NEUTRON FLUX PERTURBATION BY FOIL DETECTORS

BY

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ABSTRACT

"Neutron Flux Perturbation by Foil Detectors", by Charles Lee Dunford, a partial fulfillment of the requirements for the degree of Bachelor of Science in Physics at the Massachusetts Institute of Technology, June 1961.

There is an active interest in both science and engineering in the effect of a foil detector on the neutron flux which it is to measure. In this connection, this thesis has two objectives. The first objective is a complete compilation of the theoretical solution of the problem, and the second is a more thorough investigation of the theory than has been performed previously.

The theoretical treatment of flux perturbation follows closely that of Bothe\textsuperscript{2} with some modification of notations to conform with contemporary usage. To accompany this basic derivation, there is a summary of other approaches to the solution of this problem. There is also a summary of previous investigations which indicate that the Bothe formulas are essentially correct.

An investigation of gold and copper foils in graphite was carried out using the neutron flux in the hohlraum of the M.I.T. Reactor. The problems encountered are discussed and different experimental approaches suggested. Some curves for gold and copper foils in graphite are presented which include both flux perturbation and beta self-absorption. However, these curves have a somewhat restricted utility.
ACKNOWLEDGEMENT

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CONTENTS

Abstract ......................................................... i
Acknowledgement ............................................. ii
List of Figures .................................................. iv
List of Tables .................................................... iv

I. NEUTRON FLUX DETECTION .................................. 1
II. FLUX PERTURBATION THEORY ................................ 7
III. EXPERIMENTAL PROCEDURE ................................ 19
IV. RESULTS AND CONCLUSIONS ................................. 29

Bibliography .................................................... 45
LIST OF FIGURES

1. Neutron Flux in the Vicinity of a Foil .... 5
2. Flux Perturbation Parameter $\xi(x)$ .... 14
3. Hohlraum Sections .......................... 24
4. a) Section of the Graphite Stack ........ 25
    b) Cut Away View of the Stack ........... 25
5. $\frac{1}{4}$"d Copper Foils in Graphite ........ 32
6. $\frac{1}{2}$"d Copper Foils in Graphite ........ 33
7. 1"d Copper Foils in Graphite .............. 34
8. $\frac{1}{8}$"d Gold Foils in Graphite .......... 35
9. $\frac{1}{2}$"d Gold Foils in Graphite .......... 36
10. 1"d Gold Foils in Graphite ............... 37

LIST OF TABLES

1. Moderator Parameters ........................ 18
2. Reduced Copper Foil Data .................... 30
3. Reduced Gold Foil Data ..................... 31
4. Beta Absorption Coefficients ................ 38
I. NEUTRON FLUX DETECTION

The precise determination of a neutron flux is of considerable interest to physicists and nuclear engineers alike. However, the accurate determination of a neutron flux presents an extremely difficult problem because the neutron has no electric charge. The techniques involved in overcoming this difficulty contain several intrinsic sources of uncertainty.

There exist several standard methods for measuring a neutron flux. One method depends upon the detection of a product of a neutron induced transmutation, as exemplified by a $\text{BF}_3$ counter. Another method relies upon the detection of a charged recoil particle resulting from the elastic scattering of a neutron. These two methods can give a continuous measurement of the neutron flux. But, these methods require that rather bulky equipment be placed in the neutron flux, which is often neither feasible nor desirable.

The activation method is best suited for many purposes. This method relies upon the measurement of neutron induced radioactivity in a detector material. It is often advantageous to use this method because the detectors can be made very small. The sensitivity of the detector can be controlled by the proper choice of materials with different cross-sections. It is also possible to separate the
process of activation from that of measuring the foil ac-
tivity, so that the amount of material inserted in the flux
is small, and complex counting equipment need not be brought
to the activation area. This detection method does preclude
the use of activation detectors for instantaneous flux mea-
surement.

A material used for activation detection must
have a suitable cross-section in the energy range that is
of interest. If the neutron flux is large, then the cross-
section should be relatively small, and if the flux is small,
then a larger cross-section would be required. For reasons
that will be discussed later, a detector should be very
thin and of course, free of impurities. These requirements
set the metallurgical and chemical standards for detector
materials. The activation product should have a single
radioactive species with a known simple decay scheme. The
half-life for decay should range from several minutes to a
few days for convenient activity measurement. The most
commonly used foils are made from indium or gold. Dys-
prosium and europium compounds are used for very small
fluxes because of their very large thermal cross-sections.
Here at M.I.T., copper is widely used for several reasons,
principally because it is easy to handle and inexpensive.

The basic theory of flux detection by foil activa-
tion is relatively simple. The activity of a radionuclide
follows a simple exponential decay after the removal of the
foil from a neutron flux. The activity at any time during the irradiation can be obtained from the following equation.

\[
\frac{d N_a}{d t} = \varphi \sigma N - \lambda_a N_a
\]  

(1)

In equation (1), \(N_a\) is the number of radioactive daughter atoms, \(N\) is the number of parent atoms, \(\sigma\) is the reaction cross-section, \(\varphi\) is the neutron flux, and \(\lambda\) is the decay constant of the daughter. We assume that \(N\) is a constant during the irradiation which is reasonable since only an extremely small fraction of the parent atoms are ever transformed during a usual activation period.

