GRINDING IN A BALL MILL

by

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ABSTRACT

The purpose of this thesis is to determine the rate of grinding in a ball mill as a function of the number of revolutions of the mill and as a function of the radius of the ball used.

A total of 32 experiments were carried out. Each experiment consisted of grinding 50 grams of minus 14 plus 20 mesh quartz for a predetermined length of time, with one of four different sizes of balls. At the end of the experimental time the single ball and the material were weighed. There was no loss of material and no loss in the weight of the balls. The material was then screened on a 20 mesh screen, and the oversize was weighed. By means of a Veeder Root counter the revolutions were recorded. The ball sizes were $\frac{3}{4}$, 1, $\frac{1}{2}$ and $\frac{3}{2}$ inches in diameter.

It was concluded from the experimental work that the rate of grinding, $\frac{dW}{dr}$, of a given size fraction is directly proportional to the weight of that size fraction present and proportional to the cube of the radius of the balls being used. This is expressed mathematically in the equations 1, 2 and 3.

$$\frac{dW}{dr} = -PW$$

(1) $\frac{dW}{dr} = -\left(c_1 R^3 - c_2\right)W$

(2) $W = W_0 e^{-\left(c_1 R^3 - c_2\right)r}$

(3) $P$ a constant

$W$ weight in mesh fraction

$r$ number of revolutions

$c_1$ a constant

$c_2$ a constant

$R$ Radius of Balls

$W_0$
Equation 1 states that the rate of grinding is directly proportional to the weight of material to be ground. Equation 2 states that the rate of grinding is proportional to the cube of the radius of the ball doing the grinding. Equation 3 is the integral of equation 2, and it predicts the weight of material still unground after r revolutions.
ACKNOWLEDGEMENT

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GRINDING IN A BALL MILL

INTRODUCTION

The purpose of this thesis was to determine the general laws of impact grinding in a ball mill. The specific problems tackled were two fold. The first was to determine the rate of grinding as a function of the number of revolutions of the mill. The second was to determine the rate of grinding as a function of ball diameter. The experiment was designed to eliminate attrition grinding. Thus only one ball was used to grind during any one run.

THEORETICAL CONSIDERATION

While conducting other experiments in the field of comminution the author noted that the rate of grinding of size fraction decreased with the length of grinding. This led to an attempted explanation and finally to the experiments presented in this paper.

The rate of grinding in this paper is defined as the number of particles ground divided by the number of revolutions to grinding them. This is expressed mathematically as:
(4) \( \frac{\Delta N}{\Delta r} = \) rate

\( \Delta N \) Number of particles ground.

\( \Delta r \) Number of revolutions to grind them.

By taking the limit of equation 1 as \( r \) goes to zero:

(5) \( \frac{\Delta N}{\Delta r} \) (limit as \( r \to 0 \)) \( = \frac{dn}{dr} \)

(6) \( \frac{dn}{dr} \) Instantaneous rate of impact grinding in a ball mill.

Let us assume that every individual particle of a given mesh fraction has an equal chance of being broken per revolution of the ball mill. Let us call this probability of being broken per revolution \( P \). Then the probability of an individual particle being broken in \( dr \) revolutions is \( Pdr \). Or we can say that out of a large number of \( N \) particles, \( NPdr \) of them will be broken in \( dr \) revolutions. Thus \( N \) is decreased by \( dN \) during \( dr \) revolutions, or:

(7) \( dN = -NPdr \)

Dividing through by \( dr \):

(8) \( \frac{dn}{dr} = -PN \)
Integrating equation 6:

\[ N = N_0 e^{-Pr} \]

(9) \( N - N_0 e \) \( N_0 \) Initial number of particles.

The weight of a mesh fraction is directly proportional to the number of particles in the mesh fraction. Thus equation 9 may be written as:

\[ W - W_0 e^{-Pr} \]

(10) \( W - W_0 e \) \( W_0 \) Initial weight

This formula gives a basis for an experiment, but it demands that certain limits be placed on the experiment. First, all grinding must be done by impact and secondly that the probability of a particle being broken must be the same throughout the experiment. Also, enough particles must be ground per experiment so that statistical deviation will not require a vast number of experiments to get the mean.

Since attrition grinding occurs when one ball rolls over another, it can clearly be seen that using one ball to do the grinding will all but eliminate attrition grinding.
To determine the number of particles per run is a more difficult task. The larger the number of particles used, the smaller the statistical deviation. On the other hand increasing the number of particles, will after a point, change the probability \( P \). Since there was no literature on the subject, an arbitrary choice of 50 grams of minus 14 plus 20 mesh quartz was chosen. The experimental data indicated that \( P \) remained constant, and that there was no appreciable statistical deviation.

