Experimental Studies of Hybrid Photonic Band Gap Accelerator Structures

by

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Abstract

This thesis reports the first high power tests of a hybrid photonic band gap (PBG) accelerator structure. PBG structures can support a single electromagnetic mode, thus damping higher-order modes (HOMs) generated by wakefields. We have designed, built and successfully tested a 17.14 GHz hybrid PBG (HPBG) structure containing both dielectric and metallic elements. Dielectric elements have low loss and the potential to survive high surface electromagnetic fields.

The HPG structure was constructed as a triangular lattice array with sapphire rods inside and copper rods outside sandwiched between copper plates. The lattice parameter and the rod pattern were adjusted to excite a high-Q $\text{TM}_{02}$ mode and to suppress HOMs. This overmoded operation is a unique and novel feature of the hybrid design. The design included the birefringence of sapphire. Simulations showed relatively high surface fields at the triple point where sapphire, copper and vacuum meet as well as in any gaps between components in the clamped assembly.

Three structures were tested with later structures designed to sequentially reduce the surface electric field. The third structure used sapphire rods with pin extensions at each end and obtained the highest gradient of 19 MV/m, corresponding to a surface $E$ field of 78 MV/m, with a breakdown probability of $5 \times 10^{-1}/\text{pulse/m}$ in 45-ns pulses. Operation above 20 MV/m gradient led to runaway breakdowns with extensive light emission and eventual damage. For all three structures, multipactor light emission was observed at gradients well below the breakdown threshold. Breakdown damage was found at the triple point where surface fields peaked. The deposition of copper onto sapphire resulting from breakdowns might eventually degrade the cavity quality. This research indicated that multipactor triggered at the triple point limited the operational gradient of the hybrid structure.

These experiments represent the first high power tests of a hybrid PBG structure. The gradient achieved of 19 MV/m is the highest achieved with a dielectric structure. The gradient was found to be limited by multipactor and breakdown. The overmoded cavity with relatively large beam apertures might still find applications at high frequency or in high current transmission.
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Chapter 1

Introduction

For the last century, high energy particle accelerators have led the energy frontier contributing new particle discoveries and exploring the fundamental universe [18,19]. In 2012, the discovery of a Higgs boson marked a great physics breakthrough of the 21st century [20,21]. The accomplishment, ending a more-than-five-decade search, is attributed to the incredible performance of the ATLAS and CMS detectors on the Large Hadron Collider (LHC) at CERN [22,23]. The happy ending resulted in a beautiful photo, Figure 1-1, and two joint Nobel Laureates, François Englert and Peter W. Higgs.

Figure 1-1: A Higgs boson decays to 4 leptons in this collision recorded by the ATLAS detector on May 18, 2012 [1].
This is not the end. Searching for new particles, making more precise measurements and determining accurate properties of the fundamental particles [24] requires a collider with even higher energy and luminosity. More advanced concepts and technologies are under development, including updates to the current energy frontier colliders like LHC, constructions of new colliders with advanced technologies, or proof-of-principle tests of advanced far future accelerator concepts, such as plasma accelerators, dielectric accelerators, etc [25]. All approaches are aiming at higher luminosity and higher energy gain with shorter accelerating length, i.e., higher accelerating gradient $E_G$. Many limitations stand in the way of achieving the goals. Opportunities come with challenges.

1.1 High Gradient RF Accelerators

In 1870, the discovery of the cathode rays by William Crookes opened the door to accelerating charged particles with electromagnetic fields. Fifty years later, Wideröe realized an 1 MHz, 25 kV oscillator to demonstrate Ising’s proposal of acceleration using time-varying field (ac) instead of static high voltage (dc) [26]. This set up the timeline for alternating field acceleration, or the “true” accelerator, based on which the radio-frequency (rf) accelerators have been developed until today. Without exception, modern high energy colliders are all rf accelerators, and most of them contain a linear accelerator or “LINAC”. Today, the LINAC is still an active area of accelerator research. The requirement for higher collider performance with higher gradient and/or luminosity has encountered a few bottlenecks.

1.1.1 Accelerating Gradient

The higher the energy of the collision event, the more mysterious phenomena are revealed. A proposal using a storage ring to collide beams, achieving higher center of mass (CM) energy, producing high-energy collisions between particles, started the energy ramp of colliders in the late 1950’s [27, 28]. The center of mass energy for a
A head-on collision is

$$E_{CM} = \left[ 2E_1E_2 + (m_1^2 + m_2^2)c^4 + 2\sqrt{E_1^2 - m_1^2c^4}\sqrt{E_2^2 - m_2^2c^4} \right]^{1/2},$$ \hspace{1cm} (1.1)

where $m_1$, $m_2$ and $E_1$, $E_2$ are the rest masses and the energies of the two particles, respectively. The constant $c$ is the speed of light.

After more than fifty years’ development, today, the LHC can collide protons up to 13 TeV of the collision energy [29]. More intriguingly, we expect the development of an electron-positron collider with similar terascale energies. These cleaner, simpler collisions can reveal more underlying physics than more complex collisions between hadrons. One proposed international accelerator project, the Compact Linear Collider (CLIC) is aiming at a nominal total energy of 3 TeV. Figure 1-2 depicts the energy evolution of the major colliders over the decades [2].

![Figure 1-2: Energy evolution of the major colliders over the decades [2].](image)

The facility size and the cost have grown too, by several orders of magnitude along with the energy growth. This has resulted in high gradient accelerators always attracting enthusiastic endeavors to research technology improvements as well as principle innovations. Theoretically the upper bound of the gradient is the pair-production threshold of $\sim 10^{18}$ V/m, above which $e^+e^-$ pairs are produced, quenching
the field [30]. This theoretical limit, however, is far greater than what has been practically achieved. The potential of over GeV/m-gradients in dielectric structures or plasma wakefield accelerators [2] is still under proof-of-principle development and is beyond 20 years in the future. Alternatively, colliders built with lossless superconducting (SC) cavities could be limited in gradient by the surface magnetic field exceeding the critical magnetic field of the SC material. The International Linear Collider (ILC) using superconducting cavities is aiming at a gradient of 31.5 MV/m. With an accelerating length of 31 kilometers, particles can reach a colliding energy of 500 GeV. To compare, the design gradient of CLIC using room temperature metallic cavities is 100 MV/m, requiring 48 km in total length to reach an energy of 3 TeV [31].

Conventional rf accelerators made of metallic cavities have reached an accelerating gradient of over 100 MV/m [30]. The gradient of a room-temperature, metallic-cavity accelerator is limited by material damage and voltage breakdowns which are believed to be strongly related to the maximum electromagnetic (EM) field that the surface can tolerate. High EM fields cause surface damage by field emission, multipactor, breakdown, and cyclic fatigue from pulsed heating. Those effects are not separate but entangled and sometimes caused by one another. The mask covering those phenomena associated with breakdown is not at all transparent. Theoretical models have been built and experiments have been conducted during the past few decades to explore the mystery. Details about this exploration will be discussed in the following chapter.

1.1.2 Luminosity and Wakefields

More collision events provide higher possibility for a rare particle physics phenomenon to happen. The event rate $dN_{\text{event}}/dt$ in a collider is proportional to a known figure of merit, the beam luminosity, $L$ [2]:

$$\frac{dN_{\text{event}}}{dt} = L \cdot \sigma_{\text{int}}$$ \hspace{1cm} (1.2)

Here $\sigma_{\text{int}}$ is the interaction cross-section, which decreases with increased energy.

If the colliding beam contains $n_b$ bunches of $N$ particles with repetition frequency
\[ f_{\text{rep}}, \text{and effective overlap area } A, \text{the luminosity is} \]

\[ L = f_{\text{rep}} \frac{N^2}{A}. \]  

(1.3)

In a simple case of two identical gaussian beams with rms widths \( \sigma_x \) and \( \sigma_y \), the area is approximately \( A = 4\pi\sigma_x\sigma_y \).

Again, along with the peak energy growth in modern colliders, impressive progress of increasing peak luminosity has been achieved during the last fifty years, as shown in Figure 1-3.

![Figure 1-3: Luminosity evolution of the major colliders over the decades [2].](image)

More progress is required. A number of methods can be used to increase the luminosity. Increasing the repetition rate is limited by the average power of the source. The number of beam bunches within a pulse, and/or the charge of a single bunch can be increased, too. However, both will excite stronger wakefields, the former for long-range and the latter for short-range wakefields. For an accelerator operating at the fundamental mode, wakefields occur as higher-order-modes (HOMs), which are able to produce deleterious side effects to the accelerated beam. Theoretical studies and special mechanical innovations have been developed over the decades to try to suppress wakefields, including some incorporated in the novel design of this thesis.
1.2 Introduction to Photonic Band Gap Structures

Development of a high gradient accelerator requires fundamental understanding of the gradient limitations. Meanwhile, in order to meet the luminosity requirement, special design of the conventional accelerator cavity is needed to overcome the wakefield problem. In this thesis, a novel accelerator cavity design, called a photonic band gap (PBG) structure [32], using a frequency selective photonic feature to suppress wakefields, is to be discussed.

Photonic band gap structures, or photonic crystals, can be formed by periodically varied dielectric constants in one-dimensional (1D), two-dimensional (2D), or three-dimensional (3D) space, as shown in Figure 1-4 [3].

![Figure 1-4: Examples of photonic crystals in one, two and three dimensions. Different colors represents different dielectric constants in space [3].](image)

Frequency selectivity can be performed on the electromagnetic wave by incorporating periodic structural elements of sub-wavelength sizes, i.e. a metamaterial structure. The effect of the spatially periodic structure on the EM wave is analogous to the effect that semiconductors have on electrons. Band gaps are formed for particular frequency ranges that can confine the EM waves in 1D, 2D or 3D in the PBG structure. To construct an accelerator cavity, a 2D PBG structure is employed. Periodic arrays can be constructed by arranging elements of metallic or dielectric materials in the transverse direction. By removing a few central elements of the periodic structure, a "defect" can be formed to confine EM field, i.e., an accelerator cavity.
is built. The frequency selectivity allows only waves of the right frequency to be confined. All the other waves propagate out through the transverse PBG array. By manipulating the dimension and the arrangement pattern of the periodic structure, the cavity can confine only the fundamental accelerating mode and damp all the other unwanted HOMs away.

Previous research has demonstrated the confinement of the accelerating mode. A six-cell traveling-wave (TW) metallic PBG structure was built and tested at MIT. This structure confined a TM$_{01}$ mode to successfully accelerate electrons at a gradient of 35 MV/m [33]. After the proof-of-principle demonstration, experiment and simulation were carried out by succeeding researchers to investigate the wakefield damping properties of the metallic PBG structure [14, 15, 34]. In order to get high gradient, new standing-wave PBG structures were designed to put into high power testing. Accordingly, two PBG structures were high power tested at SLAC to investigate the breakdown phenomena at X-Band, 11.424 GHz [35, 36]. Generally, the size of an accelerator cavity is scaled with the operation frequency as $\sim 1/f^2$ (or the wavelength, $\sim \lambda^2$), and the volume is scaled as $\sim 1/f^3$ ($\sim \lambda^3$). In addition, the higher the frequency, the lower input microwave power is required for the same accelerating mode. To investigate the breakdown phenomenon of the PBG structure at a higher frequency, a metallic PBG structure was designed and built to put into high power testing at MIT at Ku-Band, 17.14 GHz. Comparable gradients and breakdown probabilities were achieved by the metallic PBG structures and conventional disk-loaded waveguide (DLWG) cavities [15].

1.3 Motivated Research and Outline of Thesis

All of the PBG structures previously tested are made of metal, oxygen-free high-conductivity copper (OFHC). The metallic photonic band gap has a cut-off frequency, which allows a confinement of the fundamental accelerating TM$_{01}$ mode. A design point can be chosen to be located on the upper edge of the lowest band gap. Ideally, all the HOMs are eliminated by the frequency selectivity. In reality, dangerous dipole
modes, $\text{TM}_{11}$, are weakly confined in the truncated PBG structures which include only a finite number of periodic lattices. Simulations showed rise of dipole modes in the three-row metallic PBG lattice [15,34]. Even though in a metallic PBG lattice the HOMs can be damped faster than in a DLWG, the dipole modes are still a concern.

A dielectric PBG lattice, like sapphire, has a band gap map without a cut-off frequency. Thus the lowest band gap is located above the dc range. With specific geometry design a $\text{TM}_{02}$ mode can be confined as the accelerating mode, with the dipole mode sitting below the band gap so that it cannot be confined at all. This suggested a new PBG structure that employs both metallic and dielectric material to constitute a prototype accelerating cavity, which we name a Hybrid Photonic Band Gap (HPBG) structure. Due to the photonic band gap of the dielectric material, this hybrid structure uses an overmoded accelerating mode to make a larger geometry design for a higher frequency operation. In addition, lack of experimental data about breakdown phenomenon on dielectric materials, or on hybrid structures at Ku band frequency range encourages this high power testing of the novel hybrid PBG cavity.

In this thesis, Chapter 2 will discuss the theory of the mechanism underlying wakefields and gradient limitations by breakdowns, followed by a brief introduction to the photonic band gap structures and the foregoing experimental accomplishments. Calculations of detailed band gap maps of the PBG structure will be presented in Chapter 3. The cavity modes confined by the special hybrid PBG structure will be discussed in the same chapter. The process of the first HPBG experimental cavity design will be described in Chapter 4. Chapter 5 covers the preparation for the high power experiment, including the cold test, the experimental setup and methodologies to diagnose breakdowns and analyze data. Chapter 6 discusses the results of the first breakdown experiment and the post-inspection of the tested HPBG structure. Based on the first HPBG structure, Chapter 7 and Chapter 8 describe designs and experiments of two new HPBG structures that have better high power performance. In the end, conclusion and future works will be discussed in Chapter 9.
Chapter 2

Theory

This chapter covers theories to motivate, design, and analyze the breakdown experiments of the hybrid photonic band gap structure. Accelerator limits will be discussed with more detail in this chapter. Models of photonic band gap structures, for both metallic and dielectric materials, will be described, including the experimental studies of the prior PBG structures.

2.1 Wakefields and Higher Order Modes

Wakefields are parasitic electromagnetic fields (usually unwanted) generated in accelerator cavities by charged particle beams passing through. According to the effective range, wakefields are classified as short-range and long-range wakefields. Short-range wakefields are induced by the head of a single bunch of particles and affect the trailing particles in the same bunch. Short-range wakefields cause energy spread and transverse deflection of the tailing particles for off-axis beams. Long-range wakefields are generated by antecedent bunches and affect the later bunches. Long-range wakefields emerge as higher order modes (HOMs) of the cavity. Some HOMs with significant transverse fields can deflect the succeeding bunches, resulting in beam-breakup, instability, effective emittance growth, and eventually beam loss, which are extremely harmful to the quality of both the accelerated beam and the accelerator cavity [37].

To describe the effect of long-range wakefields, wake potential $W$ is used to sum
over all possible HOMs in a cavity [37,38]:

\[ W(s) = \sum_n a_n f_n(\vec{x}) e^{-i\omega_n t} e^{-\omega_n t/2Q_n}. \] (2.1)

In Eq. 2.1, the wake potential is a function of \( s \), the distance behind the drive bunch. For relativistic particles, \( s = ct \), where \( c \) is the speed of light and \( t \) is the traveling time of the leading bunch. Except for the oscillatory term, the wake potential of an \( n \)th order eigenmode is proportional to the coupling coefficient \( a_n \) and the field pattern \( f_n(\vec{x}) \), where \( \vec{x} \) is the transverse coordinate. In particular, Eq. 2.1 tells that the wake potential of an \( n \)th order mode exponentially decays as \( \exp(-\omega_n t/2Q_n) \).

The time constant \( \tau_n = 2Q_n/\omega_n \) depends on the angular frequency of the eigenmode, \( \omega_n \), and the quality factor of that mode in the cavity, \( Q_n \), of the \( n \)th eigenmode.

Eq. 2.1 also provides ways to damp long-range wakefields. Reducing the coupling effect requires specific geometry design or material selection. Extending the space between charge particle bunches will probably interfere with the luminosity requirement. We can also increase the frequency of a HOM. Generally, a higher frequency of the operational accelerating mode is preferred. The frequency of HOMs will be increased, too. However, this is limited eventually by geometric convenience and machining techniques of manufacturing the accelerator cavity. Degrading the quality factor \( Q \) of HOMs without sacrificing the quality factor of the fundamental mode seems an effective way to operate. A novel design of an accelerating cavity that can damp HOMs will be discussed in detail in Section 2.4.

### 2.2 Field Emission and Breakdown: On Metallic Surfaces

On the path to developing high-gradient accelerator cavities, the breakdown (BD) phenomenon seems to be a gigantic obstacle. Breakdown is a phenomenon of an abrupt interference of the input rf field in an accelerator cavity. Most of the time, breakdown occurs at a surface spot with relatively high surface electric or magnetic...
field, and is accompanied with local outgassing and plasma formation. Breakdown not only limits the operation gradient, but also causes irreversible damage to the surface material of the accelerator structure. Almost a half century has passed since accelerator scientists started to notice the limit and began systematic study on breakdown, but a complete understanding of breakdown mechanism is lacking.

Furthermore, preceding research is mostly related to breakdown on either a single metallic surface or a pure dielectric surface, which is already of great difficulty. Inclusion of both dielectric and metallic materials generates many more complications. In this section, theories about field emission and breakdown will be described, including studies on both metallic and dielectric materials.

2.2.1 Surface Electric and Magnetic Fields

Normally, high accelerating gradient results in high surface electric and magnetic fields on an accelerator cavity. The gradient limit is actually the limitation of the highest electric and magnetic fields the cavity surface can tolerate.

Theoretically, surface electric field should sit below either the $e^+e^-$ pair-production threshold ($10^{18}$ V/m), or the stress a surface material can withstand before ionization with presence of nearby boundary ($10^{10}$ V/m) [30]. In reality, the highest electric field achievable on a metallic surface is far below those ideal thresholds. Sufficiently high electric fields can excite a great deal of field emission current and finally trigger a breakdown. The exact value of surface electric field needed to initiate breakdown is unknown; some early study showed a breakdown limit of surface electric field to be $\sim 450$ MV/m [39]. Varied conditions, such as geometry design, material selection and surface polishing, may lead to a different ratio of $E_{surf}/E_G$, where $E_{surf}$ is the surface electric field and $E_G$ is the accelerating gradient. Optimization to increase the gradient with the same surface electric field is a main target. Most accelerator designs give a ratio of $E_{surf}/E_G \sim 2.2$. To achieve a ratio of $E_{surf}/E_G < 2$ seems difficult. Dependence of breakdown limit on the frequency of the rf field [5,40] is also believed to be such that, in general, the higher the operation frequency, the higher electric field the surface can tolerate. This scale may not work, however, at very high
frequency such as 21, 30 or 39 GHz [39].

Surface magnetic field is involved in the gradient limitation through pulsed heating and cyclic fatigue. For each rf cycle, surface current induced by the magnetic field ohmically heats the thin metallic surface layer during the short rf pulse. The heat is rapidly dissipated and this thin layer suffers a fast temperature rise and a cool-down effect after the rf pulse. Stress is generated on the surface during each cycle, resulting in cyclic fatigue. S. V. Kuzikov and M. E. Plotkin proposed a "grain" model that relates the irreversible destruction of the copper surface to the broken links between grains in the copper crystalline structure near the surface. A higher temperature rise, from a higher magnetic field and a longer pulse length (within the relevant pulse length range, 10-1000 ns), applies more stress to the metallic surface, leading to a shorter life time of an accelerator cavity [41]. Aiming at millions of operation rf cycles, accelerator cavities are designed to receive as low magnetic field as possible. However, lowering the magnetic field also lowers the possible accelerating gradient.

2.2.2 Fowler-Nordheim Field Emission

Field emission (FE), believed to be a cause of prebreakdown or breakdown phenomena, has been intensely studied. R. H. Fowler and L. Nordheim considered quantum mechanical tunneling of Fermi-Dirac statistically assembled electrons out of a perfect clean metallic surface [42]. In their model, they assumed a modified field potential barrier, \( V(x) \), in the vicinity of the surface with an applied external field to be:

\[
V(x) = \begin{cases} 
-W_a, & x < 0 \\
-eEx - e^2/4x, & x > 0
\end{cases}
\]

(2.2)

where \( E \) is the applied surface electric field and \( x \) is the distance from the interface to vacuum. The term \(-e^2/4x\) is from the interaction of the emitted electron with its image charge and \(-W_a\) is the potential energy of an electron inside the metal. The potential diagram is shown in Figure 2-1.
Consider free electrons in a metal obeying Fermi-Dirac statistics. Electrons tunnel out of a metallic surface to form a "Field Emission" (FE) current. The current density $j_F$ can be derived by WKB approximation. For low temperature ($T \leq 300^\circ \text{K}$),

$$j_F = \frac{1.54 \times 10^{-6} \times 10^{4.5 \phi^{-0.5}} E^2}{\phi} \exp \left( -\frac{6.53 \times 10^9 \phi^{1.5}}{E} \right),$$

where $E$, in V/m, is the applied dc field on the surface, and $\phi$, in eV, is the work function of the material [5].

In practice, even the smoothest surface of an accelerator structure contains microscopic imperfections which are called "emitters". The localized surface field $E_m$ could be enhanced by those imperfections, making the surface spot vulnerable to field emission. With an enhanced electric field, much larger-than-normal emission current can be observed. The enhancement factor $\beta$ is defined as

$$\beta = \frac{E_m}{E}.$$  \hspace{1cm} (2.4)

F. Rohrbach calculated $\beta$ of several typical (and idealized) emitter shapes, as shown in Figure 2-2 [4]. Assuming $A_e$ as an effective area of the emitter and inserting $\beta$ into Eq. 2.3, J. W. Wang and G. A. Loew obtained a formula for enhanced field emission
Figure 2-2: Field enhancement factor $\beta$ for idealized metallic microprotrusion geometries, plotted versus geometry parameters $h/\rho$ or $h/k$ [4, 5].

(EFE) current [5]:

$$j_F = \frac{1.54 \times 10^{-6} \times 10^{4.52\phi^{-0.5}} \beta^2 E^2}{\phi} \exp \left(-\frac{6.53 \times 10^9 \phi^{1.5}}{\beta E}\right), \quad (2.5)$$

$$I_F = \frac{1.54 \times 10^{-6} \times 10^{4.52\phi^{-0.5}} \alpha E_0^2}{\phi} \exp \left(-\frac{6.53 \times 10^9 \phi^{1.5}}{\beta E}\right). \quad (2.6)$$

The numerical value of $\beta$ can be obtained from a Fowler-Nordheim plot, which plots $I_F/E^2$ versus $1/E$ in a semilog scale. The slope of line gives an evaluation of $\beta$ as shown in Eq. 2.7:

$$\frac{d\left(\log_{10}(I_F/E^2)\right)}{d(1/E)} = -\frac{2.84 \times 10^9 \phi^{1.5}}{\beta}. \quad (2.7)$$

For an rf field of the form $E_0 \sin \omega t$, the average field emission current, shown in Eq. 2.8 can be obtained by time-averaging of Eq. 2.6 over an rf period $T$:

$$\bar{I}_F = \frac{5.7 \times 10^{-12} \times 10^{4.52\phi^{-0.5}} \alpha E_0^2}{\phi^{1.75}} \exp \left(-\frac{6.53 \times 10^9 \phi^{1.5}}{\beta E_0}\right). \quad (2.8)$$
Again, $\beta$ can be obtained from:

$$\frac{d(\log_{10}(I_F/E_0^{2.5}))}{d(1/E_0)} = -\frac{2.84 \times 10^9 \phi^{1.5}}{\beta}. \quad (2.9)$$

With the assumption of sharp protrusions as shown in Figure 2-2, foregoing experimental data suggested a typical value of the enhancement factor $\beta$ falling between 40 and 100. In practice the FE current becomes measurable when the local enhanced field reaches a scale of $\sim$GV/m, which means with the typical value of $\beta$, the applied surface field is at a scale of a few tens of MV/m [5]. More recent investigations on surface emission showed that instead of a high $\beta$, the low work function $\phi$ might be responsible for the electron emission in high gradient rf cavities [43]. Some studies found that if taking into account a triple-point boundary where metal, dielectric, and vacuum interact, a much higher $\beta$ should be considered, which will be discussed in Subsection 2.3.4.

### 2.2.3 Breakdown Criteria: The Kilpatrick Limit and Loew-Wang Scaling

Dark current is defined as the electron emission current without beam loading in accelerator cavities. The idea that FE is the initial cause of breakdown is of little doubt, but the clear sequences of triggering a breakdown, a “runaway” condition from a stable field emission phenomenon is not well understood. With a large $\beta$, the local field on a microscopic emitter can approach a scale of a few GV/m, leading to a current density of $10^{11}$ A/m$^2$. Upon the high current density, the ohmic heat on the emitter can vaporize part of the surface material, which will be described in Subsection 2.2.4. The neutral gas can be ionized by the FE electrons, leading to plasma formation and ion generation. Secondary electron emission (SEE) and electron multipactor can be generated by the positive sheath and the bombardment of ions and electrons, thus more FE electrons, more plasma, and more ions. Eventually breakdown is triggered with abrupt rf power dissipation or reflection. An instantaneous increase of the dark current can be observed, accompanied most of the time by a visible flashover.
condition is modified due to the meltdown process and the metal droplets. Evidence of rf power reflection, sudden rise of dark current, scintillation, and surface condition change is abundant from decades of breakdown experiments. However, neither exact explanation of the breakdown phenomenon, nor the triggering threshold, is known.

W. D. Kilpatrick thought regular field emission could be enhanced to cascade electron emission by ion bombardment [40]. An empirical relation between the breakdown threshold of the electric field, $E$, in V/cm, and the maximum ion energy, $W$, in eV, is shown in Eq. 2.10:

$$WE^2 \exp\left(-\frac{K_1}{E}\right) = K_2,$$

(2.10)

where $K_1 = 1.7 \times 10^6$ V/cm and $K_2 = 1.8 \times 10^{14}$. Kilpatrick considered ions traveling across a gap formed by two electrodes, or two parallel plates. The maximum energy of the ion is limited by the applied frequency. Therefore the Kilpatrick criterion can be derived to obtain the relation between the operating frequency and the electric field threshold of breakdown [5], as shown in Eqs. 2.11 and 2.12:

$$f = 1.64E^2 \exp(-8.5/E),$$

(2.11)

where $f$ is in MHz and $E$ is in MV/m; or

$$E \exp\left(-4.25/E\right) = 24.7f^{1/2},$$

(2.12)

where $f$ is in GHz and $E$ is in MV/m. In Eqs. 2.11 and 2.12, the ion is assumed to be hydrogen.

The Kilpatrick criterion sets up a threshold surface field of 88 MV/m for 11.5 GHz frequency. G. A. Loew and J. W. Wang measured a surface field of 310 MV/m at 2.856 GHz and 500 MV/m at 11.424 GHz on copper, after aggressive rf processing [44]. They tried to scale the Kilpatrick’s criterion by bringing in rf pulse length in Eqs. 2.11 and 2.12. Their scaling factor is shown in Eq. 2.13:

$$E \propto \frac{f^{1/2}}{t^{1/4}}.$$

(2.13)
And a new criterion for the surface $E$ field is given [5], as shown in Eq. 2.14:

$$ E = 220f^{1/3}, $$

(2.14)

where $f$ is in GHz and $E$ is in MV/m.

More recent experimental data violated the Kilpatrick limit and Loew-Wang scaling. At higher frequency, such as 17, 21, 30, and 39 GHz, experimental data showed an absolute surface field limit of $\sim 300 - 400$ MV/m for the copper surface, with no significant frequency dependence [39,45-47].

