the experimental studies produced relations which demonstrated the influence of the surface 'roughness' on the interface resistance. However, only by referring to the mechanism behind the so-called 'smooth' and 'rough' surfaces can these terms be identified and quantified in relation to the soil at the interface.

Dickey (1966) (see Lambe and Whitman—1969), measured the profile of a 'smooth' quartz surface as shown in Figure 6.7a. Figure 6.7 is used to demonstrate the idea that the mechanism of shear resistance is a function of the contact detail, as presented in Section 6.2.1. A circle the size of a medium sand grain was drawn using the horizontal scale and added to the presented figure. Note that the five marked divisions of the vertical scale, which represent 10^{-5} inch, are actually about 1/2000 the thickness of the line which describes the border of the sand grain. Figures 6.7b and 6.7c were added in order to demonstrate the relativity of the grain size to the surface profile. In Figure 6.7b the sand grain and the surface profile are both drawn to the same scale, with the quartz surface appearing as a 'smooth' horizontal line. Figure 6.7c was drawn using the average contours of the surface profile in Figure 6.7a, as if the vertical scale of Figure 6.7a was identical to the horizontal one.

The conclusions drawn from the comparison of the size of the sand grain to the surface details of a contact body, as shown in Figures 6.7a, b, c are:

1. The fact that the smooth—appearing surface of the quartz is actually composed
of numerous asperities supports the adhesion theory of friction (see Section 6.2.1).

2. When the sand grain and the surface asperities are viewed at the same scale, the quartz surface may be considered as smooth and horizontal. The influence exerted by the asperities of such a surface is represented by its friction coefficient \(\tan\delta\). For an analysis such as that of the S.G. model, in which the mechanical relations between a grain and a surface are examined, the smooth surface can be represented as a horizontal line with a friction coefficient.

3. When a sand grain and a rough surface are compared at the same scale as shown in Figure 6.7c, the surface can no longer be considered as a horizontal plane when the contact normal is not normal to the direction of movement. This occurs when the wavelength of the surface roughness is equal to or greater than the average grain diameter \(D_{50}\). Assuming that the plane of Figure 6.7c is moving from right to left, the sand grain will be in contact with a plane tilting at a \(10^\circ\) angle from the direction of the movement.

In general, the rougher the surface, the greater the interface friction angle. Based on the above concept of a "tilting" contact plane, the S.G. model can thus be expanded to account for the surface roughness by considering a contact which would impede the movement of the sphere up the groove. The relationship between the contact orientation and the surface friction coefficient can then be established. The surface orientation and the contact location (which is a function of the grain size) can be used to calculate the height of the equivalent asperity. The relations between the calculated friction coefficient and the height of the asperity can then be compared to experimental data.

(b) *The modified S.G. model for spherical particles.*
Figure 6.8a presents a sphere in a groove subjected to a 'rough' surface, as previously discussed. The angle between the plane of the movement and the plane in contact is denoted by \( \alpha \) (the roughness angle).

The general limiting equilibrium equation for the sphere in the groove was obtained in part (d) of Section 6.2.2. By referring to moment equilibrium around point \( C \), the relations of \( \tan \delta_s = \frac{d}{e} \) were obtained (Equation 6.7a). In the general case of Figure 6.8a the geometrical relations are: \( e = r + r \cdot \sin \beta \cdot \cos(\theta + \alpha) \) and \( d = r \cdot \sin \beta \cdot \sin(\theta + \alpha) \). Use of these relations leads to:

\[
\tan \delta_s = \frac{d}{e} = \frac{\sin \beta \cdot \sin(\theta + \alpha)}{1 + \sin \beta \cdot \cos(\theta + \alpha)} \tag{6.9}
\]

For \( \alpha = 0^\circ \), Equation 6.9 becomes identical to Equation 6.7a, which is applicable to the general case of a 'smooth' contacting surface.

The roughness of the surface can be estimated by calculating the height of the asperity needed to create a contact point at 'O' for a plane in the \( \alpha \) direction, shown in Figure 6.8b. This height is relative to the particle diameter \( d \):

\[
h = \frac{d}{2} (1 - \cos \alpha) \tag{6.10}
\]

For practical purposes, \( D_{50} \) (the median diameter, by weight) can be used as the particle diameter \( d \); \( h \) then expresses roughness, such that Equation 6.10 can be rewritten as:

\[
R = 0.5 \cdot D_{50} \cdot (1 - \cos \alpha) \tag{6.10a}
\]

In order to obtain a dimensionless roughness coefficient, the relation of Equation 6.10a can be normalized with respect to the grain size such that the
normalized roughness is:

\[ R_n = \frac{R}{D_{50}} = 0.5(1 - \cos \alpha) \]  

(6.11)

The relations of Equation 6.9 between \( \tan \delta_s \) and the roughness angle \( \alpha^o \) are presented in Figure 6.9 as a set of 3 curves labeled 'spherical particles'. These curves were constructed for \( \theta = 30^\circ \) and 3 different \( \beta \) values. Two additional scales were added to Fig. 6.9: (1) friction angle expressed in degrees and (2) normalized roughness, as described in Equation 6.11.

For the purpose of practical demonstration, it can be stated that the relations of Figure 6.9 are those of a dense round sand with an internal friction angle of 45°. In this case it can be assumed that the maximum coefficient of shear resistance along the interface is 1.0 (\( \tan \delta_s = \tan \phi = \tan 45^\circ \)). The scale of \( \tan \delta_s \) will therefore also provide the ratio of \( \tan \delta_s / \tan \phi \). The three curves for spherical particles begin with an interface friction coefficient of approximately 0.25. This value represents the transition from sliding to rolling of the particle when subjected to a ‘smooth’ surface, and can also be obtained from Figure 6.2b for the appropriate \( \theta \) and \( \beta \) values. As the roughness increases, the anticipated friction coefficient increases as well. The case of \( \beta = 90^\circ \) remains the upper limit for which a surface with a normalized roughness of 0.25 causes shear along the interface equivalent to the internal shear of the soil itself. Assuming medium sand grains of \( D_{50} = 0.3\text{mm} \), this normalized roughness can be translated into asperities of about 75\( \mu \text{m} \).

(c) The modified S.G. model for non-spherical particles

Figure 6.8c presents an ellipse between two planes subjected to a ‘rough’ surface. This modification refers to the method of analysis developed in part (g) of Section 6.2.2. The analysis was then restricted to the upper limit case in which
\( \beta = 90^\circ \). The change to a contact surface at an inclination of \( \alpha \) degrees can be viewed (for \( \beta = 90^\circ \)) as the same problem with a new \( \theta_n \) angle, for which \( \theta_n = \alpha + \theta \).

The solution will therefore be identical to Equation 6.8, with \( \theta_n \) replacing \( \theta \):

\[
\tg \delta_s = \frac{b_e}{a_e} = \frac{R_a^2 \cdot \tg(\theta+\alpha)}{1 + \sqrt{R_a^2 \cdot \tg^2(\theta+\alpha) + 1}}
\]  

(6.12)

where for \( \beta = 90^\circ \), \( \tg \delta_s = \tg \phi \mu \)

(d) The general equations of the S.G. model

With the assumption of a circular cross-section Equation 6.12 can be modified for the two general equations of the S.G. model, considering the particle shape, the roughness angle \( \alpha \), the inclination \( \theta \), and the density \( \beta \).

By multiplying Equation 6.12 by the ratio of Eq. 6.9 over Eq. 6.7, the general equation for the interface friction is obtained.

\[
\tg \delta_s = \frac{R_a^2 \cdot \tg(\theta+\alpha)}{1 + \sqrt{R_a^2 \cdot \tg^2(\theta+\alpha) + 1}} \cdot \frac{\sin \beta \cdot [1 + \cos(\theta+\alpha)]}{1 + \cos(\theta+\alpha) \sin \beta}
\]  

(6.13)

By multiplying Equation 6.12 by the ratio of Eq. 6.4 over Eq. 6.2 the general equation for the interparticle friction is obtained.

\[
\tg \phi \mu = \frac{R_a^2 \cdot \tg \theta}{1 + \sqrt{R_a^2 \cdot \tg^2 \theta + 1}} \cdot \frac{(1 + \cos \theta)}{(\cos \theta + \sin \beta) \sqrt{\tg^2(90^\circ - \beta) + 1}}
\]  

(6.14)

For \( R_a = 1 \), \( \beta = 90^\circ \) and \( \alpha = 0^\circ \), Equations 6.13 and 6.14 reduce to Eqs. 6.2 and 6.6:

\[
\tg \delta_s = \tg \phi \mu = \frac{\sin \theta}{1 + \cos \theta}
\]
The interface roughness can be estimated in a manner similar to that previously developed for spherical particles. It is unnecessary (and impractical) to modify the roughness Equations 6.10a and 6.11 for geometrical shapes other than spheres, since the value of \( D_{50} \) is influenced by particle shape. This influence will be reflected in the obtained relations.

The relations of Equation 6.12 between \( \tan \delta_s \) and the roughness angle \( \alpha^o \) are presented in Figure 6.9 as a set of 2 curves labeled 'elongated particles'. The curves were constructed for the assumed dense sand with \( \theta = 30^o \) and two \( R_a \) ratios, 1.25 and 1.50. As the relations of Eq. 6.12 represent the upper limit \( (\beta = 90^o) \), most particle assemblies in which \( \theta = 30^o \) will have a value of \( \beta \) which will be less than \( 90^o \), and therefore friction values which will be smaller than those of the presented relations.

The elongated particle curves clearly demonstrate the influence of the particle shape on the interface friction resistance. Both curves have an interface friction coefficient of approximately 0.5 for the smooth case \( (\alpha = 0^o) \). This means that even a smooth surface would mobilize about 50% of the internal friction of the soil. Relatively little roughness (of about \( R_n = 0.1 \)) would mobilize an interface friction resistance equal to the shear strength of the soil \( (\delta_s = \phi = 45^o) \).

(e) Discussion

A comparison can be made between an assembly of spherical and an assembly of elongated particles for the 2-D case \( (\beta = 90^o, \theta = 30^o, \phi = 45^o \) and \( D_{50} = 0.3 \)mm in both cases). For the spherical particles, asperities need have a height of at least 75\( \mu \)m in order for full shear to be mobilized. For elongated particles, asperities need only have a height 45\( \mu \)m and 15\( \mu \)m (for \( R_a = 1.25 \) and 1.50 respectively) in order to mobilize the same shear strength.

In summary, any measure of absolute roughness based on asperity height which disregards the particle size and shape is meaningless when used to describe a
surface as being 'smooth' or 'rough'. This is demonstrated in the following examples:

(1) Referring to the upper limit case of $\beta = 90^\circ$ and spherical particles; a surface with a roughness angle of $60^\circ$ (equal to a normalized roughness of $R_n = 0.25$) mobilizes the full resistance of the soil internal friction angle ($\phi = 45^\circ$). Relating these spheres to medium-sized silt particles of 0.01 mm, the surface must have asperities at least $2.5\mu m$ high in order to mobilize the full shear strength of the silt, and therefore to be considered a 'rough' surface.

Asperities of the same height will have a much smaller effect on medium-sized sand particles of 0.3 mm, where only about 30% of the internal shear strength of the soil would be mobilized ($\tan \delta_s \cong 0.3$). The same contacting body will now be considered to have a 'smooth' surface. An asperity height of at least $75\mu m$ is required to mobilize the full internal shear resistance of the sand. This would then characterize the surface as 'rough' for both the silt and the sand particles.

The above example clarifies test results such as Potyondy's (1961). (I) For an interface shear test between a 'smooth' metal surface and sand ($D_{50} = 0.6mm$), Potyondy reported interface to internal shear angle ratios of $\delta/\phi \cong 0.56$. For the same surfaces with silt ($D_{50} = 0.02mm$), the results were $\delta/\phi \cong 0.75$. (II) For 'smooth' and 'rough' concrete surfaces and the sand, Potyondy reported ratios of 0.89 and 0.98, respectively. The same 'smooth' and 'rough' surfaces with the silt resulted in ratios of 1.00 and 1.00, showing that even the 'smooth' concrete was rough enough to shear the silt.

Potyondy's results also imply that the type of material in contact (e.g. metal, concrete, wood) is less important than its surface appearance, as will be shown in Section 8.5.

(2) For a spherical soil assembly, the asperity height which will mobilize a friction along the interface of about 50% of the soil internal shear resistance will
mobilize the full strength of a soil of approximately the same average grain size consisting of elongated particles, having a ratio of long over short axis greater than 1.50.

The roughness factor, introduced in this section as part of the expansion of the S.G. model, will be further described in Chapter 8, where it is used in comparison with detailed experimental data.

6.3.3 Parameters which are determined by soil characteristics

Those controlling parameters of the interface shear resistance which are determined by the soil characteristics can be subdivided into four groups: (a) relative density; (b) appearance and composition of the particles; (c) moisture content; and (d) stress-strain behavior. Table 6.2 presents data which describe these soil parameters as they relate to granular soils and different sphere packings. The data are utilized to demonstrate the relations between the parameters and the S.G. model.

(a) Relative Density

Relative density is a reflection of unit weight, and is a useful way to characterize the density of granular soils (see comment 2, Table 6.2). It is also closely related to the stress-strain behavior, and therefore to the strength of a soil. The four modes of sphere packings described in Table 6.2 present the entire range of possible relative densities, varying from very loose to very dense.

One of the parameters of the S.G. model is the angle $\beta$, the angle of the groove. This angle decreases as the relative density increases. The maximal $\beta$ angle of $90^\circ$ can exist in the very loose and medium dense packings, when one grain ‘rides’ over another, resembling the case of 2-D rods rolling over each other. This results in the maximal resistance to sliding. As the packing density increases, the spheres are forced
into the void between neighboring spheres, and the $\beta$ angle decreases. It is important to note, however, that for well-graded material ($C_u > 4$), the distance between two spheres can decrease, resulting in a denser material with higher $\beta$ values. The overall correlation between density and $\beta$ is such that loose material has high $\beta$ values while denser, uniform (poorly graded) material has lower $\beta$ values. A well-graded dense material will have a higher $\beta$ value than poorly-graded material of the same relative density.

Based on the data of Table 6.2 and the above assertions, it may be assumed that all possible packings of granular material will lie in the range of $30^\circ \leq \beta \leq 90^\circ$, while round, poorly-graded particles will lie in the range of $45^\circ \leq \beta \leq 90^\circ$. Using Figure 6.2b, it can be concluded that when the contacting surface is smooth the influence of the soil density, as reflected by $\beta$, is small. This influence increases with an increase of $\theta$. However, for the possible $\theta$ angles (those smaller than $45^\circ + \phi/2$, as will be discussed in Chapter 8), we do not expect to see major differences in the interface shear resistance when a 'smooth' contacting body slides along loose or dense soils. The same conclusion is confirmed for 'rough' surfaces by using the relations of Figure 6.7. Moreover, when the small influence of the soil density on the interface sliding shear resistance is examined through the S.G. model, higher resistance values are predicted for the looser material ($\beta$ approaches $90^\circ$). This conclusion is confirmed experimentally, and is in agreement with Rowe's stress-dilatancy theory, as will be discussed in Sections 8.3 and 8.5.

The above conclusions are valid so long as shear does not take place in the soil. The values of $\tan\delta_s$ of the S.G. model, as presented in Table 6.2, are limited by the internal soil shear resistance ($\tan\phi'$). For that reason, the total shear resistance of very rough surfaces with dense materials is greater than that of the loose material; e.g. the orthorhombic and the rhombohedral packings have relative densities of 46% and 100% respectively. The interface friction coefficient of these materials sliding along a smooth
surface will be approximately 0.28 for the loose material and 0.18 for the dense. However, for a rough surface having a roughness angle $\alpha = 45^\circ$, shear will be initiated in the looser soil, as the expected shear resistance coefficient $\tan \theta = 0.77$ is higher than the internal shear resistance, $\tan \phi' = 0.67$. The dense material, on the other hand, exhibits a shear resistance coefficient of approximately 0.56, which will be smaller than its internal shear resistance.

The results obtained from the above example should remain qualitative only at this stage, and be restricted to the $\beta$ and $\theta$ values used in Table 6.2. As the $\beta$ angle is shown to be of less significance, it is clear that the $\theta$ values (along with the particle shape) actually determine the interface shear resistance. The influence of the angle $\theta$ will be discussed in Section 6.3.4, and the possible $\theta$ angles in Section 8.2.

(b) Appearance and Composition

Appearance

The appearance of a soil particle is determined by its geometry, surface texture, and color. The geometry of a soil particle is in turn determined by its size and shape. Particle shape can best be specified by using the concepts of sphericity and roundness as defined by Wadell (1935). Lucks (1970) further developed the evaluations of sphericity and roundness, and presented ways in which the particle shape can be measured directly by using orthogonal micro-photographs, and indirectly by using the porosity, which results from a standard technique of deposition. These methods are beyond the scope of this work, but may be useful in future developments and experimental work. Usually, the shape of the soil particles is determined visually, and is described by referring to the degree of roundness. The degree of particle roundness is divided into five classes (Pettijohn -1949); well rounded, rounded subrounded, subangular and angular. Even though the degree of roundness refers to
the sharpness of the edges and corners of a particle and, therefore, only indirectly to its shape, it is useful for the present purpose of the S.G. model. The size of the soil particles is given by grain size curves obtained through sieve analysis, or by referring to the median grain diameter (by weight) $D_{50}$, and the uniformity coefficient $C_u$.

The analysis of the S.G. model demonstrated the influence of the grain shape by considering elliptical vs. round particles. Geometrical relations control the shear resistance and the calculated interface friction coefficient. The ellipse can therefore accurately reflect the change in these relations, and can be utilized to predict the influence of the actual grain shape on the interface friction. It is assumed that calculated values using spherical vs. elliptical particles (of various degrees) will accurately predict the expected trend of experimental data. Such a comparison is presented in Chapter 8.

The shape of the grain has a major effect on the ability of the particles to roll, and influences the present analysis of friction along an interface in two closely related ways: (1) it facilitates direct predictions of the interface coefficient as suggested by the S.G. model, and (2) it affects the upper limit of possible friction along a rough surface. This value is equal to the internal shear resistance of the soil. (For the influence of particle shape on the strength of granular material, see Lucks –1970, Koerner –1970, and Section 8.2).

Most common sands are quartzic, consisting mostly of silica. Vesic and Clough (1968) investigated the behavior of a medium–grained uniform quartz sand under high stresses. The researchers found that at very low pressure (i.e. below 1 kg/cm²) there is very little crushing, as the sand particles are relatively free to move with respect to each other. As the mean normal stress increases, crushing becomes more pronounced. It appears to be most intense in the elevated pressure range (10 to 100 kg/cm²) until the so–called breakdown stress is reached, where all effects of the initial void ratio of the sand are eliminated.
Vesic and Clough monitored the particle size distribution curves prior to and after triaxial compressive tests at various confining pressures. The tested sand contained originally 40% (by weight) of particles finer than 0.4mm and about 1% finer than 0.1mm. Under confining (hydrostatic) pressure of 633 kg/cm², it was changed to 70% and 20% respectively. After test to failure under the same confining stresses, the proportions increased to 87% and 45% respectively. These results demonstrated the change in the grain size and the distribution which affects the soil's stress-strain behavior, strength and compressibility, as will be discussed later. The influence of particle crushing on the grain shape was photographed and measured by Lucks (1970). Using Krumbein's (1944) visual classification (in which roundness ranges from a minimum of 0.1 to a maximum of 0.9), Lucks found that crushed quartz had a roundness of about 0.3 compared to 0.65 for Ottawa sand. This is comparable to the angular crushed quartz particles vs. rounded Ottawa sand particles of Pettijohn's degree of roundness.

As previously discussed, geometrical relations control the calculated interface friction coefficient. The absolute size of the particles comes into consideration only when determining the degree of surface roughness (as was presented in Section 6.3.2).

Two additional parameters are factors in the appearance of a soil particle: color and surface texture. Our interest in color arises because of its relation to the type of mineral of which the grain is composed. The surface texture influences the intergranular friction coefficient, and is also related to the grain composition. The influence of the composition of the soil on the relevant parameters is discussed in the following section.

Composition

The parameters which control the interface shear resistance are affected by the mineral composition of the soil in two ways: (1) through the intergranular friction
angle and (2) through the influence of the mineral strength on the grain shape and the soil strength in relation to the confining and loading stresses.

The value of the intergranular friction angle $\phi_\mu$ depends on the nature of the mineral and the roughness of its surface (Proctor and Barton -1974). The friction of minerals has not been studied sufficiently and contradictory data exists. A quick review of some of the data is presented in the following section, and further discussion of the subject appears in Section 8.3.4.

Skinner (1969) measured the interparticle friction coefficient of spheres of different materials and sizes, dry and flooded. In all cases, an increase in the contact load resulted in an increase in the interparticle friction coefficient. The increase was especially profound in the case of the smaller and harder spheres. The following relations, which were established for dry glass ballotini (1 and 3 mm in diameter), demonstrate the phenomenon:

$$\phi_\mu^o = \tan^{-1}(\log N)/20$$

(6.15)

where:

$$N = \text{Normal contact load in grams.}$$

The flooded spheres had an interparticle friction angle of about 10 times that of the dry spheres under the same particle load.

Rowe (1962) investigated the interparticle friction angles of quartz and some sands. Rowe's results show that $\phi_\mu$ is affected by the size of the particle involved in the test. Based on this data, the relation between $\phi_\mu$ and the size of quartz particles in water was found to be:

$$\phi_\mu = 22.7^\circ - 5.16 \cdot \log d$$

(6.16)
where:

\[ d = \text{particle size in mm for } d \leq 1 \text{ mm}. \]

For particles varying in size from silt (approximately 0.02 mm) to coarse sand (approximately 0.7 mm), \( \phi_\mu \) varies between 31.5° and 23.5°. The decreasing interparticle friction coefficient (\( tg\phi_\mu \) varies from 0.61 to 0.43) with the increase in particle size implies that the particle size has some direct effect on the interparticle friction. It may be assumed that the contact force can be affected by the particle size. However, Rowe reported that the friction angle \( \phi_\mu \) remained constant within ± 1° as the pressure ranged from 2 to 100 psi. Tests of three different sands led to results similar to those obtained for the quartz particles. Sands varying from medium to fine graded and coarse sands resulted in \( \phi_\mu \) values from 26° to 23°, which correspond to a friction coefficient variation of 0.49 to 0.42, respectively.

Lambe and Whitman (1969) summarized data obtained by Bromwell (1966) and Dickey (1966) for quartz under varying conditions of surface cleanliness, humidity and surface roughness. They concluded that as the surfaces roughness increases, the effect of the cleaning procedure on the interparticle friction decreases. 'Rough' surfaces of 60 \( \mu \) inch give the same value of \( tg\phi_\mu \) \( (tg \ 26^\circ = 0.5) \), independent of surface cleanliness. They further concluded that since all quartz particles in natural soils have rough surfaces, the interparticle friction coefficient of quartz is essentially constant and equal to 0.50.

Proctor and Barton (1974) investigated the interparticle friction coefficient of quartz based on particle-to-particle contact and particle-to-plane contact, under saturated and dry conditions. A comparison between their results and data compiled by them from other sources suggested the following:

1. The values of \( \phi_\mu \) measured at a single contact point are in acceptable agreement (within limits of experimental error) with the values obtained in
other studies.

(2) The reliability of the values obtained under ‘dry’ conditions are questionable, especially if the surfaces are smooth.

(3) The techniques which employ the measurements of $\phi_{\mu}$ for a mass of particles are preferable (especially the method described by Rowe –1962).

An attempt has been made to explain the relation between the grain size and the interparticle stress to $\phi_{\mu}$, using the Hertz theory (see Richart et al. –1970). No clear correlation between this approach and experimental data has been found. Section 8.3.4 (e) presents additional data from Koerner (1970) which suggest that the increase of $\phi_{\mu}$ with the decrease in particle size is due to the rapid increase of the surface area in the fine material. At the present stage, however, a look at the entire range of possible $\phi_{\mu}$ values leads to some practical conclusions regarding $\phi_{\mu}$ and the analysis of the S.G. model.

The above conclusions of Proctor and Barton confirm the observations of Lambe and Whitman. From a practical viewpoint, the range of $21^\circ \leq \phi_{\mu} \leq 31^\circ$ can be accepted for quartz particles, dry (naturally dry) or saturated. This range includes the effect of grain size and contact stresses, as well as the data obtained and compiled by Proctor and Barton. The corresponding interparticle friction coefficient varies between 0.38 and 0.60 with a representative value of 0.5 ($\pm$ 0.1), which matches the value suggested by Lambe and Whitman. These values refer to actual measurements which require movement between particles or between particles and plane. The stated $\phi_{\mu}$ values refer to kinetic friction, and therefore are in the lower range of values when related to the static friction, which is considered in the limiting equilibrium state. (For a possible reason why dynamic friction is lower than static friction, see Bowden and Tabor –1945, Proctor and Barton –1974).

The effect of the intergranular friction coefficient on the analysis of the S.G.
model (for spherical particles) is demonstrated in Figure 6.2a (based on Equation 6.4). It was previously argued that $45^\circ \leq \beta \leq 90^\circ$ and $\theta < 60^\circ$. Based on Fig. 6.2a, it is obvious that for this range of values, an interparticle friction coefficient of $\tan \phi \mu \approx 0.5$ will define the conditions of mostly rolling for the spherical particles. The effect of the intergranular friction coefficient on the analysis of the S.G. model can be further assessed by examining the rolling conditions of non-spherical particles, as shown in Figure 6.2c (based on Equation 6.8). Fig. 6.2c is drawn for $\beta = 90^\circ$, and therefore represents the upper possible limit of impedance to motion. Considering $\theta = 30^\circ$, an interparticle friction coefficient of $\tan \phi \mu = 0.5$ will define the conditions for rolling of ellipsoids (with $R_a = 1.25$). If particles of greater angularity are considered for conditions of $\beta < 90^\circ$ and $\theta > 30^\circ$, the average (dynamic) value of $\tan \phi \mu = 0.5$, which was previously obtained, would define the condition of rolling for most of the particles.

Based on the S.G. model, it can therefore be concluded that the existing interparticle friction coefficient of quartzic sands indicates that rolling is the dominant interparticle motion. Determination of the mode of particle rolling during shear requires additional data, which are presented in Section 8.3.

McClelland (1974) brought examples of carbonate sands which consist of ooliths (rounded and highly polished particles of calcium carbonate) in the fine to medium size range. Calcium carbonate has a mineral hardness of 3 as compared to 7 for silica sand. Such sands have smaller interparticle friction coefficients, break and crush under smaller confining stresses, and present substantially lower internal friction resistances.

(c) **Moisture content**

The moisture content of the soil was recognized as an experimental factor in tests of interparticle friction coefficients (e.g. Skinner –1969) and in direct shear of interfaces (e.g. Potyondy –1961).
It was previously shown that flooded glass spheres have an interparticle friction coefficient about ten times greater than that of dry ones. The 'moisture influence' on the interparticle friction is controlled by the surface roughness. The surface roughness of natural sands was found to be sufficiently high so that no significant influence of the moisture on $\phi$ values could be detected. For that reason, the range of interparticle friction coefficients for quartzic sand was taken to be valid for sands naturally dried to flooded.

The overall interface shear resistance can be affected through the influence of pore pressures on the forces described by the S.G. model. Shear of interface in undrained conditions would result in a build up of pore pressure (see e.g. Moller and Bergdahl –1981, for dynamic pore pressure during pile driving in fine sand). An unknown pore pressure would invalidate the S.G. model, which is based on effective forces acting between the grains and between the grain and the plane (forces $N$ and $P$ of Figure 6.1). It would also lead to unknown shear resistance of the soil.

The results of the S.G. model are therefore relevant in drained conditions only.

(d) Stress–Strain Behavior

Shear of soil along the interface and/or within the soil mass is affected by the soil stress–strain behavior through: (1) volume change (especially dilatancy) and (2) shear strength.

A detailed analysis accounting for these factors must be related to a specific problem. The stress state within a soil plug is analyzed in Chapter 7. Some factors concerning the stress–strain behavior of granular materials, the S.G. model, and the interface mechanism are discussed briefly.

Due to the displacement of the contacting body, dilation and/or high stresses can develop along the interface (e.g. passive stresses due to wall movement, dilation due to pile penetration). The magnitude of the stresses has no relation to the S.G.
model; however, the stresses determine the friction resistance and its ultimate value, the shear strength of the soil.

Vesic and Clough (1968) investigated the behavior of granular materials under high stresses. They found that in the elevated pressure range (10 to 100 kg/cm\(^2\), as previously discussed) where crushing of sand grains appears to be the most intense, the strength envelopes are all curved, and the secant angle of shear resistance (\(\phi_s\)) in this range decreases in inverse proportion to the logarithm of the mean normal stress until it reaches the value of the angle of interparticle friction at the breakdown stress. This enormous reduction in the shear strength of the soil has a major effect on the maximum possible friction along the interface. Accounting for this effect in the analysis of friction along a pile shaft explains the limiting value of skin (and end bearing) capacity with depth, as was demonstrated by Zeitlen and Paikowsky (1982).

Under the high confining stresses, when crushing becomes pronounced, the dilatancy effect gradually disappears (Vesic and Clough - 1968). Under the low confining stresses (less than 10 kg/cm\(^2\)) which prevail in most shear interface tests, conditions which lead to shear of the soil would be accompanied by dilatancy and increase of the normal stresses along the interface. The relation between the normal stresses on the interface and the maximum possible soil shear resistance is discussed in detail in Chapter 7.

6.3.4 Parameters which are determined by the interface conditions.

The interface is formed by the soil and the contacting body. Parameters which are controlled by the qualities of either one were discussed in the previous sections. The parameters which are determined mutually (by the conditions at the interface), namely the stresses and the rate of shear, will be discussed herein.

(a) Stresses
It was originally assumed (see Section 6.2.2a — the underlying concept of the S.G. model) that the direction of the groove will be normal to the major principal stress, coinciding with the direction of the force normal to the contact for the case of $\beta = 90^\circ$. This assumption was based on the data presented in Chapter 5. Soil particles were found to rotate such that the maximum number of contacts (and maximum area at each contact) will be developed normal to the major principal stress orientation. This phenomenon led to 'paths' of contacts which followed the principal stress trajectory, reoriented themselves under stress rotation, and were dispersed following shear.

The analysis of the S.G. model is undertaken for a single soil particle. However, by using parameters which reflect the state of an average particle, a representation of the entire soil layer along the interface is obtained. A detailed analysis of a particular packing of particles would lack the ability to truly represent the average contact orientation ($\theta$). The $\theta$ angles in Table 6.2 are based on the analyzed models and their assumptions. These angles indicate possible contact orientations which fall within the expected range, as will be shown later.

In order to obtain the best representative direction of the groove $\theta$ (which determines the direction of the interparticle force $N$ as well as the contact force with the plane $P$), an analysis based on continuum mechanics should be performed. Such an analysis should consider: (1) the boundary conditions of the specific problem (i.e. stress and geometry), and (2) the possible behavior of the particulate media (i.e. the systematic arching theory, see Section 5.4). It would then show the possible stress trajectories.

This analysis is performed in Chapter 7 for the problem of the inner soil plug, and for a general assessment of the $\theta$ angle in Section 8.2. The S.G. model does not depend on the absolute magnitude of the stresses, and requires the determination of the
stress orientation only. However, for the complete solution of the problem both stress directions and magnitudes are required.

In summary, the stress state of a chosen analyzed problem will determine the principal stress trajectories, and therefore the stresses acting on the interface. The orientation of these stresses, as determined from an analysis based on continuum mechanics of the chosen problem, should be used in order to assess the interface shear resistance as obtained by the S.G. model.

(b) Rate of Shear

The S.G. model is based on the analysis of a rigid massless sphere under limiting static equilibrium conditions. It should be applicable to the peak 'static' resistance prior to displacement.

The effect of shearing rate on the soil interface behavior was explored by Lemos (1986), Desai et al. (1985), and others. Usually, those tests were carried out by applying different displacement rates to the contacting body and measuring their influence on the shear resistance. Brumund and Leonards (1973) measured the interface shear resistance between soil and a rod subjected to an impact load.

The applicability of the interface friction coefficient as obtained from the S.G. model to the different shear rates should be examined.

As previously noted, the interparticle friction coefficient is measured during relative displacements. Therefore, it is smaller than the 'peak' static coefficient. However, both indicate interparticle rolling within quartzic sand. As long as the contact body slides over the soil grains and there is no shear in the soil, the S.G. model

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1The fact that the normal force at the contact (\(N_{xyz}\)) differs from the normal force to the groove (\(N_{yz}\) in the direction of \(\sigma_1\)), was already considered in the analysis of the S.G. model (see Section 6.2.2c)
is applicable to any displacement rate of the contacting body, as its motion does not induce inertia forces in the soil mass. Some modifications of the values obtained from the S.G. model are required when the indirect effect of the displacement and the normal confining stress is considered.

Under low stresses (below 1 kg/cm², as noted in Section 6.3.3b), some rearrangement of soil particles is expected to take place with the movement of the contacting body. Such rearrangement leads to soil densification, which is expressed by smaller β values of the S.G. model. This indicates a reduction of the interface friction coefficient τgδs with the displacement. Under higher pressures, and especially in the range where dilatancy is constrained (above 10 kg/cm²), the friction coefficient will not peak and τgδs may remain constant or increase under displacement. The increase may occur due to crushing of particles (see Section 6.3.3b), which leads to better-graded material. As explained in Section 6.3.3a, a better-graded material would indicate higher β values, leading to higher interface friction coefficients. In both cases, the expected decrease or increase in the values of τgδs due to movement are restricted by the influence of a small possible change in the value of β.

The above discussion holds even for the case in which the contacting body accelerates as a rigid body. However, under impact, in which a wave propagates along the contacting body (e.g. penetration of a pile wall during driving), the use of the S.G. model may not be applicable. Under such conditions, the soil may be strained due to propagation of shear waves for which the analysis of a single grain may be limited.

Chapter 8 examines the different parameters of the S.G. model in light of experimental results, and contains further discussion of some of the above subjects.
6.4 SUMMARY AND CONCLUSIONS OF THE S.G. MODEL

1. A mechanical (S.G. – Sphere in the Groove) model of a single particle which considers the micro behavior of granular material (Chapter 5) is introduced. The S.G. model is currently the only available tool for modeling the mechanism of granular soil/interface shear resistance.

2. The S.G. model examines a single rigid soil grain (sphere or ellipse) supported by adjacent soil particles (frictional groove) and subjected at an additional contact point to normal and traction forces. These forces can result from a moving frictional plane (modeling the body in contact along an interface) or from the motion of an adjacent soil particle (modeling conditions within granular material).

3. Based on observations of granular material behavior, the S.G. model assumes that the grain will roll or slide along the groove between the supporting particles.

4. A limiting equilibrium analysis is applied to the interface/interparticle contacts, examining the possibility of sliding or rolling in both locations.

5. The following parameters are considered in the S.G. model:

   \[ \beta \] – Angle of the groove (measured from center line to wall).

   Determined by the soil density and grading. Can be evaluated through different uniform sphere packings \((45^\circ \leq \beta \leq 90^\circ)\) and/or experimental data.