**APPARATUS AND EXPERIMENTAL PROCEDURE**

The ball mill was a right angle cylinder with an inside radius of 198 mm. and a length of 70 mm. The sides were made of plate glass. The mill was driven at an average speed of 27 revolutions per minute.

The material used was minus 14 plus 20 mesh rose quartz. The material was first screened 20 minutes on Tyler screens with 200 grams of the quartz ending up on the 20 mesh screen. Next, 100 grams from the first screening was placed on the 20 mesh screen and screened for 30 minutes. Quartz was used because it was cheap and easy to get, and because it had no preferential breakage.
The balls used were Atlas ball-bearings. The diameters of the balls were 3/4, 1, 1\(\frac{1}{4}\) and 1\(\frac{3}{5}\) inches. The weight of the balls were respectively: 27.7 grams, 66.7 grams, 128 grams and 222 grams. Ball-bearings were used because they were the only steel spheres that could be found. The maximum range in diameter was 2:1 whereas the maximum range in weight was 8:1.

A Veeder Root counter was installed on the ball mill frame to count the revolutions.

The screens used were the regular Tyler type screens used in conjunction with a ro-tap.

The procedure used is as follows: 50 grams of quartz were placed in the mill, and the ball to do the grinding was then added. The mill was closed and the material was ground for the predetermined length of time.

When the grinding time was finished the number of revolutions were recorded, and then the mill was opened and the material was weighed. The material was then screened, and the plus 20 mesh material was weighed.
The method of screening was as follows: The material to be screened was placed on the 20 mesh screen and then shaken 25 times, striking the screen sharply against the opposite hand once each shake. The screen was then rotated 90 degrees and shaken 25 more times. This process was continued till a total of 100 shakes had been applied. As mentioned in the above paragraph the plus 20 mesh material was then weighed.

INTERPRETATION OF RESULTS

Equation 10 is the key to the interpretation of the results.

\[ -Pr \]

\[ W - W_0 e \]

\( W_0 \) is 50 grams of minus 14 plus 20 mesh quartz. The quartz was placed in the mill and ground. The number of revolutions, \( r \), was recorded, and the weight, in grams, of material still plus 20 mesh, \( W \), was also recorded. When simultaneous values of \( W, W_0 \) and \( r \) are substituted in equation 10, \( P \) can be calculated. In figure 1, \( \log W \) was plotted against the number of revolution, \( r \), for a given ball size. The best straight line was then drawn through these points.
By reading the r intercept where $W$ is equal to $\frac{1}{2}W_0$
and substituting these values in equation 10, $P$ can be
calculated. This method automatically averages the value
$P$. Using this method $P$ was calculated for all 4 ball sizes.

Equation 10 predicts a straight line for all data
plotted on semi-log paper. It should be noted that in figure
1, all the data came out on a straight line as was predicted.

To determine the effect of ball radius on the grinding
rate a graphical method of analysis was used. On log-log
graph paper log $P$ was plotted against log $R$. Since this log-
log plot, figure 2., showed a straight line, it indicated that
$P$ was a power function of $R$. In figure 3, $P$ is plotted as
integer power functions of $R$. When $P$ is plotted against $R^3$
a straight line results, indicating that.

$$P = c_1 R^3 + c_2$$

$c_1$ and $c_2$ are empirical
constants.

By substituting equation 11 in equation 10 we have:

$$W = W_0 e^{-(c_1 R^3 + c_2)}$$
Differentiation equation 12:

$$\frac{dW}{dt} = -(c_1R^3 + c_2)W$$

**CONCLUSIONS**

Equation 13 predicts accurately any point on the 4 lines in figure 1. All the data taken during the experiment is plotted in figure 1. This means that all conclusions drawn must be interpretations of equation 13.

From equation 13 the following two conclusions may be drawn:

1. The rate of grinding of a given size fraction is independent of $r$, but directly proportional to the amount of material in that size fraction.

2. The rate of grinding of a given size fraction is proportional to the cube of the radius of the ball used to do the grinding, providing that all of the balls used are of the same material.
POSSIBLE FUTURE WORK

I would suggest that experiments be carried out to determine the effect of varying the particle size while holding the ball size constant. Also I would suggest that the density of the ball be varied while holding ball size and particle size constant.

