High-Precision
Planar Magnetic Levitation
by
Won-jong Kim

Submitted to the Department of Electrical Engineering and Computer Science
on May 12, 1997, in partial fulfilment of the
requirements for the degree of
Doctor of Philosophy in Electrical Engineering and Computer Science

Abstract
This thesis presents the design and implementation of a high-precision magnetically
levitated stage with large planar motion capability. This stage is the first which is
capable of providing all the motions required for photolithography in semiconductor
manufacturing with only one moving part, namely the platen. The platen is driven in
all six-degree-of-freedom motions with small adjustments for focusing and alignment
and with large planar motions for positioning across the wafer surface. The underlying
electromechanical modeling and analysis, mechanical and electrical design, and real-
time control of such a high-precision planar magnetic levitator are presented.

The platen is levitated without contact by four novel permanent-magnet linear
motors that provide both suspension and drive forces. The linear motors consist of
Halbach-type magnet arrays attached to the underside of the levitated platen, and
coil sets attached to the fixed machine platform. Since all the motor coils are fixed,
no wires need to be connected to the moving part. The platen mass of 5.6 kg is
supported against gravity by the combined forces of the four motors. Each motor
consumes about 5.4 W to lift the platen. Two of the motors drive the stage in the
x-direction, and the two other motors drive in the y-direction. The motor forces
are coordinated appropriately to control the remaining four degrees of freedom. The
present design has a travel of 50 mm in x and y, a travel of 400 µm in z, and is
capable of milliradian-scale rotations about each of these three axes.

The stage position in the plane is measured with three laser interferometers with
sub-nanometer resolution. The stage position out of the plane is measured by three
capacitance probes with nanometer resolution. The stage operates with a position
noise of 5 nm rms in x and y, and is demonstrating acceleration capabilities in excess
of 10 m/s² (1 g). The control bandwidth of the system is 50 Hz. This design can
readily be scaled to travel on the order of 300 mm for the future needs of lithographic
systems.
Thesis Supervisor: David L. Trumper
Title: Rockwell International Associate Professor
Acknowledgments

First of all, I would like to thank my thesis advisor Professor David L. Trumper for his consistent support and encouragement for years. He offered me the privilege to be his first doctoral student. I cannot forget the excitement when he suggested this challenging magnetic levitation project. I admire his competence throughout electrical and mechanical engineering disciplines; what I learned from him includes everything theoretical and practical from circuit design to precision machine design.

I am grateful to Professor Jeffrey H. Lang for his serving on my thesis committee and his commitment throughout my doctoral study here at MIT. He taught me much about electric machinery and electromechanical system analysis and control. As my thesis committee members, Professor James L. Kirtley, Jr. and Professor George C. Verghese have been showing their great interests since the earliest stage of my doctoral work. I appreciated their taking time to read my thesis and giving me precious comments.

I wish to thank Professor Hermann A. Haus. As his last teaching assistant at MIT, I gained invaluable experience in teaching the students and interacting with the teaching staff. I learned from him how to organize such a big course as well as how to teach the material. Professor Terry P. Orlando was always kind and approachable, and willing to support me in preparation for my Area Exam and Appendix C in this thesis.

As anyone may imagine, this level of doctoral work would not have been successful without my colleagues’ cooperation. In particular, I should acknowledge Mark E. Williams’ sharing ideas with me in hardware construction and software development. I could save much time by avoiding many time-consuming debugging processes. As a Bachelor’s student, Dean A. Sheppard assisted me in the fabrication of the honeycomb sandwich platen. I thank everyone else in the Precision Motion Control Lab for the friendly environments they made. I also thank my friends at the Laboratory for Electromagnetic and Electric Systems, Dr. John Ofori-Tenkorang for his insight on
the Halbach magnet array, and Marc T. Thompson for his helping me use their Ansoft three-dimensional finite element software.

There are MIT research specialists who gave me essential help in hardware construction. Mr. Fred Cote at the Laboratory for Manufacturing and Productivity machine shop has always been helpful and resourceful. I learned a significant amount of precision machining from him. Professor Paul A. Lagace in the Department of Aeronautics and Astronautics and Mr. Albert Supple at Technology Laboratory for Advanced Composites allowed Dean and me to use their autoclave facility to construct honeycomb sandwich panels for the magnetically levitated platen. Mr. Peter Morley at Laboratory for Nuclear Science machine shop did a great job in platen machining. I must commend machinists outside MIT, Mr. Mike Gaudett at WireWinders, Inc. for winding coils and Mr. Charlie Bell at Eastern Tool Corp. for machining stator blocks.

My special appreciation goes to our research sponsor, the Sandia National Laboratories, especially Mr. Stew Kohler, Mr. Tony Smith and Mr. John Wronosky. I got a great help from them in power amplifier circuit design and in software development. They also provided layouts of power amplifier printed circuit boards and a back plane.

Finally, I can never thank my wife Hyun-Jung enough for her patience and understanding. She has always been a great friend and wonderful counselor of mine. I would like to thank my parents, Mr. and Mrs. Duck-nam Kim, and Hyun-Jung’s parents, Mr. and Dr. Han Keun Kim. Without their prayer and support, this work would not have been possible.

This work was supported in part by a contract from Sandia National Laboratories under Subcontract No. AH-4243 and in part by a Scholarship from Korean Ministry of Education to the author and a National Science Foundation Presidential Young Investigator award to Professor Trumper.
## Contents

List of Figures ............................................. 15

List of Tables .............................................. 22

Symbols ....................................................... 23

1 Introduction .............................................. 31

1.1 High-Precision Planar Motion Control .................. 31

1.2 The Planar Magnetic Levitator ......................... 33

1.3 Thesis Overview ......................................... 35

1.4 Contributions of the Thesis .............................. 37

1 Review of Prior Art ........................................ 38

2 Levitation Techniques .................................... 40

2.1 Levitation in the Physical World ....................... 41

2.1.1 Aerodynamic Levitation ............................... 41

2.1.2 Magnetic Levitation .................................... 42

2.1.3 Electrodynamic Levitation ........................... 43

2.1.4 Superconducting Levitation .......................... 44

2.2 Magnetic Suspension and Levitation ................... 44

2.2.1 Single-Degree-of-Freedom Suspension—Ball Suspension System ........................................... 46
2.2.2 Multiple-Degree-of-Freedom Suspension—Magnetic Bearings, Wind Tunnel Balances, and Gyroscopes ........................................................................ 49
2.2.3 One-Dimensional Levitation—Maglev Vehicles and Positioner ...................... 50
2.2.4 Two-Dimensional Levitation—Objective of the Thesis .................................... 51

3 Technical Trends in Motion Control .................................................................... 53
  3.1 Conventional Motors ........................................................................................... 54
    3.1.1 DC Motors ................................................................................................... 55
    3.1.2 Synchronous Motors .................................................................................... 55
    3.1.3 Induction Motors ......................................................................................... 57
    3.1.4 Variable-Reluctance Motors ...................................................................... 58
  3.2 Actuators, Bearings, and Sensors for Positioners .............................................. 59
    3.2.1 Actuators ..................................................................................................... 59
    3.2.2 Bearings ...................................................................................................... 61
    3.2.3 Sensors ........................................................................................................ 62
  3.3 Prior Art for Planar Positioners .......................................................................... 64
    3.3.1 Variable-Reluctance Types ......................................................................... 65
    3.3.2 Permanent-Magnet Types ........................................................................... 68
    3.3.3 Permanent-Magnet Matrix Types ................................................................. 72
  3.4 Summary of Part I .............................................................................................. 74

II Analysis .................................................................................................................. 78

4 Continuum Electromechanical Analysis .................................................................. 80
  4.1 Review of Electromagnetic Field Theory ........................................................... 81
    4.1.1 Magnetoquasistatic (MQS) Approximation ............................................... 81
    4.1.2 Vector Potential and Equivalent Current Model ......................................... 83
    4.1.3 Poisson Equation for Vector Potential ....................................................... 84
    4.1.4 Maxwell Stress Tensor ................................................................................ 84
4.1.5 Fourier Series Representation ........................................... 85
4.2 Field Solutions ................................................................. 87
  4.2.1 Field due to Magnet ..................................................... 88
  4.2.2 Field due to Stator Current ......................................... 91
  4.2.3 Transfer Relations ...................................................... 92
  4.2.4 Total Field ............................................................... 94
4.3 Force ................................................................................. 94

5 Analysis for Electromagnetic System ........................................... 97
  5.1 Single-Sided Field Sources ............................................... 97
    5.1.1 Halbach Magnet Array ............................................... 98
    5.1.2 Halbach Array Harmonics .......................................... 99
    5.1.3 Triangular Winding .................................................. 101
  5.2 Electrical Terminal Relation of Stator .................................. 108
    5.2.1 Vector Potential inside Stator .................................... 108
    5.2.2 Stator Flux Linkage .................................................. 109
    5.2.3 Self-Inductance ...................................................... 111
    5.2.4 Back Electromotive Force ......................................... 112
    5.2.5 Power Balance ....................................................... 114
  5.3 Force Ripple with Respect to Phase Number ......................... 116
    5.3.1 Fourier Harmonics of Stator Current ........................... 116
    5.3.2 Force Ripple .......................................................... 118
  5.4 Experimental Verifications ................................................ 120
  5.5 Summary of Part II .......................................................... 122

III Design ................................................................................. 124

6 Conceptual Designs ............................................................... 126
  6.1 Design Considerations ..................................................... 126

8
6.1.1 Performance Goals Revisited .................................. 127
6.1.2 Selection of Actuators ......................................... 128
6.1.3 Mass of the Platen ............................................. 128
6.1.4 Power Consumption ........................................... 129
6.2 Design Concepts .................................................... 130
6.2.1 Multiple-Moving-Part Types ................................. 130
6.2.2 One-Moving-Part Types ...................................... 133
6.3 Selection for Prototyping .......................................... 149
6.3.1 Selection Criteria .............................................. 149
6.3.2 Suggestions for Implementation ............................ 151
6.4 Selected Design Concept ......................................... 153

7 Electromagnetic Design ................................................. 156
7.1 Magnet Arrays ....................................................... 157
7.1.1 Permanent-Magnet Material ................................. 157
7.1.2 Magnet Specifications .......................................... 159
7.1.3 Design of Magnet Arrays ..................................... 160
7.1.4 Fabrication of Halbach Magnet Arrays .................... 161
7.2 Stators ............................................................... 170
7.2.1 Discussions on Winding Structure ......................... 171
7.2.2 Electrical Design Parameters ............................... 172
7.2.3 Winding Fabrication .......................................... 174
7.3 Power Amplifier ..................................................... 176
7.3.1 Power OP Amp ................................................. 176
7.3.2 Power Supplies ................................................ 177
7.3.3 Design Characteristics ....................................... 178
7.4 Instrumentation Structure ....................................... 180
7.4.1 VMEbus .......................................................... 181
7.4.2 Digital Signal Processor ...................................... 184
7.4.3 Sensors ......................................................... 185
7.4.4 A/D and D/A Converter Boards ...................... 187
7.4.5 Miscellanies ................................................... 190

7.5 Software ......................................................... 191
7.5.1 Control Routine .............................................. 192
7.5.2 User Interface Routine .................................... 194

8 Mechanical Design ............................................. 196
8.1 Platen .......................................................... 196
  8.1.1 Optimal Design of Honeycomb Sandwich Panel ......... 197
  8.1.2 Sandwich Panel Fabrication .............................. 200
  8.1.3 Machining Process ........................................ 202
  8.1.4 Damping ..................................................... 205
  8.1.5 Resonant Frequency ....................................... 208
  8.1.6 Magnet Placing ............................................ 213
  8.1.7 Mirror Mounting .......................................... 218

8.2 Stators ........................................................ 218
  8.2.1 Capacitance Probe Mounting ............................. 218
  8.2.2 Stator Core ............................................... 219
  8.2.3 Winding Assembly .......................................... 226

8.3 Packaging and Assembly ................................... 228
  8.3.1 Mounting Table ........................................... 228
  8.3.2 Stators ..................................................... 230
  8.3.3 Metrology Devices ....................................... 230
  8.3.4 Bumpers .................................................... 230
  8.3.5 Mechanical Tolerances .................................. 231

8.4 Summary of Part III .......................................... 231
IV  Dynamics and Control 233

9  Modeling and Dynamic Behavior 235
   9.1  Electromechanical Parameters .................................. 235
       9.1.1  Mass and Inertia Tensor of the Platen .................. 235
       9.1.2  Winding Resistance and Self-Inductance ................. 238
       9.1.3  Specifications of the Levitator ......................... 240
   9.2  Decoupled Equations of Motion ................................ 242
       9.2.1  DQ Decomposition ........................................ 242
       9.2.2  Linearized Force Equations ............................. 244
       9.2.3  Vertical Equations of Motion ........................... 246
       9.2.4  Lateral Equations of Motion ............................ 246
   9.3  State-Space Equations of Motion ................................ 247
       9.3.1  Euler Angles ............................................. 248
       9.3.2  Forces and Torques ...................................... 250
       9.3.3  Linearized Equations of Motion ......................... 251
       9.3.4  Sensor Equations .......................................... 257

10  Control 261
   10.1  Force Allocation ............................................... 263
       10.1.1  Commutation Law ......................................... 263
       10.1.2  Vertical Force Allocation ............................... 265
       10.1.3  Lateral Force Allocation ................................ 267
       10.1.4  Modal-Decomposed Force Transformation ................. 269
   10.2  Decoupled Control ............................................... 270
       10.2.1  Sampling Rate ............................................ 272
       10.2.2  Vertical Mode Control ................................... 273
       10.2.3  Lateral Mode Control .................................... 285
       10.2.4  Position Noise ............................................ 285
10.3 Multivariable Control ........................................ 292
10.3.1 Multivariable Linear Quadratic Control .......... 296
10.3.2 Time-Optimal Control ................................. 298
10.4 Demonstrations .............................................. 302
10.5 Summary of Part IV ........................................ 304

11 Conclusions and Suggestions of Future Work .......... 305
11.1 Conclusions .................................................. 305
11.2 Suggestions for Future Work ............................. 308
   11.2.1 Incremental Improvements of the Current Design .. 308
   11.2.2 Accommodating Larger Wafers—Scaling Issues ... 309

Appendices .......................................................... 312

A Planar Levitator with Superimposed Halbach Magnet Matrix 313
   A.1 Magnet Matrices—Prior Art .............................. 313
   A.2 Conception of Superimposed Halbach Magnet Matrix .. 314
   A.3 Analysis of Superimposed Halbach Matrix ............ 316
      A.3.1 Transfer Relations .................................. 316
   A.4 Force and Commutation Law ............................ 317
   A.5 Planar Levitator Concept with Superimposed Halbach Magnet Matrix 318