If equation (1) is solved with the initial condition that there are no daughter atoms at time zero, then the number of daughter atoms as a function of time is given by equation (2). Now if the foil is irradiated for a time \(T_a\), then the number of daughter atoms is given by equation (2A). If the foil is removed from the neutron flux at time \(T_a\), then the number of daughter atoms as a function of time is given by equation (2B).

\[
N_a(t) = \frac{N \sigma \varphi}{\lambda_a} \left(1 - e^{-\lambda_a t}\right)
\]  

(2)

\[
N_a(T_a) = \frac{N \sigma \varphi}{\lambda_a} \left(1 - e^{-\lambda_a T_a}\right)
\]  

(2A)

\[
N_a(t^*) = \frac{N \sigma \varphi}{\lambda_a} \left(1 - e^{-\lambda_a T_a}\right) e^{-\lambda_a t^*}
\]  

(2B)
Now let us see how we account for the counting procedure. Assume that the counting begins at a time \( T_d \) and continues for a time \( T_c \). Then the number of daughter atoms decaying during the counting time is given by equation (3). The measured activity will then be given by equation (3A), if we assume that the counter has an efficiency \( \epsilon \).

\[
\Delta N_a = \frac{N \sigma \varphi}{\lambda_\alpha} \left( 1 - e^{-\lambda_\alpha T_a} \right) \left( e^{-\lambda_a T_0} - e^{-\lambda_a [T_0 + T_c]} \right) \quad (3)
\]

\[
A_{MEAS} = \epsilon \lambda_\alpha \Delta N_a = \epsilon N \sigma \varphi \left( 1 - e^{-\lambda_\alpha T_a} \right) \left( 1 - e^{-\lambda_a T_c} \right) e^{-\lambda_a T_0} \quad (3A)
\]

The group of terms \( N \sigma \varphi \) can be defined as the saturation activity, \( A_{sat} \), and this quantity is proportional to the neutron flux. The saturated activity can be determined from equation (3B) which is just another form of (3A).

\[
A_{SAT} = \frac{A_{MEAS}}{\epsilon \left( e^{-\lambda_\alpha T_0} \right) \left( 1 - e^{-\lambda_a T_c} \right) \left( 1 - e^{-\lambda_a T_a} \right)} \quad (3B)
\]

For absolute flux measurements, the efficiency must be determined for the particular counting instrument used. Various other small corrections to the measured activity should be made. These corrections can be found in any book about radiation detection and so will not be discussed here.

There is a particular phenomenon associated with neutron flux detection which will be the topic of the paper. Briefly the problem is this. When a foil detector is put in a neutron flux, it tends to disturb the flux which we wish to measure. So the problem becomes, what correction...
must be applied to the measured activity to account for the change in flux caused by the foil's insertion?

Following the conventions set forth in Sola’s paper, I shall describe the entire phenomenon as flux perturbation and define a quantity $P$, such that when $\psi_o$ is the unperturbed flux, and $\psi$ is the measured flux, then $P$ is defined by equation (4). In other words, $P$ is a correction factor to be applied to the measured flux in order to obtain

$$P = \frac{\psi}{\psi_o} = \frac{A_{SAT}}{A_{SAT_o}}$$

the corrected activity corresponding to the unperturbed flux.

In order to give a little more insight into this problem, let's identify two distinct phenomena which are lumped into the single term, flux perturbation. These two problems are usually analyzed separately but can not be conveniently separated in practice. The first phenomenon is known as flux depression and is caused by the foil acting as a neutron sink in the midst of a region that has a uniform neutron flux. An example of this region might be a reactor moderator which can be considered as uniformly producing thermal neutrons. This phenomenon can be associated with the quantity $\frac{\psi}{\psi_o}$ which we shall call $H$, the flux depression factor.

![Figure 1](image-url)
depression factor.

The interior of the foil, on the average, sees a flux that is smaller than the surface flux. This is due to the absorption of neutrons by the outer layers of foil atoms, while no thermal neutrons are produced. This phenomenon is known as self-protection or self-absorption. It can be described by the ratio \( \frac{\phi_{\text{ave}}}{\phi_s} \), which will be called \( G \), the self-protection factor. Now \( \frac{\phi_{\text{ave}}}{\phi_s} \) is directly related to the measured activity. Then \( P \) is given by \( P = GH \), and the correction is given by equation (5). The saturation activity

\[
A_{\text{MEAS}} = P \times A_{\text{MEAS}_0} = G \times H \times A_{\text{MEAS}_0} \quad (5)
\]

corresponding to \( \gamma \) can be obtained by the theory previously outlined. I shall now explore the analytical determination of \( G \) and \( H \) and so \( P \).
II. FLUX PERTURBATION THEORY

As stated previously, the induced activity of a foil placed in a neutron flux will be affected by several properties of the flux and the surrounding medium. The activity will depend upon the neutron density, the energy distribution, the directional properties of the flux, and the perturbations caused by the foil detector. The first published attack on this problem came from Bothe\textsuperscript{2} in 1943. I shall follow the general outline of his analysis.

The analysis is based in a one group model of the neutron energy spectrum. So we shall assume a constant neutron velocity $\mathbf{v}$. Let $\mathcal{A}$ be the activation of the foil. Now if $K$ is the neutron intensity in a unit solid angle, then $\eta$, the neutron density is given by equation (6).

$$\eta = \frac{1}{V} \int K \ d\Omega$$

If this integral is independent of spatial orientation, then it can be shown that the activation of a foil is independent of the directional properties of the neutron flux, but only if the foil is a weak absorber of neutrons.

Now consider the absorption by a finite disk shaped foil of thickness $\Sigma$ and a neutron absorption coefficient $\mu$. The activation of this disk, that is, the number of daughter atoms produced, is given by equation (7).
\[ A = \int \kappa |\cos \theta| (1 - e^{-\frac{\mu \delta}{|\cos \theta|}}) \, d\Omega \]  

This integral will be independent of \( \theta \) for a unidirectional beam if \( \frac{\mu \delta}{|\cos \theta|} \ll 1 \).

It is also possible to integrate this expression for some heterogeneous fields. Assume that \( \kappa \) is symmetric about some axis fixed in space. Let \( \omega \) be the angular parameter of the flux intensity distribution. If we find that the angular distribution of the flux is of the form \( k(\omega) = k_0 + f_{\omega\omega}(\omega \omega) \), then the probe activation will be independent of direction. The results are given in equations (8) and (9).

\[ \tau = \frac{1}{V} \int k_0 \, d\Omega = \frac{4\pi k_0}{V} \]  

\[ A = 4\pi k_0 \left[ 1 - e^{-\mu \delta} (1 - \mu \delta) + (\mu \delta)^2 E_1(-\mu \delta) \right] \]  

The function \( E_1(x) \) is the exponential integral defined by \( \int_{x}^{\infty} \frac{e^{-u}}{u} \, du \). This integral is tabulated by Jahnke & Emde\(^3\). If we now take the case where \( \mu \delta \ll 1 \) the expression for the activation becomes simplified and is presented in equation (10). It is known that for a scattering medium

\[ A = 4\pi k_0 \mu \delta = n V \mu \delta \]
that is only weakly absorbing, the distribution function $K(\omega)$ is $K_0 - K_\omega \omega \omega$ and so is described by the case just discussed.