2.2.4 A New Model: Breakdown by Field Emission Current Heating

A. Grudiev proposed a new model to predict breakdown from field emission [48]. This model is based on the ohmic heat generated by the field emission (FE) current to heat the surface protrusion to the melting point. The current density $j_F$ follows the Fowler-Nordheim field emission formula Eq. 2.5. The heat flux generated by $j_F$ obeys the heat conduction equation in one dimension, as shown in Eq. 2.15:

$$ C_V \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + j_F^2 \rho_{res}. $$

(2.15)

Here $C_V$ is the volumetric heat capacity, $k$ is the thermal conductivity, and $\rho_{res}$ is the electrical resistivity that changes with temperature. At a sufficiently high temperature $T$, $\rho_{res}$ follows a linear approximation of the Bloch-Grueneisen scaling:

$$ \rho_{res} = \rho_{res0} \frac{T}{T_0}. $$

(2.16)

$\rho_{res0}$ is resistivity at room temperature $T_0$. For all the surface perturbation shapes shown in Figure 2-2, assume the enhancement factor $\beta$ of the protrusion to be

$$ \beta \cong \frac{h}{r}. $$

(2.17)
where \( h \) is the height and \( r \) is the radius (referred to as \( \rho \) in Figure 2-2). Solving Eq. 2.15 in steady state with Bloch-Grueneisen scaling yields an expression of the FE current density \( J_{Fm} \) required to heat the end of the protrusion up to a temperature \( T_m \).

\[
J_{Fm} = \sqrt{\frac{kT_0}{h^2 \rho_{\text{res0}}}} \arccos \frac{T_0}{T_m}. \tag{2.18}
\]

Considering in time domain, solving Eq. 2.15 in linear approximation yields a transient time \( \tau_m \) required to heat the end of the protrusion up to a temperature \( T_m \):

\[
\tau_m = \frac{C_V T_0}{J_{Fm} \rho_{\text{res}}} \ln \frac{T_m}{T_0}. \tag{2.19}
\]

Substituting Eq. 2.18 into Eq. 2.19 yields

\[
\tau_m = \frac{C_V h^2 \ln \frac{T_m}{T_0}}{k} / \arccos^2 \frac{T_0}{T_m}. \tag{2.20}
\]

Let us set the final temperature \( T_m \) to be the melting temperature of copper, the transient time \( \tau_m \) to be the rf pulse length (e.g., \( \sim 100 \) ns), and a typical surface field \( E \) as 200 – 300 MV/m. Using the copper parameters listed in Table 2.1, solving

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>Thermal Conductivity</td>
<td>400</td>
<td>W/(m·K)</td>
</tr>
<tr>
<td>( C_V )</td>
<td>Volumetric Heat Capacity</td>
<td>( 3.45 \times 10^6 )</td>
<td>J/(m³·K)</td>
</tr>
<tr>
<td>( \rho_{\text{res0}} )</td>
<td>Resistivity at 300 K</td>
<td>( 1.7 \times 10^{-8} )</td>
<td>Ω·m</td>
</tr>
<tr>
<td>( T_m )</td>
<td>Melting Temperature</td>
<td>1358</td>
<td>K</td>
</tr>
</tbody>
</table>

Eq. 2.20 yields the numerical value of \( h \). Inserting \( h \) back to Eq. 2.18 gives \( J_{Fm} \).

Knowing \( J_{Fm} \), solving Eq. 2.5 where \( \phi \) is usually assumed to be 4.5 eV for copper gives \( \beta \), which can be inserted into Eq. 2.17 to give \( r \). Those estimated values are:

\[ h \sim 1 \, \mu m, \ r \sim 17 - 25 \, nm, \ J_{Fm} \sim 36 \, A/\mu m^2, \ \beta \sim 40 - 60. \]
Those values are in good agreement with available experimental data. In particular, the enhancement field is $\sim 10 \text{ GV/m}$ for copper, consistent with the popular assumption of surface field threshold for breakdown on copper surface.

### 2.2.5 Pulsed Heating and Cyclic Fatigue

For metallic accelerator cavities, a high magnetic field can ohmically heat the surface, resulting in a possible high temperature rise within a short time. D. P. Pritzkau derived a formula to calculate the temperature rise by rf power $P$ dissipated on the infinite vacuum-material interface area $A$, as shown in Eq. 2.21 [49]:

$$
\Delta T = \frac{1}{\rho c_e \sqrt{\pi \alpha_d}} \int_0^t \frac{dt'}{\sqrt{t - t'}} \frac{dP(t')}{dA}.
$$

(2.21)

The power dissipation due to the rf magnetic field is:

$$
\frac{dP(t)}{dA} = \frac{1}{2} R_s |H(t)|^2,
$$

(2.22)

where $R_s = \sqrt{\mu f \rho_{res}}$ is the surface resistivity, varying with frequency. Inserting $R_s$ into Eq. 2.21 gives

$$
\Delta T = \frac{1}{2 \rho c_e \sqrt{\pi \alpha_d}} \int_0^t \frac{dt'}{\sqrt{t - t'}} R_s |H(t')|^2.
$$

(2.23)

The material properties used for the temperature rise are summarized in Table 2.2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Copper Permeability</td>
<td>$1.256 \times 10^{-6}$</td>
<td>H/m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>$8.95 \times 10^3$</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$c_e$</td>
<td>Specific Heat at Constant Strain</td>
<td>385</td>
<td>J/(kg·K)</td>
</tr>
<tr>
<td>$\alpha_d$</td>
<td>Thermal Diffusivity</td>
<td>$1.1 \times 10^{-4}$</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$\rho_{res0}$</td>
<td>Resistivity at 300 K</td>
<td>$1.7 \times 10^{-8}$</td>
<td>$\Omega$·m</td>
</tr>
</tbody>
</table>

If the induced cyclic stress by pulsed heating is beyond some threshold, damage occurs on the metal surface in the form of surface roughening and micro-cracks.
Cracks propagate along the surface, eventually leading to an increase of resistivity of the surface and a degradation of the quality of the accelerator cavity. This degradation indicates a failure of the cavity, setting a lifetime of the structure. Experiments showed that for a fully annealed oxygen-free copper (OFE) surface, damage occurred after $56 \times 10^6$ pulses at a temperature rise of $120 \pm 10$ K with 1.25-µs pulse length and after $86 \times 10^6$ pulses at a temperature rise of $82 \pm 3$ K with 1.5-µs pulse length [50]. In addition, local melting of the surface was observed for $56 \times 10^6$ pulses at a temperature rise of 250 K [50]. Note that those numbers of pulses are not the minimum number for damage emergence. A detailed theoretical discussion is described in [49,50].

Following Pritzkau's experiments, S. V. Kuzikov and M. E. Plotkin developed a "grain" model for thermal fatigue, based on the quasi-elastic interaction between neighboring grains in the metal due to thermal expansion of the skin layer [41]. Heating of the surface is caused by a magnetic field penetrating into the rf skin layer of the metal. A copper crystal has a face-centered cubic lattice with atoms connected to each other. Pulsed heating applies stress on the lattice, resulting in a probability of breaking the links between neighboring atoms. In reality, the copper surface is composed of many grains. Within the frequency range and the pulse length range that are popular in colliders, the size of each grain is larger than the thermal skin layer. Expansion caused by pulse heating in the skin layer makes grains start to push and bring extra elastic forces, which increases the probability to break the links between atoms. Kuzikov and Plotkin derived a formula to estimate the number of pulses $N_f$ needed to damage a copper surface, as shown in Eq. 2.24 [41]:

$$N_f = \frac{C}{\exp(\zeta \sqrt{\tau} \Delta T^2) - 1},$$

(2.24)

where $\tau$ is the pulse length. $\zeta$ is related to Young's modulus and Poisson coefficient of the material, and $C$ is related to a certain critical value of the probability to break links. Both can be fitted with experimental data. Eq. 2.24 predicts a lifetime of a copper cavity under a specific temperature $\Delta T$ with a pulse length $\tau$. This brings
the condition for a “fixed” life time:

$$\Delta T^2 \cdot \tau^{1/2} = \text{const.}$$

Eq. 2.25 shows a possibility to keep a similar life time by trading off $\Delta T$ and $\tau$.

2.3 Field Emission and Breakdown: Inclusion of a Dielectric

Research of breakdown on a dielectric surface has been motivated mostly by the limitation on high-power microwave (HPM) sources. Breakdowns at the vacuum-dielectric interfaces of the HPM windows limit the power capacity. It is widely believed that breakdown at the vacuum-dielectric interface is triggered by multipactor, which is easily to be initiated with the presence of a dielectric, especially at a triple point where vacuum, dielectric and metal meet. A detailed discussion about multipactor will be given in this section, including a model of the electric field and field emission near a triple point.

2.3.1 Multipactor

Multipactor is an electron multiplication process happening on surfaces exposed to rf fields. Electrons, striking the surface with impact energy $E_i$, cause the surface to emit secondary electrons. The secondary electron yield, or the secondary electron emission (SEE) ratio, $\delta$, is defined as:

$$\delta = \frac{\text{the number of electrons emitted by the surface}}{\text{the number of electrons impacting the surface}}.$$  (2.26)

Multipactor occurs when $\delta > 1$ [6,51]. Secondary electrons can be accelerated by the rf field and impact the surface again, and yield more electrons, and so on. Finally, electron avalanche occurs and breakdown can be triggered.

Multipactor can take place for a wide range of frequencies and in a wide variation
of geometries [7]. It has been frequently observed in microwave systems such as rf windows and accelerator structures. Multipactor can dissipate considerable amounts of power fed into rf cavities. Interaction of the multipactor with an rf cavity can detune or degrade the cavity. Multipactor discharge can heat the surface, increase power loss and possibly cause damage. Breakdown can be triggered by multipactor at the vacuum-dielectric interface and realized by plasma avalanche in the ambient desorbed or evaporated gas layer above the dielectric [52]. Despite all of the above disadvantages, applications of multipactor have been developed, too. For example, multipactor can be used in electron gun technologies and in protection of sensitive receivers [7].

The complete mechanism of multipactor is unknown. The first systematic study may come from Gill and von Engel in the 1940s [7, 53]. Gill and von Engel introduced a parameter $k$ to represent the impact velocity of the primary electron relative to the emission velocity of the secondary electron. To avoid complication of a random velocity distribution of the emission, Gill and von Engel assumed $k$ to be constant [7, 53]. J. R. M. Vaughan advanced an alternative theory based on a more realistic assumption of a monoenergetic nonzero initial velocity [6]. Vaughan gave an empirical formula relating $\delta$ to the impact energy $E_i$. A generic shape of $\delta$ versus $E_i$ is shown in Figure 2-3. In Figure 2-3, $E_1$ and $E_2$ are called the first and the second crossover

![Figure 2-3: Dependence of the secondary electron yield $\delta$ on the impact energy $E_i$. The curve below has an impact angle of 0° and the above has an impact angle of 60° [6].](image-url)
energy, respectively, where \( \delta = 1 \). The two parameters \( \delta_{\text{max}} \) and \( E_{\text{max}} \) are material based and dependent on the impact angle. If the impact is at an angle \( \theta \) to the normal, the empirical formulas are:

\[
V_{\text{max}\theta} = V_{\text{max}0}(1 + k_s \theta^2 / \pi), \quad \text{(2.27)}
\]

\[
\delta_{\text{max}\theta} = \delta_{\text{max}0}(1 + k_s \theta^2 / 2\pi), \quad \text{(2.28)}
\]

where \( E_i = eV_i \). \( k_s \) is a "smoothness factor" for the surface, falling in the range from 0 to 2 depending on the surface material and polishing. In the absence of specific data, a typical dull surface should be assigned \( k_s = 1 \).

For a low-voltage region,

\[
\frac{\delta(\theta)}{\delta_{\text{max}}(\theta)} = (ve^{1-v})^k, \quad \text{(2.29)}
\]

where

\[
v = \frac{V_i - V_0}{V_{\text{max}}(\theta) - V_0}, \quad V_0 = 12.5 \text{ V}, \quad \text{(2.30)}
\]

and

\[
\begin{cases} 
  k = k_1 = 0.62, & \text{for } v < 1 \\
  k = k_2 = 0.25, & \text{for } v > 1 
\end{cases} \quad \text{(2.31)}
\]

For a high-voltage region,

\[
\frac{\delta(\theta)}{\delta_{\text{max}}(\theta)} = \frac{1.113}{(V_i/V_{\text{max}})^{0.35}}, \quad \text{(2.32)}
\]

### 2.3.2 Two-Surface Multipactor on Metals

Based on Vaughan's empirical formulas, Eqs. 2.27 and 2.28, R. A. Kishek built models to estimate multipactor on metals and dielectrics [7]. For metals, Kishek assumed a simplified geometry of the cavity to be two parallel planes. For this special geometry, to sustain multipactor, electrons just released from one plate must be strongly accelerated to reach the other plate, and do so at (or close to) a time
when the field has reversed in order that the secondary electrons can be accelerated as well. This requires a transit time to be near an odd number, $N$, of $1/2$ rf cycles. This transit time condition relates the voltage and the frequency of the rf field to the geometry. Assume a gap width $D$ to separate the two parallel metallic surfaces. A perpendicular rf field applied on the surface is of the form $(V_{g0}/D)\sin(2\pi ft + \theta)$, where $f$ is the frequency and $V_{g0}$ is the amplitude of the voltage across the gap. Assume that upon impact, the secondary electrons are released with a monoenergetic nonzero initial energy $E_0$, the voltage boundaries $V_{g\min}$ and $V_{g\max}$ can be derived [7]:

$$V_{g\min} = \frac{22480(fD)^2 - (N\pi fD)\sqrt{44960E_0}}{(N\pi)^2 + 4},$$

(2.33)

$$V_{g\max} = \frac{22480}{N\pi} (fD)^2,$$

(2.34)

where voltages are in volts, the frequency $f$ is in GHz, the gap width $D$ is in cm, and the secondary electron emission energy $E_0$ is in eV. No simple closed-form solution of $V_{g\max}$ exists for nonzero $E_0$. $V_{g\max0}$ is the special case for $V_{g\max}$ at $E_0 = 0$. Figure 2-4, plotted by Kishek, depicts a universal curve to determine the upper limiting voltage $V_{g\max}$. Susceptibility curves, mapping the regions of the external parameter space in

![Figure 2-4](image)

Figure 2-4: Universal curve to determine the upper limiting voltage, $V_{g\max}$, for a first-order ($N=1$), two-surface multipactor assuming a general nonzero monoenergetic emission energy $E_0$ (in eV). Here the frequency $f$ is in GHz and the gap separation $D$ is in cm [7].

which multipactor is possible, are shown in Figure 2-5 [7].
Figure 2-5: Susceptibility curve for a two-surface multipactor showing the effects of nonzero emission velocity and surface materials. This example is constructed for oxygen-free copper ($\delta_{\text{max}} = 1.3; E_{\text{max}} = 600$ eV) and alumina ($\delta_{\text{max}} = 6.5; E_{\text{max}} = 1300$ eV), assuming a monoenergetic emission energy, $E_0 = 2$ eV (−); $5$ eV(−−); and $10$ eV(...). For boundary with vertical lines, the solid line represents the maximum possible impact energy to be equal to the first crossover energy $E_1$. The dotted line represents the minimum possible impact energy to be $1.33E_1$. $1.33E_1$ is the simulated energy above which the multipactor discharge appears sporadically [7].

For resonant cavities, loading and detuning by multipactor lead to saturation of the multipactor discharge on metals. Space charge also plays a role in the saturation of multipactor. 1%-50% of the input rf power can be dissipated in multipactor [7].

### 2.3.3 Multipactor on a Dielectric

Dielectric surfaces are capable of supporting charges, therefore multipactor is allowed on a single dielectric surface. Charges on the dielectric surface create dc electric fields that restore the primary electrons to the same dielectric surface. In most cases the rf electric field is parallel to the dielectric surface. Thus the restored primaries gain energy from the rf field, striking the surface with higher impact energies. If for an impact angle $\theta$, the impact energy $E_i$ falls within the first and second crossover points, $E_1$ and $E_2$, respectively, the secondary electron yield $\delta > 1$, and multipactor occurs on the single dielectric surface. Table 2.3 lists typical values of $E_1$ and $E_2$ for dielectric materials commonly used in HPM rf windows. $\delta_{\text{max}0}$ and $E_{\text{max}0}$ represent values of an impact angle of $0^\circ$. 
Table 2.3: Typical secondary electron emission parameters for materials commonly used in rf windows [8,17].

<table>
<thead>
<tr>
<th>Material</th>
<th>$\delta_{\text{max}}$</th>
<th>$E_{\text{max}}$ (eV)</th>
<th>$E_1/E_{\text{max}}$</th>
<th>$E_2/E_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Al}_2\text{O}_3$ (alumina)</td>
<td>1.5–9</td>
<td>350–1300</td>
<td>0.23–0.011</td>
<td>10.2–24.5</td>
</tr>
<tr>
<td>$\text{SiO}_2$ (quartz)</td>
<td>2.4</td>
<td>400</td>
<td>0.099</td>
<td>14.1</td>
</tr>
<tr>
<td>Quartz-glass</td>
<td>2.9</td>
<td>420</td>
<td>0.072</td>
<td>15.6</td>
</tr>
<tr>
<td>Technical glasses</td>
<td>2–3</td>
<td>300–420</td>
<td>0.136–0.068</td>
<td>12.6–15.9</td>
</tr>
<tr>
<td>Pyrex</td>
<td>2.3</td>
<td>340–400</td>
<td>0.107</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Figure 2-6 shows the single-surface model built by Kishek [7]. The parallel rf field $E_{\text{rf}}$ and normal dc electric field $E_{\text{dc}}$ are shown in the model. $E_{\text{rf}}$ is the rf field, $E_{\text{dc}}$ is the dc field created by the surface charge on the dielectric. $V_0$ and $\phi$ are the initial energy and angle of a primary electron. $\theta$ is the impact angle [7].

Figure 2-6: Model of a single-surface multipactor in a parallel rf and normal dc electric fields. $E_{\text{dc}}$ is dc field created by the surface charge on the dielectric. $E_{\text{rf}}$ is the rf field. $V_0$ and $\phi$ are the initial energy and angle of a primary electron. $\theta$ is the impact angle [7].

$E_{\text{rf}} = E_{\text{rf}0} \sin(\omega t + \phi_0) = E_{\text{rf}0} \sin(2\pi ft + \phi_0)$, where the initial phase $\phi_0$ is assumed to be uniformly distributed. Solving the equations of motion for electron gives the $x$ and $y$ components, $V_{ix}$ and $V_{iy}$, respectively, of the impact energy $V_i = eE_i$ [8]:

\[
V_{ix} = \frac{1}{2} m v_0^2 \sin^2 \phi, \tag{2.35}
\]
\[
V_{iy} = \frac{1}{2m} \left( \frac{eE_{\text{rf}0}}{\omega} \right)^2 \left\{ \cos \left[ \frac{2mv_0 \sin \phi}{e(E_{\text{dc}}/\omega)} + \phi_0 \right] - \cos \phi_0 + \frac{mv_0 \cos \phi}{e(E_{\text{rf}0}/\omega)} \right\}^2, \tag{2.36}
\]

The impact angle is

\[
\theta = \arctan \left( \frac{\sqrt{V_{iy}}}{V_{ix}} \right). \tag{2.37}
\]
With the impact energy $V_i$ and angle $\theta$, the yield is determined by Eqs. 2.27 and 2.28. Figure 2-7 shows the susceptibility curve for a single-surface multipactor on a dielectric as a function of the dc and rf electric fields \[8\]. In Figure 2-7, the lower boundary corresponds to the first crossover energy $E_1$, and the upper boundary corresponds to the second crossover energy $E_2$. In Figure 2-7, the dc electric field is assumed to be static, i.e., the charge on the dielectric surface is unchanged. In reality, the secondary emission process charges the surface even more. To perform an accurate calculation, a fully dynamic model, accounting for both the variation of the surface charge and the rf field change due to the loading change by multipactor, has to be involved. The transit time condition is absent in multipactor on a dielectric. This greatly widens the parameter space for multipactor, making more susceptible for multipactor occurring on the dielectric surfaces than the metallic ones \[7\].

Distinguished from multipactor on metals, the saturation mechanism for multipactor on a dielectric is dominated by space charge forces. Kishek estimated that by multipactor, the power deposited on the dielectric surface was of the order of 0.5%-1% of the rf power \[7\]. In a recent experiment, J. G. Power, W. Gai et al. observed that with strong normal and tangential rf electric fields, the fraction of power absorbed at
saturation is an increasing function of the rf power [54]. An absorbed power of \( \sim 50\% \) by multipactor in an alumina-based dielectric-loaded accelerating (DLA) structure was detected.

O. V. Sinitsyn, G. S. Nusinovich, and T. M. Antonsen, Jr. proposed a self-consistent non-stationary two-dimensional model of multipactor in DLA structures [9]. Based on Vaughan’s model, Sinitsyn et al. used the Monte Carlo algorithm to solve the equations of motion for electron with presence of dc and rf fields in the model shown in Figure 2-8a, resulting in the particle trajectories with random initial conditions shown in Figure 2-8b.

![Cross section of the 2D DLA model](image1)

Figure 2-8: (a) Cross section of a cylindrical DLA structure. 1 is the vacuum region, 2 is the dielectric, \( r_d \) is the radius of the dielectric, \( r_w \) is the radius of the waveguide wall, and \( r_n' \) is the location of the charged layer. (b) Sample particle trajectories in the presence of both rf and dc fields of equal amplitudes with \( E_{dc} = 7 \text{ MV/m} \) [9].

The averaged normalized radial coordinate of the particles as a function of the normalized time indicates that with time, the effect of dc field increases, and particles move away from the center until most of them concentrate in a thin layer near the dielectric surface, which is the saturation. At saturation the average particle energy falls between the first crossover energy \( E_1 \) and the maximum yield energy \( E_{max} \). Figure 2-9 shows the power loss by multipactor as a function of the rf power, which demonstrates the increasing function of the loss factor with the input rf power.

M. N. Buyanova et. al. demonstrated that multipactor can arise on a dielectric
surface in strong electromagnetic fields without any external static fields [55]. It is the ponderomotive force, generated by the gradient of the high frequency electromagnetic field itself, to provide the restoring force for the emitted electron to return to the dielectric surface, therefore causing secondary electron emission. The occurrence of multipactor depends on the amplitude of the high frequency electric field on the dielectric surface and the presence of the interference field structure near the surface.

2.3.4 Electric Field near a Triple Point

A triple point is a junction of metal, dielectric and vacuum. In the presence of high electric field, the intersection is a favorable location for electron emission and vulnerable to rf breakdowns. The localized electric field can be strongly enhanced by the sharp geometry of the interface, leading to a significant electron emission. Those primary electrons are able to produce rapid multipactor either through ionization of neutrals, or through secondary emission from the dielectric surface. Following the electron avalanche, breakdown can be initiated on the dielectric and damage can occur on the structure [6, 51, 56-60].

The field emission mechanism is unclear due to the complicated entangled situation. Inclusion of a dielectric material may modify the work function of the metallic surface. Instead of the potential assumed by Fowler and Nordheim, as shown in
Figure 2-1, the potential of field emission through a dielectric is modified, leading to change of field emission, as shown in Figure 2-10 [10]. The geometric complica-

![Figure 2-10](image)

(a) Through thick insulator (b) Through insulator with surface charging

Figure 2-10: Field emission through insulator. Shaded regions represent filled conduction band states in metal and filled valence band states in insulator. Image charge effects are omitted [10].

tion and different surface conditions bring in tremendous uncertainties, in addition to the space charge effect on a triple point or the dielectric anisotropy. Simplified models with varied geometry regimes have been analyzed by K. D. Bergeron [56], L. Schächter [58], R. A. Anderson and J. P. Brainard [57] and N. M. Jordan [11]. Here we show a model built by Jordan that concentrates on the localized field enhancement near the triple point.

Jordan considered a 2D model of a triple point, as shown in Figure 2-11 [11]. In Figure 2-11, the metal-vacuum interface is on the positive x axis. The dielectric-vacuum interface lies at an angle \( \theta \) from the positive y axis and the metal-dielectric interface lies at an angle \( \alpha \) from the positive x axis. The angle subtended by the vacuum is \( p_v = \theta + \pi/2 \), the angle subtended by the metal is \( p_m = \alpha \), and the angle subtended by the dielectric is \( p_d = 2\pi - p_v - p_m = 3\pi/2 - \theta - \alpha \). Electric field in the vacuum is a function of \( r \) and \( \phi \). \( \theta \) and \( \phi \) are positive going counterclockwise. \( \alpha \) is positive going clockwise. Assuming the potential \( \Phi \) to be zero on the metal surface and positive at large \( y \), in cylindrical coordinate system \((r, \phi)\), the potential in the vacuum region is

\[
\Phi(r, \phi) = r^{\nu} \sin(\nu \phi), \quad 0 < \phi < \pi/2 + \theta, \tag{2.38}
\]
where \( \nu \) is the index to be determined. In the dielectric region, the potential reads

\[
\Phi(r, \phi) = r^\nu \frac{\sin\{\nu[(\pi/2 + \theta) - (2\pi - \alpha)]\}}{\sin\{\nu[(\pi/2 + \theta) - (2\pi - \alpha)]\}} \sin[\nu(\pi/2 + \theta)], \quad 0 < \phi < \pi/2 + \theta, \quad (2.39)
\]

which ensures continuity of \( \Phi \) on the boundary. And \( \nu \) can be determined by the continuity of \( \varepsilon \frac{\partial \Phi}{\partial \phi} \):

\[
\varepsilon_r \cot \left[ \nu \left( \frac{3\pi}{2} - \alpha - \theta \right) \right] \tan \left[ \nu \left( \frac{\pi}{2} + \theta \right) \right] = -1. \quad (2.40)
\]

where \( \varepsilon_r \) is the relative dielectric constant. Once \( \nu \) is determined from a specification of \( \varepsilon_r, \alpha, \) and \( \theta \), the electric field \( \mathbf{E} = -\nabla \Phi \) is determined:

\[
\mathbf{E} = -E_0 \left( \frac{r}{a} \right)^\delta [\sin(\nu \phi) \mathbf{x} + \cos(\nu \phi) \mathbf{y}], \quad (2.41)
\]

In Cartesian coordinates, Eq. 2.41 reads:

\[
\mathbf{E} = -E_0 \left( \frac{r}{a} \right)^\delta [\sin(\delta \phi) \mathbf{x} + \cos(\delta \phi) \mathbf{y}], \quad (2.42)
\]

where \( E_0 \) and \( a \) are the scale factors of the electric field and the distance, and \( \delta \) is the field index:

\[
\delta = \nu - 1. \quad (2.43)
\]
If $\delta < 0$, approaching the triple point, the electric field diverges, which means a strong enhancement of the electric field by the geometric junction.

Bergeron [56] and Jordan [11] studied the case of $\alpha = \pi$. In this case, derived from Eq. 2.40 and 2.43, for small $\theta$, the asymptotic approximation of $\delta$ is

$$\delta \approx \frac{2\theta}{\pi} \left( \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \right); \quad \alpha = \pi, \quad |\theta| \ll 1, \quad (2.44)$$

Eq. 2.44 tells that for a small $\theta$, $\delta$ changes sign with $\theta$. Figure 2-12 shows the electric field near the triple point with $\alpha = \pi$, calculated by Jordan using a numerical code [11].

Figure 2-12: Electric fields near the triple point for (a) $\theta < 0$ and (b) $\theta > 0$ [11].

Eq. 2.42 shows that the $x$ component of the electric field changes sign with $\delta$, and therefore with $\theta$, which is consistent with the shape of electric fields shown in Figure 2-12. For $\theta < 0$, the electric field tends to attract electrons to the dielectric surface. Electrons accelerated to the dielectric are more likely to pull out secondary electrons. For $\theta > 0$, in Figure 2-12b, the electric field tends to repel electrons from the dielectric and is less likely to trigger multipactor. Those are consistent with experimental data about the triple point [56]. Trajectories of seed electrons have been calculated by Jordan, too, for both cases ($\theta < 0$ and $\theta > 0$). For $\theta < 0$, the impact energy of the seed electron is more likely to fall between the first and second crossover energies. The approximate range of the angle $\theta$ most likely to produce an
avalanche of secondary electron emission is estimated [11]:

\[ 0 > \theta > -9.1^\circ \times \left( \frac{\varepsilon_r + 1}{\varepsilon_r} \right) \times \sqrt{\frac{E_{om}/(1 \text{ eV})}{E_1/(40 \text{ eV})}}, \]

(2.45)

where \( E_{om} \) (of order a couple eV’s) is the perpendicular energy with which a secondary electron is ejected from the dielectric surface. \( E_1 \) is the first crossover energy.