B Tubular Linear Motor ........................................... 321
   B.1 Tubular Motor Model ...................................... 321
   B.2 Mathematical Preliminaries ............................. 323
      B.2.1 Modified Bessel Functions [Arf85] ................. 323
      B.2.2 Variation of Parameters [RiR68] ................... 324
   B.3 Field Solutions .......................................... 325
      B.3.1 Solution to Poisson Equation ...................... 325
B.3.2 Field due to Stator Current ........................................ 326
B.3.3 Field due to Magnet ................................................. 329
B.3.4 Approximate Stator Field Solution ................................. 330
B.3.5 Transfer Relations in the Air Gap .................................. 332
B.4 Boundary Conditions and Transfer Relations ......................... 332
B.5 Parametric Analysis of a Tubular Motor ............................... 333
B.6 Implementation of a Tubular Motor .................................... 335

C Superconducting Levitation—A Case Study ............................. 338
C.1 Introduction .............................................................. 339
C.2 Brief History of Superconductivity .................................... 342
C.3 Phenomena and Characteristics ....................................... 343
  C.3.1 Earnshaw’s Theorem ................................................ 345
  C.3.2 Meissner Effect .................................................... 346
  C.3.3 Flux Pinning ........................................................ 349
  C.3.4 Bean’s Critical State Model ...................................... 350
  C.3.5 Hysteresis in Type II Superconductors .......................... 351
  C.3.6 Levitation and Lateral Force ...................................... 354
C.4 Current Researches on High-temperature Superconducting Suspension 356
  C.4.1 Ma, et al.—Superconducting Bearing, Torque Coupler, and Damper 356
  C.4.2 Delprete, et al.—High-Speed Induction Motor with Superconducting Bearings ........................................... 362
  C.4.3 Goodall, et al.—Superconducting Magnet ........................ 367
C.5 Discussions .................................................................... 372

D Code ................................................................................. 375
D.1 Real-Time Control Code .................................................. 375
  D.1.1 Decoupled Lead-Lag Compensators ............................... 375
  D.1.2 Multivariable Linear Quadratic Regulator ....................... 382
D.1.3 Demonstration Routines ........................................ 382
D.2 MATLAB Simulation Code ......................................... 386
  D.2.1 Decoupled Lead-Lag Compensators ....................... 386
  D.2.2 Self-Inductance ............................................. 388
  D.2.3 Force and Force Ripple .................................... 389

Bibliography .......................................................... 392
List of Figures

1-1  Schematic wafer stepper realized with a six-degree-of-freedom magnetically levitated stage ........................................ 32
1-2  The six-degree-of-freedom magnetic levitator with planar motion capability .......................................................... 34
1-3  Perspective view of the magnetically levitated stage .............. 34
1-4  Force components acting on magnet-array centers of masses ...... 35

2-1  Toy to demonstrate aerodynamic levitation (after [Bra89]) .... 41
2-2  Levitation of molten metal by electrodynamic levitation (after [Jay81]) .............................................................. 43
2-3  A current loop is levitated by repulsion force due to its image current loop induced inside the superconductor .................. 44
2-4  Ball suspension system with an electromagnet. The vertical position of the ball is fed back by the light source, and stabilized at the reference position by the controller .............................................. 47
2-5  One-dimensional magnetically levitated stage by Mark Williams ... 52

3-1  Sawyer motor stage (after [Saw68]) ........................................ 66
3-2  Magnetically suspended stepping motor (after [HK89]) ........ 67
3-3  Plane view of slider teeth of Higuchi motor ............................ 68
3-4  Electromagnetically driven wafer stage (after [BGK89]) ........ 69
3-5  Brushless DC planar motor (after [Gal85]) ............................ 71
3-6  Top and side views of Asakawa motor (after [Asa85]) ............ 73
3-7 Prospective view of Hinds motor (after [Hin87]) .................................. 75
3-8 Bottom view of Hinds motor's platen (after [Hin87]) .......................... 76
3-9 Top view of Hinds motor's magnet matrix (after [Hin87]) ..................... 76
4-1 Linear motor model described in infinite complex Fourier series. Free space is assumed except for the shaded regions for magnet and current. 87
5-1 Linear Halbach magnet array with its magnetization ............................. 99
5-2 (a) Linear Halbach magnet array (b) vertical and (c) lateral magnetization components with respect to z ..................................................... 100
5-3 Fourier coefficients of Halbach magnet array flux density ................... 101
5-4 Flux lines of a square Halbach magnet array ........................................ 102
5-5 Electromagnetic dual of Halbach array (a) Halbach magnet array (b) Equivalent current model (c) triangular winding pattern ......................... 103
5-6 Triangular winding pattern with a phase filled with uniform current density ........................................................................................................... 103
5-7 Two-phase operation with superposition of two phases with 90° apart with each other ............................................................................................ 107
5-8 Imaginary boundary (dashed line at $x = X$) inside stator ...................... 109
5-9 Closed contour for magnetic flux, self-inductance, and back emf calcu-
lation ............................................................................................................... 110
5-10 Spatial current density distribution with six-phase winding as a func-
tion of spatial angle $\delta = 2\pi z/l$ ................................................................. 117
5-11 Back emf wave form acquired with a prototype linear motor ............... 122
6-1 Crossed-axis design ................................................................................. 130
6-2 Gantry design .......................................................................................... 132
6-3 Legend ....................................................................................................... 133
6-4 Flying puck design (moving winding) ..................................................... 135
6-5 Cross-sectional view of the flying puck design ....................................... 136
6-6 Four two-sided motors design (platen) .......................... 136
6-7 Four two-sided motors design (top-surface of stators) ........ 137
6-8 Rotational motion generation with four two-sided motors ........ 138
6-9 Three one-sided motors design ..................................... 140
6-10 Translational motion generation with three one-sided motors .... 141
6-11 Cross-sectional view of one motor in the three one-sided motors design 141
6-12 One-sided motors on the bottom design (platen) .............. 143
6-13 One-sided motors on the bottom design (top view of stators) .... 144
6-14 Orthogonal two-sided motors design (platen) .................... 145
6-15 Orthogonal two-sided motors design (bottom view of top stators) 146
6-16 Puck with hole design (perspective view) ....................... 147
6-17 Puck with hole design ............................................. 148
6-18 Selected design for prototyping .................................... 154

7-1 Dimension of a magnet chip ........................................ 159
7-2 Toolings for magnet array fabrication ............................. 162
7-3 Magnet chip and magnet array consisting of forty-five such magnet chips. Each drawing shows top and side views. .................. 163
7-4 Tooling to make two magnet rows—bottom part ................. 164
7-5 Tooling to make two magnet rows—top part ..................... 165
7-6 Magnet alignment in the tooling before cap screws are tightened .. 166
7-7 Tooling to make a whole magnet array—lid ....................... 166
7-8 Tooling to make a whole magnet array—bottom part .......... 167
7-9 Tooling to make a whole magnet array—top part ............... 168
7-10 Lettering for magnet arrays. The drawing shows the bottom view of the platen. The letters N and S indicate that the nearest poles from the edges are North and South, respectively. ..................... 169
7-11 Stator winding .................................................... 172
7-12 Tooling for winding fabrication ................................. 174
7-13 Dimensions of the winding tooling ........................................ 175
7-14 Power amplifier circuit ......................................................... 179
7-15 Power amplifier circuit board ................................................. 180
7-16 Instrumentation structure ...................................................... 182
7-17 Laser interferometry metrology with the platen ......................... 188

8-1 Top and side views of the sandwich panel ................................ 203
8-2 Side and bottom views of the sandwich panel ............................. 204
8-3 Alignment cross ................................................................. 206
8-4 Dimension of the alignment cross ............................................. 207
8-5 Perspective view of the sandwich panel after fabrication ............. 208
8-6 Exploded view of the platen .................................................... 209
8-7 Test rig for resonant frequency and damping ratio ..................... 210
8-8 Accelerometer positions on the bottom side of the platen. The coordinates are relative to the lower left corner of the platen ....................... 210
8-9 Power spectra of impulse responses with (solid) and without (dashed) damping ................................................................. 211
8-10 Time plot of impulse responses with (solid) and without (dashed) damping ................................................................. 212
8-11 Placing magnet arrays with the alignment cross ....................... 214
8-12 Average heights of magnet-array and capacitance-probe-target surfaces on the bottom side of the platen. The reference plane is the one which encloses four top surfaces of mirror pads on the other side. ........... 216
8-13 Bottom view of the platen ...................................................... 217
8-14 Side view of the stage indicating the nominal sensor gap of 500 μm, and the nominal actuator gap of 250 μm. ............................ 219
8-15 Cross-sectional view of the stator ............................................ 220
8-16 Top view of the stator core .................................................... 222
8-17 Side view of the stator core .................................................... 223
8-18 End view of the stator core ........................................ 224
8-19 Stator loose part .................................................. 225
8-20 Winding assembly .................................................. 226
8-21 Top view of a motor stator with a capacitance probe mounted on its rail 227
8-22 Top and side views of stationary parts on the mounting table except
for the metrology devices ............................................. 229

9-1 A rectangular prism with indication of its center of mass .......... 237
9-2 Two parallel axes attached on a rigid body with origins at its center
of mass and at an arbitrary point A ................................... 238
9-3 Self-inductance measurement ...................................... 239
9-4 DQ frame attached to the platen .................................. 243
9-5 Suspension of the platen in dynamic equilibrium. Only peaks of wind-
ing currents are indicated as dots and crosses. ...................... 244

10-1 Control loop of the planar magnetic levitator system ............ 262
10-2 Free body diagram for force allocation ............................ 265
10-3 Decoupled lead-lag controller for z ............................. 276
10-4 Root locus for z ...................................................... 276
10-5 3-μm vertical step responses by real measurement (solid) and by MAT-
LAB simulation (dashed) ............................................... 277
10-6 Loop transmission for z ............................................. 279
10-7 Closed-loop Bode plot for z .......................................... 280
10-8 5-μm step response in z with perturbed motions in the other five axes 281
10-9 Loop transmission for ψ and θ ...................................... 282
10-1050-μrad step response in ψ with perturbed motions in the other five axes 283
10-1150-μrad step response in θ with perturbed motions in the other five axes 284
10-12 Loop transmission for x and y ..................................... 286
10-13 Loop transmission for φ ........................................... 287
10-145-μm step response in \( x \) with perturbed motions in the other five axes 288
10-155-μm step response in \( y \) with perturbed motions in the other five axes 289
10-1650-μrad step response in \( \phi \) with perturbed motions in the other five axes 290
10-17Position noises in six-degree-of-freedom position regulation with the six decoupled lead-lag compensators .................. 291
10-18Position noise with the optical table air legs pressurized. Platen is resting on stators with control turned off .................. 293
10-19Position noise with the optical table air legs not pressurized. Platen is resting on stators with control turned off .................. 294
10-20Position fluctuation due to the laser wave length change induced by air movement ........................................ 295
10-21Position noises in six-degree-of-freedom position regulation with the linear quadratic regulator for the lateral modes .................. 299
10-2220-mm repetitive steps in \( y \) with its velocity profile \( (v) \) and perturbed motions in \( z \) and \( x \) .......................... 302

A-1 Conventional planar motor magnet matrices (a) Asakawa (b) Hinds (c) Ebihara .................................................. 314
A-2 Magnet matrix for planar motors .................................. 315
A-3 Flying puck design with superimposed Halbach magnet matrix with interweaved windings .................................. 319
A-4 Superimposed Halbach magnet matrix on bottom of the platen .... 320

B-1 Fabrication of radially magnetized tubular magnet .................. 322
B-2 Tubular linear motor model with a tubular Halbach array and three-phase windings .................................. 322
B-3 Permanent-magnet tubular motor stage by Michael Berhan .......... 336

C-1 Critical surface of a generic superconductor .................. 340
C-2 History of superconducting critical temperatures (after [BB94]) .... 344
C-3 Low-temperature behaviors of superconductor and perfect conductor
(after [OD91]) .................................................. 346
C-4 Image method for Meissner effect (after [OD91]) .......................... 347
C-5 Magnetization versus applied magnetic field ................................. 348
C-6 Bean's critical state model ............................................... 352
C-7 Hysteretic curves for high-temperature superconductors ............... 353
C-8 Flux lines causing levitation and lateral stability of magnet over type
II superconductor (after [HGJ+88]) ...................................... 354
C-9 No hysteretic loss due to axial symmetry .................................. 357
C-10 Hybrid superconducting magnetic bearing ............................... 358
C-11 Magnet-superconductor torque coupler ................................. 360
C-12 Superconducting damper ............................................... 362
C-13 High-speed induction motor using high-temperature superconductor
bearings ................................................................. 363
C-14 Levitation force as a function of vertical position ....................... 365
C-15 State-space model of a superconducting magnet ....................... 370
C-16 Test magnet .................................................................. 370
C-17 Impedance plots with respect to frequency variation .................. 371
List of Tables

5.1 Ripple force with various phases with a current density of $1.5 \times 10^6$ A/m$^2$ 119

7.1 Specifications of NdFeB and ferrite magnets .......................... 160
7.2 Magnet array dimensions ............................................. 169
7.3 Modules in the VMEbus rack ........................................ 183
7.4 Selected memory map in Pentek 4284 ................................ 184
7.5 DATEL D/A converter board channel assignment ................. 190

9.1 Self-inductance of the phase windings without the effect of the magnet arrays .................................................. 240
9.2 Self-inductance of the phase windings with the effect of the magnet arrays .................................................. 240

B.1 Forces of tubular and linear motors (N/cell) ....................... 334
Symbols

