A Bayesian Semiparametric Competing Risk Model with Unobserved Heterogeneity

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Abstract

This paper generalizes existing econometric models for censored competing risks by introducing a new flexible specification based on a piecewise linear baseline hazard, time-varying regressors, and unobserved individual heterogeneity distributed as an infinite mixture of Generalized Inverse Gaussian (GIG) densities, nesting the gamma kernel as a special case. A common correlated latent time effect induces dependence among risks. Our model is based on underlying latent exit decisions in continuous time while only a time interval containing the exit time is observed, as is common in economic data. We do not make the simplifying assumption of discretizing exit decisions – our competing risk model setup allows for latent exit times of different risk types to be realized within the same time period. In this setting, we derive a tractable likelihood based on scaled GIG Laplace transforms and their higher-order derivatives. We apply our approach to analyzing the determinants of unemployment duration with exits to previous or new jobs among unemployment insurance recipients on nationally representative individual-level survey data from the U.S. Department of Labor. Our approach allows us to conduct a counterfactual policy experiment by changing the replacement rate: we find that the impact of its change on the probability of exit from unemployment is inelastic.

JEL: C11, C13, C14, C41, J64
Keywords: competing risk model, Bayesian semiparametric model, unemployment insurance.

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1. Introduction

In an economic competing risk (CR) model with censoring, an individual is associated with a current state (e.g. of being unemployed) with the possibility to exit to one of several different states (e.g. back to their previous job or to a new job). However, only one such exit is observed for some individuals, while other (censored) individuals are never observed to exit their current state. The state duration before exit to each potential new state or censoring time is modeled with a separate latent variable but only the shortest such duration is actually observed for each individual. A key ingredient of CR models is the survival function which captures the probability that the individual will remain the current state beyond a any given time. CR analysis then typically seeks to determine the impact of observable characteristics of the individual and the various states on the survival function that can lead to policy recommendations.\(^\text{5}\)

In this paper we introduce a new flexible model specification for the competing risk model, extending several strands of econometric and statistical literature on duration analysis. Our model encompasses three key features: (i) we estimate non-parametrically the density of unobserved individual heterogeneity; (ii) we model correlations between the different risk types; (iii) we allow for multiple latent exits within a time period with interval outcome data. Our model nests the single exit type (so-called duration model) as a special case. We apply our method to analyzing the determinants of unemployment duration among unemployment insurance recipients using data from the U.S. Department of Labor. We conduct a counterfactual experiment by changing the replacement rate. The counterfactual results show the impact of changing key policy variables such as the replacement rate on the survival function.

We will introduce each model feature in turn and discuss the advantages of using our model over the existing alternatives. First, our model provides a flexible approach to controlling for unobserved heterogeneity in competing risk data. Unobserved heterogeneity refers to any differences in the distributions of the dependent variables remaining after controlling for the effect of observable variables. It typically has two sources: the misspecification of the functional form of the econometric model and the omission of important but perhaps unobservable variables from the conditioning set. As an example of the latter, more motivated individuals may exit unemployment more quickly because they put more effort into the search for a new job.

It is well established that failure to account for unobserved heterogeneity biases the estimated hazard rate and the proportional effects of explanatory variables on the population hazard (Lancaster 1979, 1990). Lancaster’s (1979) Mixed Proportional Hazard (MPH) model generalizes Cox’s (1972) Proportional Hazard model with an explicit model component for unobserved heterogeneity. A number of semi-parametric

\(^5\)Applications of CR models in economics include analyzing unemployment duration (Flinn and Heckman, 1982; Katz and Meyer, 1990; Tysse and Vaage, 1999; Alba-Ramírez, Arranz, and Muñoz-Bullón, 2007), Ph.D. completion (Booth and Satchell, 1995), teacher turnover (Dolton and van der Klaauw, 1999), studies of age at marriage or cohabitation (Berrington and Diamond; 2000), mortgage termination (Deng, Quigley, and Van Order, 2000), school dropout decisions (Jakobsen and Rosholm, 2003), and manufacturing firms’ exits from the market (Esteve-Perez, Sanchis-Llopis, and Sanchis-Llopis, 2010). A comprehensive overview is given in Van den Berg (2001).
estimators for the MPH model have been proposed following Elbers and Ridder (1982) proof of MPH semi-parametric identification. Heckman and Singer (1984) consider the nonparametric maximum likelihood (NPML) estimator of the MPH model with a parametric baseline hazard. Using the results of Kiefer and Wolfowitz (1956), they approximate the unobserved heterogeneity with a discrete mixture. However, their estimator is not \( \sqrt{N} \)-consistent and encounters substantial computational challenges, including in calculation of standard errors. Honoré (1990) suggests another estimator with a Weibull baseline that does not require specifying the unobserved heterogeneity distribution.

Han and Hausman (1990) and Meyer (1990) propose an estimator for piecewise-constant baseline hazard and gamma distributed unobserved heterogeneity. Horowitz (1999) proposed a nonparametric estimator for both the baseline hazard and the distribution of the unobserved heterogeneity, under the assumption of constant time-invariant regressors. Hausman and Woutersen (2012) show that a nonparametric estimator of the baseline hazard with gamma heterogeneity yields inconsistent estimates for all parameters and functions if the true mixing distribution is not a gamma, stressing the importance of avoiding parametric assumptions on the unobserved heterogeneity. They propose an estimator for the MPH model with time-varying regressors and nonparametric unobserved heterogeneity, without estimating the heterogeneity form explicitly. We note that the above mentioned semiparametric variants of the MPH model feature only one exit type from a given state, such as exiting from being unemployed to being employed.

As the second key feature of our model, our approach allows for correlations between the different risks in the CR model environment, even in the presence of the flexible individual heterogeneity infinite GIG mixture model component. This is important since the determinants of exit can differ depending on the risk type while being correlated across the risk types, and thus our approach provides additional information to the analyst compared to single-risk duration models.

Third, in our application, we deal with interval outcome data as is common in economics and other social sciences. Even though the underlying exit decision model is set in continuous time, only the broader period in which exit occurred is available to the researcher. Our data contain the week of exit from unemployment. Based on scaled GIG Laplace transforms and their higher-order derivatives, we provide a complete likelihood specification allowing for multiple latent exits within a single time period which is more realistic than simplifying the analysis by assuming that only one latent exit can occur in a given time period. Thus, we do not rule out by assumption an individual contemplating a new job offer versus the return to previous job within any given week.

The combination of these three features makes our model unique in the literature. To the best of our knowledge, previous work in the given model environment has always focused only on a subset of these features. Indeed, it is difficult to combine all three features in one model but we show that given the analytical form of the model provided in this paper, the model can be implemented in a user-friendly way via a Bayesian nonparametric approach. One of the key benefits of Bayesian Markov chain Monte Carlo (MCMC) methods that we utilize is their ability to factorize a complicated joint likelihood model into
a sequence of conditional tractable models, so-called Gibbs blocks, and by sampling each in turn deliver outcomes from the joint model. We detail this approach for our proposed model.

The bulk of the literature on CR model development is concentrated in the natural sciences. A recent overview of CR modeling in biostatistics is provided by Beyersmann, Schumacher, and Allignol (2012), and in medical research by Pintilie (2006). The associated estimation methods typically rely on continuous time data for the exact point of exit. In contrast, we observe only discrete time intervals within which latent exits occur.

Competing risk models suitable for economic interval outcome data have been proposed in various forms. Han and Hausman (1990), Fallick (1991), Sueyoshi (1992), and McCall (1996) provide model specifications either without or with parametric individual heterogeneity. Butler, Anderson, and Burkhauser (1989) propose a semiparametric CR model controlling for the correlation between unobserved heterogeneity components in each state, with quadratic time dependence. Bierens and Carvalho (2007) consider Weibull baseline hazards and common flexible unobserved heterogeneity. Canals-Cerdá and Gurmu (2007) approximate unobserved heterogeneity distribution with Laguerre polynomials. They find that model selection rules (BIC, HQIC, and AIC) perform worse in determining the polynomial order than a naive approach of controlling for unobserved heterogeneity using simple models with a small number of points of support or a polynomial of small degree. Van den Berg, van Lomwel, and van Ours (2008) consider a model with nonparametric unobserved heterogeneity terms that is based on discrete time counts. Although the model can be derived as a time-aggregated version of an underlying continuous-time model, the latter is different from the continuous-time mixed proportional hazard model.

The literature on Bayesian nonparametric methods in the CR environment has been scant and, to our knowledge, has only been used in biostatistics for estimation of other objects of interest than individual heterogeneity. Variants of Bayesian Dirichlet Process analysis have been used by Gasbarra and Karia (2000) for estimating nonparametrically the overall hazard rate and in Salinas-Torres, Pereira, and Tiwari (2002) and Polpo and Sinha (2011) for the vector of risk-specific cumulative incidence functions. De Blasi and Hjort (2007) specify a beta-process prior for the baseline hazard, with asymptotic properties analyzed in De Blasi and Hjort (2009).

Identification results under various assumptions were established by Heckman and Honoré (1989), Sueyoshi (1992), Abbrin and van den Berg (2003), and Lee and Lewbel (2012). In general, there have been three different approaches to identification (Honoré and Lleras-Muney, 2006): (a) to make no additional assumptions beyond the latent competing risk structure and estimate bounds on the objects of interest; (b), assume that the risks are independent conditional on a set of observed covariates and deal with a multiple duration models environment; and (c), to specify a parametric or semi-parametric model conditional on the covariates. Here we take the last approach. In particular, we do not assume that the risks are independent conditional on the observed covariates.

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6In the single-risk, duration model case, Bayesian analysis with economics application was undertaken by Ruggiero (1994), Florens, Mouchart, and Rolin (1999), Campolieti (2001), Psarman (2004), and Li (2007).
We develop our model along with a number of more restrictive benchmark models under a variety of assumptions on the heterogeneity type, as summarized below:

<table>
<thead>
<tr>
<th>Overview of Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heterogeneity type</strong></td>
</tr>
<tr>
<td>No heterogeneity</td>
</tr>
<tr>
<td>Parametric GIG</td>
</tr>
<tr>
<td>Parametric gamma</td>
</tr>
<tr>
<td>Flexible GIG mixture</td>
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<tr>
<td>Flexible gamma mixture</td>
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The remainder of the paper is organized as follows. Section 2 establishes the assumptions and building block results for a single risk duration model. Section 3 introduces assumptions and results for the competing risk model. Section 4 details our application and the counterfactual experiment and Section 5 concludes. Proofs or all theorems and additional empirical results are provided in the Appendix. The Online Appendix contains further results.

2. Single Risk Duration Model

We will first define a number of model building blocks along with a set of assumptions and then state a set of results pertaining to the single exit type case. For a continuous time variable $\tau$, define the hazard rate $\lambda_i(\tau)$ as the failure rate at time $\tau$ conditional upon survival to time $\tau$, $\lambda_i(\tau) = \lim_{\delta \to 0} Pr(\tau < t_i < \tau + \delta)/\delta$ and denote the integrated hazard by:

\[
(2.1) \quad \Lambda_i(\tau) = \int_0^\tau \lambda_i(u)du
\]

The survivor function $S_i(\tau)$ and the distribution function $F_{i\tau}(\tau)$ of $\tau$ are defined by the following relations:

\[
(2.2) \quad \Lambda_i(\tau) = -\log (S_i(\tau))
\]

\[
(2.3) \quad S_i(\tau) = \exp (-\Lambda_i(\tau))
\]

\[
(2.4) \quad F_{i\tau}(\tau) = 1 - S_i(\tau)
\]

\[
(2.5) \quad f_{i\tau}(\tau) = -S_i'(\tau) = \exp (-\Lambda_i(\tau)) \lambda_i(\tau)
\]

Moreover, using (2.4), it follows that

\[
S_i(\tau) = 1 - F_{i\tau}(\tau)
\]

\[
= 1 - \int_{-\infty}^\tau f_{i\tau}(\tau)d\tau
\]

\[
= \int_\tau^\infty f_{i\tau}(\tau)d\tau
\]

\[
(2.6)
\]
Denote by $t_i$ the time period in which an individual $i$ was observed to exit from a given state into another state.