CALCULATIONS AND DATA

The following is the method used to calculate $P$.

\[(10) \quad W = W_0 e^{-Pr}\]

Taking the log of both sides:

\[(14) \quad \ln W = -Pr \]

Dividing through by $r$:

\[(15) \quad P = \frac{1}{r} \ln \frac{W}{W_0}\]
From the plot of the 1 inch ball in figure 1., r equals 800 revolutions when W equals 25 grams. Substituting these figures in equation 15 yields:

\[(16) \quad P = 1 \cdot \frac{1}{500} \cdot 0.65 \times 10^{-4}\]
Similiar calculations for the other ball sizes are tabulated in table 1.

<table>
<thead>
<tr>
<th>Ball Size</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4 inch</td>
<td>4.07 x 10^{-4}</td>
</tr>
<tr>
<td>1 inch</td>
<td>8.65 x 10^{-4}</td>
</tr>
<tr>
<td>1 1/4 inch</td>
<td>14.9 x 10^{-4}</td>
</tr>
<tr>
<td>1 1/2 inch</td>
<td>23.6 x 10^{-4}</td>
</tr>
</tbody>
</table>

By examining equation 11 in conjunction with figure 3, it is obvious that:

\[ c_1 = 7.34 \times 10^{-4} \]
\[ c_2 = 1.31 \times 10^{-4} \]

Thus equation 13 becomes:

\[
(17) \quad \frac{dW}{dr} \left. W_0 e^{-((7.34R^3 \pm 1.31) r \times 10^{-4})} \right|
\]
<table>
<thead>
<tr>
<th>Ball Size</th>
<th>Time</th>
<th>Total Revolutions</th>
<th>Plus 20 Mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4 inch</td>
<td>30 min.</td>
<td>725</td>
<td>36 grams</td>
</tr>
<tr>
<td>3/4</td>
<td>60</td>
<td>1380</td>
<td>29</td>
</tr>
<tr>
<td>3/4</td>
<td>90</td>
<td>2010</td>
<td>22</td>
</tr>
<tr>
<td>3/4</td>
<td>120</td>
<td>2830</td>
<td>14.17</td>
</tr>
<tr>
<td>3/4</td>
<td>30</td>
<td>771</td>
<td>36.15</td>
</tr>
<tr>
<td>3/4</td>
<td>60</td>
<td>1528</td>
<td>27.3</td>
</tr>
<tr>
<td>3/4</td>
<td>90</td>
<td>2203</td>
<td>22.07</td>
</tr>
<tr>
<td>3/4</td>
<td>120</td>
<td>2901</td>
<td>14.37</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>743</td>
<td>26.2</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
<td>2155</td>
<td>7.7</td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>2891</td>
<td>3.5</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>630</td>
<td>29.4</td>
</tr>
<tr>
<td>1</td>
<td>80</td>
<td>1400</td>
<td>15.0</td>
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<td>2120</td>
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<td>2840</td>
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<td>1.81</td>
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<td>.41</td>
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<td>387</td>
<td>21.5</td>
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<tr>
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<td>30</td>
<td>734</td>
<td>7.77</td>
</tr>
<tr>
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<td>45</td>
<td>1168</td>
<td>3.20</td>
</tr>
<tr>
<td>1 3/8</td>
<td>60</td>
<td>1503</td>
<td>.31</td>
</tr>
<tr>
<td>1 3/8</td>
<td>90</td>
<td>2264</td>
<td>.11</td>
</tr>
<tr>
<td>1 3/8</td>
<td>30</td>
<td>724</td>
<td>9.5</td>
</tr>
<tr>
<td>1 3/8</td>
<td>60</td>
<td>1520</td>
<td>1.29</td>
</tr>
</tbody>
</table>
$W_0 = 50$ grams of plus 20 mesh quartz

This is a plot of $\log W$ vs. $r$, showing that the data follows the general equation, $W = W_0 e^{-Pr}$ for each ball size. The generalized equation:

$$W = W_0 e^{-\left(7.54R^{0.7} + 1.5\right)r \cdot 10^4}$$

describes all four curves. The $R$ used in this equation is the diameter of the ball.
This is a plot of $P$ vs. $R$, showing that $P$ is a power function of $R$. This plot is the graphical solution to the relationship between $P$ and $R$.

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Figure 2

Radius of Ball x 2 \((Rx2)\) (in inches)
Plot of \( P \) vs. \( R \), showing that \( P = c_1 R^2 \neq c_2 \).

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Figure 3.