Now let's examine an even function of $\cos \omega$, namely $K(\omega) = K_m \cos^2 \omega$. Then the neutron density is simply given by equation (11). Now the evaluation of the activity is much more complicated. Bothe's results are given below.

$$n = \frac{K_m}{V} \int \cos^2 \omega \, d\tau = \frac{4}{3} \frac{K_m}{V} \quad (11)$$

$$A = 2K_m \int_0^{2\pi} d\psi \int_0^1 \cos^2 \omega \left(1 - e^{-i\alpha/\theta}\right) \cos \theta \, d(\omega \theta) \quad (12)$$

$$\cos \omega = \cos \psi \cos \theta - \sin \theta \sin \psi \cos \psi \sin \psi$$

$$A = \frac{1}{2} K_m \left(2 \cos^2 \psi - \sin^2 \psi\right) \left[1 - e^{-\mu \delta (1 - \ldots)} + (\delta \mu)^2 E_1(\mu \delta) + \ldots\right] \quad (13)$$

So the activation does indeed depend on the angle $\psi$ between the surface normal and the distribution axis of the neutron flux. The activation is a maximum when $\psi = 0$ and a minimum when $\psi = \pi$. However, if $\mu \delta \ll 1$, equation (13) reduces to equation (14). In general, it can be shown that

$$A = \frac{4}{3} K_m \mu \delta = n \sqrt{\mu \delta} \quad (14)$$

the activation of a probe is independent of the directional properties if $\mu \delta \ll 1$. Bothe has performed some calcu-
tions using these results. The computations indicated that errors up to 7% were introduced in the activation due to the directional properties of the flux if the target absorbs up to 2% of the incoming beam.

Let's now consider a disk shaped probe in an isotropic field. Let $\mathcal{X}$ be the number of neutrons crossing a unit surface area per unit time, and let $\alpha$ be the neutron capture probability in the disk. These new quantities are given in equations (15) and (16).

\[ \mathcal{X} = \int k_0 |\cos \theta| d\Omega = 2\pi k_0 = \frac{1}{\delta} n V \quad (15) \]

\[ \alpha = \frac{2A}{nV} = 1 - e^{-\mu \delta (1-\mu \delta)} + (\mu \delta)^2 E_1(-\mu \delta) \quad (16) \]

Now we are ready to proceed with the evaluation of the flux depression factor. Assume that the probe is a sphere of radius $R$ and that the probe is in an infinite scattering medium with a diffusion length $L = \sqrt{\frac{1}{\kappa_s} \frac{\lambda_s}{\lambda_s}}$. Assume also that the neutron density in the medium is isotropic.

**CASE I** \( R \gg \lambda_s \)

Consider the diffusion equation where $n_0$ is the uniform production term.

\[ \nabla^2 n - \frac{n}{L^2} = n_0 \quad (17) \]
If we consider the center of the probe as the origin of the coordinate system, the solution in spherical coordinates is given by equation (18).

\[ n = n_o - \frac{a}{L} e^{-\frac{a}{L}} \]  

\[ \frac{dn}{d\rho} = \frac{a}{L} \left[ \frac{1}{\rho} + \frac{1}{L} \right] e^{-\frac{a}{L}} \]  

Diffusion theory gives the following expressions for the neutron current at a point.

\[ J_+ = \frac{V}{4} \left[ n(\rho) + \frac{a}{3} \lambda_s \frac{dn}{d\rho} \right] \]  

\[ J_- = \frac{V}{4} \left[ n(\rho) - \frac{a}{3} \lambda_s \frac{dn}{d\rho} \right] \]  

Now the albedo, which is defined as \( J_- / J_+ \), is only the non-absorption probability, \( 1 - \alpha \).

\[ \alpha = \frac{n_R - \frac{a}{3} \lambda_s \left( \frac{dn}{d\rho} \right)_R}{n_R + \frac{a}{3} \lambda_s \left( \frac{dn}{d\rho} \right)_R} \]  

The unknown \( \alpha \) in equation (18) can now be evaluated by substituting the expressions for \( n_R \) and \( \left( \frac{dn}{d\rho} \right)_R \) into the albedo equation. The solution for \( \alpha \) is presented in equation (21).

\[ \alpha = \frac{\frac{3}{2} a R}{\lambda_s} \frac{R}{\lambda_s} \frac{R n_o e^{\frac{R}{L}}}{\frac{3}{2} a R \left( \frac{R}{\lambda_s} \right) + (2 - \alpha) \left( 1 + \frac{R}{L} \right)} \]
If there were no absorption by the sphere, the neutron current would be given by $J_{+0} = \frac{m_0 V}{4}$. Combining equations (18), (19), and (21), the flux depression factor $H$ can be defined and evaluated. The result is given in equation (22).

$$H = \frac{J_+}{J_{+0}} = \frac{1}{1 + \frac{1}{2} \alpha \left[ \frac{3L}{\lambda_s (R+L)} - 1 \right]} \quad (22)$$

In certain useful special cases, the following simplified expressions can be used as corrections. Bothe proposed a slight change be made in these formulas, so that they would be applicable to the case of a disk shaped foil. He suggested that $R$ be replaced by $2R/3$.

$$H = \frac{1}{1 + \frac{3}{4} \alpha \frac{R}{\lambda_s}} \quad L \gg R \gg \lambda_s \quad (23)$$

$$H = \frac{1}{1 + \alpha \left[ \frac{3}{4} \lambda_s - \frac{1}{4} \right]} \quad R \gg L \gg \lambda_s \quad (24)$$

proposed a slight change be made in these formulas, so that they would be applicable to the case of a disk shaped foil. He suggested that $R$ be replaced by $2R/3$.