**ASSUMPTION (A1).** The data $\{t_i\}_{i=1}^N$ consists of single spells censored at time $T$ and drawn from a single risk process.

**ASSUMPTION (A2).** The hazard rate is parameterized as

$$(2.7) \quad \lambda_i(\tau) = \lambda_0(\tau) \exp(X_i(\tau)\beta + V_i)$$

where $\lambda_0(\tau)$ is the baseline hazard, $X_i(\tau)$ are observed covariates that are allowed to vary over time, $\beta$ are model parameters, and $V_i$ is an unobserved heterogeneity component.

Hence, using (2.1) and (2.7) the integrated hazard is given by,

$$(2.8) \quad \Lambda_i(\tau) = \int_0^\tau \lambda_0(u) \exp(X_i(u)\beta + V_i) \, du$$

**ASSUMPTION (A3).** The baseline hazard $\lambda_0(u)$ and the values of the covariates are constant for each time period $t$.

Assumptions 1 and 2 are common in the literature. Assumption A3 is based on Han and Hausman (1990).

Given Assumption A3, instead of $\lambda_i(\tau)$ we can consider the integrated baseline hazard in the form

$$(2.9) \quad \mu_{0j} = \int_{j-1}^j \lambda_0(u) \, du,$$

where we denote the vector $(\mu_{01}, \ldots, \mu_{0T})$ by $\mu_0$.

Denote the probability of the exit event in time period $t$ by $P(t_i = t)$. Conditional on $V_i$, for outcomes that are not censored ($t_i \leq T$),

$$(2.10) \quad P(t_i = t) = F_i(\tau(t)) - F_i(\tau(t-1)) = (1 - S_i(t)) - (1 - S_i(t-1)) = S_i(t-1) - S_i(t)$$

When the duration observations are censored at $T$,

$$(2.11) \quad P(t_i > T) = 1 - F_i(\tau(T)) = S_i(T)$$

The following result is familiar in the literature and we include it here for the sake of completeness as a benchmark of comparison for the new competing risk model developed in the next Section.

**RESULT 1.** Under Assumptions A1–A3, conditional on $V_i$, for uncensored observations

$$(2.12) \quad P(t_i = t|V_i) = \exp\left(-\sum_{j=1}^{t-1} \mu_{0j} \exp(X_{ij}\beta + V_i)\right) - \exp\left(-\sum_{j=1}^t \mu_{0j} \exp(X_{ij}\beta + V_i)\right)$$
and for the censored case

\[ P(t_i > T|V_i) = \exp \left( -\sum_{j=1}^{T} \mu_{ij} \exp (X_{ij}\beta + V_i) \right) \]  

2.1. Parametric Heterogeneity

**ASSUMPTION (A4).** Let

\[ v_i \equiv \exp(V_i) \sim G(v) \]

where \( G(v) \) is a generic probability measure with density \( g(v) \).

Using the notation established in Assumption A4 and equation (2.8),

\[ \Lambda_i(\tau) = \int_0^\tau \lambda_0(u) \exp (X_i(u)\beta + V_i) \, du \]

(2.14)

For notational convenience, we will use subscripts for the time index and denote by \( \Lambda_{it} \) the quantity \( \Lambda_i(\tau) \) at the end of the time period \( t \), and similarly for other variables. Due to Assumption A3, (2.9), and (2.14), we have

\[ \tilde{\Lambda}_{it} = \sum_{j=1}^{t} \mu_{ij} \exp (X_{ij}\beta) \]

(2.15)

and

\[ \Lambda_{it} = v_i \tilde{\Lambda}_{it} \]

(2.16)

If \( v \) is a random variable with probability density function \( g(v) \) then the Laplace transform of \( g(v) \) evaluated at \( s \in \mathbb{R} \) is defined as

\[ \mathcal{L}(s) \equiv E_v[\exp(-vs)] \]

(2.17)

Using (2.3), (2.16), and (2.17), the expectation of the survival function can be linked to the Laplace transform of the integrated hazard function (Hougaard, 2000) as follows:

\[ E_v[S_{it}] = \mathcal{L}(\tilde{\Lambda}_{it}) \]

(2.18)

Using (2.15), (2.16), and (2.18) yields the unconditional exit probability of Result 1 as follows:

**RESULT 2.** The expectation of (2.10) for the uncensored observations is

\[ E_{v_i}[P(t_i = t)] = \mathcal{L}(\tilde{\Lambda}_{i(t-1)}) - \mathcal{L}(\tilde{\Lambda}_{it}) \]  

(2.19)
and the expectation of (2.11) for the censored observations takes the form

\[ E_{v_i} [P(t_i > T)] = \mathcal{L}(\tilde{\Lambda}_{iT}) \]  

Since the individual heterogeneity term \( v_i \) defined in Assumption A4 is non-negative, a suitable family of distributions \( G(v) \) with support over \([0, \infty)\) and tractable closed-form Laplace transforms is Generalized Inverse Gaussian (GIG) class of distributions, whose special case is the gamma distribution popular in duration analysis.

**ASSUMPTION (A5a).** The unobserved heterogeneity term \( v_i \) is distributed according to the Generalized Inverse Gaussian distribution,

\[ G(v) = G^{GIG}(v; \kappa, \varphi, \theta) \]

The GIG has the density

\[ g^{GIG}(v; \kappa, \varphi, \theta) = \frac{2^{\kappa-1}}{K_{\kappa}(\varphi)} \frac{\theta^{\kappa-1}}{\varphi^{\kappa}} \exp\left\{ -\theta v - \frac{\varphi^2}{4\theta v} \right\} \]

for \( \varphi, \theta > 0, \kappa \in \mathbb{R} \), where \( K_{\kappa}(\varphi) \) is the modified Bessel function of the second kind of order \( \kappa \) evaluated at \( \varphi \) (Hougaard, 2000). The GIG Laplace transform is given by

\[ \mathcal{L}^{GIG}(s; \kappa, \varphi, \theta) = (1 + \frac{s}{\theta})^{-\kappa/2} K_{\kappa} \left( \varphi \left(1 + \frac{s}{\theta} \right)^{1/2} \right) \]

The GIG family includes as special cases the gamma distribution for \( \varphi = 0 \), the inverse gamma distribution for \( \theta = 0 \), and the inverse Gaussian distribution for \( \kappa = -\frac{1}{2} \), among others.

Application of the Laplace transform of the GIG distribution (2.22) in Result 2 yields the following result that appears to not have been previously stated in the literature:

**RESULT 3.** Under the Assumptions A1–A4, and A5a

\[ E^{GIG}_{v_i} [P(t_i = t)] = \left( 1 + \frac{1}{\theta} \tilde{\Lambda}_{i(t-1)} \right)^{-\kappa/2} K_{\kappa} \left( \varphi \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{i(t-1)} \right)^{1/2} \right) \]

\[ \mathcal{L}^{GIG}(s; \kappa, \varphi, \theta) = (1 + \frac{s}{\theta})^{-\kappa/2} K_{\kappa} \left( \varphi \left(1 + \frac{s}{\theta} \right)^{1/2} \right) \]

and for the censored observations

\[ E^{GIG}_{v_i} [P(t_i > T)] = \left( 1 + \frac{1}{\theta} \tilde{\Lambda}_{iT} \right)^{-\kappa/2} K_{\kappa} \left( \varphi \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{iT} \right)^{1/2} \right) \]
A special case of the GIG distribution is the gamma distribution, obtained from the GIG density function (2.21) when $\varphi = 0$. We use the gamma distribution for $v_i$ as a benchmark model under the following alternative to Assumption 5a.

**ASSUMPTION (A5b).** The unobserved heterogeneity term $v_i$ is distributed according to the gamma distribution,

$$G(v) = G^G(v; \gamma, \theta)$$

The gamma density is parameterized as

(2.25) $$g^G(v; \gamma, \theta) = \frac{\theta}{\Gamma(\gamma)} (\theta v)^{\gamma-1} \exp(-\theta v)$$

and its Laplace transform is given by

(2.26) $$L^G(s; \gamma, \theta) = (1 + s/\theta)^{-\gamma}$$

In the gamma density (2.25) the parameter $\gamma > 0$ corresponds to the GIG parameter $\kappa \in \mathbb{R}$ in (2.21) restricted to the positive part of the real line. Using the gamma distribution in place of the GIG constitutes a special case of Result 3:

**RESULT 4.** Under the Assumptions A1–A4, and A5b,

(2.27) $$E^G_{v_i}[P(t_i = t)] = \left(1 + \frac{1}{\theta} \tilde{\Lambda}_{i(t-1)}\right)^{-\gamma} - \left(1 + \frac{1}{\theta} \tilde{\Lambda}_t\right)^{-\gamma}$$

and

(2.28) $$E^G_{v_i}[P(t_i > t)] = \left(1 + \frac{1}{\theta} \tilde{\Lambda}_t\right)^{-\gamma}$$

Result 4 was obtained in Han and Hausman (1990) and Meyer (1990).

In both gamma and GIG distributions, the scale parameter $\theta$ performs the same role. Specifically, for any $c \in \mathbb{R}_+$, if $v \sim G^G(v; \gamma, \theta)$ then $cv \sim G^G(cv; \gamma, \theta/c)$, and if $v \sim G^{GIG}(v; \kappa, \varphi, \theta)$ then $cv \sim G^{GIG}(cv; \kappa, \varphi, \theta/c)$. Due to this property, $c$ and hence its inverse $s \equiv c^{-1}$ are not separately identified from $\theta$ in the Laplace transform expressions (2.22) and (2.26). Since all likelihood expressions are evaluated at $s = \tilde{\Lambda}_{i(t)}$ which is proportional to $\mu_{0j}$ for all $j$, as specified in (2.15), any change in $\theta$ only rescales the baseline hazard parameters $\mu_{0j}$, leaving the likelihood unchanged. Hence, $\theta$ needs to be normalized to identify $\mu_{0j}$. In the gamma case, typically this normalization takes the form $\theta = \gamma$ so that $E[v] = 1$. We use the equivalent normalization for the GIG case in order to nest the normalized gamma as a special case and to maintain the moment restriction $E[v] = 1$. 
2.2. Flexible Heterogeneity

We now depart from the parametric form of the unobserved heterogeneity and instead consider a non-parametric infinite mixture for the distribution of $v_i$, as formulated in the following assumption.

**ASSUMPTION (A6).** The prior for $v_i$ takes the form of the hierarchical model

$$
t_i \sim F(v_i)
$$

$$
v_i | G \sim G
$$

$$
G \sim DP(G_0, \alpha)
$$

$$
\alpha \sim g_{\mathcal{G}}(a_0, b_0)
$$

$$
E[v_i] = 1
$$

In Assumption A6, $G$ is a random probability measure distributed according to a Dirichlet Process (DP) prior (Hirano, 2002; Chib and Hamilton, 2002). The DP prior is indexed by two hyperparameters: a so-called baseline distribution $G_0$ that defines the “location” of the DP prior, and a positive scalar precision parameter $\alpha$. The distribution $G_0$ may be viewed as the prior that would be used in a typical parametric analysis. The flexibility of the DP mixture model environment stems from allowing $G$ to stochastically deviate from $G_0$. The precision parameter $\alpha$ determines the concentration of the prior for $G$ around the DP prior location $G_0$ and thus measures the strength of belief in $G_0$. For large values of $\alpha$, a sampled $G$ is very likely to be close to $G_0$, and vice versa. Assumption A6 is then completed by specifying the baseline measure $G_0$. We consider two cases:

**ASSUMPTION (A7a).** In Assumption A6,

$$
G_0 = G_{\mathcal{G}}^{GIG}(\kappa, \phi, \theta)
$$

Implementation of the GIG mixture model under Assumptions A1–A3, A6, and A7a uses the probabilities (2.12), (2.13), (2.23) and (2.24). Further implementation details are given in the Appendix.