To give a summary on Jordan’s model, a sharp negative \( \theta \) gives a large enhancement factor \( \beta \) and is more likely to cause electron multipactor, thus is more vulnerable to initiating breakdown near the triple point. Jordan’s calculation ignored dielectric charging completely. Charging on the dielectric surface, even for \( \theta > 0 \), could easily lead to field enhancement and electron avalanche. Difficulties to include space charge effect and field enhancement by surface materials are avoided, too. In general, a triple point is a geometry vulnerable to triggering secondary electron emission and initiate breakdowns.

### 2.3.5 Multipactor Triggered Breakdown

Detection of spectral emissions from hydrogen atoms and carbon ions has proved that final breakdowns are triggered in the desorbed or evaporated gas cloud by multipactor above the dielectric surface [12, 61–63]. During a flashover of HPM window breakdown, A. A. Neuber et al. observed four distinct phases: phase 0, field emission from triple point; phase 1, subsequent electron avalanche developed by secondary electron emission from the dielectric surface; phase 2, a local pressure buildup due to electron stimulated outgassing; and phase 3, finally followed by gaseous ionization and breakdown [12]. Figure 2-13 shows the processes. The electron-induced outgassing can form a gas (multi)layer. A secondary electron is likely to be produced within the gas layer, which changes the effect of the secondary electron emission. The induced gas desorption causes a local rise in gas density above the dielectric surface. Neuber observed a sharp rise of flashover current within 2-3 ns for phase 1. Phase 2 followed with two orders of magnitude slower rise-time, varying from 50 ns to 500 ns in duration. Transiting to phase 3 exhibited a nanosecond rise-time [12]. Neu-
Figure 2-13: Sketch of physical processes involved in surface flashover and generic measurement signals [12].

ber’s simulation showed a saturated SEE avalanche established with the transition of phase 1 to phase 2. So phase 2 was characterized by a slowly rising gas pressure and a positive space charge formation. The positive space charge started to enhance the electric field at the transition point from phase 2 to phase 3. Finally breakdown was triggered. The mechanism was built for a unipolar pulsed or self-breakdown surface flashover. Microwave surface flashover was found to be similar to this mechanism as long as the frequency was below ~ 20 GHz [12].

Multipactor can be influenced by dc electric or magnetic fields [64]. Modifying geometry to alter the trajectories seems to be effective to suppress multipactor. Periodic surface profiles, such as periodic rectangular grooving or triangular surfaces grooving, have been demonstrated to effectively suppress HPM multipactor at the vacuum side of the dielectric window [52, 65]. For dielectric materials, thin film coating on the surface with special material and other surface features can greatly suppress the surface multipactor. State-of-the-art coating technique is of great help. Recent research showed a much higher rf power transmitted by a TiN-coated alumina window than non-coated one, demonstrating suppression of the surface multipactor [66, 67]. Those
high power tests have reached an accelerating gradient of 15 MV/m in a gap-free dielectric-loaded accelerator (DLA) structure without rf breakdown.

2.4 Photonic Band Gap

A Photonic Band Gap (PBG) lattice, formed by periodically varied elements of different materials in space, has a frequency selectivity. By imposing different reflection and transmission properties on electromagnetic waves, PBG lattice arrays can confine or propagate EM waves with different frequencies. Lattice elements can be constructed by metallic or dielectric materials. In 1993, S. Schultz, D. R. Smith and N. Kroll proposed a novel microwave resonator formed by a PBG structure, using frequency selectivity to confine rf fields for acceleration [68]. Since then, attentions have been drawn, theories and experiments have been advanced to explore this new structure.

2.4.1 2D Lattice and Band Gap

Resembling disk-loaded waveguide (DLWG) cavities, PBG structures use transverse photonic lattices to confine an accelerating mode. To form a PBG lattice, metallic or dielectric rods are arranged periodically in space. Two parameters, the radius of the rod, \( a \), and the lattice spacing, \( b \), are used to characterize the PBG lattice. There are two kinds of lattices used in 2D PBG structures: the square lattice and the triangular lattice. Figure 2-14 shows the two kinds of lattice and their reciprocal lattices.

Dispersion relation of a PBG lattice can be calculated, either by analytical method or by employing codes or commercial softwares [3, 13, 69]. Figure 2-15 shows the dispersion relation over the irreducible Brillouin zone of the triangular PBG lattice. In Figure 2-15, a band gap, painted by yellow, shows no propagation of EM waves with frequencies falling in this range. Dispersion relation versus the lattice ratio \( a/b \) can be summarized to depict a band gap map of a PBG lattice. Figure 2-16 shows the band gap map of the two lattice types of metallic PBG (MPBG) structure. Both metallic lattices have a cut-off frequency in the band gap map, i.e., the lowest band gap starts from the dc range. This band gap map is able to confine the fundamental
Figure 2-14: Square and triangular lattices and their reciprocal lattices. The dashed line draws the preliminary lattice. Irreducible Brillouin zones are shaded in (c) and (d) [13, 14].

Figure 2-15: Dispersion relation including seven modes of the lowest normalized frequency of the triangular PBG lattice with \( a/b = 0.2 \), yellow shadow indicates the band gap [15].
Figure 2-16: Global band gap maps for TM modes versus $a/b$ ratio for (a) the square lattice, and (b) the triangular lattice. The solid dot in (b) indicates the design point for the MIT 17 GHz Traveling-Wave (TW) PBG structure [13].

The TM$_{01}$ mode, which will be discussed in the following subsection.

### 2.4.2 Metallic PBG Structures

For all of the metallic PBG structures mentioned in section 1.2, the triangular lattice has been chosen to create the PBG structure. A triangular lattice resembles a traditional cylindrical cavity with more azimuthal uniformity than a square lattice. In addition, the sharp variation of the electric field profile at the lattice corner and the azimuthal asymmetry make the square lattice not applicable for a practical accelerator structure. Since the band gap map of the metallic triangular lattice has a cut-off frequency, the lowest mode, the TM$_{01}$ mode, can be confined. To confine an rf wave in the lattice, a defect should be created by removing a few central rods from the lattice array. Meanwhile, an infinite number of rods is not practical, i.e., the PBG lattice is truncated in practice. For copper, two or three hexagonal rows of rods are able to provide enough confinement of the TM$_{01}$ mode. Figure 2-17 shows a metallic PBG structure and the TM$_{01}$ mode it confines [15]. The PBG lattice array consists of three hexagonal rows of copper rods with one central rod removed forming a defect cavity.

The first demonstration of acceleration using a PBG structure was done by E. Smirnova at MIT [33]. The structure was a traveling-wave (TW) accelerator, con-
Figure 2-17: Metallic PBG structure with defect and truncated 3 rows of rods and confined TM$_{01}$ mode. Color map shows the normalized magnitude of the electric field simulated by HFSS [15].

structured by six PBG cells, as shown in Figure 2-18.

Figure 2-18: Photographs of the six cell traveling wave PBG accelerator formed by three rows of copper rods [14].

Wakefields in the TW PBG structure were studied by R. A. Marsh, B. J. Munroe and M. Hu [14, 15, 34]. In general, effective damping of HOMs is performed by the metallic PBG structure. Subsequently, in order to investigate the performance with high-gradient operation, breakdown experiments on metallic PBG structures have been performed both at SLAC for X band, 11.424 GHz and at MIT for Ku band, 17.14 GHz [15, 35, 36]. Figure 2-19 illustrates two examples of metallic PBG structures [15].
With detail discussed in [14, 15], the breakdown performance between those PBG
structures and the conventional disk-load-waveguide are comparable.

2.4.3 Dielectric PBG Structure

Dielectric materials are alternatives of the metallic rods in PBG structures. The
finite permittivity of a dielectric gives a band gap map without a cut-off frequency;
the bottom of the lowest band gap is higher than the dc range, as shown in Figure 2-
20 [3]. Therefore a higher accelerating mode, such as the TM02 mode, can be confined

Figure 2-20: The band gap map of an finite triangular lattice of dielectric rods. The
dielectric constant $\varepsilon_r = 11.4$ [3].
in the dielectric PBG lattice array. This allows an overmoded accelerator structure that does not confine the dangerous dipole TM_{11} mode.

### 2.4.4 Introduction to Sapphire

Sapphire, with very low dielectric tangent loss, has been widely applied in microwave engineering. It is a rhombohedral crystal form of Aluminum Oxide (Al_{2}O_{3}). Sapphire has an optical anisotropy, which gives different permittivities depending on the orientation of the optical-axis with respect to the incident electromagnetic waves. Besides, the dielectric constant of sapphire varies with the rf frequency.

In this thesis, sapphire is used to form a dielectric PBG lattice array to confine the TM_{02} mode. Calculation of the band gap map of sapphire will be shown in Chapter 3, as well as the arrangement of the sapphire rods made to provide enough confinement of the accelerating mode. A novel hybrid photonic band gap (HPBG) structure, using both sapphire rods and copper rods has been built and tested in breakdown experiments.
A dielectric PBG (DPBG) lattice can have a very different band gap map from a metallic PBG lattice. In this chapter, the band gap maps of the square and the triangular photonic crystal lattice formed by sapphire rods are calculated. Based on the band gap map, the eigenmodes that can be confined in a defect cavity of a simple two-dimensional (2D) dielectric PBG structure are simulated.

3.1 Optical Properties of Sapphire

Sapphire is chosen as our dielectric material due to its low dielectric loss. Sapphire is an anisotropic crystal whose dielectric constant, or refractive index, varies due to the polarization and the propagation direction of the incident light with respect to the optical axis of the crystal. The crystal form and the typical orientation of sapphire are shown in Figure 3-1. The anisotropy gives different values of the permittivity due to the different orientation of a sapphire rod with respect to the incident electromagnetic (EM) waves. For our structure, the orientation of the sapphire rod makes the optical axis C parallel to the longitudinal accelerating z axis. In addition, the permittivity changes as a function of the frequency of the incident wave. At 17 GHz with the above orientation, the permittivity of sapphire is \( \varepsilon_r = [9.398, 9.398, 11.587] \) by interpolating...
data from [70]. The dielectric loss is very low, and varies with direction, too. The loss tangent is \( \tan \delta = [3, 3, 8.6] \times 10^{-5} \).

### 3.2 Numerical Methods to Calculate Eigenmodes for DPBG Lattices

In this section, we consider two kinds of lattice, the square lattice and the triangular lattice, as shown in Figure 2-14, with the lattice elements made of sapphire. For a periodic 2D lattice, the boundary condition requires:

\[
\sigma(\vec{x}_\perp + T_{mn}) = \sigma \vec{x}_\perp, \tag{3.1}
\]

where \( \sigma \) is the conductivity profile and \( \vec{x}_\perp \) is the transverse coordinate, \( \vec{x}_\perp = x\hat{e}_x + y\hat{e}_y \). \( T_{mn} \) is the periodicity vector of the form:

\[
T_{mn}^s = mb\hat{e}_x + nb\hat{e}_y, \text{ square lattice,} \tag{3.2}
\]

\[
T_{mn}^t = \left( m + \frac{n}{2} \right) b\hat{e}_x + \frac{\sqrt{3}}{2} nb\hat{e}_y, \text{ triangular lattice,} \tag{3.3}
\]

where \( m \) and \( n \) are integers.

Decomposition of the Maxwell's Equations shows that the field components in
TM and TE modes, both in transverse and longitudinal directions, can be expressed through the axial component of the electric (magnetic) field. The field $\psi$ is a function of the transverse coordinate $\bar{x}_\perp$, the longitudinal wavevector $k_z$, and the frequency $\omega$. Fourier transforming $\psi(\bar{x}_\perp, z, t)$ to the longitudinal coordinate $z$ and time $t$ yields:

$$\phi(\bar{x}_\perp, k_z, \omega) = \int \int \psi(\bar{x}_\perp, z, t) e^{i(k_z z - \omega t)} dz dt.$$  \tag{3.4}

The periodicity of the optical profile allows to solve the equation of the Bloch form:

$$\psi(\bar{x}_\perp + \bar{T}_{mn}) = \psi(\bar{x}_\perp) e^{i\bar{k}_\perp \cdot \bar{T}_{mn}},$$  \tag{3.5}

where $\bar{k}_\perp = k_x \hat{e}_x + k_y \hat{e}_y$ is the transverse wave vector. Here $\psi(\bar{x}_\perp)$ is the wave in the unit cell shown in Figure 2-14. Correspondingly, we can vary the transverse wave vector $\bar{k}_\perp$ in the Brillouin zone of the reciprocal lattice of the two lattice configurations. In the Brillouin zone, we have:

$$|x| \leq \frac{b}{2}, |y| \leq \frac{b}{2}, \text{ square lattice,}$$  \tag{3.6}

$$\left|x - \frac{y}{\sqrt{3}}\right| \leq \frac{b}{2}, |y| \leq \frac{\sqrt{3}}{4b}, \text{ triangular lattice.}$$  \tag{3.7}

Inserting the periodic translational symmetry Eqs. 3.2 and 3.3 into Eq. 3.5 reads:

$$\begin{cases} 
\psi(x = b/2, y) = e^{ik_x b} \psi(x = -b/2, y) \\
\psi(x, y = b/2) = e^{ik_y b} \psi(x, y = -b/2) 
\end{cases}, \text{ square lattice,}$$  \tag{3.8}

$$\begin{cases} 
\psi(x = \frac{b}{2} + \frac{y}{\sqrt{3}}, y) = e^{ik_x b} \psi(x = -\frac{b}{2} + \frac{y}{\sqrt{3}}, y) \\
\psi(x, y = \sqrt{3}b/4) = e^{ik_x b + ik_y \sqrt{3}b/2} \psi(x = -\frac{b}{2}, y = -\sqrt{3}b/4) 
\end{cases}, \text{ triangular lattice.}$$  \tag{3.9}

Solving the eigenvalue problem of Eqs. 3.8 and 3.9 we can find the dispersion relation as a function of the transverse wave vector $\bar{k}_\perp$.

In this chapter, a commercial finite element method solver for electromagnetic
structures from Ansys, Inc., the high frequency structural simulator (HFSS) [71] is employed to solve the wave equation [69]. In HFSS, a thin vacuum layer with a sapphire rod in the center was built to model the 2D unit cell of the infinite square lattice and the infinite triangular lattice with different lattice ratios. Figure 3-2 shows the HFSS models, both with a lattice ratio $a/b = 0.35$.

![HFSS models](image)

(a) Square lattice  
(b) Triangular lattice

Figure 3-2: HFSS models of the unit cell of (a) a square lattice, and (b) a triangular lattice of a DPBG crystal. The round darker center represents the sapphire rod and the relatively lighter extra part is vacuum. Both have a lattice ratio $a/b = 0.35$. By assigning the phase advance between the two pairs of boundaries, the dispersion relation of an infinite lattice can be obtained.

Specific boundary conditions are assigned to derive the dispersion relation of the infinite lattice array. In accelerators, the accelerating mode is a TM mode of which the accelerating electric field lies on the longitudinal $z$ axis. "Perfect $E$" boundaries are assigned on the top and bottom surfaces of the unit cell model to limit the solved eigenmodes to be TM modes. For TE modes, the top and bottom surfaces are "Perfect $H$" boundaries.

To include all the eigenmode solutions in the unit cell, we need $\vec{k}_\perp$ to spread over the Brillouin zone. For each $\vec{k}_\perp = k_x \hat{e}_x + k_y \hat{e}_y$, there exists one specific phase advance for each boundary pair in the transverse directions. HFSS allows to define a "Master-and-Slave" boundary pair to force a finite phase advance over a specific direction. The variations and relations of the phase advance pair, $\phi_1$ and $\phi_2$, corresponding to necessary values of the $\vec{k}_\perp$ over the Brillouin zone are listed in Table 3.1 for the square
lattice and in Table 3.2 for the triangular lattice.

Table 3.1: Variations and relations of the phase advance pair of the square lattice

<table>
<thead>
<tr>
<th>Brillouin zone</th>
<th>$k_x$</th>
<th>$k_y$</th>
<th>$\phi_1 = k_x b$</th>
<th>$\phi_2 = k_y b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma \rightarrow M$</td>
<td>$0 \rightarrow \pi/b$</td>
<td>$0 \rightarrow \pi/b$</td>
<td>$0 \rightarrow \pi$</td>
<td>$0 \rightarrow \pi$</td>
</tr>
<tr>
<td>$M \rightarrow X$</td>
<td>$\pi/b$</td>
<td>$\pi/b \rightarrow 0$</td>
<td>$\pi$</td>
<td>$\pi \rightarrow 0$</td>
</tr>
<tr>
<td>$X \rightarrow \Gamma$</td>
<td>$\pi/b \rightarrow 0$</td>
<td>$0$</td>
<td>$\pi \rightarrow 0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Table 3.2: Variations and relations of the phase advance pair of the triangular lattice

<table>
<thead>
<tr>
<th>Brillouin zone</th>
<th>$k_x$</th>
<th>$k_y$</th>
<th>$\phi_1 = k_x b/2$</th>
<th>$\phi_2 = k_y b/2 + k_y \sqrt{3}b/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma \rightarrow X$</td>
<td>$0$</td>
<td>$0 \rightarrow 2\pi/\sqrt{3}b$</td>
<td>$0$</td>
<td>$0 \rightarrow \pi$</td>
</tr>
<tr>
<td>$X \rightarrow J$</td>
<td>$0 \rightarrow 2\pi/3b$</td>
<td>$2\pi/\sqrt{3}b$</td>
<td>$0 \rightarrow 2\pi/3$</td>
<td>$\pi \rightarrow 4\pi/3$</td>
</tr>
<tr>
<td>$J \rightarrow \Gamma$</td>
<td>$2\pi/3b \rightarrow 0$</td>
<td>$2\pi/\sqrt{3}b \rightarrow 0$</td>
<td>$2\pi/3 \rightarrow 0$</td>
<td>$4\pi/3 \rightarrow 0$</td>
</tr>
</tbody>
</table>

3.3 Band Gap Map of the Square Lattice

3.3.1 Dispersion Relation of TM Modes

By “Perfect E” boundary condition assigned on the top and bottom surfaces of the thin HFSS layer for TM modes, the electric field vector lies parallel to the longitudinal $z$ direction, as shown in Figure 3-3. Figure 3-3c shows the normalized color bar that is applied to all the field plots in this chapter. Taking into account the birefringence,

Figure 3-3: The vector of the electric field of the second lowest order of propagating TM mode with phase shift $\phi_1 = 0$, $\phi_2 = 0$, ($\Gamma$ point), for the lattice ratio $a/b = 0.35$ of the square lattice.
the field vector assumes an effective dielectric constant \( \varepsilon_{r,\text{eff}} = 11.587 \) to simplify the anisotropic \( \varepsilon_r = [9.398, 9.398, 11.587] \) in a 2D simulation for TM modes. In a 3D simulation, except for the on-axis electric field, the vector of the electric field everywhere else is not perfectly parallel to the \( z \) axis, which necessitates the usage of an isotropic permittivity. The 2D field pattern changes with the phase advance and the lattice ratio \( a/b \), as illustrated in Figure 3-4.

![Figure 3-4](image)

\((a) a/b = 0.1, \phi_1 = \pi, \phi_2 = \pi\) \hspace{1cm} \((b) a/b = 0.35, \phi_1 = \pi, \phi_2 = \pi\)

\((c) a/b = 0.1, \phi_1 = \pi, \phi_2 = 0\) \hspace{1cm} \((d) a/b = 0.35, \phi_1 = \pi, \phi_2 = 0\)

Figure 3-4: The complex magnitude of the electric field of the second lowest order of propagating TM modes for the square lattice.

A dispersion relation of the infinite PBG lattice can be plotted as the normalized frequency \( f_{\text{norm}} = \omega b/2\pi c \) as a function of the wave vector \( k_\perp \) over the Brillouin zone \((\Gamma \rightarrow M \rightarrow X \rightarrow \Gamma)\). Figure 3-5 shows the dispersion relation of the square lattice with two lattice ratios, \( a/b = 0.05 \) and \( a/b = 0.4 \). Lack of propagation of any mode over the entire wave vector range gives a photonic band gap. For example, no band gap occurs for the lattice ratio \( a/b = 0.05 \), as shown in Figure 3-5a. When \( a/b \) increases to 0.4, as shown in Figure 3-5b, there emerge three band gaps, \([0.214, 0.227] \omega b/2\pi c\), \([0.351, 0.383] \omega b/2\pi c\), and \([0.523, 0.556] \omega b/2\pi c\). Significantly in contrast with the
band gap map of the metallic PBG structure, the dielectric band gap map has no cut-off frequency. The bottom of the lowest band gap is above the dc range, leading to a possibility of an overmoded operation. By selection of the frequency range, the fundamental TM01 mode and the dangerous dipole mode, the TM11 mode, can be set below the bottom of the first band gap map. They will naturally propagate out through a dielectric PBG lattice. Instead, a higher TM mode, such as the TM02 mode, can be used as the accelerating mode. Therefore we do not need to worry about damping the dangerous dipole mode.

### 3.3.2 Dispersion Relation of TE Modes

By applying “Perfect H” boundary conditions on the top and bottom surfaces of the unit cell model in HFSS, we limit the eigenmode solutions to TE modes, where the \( E \) field vector is lying on the transverse \((xy)\) plane, which makes the effective permittivity \( \varepsilon_{r,e}\text{ff} = 9.398 \), as shown in Figure 3-6. The dispersion relation of the TE modes gives no band gap for most lattice ratios. Figure 3-7a gives an example. Only a very narrow band gap occurs with \( a/b = 0.35 \) for a high frequency range, \([0.725, 0.732]\ \omega b/2\pi c\), as shown in Figure 3-7.
Figure 3-6: The vector of the electric field of the second lowest order of propagating TE modes with phase shift $\phi_1 = 0, \phi_2 = 0$, (I' point), for the lattice ratio $a/b = 0.35$ of the square lattice.

Figure 3-7: Dispersion relation of the lowest TE eigenmodes as a function of $\mathbf{k}_\perp$ for (a) the lattice ratio $a/b = 0.25$, and (b) the lattice ratio $a/b = 0.35$, of the square lattice.
3.3.3 Band Gap Map

A band gap map of a PBG lattice type can be obtained by plotting the band gap versus the lattice ratio \( a/b \) [3]. Figure 3-8 shows the band gap map of the square lattice formed by sapphire rods with a permittivity \( \varepsilon_r = [9.398, 9.398, 11.587] \). The lowest three TM band gaps are indicated by the yellow islands enclosed by solid blue lines. One narrow TE band gap appears in a relatively high frequency range.

![Band Gap Map](image)

Figure 3-8: The band gap map of the square lattice formed by sapphire rods with dielectric constant \( \varepsilon_r = [9.398, 9.398, 11.587] \).

3.4 Band Gap Map of the Triangular Lattice

3.4.1 Dispersion Relation of TM Modes

In this section, the aforementioned calculations for the square lattice are applied instead to the triangular lattice formed by sapphire rods. Figure 3-9 shows the complex magnitude of the electric field of a triangular lattice with a lattice ratio \( a/b = 0.2 \) and \( a/b = 0.35 \). We obtain the dispersion relation shown in Figure 3-10. One narrow band gap \([0.566, 0.576] \omega b/2\pi c \) occurs for \( a/b = 0.05 \). For \( a/b = 0.35 \), three band gaps, \([0.214, 0.271] \omega b/2\pi c \), \([0.380, 0.467] \omega b/2\pi c \), and \([0.568, 0.644] \omega b/2\pi c \), are
3.4.2 Dispersion Relation of TE Modes

For the triangular lattice, the dispersion relation of the TE modes of a lattice ratio $a/b = 0.2$ shows no band gap, as shown in Figure 3-11. For $a/b = 0.35$, there exists one narrow band gap, $[0.337, 0.355] \omega b/2\pi c$, as shown in Figure 3-11b.

3.4.3 Band Gap Map

Varying the lattice ratio $a/b$ we obtain the band gap map of the triangular DPBG lattice with $\varepsilon_r = [9.398, 9.398, 11.587]$, as shown in Figure 3-12.

3.5 TM Modes in a 2D DPBG structure

The metallic photonic band gap (MPBG) structures that have been high power tested are all operated at the fundamental TM$_{01}$ mode, based on that the band gap map has a cut-off frequency. Sapphire, with a permittivity $\varepsilon_r = [9.398, 9.398, 11.587]$, for both
Figure 3-10: Dispersion relation of the lowest TM eigenmodes as a function of $\vec{k}_\perp$ for (a) the lattice ratio $a/b = 0.05$, and (b) the lattice ratio $a/b = 0.35$, of the triangular lattice.

Figure 3-11: Dispersion relation of the lowest TE eigenmodes as a function of $\vec{k}_\perp$ for (a) the lattice ratio $a/b = 0.2$, and (b) the lattice ratio $a/b = 0.35$, of the triangular lattice.
the square lattice and the triangular lattice, has no cut-off frequency. By employing a proper geometric design, we can operate the accelerator cavity at a higher order mode, the TM$_{02}$ mode. With this overmoded operation, the mechanical design should result in a bigger, thus an easier-machined geometry, since overmoded operation resonates at a higher frequency with the same geometry size. Between the two lattice types, a triangular lattice is chosen due to more uniformity in the azimuthal direction for the TM$_{0n}$ mode and less vulnerability to excite dipole or quadrupole TM modes with its sextuple symmetry [13,72]. All of the preceding MPBG structures have been made of a triangular lattice.

To construct a dielectric PBG (DPBG) cavity, the periodic triangular lattice array is formed by a few hexagonal rows of the dielectric (sapphire) rods in the transverse $xy$ plane. The lattice array is sandwiched between two metal plates. The first few center hexagonal rows are removed to form the defect cavity. Similar to the simulation applied in the band gap map calculation, a thin layer of sapphire rods surrounded by vacuum is built to form a 2D DPG cavity model in HFSS. The layer is sandwiched by “Perfect $E$” boundaries on the top and bottom surfaces to force TM modes resonating ($k_z = 0$). Figure 3-13 shows a 2D cavity model with four hexagonal rows of sapphire rods forming a triangular lattice array, where $a$ is the radius of the sapphire rod, $b$
Figure 3-13: The HFSS model of the 2D, 4-row, DPBG cavity.

is the lattice constant, and $r_{eff}$ is the effective radius that matches the defect to a circular waveguide with a radius $r_{eff}$. The defect is formed by removing the first four hexagonal rows of sapphire rods (thirty-seven rods in total). "Perfectly Matched Layers (PML)" boundaries are assigned on the outer walls to model the unbounded boundary condition. Due to the anisotropic dielectric constant of sapphire, the field profile in the 2D model is not the same as in a 3D structure. However, investigating resonant modes in a simple 2D model gives a direction to the 3D design.

With the model shown in Figure 3-13, TM eigenmodes are simulated in HFSS. The normalized frequency $f_{norm} = \omega b / 2\pi c$ of a TM mode is plotted versus the lattice ratio $a/b$, as shown in Figure 3-14. The TM band gap map of the triangular lattice is also showed to indicate a clear location of the confinement. The electric field profile of the accelerating TM$_{02}$ mode in the DPBG structure is shown in Figure 3-15, with comparison of the the TM$_{01}$ mode in the MPBG structure, and the TM$_{01}$ mode in a disk-loaded waveguide (DLWG) cavity.

In Figure 3-14, the design point of a DPBG structure is shown at around $a/b = 0.35$, where the TM$_{02}$ mode sits in the middle of the lowest band gap, resulting in a high radiative quality factor $Q_{rad}$. Only a few other modes, the TM$_{21}$ and the TM$_{31,a}$, TM$_{31,b}$ are involved in the same band gap. Figure 3-16 shows the electric field profile of the other three modes. Different from the DLWG cavity with a degeneracy of the two TM$_{31}$ modes, the defect cavity of the triangular lattice has a six-fold symmetry,
Figure 3-14: The simulated normalized frequency of a few TM modes in the 2D DPBG structure model shown in Figure 3-13. The TM band gaps are enclosed by solid blue lines. The red circle indicates the design point.