All of the symbols used in this thesis are listed below together with the chapter where they are first used or defined. Vector quantities and matrices are denoted with bold faces. SI units are given for all dimensional quantities.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Index</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>stator/magnet array number 1</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>stator/magnet array number 2</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>stator/magnet array number 3</td>
</tr>
<tr>
<td>IV</td>
<td>1</td>
<td>stator/magnet array number 4</td>
</tr>
<tr>
<td>$A$</td>
<td>4</td>
<td>vector potential (T-m)</td>
</tr>
<tr>
<td>$A$</td>
<td>3</td>
<td>the first phase</td>
</tr>
<tr>
<td>$A$</td>
<td>10</td>
<td>system matrix</td>
</tr>
<tr>
<td>$B$</td>
<td>3</td>
<td>magnetic flux density (T)</td>
</tr>
<tr>
<td>$B$</td>
<td>3</td>
<td>the second phase</td>
</tr>
<tr>
<td>$B$</td>
<td>10</td>
<td>input matrix</td>
</tr>
<tr>
<td>$B_r$</td>
<td>5</td>
<td>remanence (T)</td>
</tr>
<tr>
<td>$C$</td>
<td>3</td>
<td>the third phase</td>
</tr>
<tr>
<td>$C$</td>
<td>5</td>
<td>contour for integration (m)</td>
</tr>
<tr>
<td>$C$</td>
<td>10</td>
<td>input matrix</td>
</tr>
<tr>
<td>$D$</td>
<td>4</td>
<td>electric displacement (C/m²)</td>
</tr>
<tr>
<td>$D$</td>
<td>8</td>
<td>bending stiffness (N/m)</td>
</tr>
<tr>
<td>$d$</td>
<td>8</td>
<td>core thickness plus thickness of one face (m)</td>
</tr>
<tr>
<td>$d$</td>
<td>9</td>
<td>distance (m)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Number</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>electric field intensity</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
<td>modulus</td>
</tr>
<tr>
<td>F</td>
<td>3</td>
<td>force density</td>
</tr>
<tr>
<td>f</td>
<td>4</td>
<td>vector of force</td>
</tr>
<tr>
<td>f</td>
<td>1</td>
<td>force</td>
</tr>
<tr>
<td>G</td>
<td>5</td>
<td>motor geometry constant</td>
</tr>
<tr>
<td>g</td>
<td>5</td>
<td>gravitational constant</td>
</tr>
<tr>
<td>H</td>
<td>4</td>
<td>magnetic field intensity</td>
</tr>
<tr>
<td>I</td>
<td>9</td>
<td>inertia tensor</td>
</tr>
<tr>
<td>I</td>
<td>5</td>
<td>winding current</td>
</tr>
<tr>
<td>I</td>
<td>9</td>
<td>inertia</td>
</tr>
<tr>
<td>i</td>
<td>4</td>
<td>unit vector</td>
</tr>
<tr>
<td>i</td>
<td>9</td>
<td>vector of current</td>
</tr>
<tr>
<td>i</td>
<td>2</td>
<td>current</td>
</tr>
<tr>
<td>J</td>
<td>3</td>
<td>current density</td>
</tr>
<tr>
<td>J</td>
<td>9</td>
<td>square root of the 2 × 2 identity matrix</td>
</tr>
<tr>
<td>j</td>
<td>4</td>
<td>the imaginary unit</td>
</tr>
<tr>
<td>K</td>
<td>4</td>
<td>surface current density</td>
</tr>
<tr>
<td>k</td>
<td>4</td>
<td>spatial wave number</td>
</tr>
<tr>
<td>L</td>
<td>9</td>
<td>vector of angular momentum</td>
</tr>
<tr>
<td>L</td>
<td>1</td>
<td>inductance</td>
</tr>
<tr>
<td>L</td>
<td>9</td>
<td>angular momentum</td>
</tr>
<tr>
<td>l</td>
<td>4</td>
<td>spatial wavelength</td>
</tr>
<tr>
<td>l</td>
<td>10</td>
<td>moment arm</td>
</tr>
<tr>
<td>M</td>
<td>4</td>
<td>magnetization</td>
</tr>
<tr>
<td>M</td>
<td>1</td>
<td>mass</td>
</tr>
<tr>
<td>Nₘ</td>
<td>6</td>
<td>number of magnet pitches</td>
</tr>
<tr>
<td>n</td>
<td>4</td>
<td>normal vector to a surface</td>
</tr>
<tr>
<td>Symbol</td>
<td>Value</td>
<td>Definition</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>------------------------------------</td>
</tr>
<tr>
<td>( p )</td>
<td>9</td>
<td>linear momentum (kg-m/s)</td>
</tr>
<tr>
<td>( P )</td>
<td>5</td>
<td>power (W)</td>
</tr>
<tr>
<td>( p )</td>
<td>5</td>
<td>integer representing the successive winding cycles</td>
</tr>
<tr>
<td>( p )</td>
<td>9</td>
<td>angular velocity around the ( x )-axis (rad/s)</td>
</tr>
<tr>
<td>( q )</td>
<td>5</td>
<td>number of phases</td>
</tr>
<tr>
<td>( q )</td>
<td>9</td>
<td>angular velocity around the ( y )-axis (rad/s)</td>
</tr>
<tr>
<td>( R )</td>
<td>9</td>
<td>displacement vector (m)</td>
</tr>
<tr>
<td>( R )</td>
<td>5</td>
<td>resistance (( \Omega ))</td>
</tr>
<tr>
<td>( r )</td>
<td>9</td>
<td>angular velocity around the ( z )-axis (rad/s)</td>
</tr>
<tr>
<td>( S )</td>
<td>4</td>
<td>surface for integration (m(^2))</td>
</tr>
<tr>
<td>( T )</td>
<td>4</td>
<td>Maxwell stress tensor (N/m(^2))</td>
</tr>
<tr>
<td>( T_{32} )</td>
<td>10</td>
<td>inverse Blondel-Park transformation</td>
</tr>
<tr>
<td>( T )</td>
<td>4</td>
<td>temporal period (s)</td>
</tr>
<tr>
<td>( T_s )</td>
<td>5</td>
<td>sampling period (s)</td>
</tr>
<tr>
<td>( t )</td>
<td>1</td>
<td>time (s)</td>
</tr>
<tr>
<td>( t )</td>
<td>8</td>
<td>face thickness (m)</td>
</tr>
<tr>
<td>( u )</td>
<td>10</td>
<td>input vector</td>
</tr>
<tr>
<td>( u )</td>
<td>9</td>
<td>velocity along the ( x )-axis (m/s)</td>
</tr>
<tr>
<td>( u )</td>
<td>10</td>
<td>input</td>
</tr>
<tr>
<td>( V )</td>
<td>5</td>
<td>phase voltage (V)</td>
</tr>
<tr>
<td>( V )</td>
<td>10</td>
<td>performance index</td>
</tr>
<tr>
<td>( v )</td>
<td>4</td>
<td>velocity (m/s)</td>
</tr>
<tr>
<td>( v )</td>
<td>9</td>
<td>velocity along the ( y )-axis (m/s)</td>
</tr>
<tr>
<td>( W' )</td>
<td>1</td>
<td>coenergy (J)</td>
</tr>
<tr>
<td>( w )</td>
<td>5</td>
<td>depth of the stator winding (m)</td>
</tr>
<tr>
<td>( w )</td>
<td>8</td>
<td>total mass per unit area (kg/m(^2))</td>
</tr>
<tr>
<td>( w )</td>
<td>9</td>
<td>velocity along the ( z )-axis (m/s)</td>
</tr>
<tr>
<td>( X )</td>
<td>9</td>
<td>displacement in ( x )-axis (m)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Number</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>x</td>
<td>10</td>
<td>state vector</td>
</tr>
<tr>
<td>x</td>
<td>1</td>
<td>the first coordinate (m)</td>
</tr>
<tr>
<td>Y</td>
<td>9</td>
<td>displacement in y-axis (m)</td>
</tr>
<tr>
<td>y</td>
<td>10</td>
<td>output vector</td>
</tr>
<tr>
<td>y</td>
<td>1</td>
<td>the second coordinate (m)</td>
</tr>
<tr>
<td>Z</td>
<td>9</td>
<td>displacement in z-axis (m)</td>
</tr>
<tr>
<td>z</td>
<td>1</td>
<td>the third coordinate (m)</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>4</td>
<td>magnet thickness (m)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4</td>
<td>absolute value of spatial wave number (1/m)</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>4</td>
<td>winding thickness (m)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>5</td>
<td>spatial angle (rad)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>9</td>
<td>damping ratio</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>4</td>
<td>turn density (1/m²)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>6</td>
<td>angle (rad)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>9</td>
<td>the second Euler angle (rad)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>9</td>
<td>vector of magnetic flux linkage (Wb)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>5</td>
<td>magnetic flux linkage (Wb)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>4</td>
<td>permeability (H/m)</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>4</td>
<td>permeability of free space (= $4\pi \times 10^{-7}$ H/m)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>4</td>
<td>charge density (C/m³)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>4</td>
<td>conductivity (S/m)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>9</td>
<td>vector of torque (N-m)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>4</td>
<td>volume for integration (m³)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>9</td>
<td>torque (N-m)</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>5</td>
<td>magnetic flux (Wb)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>9</td>
<td>the first Euler angle (rad)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>9</td>
<td>the third Euler angle (rad)</td>
</tr>
<tr>
<td>$\chi_m$</td>
<td>4</td>
<td>magnetic susceptibility</td>
</tr>
</tbody>
</table>
\( \omega \quad 4 \) angular frequency (rad/s)

The accents are

- 1 indication of a nominal value
- 1 indication of small signal value
- 4 indication of a complex Fourier coefficient with time dependence
- 4 indication of a complex Fourier coefficient

The superscripts are

\[ a \quad 4 \] indication of surface \((a)\)
\[ b \quad 4 \] indication of surface \((b)\)
\[ c \quad 4 \] indication of surface \((c)\)
\[ d \quad 4 \] indication of surface \((d)\)
\[ e \quad 4 \] indication of surface \((e)\)
\[ f \quad 4 \] indication of surface \((f)\)
\[ g \quad 4 \] indication of surface \((g)\)
\[ h \quad 4 \] indication of surface \((h)\)
\[ p \quad 4 \] indication of surface \((p)\)
\[ q \quad 4 \] indication of surface \((q)\)
\[ T \quad 9 \] indication of algebraic transposition
\[ ^* \quad 4 \] indication of the complex conjugate
\[ ' \quad 1 \] indication of return phase
\[ ^t \quad 4 \] indication of coordinates in the platen frame

The subscripts are
0 4 indication of a nominal value
1 1 indication of motor/magnet array number 1
2 1 indication of motor/magnet array number 2
3 1 indication of motor/magnet array number 3
4 1 indication of motor/magnet array number 4
A 4 indication of phase A
a 5 indication of phase a
B 4 indication of phase B
b 5 indication of phase b
C 4 indication of phase C
c 8 indication of a sandwich core
c 10 indication of a capacitance probe
D 9 indication of a direct-axis
E 5 indication of an electrical quantity
e 4 indication of an equivalent quantity
f 4 indication of free current
f 8 indication of a sandwich face
h 4 indication of a homogeneous solution
l 10 indication of a long moment arm
M 5 indication of a mechanical quantity
n 4 indication of the nth order Fourier coefficient
p 4 indication of a particular solution
p 7 indication of a peak value
Q 9 indication of a quadrature-axis
S 5 indication of a source or supply
s 10 indication of a short moment arm
x 4 indication of the x-component of a vector quantity
y 4 indication of the y-component of a vector quantity
\[ z \quad 4 \] indication of the \( z\)-component of a vector quantity

\[ z \quad 10 \] indication of a vertical moment arm

\[ \alpha \quad 9 \] indication of phase \( \alpha \)

\[ \beta \quad 9 \] indication of phase \( \beta \)

The left superscripts are

\[ M \quad 4 \] indication of a quantity related to the magnet

\[ S \quad 4 \] indication of a quantity related to the stator

The left subscripts are

\[ U \quad 4 \] indication of a region above a boundary

\[ L \quad 4 \] indication of a region below a boundary

The components of a vector or a matrix are

\[ []_j \quad 4 \] \( j\)th component of the vector in the bracket

\[ [.]_{ij} \quad 4 \] \((i, j)\)th component of the matrix in the bracket
A meaningful presentation to engineers must interweave and interrelate mathematical concepts, physical characteristics, the modeling process, and the establishment of a physical “feel” for the world of reality.

H. H. Woodson and J. R. Melcher
Chapter 1

Introduction

1.1 High-Precision Planar Motion Control

The control of motion in the near-vicinity of a plane is an important task in many precision machines, for example, wafer steppers, surface profilometers, and scanned probe microscopes. In the case of wafer stepper stages, which are the primary focus of the thesis, the motion control stage must provide travel over relatively large displacements (on the order of hundreds of millimeters) in two planar degrees of freedom, small displacements (on the order of hundreds of micrometers) in the direction normal to the plane, as well as small rotational displacements (on the order of milliradians) about three orthogonal axes.

Figure 1-1 shows how a wafer stepper works. The wafer stepper is operated in step, expose, and repeat sequence to position the wafer under the lens for lithography. The die site on the wafer is exposed by patterned ultraviolet light as defined by the mask. The one-step distance depends on the dimension of the die sites whose typical lateral dimension is on the order of 20 mm. As the time duration for moving the wafer from one die site to another heavily affects the throughput, faster positioning speed is desirable. In steppers, a die site under the lithographic lens is exposed while the wafer is at a standstill. There are also advanced step-and-scan type lithography
tools where exposure occurs on the fly. So, precise position control is very important in the current and future deep-submicron lithography technology.

Magnetic levitation is an enabling technology for high-precision motion control. This thesis presents the design of a stage which can provide the above ranges of motion with only one magnetically levitated moving part. This approach promises significant advantages for the wafer stepper application. Among these advantages are: (1) One moving part can be designed to have high natural frequencies and thus can be moved rapidly vis-a-vis multi-element stages which have more complex dynamics; this
allows increased machine throughput. (2) The accuracy of a magnetically levitated stage is not limited by its bearing surfaces, and thus our design can scale with the decreasing feature sizes of next-generation integrated circuits. (3) A fully levitated stage requires no precision bearing surfaces, which thereby reduces fabrication cost. (4) A levitated stage requires no lubricants, and does not generate wear particles, and is thus highly suited for clean-room or vacuum environments. (5) By eliminating complex mechanical elements the stage fabrication costs are reduced and the stage reliability is increased.

In the following section, I provide an overview of the performance goals and working principles of the planar magnetic levitator with only one moving part. Detailed discussions on analysis, design, and control issues follow in the subsequent chapters.

1.2 The Planar Magnetic Levitator

Figure 1-2 shows the prototype magnetically levitated stage. The planar coverage of the stage is $50 \times 50$ mm, which can readily be scaled to accommodate the next generation 300-mm wafers. For focusing and alignment, the vertical travel range is $\pm 200 \ \mu m$ and the angular ranges of the stages are $\pm 600 \ \mu rad$ with 1-\mu rad angular position noise. Thus, all motions required for lithography can be supplied by this single stage. We aim for a design goal to step 20 mm and settle to the required position noise (20 nm) in under 200 ms. The position noise should be maintained during an approximately 300 ms die-site exposure time. The design is also applicable to the newer classes of step-and-scan lithography stages.

The magnetic levitator contains four three-phase linear permanent-magnet motors as labeled in Figure 1-3. Each linear motor can generate suspension (vertical) force as well as drive (lateral) force. With an orthogonal arrangement of the motors as in Figures 1-3–1-4, the platen generates all six-degree-of-freedom motions for focusing and alignment and large two-dimensional step and scanning motions for high-precision
Figure 1-2: The six-degree-of-freedom magnetic levitator with planar motion capability.

Figure 1-3: Perspective view of the magnetically levitated stage.
Figure 1-4: Force components acting on magnet-array centers of masses

positioning as a wafer stepper stage in semiconductor manufacturing. For example, we activate positive $f_{1z}$ and $f_{3z}$ in Figure 1-4 to get a motion in the $+x$-direction. If $f_{1z}$ and $f_{3z}$ are in opposite directions, we get a rotational motion around the $z'$-axis. To generate a positive rotation around the $x'$-axis, we drive with positive $f_{1z}$ and $f_{2z}$ and negative $f_{3z}$ and $f_{4z}$. Motions in the three other degrees of freedom are generated in a similar fashion.