**ASSUMPTION (A7b).** In Assumption A6,

$$
G_0 = G^{\mathcal{G}}(\gamma, \theta)
$$

Under Assumptions A6 and A7b, as a special limit case, putting all the prior probability on the baseline distribution $G_0$ by setting $\alpha \to \infty$ would result in forcing $G = G_0 = G^{\mathcal{G}}(\gamma, \theta)$ which yields the parametric model of Han and Hausman (1990). Here we allow $\alpha$ and hence $G$ to vary stochastically, but the Han and Hausman (1990) specification is nested in our model which could potentially be supported by the data. Furthermore, the gamma baseline (2.30) results as a special case of the GIG baseline (2.29) under Assumptions A6 and A7a for the hyperparameter value $\phi = 0$. Hence, both the gamma flexible case with $G \sim DP(G^{\mathcal{G}}, \alpha)$ and the parametric benchmark Han and Hausman (1990) case with $G = G^{\mathcal{G}}$ are nested within our full GIG mixture model specification.
3. Competing Risk Model

We will now generalize the results from the single-risk case to the competing risk (CR) environment with several different potential types of exit. Let the risk type be indexed by $k = 1, \ldots, K$ and denote by $t^*_k$ the lifetime associated with $k$. Define the latent failure (or exit) times as $\tau^*_1, \ldots, \tau^*_K$ corresponding to each risk type $k$, for each individual $i$. Define their minimum by

$$\tau_i \equiv \min(\tau^*_1, \ldots, \tau^*_K)$$

In our CR model for interval outcome data, $\tau_i$ is not directly observed. Instead, the observed quantity is the time interval $[t-1, t)$ labeled as “$t$” which contains $\tau_i$. This is in contrast to a large class of other types of CR models where the exact failure time $\tau_i$ is directly observed, as is typical in biostatistics. Intrinsically, the lifetimes of other risk types, $\tau^*_j$ for $j \neq k$, and their corresponding time intervals, remain unobserved. For two risk types with $K = 2$, this yields the probability of exit at time $t$ of the form

$$P(t_{1i} = t, t_{2i} > t_{1i}) = \int_{t-1}^{t} \int_{u_1}^{t} f(u_1, u_2)du_2du_1 + \int_{t-1}^{t} \int_{u_1}^{\infty} f(u_1, u_2)du_2du_1$$

The first right-hand side term in (3.1) gives the probability that the second latent exit time occurred within the same time interval $t$ as the first latent exit time. The second right-hand side term in (3.1) is then the probability that the second latent exit time occurred in a later time interval than $t$. A key difficulty with evaluating (3.1), precluding direct factorization, is the presence of the outer integrand $u_1$ in the lower bound in the inner integral of the first term. We deal with this issue and derive a closed-form solution for (3.1), under various assumptions on the latent model components. In particular, the joint density $f(u_1, u_2)$ is obtained as a function of covariates and unobserved heterogeneity from the parameterization of risk-specific hazard functions, in a direct analogy to the single-risk case. Previous work using CR interval outcome data has either bypassed this link (e.g. by assuming a multivariate Gaussian density for $f(u_1, u_2)$) or employed a discrete time approximation whereby only one exit type can occur per any one time period. Our model explicitly accounts for the continuous-time nature of the exit decisions. The statistical background for the stochastic environment of our CR model is given in the Appendix.

For clarity of exposition, the numbering of the Assumptions and Theorems in this Section provides a direct counterpart to the Assumptions and Results of the single-risk case in the previous Section. We first treat the parametric case under the GIG and gamma distributions of unobserved heterogeneity, adding a common latent component for all risk types, and then proceed to infinite mixture modeling.

**ASSUMPTION (B1).** The data consists of single spell data, drawn from a process with two risks $k = 1, 2$, and is censored at at $T_k$.

Assumption B1 readily generalizes to an arbitrary number of risks. Without loss of generality, suppose that the failure type is of type 1 so that $t_{1i} = \min(t_{1i}, t_{2i})$. 
ASSUMPTION (B2). The risk-specific hazard rate is parameterized as
\[ \lambda_{ki}(\tau) = \lambda_0(\tau) \exp(X_i(\tau)\beta_k + V_{ki} + \zeta_k(\tau)) \]
where \( \lambda_0(\tau) \) is the baseline hazard, \( X_i(\tau) \) are covariates that are allowed to vary over time, \( \beta_k \) are model parameters, \( V_{ki} \) is an unobserved heterogeneity component, and \( \zeta_k(\tau) \) is a common correlated component.

ASSUMPTION (B3). For each \( k \), the baseline hazard \( \lambda_0(u) \) and the values of the covariates are constant for each time period \( t \).

The probability (3.1), conditional on \((V_i, \zeta)\) and a set of covariates, is
\[
P(t_{i1} = t, t_{2i} > t_{i1}|V_i, \zeta) = \int_{t-1}^t \int_{u_1}^t f(u_1, u_2|V_i, \zeta) du_2 du_1 + \int_t^\infty f(u_1, u_2|V_i, \zeta) du_2 du_1
\]
We derive a closed-form solution to (3.2) in the following Theorem which extends Result 1 to our CR model environment.

THEOREM 1. Under Assumptions A1–A4, conditional on the latent vector \((V_i, \zeta)\) and a set of covariates,
\[
P(t_{i1} = t, t_{2i} > t_{i1}|V_i, \zeta) = S_{2i(t-1)} S_{1i(t-1)} \lambda_{1it}(\lambda_{2it} + \lambda_{1it})^{-1}
\times [1 - \exp(- (\lambda_{2it} + \lambda_{1it}))]
\]
for uncensored observations, and
\[
P(t_{i1} > T, t_{2i} > T|V_i, \zeta) = (1 - F_{1iT})(1 - F_{2iT})
\]
for censored observations.

The proof is provided in the Appendix. The log-likelihood follows immediately from (3.4) as
\[
\ln P(t_{i1} = t, t_{2i} > t_{i1}|V_i, \zeta) = -\Lambda_{2i(t-1)} - \Lambda_{1i(t-1)} + \log(\lambda_{1it}) - \log(\lambda_{2it} + \lambda_{1it})
\]
\[
+ \log(1 - \exp(- (\lambda_{2it} + \lambda_{1it})))
\]

3.1. Parametric Heterogeneity in the CR Model

ASSUMPTION (B4). Let
\[ v_{ki} \equiv \exp(V_{ki}) \sim G_k(v_k) \]
where \( G_k(v_k) \) is a generic probability measure with density \( g_k(v_k) \). Assume \( \zeta_k(\tau) \) is piece-wise constant with values changing at the start of each time period \( t \). Denote such values \( \zeta_{kt} \) and their \( T \)-vector by \( \zeta_k \).

The prior for \( \zeta_{kt} \) is
\[
(\zeta_{1t}, \zeta_{2t}) \sim N\left(0, \begin{bmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_2 \sigma_1 & \sigma_2^2 \end{bmatrix}\right)
\]
with hyperparameters \( \rho, \sigma_1, \sigma_2 \).
As in the single-risk case, we consider two alternative forms of the distribution of unobserved heterogeneity \( G(v) \) in Assumption B4. The first form is parametric given either by the GIG or gamma density.

Here we provide new results regarding the model likelihood for the model B1–B4. These will be used in the nonparametric mixture model. This approach is different from Han and Hausman (1990) who considered the truncated multivariate Normal likelihood.

For the expected likelihood, we have two new expression for the expected probability of (3.3): one based on a quadrature, and another one with a series expansion without the need for a quadrature. The following Theorem extends Result 2 into the CR model environment.

**THEOREM 2.** Under Assumptions A1–A4,

\[
E_v P(t_{1i} = t, t_{2i} > t_{1i}) = \tilde{\lambda}_{1it} \int_0^1 L_2 \left( \tilde{\Lambda}_{2i(t-1)} + \tilde{\lambda}_{2it}s_1 \right) L_1^{(1)} \left( \tilde{\Lambda}_{1i(t-1)} + \tilde{\lambda}_{1it}s_1 \right) ds_1
\]

or

\[
E_v P(t_{1i} = t, t_{2i} > t_{1i}) = \sum_{r_2=0}^{\infty} \sum_{r_1=0}^{\infty} \frac{(-1)^{2r_1+2r_2+1}}{r_2! r_1! (r_2 + r_1 + 1)} \tilde{\lambda}_{1it}^{r_1+1} \tilde{\lambda}_{2it}^{r_2} \times L_1^{(r_1+1)} \left( \tilde{\Lambda}_{1i(t-1)} \right) L_2^{(r_2)} \left( \tilde{\Lambda}_{2i(t-1)} \right)
\]

for uncensored observations, and

\[
E_v P(t_{1i} > T, t_{2i} > T) = L_1 \left( \tilde{\Lambda}_{1iT} \right) L_2 \left( \tilde{\Lambda}_{2iT} \right)
\]

for censored observations, where \( L_k^{(r)} (s) \) is the \( r \)-th derivative of the Laplace transform.

The proof is given in the Appendix. Theorem 2 is derived for a generic distribution of the unobserved individual heterogeneity term \( v_i \) and provides a direct extension of (2.19) and (2.20) to the competing risk model environment. Specific alternative distributional assumptions with corresponding likelihood expressions are provided next.

**ASSUMPTION (B5a).** The unobserved heterogeneity term \( v_i \) is distributed according to the Generalized Inverse Gaussian distribution,

\[ G(v) = G^{GIG}(v; \kappa, \phi, \theta) \]

**ASSUMPTION (B5b).** The unobserved heterogeneity term \( v_i \) is distributed according to the gamma distribution,

\[ G(v) = G^{G}(v; \gamma, \theta) \]

The derivatives of the Laplace transform in (3.6) and (3.7) Theorem 2 depend on the functional form of the density kernel of \( v_i \), given in Assumptions B5a and B5b. The formulas for the derivatives of arbitrary order of the Laplace transform for the GIG or gamma densities do not appear to be available in the literature; we derive them in the Appendix. Using those expressions in Theorem 2 yields the following two Corollaries, extending Result 3 and 4, respectively, from the single-risk case to the competing risk model environment.
Corollary 1 (to Theorem 2). Under Assumptions B1–B4 and B5a, the functional forms of Theorem 2 are given in (7.43), (7.44), and (7.45) in the Appendix.

Corollary 2 (to Theorem 2). Under Assumptions B1–B4 and B5b, the functional forms of Theorem 2 are given in (7.47), (7.48), and (7.49) in the Appendix.

3.2. Flexible Heterogeneity in the CR Model

We will now proceed from the parametric case to an infinite mixture model for the distribution of $v_{ik}$.