   - \(\beta\) approaches \(90^\circ\) for loose material.
   - \(\beta\) decreases for denser material.
   - \(\beta\) seems to increase for a better–graded material compared to a poorly–graded one of the same density.
   - \(\beta\) can be less than \(45^\circ\) for non–spherical dense material.
\( \theta \) — Inclination of the groove (measured on the groove plane of symmetry, between the direction of the contacting body movement and the plane tangential to the interparticle contact). \( \theta \) may be determined in several ways (to be investigated in Section 8.2):

- by determining the major principal stress trajectory through evaluation of the state of stress of the considered problem and/or experimental results
- by referring to the minimum energy criterion to obtain preferred contact orientation
- in general; \( 45^\circ - \phi / 2 \leq \theta \leq 45^\circ + \phi / 2 \)

\( l, s, R_a \) — Dimensions of the grain, considering the grain shape. Longer and shorter ellipse axes and their ratio \( R_a = l/s \). For spheres, \( R_a = 1 \).

\( \alpha \) — Angle of roughness (measured between the direction of the contacting body movement and the tangent to the contact between the plane and the grain). It is determined by the roughness of the contacting body surface, and can be translated into asperities in relation to the grain size \( (R_n = \text{normalized}) \) roughness; Eq. 6.11).

6. Equation 6.14 describes the boundary between sliding and rolling at the interparticle contact \( \text{[tg} \phi_\mu = f(R_a, \theta, \beta) \text{]} \). When the actual intergranular friction is equal to the calculated one, sliding takes place along the interparticle contact (the groove). When the actual intergranular friction is greater than that calculated by Eq. 6.14 for the given conditions, only rolling
is possible at the interparticle contact.

7. A review of the actual interparticle friction angle of quartzic sand suggests that a practical range of $21^\circ \leq \phi_{\mu} \leq 31^\circ$ can be accepted, with a representative intergranular friction coefficient of $tg\phi_{\mu} = 0.5$. Additional information is required to examine possible effects of normal force and/or grain size. The subject is further considered in Chapter 8.

8. Measured conditions of soil shear are used for comparing the results of Equation 6.14 and the actual interparticle friction. These analyses (conducted in Chapter 8) and some simplified examples in the present chapter suggest that:

- prior to shear, the actual interparticle friction is greater than the calculated sliding/rolling boundary of Eq. 6.14, producing the conditions of soil rolling in all dominant zones of stress/deformation concentrations under the particular loading state.

- during shear, the dominant zone includes some sliding of particles, which provides most of the shear resistance. These particles need not be at the same location at the same time.

9. Equation 6.13 describes the boundary between sliding and rolling at the interface contact [$tg\delta_s = f(R_a, \theta, \beta, \alpha)$]. As long as the actual interparticle friction coefficient is greater than that calculated by Eq. 6.14, the interface friction coefficient of Eq. 6.13 ($tg\delta_s$) describes the sliding resistance of a body in contact with granular material along the interface. For the conditions where the actual $tg\phi_{\mu}$ equals that calculated by Eq. 6.14 the value $tg\delta_s$ should be equal to the shear resistance of the soil, $tg\phi$. The value of $tg\delta_s$ in Eq. 6.13 is limited, therefore, by the shear strength of the soil, $tg\phi$. 
GRANULAR-SOIL/INTERFACE SHEAR RESISTANCE

The Micro Approach to the Behavior of Granular Material
Chapter 5

1. The individual grains rotate and shift to the most stable condition as a reaction to the applied load. This movement is achieved mainly by rolling of the grains.
2. The preferred arrangement is obtained by orientation of the long axes of the particles parallel to the direction of the major principal stress. As a result, a concentration of contacts in the preferred direction forms a column-like load path which transfers the increasing axial stress along its trajectory.
3. The study of an ideal rigid disc model using simplified arching approach, provided a very good fit to the silo theory. The model explains the physical phenomena and the behavior of the granular material in a silo, and is in agreement with the aforementioned behavior of granular material under loading.

The Sphere in the Groove (S.G.) Model
Section 6.2

1. Determination of the limiting equilibrium for rolling/sliding of particles along the intergranular and interface planes.
2. Parameters: \( \beta, \phi, R_a, \alpha, \theta \)
3. The S.G. model predicts the interface sliding coefficient \( (t_g) \), limited by a maximum value corresponding to initiation of shear in the soil.

The Parameters of the S.G. Model
Sections 6.3, 8.2

Soil Characteristics
- \( \beta \) - Angle of the groove. Determined by the soil density and gradation. Evaluated through different sphere packings.
- \( \phi \) - Interparticle friction angle. Determined by the type of granular material.
- \( R_a \) - The shape of the grain. Determined by the equivalent longer and shorter ellipse axes \( (R_a = 1/\varepsilon, \text{ for spheres } R_a = 1) \)

Contacting Body
- \( \alpha \) - Angle of roughness. Determined by the roughness of the contacting body surface. Expressed by the normalized roughness \( (R_n) \), for grain size consideration.

Interface Conditions:
- \( \theta \) - Inclination of the groove. Determined by: (a) Inclination of the intergranular plane based on minimum energy approach. Enables evaluation of the minimal sliding resistance. (b) Major principal stress trajectory based on continuum approach. Enables evaluation of the average groove inclination for the state of stress of the considered problem.

The Stress State of the Inner-Soil Plug
Chapter 7

1. Using the 'soil-arching' approach, the possible principal stress trajectories become continuous curved lines in the shape of a catenary, which may be simplified to segments of a circle.
2. The major principal stress \( (\sigma_1) \) orientation is described by the angle \( \psi \).
3. \( \psi = \hat{f}(\phi, \sigma_1) \)
4. \( K = \hat{f}(\phi, \sigma_1) \); the ratio between the horizontal and vertical stresses along the interface \( (K_i = \sigma_1/\sigma_v) \).
5. \( 45^\circ < \psi < 90^\circ \) : 
   - \( \delta = 0, \psi = 90^\circ \); \( K_i = K_0 = \tan(45^\circ - \phi/2) \)
   - \( \delta = \phi, \psi = 45^\circ + \phi/2 \); \( K_i = 1/(1 + 2\tan^2 \phi) \)
   - \( \psi = 45^\circ \) \( \Rightarrow K_i = 1 \)
6. \( 0^\circ < \psi < 45^\circ \) : 
   - \( \delta = 0, \psi = 0 \); \( K_i = K_0 = \tan(45^\circ + \phi/2) \)
   - \( \delta = \delta_p(\phi), \psi = 45^\circ - \phi/2 \); \( K_i = 1 + 2\tan^2 \phi \)
TABLE 6.2 Parameters for sphere packings and granular soil

1. $D_R$ was calculated using $e_{\text{max}} = 0.91$ and $e_{\text{min}} = 0.35$

2. $\varphi_d$ was based on $G_s = 2.65$

3. For $\beta = 90^\circ$ there is only one interparticle contact point at $G$. The two $\beta$ values of $45^\circ$ and $60^\circ$ for the F.C.C. assembly refer to the analyses of Rowe (1962) and Scott (1963), respectively. According to Rowe (1964), this value respresents possible lower energy dissipation and therefore is the ‘right’ angle.

4. When $R_a \neq 1.0$ the particles are nonspherical, and the description of the sphere packings is used for comparison only. The values for $\beta \neq 90^\circ$ were obtained by using Equation 6.12b. The numbers in parameters refers to $\beta = 90^\circ$.

5. Based on different correlations between $D_R$ and $\phi'$ (Paikowsky –1982), compared to representative values of $\phi'$ as suggested by Dimtiz (1972) for different $\varphi_d$ values.

6. Based on the average of several tests.
TABLE 6.2 Parameters for sphere packings and granular soil

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Note: The table continues with more parameters and values not visible in the image.
The Sphere in the Groove (S.G) Model: (a) An Isometric Presentation with Angles Detailed (b) A Sphere Between Two Planes ($\beta=90^\circ$) (c) A Sphere in a Groove ($\beta\neq90^\circ$).
Results of the Sphere in the Groove Model for Limiting Equilibria Conditions of:

(a) The Interparticle Friction
(b) The Interface Friction
(c) The Interface and the Interparticle Friction for \( \beta = 90^\circ \), Considering Elongated Particles.
**SPHERE**

- \( c = r(1 - \sin \beta) \)
- \( r - c = rsin \beta \)
- \( a = r(\sin \beta + \cos \theta) \)
- \( b = rsin \theta \)
- \( d = rsin \beta \sin \theta \)
- \( e = r(1 + \sin \beta \cos \theta) \)

**ELLIPSE**

- \( a_e = \frac{s}{\sqrt{R_a^2 \tan^2 \theta + 1}} - s \)
- \( b_e = \frac{1R_a \tan \theta}{\sqrt{R_a^2 \tan^2 \theta + 1}} \)

For a sphere: \( R_a = 1, r = s = l \) and \( a_e = a, b_e = b \)

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Fig. 6.3: Geometrical Relations for the Sphere in the Groove Model (Smooth Surface).
Extreme Conditions for the Sphere in the Groove (a) The Groove Normal to the Interface where No Rolling can take place (b) The Groove Parallel to the Interface where No Sliding can take place.

The Sphere in the Groove Model: Details of Force and Geometrical Relations for the Analysis of Moment Equilibrium around the Contact Points C.

Details of Forces and Geometrical Relations for an Elliptical Particle Between Two Planes ($\beta=90^\circ$).
Fig. 6.7:
(a) Profile of 'Smooth' Quartz Surface Traces. Scale: Vertical, 1 Division=2x10^-5 inch.; Horizontal, 1 division=2x10^-5 inch (from Dickey -1966)
(b) The Profile Described in (a) for Vertical = Horizontal Scale.
(c) Example of Profile of 'Rough' Surface in which the Vertical and the Horizontal Scales are Equal.
The Sphere in the Groove Model for Rough Surfaces.

(a) Geometrical Relations for Moment Equilibrium of a Sphere
(b) Evaluation of the Asperity Height.
(c) Geometrical Relations for Moment Equilibrium of an Ellipse

\[ e = r + rsin\beta\cos(\theta + \alpha) \]
\[ d = rsin\beta\sin(\theta + \alpha) \]
\[ c = r(1 - \sin\beta) \]

\[ h = \frac{d}{2} (1 - \cos\alpha) \]

**Fig. 6.8:** The Sphere in the Groove Model for Rough Surfaces.
(a) Geometrical Relations for Moment Equilibrium of a Sphere
(b) Evaluation of the Asperity Height.
(c) Geometrical Relations for Moment Equilibrium of an Ellipse
Fig. 6.9: Results of the Sphere in the Groove Model for Limiting Equilibria Conditions of the Interface Friction, Considering the Surface Roughness of the Contacting Body along the Interface.
CHAPTER 7
THE STRESS STATE OF THE INNER SOIL PLUG

7.1 INTRODUCTION

The behavior of a soil plug under static loads was analyzed using the ‘silo approach’ in Chapter 4. A review of the underlying assumptions and the inherent simplifications of this analysis was presented in Section 4.3. The results of comparisons between the silo approach analysis and experimental data were discussed in Section 4.8.

The assumptions of the silo analysis are reconsidered in light of the experimental data and the previously developed model for granular soil/interface friction resistance.

(a) Concerning the interface friction coefficient:

The roughness of the surface along a pipe pile is assumed to be constant. The density of the inner sand changes, due to plug compaction. However, based on the data of Chapter 3 and the discussion in Section 6.3.3, the changes in the relative density do not significantly influence the interface friction. For a given sand type and surface roughness, the interface friction coefficient will therefore be influenced mainly by the orientation of the principal stresses.

(b) Concerning the coefficient of lateral stress:

The assumption of a constant K (z and r independent) along the soil plug, leads to constant horizontal and vertical stresses across any given depth. The following observations can be made regarding this assumption:

1. The axisymmetric condition in which the soil cylinder exists determines that the horizontal and vertical stresses along the center line ($\sigma_h/r = 0$, $\sigma_z/r = 0$) are principal stresses.

2. The horizontal stresses acting normal to the pile wall cannot be principal
stresses (by definition), due to the soil/pile wall frictional stresses.

3. Consequently, the direction of the principal stresses rotates and the magnitudes of the horizontal and vertical stresses change along any horizontal cross-section of the soil plug as one moves outward from the center line to the pile wall.

4. Analyses of experimental data using the silo approach have led to $K$ values which depend on the pile diameter (see Figure 4.10).

(c) Concerning the material qualities:

No assumptions were made concerning the material qualities other than those inherent in the above two assumptions. Therefore, the material may be treated either as a continuum, for which elastic or plastic behavior can be assumed, or as a particulate media, where the arching mechanism can be applied. The results of each assumption should be reviewed in light of experimental data.

The stress state of the inner soil plug must, therefore, be investigated. To this end, elastic and plastic solutions are briefly reviewed. Taking these solutions into consideration, along with the granular material behavior (see Section 5.4), stress trajectories are suggested. These trajectories are then used to calculate the stress state of the soil plug.
7.2 ELASTIC SOLUTION

Major simplifications must be made in order to obtain a closed form solution. The inner soil is considered as an isotropic circular cylinder, subjected to the following conditions (see Figure 7.1a):

1. All-around uniform lateral pressure \( \tilde{\omega} \)
2. A uniform pressure \( p_1 \) at one end (\( R = 0 \)) and \( p_2 \) at the other end (\( R = L \))
3. Body forces are omitted
4. No torsion or rotation takes place
5. A constant coefficient of friction (\( f \)) is assumed along the boundary.

The problem differs from plane elasticity by the fact that certain shearing stresses do not vanish, and an antiplane solution is required. Milne–Thomson (1962) offers a solution to this problem (using Filon –1937). The lines of principal stress are described by:

\[
\rho^{3/2} \cos^3\theta = \text{constant} \quad \text{(see Figure 7.1b)}
\]

where: \( \rho, \theta \) — a radius vector with origin at \( J \) \((0,0,R_o)\).

Regarding size and shape, the lines are independent of \( p_1, p_2 \) and \( \tilde{\omega} \). However, their positions depend on the isotropic point \( J \) (where \( \sigma_1 = \sigma_2 = \sigma_3 \)). The point \( J \) is inside the cylinder for \( p_1 > \tilde{\omega} > p_2 \). In the case of the soil plug, \( p_2 = 0 \). However, the condition of \( p_1 > \tilde{\omega} \) guarantees that \( K < 1 \). The lines of the above equation are plotted in Figure 7.1c. The solution of the stresses in the cylinder can be used to determine the location of the greatest stress difference as a criterion for failure. The difference between the principal stresses is: \( \sigma_1 - \sigma_2 = 2\tilde{\omega}\cdot\rho/a \). This means that the largest stress difference is first expected at the location where \( \rho \) (the distance from \( J \)) is the greatest. That occurs at points on the circumference where \( p_1 \) is applied (i.e. the bottom of the soil plug will yield first).
7.3 PLASTIC SOLUTION

Sokolovski’s (1960) solutions of the equilibrium and critical state equations in the critical zone allow construction of the slip surfaces. Considering plane strain problems, the lines of slip form a system of two families which are generally non-orthogonal. The tangents to the slip lines are inclined to the principal axes at the angles $\pm 45 - \phi / 2$. Fig. 7.2 shows the slip lines of a massless medium pushed between two walls. The medium is cohesionless with an internal friction angle $\phi = 20^\circ$ and interface friction angle $\delta = 15^\circ$. Based on the slip lines in the inner field (above the active wedge), a trajectory of the minor principal stress was constructed.

Generically, this trajectory is similar to the one obtained in Fig. 7.2 by the elastic solution. Both trajectories are consistent with:

(1) K values smaller than 1, and

(2) assumptions leading to a shape which has no special relevance to the analyzed particulate media, but is applicable to any frictional continuum under elastic or plastic states.

---

There are some fundamental differences between the plane strain and the axisymmetric cases. For the present example, based on the analysis of Sec. 5.4.4c, it can be assumed that the trajectories under static loading will be identical for both cases.
THE ‘SOIL ARCHING’ APPROACH

7.4.1 Underlying Concept

The analysis of the ‘silo approach’ in Chapter 4 led to the conclusion that the so-called ‘arching effect’ controls the mechanism of load transfer in the silo. This conclusion was supported by the micro behavior of the granular material, as revealed by the measurements of Chapter 3, and in Chapter 5. A detailed analysis of the silo problem using the ‘systematic arching theory’ (Section 5.4.1), applied to a disc model, provided a very good fit to the silo analysis and experimental data. Based on the previous discussion and solutions, the stress trajectories are arches connecting the center line to the soil/plug interface.

The following developments discuss the different possibilities and their consequences, assisted by previous works of Krynine (1945), Stefanoff and Boshinov (1977) and Handy (1985). Table 7.1 summarizes this information.

7.4.2 The Possible Trajectories

Two possible relative soil/wall movements can take place: (I) ‘Active’ (associated with ‘active arching’), in which the soil settles in respect to the walls. Examples are soil between retaining walls and backfill of trench excavation, i.e., the ‘standard’ silo case. (II) ‘Passive’, in which the walls move downwards in respect to the soil (or the soil is pushed upwards), e.g. open pile penetration.

Two possible continuous arch trajectories for the principal stresses are presented in Figure 7.3a. The stresses acting on the soil and pile interface elements are described by the two free body diagrams denoted by B.

The pile moves downwards in respect to the soil. The shear stresses are

---

2The subsequent developments concern the stress ratio and the principal stress rotation at the interface only. For the sake of a simplified presentation, the shape of the trajectory is taken as a segment of a circle having its center along the center–line. The physical requirements and the consequent expected trajectory shape are discussed in Section 7.5.
therefore acting downwards on the soil and upwards on the pile. Mohr circle
description of the state of stresses at a point along the interface is shown in Figure
7.3c. Knowing the shear stress direction at the interface, and assuming (for the
demonstration only) a ‘rough’ surface along this plane (\(\delta = \phi\)), the location of the pole
may be determined at the Mohr circle (P — origin of planes). The directions of the
principal stresses at the interface are then identified. These directions lead to the two
possible stress trajectories of Figure 7.3. Assuming a vertical direction for the major
principal stress along the center line, the trajectory of the minor principal stress is then
convex (upwards), similar to those in Figs. 7.1 and 7.2. When the major principal
stress along the center line is assumed to be in the horizontal direction, the trajectory
of the major principal stress is concave (downward).

Upwards shear stresses act on the soil in the ‘active’ arching condition,
resulting in a convex major principal stress trajectory (identical to the minor principal
stress trajectory of Figure 7.3). Based on the granular material behavior as described
in Chapter 5, it is assumed that a ‘path’ of contacts will be created in this direction,
‘supporting’ the load above it. This is actually the mechanism which develops in silos,
as shown in the radiograph of Figure 5.14.

In the condition of pile plugging shown in Figure 7.3, the soil is being "pushed
upwards" in a ‘passive arching’ mode, in which a supporting arch made of grain
contacts is oriented concavely (downwards), in the major principal stress direction.
This is supported by the arch shapes formed in an overdriven sampler (Figs. 3.25 a,b,
3.27 a, b, and 3.28a) or under closed—ended piles (Figs. 3.28, 3.29).

Both possible trajectories of Figure 7.3 provide the stress state at the
boundaries, and they ‘complement’ each other, as will be shown. A general solution for
the state of stress at the boundaries is developed as a function of the ‘arching
angle\(^3\). It should be noted that the analyzed arches are actually a cross-section of a ‘spherical cap’ (as the problem of the soil plug is axisymmetric), shown as a segment of a circle.

7.4.3 The Stress State Along the Interface

The normal and shear stresses along the interface \((\sigma_i, r_i)\) can be found from force equilibrium on a boundary triangular element, as shown in Figure 7.4:

\[
\sigma_{hi} = \sigma_3 \cdot \sin^2 \psi + \sigma_1 \cdot \cos^2 \psi \tag{7.1}
\]

\[
r_i = (\sigma_1 - \sigma_3) \cdot \sin \psi \cdot \cos \psi \tag{7.2}
\]

The ratio between the horizontal (normal) stress at the interface and the major principal stress is:

\[
\frac{\sigma_{hi}}{\sigma_1} = \frac{\sigma_3}{\sigma_1} \cdot \sin^2 \psi + \cos^2 \psi \tag{7.3}
\]

As identical shear stresses are acting on normal planes (from equilibrium requirement), Mohr’s circle symmetry can be used: \(\sigma_1 - \sigma_v = \sigma_h - \sigma_3\). Note that this equation also holds for \(\psi < (45 - \phi/2)\), where \(\sigma_h > \sigma_v\) and \(\sigma_1 - \sigma_h = \sigma_v - \sigma_3\). Substituting for \(\sigma_h\) in Equation 7.3 gives:

\[
\frac{\sigma_v}{\sigma_1} = \frac{\sigma_3}{\sigma_1} \cdot \cos^2 \psi + \sin^2 \psi \tag{7.4}
\]

\(^3\)The arching angle \(\psi\) generally varies, and is used as a variable in the analysis of Chapter 9. In the following developments, we are interested in its value along the interface, which is referred to as \(\psi\) (denoted as \(\psi_i\) in Chapter 9, where \(\psi\) becomes variable).
Dividing Equation 7.3 by Equation 7.4 leads to $K_i$, the ratio between the horizontal to the vertical stresses at the interface:

$$K_i = \frac{\sigma_{hi}}{\sigma_{vi}} = \frac{\sigma_3 \cdot \sin^2 \psi + \cos^2 \psi}{\sigma_1 \cdot \cos^2 \psi + \sin^2 \psi} \quad (7.5)$$

Equation (7.5) is a general equation which describes the ratio between horizontal to vertical stresses as a function of the principal stress rotation, expressed through the angle $\psi$. The angle $\psi$ is measured between the horizontal plane and the plane on which the minor principal stress acts, or between the vertical plane and the plane on which the major principal stress acts. It should be emphasized that Equation 7.5 is valid also for the conditions prior to failure, as long as the pile is displaced downwards in relation to the soil. The previous assumption which associated the failure plane with the pile–soil interface was made for the illustration, as we are usually interested in that condition.

For frictional soil (where the internal friction angle $\phi$ is known) and Mohr–Coulomb failure criterion, the ratio between the principal stresses at failure is given by: $\frac{\sigma_1}{\sigma_3} = K_p = \tan^2(45^\circ + \phi/2)$ or $\frac{\sigma_3}{\sigma_1} = K_a = \frac{1}{K_p} = \tan^2(45^\circ - \phi/2)$. Substituting these relations into Equation 7.5 gives:

$$K_i = \frac{K_a \cdot \sin^2 \psi + \cos^2 \psi}{K_a \cdot \cos^2 \psi + \sin^2 \psi} \quad (7.6)$$

Equation 7.6 holds for any possible $\psi \left(0^\circ \leq \psi \leq 90^\circ\right)$. Examination of Equation 7.6 and the corresponding trajectories in the permissible range of $\psi$ is presented in Figure 7.5.

1. For $\psi = 90^\circ$, $K_i = K_a$. This means that we have a horizontal $\sigma_3$ trajectory
for the state of stresses in which the major principal stress is vertical. This situation could exist if there was no friction at the boundaries. The complementary trajectory ($\sigma_1$) is a half circle with a diameter equal to that of the plug. The minor principal stress (vertical at C.L.) becomes the normal stress at the interface. The material then must allow a complete rotation of the principal stresses ($90^\circ$) with frictionless boundaries, a non–realistic requirement.

2. An ideal sampler penetration is assumed, in which the normally consolidated sand remains untouched. Using Equation 7.6 for $K_i \equiv K_0 \equiv 1 - \sin \phi$ for $\phi = 30^\circ$ leads to $\psi = 60^\circ$. A moderate trajectory with a radius of $R \equiv 1.2 \cdot ID$ is obtained. The complementary trajectory (for the state of stress in which the horizontal stress at the center line is the major principal stress) has a substantial curvature with a radius of $R \equiv 0.55 \cdot ID$.

3. In a transition case in which $\psi = 45^\circ$, the vertical and horizontal stresses at the boundary are identical ($K_i = 1$); both trajectories have the same radius, $R = ID/2$. This state will be further discussed in the following section (7.4.4).

4. For $\psi$ angles smaller than $45^\circ$, $K_i > 1$. For example, for $K_i = 2$, $\psi = 23.2^\circ$. The trajectory which describes the stress variation for the case of horizontal major principal stress at the center line requires a moderate curvature with $R = 1.2 \cdot ID$. Its complement requires then a substantial curvature of $R \equiv 0.55 \cdot ID$.

5. For $\psi = 0^\circ$, $K_i = K_p$ and we have a horizontal $\sigma_1$ trajectory. This could have taken place if there was no friction at the boundary. Due to frictionless boundaries, however, a vertical load would not have been able to impose a state of stress in which the major principal stress at the center line is horizontal. The complementary trajectory is a half circle, as in the case of $\psi = 90^\circ$. 
7.4.4 Possible Stress Conditions in the Soil Plug

The values of $K_i$, $\psi$, and their associated trajectories presented the possible range of the stress conditions. There is a need, however, to examine the validity of the different conditions and the mode of failure which takes place while the soil is displaced in relation to the pile (see Table 7.1 for clarification).

(a) $45^\circ < \psi \leq 90^\circ$; $K_i < 1$

The expected natural intergranular structure for resisting the upward forces by arching coincides with the shape of the major principal stress trajectory (concave spherical cap). At initial penetration the circumferential friction forces are limited. Under dynamic driving conditions and in piles with a large area ratio, these arches cannot develop the required resistance to the forces which push the soil upwards, and collapse (see Chapter 5 for the collapse of the granular arches during flow in a silo and Figures 3.23, 3.24). This situation is associated with a major principal stress along the center line (vertical), $45^\circ < \psi \leq 90^\circ$ and $K_i < 1$.

(1) $\delta = 0$.

For this case, $\psi = 90^\circ$, $K_i = K_a$, and failure takes place along planes at angles of $\pm 45 + \phi/2$, as in the regular active case (see Figure 7.6a). A zero friction boundary condition is required to maintain this state. The soil slips along the pile interface, but no failure takes place along this plane. This situation is not realistic, for two reasons: (1) the pile–soil interface is not frictionless; (2) the soil inside the pile is not likely to be in an active state. It is pushed into the pile from an initial state (prior to driving) of $K = K_0 > K_a$. This condition is approximated using $\psi = 60^\circ$, as shown in Figure 7.5.

(2) $\delta = \phi$

The conditions of a soil element under failure are described by Figure 7.6b. In
order to ensure failure in the soil along the interface, the value of $\psi$ must be $\psi = 45 + \phi/2$, for which Equation 7.6 reduces to:

$$K_i = \frac{1 - \sin^2 \phi}{1 + \sin^2 \phi} = \frac{1}{1 + \frac{1}{2\tan^2 \phi}}$$  \hspace{1cm} (7.7)

Equation 7.7 can also be obtained directly by using Figure 7.6b for the geometrical relations between $\sigma_h$ and $\sigma_v$. The rotation of the principal stresses and the failure planes for the particular case of $\phi = \delta = 30^\circ$ are presented in Figure 7.7.

The above assumptions lead to the conclusion that if $\delta = \phi$ and the trajectories can be described as segments of a sphere then the radius of the trajectory must be equal to:

$$\frac{r}{ID} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\cos \phi/2 - \sin \phi/2}$$ \hspace{1cm} (7.8)

and the ratio between $K_i$ to $K_o$ (assuming $K_o = 1 - \sin \phi$) is:

$$\frac{K_i}{K_o} = \frac{1 + \sin \phi}{1 + \sin^2 \phi}$$ \hspace{1cm} (7.9)

This equation gives a value of about 1.2 for a wide range of possible $\phi$ angles.

(3) $\delta \leq \phi$

The following analysis deals with the assumptions of $45^\circ < \psi < 90^\circ$ and $\delta \leq \phi$ for different possible failure patterns:

(1) The soil mass fails ($\sigma_1$ acts vertically along the center line), while simultaneously the stresses along the interface are at the limited friction of $\tau_i/\sigma_1 = \tan \delta$. This state of stress is described by Figure 7.6c, where the
presented stresses have the following relations:

\[ \frac{\tau_i}{\sigma_h - \sigma_3} = \tan \psi \]  
(i) \hspace{1cm} \text{(7.10)}

\[ \frac{\tau_i}{\sigma_{hi}} = \tan \delta \]  
(ii)

Substituting 7.10(i) into 7.10(ii) gives:

\[ \frac{1}{\tan \delta} \cdot (1 - \frac{\sigma_3}{\sigma_h}) = \cot \psi \]

Dividing Equation 7.1 by \( \sigma_3 \) and substituting in the above leads to:

\[ \tan \delta = \tan \psi \cdot (1 - \frac{1}{\sin^2 \psi + K_p \cdot \cos^2 \psi}) \]  
(7.11)

Using Equation 7.11 for known \( \phi \) and \( \delta \) angles, enables calculation of the angle \( \psi \). Two values of \( \psi \) satisfy Equation 7.11. At this stage we are interested in the range of \( 45 + \phi/2 \leq \psi < 90^\circ \). The ratio between the horizontal to the vertical stresses at the boundary (\( K_1 \)) can then be evaluated using Equation 7.6, which is valid for any \( \psi \). For the singular case of \( \delta = \phi \), discussed above, Equation 7.11 gives \( \psi = 45 + \phi/2 \), and when substituted into Equation 6.18 gives \( K_1 = 1/(1 + 2\tan^2 \phi) \).

Figure 7.8 presents the relations between \( K_1 \) and \( \phi \) for various \( \delta \) angles. For \( 45 + \phi/2 \leq \psi \leq 90^\circ \) Equation 7.6 can be simplified to:

\[ K_1 \leq K_a + \cot^2 \psi \]  
(7.6a)

This equation still requires the evaluation of \( \psi \) using Equation 7.11. Further crude simplification can be obtained by using the following equation:
\( K_i \equiv K_a + 0.03 \)  \hspace{1cm} (7.12)

Equation 7.12 seems to give reasonable values for the range of \( 28^\circ \leq \phi \leq 36^\circ \) and \( \delta \) between \( \left( \frac{1}{2} \right) \phi \) and \( \left( \frac{2}{3} \right) \phi \). Comparison of Equation 7.11 to Equation 7.6a gives \( \psi \) of about \( 80^\circ \) as an approximation for the abovementioned range.

II) Slip or failure of soil along the interface. As opposed to simultaneous failure along the interface and within the soil mass, failure can take place at one of the zones only. If failure takes place in the soil mass before the limited friction is obtained, the friction angle, which can not exceed \( \phi \), forces a failure plane parallel to the interface. This case is equivalent to the one discussed in Section (2) for which \( \delta = \phi \). If failure or slippage takes place along the interface before the entire soil mass fails, two possible stress relations can be described:

(II–1) Minimal state of stress, in which the envelope of the interface friction angle is tangential to the Mohr circle, as shown in Figure 7.6e. At smaller stresses, no failure can take place at the interface or within the soil. Using Equation 7.7 with the replacement of \( \phi \) by \( \delta \) (see the similarity between Figures 7.6e and 7.6b) enables calculation of \( K_i \) for this case:

\[
K_i = K_\delta = \frac{1}{1 + \frac{\sin^2 \delta}{\sin^2 \phi}} = \frac{1}{1 + \frac{1}{2 \tan^2 \delta}} \hspace{1cm} (7.13)
\]

The principal stress rotation under this state is \( \psi = 45^\circ + \delta/2 \). The value of \( K_\delta \) for different \( \delta \) angles can be obtained from Figure 7.8, using the horizontal scale as \( \delta \) instead of \( \phi \) and the curve of \( \delta = \phi \). As \( \delta < \phi \), \( K_i(\delta) > K_i(\phi) \) by the ratio of:

\[
\left( \frac{1 + 2 \tan^2 \phi}{1 + 2 \tan^2(\delta/\phi)} \right) \cdot \phi
\]
(II-2) The intermediate state of stress, in which the limited friction along the interface is met before a general soil failure takes place, is shown in Figure 7.6d. This condition is similar to that of Figure 7.6c for a reduced fictitious angle \( \phi^* = \alpha \cdot \phi \), where \( \phi^* \) is the angle which would describe an envelope tangential to the existing boundary state of stress, as shown.

The principal stress rotation \( \psi \) can be calculated using Equation 7.11 for which \( \phi \) is substituted by \( \phi^* \). The ratio \( K_1 \) can then be calculated using Equation 7.6 with the above \( \psi \) and \( \phi^* \) angles. The ratio \( K_1 \) for this case is limited between \( \delta \) as a maximum ratio and \( K_1 \) for the combination of \( \delta \) and \( \phi \) (Figure 7.5) as a minimum ratio (which cannot be smaller than \( K_a \)).

Examples of the various alternatives are presented in Figure 7.6f:

1. State of stress before any failure takes place.
2. Possible failure along the interface. Using Equation 7.13 for \( \delta = 20^\circ \) gives \( K_\delta = 0.791 \) (\( \psi = 55^\circ \)).
3. Possible failure along the interface, not as a marginal case. Using \( \phi^* = 23^\circ \) and \( \delta = 20^\circ \) in Equations 7.11 and 7.6 leads to \( \psi = 69.09^\circ \) and \( K_1 = 0.549 \), respectively.
4. Simultaneous failure in the soil mass and along the interface. Using the above procedure or Figure 7.8 for \( \phi = 30^\circ \) and \( \delta = 20^\circ \) leads to \( K_1 = 0.371 \) (\( \psi = 78.35^\circ \)).
5. For the completion of the example, we can assume the existence of a rough surface for which \( \phi = \delta = 30^\circ \), which leads to \( K_1 = 0.600 \) (\( \psi = 60^\circ \)).

\[ \psi = 45^\circ ; K_1 = 1 \]

Further penetration (beyond the initial stage) is expected to increase the upwards pushing forces only to a maximal value, corresponding to the critical depth (at which the bearing capacity stresses are maximal and do not increase more with the
The resisting forces, however, increase continuously with the increase in plug length, a process which is closely related to a gradual increase in the lateral stresses. At a certain plug length, the lateral confinement will bring about a special state of stress in which the 'passive' arch will not be destroyed and the vertical stress will be equal to the horizontal stress at the interface ($K_i = 1$). This transition state is distinguished by $\psi = 45^\circ$ with identical arches for both trajectories, as shown in Figure 7.9a. It is assumed that a transition layer with a 'lens' shape will be created, in which hydrostatic stresses will exist ($\sigma_1 = \sigma_3$).

Further build-up of stresses will enable the mobilization of the full 'passive arch', for which $K_i > 1$ (see Figure 3.25b).

\[(c) \quad 0^\circ \leq \psi < 45^\circ; \quad K_i > 1\]

Following the build-up of the aforementioned transition layer, conditions for the development of a stable 'passive' arch are created. In this state, the soil plug mobilizes sufficient resistance to react to the upwards pushing forces and to allow the lateral confinement to develop 'passive' arches of contacts along the major principal stress trajectories, as shown in Figure 3.25b or under closed piles in Fig. 3.18b. These arches may transfer horizontal stresses at the interface (due to dilation) which are greater than the vertical stresses. This stress state is associated with a horizontal major principal stress along the center line, and $\psi$ in the range of $0^\circ \leq \psi < 45^\circ$ for $K_i > 1$.

1. $\delta_p(\phi)$ Limiting interface friction, as a function of the internal friction angle

A rough surface for which the friction angle is identical to the internal friction angle $\phi$ is assumed. In order to ensure failure along the interface, the stress state must be that of Figure 7.6b, where simultaneous failure takes place along the interface and in the soil mass. However, in Figure 7.6b, $K_i < 1$ and $\psi = 45 + \phi/2$. If, under the
same assumption, we impose failure for the case of $K_i > 1$ and $\psi < 45^\circ$, the stress state of Figure 7.9b is obtained. In this state, failure takes place in the soil and $\psi = 45 - \phi/2$, for which Equation 7.6 reduces to:

$$K_i = \frac{1}{1 - \sin^2 \phi} = 1 + 2 \tan^2 \phi$$  \hspace{1cm} (7.14)

Equation 7.14 can be directly obtained by using Figure 7.9b for the geometrical relations between $\sigma_h$ and $\sigma_v$. It is the inverse of Equation 7.7 (Equation 7.14 = $1$/Equation 7.7).