1.3 Thesis Overview

The thesis consists of four parts: Review of Prior Art; Analysis; Design; and Dynamics and Control of the high-precision planar magnetic levitator. Appendices follow the main body of the thesis.

Part I of the thesis provides a literature review for levitation techniques and
precision motion control as background for this work. In Chapter 2, we discuss various methodologies in the broad field of levitation. The focus there is magnetic suspension and levitation. Chapter 3 includes a review of conventional actuators and sensors in motion control systems. Prior precision planar positioning systems are also introduced.

In Part II, I concentrate on the electromechanical analysis of the permanent-magnet linear motors that are essential parts of the planar levitation system. To facilitate the design, continuum and lumped electromechanical analyses are developed. Continuum electromechanical analysis for field solutions and force capacity is given in Chapter 4. Lumped analysis for the electrical terminal relation of the linear motor is the main subject of Chapter 5. Fourier analysis for magnet arrays and multiphase surface wound stators predicts the force ripple which should be minimized for precision motion control.

Part III covers design concepts, and electromagnetic and mechanical design of the levitator. In the conceptual design phase (Chapter 6), several candidate concepts are generated and the best concept for prototyping is determined. Chapter 7 presents electromagnetic design of magnet arrays and stators of the linear motor part of the levitator, power amplifiers, and the overall instrumentation. Design and construction of mechanical parts are the main issues in Chapter 8. Packaging and assembly of the mechanical parts is also described in detail.

Part IV presents the dynamics and control of the levitator. I derive linearized dynamic models of the levitator in Chapter 9. A DQ-decomposition theory is applied to decouple the system dynamics into vertical and lateral dynamics. Chapter 10 gives testing results and classical and modern control designs for the stage.

In the last chapter of the thesis (Chapter 11), we discuss the achieved performance of the levitator. I suggest improvements for the current design and discuss scaling issues to accommodate larger next-generation wafers.

In the Appendices, I present a new magnetic levitator concept with a superimposed
magnet matrix, analyses for permanent-magnet tubular linear motors, a case study of superconducting levitation, and the real-time control code and MATLAB code.

1.4 Contributions of the Thesis

The realization of the world’s first planar magnetic levitator is the main contribution of this thesis work. This high-precision magnetically levitated stage with large planar motion capability is the first stage capable of providing all the motions required for photolithography in semiconductor manufacturing with only one moving part.

More specific contributions of the thesis are the analysis, design, and control of such a levitator as follows. (1) Development of analytical tools predicting electromechanical properties: derivation of transfer relations and field solutions using magnetic flux density and vector potential for linear motors and tubular motors; electrical terminal relations—stator flux linkage, self-inductance and back emf; force ripple in Halbach linear motors. (2) Electromagnetic and mechanical design for a planar magnetic levitator: generation of several conceptual designs; single-sided field designs—triangular windings and two-dimensional Halbach magnet matrix; metrology, sensor mounting, and arrangements of actuators; detailed construction methodology including Halbach magnet array fabrication. (3) Experimental verification of nanometer-level planar position control: decoupled real-time controllers for six-degree-of-freedom stabilization and large two-dimensional motions; development of a planar-motor version of the DQ-decomposition theory to decouple vertical and lateral dynamics; development of a multivariable linear quadratic controller.
Part I

Review of Prior Art
In the Review of Prior Art part of this thesis, I present a literature and patent review for the technical trends in levitation and precision motion control. Chapter 2 gives historical background material on various levitation schemes with an emphasis on magnetic levitation. We conclude there is no successful previous work to achieve six-degree-of-freedom magnetic levitation with large two-dimensional motion capability with only one moving part.

In the first two sections of Chapter 3, we discuss actuators, bearings, and sensors frequently used in motion control. A linear multi-phase permanent-magnet synchronous motor is one the most suitable actuators for suspension and propulsion purposes. I introduce existing planar positioners with concentration on one-moving part designs with no mechanical contact.
Chapter 2

Levitation Techniques

There are many systems in everyday life that utilize levitation phenomena. We may take an airplane for one of the most common examples for aerodynamic levitation. The other example being commercialized is a maglev train [Jay81, KT69, Lai77, Sie94]. A recent application is a flywheel energy storage using superconducting levitation [NHT+95]. We can derive a common feature out of these applications: There is no mechanical contact between the body of interest and the fixed frame that could be the earth, a rail, or a housing, respectively. I classify these levitation phenomena into four classes—aerodynamic levitation, magnetic levitation, electrodynamic levitation, and superconducting levitation. In the first section of this chapter, we review background materials for these levitation techniques. Good survey materials for levitation techniques include [Gea64, Jay81, Jay82, Tru90]. Brandt gives a survey of intrinsically stable levitation including acoustic levitation and optical levitation and concentrates on superconducting levitation [Bra89].

In the second section of this chapter, we concentrate on the prior techniques in magnetic suspension and levitation. Geary gives a vast number of references on magnetic levitation and its early development [Gea64]. Jayawant gives a broad treatment of levitation phenomenon with a concentration on applications of maglev trains [Jay81]. Design, dynamics, and control issues of magnetic levitation, primarily
on maglev trains, can be found in [Sin87]. Trumper gives another good review for general magnetic levitation techniques for precision motion control [Tru90].

2.1 Levitation in the Physical World

2.1.1 Aerodynamic Levitation

Figure 2-1 reminds us of a toy with which we suspend a ping-pong ball by giving out breath through a pipe. The air jet coming through the nozzle at the end of the pipe can have a sufficiently large velocity to give a drag force which can compensate the weight of the ball. In the direction of the air jet, the drag force decreases as the height of the ball increases. So, the motion is stable in this direction. The tilted and off-centered air-jet direction provides a stable lift force for the levitated ball. Due to this asymmetry, velocities of the air at the left and right sides of the sphere are different. The ball has a force that comes from the Bernoulli pressure $-\rho v^2/2$ in air with density $\rho$ and local velocity $v$. So, the ball can be stabilized and the suspension height is adjustable by changing the velocity of the air jet. An airplane wing design uses this Bernoulli pressure to fly.

Hovercrafts use reaction force with controlling air pressure to stabilize and propel the vehicle. In smaller-scale applications, a vacuum-preloaded aerostatic bearing uses the same action-reaction principle. With three or more such aerostatic bearings, we can control three degrees of freedom. This scheme is used by Asakawa [Asa85] in his
planar actuator and some other state-of-the-art positioners, which are described in more detail in the next chapter.

2.1.2 Magnetic Levitation

A nineteenth century English minister and natural philosopher Samuel Earnshaw (1805–1888, [Sco59] for a brief biography) stated a fundamental proposition on passive electromagnetic stability [Ear42]. A modern statement of the Earnshaw’s theorem can be found in [Str41] and other texts. Here is a magnetic version of the statement.

A magnetized body placed in an magnetostatic field cannot be maintained in stable equilibrium in systems under the influence of the magnetic force alone.

The magnetic potential satisfies the Laplace equation, so it is a harmonic function. It has no local minima or maxima by the maximum principle [Ahl79]. Therefore, there is no static equilibrium for stable suspension of the magnetized body. However, the Earnshaw’s theorem has nothing to do with the dynamic equilibrium. This is exactly how magnetic suspension and levitation systems work, but they need external energy for control. So, we need controlled actuators to stabilize the levitated body. There is another way to evade Earnshaw’s theorem, which is use of diamagnetic material ($\mu < 0$). This is the case of superconducting levitation. A case study of superconducting levitation are presented in Appendix C.

Recent developments in magnetic material and power electronic devices and circuits make magnetic levitation more attractive. The magnetically levitated (maglev) trains are the most mature and popular applications in this category. The contactless magnetic bearings come into being viable applications in the area of high-speed rotational machinery [O’C92]. A recent application under development is flywheel energy storage, which contains a flywheel suspended in a housing [Bas82, XCM+95]. All these conventional magnetic levitation systems need active feedback control to give
Figure 2-2: Levitation of molten metal by electrodynamic levitation (after [Jay81])

stability to the system. A more extensive review of magnetic levitation is given in the next section.

2.1.3 Electrodynamic Levitation

From Earnshaw's theorem, six-degree-of-freedom electrostatic levitation in free space is inherently unstable because the dielectric constant of any material $\varepsilon$ is always greater than that of free space $\varepsilon_0$. For system stability, thus, an electrodynamic levitation system needs AC actuators as in magnetic levitation. A common electrodynamic levitator is an eddy current levitator. Lenz's law dictates that eddy current is generated to resist a time-varying external field induced by a current loop. In reality, distribution of the eddy current is complicated and depending on the frequency and height of the current loop and the conductivity of the conductor. We can show a linear induction motor as an eddy current levitator can generate both suspension and drive forces. However, there are limitations in the induction levitation. This will be discussed in more detail in Chapter 3.

Another example of electrodynamic levitation is levitation of molten metal as in Figure 2-2. AC current at radio frequency flows through the cooper tubes, which are water-cooled. The counter winding above the liquid metal is introduced to stabilize the metal. The induced eddy current in the liquid metal maintains levitation and its shape. Electrodynamic levitation is also used in the design of some maglev vehicles in transportation.
Figure 2-3: A current loop is levitated by repulsion force due to its image current loop induced inside the superconductor.

### 2.1.4 Superconducting Levitation

Diamagnetism is a macroscopic phenomenon of Faraday’s law and Lenz’s law acting on the atomic level. The induced current opposing a field change is dispersive in normal resistive conductors. Hence, they have very weak diamagnetism, if any. If a superconductor is merely a lossless conductor, we could not take advantage of it for levitation. Yet, a superconductor is more than a lossless conductor; it is also a diamagnet. Figure 2-3 shows schematic diagram of superconducting levitation. This diamagnetic feature was what confused people in the early stage of superconductivity. A superconductor expels existing field even when the cooling process is applied under the external field (Meissner effect—zero relative permeability or zero flux density inside superconductor). On the contrary, a perfect conductor is expected to freeze-in, and not to expel the existing field. There have been many research efforts in superconducting levitation. I provide a case study of recent researches [MMLC93, DGM+92, GMEA+95] in Appendix C.

### 2.2 Magnetic Suspension and Levitation

We have discussed various levitation phenomena in the physical world. Magnetic levitation has advantages over other levitation techniques given in the previous section for precision motion control applications. (1) Electrodynamic levitation uses repulsive force due to eddy current to lift the levitator. So, power loss and heat generation in
the conducting body are not avoidable. The thermal expansion error due to the heat generation is detrimental for precision position control. (2) The magnetic levitation can be realized less expensive by compared with superconducting levitation, as it is still very costly to manufacture superconducting material. A superconducting levitation system also needs a cryocooler that increases its operational cost. The stiffness of superconducting levitation system is generally low and it usually uses additional electromagnets for more stability and precise motion control. (3) We can achieve very stiff and stable position control with air-vacuum chucks in aerostatic levitation. The critical disadvantage is that they cannot be readily used in vacuum-chamber environments, which becomes more frequent in semiconductor manufacturing. Thus, we decide to employ magnetic levitation for our application.

In the literature, there is a tendency to use the terminologies, levitate and suspend interchangeably. However, according to the American Heritage Dictionary:

- levitate: to rise or cause to rise into the air and float in apparent defiance of gravity

- suspend: to support or keep from falling without apparent attachment, as by buoyancy

We could extract a fine meaning difference with existence of significant 'floating' movement. In this thesis, I distinguish them for classification purpose. I use suspension in the cases where the lateral positions of the suspended body in space are stabilized and regulated. It could be either single-degree-of-freedom (DOF) suspension, if all but single-degree-of-freedom stability are granted or not of interest, or multiple-degree-of-freedom suspension if more than one degree-of-freedom motion should be stabilized and regulated. So, a single-axis magnetically levitated rate gyroscope is classified as a multiple-degree-of-freedom suspension device by this definition. I specifically use the word levitation only in the case there is significant large-movement capacity in one or two lateral directions (one-dimensional levitation and two-dimensional levitation). In
other words, a levitator has capabilities in propulsion as well as in suspension. The ratio of the relative magnitudes between large movements in propulsion and small movements in suspension could be more than 100, even if absolute magnitudes of the movements depend on specific applications. So, there should be no confusion between the meanings of the terms, magnetic suspension and magnetic levitation. Now we define the magnetic levitation as:

a stable hovering state of a body without any mechanical contact to a reference frame by magnetic attraction or repulsion force which cancels the body’s weight

In the rest of this section, I give fundamental materials and prior art reviews for magnetic suspension and levitation technology.

2.2.1 Single-Degree-of-Freedom Suspension—Ball Suspension System

It is believed that the earliest magnetic suspension system documented to properly operate was done by F. T. Holmes at the University of Virginia [Hol37a, Hol37b]. He used a vacuum tube circuit to implement a proportional controller to control an electromagnet to suspend a vertical ferromagnetic needle. The position of the needle was transduced optically. His experiment was the first of many research efforts at the University of Virginia through the 1960’s [Bea63]. A detailed bibliographical review of these works can be found in [Tru90]. Cho, et al. presented a sliding mode control of a single-degree-of-freedom ball suspension system, which is one of the most recent control efforts for magnetic suspension systems [CKS93].

Figure 2-4 shows the schematic diagram of the ball suspension system [WM68]. The steel ball with mass $M$ is suspended under the electromagnet. The electromagnet generates attraction force to carry the weight of the ball. The vertical motion is inherently unstable, so we need a feedback control to stabilize it. For this purpose,
Figure 2-4: Ball suspension system with an electromagnet. The vertical position of the ball is fed back by the light source, and stabilized at the reference position by the controller.

The system has a position sensor, a controller and an amplifier. Since the whole system wants to decrease the reluctance of its magnetic circuit and its magnetic energy, the two lateral degree-of-freedom motions of the ball are inherently stable. This is the case because the magnetic flux density is densest on the axis of the electromagnet. We do not care about three rotational degrees of freedom.\textsuperscript{1} Any perturbed rotational motions will die off due to eddy current loss in the steel ball. Thus, we call this system a single-degree-of-freedom suspension system since we can control the vertical position of the ball by setting reference position in the controller. We closely follow the approach given in Example 5.1.3 in [WM68] for the analysis of this single-degree-of-freedom ball suspension system.

\textbf{Nonlinear Suspension Model}

We can model the inductance $L$ of this system as a function of the vertical position $x$ of the ball. The inductance is largest at $x = 0$ and approaches a constant as $x$ tends

\textsuperscript{1}Where the ball has magnetic hysteresis, these rotational motions can be coupled to translation in some complex ways.
to infinity. So, we can model the inductance as

$$L(x) = L_1 + \frac{L_0}{1 + x/a}, \quad (2.1)$$

where $L_1$, $L_0$ and $a$ are positive constants. The magnetic coenergy is with the coil current $i$,

$$W'(i, x) = \frac{1}{2} L(x) i^2 = \frac{1}{2} \left( L_1 + \frac{L_0}{1 + x/a} \right) i^2, \quad (2.2)$$

and the force of electrical origin is

$$f = \frac{\partial W'}{\partial x} = -\frac{1}{2a} \frac{L_0}{(1 + x/a)^2} i^2. \quad (2.3)$$

When the ball is in equilibrium, this force cancels the weight of the ball,

$$M g = \frac{1}{2a} \frac{L_0}{(1 + \bar{x}/a)^2} \bar{i}^2, \quad (2.4)$$

where $\bar{i}$ is the nominal bias current, and $\bar{x}$ is the equilibrium position.