**ASSUMPTION (B6).** The prior for $v_i$ is specified as the hierarchical model

\[
\begin{align*}
t_i & \sim F(\cdot; v_i|\zeta) \\
v_{ki}|G_k & \sim G_k \\
G_k & \sim DP(G_0, \alpha_k) \\
\alpha_k & \sim g^G(a_0, b_0) \\
E[v_{ki}] &= 1
\end{align*}
\]

The roles of the individual model components are described in Assumption A6 and generalize to the CR framework. Similarly to the single risk model environment, we consider two cases for the functional form of the baseline measure $G_0$:

**ASSUMPTION (B7a).** In Assumption B6,

\[G_{0k} = G^{GIG}(\kappa_k, \phi_k, \theta_k)\]

**ASSUMPTION (B7b).** In Assumption B6,

\[G_{0k} = G^G(\gamma_k, \theta_k)\]

Implementation of the mixture models under Assumptions B1–B3, B6, and B7a or B7b uses the probabilities derived in Theorem 1, Corollary 1, and Corollary 2. Further implementation details are given in the next Section and in the Appendix.

Cox (1962) and Tsiatis (1975) state that the simple competing risks model with no regressors is not identified. In particular, any competing risk model with correlated risks is observationally equivalent to some other competing risks model with independent risks. Heckman and Honoré (1989) show how the introduction of covariates allows identification of a large class of dependent competing risks models without invoking distributional assumptions. Nonetheless, normalization assumptions are necessary for parameter identification. The normalization constraints generalize directly from the single-risk case and we impose them for each risk type.
4. MCMC Posterior Sampling

4.1. Single Risk Model

For the implementation of the Dirichlet Process Mixture model (Assumptions A6 and A7a,b) we used the Bayesian generalized Pólya urn scheme (Neal 2000 Algorithm 2; West, Müller, and Escobar, 1994; Bush and MacEachern, 1996). The posterior sampling of \( v_i \) takes the form

\[
v_i | v_{-i} \sim q_0 H_i + \sum_{j=1, j \neq i}^N q_{ij} \delta_{v_j}
\]

\[
q_0 \propto \alpha \int h(t_i|v) dG_0(v)
\]

\[
q_{ij} \propto h(t_i|v_j)
\]

where \( H_i \) is the posterior for \( v_i \) based on the prior \( G_0(v) \) and the single observation \( t_i \) with likelihood \( h(t_i|v_i) \), while \( \delta_{v_j} \) is the distribution concentrated at the single point \( v_j \). Implementation of the GIG mixture model (Assumptions A1–A3, A6, and A7a) uses (2.12) and (2.13) for \( h(t_i|v) \), while \( \int h(t_i|v) dG_0(v) \) is given by (2.23) and (2.24). The gamma mixture model (Assumptions A1–A3, A6, and A7b) uses (2.12), (2.13), (2.27) and (2.28), respectively. The remaining model parameters were sampled in standard Gibbs blocks using Hybrid Monte Carlo (Neal 2010) with diffuse priors unless stated otherwise above. The reported posterior means were obtained from Markov Chain Monte Carlo (MCMC) chains of total length of 30,000 steps with a 10,000 burn-in section. All models were implemented using the Intel Fortran 95 compiler on a 2.8GHz Unix machine under serial compilation. For a sample of 15,398 individuals, the single-risk model implementation took approximately 3 hours for \( T = 6 \), 4 hours for \( T = 13 \), and 6 hours for \( T = 24 \) to run.

In the gamma mixture model we found that the probability mass of the individual heterogeneity component was accumulating at zero, with a thin right tail diverging to infinity, leading to a degenerate outcome. We believe this to be an artefact of the gamma density kernel shape with mode at zero for mean less than or equal to one. In contrast, for the GIG density under the Assumptions A6 and A7a, we obtained a well-defined stable nonparametric heterogeneity clustering without the degenerative tendencies of the gamma. We attribute this outcome to the more flexible functional form of the GIG with a well-defined mode at a strictly positive value for \( v \) for mean values smaller than one. The results are discussed in our application below.

4.2. CR model

Similarly to the single-risk case, for the implementation of the Dirichlet Process Mixture model in the competing risk environment (Assumptions B6 and B7a,b) we also used the Bayesian generalized Pólya
urn scheme. The posterior sampling of $v_{ki}$ takes the form
\[ v_{ki} | v_{-ki} \sim q_{k0} H_{ki} + \sum_{j=1, j\neq i}^{N} q_{kij} \delta_{v_j} \]
\[ q_{k0} \propto \alpha_k \int h(t_i | v) dG_0(v) \]
\[ q_{kij} \propto h(t_i | v_{kij}) \]
where $H_{ki}$ is the posterior for $v_{ki}$ based on the prior $G_0(v)$ and the single observation $t_i$ with likelihood $h(t_i | v_i)$, while $\delta_{v_j}$ is the distribution concentrated at the single point $v_j$.

Implementation of the GIG mixture model (Assumptions B1–B3, B6, and B7a) uses (3.3) and (3.4) for $h(t_i | v)$ from Theorem 1, and (7.43) and (7.45) for $\int h(t_i | v) dG_0(v)$, as derived in Corollary 1 to Theorem 2. The gamma mixture model (Assumptions B1–B3, B6, and B7b) uses (7.47) and (7.49) for the latter integral, from Corollary 2 to Theorem 2.

The remaining model parameters were sampled in standard Gibbs blocks using Hybrid Monte Carlo (Neal 2010) with diffuse priors unless stated otherwise above. The reported posterior means were obtained from Markov Chain Monte Carlo (MCMC) chains of total length of 30,000 steps with a 10,000 burn-in section. All models were implemented using the Intel Fortran 95 compiler on a 2.8GHz Unix machine under serial compilation. For a sample of 1,317 individuals, the CR model implementation took approximately 2 hours for $T = 6$, 6 hours for $T = 13$, and 13 hours for $T = 24$.

In the gamma mixture model we obtained a similar degenerate outcome to the single-risk case, with probability mass of the individual heterogeneity component accumulating at zero and a thin right tail diverging to infinity. For the GIG mixture we obtained a well-defined stable nonparametric heterogeneity clustering. As mentioned above, we attribute this outcome to the restrictive shape properties of the gamma density kernel with mode at zero for mean less than or equal to one, while the more flexible GIG has a well-defined non-zero mode for the whole non-zero mean range. The results of the implementation are discussed in the application section below.

5. Application

Since its introduction in 1935 as part of Roosevelt’s Social Security Act, unemployment insurance (UI) benefits provide partial insurance to workers who become unemployed. Most states offer unemployment insurance for up to 26 weeks. Neoclassical economic thought suggests that higher benefits also lead to reduced incentives to search for a job, thus prolonging the period of time an individual spends out of employment (Mulligan, 2012). As a result policy makers have placed increased emphasis on reforming the UI system by rewarding personal responsibility rather than bad luck. This has lead to a shift away from the unqualified provision of UI benefits towards a system that is search intensive, making benefits conditional on providing evidence that the potential recipient engaged in a certain minimum amount of job search. Additionally, schemes whereby individuals are provided one-off grants that attempt to alleviate
temporary hardship rather than longer term UI benefits are advocated. At the same time the recent economic crisis has forced policy makers to extend the duration of unemployment insurance benefits for up to 99 weeks to help workers deal with the prolonged economic downturn and high unemployment rates in some states.\footnote{The unemployment extension legislation is set to expire on January 1, 2013.}

Applied economists require econometric tools to accurately estimate the impact of unemployment insurance on the duration of unemployment, while accounting for state unemployment rates, generosity of unemployment insurance benefits and workers’ observed and unobserved heterogeneity. In this section we apply our approach to analyzing the determinants of unemployment duration among unemployment insurance recipients. We will stress the importance of relaxing the parametric assumptions of the econometric models and accounting for correlations in the competing exit choices faced by workers. One of the major advantages of our approach is that it is possible to simulate counterfactual policy changes which can inform policy makers on the relative merits of various changes that may be contemplated. We will illustrate this feature by evaluating the impact of a change in the replacement rate on the duration of unemployment.

5.1. Data

We use data from the Needels et. al. (2001) report submitted to the U.S. Department of Labor that is based on a nationally representative sample built from individual-level surveys of unemployment insurance (UI) recipients in 25 states between 1998 and 2001. Candidates for the survey are selected on the basis of administrative records and are sampled from the pool of unemployed individuals that started collecting UI benefits at some point during the year 1998.

We are interested in analyzing the effect of unemployment insurance on the duration of unemployment. The duration of unemployment is measured in weeks. At the time of the survey and from the states that were included in the survey only two states provided UI benefits for a maximum of 30 weeks, the rest providing UI benefits for a maximum of 26 weeks. Theoretical models of the impact of UI benefits on unemployment duration, such as Mortensen (1977) and Moffitt and Nicholson (1982) make precise predictions for the shape of the hazard rate at the time of benefit exhaustion. These models predict an increasing hazard up to the point of benefit exhaustion and a flat one afterwards. We limit our study to the first 24 weeks of unemployment due to the recognized change in behavior in week 26 when UI benefits cease for a significant part of the sample (see, e.g., Han and Hausman, 1990), and which would affect the econometric model in a substantial fashion.

The data contain individual-level information about labor market and other activities from the time the person entered the UI system through the time of the interview. The data include information about the individual’s pre-UI job, other income or assistance received, and demographic information. We use two indicator variables, race (defined as an indicator for black) and age (defined as an indicator for over 50). We further use the replacement rate, which is the weekly benefit amount divided by the UI recipient’s base
period earnings. Lastly, we utilize the state unemployment rate of the state from which the individual received UI benefits during the period in which the individual filed for benefits. This variable changes over time. The Needels et. al. (2001) data shares certain similarities to the PSID dataset used in Han and Hausman (1990). The UI recipients are mostly white, young, poorly educated workers who find themselves below or very near the poverty line.\footnote{Note that the labor market conditions captured in this dataset are substantially different than the ones experienced today. According to the Bureau of Labor Statistics (BLS), the latest figures broken down by state for September 2012 show the mean state unemployment rate is 7.5\% and varies between 3\% and 11.8\%. In contrast, in our dataset the state unemployment rate is approximately 4.5\%.}

Below we will estimate a single risk duration model for the duration to re-employment and also a competing risk duration model for the duration to re-employment using our proposed approach, which treats new jobs and recalls to previous jobs separately while allowing for correlations between the two types of risk. For the single risk model we use a sample consisting of 15,398 individuals. Summary statistics for this sample are given in Table 1. For a subset of 1,317 of these individuals we also know whether they were recalled to a previous job or whether they found a new job. Individuals with recall status equal to zero correspond to those that were being told by their previous employer that they will not be recalled to their previous job. UI recipients with recall status equal to 1 correspond to those who expected to be recalled but started receiving UI benefits without any firm assurance that they would or individuals that expected to be recalled based on some explicit information they received. We denote individuals which are recalled to a previous job as individuals with risk type 1, while those who are not as individuals with risk type 2. Summary statistics for this subsample is given in Table 2. We note a marked difference in the unemployment durations of these two groups of individuals. Figure 1 provides plots of the number of individuals who exited in each time period, shown separately for recalls to previous jobs (risk type 1) and for entries into new jobs (risk type 2). Individuals who had reasonable expectations to be recalled to a previous job exit unemployment much faster in the first few weeks after they lose their job but conditional of not having been recalled by week 8 their exit pattern resembles that of the other individuals.