**CASE II** \( R \ll \lambda_s \)

This case does not lend itself to a similar analysis. However, after using some physical intuitive reasoning, Bothe presents this result.

$$H = \frac{1}{1 + 0.34 \lambda_s \frac{R}{\lambda_s}} \quad (25)$$
Bothe performed some brief experiments to test his proposed theory. He used thin foils of dysprosium oxide and irradiated them in both paraffin and aluminum. He found that within his experimental error, the results gave good agreement with his theory for foils down to $2\lambda$, and that for $R \gg \lambda$, the results were in excellent agreement. More experimental investigations of flux perturbation were carried out by Goodman, Darcy, and Paine, here at M.I.T. in 1947. Their results indicate that there is good agreement with Bothe's formulation, for indium foils in light water with radii as small as one scattering free path in light water.

When the transport theory modification of diffusion theory was applied by Tittle, he found that Bothe's experiment data gave better agreement with the theory. Tittle also found that Bothe's formulas for a spherical probe need not be modified to give the correct results for a disk shaped probe. It would appear from this result that the effects of a sphere and a disk of the same radius on the flux are the same. The transport theory modification of the diffusion theory parameters are presented in equation (26), and Tittle's modification to Bothe's formula (22) is given in equation (27). The parameter $\alpha$ is given and redefined as $\int -e^{-x}\left(1-x\right) + x^2 E_1(-x)$, where $x$ is the product of the macroscopic absorption cross-section and the thickness of the foil. Figure #2 is a graph of $\alpha$ as a function of $x$. 

-13-
FLUX PERTURBATION PARAMETER $\alpha(x)$

\[ x = \sum a \cdot t \]
Subsequent measurements by Sola, and separately by Klema and Ritchie\(^6\), found that Bothe's formulas, as modified by Tittle, gave good agreement for the flux perturbation of a indium foil in graphite, and reasonably good results for gold foils in graphite.

The associated problem of self-protection has been treated by several men for various geometries. For the case of interest, namely the limit of very thin foils, Sola finds that the self-protection factor is given reasonably well by the following expression.

\[
G = \frac{1 - e^{-x(1-x)} + x^2 E_i(-x)}{2x} = \frac{\sigma}{\Sigma x} \quad (28)
\]

Another approach to the problem of flux perturbation has been presented by Skyrme\(^7\), who treated the problem using one speed transport theory. Considering the foil as a first order perturbation, Skyrme derived equation (29).
In this expression, $x$ has the usual meaning, $g$ is the product of the macroscopic cross-section and the foil radius. $A(g)$ and $L_1$ are parametric functions which may be found in graphical form in Sola's paper. $D_1$ is so small that it may be neglected for most computational purposes. Equation (29) is valid under the conditions that $X \ll 1$ and $x \ll 4g$. It might be useful here to mention how the macroscopic absorption cross-section of the foil detector should be evaluated. It has been found by workers in the field that the best cross-section to use for thermal neutron detectors is not the standard 2200 meter per sec. cross-section but the Maxwell-Boltzman averaged cross-section which can be computed using equation (30). This averaged cross-section is used because it has been found that the energy spectrum of neutrons in a moderator is very nearly Maxwell-Boltzman. So any cross-section used should be an effective cross-section that has been averaged over the Maxwell-Boltzman distribution.

An attempt has been made in the recent literature to present a single theory which is valid for all sizes of disk shaped detectors in any medium. Ritchie & Eldridge present such a theory for consideration. First they put forth a modified version of Bothe's and Skyrme's theories. The self-protection factor $G$ is modified and hence the expression for $\alpha$. These changes are given in equation (31).
Botho's modified formula with the self-protection factor included is given by equation (31).

\[
\frac{\alpha}{\lambda} = \frac{1}{2} - E_3(x)
\]

\[
E_3(x) = \int_{-1}^{1} \frac{c}{u^2} \, du
\]

They then proceed to derive an expression for the flux perturbation by a foil using a variational technique on one speed transport theory. The result is obtained in terms of several parametric functions which are given as graphs in their paper. Ritchie and Eldridge then show that both Bothe's and Skyrme's are just special cases of their general equations.

A report published in 1954 by Fitch and Drummond contains some extensive results for flux perturbation in light water. They found good agreement with Bothe's formulas for indium foils in light water. This report would be extremely useful for someone interested in the flux perturbation caused by a wire. Fitch and Drummond developed a formula for flux perturbation by a cylindrical rod, thermal neutron detector. The result is quite complicated and will not be presented here. The report then gives the results of an investigation of indium rods in light water, lead-indium.
wires in light water near a uranium boundary. Curves are presented for these cases.

The table below gives the values of important parameters for common moderating media in which the detector might be placed. From this table, one can see that the flux depression would be most prominent in paraffin and light water since they have small values of the transport mean free path. A much smaller effect would be observed in heavy water, beryllium, and graphite.

<table>
<thead>
<tr>
<th>Medium</th>
<th>L</th>
<th>λ tr</th>
<th>λ a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paraffin</td>
<td>2.42cm.</td>
<td>0.395</td>
<td>44.9cm.</td>
</tr>
<tr>
<td>H₂O</td>
<td>2.76</td>
<td>0.425</td>
<td>53.8</td>
</tr>
<tr>
<td>D₂O</td>
<td>171</td>
<td>2.4</td>
<td>31,000</td>
</tr>
<tr>
<td>Be</td>
<td>31</td>
<td>2.6</td>
<td>1,100</td>
</tr>
<tr>
<td>Graphite</td>
<td>50.2</td>
<td>2.7</td>
<td>2,800</td>
</tr>
</tbody>
</table>
III. EXPERIMENTAL PROCEDURE

This thesis was designed to obtain more comprehensive data on flux perturbation in order to make some judgement as to the validity of current theories. A simple method of operation was first investigated. Two radium-beryllium neutron sources, each with a strength of approximately $10^6$ neutrons per second were placed symmetrically in the base of the M.I.T. Sigma Pile. Then foils of varying diameters and thicknesses were to be irradiated in some convenient slot in the pile stringers. Each foil was to be irradiated in the same position in the pile, for the same length of time. Then the activity of each foil was to be measured using an end-windowed beta counter.