**Linearized Suspension Model**

We can set the suspension height $\bar{x}$ by adjusting the bias current $\bar{i}$ with the relationship (2.4). Now, we develop a small signal model for the suspension system. Let $\bar{x}$ and $\bar{i}$ be the perturbed position and current. That is,

$$x = \bar{x} + \bar{x}, \quad (2.5)$$

$$i = \bar{i} + \bar{i}. \quad (2.6)$$

By using a Taylor series expansion, the linearized force equation is

$$f = -\frac{L_0}{2a} \left[ \frac{\bar{i}^2}{(1 + \bar{x}/a)^2} - \frac{2\bar{i}^2 \bar{x}}{a(1 + \bar{x}/a)^3} + \frac{2\bar{i} \bar{i}}{(1 + \bar{x}/a)^2} \right] \quad (2.7)$$

So, the linearized incremental equation of motion using (2.4) is

$$M \frac{d^2 \bar{x}}{dt^2} - \frac{L_0 \bar{i}^2}{a^2(1 + \bar{x}/a)^3} \bar{x} = -\frac{L_0 \bar{i} \bar{i}}{a(1 + \bar{x}/a)^2}. \quad (2.8)$$
The above small signal dynamic equation is the starting point for modeling and control of the suspension system. Since the coefficient of the $\ddot{x}$-term is a negative number, the system is unstable in the vertical direction as expected. Now, the small signal current $\tilde{i}$ is the control variable. We can implement a classical controller (for example, a PD controller) to stabilize the system dynamics and to give a sufficient damping to the system. The derivation of the linearized equations of motion for the planar levitator given in Chapter 9 follows a conceptually similar approach as in this example.

2.2.2 Multiple-Degree-of-Freedom Suspension—Magnetic Bearings, Wind Tunnel Balances, and Gyroscopes

Active rotary magnetic bearings are replacing conventional mechanical bearings or fluid film bearings in some applications. The advantages of the magnetic bearings include (1) no friction or wear, (2) no power loss or heat generation in the bearings, and (3) high rotational speed. Many magnetic bearing system have mechanical bearings for back-up purpose, which are not used in normal operations. There are good survey papers on magnetic bearings [Sch90, Ble92].

A typical rotary magnetic bearing system suspends the rotor shaft in five degrees of freedom; it does not confine the rotational motion around the symmetry axis of the shaft. Thus, typical magnetic bearings have radial and thrust bearings ([O'C92] and other references in this section). So, in the magnetic bearing literature, radial and thrust force control is an important issue [ME92, MFO87]. The electromechanical analysis given in the previous section for a single-degree-of-freedom suspension system can be extended to a multiple-degree-of-freedom suspension system. Control of a rotational axis with magnetic bearings is a challenging task since it is an unstable multivariable dynamic system. Moreover, a usual high-frequency bending mode of the shaft requires a high position sampling rate ($\sim 100$ Hz). Yates and Williams even suggests a multiprocessor controller may be required [YW88].
Other significant applications in aerospace engineering are wind tunnel magnetic balances and magnetic suspension gyroscopes. A wind tunnel is usually used in research for aerodynamic behaviors (such as aerodynamic coefficients identification) of a prototype aeronautic vehicle. Covert provides a good review and classification of the magnetic suspension systems for wind tunnel purpose [Cov88]. It is known that the first successful application in wind tunnels was achieved by Laurenceau and Tournier in France in 1954. Without any mechanical connection between the model and the wind tunnel, the airflow is not disturbed. This simulates a real situation more accurately. By controlling forces exerted to the model, balances in conventional wind tunnel testing are unnecessary and simpler testing setups are possible. Magnetic suspension of a wind tunnel model gets more interest as more accurate data are needed for a complicated flight conditions. Suspension systems for gyroscopes developed at the Charles Stark Draper Laboratory up to 1970's are introduced in [FGO74]. They report pendulous accelerometers with magnetic suspension have some 0.1-μrad angular uncertainties. On the other hand, systems without magnetic suspension could have angular uncertainties as large as 0.4 mrad depending on the angle between the output axis and the horizontal reference.

2.2.3 One-Dimensional Levitation—Maglev Vehicles and Positioner

Magnetic levitation reminds us of a large-scale application as in maglev trains. The maglev trains classified in two categories: the first is an attractive type as in German developments and the other is a repulsive type as in Japanese maglev trains (which uses electrodynamic levitation in the strict sense). In the German model by Siemens, the vehicle encloses the rail partially. The Japanese model by Japan National Railways employs superconducting magnets for suspension and linear synchronous motors for propulsion. The clearance between the magnet and the rail can be relatively large, which leads to a large manufacturing tolerance. There are a vast number of publica-
tions on maglev design and control. [HRC93, SMKT93] are among the recent research efforts in that direction.

Many references including [Lai77, Jay81, Sin87] deal with various design alternatives for maglev trains. Especially, [Sin87] covers operational and control issues. In the United States, Kolm and Thornton at MIT initiated the Magneplane scheme in 1969 [KT69]. Since then, there have been many ups and downs in the commercial maglev train project, as well as debates on the safety and economy. A recent article reports a proposal for a 14-mile maglev route between Orlando airport and the EPCOT center at Walt Disney World, Orlando, Florida. Maglev trains are being developed or surveyed in other countries including Korea [KSCK92] and Canada [Sle77]. One of the first and notable commercial application of a maglev train is being constructed in Germany to cover 283 km between Hamburg and Berlin in less than 1 hour [Sie94]. The maglev train, Transrapid, is expected to begin service in 2005.

A one-main-axis magnetically levitated stage for precision position control has been constructed by Mark Williams in our research group, which uses electromagnets to control the motion of a 13.5-kg platen in five (three rotational and two translational) degrees of freedom and a permanent-magnet linear motor to control motion in the sixth degree of freedom (Figure 2-5) [WTH93]. For fine focusing, the stage can provide 400 μm of travel normal to the wafer surface as well as milliradian rotations around three axes. The linear motor consists of a permanent-magnet array attached to the underside of the platen and a linear ironless six-phase stator fixed in the machine frame. This stage is intended to be mounted on a conventional mechanical linear slide used to provide 200 mm of travel in the y-direction.

2.2.4 Two-Dimensional Levitation—Objective of the Thesis

We have discussed various schemes for magnetic suspension systems and one-main-axis magnetic levitation systems, hereafter referred to as one-dimensional levitator
systems. A one-dimensional magnetic levitator provides a long-range travel in one direction perpendicular to the suspension direction. On the other hand, a two-dimensional, or planar, magnetic levitator provides long-range planar travel in any direction perpendicular to the suspension direction. A simple solution to realize two-dimensional motions with magnetic levitation may be stacking a one-dimensional magnetic levitator orthogonally on top of another one-dimensional magnetic levitator. However, this is not a two-dimensional magnetic levitation system in the strict sense because there is more than one moving part in the whole system. For recapitulation, the objective of this thesis work is to implement a six-degree-of-freedom magnetic levitator which has large planar motion capabilities with only one moving part.

In the following chapter, we review prior art in two-dimensional hybrid positioning systems. They are hybrid in the sense that many systems use aerostatic bearings for suspension, which is an inexpensive and reliable solution for some applications. Even if all the systems use some sorts of linear motors, no one has tried to interweave two-dimensional propulsion with magnetic suspension as far as I investigated.
Chapter 3

Technical Trends in Motion Control

The semiconductor industry is evolving rapidly. For instance, dynamic memory cell density is quadrupled every three-year period; the mass-production of 256M DRAMs is expected to begin soon. The development of high-precision equipment, such as wafer steppers, mask aligners, and wire bonders for semiconductor processes has accelerated to keep up with fabrication of larger and denser devices. Many traditional mechanical stages for $x$-$y$ positioning have either crossed-axis type or gantry-type configurations. In wafer steppers in semiconductor manufacturing the $x$-axis linear stage is typically driven by some sort of $y$-axis stage. The mechanical bearings for high-precision planar position control should be very accurate, so they are expensive to make. Some such systems also use a piezoelectric fine motion stage on top of a coarse $x$-$y$ mechanical stage. Therefore, the whole system can be complicated and expensive and have complex dynamics, which limits high-speed operation. Moreover, the vertical ($z$-axis) motions for focusing are generated by other independent mechanical means.

In this chapter, I provide a broad survey of the current technology of such $x$-$y$ stages. Some of them are currently used in photolithography systems. Even though
some are not intended for use in the semiconductor industry, they are listed here because of their conceptual importance. Before discussing the prior art, descriptions of conventional actuators, bearings, and sensors for precision positioners are given. In the following sections technical trends and brief introductions of such positioners are given.

3.1 Conventional Motors

In this section, I give brief descriptions of conventional motors: synchronous motors, induction motors, and variable-reluctance motors.\(^1\) We concentrate on their characteristics for precision motion control capabilities. General texts that cover these kinds of motor categories are [FKU90, KWS95]. [Leo96] is a more advanced text especially for machine control. On the basis of the characteristics of each motor type, I will propose a motor category for realization of our system. Any motors with brushes, such as DC motors, are eliminated from considerations for implementation because their inherent mechanical contacts prevent control of vertical motions.

As we justify in later sections, we use linear versions of conventional rotary motors, i.e., linear motors. A linear motor can be conceptually constructed by cutting off the rotary counterpart and opening it up to be flat [BN85]. The operational principle of linear machines is basically the same as that of conventional rotary machines. We usually ignore geometric characteristics that exist only in linear machines, such as end effects. However, a linear motor is less power efficient comparing with its rotary counterpart. It consumes power in non-effective portions of the stator or platen unless we switch them off.

\(^1\)A variable-reluctance motor is also a synchronous motor in the sense that there is no slip. In this section, however, we use the term synchronous motors only for wound-field and permanent-magnet synchronous motors just for classification purpose.
3.1.1 DC Motors

The conceptually simplest electric machine is a so-called DC motor. It has brushes and a mechanical commutator that act as a mechanical inverter. The field magnetomotive force (mmf) and armature mmf are maintained orthogonal to each other, and the axis for the armature mmf is fixed in space. So, a DC motor has a simple decoupled model and its control task is easier than that of AC machines. Before the advent of microcomputers, such DC motors were the most common type used for motion control purposes. However, a DC motor is bulkier and heavier than AC counterparts, and it needs periodic maintenance for its mechanical parts. So, the operational cost of a DC motor is generally higher. Another drawback of DC motors is that the brush wear cannot be avoided. The generation of wear particles leaves the DC motors out of applications in vacuum-chamber environments. Thus, a DC motor with brushes is ruled out for the candidate actuator of our application.

3.1.2 Synchronous Motors

In a wound-field synchronous motor, the rotor winding acts as source of a DC field, and the stator windings as armature windings. A synchronous motor has its name because the steady-state speed is only proportional to the electrical frequency in the armature excitation. The rotor rotates at the same speed with the rotating resultant armature flux wave. In other words, the synchronous speed does not vary with load change, but the torque angle changes. The torque angle is defined by the phase difference between the mmf of DC field winding and the resultant air-gap flux per pole. Synchronism is kept until the torque angle reaches 90°. Then, an instability, known as a pulling-out condition, occurs. A synchronous motor has important dynamic characteristics, called hunting, when the rotor's mechanical or electrical speed changes rapidly. The rotor shows an underdamped oscillation around the new torque angle.

A wound-field synchronous motor needs two excitations for magnetic field sources:
one for the rotor and the other for the stator. This fact makes the motor structure complicated. Moreover, in magnetically levitated systems, wires to the rotor (or the platen in linear stages) are problematic. Power dissipation due to the current in the rotor winding is another reason to keep synchronous motors with field windings from our application. This is because the thermal expansion due to temperature rise can do much harm to the accuracy of the stage.

Fortunately, we can solve most of the above problems by replacing the field windings with permanent-magnet arrays. Because there is no power loss in the permanent magnets\(^2\), the efficiency is improved in general. Using permanent magnets also leads to a more compact design. We can thereby reduce the heat load and the thermal expansion error in the platen. The magnet array acts as a source of platen field, which eliminates problems with both umbilical cables and brushes.

There are two kinds of permanent-magnet synchronous motors. The more recent kind is an interior magnet motor. The magnets are buried inside the rotor iron. The air gap is smaller and the air-gap flux can be made even larger than the magnet remanence with a proper arrangement. The torque exists in the form of magnet-stator reaction and in the form of variable reluctance. So, in the strict sense this kind of machine is a hybrid type with saliency. The other kind is a surface magnet motor, which is more conventional. It has a smooth and relatively large air gap. Even though the efficiency is lower, the cogging force, which we want to avoid in precision motion control applications, is minimal. The stator of a permanent-magnet synchronous motor can be slotted or slotless. To avoid any saliency in the stator, we make it as a slotless surface-wound type. The force ripple in this surface-wound surface-magnet linear synchronous motor is estimated in Chapter 5. This proves to be negligible; manufacturing errors in the motor fabrication will override this force ripple error.

One of the biggest drawbacks of a permanent-magnet synchronous motor is high

\(^2\)except for negligible eddy current loss due to the armature commutation
manufacturing cost because the price of rare-earth permanent-magnet material is still relatively high. However, this is not a limiting factor for the wafer stepper application, and the magnet cost is expected to decrease with improvement of the rare-earth permanent-magnet material and process. Due to the high-remanence field of permanent magnets, fabrication into magnet arrays is a non-trivial task due to large repulsive forces among magnets. However, the material’s high coercive force prevents demagnetization and makes it possible to design a permanent-magnet motor with a larger air gap.

3.1.3 Induction Motors

An induction motor has shorted windings (for wound-rotor motors) or conductors (for squirrel-cage motors) on the rotor surface instead of driven rotor windings in the synchronous motor case. The rotating armature commutation generates induced current on the rotor surface by magnetic induction. The heat generation due to the induced current causes thermal expansion of the rotor, which is problematic in high-precision control applications. The induced current reacts with the air-gap flux wave due to the armature field. Since the magnitude of the induced current depends on the relative speed between the rotor and the stator, the speed of the rotor varies with load. Frequently, the starting torque of an induction motor is not sufficient. So, variable rotor resistance or double-squirrel-cage rotor bars are usually used to increase starting torque. We define the slip of the rotor to be the difference of the synchronous speed of the stator field and the rotor mechanical speed as a fraction of synchronous speed.