The rotation of the principal stresses and the failure planes for the particular case of $\phi = 30^\circ$ are presented in Figure 7.10. The radius of the trajectory (concave, downwards) can be calculated by Equation 7.8. For the particular case of Figure 7.10, this radius is identical to that of Figure 7.7, being $\phi$ and ID dependent only. Figure 7.9b shows that a unique angle $\delta$ [denoted as $\delta_p(\phi)$] ensures failure along the interface. Under these conditions, the shear stress along the interface is equal to the maximal possible shear stress according to the Mohr-Coulomb failure criterion (denoted by $\tau_f$). It is reasonable to assume that the shear stress along the interface will not exceed $\tau_f$, and therefore $\delta_p(\phi)$ serves as an upper limit interface friction angle. This assumption is further considered in Chapter 8, where it is supported by experimental data.

The relation between $\delta_p(\phi)$ to the friction angle $\phi$, can be obtained in the following way: substituting Equations 7.1 and 7.2 into the condition of $\tau_i/\sigma_i = \tan \delta_p(\phi)$, then using the angle $\psi = 45 - \phi/2$ and the ratio of $\sigma_i/\sigma_3 = K_p$, leads to the following expression:

$$\delta_p(\phi) = \tan^{-1} \left( \frac{K_p - 1}{(K_p - 1) \cdot \sin \phi + (K_p + 1)} \right) \cdot \left[ 1 - 2 \sin^2 (\phi/2) \right]$$  \hspace{1cm} (7.15)

The relations of Equation 7.15 can be checked by substituting $\delta_p(\phi)$ for $\delta$ in Equation
The meaning of the calculated unique $\delta_p(\phi)$ is revealed by the following study concerning the nature of Equation 7.15 and its applicability.

Values of $\delta_p(\phi)$ and $K_i$ as a function of different internal friction angles $\phi$ were calculated using Equations 7.15 and 7.5 respectively. These values, along with the ratios of $\delta_p(\phi)/\phi$ and $\tan \delta_p(\phi)$, are presented in Table 7.2.

Figure 7.11 presents the relations between $\phi$ and $\delta_p(\phi)$ based on Equation 7.15. Although the entire possible range of internal friction angles was taken into account ($20^\circ \leq \phi \leq 45^\circ$), it is more practical to focus on the limited common range for which $\phi$ varies between 30 and 40 degrees. Two observations can be made regarding the obtained relations: (a) The angle $\delta_p(\phi)$ does not increase as $\phi$ increases (monotonically) but reaches a maximum value of $\delta_p(\phi) = 19.5^\circ$ for $\phi = 35^\circ$. (b) For the above range, a small variation takes place in the interface friction angle, where $19.1^\circ \leq \delta_p(\phi) \leq 19.5^\circ$; the friction coefficient varies only slightly with the change in $\phi$.

Figure 7.12 presents the relations between the friction coefficient $\tan \delta_p(\phi)$ and the soil shear friction angle $\phi$. For $30^\circ \leq \phi \leq 40^\circ$ the friction coefficient is $\tan \delta_p(\phi) = 0.35$; (with the range of accuracy of $\pm 0.005$). The obtained results are based on the theoretical approach previously presented. In practice, the interface friction angle $\delta_p(\phi)$ is expressed by the ratio of $\delta_p(\phi)/\phi$. Figure 7.13 presents the relations between $\delta_p(\phi)/\phi$ and $\phi$. The ‘exact’ results are based on Equation 7.15 and presented as plus signs calculated for intervals of half a degree. The continuous line is the linear best fit using the minimum square approach, from which the following relations have been obtained:

$$\delta_p(\phi)/\phi = 1.12 - 0.016\phi^\circ$$ (7.16)
In the practical range of $30^\circ \leq \phi \leq 40^\circ$, the ratio of $\delta_p(\phi)/\phi$ varies between 0.48 and 0.64, which is the empirical ratio usually quoted (see Section 6.1), where 

$\frac{1}{2} \leq \delta/\phi \leq \frac{2}{3}$. The average of the $\delta/\phi$ ratio for $25^\circ \leq \phi \leq 40^\circ$ is $\delta/\phi = 0.6 \pm 0.1$. 

As $\delta$ and $K$ are both functions of $\phi$, a relation between $\delta_p(\phi)/\phi$ and $K_i$ can be obtained, as shown in Figure 7.14.

The main observation regarding the obtained results is that the theoretical approach (subjected to the specified assumptions) resulted in $\delta$ values and relations between interface and internal friction angles which fit the common practical range nicely, based on experimental data.

(2) $\delta < \delta_p(\phi)$ Small interface friction angles

The previous section revealed that if a state of stresses exists within the plug such that the major principal stress acts horizontally at the center-line, the friction along the interface will be limited by the calculated interface friction angle $\delta_p(\phi)$ (Equation 7.15). If the existing interface has a friction angle which is greater than or equal to $\delta_p(\phi)$, the failure along the interface will occur according to the calculated limiting friction angle, $\delta_p(\phi)$. If the interface friction angle is less than the calculated one, theoretically the state of stress can be one of the three possibilities described in Section a–3 (Figure 7.6c,d,e), for $\psi$ in the range of $0^\circ$ to $45^\circ$. As discussed previously, however, the development of the 'passive arch' is associated with soil-dilation, and therefore the possibility of shear along the interface without failure in the soil mass is not realistic.

For the conditions of simultaneous failure along the interface and within the soil mass, $\psi$ can be calculated using Equation 7.11 (see Appendix III Table B for values of $\phi$ in the range of $20^\circ$ to $45^\circ$). $K_i$ can then be calculated using Equation 7.6.
(d) Summary

The above possible stress conditions in the soil plug are summarized in Table 7.1 and Figure 7.15. Table 7.1 outlines the different possibilities of principal stress rotations with different combinations of internal friction angles $\phi$ and interface friction angles $\delta$. The ratio between the horizontal and the vertical stresses along the interface ($K_i$), as a function of the different possibilities of stress rotations and $\phi$ and $\delta$ angles, is shown in Figure 7.15.
7.5 THE SHAPE AND MECHANISM OF THE ARCH

The spherical caps (see Fig. 9.1 for isometric presentation), shown in cross-section as arch elements (Fig. 7.3), are bounded by surfaces representing planes of zero shearing stresses. Thus, moment equilibrium requires that the stresses be constant throughout the arch, following then the shape of the trajectory.

The natural soil structure resisting the downward movement of the pile (equivalent to the upward movement of the soil) is a concave arch (downward). This was supported by measurements and observations such as the radiograph arch of Fig. 5.14 (for the flow downwards) and the arches formed in and under overdriven samplers and closed-ended piles (Figs. 3.25 a,b, 3.27 a,b, and 3.28 a,b) for the relative flow upwards. The formation and reformation of rupture surfaces, referred to as ‘dynamic arch’ was observed by various researchers (e.g. Perry and Handley—1967) investigating the flow of granular material discharged from model hoppers. The formation of the arch has been shown to be linked with cyclic peak stresses on the walls of the bunker as the rupture surface forms. The stress then decays as the rupture surface widens and extends (Perry and Handley—1967). A later peak in the wall stresses occurs as the next rupture surface forms. The increase in the wall stress is accompanied by a decrease in the pressure within the soil mass, just above the dynamic arch.

As mentioned earlier, the exact shape of the entire arch is of less importance than the conditions along the inner wall interface as described earlier. It is noticeable, however, that the arches in a soil mass assume a higher parabolic shape coinciding with high \( \psi \) angles (e.g. Figs. 3.28 a,b and 3.29), which suggests that within the soil mass, \( K_i < 1 \). The arches inside the tubes or silos (e.g. Fig. 3.25b) are ‘flatter’, coinciding with low \( \psi \) angles which suggest that, due to the pile confinement, \( K_i > 1 \).

The relations between stresses, strains and deformation before and after the arch formation can be investigated with the help of data such as those of Airey et al. (1985) and those relating to flow in silos, mentioned above. This kind of analysis is,
however, beyond the scope of this study. However, a practical solution will be suggested, based on a simplified approach (Chapter 9).

The flow of the soil inside the pile differs from that of the material in the silo. During discharge, the material in the silo (above the arch) undergoes very small strains (less than 1% ; note that the opening of a hopper in silos is much smaller than the bin’s cross-section), and moves downwards in a slow motion, similar to a rigid body motion (Bransby and Blair—Fish — 1973). In the pile, on the other hand, it is expected that the failure of the concave arch and the continuation of the flow upwards (walls downwards) leads to reverse shapes of convex (upwards) arches, as shown in Figs. 3.23 and 3.24. Assuming these arches are of uniform density and thickness, and thus of uniform weight throughout, the shape of the arch will be that of a catenary, the shape taken by a chain held at the ends. Under most states of soil entering the pile ($45^\circ < \psi \leq 90^\circ$), the arch would therefore be depicted as the trajectory of the minor principal stress, with a convex (upwards) shape. It is assumed that with the increase of internal friction, full plugging will take place when a ‘passive’ arch will be mobilized in the direction of the major principal stress. This arch follows the natural path of maximum contacts between the grains.

The stress conditions within the arch itself are those of a ‘true’ triaxial compression test. The soil is compressed along the arch axes by the major principal stresses, normal to the arch by the minor principal stress, and along the arch circumference ($\sigma_\theta$) by the intermediate principal stress. This results in a dilation, which explains the large horizontal stresses and the development of large frictional forces along the interface. The dilation of the arch as a mechanism of support explains two phenomena: (1) the ‘arching’ is profound in dense sands; (2) the dilation results in a zone of decreased density which is reflected as a lighter area in radiographs, as shown in Figure 5.14. The strength of the arch depends on the compressive strength of the soil. This explains why the arching mechanism is more effective in smaller spaces,
where the strength of the arch is sufficient to hold the column of soil which it supports.

The shape of the convex arch can be described by the equation of the catenary:

\[ z = \frac{a}{2} \left( e^{x/a} + e^{-x/a} \right) \]  \hspace{1cm} (7.17)

in which:

- \( a \) - coefficient
- \( x \) - relative distance from the center line, having the limits of \( \pm 1 \)
  
  (for \( \pm B/2 \)).

The slope of the catenary is obtained by differentiating Equation 7.17 with respect to \( x \):

\[ \frac{dz}{dx} = \frac{1}{2} \left( e^{x/a} - e^{-x/a} \right) \]  \hspace{1cm} (7.18)

Assuming that both arches are catenaries, then the major principal stress trajectory (concave, downwards) \( \frac{dz}{dx} = \tan \psi \). For the minor principal stress trajectory (convex, upwards) \( \frac{dz}{dx} = -\cot \psi = -\tan(90 - \psi) \).

The angle \( \psi \) can be calculated for given \( \phi \) and \( \delta \) values, using Equation 7.11. Equation 7.11 will provide both possible solutions (\( 45^\circ < \psi < 90^\circ \) and \( 0^\circ < \psi < 45^\circ \)). Knowing \( \psi \), the coefficient 'a' can be calculated, using Equation 7.18 for the boundaries of \( x = 1 \). The shape of the catenary can then be given by Equation 7.17.

Appendix III (A,B) presents values of \( \psi \), \( K_1 \) and 'a' for various combinations of \( \phi \) and \( \delta \) angles. Appendix III (A) is for the range of \( 45^\circ < \psi < 90^\circ \) (\( K_1 < 1 \)) and Appendix III (B) for \( 0^\circ < \psi < 45^\circ \), limited by \( \delta_{\text{max}} = \delta_p(\phi) \) (\( K_1 > 1 \)). In both tables 'a' is calculated for the concave (downwards) catenary. The actual range of \( \psi \) in the tables varies between \( 45 + \phi/2 \leq \psi < 90^\circ \) (as \( \psi = 45 + \phi/2 \) for \( \delta = \phi \)) and
$0 \leq \psi \leq 45 - \phi/2$ [as $\psi = 45 - \phi/2$ for $\delta = \delta_\phi(\phi)$].

Figure 7.16 presents different catenaries compared with circular arches (in dashed lines) for various $\psi$ angles. The comparison between the arches and the catenaries shows that in the range of $0^\circ \leq \psi \leq 45^\circ - \phi/2$ for which $K_1 > 1$, there is good agreement between the catenary and the concave arch, which is assumed to be the 'correct' arch. In the range of $45^\circ < \psi \leq 45^\circ + \phi/2$ for which $K_1 < 1$, there is good agreement between the catenary and the convex arch, which is assumed to be the 'correct' arch for this range. Fairly good agreement exists between the circular arch and the catenary for $\psi = 45^\circ$. It can therefore be concluded that for further analysis (developed in Chapter 9), the catenary can be substituted by a circular arch, assuming a concave shape for $K_1 > 1$ and a convex shape for $K_1 < 1$. 
7.6 SUMMARY AND CONCLUSIONS

1. The roughness of the pile wall is assumed to be constant. The densification of the soil and its influence on the interface friction are both limited. For given soil and pile conditions, the pile wall/soil interface friction coefficient is therefore highly dependent on the interparticle contact orientation (the groove inclination, \( \theta \) of the S.G. model), which may be determined by the major principal stress trajectory of the inner soil plug.

2. The analysis of the soil plug using the 'silo approach' (Chapter 4) indicated that the ratio between the horizontal to the vertical stresses at the interface and the interface friction coefficient (\( \tan \delta \)) control the interface friction, and therefore the plugging of the pile. Both factors are determined by the stress trajectories.

3. The soil plug analysis using the 'silo approach' was developed under assumptions which required reconsideration in light of experimental results.
   - A constant K was assumed. The analysis of experimental data indicated the dependence of K on the soil height and the pile diameter. K in excess of 1.0 was back-calculated for many experiments (Fig. 4.10).
   - Constant average vertical and horizontal stresses across any given depth were a result of this assumed constant K. The axisymmetric conditions determine that the horizontal and vertical stresses along the plug center line are principal stresses. Friction takes place along the interface; therefore, the horizontal stresses acting normal to the wall cannot be principal stresses. Rotation of the principal stresses is expected to take place when moving outward from the center line to the interface.

4. The experimental results which were described by Kishida et al. (1985) could be explained only by an arching mechanism [see Section 4.8 (7)]. An arch follows the path of maximum intergranular contacts, and assumes the shape of
the principal stress trajectory (the micro mechanism of the granular material behavior was reviewed in Chapter 5). The arch is therefore bounded by surfaces representing planes of zero shearing stresses, creating stress conditions similar to those of a ‘true’ triaxial compression test within the arch. The soil is compressed along the arch by the major principal stress, and confined by the minor and intermediate principal stresses. This compression is followed by soil dilation, which exerts large stresses on the pile wall, and explains the conditions in which the horizontal stresses are greater than the vertical stresses at the boundaries.

Use of the ‘systematic arching theory’ on a mass of uniform discs in a silo provided a good fit to the silo theory and experimental results, proving the arching approach analytically.

5. Elasticity theory was used to obtain the stress trajectories of a simplified plug problem. The obtained solution (Fig. 7.1) shows that the major principal stress acts along the center line. The minor principal stress trajectory is horizontal at the center line, curving down towards the boundaries.

6. Plasticity theory was utilized by Sokolovski (1960) to define the critical state conditions. Sokolovski’s approach was used to obtain the slip surfaces of a simplified plug problem. The obtained solution (Fig. 7.2) shows trajectories similar to those of Fig. 7.1.

7. Refined solutions based on either of the above procedures can be obtained for more realistic soil plug conditions. Such solutions, however, have two main limitations:
   - The procedure requires a numerical analysis for each set of conditions (e.g. F.E. analysis for the elastic problem and F.D. solution for Sokolovski’s equations of slip planes).
   - The solutions cannot explain the experimental results in which the
horizontal stress at the interface is greater than the vertical stress along the interface \((K_i > 1)\).

An F.E. analysis was carried out (Hight, 1987) with the intent of providing an explanation of experimental results, similar to those described by Kishida. Allowing for dilation in ‘shear bands’ along the interface did not provide the required answers. Only excessive dilation throughout the soil mass provided results which could explain the experimental data. As was discussed previously (Section 4.8), such conditions do not exist in active arching (in which the disc supporting the soil is moved downwards, ‘away’ from the soil), and therefore are not acceptable for a satisfactory explanation.

8. A ‘soil arching’ approach was developed along the following lines:

- The downwards movement of the pile walls relative to the soil determines the shear stress direction at the interface: downwards on the soil and upwards on the pile.

- Use of the Mohr–Coulomb failure criterion for the soil and the interface, and knowledge of the shear stress direction at the interface allows description of the Mohr circle of stresses and determination of the principal stress orientation at the interface.

- Knowing that

  (a) The horizontal and vertical stresses along the center line (of the soil cylinder) are principal stresses, and

  (b) The principal stress trajectories are continuous curved lines, as obtained by the elastic and plastic solutions

allows two possible trajectories for the major and minor principal stresses to be constructed.

- The main purpose of constructing the trajectories is to obtain the possible stress state at the interface. The actual trajectory shape is of
lesser importance (at this stage), and therefore a curved line, which is a segment of a circular arch, is used for simplicity.

- The principal stress orientation is described by the angle $\psi$, measured between the horizontal plane and the plane on which the minor principal stress acts, or between the vertical plane and the plane on which the major principal stress acts.

9. The stress relations along the interface were described in Eq. 7.6 as $K_i = f(\phi, \psi)$, leading to the following possible stress conditions (see Table 7.1):

- $45^\circ < \psi < 90^\circ$, $K_i < 1$
  
  for $\delta = 0^\circ$, $\psi = 90^\circ$  
  $$K_i = K_a = \tan^2(45 - \phi/2)$$
  
  for $\delta = \phi$, $\psi = 45 + \phi/2$  
  $$K_i = 1/2 \tan 2\phi$$
  
  for $0^\circ < \delta < \phi$
  
  for failure of soil and interface: $K_i \geq K_a + 0.03$, $\psi$ from Eq. 7.11
  
  for soil slip along interface only: $K_a < K_i < K_\delta = 1/(1+2\tan^2\delta)$
  
  $-$ see Fig. 7.8 for $K_i = f(\phi, \delta)$

- $\psi = 45^\circ$, $K_i = 1$

- $0^\circ < \psi < 45^\circ$, $K_i > 1$
  
  for $\delta = 0^\circ$, $\psi = 0^\circ$  
  $$K_i = K_p = \tan^2(45 + \phi/2)$$
  
  for $\delta = \phi$, $\psi = 45 - \phi/2$, $\delta$ is limited by $\delta_p(\phi) = f(\phi)$ (Eq. 7.15)
  
  $$K_i = 1+2\tan^2\phi$$
  
  for $0^\circ < \delta < \delta_p(\phi)$
  
  for failure of soil and interface: $\psi = f(\phi, \delta)$ (Eq. 7.11)
  
  $\rightarrow K_i = f(\psi, \phi)$ (Eq. 7.6)

slip or failure along the interface only is not realistic.

10. The possible conditions at the boundary ($K_a \leq K_i \leq K_p$) for the entire range of principal stress rotation ($0^\circ \leq \psi \leq 90^\circ$) can be explained by either trajectory, as they complement each other. Practically, however, the shape of the
trajectory and its mechanism suggests that each is associated with a certain stress state in the plug, and therefore with one range of principal stress rotation.

- For $0^\circ \leq \psi \leq 45^\circ$, the convex trajectory of $\sigma_3$ would supply the appropriate conditions for this state ($K_a < K_i < 1$). The soil is under conditions of a 'true' triaxial extension test (loading), and the volume change can be one of contraction only; $\sigma_1$ is the vertical stress along the center line.
- For $45^\circ \leq \psi \leq 90^\circ$, the concave trajectory of $\sigma_1$ would supply the appropriate conditions for this state ($1 < K_i < K_p$). The soil is under the conditions of a 'true' triaxial compression test (loading) and dilation takes place, explaining the possible development of high stresses normal to the interface; $\sigma_1$ is the horizontal stress along the center line.

The shape of the convex arches is that of a catenary, described by Eq. 7.17. Values of the constant 'a' for various $\phi, \delta$ combinations (for both ranges of $\psi$) are presented in Appendix III, Tables A and B. The 'a' values in Appendix III were calculated for the concave trajectory.

12. Comparison of of the catenary shape to the circular arch in the relevant ranges ($0^\circ \leq \psi \leq 45 - \phi/2$ for the convex arch and $45 - \phi/2 \leq \psi < 90^\circ$ for the concave arch) suggests that the circular arch may be used instead of the catenary. This fact simplifies the mathematical calculations of the arch description (to be used in Chapter 9).

13. The relations of Chapter 7 were developed for the inner soil plug. They are relevant, however, to similar problems in soil mechanics (e.g. the outside wall of the pile). Due to the symmetry of the problem, the center line is a principal stress axis. Such conditions also exist in the undisturbed soil, away from the
outer wall of the pile. The rotation of the stresses from the center line to the inner wall are the same as those from the undisturbed soil to the outer wall. The inverse of the shear forces in Fig. 7.3 will simulate the conditions of the active case, which is the mirror image of the analysis developed in Chapter 7. The trajectories are reversed (the concave trajectory will become that of $\sigma_3$), as are the equations.
Now

7.4.4

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Table 7.2 Values of the Limiting Interface Friction Angle \( \delta_p \), and the Associated Lateral Earth Pressure Coefficient \( K_i \) for Different Values of the Soil Shear Angle \( \phi \).
Elastic Solution for Pushing an Isotropic Circular Cylinder in all-around Confinement (a) The Analyzed Problem (b) Lines of Principal Stress (c) Major and Minor Principal Stresses after Milne—Thomson—1962.

Fig. 7.1:
Fig. 7.2: Plastic Solution for the Slip Lines of Massless Media ($\phi = 20^\circ$, $\delta = 15^\circ$), Pushed Between Two Walls
The 'Arching Approach' to the State of Stress in the Inner Soil Plug (a) Continuous Possible Arches Defined by the Principal Stress Trajectory (b) Free Body Diagram (c) Mohr Circle for the State of Stress Along the Interface
Fig. 7.4: Force Equilibrium on an Interface Boundary Element
Fig. 7.5: The Range of Possible Major and Minor Principal Stress Trajectories \(0 \leq \psi \leq 90^\circ\), Presented as Segments of Circular Arches
Fig 7.6:

Possible Stress States in the Inner Soil Plug for 45°≤ψ≤90°

p - Origin of Planes
fp - Failure Plane
p — Origin of Planes

pp — Principal Stress Plane

\( f_p \) — Failure Plane

**Fig. 7.7:** Rotation of Principal Stresses and Failure Planes for Major Principal Stresses Along the Center Line, and \( \delta = \phi \)
Fig. 7.8: Relations Between Horizontal and Vertical Stresses along the Interface as a Function of the Internal Friction Angle ($\phi$) and the Interface Friction Angle ($\delta$) for the Range of $45 + \phi/2 \leq \psi \leq 90^\circ$
Possible Stress States in the Inner Soil Plug for $0 \leq \psi \leq 45^\circ$
Fig. 7.10: Rotation of Principal Stresses and Failure Planes for Minor Principal Stress along the Center Line, and \( \delta = \delta_p(\phi) \)

- **p** - Origin of Planes
- **pp** - Principal Stress Plane
- **fp** - Failure Plane
Fig. 7.11: Relations Between $\phi$ and $\delta_p(\phi)$

Fig. 7.12: Relations Between $\phi$ and $\tan \delta_p(\phi)$
Fig. 7.13: Relations Between $\phi$ and $\delta_p(\phi)/\phi$. The +++ line shows the exact relations and the continuous line shows the approximated relations of Eq. 6.28.

Fig. 7.14: Relations Between $\delta_p(\phi)/\phi$ and $K_i$ Along the Interface.
Fig. 7.15: Relations Between Horizontal and Vertical Stresses along the Interface as a Function of the Internal Friction Angle (\(\phi\)) and the Interface Friction Angle (\(\delta\)) for the Range of \(0 \leq \psi \leq 90^\circ\)
Catenaries and Circular Arches for Different $\psi$ Angles

$\psi = 0^\circ$
$K_1 = K_p$

$(\phi = 30^\circ)$
$\psi = 23.2^\circ$
$K_1 = 2$

$\psi = 45^\circ$
$K_1 = 1$

$(\phi = 30^\circ)$
$\psi = 60^\circ$
$K_1 = 0.6 \leq 1.2K_0$

$\psi = 90^\circ$
$K_1 = K_a$

Fig. 7.16
CHAPTER 8
CRITICAL REVIEW OF EXPERIMENTAL DATA IN LIGHT OF THE 
PROPOSED INTERFACE SHEAR RESISTANCE MECHANISM

8.1 INTRODUCTION

The analysis of the granular soil-interface shear resistance mechanism consists of two parts: (1) a model of a single particle which considers the micro behavior of granular material (presented in Chapter 6), and (2) a stress solution for the inner-soil plug (presented in Chapter 7) which considers the arching phenomenon, utilizing a continuum representation of stress trajectories and failure criteria.

This chapter examines the proposed interface shear resistance mechanism in light of available experimental data. Before undertaking such an examination, three fundamental questions must be confronted:

(1) Is it necessary to use the presented analysis?

(2) What is the validity of using a single grain-element model to simulate a soil mass?

(3) How valid are assumptions such as those made in Section 7.4.4 (part c-1) concerning the 'limiting friction' angle along the interface?

The need to examine granular material at the micro level is obvious. The two fundamental parameters that control the mechanism of soil plugging in granular material (the interface stresses and the interface friction coefficient) can be explained only through the load transfer behavior of individual grains and the shear mechanism in an assembly of particles. Solutions utilizing the 'standard' tools such as F.E. analysis can, undoubtedly, be manipulated to yield results which agree with experimental data. For example, results of experiments similar to those carried out by Kishida and Isemoto (1977, see Fig. 3.37) were analyzed by some researchers using the F.E.M. (according to Hight – 1987). In order to match the analysis results to the experimental data, dilation was first assumed to take place in a 'shear band' along the
interface (see for example Vardoulakis et al. -1978, 1981). Since this approach did not produce the correct results, dilation was assumed to take place in the entire soil mass to the extent where agreement between the experimental data and the F.E. analysis was reached.

The possibility of using a distinct element method (e.g. Cundall and Strack -1979, Petrakis and Dobry -1987) was also considered, and it will most likely provide a significant modeling tool in the future (Scott -1987). However, it is believed that simple models, based on observation of the granular material, are a prerequisite for any type of analysis. Only by focusing on the "why" rather than on the "how" can a better understanding of soil behavior be arrived at, leading to more "correct" analyses of soil mechanics problems.

The second question (relating the single grain to the soil mass) now arises. Even though the analysis of the S.G. model is undertaken for a single particle, all possibilities for different packings and intergranular contacts can be obtained with different combinations of the parameters $\beta$ and $\theta$. By using representative values which reflect the state of many particles, it is assumed that the model would provide a valid representation of the soil mass. For that reason, the surface roughness is related to the median particle size by weight $D_{50}$, and the particle shape is represented by equivalent geometrical relations. Even with a good match to experimental results, improvements can undoubtedly be made by using a probabilistic approach to the representative values. All deterministic values, including the interparticle friction coefficient and the interparticle contact orientations, can (and maybe should, in the future) be replaced with appropriate probability distribution functions (e.g. see Figs. 5.16 and 5.20). It should also be borne in mind that a cubic meter of sand contains approximately $10^9$ grains. An approach utilizing a mechanical model of a single grain with representative values of the soil mass may result in a better practical solution than a discrete analysis of a few hundreded particles (mostly 2-D discs), which is about
the upper possible limit of today. (This argument is not intended to reduce the importance of the discrete element analysis, as much as to support the idea of the S.G. model.)

Another major factor that facilitates transfer from the micro to the macro level is the way in which $\theta$ is perceived (as will be discussed further in this introduction). For additional discussion on the application of failure theory of a simplified model to cohesionless soil, see Scott (1963, Section 7–2).

The third question, regarding the validity of assumptions concerning the ‘limiting friction’ angle along the interface, can be debated on different levels, from the theory of strength of materials to interpretation of direct shear tests (Hansen –1961, Morgenstern and Tchalenko –1967, Konishi –1978, Potts et al. –1987, Jewell and Wroth –1987). The assumptions made in section 7.4.4 are supported to some extent by these references. However, as no research has been undertaken to investigate specifically the interface testing in a direct shear box, which is beyond the scope of the present work, these assumptions are intuitive, and will be checked in a limited way in the present chapter, while further work on the subject is required.

The theta parameter ($\theta$) of the S.G. model describes the angle between the orientation of the groove and the orientation of the moving plane. The groove represents the possible interparticle contacts, and the moving plane represents the applied forces acting on this grain. For the 2–D case ($\beta = 90^\circ$), the groove orientation coincides with the interparticle contact plane, normal to the interparticle force. For the 3–D case, the resultant of the interparticle forces is normal to the groove orientation. The forces $T$ and $P$ (Figure 6.1) acting on a grain can result from an external moving plane along an interface, or from an adjacent soil grain.

Based on the micro behavior of granular material, it was previously stated that in order to relate statistically the forces acting on a single grain to the entire soil mass, the direction of the groove was chosen to be normal to the major principal stress.
[see Sections 6.1 (Introduction) and 6.2.2a (The Underlying Concept)].

The results of the S.G. model depend on the above assumption, which is examined in the next section (8.2) in relation to: (a) the principal stress trajectory of the continuum approach (Chapter 7); (b) the minimum energy solution for the preferred intergranular contact; and (c) packings of spheres and rods. Once these possible \( \theta \) angles are established, they are used to examine the S.G. model in relation to experimental data.

Section 8.3 compares the major principal stress orientation in failure, as suggested by the S.G. model, to that of the continuum approach and experimental data. By assuming that the ‘moving plane’ represents the applied forces of an adjacent grain, the S.G. model is checked as a failure criterion of granular material. This condition is further checked in Section 8.5 for the state in which a ‘rough’ contact plane shears the soil along the interface.

Section 8.4 examines data relating the major principal stress orientation to applied loading on granular material. Sections 8.5 and 8.6 use the different possibilities of the \( \theta \) angles to compare calculated interface shear resistances with measured ones.
8.2 THE PARAMETER $\theta$ OF THE S.G. MODEL

8.2.1 The principal stress trajectory and the S.G. model

The principal stress trajectory, described by the arching angle $\psi$ and the groove orientation $\theta$ of the S.G. model, are presented in Figure 8.1. This figure suggests that for the above assumed orientation, $\theta = \psi$.

The S.G. model predicts the interface shear resistance to sliding of the moving plane on the soil particles ($\tan \delta_s$). This value is limited by the shear strength of the soil mass, which is described by the Mohr–Coulomb failure criterion in a continuum approach. However, the S.G. model suggests that the shear strength of the soil may be examined by applying the limiting equilibrium condition along the interparticle plane of contact (the groove), as expressed by the relations for $\tan \phi \mu$ , considering $R_a$, $\beta$ and $\theta$. This may be examined by using data of soil mass shear resistance and principal stress orientation.

Although the subject is not required for the present work, review of the interparticle limiting equilibrium as a failure criterion is of great interest to soil mechanics, and will be examined in Section 8.3.

The description of the major principal stress trajectory using the angle $\psi$ was obtained for the shear conditions by utilizing the Mohr–Coulomb failure criterion. The analysis of Chapter 7 (see Table 7.1) showed that within the possible range of $0^\circ \leq \psi \leq 90^\circ$, the practical limits are:

$$
(1) \quad \text{for } K_i < 1 \text{ and } \delta = \phi' , \psi = \psi_{\text{max}} = 45 + \phi/2 \\
(2) \quad \text{for } K_i > 1 \text{ and } \delta > \delta_p(\phi) , \psi = \psi_{\text{min}} = 45 - \phi/2
$$

The orientation of the groove in the S.G. model is then expected to be restricted by the range of:
45 - \phi/2 \leq \theta \leq 45 + \phi/2 \quad (8.1)

The lower boundary refers to the special conditions of K_i > 1, and is subjected to the 'limiting friction' assumption of Section 7.4.4. The upper boundary for K_i < 1 will be checked to demonstrate the compatibility of the S.G. model criterion of soil failure to measured values.

8.2.2 Minimum energy solution for the preferred intergranular contract

For no friction along the interface (tg\delta_s = 0), Chapter 7 shows \psi = 0 and the S.G. model shows \theta = 0 (only rolling is possible). Practically, however, some friction always exists between the 'moving plane' and the soil along the interface. The above section suggests that this minimum friction will coincide with \psi = \psi_{\text{min}} = 45 - \phi/2, based on the continuum approach. Another way to examine this low friction is through a minimum energy solution for the 'micro' behavior.

Under any friction traction along the interface, the soil is strained. Small strains in soil are associated with 'elastic' deformation within the 'elastic' stress-strain relations. From the examination of the sphere condition in the S.G. model, it is clear that such deformations are possible as long as the sphere does not move up the groove. An evaluation of the 'minimum' practical friction utilizes the assumption of a 'smooth' surface (the traction is in the orientation of the displacement) on the verge of movement.

The S.G. model is based on the limiting equilibrium of the sphere. Upon initiation of movement, the relations between the intergranular forces at the contact point are \( F = N \cdot \text{tg} \phi \mu \). Figure 8.2a shows the position of the sphere, assuming an infinitesimal displacement in the groove. If the plane is free to displace upwards (e.g., loading of soil/interface in direct or ring shear apparatus), and the grains do not deform or crush, an infinitesimal horizontal displacement \( \delta_y \) will be accompanied by an
upward displacement of the plane \( \delta_z \), where:

\[
\frac{\delta_z}{\delta_y} = \tan \theta \tag{8.2}
\]

The movement of the sphere upwards is made possible by a net horizontal force \( F_y \) and is opposed by the net force \( F_z \), as denoted in Figure 8.2b. The ratio between the two under this displacement is identical to the force ratio of friction of inclined planes:

\[
\frac{F_y}{F_z} = \tan (\phi_\mu + \theta) \tag{8.3}
\]

The ratio of energy supplied to energy transmitted can be calculated using Equations 8.2 and 8.3:

\[
E_R = \frac{F_y \cdot \delta_y}{F_z \cdot \delta_z} = \tan (\phi_\mu + \theta) \cdot \frac{1}{\tan \theta} \tag{8.4}
\]

For conditions of no intergranular friction \( (\phi_\mu = 0) \), there is no loss of energy by heat (friction), and \( E_R = 1 \). The movement of the sphere is that for which the energy ratio is minimal, and is obtained by differentiating \( E_R \) in respect to \( \theta \) and equating it to zero:

\[
\frac{dE_R}{d\theta} = \frac{\tan \theta \cdot \sec^2 (\phi_\mu + \theta) - \tan (\phi_\mu + \theta) \cdot \sec^2 \theta}{\tan^2 \theta} = 0
\]

which is solved by:

\[
\theta = 45 - \phi_\mu / 2 \tag{8.5}
\]
The practical meaning of Eq. 8.5 is that ‘smooth’ surfaces that do not induce shear in the soil will not induce a principal stress rotation (equal to interparticle contact orientation) greater than $45 - \frac{\phi_\mu}{2}$. Noticeable strains in the ‘elastic’ zone of the soil (say $\gamma \leq 1\%$) are expected to be associated with this value.

When trying to predict the interface friction coefficient, the $\theta$ value of Eq. 8.5 can be used in order to assess the ‘minimum’ friction which will not induce shear in the soil itself. Theoretically (and probably practically in a controlled laboratory environment), smoother surfaces may be obtained; however, it seems reasonable to expect that this value will distinguish between one state and the other.