The important reactor moderating media are of most interest as parameters in this study. Graphite is the easiest material to handle and so the first attempts to obtain data were in graphite. The Sigma Pile was used for this purpose. Upon completion of the first phase, a subcritical tank was to be used for the investigation of light and heavy water. Heavy water was to be used in the final phase because special handling methods were needed, as heavy water has to be sealed in a tank with a helium atmosphere.

The first material to be irradiated in the Sigma Pile was copper. This material is commonly used here at M.I.T. because of its metallurgical properties and its availability.
in many different thicknesses. Copper foils do present several difficulties in their use as foil detectors.

Natural copper consists of two isotopes, Cu$^{63}$ and Cu$^{65}$ with respective abundances of 69.1% and 30.9%. Each of these isotopes have similar absorption cross-sections. The tabulated values are 4.5 barns for Cu$^{63}$ and 2.2 barns for Cu$^{65}$. However the activation products have half-lives which differ considerably. The copper 63 product has a half-life of 12.8 hours and the copper 65 product has a half-life of 5.15 minutes. For moderate activation times and flux levels, the copper 63 can be used as the detector, if certain procedures are followed. The activity from copper 65 will tend to saturate during the irradiation because of the short half-life, but the activity of the copper 63 will continue to build up. Activity measured right after exposure will be composed of a mixture of radiations from the two isotopes. But if one waits for about two hours before counting the foil, then the activity due to copper 65 will be negligible because of its short half-life.

Copper foils one inch in diameter and twenty mils thick were irradiated in a slot as close to the sources as possible, for times varying from several minutes to several hours. No appreciable increase in activity of the foils (above background) was detected. It was then decided to investigate the activity which could be obtained using gold foils. Gold is a more commonly used foil detector because it has a cross-section of 98.8 barns.

-20-
The radioactive product has a long half-life, about 2.7 days, so that the activity increase in gold is not nearly as large as a ratio of the cross-sections would seem to indicate. Several attempts were made to irradiate some gold foils one half inch in diameter and ten mils thick. Irradiation times from two hours to one day were tried, but still the beta counter could not detect any increase in activity in the foils. Finally a gold foil one inch in diameter and ten mils thick was inserted in the pile as close to the source as possible and left for six days. This time there was a significant increase in the activity of the foil.

This result and several other factors caused the abandonment of this approach to the problem. First, only large gold foils could be irradiated in this manner and still get measurable activity. Since only symmetrical positions in the pile could be used so as to eliminate flux variations, long irradiation times would preclude collection of sufficient data in a reasonable length of time. It must be realized that at a distance of only a few centimeters from these two neutron sources, the flux is relatively small. On top of this, the counter being used had an efficiency of only 2 or 3 percent. So a much larger flux is needed.

Since this experiment was being performed to investigate the flux perturbation caused by thermal neutrons, large discrepancies would result from irradiation too close
to the source. Near the source, the neutron spectrum will not yet be fully thermalized. Thus resonance and epithermal capture by the absorbing foils would introduce considerable error. Yet in order to obtain sufficient induced activity, one would have to place the foil close to the source.

An experiment was performed to find out how well the neutrons were thermalized at various distances from the sources. Cadmium covered, one inch diameter gold foils were placed in slots near the center of each of four stringers, each stringer being successively further away from the sources. Bare gold foils were placed in symmetric positions in the stringers. Each gold foil was identical with all the other gold foils. They were irradiated for six days and then counted with an end windowed beta counter. The cadmium ratios (order of magnitude only) were found successively to be 4.8, 8.7, 12.5, and 13.4. These results indicated that the neutrons were not well thermalized, as a well thermalized spectrum would give a ratio of better than 30. Then it was decided that a new method had to be found.

It was then decided that the M.I.T. Reactor might be used to obtain a larger thermal neutron flux. The most suitable place to conduct this experiment was the top of the hohlraum. The flux at the top of the hohlraum is of the order of \(5 \times 10^5\) neutrons per square centimeter per sec. when the reactor power is 80 KW. The hohlraum is a hollow graphite enclosed volume at the end of the thermal column.
The top of the hohlraum is covered by an aluminum plate. On top of the plate, 48 inch graphite stringers were stacked as shown in figures #3 and #4. Each of the numbered stringers in figure #4 had four, one inch diameter slots milled in their upper surface. Each slot was 100 mils deep. These slots were to hold the test foils. In order to get an isotropic flux, a cadmium sheet was inserted between the aluminum plate and the graphite stack so as to cut down the streaming from the reactor thermal column. Of course this strong absorber would tend to distort the flux, but the corrections made using the monitor foils would correct this. Since this direct feed of neutrons was cut out, we had to rely on neutrons diffusing into the stack from the side.

One problem concerned with the irradiation of many foils at one time is the variation of the neutron flux throughout the graphite stack. An attempt to provide a correction for this uncertainty caused the insertion of the monitor foils. These foils were taped to the graphite stringers, opposite the test foil it was to monitor. Each foil was of identical material, cross-section area, and thickness. Thus many corrections, including flux perturbation, which are the same for identical foils could be neglected. Since we are only concerned with relative activity in the monitor foils, and the flux perturbation and other corrections were identical, all the corrections would cancel out. The problem of mutual flux interaction
SECTION OF GRAPHITE STACK

- MONITOR FOILS
- TEST FOILS

GRAPHITE STRINGERS ARE 4" x 4" x 48"

SHADED AREAS REPRESENT PERMANENT WALL

Figure #4 A

TO REACTOR

CAesium SHEET

CUT AWAY VIEW OF GRAPHITE STACK

Figure #4 B

- 25 -
was considered. Since it was not feasible to separate the test and monitor foils by even one diffusion length (50.2 centimeters in graphite) the possibility of mutual interaction existed. It was hoped that by symmetrical arrangement of the foils, the interaction between all the foils would be about the same and so could be neglected.