Figure 3-15: Comparison of the electric field pattern of (a) the TM$_{02}$ mode in a DPBG cavity, (b) the TM$_{01}$ mode in a MPBG cavity, and (c) the TM$_{01}$ mode in a DLWG cavity.
resulting in two non-degenerate TM$_{31}$ modes, as shown in Figures 3-16b and 3-16c.

To benefit HOM damping, a larger $a/b$ ratio, for example, $a/b \sim 0.4$, is preferred. However, the convenience of the mechanical design requires a not-too-large $a/b$ ratio to make easier arrangement of the rods in space. Thus $a/b \sim 0.35$ is chosen to be the design point.

### 3.6 Radiative $Q_{\text{rad}}$

For a PBG lattice array with an infinite number of rods, the confined modes will be completely prohibited to propagate through the lattice. In reality, however, only a finite number of rods can be supplied, resulting in a truncated lattice array. Even waves whose frequencies are in the band gap will inevitably radiate out through the truncated lattice array. The radiation loss of the confined mode gives a cavity quality factor, which is called the radiative $Q_{\text{rad}}$ or the diffractive $Q_{\text{diff}}$, as a well-known figure of merit of an accelerator cavity. Figure 3-17 plots $Q_{\text{rad}}$ versus the number of hexagonal rows of the sapphire rods to construct the cavity. There are other losses such as the ohmic loss ($Q_{\text{ohm}}$) from copper and the dielectric absorption loss ($Q_{\text{dieI}}$) from sapphire. All the other lossy sources can be removed by correspondingly assigned material properties or boundary conditions in the 2D HFSS simulations.

A requirement of $Q \geq 10^3$ asks for at least five hexagonal rows of sapphire rods (in total 180 rods), if the defect created by removing the four center hexagonal rows
of rods. This is an enormous number taking into account the endeavor to arrange and align those rods. The number of three or four hexagonal rows of rods is practically acceptable. The loss in $Q_{rad}$ of the TM$_{02}$ mode ($Q_{rad,02}$) can be compensated and even improved by other means described in the following section. Table 3.3 lists the

Table 3.3: $Q$ of the four 2D TM Modes in a 4-row DPBG structure.

<table>
<thead>
<tr>
<th>TM$_{02}$</th>
<th>TM$_{21}$</th>
<th>TM$_{31,a}$</th>
<th>TM$_{31,b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{02}$</td>
<td>$Q_{21}$</td>
<td>$Q_{31a}$</td>
<td>$Q_{31b}$</td>
</tr>
<tr>
<td>370</td>
<td>710</td>
<td>230</td>
<td>80</td>
</tr>
</tbody>
</table>

quality factors of the four TM modes confined by a four-row pure dielectric PBG (DPBG) structure in the 2D simulation. Here $Q$ of the structure is the total $Q$:

$$\frac{1}{Q_{tot}} = \frac{1}{Q_{rad}} + \frac{1}{Q_{die}}.$$  \hspace{1cm} (3.10)

We exclude $Q_{ohm}$ by imposing perfect conducting boundaries on the top and bottom surfaces of the 2D model. Extracting from Figure 3-17, for the four-row DPBG structure, $Q_{rad,02} \sim 400$. Inclusion of the dielectric loss gives $Q_{tot,02} \sim 370$, implying $Q_{die}$ is small. In the following, except where explicitly stated, the dielectric loss is always included in the simulation.

Table 3.3 suggests the simple four-row pure DPBG structure may not be a good
candidate with a low $Q_{rad,02}$ and relatively high $Q_{rad,21}$ and $Q_{rad,31a}$. Usually for a good damping on other modes, the operation mode should be one or two orders of magnitude higher. One advantage of the DPBG structure with overmoded operation is that the quadrupole or sextupole modes are far less harmful than the dipole TM$_{11}$ mode. Moreover, by arrangement of the rods pattern, we can obtain a great enhancement of the quality factor $Q_{rad}$ of the accelerating TM$_{02}$ mode, and meanwhile decrease $Qs$ of the other unwanted modes, which will be explained in the next section.

### 3.7 Damping Channels and the Hybrid PBG Structure

The TM$_{02}$ mode in the defect in a pure DPBG structure is hexagonal at its periphery due to the shape of the triangular lattice. Comparing with the MPBG structure and the DIWG structure, making the TM$_{02}$ mode more azimuthally uniform could give a higher $Q_{rad,02}$. Figure 3-18 shows a new configuration of three rows of sapphire rods with damping channels, including the TM$_{02}$ mode it confined. $Q_{rad,02}$ of this new configuration with less sapphire rods is enhanced by almost a factor of twenty, as shown in Table 3.4 comparing of the quality factors of the four modes with a 4-row DPBG structure. Optimization to arrange the pattern could potentially increase $Q_{rad}$
Table 3.4: Comparison of $Q$s of 2D TM Modes between a pure 4-row PBG structure and a pure 3-row PBG structure with damping channels.

<table>
<thead>
<tr>
<th>Structure</th>
<th>TM_{02} $Q_{02}$</th>
<th>TM_{21} $Q_{21}$</th>
<th>TM_{31,a} $Q_{31a}$</th>
<th>TM_{31,b} $Q_{31b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-row</td>
<td>370</td>
<td>710</td>
<td>230</td>
<td>80</td>
</tr>
<tr>
<td>3-row damping</td>
<td>7300</td>
<td>3600</td>
<td>1600</td>
<td>480</td>
</tr>
</tbody>
</table>

of the accelerating mode by more than a hundred times [73].

In addition, the removal of rods creates damping channels for the unwanted modes. Some studies are shown in [74]. For our structure, Figure 3-18c shows clearly a leakage of the TM_{31,b} mode. Table 3.4 shows that $Q_{rad,02}$ is dominant over the other three modes.

Reflection of microwaves from metallic rods is much higher than from dielectric rods. By adding one row of metallic rods outside the sapphire rods, the confinement of the TM_{02} mode is improved even more. The PBG structure constructed by both metallic rods and dielectric rods is called a hybrid metallic-dielectric PBG (HPBG) structure. The outermost metallic rods have little influence on the field profile of the TM_{02} mode, since the high field of the accelerating mode is concentrated within the defect. For the other modes, the metallic rods help suppress the field of some modes. Figure 3-19 shows four configurations of the rod pattern of a 4-row HPBG structure constructed by different numbers of rods and damping channels. Each model is named under the number of metallic rods (Mxx) and dielectric rods (Dxx) used. Metallic rods are simulated with empty geometry surrounded by a “Finite Conductivity” boundary, which is valid due to the skin depth of copper at 17.14 GHz ($\delta \sim 0.5$ $\mu$m is much shorter than the wavelength $\lambda = c/f = 1.75$ cm).

Including the ohmic loss ($Q_{ohm}$) from the copper rods, $Q_{tot}$ is

$$\frac{1}{Q_{tot}} = \frac{1}{Q_{rad}} + \frac{1}{Q_{die}} + \frac{1}{Q_{ohm}}.$$  \hspace{1cm} (3.11)

The dielectric loss is very small due to the high quality of sapphire. The ohmic loss is small as well on a small number of copper rods. So $Q_{tot}$ is primarily determined by
Figure 3-19: The 2D models of four hybrid dielectric-metallic PBG (HPBG) structures.

Table 3.5 gives the $Q_{tot}$s of the modes in the four hybrid models shown in Figure 3-19 and the two aforementioned DPBG structures, the 4-row DPBG structure and the 3-row DPBG structure with damping channels. A large improvement of $Q_{02}$ by adding copper rods is achieved. $Q_{21}$ is also enhanced, but fortunately, the hybrid structures give no confinement of the TM$_{31,b}$ mode in the near frequency range. A complete leakage of the TM$_{31,b}$ mode from the HM6D57 structure is shown in Figure 3-20. A clamped design of the experimental HPBG structure allows cold tests of all the four rod patterns. The three-dimensional simulation and the cold tests, described in Chapter 4 and Chapter 5 will give the final choice from the four configurations.
Table 3.5: Comparison of Qs of the 2D TM Modes between the DPBG structures and the four HPBG structures.

<table>
<thead>
<tr>
<th>Mode</th>
<th>TM_{02}</th>
<th>TM_{21}</th>
<th>TM_{31,a}</th>
<th>TM_{31,b}</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-row DPBG</td>
<td>370</td>
<td>710</td>
<td>230</td>
<td>80</td>
</tr>
<tr>
<td>3-row DPBG damping</td>
<td>7.3 \times 10^3</td>
<td>3.6 \times 10^3</td>
<td>1.6 \times 10^3</td>
<td>4.8 \times 10^2</td>
</tr>
<tr>
<td>HM24D60</td>
<td>2.4 \times 10^4</td>
<td>2.4 \times 10^4</td>
<td>-</td>
<td>1.2 \times 10^4</td>
</tr>
<tr>
<td>HM12D60</td>
<td>1.1 \times 10^4</td>
<td>2.6 \times 10^3</td>
<td>-</td>
<td>1.2 \times 10^3</td>
</tr>
<tr>
<td>HM6D60</td>
<td>7.6 \times 10^3</td>
<td>2.9 \times 10^3</td>
<td>-</td>
<td>6.5 \times 10^2</td>
</tr>
<tr>
<td>HM6D57</td>
<td>5.9 \times 10^3</td>
<td>1.4 \times 10^3</td>
<td>-</td>
<td>40</td>
</tr>
</tbody>
</table>

Figure 3-20: The electric field profile of the TM_{31,b} mode in the HM6D57 structure.

3.8 Conclusion of the 2D Simulation

Taking into consideration of the birefringence of sapphire, the new band gap maps of both the square lattice and the triangular lattice are calculated using HFSS. Because the band gap map has no cut-off frequency, an overmoded operation is allowed and the TM_{02} mode is chosen to be the accelerating mode in a dielectric PBG structure. A dielectric PBG cavity uses sapphire rods to form the triangular lattice array in the transverse xy plane. A defect is formed by removing the center four hexagonal rows of sapphire rods.

A simple 2D cavity model is built in HFSS to start investigating the quality factor $Q$ of each TM mode confined in the lowest band gap. $Q_{rad}$ of the accelerating mode increases with the number of sapphire rods. In practice, a 3-row or 4-row DPBG cavity is mechanically acceptable but with poor confinement of the operation mode. Creating damping channels and inclusion of copper rods on the outermost row provide large enhancements in confining the TM_{02} mode and helps suppress some HOM as
well. To fit our purpose of high power testing of the dielectric material, a higher $Q_{02}$ is of priority. The four patterns of the hybrid metallic-dielectric PBG structure are candidates for a future design. A detailed three-dimensional design for the breakdown experiment based on the HPBG models is described in Chapter 4.
Chapter 4

Design of the HPBG Structure for the Breakdown Experiment at 17 GHz

4.1 Introduction

The performance of an accelerator structure, constructed by different materials with different geometry designs, undergoing high input power level is essential for future high gradient colliders. To conveniently and comparably test the high power properties of a specific cavity, SLAC developed a standard procedure of testing a single-cell, Standing-Wave (SW) cavity with two side disk-loaded-waveguide (DLWG) cavities to couple the high field only in the central test cavity [75,76]. A mode launcher is used to convert the regular waveguide mode, the TE$_{10}$ mode, to the fundamental accelerating mode, the TM$_{01}$ mode, into the SW test stand, as shown in Figure 4-1. The mode launcher is scalable with the microwave frequency. To employ the 17.14-GHz klystron made by Haimson Research Corporation (HRC) at MIT Accelerator Laboratory, a scaled mode launcher was built to couple the rf power from WR-62 waveguide.

Utilizing the SW test stand, four preceding metallic photonic band gap (MPBG) structures were built and high power tested: the X-Band PBG-R structure with round
copper rods (at SLAC), the X-Band PBG-E structure replacing the first row of PBG-R with elliptical rods (at SLAC), the Ku-Band, MIT-PBG and MIT-PBG2 structures with round rods [15, 35, 36]. The MIT-PBG-2 structure will be referred in the rest of the thesis as MPBG-MIT-2 for comparison with the HPBG structure. The former X-band MPBG structures were brazed into one piece. Tuning stubs were used to tune the volume of the two side cells to achieve resonance. The same concept failed when applied to the Ku-band structures. Instead, a new clamped [77] structure was chosen. For a clamped structure the input and output coupling cell and the central cell are machined separately and clamped together. A resonant measurement can be applied to any single cell, based on which independent corrections can be applied to each cell separately.

The novel hybrid metallic-dielectric PBG (HPBG) structure followed the same experimental principle to do a single cell testing using the SW test stand. Brazing sapphire rods to the copper plate is difficult. Instead, both dielectric rods and metallic rods were inserted into the holes machined on two copper plates. This could potentially lead to a poor connection between the rods and the copper plates, and therefore affecting the breakdown performance in high power testing. However, the flexibility to arrange rods allowed to test all of the four rod patterns shown in Figure 3-19. In the following sections, we begin with the final design result of the first
HPBG structure for the breakdown experiment. Details leading to this result will be described after.

4.2 The HFSS Model

HFSS was used to complete the experimental design. As described in Chapter 3, a full three-dimensional (3D) model was required with an anisotropic permittivity \( \varepsilon_r = [9.398, 9.398, 11.587] \) assigned to the sapphire rods at 17 GHz. Figure 4-2 shows the HFSS model of the HPBG structure used in the 3D simulation.

![HFSS model of HPBG structure](image)

Figure 4-2: The SW model of the HPBG structure in the 3D simulation, including (a) the drawing defining the hexagonal rows of rods, (b) the top view, (c) the side view, and (d) the projected view of the model. Vacuum is partially transparent. Sapphire rods (purple) and copper rods (green) are sandwiched between two "Finite Conductivity-Copper" boundaries to form a HPBG cell. The input and output DLWG cells couple the high power into the HPBG cell. The input waveguide (\( \phi = 15.24 \) mm or 0.6 inch) is connected to the mode launcher to receive the TM_{01} mode. At the end, waves are reflected back from the cutoff section. Power is coupled in each cell through the iris.
Figure 4-3 shows the side view of the model with parameters to specify the structure. Values of those parameters are listed in Table 4.1. Among those parameters, the radius of the input waveguide ($R_{WG}$), the thicknesses and the major radii of the coupling iris ($t_{cp}$, $a_{cp}$), the input iris ($t_{i}$, $a_{i}$) and the output iris ($t_{o}$, $a_{o}$), the radii of the input and output iris apertures ($r_{i}$, $r_{o}$), and the heights of the three cells ($l_{vac}$, $d_{i}$, $d_{o}$), are fixed. The radius of the end cutoff section ($r_{end}$) is easily chosen to be smaller than the cut-off waveguide of 17.14 GHz. The radii of the fillets of the cavity edges are all the same as $R_f$. All the other parameters can be modified to obtain the resonant frequency with a higher quality and lower surface fields. Details are discussed in the following sections. The radii to round the cavity edges are all the same ($R_f = 0.2$ mm).

4.3 Lattice Constant $b$ and Radius of the Sapphire Rod $a$

Chapter 3 with 2D simulation provides a design point of the lattice ratio $a/b \sim 0.35$. For the TM$_{nm}$ mode, Eqs. 4.1 and 4.2 relate the resonant frequency $f_{c, nm}$ of a
Table 4.1: Parameters of the final design of the first HPBG structure.

<table>
<thead>
<tr>
<th>Rods</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1.58</td>
</tr>
<tr>
<td>$b$</td>
<td>4.48</td>
</tr>
<tr>
<td>$l_{rod}$</td>
<td>11.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Central Cell</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>3</td>
</tr>
<tr>
<td>$l_{vac}$</td>
<td>5.68</td>
</tr>
<tr>
<td>$R_{fil}$</td>
<td>2.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Irises</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_i = t_o = t_{cp}$</td>
<td>3.07</td>
</tr>
<tr>
<td>$a_i = a_o = a_{cp}$</td>
<td>2.27</td>
</tr>
<tr>
<td>$r_i = r_o$</td>
<td>3.765</td>
</tr>
<tr>
<td>$r_{cp}$</td>
<td>4.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matching Cells</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_i = d_o$</td>
<td>5.68</td>
</tr>
<tr>
<td>$R_i$</td>
<td>7.88</td>
</tr>
<tr>
<td>$R_o$</td>
<td>7.88</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WG &amp; End</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{WG}$</td>
<td>15.24</td>
</tr>
<tr>
<td>$r_{end}$</td>
<td>4.23</td>
</tr>
<tr>
<td>$R_{end}$</td>
<td>2</td>
</tr>
</tbody>
</table>

A cylindrical cavity to the radius $r$.

\[ k_c = p_{nm}/r, \quad (4.1) \]
\[ f_{c,nm} = \frac{k_c}{2\pi \sqrt{\mu\varepsilon}} = \frac{p_{nm}}{2\pi a \sqrt{\mu\varepsilon}}, \quad (4.2) \]

where $k_c$ is the propagation vector, $p_{nm}$ is the $m$th zero of the Bessel function $J_n(x)$, and $\mu, \varepsilon$ are the permittivity and permeability of the material. For the TM$_{02}$ mode, $p_{02} = 5.520$. The frequency is required to fit the resonant frequency of the HRC klystron at $f_c = 17.14 \pm 0.01$ GHz. Replacing $r$ with $r_{eff}$ and solving Eqs. 4.1 and 4.2
yields:
\[ r_{\text{eff}} \sim \frac{P_{02} c}{2\pi f_c} = 0.0154 \text{ m}, \]
\[ \frac{\sqrt{3}}{2} (n_{\text{row}} - 1) b = r_{\text{eff}}, b \sim 0.0044 \text{ m} = 4.4 \text{ mm}, \]
\[ a \sim 0.35 \times 4.4 = 1.54 \text{ mm}. \]

To confine the high field completely in the defect area within the innermost row of rods, the circumference of the defect should be sitting on the edge of the TM_{02} mode, so the formula is adjusted:
\[ \frac{\sqrt{3}}{2} (n_{\text{row}} - 1) b - \frac{a}{2} = r_{\text{eff}}, \]
\[ b \sim 4.68 \text{ mm}. \]

Therefore the range of the lattice constant should be \( b \in [4.4, 4.68] \text{ mm} \). In 3D, due to the vectors of the electric field with different angles to the c-axis of the sapphire rods, the birefringence makes the wave see a different permittivity. The field can feel any permittivity number in the range within \([9.398, 11.587]\). A complete 3D simulation of the HPBG structure including the matching cells and the central PBG cell, is needed to determine the lattice constant \( b \) and the lattice ratio \( a/b \). With \( a/b \sim 0.35 \), varying \( b \) in the range \([4.47, 4.49]\) mm gave a resonant frequency close to the klystron frequency with a good confinement. More accurate adjustments involved the geometry dimensions of the two side coupling cells \( R_e, R_o \) and the aperture of the input coupling iris \( r_{cp} \), which could not be accomplished until the end of the simulation. We started with \( b \sim [4.47, 4.49] \text{ mm}, a/b \sim 0.35, \) and \( a \sim [1.56, 1.58] \text{ mm} \). A stronger confinement favored a larger radius, leading to \( a = 1.58 \text{ mm} \).

### 4.4 Irises

The iris was shaped elliptically to reduce the electric and magnetic field on its surface. SLAC had done a lot of work to optimize the geometry and the material of the
irises [78–82]. For an X-band structure at 11.424 GHz, the dimensions are the thickness \( t = 4.6 \) mm and the major radius \( a = 3.4 \) mm. For 17.14 GHz whose vacuum wavelength is smaller by a factor of \( \sim 2/3 \), the iris is scaled to have \( t = 3.07 \) mm and \( a = 2.27 \) mm.

4.5 The Length of the Sapphire Rod \( l_{rod} \)

The vacuum wavelength of 17.14 GHz is \( \lambda \sim 17.5 \) mm. For a \( \pi \) mode, the length of the periodic cavity is \( D = \lambda/2 = 8.75 \) mm. The cell height without the iris should be \( l_{vac} = D - t = 5.68 \) mm, as shown in Table 4.1, which sets up the lower limit of the length of a sapphire rod.

The thickness of the iris provides an upper limit of the length of the inserted sapphire rods. For our single-cell test stand, the insertion length \( d \) of the sapphire rods into the holes on the copper plates, has to be smaller than the thickness of the iris. In order to stabilize the clamped structure and ensure the alignment, an as-long-as-possible insertion length was preferred, leading to \( d = 3 \) mm and \( l_{rod} = 11.68 \) mm. Figure 4-4 shows a real sapphire rod made by Insaco Inc. [16] with an indication of the C-axis.

![Figure 4-4: A sapphire rod.](image)

4.6 Rod Patterns

To simplify the 3D model of the SW test stand involving three cells, a periodic one-cell model to simulate only the central HPBG cell was built in HFSS. This was verified because the high field is in the central HPBG cell only. Simulations of the periodic model were performed to the four rod patterns shown in Figure 3-19: HM24D60, HM12D60, HM6D60 and HM6D60.
4.6.1 The Symmetric Model and the Mesh Detail

Figures 3-19a and 3-19b show a six-fold symmetry of the HM24D60 and the HM12D60 pattern; thus a 60° wedge is enough to model the 360° structure with appropriated assigned boundary conditions. In addition, the azimuthal symmetry of the TM02 mode can reduce the simulation of the 60° wedge to a 30° wedge. This was realized by imposing "Perfect H" boundaries on both sides of the 30° wedge. Figure 4-5a shows the 30° wedge of the HM24D60 structure in HFSS. In Figure 4-5a, boundary conditions are indicated in italic. Rods 1, 2 and 3 are numbered to be referred to in the following sections. By assigning a 180° (π) phase advance along the z axis from the entrance iris end to the exit iris end, as shown in Figure 4-5a, the periodic one-cell model forced a π mode standing wave to be built, resulting in modeling a multi-cell periodic structure for an accelerator. The coordinate system is shown in red in Figure 4-5a, with its origin at the center of the HPBG cavity. The empty white circles have "Finite Conductivity-Copper" boundaries to define the copper rods. Strictly speaking, any insertion length \( d > t/2 \) is not a periodic accelerating cell in...
a collider. However, in order to resemble the HPBG cell in the SW test stand, the periodic model should hold the same parameters as in the test stand. The effect of the insertion length will be discussed later in Section 4.8. A fillet on Rod 1, the innermost rod, was built to reduce the electromagnetic field on the triple point interface, which will be discussed later in Section 4.9.

For the periodic one-cell model, eigenmode simulation was performed in HFSS to calculate the field. A “Perfectly-Matched-Layer (PML)” boundary was assigned on the outer edge of the wedge. HFSS eigenmode simulation converges on the resonant frequency of each eigenmode of each pass. The field error (uncertainty) is larger than the frequency error (uncertainty). A maximum mesh length of 0.25 mm was assigned on the surface of Rod 1 to shorten the refinement by each simulating pass and obtain higher accuracy of the field on the fillets. To obtain an accurate field calculation, a maximum frequency error of 0.1% to finish the simulation was assigned, which resulted in a ~ 0.05% frequency uncertainty and ~ 650,000 tetrahedra built in the periodic model of the HM24D60 structure, as shown in Figure 4-5b. Similarly, a 30° wedge of HM12D60 was modeled in HFSS with the same boundary assignments as HM24D60. The final result reached a convergent frequency with an uncertainty of 0.2% with ~ 700,000 tetrahedra.

For HM6D60 and HM6D57, a 60° wedge was required to complete a simulation of the 360° structure due to the three-fold symmetry. The periodic model of the HM6D60 pattern in HFSS is shown in Figure 4-6a. For HM6D60, over 750,000 tetrahedra were built to get a convergence on frequency with an uncertainty about ~ 1%, as shown in Figure 4-6b. For HM6D57, the number of tetrahedra reached over 780,000 to get an uncertainty of ~ 2% on frequency. Better accuracy requires much more simulating hours.

4.6.2 Resonant Frequency $f_0$ and $Q$

By starting with $b = 4.48$ mm and $a = 1.58$ mm, the resonant frequency $f_0$ of the TM$_{02}$ mode was calculated for the four patterns. In the HPBG structure, power dissipation comes from three sources, the radiation loss through the PBG lattice, the
ohmic heat loss and the dielectric loss. The quality factor $Q$ is:

$$\frac{1}{Q_{\text{tot}}} = \frac{1}{Q_{\text{rad}}} + \frac{1}{Q_{\text{die}}} + \frac{1}{Q_{\text{ohm}}}.$$  \hspace{1cm} (4.3)

By turning off other lossy mechanisms, we were able to calculate $Q$ of each loss independently. Table 4.2 shows the result of the resonant frequency $f_0$ and the individual $Q$ of the four patterns. The periodic one-cell model had field nulls strictly at the iris ends. For the SW model, however, the field in the central cell distributes out to the coupling cells, leading to not a strict $\pi$ mode with a longer wavelength. Thus the frequency in the periodic model should be higher than 17.14 GHz to resonate the SW structure at the right frequency. Removing copper rods decreased the frequency as expected. The highest $Q_{\text{rad}}$ occurred for HM24D60, consistent with Section 3.7. For 3D simulations, $Q_{\text{rad}}$ is much lower than in the 2D model. A big gap of the $Q_{\text{rad}}$
between HM24D60 and HM12D60 verified the importance of the symmetry pattern to keep the field profile of the TM$_{02}$ mode more uniform in the azimuthal direction. In addition, a change in rod pattern could potentially result in a slight change in the band gap map, especially the gap width, which changes $Q_{\text{rad}}$. With copper plates to sandwich the sapphire rods, the ohmic heat loss in the 3D model was much higher than in the 2D simulation. $Q_{\text{rad}}$ was required to be larger than $Q_{\text{ohm}}$. The HM24D60 pattern was the only one that met the requirement, which would be employed in the experimental design and will be discussed in the following sections.

4.7 $E$ and $H$ Fields of the TM$_{02}$ Mode

The electric and magnetic field maps on the $xy$ plane and on the $yz$ plane are shown in Figure 4-7. Clearly, the $E$ field map on the $xy$ plane shows the overmoded field pattern and the map on the $yz$ plane shows a phase advance of $\pi$. The field magnitude is normalized to an accelerating gradient $E_G = 100$ MV/m. $E_G$ is defined as an integration of the axial accelerating field $E_z$ with the spatial variation over the cavity length $L$, as shown in Eq. 4.4 [37]:

$$E_G = \frac{1}{L} \int_{L/2}^{L/2} E_z \cos (\omega z/c)dz.$$ (4.4)

Figure 4-8a shows the axial accelerating field ($E_z$) and the magnetic field profile. With $E_G = 100$ MV/m, the highest axial $E$ field was $\sim 200$ MV/m. Figure 4-8b plots the electric field vectors on the $xy$ plane, with zoom in plot inside Rod 1, which shows the orientation of the $E$ field is not always parallel to the $z$ axis (the C-axis of the sapphire rods), verifying the discussion about the birefringence in Chapter 3.

4.8 Insertion Length $d$

To simplify the assembly we chose a long end ($d = 3$ mm) of the sapphire rod to be inserted into the copper plate (see Figure 4-3 for the parameter definition). However,
Figure 4.7: The electric and magnetic field maps on the xy plane and on the yz plane of the periodic one-cell model for HM24D60.

Our simulations showed the shorter the insertion length \( d \), the lower the maximum surface field. A comparison of three cases: \( d = 0 \) mm, \( d = 1.5 \) mm and \( d = 3.0 \) mm, all without fillets, will be discussed to investigate the influence of the insertion length on the fields of the HM24D60 structure. For those simulations, to shorten the calculating time, the convergent requirement was \( \sim 1\% \) and 100,000 to 200,000 tetrahedra were built in the periodic model.

4.8.1 Resonant Frequency \( f_0 \) and \( Q \)

Table 4.3 lists the resonant frequencies \( f_0 \) and the quality factor \( Q \) of the accelerating TM\(_{02}\) mode of the three insertion lengths. Frequency may change primarily due to
Figure 4-8: (a) The axial electric and magnetic field profile, and (b) the E field vector on the $xy$ plane of the periodic one-cell model for HM24D60.