5.2. Single Risk Duration Model with Flexible Heterogeneity

Estimation results of the semiparametric duration model with a flexible form of unobserved heterogeneity under GIG mixing (Assumptions A1–A3, A6, and A7a) are presented in Table 3. In addition to the above mentioned censoring at $T = 24$ weeks, we also include the benchmark cases where we censor at $T = 6$ and $T = 13$. All of our variables (state unemployment rate, race, age, and replacement rate) are estimated to have a negative significant impact on the hazard rate of exiting unemployment. Recall however, that when comparing the estimates of the $\beta$ coefficients, the scaling changes depending on the variance of the estimated heterogeneity distribution. Thus, the ratios of the coefficients should be compared, as opposed to their absolute values. This makes the interpretation of the coefficients less transparent. We note however that the coefficient estimates obtained from the flexible model are substantively different than those obtained from the parametric model. As discussed above, the parametric restriction on the
heterogeneity distribution can lead to inconsistent estimates if the true mixing distribution does not exactly correspond to the parametric specification.

We would expect to obtain similar results irrespective of the truncation point and thus the coefficients obtained for the models truncated at 6, 13, and 24 weeks to be very similar. While the coefficient ratios are not constant they tend to be relatively similar. The one exception comes from the ratios involving the replacement rate, in particular for the model with censoring at 24 periods. This is similar to the results in Hausman and Woutersen (2012) and might be explained by behavioral changes as individuals approach the date of UI exhaustion.

The estimated distribution of the unobserved individual heterogeneity is presented in Figure 2. The estimated distributions can only be very roughly approximated by the gamma distribution. While in all three cases the mode of the estimated distribution of the latent heterogeneity term $V_i = \log(v_i)$ is negative, as we increase the number of periods used in the estimation the distribution acquires a more pronounced left tail. This indicates that as we observe individuals over a longer period of time the model captures a larger extent the part of unobserved heterogeneity which prevents workers from finding employment and thus becomes indicative of the propensity for long term unemployment.

The survival function estimates along with 95% confidence bands are presented in Figure 3, featuring the anticipated downward sloping shape. The smoothing parameter $\alpha$ of the Dirichlet Process (DP) Mixture model introduced in Assumption A6 controls the extent to which the DP draws mixture distributions that are more or less "similar" to the baseline parametric distribution $G_0$. In the limiting case of $\alpha \to \infty$ the mixture distribution becomes equivalent to $G_0$, while in the other extreme $\alpha \to 0$ the mixture distribution limits to a convolution of density kernels centered at each data point without any influence of the DP prior. The posterior distribution estimates of $\alpha$ are plotted in Figure OA1 in the Online Appendix. The distributions are concentrated around a well-defined mode with a value of less than 1 indicating a strong influence of data relative to the baseline prior distribution thereby providing a high degree of support in favor of our nonparametric approach.

In the Online Appendix we also present estimation results for two benchmark parametric models. Estimation results of a model with parametric gamma heterogeneity (Han and Hausman, 1990; Meyer 1990), as specified in Assumptions A1–A4 and A5b, are given in Table OA1. In Table OA2 we present estimation results for another benchmark model with parametric GIG heterogeneity (Assumptions A1–A4 and A5a).

5.3. Competing Risk Model with Flexible Heterogeneity

We now present the results of our newly proposed competing risk model with a flexible form of unobserved heterogeneity using GIG mixing and correlated risks (Assumptions B1-B3, B6, and B7a). Recall that in our example risk 1 corresponds to the event that a worker is recalled to a previous job, while risk 2 corresponds to the event that she finds a new job which is different from the previous one. We present
the estimated coefficients in Table 4. For all three censoring times \((T = 6, 13, 24)\), the partial effects of race and age are not statistically significant, with a few isolated exceptions. This could be due to smaller sample size available for the competing risk case as opposed to the single-risk case, with the former consisting of less than 10% of observations of the latter. For all three censoring times, the relative influence of the replacement rate is declining from \(T = 13\) to \(T = 24\) indicating the impact of benefit exhaustion. It is significant to notice that while the effect of age in the two risk types is comparable, the estimated effect of race differs. The probability of being recalled to a previous job is substantially lower for black workers, thus being potentially indicative of discrimination in the labor market.

The estimated density\(^9\) of unobserved heterogeneity \(V_{ik}\) is shown in Figure 4 for the GIG mixture model for both risk types \(k = 1, 2\), each centered at the time average of the risk-specific latent common time effect \(\zeta_{kt}\) to reflect the overall influence of the unobserved heterogeneity component \(\zeta_{kt} + V_{ik}\). The differences between the density of unobserved individual heterogeneity further highlight the importance of distinguishing between the different risk types in the competing risk model environment as compared to the single-risk duration case. In particular the two distributions of are distinct and well-separated indicating that conditional on observed covariates there is a significant degree of sorting between workers recalled to a previous job and those who are not. While the mode of both distributions is negative, workers who are recalled to a previous job possess latent attributes that make them more desirable that workers who are not.

Figure 6 shows the estimated correlation structure of the latent time variables \(\zeta_{kt}\) common to all individuals, defined in Assumption B4, for the GIG mixture model in terms of the estimated densities for the variances \(\sigma^2_1, \sigma^2_2\), and the correlation coefficient \(\rho\) between \(\zeta_{1t}\) and \(\zeta_{2t}\) for the two different risk types. Interestingly, most probability mass for the density of \(\rho\) is negative for \(T = 6\), around zero for \(T = 13\) and positive for \(T = 24\). This suggests a negative correlation of common shocks for recalls versus new jobs for the first several weeks of unemployment with a subsequent correlation reversal in later time periods. This finding explains the exit counts shown in Figure 1 for recalls and new jobs, with high ratios of recalls to new jobs for the first few weeks abetting to parity around week six.

The survival function estimates along with 95% confidence bands are presented in Figure 5 for both types of risks. The differences in the shapes for the first few weeks are striking and also indicative of the differences in exit rates between workers recalled to a previous job and those who are not in the first few weeks after they lose their job. The estimated survival function for workers who are recalled to their previous job is convex while that for workers who are not recalled is concave, indicating a much slower overall re-integration into the labor market. This appears to confer a long term advantage with the overall probability of being unemployed being substantially higher for workers looking for a new job.

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\(^9\) In Table 4 we report the estimated GIG mixture model coefficients but as these enter all mixing kernel moments their interpretation is not immediate. Hence it appears more informative to examine the resulting mixture density estimate.
The posterior distribution estimates of the smoothing parameter \( \alpha \) of the Dirichlet Process Mixture model (Assumption B6) are plotted in Figure OA2 in the Online Appendix. The distributions are concentrated around a well-defined mode of value less than five, indicating a strong influence of data relative to the baseline prior distribution, albeit with a relatively more influential prior than in the single-risk case.

It is informative to contrast the estimates from our preferred model with those obtained under different modeling assumptions on the unobserved heterogeneity which are detailed in the Online Appendix. Table OA3 presents the estimated coefficients from a model that ignores the presence of individual heterogeneity, Table OA4 corresponds to a model which assumes parametric gamma distributed heterogeneity, Table OA5 estimates a competing risk model with parametric GIG heterogeneity, and Table OA6 presents the estimation results from a flexible model which estimates the unobserved semi-parametrically using an infinite mixture of GIG distributions but also further imposes the assumption of independence between the different risks. In Table OA7 we present estimation results from our single risk GIG mixture model but applied to the subsample of observations which records the outcomes for the two competing risks.

Given the large number of parameters to be considered it is helpful to compare these different models in a graphical setting. In order to facilitate the comparison between the model which pools the two risks and the models which do not we can combine the two risks into a common survival function, as discussed in the Appendix. Thus, in Figure 7 we compare the estimated survival function of our CR GIG mixture model (Assumptions 1–3, B6 and B7a, labeled as "CR full") with its estimates in three restricted model versions: 1) the parametric GIG case (labeled as "CR param"), 2) the independent risks case where we estimate a single-risk model separately for each risk type of data and then merge their survival functions ex-post (labeled as "CR indep"); 3) the case without individual unobserved heterogeneity under the restriction \( V_{ik} = 0 \) (labeled as "CR no ihet"); and 4) the single-risk case where we do not distinguish between risk types in the competing risk data (labeled as "SR full").

Two features are particularly significant. First, we notice that if we enforce the assumption of independence of the two risk types, the resulting common survival function is severely downward biased. The magnitude of the bias dominates the other modeling choices which we make on the specification of unobserved heterogeneity. This could be due to the distributional effects of the risk correlations (Figure 7) that are absent in the independent risk model. Second, we plot the confidence bounds for our proposed model which allows for a flexible specification of the unobserved heterogeneity but also for correlated competing risks. We notice that all other more restrictive specifications are downward biased and the differences become statistically significant as the number of time periods increases.

5.4. Counterfactual Policy Evaluation

One of the advantages of our model consists in the explicit estimation of the unobserved heterogeneity components which enables us to evaluate the effectiveness of counterfactual policy experiments taking into account the distributional effects of individual heterogeneity. As discussed above, one of the main policy questions currently faced by economists is the extent to which the generosity of unemployment
insurance benefits impacts the workers’ incentives to find employment once their lose their job. On the one hand, more generous benefits are expensive to provide given the ongoing debt crisis and may actually prove detrimental in the long run as they may erode workers’ incentives to find a job quickly. Thus they would ultimately contribute to increasing long run unemployment. On the other hand, low levels of unemployment insurance benefits can make unemployment very difficult for many low income families. Poverty can also have a negative effect on their ability to find employment since job search is costly and in the absence of unemployment insurance benefits many workers may find themselves unable to support their families while also searching for an adequate job. As a result workers may end up underemployed or leave the labor market altogether. The relative magnitude of the impact of incentives over poverty is an empirical question and a counterfactual analysis using model estimates can provide some evidence in this debate.

In the context of our model we can consider changing the replacement rate in order to investigate its impact on the probability of exit from unemployment as captured by the survival function. We can perform this policy counterfactual using both the single risk and the competing risk model. For clarity, we combine the two risk types in the CR model into a common survival function as described in the Appendix. The counterfactual experiment consists in increasing and decreasing the replacement rate by 10%. We present counterfactual results from our preferred specification which flexibly models the unobserved heterogeneity as an infinite GIG mixture. The estimated and counterfactual survival curves under the two scenarios are presented in Figure 8 for the single risk dataset and in Figure 9 for the competing risk subsample.

Both Figures show that the survival function moves in the anticipated direction: for a replacement rate decrease the probability of staying unemployed is lower, and for replacement rate increase the probability of continued unemployment is higher. However, the changes are relatively small. For example, for $T = 24$ in the final period the survival function changes by $-2.8\%$ and $2.4\%$ for the CR data, respectively. This suggests that while the estimated impact of a change in unemployment benefit generosity has the sign predicted by economic theory, the magnitude of the impact on the probability of unemployment exit is inelastic. Policy makers may thus wish to consider the extent to which cutting unemployment benefits may ultimately influence an unemployed worker’s welfare.

6. Conclusion

We introduced a new flexible model specification for the competing risk model with piecewise linear baseline hazard, time-varying regressors, risk-specific unobserved individual heterogeneity distributed as an infinite mixture of density kernels, and a common correlated latent effect. Unobserved individual heterogeneity is assumed to be distributed according a Bayesian Dirichlet Process mixture model with a data-driven stochastic number of mixture components estimated along with other model parameters. We derive a tractable likelihood for Generalized Inverse Gaussian (GIG) mixing based on scaled GIG Laplace transforms and their higher-order derivatives. We find that mixing under a special case of the GIG, the
gamma kernel, leads to degenerate outcomes in nonparametric mixtures motivating the use of the more flexible GIG. We apply our approach to analyzing unemployment duration with exits to previous or new jobs among unemployment insurance recipients on nationally representative individual-level survey data from the U.S. Department of Labor. We also conduct a counterfactual policy experiment that changes the replacement rate and find that the extent to which cuts in unemployment benefits incentivize unemployed workers is relatively very small.