The experiment was performed in three separate runs, one run for each of three different materials. The first run consisted of fifteen copper foils with thicknesses of 5, 10, 20, 32, and 40 mils, and with diameters of \( \frac{1}{4}, \frac{1}{3}, \) and 1 inch. Since not all these thicknesses were available, some test foils were made by stacking two foils together. These foils were exposed for 45 minutes in a flux of about \( 5 \times 10^6 \) neutrons/cm.\(^2\)-sec., with the reactor operating at 80 kW. The second run consisted of fifteen gold foils with diameters of \( \frac{1}{2}, \frac{1}{3}, \) and 1 inch, with thicknesses of 2, 5, 10, 20, and 30 mils. The same flux conditions existed but the foils were exposed for just thirty minutes. For the third run, indium foils were inserted in the graphite. These fifteen indium foils were \( \frac{1}{2}, \frac{1}{3}, \) and 1 inch in diameter, and 5, 10, 20, 30, and 40 mils thick. This run was made for 15 minutes at a reactor power of 50 kW. So the average flux for this run was less than for the first two.

In each case the monitor foils used were \( \frac{1}{8} \) inch, 20 mil copper foils. The activity of the foils produced in these three runs was sufficient to be measured. The copper foils
were easiest to handle. The activity of the individual foils was a little larger than expected. After about 36 hours waiting time, the activity had decayed to a reasonable level for counting. The gold run presented a little more difficulty because of the long half-life of the irradiation product. In fact, it was more than a week before the gold activity had dropped to a suitable counting level. The indium foils, with their combination of large absorption cross-section and short half-life, caused the greatest headaches. The radiation from all the foils was so intense after the irradiation that the foils had to be shielded and could not be counted. By the following morning they were too cold to count. A second run with indium was made. This time the counting was started in five hours. The activity of the various foils differed widely, and so the colder foils tended to slow down the counting procedure. Meanwhile, with such a short half-life, the other foils would cool too much before they could be counted. Therefore, the requisite number of counts for good statistics were not recorded for all the indium foils. Hence the data for this material can not be presented in this paper.

The apparatus used for counting the foil activity was a Baird-Atomic automatic sample changer used in conjunction with a proportional flow counter. In order to obtain good statistics (1%), at least nine thousand counts are required. The counter has a dead time of about one microminute,
which gave a very small correction to the counting rates. In order to check on possible irregularities in counting by the apparatus, at the start of each pass a standard barium source was counted. A comparison of these measured times indicate that the counting apparatus was counting consistently throughout the experiment. At the conclusion of each pass, a background count was taken and from this count, the measured activity can be corrected for radiation background.
IV. RESULTS AND CONCLUSIONS

The results for the gold and copper runs are tabulated in tables #2 and #3, and are also presented in the form of graphs of saturation activity per unit thickness against foil thickness for each foil diameter. When analyzing the data, a problem appeared that was associated with the counting technique used. This phenomenon is beta self-absorption by the test foil. This occurs because beta particles are readily degraded in energy and captured in interactions with the atomic electrons and nuclei in the test foil itself. The result is that all the radiations from the decaying atoms do not escape from the foil. The various other beta counting problems are briefly covered in Price\textsuperscript{10}, but since our counting geometry was constant, these other corrections were either negligible or a constant factor for all foils.

Price presents a crude calculation for determining a correction factor for beta self-absorption in a source. This correction factor is given in equation (33).

\[
\int_t^{\infty} = \frac{1}{\nu \tau} \left( 1 - e^{-\mu t} \right)
\]

(33)

In this expression, \( \mu \) is the beta absorption coefficient for the material and \( t \) is the source thickness. The various values of \( \mu \) are not readily available and are dependent on the maximum energy of the beta ray spectrum. Evans\textsuperscript{11} presents
### TABLE # 2
#### COPPER

<table>
<thead>
<tr>
<th>FOIL</th>
<th>COUNT</th>
<th>COUNTING TIME (min.)</th>
<th>COUNT LESS BACKGROUND</th>
<th>$A_{sat}$</th>
<th>$A_{sat}/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/2&quot; )</td>
<td>0.0055&quot;</td>
<td>9000</td>
<td>21.55</td>
<td>8718</td>
<td>0.97 x 10^3</td>
</tr>
<tr>
<td>( 1/2&quot; )</td>
<td>0.0110&quot;</td>
<td>9000</td>
<td>15.98</td>
<td>8791</td>
<td>1.14</td>
</tr>
<tr>
<td>( 1/4&quot; )</td>
<td>0.0210&quot;</td>
<td>9000</td>
<td>16.53</td>
<td>8784</td>
<td>1.13</td>
</tr>
<tr>
<td>( 3/8&quot; )</td>
<td>0.0334&quot;</td>
<td>9000</td>
<td>17.56</td>
<td>8770</td>
<td>1.13</td>
</tr>
<tr>
<td>( 1/4&quot; )</td>
<td>0.0415&quot;</td>
<td>9000</td>
<td>22.60</td>
<td>8704</td>
<td>1.28</td>
</tr>
<tr>
<td>( 1/3&quot; )</td>
<td>0.0056&quot;</td>
<td>9000</td>
<td>6.40</td>
<td>8916</td>
<td>3.68</td>
</tr>
<tr>
<td>( 1/3&quot; )</td>
<td>0.0110&quot;</td>
<td>9000</td>
<td>5.56</td>
<td>8927</td>
<td>3.56</td>
</tr>
<tr>
<td>( 1/8&quot; )</td>
<td>0.0208&quot;</td>
<td>9000</td>
<td>6.54</td>
<td>8914</td>
<td>3.55</td>
</tr>
<tr>
<td>( 1/8&quot; )</td>
<td>0.0328&quot;</td>
<td>9000</td>
<td>8.50</td>
<td>8889</td>
<td>4.38</td>
</tr>
<tr>
<td>( 1/8&quot; )</td>
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<td>5.77</td>
<td>8925</td>
<td>5.11</td>
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<tr>
<td>1&quot;</td>
<td>0.0055&quot;</td>
<td>9000</td>
<td>1.32</td>
<td>8983</td>
<td>15.6</td>
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<tr>
<td>1&quot;</td>
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<td>9000</td>
<td>2.55</td>
<td>8967</td>
<td>11.4</td>
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<tr>
<td>1&quot;</td>
<td>0.0203&quot;</td>
<td>9000</td>
<td>4.21</td>
<td>8945</td>
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<tr>
<td>1&quot;</td>
<td>0.0317&quot;</td>
<td>9000</td>
<td>3.07</td>
<td>8960</td>
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<tr>
<td>1&quot;</td>
<td>0.0406&quot;</td>
<td>9000</td>
<td>2.60</td>
<td>8966</td>
<td>12.9</td>
</tr>
</tbody>
</table>