Table 4.3: Resonant frequency $f_0$ and $Q$ of each insertion case.

<table>
<thead>
<tr>
<th>$d$ (mm)</th>
<th>$f_0$ (GHz)</th>
<th>$Q_0$</th>
<th>$Q_{rad}$</th>
<th>$Q_{die}$</th>
<th>$Q_{oh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17.2877</td>
<td>$8.0 \times 10^3$</td>
<td>$6.2 \times 10^4$</td>
<td>$3.8 \times 10^4$</td>
<td>$1.2 \times 10^4$</td>
</tr>
<tr>
<td>1.5</td>
<td>17.1953</td>
<td>$4.7 \times 10^3$</td>
<td>$1.7 \times 10^4$</td>
<td>$3.1 \times 10^4$</td>
<td>$8.0 \times 10^3$</td>
</tr>
<tr>
<td>3.0</td>
<td>17.0703</td>
<td>$3.6 \times 10^3$</td>
<td>$1.4 \times 10^4$</td>
<td>$2.8 \times 10^4$</td>
<td>$5.8 \times 10^3$</td>
</tr>
</tbody>
</table>

the direction of the field vector with respect to the optical axis of the rods. A decrease of $Q_{rad}$, from $d = 0$ to $d = 1.5$ may be caused by the radiative loss from the part of the sapphire rods inserted into the copper plate. Microwaves not only propagate out through the photonic lattice, but also dissipate from the dielectric end, if not stopped by the metal as in the $d = 0$ case. Intuitively, the longer the sapphire rods, the larger the dielectric loss. In all the three cases $Q_{oh}$ is the dominant term to determine $Q_{tot}$.

### 4.8.2 Surface $E$ and $H$ Fields

The surface $E$ and $H$ field of each insertion case was quite different, as shown in Figure 4-9. Again, all the field magnitudes are normalized to $E_G = 100$ MV/m. The longer the insertion length, the larger the surface field maxima. A longer insertion always brings a large enhancement of the field along the triple edge. The highest surface $E$ and $H$ fields corresponding to $E_G = 100$ MV/m for each insertion length are listed in Table 4.4. A low surface field favored as short as possible an insertion.
Figure 4-9: The surface electric and magnetic fields of the three insertion lengths.
Table 4.4: Comparison of the maxima of the surface $E$ and $H$ fields with different insertion lengths.

<table>
<thead>
<tr>
<th>$d$ (mm)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$H_{surf}$ (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>235</td>
<td>588</td>
</tr>
<tr>
<td>1.5</td>
<td>640</td>
<td>1987</td>
</tr>
<tr>
<td>3.0</td>
<td>888</td>
<td>3911</td>
</tr>
</tbody>
</table>

length. The values of the $E$ and $H$ fields for $d = 1.5$mm and $d = 3$ mm were much higher than regular DLWG structures and the MPBG structure at 17 GHz. Only when $d = 0$, the surface fields were comparable. However in practice for the first HPBG structure, we need to prioritize the mechanical realization to stabilize and align the rods between the copper plates; thus $d = 3$ mm was kept and fillets were used on Rod 1 to reduce the field, of which details will be discussed later. Due to the insertion length, we refer to the hybrid structure as the HPBG-d3 structure in the rest of this thesis. Field maps on the innermost two rows are shown in Figure 4-10 for $d = 3$ mm. Log scale is chosen based on the large variation of the field magnitude. Both $E$ and $H$ fields have the maxima at the triple-point edge of Rod 1 and drop quickly away from the interface. The surface field is quite small on the other rods.

4.9 Fillets

In this section, the effect of the fillet on the innermost rod impacting the field distribution will be discussed in detail. Comparison between $R_{fil} = 0$ (no fillet), $R_{fil} = 1$ mm and $R_{fil} = 2.25$ mm will be made. Figure 4-11a shows the smaller fillet model of $R_{fil} = 1$ mm in HFSS. The $R_{fil} = 0$ mm case is shown in Figure 4-9e and Figure 4-9f, and the $R_{fil} = 2.25$ mm case is shown in Figure 4-5a and here in Figure 4-11b.

4.9.1 Resonant Frequency $f_0$ and $Q$

The fillet size influenced the direction of the field vectors along Rod 1, the innermost rod (see Figure 4-5a for the rod number definition), thus made the field see different dielectric constant in the structure, leading to a different resonant frequency and a
Figure 4-10: The surface electric and magnetic fields on Rod 1, 2, and 3, for $d = 3$ mm, in log scale.

Figure 4-11: The periodic model of (a) $R_{fil} = 1$ mm, and (b) $R_{fil} = 2.25$ mm of the HM24D60 pattern. Lines on (b) illustrate the geometries used in Section 4.9.2.
different quality factor, as listed in Table 4.5. The trend of the $Q$ change already

<table>
<thead>
<tr>
<th>$R_{fil}$ (mm)</th>
<th>$f_0$ (GHz)</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17.0703</td>
<td>$3.6 \times 10^3$</td>
</tr>
<tr>
<td>1</td>
<td>17.3082</td>
<td>$4.1 \times 10^3$</td>
</tr>
<tr>
<td>2.25</td>
<td>17.2322</td>
<td>$5.3 \times 10^3$</td>
</tr>
</tbody>
</table>

prefers to a larger fillet size.

4.9.2 Surface $E$ and $H$ Fields

Figure 4-12 shows the surface electric and magnetic fields $R_{fil} = 1$ mm and $R_{fil} = 2.25$

![Electric Field](image)

![Magnetic Field](image)

Figure 4-12: The surface electric and magnetic fields for the fillet size of $R_{fil} = 1$ mm and $R_{fil} = 2.25$ mm.

mm, which can be compared with Figures 4-9e and 4-9f for $R_{fil} = 0$ mm. The
comparison of the highest $E$ and $H$ field normalized to $E_G = 100$ MV/m of each fillet size is listed in Table 4.6. Shown in Figure 4-11b are the three lines used to compare the fields. All the three lines are on the plane of the copper plate. Line 1 goes through the surface center; Line 2 goes through the center of Rod 3; and Line 3 is on the symmetric edge. The surface $E$ and $H$ field profiles along the three lines of the three fillet sizes are plotted in Figure 4-13. Figure 4-13 show the reduction of

<table>
<thead>
<tr>
<th>$R_{fil}$ (mm)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$H_{surf}$ (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>888</td>
<td>3911</td>
</tr>
<tr>
<td>1.0</td>
<td>841</td>
<td>1488</td>
</tr>
<tr>
<td>2.25</td>
<td>521</td>
<td>2320</td>
</tr>
</tbody>
</table>

Table 4.6: Comparison of the maximum magnitude of the surface $E$ and $H$ fields of each fillet case.

Figure 4-13: The electric and magnetic field profile along the three lines for the three fillet models.
the surface $E$ field on the sapphire rods by the fillets. With $R_{fil} = 2.25$ mm, the maximum surface field was mitigated to the triple point edge on Rod 2 due to the large fillet size. The $E$ field maximum is reduced to 521 MV/m.

### 4.9.3 Fields on the Rods and on the Fillets

Figure 4-14 shows in log scale the field profile on Rod 1, 2 and 3 and on the fillet, for $R_{fil} = 2.25$ mm. Values of those fields are listed in Table 4.7, showing the mitigation of the high field from Rod 1 to Rod 2. The high surface field gave a ratio $E_{surf}/E_G \sim 5.2 : 1$, which was much larger than most of other high-gradient accelerator structures, leaving a disadvantage of the HPBG-d3 design. All the high fields were at the triple point, which was consistent with the theory.

Figure 4-14: The detailed field profile on Rod 1, 2 and 3 and on the fillet, for $R_{fil} = 2.25$ mm
Table 4.7: Comparison of the maximum $E$ and $H$ field on Rod 1, 2 and 3 and on the fillet, for $R_{fil} = 2.25$ mm.

<table>
<thead>
<tr>
<th>Surface</th>
<th>$E_{\text{max}}$ (MV/m)</th>
<th>$H_{\text{max}}$ (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet</td>
<td>430</td>
<td>2320</td>
</tr>
<tr>
<td>Rod 1</td>
<td>75</td>
<td>480</td>
</tr>
<tr>
<td>Rod 2</td>
<td>521</td>
<td>2090</td>
</tr>
<tr>
<td>Rod 3</td>
<td>462</td>
<td>1174</td>
</tr>
</tbody>
</table>

4.10 $E$ and $H$ Fields on Irises

With parameters defining the central HPBG cell settled in the foregoing sections, Figure 4-15 plots the electric and magnetic fields on the irises of the central HPBG cell. The highest electric field was $|E| \sim 250$ MV/m, of the same order as the maximum accelerating field on axis. The maximum magnetic field was $|H| \sim 400$ kA/m. Those fields were used to calibrate the input power in the SW model of the HPBG-d3 structure.

Figure 4-15: The electric and magnetic fields on the surfaces of the two irises of the periodic one-cell model for HM24D60.
4.11 $R_i, R_o, r_{cp}$

The other adjustable parameters are the radii of the two matching cells, $R_i$ of the input coupling cell and $R_o$ of the output coupling cell, and the radius of the coupling iris, $r_{cp}$, as shown in Figure 4-3. The radii of the side cells can slightly tune the resonant frequency of the SW structure and insure a good coupling of the rf power. The coupling coefficient is defined as $\beta = Q_0/Q_{ext}$, where $Q_0$ is the internal quality factor for our structure and $Q_{ext}$ represents the external quality factor connected to the structure. $\beta$ can be adjusted by the radius $r_{cp}$ of the aperture of the input coupling iris. If we do a frequency sweep to our structure, changing $r_{cp}$ will make the circle in the Smith chart move up and down. A critical coupling, with resonant frequency sitting in the center of the Smith chart where $\beta = 1$ is wanted. Varying $r_{cp}$ will slightly change the resonant frequency of the whole structure. The three parameters are entangled. Careful adjustments were made to finalize the experimental design. Those values are listed in Table 4.1.

4.12 Design Results of the HPBG-d3 Structure

The final simulation of a 30° wedge of the model shown in Figure 4-2 was performed by a “Driven Modal” simulation in HFSS. Instead of the “PML” boundary in the eigenmode simulation, a “Radiation Boundary” was assigned on the outer periphery. One “PML” was assigned at the end of the cut-off waveguide. An exciting wave port was assigned to the front of the input waveguide. To facilitate the calculation duration, a maximum length of 0.25 mm of the mesh element was assigned on the surface of the innermost rod. A simulation uncertainty of ±0.05% on $S$ parameter was required to complete the simulation. The final result achieved an uncertainty of 0.05% with convergence on a 889,000 tetrahedra-meshed model. The mesh plot is shown in Figure 4-16. Mesh is concentrated at the high field region.
4.12.1 Resonant Frequency $f_0$ and $Q$

Figure 4-17 shows the $S_{11}$ and the Smith chart of the experimental design of the HPBG-d3 structure, with parameters summarized in Table 4.1. Figure 4-17a shows

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{s11_magnitude}
\caption{(a) The magnitude of $S_{11}$}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{smith_chart}
\caption{(b) The Smith chart of $S_{11}$}
\end{figure}

the resonant frequency right at $f_0 = 17.1446$ GHz. The loaded quality factor, $Q_L$, can be calculated by extracting the 3-dB frequency width (Full Width of Half Maximum,
FWHM), and for a critical coupling, \( Q_0 \) can be derived:

\[
Q_L = \frac{f_0}{f_{3dB,2} - f_{3dB,1}},
\]

\[
\beta = \frac{Q_0}{Q_{ext}} \sim 1,
\]

\[
\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}},
\]

\[
Q_0 = 2 \times Q_L.
\]

The three values are listed in Table 4.8. Here \( Q_0 \) is the \( Q_{tot} (Q_{tot} = 5.3 \times 10^3) \) in the periodic one-cell simulation. Taking into account the ohmic loss from the matching cells, they are consistent.

Table 4.8: Frequency and \( Q \) of the SW Model in HFSS simulation.

<table>
<thead>
<tr>
<th>( f_0 ) (GHz)</th>
<th>( S_{11} ) (dB)</th>
<th>( Q_L )</th>
<th>( \beta )</th>
<th>( Q_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.1446</td>
<td>-51</td>
<td>2230</td>
<td>( \sim 1 )</td>
<td>( 4.4 \times 10^3 )</td>
</tr>
</tbody>
</table>

4.12.2 Input Power Calibration and Shunt Impedance

The accelerating gradient in the SW structure should be calibrated using the value in the periodic simulation. In order to get in the “Driven Modal” simulation of the SW structure the same gradient as in the eigenmode simulation of the periodic structure, we need to calibrate the gradient with respect to the surface fields. The surface fields on the irises besides the HPBG cell were chosen because they were located closest to the accelerating axis. Due to the different simulation models of the SW structure and the periodic structure, it was not possible to match both \( E \) and \( H \) fields simultaneously. After comparison of all the surface fields, the electric field on the irises was chosen. Adjusting the power feed in the SW model with the same electric field on the irises as in the periodic model shown in Figure 4-15, we obtained the same accelerating gradient \( E_G = 100 \) MV/m. To generate a surface \( E \) field of 260 MV/m on the irises, the input power provided to the 30° wedge was \( P_{in,1/12} = 930.5 \) kW. For a full 360° structure, the input power required in total was \( P_{in} = 11 \) MW.
Considering the gradient in the HPBG-d3 cell, the shunt impedance was [37]:

\[ R_s = \frac{E_G^2 L}{P_m} \approx 7.95 \text{ MΩ/m}, \]

where \( L = 8.75 \text{ mm} \) is the length of the central cell, or the wavelength of the \( \pi \) mode. The shunt impedance was smaller comparing with the DLWG structures or the metallic PBG structures. It was caused by 1) the overmoded operation, and 2) the larger geometric design.

### 4.12.3 \( E \) and \( H \) Fields

The normalized magnitude of the electric field profile along the accelerating axis is plotted in Figure 4-18, with comparison of the field profile in the periodic model. The

![Normalized Magnitude of Electric Field](image)

Figure 4-18: Comparison of the normalized magnitude of the electric field on z-axis, between the SW test stand model and the periodic model in HFSS.

...local maxima of the field in the side cells were only a quarter of the peak field in the central HPBG cell. Unlike the \( E \) field in the periodic model, the \( E \) field in the HPBG cell of the SW model does not have a null, which forces us to use the surface field to calibrate the gradient, as the aforementioned.

Figure 4-19 plots the field maps on the \( xy \) plane and Figure 4-20 plots the field maps on the \( yz \) plane of the SW, HPBG-d3 structure simulated in HFSS with the final parameters listed in Table 4.1.

Surface field maps are plotted in log scale in Figure 4-21. With \( E_G = 100 \text{ MV/m} \), the highest surface fields are \( E_{surf} = 628 \text{ MV/m} \) and \( H_{surf} = 2297 \text{ kA/m} \), much
Figure 4-19: The electric and magnetic field maps on the $xy$ plane of the SW, HPBG-d3 structure in HFSS.

Figure 4-20: The electric and magnetic field maps on the $yz$ plane of the SW, HPBG-d3 structure in HFSS.
higher comparing with the DLWG (DLWG-MIT) and the MPBG-MIT structure designed for the same frequency, as listed in Table 4.9.

Table 4.9: Comparison of surface fields of the HPBG-d3, the MPBG and the DLWG structures.

<table>
<thead>
<tr>
<th>Structure</th>
<th>$E_G$ (MV/m)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$H_{surf}$ (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPBG-d3</td>
<td>100</td>
<td>628</td>
<td>2297</td>
</tr>
<tr>
<td>MPBG-MIT [15]</td>
<td>100</td>
<td>200</td>
<td>900</td>
</tr>
<tr>
<td>DLWG-MIT [15]</td>
<td>100</td>
<td>197</td>
<td>421</td>
</tr>
</tbody>
</table>

Both field maxima appeared on the metallic surface of the triple point edge surrounding Rod 2, as shown in Figure 4-14.

### 4.13 Inventor Model

The final drawing was done using Inventor [83], as the model shown in Figure 4-22. The six components were clamped together by eight clamping rods. Twenty-four metallic rods (in dark orange) and sixty sapphire rods (in white) were inserted by hand to the central rod plates. In Figure 4-22a, the microwave power is coupled from the left through the connection of the input flange made of stainless steel.
(a) Three-quarter solid model, component (b) Assembly section drawing, unit in mm, separated, power coupled from left, power coupled from bottom.

Figure 4-22: (a) A three-quarter view, and (b) a section view of the solid model of the HPBG-d3 structure designed for the high power testing. Two gray flanges are the input and output flanges made of stainless steel. Four dark orange components are the input coupling cell, the two rod plates and the output coupling cell made of OFHC copper. 24 OFHC rods and 60 sapphire rods are inserted into the rod plates.

The input coupling cell, two rod plates and the output coupling cell were made of OFHC copper. Figure 4-22b shows a section side view of the assembly drawing of the HPBG-d3 structure. Microwave is coupled from the bottom. All the metallic parts were machined by the MIT Central Machine shop.

4.14 Conclusion and Improvement Direction

A novel HPBG structure was designed for the breakdown experiments at Ku-band, 17 GHz, employing the mode launcher and the SW test stand scaled from the SLAC design. The HM24D60 pattern was chosen to build up the HPBG structure. Parameters were optimized, resulting in a correct resonant frequency $f_0 = 17.1446$ GHz and a critical coupling with $Q_L = 2230$. To prioritize the mechanical stability of the first HPBG structure designed for the high power testing, we chose a long insertion length $d = 3$ mm on both ends of the sapphire rod to be inserted into the copper plates; the structure was called the HPBG-d3 structure. The surface $E$ and $H$ fields of the HPBG-d3 structure, even after optimization of the rod length and the fillet size, were much higher than the DLWG and the MPBG-MIT structure at the same frequency.
To improve, a different lattice ratio $a/b$ could be taken into account. The 2D pattern of the arrangement of the dielectric rods can be optimized to give a huge enhancement of the quality factor [73]. A straightforward way would be using shorter rods or rods with other shapes to lower the surface fields. Improved designs will show reduction in these fields in Chapter 7 and Chapter 8.
Chapter 5

Cold Test and Experimental Setup of the HPBG Structure

The high power experiment for the novel hybrid metallic-dielectric photonic band gap (HPBG) structure is to test the breakdown properties of the PBG structure with the dielectric material. Figure 5-1 shows all the components of the first HPBG structure used for the high power testing. All the metallic parts are machined by the MIT Central Machine Shop. The sapphire rods are produced by Insaco, Inc [16].

In this chapter, the preparation before the breakdown experiment will be described, including the cold test of the clamped HPBG structure and the experimental setup at the Accelerator Laboratory located at the Plasma Science and Fusion Center (PSFC) at MIT.

5.1 Cold Test: Overmoded Excitation by $S_{21}$ Measurement

5.1.1 $S_{21}$ Measurement: Setup

As the first to use an overmoded operation with the TM$_{02}$ mode, the success and efficiency to excite the TM$_{02}$ mode in the HPBG cell is important. To demonstrate the overmoded excitation, a transmission measurement employing one of the microwave
Figure 5-1: Machined components of the HPBG structure, including (left to right) the input flange with a copper gasket; the input coupling cell; a rod plate with holes for insertion of the rods forming the upstream side of the HPBG cell; the downstream HPBG cell plate with sapphire and copper rods inserted; the output coupling cell; and the output flange. The flanges are made of stainless steel and the others are OFHC copper. 24 copper rods and 60 sapphire rods are inserted into the plate.

$S$ parameter, $S_{21}$, was designed and applied only to the central HPBG cell where the $TM_{02}$ mode was excited. Figure 5-2 shows the copper plate and the central HPBG cell assembly.

An Agilent E8363B Vector Network Analyzer (VNA) was used to measure the $S$ parameters. Two coaxial antennas, each held by a three-axis translation stage, were connected to the VNA to send and receive signals. The $S_{21}$ setup is shown in Figure 5-3. Over a frequency scan, the resonant frequency $f_0$ can be measured and each $Q$ can be calculated as the following [84]:

$$Q_L = \frac{f_0}{\Delta f}, \quad \frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}, \quad \text{(5.1)}$$

$$\frac{Q_0}{Q_{ext}} = (\beta_1 + \beta_2), \quad \frac{Q_0}{Q_L} = (1 + \beta_1 + \beta_2), \quad \text{(5.2)}$$

$$Q_0 = \frac{f_0}{\Delta f}(1 + \beta_1 + \beta_2). \quad \text{(5.3)}$$
Here $\Delta f$ is the half-power bandwidth. $\beta_1$ and $\beta_2$ are the coupling coefficients of the sending and the receiving antenna, respectively. Accurate measurement of $\beta_1$ and $\beta_2$ was beyond the current experimental setup. Demonstration of the overmoded excitation needed only a qualitative measurement. Decreasing $\beta_1$ and $\beta_2$ led to the measured $Q_L$ closer to, but a little less than $Q_0$. In practice, the antennas were kept at a distance relatively far away from the HPBG cell to minimize the coefficients but close enough to receive a good signal-to-noise ratio (SNR) of the measurement.

5.1.2 $S_{21}$ Measurement: Results

The clamped design allowed the flexibility to arrange the rod pattern by inserting different numbers of rods. Therefore the $S_{21}$ measurement was possible to be applied
to all of the four rod patterns shown in Figure 3-19. Figure 5-4 shows the plates with rods inserted for the four rod patterns. Figure 5-5 shows results of the $S_{21}$ measurements, with the detailed frequency $f_0$ and the quality factor $Q_L$ listed in Table 5.1. Within the frequency range of 11-20 GHz, for all of the four patterns, an absolutely dominant resonance of the HPBG central cell around $f_0 = 17.1$ GHz with a high $Q_L$ is shown, demonstrating the single, overmoded excitation. A slightly lower-than-17.14 GHz frequency came from the iris-coupling of the wave to the free space which resulted in a slightly longer wavelength and thus a lower frequency. Besides, the only resonance for the four patterns is the $TM_{02}$ mode, providing a potential

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$f_0$ (GHz)</th>
<th>$Q_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HM24D60</td>
<td>17.113</td>
<td>2790</td>
</tr>
<tr>
<td>HM12D60</td>
<td>17.099</td>
<td>1950</td>
</tr>
<tr>
<td>HM6D60</td>
<td>17.098</td>
<td>1950</td>
</tr>
<tr>
<td>HM6D57</td>
<td>17.096</td>
<td>1950</td>
</tr>
</tbody>
</table>

Figure 5-4: Four rod patterns of the HPBG structure.
Figure 5-5: The magnitude of $S_{21}$ of the four patterns. The red circles indicate where the TM$_{11}$ mode should be.

of wakefield damping of the HPBG structure. Those red circles indicate where the TM$_{11}$ mode should be located. At that frequency there is no resonance. This is an advantage of the capability of the HPBG cell to suppress the most dangerous dipole mode.

5.2 Cold Test: Resonant Frequency and Field Profile

5.2.1 $S_{11}$ Measurement: Setup

As selected in Section 4.6, the rod pattern HM24D60 was to be put into the high power experiment. Due to the insertion length $d = 3$ mm, we refer to the structure as HPBG-d3 in the rest of the thesis. A calibrated $S_{11}$ measurement of the resonant frequency and the quality factor was applied to the completely assembled HPBG-d3 structure. All the parts in Figure 5-1 were clamped using eight clamping rods, as
shown in Figure 5-6a, and connected to the mode launcher shown in Figure 5-6b to have the microwave power coupled in. The measurement setup is shown in Figure 5-6c.

Figure 5-6: $S_{11}$ measurement setup includes: (a) The assembled HPBG-d3 structure, (b) the mode launcher for 17 GHz, and (c) the measurement setup.

5.2.2 $S_{11}$ Measurement: Results

The detailed results of the $S_{11}$ measurement are shown in Figure 5-7 and Table 5.2. Later in next section it shows the resonance is exactly the $\pi$-mode resonance at

Figure 5-7: Results of the $S_{11}$ measurement of the HPBG-d3 structure.
Table 5.2: $f_0$ and $Q$ of the reflection measurement around the resonance at 17.14 GHz ($\pi$ mode).

| $f_0$ (GHz) | $S_{11}$ (dB) | $Q_L$ from $|S_{11}|$ | $Q_L$ from the Smith chart | $Q_0$ | $Q_{ext}$ | $\beta$ |
|-------------|---------------|------------------------|--------------------------|------|-----------|--------|
| 17.147      | -37           | 2150                   | 2460                     | 4570 | 4880      | 0.94   |

17.14 GHz. This measurement was done in air whose permittivity is slightly larger than vacuum, resulting in a resonant frequency higher by $\sim 5$ MHz than that from the simulation. The manipulation to match $S_{11}$ into the Smith chart may cause a small discrepancy of the calculated $Q_L$ between from the magnitude and from the Smith chart. We keep $Q_L = 2150$ in the future analysis. $Q_L$ and $Q_0$ for the HPBG structure are comparable to the metallic MIT-PBG structure with $Q_L = 2646$ and $Q_0 = 4609$ [15].

5.2.3 Bead Pull Measurement: Setup

Field profile of an accelerator cavity can be measured by introducing a small metallic or dielectric perturbation, usually a small bead, which we call a “bead pull measurement”. The change of the resonance or the field caused by the small perturbation is proportional to the field strength at the perturbed location. Whether a resonant measurement [85] or a non-resonant measurement [86] is applicable to a specific structure, depends on the experimental setup and the size of the bead with respect to the structure. For a resonant measurement, the perturbation should be big enough to shift the resonant frequency. For the HPBG structure, a very small dielectric bead was used and a non-resonant bead pull measurement was conducted.

In our measurement, the bead was formed by a super glue drop adhering to a thin dielectric wire (Ashaway 10/0 black monofilament 2 lb Trilene). Figure 5-8 shows the bead and the measurement setup. The wire, supported on a roller system, was connected to a VEXTA 2-phase stepping motor to precisely move the bead up and down. The HPBG-d3 structure was bolted to the roller system with an alignment to make the bead go along the accelerating axis.
The non-resonant bead pull measurement was based on the $S_{11}$ measurement. Due to the small size, the frequency shift by the perturbation is negligible. Instead, we can measure the magnitude and the phase change of $S_{11}$ at the resonant frequency caused by the bead perturbing the local field. Because the axial $H$ field is small, the change is proportional to the local axial $E$ field. Scanning the bead along the axis of the structure gives the axial $E$ field profile. Assuming an $S_{11}$ signal of the form $S = |S_0|e^{i\phi}$, the real and imaginary parts are:

$$S_r = |S_0| \cos \phi, \quad S_i = |S_0| \sin \phi. \quad (5.4)$$

The field strength can be obtained from Eq. 5.5:

$$E^2 \propto \sqrt{S_r^2 + S_i^2}, \quad \text{or} \quad |E| \propto (S_r^2 + S_i^2)^{1/4}. \quad (5.5)$$

The calculated imaginary phase versus the real phase for the HPBG-d3 structure is plotted in Figure 5-9. Figure 5-9a shows two clear paths except for the upper left where a bunch of phases are entangled, as shown in Figure 5-9b in detail. This is the offset center brought by the arbitrary path length from the VNA to the measured structure. From Figure 5-9b we set the offset center $S_{r0}$ and $S_{i0}$ (i.e., $S_{r0} \sim 3.35 \times 10^{-4}$

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and $S_{\omega} \sim -3 \times 10^{-4}$). Thus the final normalized field is obtained:

$$|E|_{\text{norm}} \propto \left[ (S_r - S_{r0})^2 + (S_i - S_{i0})^2 \right]^{1/4}.$$  \hspace{1cm} (5.6)

**5.2.4 Bead Pull Measurement: Results**

The bead pull measurement returned a $\pi$-mode field profile, as shown in Figure 5-10, with a good agreement with the simulation result. The small discrepancy in the output coupling cell may come from noise, since the magnitude of the field in the side cells is much smaller than in the central HPBG cell.
5.3 High Power Testing: Experimental Setup

The principle of the high power experiment of the HPBG structure is similar to what had been conducted with the preceding metallic PBG structures [15] at the MIT PSFC Accelerator Laboratory. Figure 5-11 is a recent photograph of the laboratory. The details will be discussed in this section.