The primary advantage of an induction motor is its simple structure and ruggedness. This is the reason it is widely used in many electric appliances. However, it has critical disadvantages for our design purpose. Because the induced current can be generated only by the speed difference, there is no perpendicular force without relative motion between the excitation field wave and the stage. Since the speed
characteristic of induction motors is complicated and nonlinear, modeling for precision position control is much more difficult. Since we do not know the angular position of the rotor magnetic axis, vector control or field oriented control is necessary [Leo96]. To make matters worse, the end effect depends also on the speed. A DC brake and additional coil are needed for high-precision speed and position control [BN85]. This accounts for the reason that we seldom find an example for a precision position control with induction motors, even though they may be employed for bulk applications like maglev-train propulsion.

3.1.4 Variable-Reluctance Motors

A variable-reluctance motor has many salient protrusions (teeth) on its rotor surface. The rotor has a tendency to align for a geometry with minimum air-gap reluctance so that the whole system stays at the minimum magnetic energy status. Feedback control is unnecessary for relatively low precision of some fraction of tooth pitch. However, without feedback, the position accuracy is dependent on the load and the step response is lightly damped. Because it is free of any kinds of windings on the rotor, there are no copper loss and little eddy current loss with a laminated rotor. So, a variable-reluctance motor is inexpensive and attractive for applications with low precision. With good position sensors and feedback, the performance of a variable-reluctance motor can be enhanced. Then, its merits—simplicity and inexpensiveness—are compromised. There is a variation of variable-reluctance motor: a switched reluctance motor. It has a salient rotor and stator and uses switched AC signal for its stator currents. It has a simpler structure and is less expensive than a permanent-magnet brushless motor. However, it is still hard to avoid the inherent pulsating cogging torque.

Typical linear variable-reluctance motors have about ten times stronger attraction force between the two halves of the machine than the drive force, which is not a big problem given the symmetry of rotary motors. Moreover, a single-sided,
unbalanced, linear variable-reluctance motor can only pull, and cannot push. So, motors single-sided variable-reluctance motors cannot be readily used for magnetic suspensions. The variable-reluctance type wafer stepper stages used by Ultratech\footnote{Ultratech Steppers, Inc., 3050 Zanker Road, San Jose, California 95134} use aerostatic bearings to compensate this strong pulling force. Another large problem in using them is how to deal with the inherent cogging force to achieve high position resolution. This cogging force comes from the saliency of the rotor and is very hard to model.

3.2 Actuators, Bearings, and Sensors for Positioners

There are many kinds of actuators, bearings, and sensors. However, we confine our discussions in this section within those which are appropriate for magnetically levitated positioners.

3.2.1 Actuators

Typical conventional systems use rack and pinions or ball screws to convert rotary motions from DC servo motors into linear motions. Although there are clever ideas to reduce backlash in ball screws, mechanical friction and lubrication remain problematic. Generally, it is not easy to obtain ultra-high position resolution due to backlash, friction and inaccurate movements in ball-screw drives. Thus, many high-precision positioners use fine-motion stages stacked on top of coarse stages. The fine-motion stages are usually driven by piezoelectric or voice-coil actuators. If we need vertical motion control for focusing in photolithography process in addition, other means like voice-coil actuators should also be used for this vertical axis. This makes the whole system large and leads to a lower resonant frequency. Fast motion control thus be-
comes more difficult. There are actuators using mechanical contacts as friction drives as found in [KDW91].

Incidentally, a voice-coil actuator has a permanent magnet to provide a DC magnetic field. The force is generated by the Lorentz force law with interaction of the magnetic field and the current in the voice coil. Thanks to their high bandwidth, the voice-coil actuators are appropriate to applications which require fast responses. So, the voice-coil actuator can give fast and high-precision motion without cogging or hysteresis in a limited linear motion range, typically of one-centimeter order.

In planar-motion-dominant applications, direct-drive linear motors look much more promising. The direct-drive motors are also free from the mechanical noise and lead error. A higher acceleration can be achieved, since they have only to drive the platen mass. So, the system inertia decreases and the bandwidth increases, compared with ball screw drives. We can improve the resolution with high precision sensors and fast feedback. There are some drawbacks of the direct-drive linear motors. First of all, they do not have static stiffness in the direction of the motion. They require a proper feedback, even when they are at standstill, to have a sufficient stiffness. The other drawback is that they are more expensive compared with ball-screw drives with conventional rotary motors in the same power range.

Economical operations and high force capabilities with linear motors become possible thanks to the advent of less expensive, high-remanence rare-earth permanent magnets. The capability of the vertical as well as the lateral forces is one of the greatest advantages of linear motor systems, if we design the system so as to be able to control the vertical motion. [GP86] suggests an integrated way to generate suspension and propulsion force with a linear synchronous unipolar motor for maglev vehicle application. This is impossible by definition with rotary motors due to cancellation of the radial force components. This capability allows us to dispense with fine vertical position control mechanism, such as voice-coil actuators.

An electromagnet is one of the most power efficient attitude control actuators.
A moderate size electromagnet (whose mass is 150 g including coil and core) with E-core lamination has some 75-N force capability. The whole system may lose stability with only single-sided electromagnets when there is a disturbance which requires opposite direction force for compensation. So, it is typical to use electromagnets in push-pull pairs. Using electromagnets in push-pull pairs can make the platen more robust from outside disturbances because they can increase stiffness by feedback control. In addition to using electromagnets, preload permanent magnets to compensate the gravity load are helpful to save power consumption. However, if multi-phase permanent-magnet linear motors are used, we can control vertical as well as lateral motions without electromagnets according to the commutation and force laws derived in Chapter 4. This leads to a simpler platen structure with no electromagnets.

3.2.2 Bearings

To achieve six-degree-of-freedom motions, a common approach is to separate lateral and vertical modes. For the lateral motion driving, three or more actuators are used to generate $x$, $y$, and yaw motions. The independent driving mechanisms for each degree of freedom require a number of bearings.

Aerostatic bearings have good characteristics of high stiffness, very low friction at the required speeds, vibration resistance, and moderate load capacity. If aerostatic bearings are used in the levitator design to suspend the weight of the platen, we may consider using aerostatic bearing-vacuum chuck pairs. Because the air pressure from nozzles is large and unbalanced, the platen can become easily unstable without the vacuum chucks. Stiff and sagging lines from air and vacuum pumps to the platen cause big troubles. The pipes and cables may introduce unmodeled dynamics which affect the overall system stability. It is also really difficult to solve the friction problems, because we cannot design a cable carrier without friction with any fixed members in two-dimensional motion systems. Moreover, aerostatic bearings are inappropriate for vacuum-chamber environments. Thus, they are excluded in our design consideration.
The magnetic bearing is one of the best candidates to meet our speed and precision requirements. Since there is no mechanical friction involved, no lubrication is necessary and no dust is generated. A fast response to command signal is obtainable. Since the accuracy is set primarily by sensors and the precision depends on metrology, the bearing surfaces can be finished to moderate tolerances. This reduces the manufacturing cost and leads to a simple mechanism. With a multi-phase motor structure, we can control vertical as well as lateral forces. That means we can focus the stage over the required range without other fine actuators. So, by using three or more linear motors, we can control all the rotational alignment and focusing motions while in suspension with only one moving part without any fine adjustment devices. Merging a multi-phase linear motor and a magnetic bearing also simplifies the manufacturing process significantly. However, significant control efforts are necessary to achieve the precision specifications and the overall system stability in six degrees of freedom.

3.2.3 Sensors

Many kinds of position sensors are used for magnetic suspension and magnetic bearing applications. Ball suspension systems as in Figure 2-4 typically use an optical sensor. The vertical position of the levitated body, here the ball, changes the intensity of the light so that we can tell the height of the ball. Such a sensor is simple and efficient but will not have the resolution required for our application. [BGK93] gives a summary and brief guidelines for position sensors for magnetic bearing applications. The survey in [Wal94] is another source for various sensors in mechatronics.

For precision position measurements in photolithography, a laser interferometric sensor is typically used. Many commercial systems use a HeNe laser head whose wavelength is 632.991 nm. The resolution depends on the wavelength of the light source and on the processing electronics. In our system, we use a system from Hewlett-Packard with a resolution of \( \lambda/1024 \), or 0.6 nm [Hew93a]. The laser interferometry
electronics allows direct digital position data in multiple of the smallest resolution of the system. So, A/D conversion is not required. To measure three degree-of-freedom motion of the stage, namely $x$, $y$, and yaw, we use three channels of laser interferometers. Since the laser interferometer can only measure a relative displacement from a reference position, we need other absolute displacement sensors.

An eddy current proximity sensor also can measure absolute gaps. It is frequently used in magnetic bearing applications because it has an excellent frequency response. The phase shift is no more than a few degrees up to 3 kHz for commercial sensor [BGK93]. The coil in the sensor is driven at a high constant frequency (0.5–2 MHz). The conducting material in the vicinity of this coil induces eddy current which generates a reaction field. So, the wire in the sensor looks as if its impedance has changed and the impedance depends on the distance between the sensor and the target. It dose not depend much on surface finish of the target nor dirt on the surface. However, one of the biggest disadvantages is the sensor can be affected by the magnetic field due to the magnet or the stator current. So, we rule out this sort of sensors for this magnetic levitation application.

A capacitance probe measures the capacitance between the target and the sensor itself. As the capacitance is inversely proportional to the distance, we can tell the distance with a well-calibrated capacitance probe. In general, capacitance probes have high resolution and good temperature- and DC-stability, i.e., low drift. The resolution also depends on the bit length of the A/D converter and on sensor noise as in every analog sensor. Thus, A/D converters with long bit length are recommended for applications of large traveling range with high precision. The capacitance depends on the surface roughness of the target, so it is recommended to use a gage with a large tip area to smooth out any local roughness of the target, which makes the sensor size large. Work on capacitance probes for magnetic bearing applications can be found in many papers such as [SDSW90]. The specific capacitance gages and electronics used in this thesis work is described in detail in Subsection 7.4.3.
3.3 Prior Art for Planar Positioners

Due to the diversity in the literature, we cannot define a single standard design for planar positioners including wafer steppers. The crossed-axis type where a stage for one axis stacks on top of the other is one of the conceptually simplest planar structures. The gantry type, a bridgelike framework which spans a distance with two motors at the end, is also a common stepper solution also frequently seen in $x$-$y$ plotters. Typical systems have two or three moving parts driven by DC motors and ball screws. Each actuator drives one degree of freedom. Many systems use ball or roller bearings for guidance and suspension. Some sophisticated systems use aerostatic bearings. If the resolution requirement is not so high, as in the case of plotters, the conventional $x$-$y$ stages work satisfactorily. To achieve a microfine position control as usual in photolithography, some systems use voice-coil or piezoelectric actuators and flexures mounted on top of coarse stages. Moreover, conventional crossed-axis stages do not have inherent rotary motion capability. So, they commonly use other independent actuators for small rotational position adjustments. These additional apparatuses make the whole system more complicated and bulkier, and lead to complex dynamics, which increases settling time and slows responses down.

Some of these systems utilize planar motors (or surface motors in some literature) that can directly generate two-dimensional motions with one moving part. Since a motion can be coupled in six degrees of freedom in a planar positioner system, a fast and sophisticated feedback control algorithm is mandatory. Also, the resolution of position sensors should be high enough for high-precision position control. One of the most prominent technical trends is to use aerostatic bearings for suspension along with planar motors to avoid mechanical contact or friction between the moving body and stators. Nevertheless, additional fine-movement actuators are generally required for fine attitude control—focusing, for instance—because the clearance of aerostatic-bearing surface should be kept on the order of micrometers. A notable drawback of aerostatic bearings is that they cannot be readily used inside a vacuum chamber.
Many researchers have studied the control of planar motions. An early example is the Sawyer motor which is a variable-reluctance type and has frequently been used without explicit position feedback [Saw68]. The reported step resolution is on the order of 250 $\mu$m (0.01”). Since the primary application is $x$-$y$ plotters, 0.01” resolution is sufficient. For precision motion control with a particular emphasis on wafer stepper stages, Hinds, et al. and Pelta improved the original version of the Sawyer motor [HN73, Pel86]. Ultratech steppers use closed-loop control of a Sawyer motor to achieve the motions required for photolithography.

Keeping pace with advances of permanent-magnet material in the last decade, planar motor structures using permanent magnets are presented in [Asa85, Hin87, EW89]. Other plane movement systems are given in [BGK89, HK89]. A recent planar motion system is by Buckley, et al. [BGK89]. It has two unipolar permanent-magnet planar motors to generate major two-dimensional motions and rotation about the normal to the stage. The wafer stage is constrained by aerostatic-bearing pads. It uses voice-coil-type actuators for fine motion control. The stages mentioned so far cannot provide focus range, or local leveling without using fine motion actuators.

In this section, we focus on existing one-moving part stages. The following are examples of such planar positioners in three major categories: variable-reluctance types, permanent-magnet types, and permanent-magnet matrix types.

### 3.3.1 Variable-Reluctance Types

**Sawyer** [Saw68]

An early planar motor was developed by Sawyer [Saw68]. Figure 3-1 shows the schematic structure. He invented his motor by conceptually superimposing two orthogonal linear variable-reluctance motors. Thus, the stator has many square protrusions (12) due to the orthogonal superimposition of tooth structures. He intended to use his motor in a chart marker, or a $x$-$y$ plotter. Relatively low resolution (0.01”)
Figure 3-1: Sawyer motor stage (after [Saw68])
was enough for those applications. So, no feedback control scheme was involved in his original invention. Using a proper commutation law in the winding currents \((A, B, C, A', B', \text{ and } C')\), we can move the platen back and forth and hold it at a stationary position. A small yaw motion is also possible under closed-loop control by operating two parallel linear motors (e.g., 27 and 29) differentially.

Typically, this kind of linear variable-reluctance motors has about ten times stronger attraction force than lateral force. The original solution of Sawyer is using aerostatic-bearing pads to cancel the attraction force and the gravity load acting on the platen. Aerostatic bearings are still usual in the current designs of Sawyer motor systems. Nevertheless, vertical motions cannot be controlled at high resolution with these aerostatic bearings.

**Higuchi, et al. [HK89]**

Another linear variable-reluctance device is Higuchi and Kawakatsu's stage [HK89]. They devised a magnetically suspended stepper motor for linear conveyer systems, \(x-y\) tables, product handling machines, and the like. Figure 3-2 shows the cross-sectional view of a one-dimensional moving system. It is a combination of a magnetic suspension system and stepper motors; suspension and propulsion are simultaneously achievable by using a two-sided motor geometry rather than using an unbalanced
Figure 3-3: Plane view of slider teeth of Higuchi motor

motor with aerostatic bearings. Controlling the currents in the upper and lower stators can offset the strong attraction force between the stators and the slider teeth. Limited two-dimensional motions are possible by putting orthogonal-axis variable-reluctance motors side by side. It has a small range for position adjustment in the direction perpendicular to the long-travel axis. Figure 3-3 shows one side of the slider. This structure is not superimposition of two orthogonal teeth as in Sawyer motor. However, activating the orthogonal-axis motors independently can generate limited planar motions. Also, this stage cannot generate vertical focusing motions with the required travel, since the small air gaps must be maintained for the proper functioning of the motor.