For the simplistic case of round quartzic particles, the obtained minimum $\theta$ is approximately 32°. The minimum $\theta$ angle of Equation 8.5, according to the energy criterion, fits the limiting boundaries based on the continuum approach of Equation 8.1. In ‘normal’ soil conditions $\phi_\mu < \phi'$, and the $\theta$ value of Equation 8.5 is greater than the minimum of Equation 8.1. For no volume change conditions $\phi'$ approaches $\phi_\mu$ (see Section 6.3.3d), and therefore both minimum $\theta$ values will be the same.

The energy approach can also be used to assess the value of $\theta$, assuming particle displacements and volume change. Fig. 8.3a describes the ‘standard’ notation of 2-D rod arrangement, or spheres in an orthorhombic packing (Rowe -1962). The deformation shown in Fig. 8.3b can be analyzed with the assistance of the geometrical relations of Fig. 8.3c, indicating that:

\[
\frac{\delta_2}{\delta_1} = \tan \varphi
\]

Equation 8.4 can be rewritten as:

\[
E_\ell = \tan(\phi_\mu + \varphi) \cdot \frac{1}{\tan \varphi}
\]
for minimum energy conditions, leading to $\varphi = 45 - \phi / 2$. Fig. 8.3d shows the relations between $\theta$ of the S.G. model and $\varphi$ to be $\theta = 90 - \varphi$, leading to a value of $\theta$ in failure of

$$\theta = 45 + \phi / 2$$  \hspace{1cm} (8.6)

For round quartzic particles, the obtained $\theta$ in shear is approximately $58^\circ$, again limited by the value of Eq. 8.1 and equal to it for no volume change conditions.

8.2.3 Sphere and rod packings

Possible rod and sphere packings were discussed in Section 5.2. Table 6.2 presents the different packing possibilities and their associated parameters, as discussed in Section 6.3. The $\theta$ values of the different packings are given in column 11 of Table 6.2. Two $\theta$ values are presented for the F.C.C. packing. $\theta = 19.5^\circ$ is related to the analysis of Thurston and Dersiewicz (1959, see Section 5.3.2 and Figure 5.10) and $\theta = 26.5^\circ$ is related to the analysis of Rowe (1962, see Section 5.3.1c and Figure 5.6). Rowe (1964) demonstrated that his analysis leads to a lower stress ratio (at failure) for the assembly, and therefore should be considered as the appropriate one (see Section 5.3.1e).

Except for the less realistic loosest packing (for which spheres or rods are on top of each other and $\theta = 90^\circ$), the average value of $\theta$ is $30.6^\circ$. This value (probably coincidentally), is in close agreement with the $\theta = 32^\circ$ suggested for quartzic sand based on Equation 8.5, which was developed for spherical particles.
8.3 FAILURE CRITERIA FOR GRANULAR MATERIAL ACCORDING TO THE S.G. MODEL

8.3.1 Introduction

Assuming that the forces T and P acting on the grain are the result of adjacent soil grain motion allows the S.G. model to be assessed as a failure criterion for granular material. In so doing, three goals are attained:

1. The assumption of $\theta$ as described earlier is checked for the failure state.
2. If the predictions for failure condition agree with the measured data, then the calculated interface shear resistance based on the S.G. model can also be used to predict the maximum resistance.
3. The S.G. model can be utilized to enhance the understanding of the basic underlying mechanism of granular material behavior.

The suitability of the S.G. model to failure conditions in granular material is examined in two stages:

1. The interparticle failure criterion is used, through the relations of $tg\phi_\mu$ vs. $\theta$, to predict principal stress rotation in failure from known interparticle friction values. If the obtained orientations (based on the condition of $F = N \cdot tg\phi_\mu$) are correct, then it can be concluded that failure was due to sliding of the particles (or at least the shear resistance was due mainly to sliding).
2. Measured shear strength and the principal stress rotation in failure are then compared to the calculated $tg\delta_s$ corresponding to the above obtained $\theta$ angle. Under these conditions of failure, the calculated shear induced by the adjacent particles ($tg\delta_s$) is expected to be the shear strength of the soil ($tg\phi'$).

It should be emphasized that only for $\beta = 90^\circ$ is the normal to the contact also normal to the groove orientation, resulting in $tg\delta_s = tg\phi_\mu$. 

8.3.2 The interparticle failure criterion

The relations between the principal stress orientation, the interparticle friction coefficient, and the shear strength of the soil are used in Figure 8.4 to compare the continuum approach, using the Mohr–Coulomb failure criterion, to the particulate approach, using the interparticle equilibrium criterion of the S.G. model.

A representative value for the interparticle friction coefficient of quartzic sand is $\tan \phi = 0.5$ (see Section 6.3.3b). Figure 8.4a shows the relations between $\tan \phi$ and $\theta$ for spherical particles. Based on these relations, the limiting interparticle equilibrium for initiation of sliding of quartzic spheres for the range of $30^\circ < \beta < 90^\circ$ is obtained for the groove orientation of $53^\circ < \theta_f < 66^\circ$, where $f$ denotes the failure state. The greater $\beta$ value represents a loose sphere packing or 2-D state, and corresponds to $\theta_f = 53^\circ$. The smaller $\beta$ value, which corresponds to $\theta_f = 66^\circ$, represents a very dense packing of well-graded spheres.

Figure 8.4b shows the relations between $\delta_b$ and $\psi$ based on the analysis of Chapter 7 for the case of $K_i < 1$. For $\delta_b = \phi$ the relations are the standard Mohr–Coulomb failure criterion of $\psi = 45 + \phi/2$, for which the shear conditions in the soil along the interface are identical to those of the entire soil mass (see Fig. 8.1c). These relations show that for $\psi = \theta$ of the above range ($55^\circ$ to $65^\circ$) the shear angle of the soil ranges between $20^\circ < \phi < 40^\circ$, which is expected for loose to dense sand, and corresponds well to the above range of $\beta$ angles.

This first general crude measure indicates that the required interparticle contact–groove orientation for sliding of quartzic sands of different densities matches the familiar relations between the orientation of the major principal stress and the failure plane, as expressed through the corelation between general shear and soil density.

\footnote{In this context, the orientation should also be referred to as the angle between the major principal stress and the normal to the failure plane, see Fig. 8.1c.}
In spite of the frequency with which principal stress directions rotate in field conditions, few experimental studies have been conducted concerning this phenomenon.

Asadi (1975) built a plane strain apparatus in which cubical samples (75mm side) were free to deform under the action of the applied stresses, while monitoring the magnitudes and directions of these stresses. Measurements of the inclination of the rupture layer with the $\sigma_1$ plane in dense Leighton–Buzzard sand samples were reported. These angles, identical to the $\psi$ angles, are plotted in Figure 8.4b and show principal stress rotations in failure ranging from $62^\circ$ to $66^\circ$, and corresponding shear angles, $\phi$, ranging from $45^\circ$ to $51^\circ$. Asadi's principal stress rotations fall within the expected range of the S.G. model for very dense sand. The expected rotation according to the Mohr–Coulomb criterion ranges from $68^\circ$ to $70.4^\circ$, which also presents a relatively nice fit to the measured values.

Roscoe et al. (1967) described test results in a simple shear apparatus capable of measuring sufficient stresses to allow determination of the magnitudes and directions of principal stresses (independently of principal strains) at all stages of a test (see Figs. 8.10, 8.11). These results were added to Figure 8.4b, and differ significantly from the relations of $\psi = 45 + \phi/2$, and from the relations of Figure 8.4a for quartzic spherical particles. The discrepancy may be explained by the relations of Figure 8.4c, where the shape of the grain is considered. Roscoe's results for the 'subangular' particles compare with the limiting equilibrium state for elliptical particles ($R_a = 1.25$) along an interparticle plane of about $45^\circ$ ($\theta_t = \psi_t = 45^\circ$) for medium dense sand ($\phi' = 30^\circ$, $\beta = 45^\circ$) and $55^\circ$ for the denser sand ($\phi = 40^\circ$, $\beta = 30^\circ$).

The differences between the experimental results are probably due to the testing apparatuses and the tested sand. Although both experiments used Leighton–Buzzard sand, Roscoe's sand was better–graded and contained larger particles, having the effect of a lower interparticle friction angle ($\phi_\mu$ decreases with the increase of $d$, see Section 6.3.3b), greater angularity, and larger $\beta$ angles ($\beta$ increases
for better-graded material, see Section 6.3.3a). The simple shear test seems also to indicate a smaller principal stress rotation in shear (reviewed in Section 8.4).

Additional data from biaxial loading of a model made of photo-elastic cylinders of different diameters (Oda and Konishi -1974a, see Figs. 8.10, 8.11) were also added to Figure 8.4b. The expected principal stress rotation based on the measured shear angle of the model \( \psi = 45 + \phi/2 \) does not fit the observed results. However, when the given interparticle friction angle of \( \phi_\mu = 22^\circ \) is used in the S.G. model for 2-D cylinders \( (\beta = 90^\circ) \), the required interparticle (groove) orientation for interparticle sliding exactly fits the measured principal stress orientation of the dense packing \( (\theta_f = \psi_f = 44^\circ) \). This value is also in relatively good agreement with the measured orientation of the loose material \( (\psi_f = 37^\circ) \).

The limited data of Fig. 8.4 suggest that: (1) the shear criterion of a continuum is insufficient to describe the shear orientation of an assembly of particles. (2) The particle moment equilibrium, considering its shape and packing as well as the interparticle orientation and friction, seems to determine the shear orientation and (3) The shear of the different materials corresponds to the interparticle sliding equilibrium equation, suggesting that the shear resistance is mobilized mainly by sliding friction. This will be discussed further in subsequent sections.

8.3.3 The shear strength and the principal stress rotation

The compliance of the underlying principles of the S.G. model with the shear mechanism of granular soils is further examined in Figure 8.5. A comparison is made between the major principal stress orientation at failure \( \psi_f \), and the measured shear strength \( \tan\phi' \), to the groove orientation \( \theta_f \) \( (\theta_f = \psi_f) \) and the interface shear resistance \( \tan\delta_s \). Since the forces acting on the grain can result either from the action of an adjacent grain or from a moving plane, under failure conditions the calculated \( \tan\delta_s \) values are expected to be equal to \( \tan\phi' \) (the S.G. model is then perceived as a granular
shear mechanism model).

This can be further explained by the fact that Figure 8.4 suggested that the interparticle failure criterion used in the S.G. model may correctly model the relations between the groove orientation in failure and the interparticle friction coefficient. If this is correct, then for the same $\theta_f$, the force/stress ratio which leads to this state $(tg\delta_b)$ is identical to the shear strength of the soil.

In addition to the data from Asadi's plane strain apparatus (1975) and Roscoe's et al. simple shear apparatus (1967), Figure 8.5 presents data from Symes (1983). Symes measured the shear strength of Ham River sand using a hollow cylinder apparatus. These data are related to dense, fine, angular quartzic sand subjected to different combinations of $\alpha$ ($\sigma_1$ orientation measured from the vertical) and $b$ values

$$b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$$

under the different loadings. The sum of the failure plane orientation and $\alpha$ was taken as the angle $\psi$, which describes the orientation of $\sigma_1$ from a plane normal to the failure plane. Symes' data of Figure 8.5 are related to $\alpha$ values ranging from 0 to 67.5° and $b$ values ranging from 0 to 1.0. These data were not included in Figure 8.4 as Symes presents values of $45^\circ + \phi/2$ ranging from 7° to 68°, which were not well understood; however, they indicated a lack of correlation between the Mohr–Coulomb anticipated failure plane and the measured one.

The data of Figure 8.5 demonstrate that a good agreement with the most simple representation of the soil grain shape in the S.G. model is obtained, especially when considering the accuracy of the measurements. Further conclusions are discussed in the next section.

8.3.4 Discussion

(a) Conclusions from Figures 8.4 and 8.5

The following conclusions are based on the comparison of Figures 8.4 and 8.5
and should be reviewed cautiously, as only limited data are available.

I. Based on Mohr–Coulomb failure criterion, the angle between the principal stress orientation and the failure plane is a function of the continuum shear angle (see Fig. 8.1c). The granular material shear angle, on the other hand, does not control the angle between the major principal stress orientation and the failure plane, but is rather a result of it. The principal stress orientation seems to be determined by the grain shape, the intergranular friction and the loading mode. This is further supported by the findings of Section 8.6, and can also be explained by the observations of Rowe (1962) and Oda (1972b), and the experimental data of Chapter 5, which strongly suggest that the shear zone is formed as a consequence of the initiation of relative movement and failure, rather than being their cause.

II. The granular material shear angle seems to be a function of the internal friction angle $\phi$, but in a different way than that suggested by Rowe. Rowe assumed that the local granular contact determines the mass shear angle (see Figs. 5.4 and 5.5), and his $\alpha$ angle is equivalent to the S.G. model $\theta$ angle during sliding and for $\beta = 90^\circ$ (see Fig. 8.1d). However, the inclination of the contact prior to the shear is determined by the principal stress orientation, such that the particles orient themselves (mostly by rolling) prior to reaching the sliding limiting equilibrium condition. The predetermined orientation which was chosen by Rowe (1962) was correct, therefore, only during sliding and/or for well–arranged sphere packings, where no ‘extraneous’ motion occurred and the shear took place as an ‘instant’ movement of two ‘blocks’ of particles. This is further supported by the observation of Rowe (1963):

"A series of triaxial compression tests and direct measurement of $\phi$... were measured independently, established that equation 10 was valid for dense sand, overconsolidated or reloaded soils... On the other hand, loose, normally consolidated soils on first loading suffered additional loss of energy in friction..."
due to deviations of $\beta$ from the preferred direction, associated with a process of rearranging of the particle assembly during deformation."

These results confirm the above suggested mechanism and comply with the discussion of Chapter 5, which explains the difference in shear strength and stiffness of different soil samples by the reorientation of particles in the pre-loaded samples (e.g. the difference between soil samples which were prepared by plunging and tapping), such that 'chains' of contacts are formed in the major principal stress direction. Once this preferred orientation was formed, then the direction of the contacts chosen by Rowe was also normal to the major principal stress orientation, showing his assumptions to be correct, therefore, only for the pre-loaded material.

III. The above assumptions are supported by the data of both Figures 8.4 and 8.5. The predicted inclination of the failure plane, based on limiting intergranular equilibrium, matches that measured, as shown in Figure 8.4. The predicted relation between the shear strength of the soil and the inclination of the failure plane seems to agree with the measured relations in Figure 8.5.

IV. The data of Figure 8.5 suggest no clear correlation between the shear strength of the soil and the relative density of the sand, as expressed by the $\beta$ angle. When this issue was discussed in Section 6.3.3a for the interface shear resistance prior to the soil failure (during sliding), it was explained that such predictions are supported by experimental data (demonstrated in Sec. 8.5). The values of Fig. 8.5 are related to the shear strength of the soil and need additional consideration, as follows.

(b) The frictional component of the shear resistance.

I. In the case of cohesionless soil composed of mainly rotund particles, the residual state may be identified with the critical state ($\phi'_r = \phi'_{cv}$). In the
critical state, the void ratio reaches a constant value (critical void ratio) at which the soil continues to deform as a frictional fluid with constant strength and a constant volume. A separation of the frictional component from the dilatational component (see e.g. Rowe –1962, Scott –1980) can therefore be achieved by determining the critical state strength. A convenient way to do so is by plotting the strength (or strength ratio) against the rate of volume change in failure \((d\varepsilon_v/d\varepsilon_i)_f\). The soil strength for the zero rate of volume change at failure is the critical state one, which is identical to the frictional component of the total shear strength. Separation of frictional and dilatational components showed that an increase in shear strength (as measured by \(\phi'\)) with density is primarily due to the dilatational component, and that work done in overcoming frictional forces (measured by \(\phi'_f\)) is almost unchanged (Bishop –1971; see also Figure 8.6a–c).

II. The critical state situation can be represented in the S.G. model by the \(\beta = 90^\circ\) condition, where the model reflects a pure frictional material (the normal to the interparticle contact is also normal to the groove), for which \(P = N\) and \(\tan\delta_s = \tan\phi'_\mu\) (see Fig. 6.1b and Section 6.2.2d). The model in that state also shows a loose material under conditions where maximum frictional forces can be developed. This state can be achieved in the rearrangement of the failure zone of the dense material after the dilatancy and the peak shear resistance. The S.G. model indicates, therefore, that the shear resistance at the critical state is equal to the frictional resistance component of the general shear, which equals the interparticle friction coefficient (for \(\beta = 90^\circ\), \(\delta_s = \phi'_\mu = \phi_{cv}\)). The critical state shear resistance and the interparticle friction will be determined
by the grain material and shape\textsuperscript{2}. The interparticle friction will also be determined by the grain size.

III. A series of samples of Ham River sand of different initial densities tested in triaxial compression was used for a plot of the type described in (I), and indicated one value of $\phi'_{cv} = 32.8^\circ$ (Bishop -1966). These data were added to Figure 8.5 for comparison with the relations of $\tan \phi'$ vs. $\psi$ of Symes (1983), also measured for the Ham River sand. The value of $\tan \phi'_{cv}$ meets the angular ($R_a = 1.25$) 'dense' ($\beta = 45^\circ$) curve at $\theta = 63^\circ$. Relating this value to the maximum frictional resistance ($\beta = 90^\circ$) shows that for $R_a = 1$ and $\beta = 90^\circ$, $\theta = 65.5^\circ$. Both angles are in very good agreement with the average of Symes' measurements of $\psi = 63.5^\circ$.

Cole (1967, see Wood et al. -1979) reported that the constant volume angle of Leighton-Buzzard sand is $\phi'_{cv} = 35^\circ$. This value was also added to Figure 8.5 and suggests similar results. However, in order to better match the analysis to the measured data it seems that greater angularity is required for the modeling of the Leighton-Buzzard sand.

IV. The following should be noted in reference to Fig. 8.5:

(a) Symes (1983) notes that the triaxial tests are often conservative in comparison to the Hollow Cylinder apparatus. The difference between his shear strength results and Bishop's $\phi'_{cv}$ may also be due in part to the difference in testing techniques.

(b) This kind of interpretation leaves the question of the influence of $\beta$ on the calculated value. The angle $\beta$ represents the angle of the groove opening, and

\textsuperscript{2}Section 6.3.3b deals with the interparticle friction coefficient, and suggests that the techniques which employ the measurements of $\phi \mu$ for a mass of particles are preferable. In this case, the interparticle friction coefficient will also be a function of the grain shape, as supported by data presented in the following section.
was correlated to different densities. As explained in (II) above, when comparing calculated \( \tan \gamma \) to \( \phi'_{cv} \), it seems reasonable to use only the case of \( \beta = 90^\circ \), where the friction angle is compared to the results of the analysis leading to the maximum friction resistance, and \( \tan \gamma = \tan \phi' \mu \). This approach is used in the next paragraph, where the grain shape remains the only factor controlling the shear resistance (for materials with the same intergranular friction angle).

V. The mechanism of the S.G. model reviewed above is further supported by the fact that the soil shear angle (\( \phi' \) and \( \phi'_f \)) is a function of the shape of the sand grains, a factor which is considered by the model (see Section 6.3.3d). Figure 8.6a–c presents experimental results from drained triaxial compression tests \( (\sigma_3 = 30 \text{ psi}) \) on saturated quartz samples of different grain shapes and identical grain size, reported by Koerner (1970). The drained shear angle (\( \phi'_d \)) is presented in Figure 8.6a, and is a function of the grain shape and the relative density. The dilatational component (\( \phi'_d \)) is presented in Figure 8.6b, and is mainly a function of the relative density. The frictional component (\( \phi'_f \)) of the shear angle (\( \phi'_d = \phi'_d + \phi'_f \)) is shown clearly to be a function of the grain shape only (Figure 8.6c). These data are compared in Figure 8.6d with predictions of the S.G. model for \( \beta = 90^\circ \) and elliptical grains. Sand which is described as rounded and subrounded matches the spherical shapes (\( R_a = 1 \)); the subangular match an ellipse of \( R_a = 1.1 \); and the angular needed some correction, from \( R_a = 1.25 \) to \( R_a = 1.185 \). The \( \theta \) angles were taken as \( \theta_i = \psi_f = 45^\circ + \phi/2 \), which seem to be a good approximation for triaxial compression test results. The friction angles of Leighton–Buzzard sand (\( \phi'_f = 35^\circ \)) and Ham River sand (\( \phi'_f = 32.8^\circ \)) were also added to Figure 8.6d with \( \theta \) angles from the average measurements of Asadi (1975) and Symes (1983), respectively, as presented in Figure 8.5. The Leighton–Buzzard sand is
described as subangular, and fits well with the subangular representative value of $R_a = 1.1$. The Ham River sand checked by Bishop (1971) was not described by him, and is assumed to be angular, as described by Symes.

VI. Additional data which compare results of interface friction resistance tests with predictions of the S.G. model are presented in Section 8.6. These results are in agreement with the calculated values for different roughness conditions of the moving plane, from the minimum value (due to the orientation shown in the next section) to the full mobilization of the soil shear strength. These data, shown in Figures 8.24, 8.25, and 8.26, provide additional support to the S.G. model consideration of failure.

(c) Conclusions

I. The present discussion suggests that a ‘micro’ limiting equilibrium analysis on a representative grain (according to the S.G. model) results in a description of failure of a granular mass. The analysis considers the average interparticle friction and grain shape, and assumes the representative interparticle contact to be normal to the major principal stress orientation. By checking the limiting equilibrium along an interparticle plane of a particle subjected to shear and normal forces along a ‘rupture’ surface, the relation between the intergranular friction and the intergranular plane orientation (normal to the major principal stress) is obtained. The combination of the shear and normal components which brought about the limiting equilibrium along the intergranular plane is assumed to be equal to the soil’s shear strength.

II. Although the S.G. model differs from Rowe’s method of analysis, both confront the same fundamental question concerning the validity of the relations between the interparticle friction coefficient and the shear strength of granular soils. This question is debated in the soil mechanics literature (e.g
Horne —1969, Skinner —1969, Ochiai —1976, Bishop and Skinner —1977, Lupini et al. —1981), and an in—depth examination of the approaches taken by the different researchers is beyond the scope of the present work.

III. The S.G. model indicates that a definite correlation exists between the interparticle friction coefficient and at least the frictional component of the shear resistance (and possibly with the overall resistance as well). While theoretically valid for all granular materials, most data presented so far relate to sand. The only data to clearly contradict these findings are those of Skinner (1969 — reviewed in Chapter 5). A critical discussion of Skinner’s data is therefore in order.

(d) Critical review of Skinner’s data.

I. Section 5.5.4 reviews the measurements of Oda and Konishi (1974) and Konishi (1978), and explains that although sliding was confined to a small number of preferred contacts, these findings do not yet contradict Rowe’s model, as only the particles along the failure ‘saw—tooth’ line are sliding. These data and the turbulent shear observed by Lupini et al. (1981) can be explained by the S.G. model. As opposed to Rowe’s model, no ‘failure’ line is expected to form by the S.G. model, but rather a zone in which most particles would roll and a few will reach the limiting equilibrium of sliding. This picture explains three facts: (1) only a few particles are sliding; (2) the rolling resistance is very small (practically negligible); thus, the sliding resistance remains the major frictional resisting mechanism; (3) a ‘zone’ of turbulent failure rather than a ‘failure line’ is formed, and probably advances in stages from one area to the other, along with the progression of the relative displacement.

II. Skinner, in attempting to disprove the relation between shear resistance and
interparticle friction, performed two types of experiments: (1) direct shear in which the lower part of the shear box was replaced by a plate; (2) direct shear on an assembly of particles. Two facts should be noticed regarding these tests: (1) all of them were performed on idealized material (glass, lead and steel balls); and (2) there is a fundamental difference between the two tests, even though the interface shear tests were performed on the same materials (e.g. glass spheres on a glass plate). The suggestion of the existence of a limiting friction angle (Section 7.4.4c) raises questions regarding the interpretation of Skinner's results, and possibly also Rowe's (1962) interparticle friction tests.

III. Figure 8.7 shows theoretical and experimental relations between $\phi$ and $\phi_{cv}$ presented by Bishop and Skinner (1977), consisting mostly of the relations presented by Skinner (1969). Skinner's data relate to results of shear box tests on a mass of spheres such as that shown in Figure 5.22. In reference to Fig. 8.7a, Bishop and Skinner concluded the following:

"Lack of direct relation between $\phi'$ and $\mu$ is associated with the complexity of particle movement in a particulate mass subject to a shear strain, which involves not only interparticle slip but particle rotation and out of plane displacements even under an overall plane strain displacement."

IV. Based on the data of Fig. 8.6d it is expected that the relations between $\phi'_{cv}$ ($= \phi_f$) and $\psi_f$ ($= \theta_f$) would match the analysis of the S.G. model for the maximal friction resistance ($\beta = 90^\circ$ and $R_\alpha \approx 1.0$). Under these conditions, the normal forces $P$ and $N$ are maximal and equal ($N$ in the direction of $\sigma_1$ is normal to the interparticle contact, not only to the groove), and $\tan \phi'_{\mu} = \tan \delta_s$ (see Section 6.2.2d for equilibrium conditions around the grain's center of gravity). As $\tan \delta_s = \tan \phi'_{\mu} = \tan \phi_{cv}$, for $\beta = 90^\circ$ and $R_\alpha = 1$ we have linear relations of $\phi'_{\mu} = \phi_{cv}$, as drawn in Fig. 8.7b. Using data from Rowe (1962), experimental relations of $\phi'_{\mu}$ vs. $\phi_{cv}$ were added to Fig. 8.7b. The data consist of $\phi'_{\mu}$ from direct measurements and $\phi_{cv}$ values from triaxial compression tests...
for zero rate of volume change in failure, as previously mentioned. Excellent agreement exists between the expected and measured relations for all different soils (silt, fine sand and coarse sand) and idealized assemblies of glass and steel spheres. The data referring to the steel spheres with an interparticle friction angle of 70° diverged somewhat from the expected relations.

V. The suggested relations, although agreeing in some points with Skinner's data, still diverge substantially in others, leaving the question of whether the disagreement is due to the testing procedure or to the lack of correlation between $\phi_\mu$ and $\phi_{cv}$.

VI. Section 7.4.4 suggests that when the normal stresses perpendicular to an interface are greater than those parallel to it ($K_i > 1$), and failure conditions exist in the soil (while forcing a plane of shear along the interface), the shear resistance along the interface is limited by the failure shear stress $\tau_f$, leading to a limiting friction angle $\delta_p(\phi)$ as described by Equation 7.15.

VII. If both assumptions are correct: $\phi_\mu = \phi_{cv}$ and $\delta_p(\phi)$ is the limiting friction angle, then Skinner's tests, which were conducted on a shear box where a plate of the same material replaced the bottom part, should agree with the calculated relations. The measured $\phi_{cv}$ should be in agreement with the calculated $\delta_p(\phi)$ for $\phi = \phi_\mu$. These relations are also plotted in Fig. 8.7b, denoted by "direct shear box on interface".

VIII. Skinner's results on 3 mm diameter glass ballotini, where the bottom part of the shear box was replaced by a plate of glass, are shown in Figure 5.23. These results were added to Fig. 8.7b and denoted by "3 mm glass dry or flooded" for $\phi_\mu = 1.1^\circ$ to $8.0^\circ$ and $\phi_\mu = 31.4^\circ$ to $42.0^\circ$, respectively. Both results are in good agreement with the calculated relations, especially for the higher $\phi_\mu$ values ($\phi_\mu > 10^\circ$).

IX. Although data to confirm the calculated relations are limited, Skinner's data
for the shear box results on the sphere assembly definitely follow the calculated trend. As expected, they fall in between the two bordering relations for $\beta = 90^\circ$ and $R_a = 1$, which would be valid when the sample is ‘free’ to develop its shear orientation, and the $\delta_p(\phi)$ for forced shear along an interface.

X. The above analysis indicates that the data upon which Skinner’s (1969) conclusions:

"... both the effective angle of shearing resistance at constant volume $\phi'_{cv}$ and at peak $\phi_{\text{max}}$ for a given initial porosity do not increase monotonically with increase in the interparticle angle of friction $\phi$."

were based may be interpreted in a different way, and do not reflect a lack of correlation between the interparticle friction coefficient and the shear resistance.

XI. Additional attention should be given to Skinner’s data concerning the assemblies with interparticle friction angles smaller than $10^\circ$. Unfortunately, Skinner (1969) did not describe the method used for the interparticle friction measurements other than "... interparticle contacts measured directly with a specially developed ‘friction apparatus’ ".

As previously reviewed (Section 6.3.3b), Skinner’s ‘dry’ coefficient of friction exhibited substantial variation:

* 1 mm diameter glass — 0.03 to 0.09
* 3 mm diameter glass — 0.03 to 0.12
* 1/8" steel balls — 0.29 to 0.66
* 3 mm diameter glass balls sliding on a glass plate — 0.02 to 0.14

While the ‘flooded’ spheres show a large variation as well (see Fig. 8.7), they are within an acceptable range in relation to the absolute magnitude.
However, variations which are up to 7 times the absolute value seem dubious, and most likely confirm Proctor and Barton's (1974) observations (see Section 6.3.3b) that the reliability of the values obtained under 'dry' conditions are questionable, especially if the surfaces are smooth\(^3\) (as in Skinner's experiments).

XII. As mentioned in Section 8.1, different measurements and analyses concerning the direct shear box support the above conclusions. Some are briefly mentioned here, and will be further discussed in section 8.4.

- Hansen (1961), regarding shear box tests on sand:

  "the rupture value of the shear stress \(\tau_{nt}\) obtained during the test (shear box) must depend not only on the normal stress \(\sigma_n\) but also on the stress \(\sigma_t\) acting on planes vertical to the shear direction."

- Morgenstern and Tchalenko (1967) regarding microscopic structures in kaolin (with a very small true cohesion) subjected to direct shear (see Fig. 8.14):

  "The sequence of microscopic shear structures shows that in both cases (samples with original bedding normal and parallel to the shearing direction), the features at peak strength result from simple shear conditions and that a continuous horizontal structure appears only towards the residual stage. Intermediate structures are interpreted in terms of kinematic restraint imposed by the configuration of the box." and "... inclination of shear planes at failure in the shear box is not in the direction of the imposed relative displacement. Slip along these planes is therefore impeded in contrast to the conditions of the triaxial test."

\(\text{\bf (e) The controlling factors for failure criterion} \)

I. Figures 8.4 and 8.5 suggest that the limiting equilibrium analysis which considers particle rolling (not considered by any of the relations in the original version of Fig. 8.7), taking into account the grain shape and interparticle friction coefficient, has promising potential to accurately predict the relations

\(^3\)See also Lambe and Whitman (1969, Chapter 6) for the effect of surface water, cleanliness, and roughness on the interparticle friction,
between the interparticle friction coefficient and the orientation of the failure
zone, and between this orientation and the shear strength of the material.

II. Figures 8.6 and 8.7 verify that the frictional component of the shear resistance
of granular material is controlled mainly by the interparticle friction angle. A
model which considers both rolling and sliding can therefore correctly predict
the mechanisms and shear resistance under different stages of loading, prior to
and during shear.

III. The presented analysis and data can shed some light on the inconclusive
review of Section 6.3.3b regarding the interparticle friction coefficient of
quartzic materials. The present section suggests that these data showing the
interparticle friction coefficient to be a function of the grain size are probably
reliable, as no other correlation between the shear strength and the grain size
can be explained theoretically and/or experimentally. The relations calculated
in Equation 6.16 for the increase in the interparticle friction with the decrease
in the particle size (based on data from Rowe –1962) are supported by Proctor
and Barton (1974). Additional data with the same trend (with some
differences in the absolute values — probably due to the difference in the grain
shape) are shown in Figure 8.8. Fig. 8.8 presents the relations between the
frictional component of the shear resistance ($\phi_f$) vs. $D_{10}$ for quartzic particles
(see details in the Table of Fig. 8.8), obtained by Koerner (1970) in triaxial
compression tests. Assuming direct relations between $\phi$ and $\phi_f^4$ (as suggested
by the S.G. model), Koerner concluded his findings with the following:

"Explanation of $\phi_f$ increasing as $D_{10}$ decreases is difficult if a mineralogical constant
for physical friction is insisted upon... it appears that $\phi\mu$ (rather than being a
mineralogical constant) increases with decreasing particle size" and "therefore it

---

4King and Dickin (ASCE JSMFD –Sept. 1970) also show that triaxial tests on dense
samples lead to $\phi_f$ values approximately equal to $\phi\mu$, and $\phi_f$ varies but lies within 2o of
$\phi_{cv}$. 
appears that $\phi \mu$ (the physical friction component contained in $\phi_f$) may not be the inherent and constant mineral property as often postulated. It seems conceivable that due to the rapid increase in surface area, of these fine particles, increased surface energy might give rise to higher $\phi \mu$ values. Note that along the $D_{10}$ axis (of Fig. 8.8), the curve appears to be approaching an asymptotic value of $\phi_f$ about 26°.

The value of $\tan \phi \mu = 0.5$, chosen as a representative value in section 6.3.3b, is in good agreement with the asymptotic value of Fig. 6.33. Further use of the S.G. model should consider the material grain size through its influence on the interparticle friction coefficient.
8.4 SIMPLE SHEAR AND PRINCIPAL STRESS ROTATION

8.4.1 Introduction

Data concerning principal stress rotation in failure were discussed in Section 8.3. Figure 8.9a presents the outside and inside deformations in simple shear of soil within and adjacent to an axially loaded pile (based on the approach of Randolph and Wroth —1981, and the analysis of Chapter 7). The soil is brought to a state of failure by the action of strain increments, whose principal directions are not parallel to the original directions of the principal stress axes throughout the process of shearing. In the laboratory, the Cambridge simple shear test device has been developed (Roscoe —1953) in order to test the soil under conditions such as those shown in Figure 8.9b.

The applied normal stresses in the simple shear test ($\sigma_{yy}$) correspond to the radial stress $\sigma_r$, normal to the pile wall. Before application of shear, both stresses ($\sigma_{yy}$ and $\sigma_{xx}$) are principal stresses. However, $\sigma_{yy}$ may be a major or minor principal stress, depending on the stress history of the sample (namely, its O.C.R.). The Cambridge simple shear device is the only one which provides information on the principal stress rotation during loading in simple shear. Referring to Fig. 8.9, the measured angle of the principal stress rotation in the Cambridge apparatus ($\psi$) is identical to the angle $\psi$ used in this study to describe the principal stress rotation. Some of these data are presented and discussed below.

8.4.2 Principal stress rotation under small strains

Equation 8.5 describes the preferred interparticle contacts based on minimum energy consideration. It is expected that under a sliding motion of the moving plane, contacts will be formed in this orientation as long as only minimal shear strain will be induced within the soil mass, due to the low sliding interface shear stresses. This situation seems to be similar to the first stages of the simple shear loading under small displacements. Figs. 8.10 and 8.11 present the results of a simple shear test on dense
and loose sand, reported by Roscoe et al. (1967). These figures show that the lowest recorded principal stress rotation under the minimum meaningful displacement is \( \psi \leq 20^\circ \). Fig. 8.12 presents rotation of principal stresses from 6 tests on Ottawa sand in loose to medium dense states, reported by Budhu (1988). The measurements were performed on a new simple shear device and show variation of principal stress rotation for very small strains, from \( \psi = 25^\circ \) to \( 35^\circ \).

Admittedly crude (but the only data available), the presented measurements of the simple shear apparatus suggest that under very small strains, which are associated with the smallest displacement, the measured principal stress orientation is within crude proximity to that suggested in Equation 8.5. These data reinforce the previously suggested view that due to sliding of the interface, the interparticle contact \( \theta = \psi \) can be estimated using Eq. 8.5. Comparisons between this approach and measured data of a ‘sliding’ plane are presented in Section 8.5.