7. Appendix: Proofs and Derivations

7.1. CR Stochastic Environment

Consider the CR model setup with interval outcome data and latent exit times, as described in the main text. In this section we will initially omit the subscripts \(i\) and \(t\) and also covariates and heterogeneity variables to focus on the general model, without loss of generality. We will then include these elements into the model as needed. Denote the latent exit time variables by \(\tau^* = (\tau_1^*, \ldots, \tau_K^*)\) while the time integration variables by \(u = (u_1, \ldots, u_K)\), assumed conditionally independent.

The cause-specific hazard function for the \(k\)-th cause, which is the hazard from failing from a given cause in the presence of the competing risks, is defined as

\[
\lambda_k(u_k) = \lim_{h \to 0} \frac{\Pr(u_k < \tau_k^* \leq u_k + h; k|\tau_k^* > u_k)}{h}
\]

The joint hazard from all causes is

\[
\lambda(u) = \lim_{h \to 0} \frac{\Pr(u < \tau^* \leq u + h|\tau > u)}{h} = \sum_{k=1}^{K} \lambda_k(u_k)
\]

where all inequalities are defined element-wise. The cause-specific integrated hazard is

\[
\Lambda_k(\tau_k^*) = \int_{0}^{\tau_k^*} \lambda_k(u_k)du_k
\]

and the joint integrated hazard is

\[
\Lambda(\tau^*) = \int_{0}^{\tau^*} \lambda(u)du = \int_{0}^{\tau} \sum_{k=1}^{K} \lambda_k(u_k)du = \sum_{k=1}^{K} \int_{0}^{\tau} \lambda_k(u_k)du_k = \sum_{k=1}^{K} \Lambda_k(\tau_k^*)
\]

The joint survival function is

\[
S(u) = \Pr(\tau^* > u) = \exp(-\Lambda(u))
\]
which is the complement of the probability of failure from any cause up to time \( \tau \) given by the overall cumulative distribution function

\[
F(u) = \Pr(\tau^* \leq u) = 1 - S(u)
\]

For ease of exposition, we will focus on the case of two risk types with \( K = 2 \). The joint density of failure at time \( u \) is thus given by

\[
f(u_1, u_2) = \frac{\partial^2 F(u_1, u_2)}{\partial u_1 \partial u_2}
= -\frac{\partial^2 S(u_1, u_2)}{\partial u_1 \partial u_2}
= -\frac{\partial^2 \exp \left( -\Lambda_1(u_1) - \Lambda_2(u_2) \right)}{\partial u_1 \partial u_2}
= \exp \left( -\Lambda_1(u_1) - \Lambda_2(u_2) \right) \lambda_1(u_1) \lambda_2(u_2)
\]

(7.4)

Equation (7.4) links \( f(u_1, u_2) \) with the risk-specific hazard functions. Parametrization of the latter in terms of covariates and unobserved heterogeneity \( (V, \zeta) \) is given by Assumption B2. We will now invoke this Assumption and reintroduce \( (V, \zeta) \), while suppressing notational conditioning on the covariates \( X \) without loss of generality.

Note that conditional on \( X \) the failure times \( u_1 \) and \( u_2 \) are dependent since \( \zeta_1t \) and \( \zeta_2t \) are correlated. However, conditional on \( X, V, \zeta \) the failure times \( u_1 \) and \( u_2 \) are independent. Hence \( f(u_1, u_2|V, \zeta) \) can be factorized into the product

\[
f(u_1, u_2|V, \zeta) = f(u_1|V, \zeta)f(u_2|V, \zeta)
\]

From (7.4) it follows that

\[
f(u_k|V, \zeta) = \exp \left( -\Lambda_k(u_k) \right) \lambda_k(u_k)
\]

(7.5)

Define the function

\[
S_k(u_k) \equiv \exp \left( -\Lambda_k(u_k) \right)
\]

(7.6)

for \( k \in \{1, 2\} \). From (7.5) and (7.6) we have,

\[
f(u_k|V, \zeta) = S_k(u_k)\lambda_k(u_k)
\]

(7.7)

From (7.1), (7.6), and (7.7) it follows that

\[
\int_{t-1}^{t} f(u_k|V, \zeta) du_k = S_k(t-1) - S_k t
\]

(7.8)

The density (7.7) should not be confused with the so-called subdensity function \( f_j(u_j) = S(u = u_j)\lambda_j(u_j) \) that is sometimes used in CR analysis. Moreover, the function \( S_k(u_k) \) defined in (7.6) does not, in general,
have the survival function interpretation for $K > 1$. Nonetheless, examining (7.2), (7.3), and (7.6) reveals that the product of $S_k(u_k)$ over $k$ equals the joint survival function:

$$S(u) = \prod_{k=1}^{K} S_k(u_k)$$

(for further details of interpretation of functions with survival-like properties see e.g. Porta, Gomez, and Calle 2008). In general, the unconditional product form of (7.9) characterizes independent risks. However, dependence among risks can be introduced by conditioning each $S_k(u_k)$ on variables correlated across the risk types.

### 7.2. Competing Risk Model: Conditional Likelihood

From (3.2),

$$P(t_{1i} = t, t_{2i} > t_{1i}|V_i, \zeta) = A + B$$

where

$$A = \int_{t-1}^{t} \int_{u_1}^{t} f(u_1, u_2|V_i, \zeta) du_2 du_1$$

$$B = \int_{t-1}^{t} \int_{t}^{\infty} f(u_1, u_2|V_i, \zeta) du_2 du_1$$

The expression $A$ is more difficult to evaluate than $B$ since in $A$ the lower bound $u_1$ of the inner integral is an argument of the outer integral. In contrast, the two integrals in $B$ are independent of each other and hence can be factorized.

Thus,

$$A = \int_{t-1}^{t} \int_{u_1}^{t} f_{it}(u_1|V_{1i}, \zeta_{it}) f_{it}(u_2|V_{2i}, \zeta_{2it}) du_2 du_1$$

$$= \int_{t-1}^{t} \left[ \int_{u_1}^{t} f_{it}(u_2|V_{2i}, \zeta_{2it}) du_2 \right] f_{it}(u_1|V_{1i}, \zeta_{it}) du_1$$

where $u_k \in [t-1, t)$ for $k \in \{1, 2\}$. For the inner integral in (7.11), using (7.8)

$$f_{it}(u_2|V_{2i}, \zeta_{2it}) = \exp\left(-\Lambda_{ki}(t) - s_k \lambda_{kit}\right) \lambda_{kit}$$

Similarly,

$$S_{ki}(u_j) = S_{ki(t-1)} \exp\left(-s_j \lambda_{kit}\right)$$
for \(k, j \in \{1, 2\}\). Using (7.12), and integration by substitution with (7.13) for \(k = 1\) and with (7.14) for \(k = 2, j = 1\), in (7.11) yields

\[
A = \int_{t-1}^t [S_{2t}(u_1) - S_{2it}] f_{it}(u_1|V_i, \xi_t)du_1
\]

\[
= S_{2i(t-1)} \int_0^1 \exp \left( - (s_1 \lambda_{2it}) \right) \exp \left( - \Lambda_{1i(t-1)} \right) \exp \left( - s_1 \lambda_{1it} \right) \lambda_{1it}ds_1
\]

\[
- S_{2it} \int_0^1 \exp \left( - \Lambda_{1i(t-1)} \right) \exp \left( - s_1 \lambda_{1it} \right) \lambda_{1it}ds_1
\]

(7.15)

where

\[
A_{11} = S_{2i(t-1)} \int_0^1 \exp \left( - (s_1 \lambda_{2it}) \right) \exp \left( - \Lambda_{1i(t-1)} \right) \exp \left( - s_1 \lambda_{1it} \right) \lambda_{1it}ds_1
\]

\[
= S_{2i(t-1)} S_{1i(t-1)} \lambda_{1it} \lambda_{2it} + \lambda_{1it})^{-1} \int_0^1 \exp \left( - (s_1 \lambda_{2it} + \lambda_{1it}) \right) \lambda_{2it} + \lambda_{1it})ds_1
\]

(7.16)

\[
A_{12} = - S_{2it} \int_0^1 \exp \left( - \Lambda_{1i(t-1)} \right) \exp \left( - s_1 \lambda_{1it} \right) \lambda_{1it}ds_1
\]

\[
= - S_{2it} S_{1i(t-1)} \int_0^1 \exp \left( - s_1 \lambda_{1it} \right) \lambda_{1it}ds_1
\]

(7.17)

Using (7.16) and (7.17) in (7.15) yields

(7.18)

\[
A = S_{2it} S_{1it} \left\{ 1 - \exp(\lambda_{1it} - \lambda_{1it}(\lambda_{2it} + \lambda_{1it})^{-1} \left[ 1 - \exp (\lambda_{2it} + \lambda_{1it}) \right] \right\}
\]

The expression for \(B\) of (3.2) is given by

\[
B = [F_{1it} - F_{1i(t-1)}] \left[ 1 - F_{2it} \right]
\]

\[
= [S_{1i(t-1)} - S_{1it}] S_{2it}
\]

\[
= S_{1i(t-1)} S_{2it} - S_{1it} S_{2it}
\]

(7.19)

Combining (7.18) and (7.19) in (7.10) yields

\[
P(t_1 | t, t_2 > t_1| V_i, \xi) = S_{2i(t-1)} S_{1i(t-1)} \lambda_{1it} (\lambda_{2it} + \lambda_{1it})^{-1} \times \left[ 1 - \exp (- (\lambda_{2it} + \lambda_{1it})) \right]
\]

with the resulting log-likelihood

\[
\ln P(t_1 | t, t_2 > t_1| V_i, \xi) = - \Lambda_{2i(t-1)} - \Lambda_{1i(t-1)} + \log(\lambda_{1it}) - \log (\lambda_{2it} + \lambda_{1it})
\]

\[
+ \log (1 - \exp (- (\lambda_{2it} + \lambda_{1it})))
\]
7.3. Competing Risk Model: Integrated Likelihood

7.3.1. Quadrature Version

Here we derive an expression for the expectation of the exit probability (3.2) with respect to unobserved heterogeneity for each risk type, based on a simple quadrature. Taking the expectation of (3.2) yields

\[
E_v P(t_{1i} = t, t_{2i} > t_{1i}) = E_v \int_{t_{1i}}^{t} \int_{u_1}^{\infty} f(u_1, u_2 | V_i, \zeta_i) du_2 du_1 \\
+ E_v \int_{t_{1i}}^{t} \int_{u_1}^{\infty} f(u_1, u_2 | V_i, \zeta_i) du_2 du_1 \\
= E_v \int_{t_{1i}}^{t} \int_{u_1}^{\infty} f(u_1, u_2 | V_i, \zeta_i) du_2 du_1 \\
= E_v \int_{t_{1i}}^{t} \int_{u_1}^{\infty} f_{it}(u_1 | V_{1i}, \zeta_{1i}) f_{it}(u_2 | V_{2i}, \zeta_{2i}) du_2 du_1 \\
= E_v \int_{t_{1i}}^{t} \int_{u_1}^{\infty} f_{it}(u_2 | V_{2i}, \zeta_{2i}) du_2 f_{it}(u_1 | V_{1i}, \zeta_{1i}) du_1 \\
= \int_{t_{1i}}^{t} E_{v_{2i}} [S_{2i}(u_1)] E_{v_{1i}} [f_{it}(u_1 | V_{1i}, \zeta_{1i})] du_1
\]

(7.20)

From (7.6),

\[
E_{v_{2i}} [S_{2i}(u_1)] = \mathcal{L}_2 (\bar{\lambda}_{2i}(u_1))
\]

(7.21)