Figure 5-11: Photograph of the MIT Accelerator Laboratory.
5.3.1 The HRC Klystron

The high power microwave (HPM) for the breakdown experiment is provided by a traveling-wave (TW) relativistic klystron located at the MIT Accelerator Laboratory. The klystron, designed by Haimson Research Corporation (HRC), can produce 10-1000 ns square microwave pulses with an output power up to 25 MW. The central frequency is around 17.145 GHz with a bandwidth of ~20 MHz. The operation parameters of the HRC klystron are listed in Table 5.3. The initial signal is generated by an Agilent E8257D PSG Microwave Analog Signal Generator. After a modulation by a Hewlett Packard 11720A Pulse Modulator and a pre-amplification by an HRC solid-state amplifier, the signal is sent to the klystron. By interaction with the electron beam in the klystron, the final amplification, the klystron gain, can go up to 75 dB. The ~500-kV, ~90-Amp electron beam is generated by a Pierce electron gun. The gun is driven by a modulator which is capable of driving three electron guns simultaneously at 500 kV. The modulator generates a high-voltage (HV) pulse with a flat top of ~1 μs. Timing is adjustable to set the microwave pulse in the center of the HV flat top to ensure the amplification and a flat top of the output microwave pulse. Two pairs of steering coils are added on the klystron near the electron gun to steer the electron beam and protect the klystron from possible beam interception and improve the stability and the efficiency.

Table 5.3: Operating parameters for MIT HRC TW relativistic klystron.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_c )</td>
<td>Central Frequency</td>
<td>17.145</td>
<td>GHz</td>
</tr>
<tr>
<td>BW</td>
<td>Bandwidth</td>
<td>~20</td>
<td>MHz</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Pulse Length</td>
<td>10-1000</td>
<td>ns</td>
</tr>
<tr>
<td>( P_{\text{max}} )</td>
<td>Maximum Output rf Power</td>
<td>25</td>
<td>MW</td>
</tr>
<tr>
<td>( G )</td>
<td>Gain</td>
<td>~60 - 70</td>
<td>dB</td>
</tr>
</tbody>
</table>

5.3.2 The Standing-Wave Test Stand

The output rf power of the HRC klystron is divided into two waveguide arms. The power in the two arms is combined and transmitted to the standing-wave (SW) test
stand for the high power testing of the HPBG structure. The HPBG structure is put into a vacuum chamber with its inside pressure kept at $10^{-8}$ Torr using an ion pump during the high power test. A 4.4-dB hybrid between the klystron and the test stand prevents the power from reflecting back to the klystron. Besides, an rf window is used to isolate the vacuum system between the klystron and the test stand. The peak power allowed by the rf window is $\sim 10$ MW. Given the 4.4-dB attenuation from the hybrid, the maximum rf power transmitted to the SW test stand is 3.63 MW. The schematic power flow is shown in Figure 5-12. Microwave power is coupled into the HPBG structure through the mode launcher connected to the waveguide mounted on the top of the SW test stand, as shown in Figure 5-13.

5.3.3 Diagnostics

Figure 5-14 schematically shows the diagnostics used for the breakdown experiment of the HPBG structure. For the breakdown experiment, both the incident (forward) power to the HPBG structure and the reflected (reverse) power from the structure must be detected. The gradient coupled in the HPBG structure can be calculated from the forward power. The reverse signal is used to check if the microwave frequency...
is at the right resonant frequency. Due to the critical coupling of the HPBG structure, there will be little reflection if the frequency is at resonance. A single high-directivity waveguide directional coupler is connected to the waveguide to separate the forward and the reverse microwave power. The connection is located close to the SW test stand to ensure accurate detection. Each of the two arms of the directional coupler imposes a 65-dB attenuation on the power. Through the directional coupler, each of the forward and the reflected microwave power is detected by a Hewlett Packard 8473B low-barrier Schottky diode individually. Both of the two voltage signals generated by the two diodes are coupled into an oscilloscope. For the HPBG experiment, an Agilent Infiniium DSO-S 404A High-Definition Oscilloscope is employed.

Two Faraday cups, made of OFHC copper, are attached to the HPBG structure to monitor the dark current generated by each input shot of the microwave. The downstream Faraday cup is bolted to the output flange of the HPBG structure. The upstream Faraday cup is bolted to the other end of the mode launcher. Both Faraday cups are electrically isolated from the HPBG structure and grounded to the body of the vacuum chamber. Each Faraday cup is connected to a BNC cable with a 50-Ω load to couple the current signal to two channels on the same oscilloscope. During the experiment, the downstream Faraday cup functioned properly. However, a lot of noise on the upstream Faraday cup led to a less useful signal. The timing information of a breakdown with respect to the incident power can be extracted from the dark current signal.
Figure 5-14: Diagnostics used for the high power testing of the HPBG structure.

The open PBG cavity makes a direct optical diagnostic possible. A video camera connected to a TV is used to visually observe the light flash generated in the HPBG cell in real-time. A Thorlabs 8050M-GE 8 Megapixel Monochrome Scientific charge-coupled device (CCD) Camera [87] is used to captured the light in the HPBG cell. Figure 5-15 shows the setup of the two cameras. Figure 5-16a shows a picture captured during a flash in the central HPBG cell from the TV connected to the video camera and Figure 5-16b shows a photo taken by the CCD camera with only ambient light,
i.e., no light emitted by the HPBG structure. Irises can be spotted by the CCD. The view of the CCD camera is turned by 90° with respect to the setup of the HPBG structure, resulting in a horizontal central cell in the CCD view and easily rotate images to align with experimental setup. For the experiments of the second and the third HPBG structure discussed later, a Thorlab silicon Avalanche Photodiode APD130A2 [87] is attached to the window of the SW test stand to measure the amount of light emitted by the structure for every shot, as shown in Figure 5-14.

If there is a breakdown event during the rf shot, three changes of the foregoing diagnostics can be observed: a sudden rise of the reflected rf power; a much higher dark current on both Faraday cup channels; and a bright light seen by both the video camera and the CCD camera.

5.4 Data Analysis

The methodology to analyze the experimental data of the high power testing for the HPBG structure follows a similar procedure as those used for the tests of the preceding MPBG structures at SLAC and MIT [15]. The power coupled in the PBG structure can be obtained from the incident (forward) power signal. Comparing with the HFSS simulation, the gradient, the surface electric and magnetic fields, and the temperature rise can be calculated. Breakdown events can be determined by the reverse power
signal and the dark current signal. Combining with the gradient information, the breakdown rate (BDR), or the breakdown probability at a specific gradient can be calculated.

For our experimental setup, the incident power $P_{in}(t)$ can be calculated from the diode signal using the calibration formula of the Schottky diode combining with the total attenuation added. To show the process, we assume an artificial incident power with the shape shown in Figure 5-17. In Figure 5-17, the calculated structure power

![Figure 5-17: The artificial incident power trace, the calculated structure power and the gradient in the HPBG cell.](image)

and the gradient are shown, too. The procedure will be described in this section.

The PBG structure is actually a damped resonator with a quality factor $Q_L$. The first flat top in the incident power has a higher power to make the coupled power in the structure rise up to reach a steady state. Thus a flat top of the structure power can be maintained during the second top of the input power. We assume the incident electric field has a form $E_{in}(t) = E_0(t)e^{i\omega t}$, where the magnitude $E_0(t) = \sqrt{P_{in}(t)}$. The field response in the HPBG structure follows the damped oscillation equation as Eq. 5.7:

$$\frac{d^2 E(t)}{dt^2} + \frac{\omega_0}{Q_L} \frac{dE(t)}{dt} + \omega_0^2 E(t) = E_0 e^{i\omega t}. \quad (5.7)$$
The steady state solution reads:

\[ E(t) = A e^{i\omega t}, \quad (5.8) \]

\[ A = \frac{E_0}{\omega_0^2 - \omega^2 + \frac{i\omega Q_L}{Q_L}}. \quad (5.9) \]

The field amplitude has a maximum of \( |A|_{\text{max}} = E_0 Q_L / \omega_0^2 \) at \( \omega = \omega_0 \). For our experiment, the frequency is always kept at resonance. Because the input pulse is finite in time, the spectrum of the input electric field has components of other frequencies. The response to other frequencies, \( A(f) \), is a fraction of the maximum amplitude \( |A|_{\text{max}} \). Replacing \( \omega \) with the frequency \( f \) in Eq. 5.9, a normalized field response as a function of \( f \) can be obtained:

\[ A_{\text{norm}} = \frac{A(f)}{|A|_{\text{max}}} = \frac{f_0^2}{Q_L f_0^2 - f^2 + \frac{i\omega Q_L}{Q_L}}. \quad (5.10) \]

Using Fourier Transform, we can obtain the spectrum of the input field \( \tilde{E}_{\text{in}}(f) \) which can be convolved with the frequency response \( A_{\text{norm}} \) to calculate the field coupled in the structure. However, since the effective IF bandwidth (4-6 GHz) of the oscilloscopes is lower than the resonant frequency around 17 GHz, to calculate the field response we have to shift both the spectrum of the input power and the frequency response in the frequency domain to be centered at a frequency lower than or equal to the IF frequency \( f_C \). To validate the manipulation, we have to keep the entire spectrum shifted in parallel, i.e., to use \( f_{\text{shift}} = f - f_C \) in Eq. 5.10. Figure 5-18 shows the shifted spectrum of the artificial signal and the normalized frequency response. The convolution in the time domain is a multiplication in the frequency domain, as shown in Eq. 5.11:

\[ \tilde{E}_{\text{str}}(f) = \tilde{E}_{\text{in}}(f) \times A_{\text{norm}}(f). \quad (5.11) \]

The time-domain response of the coupled electric field in the cavity \( E_{\text{str}}(t) \) can be calculated by reversing Fourier transforming \( \tilde{E}_{\text{str}} \). The structure power then is obtained:

\[ P_{\text{str}}(t) = |E_{\text{str}}(t)|^2. \quad (5.12) \]
Calibrating with the simulation results in Chapter 4, the gradient in the central HPBG cell is obtained, as shown in Figure 5-17. The pulse length for a steady-state flat top, the average power and the average gradient can be extracted from the structure power trace and the gradient trace. The surface electric and magnetic fields can also be calculated from the structure power. Thus the temperature rise at the highest $H$ field spot is evaluated using Eq. 2.23. The entire analysis procedure is done by a series of scripts written in MATLAB [88].
Chapter 6

Breakdown Experiment of the HPBG-d3 Structure at 17 GHz: Results

The first high power tests of the HPBG-d3 structure lasted for about one month, including 458,000 high power shots in total. Figure 6-1 shows the summarized results of the breakdown experiment of the HPBG-d3 structure at 17 GHz at MIT, with details to be discussed in this chapter.

6.1 Initial Phase (Phase 1)

We processed the structure in Phase 1, starting at a low gradient level of \( \sim 1 \) MV/m. Input shots with long pulse length (200-300 ns) were used to determine the resonant frequency at the beginning stage. After that, we gradually increased the input power, neglecting the large dark current (DC) and the high breakdown rate, until the gradient reached \( \sim 14 \) MV/m. The pulse length of the input power was kept at 100 ns for all the input shots. Breakdown started to happen on every shot at a gradient \( E_C \geq 3 \) MV/m. It was not useful to calculate the breakdown probability during the processing phase.
Figure 6-1: Summarized results of the high power tests of the HPBG-d3 structure of (a) gradient (blue) and the flat-top length of the structure power (red), and (b) gradient (blue) and the magnitude of dark current from the downstream Faraday cup (green). The black circles at the top of each figure indicate each end of data for one day’s run, with each date detailed around in (b). Gradient is calculated only in the central HPBG cell by the forward power.
6.1.1 The Histories

Figure 6-2 shows during processing the typical traces of the forward power, the reverse power, the calculated structure power and the dark current from the downstream Faraday cup in Phase 1. Here the value of the reverse trace is not accurate due to the lack of calibration on either the diode or the attenuator added or removed. Only the shape was used to determine the resonant frequency and breakdown emergence. This applies to all the reverse traces described in this thesis. The structure power was calculated only by the forward power, without consideration of the absorption by multipactor. In Figure 6-2, the shape of the structure power calculated by Eq. 5.11 is close to triangular and does not have a flat top. This came from the short pulse length.
(100 ns) of the input power. We estimated the structure power as equal to 75% of the maximum structure power, which yielded a ~ 40-ns flat top of the structure power. Gradient was averaged over this 40-ns flat top. The dark current trace was the signal of the downstream Faraday cup, shown in Figure 5-14. The shape of the dark current in Phase 1 was a single high spike regardless of whether breakdown occurred or not. Because of the relatively large dark current and lack of the shape variation for every shot, breakdown was not distinguishable from the rise of the dark current used in the high power tests of the metallic PBG (MPBG) structures at both SLAC and MIT. Breakdown was detected from a sudden reflection of the reverse power. When there was a breakdown, the reverse power trace also coincided with the dark current trace for most of the shots, which meant when the peak of the dark current moved inward, especially inside the input power, breakdown happened.

6.1.2 Dark Current and Light Emission

Figure 6-3 plots the the dark current peak and the delay time between the peak and the start of the input power as a function of gradient during one day of test (Jan. 14), showing a proportionality between the dark current and the gradient level. The dark current always rose at a delay time after the input power began, as shown in the traces in Figure 6-2. The delay time decreases monotonically as gradient. For each specific gradient, the dark current peak spreads over a range of a few mA and the delay time spreads over a range of 10-20 ns. The range of the delay times is
about 50 – 80 ns, which is consistent with Neuber’s observation for the buildup of multipactor and electron avalanche [12].

In the preceding tests of the metallic PBG structures tested at MIT, the dark current was in a range of 0.1 – 1.0 mA among a gradient range of 20 – 60 MV/m (for the MPBG-MIT-1 structure, referred to as MIT-PBG-1 in [15]). The dark current of the HPBG-d3 structure was much higher at a lower gradient level. Two sources may be responsible: (1) the high surface $E$ field ratio, as compared in Table 4.9, and (2) the triple point to enhance the surface field of the HPBG-d3 structure, which is to be discussed in Subsection 6.1.3.

For the MPBG structure, the surface $E$ field peaked at the irises, where the breakdown events were triggered. Figure 6-4 shows the CCD images of the central cell of the MPBG-MIT-2 structure (referred to as MIT-PBG-2 in [15]) for comparison between the nonbreakdown and breakdown pulses. Figure 6-4a is the nonbreakdown pulse with only background light and the central black area representing one iris. If there is a breakdown event, a spark on the iris is captured by the CCD camera as shown in Figure 6-4b, which is exactly as expected.

However, the highest surface electric field of the HPBG-d3 structure was at the triple-point edge on Rod 2 identified in Figure 4-5a. Figure 6-5a shows the central hybrid cell of the HPBG-d3 structure at a lower gradient. There is only background light shining the copper rods. At a higher gradient level ($E_G > 5.7$ MV/m), on every shot with or without breakdowns, there was light emitted from the sapphire rods, as shown in Figure 6-5b. This was an evidence of plasma formation along the
Figure 6-5: CCD images of the central cell of the HPBG-d3 structure, showing when $E_G > 5.7$ MV/m, even without breakdown, there is light emission on the sapphire rods. That meant there was multipactor triggered on the dielectric surface which caused consequent procedures, such as electron avalanche, plasma formation, surface material outgassing and eventually breakdown, as described in Section 2.3.5. The resolution of the CCD camera could not pin-point the exact location of the light emission. Based on the HFSS simulation, it is most likely the brightest rod was the one with the highest surface electric field (Rod 2 in Figure 4-5a).

### 6.1.3 Triple Point Analysis

In the HPBG-d3 structure, there are three cases of the triple-point interface, with their schematic 2D geometries shown in Figure 6-6. The angles are defined according to Figure 2-11 in Section 2.3.4. For case (a) normal insertion, the angle subtended by the dielectric is $\theta_d = \pi$; the angle subtended by the metal is $\theta_m = \alpha = \pi/2$, $\theta = 0$. 

Figure 6-6: Three cases of the triple point in the HPBG-d3 structure.
Eq. 2.40 reads:
\[
\varepsilon_r \cot(\nu \pi) \tan\left(\frac{\nu \pi}{2}\right) = -1.
\] (6.1)

Using double-angle formulae, solving Eq. 6.1 yields:
\[
\frac{1}{2} \left[ 1 - \tan^2\left(\frac{\nu \pi}{2}\right) \right] = -\frac{1}{\varepsilon_r}.
\] (6.2)

Setting \(\varepsilon_r = 11.587\), solving Eq. 6.5 gives:
\[
\tan\left(\frac{\nu \pi}{2}\right) = \sqrt{1 + \frac{2}{\varepsilon_r}},
\] (6.3)
\[
\nu = 0.5253.
\] (6.4)

From Eq. 2.43, the field index is:
\[
\delta = \nu - 1 = -0.4747 < 0,
\] (6.5)

which generates a divergent field at the triple point.

For case (b), with a fillet on Rod 1, the triple-point angle \(\theta\) is negative, approaching \(-\pi/2\). At the triple point it yields \(\theta = -\pi/2, p_d = \pi\) and \(p_m = \alpha = \pi\). Eq. 2.40 yields Eq. 6.1 again. In addition, with the negative \(\theta\), the triple point is more susceptible to multipactor. For case (c), it requires to solve the transcendental equation Eq. 2.40. Since the edge of the fillets imprinted on Rod 2 and Rod 3 yield different angles, it is complicated to determine the exact value of \(\theta\). However the imprint of Rod 2 and Rod 3 include the normal insertion case, which already, gives a divergent field at the triple point. Therefore, a huge enhancement of the field is presented at the triple point edge on the inner rows of rods.

6.1.4 Phase 1: Summary

Phase 1 was the conditioning phase including 222,000 shots. During Phase 1, the highest gradient was 14.4 MV/m, corresponding to a surface \(E\) field of 95.9 MV/m. The temperature rise was low, around a few K to \(\sim 14\) K. The highest temperature
rise was 13.6 K.

6.2 Phase 2

The HPBG-d3 structure was greatly processed after ~222,000 of high power shots, indicated by much lower dark currents at the same gradient as in Phase 1. We called the after-processing phase Phase 2, including 236,000 shots. For the first three days’ operation, the pulse length of the input power was 100 ns, resulting in a 40-ns flat top of the structure power in the HPBG-d3 structure. Later, longer input pulses, of 160 ns and 210 ns, were fed into the HPBG-d3 structure, resulting in real flat tops of 80 ns and 130 ns of the structure power, respectively.

6.2.1 Traces and Dark Current

In Phase 2, for each new gradient level, the structure needed ~ 10⁴ shots to be re-conditioned. During the reprocessing, as shown in Figure 6-7, the dark current decreased gradually. The reduction of the peak value was accompanied by a change of the dark current shape, from a single narrow negative peak similar to those seen in Phase 1, to a final shape with a leading edge of a narrow negative peak and a wider main part in time scale. Figure 6-8 shows the typical traces with the dark current shape change. The peak value of the dark current, even during reprocessing, was much lower than in Phase 1. After reprocessing, the peak value of the dark current

![Figure 6-7: Reprocessing in Phase 2, during Shot 222,000 and 300,000.](image-url)
Figure 6-8: Typical traces of the forward power, the reverse power, the calculated structure power and the dark current from the downstream Faraday cup of (a) Shot 284,238 (Jan. 23), with the average gradient $E_G = 4.92$ MV/m, during reprocessing, (b) Shot 293,238 (Jan. 23), with $E_G = 5.12$ MV/m, after reprocessing, no breakdown, and (c) Shot 294,328 (Jan. 23), with $E_G = 5.01$ MV/m, after reprocessing, breakdown.
stayed at a level of an order of magnitude lower than in Phase 1 or at the beginning of the reprocessing. The structure came to a quasi-steady state with a constant breakdown probability. Breakdown was determined when a reflection was observed on the reverse. Compare Figure 6-8c with Figure 6-8b, where, for a breakdown, a lower dark current appears; this is in complete contrast with what happened to the MPBG structures tested at MIT or SLAC. This may be caused by breakdown truncating the multipactor progress, resulting in a lower dark current.

Figure 6-9 shows traces for a longer (160 ns) input pulse, resulted in a real 80-ns flat top of the calculated structure power. Three peaks of the dark current occurred in

![Figure 6-9: Typical traces of the forward power, the reverse power, the calculated structure power and the dark current from the downstream Faraday cup of (a) Shot 434,578 (Jan. 26), with $E_G = 8.55$ MV/m, no breakdown, and (b) Shot 434,628 (Jan. 26), with $E_G = 8.33$ MV/m, breakdown. The pulse length of the structure power was 80 ns.](image)

some traces. Breakdown happened when the main part of the dark current overlapped
in time with the input power. However, opposite to the shorter pulses (100 ns), the dark current for breakdown was higher for this longer pulse.

Figure 6-10 plots the magnitude of dark current after reprocessing as a function of gradient. The absolute peak value of the dark current was much lower than in Phase 1 and was still proportional to gradient.

### 6.3 Breakdown Probability

The breakdown probability was calculated for each gradient after reprocessing. The detailed breakdown probabilities in Phase 2 are summarized in Table 6.1 and plotted as a function of gradient in Figure 6-11. The error bar of the breakdown probability was statistically calculated from the Poisson distribution. For $E_G = 5.7$ MV/m, for

![Figure 6-10: The absolute value of the dark current peak as a function of gradient after reprocessing.](image)

<table>
<thead>
<tr>
<th>Date</th>
<th>Structure power</th>
<th>Gradient (MV/m)</th>
<th>BDR (/pulse/m)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$H_{surf}$ (kA/m)</th>
<th>$\Delta T$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 23</td>
<td>40</td>
<td>9.0</td>
<td>$2.91 \times 10^{-1}$</td>
<td>59.8</td>
<td>218.5</td>
<td>5.1</td>
</tr>
<tr>
<td>Jan. 23</td>
<td>40</td>
<td>5.2</td>
<td>2.03</td>
<td>34.3</td>
<td>125.4</td>
<td>1.7</td>
</tr>
<tr>
<td>Jan. 25</td>
<td>80</td>
<td>6.8</td>
<td>$4.23 \times 10^{-2}$</td>
<td>44.0</td>
<td>160.7</td>
<td>3.8</td>
</tr>
<tr>
<td>Jan. 25</td>
<td>80</td>
<td>5.7</td>
<td>$1.33 \times 10^{-2}$</td>
<td>36.9</td>
<td>134.8</td>
<td>2.7</td>
</tr>
<tr>
<td>Jan. 26</td>
<td>80</td>
<td>8.8</td>
<td>$1.28 \times 10^{-1}$</td>
<td>56.8</td>
<td>207.7</td>
<td>6.3</td>
</tr>
<tr>
<td>Jan. 26</td>
<td>130</td>
<td>8.7</td>
<td>$2.03 \times 10^{1}$</td>
<td>57.9</td>
<td>211.6</td>
<td>7.9</td>
</tr>
<tr>
<td>Jan. 26</td>
<td>130</td>
<td>7.4</td>
<td>$2.03 \times 10^{1}$</td>
<td>48.1</td>
<td>175.9</td>
<td>5.6</td>
</tr>
</tbody>
</table>

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two hours' running of about $8.6 \times 10^3$ shots, no breakdown happened. Thus a single breakdown event was assigned, resulting in an error bar of the same value, which was indicated by an arrow going down to zero in the log scale plot, as shown in Figure 6-11.

![Figure 6-11: Breakdown probability as a function of gradient for the HPBG-d3 structure.](image)

For the 40-ns structure power, the rate went down with more input pulses, indicating the structure was experiencing more conditioning. The highest gradient was 9.0 MV/m with a breakdown probability of $2.91 \times 10^{-1}$ per pulse per meter; we describe the breakdown probability in unit length to take into account of the length of a real accelerator. The highest gradient achieved in a 80-ns structure power was 8.8 MV/m, with a breakdown probability of $1.28 \times 10^{-1}$ per pulse per meter. Increasing the gradient to 9.6 MV/m resulted in a runaway condition with chains of breakdowns, an order of magnitude higher dark currents, and no sign of conditioning after $\sim 2.7 \times 10^4$ input shots. With the 130-ns structure power, the breakdown rate at a gradient of 8.7 MV/m was $2 \times 10^1$, two order of magnitude higher than the 80-ns case. This might be caused by the higher probability to trigger breakdown with longer time to
build up multipactor.

The highest surface electric and magnetic fields and the highest temperature rise for each shot are plotted in Figure 6-12, including both phases.

![Figure 6-12: The highest surface electric (blue) and magnetic (red) fields and the highest temperature rise (green) for each shot in the high power tests of the HPBG-d3 structure.](image)

6.4 Post-test

After the high power testing, the HPBG-d3 structure was removed from the vacuum chamber for damage inspection.

6.4.1 Post-test Cold Test

The post-test cold test results of the $S_{11}$ measurement are shown in Figure 6-13 and Table 6.2, with comparison to the structure before the high power testing and the structure with the same metallic plates but a new set of sapphire rods. An asymmetric deformation was observed for the magnitude of $S_{11}$ in Figure 6-13a, corresponding to a distortion on the Smith chart shown in Figure 6-13b. A small reduction of $Q_L$ was
Figure 6-13: Cold test results after high power experiment including (a) the magnitude, and (b) the Smith chart of $S_{11}$, and (c) the axial field profile of the HPBG-d3 structure. The $S_{11}$ magnitude is compared with the cold test before the high power testing and the same rods plates with new sapphire rods.

measured, too, as shown in Table 6.2. Changing to a new set of sapphire rods led to a higher $Q_L$, indicating that the measured change in $Q_L$ was within our experimental measurement accuracy and was therefore the result of damage to the sapphire rods.

The bead pull measurement is shown in Figure 6-13c.

Table 6.2: Comparison of the resonant frequency $f_0$ and the quality factor $Q_L$ of the HPBG-d3 structure before and after the high power testing.

<table>
<thead>
<tr>
<th></th>
<th>$f_0$ (GHz)</th>
<th>$S_{11}$ (dB)</th>
<th>$Q_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Test</td>
<td>17.147</td>
<td>-37</td>
<td>2150</td>
</tr>
<tr>
<td>After Test</td>
<td>17.149</td>
<td>-23</td>
<td>2010</td>
</tr>
<tr>
<td>After, with New Sapphire Rods</td>
<td>17.141</td>
<td>-24</td>
<td>2170</td>
</tr>
</tbody>
</table>
6.4.2 Autopsy

Due to the clamped design, the structure was disassembled without suffering any additional damage. A Canon PC1234 camera and a Dino-Lite Pro optical microscope [89] were used to investigate the structure.

For the iris surface of the input coupling cell and the upstream plate of the HPBG cell, no obvious damage was observed. However, shown in Figure 6-14 were some black marks observed on the iris of the downstream central plate.

Figure 6-14: The microscope image (50X) of the iris on the downstream central plate with some black marks.

Figure 6-15 shows the locations of breakdown damage observed on the plate and the sapphire rods of the HPBG-d3 structure. Simulations gave the highest field occurring at the triple-point edge of Location 1 in Figure 6-15, where we discovered the most significant damage. Discoloration happened on the edge of the hole on the downstream copper plate at Location 1, as shown in Figures 6-16b and 6-16c. Metallization and a more transparent erosion belt occurred along the triple-point interface on the rod, as shown in Figures 6-16d and 6-16e. This damage was the only
obvious breakdown spot. Similar marks were not found on the other five analogous holes or rods.

![Image](image.png)

(a) Breakdown damage

(b) Plate, 50X

(c) Plate, 230X

(d) Rod, 50X

(e) Rod, 230X

Figure 6-16: Breakdown damage at the triple-point edge with microscope images of the damaged plate and the sapphire rod.

The sapphire rod at Location 2 had a copper line surrounding the surface, as shown in Figure 6-17a. The sapphire rod on Location 3 had a more transparent erosion belt along the triple-point edge; corresponding black marks on the edge of the hole were observed on the copper plate, as shown in Figure 6-17b.