3.3.2 Permanent-Magnet Types

The permanent-magnet type stages have permanent magnets as sources of magnetic field intensity and windings for current density. By the Lorentz force equation, \( \mathbf{F} = \mathbf{J} \times \mathbf{B} \), we can generate forces for motion by controlling the current in the winding. In this section, only a unipolar field (one magnet rather than a magnet array) is used. Therefore, they can control neither vertical motions nor pitch and rolling motions. All designs in this category use aerostatic bearings or other means for suspension and
Figure 3-4: Electromagnetically driven wafer stage (after [BGK89])

controlling rotational motions.

Buckley, et al. [BGK89]

Figure 3-4 is the original design for the SVGL\textsuperscript{4} Micrascan system. It has two unipolar permanent-magnet planar motors, which have two orthogonal coil assemblies, to generate major two-dimensional motions. Independent activation of a coil of each coil assembly can generate $x$- or $y$-directional motions by the Lorentz force law. So, a linear combination of the two orthogonal motions gives a planar motion. Yaw mo-

\textsuperscript{4}Silicon Valley Group Lithography Systems, Inc., 77 Danbury Road, Wilton, Connecticut 06897
tions are also possible because there are two such coil assemblies. That is, a torque around the vertical axis can be generated by differential excitation in the two coil sets. Each of three aerostatic bearings supports the stage against the back plane structure and has a focus actuator. The stage also incorporates a flexure and a $z$-direction voice-coil actuator with $\pm 0.25$ mm travel. The focus actuators, driven differentially, enable small tilt adjustment so that roll and pitch motions can be generated. So, this design can achieve six-degree-of-freedom motion control with only one moving part. The platen has an aluminum honeycomb structure inside for structural strength with light weight.

**Galburt [Gal85]**

This system [Gal85] (Figure 3-5) has a brushless DC motor with permanent-magnet assemblies. The mover has four coil assemblies; two (110 and 114) for $x$-directional motions, and other two (112 and 116) for $y$-directional motions. There are four closed magnetic flux paths in the stator. The member 92 is one of them. Four corner piece permanent magnets (108 is one of them) serve as flux sources for the magnetic circuits. If we drive the currents in the four coil assemblies differentially or in synchronism, three degree-of-freedom motions ($x$, $y$, and yaw) can be generated with four motors. The mover has no fine motion actuator as in the previous design and is not intended to control other three-degree-of-freedom motions. The mover is supported by four aerostatic bearings on each corner (138 and 140) and three vertical degrees of freedom of the mover are controlled by four leveling actuators (144). The invention was assigned to the Perkin-Elmer Corporation\(^5\).

**Tomita, et al. [TK95]**

Tomita and Koyanagawa’s planar motor consists of three permanent-magnet linear

\(^5\)Perkin-Elmer’s photolithography division was subsequently acquired by Silicon Valley Group, Inc.
Figure 3-5: Brushless DC planar motor (after [Gal85])
motors for $x$-, $y$- and yaw motion generation. One permanent-magnet linear motor is arranged to be perpendicular to the other two, which are parallel to each other. The latter two linear motors generate $y$- and yaw motion and the first one generates $x$ motion. So, it is conceptually similar to Figures 6-12 and 6-13. It uses three aerostatic bearings to suspend the platen. They report the positioning resolution in $x$- and $y$-directions is 10–20 nm and the yawing accuracy, 0.08 μrad.

### 3.3.3 Permanent-Magnet Matrix Types

The Sawyer motor inspired the designs of Asakawa [Asa85] and Hinds [Hin87]. They are characterized by a large number of permanent-magnet cubes which form checkerboard-like stators. On the contrary, the Sawyer motor has iron protrusions. The platens carry properly sized winding assemblies which interact with the magnets to generate planar motions. The stator of Hind’s motor is conceptually constructed by superimposing two orthogonal linear permanent-magnet motors’ stators. His wafer stage is suspended by aerostatic-bearing pads. The permanent-magnet matrix and winding current interact to generate planar motions. These designs use aerostatic bearings for suspension. They have magnet matrices as seen in Figure A-1 in Appendix A. So, a large number of permanent-magnet teeth are required on the stator. There is also a problem to assemble high-remanence permanent-magnet pieces against their large repulsion forces.

**Asakawa** [Asa85]

The Asakawa motor (Figure 3-6) is the first kind of permanent-magnet planar motors with aerostatic bearings. Besides the magnet matrices in Figure A-1, there are four coils (53–56) on the platen to interact the magnets. Yaw, $x$, and $y$ movements can be controlled by selective energization of the four coils. However, the available forces are varied with the relative position between the coils and the magnet matrix, and
Figure 3-6: Top and side views of Asakawa motor (after [Asa85])
they are nonlinear with position.

**Hinds [Hin87]**

Figure 3-7 is a perspective view of Hinds motor. Figure 3-8 shows the bottom side of the platen that carries winding assemblies. Although having more magnets than Asakawa's, Hinds' magnet matrix is a natural superimposition of two orthogonal linear magnet arrays. So, the planar motion generation is nothing more than a simultaneous operation of two linear motions. He claims that a simple PID control can be applied to his system. The thrust is constant, for equal pole areas are covered by the six-phase windings (64–67 in Figure 3-9) at any instant of time. So, the motor system expects neither cogging nor nonlinear interactions. The activation of the windings 65 and 67 generates \( x \)-directional motions. And the windings 64 and 66 are for \( y \)-directional motions. Yaw motion is possible by differential operations of the four windings. But, only half the winding area is on the effective side, which makes the motor inefficient in power consumption. Hinds also suggested a version of his motor system in a moving magnet–stationary winding.

### 3.4 Summary of Part I

Having reviewed the prior art of levitation techniques and planar positioning systems in this part, we believe a novel planar magnetic levitator is one of the most promising candidates for the high-precision motion control applications. The candidate actuator for the magnetic levitator is a surface-wound surface-permanent-magnet linear motor. The choice of induction motors is eliminated since they dissipate heat in the moving part, and are generally hard to control. Variable-reluctance motors, such as stepper motors, have inevitable cogging forces. Multi-phase linear permanent-magnet motors have capabilities to generate suspension force as well as drive force so that we do not need any other actuators for suspension and small position adjustments. Error
Figure 3-7: Prospective view of Hinds motor (after [Hin87])
FIG. 2

Figure 3-8: Bottom view of Hinds motor's platen (after [Hin87])

FIG. 3

Figure 3-9: Top view of Hinds motor's magnet matrix (after [Hin87])
motions due to cogging and other nonlinear effects are intended to be minimal.

It is a technical trend that the industry comes to use planar motors for two-dimensional positioning applications more frequently than ever. Earlier inventions like the Sawyer motor are tightly constrained to a plane. The stages mentioned in the last section cannot provide focus range, or local leveling without using fine motion actuators. However, the magnetically levitated stage presented in this thesis can generate all six-degree-of-freedom motions required for focusing and alignment, and large planar motions for positioning using only one magnetically levitated moving part. In the forthcoming chapters, analysis, design, and control for this magnetic levitator will be fully discussed.
Part II

Analysis
Since a linear permanent-magnet motor is an integral component of the levitator, we concentrate on analyzing it throughout Part II. In Chapter 4, we compute drive and suspension forces with field solutions using the Maxwell stress tensor. Major results in this part include the unified modeling of the winding and magnet array, the derivation of the electrical terminal relation, back electromotive force (emf) calculation, self-inductance estimation, and power balance. A winding pattern and a magnet matrix with single-sided field was devised and published [TKW94, TKW96b].

In Chapter 5, the analysis of the Halbach magnet array and its advantages are discussed. In conjunction with the continuum electromechanical analysis, the electrical terminal relation of the stator as a lumped-parameter model is given. This lumped model is useful to determine the electrical behavior of the stator and to design power amplifier circuits to drive the motors. We evaluate the performance of linear motors in terms of force ripple as a function of the number of phases. I decide to build a three-phase motor on the basis of this evaluation.
Chapter 4

Continuum Electromechanical Analysis

The key elements of any magnetically levitated stage are the actuators which apply controlled forces in order to stabilize the position of the stage. In this chapter, a unified methodology utilizing transfer relations [Mel81] is developed for linear permanent-magnet machines. This approach treats magnetized material, conductor sheets, and winding currents in a uniform fashion. Further, the results derived herein can be used to analyze other types of electric machines. For instance, the linear motors in the levitator I develop are iron-free in order to remain compatible with magnetic suspension bearings. However, the transfer relations we present herein can be applied in the context of an iron-backed motor simply by changing boundary conditions. This modified analysis is currently in progress in our lab at MIT by Michael Liebman, a Master’s student.

We describe our magnetic levitator with a continuum electromechanical model in Chapter 4. There are three crucial reasons for introducing this continuum or distributed model. First, the magnetic force is distributed throughout the region where the magnet array and the stator interact electromagnetically. Second, the continuum description of the electromagnetic system is necessary to deal with the
magnetic diffusion phenomenon and eddy current loss in the stator core and the magnet arrays. Third, the description of the field in Fourier series enables us to determine higher order force components in order to estimate force ripple.

Since the stage requires significant planar travel, I focus the design effort on the development of linear motors which are compatible with the motion requirements stated in Chapter 1. As will be discussed in detail in Chapter 6, a promising motor structure utilizes permanent magnets on the moving stage which are driven by stator coils in the fixed machine base. In order to choose the best motor topology, I have developed a set of analytical tools which allow the optimization of the linear motor design in terms of criteria such as packaging, force density, power efficiency, and winding pattern. These tools are presented in subsequent sections.

4.1 Review of Electromagnetic Field Theory

We give fundamentals of the underlying electromagnetic field theory to develop the analysis in this section.

4.1.1 Magnetoquasistatic (MQS) Approximation

The magnetic field originating from currents or permanent-magnets prevails in many electric machine systems. So, to avoid unnecessary complications, we can practically make an approximation that the system is magnetoquasistatic (MQS) [HM89]. Under the MQS approximation, time variation of electric field related quantities becomes insignificant. In other words, the displacement current term in the general form of Maxwell’s equations is simply dropped out. In this case, the governing equations for electric displacement $\mathbf{D}$, electric field intensity $\mathbf{E}$, magnetic field intensity $\mathbf{H}$, and magnetic flux density $\mathbf{B}$ simplify to

$$\nabla \times \mathbf{H} = \mathbf{J}_f \tag{4.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{4.2}$$
\[ \nabla \cdot \mathbf{J}_f = 0 \quad (4.3) \]
\[ \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \quad (4.4) \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (4.5) \]

where \( \mathbf{J}_f \) is free volume current density due to the movement of free charges. The permeability of free space is \( \mu_0 = 4\pi \times 10^{-7} \) H/m. We assume the permeability to be \( \mu_0 \) everywhere, since our design is iron-free. This assumption is reasonable even for permanent-magnet material like neodymium-iron-boron (NdFeB) since its relative permeability is near unity. The magnetization \( \mathbf{M} \) is a macroscopic representation of the effect of microscopic magnetic dipoles. A constitutive law for \( \mathbf{M} \) for linear and isotropic material is

\[ \mathbf{M} = \chi_m \mathbf{H}, \quad (4.6) \]

where \( \chi_m \) is the magnetic susceptibility. We define the permeability of such media with the relation, \( \mu = \mu_0 (1 + \chi_m) \). We can now describe the constitutive law of a linear and isotropic material as

\[ \mathbf{B} = \mu \mathbf{H}. \quad (4.7) \]

The constitutive law for conduction induced by electric field (Ohm’s law) is

\[ \mathbf{J}_f = \sigma \mathbf{E}, \quad (4.8) \]

where \( \sigma \) is the conductivity of the material.\(^1\)

The boundary conditions which derive from (4.1) and (4.2) can be written as \( \mathbf{n} \times [\mathbf{B}^a - \mathbf{B}^b] = \mu_0 \mathbf{K}_f + \mathbf{n} \times [\mu_0 \mathbf{M}^a - \mu_0 \mathbf{M}^b] \) and \( \mathbf{n} \cdot [\mathbf{B}^a - \mathbf{B}^b] = 0. \) Here, variables denoted \( a \) and \( b \) represent the associated quantities evaluated on opposite sides of a boundary. The vector \( \mathbf{n} \) is normal to the boundary and points into the side labeled \( a \). The variable \( \mathbf{K}_f \) is a free surface current density directed along the boundary. We further define \( \mathbf{K}_e = \mathbf{n} \times [\mathbf{M}^a - \mathbf{M}^b]; \) this can be thought of as an equivalent surface current density which represents step discontinuities in the magnetization tangential to the boundary.

---

\(^1\)In its own frame, otherwise \( \mathbf{J}_f = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}). \)
4.1.2 Vector Potential and Equivalent Current Model

In earlier work [TWN93], the field and force characteristics of a linear motor were analyzed via the magnetic scalar potential. Since the scalar potential is not unique in current-carrying volumes, the analysis therein uses a Green’s function approach. In contrast, in the present work I use the magnetic vector potential since it is valid throughout both the permanent magnet and the current-carrying regions of the linear motor model. Use of the vector potential also simplifies the calculation of flux linkage in order to calculate coil inductance and back emf.

Recalling the divergence of the curl of a vector is identically zero\(^2\), the magnetic vector potential \( \mathbf{A} \) can be defined as

\[
\mathbf{B} \equiv \nabla \times \mathbf{A}.
\]

By the Biot-Savart law and the above definition, the magnetic vector potential is written in the following integral form [LC70].

\[
\mathbf{A} = \frac{\mu_0}{4\pi} \int_{\tau} \mathbf{J}_f + \nabla \times \mathbf{M} \frac{d\tau}{r} + \frac{\mu_0}{4\pi} \int_{S} \mathbf{K}_f + \mathbf{M} \times \mathbf{n} \frac{da}{r},
\]

where \( S \) is a surface enclosing a source volume \( \tau \), and \( \mathbf{n} \) is a normal vector to the surface. The distance between the source and observing points is denoted as \( r \). By examining these terms, we can define an equivalent volume current density \( \mathbf{J}_e = \nabla \times \mathbf{M} \), and an equivalent surface current density \( \mathbf{K}_e = \mathbf{M} \times \mathbf{n} \). The equivalent currents can thereby represent the material magnetization, but they are not really microscopic currents. So, \( \mathbf{A} \) and further \( \mathbf{B} \) can always be calculated using the equivalent currents as if they replaced the magnetized material. From \( \mathbf{J}_e = \nabla \times \mathbf{M} \), we know \( \nabla \cdot \mathbf{J}_e \) is always zero. That is, all of these equivalent currents are solenoidal.