**8.4.3 Additional relevant observations concerning the simple shear test device**

1. Figs. 8.10 and 8.11 also present test results of dense and loose models made of assemblies of photo-elastic cylinders (Oda and Konishi -1974a,b). These results are very similar to those presented by Roscoe, and verify again the assumption of principal stress normality to the interparticle contact.

2. Roscoe’s results clearly indicate that the orientation of the plane of maximum obliquity is not horizontal during the simple shear test (see variation of \( \omega \) with the increase of \( \gamma_{yz} \) in Figs. 8.10b and 8.11b). As mentioned earlier, the state of stress in the direct shear test was found during shear to be similar to that of the simple shear test (Morgenstern and Tchalenko -1967, Potts et al. -1987, and others). These combined facts further support the previous assumption regarding the limiting friction angle along the interface and the interpretation of Skinner’s data of Section 8.3. Figure 8.13 presents the rupture zone in sand
(sketch of a radiograph) for different sands tested in the Cambridge and the NGI (circular samples) apparatuses (Wood and Budhu –1980). Fig. 8.14 presents a sequence of structures developed in Kaolin tested in a direct shear apparatus (Morgenstern and Tchalenko –1967). The visual evidence for the divergence of the rupture zone from the orientation of the applied displacement supports the above conclusions, and explains the possibility of limiting friction along the interface while rupture takes place in the soil itself. For further discussion concerning the plane of maximum obliquity and the plane of maximum shear stress in the simple shear test, see Ochiai (1976).

3. Linear relations between $t\text{g}\psi$ and the ratio of shear to normal stresses ($\tau_{yx}/\sigma_{yy}$) were suggested by Wood et al. (1979) (see also Airey et al. –1985). These relations, although supported mainly by data of non–failure conditions, are used by different researchers (e.g. Airey et al. –1985, Ochiai –1976) as a basis for a failure criterion. A review of these relations suggests that the interpretation of the ratio $\tau_{yx}/\sigma_{yy}$ vs. the shear strain depends on the assumptions made for the correction of the measured forces and eccentricities (Wood et al. –1979). It is believed that if equilibrium of forces and moments was used in the data interpretation rather than equilibrium of forces only, the above relations would not have been linear. For that reason these relations are not used for comparison with the expected principal stress rotation.

4. From the data of Figs. 8.4, 8.5, 8.10 and 8.22, it seems that the measured principal stress orientations during failure in the Cambridge simple shear apparatus are lower than the other available data.
8.5 PREDICTIONS AND MEASUREMENTS OF THE INTERFACE SHEAR RESISTANCE OF SLIDING SMOOTH SURFACES

8.5.1 Introduction

The preferred intergranular contact orientation according to the minimum energy solution of Section 8.2.2 is \( \theta = 45 - \phi / 2 \) (Eq. 8.5). This value was supported by the measured principal stress rotation of the simple shear test under small strains (Section 8.4.2). It is expected, therefore, that surfaces which slide over and do not induce shear in the soil mass will only induce small strains, such that the particles in contact will be reoriented into angles equal to or less than those calculated according to the minimum energy solution.

8.5.2 The prediction of the S.G. model

For quartzic sand subjected to a smooth moving plane, the parameters for the S.G. model are:

\[
\phi \approx 26^\circ \rightarrow \theta \approx 32^\circ, \quad \alpha = 0.
\]

Table 8.1 summarizes the S.G. model predictions for the friction coefficient of smooth surfaces on quartzic sand using the parameters in Equation 6.13 for different grain shapes (\( 1 \leq R_a \leq 1.25 \) for well-rounded to angular) and densities (\( 45^\circ \leq \beta \leq 90^\circ \) for dense to loose packings, according to Table 6.2).

8.5.3 Experimental results from the ring torsion apparatus

(a) The Apparatus

Yoshimi and Kishida (1981a,b) used a simple shear ring torsion apparatus (Yoshimi and Oh–oka –1973) to measure the friction between sand and metal surfaces (see Fig. 8.15). The ring–shaped metal specimens are 24 mm wide with a 240 mm inside
diameter. The sand specimens are 9 mm to 42 mm high, and were subjected to normal load and circumferential shearing stress. As the container of the soil specimen consisted of a stack of lubricated rings, it could follow the shearing deformation of the sand while maintaining a nearly constant width of the soil specimen. Thus, the soil specimen deformed under nearly plane strain conditions with the ability to form progressive failure, as the specimen has no 'ends' in the circumferential direction.

\textbf{(b) The Tests}

Use of metal surfaces of different roughness and sands of different types and densities allowed Yoshimi and Kishida to check the influence of both on the interface friction. The surface roughness of their measurements is related to peak to peak value along a distance of 2.5 mm. This type of measurement does not match the roughness concept of the S.G. model expressed through Equation 6.10a, as the grain size of the tested sand was approximately 0.2 mm and the roughness was measured by Yoshimi and Kishida along a distance of approximately 12 times that length. The measurements using the lowest surface roughness refer, however, to a 'smooth' surface (see Section 6.32 and Fig. 6.7), and are summarized in Table 8.2. These values may be compared with the S.G. model predictions of Table 8.1, and are discussed below in detail.

\textbf{(c) Test results from rounded sand.}

Figure 8.16a presents the measured interface friction coefficients for shear between quartzic (90\% quartz, $\phi\mu = 26^\circ$), rounded$^5$, Toyoura sands of different densities and metal surfaces of different roughness (Yoshimi and Kishida 1981a). The data of Figure

\footnote{Toyoura sand is also described as subrounded (Uesugi and Kishida -1986b) and of only 80\% quartz (Uesugi and Kishida -1986a) which may have resulted from using a different grain gradation.}
8.16a indicate the following:

I. The measured friction coefficient of the smooth surfaces fits excellently with the calculated values of the S.G. model for round particles \((R_a = 1)\).

II. The measured friction coefficient for the different sand densities matches the expected behavior based on the S.G. model. The different predictions for dense \((\beta = 45^\circ)\) to loose \((\beta = 90^\circ)\) sands were plotted in Figure 8.16a, along with lines connecting the average values of the measured data according to the different sand densities. The densest sand \((D_r = 90\%)\) has the lowest friction resistance, and corresponds to the prediction of the most dense possible packing of spherical particles \((\beta = 45^\circ\), see Table 6.2). The loosest sand \((D_r = 40\%)\) exhibits the greatest frictional resistance, and corresponds to the predicted friction of the loosest possible packing \((\beta = 90^\circ)\). The medium dense sand \((D_r = 60\%)\) exhibits intermediate resistance, which corresponds to the predictions for the medium dense packing \((\beta = 60^\circ)\).

III. The lines which were added to Fig. 8.16a, describing the frictional resistance according to the different sand densities, clearly indicate that the above observed friction relations continue to be consistent until the surface roughness becomes great enough to induce shear in the soil mass itself\(^6\); i.e. the densest soil continues to show lower frictional resistance than the loosest or the medium dense soil.

IV. Data related to surfaces other than steel (brass, aluminum, wood and concrete) were also added by Yoshimi and Kishida to the presented relations of Fig. 8.16a. These data clearly indicate that the material of the body in

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\(^6\)According to Uesugi and Kishida –1984 (referring to Yajima et al. –1984), the internal friction angle of Toyoura sand from triaxial compression tests is \(\phi' = 42^\circ\). The plane strain tests, however, usually show a shear strength of approximately 10% less (Parry –1971, Bowles –1977).
contact has no influence on the frictional resistance, other than through the roughness of its surface.

\[
(d) \textit{Test results from angular sand}
\]

Figure 8.16b describes the relations between the surface roughness and the interface friction coefficient for Tonegawa sand (Yoshimi and Kishida –1981b). This sand consists mainly of 23\% quartz, 21\% chert and 44\% dark minerals. Considering the fact that chert is a siliceous rock, the dark minerals are most likely igneous, and therefore will have a hardness of about 6 (compared to 7 of quartz). Since the soil particles of Tonegawa sand are slightly larger than those of Toyoura, but have a rougher grain surface, it seemed reasonable to assume that the interparticle friction angle of this sand will also be approximately $\phi_\mu = 26^\circ$. Some back calculation of an analysis presented by Yoshimi and Kishida (1981a) supports the above value, and indicates that they used a $\phi_\mu$ value of approximately $24.5^\circ$. Tonegawa sand is described as ‘angular’; the measured results of Fig. 8.16b are therefore compared to the S.G. model predictions for angular grain shape ($R_a = 1.2$).

The comparison between the calculated values according to the S.G. model and the measured values, as shown in Tables 8.1, 8.2 and Fig. 8.16b, further support the above assumptions derived from Fig. 8.16a. The predicted results considering the grain shape show excellent agreement with the experimental measurements. Again, the higher density sand consistently exhibits lower values of friction than the medium or loose sand, as long as the soil does not shear. For the very rough surfaces, where the soil is sheared, the denser sand exhibits a greater shear resistance than the medium and the loose sand, due to differences in the shear mechanism (the effect of the dilatational component on the friction angle).
(e) The sliding and shearing mechanism of interfaces

The different shear mechanisms of ‘sliding’ and ‘shearing’ of interfaces, as suggested by the S.G. model, are elucidated by the observations of Fig. 8.17 concerning the interface shear tests of Tonegawa sand and steel surfaces (Yoshimi and Kishida—1981b).

I. Fig. 8.17a shows that the friction coefficient increases with the surface roughness (quantitative comparisons to the S.G. model predictions will be presented in the next section).

II. ‘Smooth’ surfaces (R_{max} \leq 31\mu m) do not cause dilation, because they slide over the particles. ‘Rough’ surfaces, (R_{max} \geq 220\mu m) cause soil dilation, due to shear of the soil mass.

III. Fig. 8.17a shows that all surfaces induce strains in the soil. The detail of Fig. 8.17a suggests that even the very smooth surface (R_{max} = 3\mu m) induced a shear strain of\gamma = x/h \leq 1.5\% prior to sliding. The initial displacement induces rearrangement of particles to a preferred orientation, which induces minimal strains in the soil without shearing the soil mass, and is in agreement with the previously presented measurements of the DSS under very small strains. This induced strain across the sample, as shown in Fig. 8.17b, supports the idea of using a preferred interparticle contact orientation according to the minimum energy principle, which was suggested in Eq. 8.5 and used in the calculations for the values of Table. 8.1. Although for very smooth surfaces the principal stress rotation can be smaller than that of Eq. 8.5, the ‘practical’ smooth surfaces used according to the machinists’ notation

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7The accuracy of the data is questionable due to the limitations of the X-ray method (see for example Roscoe et al. —1963 and James —1973). Considering the size of the lead marker (\approx 0.75mm diameter) and the height of the sampled soil (h = 22mm), a shear strain of 1.5\% corresponds to a maximum movement of 0.33mm, which is less than half of the maximum grain size (\approx 0.71mm) or the lead marker (see Fig. 8.17c for example of an actual X-ray photograph)
seemed to agree with the suggested values. This observation is further supported in the next section. The only measurement point (one) of a lower friction angle is reported by Uesugi and Kishida (1984, referring to publications by Yajima et al. 1984), where an interface friction coefficient of $\tan \delta_s = 0.14$ was measured for Toyoura sand and a smooth surface with $R_{max} = 0.6 \mu m$, measured along an interval of $L_{max} = 0.2 mm$. Furthermore, the detail of Fig. 8.17a (for strains $\gamma \leq 1.5\%$) suggests that the initial 'preferred' interparticle contact orientation according to the minimum energy solution can be used for all surface roughnesses. A quantitative comparison between measured and calculated values will be presented in the next section.

IV. The very smooth surfaces exhibit a well-defined stick-slip behavior, for surface roughnesses of $R_{max} \leq 5 \mu m$ (see Fig. 8.17a). This type of behavior reflects the difference between the static friction of the surfaces and the lower kinematic friction during the slip itself. In general, when referring to the 'micro' theory of friction, the friction between surfaces depends on three main factors: (a) the intimacy of contact, which depends on the cleanliness of the surfaces; (b) the yield (flow) stress, which determines the areas of real contact between the asperities (the Adhesion Theory of Friction); and (c) the shear strength, which determines the strength of the formed junctions. The experimental variations which alter one or more of these factors will be reflected to a certain extent by the measured friction. The torque in the described experiments was applied in such a way that the metal surface moved at the extremely low rate of 0.6 mm per minute. Under such conditions, the resultant strength of the junctions formed is greater than that occurring at a higher speed of sliding. This means that for our way of friction representation, the static friction is greater than the kinematic friction (for references supporting this observation, see Bowden and Tabor 1945). Therefore,
because the sliding surface has a certain degree of elastic freedom, the motion is not continuous, but rather intermittent, and proceeds by a process of stick-slip. This motion clearly depends on the mechanical properties of the system; even when the moving parts are extremely rigid, the surface irregularities may be capable of microscopic elastic deformation, as was measured by Khaikin et al. (1939, refer to Bowden and Tabor—1945) using quartz crystals.

V. The solid symbols with arrows in Fig. 8.17b, indicate horizontal displacement of the metal ring from the initial position, while the open symbols show the displacement of the lead markers within the sand. The smooth surface shows that slip (denoted by \( x_s \)) occurs after a very small straining of the soil mass (\( \approx 1\% \)), while for the very rough surfaces, there is no slip at all (\( x_s = 0 \)), and shear takes place in the soil itself. The soil was first strained uniformly to about \( \gamma \approx 23\% \). A zone of rupture was then observed between the top lead marker and the next one down. Thus, the distance between the two markers, about 2.5 mm (about 10 times the mean grain size of the sand), gives an upper limit for the shear zone thickness. This is in good agreement with the measured shear bands within the soil mass (e.g. Vardoulakis et al. —1981) which, due to the opposite relative movement on both sides of the shear band, are twice as thick as that along the interface (approximately 20 \( D_{50} \approx 5\) mm), also calculated for the arch of Fig. 5.14 (\( \approx 4\) mm).

VI. An additional set of constant normal stress tests between smooth-surfaced steel and Toyoura sand at a relative density of 65% was found to be independent of the normal stress over the range of 0.5 to 1.6 kg/cm\(^2\) (Yoshimi and Kishida —1981b). These results are in agreement with the predictions of the S.G. model concerning the independence of the friction coefficient of the normal stress, as long as the stress does not alter the shape and/or the size of
the particles. The subject is further discussed in Section 8.7.2.

(f) **Test results from subrounded sand.**

Figure 8.18a describes the relations between the surface roughness and the interface friction coefficient for Nigata sand (Yoshimi and Kishida –1981b). This sand consists mainly of 58% quartz, 12% chert and 17% dark minerals. The same arguments previously raised concerning the Tonegawa sand interparticle friction coefficient seem to be valid for Nigata sand as well. Nigata sand is described as subrounded, with a smooth grain surface. The measured results of Fig. 8.18a are therefore compared to the S.G. model predictions for \( R_a = 1.1 \).

The comparison between the calculated values according to the S.G. model and the measured values as shown in Tables 8.1, 8.2 and Fig. 8.18a show once more a nice match between the grain shape description and the relative density, and the S.G. model representation using the parameters \( R_a \) and \( \beta \), respectively.

Even though the experimental data of Fig. 8.18a contain less information than those from the rounded and angular sands, they complete nicely the comparisons between the measured and predicted values.

(g) **Maximum and residual interface friction coefficients.**

The data and the range of predictions of Figs. 8.16a,b and 8.18a are presented in Figs. 8.19a,b,c respectively, and denoted as 'maximum'. Figs. 8.19a',b', and c' present the residual values of the interface friction coefficients of the same tests, as obtained by Yoshimi and Kishida (1981a).

I. No details are provided regarding the 'residual' test results. Comparison between the data of Fig. 8.19a to those of Fig. 8.17 suggests that the residual state of Toyoura sand of \( D_{50} = 60\% \) under a smooth surface corresponds to a shear strain of \( \gamma \approx 54\% \).
II. The residual friction coefficient of the ‘smooth’ surfaces is about 20% lower than the maximum friction coefficient. The residual and the maximum friction coefficients of the ‘rough’ surfaces are about the same.

III. The above observations may be explained in the following way: (a) Fig. 8.17 suggests that the maximum resistance of the smooth surfaces occurs when the shearing force overcomes the ‘static’ friction coefficient. Thereafter, the slow movement of the surface creates a stick–slip sequence mechanism, which is identical to a sequence of cycling loading where each of the ‘stick’ peaks strains the soil (see Fig. 8.17b), which is then ‘released’ under the ‘slip’. This load–unload mechanism (a result of the particular test) densifies the soil and gradually reduces the interface shear resistance. This kind of process is in general agreement with the expected behavior suggested by the S.G. model (Section 6.3.4b). The soil densification increases the $\beta$ value, and therefore decreases the friction coefficient. However, although they are in the correct trend, the absolute S. G. model values of Fig. 8.19 do not seem to fully correspond quantitatively. (b) Fig. 8.17 suggests that for the rough surfaces, shear takes place in the soil itself. With the increase in the soil straining, the soil dilates (compare Fig. 8.17a to 8.17b) until at the ‘peak’ value, rupture takes place in the soil itself. Up to the peak, the dilation can be explained by an increasing $\beta$ value (decreasing density) and increasing friction. The shear strength of the soil is obtained at the peak ($\gamma \approx 20\%$), and does not change with additional shear.

IV. The ‘residual’ test result of Figs. 8.19a’,b’,c’ reinforces the aforementioned observations that the most dense soil provides the lowest friction for smooth surfaces, which increases for the less dense material.
8.5.4 Discussion and Conclusions

Fig. 8.18b presents the data of all tests and the predictions of the S.G. model for 'smooth' surfaces on quartzic sand ($\theta = 32^\circ$). Each of the lines represent a grain shape according to its $R_a$ value, and the arrow indicates the possible range from $\beta = 45^\circ$ (the lowest for the dense material) to $\beta = 90^\circ$ (the highest for the most loose material). The data of Fig. 8.18b and the previous discussion suggest the following:

I. The measurements of friction with smooth surfaces show good agreement with the predictions of the S.G. model. This supports the following concepts, which the model utilizes:

a. Consideration of the limiting equilibrium of a single grain under various conditions, with the appropriate parameters, resulted in very good agreement with experimental data, supporting this type of approach to soil mass modeling.

b. The importance of the grain shape, as it controls the geometry, and therefore the movement, of the soil grains and the strength of the soil mass.

c. The dependence of the friction coefficient on the surface roughness.

d. The representation of the 'smooth' surface at the contact with the soil grain as tangential traction coinciding with the direction of the plane movement.

e. The development of a preferred contact orientation, according to the minimum energy principle.

II. The ring torsion apparatus 'fits' the conditions which were assumed by the S.G. model.

a. The simple shear conditions (without the 'ends' effect) are such that the grains move along the direction of the moving plane. This is identical to the uniaxial movement assumptions of the S.G. model.
b. The plane of shear develops under 'free' conditions, depending on the roughness of the surface or the strength of the soil.

c. The moving plane is 'on' the soil mass and does not support it, such that a better distinction is observed between sliding and shearing.

III. The S.G. model predictions of higher frictional resistance with looser granular materials (with all other parameters being constant) were proven to be correct by the experimental data. These predictions contradict the common view of soil shear resistance, and therefore require some interpretation.

Section 8.3.4b discussed the frictional component of the shear resistance. It was shown that the dilatational component is the only cause for the higher shear resistance of dense material, as opposed to loose material. The available data are based, however, on the separation of the dilatational component from the frictional component of the total shear resistance, and therefore do not necessarily reflect the state of small strains.

Under small strains, before shear takes place, Eq. 8.5 and the available DSS measurements suggest that initial principal stress rotation would be independent of the density (the development of Eq. 8.5 will be correct for any $\beta$ value). The greater frictional resistance for higher $\beta$ values is due to the fact that the resultant of the forces normal to the interparticle planes is larger for looser material, producing higher friction and greater resistance to motion.

This can be explained by the 'freedom' that the particles may have in the loose packing to rearrange themselves in a preferred orientation in order to transfer the load, as opposed to the restricted particle configuration in the dense array.

Rowe (1962), confronted with the situation where only the dense material matched his analysis [see Section 8.3.4(II)], found it necessary to introduce a value of $\phi_f^*$ (which is not constant) in place of $\phi_\mu$, "in order to satisfy the equation" (for
minimum energy). His analysis of a series of tests on overconsolidated (O.C.R. = 2) medium fine sand of different densities is presented in Figure 8.20. This type of analysis resulted in $\phi_f^*$ increasing from $\phi_\mu$ at the minimum porosity (maximum density) to $\phi_{cv}$ at the maximum porosity (loosest packing). Rowe's $\phi_f$ was intentionally denoted in this study as $\phi_f^*$, because it is clear from Fig. 8.20 that Rowe's $\phi_f^*$ is not only a frictional component, but includes additional loss of energy, termed by Rowe "internal energy spent in rearrangement".

Although Rowe's results of Fig. 8.20 support the above findings that friction decreases as density increases, they seem to be a combination of: (a) the real decrease of friction with increase in density, and (b) the inability of Rowe's analysis to consider the true interparticle contact orientation, as explained earlier, and therefore reflect the need to 'fit' the results of the loose material to his analysis. It should also be emphasized that Rowe's presentation of Fig. 8.20 contradicts (a) the data shown in Fig. 8.7 for dense material (in which $\phi_{cv} = \phi_\mu$), extracted from Rowe's experiments, and (b) the basic concept of the constant volume — critical state shear angle ($\phi'_{cv}$) which is independent of the soil density.

The discrepancies and similarities between the results of this study and those of Rowe's may be explained by the fact that the higher frictional resistance of the loose material is valid as long as a rupture zone does not develop within the soil mass. If this occurs, the rearrangement of the dense particles will be similar to that of the loose particles, resulting in an identical $\phi_{cv}$.

Under shear, the situation is different, however. The S.G. model shows that for the same interparticle friction, a greater principal stress rotation ($\theta$) will take place in the dense material than in the loose material, (see Fig. 6.2a), as confirmed by the experimental results of the next section. When both factors are now considered (see Fig. 6.2b), the higher $\theta$ with the lower $\beta$ result in a frictional resistance similar to that of the lower $\theta$ and the higher $\beta$ value of failure of a loose material: e.g. for quartzic
sand with $\tan \phi = 0.5$:

1. loose, $\beta = 90^\circ \rightarrow \theta = 53^\circ \rightarrow \tan \delta_s = 0.50$
2. dense, $\beta = 45^\circ \rightarrow \theta = 60^\circ \rightarrow \tan \delta_s = 0.47$

where practically, the particles in the loose packing can not ‘ride over’ each other and $\beta$ must be less than $90^\circ$, leading to identical frictional resistances.

The fit of the interface friction test results to the S.G. model analysis in light of the above interpretation justifies the approach and findings of Fig. 8.6, in which the grain shape becomes the major factor determining the frictional resistance of the granular material (with constant interparticle friction coefficient). The arbitrary introduction of $\phi^*_f$ by Rowe is thus unnecessary, and does not reflect a ‘true’ decrease of friction with increase in density.

The reviewed analysis and the test results of this study, therefore, do not contradict the common view of soil shear resistance. The seeming discrepancy is due to the fact that the frictional component of ‘sliding’ shear resistance (under small strains) was never examined. This may have led Yoshimi and Kishida (1981a) to comment in relation to Fig. 8.16a that:

"the coefficient of friction is essentially governed by the surface roughness with the relative density of the sand and the surface material playing a negligibly minor part" and in relation to Fig. 8.18b "the kind of sand has little influence on the coefficient of friction if $R_{\text{max}}$ exceeds about $20 \mu m$. For a smoother surface ($R_{\text{max}} < 20 \mu m$), however, the kind of sand seems to make some differences..."
8.6 PREDICTIONS AND MEASUREMENTS OF THE INTERFACE SHEAR RESISTANCE OF ROUGH SURFACES

8.6.1 Introduction

The S.G. model was shown to provide nice predictions of friction measurements along smooth surfaces (Section 8.5). The comparison between predicted to measured friction coefficients along rough surfaces entails two additional requirements: (a) Tests on rough surfaces with measurements matching the roughness definition of Section 6.3.2. Such tests were reported by Uesugi and Kishida (1986a,b) and Kishida and Uesugi (1987), and will be used in this section. (b) Evaluation of the $\theta$ angle. While the parameters, $R$, $\beta$ and $\alpha$ are controlled by the soil and surface roughness, the variation of $\theta$ remains questionable. Equation 8.5 suggests that for soil straining prior to shear, $\theta \approx 45^\circ - \phi \mu /2$. Fig. 8.17a and the predictions of Section 8.5 suggest that this value is applicable to all cases, regardless of the surface roughness.

The analysis of this section will be conducted in two stages: (1) measurements vs. predictions for different roughnesses with variation of $\theta$, and (2) simplified analyses based on the results of the first stage.

8.6.2 The predictions of the S.G. model

Table 8.3 summarizes the S.G. model predictions (using Eq. 6.13) for the friction coefficient of surfaces of different roughness and sands of different grain shapes and densities. For a first approximation, the angle $\theta$ is assumed to be $32^\circ$. It is expected that this value correctly represents the principal stress orientation prior to failure.

8.6.3 Experimental results from the simple shear apparatus

(a) The Apparatus

Uesugi and Kishida (1986a) and Kishida and Uesugi (1987) used a rectangular
simple shear apparatus to measure the friction between sand and metal surfaces, as shown in Fig. 8.21A. This apparatus was made from a container consisting of stacked aluminum plates with internal openings of 40 x 10 cm, which is the contact area between the sand sample (29 mm high) and the steel specimen under it. A constant normal load was applied to the sand and a tangential load was applied to the steel specimen. The steel specimen is longer than the contact surface, so that even during sliding the contact area remains constant. The tangential load was corrected for the frictional resistance between the steel specimen and the teflon plate which supported it. During the tests, the displacements of $\delta_a$, $\delta_b$ and $\delta_c$ were recorded [see Fig. 8.21A(b)], from which the displacements due to shear deformation of the sand and to sliding between the steel and the dry sand could be calculated.

Uesugi and Kishida (1986b) used a similar apparatus with smaller dimensions, as shown in Fig. 8.21B(a). The sand height was 22 mm and the contact area was 10 x 4 cm. The same apparatus was used with a rigid box, with an 18mm x 10cm x 4cm opening for shear box type friction tests. The applied normal load was also kept constant during the test, and the tangential load was applied to the steel specimen. The displacement measurements of the direct and simple type of shear tests are shown in Fig. 8.21B(b).

(b) The tests and the available data

The three sets of tests report surface roughness measurements along 0.2 mm and 2.5 mm segments. Fig. 8.22 demonstrates the meaning of the difference in the two measurements for sand with a mean grain size particle of 0.2 mm. The roughness measurements along a 2.5 mm scale do not reflect the orientation of the contact between a 0.2 mm diameter grain and the metal (Fig. 8.22a). A measurement along 0.2 mm reflects the slope actually in contact (Fig. 8.22b), and matches the definitions of section 6.3.2(b) for the interface roughness of the S.G. model. Uesugi and Kishida
(1986b) report different roughness measurements according to the mean grain size, such that $L_{\text{max}} = D_{50}$ which can be used for comparison with the S.G. model analysis (see Fig. 6.7 and Section 6.3.2 for the 'roughness' concept).

The friction coefficient parameter was designated by the researchers as $\mu_y$, and refers to the coefficient of friction at yield, as shown in Fig. 8.23. This value is identical to the 'maximum coefficient of friction' reported by Yoshimi and Kishida (1981a,b), and is the one which is used for comparison with the S.G. model designated as $\tan \delta_s$ (see Fig. 8.19 and discussion of Section 8.5.3g for maximum vs. residual values).

Uesugi and Kishida (1986a) reported the results of 57 tests on dense material ($D_r \approx 90\%$), conducted using the large simple shear apparatus of Fig. 8.21A. The tests were aimed at checking the influence of four different factors on the friction coefficient: (1) The sand type, using 3 types of sand and glass beads; (2) the surface roughness; (3) the normal stresses, ranging from 1 kg/cm$^2$ to 10 kg/cm$^2$ and (4) the mean grain size, checking grains at sizes of 0.55–0.62 mm and 0.15–0.19 mm. The displacements due to shear deformation and due to sliding were measured, as previously explained [Fig. 8.21A(b)].

Uesugi and Kishida (1986b) reported the results of 78 tests on dense sand ($D_r = 87\%–105\%$) and 10 tests on medium loose sand ($D_r = 44\%–49\%$). The tests were conducted on the small apparatus of Fig. 8.21B, utilized as a simple shear device for 80 tests and as a shear box for 8 tests. The tests were aimed at checking the influence of the following factors: (1) the surface roughness of the steel; (2) the mean grain size $D_{50}$ (3) the sand type; (4) the test type (simple shear vs. shear box); and (5) the uniformity coefficient. The angularity of the particles was evaluated by modified roundness, and the concept of normalized roughness was proposed as an evaluation of the relative roughness of the sand–steel interface.

Kishida and Uesugi (1987) conducted a review of interface shear testing
apparatuses and a comparison between the ring torsion apparatus, the simple shear apparatus, and the direct shear box. The presented test results are those of the experiments described by Yoshimi and Kishida (1981a,b) and Uesugi and Kishida (1986a,b).

The various test results will be used in the following sections for comparison between measured and predicted results for shear of rough surfaces and in section 8.7 for the influence of the various factors.

8.6.4 Consideration of principal stress variation

When only the contacting body surface roughness varies and the other parameters (soil type and density) are unchanged, the increase in the roughness is expected to result in an increase in the ratio between the shear to the normal stress along the interface, accompanied by the appropriate principal stress rotation/contact orientations (see Fig. 6.9 for $\tan \delta_s$ vs. $\alpha$ and Fig. 6.2 for $\tan \delta_n$ vs. $\theta$). The present section follows the test results of different sands by assigning $R_a$, $\beta$ and $\alpha$ values according to the given conditions, and varying $\theta$ starting from the values of Eq. 8.5.

(a) Test results from rounded sand

Table 8.4a summarizes results of interface shear tests reported by Uesugi and Kishida (1986b). These data are presented in Fig. 8.24a, demonstrating the effect of surface roughness on the interface friction coefficient. The data of Table 8.4a and Fig. 8.24a are from Toyoura dense sand ($D_{10} = 84\%–98\%$), tested in the smaller simple shear apparatus (Fig. 8.21B), under a constant normal stress of $\sigma_n = 1$ kg/cm$^2$. The sand had a range of grain diameters from 0.11 to 0.25 mm with a median $D_{50} = 0.18$mm and $C_u = 1.4$. Additional data on Toyoura sand and analysis of the results related to smooth surfaces are discussed in Section 8.5.3 and presented in Fig. 8.16a in relation to roughness measurements along 2.5 mm. Data on roughness measurements
along segments of 0.2 mm and 2.5 mm on the same surfaces were reported by Uesugi and Kishida (1986a). Average values of 19 tests show that the relation between the two is:

$$\frac{R_{\text{max}}(L = 2.5\text{mm})}{R_{\text{max}}(L = 0.2\text{mm})} = 2.13 \pm 0.31$$

(actual range from 1.27 to 2.69).

The data of Fig. 8.16a for the test results of the ring shear apparatus on smooth surfaces ($R_{\text{max}} = 3$ to $4\mu\text{m}$) were added to Fig. 8.24a, using the ratio of Eq. 8.7, and show an acceptable fit (note that the results for the dense sand are in the lower part of the marked range). The cluster of test points next to it ($R_n \approx 19 \times 10^{-3}$) shows a large variation in $\tan \delta_s$ (from 0.27 to 0.35) with the lower range being lower than expected in relation to the ring shear apparatus results (considering the increase in roughness). This is believed to be a typical trend, a result of the difference in testing apparatuses, and will be further discussed in Section 8.7.

A set of relations between the normalized roughness and the interface friction coefficient was calculated using the S.G. model and added to Fig. 8.24a. The parameters $R_a = 1$ and $\beta = 45^\circ$ were used for all calculations, based on the sand description and the results of the Toyoura sand analysis of Section 8.5. The roughness angles $\alpha$ were calculated using Equation 6.11 for the normalized roughness values $R_n$.

An observation of the calculated relations and the experimental data indicates the following:

I. Predictions of the increase in friction coefficient with the increase in the principal stress rotation/contact orientation match the measured data.

II. The range of $\theta$ values ($28^\circ$ to $55^\circ$) matches very well the principal stress rotation $\psi$ measured by Roscoe et al. (1967) during DSS tests on dense sand ($20^\circ$ to $55^\circ$, Fig. 8.10). As previously discussed, the lower $\theta$ values depend on
the finish of the 'smooth' surface. This range of values is within that of Eq. 8.1, and closely matches that of Eqs. 8.5 and 8.6 (32° to 58°).

III. The predicted values of Eq. 8.5 (θ = 32°, ± 20%) match the measured data to about 85% of the maximum shear stress ratio. This is in good agreement with the hypothesis supporting Eq. 8.5 (see Sec. 8.2.2) and the observations of Yoshimi and Kishida (1981b):

"...until the shear stress exceeds about 85% of the maximum value, both \( x_p \) (shear zone displacement) and \( x_s \) (slip distance) remain at 0: the sand deforms uniformly throughout its height."

(b) Test results from angular sand

Table 8.4b summarizes results of interface shear tests reported by Uesugi and Kishida (1986b). These data are presented in Fig. 8.25a, demonstrating the effect of surface roughness on the interface friction coefficient. The data are from Fujigawa dense sand (\( D_R = 87\% - 105\% \)), tested in the smaller simple shear apparatus and the direct shear box (4 tests, not included in Table 8.4b) under a constant normal stress \( \sigma_n = 1\,\text{kg/cm}^2 \). Fujigawa sand contains quartz, feldspar and magnetite (unspecified quantities, Uesugi and Kishida -1986a) and is described as angular sand. On a scale of 0 (platey particle) to 1.0 (sphere), using a modified roundness, Uesugi and Kishida (1986b) obtained for Fujigawa sand the value \( R = 0.19 \). Unfortunately, quantitative comparisons could not be established between \( R_a \) of the S.G. model and the modified roundness \( R \). The analysis shows that for both angular sands (Fujigawa and Seto), \( R_a = 1.25 \) provided acceptable predictions, and both had similar \( R \) values of 0.19 and 0.17, respectively, compared with 0.27 of Toyoura sand. Fujigawa sand was used in the following four grain size ranges: (1) 1.68 to 2.00 mm with \( D_{50} = 1.82 \,\text{mm}, C_u = 1.1 \); (2) 0.50 to 0.59 mm, with \( D_{50} = 0.54 \,\text{mm}, C_u = 1.1 \); (3) 0.07 to 2.00 mm, with \( D_{50} = 0.54, C_u = 5.1 \); and (4) 0.15 to 0.18 mm, with \( D_{50} = 0.16 \,\text{mm}, C_u = 1.1 \). The normalized roughness values of Table 8.4b and Fig. 8.25 refer to each of the \( D_{50} \) values.
according to the gauge length L, shown in Fig. 8.25a [e.g. \( R_n \) for the sand with \( D_{50} = 1.82 \text{ mm} \) was obtained by \( R_n = R_{\text{max}} (L = 2.0\text{mm})/D_{50} = 1.82\text{mm} \)].

A set of relations between the normalized roughness and the interface friction coefficient was calculated using the S.G. model and added to Fig. 8.25a. The parameters \( R_a = 1.25 \) and \( \beta = 45^\circ \) were used for all calculations, based on the sand description.

An observation of the calculated relations and the experimental data indicates the following:

I. Predictions of the increase in the stress ratio along the interface (friction coefficient) with the increase in the principal stress rotation (contact orientation) match the measured data.