A number of rods were metallized with spotty copper on the surface exposed to high power microwaves, with an example shown in Figure 6-18. The deposition of copper onto the sapphire surface might eventually change the cavity quality.
6.5 Conclusion

The HPBG-d3 structure was put into high power test at MIT at 17 GHz for about 460,000 microwave shots. The highest gradient achievable in a 40-ns flat-top length of the structure power was 9.0 MV/m, with a breakdown probability of $2.91 \times 10^{-1}$ per pulse per meter. The highest gradient achievable in an 80-ns flat-top length of the structure power was 8.7 MV/m, with a breakdown probability of $1.28 \times 10^{-1}$ per pulse per meter. This gradient corresponded to a highest surface electric field of 57.9 MV/m and a highest surface magnetic field of 212 kA/m. A long-time rf conditioning was required to process the structure which greatly reduced the magnitude of the dark current. Comparing with the preceding metallic PBG structures tested at MIT at Ku band, the magnitude of the dark current was much higher in the HPBG-d3 structure. Light emission was observed on every shot with a gradient higher than 5.7 MV/m, indicating the occurrence of multipactor, and plasma formation.

A reduction of the quality factor $Q$ was measured in the cold test after the high power testing. Post-test images showed damage happening at one of the triple points where the highest surface electric field was located. Deposition of copper onto the surface of the sapphire rods was observed, which might potentially change the rf
properties of the accelerator structure. The design of the HPBG-d3 structure gave a much higher surface electromagnetic field than seen in the metallic PBG structures. Results from a new design, with lower surface fields, aimed at improving the high power performance of the HPBG structure, will be shown in the next chapter.
Chapter 7

Design and Experimental Results of the HPBG-d025 Structure

The design in Chapter 4 showed a significant field enhancement at the triple point due to the long insertion of the sapphire rods into the copper plates. Simulations indicate that with shorter insertion length, the surface fields should be lower. $d = 0$ mm returns an ideal case with $E_{\text{surf}} = 230$ MV/m and $H_{\text{surf}} = 590$ kA/m at $E_G = 100$ MV/m, but is not practical. To approach this case, a new design of the hybrid PBG structure using much shorter sapphire rods will be described in this chapter.

7.1 Design

To lower the fields at the triple point, shorter rods, with insertion length $d = 0.25$ mm were employed on the inner two rows for a new design. To assemble this structure more easily, the third row of sapphire rods was kept with $d = 3$ mm. This new design will be referred to as the HPBG-d025 structure. Simulations with the lattice of $a = 1.58$ mm and $b = 4.49$ mm gave a resonance at the right frequency. The periodic single cell HFSS simulation required 746,000 tetrahedra to limit the frequency uncertainty to within 0.04%. The results are listed in Table 7.1. For the single cell simulation, the ratio of the highest surface field to the gradient was lowered to $E_{\text{surf}}/E_G = 3.8$, which was a significant improvement comparing with the HPBG-d3 structure.
Table 7.1: HFSS results of the HPBG-d025 structure of the periodic single cell simulation.

<table>
<thead>
<tr>
<th>$f_0$ (GHz)</th>
<th>$Q_0$</th>
<th>$E_G$ (MV/m)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$I_{surf}$ (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.2338</td>
<td>6230</td>
<td>100</td>
<td>385</td>
<td>1108</td>
</tr>
</tbody>
</table>

For the HFSS simulation of the standing-wave (SW) model, due to the higher $Q$ of the HPBG-d025 structure, a smaller coupling aperture was needed to better couple power into the central cell. As shown in Figure 4-3, choosing $r_p = 4.17$ mm and $b = 4.49$ mm while leaving the rest of the parameters the same as in Table 4.1 returned a good coupling in the 3D simulation. The SW simulation required 1,049,000 tetrahedra to give a result with a frequency uncertainty of $\sim 0.03\%$, with the results shown in Figure 7-1 and Table 7.2.

![Figure 7-1](image)

(a) The magnitude of $S_{11}$  
(b) The Smith chart of $S_{11}$

Figure 7-1: (a) The magnitude, and (b) the Smith chart of $S_{11}$ of the SW, HPBG-d025 structure simulated in HFSS.

Table 7.2: Resonant frequency and $Q$ of the SW model of the HPBG-d025 structure in HFSS simulation.

<table>
<thead>
<tr>
<th>$f_0$ (GHz)</th>
<th>$S_{11}$ (dB)</th>
<th>$Q_L$</th>
<th>$\beta$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.1366</td>
<td>-39</td>
<td>3120</td>
<td>$\sim 1$</td>
<td>$6.2 \times 10^3$</td>
</tr>
</tbody>
</table>

Calibrating the input power with fields on irises, the input power required for $E_G = 100$ MV/m was $P_{in} = 6.5$ MW and the shunt impedance was $R_s = E_G^2 L / P_{in} \approx$
13.4 MΩ/m. Figure 7-2 shows the field distribution on the surface of the HPBG-d025 structure, with the peak values listed in Table 7.3.

![E Field](image)

(a) E field

![H Field](image)

(b) H field

Figure 7-2: The simulated surface electric and magnetic fields of the SW model of the HPBG-d025 structure.

Table 7.3: Simulated peak surface fields of the SW, HPBG-d025 structure.

<table>
<thead>
<tr>
<th>Structure</th>
<th>$E_G$ (MV/m)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$H_{surf}$ (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPBG-d025</td>
<td>100</td>
<td>434</td>
<td>1335</td>
</tr>
</tbody>
</table>

### 7.2 Experimental Results of Cold Test

The sapphire rods were produced by Insaco, Inc [16]. All the metallic parts were machined by the MIT Central Machine Shop. Figure 7-3 shows the rod plate and the rods inserted of the HPBG-d025 structure.

Figures 7-4 shows the resonance and the accelerating field profile of the cold test measurement, with results listed in Table 7.4. The axial field distribution agreed well with the HFSS simulation.
Figure 7-3: (a) The rod plate, and (b) the plate with rods inserted of the HPBG-d025 structure.

Figure 7-4: Cold test results of (a) the magnitude of $S_{11}$, (b) the Smith chart of $S_{11}$, and (c) the axial $E$ field profile of the HPBG-d025 structure.
Table 7.4: Cold test results of the HPBG-d025 structure.

| $f_0$ (GHz) | $S_{11}$ (dB) from $|S_{11}|$ | $Q_L$ | $Q_L$ from the Smith chart | $Q_0$ | $Q_{ext}$ | $\beta$ |
|------------|-----------------------------|------|-----------------------------|------|-----------|-------|
| 17.140     | -26                         | 2980 | 3840                        | 6770 | 8070      | 0.84  |

7.3 Experimental Results: High Power Tests

The high power tests of the HPBG-d025 structure lasted for about two weeks in June 2015; in total $\sim 212,000$ shots were fed into the structure at 17 GHz. Figure 7-5 shows the summarized results. Again, gradient is calculated only in the high power HPBG cell by the forward power.

7.3.1 Traces

The first few thousand shots were set at a low power level in different pulse lengths to find the resonant frequency; thus no gradient was calculated for those shots. After determination of the resonance, we began to increase the power gradually. Before processing, at a very low input power, we saw some dark current rising abruptly with a much higher negative peak, usually after the input power. The reverse power did not perform any change. The comparison of two traces with low and high dark currents is shown in Figure 7-6. The same as in Chapter 6, the absolute value of the reverse power was not accurate; only the shape was used to determine the resonance and breakdown.

In order to process the structure efficiently, we started to raise the input power to very high levels ($E_G \sim 15$ MV/m and $E_G \sim 20$ MV/m) after only a few ten thousand shots. After the rapid processing, the dark current, comparing with the former results of the same gradient level, was much smaller. Therefore the structure had been conditioned by this brutal process. After being bombarded by a few thousand shots at gradients over 20 MV/m, the HPBG-d025 structure exhibited a much lower dark current than before. Therefore we were able to go to a higher gradient with a lower dark current. Figure 7-7a shows a typical pulse after processing at $E_G = 8.24$
Figure 7-5: Summarized results of the high power tests of the HPBG-d025 structure for (a) gradient (blue) and the flat-top length of the structure power (red), and (b) gradient (blue) and the magnitude of dark current (green) from the downstream Faraday cup. The black circles at the top of each figure indicate each end of data for one day’s run, with each date detailed in (b). Gradient is calculated only in the HPBG cell by the forward power.
Figure 7-6: During processing, typical traces of the forward power, the reverse power, the calculated structure power and the dark current from the downstream Faraday cup of (a) Shot 88,633 (Jun. 12), with the average gradient \( E_G = 3.73 \text{ MV/m} \), no breakdown, low dark current, and (b) Shot 88563 (Jun. 12), with \( E_G = 3.78 \text{ MV/m} \), no breakdown, high dark current.

MV/m without breakdown. In the high power testing of the HPBG-d025 structure, a photodiode was attached to the window of the test-stand chamber to detect the light emitted by the structure. The photodiode signal indicated relatively the amount of light emitted during each shot. After processing, the dark current had a few narrow peaks in the leading edge and followed by a negative main part, as in Figure 7-7a.

Raising the input power gradually, the resonance looked worse, indicated from the reverse trace with a smaller second rabbit ear, as shown in Figure 7-7b. Surprisingly, at gradients higher than \( \sim 12 \text{ MV/m} \), the main part of the dark current became positive, as shown in Figure 7-7b. However, if breakdown happened, it became negative again and a significant rise on the photodiode occurred, as shown in Figure 7-7c. The details of the dark current will be discussed in the next subsection.

After the rapid processing, the breakdown probability (or the breakdown rate
Figure 7-7: After processing, typical traces of the forward power, the reverse power, the calculated structure power, the dark current from the downstream Faraday cup and the photodiode signal of (a) Shot 130,329 (Jun. 13), with the average gradient $E_G = 8.24$ MV/m, no breakdown, the low dark current has a leading edge and a negative main part, low photodiode signal; (b) Shot 136,907 (Jun. 13), $E_G = 17.7$ MV/m, no breakdown, low dark current but a positive main part, low photodiode signal; (c) Shot 135,455 (Jun. 13), $E_G = 20.4$ MV/m, breakdown, high dark current with a negative main part, high photodiode signal.
(BDR)) of two gradient levels were calculated. The input power level was raised again to the extent with every shot triggering a breakdown. After about 7,500 shots of such breakdowns, suddenly, a runaway situation happened which saturated the photodiode and emitted much more light with a much brighter picture on the CCD camera. The pressure rise of the ion pump connecting to the vacuum chamber was detected, indicating the local outgassing buildup. Figures 7-8a and 7-8b compare the traces between a normal breakdown and the runaway situation. The saturation and

![Graph](image)

(a) $E_G = 20.4$ MV/m, breakdown

(b) $E_G = 20.3$ MV/m, breakdown, runaway, photodiode saturated

Figure 7-8: Traces of the forward power, the reverse power, the calculated structure power, the dark current from the downstream Faraday cup and the photodiode signal of (a) Shot 202,982 (Jun. 16), $E_G = 20.4$ MV/m, normal breakdown, high dark current and high but unsaturated photodiode signal; and (b) Shot 203,175 (Jun. 16), $E_G = 20.3$ MV/m, runaway situation, relatively low dark current, photodiode saturated with a high flat top.

breakdown continued and the structure showed no recovery even at lower gradients, indicating the permanent destruction of the structure. The high power testing stopped
after the runaway situation.

### 7.3.2 Dark Current and Light Emission

Despite some breakdown events, the amplitude of the dark current was proportional to gradient, for both the leading part and the main part, as shown in Figure 7-9a. The sign change of the dark current main part happened at gradients higher than $E_G \sim 12.2$ MV/m. Shown in Figure 7-9b is the delay time between the peak of the dark current and the start of the input power as a function of gradient. It included both the leading part and the main part. At $E_G < 10$ MV/m, the delay time decreases monotonically to gradient. However, at higher gradients, the delay time shows little variance. It is possible if the gradient exceeds a particular value, increasing gradient does not help facilitate multipactor.

Figure 7-10 plots the amplitude of the photodiode signal as a function of gradient, neglecting all breakdown pulses. The photodiode amplitude has a parabolic dependence on gradient. O. V. Sinitsyn calculated the multipactor induced power loss and demonstrated a squared dependence of the loss power at saturation on the rf amplitude [9], as shown in Figure 2-9. $E_G$ is proportional to the rf amplitude. The voltage of the photodiode is proportional to the power of the light [87]. That means Figure 7-10 agrees well with the calculation by Sinitsyn.

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![Figure 7-9: (a) The peak value of dark current, and (b) the delay time between the DC peak and the start of the input power as a function of gradient, including both the leading part and the main part, during one day of test on Jun. 13th.](image-url)

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Figure 7-10: The amplitude of the photodiode signal, in mA, versus gradient, in MV/m, during one day of testing on Jun. 13th.

Figure 7-11 displays the central HPBG cell for four situations: (a) a clear view of the hybrid cell, (b) operation at low gradients, no light emitted, the stray light view showing only the copper rods, (c) normal light emission, a few sapphire rods emitting light, and (d) the runaway situation with more rods emitting extensive light. For the HPBG-d025 structure, above $E_G = 7.4$ MV/m, light can be detected by the CCD camera for every shot. Recalling for the HPBG-d3 structure, light was visualized by the CCD camera when $E_G > 5.7$ MV/m. The gradient is $E_{G,d025}/E_{G,d3} \approx 1.3$. The inverse ratio of the highest surface $E$ field is $E_{surf,d3}/E_{surf,d025} = 628/434 \approx 1.4$. They are comparable. Again, light emission at gradients below the breakdown threshold
indicated multipactor triggering and low level of plasma formation on the sapphire rods.

### 7.3.3 Breakdown Probability

The detailed breakdown probabilities of the HPBG-d025 structure are summarized in Table 7.5 and Figure 7-12, plotted as a function of gradient. At $E_G = 11.8$ MV/m,

Table 7.5: Breakdown probabilities of the HPBG-d025 structure.

<table>
<thead>
<tr>
<th>Date</th>
<th>Structure Power</th>
<th>Gradient</th>
<th>BDR</th>
<th>$E_{surf}$</th>
<th>$H_{surf}$</th>
<th>$\Delta T$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>flat-top length (ns)</td>
<td>(MV/m)</td>
<td>(/pulse/m)</td>
<td>(MV/m)</td>
<td>(kA/m)</td>
<td>(K)</td>
</tr>
<tr>
<td>Jun. 13</td>
<td>50</td>
<td>19.2</td>
<td>1.92</td>
<td>88.1</td>
<td>271.1</td>
<td>8.6</td>
</tr>
<tr>
<td>Jun. 15</td>
<td>50</td>
<td>16.8</td>
<td>1.28</td>
<td>77.1</td>
<td>237.1</td>
<td>6.7</td>
</tr>
<tr>
<td>Jun. 16</td>
<td>50</td>
<td>11.8</td>
<td>$8.55 \times 10^{-3}$</td>
<td>53.9</td>
<td>164.6</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Figure 7-12: Breakdown probability as a function of gradient of the HPBG-d025 structure.

for three hours' operation with $1.3 \times 10^4$ pulses, no breakdown happened. Thus a single breakdown event was assigned, resulting in an error bar of the same value, which was indicated by an arrow going down to zero in the log scale plot, as shown.
in Figure 7-12. The highest achievable gradient was about 19 MV/m for the HPBG-d025 structure with a breakdown probability of 1.92 per pulse per meter. Lowering the gradient reduced the breakdown rate, as expected.

### 7.3.4 Summary

The HPBG-d025 structure gained rapid processing with high power pulses. After conditioning, the structure could be operated at a gradient level much higher than the HPBG-d3 structure. The highest gradient without runaway was 19 MV/m in 50-ns pulses, with a BDR of 1.92 per pulse per meter. The maximum surface fields and the temperature rise are plotted in Figure 7-13. Light emission on every shot at gradients higher than 7.4 MV/m indicated multipactor generation and plasma formation in the hybrid structure.

![Figure 7-13: The highest surface electric (blue) and magnetic (red) fields and the highest temperature rise (green) for each shot in the high power tests of the HPBG-d025 structure.](image)

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7.4 Post-Test

7.4.1 Post-Test Cold Test

The cold test results after high power experiment of the HPBG-d025 structure, as shown in Figure 7-14 and Table 7.6, indicated no degradation in $Q$ or in the Smith chart and no deformation in the field profile, as compared with the same structure before the high power testing.

![Graphs and Smith chart](image)

(a) The magnitude of $S_{11}$  
(b) The Smith chart of $S_{11}$ after test 
(c) Comparison of the axial accelerating field profile

Figure 7-14: (a) The magnitude, and (b) the Smith chart of $S_{11}$, and (c) the field profile of the HPBG-d025 structure after the high power testing. The magnitude is compared with the cold test before the high power testing.

7.4.2 Autopsy

As inspections done to the HPBG-d3 structure, the Canon PC1234 camera and the Dino-Lite Pro optical microscope [89] were used for the HPBG-d025 structure. All
Table 7.6: Comparison of $f_0$ and $Q_L$ of the HPBG-d025 structure before and after the high power testing.

|                | $f_0$ (GHz) | $S_{11}$ (dB) | $Q_L$ (from $|S_{11}|$) | $Q_L$ (from the Smith chart) | $Q_0$ | $Q_{ext}$ | $\beta$ |
|----------------|-------------|---------------|--------------------------|-------------------------------|-------|-----------|---------|
| Before Test    | 17.140      | -26           | 2980                     | 3840                          | 6770  | 8070      | 0.84    |
| After Test     | 17.141      | -24           | 3160                     | 3710                          | 6590  | 7930      | 0.83    |

the three irises, one on the input coupling cell and two on the central rod plates, showed no damage on the surface, as photographed in Figure 7-15.

![Optical microscope images](image)

(a) Coupling iris, 30X (b) Upstream central plate, (c) Downstream central plate, 30X

Figure 7-15: Optical microscope images of the irises on the input coupling cell and the central plates.

Beside copper deposition happened to a few sapphire rods, severe breakdown damage occurred at the triple-point edge of the innermost row, as locations shown in Figure 7-16.

![Locations of breakdown damage](image)

Figure 7-16: Locations of breakdown damage of the HPBG-d025 structure.

Discoloration and erosion were observed on the downstream copper plate and the sapphire rod at Location 1, as shown in Figures 7-17a to 7-17d. The sapphire rod on
Figure 7-17: Damage at the triple-point edge with microscope images showing erosions on both the downstream plate and the sapphire rod at Location 1 of the HPBG-d025 structure.

Location 2 showed a crack on the end, as shown in Figure 7-18. Metallization was observed along the crack on the top of the rod. The two locations are possibly those with brighter light in Figure 7-11d. It is plausible that the runaway situation cracked the rod which resulted in much worse performance of the HPBG-d025 structure even at lower gradients.

### 7.5 Conclusion

The HPBG-d025 structure reduced the surface electric and magnetic fields by employing a much shorter insertion length of a quarter millimeter on the innermost two rows of sapphire rods. This also improved the quality factor $Q$. The HPBG-d025 structure, with a good resonance at 17.14 GHz, was put into high power test at MIT to see a total of $\sim$212,000 high power shots. The highest gradient achievable in a 50-ns structure power was 19.2 MV/m, with a breakdown probability of 1.92 per
pulse per meter. This gradient corresponded to a highest surface electric field of 88.1 MV/m.

This time the structure gained rapid processing by fed in shots with a high input power corresponding to $E_G > 20$ MV/m. Gradient over 20 MV/m resulted in severe breakdown on every shot, and finally led to permanent damage to the structure after a few thousand shots. Thus the rapid processing should be operated with limited shots.

The cold test after the high power tests showed no degradation on the resonance or the quality factor. However, images of the sapphire rods and the downstream copper plate showed breakdown damage, including discoloration on the copper plate, erosion and crack on the sapphire rods. Both happened at the triple point of the innermost rods, where surface fields peaked. The improvement of doubling the operating gradient in the HPBG-d025 structure to the HPBG-d3 structure has proved a success of the shorter insertion design.
Chapter 8

Design and Experimental Results of the HPBG-Pin Structure

To reduce the surface electric field more, in this chapter, a new design of the hybrid PBG structure employing sapphire rods other than a regular cylindrical shape will be described.

8.1 Design

This design employed a new rolling-pin shaped sapphire rod in the innermost two rows to lower the high fields at the triple interface. Figure 8-1 shows the drawing of the new rod with the rolling-pin shape. The rolling-pin rod contained a main cylindrical part exposed to vacuum in the hybrid cell and two small projections (pin part, 1 mm diameter by 1 mm long) extending into the copper plate. The main part
was \( a = 1.58 \) radius by 5.68 mm long. The size of the pin part was small enough not to affect the field distribution in the main part and big enough not to be easily cracked during assembly. This rolling-pin shape resembled the "butt joint" (insertion length \( d = 0 \) mm) connection, thus was capable to reduce the surface fields.

Figure 8-2 shows the top view of the HFSS periodic model of the HPBG cell with new sapphire rods, which will be referred to as the HPBG-Pin structure in the rest of this thesis. In Figure 8-2, rolling-pin shaped sapphire rods are in blue; regular sapphire rods are in purple; and copper rods are in green. The insertion length of the regular sapphire rods and the copper rods was \( d = 1 \) mm. The configuration in

![Figure 8-2: Top view of the HFSS model of the HPBG-Pin structure. Rolling-pin shaped sapphire rods are in blue. Regular sapphire rods are in purple. Copper rods are in green.](image)

Figure 8-2 was to optimize the surface electric and magnetic fields. Changing to other configurations resulted in a higher surface \( H \) field. The lattice spacing was \( b = 4.48 \) mm. The single cell simulation resulted in a resonant frequency uncertainty of 0.06% with 1,128,000 tetrahedra mesh. The surface field distributions including the details on the sapphire rods of the periodic HPBG-Pin model are shown in Figure 8-3, with the peak values listed in Table 8.1. The highest \( E \) field was on the irises instead of

<table>
<thead>
<tr>
<th>( f_0 ) (GHz)</th>
<th>( Q_0 )</th>
<th>( E_G ) (MV/m)</th>
<th>( E_{surf} ) (MV/m)</th>
<th>( H_{surf} ) (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.2267</td>
<td>8390</td>
<td>100</td>
<td>220</td>
<td>1429</td>
</tr>
</tbody>
</table>

Table 8.1: Simulated peak surface fields of the periodic single cell model of the HPBG-Pin structure.
Figure 8-3: The simulated surface electric and magnetic field maps of the periodic structure of the HPPB-G-Pin structure.

at the triple point of the innermost sapphire rod. The surface $H$ field peaked at the connection between the pin part and the main part of the innermost rod. Comparing with the HPBG-d025 structure, the peak surface $E$ field was lowered by a factor of 2/3, giving a ratio of $E_{surf}/E_G = 2.2$, comparable to the metallic PBG (MPBG) structure at 17 GHz [15]. The peak surface magnetic field was a little higher than HPBG-d025. From the foregoing experiments, it was speculated that the limitation of the high power performance was the surface electric field, thus the improvement of the rolling-pin design was expected to yield an increase in the achievable gradient.

In terms of the design of the Standing-Wave (SW) structure, as shown in Figure 4-3 and Table 4.1, the sizes of the two side cavities had to be changed to $R_i = 7.82$ mm
and $R_o = 7.84$ mm and the aperture of the coupling iris had to be shrunk to $r_{cp} = 3.93$ to provide the correct resonant frequency and good coupling. The final HFSS mesh included 1,072,000 tetrahedra, with an $S$ parameter uncertainty of 0.06%. Figure 8-4 and Table 8.2 show the HFSS simulation results of the SW design.

![Figure 8-4: (a) The magnitude of $S_{11}$ and (b) The Smith chart of $S_{11}$](image)

Figure 8-4: (a) The magnitude, and (b) the Smith chart of $S_{11}$ of the SW, HPBG-Pin structure simulated in HFSS.

<table>
<thead>
<tr>
<th>$f_0$ (GHz)</th>
<th>$S_{11}$ (dB)</th>
<th>$Q_L$</th>
<th>$\beta$</th>
<th>$Q_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.1379</td>
<td>-39</td>
<td>3990</td>
<td>$\sim$1</td>
<td>$8.0 \times 10^3$</td>
</tr>
</tbody>
</table>

Normalized to $E_G = 100$ MV/m, the input power required was $P_{in} = 4.6$ MW, and the shunt impedance was $R_s = E_G^2 L / P_{in} \approx 19.0$ M$\Omega$/m. Figure 8-5 shows the surface field distributions of the HPBG-Pin structure, with the peak values listed in Table 8.3.

Table 8.3: Surface field peaks of the SW, HPBG-Pin structure.

<table>
<thead>
<tr>
<th>Structure</th>
<th>$E_G$ (MV/m)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$H_{surf}$ (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPBG-Pin</td>
<td>100</td>
<td>220</td>
<td>1436</td>
</tr>
</tbody>
</table>
Figure 8-5: The simulated surface electric and magnetic field maps of the SW, HPBG-Pin structure.

8.2 Experimental Results of Cold Test

8.2.1 Rods Produced

The ideal design of the rolling-pin rod, as shown in Figure 8-1, had a sharp connection between the pin part and the main part. While the rolling-pin rods produced by Gavish, Inc [90], had a 0.13-mm rounded connection. Besides, the edge of the main rod was chamfered with a radius of about 2 mm. Figure 8-6 shows the produced rolling-pin rod, with an image under a comparator showing the rounded connection and the chamfered edge.

Figure 8-6: (a) Rolling-pin sapphire rod and (b) Comparator image

Figure 8-6: (a) The rolling-pin sapphire rod made by Gavish, Inc., and (b) a comparator image showing the rounded connection and the chamfered edge.
In addition, aligning the central cell with \( d = 1 \) mm depth resulted in cracking the rolling-pin rods. We had to choose the regular rods produced by Insaco Inc. [16] with an insertion depth of \( d = 3 \) mm. The new HFSS simulation reflecting the change of the shape of the rolling-pin rods and the insertion length of the regular rods is shown in Figure 8-7, with the peak values listed in Table 8.4.

![Figure 8-7: The simulated surface electric and magnetic field maps of the modified SW, HPBG-Pin structure.](image)

Table 8.4: Resonance and peak surface fields of the modified SW, HPBG-Pin (rounded) structure.

<table>
<thead>
<tr>
<th>Structure</th>
<th>( f_0 ) (GHz)</th>
<th>( S_{11} ) (dB)</th>
<th>( Q_L )</th>
<th>( E_G ) (MV/m)</th>
<th>( E_{surf} ) (MV/m)</th>
<th>( H_{surf} ) (kA/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPBG-Pin (rounded)</td>
<td>17.1399</td>
<td>-34</td>
<td>4080</td>
<td>100</td>
<td>391</td>
<td>1016</td>
</tr>
</tbody>
</table>

With the modification, the surface field ratio \( E_{surf}/E_G \) was enhanced to \( \sim 3.9:1 \), still slightly better than the HPBG-d025 structure. The location of the peak \( E \) field migrated to the triple point of the chamfered edge on the innermost sapphire rod. The magnetic field was reduced and peaked at the rounded connection shown in Figure 8-6b. The power required for \( E_G = 100 \) MV/m was 4.54 MW, leaving a shunt...
impedance of $R_s = 19.2 \, \text{MΩ/m}$. Table 8.5 compares the peak surface electromagnetic fields of those hybrid PBG structures. Even with the rounded rods, the HPBG-Pin structure was showing improvement for both fields. In the following, we refer to the experimental structure with the rounded rods as the HPBG-Pin structure.

Table 8.5: Comparison of the surface $E$ and $H$ fields of the HPBG structures.