* $A_{sat}/t$ is given in counts per minute per inch

** $A_{sat}$ is given in counts per minute
<table>
<thead>
<tr>
<th>FOIL D x t</th>
<th>COUNT</th>
<th>COUNTING TIME (min.)</th>
<th>COUNT LESS BACKGROUND</th>
<th>( A_{\text{sat}} )</th>
<th>( A_{\text{sat}}/t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8&quot; .0022&quot; 30000</td>
<td>22.77</td>
<td>29702</td>
<td>2.36 ( \times 10^5 )</td>
<td>10.70 .70 ( \times 10^7 )</td>
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</tr>
<tr>
<td>1/8&quot; .0050&quot; 30000</td>
<td>26.66</td>
<td>29651</td>
<td>2.35</td>
<td>4.71 .38</td>
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</tr>
<tr>
<td>1/8&quot; .0110&quot; 30000</td>
<td>19.25</td>
<td>29748</td>
<td>2.62</td>
<td>2.38 .20</td>
<td></td>
</tr>
<tr>
<td>1/4&quot; .0212&quot; 30000</td>
<td>35.21</td>
<td>29539</td>
<td>2.50</td>
<td>1.18 .18</td>
<td></td>
</tr>
<tr>
<td>1/4&quot; .0327&quot; 30000</td>
<td>31.72</td>
<td>29585</td>
<td>2.53</td>
<td>0.77 .17</td>
<td></td>
</tr>
<tr>
<td>1/2&quot; .0022&quot; 30000</td>
<td>7.99</td>
<td>29895</td>
<td>8.82</td>
<td>40.1 1.8</td>
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<td>1/2&quot; .0050&quot; 30000</td>
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<td>29909</td>
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<td></td>
</tr>
<tr>
<td>1/2&quot; .0100&quot; 30000</td>
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<td>29913</td>
<td>10.0</td>
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<td></td>
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<td>1/2&quot; .0204&quot; 30000</td>
<td>7.34</td>
<td>29904</td>
<td>8.23</td>
<td>4.03 .36</td>
<td></td>
</tr>
<tr>
<td>1/2&quot; .0310&quot; 30000</td>
<td>5.00</td>
<td>29935</td>
<td>9.13</td>
<td>2.94 .25</td>
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<tr>
<td>1&quot; .0022&quot; 30000</td>
<td>2.81</td>
<td>29963</td>
<td>32.6</td>
<td>148.0 18</td>
<td></td>
</tr>
<tr>
<td>1&quot; .0052&quot; 30000</td>
<td>1.81</td>
<td>29976</td>
<td>31.8</td>
<td>61.3 4.4</td>
<td></td>
</tr>
<tr>
<td>1&quot; .0100&quot; 30000</td>
<td>3.48</td>
<td>29954</td>
<td>23.0</td>
<td>23.0 1.2</td>
<td></td>
</tr>
<tr>
<td>1&quot; .0204&quot; 30000</td>
<td>3.01</td>
<td>29961</td>
<td>31.0</td>
<td>15.2 0.8</td>
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<tr>
<td>1&quot; .0307&quot; 30000</td>
<td>4.86</td>
<td>29936</td>
<td>22.7</td>
<td>6.48 .76</td>
<td></td>
</tr>
</tbody>
</table>

* \( A_{\text{sat}}/t \) is given in counts per minute per inch

** \( A_{\text{sat}} \) is given in counts per minute
\frac{A_{\text{sat}}}{t} vs. \log_{10}\text{Counts/Minute - Inch}

Thickness (t) Mil
1"d COPPER FOILS IN GRAPHITE

Figure #7

$A_{\text{sat}}/t$ vs. Thickness (t) MILS

- 34 -
\[ \frac{A_{sat}}{t} \text{ Count/min-inch} \]

Thickness (t) Mils

Figure 8
1"d GOLD FOILS IN GRAPHITE

Figure #10

Asat / t vs. 10^7 Counts/Minute - Inch

THICKNESS (t) MILS
one of several correlation equations for determining a value for $\mu$. In equation (34), the density, $\rho$, is measured in grams per cm$^3$, $E_m$ is the maximum energy of the beta ray spectrum measured in MeV, and $\mu$ is the absorption coefficient given in inverse centimeters. Below is the result of some calculations.

Table #4

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$</th>
<th>$E_m$</th>
<th>$\mu$</th>
<th>$Z_{\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>8.93</td>
<td>0.57</td>
<td>288</td>
<td>1.37 mils</td>
</tr>
<tr>
<td>Gold</td>
<td>19.32</td>
<td>1.37</td>
<td>230</td>
<td>1.71</td>
</tr>
<tr>
<td>Indium</td>
<td>7.28</td>
<td>1.00</td>
<td>123</td>
<td>3.20</td>
</tr>
</tbody>
</table>

Now, since the foils with which we are dealing are several mean free paths thick (except the 2 mil gold foil), the correction according to Price's formula would be proportional to the foil thickness. Since this correction is so crude and so large compared to the effect associated with flux perturbation, no satisfactory separation of the two effects could be made. Therefore the data is presented with the beta self-absorption included.

In order to obtain meaningful results, all the foils should be exposed to the same flux-time. However, it was known that the flux was not constant throughout the test
area. For each test material, it was possible to relate the flux at any foil position to that at another foil position using the relative activity of copper monitor foils. These monitor foils were described in a previous section. The same foil position was selected for each of the two runs and all measured activities were normalized to that position.