Using (7.5),

\[
E_{v_{1i}} [f_{it}(u_1 | V_{1i}, \zeta_{1i})] = E_{v_{1i}} [\exp (-\lambda_{1i}(u_1)) \lambda_{1i}(u_1)] \\
= \tilde{\lambda}_{1i}(u_1) E_{v_{1i}} [\exp (-v_{1i} \bar{\lambda}_{1i}(u_1)) v_{1i}] \\
= -\tilde{\lambda}_{1i}(u_1) \mathcal{L}_1^{(1)} (\bar{\lambda}_{1i}(u_1))
\]

(7.22)

where \( \mathcal{L}_1^{(1)} (s) \) is the first derivative of the Laplace transform \( \mathcal{L}(s) \) evaluated at \( s \). Using (7.21) and (7.22) in (7.20) yields

\[
E_v P(t_{1i} = t, t_{2i} > t_{1i}) = - \int_{t_{1i}}^{t} \tilde{\lambda}_{1i}(u_1) \mathcal{L}_2 (\bar{\lambda}_{2i}(u_1)) \mathcal{L}_1^{(1)} (\bar{\lambda}_{1i}(u_1)) du_1
\]

(7.23)

Letting again \( s_k = u_k - (t - 1), s_k \in [0, 1), k \in \{1, 2\}, \) and using piecewise constancy of \( \lambda_{ki}(\cdot) \) and piecewise linearity of \( \Lambda_{ki}(\cdot) \), following a change of variables (7.23) becomes

\[
E_v P(t_{1i} = t, t_{2i} > t_{1i}) = -\tilde{\lambda}_{1it} \int_0^1 \mathcal{L}_2 (\tilde{\bar{\lambda}}_{2i(t-1)} + \tilde{\lambda}_{2it} s_1) \mathcal{L}_1^{(1)} (\tilde{\bar{\lambda}}_{1i(t-1)} + \tilde{\lambda}_{1it} s_1) ds_1
\]

(7.24)
7.3.2. Series Expansion

The series expansion expression for the expectation of (3.2) can be derived as follows. Using (7.10) and taking expectations,

\[ E_v P(t_{11} = t, \ t_{2t} > t_{11}) = E_v \int_{t-1}^{t} \int_{u_1}^{t} f(u_1, u_2 | V_i, \xi_i) du_2 du_1 \]

\[ + E_v \int_{t-1}^{t} \int_{t}^{\infty} f(u_1, u_2 | V_i, \xi) du_2 du_1 \]

(7.25)

\[ = E_v A + E_v B \]

From (7.11),

\[ E_v A = \int_{t-1}^{t} E_{v_{21}} \left[ \int_{u_1}^{t} f_{st}(u_2 | V_{2i}, \xi_{2i}) du_2 \right] E_{v_1i} \left[ f_{st}(u_1 | V_{1i}, \xi_{1i}) \right] du_1 \]

(7.26)

For the expectation of the inner integral,

\[ E_{v_{21}} \left[ \int_{u_1}^{t} f_{st}(u_2 | V_{2i}, \xi_{2i}) du_2 \right] = E_{v_{21}} \left[ S_{2i} (u_1) - S_{2it} \right] \]

(7.27)

with the first right-hand side term intentionally not converted to the Laplace form in order to facilitate subsequent series expansion. Using (7.27) in (7.26),

\[ E_v A = \int_{t-1}^{t} \left[ E_{v_{21}} \left[ S_{2i} (u_1) \right] - \mathcal{L}_2 \left( \tilde{\Lambda}_{2it} \right) \right] E_{v_1i} \left[ f_{st}(u_1 | V_{1i}, \xi_{1i}) \right] du_1 \]

\[ = \int_{t-1}^{t} E_{v_{21}} \left[ S_{2i} (u_1) \right] E_{v_1i} \left[ f_{st}(u_1 | V_{1i}, \xi_{1i}) \right] du_1 \]

\[ - \mathcal{L}_2 \left( \tilde{\Lambda}_{2it} \right) E_{v_1i} \int_{t-1}^{t} f_{st}(u_1 | V_{1i}, \xi_{1i}) du_1 \]

(7.28)

\[ = E_v A_1 + E_v A_2 \]

Substituting with (7.6) and (7.7),

\[ E_v A_1 = \int_{t-1}^{t} E_{v_{21}} \left[ S_{2i} (u_1) \right] E_{v_1i} \left[ f_{st}(u_1 | V_{1i}, \xi_{1i}) \right] du_1 \]

\[ = E_{v_{21}} E_{v_1i} \int_{t-1}^{t} S_{2i} (u_1) \left[ f_{st}(u_1 | V_{1i}, \xi_{1i}) \right] du_1 \]

\[ = E_{v_{21}} E_{v_1i} \int_{t-1}^{t} \exp (-\Lambda_{2i} (u_1)) \exp (-\Lambda_{1i} (u_1)) \lambda_{1i}(u_1) du_1 \]

(7.29)

\[ = E_{v_{21}} E_{v_1i} \int_{t-1}^{t} \exp (-v_{2i} \tilde{\Lambda}_{2i} (u_1)) \exp (-v_{1i} \tilde{\Lambda}_{1i} (u_1)) \nu_{1i} \tilde{\Lambda}_{1i}(u_1) du_1 \]
Using integration by substitution with \( s_k = u_k - (t - 1) \) in (7.29) and piecewise constancy of \( \tilde{\lambda}_{ki} (s_k) \) for \( s_k \in [0, 1], k \in \{1, 2\}, \)

\[
E_v A_1 = E_{v_2} E_{v_1} \exp \left( -v_2 \tilde{\lambda}_{2i(t-1)} \right) \exp \left( -v_1 \tilde{\lambda}_{1i(t-1)} \right) \\
\times \int_0^t \exp \left( -v_2 s_1 \tilde{\lambda}_{2i} \right) \exp \left( -v_1 s_1 \tilde{\lambda}_{1i} \right) v_{1i} d\tau_1 ds_1 \\
= E_{v_2} E_{v_1} \exp \left( -v_2 \tilde{\lambda}_{2i(t-1)} \right) \\
\times \int_0^t \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{r_2!} \left( v_{2i} \tilde{\lambda}_{2i} \right)^{r_2} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1}}{r_1!} \left( v_{1i} \tilde{\lambda}_{1i} \right)^{r_1} v_{1i} \tilde{\lambda}_{1i} d\tau_1 ds_1 \\
= \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{r_2!} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1}}{r_1!} E_{v_1} \left[ A_{1i} \right] E_{v_2} \left[ A_{12} \right] A_{13} \\
(7.30)
\]

where

\[
E_{v_1} \left[ A_{1i} \right] = \tilde{\lambda}_{1i}^{r_1+1} E_{v_1} \left[ \exp \left( -v_1 \tilde{\lambda}_{1i(t-1)} \right) v_{1i}^{r_1+1} \right] \\
(7.31)
\]

\[
E_{v_2} \left[ A_{12} \right] = \tilde{\lambda}_{2i}^{r_2} E_{v_2} \left[ \exp \left( -v_2 \tilde{\lambda}_{2i(t-1)} \right) v_{2i}^{r_2} \right] \\
(7.32)
\]

\[
E_{v_1} \left[ A_{13} \right] = \int_0^t s_{1i}^{r_2+r_1} d\tau_1 \\
= \frac{1}{r_2 + r_1 + 1} \\
(7.33)
\]

whereby the time dimension of the previous quadrature has been parsed through following the series expansion linearization and integrated out in the remaining polynomial term in (7.35). Combining (7.32) and (7.34) and (7.35) in (7.30) results in

\[
E_v A_1 = \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{r_2!} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1}}{r_1!} \frac{(-1)^{r_1+r_2+1}}{r_2 + r_1 + 1} \tilde{\lambda}_{1i}^{r_1+1} \tilde{\lambda}_{2i}^{r_2} \\
\times \mathcal{L}_1^{(r_1+1)} \left( \tilde{\lambda}_{1i(t-1)} \right) \mathcal{L}_2^{(r_2)} \left( \tilde{\lambda}_{2i(t-1)} \right) \\
(7.34)
\]

For the second part of (7.28), using (2.5),

\[
E_v A_2 = -\mathcal{L}_2 \left( \tilde{\lambda}_{2i} \right) E_{v_1} \int_{t-1}^t f_{it}(u_1|V_{1i}, \zeta_{1i}) du_1 \\
= -\mathcal{L}_2 \left( \tilde{\lambda}_{2i} \right) E_{v_1} \left[ S_{1i(t-1)} - S_{1it} \right] \\
= -\mathcal{L}_2 \left( \tilde{\lambda}_{2i} \right) \left[ \mathcal{L}_1 \left( \tilde{\lambda}_{1i(t-1)} \right) - \mathcal{L}_1 \left( \tilde{\lambda}_{1it} \right) \right] \\
(7.35)
\]
Collecting (7.36) and (7.37) in (7.28) yields

\[
E_v A = \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{r_2!} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1}}{r_1!} \frac{1}{r_2 + r_1 + 1} \tilde{\lambda}_{1i}^{r_1+1} \tilde{\lambda}_{2i}^{r_2} \\
\times \mathcal{L}_1^{(r_1+1)}(\tilde{\Lambda}_{1i(t-1)}) \mathcal{L}_2^{(r_2)}(\tilde{\Lambda}_{2i(t-1)}) \\
- \mathcal{L}_2(\tilde{\Lambda}_{2i}) \left[ \mathcal{L}_1(\tilde{\Lambda}_{1i(t-1)}) - \mathcal{L}_1(\tilde{\Lambda}_{1i}) \right]
\]

(7.38)

The expectation expression for \( B \) in (7.25) is

\[
E_v B = \int_t^{t-1} E_{v_2} \left[ \int_t^\infty f_{it}(u_2|V_{2i}, \zeta_{2i}) du_2 \right] E_{v_1} \left[ f_{it}(u_1|V_{1i}, \zeta_{1i}) \right] du_1 \\
= E_{v_2} [S_{2it}] E_{v_1} [S_{1i(t-1)} - S_{1i}] \\
= \mathcal{L}_2(\tilde{\Lambda}_{2i}) \left[ \mathcal{L}_1(\tilde{\Lambda}_{1i(t-1)}) - \mathcal{L}_1(\tilde{\Lambda}_{1i}) \right]
\]

(7.39)

Substituting (7.38) and (7.39) into (7.25) yields

\[
E_v P(t_1 = t, t_2 > t_1) = \sum_{r_2=0}^{\infty} \sum_{r_1=0}^{\infty} \frac{(-1)^{2r_1+2r_2+1}}{r_2!r_1!(r_2 + r_1 + 1)} \tilde{\lambda}_{1i}^{r_1+1} \tilde{\lambda}_{2i}^{r_2} \\
\times \mathcal{L}_1^{(r_1+1)}(\tilde{\Lambda}_{1i(t-1)}) \mathcal{L}_2^{(r_2)}(\tilde{\Lambda}_{2i(t-1)})
\]

(7.40)

7.3.3. Derivatives of the Laplace transform

In general,

\[
\mathcal{L}^{(r)}(s) = (-1)^r \int v^r \exp(-sv) g(v) dv
\]

(7.41)