II. Comparison of the relations of Fig. 8.25 for the angular sand to those of Fig. 8.24 for the rounded sand verifies the S.G. model principles and predictions in the following way:

- The principle of normalized roughness was discussed in Section 6.3.2. Fig. 6.9 presented the expected results for soils of different grain shape under the same roughness angle (or normalized roughness). Observation of the relations of Fig. 6.9 and use of the values of Table 8.3 suggests that a surface with a normalized roughness of 0.10 will be associated with a stress ratio (interface friction coefficient) of 0.526 for dense round sand, which approximately matches the results of Fig. 8.24. The same normalized roughness is associated with a stress ratio of 0.707 for dense angular sand, which approximately matches the results of Fig. 8.25. Correct consideration of the surface roughness and the grain shape led, therefore, to excellent prediction of the experimental data, demonstrating the importance of the different factors. Furthermore,
section 8.7 suggests simplified relations for the S.G. model (see Eq. 8.8, Fig. 8.27 and Table 8.5), which for the same normalized roughness \((R_n = 0.1)\) anticipate a stress ratio of 0.599 for round sand and 0.825 for angular sand. This means that although both sands have the same shear strength\(^8\), the same normalized roughness that mobilized the full shear strength of the angular sand mobilized only about 70\% of the shear strength of the rounded sand.

- The principal stress rotation of the angular sand in failure \((\theta_f \approx 45^\circ)\) is smaller than that of the rounded sand in failure \((\theta \approx 55^\circ)\), even though both have the same shear strength. The way in which it is predicted by the S.G. model can be explained by following Figs. 6.9 and 6.2c. For the same roughness, the angular soil stress ratio (friction resistance) is greater than that of the rounded soil. Fig. 6.2c shows that for the same shear resistance, angular sand requires a smaller interparticle orientation than rounded sand. Therefore, the lesser roughness, which mobilized the full shear strength of the angular sand, is associated with a smaller principal stress rotation/interparticle orientation than in the rounded sand.

I. The relations between the frictional resistance (under failure in the soil or along the interface) and the soil type, and between the principal stress direction/contact orientation and the soil type, were discussed earlier, while supporting different observations and assumptions (see Sections 8.3.4a and 8.3.4e). The present data suggest again that failure is a function of the interparticle contact orientation,

---

\(^8\)The average shear strength ratio from DSS tests on Toyoura and Fujigawa sands are 0.863 and 0.865 for 3 tests \((D_R = 97\%)\) and 4 tests \((D_R = 94\%)\), respectively (Uesugi and Kishida –1986b).
which is associated with the principal stress rotation, both being governed by the grain shape, the interparticle friction, and the loading conditions. This reaffirms the conclusions derived earlier from Figs. 8.4 and 8.5, that the shear angle does not control the major principal stress orientation, but is rather a result of it, determined by the above factors.

As the tests of Figs. 8.24 and 8.25 were conducted under the same loading conditions and it is assumed that the sands have the same interparticle friction (at least for the grain sizes of $D_{50} = 0.18$ mm and $D_{50} = 0.20$ mm), the mechanism behind the results may be suggested. Angular particles encounter greater impedance to rotation under the applied load, developing greater interparticle friction along a contact (see Fig. 6.2c for illustration), in comparison to a round particle with the same interparticle orientation (see Figs. 6.2c, 6.5 and 6.6 for illustration). The shear, therefore, will be mobilized under a smaller principal stress rotation, even though it is at the same stress ratio.

II. The same relations support two additional ideas, which were previously presented regarding relations between loose and dense materials of the same grain shape. (1) A loose material (prior to shear) allows 'better' interparticle contact orientation (a more efficient load transfer, as expressed by the parameter $\beta$). This results in a greater interparticle friction, reflected as a higher friction coefficient along smooth interfaces (at the same loading for both packings). (2) For shear in soil, under a constant volume state, the denser material requires a smaller principal stress rotation/contact orientation to mobilize the full shear strength of the soil which equals the frictional resistance of the loose material. This shear strength is obtained at smaller angles of principal stress rotation/contact orientation, resulting in equal frictional resistances under different contact orientations.

III. Fig. 8.26a shows a similar analysis for another type of angular sand (Seto sand), as previously mentioned. Uesugi and Kishida (1986b) provide only limited
information about this sand. Its strength is somewhat less (0.82 stress ratio) in comparison to the strength of Toyoura and Fujigawa sands (0.86). Although described as "very angular", its modified roundness of 0.17 is very similar to that of Fujigawa sand at 0.19. The S.G. model analysis was performed using the same parameters as those used for Fujigawa sand in Fig. 8.25. Although the sand reaches failure under a lesser normalized roughness (supporting the observation of very angular), a generally good agreement exists between the test results and the predictions of Figs. 8.26 and 8.25, in comparison with the round sand of Fig. 8.24. The analysis and the measurements of Fig. 8.26 therefore, add weight to the above discussion regarding Fig. 8.25.

8.6.5 Simplified analysis for shear resistance of rough surfaces

(a) The controlling factors

Figs. 8.24a, 8.25a and 8.26a compared the measured data to the S.G. model predictions, considering the principal stress rotation, the density, the particle shape and the surface roughness. These figures suggest a gradual change in principal stress rotation up to failure. The total variation, from smooth surfaces to failure, covers a range of about 25° (from ≈ 25° to ≈ 50°), which is in good agreement with the DSS measurements (Figs. 8.10, 8.11, 8.12). Figs. 6.2 and 6.9, supported by the data of Sections 8.3 to 8.6, suggest that the grain shape and the interface roughness are the major factors which control the interface friction coefficient. The density influence (represented by the $\beta$ parameter) is very limited, and the fact that $\theta$ varies within a small range only (described above) reduces its influence as well.

(b) The required roughness range

Fig. 8.16a shows a range of roughness for construction materials suggested by Yoshimi and Kishida (1981a). The roughness ranging from about 10 to 150 $\mu$m along a
gauge length of 2.5 mm is equivalent to a roughness of about 5 to 70 μm along a gauge length of 0.2mm (using Eq. 8.7). These roughness values are translated for Toyoura sand into the normalized roughness $R_n$, ranging from 0.025 to 0.40. This is presented in Fig. 8.24b, and indicates that the different construction materials would cover all roughness possibilities to failure except for those of ‘smooth’ surfaces. According to Uesugi and Kishida (1984), the roughness of full-scale steel pipe piles driven into sand, was measured by Sumikin Weld Pipe Co. Ltd. (Japan) along a gauge length of 0.2 mm. $R_{max}$ ranged from 7μm to 25μm. These measurements are in agreement with the grain size of Toyoura sand, and are presented in Fig. 8.24b as normalized roughness ranging from 0.04 to 0.14. This range could suggest that the roughness of the pile surface is insufficient to mobilize the full shear strength of the soil mass. Uesugi and Kishida (1984) explain the measured roughness range as the abrasive wear of the pile by the sand, and suggest that the surface of a steel pipe pile driven into sand deposits cannot be considered a rough surface. It is of interest to know what sand type (minerals, grain shape and grain size) caused the above abrasion as otherwise such conclusions cannot be made. It may well be that, having grains finer than 0.2mm, this sand was sheared by the pile wall; however, its friction created a surface which would not shear sand with grains like Toyoura sand. Another possibility is that the pile was driven deep into dense sand, resulting in contact stresses (especially under the point) in excess of 10kg/cm², causing crushing of grains and creating finer and more angular sand.

Regardless whether the above speculations are correct or not, the suggested roughness ranges of Fig. 8.24b indicate that construction materials (including pipe piles) cannot be considered to be ‘smooth’ surfaces, and may have sufficient roughness to induce shear in the soil.

\textit{(c) The requirements of the simplified analysis}

In relation to the presented data and the above discussion, it seems practical
to develop simplified relations having the following requirements: (a) Consideration of normalized roughness and grain shape; (b) Good agreement with measurements for surfaces 'rougher' than the smooth ones, all the way to the shearing strength of the soil; (c) As the influence of the density decreases with the increase in the normalized roughness, an average ratio would be acceptable for both $\theta$ and $\beta$.

(d) Possible simplified relations

Four possible simplified relations were checked on the following basis:

I. The average density ($\beta = 60^\circ$) and the preferred intergranular contact, $\theta = 32^\circ$ (according to Eq. 8.5), are expected to provide a nice match beyond the very 'smooth' zone.

II. $\beta = 90^\circ$ and $\theta = 32^\circ$ for consideration of the upper 'acceptable' boundary of resistance. 

III. Combination of the above, lower contact orientation ($\theta = 25^\circ$) and higher interparticle friction resistance ($\beta = 90^\circ$).

IV. $\beta = 45^\circ$ and $\theta = 32^\circ$ for consideration of the lower 'acceptable' boundary. 

(for the relations of $\theta = 32^\circ$, and different $\beta$, $R_a$ and $R_n$ values, see Table 8.3).

All four relations were plotted in Figs. 8.24b, 8.25b and 8.26b for normalized roughness vs. interface friction coefficient of Toyoura, Fujigawa and Seto sand respectively. Additional data for Toyoura sand were added to Fig. 8.24b to enable a better judgment of the proposed relations. These data include test results on medium loose sand (Uesugi and Kishida –1986b), the data of smooth surfaces of Yoshimi and Kishida (1981a) and test results of Yajima (1984), reported by Uesugi and Kishida (1984).

The four relations present the following:

- $\beta = 90^\circ$, $\theta = 32^\circ$; represents upper boundary for all 3 sands (except of one
questionable data point in Fig. 8.24b).

- \( \beta = 45^\circ, \theta = 32^\circ \); represents lower boundary for all sands for the relations of the rougher surfaces of \( R_n > 0.05 \).
- \( \beta = 60^\circ, \theta = 32^\circ \); presents good agreement with the data; somewhat overestimates the smooth surfaces (as expected), and slightly underestimates the shear resistance at the rough surfaces.
- \( \beta = 90^\circ, \theta = 25^\circ \); provides very good agreement with most of the data in all ranges, except for the 'smooth' surfaces of Toyoura sand (Fig. 8.24b), which are lower than expected, as explained above, and lie within the zone of normalized roughness smaller than the range for construction materials and steel pipe piles.

\((e)\) The suggested simplified relations

The relations of \( \beta = 90^\circ \) and \( \theta = 25^\circ \) seem to provide a very good estimation for the interface friction coefficient of quartzic sands and surfaces, considering the surface roughness and the grain size (through \( R_n \)) and the grain shape (through \( R_a \)). The simplified conditions of \( \beta = 90^\circ \) allow reduction of the general relations of Equation 6.13 to those of Equation 6.12:

\[ \tan \delta_s = \frac{R_n^2 \cdot \tan(\theta + \alpha)}{1 + \sqrt{R_a^2 \cdot \tan^2(\theta + \alpha) + 1}} \]

and for \( \theta = 25^\circ \)

\[ \tan \delta_s = \frac{R_n^2 \cdot \tan(25^\circ + \alpha)}{1 + \sqrt{R_a^2 \cdot \tan^2(25^\circ + \alpha) + 1}} \]  

(8.8)

The relations of Eq. 8.8 for different grain shapes and surfaces of various roughness are summarized in Table 8.5. Figure 8.27 presents a comparison between
the data of Figs. 8.24b, 8.25b and 8.26b with the relations of Eq. 8.8 only. A detailed comparison with the measured data is presented in Tables 8.4a and 8.4b for Toyoura and Fujigawa sands, respectively. Except for the smooth surfaces \((R_n < 0.020)\) excellent agreement exists between the simplified relations and the measured data. Even though the data of Fig. 8.27 refer to many tests of various sands, additional data of different quartzic sands will be required in the future for further validation of these simplified relations.
8.7 THE EFFECT OF VARIOUS FACTORS ON THE INTERFACE SHEAR RESISTANCE AS DETERMINED THROUGH PREDICTIONS AND MEASUREMENTS

8.7.1 Introduction

The parameters which control the interface shear resistance were sorted into three groups in Section 6.1, and discussed in Section 6.3 in relation to the S.G. model. The recent review of experimental data also included a discussion of many of these parameters.

Additional parameters are suggested in the literature as also possibly controlling the interface shear resistance. Many of these do not necessarily reflect the mechanism along the interface. Several will be presented here, and will be used to demonstrate the possibility of using the S.G. model to understand and analyze the interface friction mechanism.

8.7.2 Stresses normal to the interface

The S.G. model suggests no influence of the normal stresses on the interface friction coefficient other than that due to crushing of particles, altering their shapes and size. An early report by Yoshimi and Kishida (1981b, see Section 6.5.5c), which did not contain testing details, confirmed the above assumption for normal stresses ranging from 0.5kg/cm² to 1.6kg/cm².

Uesugi and Kishida (1986a) used the large simple shear device (Fig. 8.20) to measure the influence of the normal stresses on the interface friction coefficient. Their results from 15 tests on a surface roughness of approximately 10μm are presented in Fig. 8.28. The table in Fig. 8.28 was subdivided into 4 groups according to the normal stresses, ranging from 0.8kg/cm² (78KPa) to 40kg/cm² (3920Kpa). The average roughness of each group was used to calculate the normalized roughness (For D₅₀ = 0.19mm) and the roughness angle (α). These values (presented in the Table of Fig.)
8.28) were used in Eq. 8.8 with $R_a = 1$ to predict the interface friction coefficient according to the simplified S.G. approach. The predicted results show excellent agreement with the measurements and an exact match to the average numbers. Four additional tests on a rougher surface (roughness of 59$\mu$m), also did not show an influence of the normal stresses on the interface friction coefficient.

Uesugi and Kishida observed crushed particles along the contact surface after the friction tests under high normal stresses. They were able to quantify the degree of particle crushing by the weight of particles finer than 0.105mm (presented in the column denoted $W_{105}$ in the Table of Fig. 8.28), since the sand had been sieved prior to testing to remove all particles finer than 0.105 mm. Uesugi and Kishida suggest that the degree of particle crushing is proportional to the sliding distance, such that particle crushing is correlated to the energy consumed along the sliding surface.

8.7.3 Grain size and surface roughness

The S.G. model suggests that because geometrical relations control the particle movement, the absolute grain size has no effect on the interface friction coefficient other than through: (1) influencing the interparticle friction angle, and (2) scaling of the surface roughness. The ability to distinguish between the different factors (effect of soil and effect of surface roughness) seems to be the major obstacle of all interface experiments, including the detailed and careful studies of Uesugi and Kishida (1986a), who concluded:

"There can be a number of reasons, including the shape and the major minerals of the sand particles for the significance of sand type on frictional coefficient." and "Although the influence of sand type could not be fully worked out, there is a possibility that the effect of sand type can be indirectly correlated with frictional coefficient by means of the shear strength of sand" and "Further works, however, are required to estimate the sand type effects quantitatively."

It is believed that the S.G. model gives the correct answers, which will be demonstrated herein.
Fig. 8.25 presented the experimental results of Fujigawa sand of 3 different median grain sizes \(D_{50} = 0.16, 0.54, 1.82\text{mm}\). The roughness measurements along the relevant gauge length led to a normalized roughness, which showed excellent agreement with the test results when used in the S.G. model analyses. Uesugi and Kishida (1986b), when studying the coefficient of friction, plotted it against the roughness measured along a gauge of 0.2mm and obtained the relations shown in Fig. 8.29a. A linear regression of data for each \(D_{50}\) gave straight lines. The researchers concluded that at the same value of \(R_{\text{max}}\) (e.g., at \(R_{\text{max}} = 20\mu\text{m}\)), smaller values of \(\mu_y\) are measured for larger \(D_{50}\) values. The explanation for this was that, qualitatively, the results are consistent with those of Rowe (1962) for friction between quartz particles and a quartz block. Subsequently, after showing the normalized data of Fig. 8.25 the researchers concluded that: 

"...on the other hand...the influence of \(D_{50}\) is insignificant."

Fig. 8.29b presents the data of Uesugi and Kishida with the predictions of the S.G. model considering principal stress rotation (b–1), and simplified average relations (b–2,b–3). It is clear that all three attempts failed to predict the data of all grain sizes. The consideration of principal stress rotation brought good agreement with the sand of \(D_{50} = 0.16\text{mm}\), but not for the others. The other ‘average’ predictions did not match the data for the rough surfaces.

This all may be explained by relating the surface roughness to the grain size, as suggested by the S.G. model. Replotting the data of Fig. 8.29 (see Table 8.4b) on the ‘proper’ scale of roughness appropriate to each grain size led to the relations of Fig. 8.30a,b,c, corresponding to \(D_{50} = 0.16, 0.54, 1.82\text{mm}\). Two analyses are then presented: (1) the S.G. model, considering the principal stress rotation and (2) the simplified S.G. model relations of Eq. 8.8. Both analyses are shown now to have excellent agreement with the measured data. The absolute measured roughness is demonstrated to have no clear effect when not correlated to the sand grain size. From Fig. 8.30c, it is also
evident that the surfaces tested were not 'rough' enough to induce shear in the large grain-sized sand. Using the simplified relations of Eq. 8.8, it is possible to assess that with $R_{max} = 190\mu m$ measured along $L = 2mm$, this soil would have been sheared. This value is approximately equivalent to $R_{max} = 83\mu m$ measured along $L = 0.2mm$, which is more than twice the maximum roughness of the measured data in Fig. 8.29a, explaining why this soil did not fail.

### 8.7.4 Test Type

The most commonly used apparatus for interface friction tests is the direct shear box, where the lower part is replaced by a plate (e.g. Potyondy —1961, Butterfield and Andrawes —1972, Lemos —1986), or other apparatuses utilizing the same principle (e.g. Desai et al. —1985, Uesugi and Kishida —1986b). Sections 8.3 and 8.4 already reviewed and analyzed the limitations of the shear box for the study of soil shear and interface shear (see Fig. 8.14). Additional analysis (not presented here) showed that the friction test results in the shear box are controlled by the geometry of the box, mainly the proportion between the interface area and the cross-sectional area (normal to the interface and to the applied tangential load). A detailed analysis and review of data is beyond the scope of the present work. However, the principle behind the limitations of the different tests will be briefly presented herein.

The nature of the contact at the interface between the soil and the moving body surface is, in most cases (e.g. static loading of a pile), such that the body displacement induces shear along the interface. The S.G. model, therefore, assumed initiation of motion by the plane/sphere friction, which induces countering interparticle friction. This situation is simulated correctly by the ring shear apparatus, which allows 'simple shear' conditions without 'end effects'. Any test which induces loading of the soil not only through the interface by definition imposes stress conditions different than those required. The simple shear apparatus is the next best
way to simulate field conditions. However, its end effects undoubtedly impose non-uniform stress conditions which promote progressive failure, leading to a smaller soil shear strength (Lucks et al. –1972, Budhu –1984, Airey et al. –1985). The shear box drastically changes the field conditions by loading the soil (tangentially) not through the interface only, and by imposing a failure plane (when, instead of sliding, shear takes place in the soil). The data of Fig. 8.14 suggest that in order to shear a soil along the interface, large deformations are required. During these deformations, the soil is weakened by rupture zones, and only then sheared along the interface. Larger deformations and lower strength are therefore expected in direct shear tests as opposed to simple shear tests, and in a rectangular (or annular) simple shear as opposed to the ring simple shear.

The results of different tests carried out by Uesugi and Kishida (1981a,b and 1986a,b), and those presented by Kishida and Uesugi (1987), are presented in Fig. 8.31 to demonstrate the above expected trends. Fig. 8.31a clearly shows the lower simple shear results in relation to the ring torsion test. Figs. 8.31b,c show the results obtained from a direct interface test to be mostly lower in relation to a simple shear test. The only obvious place where the results were comparable is beyond the peak strength (Fig. 8.31b). Fig. 8.31d shows the interface displacement in simple and direct shear tests (see the notations in Fig. 8.21B). The displacement \( \delta' \) in the direct shear tests includes the displacement due to the deformation of the sand within the frame as well as the sliding displacement of the interface. Kishida and Uesugi therefore concluded, regarding \( \delta' \):

"This value would be influenced by the height of the sand mass. These relationships by direct shear tests are not suitable for modeling the sand–steel interface behavior. Direct shear interface tests could be used only to obtain the peak i.e the coefficient of interface friction"
### TABLE 8.1: Friction Coefficient of Smooth Surfaces ($\alpha = 0^\circ$) and Quartzic Sand ($\theta = 32^\circ$) of Different Densities and Particle Shape, According to the S.G. Model.

<table>
<thead>
<tr>
<th>Description &gt;</th>
<th>Well-Rounded to Sub-Rounded</th>
<th>Sub-Angular to Angular</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta &gt; Ra$</td>
<td>1.00 1.10 1.15 1.20 1.25</td>
<td></td>
</tr>
<tr>
<td>Dense $45^\circ$</td>
<td>0.234 0.279 0.303 0.327 0.352</td>
<td></td>
</tr>
<tr>
<td>Medium $60^\circ$</td>
<td>0.265 0.315 0.342 0.369 0.397</td>
<td></td>
</tr>
<tr>
<td>Loose $90^\circ$</td>
<td>0.287 0.342 0.370 0.400 0.430</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.262 0.312 0.338 0.365 0.393</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 8.2: Measured Friction Coefficient of Smooth Surfaces and Different Sands (Yoshimi and Kishida -1981a,b)

<table>
<thead>
<tr>
<th>Description &gt;</th>
<th>Rounded Quartzic</th>
<th>Subrounded 60% Quartz</th>
<th>Angular diff mins.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma &gt; D&amp;R$ Type</td>
<td>Toyoura</td>
<td>Nigata</td>
<td>Tonegwa</td>
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<tr>
<td>Dense $90%$</td>
<td>0.25</td>
<td>---</td>
<td>0.31</td>
</tr>
<tr>
<td>Medium $60%$</td>
<td>0.27</td>
<td>0.33</td>
<td>0.38</td>
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<tr>
<td>Loose $40%$</td>
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<td>---</td>
<td>0.34</td>
</tr>
<tr>
<td>Average</td>
<td>0.27</td>
<td></td>
<td>0.34</td>
</tr>
</tbody>
</table>
TABLE 8.3: Friction Coefficient ($\text{tg}\delta_s$) of Surfaces of Different Roughnesses ($\alpha$) and Quartzic Sand ($\theta = 32^\circ$) of Different Densities ($\beta$) and Particle Shapes ($R_a$), According to the S.G. Model

<table>
<thead>
<tr>
<th>Roughness Angle $\alpha^\circ$</th>
<th>$R_a$</th>
<th>$\beta^\circ$</th>
<th>$R_n$</th>
<th>0</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>90</td>
<td>0.287</td>
<td>0.399</td>
<td>0.553</td>
<td>0.686</td>
<td>0.804</td>
<td>0.918</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>60</td>
<td>0.265</td>
<td>0.366</td>
<td>0.502</td>
<td>0.616</td>
<td>0.713</td>
<td>0.804</td>
</tr>
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<td>45</td>
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<td></td>
<td></td>
<td>90</td>
<td>0.342</td>
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<td>0.638</td>
<td>0.779</td>
<td>0.901</td>
<td>1.018</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td>60</td>
<td>0.397</td>
<td>0.431</td>
<td>0.580</td>
<td>0.700</td>
<td>0.800</td>
<td>0.891</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>45</td>
<td>0.279</td>
<td>0.378</td>
<td>0.502</td>
<td>0.598</td>
<td>0.672</td>
<td>0.737</td>
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<td></td>
<td>90</td>
<td>0.430</td>
<td>0.581</td>
<td>0.770</td>
<td>0.922</td>
<td>1.049</td>
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<td>60</td>
<td>0.397</td>
<td>0.533</td>
<td>0.700</td>
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<td>0.931</td>
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<td>45</td>
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<td>0.468</td>
<td>0.607</td>
<td>0.707</td>
<td>0.782</td>
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Notes: Values in parentheses refer to ratios of ($\theta + \alpha$) > 90°
TABLE 8.4a: Measured Friction Coefficient of Toyoura Rounded Sand for Surfaces of Different Roughnesses (Uesugi and Kishida 1986b) Compared to the Simplified Predictions of the S.G. Model, According to Eq. 8.8

<table>
<thead>
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<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR%</td>
<td>D50 (mm)</td>
<td>Rn</td>
<td>α°</td>
<td>tgδs</td>
<td># of tests</td>
<td>Range of measurements</td>
<td>S.G. tgδs</td>
<td>Ratio</td>
</tr>
<tr>
<td>93</td>
<td>0.18</td>
<td>0.018</td>
<td>15.4</td>
<td>0.29</td>
<td>3</td>
<td>0.27-0.31</td>
<td>0.368</td>
<td>1.27</td>
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<tr>
<td>86</td>
<td>0.18</td>
<td>0.020</td>
<td>16.3</td>
<td>0.33</td>
<td>3</td>
<td>0.31-0.35</td>
<td>0.376</td>
<td>1.14</td>
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<tr>
<td>97</td>
<td>0.18</td>
<td>0.053</td>
<td>26.6</td>
<td>0.50</td>
<td>5</td>
<td>0.47-0.54</td>
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<tr>
<td>97</td>
<td>0.18</td>
<td>0.061</td>
<td>28.6</td>
<td>0.52</td>
<td>2</td>
<td>0.51-0.53</td>
<td>0.505</td>
<td>0.97</td>
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<td>94</td>
<td>0.18</td>
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<td>0.55</td>
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<tr>
<td>94</td>
<td>0.18</td>
<td>0.106</td>
<td>38.0</td>
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<td>4</td>
<td>0.61-0.63</td>
<td>0.613</td>
<td>0.99</td>
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<tr>
<td>97</td>
<td>0.18</td>
<td>0.129</td>
<td>42.1</td>
<td>0.64</td>
<td>4</td>
<td>0.63-0.66</td>
<td>0.663</td>
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<tr>
<td>91</td>
<td>0.18</td>
<td>0.163</td>
<td>47.7</td>
<td>0.79</td>
<td>5</td>
<td>0.74-0.83</td>
<td>0.735</td>
<td>0.93</td>
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<td>0.18</td>
<td>0.222</td>
<td>56.2</td>
<td>0.84</td>
<td>6</td>
<td>0.83-0.87</td>
<td>0.857</td>
<td>1.02</td>
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</table>

Columns:
1) Relative density - average of tests (column 6)
2) D50 - Median (by weight) grain size in mm.
3) Rn - Normalized Roughness = Rmax(L / D50)/D50.
   L = 0.15mm for D50 = 0.16mm
   L = 0.20mm for D50 = 0.18mm
   L = 0.50mm for D50 = 0.54mm
   L = 2.00mm for D50 = 1.82mm
4) α - Roughness Angle using Eq. 6.11 α = cos⁻¹(1 - 2Rn)
5) tgδs - Measured interface friction - average of tests (column 6)
6) Number of tests.
7) Range of the measured interface friction coefficient.
8) S.G. tgδs - Calculated friction coefficient using the simplified predictions of the S.G. model, according to Eq. 8.8.
9) Ratio of calculated over measured friction coefficient.
TABLE 8.4b: Measured Friction Coefficient of Fujigawa Angular Sand for Surfaces of Different Roughnesses (Uesugi and Kishida -1986b) Compared to the Simplified Predictions of the S.G. model (Eq. 8.8)

<p>| | | | | | | | |</p>
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<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>DR%</td>
<td>D50 (mm)</td>
<td>Ra</td>
<td>α°</td>
<td>tgδs</td>
<td># of tests</td>
<td>Range of amounts</td>
<td>S.G. tgδs</td>
</tr>
<tr>
<td>----</td>
<td>---------</td>
<td>----</td>
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<td>102</td>
<td>0.16</td>
<td>0.018</td>
<td>15.4</td>
<td>0.49</td>
<td>1</td>
<td>0.49</td>
<td>0.541</td>
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<tr>
<td>92</td>
<td>0.16</td>
<td>0.020</td>
<td>16.3</td>
<td>0.50</td>
<td>1</td>
<td>0.50</td>
<td>0.552</td>
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<tr>
<td>95</td>
<td>0.16</td>
<td>0.098</td>
<td>36.5</td>
<td>0.79</td>
<td>1</td>
<td>0.79</td>
<td>0.820</td>
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<td>104</td>
<td>0.54</td>
<td>0.008</td>
<td>10.3</td>
<td>0.49</td>
<td>1</td>
<td>0.49</td>
<td>0.473</td>
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<tr>
<td>89</td>
<td>0.54</td>
<td>0.027</td>
<td>18.9</td>
<td>0.58</td>
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<td>0.58</td>
<td>0.587</td>
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<td>87</td>
<td>0.54</td>
<td>0.051</td>
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<td>0.61</td>
<td>0.681</td>
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<td>101</td>
<td>0.54</td>
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<td>0.90</td>
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<td>95</td>
<td>1.82</td>
<td>0.0024</td>
<td>5.6</td>
<td>0.38</td>
<td>5</td>
<td>0.34-0.42</td>
<td>0.412</td>
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<td>92</td>
<td>1.82</td>
<td>0.012</td>
<td>12.6</td>
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<td>0.43</td>
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<td>0.46-0.55</td>
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<td>105</td>
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<td>0.72</td>
<td>1</td>
<td>0.72</td>
<td>0.698</td>
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columns: 1) Relative density - average of tests (column 6)
2) D50 - Median (by weight) grain size in mm.
3) Rn - Normalized Roughness = Rmax(L ≤ D50)/D50.
   L = 0.15mm for D50 = 0.16mm
   L = 0.20mm for D50 = 0.18mm
   L = 0.50mm for D50 = 0.54mm
   L = 2.00mm for D50 = 1.82mm
4) α - Roughness Angle using Eq. 6.11 α = cos⁻¹(1 - 2Rn)
5) tgδs - Measured interface friction - average of tests (column 6)
6) Number of tests.
7) Range of the measured interface friction coefficient.
8) S.G. tgδs - Calculated friction coefficient using the simplified predictions of the S.G. model, according to Eq. 8.8.
9) Ratio of calculated over measured friction coefficient.
TABLE 8.5: Friction Coefficient (tgδ<sub>α</sub>) of Surfaces of Different Roughnesses (α) and Quartzic Sands of different Grain Shapes (Ra) According to the Simplified Relations of the S.G. Model (β = 90°, θ = 25°)

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<th>α° →</th>
<th>0</th>
<th>11.5</th>
<th>25.8</th>
<th>36.9</th>
<th>45.6</th>
<th>53.1</th>
<th>60.0</th>
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<td>Ra</td>
<td>Ra₀</td>
<td>Ra₁</td>
<td>Ra₂</td>
<td>Ra₃</td>
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<td>Ra₆</td>
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<td>0.00</td>
<td>0.222</td>
<td>0.330</td>
<td>0.475</td>
<td>0.599</td>
<td>0.708</td>
<td>0.812</td>
<td>0.916</td>
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<td>1.10</td>
<td>0.266</td>
<td>0.391</td>
<td>0.554</td>
<td>0.688</td>
<td>0.802</td>
<td>0.910</td>
<td>1.016</td>
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<td>1.15</td>
<td>0.289</td>
<td>0.423</td>
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<td>0.734</td>
<td>0.850</td>
<td>0.959</td>
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<td>1.20</td>
<td>0.313</td>
<td>0.456</td>
<td>0.636</td>
<td>0.779</td>
<td>0.898</td>
<td>1.008</td>
<td>1.116</td>
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<td>1.25</td>
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<td>0.489</td>
<td>0.678</td>
<td>0.825</td>
<td>0.946</td>
<td>1.057</td>
<td>1.165</td>
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Fig. 8.1: Comparison Between the Principal Stress Orientation of (a) the S.G. Model (b) the Trajectory of the Arching Approach (c) Mohr–Coulomb Failure Criterion under T.C State of Stress (d) Rowe's Friction of Inclined Planes Principle.
Fig. 8.2: The Infinitesimal Displacement of the Sphere in the Groove.
(a) The Movement (b) Analogy to Friction of Inclined Planes.

Fig. 8.3: Deformation of 2-D Rods or Spheres in Orthorhombic Packing.
(a) The 'Standard' Arrangement (b) The Deformed Configuration
(c) Geometrical Relations (d) Relation to the S.G. Model.
Fig. 8.4: Orientation of Principal Stresses During Shear of Spherical Particles:
(a) Using the Groove Orientation of the S.G. Model for Interparticle Limiting Equilibrium
(b) Using the Principal Stress Trajectory Based on the Mohr–Coulomb Failure Criterion
(c) Consideration of the Grain Shape by the S.G. Model
Fig. 8.5: Relations of the S.G. Model Between the Groove Orientation \( \theta \) and the Interface Friction Coefficient \( \tan \delta \) for Elliptical Particles and Different \( \beta \) Angles, Compared With Experimental Results of Principal Stress Orientation \( \psi \) and Soil Friction Coefficient of Different Sands \( \tan \phi \).
Fig. 8.6: Experimental Results from Drained Triaxial Compression Tests (σ₃ = 30 psi) on Saturated Quartz Grains of Different Shapes and Identical Size, as a Function of the Relative Density:

(a) Drained Shear Angle
(b) Dilatational Component of the Shear Angle
(c) Frictional Component of the Shear Angle

(d) Frictional Resistance for β = 90° and Different Grain Shapes as Calculated Using the S.G. Model, Compared to Experimental Data

(a) Drained Shear Angle

(b) Dilatational Component of the Shear Angle

(c) Frictional Component of the Shear Angle

(d) Frictional Resistance for β = 90° and Different Grain Shapes

Leighton Buzzard, φᵥ = Cole (1967); sand
Ham River sand, φᵥ = Asadi (1975)
Ψ - Bishop (1966)
Ψ - Symes (1983)

Koerner (1970)
Fig. 8.7: Theoretical and Experimental Relations Between $\phi_\mu$ and $\phi_{cv}$:

(a) According to Skinner (1969) (See Bishop and Skinner -1976)
(b) The Above Relations Plotted with the Theoretical Relations of the S.G. Model for $\beta = 90^\circ$, $R_a = 1$, and the Limiting Friction Interface Angle (Eq. 7.15), with Data from Rowe (1962), and with Additional Data of Skinner (1969).
SOIL CHARACTERISTICS

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<tr>
<th>( d_{10}, \text{ in millimeters} )</th>
<th>CU</th>
<th>( y_{\text{min}}, \text{ in pounds per cubic foot} )</th>
<th>( y_{\text{max}}, \text{ in pounds per cubic foot} )</th>
<th>Angularity</th>
<th>Mean (6)</th>
<th>Standard deviation (7)</th>
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<td>(1)</td>
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<td>(4)</td>
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<td>(7)</td>
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<td>2.60</td>
<td>1.31</td>
<td>78.1</td>
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<td>Angular</td>
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<td>1.35</td>
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<td>0.25</td>
<td>1.25</td>
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<td>53.7</td>
<td>85.7</td>
<td>Subangular</td>
<td>0.53</td>
<td>0.14</td>
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Fig. 8.8: The Influence of the Grain Size on the Frictional Resistance of Saturated Quartz Soils (Koerner - 1970)
Fig. 8.9: Deformation in Simple Shear of:
(a) Axially Loaded Pile (b) Simple Shear Apparatus
Mohr Circle of the Simple Shear Describes:
(c) The Effective Stress  (d) The Effective Stress Increment
(e) Strain Increment for the Whole Sample
Fig. 8.10: Results of Simple Shear Tests:

(a) Stress Ratio and Average Void Ratio Change for Whole Sample, Against Shear Strain (Roscoe et al. -1967)

(b) Inclination to Horizontal of Major Principal Planes of Stress $\psi$, Stress Increment $\chi$, and Strain Increment $\xi$, as well as of Plane of Maximum Shear Stress $\beta$ and Maximum Obliquity $\omega$ (Roscoe et al. -1967)

(c) Relations Between Shear Force $S$, Shear Distortion $\gamma$, Rate of Thickness Change $\Delta H/H$, Inclination Angle of Maximum Principal Stress Axis $\psi$, and Inclination Angle of Maximum Principal Strain Increment Axis $\xi$, for Experimental Series of Dense Model (Oda and Konishi -1974b)
Fig. 8.11: Results of Simple Shear Tests:

(a) Stress Ratio and Average Void Ratio Change for Whole Sample, Against Shear Strain (Roscoe et al. 1967)

(b) Inclination to Horizontal of Major Principal Planes of Stress $\psi$, Stress Increment $\chi$, and Strain Increment $\xi$, as well as of Plane of Maximum Shear Stress $\beta$ and Maximum Obliquity $\omega$ (Roscoe et al. 1967)

(c) Relations Between Shear Force $S$, Shear Distortion $\gamma$, Rate of Thickness Change $\Delta H/H$, Inclination Angle of Maximum Principal Stress Axis $\psi$, and Inclination Angle of Maximum Principal Strain Increment Axis $\xi$, for Experimental Series of Loose Model (Oda and Konishi 1974b)

(d) Direction of Maximum Principal Stress Axis and its Relation to Preferred Direction of $N_i$ (Rosette Diagram and Schmidt's Equal Area Projection Show 2- and 3-Dimensional Distribution of $N_i$ (Oda and Konishi 1974b)
Fig. 8.12: Rotation of Principal Axes of Stress and Strain (Budhu —1988)

Fig. 8.13: Rupture Zone in Direct Simple Shear Tests (Sketches of Radiograph):
(a) NGI DSS, Leighton—Buzzard Sand
(b) Cambridge DSS, Leighton—Buzzard Sand
(c) Cambridge DSS, Fine Sand
(Wood and Budhu —1980)
Fig. 8.14: Sequence of Structures Indicating the Rupture Zone in a Direct Shear Test on Kaolin (Morganstern and Tchalenko –1967)
Ring Torsion Apparatus for Measuring Friction between Soil and Metal Surfaces (Yoshimi and Kishida –1981b)
Fig. 8.16: Comparison of Measurements of the Effect of Sand Density and Surface Roughness on the Coefficient of Friction (Yoshimi and Kishida –1981a,b) to Friction Coefficient for Smooth Surfaces and Different Densities Calculated According to the S.G. Model.
(a) Rounded to Subrounded Quartzic Sand
(b) Angular Sand of Different Minerals (23% quartz)
Fig. 8.17: Test Results from Tonegawa Sand and Steel Surfaces (Yoshimi and Kishida—1981b):
(a) Typical Test Results
(b) Sand Deformations Visualized by Radiograph
(c) X-Ray Photographs
**Fig. 8.18:** Comparison of Measurements of the Effect of Sand Type and Surface Roughness on the Coefficient of Friction (Yoshimi and Kishida -1981a,b) to Friction Coefficient for Smooth Surfaces and Different Densities Calculated According to the S.G. Model.