Let me give an example of the way in which one might use these graphs. Assume that a $\frac{1}{4}''-.005''$ gold foil is irradiated and its activity is measured with a proportional flow counter. Let's say that this measured activity is $A$. Using equation (3B), the corresponding saturation activity $A_{sat}$, can be evaluated. Now assume that we irradiated a $1''-.02''$ copper foil in the same flux at a later time. What would be the expected measured activity $A_2$? This can be easily determined using the appropriate graphs. From the way in which the graphs were normalized, it is evident that \[
\frac{A_{sat_2}}{A_{sat_1}} = \frac{A_{2}}{A_{1}} \]
where $A'_1$ is the value taken from figure #7 for a $1''-.02''$ copper foil and $A'_2$, is the value taken from figure "8 for a $\frac{1}{4}''-.005''$ gold foil. In this case \[
\frac{A_{sat_2}}{A_{sat_1}} = \frac{A_2}{A_1} = 0.012 \]
After one solves for $A_{sat_2}$, then using equation (3A) under the appropriate conditions one can predict $A_2$.

The six graphs all essentially exhibit the same characteristics. In the range from zero to ten mils thickness the saturation activity per unit thickness shows a very sharp decline. Beyond this point the decline is very slow and seems to approach some asymptotic value. This value can
not be determined without data for much thicker foils. The range of zero to ten mil thick foils corresponds to the range from zero to five beta particle mean free paths. In this range, the differential contribution to the activity by each additional differential element of the foil decreases exponentially. So one would expect to see this initial exponential decline of activity per unit thickness. Beyond ten mils or five mean free paths, the differential contribution of each additional element of thickness is negligible. In this region we have what might be called surface activity. That is, the measured activity of the foil no longer depends on its thickness, only its area. Thus one would expect the activity to be constant or the activity per unit thickness to behave like an hyperbola. Of course, flux perturbation would cause a deviation from this hyperbola.

An attempt has been made to get some useful results from the plotted data. Each curve was extrapolated to zero foil thickness. This extrapolated point should give the activity of an infinitely dilute foil. This foil would cause no flux perturbation or beta self-absorption. If a horizontal line is drawn at that intercept, a total correction factor including both factors can be estimated. This factor is just the ratio of the experimental curve to the horizontal line. This would not be a general correction factor. It would be valid only for the given foil materials and the given diameters. This correction could only be used
when the Baird-Atomic automatic sample is used as a proportional flow counter.

For reasons discussed later, the uncertainty in the location of the experimental points was rather large. This made an accurate determination of the zero thickness intercept rather difficult. At zero thickness, we know that the activity per gram should be the same for all sized foils of the same material. So the initial estimated intercepts were converted to activity per gram of material. The two sets of three values were averaged to obtain a more reliable value for the zero thickness intercept. In this manner, the horizontal line as described in the previous paragraph was obtained.

The error involved in this experiment is rather large, of the order of 7 percent. This large uncertainty is not caused by our counting procedure. The large number of counts taken for each foil give us at least $1\%$ percent statistics. Considering uncertainty in the machine printing system and time measurement, there could not be more than another 1 percent uncertainty introduced. The large uncertainty is due to uncertainty in the relative value of the flux at each foil location.

You may remember that copper monitor foils were put in the graphite to take care of this situation. However, these foils were not in the same positions as the test foils. Therefore it was necessary to plot the activities of the
monitor foils to obtain a neutron flux plot for the graphite. The cadmium sheet had introduced such large deviations in the flux pattern, that there were not enough monitor foils to get an accurate flux plot.

Longitudinal flux plots were made for each run, but there were only four points to cover the full forty-eight inches of graphite. As a result, the marked flux variation caused by the cadmium sheet was not accurately plotted. In arriving at this seven percent error, I estimated various reasonable lines through the experimental points and used the maximum deviation as my uncertainty.

A method is envisioned for surmounting this difficulty. Since the shape of a flux distribution and hence relative flux values throughout the graphite is a function of geometry and not the power level of the reactor, a run could be made with the copper monitor foils in the slots to be occupied by the test foils in later runs. In this way, if we assume that the flux is essentially constant across the foil, the relative flux at these positions can be determined directly. Since the relative flux is independent of the power level or exposure time, these results could be used to correct subsequent test runs. I had hoped to try out this method but the hohlraum facility was no longer available.

If the experiment were to be carried out in a fluid medium, a foil wheel could be used in place of the monitor foils. The foil wheel is a disk on which the test
foils can be placed. This disk is in turn rotated slowly by a motor to insure that all the foils are exposed to the same time averaged flux. If the experiment is carried out in light water a lucite disk should be used. Several authors have reported that lucite causes no flux depression in light water. Lucite is not suitable in the case of heavy water. It appears that aluminum would be the best material to use in this case. The chief drawback to this method is that the disks may become somewhat large and unwieldy, since the foils should be far enough apart to avoid mutual flux depression.

The solution of the problem of separating the beta self-absorption from the effects of flux perturbation is required before current theories can be tested. I can suggest two possible ways of doing this in future experiments. The first is simply making use of a scintillation counter for measuring the activity of the foils. Since the self-absorption of gamma rays by foils as thin as we are using is negligible, the problem does not appear. However, the beta counter is the most simple instrument for activity measurements. If the foil activities were measured with both a scintillation counter and a beta counter it should be possible to quantitatively compare the effects of both flux perturbation and beta self-absorption. The activity curves with each instrument can be extrapolated to zero and normalized. Now the flux perturbation can be detected from the scintillation counter curve and compared with theory.
The self-absorption effect can also be investigated by comparing a set of curves that come from the two counting methods.

A method for eliminating the effect of beta self-absorption can also be devised making use of a beta counter only. If instead of using a single foil, each test foil was made from a stack of identical foils, then during the counting procedure, each individual foil in the stack of foils can be counted separately. The activities of these individual foils can be summed to give the activity of the entire stack. In this way, the beta self-absorption correction is just a multiplicative constant and can be normalized out.

Future experiments investigating flux perturbations in light and heavy water are extremely desirable. It may be possible to use the tank on top of the hohlraum for these experiments. A grid can be constructed to insert in the tank to hold the foils. The grid should be made of lucite for light water and aluminum for heavy water. An initial run should be made with the monitor foils to obtain the flux pattern. It might also be possible to rig up foil wheels for use in this tank.
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