(see e.g. Hougaard, p. 498) and \( \mathcal{L}^{(r)}(s) \) exists for each \( r > c \) such that \( |g(v)| \leq K \exp(cv) \) if \( g(v) \) is piecewise continuous over its domain.
In the GIG density function (2.21), replace $\theta$ with $\theta/2$, then let $\chi = \phi^2/\theta$, and then substitute the resulting expression into (7.41) to obtain
\[
\mathcal{L}^{(r)GIG}(s) = (-1)^r \int v^r \exp(-sv) g^{GIG}(v) dv
\]
\[
= (-1)^r \int v^r \exp(-sv) \frac{(\theta/\chi)^{\kappa/2}}{2K_\kappa((\theta/\chi)^{1/2})} v^{\kappa-1} \exp \left\{ -\frac{1}{2} \left( \theta v + \frac{\chi}{v} \right) \right\} dv
\]
\[
= (-1)^r \int \frac{(\theta/\chi)^{\kappa/2}}{2K_\kappa((\theta/\chi)^{1/2})} v^{\kappa+r-1} \exp \left\{ -\frac{1}{2} \left( \theta (2s) v + \frac{\chi}{v} \right) \right\} dv
\]
\[
= (-1)^r \int \frac{2K_{\kappa+r}((\theta + 2s) \chi)^{1/2}}{2K_{\kappa+r}((\theta + 2s) \chi)^{1/2}} \frac{(\theta/\chi)^{\kappa/2}}{((\theta + 2s) / \chi)^{(\kappa+r)/2}} v^{\kappa+r-1} \exp \left\{ -\frac{1}{2} \left( \theta (2s) v + \frac{\chi}{v} \right) \right\} dv
\]
\[
= (-1)^r \int \frac{K_{\kappa+r}((\theta + 2s) \chi)^{1/2}}{K_\kappa((\theta/\chi)^{1/2})} \frac{(\theta/\chi)^{\kappa/2}}{((\theta + 2s) / \chi)^{(\kappa+r)/2}} v^{\kappa+r-1} \exp \left\{ -\frac{1}{2} \left( \theta (2s) v + \frac{\chi}{v} \right) \right\} dv
\]
Reversing the substitution with $\phi = \sqrt{\chi}$ and then replacing $\theta$ with $2\theta$ yields
\[
(7.42) \quad \mathcal{L}^{(r)GIG}(s) = (-1)^r \frac{K_{\kappa+r} \left( \phi (1 + s/\theta)^{1/2} \right)}{K_\kappa(\phi)} \left( \frac{\phi}{2\theta} \right)^r (1 + s/\theta)^{-(\kappa+r)/2}
\]
The quadrature version for the GIG then follows from (2.22), (7.24), and (7.42).

\[
E_v^{GIG} P(t_{1i} = t, t_{2i} > t_{1i}) = \frac{\tilde{\lambda}_{1it} \phi_1}{2\theta_1 K_{\kappa_1}(\phi_1) K_{\kappa_2}(\phi_2)} \times \int_0^1 \left( 1 + \frac{1}{\theta_1} \tilde{\Lambda}_{1i(t-1)} + \frac{1}{\theta_1} \tilde{\lambda}_{1it}s_1 \right)^{-(\kappa_1+1)/2}
\times \left( 1 + \frac{1}{\theta_2} \tilde{\Lambda}_{2i(t-1)} + \frac{1}{\theta_2} \tilde{\lambda}_{2it}s_1 \right)^{-\kappa_2/2}
\times K_{\kappa_1+1} \left( \phi_1 \left( 1 + \frac{1}{\theta_1} \tilde{\Lambda}_{1i(t-1)} + \frac{1}{\theta_1} \tilde{\lambda}_{1it}s_1 \right)^{1/2} \right)
\times K_{\kappa_2} \left( \phi_2 \left( 1 + \frac{1}{\theta_2} \tilde{\Lambda}_{2i(t-1)} + \frac{1}{\theta_2} \tilde{\lambda}_{2it}s_1 \right)^{1/2} \right) ds_1
\]
(7.43)
The quadrature version for the gamma then follows from \(2.26\), \(7.24\), and \(7.46\).

\[
\mathcal{L}^{(r)}(s) = (-1)^r \int v^r \exp(-sv) g^G(v) dv
\]

\[
= (-1)^r \int v^r \exp(-sv) \frac{\theta^r}{\Gamma(\gamma)} v^{\gamma-1} \exp(-\theta v) dv
\]

\[
= (-1)^r \int \theta^r \frac{1}{\Gamma(\gamma)} v^{\gamma+r-1} \exp(- (\theta + s) v) dv
\]

\[
= (-1)^r \frac{(\theta + s)^{\gamma+r}}{(\theta + s)^{\gamma+r}} \frac{\Gamma(\gamma + r)}{\Gamma(\gamma)} \theta^r \int \theta^r \frac{1}{\Gamma(\gamma)} v^{\gamma+r-1} \exp(- (\theta + s) v) dv
\]

\[
= (-1)^r \frac{\theta^r}{(\theta + s)^{\gamma+r}} \frac{\Gamma(\gamma + r)}{\Gamma(\gamma)} \theta^r \int \theta^r \frac{1}{\Gamma(\gamma)} v^{\gamma+r-1} \exp(- (\theta + s) v) dv
\]

\[
= (-1)^r \frac{\theta^r}{(\theta + s)^{\gamma+r}} \frac{\Gamma(\gamma + r)}{\Gamma(\gamma)} \theta^r \int \theta^r \frac{1}{\Gamma(\gamma)} v^{\gamma+r-1} \exp(- (\theta + s) v) dv
\]

The quadrature version for the gamma then follows from \(2.26\), \(7.24\), and \(7.46\).
\[ E_0^GP(t_{1i} = t, \ t_{2i} > t_{1i}) = \gamma_1 \tilde{\lambda}_{1it} \left( \psi \tilde{\lambda}_{1i(t-1)} + \frac{1}{\tilde{\theta}_2} \tilde{\lambda}_{2it} s_1 \right)^{-\gamma_2} \]
\[ \times \left( 1 + \frac{1}{\tilde{\theta}_1} \tilde{\lambda}_{1i(t-1)} + \frac{1}{\tilde{\theta}_1} \tilde{\lambda}_{1it} s_1 \right)^{-(\gamma_1 + 1)} ds_1 \]

(7.47)

The series version follows from (7.40) and (7.46).

\[ E_v^GP(t_{1i} = t, \ t_{2i} > t_{1i}) = \sum_{r_2=0}^{\infty} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1+r_2}}{r_1!r_2!(r_1+r_2+1)} \left( \frac{\tilde{\lambda}_{1it}}{\tilde{\theta}_1} \right)^{r_1+1} \left( \frac{\tilde{\lambda}_{2it}}{\tilde{\theta}_2} \right)^{r_2} \]
\[ \times \left( 1 + \frac{1}{\tilde{\theta}_1} \tilde{\lambda}_{1i(t-1)} \right)^{-(\gamma_1 + r_1 + 1)} \left( 1 + \frac{1}{\tilde{\theta}_2} \tilde{\lambda}_{2i(t-1)} \right)^{-(\gamma_2 + r_2)} \]
\[ \times \Gamma(\gamma_1 + r_1 + 1) [\Gamma(\gamma_1)]^{-1} \Gamma(\gamma_2 + r_2) [\Gamma(\gamma_2)]^{-1} \]

(7.48)

For the censored case,

\[ E_v^GP(t_{1i} > T, \ t_{2i} > T) = \left( 1 + \frac{1}{\tilde{\theta}_1} \tilde{\lambda}_{1iT} \right)^{-\gamma_1} \left( 1 + \frac{1}{\tilde{\theta}_2} \tilde{\lambda}_{2iT} \right)^{-\gamma_2} \]

(7.49)

Expressions (7.47), (7.48), and (7.45) are referenced in Corollary 2 to Theorem 2.
References


Table 1: Summary Statistics, Duration Data

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### Table 2: Summary Statistics, Competing Risk Data

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State unemp rate:

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- period 3: 4.583, 1.107, 2, 7.5
- period 4: 4.573, 1.104, 2, 7.8
- period 5: 4.558, 1.094, 2, 8.1
- period 6: 4.553, 1.086, 2, 8.1
- period 7: 4.556, 1.084, 2, 8.1
- period 8: 4.546, 1.075, 2, 7.4
- period 9: 4.537, 1.074, 2, 7.2
- period 10: 4.506, 1.076, 2, 7.2
- period 11: 4.502, 1.077, 2, 7.2
- period 12: 4.478, 1.086, 2, 6.9
- period 13: 4.457, 1.091, 2, 6.9
- period 14: 4.431, 1.097, 2, 6.9
- period 15: 4.423, 1.099, 2, 6.9
- period 16: 4.408, 1.096, 2, 6.9
- period 17: 4.394, 1.087, 2, 7.8
- period 18: 4.385, 1.077, 2, 7.8
- period 19: 4.374, 1.071, 2, 7.8
- period 20: 4.359, 1.064, 2, 7.8
- period 21: 4.354, 1.051, 2, 7.5
- period 22: 4.351, 1.036, 2, 7.4
- period 23: 4.361, 1.037, 2, 7.4
- period 24: 4.367, 1.03, 2, 7.4

Observations: 1,317
Table 3: New Semiparametric Duration Model, GIG Mixture

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$N = 15,491$, $Urate$ denotes the state unemployment rate, $Rrate$ denotes the replacement rate.
Table 4: New Semiparametric Competing Risk Model, GIG Mixture

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$N = 1,317$, $Urate$ denotes the state unemployment rate, $Rrate$ denotes the replacement rate.
Figure 1: Empirical Exit Count for Competing Risk Data
Figure 2: Density of individual heterogeneity component $v_i$, GIG mixture

$T = 6$

$T = 13$

$T = 24$
Figure 3: Survival function, GIG mixture

$T = 6$

$T = 13$

$T = 24$
Figure 4: Heterogeneity density, GIG mixture

$T = 6$

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Figure 5: Survival function, GIG mixture

$T = 6$

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Figure 6: Correlation structure of $\zeta_t$: density of $\sigma_1^2$, $\sigma_2^2$, and $\rho$, GIG mixture

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Figure 7: Model Comparison in Terms of Survival Functions

$T = 6$

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$T = 24$
Figure 8: Counterfactual Experiment for the Single Risk GIG Mixture

\( T = 6 \)

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8c.pdf}
\end{figure}
Figure 9: Counterfactual Experiment for the Competing Risks Model using a GIG Mixture and Combining the Risks.
Online Appendix

Figure OA1: Posterior Density of the Dirichlet Process Concentration Parameter $\alpha$, GIG mixture

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Table OA1: Duration model with parametric gamma heterogeneity (Han and Hausman, 1990)

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$N = 15,491$, $Urate$ denotes the state unemployment rate, $Rrate$ denotes the replacement rate.
Table OA2: Duration Model with Parametric GIG Heterogeneity

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$N = 15,491$, $Urate$ denotes the state unemployment rate, $Rrate$ denotes the replacement rate.
Figure OA2: Competing Risk Model, Posterior Density of the Dirichlet Process Concentration Parameter $\alpha$, Type 1 Risk (left) and Type 2 Risk (right), GIG mixture

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$T = 13$

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Table OA3: Competing Risk Model without Individual Heterogeneity

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N = 1,317, Urate denotes the state unemployment rate, Rrate denotes the replacement rate.
Table OA4: Competing Risk Model with Parametric Gamma Heterogeneity

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*N = 1,317, Urate denotes the state unemployment rate, Rrate denotes the replacement rate.*
Table OA5: Competing Risk Model with Parametric GIG Heterogeneity

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$N = 1,317$, $Urate$ denotes the state unemployment rate, $Rrate$ denotes the replacement rate.
Table OA6: Competing Risk Model with Independent Risks, GIG Mixture

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\( N = 1,317 \), Urate denotes the state unemployment rate, Rrate denotes the replacement rate.
Table OA7: Single Risk Model with Competing Risk Data, GIG mixture

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$N = 1,317$, Urate denotes the state unemployment rate, Rrate denotes the replacement rate.