(a) Subrounded Smooth-Surfaced Sand of Different Minerals (60% quartz)

(b) Different Sand Types
Fig. 8.19: Comparison of Measurements of Maximum and Residual Friction Coefficient between Steel and Dry Sand (Yoshimi and Kishida—1981b) to Friction Coefficient for Smooth Surfaces and Different Densities Calculated According to the S.G. Model.
Medium-fine sand; over-consolidated to 60 Lb./in.²; tested at σ₂ = 30 Lb./in.²; rate = 0.0033 in./min; samples 4 in. diam. ●, φ_{max} (equation (2)); +, φ_s (equation (4)); Δ, φ_f (equations (5) and (7)); ○, φ_t (equation (17)). D, difference due to energy spent on dilation; R, difference due to energy spent on remoulding; F, difference due to energy spent in friction.

Fig. 8.20: Rowe's Shear Strength Analysis:
(a) Test Results (Rowe-1962)
(b) Representation of the Shear Strength Components
Fig. 8.21A: The Simple Shear Apparatus for Measuring Friction between Sand and Metal Surfaces (Uesugi and Kishida -1986a, and Kishida and Uesugi -1987)

(a) The Test Apparatus and its Details
(b) The Measured and the Calculated Displacements
(c) Schematic Diagram of the Friction process
Fig. 8.21B: (a) Simple Shear/Shear Box Apparatus for Measuring Friction between Sand and Metal Surfaces
(b) The Displacement Measurements for the 2 Tests (Uesugi and Kishida -1986b)
Fig. 8.22: The Surface Roughness Evaluation:
(a) $R_{\text{max}}(L = 2.5\text{mm})$ (Yoshimi and Kishida -1981a,b)
(b) $R_{\text{max}}(L = 0.2\text{mm})$ (Uesugi and Kishida -1986a,b)

Fig. 8.23: Definition of Coefficient of Friction at Yield by Uesugi and Kishida (1986a,b), Designated as $\text{tg} \delta_s$ when Compared with the S.G. Model Analyses
Fig. 8.24: Comparison of Measurements of the Effect of Surface Roughness on the Coefficient of Friction to Predicted Friction Coefficient of Rough Surfaces and Rounded Sand According to the S.G. Model:
(a) Considering Rotation of Principal Stress/Interparticle Contact Orientation with Stress Ratio
(b) Relations for Simplified Predictions
Fig. 8.25: Comparison of Measurements of the Effect of Surface Roughness on the Coefficient of Friction to Predicted Friction Coefficient of Rough Surfaces and Angular Sand According to the S.G. Model:
(a) Considering Rotation of Principal Stress/Interparticle Contact Orientation with Stress Ratio
(b) Relations for Simplified Predictions
Fig. 8.26: Comparison of Measurements of the Effect of Surface Roughness on the Coefficient of Friction to Predicted Friction Coefficient of Rough Surfaces and Rounded Sand According to the S.G. Model:
(a) Considering Rotation of Principal Stress/Interparticle Contact Orientation with Stress Ratio
(b) Relations for Simplified Predictions
Comparison of Measurements of the Effect of Surface Roughness on the Coefficient of Friction to Predicted Friction Coefficient of Rough Surfaces and Different Sand Types Using Simplified S.G. Model Relations ($\theta = 25^\circ$, $\beta = 90^\circ$) and Considering the Grain Shape

(a) Toyoura Sand, Round Particles
(b) Seto Sand, Angular Particles
(c) Fujigawa Sand, Angular Particles
### Table A-3. Series C for influence of normal stress

<table>
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<th>Test No.</th>
<th>Sand Type</th>
<th>Diam. &lt; 0.2 mm (mm)</th>
<th>Diam. &lt; 2.5 mm (mm)</th>
<th>Normal Stress (kPa)</th>
<th>Rmax, min (mm)</th>
<th>Rmax, avg (mm)</th>
<th>Friction Coefficient (Eq. 8.8)</th>
<th>Slide Displacement ( \mu ) (mm)</th>
<th>Friction Coefficient ( \mu )</th>
<th>56</th>
<th>27.5</th>
<th>0.493</th>
<th>0.497</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Toyoura</td>
<td>0.19</td>
<td>11.3</td>
<td>21.8</td>
<td>0.52</td>
<td>9.15</td>
<td>58</td>
<td>27.7</td>
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<td>0.515</td>
<td></td>
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</tr>
<tr>
<td>2</td>
<td></td>
<td>0.15</td>
<td>11.9</td>
<td>22.0</td>
<td>0.52</td>
<td>9.25</td>
<td>54</td>
<td>26.9</td>
<td>0.486</td>
<td>0.465</td>
<td></td>
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<tr>
<td>3</td>
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<td>11.8</td>
<td>20.5</td>
<td>0.48</td>
<td>9.31</td>
<td>55</td>
<td>27.2</td>
<td>0.490</td>
<td>0.460</td>
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<td>0.19</td>
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<td>21.8</td>
<td>0.50</td>
<td>8.72</td>
<td>1.0</td>
<td>0.485</td>
<td>0.491</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- \( W_{PS} \): Amount of crushed particles as the weight of particles finer than 105 mm collected after the tests.
- Slide: Slide displacement \( \mu \) of friction surface at the end of the tests.

average of tests 1 - 15: msd. friction c. \( \mu = 0.485 \)

Cal. friction c. \( \mu = 0.491 \)

### Fig. 8.28: The Influence of the Normal Stress on the Interface Friction Coefficient.
Measurements by Uesugi and Kishida (1986a) and Predictions Using the Simplified Relations of the S.G. Model.
Comparison of Measurements to Analyses of the Coefficient of Friction of Different Median Grain Size Fujigawa Angular Sands and Surface Roughness Measurements along L = 0.2mm.

(a) Linear Regression Relationship for Data of Each $D_{50}$ (Uesugi and Kishida—1986b)

(b) S.G. Model:
1. Considering Principal Stress Rotation
2. Simplified Relations for $\theta = 32^\circ, \beta = 60^\circ$
3. Simplified Relations for $\theta = 32^\circ, \beta = 45^\circ$
Fig. 8.30: Comparison of Measurements (Uesugi and Kishida -1986b) to S.G. Model Predictions of the Friction Coefficients of Different Median Grain Sizes of Fujigawa Angular Sands and Suitable Surface Roughness Measurements

(a) $D_{50} = 0.16\text{mm}, \ R_{\text{max}}(L = 0.2\text{mm})$
(b) $D_{50} = 0.54\text{mm}, \ R_{\text{max}}(L = 0.5\text{mm})$
(c) $D_{50} = 1.82\text{mm}, \ R_{\text{max}}(L = 2.00\text{mm})$
The Effect of Test Type on the Measurement of the Interface Fiction Coefficient (Kishida and Uesugi -1987)

Shear Interface Tests of:
(a) Toyoura Sand in Ring and Simple Shear
(b) Fujigawa (?) sand in the large Simple and Direct Shear Apparatus
(c) Fujigawa sand in the small Simple and Direct Shear Apparatus
(d) Interface Displacement in Simple and Direct Shear Interface Test.
CHAPTER 9
SOIL PLUG RESISTANCE TO STATIC LOADS – EXPANSION OF THE SILO APPROACH TO ACCOUNT FOR THE PRINCIPAL STRESS ORIENTATION AND THE INTERFACE SHEAR MECHANISM

9.1 INTRODUCTION

The analysis of soil plugs under static loads was developed in Chapter 4 utilizing the ‘silo approach’. The assumptions and simplifications entailed in the analysis were discussed in Section 4.3, and further assessed during the examination in light of experimental data of Section 4.7.

This analysis and examination led to the conclusion that the interface shear resistance and the stress state of the inner soil control the soil plug behavior, both being determined by the micro mechanism of particulate media. The micro approach to granular material was then reviewed in Chapter 5, as was the silo analysis using arching of an assembly of uniform discs, confirming the ‘continuum’ silo approach of Chapter 4.

Based on the observations and analyses of Chapters 4 and 5, a granular soil–interface shear resistance mechanism was developed in two interconnected analyses: (a) the friction coefficient of granular material along an interface in Chapter 6 and (b) the stress state of the inner soil plug in Chapter 7 (See Section 6.1 for the relations between the analysis of Chapter 4, the review of Chapter 5 and the requirements for the analyses of Chapters 6 and 7).

The following sections utilize the developments of Chapters 6 and 7 to improve the ‘silo analysis’ of Chapter 4 and to overcome its limitations, in order to gain a reliable tool for the prediction of soil plug resistance under static loads.

Based on the granular material behavior, it was shown (Section 7.4) that as the soil is pushed upwards in a ‘passive arching’ mode, the supporting arch made of
grain contacts is oriented concavely (downwards) along the major principal stress trajectory (see Fig. 7.3 and the discussion of Section 7.5). However, as long as the strength of the arch is insufficient to support the upward load, the arch collapses and is transformed into a convex (upwards) supporting catenary. This catenary can be represented for convenience by a convex arch (Section 7.5), which is used in the following developments.

This chapter presents (in Section 9.2) the governing equations of the soil plug resistance for the convex arch (9.2.1) and for the concave arch (9.2.2). The comprehensive analysis is developed in Section 9.2.3. These analyses are then compared to experimental results and previous analyses in Section 9.3.
9.2 DEVELOPMENT OF GOVERNING EQUATIONS

9.2.1 The Convex Arch

The components for a vertical equilibrium analysis of the spherical cap shown in Figure 9.1 are considered. The cap is of infinitesimal thickness $dz$, and has the shape of the trajectory shown in Figs. 7.3 and 7.4. This cap is subjected to the action of the following stresses:

1. Major principal stress, normal to its face;
2. Minor principal stress along its trajectory, normal to the cross section;
3. Vertical stress due to the soil weight; and
4. Frictional hoop stresses along the circumferential interface.

For simplification, each of the forces that acts on the spherical cap, due to the above stresses, is calculated separately. Note that the angle $\psi$ of Chapter 7 is denoted here as $\psi_1$, while $\psi$ represents a variable between $\psi_1$ and $\pi/2$.

1. The vertical force due to the major principal stresses

A differential area of a ring in the spherical cap surface is:

$$dA = 2\pi R^2 \cos \psi \cdot d\psi$$  \hspace{1cm} (9.1)

The vertical force acting on the cap due to $\sigma_1$ is therefore:

$$P_{iz} = \int_a \left[ \sigma_1' - \left( \sigma_1' + \frac{\partial \sigma_1}{\partial z} dz \right) \right] \sin \psi \cdot dA$$

---

1The surface on which $\sigma_1$ is acting corresponds to the trajectory with the length of $(R+dz/2)d\psi$, while the stress $(\sigma_1 + \partial \sigma_1 / \partial z)$ is acting on $(R-dz/2)d\psi$. However, $dz<<R$ and $R$ is constant for every cap; therefore the total area on which the stresses are acting is the same.
\[ = 2\pi R^2 \quad \int_{\psi = \psi_1}^{\psi = \pi/2} - \frac{\partial \sigma_1'}{\partial \psi} \cdot \sin \psi \cdot \cos \psi \cdot dz \cdot d\psi \]

Integration and substitution of \( \cos \psi_1 = ID/2R \) leads to:

\[ P_{1z} = -\frac{\pi}{4} ID^2 \cdot \frac{\partial \sigma_1'}{\partial z} \cdot dz \quad (9.2) \]

which is equal to the circular cross-section times the net stress at the center line.

2. The vertical force due to the minor principal stresses

As \( \sigma_3 \) is constant along the trajectory, the net vertical force of these stresses is zero. This can be obtained from examination of an element of the cap slice with the dimensions of \( dz \times R\psi \), shown in Fig. 9.1. The vertical components of the stresses normal to \( dz \) on both sides of the element cancel each other. Note, however, that the axial force \( (\sigma_3 \cdot dz) \) considered in the analysis of the frictional hoop force (see free body diagram in Fig. 7.4)

3. The vertical force due to gravity

The weight of the cap is:

\[ P_{\gamma z} = A_c \cdot \gamma \cdot dz \]

where \( A_c \) is the surface area of the cap. Using Equation 9.1:

\[ A_c = \int_{\psi = \psi_1}^{\psi = \pi/2} 2\pi R^2 \cdot \cos \psi \cdot d\psi = 2\pi R^2 \cdot (1 - \sin \psi_1) = 2\pi R^2 \cdot \left[ 1 - \frac{1}{R} \left( \frac{ID}{2R} \right)^2 \right] \]

Noting in Fig. 9.1 that \( h = R(1-\sin \psi_1) \), \( A_c = 2\pi Rh \), which is the familiar form of the equation of a spherical cap surface area.
\[ P \gamma_z = 2\pi R [R - \sqrt{R^2 - (ID/2)^2}] \cdot \gamma dz \]

or

\[ P \gamma_z = 2\pi R^2 (1 - \sin \psi_1) \gamma dz \] \hspace{1cm} (9.3)

Note that although integration of Eq. 9.3 with respect to \( z \), from 0 to \( L \), does not exactly fit the weight of the soil cylinder, for \( 45 + \phi/2 \leq \psi_1 < 90^\circ \) the difference between the two is very small (say 7%).

4. The frictional hoop force along the interface

\[ P \tau_z = \tau_1 \cdot \pi \cdot ID \cdot dz / \sin \psi_1 \] \hspace{1cm} (9.4)

where \( \tau_1 = f_s = K_1 \cdot \sigma_v \cdot \tan \delta_s \) (Eq. 6.1), the interface shear resistance described in Chapter 6.

Note that if the curvature is considered (important for small \( L/ID \) values), then the total length on which \( \tau_1 \) acts is \( L - h \) (see Fig. 9.1), where:

\[ h = R(1 - \sin \psi_1) = ID\cdot \frac{(1 - \sin \psi_1)}{2\cos \psi_1} \]

5. The vertical equilibrium equation of the convex spherical cap.

\[ \Sigma P_z = P_{iz} + P \gamma_z + P \tau_z = 0 \] \hspace{1cm} (9.5)

Substituting Eqs. 9.2, 9.3, and 9.4 into Eq. 9.5 leads to:

\[ \frac{\partial \sigma'_1}{\partial z} = 8\left( \frac{R}{ID} \right)^2 (1 - \sin \psi_1) \gamma + \frac{4}{ID} \cdot \frac{\tau_1}{\sin \psi_1} \] \hspace{1cm} (9.6)
Equation 9.6 can be rewritten using the following substitutions:

(a) \[ K_{1a} = \sigma_3' / \sigma_1' \]

- For the conditions of \( \delta = \phi \) and simultaneous failure in soil and along the interface \( K_{1a} = K_a = \tan^2(45 - \phi/2) \)
- For the conditions of \( \delta < \phi \) and failure along the interface only the following relations are developed from Eqs. 7.1, 7.2 and 7.10 (see Fig. 9.2):

\[
K_{1a} = \frac{\sigma_3'}{\sigma_1'} = \frac{\sin \psi_1 \cdot \cos \psi_1 - \cos^2 \psi_1 \cdot \tan \delta}{\sin \psi_1 \cdot \cos \psi_1 + \sin^2 \psi_1 \cdot \tan \delta} \tag{9.7}
\]

Note that for the concave arch \( \delta = \delta_p(\phi) \) of Table B, and again \( K_{1a} = K_a \).

The relations of Eq. 9.7 can be checked using Eq. 7.8 for the conditions of \( \delta = \phi \), and Eq. 7.11 (see Appendix III, Table A) for \( \psi \) substituted into Eq. 9.7, resulting in \( K_{1a} = K_a \).

(b) \[ K_{2a} = \frac{r_1}{\sigma_1'} \cdot \frac{4}{ID} \cdot \frac{1}{\sin \psi_1} \]

The relations of Eq. 7.10(ii) can be rewritten with the relations of Eq. 7.1 in the following way:

\[ \tau_1 = \sigma h_1 \cdot \tan \delta = (\sigma_3 \sin^2 \psi_1 + \sigma_1 \cos^2 \psi_1) \cdot \tan \delta \]

For the conditions of \( \delta \leq \phi \), \( K_{1a} = \sigma_3' / \sigma_1' \), therefore:

\[ \tau_1 = \sigma_1' (K_{1a} \cdot \sin^2 \psi_1 + \cos^2 \psi_1) \tan \delta, \text{ and therefore:} \]

\[ K_{2a} = \frac{4}{ID \cdot \sin \psi_1} \cdot (K_{1a} \cdot \sin^2 \psi_1 + \cos^2 \psi_1) \cdot \tan \delta \tag{9.8} \]
Note that $K_{2a}$ has dimensions of $\frac{1}{L}$

\[(c) \quad K_{3a} = 8 \cdot \left(\frac{R}{T_o}\right)^2 (1 - \sin \psi_1) \gamma = \frac{2\gamma}{1 + \sin \psi_1} \quad (9.9)\]

Using the relations of $K_{1a}$, $K_{2a}$, and $K_{3a}$ Eq. 9.6 becomes:

\[\frac{\partial \sigma'_1}{\partial z} - K_{2a} \cdot \sigma'_1 = K_{3a} \quad (9.6a)\]

This is a linear first order differential equation solved in the following way:

\[\sigma_{1z'} \cdot e^{-K_{2a} \cdot z} = -\frac{K_{3a}}{K_{2a}} e^{-K_{2a} \cdot z} + \text{Const.}\]

Using the boundary condition of $\sigma'_1 = 0$ at $z = 0$ gives:

\[\text{Const.} = \frac{K_{3a}}{K_{2a}}\]

and the solution of Eq. 9.6 is:

\[\sigma_{1z'} = \frac{K_{3a}}{K_{2a}} (e^{K_{2a} \cdot z} - 1) \quad (9.10)\]

The total vertical force on a horizontal (circular) cross-section (following Fig. 9.3) is then:

\[P_{va} = \int_{A} \sigma_{1z'} \cdot \sin \psi \cdot dA\]

\[dA = 2\pi R \cdot \cos \psi \cdot R \cdot d\psi\]

\[P_{va} = \int_{\psi = \psi_1}^{\psi = \pi/2} \sigma_{1z'} \cdot 2\pi R^2 \cdot \cos \psi \cdot \sin \psi \cdot d\psi\]

Noting that $K_{1a}$, $K_{2a}$, $K_{3a}$ and therefore $\sigma_{1z'}$ are functions of $\psi_1$ only, but not a
function of the variable \( \psi \), allows the above integration to be performed which leads to:

\[
P_{va} = \pi \cdot R^2 \cdot (1 - \sin^2 \psi) \cdot \sigma_{1z}'
\]

This can be rewritten as:

\[
P_{va} = - \frac{\pi}{4} \cdot ID^2 \cdot \frac{K_{3a}}{K_{2a}} \cdot ( e^{K_{2a} \cdot z} - 1 )
\]

or

\[
P_{va} = - \frac{\pi}{4} \cdot ID^2 \cdot \sigma_{1z}' = - \sigma_{1z}' \cdot A
\]

\( A \) being the cross sectional area, \( \left( \frac{\pi}{4} \cdot ID^2 \right) \)

9.2.2 The concave arch

Figure 9.4 presents the relations between the 'active' convex arch and the 'passive' concave arch. The solution of the concave arch follows that of Section 9.2.1 for the convex arch.

1. The vertical force due to the minor principal stresses

\[
P_{3z} = \int_A \left[ \sigma_3' - (\sigma_3' + \frac{\partial \sigma_3'}{\partial z} dz) \right] \cdot \sin \psi \cdot dA
\]

\[
= 2\pi R^2 \int_{\psi = \pi/2}^{90 - \psi_1} - \frac{\partial \sigma_3'}{\partial z} \cdot \sin \psi \cdot \cos \psi \cdot dz \cdot d\psi
\]

When substituting for \( \sin \psi_1 = ID/2R \) we obtain, as expected, a result equivalent to that of Eq. 9.2 (as both caps are of the same area):

\[
P_{3z} = - \frac{T}{4} ID^2 \cdot \frac{\partial \sigma_3'}{\partial z} dz
\]
2. The vertical force due to the major principal stresses

From symmetry, and due to the face that \( \sigma_1 \) is constant through the trajectory, the net vertical force due to \( \sigma_1 \) is zero, as previously shown to be the case for the convex arch and the minor principal stresses.

3. The vertical force due to gravity

The weight of the cap is:

\[
P \gamma z = A_c \cdot \gamma \cdot dz
\]

where \( A_c \) is the surface area of the cap:

\[
A_c = \int_{\psi = 90 - \psi_1}^{\psi = \pi/2} 2\pi R^2 \cdot \cos \psi \cdot d\psi = 2\pi R^2 (1 - \cos \psi_1) \quad (9.13)
\]

\[
P \gamma z = 2\pi R^2 (1 - \cos \psi_1) \gamma dz
\]

or

\[
P \gamma z = 2\pi R \left[ R - \sqrt{R^2 - (ID/2)^2} \right] \cdot \gamma dz
\]

Note that because \( \sin \psi_1 \) of the convex arch is equal to \( \cos \psi_1 \) of the concave arch, Eq. 9.3 is identical to Eq. 9.14. The difference will be only in the value of \( \psi_1 \) for the two different cases.

4. The frictional hoop force along the interface

\[
P \tau_z = \tau_1 \cdot \pi \cdot ID \cdot dz / \cos \psi_1
\]

where \( \tau_1 = f_s = K_1 \cdot \sigma_1 \cdot t g \delta_8 \) (Eq. 6.1), the interface shear resistance described in Chapter 6.

5. The vertical equilibrium equation of the concave spherical cap
$$\Sigma P_z = P_{3z} + P\gamma_z + P\tau_z = 0 \quad (9.16)$$

Substituting Eqs. 9.12, 9.14, and 9.15 into Eq. 9.16 leads to:

$$\frac{\partial \sigma_3'}{\partial z} = 8(R)^2(1 - \cos\psi_1)\gamma + \frac{4}{ID} \cdot \frac{\tau_1}{\cos\psi_1} \quad (9.17)$$

As noted above, \(\sin\psi_1\) of the convex arch is equal to \(\cos\psi_1\) of the concave arch. Therefore, the above equation is equivalent (except for the principal stress type) to Eq. 9.6. Using the same constants as previously defined:

(a) \(K_{1p} = K_{1a} = \frac{\sigma_3'}{\sigma_1} = \frac{\sin\psi_1 \cdot \cos\psi_1 - \cos^2\psi_1 \cdot \tan\delta}{\sin^2\psi_1 \cdot \cos\psi_1 + \sin^2\psi_1 \cdot \tan\delta} \quad (9.18)\)

Note that this solution is identical to that presented before, and was therefore denoted as Eq. 9.7 as well.

(b) \(K_{2p} = \frac{\tau_1}{\sigma_3} \cdot \frac{4}{ID} \cdot \frac{1}{\cos\psi_1} \)

The relations of Eq. 7.10(ii) can be rewritten with the relation of Eq. 7.1 in the following way:

$$\tau_1 = \sigma_1 \cdot \tan\delta = (\sigma_3 \sin^2\psi_1 + \sigma_1 \cos^2\psi_1) \cdot \tan\delta$$

For the conditions of \(\delta < \phi\), \(K_{1p} = \sigma_3' / \sigma_1'\), therefore:

$$\tau_1 = \sigma_3 \left(\sin^2\psi_1 + \frac{1}{K_{1p}} \cdot \cos^2\psi_1\right) \cdot \tan\delta$$

and therefore:

$$K_{2p} = \frac{4}{ID \cdot \cos\psi_1} \cdot \left(\sin^2\psi_1 + \frac{1}{K_{1p}} \cdot \cos^2\psi_1\right) \cdot \tan\delta \quad (9.19)$$
Using the relations of $K_{1p}$, $K_{2p}$, and $K_{3p}$ Eq. 9.17 becomes:

$$\frac{\partial \sigma'_3}{\partial z} - K_{2p} \cdot \sigma'_3 = K_{3p}$$  \hspace{1cm} (9.17a)

Which is a linear first order differential equation, solved as before:

$$\sigma'_3 \cdot e^{-K_{2p} \cdot z} = \frac{K_{3p}}{K_{2p}} e^{-K_{2p} \cdot z} + \text{Const.}$$  \hspace{1cm} (9.21)

The constant can be found by referring to two possible boundary conditions. One is a simplified condition, assuming 'passive arching' all along the soil plug, and the other is discussed in the next section (9.2.3)

$$\sigma_{zz}' = 0 \text{ at } z = 0$$, leading to $\text{Const.} = K_{3p}/K_{2p}$, and;

The stress:

$$\sigma_{zz}' = \frac{K_{3p}}{K_{2p}} \left( e^{K_{2p} \cdot z} - 1 \right)$$  \hspace{1cm} (9.22)

The total vertical force on a horizontal (circular) cross-section:

$$P_{VP} = \int_{\psi = 90 - \psi_1}^{\psi = \pi/2} \sigma_{zz}' \cdot 2\pi R \cdot \cos \psi \cdot \sin \psi \cdot d\psi$$

$$= \pi \cdot R^2 \cdot \cos^2(90 - \psi_1) \cdot \sigma_{zz}'$$

which can be rewritten as:
\[ P_{vp} = - \frac{\pi}{4} \cdot ID^2 \cdot \frac{K_{3p}}{K_{2p}} \cdot (e^{K_{2p} \cdot z} - 1) \]

or

\[ P_{vp} = - A \cdot \sigma_{3z'} \]

A being the cross sectional area, \((\frac{\pi}{4} \cdot ID^2)\)

Note that this state with the above boundary conditions refers to 'passive arching' taking place all along the soil plug.

### 9.2.3 The Comprehensive Analysis of the Soil Plug Resistance

Following is the mechanism which was suggested in Chapter 7, based on the observations of Chapters 3 and 5. The concave 'passive' arch will be formed and remain stable once the plug length is sufficient to develop (through the convex catenaries) enough resistance to create a lens-shaped zone (see Fig. 7.9a), followed by a stable 'passive' arch. This is due to the fact that while the inside resistance increases with continuous soil penetration into the pile, the 'pushing' force, determined by the bearing capacity, remains approximately constant beyond the so-called 'critical depth'.

Referring to Fig. 9.5, these boundary conditions can be described by:

\[ \sigma_{3z'} = \sigma_{iz'} \text{ at } z_p = L - z_a = 0 \]

Using Eq. 9.10 and 9.21:

\[ \sigma_{iza'} = \frac{K_{3a}}{K_{2a}} (e^{K_{2a} \cdot z_a} - 1) = \sigma_3/z_p = 0 = (- \frac{K_{3p}}{K_{2p}} e^{-K_{2p} \cdot z_p} + \text{Const.}) \cdot e^{K_{2p} \cdot z_p} \]

from which:

\[ \text{Const.} = \frac{K_{3p}}{K_{2p}} + \frac{K_{3a}}{K_{2a}} (e^{K_{2a} \cdot z_a} - 1) = \frac{K_{4p}}{K_{2p}} + \sigma_{iza'} \]
leading to a solution for $\sigma_{3z}$ (Eq. 9.21) of:

$$\sigma_{3zp}' = e^{K_{2p} \cdot z_p} \left( \frac{K_{3p}}{K_{2p}} (1 - e^{-K_{2p} \cdot z_p}) + \sigma_{1za}' \right)$$

or

$$\sigma_{3zp}' = -\frac{K_{3p}}{K_{2p}} + e^{K_{2p} \cdot z_p} \left( -\frac{K_{3p}}{K_{2p}} + \sigma_{1za}' \right)$$

(9.24)

from which:

$$P_{VP} = \frac{\pi}{4} ID^2 \cdot \sigma_{3zp}'$$

(9.25)

Equations 9.24 and 9.25 have two unknowns ($z_a$ and $z_p$), and therefore require additional information. This is provided by the calculated bearing capacity with depth, or the plug resistance for the experimental work. The solution of Eqs. 9.24 and 9.25 and their practical potential will be discussed in the following sections.
9.3 THE SOIL PLUG RESISTANCE ANALYSIS IN LIGHT OF EXPERIMENTAL DATA

9.3.1 Introduction

The plugging concept presented in the previous section is suitable to any type of driving. The equations which were developed in Section 9.2 refer to quasi-static penetration, and therefore should be applied to pile pushing. Further development should consider the dynamic effects and incorporate them into the same arching concept.

Since no comprehensive data set is available in which measurements of roughness and soil type are provided so that the S.G. model can be fully utilized, the effectiveness of the proposed analysis is demonstrated through the model pile experiments of Kishida (1967a, b), which are analyzed in Appendix II.

9.3.2 The Convex Arch Analysis

The practical meaning of the equations developed through the silo approach analysis were discussed in Section 4.5. This interpretation was clarified in Sec. 4.7.5, where Kishida's model tests were used to develop the relations shown in Fig. 4.11.

As a result of its being developed independently of the granular soil properties, the silo approach has the major disadvantage of being unable to predict the controlling $K \cdot \tan \delta$ factor.

The concave and convex arch analyses (and the comprehensive combination) both require knowledge of the internal shear angle of the sand and the interface friction coefficient. The latter is obtained from the S.G. model.

Appendix II presents the available information from Kishida's model test results. Based on his data, a skin friction coefficient of $\delta = 9.5^\circ$ was calculated ($8.4^\circ$ for the closed-ended piles and $10.5^\circ$ for the open). Kishida (1967a) quoted $\delta = 15.6^\circ$ (see Sec. 4.7.5 concerning the unrealistic low skin friction values). Using the S.G.
model (following Table 8.1 for the 'smooth' surface relations) and Kishida's description of uniform, very loose sand, \( \tan \delta \) for \( \beta = 90^\circ \) is expected to vary from 0.287 for very round sand to 0.430 for very angular sand. The minimum expected interface friction angle is therefore \( 16^\circ \) to \( 23^\circ \). If the brass surface is rougher, \( \delta \) may be greater. The low skin friction coefficient obtained from the pull-out test (as noted in Sec. 4.7.5) may be due to the fact that the sand was extremely loose, and 'cave-in' around the piles was observed during pushing (see Fig. AII.1 in Appendix II). As a result, the skin friction calculated from the pull-out tests differs greatly from the actual friction developed while pushing the piles. The average internal friction coefficient of \( \phi = 33.8^\circ \) was used with a depth of \( z = 45\text{cm} \).

Calculations of the plug force according to the convex arch analysis (Eq. 9.11), considering the adjusted height of \( L - h \) [see Sec. 9.2.1(4)] for \( \delta = 16^\circ \) to \( 20^\circ \) and for \( \delta = 30^\circ \) to \( 31^\circ \) (\( \approx 0.9\phi \)) are presented in Fig. 9.6. Both analyses indicate that the larger diameter piles are unplugged. The effectiveness of the method is obvious, as \( K \) is included in the calculations. It should be emphasized that the line of \( K \cdot \tan \delta = 0.20 \) in the silo analysis (also shown in Fig. 9.6) was developed in order to fit the data [see Sec. 4.7.5(8, 9)], while similar relations were obtained from the convex arch analysis as the by-product of a standard range of \( \delta \) values mentioned above.

### 9.3.3 The Comprehensive Analysis

The arching approach is based on the principle that the concave (passive) arches will be destroyed and replaced by catenaries until enough resistance is mobilized to support a stable concave arch. The previous simple convex analysis demonstrated that if the plug length is considered equal to the depth of penetration, then for the unplugged piles the convex arch will not mobilize enough resistance to overcome the bearing capacity.

A better insight into this phenomenon is achieved by analyzing each of the
plugs which were developed in Kishida's model piles by their length and average internal friction angle, utilizing the comprehensive analysis of Eq. 9.24. Additional information is provided by the plug capacity, which is essentially the bearing capacity multiplied by its area (see the line denoted by $Q_p = 2.3A$ in Fig. 9.6).

The results of these analyses are presented in Fig. 9.7 as relations between the pile diameter and the normalized length of the concave arch required in order to match the plug capacity. This was calculated for the entire possible range of $\delta = 16^\circ$ to $\delta = \phi$. It also should be noted that for the concave–passive arch, $\delta$ was limited by $\delta \leq \delta_p(\phi) \approx 19.5^\circ$, and the height was corrected for the height of both arches.

The results of Fig. 9.7 clearly distinguish between the plugged and unplugged piles on the basis of this suggested mechanism. For the $\delta$ values close to $\phi$ (as was found to be the case in the previous analysis), the unplugged pile showed that no concave arch developed there at all.
9.4 SUMMARY AND CONCLUSIONS

This chapter presented a comprehensive solution to the plugging problem, which can be further developed for dynamic conditions. The various tools that were developed in this study, including the S.G. model for prediction of the interface friction coefficient, provide fundamental, systematic, and logical solutions to the pile plugging problem, and hold great promise.