<table>
<thead>
<tr>
<th>Structure</th>
<th>$E_G$ (MV/m)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$H_{surf}$ (kA/m)</th>
<th>$P_{in}$ (MW)</th>
<th>$R_s$ (MΩ/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPBG-d3</td>
<td>100</td>
<td>628</td>
<td>2297</td>
<td>11</td>
<td>7.95</td>
</tr>
<tr>
<td>HPBG-d025</td>
<td>100</td>
<td>434</td>
<td>1335</td>
<td>6.5</td>
<td>13.4</td>
</tr>
<tr>
<td>HPBG-Pin (ideal)</td>
<td>100</td>
<td>220</td>
<td>1436</td>
<td>4.6</td>
<td>19.0</td>
</tr>
<tr>
<td>HPBG-Pin (rounded)</td>
<td>100</td>
<td>391</td>
<td>1016</td>
<td>4.5</td>
<td>19.2</td>
</tr>
</tbody>
</table>

To practically accommodate the rounded rods into holes on the copper plates, not only the size of the hole should be larger than the pin part, but also the edge of the hole should be filleted with a larger-than-0.13 mm radius, which left a gap of a triple point around the connection of the pin part. Simulations on some simpler geometry showed the possibility of a huge enhancement factor (∼35 for a fillet gap with a radius of 0.2 mm) on the local electric field. In addition, vacuum gaps existed between the top of the main part and the surface of the metallic plates. A gap of 0.1 mm may add up an enhancement factor of ∼7 and result in surface field peaking along the rounded pin end too. Those gaps were inevitable for the current design. Thus we considered partially coating the rolling-pin rods. In Figure 8-8a, the blue line depicts the surface to be coated with copper. Figures 8-8b and 8-8c show the coated rolling-pin rods.

Unfortunately, applying the coated rods in the HPBG cell did not yield a reasonable resonance in the cold test. This might be caused by the coating along the chamfered edges, which could affect the resonant properties of the hybrid cavity. Or, the inevitable gaps between the top of the main part and the copper plates could bring in field concentration, and thus changing the resonance. In the future, to get rid of those defects, rods with sharp edges and better coating should be produced. Alternatively, sapphire rods could be brazed on the copper plates. In the experiment
Figure 8-8: The partially coated rolling-pin rod. The blue line in (a) shows which surface is coated. (b) and (c) show the copper-coated rolling-pin rod.

discussed in this chapter, uncoated, non-brazed sapphire rods were employed in the high power test.

8.2.2 Experimental Results: Cold Test

Figure 8-9 shows the rod plate and the rods inserted of the HPBG-Pin structure, with metallic parts machined by the MIT Central Machine Shop. Figure 8-10 shows the resonance and the accelerating field profile of the cold test measurement, with results listed in Table 8.6.
Table 8.6: Cold test results of the HPBG-Pin structure.

| $f_0$ (GHz) | $S_{11}$ (dB) | $Q_L$ | $Q_L$ from $|S_{11}|$ | $Q_L$ from the Smith chart | $Q_{ext}$ | $\beta$ |
|-------------|---------------|-------|------------------------|-----------------------------|-----------|--------|
| 17.144      | -21           | 3390  | 3940                   | 6910                        | 8790      | 0.79   |

Figure 8-10: Cold test results of (a) the magnitude of $S_{11}$, (b) the Smith chart of $S_{11}$, and (c) the axial $E$ field profile of the HPBG-Pin structure.

8.3 Experimental Results: High Power Tests

The high power tests of the HPBG-Pin structure lasted for 7 days with ~ 150,000 shots fed into the structure at 17 GHz. Figure 8-11 shows the summarized results. Again, gradient is calculated only in the high power HPBG cell.

8.3.1 Traces, Dark Current and Light Emission

Based on the high power tests of the HPBG-d025 structure, this time much higher power (corresponding to $E_G = 25$ MV/m) was put into the HPBG-Pin structure
Figure 8-11: Summarized results of the high power testing of the HPBG-Pin structure for (a) gradient (blue) and the flat-top length of the structure power (red), and (b) gradient (blue) and the magnitude of dark current (green) from the downstream Faraday cup. The black circles at the top of each figure indicate each end of data for one day's run, with each date detailed in (b). Gradient is calculated only in the HPBG cell by the forward power.
at the beginning phase in order to fast-condition the structure. The dark current (DC) of this stage was just a single sharp negative spike, proportional to gradient, similar to its two antecedents. The reverse power was not quantitatively useful; its shape was used to estimate the coupling and breakdown. Dark current was also used to determine the breakdown event. It started to breakdown on every shot at a low gradient level (~5 MV/m). Figure 8-12 shows a typical pulse during processing.

![Figure 8-12](image)

Figure 8-12: During processing, a typical pulse shows the traces of the forward power, the reverse power, the calculated structure power, the dark current from the downstream Faraday cup and the photodiode signal of Shot 11,145 (Sept. 14), with the average gradient $E_G = 5.58$ MV/m. Breakdown, high spike of the dark current, no photodiode signal.

Figure 8-13 plots the magnitude of dark current and the delay time between the dark current peak and the start of the input pulse as a function of gradient. The dark current is proportional to gradient, and the delay time is monotonically decreasing as gradient.

Figure 8-14a shows a clear view of the hybrid PBG cell with ambient light taken by the CCD camera. During operation, the test vacuum chamber was masked with black cover to reduce the background light, resulting in a CCD photo shown in Figure 8-14b.

In order to facilitate the conditioning progress, the gradient level was raised to 35 MV/m. Even at this high power, the structure had been kept steady with acceptable dark current (~30 mA) and no severe breakdown or runaway light emission was seen for ~$1 \times 10^4$ shots. Due to the loss of data acquisition of the traces, the number of pulses was estimated from operation at a repetition rate of 2 Hz for about one and half hours. During the data loss, the CCD camera captured some highlight flashes,
as shown in Figure 8-14c. Before this highlight, light captured by the CCD was hard to tell. After a few shots, the captured light started to dim and fade away, as shown in Figure 8-14d. This situation was kept for about three thousand shots, until a few runaway explosions occurred. A lot of rods were emitting light, as shown in Figures 8-14e to 8-14g. Traces in Figure 8-15 display an ultrahigh dark current pulse after the restoration of data acquisition at a similarly high gradient. Even though the input power was lowered immediately after the discovery of the runaway situation and the structure seemed to have recovered, it was highly possible the structure was damaged during the first occurrence of the runaway situation. From then on, light was visualized by the CCD camera from a lower gradient of $E_G = 9.3$ MV/m. Most CCD photos had a dim flash as shown in Figure 8-14d. Figure 8-14h is the severe runaway situation to be discussed later.

The processing started to show effectiveness after around $9 \times 10^4$ shots, resulting in lower and lower dark current at the same gradient, as shown in Figure 8-11b from Sept. 17. For a new gradient, it might need around two thousand shots to reprocess the structure. Figure 8-16 compares the traces during and after the reprocessing at a similar gradient. The high spike of the dark current in the beginning was gradually smoothed out, leaving only the main part. If the spike happened within the input shot, there would be a sudden reflection on the reverse power trace, apparently breaking

![Graphs](image-url)
(a) Clear view of the HPBG cell
(b) Stray light view showing copper rods, no light emission
(c) Light emission on a few sapphire rods, the brightest may be one innermost rod (Sept. 16).
(d) The light starts to dim (Sept. 16)
(e) Runaway situation 1 with bright light, four more rods emit light (Sept. 16).
(f) Runaway situation 1 with bright light, two more rods emit light (Sept. 16).
(g) Runaway situation 1 with bright light, a lot of rods emit light (Sept. 16).
(h) Runaway situation 2 (Sept. 23)

Figure 8-14: CCD images of the central cell of the HPBG-Pin structure during the high power tests.

Figure 8-15: Before processing, an ultrahigh dark current pulse shows the traces of the forward power, the reverse power, the calculated structure power, the dark current from the downstream Faraday cup and the photodiode signal of Shot 63,472 (Sept. 16), $E_G = 33.0$ MV/m.
down. If the spike rose after the input pulse, the reverse power showed no change.

![Graph of Shot 110,914 (Sept. 18)](attachment:graph1.png)

(a) $E_G = 14.1$ MV/m, during reprocessing

![Graph of Shot 116,574 (Sept. 18)](attachment:graph2.png)

(b) $E_G = 13.8$ MV/m, after reprocessing

Figure 8-16: Comparison of the forward power, the reverse power, the calculated structure power, the dark current from the downstream Faraday cup and the photodiode signal between (a) Shot 110,914, during reprocessing (Sept. 16), with the average gradient $E_G = 14.1$ MV/m; and (b) Shot 116,574 (Sept. 18), after reprocessing, $E_G = 13.8$ MV/m.

From this time on, for a steady operational gradient, the reverse power after reprocessing seldom changed. Instead, at a high gradient level of $E_G = 19$ MV/m, some pulses showed a sudden rise of the dark current, the same as Figure 8-16a. We defined a breakdown if the dark current had a sudden rise, even without any change of the reverse power. Apparently, the rise of the dark current indicated that more electrons had been generated. The timing of the sudden rise was after the input pulse, and thus no change occurred on the reverse trace. However, breakdown damage still emerged. Evidence for this was the severe runaway situation at a higher gradient ($E_G \sim 20$ MV/m) on September 23th. After about 80 breakdowns defined by the afterward rise of the dark current, the severe breakdown happened again which saturated the photodiode and led to the final failure of the HPBG-Pin structure. Similar to the HPBG-d025 structure, the HPBG-Pin structure could not recover even
at much lower gradients. Figure 8-17 shows the saturation and Figure 8-14h shows the

Figure 8-17: Traces of the runaway situation with saturated photodiode signal, showing the forward power, the reverse power, the calculated structure power, the dark current from the downstream Faraday cup and the photodiode signal of Shot 165,627 (Sept. 23), $E_G = 19.6$ MV/m.

light explosion filling the cavity space. Pressure rise from the ion pump connected to the vacuum chamber was observed, indicating outgassing from the HPBG structure.

To compare, for $E_G = 19.1$ MV/m, we plot the dark current and the photodiode signal into one figure for both non-breakdown and breakdown traces in Figure 8-18. For a nonbreakdown pulse, the dark current contained a leading edge and a main part with a slower rise and decay. The highest value in the main part was around 1 mA. For breakdown pulses, a sudden afterward rise of the dark current added upon the normal dark current. The peak was more than 8 mA. For most of the breakdown traces, the delay time between the peak of the dark current and the start of the input power was around 150 ns, as shown in Figure 8-18. Figure 8-19 shows the charge of the dark current as a function of the absolute value of the dark current peak. Clearly,
Figure 8-19: Charge of the dark current as a function of the dark current peak at $E_G = 19.1$ MV/m during one day test of Sept. 18.

the charge reaching the Faraday cup during a breakdown event was twice or three times of that of the normal pulse, as shown in Figure 8-19.

### 8.3.2 Breakdown Probability of the HPBG-Pin Structure and Comparison of the PBG structures

**Breakdown Probability of the HPBG-Pin Structure**

The detailed breakdown probabilities are summarized in Table 8.7. At $E_G = 14.1$ MV/m, for about $8 \times 10^4$ shots, no breakdown happened. Thus a single breakdown event was assigned. The highest achievable gradient was about 19.1 MV/m for the HPBG-Pin structure with a breakdown probability of $5.04 \times 10^{-1}$ per pulse per meter.

**Table 8.7: Breakdown probabilities of the HPBG-Pin structure.**

<table>
<thead>
<tr>
<th>Date</th>
<th>Structure Power flat-top length (ns)</th>
<th>Gradient (MV/m)</th>
<th>BDR (/pulse/m)</th>
<th>$E_{surf}$ (MV/m)</th>
<th>$H_{surf}$ (kA/m)</th>
<th>$\Delta T$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept. 18</td>
<td>45</td>
<td>14.1</td>
<td>$1.42 \times 10^{-2}$</td>
<td>58.2</td>
<td>151.3</td>
<td>2.6</td>
</tr>
<tr>
<td>Sept. 18</td>
<td>45</td>
<td>19.1</td>
<td>$5.04 \times 10^{-1}$</td>
<td>78.3</td>
<td>203.5</td>
<td>4.9</td>
</tr>
</tbody>
</table>

**Comparison of the Three HPBG structures**

Figures 8-20 and 8-21 show comparisons of the breakdown probabilities as functions of gradient $E_G$, the highest surface electric field $E_{surf}$ among the three hybrid PBG structures: the hybrid PBG structure with insertion length $d = 3$ mm (HPBG-d3),

192
Figure 8-20: Breakdown probability of the HPBG-d3, the HPBG-d025, and the HPBG-Pin structures as a function of gradient.
Figure 8-21: Breakdown probability of the HPBG-d3, the HPBG-d025 and the HPBG-Pin structure as functions of (a) the highest surface electric field, and (b) the maximum surface temperature rise.
the hybrid PBG structure with insertion length \( d = 0.25 \) mm (HPBG-d025), and the hybrid PBG structure with rolling-pin shaped rods (HPBG-Pin), with different flat-top length of the structure power. The error bar of the breakdown probability is calculated statistically based on the Poisson distribution. For HPGBG-d3, only the breakdown rates in 80-ns pulses are chosen since those points reflect the high power performance after being fully processed. Data for the other structures are limited by the operation pulse length when running the experiment.

From Figure 8-21 we can tell the breakdown probability of the HPGBG structure is more relevant to the surface electric field, so is the highest gradient achievable.

**Comparison with the Metallic PBG Structure**

Figures 8-22 and 8-23 show comparisons of the breakdown probabilities as functions of gradient \( E_G \), the highest surface electric field \( E_{surf} \) and the highest temperature rise \( \Delta T \), among the four PBG structures tested at MIT at 17 GHz: the HPGBG-d3 structure, the HPGBG-d025 structure, the HPGBG-Pin structure and the metallic PBG structure MPBG-MIT-2 (referred to as MIT-PBG-2 in [15]). The high power testing of the HPGBG structures achieved a much lower gradient \( (E_{Gmax,HPBG} = 19 \text{ MV/m}) \) than that of the MPBG structure \( (E_{Gmax,MPBG} = 89 \text{ MV/m}) \), with a comparable breakdown probability. The achievable surface \( E \) field of the HPGBG structure is about 90 MV/m, which is half of that achieved by the MPBG-MIT-2 structure. The limit on the achievable gradient of the HPGBG structures results from the high surface \( E \) field which is in addition enhanced by the the triple point and the gap-enhancement between components from the clamped assembly. Besides, inclusion of a dielectric makes the system vulnerable to multipactor, and thus more easily to trigger breakdowns.

**8.3.3 Summary**

The HPGBG-Pin structure gained rapid processing with high power pulses fed in and it was possible to have breakdown damage at a gradient higher than 30 MV/m due
Figure 8-22: Breakdown probability of the HPBG-d3, the HPBG-d025, the HPBG-Pin and the MPBG-MIT-2 structures as a function of gradient.
Figure 8-23: Breakdown probability of the HPBG-d3, the HPBG-d025, the HPBG-Pin structure and the MPBG-MIT-2 structures as functions of (a) the highest surface electric field, and (b) the maximum surface temperature rise.
to the occurrence of the first runaway situation. The power was lowered down and the structure seemed recovered. After that, the highest gradient without runaway was 19.1 MV/m in 45-ns structure power, with a breakdown rate of $5 \times 10^{-1}$ per pulse per meter. The maximum surface fields and the temperature rise are plotted in Figure 8-24. Indication of multipactor and plasma formation can be shown by

![Figure 8-24: The highest surface electric (blue) and magnetic (red) fields and the highest temperature rise (green) for each shot in the high power tests of the HPBG-Pin structure.](image)

the light emitted on the sapphire rods for all the shots at $E_G > 9.3$ MV/m, with or without breakdowns. For HPBG-d025 structure, the gradient threshold of light emission was 7.4 MV/m. The ratio was $E_{G,Pin}/E_{G,d025} \sim 1.3$, while the ratio of the surface electric field normalized to $E_G = 100$ MV/m is $E_{Surf,d025}/E_{Surf,Pin} \sim 1.1$; they are consistent.
8.4 Post-Test

8.4.1 Post-Test Cold Test

The cold test results after the high power experiment of the HPBG-Pin structure, as shown in Figure 8-25 and Table 8.8, indicated no degradation in $Q$ and no deformation in the field profile, as compared with the same structure before the high power testing.

Table 8.8: Comparison of $f_0$ and $Q_L$ of the HPBG-Pin structure before and after the high power testing.

|          | $f_0$ (GHz) | $S_{11}$ (dB) | $Q_L$   | $Q_L$ from $|S_{11}|$ | $Q_0$ from the Smith chart | $Q_{ext}$ | $\beta$ |
|----------|-------------|---------------|---------|----------------------|----------------------------|-----------|---------|
| Before Test | 17.144      | -21           | 390     | 3940                 | 6910                       | 8790      | 0.79    |
| After Test | 17.145      | -21           | 3610    | 4220                 | 7350                       | 9460      | 0.83    |

8.4.2 Autopsy

This disassembled HPBG-Pin structure showed no damage on the irises. Again, all damage was on the sapphire rods interfacing the copper plates, especially, on the rounded connection between the pin part and the main part of the rolling-pin rods, as shown in Figure 8-6b. Figure 8-26 shows all the seven locations where obvious breakdown damage was observed. All of those rods can be roughly identified in Figure 8-14. Among those seven sites, all the six damaged connections were discovered on the upstream part of the rods, except for Location 6, which had damage on the downstream end. Figure 8-27 shows the microscope photos of the damaged rod and the plate on Location 1. It has a black circle along the rounded connection. Erosion around the corresponding edge of the small hole on the copper plate was observed too.

Except for Location 2, all the other locations had similar breakdown damage patterns as shown in Figure 8-27. For Location 2, a more severe crack happened on the rolling-pin sapphire rod, as shown in Figure 8-28. The runaway situation showed
extensive light emission along the rod of Location 2. The similar situation happened to the HPBG-d025 structure. So as speculated in Section 7.4, it was the crack that physically ruptured the sapphire crystal to bring in the runaway situation thus failed the HPBG structure.

Besides, most of the sapphire rods were found with partially metallization on the surface, especially for those on the innermost two rows near those breakdown locations.

The connection periphery was not where the surface electric field but the magnetic field climaxed. However, the calculated highest temperature rise was only around 18 K, which was too low to destroy sapphire. Subsection 8.2.1 mentioned the possible enhancement of the $E$ field by the gaps between the sapphire rods and the copper plates. Those gaps are inevitable and are highly possible to enhance the surface $E$.
Figure 8-26: Locations of the breakdown damage of the HPBG-Pin structure.

Figure 8-27: Breakdown damage on the connecting periphery of the rolling-pin sapphire rod and on the upstream copper plate of the HPBG-Pin structure.
Figure 8-28: Crack on the rolling-pin sapphire rod and a more severe discoloration on the edge of the hole of the upstream copper plate of the HPBG-Pin structure.

field greatly, resulting in damage to both the sapphire rods and the copper plates.

8.5 Conclusion

The ideal design of the HPBG-Pin structure was aimed at a similar $E_{surf}$-to-$E_G$ ratio ($\sim 2.2$) as the metallic PBG structure by employing the novel rolling-pin-shaped sapphire rods in the inner rows of the PBG lattice. However, in practice, the rolling-pin rods with rounded connections and chamfered edges raised the ratio to 3.9, still better than the HPBG-d025 structure. Gaps between the sapphire rods and the copper plates would enhance the surface $E$ field even more. Partially coated rods might get rid of those gaps. Using those coated rods resulted in no resonance in the designed frequency range. So uncoated rolling-pin rods were used in the high power test of the HPBG-Pin structure.

The high power testing of the HPBG-Pin structure involved $\sim 150,000$ high power microwave shots. The HPBG-Pin structure could be operated in a 45-ns structure.
power at a gradient of $E_G = 14 \text{ MV/m}$ for $8 \times 10^3$ shots without breakdowns. The highest gradient achievable in a 45-ns structure power was 19.1 MV/m, with a breakdown probability of $5 \times 10^{-1}$ per pulse per meter, which was an improvement comparing with its two forgoers. Gradient around 20 MV/m could be kept for 4,000 shots while finally leading to runaway situation that permanently damaged the structure. The post-test cold test showed no degradation on the quality factor. However, erosion and crack on the sapphire rods were observed. In the future, if rods could be produced with better quality and correctly coated and brazed on the copper surface, better high power performance of the HPBG-Pin structure could be expected.
Chapter 9

Discussion and Future Work

9.1 Summary of Results and Discussions

This thesis reports the first high power tests of a hybrid photonic band gap (HPBG) accelerator structure. Photonic band gap (PBG) accelerator structures have the attractive feature that they are designed to support a single electromagnetic mode, thus reducing or eliminating unwanted modes generated by wakefields in an accelerator. A number of pure metallic PBG structures have been tested for both room temperature and superconducting accelerator applications. We have designed, built and successfully tested at high power a hybrid PBG structure. The hybrid structure contains both dielectric and metallic elements. Dielectric elements have the advantage of low loss and have the potential to survive very high surface electric and magnetic fields.

The hybrid PBG structure was constructed as a triangular array of rods sandwiched between two flat copper plates. Rods were removed from the cavity center to form a defect cavity to confine the $\text{TM}_{02}$ mode at 17.14 GHz. The PBG rod array consisted of inner rows with a total of 60 sapphire rods plus a row of 24 copper rods added at the outside to provide a higher overall cavity $Q$. The diameter and the spacing of the sapphire rods were adjusted to excite only the $\text{TM}_{02}$ mode and to suppress higher-order modes (HOMs). The lower order $\text{TM}_{01}$ mode is not confined in the structure, neither is the $\text{TM}_{11}$ dipole mode. This overmoded operation is a novel and unique feature of the hybrid cavity design. All designs were simulated by
3D design codes and included the birefringence of the sapphire material. Simulations of the hybrid structures showed relatively high surface fields at the triple point where sapphire, copper and vacuum meet as well as in any gaps between the rods and the metal plates.

Preceding studies [7,12] showed that the dielectric system would be more susceptible to multipactor thus to a higher breakdown probability at a lower accelerating gradient. Inclusion of a triple point [11] in the hybrid structure would enhance the surface electromagnetic fields, resulting in an even lower gradient limit. In order to explore the breakdown properties of the hybrid PBG accelerator, high power tests were conducted on a test stand with a power capability of over 4 MW at 17.1 GHz, sufficient to test the structures to well above 100 MV/m gradient. The tests were conducted on a three-cell structure consisting of the hybrid PBG cell between conventional disk-loaded-waveguide cells. The hybrid cells were fabricated by clamping the rods between the copper plates, allowing for simple fabrication and tuning. Three HPBG structures were tested.

The first HPBG structure, the HPBG-d3 structure with an insertion length of 3 mm of the sapphire rods into the copper plates, resulted in a high ratio of the surface electric field to the gradient due to the triple-point enhancement. Effective reduction of the surface fields was achieved by rounding the contacting edge of the copper holes, leading to a final ratio of \( E_{surf}/E_G \sim 6.3 \). The transmission cold test of the HPBG cell resulted in a single resonant frequency close to the operation frequency, which demonstrated the overmoded coupling and the HOM damping. The reflection cold test attained excellent agreement of the frequency and the quality factor \( Q \) with the HFSS simulation, which verified the theoretical design and the methodology to calculate the dielectric and the hybrid system. The high power experiment of the HPBG-d3 structure included 458,000 high power pulses, reaching a relatively low gradient threshold at 9.0 MV/m for a 40-ns structure power, with a breakdown probability of 2.91 \times 10^{-1} \ per pulse per meter. The highest gradient achievable in a 80-ns structure power was 8.7 MV/m, corresponding to a surface \( E \) field of 57.9 MV/m, with a breakdown probability of 1.28 \times 10^{-1} \ per pulse per meter.
High-gradient processing during the experiment yielded effectiveness in reducing the multipactor effect, which was applied and improved with rapid processing in the following experiments.

To reduce the high surface fields caused by the long insertion, the HPBG-d025 structure adapted an as-short-as-possible insertion length of $d = 0.25$ mm, which led to an $E_{surf}/E_G$ ratio of 4.3. The high power tests comprised 212,000 shots to the hybrid cell. After rapid high power processing, the operational gradient achievable by this shorter insertion design was 19.2 MV/m, corresponding to a surface $E$ field of 88.1 MV/m, with a breakdown probability of 1.92 per pulse per meter for a 50-ns pulse.

In order to get rid of the enhancement from the insertion, a new structure, called the HPBG-Pin structure, employing rolling-pin shaped sapphire rods was designed. The theoretical calculation could reduce the surface $E$ field ratio to 2.2, similar to the metallic PBG structure. As manufactured, the rolling-pin rod had a rounded connection in the pin part and a chamfered edge in the main part, which resulted in a surface $E$ field ratio to 3.9. In addition, inevitable gaps existed on the rounded connection and between the top of the main part of the sapphire rods to the surface of the copper plates. Those gaps could add large enhancements to the local fields. Coated rods were tried, but resulted in no resonance in the correct frequency range. Thus the uncoated rods were used in the high power test including 150,000 shots. The HPBG-Pin structure could be operated at a gradient of $E_G = 14.1$ MV/m without breakdown occurrence for about $8 \times 10^3$ shots. The highest operational gradient attained by the HPBG-Pin structure was 19.1 MV/m, corresponding to a surface $E$ field of 78.3 MV/m, with a breakdown probability of $5.04 \times 10^{-1}$ per pulse per meter for a 45-ns structure power.

For both of the HPBG-d025 and the HPBG-Pin structure, a runaway situation occurred at a gradient of 20 MV/m. A few severe breakdowns happened with saturation on the photodiode and a lot of light emission. The structures could not recover even at much lower gradients, marking permanent damage in the hybrid cavities.

Optical diagnostics of a CCD camera and a photodiode were introduced in the
diagnostic system. Light emitted from the sapphire rods was observed for all of the three hybrid structures on every shot with a gradient higher than a threshold corresponding to a surface $E$ field of $\sim 30 \text{ MV/m}$. This indicated the emergence of multipactor and plasma formation. Breakdown damage was found at the triple point interface where the peak surface electric field was located, or the greatest field enhancement occurred. Discoloration and erosion were observed on both the sapphire rods and the copper plates. Metallization of copper onto the surface of the dielectric rods could potentially degrade the performance and the quality of the hybrid structure, and thus may be a disadvantage of the hybrid design. A more severe damage, a crack happened on one innermost sapphire rod for both the HPBG-d025 structure and the HPBG-Pin structure. They both experienced the runaway situation. Therefore it is possible that the runaway could be triggered by the crack which could lead to the permanent degradation of the hybrid structure.

The high power breakdown experiments have revealed the role of the multipactor in triggering the dielectric breakdowns and limiting the operational gradient for an accelerator structure involving a dielectric. For each hybrid structure, the dark current value after processing could be much lower than before, indicating the conditioning progress to reduce the multipactor effect. It is plausible the processing can smooth the surface roughness to generate a lower surface field or reduce the secondary electron yield of the dielectric surface. Thus rapid processing is applicable to the dielectric accelerator cavity. The sapphire rods produced by the commercial companies showed defects and also a fine ground finish on all surfaces. Optical processing may help with the performance of the sapphire rods in the high power testing.

Breakdown probability is mostly related to the surface electric field. The triple point interface involved in the hybrid design could greatly enhance the surface fields, leading to a high ratio of the surface $E$ field to the gradient, which is the major limitation of the hybrid PBG structures in high power operation. Inevitable gaps at the triple points, caused by the clamped assembly, added extra enhancement factors on the local fields, some of which could be great. This could be avoided by coating the sapphire rods and brazing the dielectric rods onto the copper plates.
This thesis is the first ever demonstration of the overmoded operation, and the only high power test of the hybrid photonic band gap structure. The gradient achieved of 19 MeV/m is well below that of metallic structures but is the highest gradient achieved with a dielectric structure. The overmoded cavity design and the wakefield damping capability of the HPBG structure could still be very useful in some applications.

9.2 Future Work

Following the current hybrid PBG structure, optimized design could be performed taking into consideration of the cost and time consumption. To get a higher gradient, a brazed structure with coated rods should be applied to get rid of the enhancement on surface fields caused by the gaps at the triple points. In addition, titanium nitride could be coated on the surface of the sapphire rods to suppress multipactor.

In order to study the multipactor and the dielectric breakdown, a much simpler, pure dielectric cavity is recommended. Further, pure dielectric PBG cavities can be built to use the current methodology and diagnostics for high power tests. The PBG structure, can be either the similar design with dielectric rods to form the PBG lattice, or a reverse structure using vacuum holes on the dielectric substrate to form the PBG lattice. Exploration of the dielectric or hybrid structures in high power operation of a high frequency band would potentially benefit the accelerator community.
Bibliography