The limited data presented in Section 9.3 indicate the effectiveness and potential of the solutions. This pioneering approach, however, needs to be supplemented with additional work.
Fig. 9.1: The Convex Arch Analysis
from equilibrium

\[ \sigma_{hi} = \sigma_3 \cdot \sin^2 \psi_i + \sigma_1 \cdot \cos^2 \psi_i \quad (7.2) \]

\[ \tau_i = (\sigma_1 - \sigma_3) \cdot \sin \psi \cdot \cos \psi_i \quad (7.1) \]

\[ \frac{\tau_i}{\sigma_{hi}} = \tan \delta \quad (7.10) \text{ (ii)} \]

Substituting 7.1 and 7.2 into 7.10(ii) leads to Eq. 9.7

Fig. 9.2: The Relations between the Minor and Major Principal Stresses for \( \delta = \phi \) and Failure along the Interface
Fig. 9.3: Analysis for the Total Vertical Force
Fig. 9.4: The Relations between the Convex 'Active' and the Concave 'Passive' Arches and the Concave Arch Analysis
The Transition from 'Active' to 'Passive' Arching

Fig. 9.5: The Transition from 'Active' to 'Passive' Arching
Fig. 9.6: Determination of Plugging Condition for Experimental Data Using the Silo Approach (see Fig. 4.11) and the Convex Arch Analysis

- INSIDE DIAMETER - ID (cm)
- Plugged Unplugged
- Specific recovery ratio \( \gamma \) (%)
- Unplugged Piles
- Plugged Piles

- \( A = \pi \cdot ID^2 / 4 \)
- \( \gamma = 1.49 \text{ gr/cm}^3 \)
- \( z = 45.0 \text{ cm} \)
Fig. 9.7: Determination of Plugging Condition Using the Comprehensive Analysis of the Soil Plug Resistance
CHAPTER 10
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

10.1 SUMMARY AND CONCLUSIONS

Each of the preceding chapters contained a summary and conclusion section. A brief summary of the issues laid out in the Statement of Problem and Method of Solution (Sections 1.3 and 1.4) is presented herein.

1. Effects of Pile Plugging

Assuming that plugged piles behave like closed-ended piles, the effect of the penetration mode transition from unplugged to plugged is examined in Chapter 2 for the following aspects: ultimate static capacity, time-dependent pile capacity and dynamic behavior and analysis.

Pile plugging was found to have the following effects:

(a) Marked contribution to the capacity of piles driven in sand.
(b) Significant effect on the gain in capacity with time for piles driven in clay.
(c) Effect on the behavior of piles during installation, causing it to differ from that described by the models commonly used to predict and analyze pile driving.

Based on the above effects, the interest in the phenomenon is focused on:

(I) How to predict plugging of piles in sand and/or assess the inner soil capacity after installation.
(II) Determination of the extent of the pile plugging phenomenon in clays.
(III) Acquiring a basic understanding of the plugging mechanism, which is a requirement for the solution to the plugging problem.
2. Review and Interpretation of Existing Measurements

An analogy is established between open-ended piles and soil samplers. Data related to penetration of soil samplers, small-scale model piles and full scale piles are then examined in Chapter 3, and lead to the following conclusions:

(a) A good indication of plugging is provided by the specific (incremental) recovery ratio \(\gamma = \frac{dL}{dD}\).

(b) The offshore pile plugging phenomenon in predominantly clay profiles is of frequent occurrence, and has much greater significance than that presently accorded it by the profession. This lack of recognition is due mainly to misinterpretation of data.

(c) The mechanism of pile plugging during quasi-static penetration differs from that during dynamic penetration. However, both appear to be consistent phenomena (for given conditions); hence, plugging prediction methods are conceivably possible to develop.

(d) A certain mechanism transforms the pile from unplugged to plugged. In sands, this mechanism creates a zone of compressed soil at the pile tip.

The above factors indicate that an accurate analysis of plugging in sands requires the identification and evaluation of the major parameters controlling the observed mechanism.

3. The Silo Approach

Identification of the parameters controlling plugging in sand is achieved by a simplified analysis in Chapter 4 using the ‘Silo Approach’. Investigation of the governing equations of that analysis and comparison with experimental results in light of its underlying assumptions indicates the following:

(a) The plug resistance is controlled by: \(K = \) the ratio between horizontal to vertical stresses along the pile interface) and \(\tan \delta = \) the interface friction
481

coefficient).

(b) No assumptions are made in the ‘Silo Approach’ regarding the soil behavior, which is therefore reflected through the calculated K values.

(c) The increase of K with decrease in plug diameter reflects an increase in horizontal stresses, which may result from soil dilatancy and/or arching.

(d) Test results and the micro behavior of granular soil show arching to be the mechanism which influences the K value.

Based on the above findings, it was concluded that a refined plug analysis must entail:

(I) Accurate prediction of the interface shear resistance

(II) A solution for the K value which satisfies the stress state of the soil plug and accounts for the arching mechanism.

Both of the above requirements for the controlling parameters are believed to be determined by the particulate behavior of cohesionless soil.

4. The Micro Behavior of Granular Material

As both controlling parameters seem to be functions of the micro behavior of the soil, a review of relevant information is presented in Chapter 5. This review is presented in order to obtain the knowledge needed to develop and support solutions for the controlling parameters, and to examine the possibilities of explaining the silo approach by the particulate arching mechanism. The following conclusions were arrived at:

(a) The individual grains move and rotate to the most stable configuration as a reaction to an applied load. The preferred arrangement is achieved by orientation of the long axes of the particles perpendicular to the direction of the major principal stress.

(b) As a result of this orientation, the contact area normal to the major principal
stresses increases. The concentration of contacts in the preferred direction forms a column-like load path, which transfers the increasing axial stress along its trajectory.

(c) Particle rolling is the major microscopic deformation mechanism and there are relatively few contacts at which sliding is dominant even when the assembly reaches the overall failure state.

(d) Behavior of the granular material under loading is in agreement with the presented rigid disc model for the silo analysis. The preferred stress transference, as expressed by the arching hypothesis, was fully supported by observations of the micro-structure behavior.

The above observations led to the conclusions that:

(I) Models which simulate granular material as an assembly of individual particles must allow for rolling to occur. Moreover, analysis of friction of granular material along an interface must consider both the rolling and sliding mechanisms and their interactions.

(II) Analysis of granular soil (under the examined plugging conditions) must take into consideration the particle reorientation and the preferred stress path in the direction of the major principal stress trajectory.

5. The Sphere in the Groove (S.G.) Model for Granular Soil Interface Shear Resistance

(a) A mechanical model of a single rigid soil grain (sphere or ellipse) which considers the micro behavior of granular material is introduced in Chapter 6. The grain is supported by adjacent soil particles (frictional groove), and subjected at an additional contact point to normal and traction forces. These forces may result from a moving frictional plane (modeling the body in contact along an interface),
or from the motion of an adjacent particle (modeling a rupture zone within the granular material).

(b) Based on observations of granular material behavior, the S.G. model assumes that the grain will roll or slide along the groove between the supporting particles. A limiting equilibrium analysis is applied to the interface/interparticle contacts, examining the possibility of sliding or rolling in both locations.

(c) The following parameters are considered in the S.G. model:

- $\beta$ - Angle of the groove, correlated to soil density and grading.

- $\theta$ - Inclination of the groove, correlated to the interparticle contact which may be evaluated in several ways:
  (i) By determining the major principal stress trajectory of the stress state of the considered problem and/or experimental results;
  (ii) By utilizing the minimum energy criterion in order to obtain the preferred contact orientation.

- $l_s, R_a$ - Dimensions of the grain. This allows consideration of the grain shape through the ratio of the longer to shorter ellipse axes, $R_a = l/s$.

- $\alpha$ - Angle of roughness. This is correlated to the roughness of the contacting body surface through measurements of asperities along a gauge length equal to the median grain size diameter.

(d) Consideration of the limiting equilibrium at the interparticle contact resulted in relations of $\tan \phi \mu = f(R_a, \theta, \beta)$ [Eq. 6.14]. When the actual intergranular friction
equals that calculated, sliding takes place along the interparticle contact (the groove). When the actual intergranular friction is greater than that calculated for given conditions, only rolling is possible at the interparticle contact.

(e) Considering a representative intergranular friction coefficient of quartzic sand of \( \tan \phi = 0.5 \), then:

- Prior to shear, the actual interparticle friction is greater than the calculated sliding/rolling boundary value, producing the conditions of soil rolling in all zones.
- During shear, the rupture zone contains sliding particles, which provide most of the shear resistance.

(f) Consideration of the limiting equilibrium at the interface contact between the moving body surface and the soil particles resulted in the relations of \( \tan \delta_s = f(R_s, \delta, \beta, \alpha) \) [Eq. 6.13]. As long as the interparticle friction coefficient is greater than that calculated by Eq. 6.14, the interface friction coefficient \( \tan \delta_s \) describes the sliding resistance of a body in contact with granular material along the interface. For the conditions of sliding along the interparticle contact, the value of \( \tan \delta_s \) should be equal to the shear resistance of the soil, \( \tan \phi' \). The value of \( \tan \delta_s \) is limited, therefore, by the shear strength of the soil, \( \tan \phi \).

6. The Stress State of the Inner Soil

The stress state of the inner soil plug is investigated in Chapter 7 for the following purposes:

(a) To obtain the possible principal stress trajectories in order to assess the principal stress rotation, and to be able to combine the micro and the macro behavior through the arching theory.

(b) To obtain a solution for the stress ratio \( K_1 \) as a function of the interface friction coefficient \( \delta \), the shear strength of the soil \( \phi \), and the principal stress rotation
The above analyses yielded the following:

(I) The relative pile/soil displacement dictates the shear stress direction along the interface. Considering the axisymmetric conditions at the plug and the nature of the stress trajectories, resulted in two possible complementary trajectories for the major and minor principal stresses.

(II) The concave (downward) trajectory depicts the major principal stress rotation, and is the natural shape of the intergranular contacts to follow for the 'arching' process of load transfer. The convex (upwards) trajectory depicts the minor principal stress rotation, and assumes the shape of a catenary following the collapse of the 'natural' concave dynamic arch.

(III) Expressions for K values were developed for the different stress states (see Table 7.1 and Fig. 7.15).

(IV) A limiting interface friction angle $\delta_p(\phi)$ is suggested for the case in which the normal stress perpendicular to an interface is greater than that parallel to it ($K_i > 1$), such that the stress along the interface will not exceed the maximum shear stress along the failure plane within the soil mass.

7. Comparison Between the S.G. Model Predictions and Experimental Data

The proposed granular shear mechanism and the S.G. model predictions are examined in Chapter 8 for three situations:

(a) The S.G. model is used as a failure criterion for granular material (Section 8.3).

The general conclusions derived from this consideration are:

(I) A 'micro' limiting analysis on a representative grain results in a description of failure of a granular mass. Consideration of particle rolling, grain shape, and the interparticle friction coefficient holds
promising potential to accurately predict the relations between the interparticle friction coefficient and the orientation of the failure zone, and between this orientation and the shear strength of the material.

(II) The S.G. model explains three fundamental facts concerning granular material behavior:

(i) Only some particles are sliding
(ii) The rolling resistance is very small; the sliding resistance, therefore, remains the major frictional resisting mechanism
(iii) A 'zone' of turbulent failure, rather than a 'failure line' (saw-tooth line) is formed, and probably advances in stages from one area to another, along with the progression of the relative intergranular displacement.

(III) Skinner's experiments, ostensibly disproving the relation between shear resistance and interparticle friction, may be interpreted in a different way (based on the limiting friction angle), and do not reflect a lack of correlation between the two.

(IV) Rowe's stress dilatancy theory fails to consider the interparticle rolling and preferred interparticle contact orientation, and hence is applicable only to dense, pre-loaded material. The use of the $\phi_r^*$ value in place of $\phi_\mu$ is therefore unnecessary.

(V) Use of the S.G. model as a failure criterion was undertaken only in order to examine its underlying assumptions. Further work on and close examination of the subject are required.

(b) Predictions of the S.G. model are compared to measurements of the interface shear resistance of sliding smooth surfaces (Section 8.5). These comparisons led to several conclusions:

(I) The measurements of friction with smooth surfaces show good agreement
with the predictions of the S.G. model, supporting the following concepts:

(i) Limiting equilibrium consideration of a single grain

(ii) The importance of the grain shape as a major factor controlling its movement, and therefore the strength of the soil

(iii) The representation of the 'smooth' surfaces as normal and tangential traction coinciding with the direction of the plane movement

(iv) The development of a preferred contact orientation according to the minimum energy principle

(c) Predictions of the S.G. model are compared to measurements of the interface shear resistance of rough surfaces (Section 8.6)

These comparisons led to excellent agreement between the calculated and measured values, with the following conclusions:

(I) Predictions of the increase in friction coefficient with the increase in the principal stress rotation/contact orientation match the measured data and comply with the Cambridge DSS measurements of the principal stresses rotation during simple shear

(II) the principle of normalized roughness, considering the median grain size and the correlation of the slope of the plane in contact with the asperity height measurements, resulted in very good agreement with experimental results.

(III) Consideration of the factors that dictate the soil shear mechanism, namely the surface roughness, grain shape, interparticle friction and soil density, resulted in excellent agreement with experimental data.

(IV) Isolation of the two dominant factors (grain shape and surface roughness) resulted in simplified relations for interface shear with
quartzic sand (Eq. 8.8), which fit experimental results well (Fig. 8.27).

8. The Inner Soil Resistance

The incorporation of the S.G. and Arching concepts (Chapters 6 and 7) into the Silo Approach (Chapter 4) was undertaken in Chapter 9 to produce a comprehensive analysis for the inner soil resistance, consisting of the following stages:

(a) Determination of the mechanism of plug formation: The natural concave supporting arch follows the major principal stress trajectory upon downward movement of the pile wall (under the condition of 'passive arching'). As long as the strength of the arch is insufficient to support the upward load it collapses, and is transformed into a convex (upwards) catenary.

(b) Development of a governing equation which considers the possible arches and the different possible states of convex arch and stable concave arch.

The proposed mechanism is in agreement with the various components which were investigated and developed in the preceding stages (1 to 8), and shows good agreement with experimental data, thereby supporting the suggested mechanism and the developed analyses.
10.2 RECOMMENDATIONS

Due to the complexity of the plugging phenomenon and the difficulties in identifying when and whether it occurs, only a limited amount of research has been devoted to the subject, despite its significance. This study, being one of the first attempts to systematically confront the plugging problem, paves the way for further investigation of this complex phenomenon.

The following recommendations fall into two categories: (I) Referring to the plugging problem as a whole, and (II) Referring to specific developments which were introduced in the course of this research.

1. The lack of recognition of the significance of the significance of the pile plugging phenomenon to offshore pile driving calls for three recommendations:

(a) Continuous plugging measurements should be obtained during driving. Such measurements, taken at minimal additional cost, enable calculation of the specific recovery ratio \( \gamma \), and hence accurately identify the occurrence of plugging, providing the option of either assessing its consequences or avoiding it completely.

(b) The assessment of the gain in capacity with time for piles which become plugged in clays can be developed by using tools which are more sophisticated than the estimation in Section 2.3. This assessment should also include the ability to predict the gain in capacity when the pile penetrates in a plugged mode only along a certain fraction of its entire penetration.

(c) To develop appropriate tools to correctly incorporate the plugging phenomenon into the dynamic analysis of offshore piles during both design and construction. Such tools should consider the nature of pile driving in clays and in sands, each separately.

(I) The gain in capacity due to radial consolidation must be considered in
clays, as all identified plugging in clays (Section 3.2.5c) occurred following an intermission in driving due to splicing, and hence, gain in capacity due to pore pressure dissipation.

(II) The build-up of the dynamic arch in sands must be considered where the strength of the arch and the plug resistance are evaluated in relation to the propagating stresses along the pile. The two bodies (the pile and the inner soil) should therefore be analyzed separately under each blow, and then in combination, until stable arch formation is possible and plugging takes place.

2. The identified plugging mechanism and its analysis may be supported and refined by additional work in several relevant areas:

   (a) Experimental laboratory work on model piles driven and pushed into sand, where the build-up and destruction of the arches can be monitored and compared to the theory.

   (b) Improvement of the theory which considers the granular flow and the arch build-up. Such analyses can consider the strength of the arch by examining the granular material compression strength under the given conditions.

   (c) Findings from the above two stages must be incorporated into the previously mentioned dynamic analysis.

3. The S.G. model is currently the only available tool for modeling the mechanism of granular material/interface shear resistance. Additional work can be done in many areas related to this model:

   (a) Improvement of the model for probabilistic consideration, in which the deterministic parameters are substituted by probability distribution functions based on existing measurements of principal stress orientation, interparticle
friction coefficient, etc.

(b) Establishing a correlation between the angularity of the grain, as expressed by the \( R_a \) parameter, and measurements of particle shape. Utilizing an indirect method based upon the porosity of the soil, which results from a standard technique of deposition (Lucks –1970), seems to be most appropriate.

(c) Laboratory and field experiments can be performed in order to check and correlate further the various parameters with the field conditions. The interrelation between the influence of friction on the surface finish and on crushing of grains can also be investigated.

(d) The S.G. model can also be improved to account for conditions of dilation and/or dynamic displacement of the moving body.

(e) The development of the concept of a limited friction coefficient \( \tan \delta_p(\phi) \), Section 7.4.4c] must be examined experimentally. The example concerning Skinner's experiments (Section 8.3.4d) demonstrated the importance of the subject.

(f) The utilization of the S.G. model as a failure criterion holds great potential for further elucidation of the fundamental mechanism of deformation and shear. Additional experimental and theoretical work based on the material presented in Section 8.3 will lead to a better understanding of the mechanism of the behavior of granular material.
A FINAL NOTE

1. Certain problems in geomechanics can only be approached by considering the fundamental nature of the soil resulting from its character as an assembly of individual particles.

2. Emphasis on Why soil behaves as it does, rather than on How it behaves under specific conditions, will lead to improved modeling of soils.
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ASCE = American Society of Civil Engineers
ASME = American Society of Mechanical Engineers
ASTM = American Society for Testing and Materials
BCP = Bearing Capacity of Piles
CE = Civil Engineering
EERC = Earthquake Engineering Research Center
ICSMFE = International Conference on Soil Mechanics and Foundation Engineering
JGED = Journal of Geotechnical Engineering
JSMFD = Journal of the Soil Mechanics and Foundations Division
JSP = Journal of Sedimentary Petrology
MIT = Massachusetts Institute of Technology
NAVFAC = Naval Facilities Engineering Command
NGI = Norwegian Geotechnical Institute
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APPENDIX I
AN EVALUATION OF THE EFFECT OF A SOIL PLUG UPON THE STATIC CAPACITY OF THE PILE

I–1) SCOPE:
This appendix presents a rough evaluation of the effect of a soil plug upon the static capacity of a pile. The analysis is carried out for two relations:

(a) The point resistance \((Q_p)\) over the total pile capacity \((Q_t)\). This analysis is performed for closed-ended piles in sand and in clay, and is aimed at identifying the contribution of the end resistance/pile plugging to the total pile capacity.

(b) The total resistance of an open-ended pile \((Q_{open})\) over the total resistance of a closed-ended pile \((Q_{closed})\). This analysis is performed for open and closed-ended piles in sand and in clay. It is aimed at identifying the depth at which plugging occurs in both soils.

The results of the analyses are presented in Figure 2.2.

I–2) ASSUMPTIONS:
1. The analysis does not consider dynamic effects during driving, and is therefore applicable to quasi-static penetration only.
2. Size effect (diameter and slenderness) is assumed not to influence the pile capacity (Meyerhof –1983, Trofimenkov –1985).
3. The different load-settlement relations of the open/closed piles are not considered when referring to the pile capacity.
4. The effect of compaction on the soil due to the penetration is not considered.
5. Groundwater is assumed to be at the surface elevation.
I–3) DEFINITIONS:

Referring to Figure 2.1:

\[ t/B \geq 1/40 \text{ (see Figures 3.2, 3.3)} \]

\[ A_p = 0.25\pi \cdot B^2 \]

\[ A_t = \pi t(B - t) \leq \pi tB \cdot (1 - 1/40) \leq \pi tB \]

\[ A_p/A_t \geq 10 \]

\[ A_o = \pi BD \]

\[ A_i = \pi(B - 2t)D \leq \pi B(1 - 1/20)D \leq A_o \]

Open-ended piles:

\[ Q_p = q_p \cdot A_t \quad Q_o = \sum f_{so} \cdot A_o + \sum f_{si} \cdot A_i \]

\[ Q_{open} = Q_p + Q_o \]

Closed-ended piles:

\[ Q_p = q_p \cdot A_p \quad Q_o = \sum f_{so} \cdot A_o \]

\[ Q_{closed} = Q_p + Q_o \]

I–4) PILES IN CLAY

(a) End Resistance

\[ q_p \leq q_c = N_c \cdot C_u + \sigma_{vo} \]

where:

- \( N_c \) – bearing capacity factor
- \( C_u \) – undrained shear strength
- \( \sigma_{vo}, \sigma_{vt} \) – effective and total vertical stress
Referring to N.C. or slightly O.C. clay:

\[ N_c \geq 14.5 \, (\pm \, 30\%), \quad \text{Baligh et al. (1980)} \]
\[ C_u \geq 0.22 \, \sigma_v \nu' \quad \text{Jamiolkowski (1982)} \]

Substitution of the above into the end resistance equation leads to:

\[ q_p \geq 5.6 \cdot \gamma' \cdot D \]

(b) Skin Resistance

\[ f_{so} \approx \frac{1}{q_c} \quad 35 \quad \text{Begemann (1969)} \]
\[ f_{so} \approx C_u \quad \text{Vesic (1977)} \]

Substituting the above relations and assuming \( \gamma = 2.4 \cdot \gamma' \) leads to the average value of:

\[ f_{so} \approx \gamma' \cdot D/7.6 \]

(c) Closed-Ended Piles

\[ Q_p = \pi \gamma' \cdot D \cdot B^2 \cdot 5.6/4 \]
\[ Q_o = \pi \gamma' \cdot D \cdot B/7.6 \]

\[
\frac{Q_p}{Q_{closed}} = \frac{Q_p}{Q_p + Q_o} = \frac{1.4}{1 + (1/7.6) \, (D/B)} \tag{1.1}
\]

\[
\frac{Q_p}{Q_o} = \frac{1.4}{(1/7.6) \, (D/B)} \tag{1.2}
\]
for \( D/B \approx 11 \) \( Q_p = Q_o \)

(d) Open-Ended Piles

\[
Q_p = \pi \gamma' \cdot D \cdot B^{2.56/40}
\]

\[
Q_o = 2\pi \gamma' \cdot D \cdot B / 7.6 \quad \text{(assuming no enlargement at the tip)}
\]

\[
\frac{Q_{open}}{Q_{closed}} = \frac{0.14 + 0.26 (D/B)}{1.4 + 0.13 (D/B)} \quad \text{(I.3)}
\]

for greater wall thickness at the tip, \( f_{si} = \frac{f_{so}}{2} \) (Toolan and Fox –1977).

\[
\frac{Q_{open}}{Q_{closed}} = \frac{0.14 + 0.20 (D/B)}{1.4 + 0.13 (D/B)} \quad \text{(I.4)}
\]

I–5) PILES IN SAND

(a) End Resistance

\[
q_p = \sigma_{vo'} \cdot N_q \leq q_{pmax}
\]

for medium dense sand:

\[
D_c \leq 15 B \quad \text{Meyerhoff (1976)}
\]

\[
\phi' \leq 35^\circ \rightarrow N_q = 60 \quad \text{Zeitlen and Paikowsky (1982)}
\]

[including practical consideration of the change in \( \phi' \) due to driving (Kishida –1976)]

\[
q_p \leq 60 \cdot \gamma' \cdot D
\]

\[
q_{pmax} \leq 900 \cdot \gamma' \cdot B
\]
(b) Skin Resistance

Using McClelland's (1974) simplification:

\[ f_s \approx \gamma' \cdot D \cdot \tan \phi \quad \text{for } D > 6B \]

using, therefore:

\[ f_s = \gamma' \cdot B \cdot D \cdot \tan \phi \quad \text{for } D \leq 15B \]

\[ f_{s_{\text{max}}} = 15 \cdot \gamma' \cdot B \cdot \tan \phi \quad \text{for } D > 15B \]

(c) Closed-Ended Piles

\[ D/B \leq 15 \]

\[ Q_p = \pi \gamma' \cdot D \cdot B^2 \cdot N_q/4 \]

\[ Q_o = 0.5 \pi \gamma' \cdot D^2 \cdot B \cdot \tan \phi \]

\[ \frac{Q_o}{Q_p} = 2 \left( \tan \phi/N_q \right) (D/B) \quad \text{(1.5)} \]

\[ \frac{Q_p}{Q_{\text{closed}}} = \frac{Q_p}{Q_o + Q_p} \]

<table>
<thead>
<tr>
<th>(D/B)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>12.5</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_o/Q_p) (%)</td>
<td>0</td>
<td>5</td>
<td>12</td>
<td>23</td>
<td>29</td>
<td>35</td>
</tr>
<tr>
<td>(Q_p/Q_{\text{closed}}) (%)</td>
<td>100</td>
<td>96</td>
<td>90</td>
<td>81</td>
<td>77</td>
<td>74</td>
</tr>
</tbody>
</table>

\[ D/B > 15 \]

\[ Q_p = 3.75 \pi \gamma' \cdot B^3 \cdot N_q \]

\[ Q_o = \pi \cdot B^3 \cdot \gamma' \cdot \tan \phi' \cdot [112.5 + 15(D/B - 15)] \]

\[ \frac{Q_o}{Q_p} = 0.7 \left( \frac{1}{15} \cdot \frac{D}{B} \right) \quad \text{(1.6)} \]
(d) Open-Ended Piles

\[ D/B \leq 15 \]

\[ Q_p = \pi \gamma' \cdot D \cdot B^2 \cdot N_q \]

\[ Q_o = \pi \gamma' \cdot D^2 \cdot B \cdot tg\phi \]

\[
\frac{Q_{\text{open}}}{Q_{\text{closed}}} = \frac{1.5 + (D/B) \cdot tg\phi'}{15 + 0.5 \cdot (D/B) \cdot tg\phi'} \quad (1.7)
\]

\[ D/B \geq 15 \]

\[ Q_p = \pi \gamma' \cdot B^3 \cdot N_q \cdot 0.375 \]

\[ Q_o = 30 \cdot \gamma' \cdot \pi \cdot B^3 \cdot tg\phi' \cdot [7.5 + (D/B - 15)] \]

\[
\frac{Q_{\text{open}}}{Q_{\text{closed}}} = \frac{180.1 + 21(D/B - 15)}{303.8 + 10.5(D/B - 15)} \quad (1.8)
\]

\[ D/B \]

\[ 0 \quad 2.5 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \]

\[
\frac{Q_{\text{open}}}{Q_{\text{closed}}} \quad (\%) \]

\[ 10 \quad 20 \quad 30 \quad 46 \quad 59 \quad 80 \quad 95 \]

for \( D/B = 26.8 \) \( Q_{\text{open}} = Q_{\text{closed}} \).
APPENDIX II
ANALYSIS OF TESTS OF MODEL PILES
Data obtained from Kishida (1967a)

II–1) REMARKS
a) Estimated using interpolation or assuming B/t = 50
b) Measured from tip upwards
c) Tests 4, 5, 6
d) Estimated by Kishida
e) Estimated using extrapolation
f) Q_f was calculated by using f_savg of other experiments
g) Using q_pavg

II–2) RESERVATIONS

a) Some of the data were not clear (e.g. for B = 2.5cm), and therefore were not used
b) The relative density of D_r = 15% reported by Kishida seems to be unacceptable. However, the internal shear angle of \( \phi' = 26^\circ \) for the very loose soil state is reasonable.
c) As a result of the pile installation and the very loose state of the soil, caving in was observed around the piles; this may explain the very low pull–out results [see Fig. AII.1(a)].
THE TABLE LINES

1–4 See Figure AII.1(b) for dimensions
5 Weight of pile in kg
6–8 See Figure AII.2
9 Results of SPT-type test [see Fig. AII.1(c)]. Reported for avg. numbers from the tip upwards, as shown in Fig. AII.1(a).
10–12 Using the correlation between $N$ and $D_R$ shown in Fig. AII.2 and the following equation, which was fitted to Kishida's data:

$$\phi = 22.25 \circ + 0.2175 \cdot D_R$$

13 $e = 0.377 \cdot D_R - 0.967$
14 $\gamma_d = 2.63/(1+\ell)$
15 $W_s = \gamma_{davg} \cdot L_{avg} \cdot \pi \cdot ID^2/4$
16 $Q_{pu} =$ pull-out test result
17 $(16)-(15)$
18 $Q_f = Q_{pu} - W_{soil} - W_p = (17)-(5)$
19 Average friction
20 Average ultimate pile capacity
21 $Q_p =$ point resistance
22 $q_p =$ average point bearing capacity stress of the closed-ended piles
23 $Q_{pannulus} =$ estimated point resistance of the steel area of the open-ended piles
24 $Q_{plug} =$ The capacity of the plug $= (21)-(18)$
25 $q_{plug} = (25)/$plug cross-section
26–28 See dimensions in Fig. AII.1
<table>
<thead>
<tr>
<th>Table II.1</th>
<th>SUMMARY OF RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \text{d} (\text{cm}) )</td>
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<tr>
<td>2</td>
<td>( l (\text{cm}) )</td>
</tr>
<tr>
<td>3</td>
<td>( D/t )</td>
</tr>
<tr>
<td>4</td>
<td>( \text{ID} (\text{cm}) )</td>
</tr>
<tr>
<td>5</td>
<td>( W_d (\text{kg}) )</td>
</tr>
<tr>
<td>6</td>
<td>( L_{\text{avg}} (\text{cm}) / \text{tot} )</td>
</tr>
<tr>
<td>7</td>
<td>( L_{\text{avg}} / \text{ID} )</td>
</tr>
<tr>
<td>8</td>
<td>( D/ID )</td>
</tr>
<tr>
<td>9</td>
<td>( L_{\text{avg}} (\text{cm}) / \text{Navg} (\text{lb}) )</td>
</tr>
<tr>
<td>10</td>
<td>( \phi )</td>
</tr>
<tr>
<td>11</td>
<td>( \phi_{\text{avg}} )</td>
</tr>
<tr>
<td>12</td>
<td>( \text{W} (\text{kg}) )</td>
</tr>
<tr>
<td>13</td>
<td>( \text{avg} )</td>
</tr>
<tr>
<td>14</td>
<td>( \text{L}_{\text{avg}} (\text{cm}) / \text{Navg} (\text{lb}) )</td>
</tr>
<tr>
<td>15</td>
<td>( \text{W} (\text{kg}) )</td>
</tr>
<tr>
<td>16</td>
<td>( \text{OP/CP} )</td>
</tr>
<tr>
<td>17</td>
<td>( \text{avg} )</td>
</tr>
<tr>
<td>18</td>
<td>( \text{L}_{\text{avg}} (\text{cm}) / \text{Navg} (\text{lb}) )</td>
</tr>
<tr>
<td>19</td>
<td>( \text{W} (\text{kg}) )</td>
</tr>
<tr>
<td>20</td>
<td>( \text{OP/CP} )</td>
</tr>
<tr>
<td>21</td>
<td>( \text{avg} )</td>
</tr>
<tr>
<td>22</td>
<td>( \text{L}_{\text{avg}} (\text{cm}) / \text{Navg} (\text{lb}) )</td>
</tr>
<tr>
<td>23</td>
<td>( \text{W} (\text{kg}) )</td>
</tr>
<tr>
<td>24</td>
<td>( \text{OP/CP} )</td>
</tr>
<tr>
<td>25</td>
<td>( \text{avg} )</td>
</tr>
<tr>
<td>26</td>
<td>( \text{L}_{\text{avg}} (\text{cm}) / \text{Navg} (\text{lb}) )</td>
</tr>
<tr>
<td>27</td>
<td>( \text{W} (\text{kg}) )</td>
</tr>
<tr>
<td>28</td>
<td>( \text{OP/CP} )</td>
</tr>
</tbody>
</table>
Fig. AII.1  
(a) Caving in around the piles and the measured $L_i$, $N_i$  
(b) Pile and plug dimensions  
(c) SPT-type test

Fig. AII.2  
Correlation between $D_R$ and $N$ (Kishida 1967a)
APPENDIX III

SOLUTION FOR EQUATIONS 7.11 AND 7.18

Principal stress rotation $- \psi$; coefficient of lateral stresses at the pile wall $- K_i$ and the coefficient of the catenary $- a^{(4)}$, as a function of $\phi, \delta$ values.

Table A — for $45 + \phi/2 \leq \psi < 90^\circ$
Table B — for $0 \leq \psi \leq 45 - \phi/2$

Table A: $0 \leq \delta \leq \phi$

$\phi = \delta + \psi = 45 + \phi/2$; $K_i = 1/(1+2\tan^2\phi)$
$\delta = 0 \rightarrow \psi = 90^\circ$; $K_i = K_a$; $a = 0$

Table B: $5^\circ \leq \delta \leq \delta_p(\phi)^{(2)}$

$\phi = \delta_p(\phi) + \psi = 45 - \phi_2$; $K_i = 1 + 2\tan^2\phi$
$\delta = 0 \rightarrow \psi = 0^\circ$; $K_i = K_p$; $a \rightarrow \infty$

(1) 'a' is calculated for the concave (downwards) catenary.
(2) The value of $\delta_p(\phi)$ in Table B was rounded downwards. For exact values, see Table 7.2.
<table>
<thead>
<tr>
<th>φ</th>
<th>δ</th>
<th>ψ</th>
<th>K₁</th>
<th>a</th>
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<td>20</td>
<td>25.00</td>
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<tr>
<td>21</td>
<td>15.00</td>
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<td>0.761</td>
<td>P4</td>
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<tr>
<td>22</td>
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<td>0.622</td>
<td>0.639</td>
<td>P4</td>
</tr>
<tr>
<td>23</td>
<td>17.85</td>
<td>0.592</td>
<td>0.596</td>
<td>P4</td>
</tr>
<tr>
<td>24</td>
<td>19.51</td>
<td>0.575</td>
<td>0.541</td>
<td>P4</td>
</tr>
<tr>
<td>25</td>
<td>21.29</td>
<td>0.567</td>
<td>0.527</td>
<td>P4</td>
</tr>
<tr>
<td>26</td>
<td>23.07</td>
<td>0.556</td>
<td>0.423</td>
<td>P4</td>
</tr>
<tr>
<td>27</td>
<td>24.86</td>
<td>0.546</td>
<td>0.350</td>
<td>P4</td>
</tr>
<tr>
<td>28</td>
<td>26.65</td>
<td>0.537</td>
<td>0.306</td>
<td>P4</td>
</tr>
<tr>
<td>29</td>
<td>28.44</td>
<td>0.528</td>
<td>0.277</td>
<td>P4</td>
</tr>
<tr>
<td>30</td>
<td>30.23</td>
<td>0.520</td>
<td>0.259</td>
<td>P4</td>
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<tr>
<td>31</td>
<td>32.02</td>
<td>0.512</td>
<td>0.240</td>
<td>P4</td>
</tr>
<tr>
<td>32</td>
<td>33.81</td>
<td>0.504</td>
<td>0.227</td>
<td>P4</td>
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<tr>
<td>33</td>
<td>35.60</td>
<td>0.500</td>
<td>0.217</td>
<td>P4</td>
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<tr>
<td>34</td>
<td>37.39</td>
<td>0.502</td>
<td>0.207</td>
<td>P4</td>
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<tr>
<td>35</td>
<td>39.18</td>
<td>0.503</td>
<td>0.198</td>
<td>P4</td>
</tr>
</tbody>
</table>

Table A
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81.78
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82.96
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0.373
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79.27
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80.5?
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