\(^2\nabla \cdot (\nabla \times \mathbf{A}) \equiv 0\)
4.1.3  Poisson Equation for Vector Potential

Since the divergence of the curl of a vector is identically zero, the representation, $\mathbf{B} = \nabla \times \mathbf{A}$ automatically satisfies (4.2). Applying the curl operator to both sides of (4.1) yields $\nabla \times \mathbf{B} = \nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$. If we set the Coulomb gage, $\nabla \cdot \mathbf{A} = 0$, this leads to $\nabla \times \mathbf{B} = -\nabla^2 \mathbf{A}$. However, via (4.1) and (4.4) we also have $\nabla \times \mathbf{B} = \mu_0 (\mathbf{J}_f + \nabla \times \mathbf{M})$. Combining the last two results yields the vector Poisson equation

$$\nabla^2 \mathbf{A} = -\mu_0 (\mathbf{J}_f + \nabla \times \mathbf{M}). \quad (4.11)$$

In the above equation, $\mathbf{J}_f$ represents the stator current source term. We can think of the term $\nabla \times \mathbf{M}$ as an equivalent current which represents the magnet.

In two dimensional cases, which are usual in many practical applications, where the fields lie in an $x$-$z$ plane, the vector potential $\mathbf{A}$ is purely $y$-directed. In this case the vector Poisson equation simplifies to the scalar relationship

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) A_y = -\mu_0 \left( J_{yf} + \frac{\partial}{\partial z} M_x - \frac{\partial}{\partial x} M_z \right). \quad (4.12)$$

This scalar equation is used in the analysis which follows in the next section.

4.1.4  Maxwell Stress Tensor

The Maxwell stress tensor is a powerful tool to calculate forces and torques, since we have only to know about the field on surfaces rather than throughout the volume which the surfaces enclose. Thus, we can avoid the complicated volume integration of force density $\mathbf{F} = \mathbf{J} \times \mathbf{B}$. With Maxwell stress tensor the total force $\mathbf{f}$ on material is obtained by the following surface integral

$$f_i = \oint_S T_{ij} n_j da, \quad (4.13)$$

where $f_i$ is the force in the $i$th coordinate, $n_j$ is the $j$th component of the normal vector $\mathbf{n}$, and we adopt the Einstein summation convention for repeated indices where
if an index is repeated, the term is summed on all coordinates\textsuperscript{3}. The Maxwell stress tensor $T_{ij}$ given by the Korteweg-Helmholtz force density is

$$T_{ij} = \mu H_i H_j - \frac{\mu}{2} \delta_{ij} H_k H_k,$$

(4.14)

where $\delta_{ij}$ is the Kronecker delta\textsuperscript{4} [Mel81].

Expanding the Maxwell stress tensor in matrix form for the Cartesian coordinates yields

$$[T_{ij}] = \begin{bmatrix}
\frac{\mu}{2}(H_x^2 - H_y^2 - H_z^2) & \mu H_x H_y & \mu H_x H_z \\
\mu H_x H_y & \frac{\mu}{2}(H_y^2 - H_z^2 - H_x^2) & \mu H_y H_z \\
\mu H_x H_z & \mu H_y H_z & \frac{\mu}{2}(H_z^2 - H_x^2 - H_y^2)
\end{bmatrix}. \quad (4.15)$$

### 4.1.5 Fourier Series Representation

Since most electric machines are based on a periodic geometry\textsuperscript{5}, it is often beneficial to use Fourier series representation of the field and source quantities. Because they have finite length, this is only an approximation for linear motors; the finite motor length can be represented at the cost of higher analytical complexity, via a Fourier transform representation. For the present purposes a Fourier series representation gives the best insight into motor operation. We follow Melcher [Mel81]'s notation in this thesis. We consider a function $\Phi$ dependent sinusoidally on $z$,

$$\Phi(z, t) = Re \left\{ \tilde{\Phi}(t)e^{-j k z} \right\}, \quad (4.16)$$

where $k = 2\pi/l$ is the spatial wave number, and $l$ is the spatial wavelength of the machine.

\textsuperscript{3}For example, $f_x = \oint_S (T_{xz} n_x + T_{xy} n_y + T_{xz} n_z) \, da$.

\textsuperscript{4}$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

\textsuperscript{5}This is only approximate for linear machines, though.
If the excitation is also sinusoidal in time, the complex amplitude represents both temporal and spatial dependences via

$$\Phi(z, t) = \text{Re} \left\{ \hat{\Phi} e^{i(\omega t - kz)} \right\},$$

(4.17)

where $\omega$ is the angular frequency, which has a relationship with the temporal period $T = 2\pi/\omega$. The spatial and temporal derivative of $\Phi$ are

$$\frac{d\Phi(z, t)}{dz} = \text{Re} \left\{ -jk\hat{\Phi} e^{i(\omega t - kz)} \right\}$$

(4.18)

and

$$\frac{d\Phi(z, t)}{dt} = \text{Re} \left\{ j\omega \hat{\Phi} e^{i(\omega t - kz)} \right\}.$$  

(4.19)

A variable with a circumflex (') is a complex constant coefficient, and one with a tilde (\~) has time dependence in it.

The Fourier series representation of a function $\Phi$ periodic in $z$ with fundamental period $l$ is

$$\Phi(z, t) = \sum_{n=-\infty}^{\infty} \tilde{\Phi}_n(t) e^{-jk_nz}; \quad k_n \equiv 2n\pi/l.$$  

(4.20)

If $\Phi$ is real, then it holds that

$$\tilde{\Phi}_n^* = \tilde{\Phi}_{-n},$$  

(4.21)

where the asterisk represents the complex conjugate of the quantity. For this representation, the complex Fourier amplitudes are determined by the analysis integral.

$$\tilde{\Phi}_m = \frac{1}{l} \int_z^{z+l} \Phi(z, t) e^{jkmz} dz.$$  

(4.22)

An important identity to evaluate the spatial average of two periodic functions is the spatial averaging theorem (Section 2.15 in [Mel81]).

$$\langle AB \rangle_z = \left\langle \sum_{n=-\infty}^{\infty} \tilde{A}_n e^{-jk_nz} \sum_{m=-\infty}^{\infty} \tilde{B}_m e^{-jk mz} \right\rangle_z$$

$$= \sum_{n=-\infty}^{\infty} \tilde{A}_n \tilde{B}_{-n} = \sum_{n=-\infty}^{\infty} \tilde{A}_n \tilde{B}_n^*,$$  

(4.23)

where $\langle \cdot \rangle_z$ signifies the spatial average in $z$ over a fundamental period $l$. This is a spatial version of Parseval’s relation. We will use this identity in later sections in
Figure 4-1: Linear motor model described in infinite complex Fourier series. Free space is assumed except for the shaded regions for magnet and current.

In relation to the Maxwell stress tensor representation in order to calculate the force acting between the magnets and current layers of the motor.

4.2 Field Solutions

The geometry used to model the motor fields is shown in Figure 4-1. Here the lower shaded region of thickness $\Gamma$ represents the stator winding with $y$-directed current density $J$, which is expressed as an infinite Fourier series. The upper shaded region of thickness $\Delta$ represents the magnet array, carrying a primed coordinate frame which is displaced from the base coordinate frame by a vector $(x_0 + \Gamma)i_x + z_0i_z$. Thus $x_0$ is the motor air gap, and $z_0$ is the lateral displacement of the magnet array relative to the stator. The region outside the shaded regions is air, or free space. The spatial period, i.e., the pitch, of the motor is $l$, and the spatial wave number of the $n$th harmonic is $k_n = 2\pi n/l$. We further define $\gamma_n = |k_n|$. The motor is assumed to be of depth $w$ in the $y$ direction. End effects in this direction are neglected.
represent the surfaces at the indicated boundaries.

The magnetization of the magnet array is represented by a Fourier series in lateral (z-directed) and vertical (x-directed) magnetization components through terms $\tilde{M}_{zn}$ and $\tilde{M}_{xn}$, respectively, i.e.,

$$M = \sum_{n=-\infty}^{\infty} [M_{xn}i_x + M_{zn}i_z] = \sum_{n=-\infty}^{\infty} [\tilde{M}_{xn}e^{-jk_nz'}i_x + \tilde{M}_{zn}e^{-jk_nz'}i_z].$$  \hspace{1cm} (4.24)

In the stator current region, the free volume current density is represented as the Fourier series

$$J = \sum_{n=-\infty}^{\infty} \tilde{j}_{yn}e^{-jk_nz'}i_y.$$

The stator current is assumed here to have only a $y$-component and is constant in $x$. The vector potential and the magnetic flux density are represented as

$$A = \sum_{n=-\infty}^{\infty} A_{yn}i_y = \sum_{n=-\infty}^{\infty} \tilde{A}_{yn}e^{-jk_nz'}i_y.$$  \hspace{1cm} (4.26)

$$B = \sum_{n=-\infty}^{\infty} [B_{xn}i_x + B_{zn}i_z] = \sum_{n=-\infty}^{\infty} [\tilde{B}_{xn}e^{-jk_nz'}i_x + \tilde{B}_{zn}e^{-jk_nz'}i_z].$$  \hspace{1cm} (4.27)

$H$ is represented likewise.

### 4.2.1 Field due to Magnet

There is no free current in the magnet region in Figure 4-1. So, here we have $\nabla \times H = J_f = 0$, and $\nabla \times B = \mu_0 \nabla \times M$. The vector potential satisfies the scalar Poisson equation for $y$-component in the Cartesian coordinate

$$\left( \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial z'^2} \right) A_{yn} = -\mu_0 [\nabla \times M_n]_{y'},$$  \hspace{1cm} (4.28)

where $M_n$ is the $n$th order Fourier component of $M$. Here we use the primed coordinate frame which moves with the magnet. Taking the curl of the $n$th magnetization term gives

$$\nabla \times M_n = -jk_n \tilde{M}_{xn}e^{-jk_nz'}i_y.$$  \hspace{1cm} (4.29)

Let $A_{yn} = A_{ynp} + A_{ynh}$, where $A_{ynp}$ is the particular part of the solution (i.e., driven by $\nabla \times M_n$) and $A_{ynh}$ is the homogeneous part (i.e., satisfying (4.28) with $\nabla \times M_n = 0$.
in the volume). By definition, then, \( \left( \frac{\partial^2}{\partial z'^2} + \frac{\partial^2}{\partial z''^2} \right) A_{ynh} = 0 \). As the vector potential depends on \( e^{-jk_n x'} \), the \( \frac{\partial^2}{\partial z'^2} \) operator pulls \((-jk_n)^2\) out. Further, only the vertical component of magnetization yields \( \nabla \times \mathbf{M}_n \) in the volume. So, solving the Poisson equation for \( A_{ynp} \),

\[
\tilde{A}_{ynp} = -\frac{j \mu_0}{k_n} \tilde{M}_{zn}. \tag{4.30}
\]

Since the potential depends on hyperbolic trigonometric functions in \( x \) (solutions to Laplace equation in Cartesian coordinates), the homogeneous solution inside the magnet region takes the following form\(^6\)

\[
\tilde{A}_{ynh} = \left( \tilde{A}_{yn}^b + \frac{j \mu_0}{k_n} \tilde{M}_{zn} \right) \frac{\sinh k_n x'}{\sinh k_n \Delta} - \left( \tilde{A}_{yn}^c + \frac{j \mu_0}{k_n} \tilde{M}_{zn} \right) \frac{\sinh k_n (x' - \Delta)}{\sinh k_n \Delta}. \tag{4.31}
\]

From the definition of the vector potential \( \mathbf{B} \equiv \nabla \times \mathbf{A} \),

\[
\tilde{B}_{xn} = -\frac{\partial}{\partial z'} \tilde{A}_{yn} = j k_n \tilde{A}_{yn}, \tag{4.32}
\]

and

\[
\tilde{B}_{zn} = \frac{\partial}{\partial z'} \tilde{A}_{yn}. \tag{4.33}
\]

Now applying (4.33) to (4.31) gives

\[
\tilde{B}_{zn} = k_n \left( \tilde{A}_{yn}^b + \frac{j \mu_0}{k_n} \tilde{M}_{zn} \right) \frac{\cosh k_n x'}{\sinh k_n \Delta} - k_n \left( \tilde{A}_{yn}^c + \frac{j \mu_0}{k_n} \tilde{M}_{zn} \right) \frac{\cosh k_n (x' - \Delta)}{\sinh k_n \Delta}. \tag{4.34}
\]

Evaluated at the boundaries (b) (at \( x' = \Delta \)) and (c) (at \( x' = 0 \)), the transfer relations are thus expressed as

\[
\begin{bmatrix}
\tilde{B}_{zn}^b \\
\tilde{B}_{zn}^c
\end{bmatrix} = k_n \begin{bmatrix}
\coth k_n \Delta & \frac{-1}{\sinh k_n \Delta} \\
\frac{1}{\sinh k_n \Delta} & -\coth k_n \Delta
\end{bmatrix} \begin{bmatrix}
\tilde{A}_{yn}^b \\
\tilde{A}_{yn}^c
\end{bmatrix} + \begin{bmatrix}
\frac{\cosh k_n \Delta - 1}{\sinh k_n \Delta} \\
-\frac{\cosh k_n \Delta - 1}{\sinh k_n \Delta}
\end{bmatrix} j \mu_0 \tilde{M}_{zn}. \tag{4.35}
\]

Using the limits \( \lim_{x \to \pm \infty} \coth x = \pm 1 \) and \( \lim_{x \to \pm \infty} \sinh x = \pm \infty \) (The order of the signs is significant.), the transfer relations for the half-infinite regions above the

\(^6\)We can check the homogeneous solution as this. From \( \tilde{A}_{ynh} = \tilde{A}_{yn} - \tilde{A}_{ynp} = \tilde{A}_{yn} + \frac{j \mu_0}{k_n} \tilde{M}_{zn} \), \( \tilde{A}_{ynh} = \left( \tilde{A}_{yn}^b + \frac{j \mu_0}{k_n} \tilde{M}_{zn} \right) \) at the boundary (b) (at \( x' = \Delta \)), and \( \tilde{A}_{ynh} = \left( \tilde{A}_{yn}^c + \frac{j \mu_0}{k_n} \tilde{M}_{zn} \right) \) at the boundary (c) (at \( x' = 0 \)). So, the homogeneous solution satisfies the Laplace equation and boundary conditions.
surface (a) and below the surface (d) can be derived from (4.35) as

\[
\tilde{B}_z^a = -\gamma_n \tilde{A}_y^a \quad (4.36)
\]
\[
\tilde{B}_z^d = \gamma_n \tilde{A}_y^d. \quad (4.37)
\]

Since there is no impulse of field (i.e., no current doublet) anywhere, the vector potential is continuous at the boundaries, i.e.,

\[
\tilde{A}_y^a = \tilde{A}_y^b \quad (4.38)
\]
\[
\tilde{A}_y^c = \tilde{A}_y^d. \quad (4.39)
\]

The equivalent surface current densities at the boundaries (b) and (c) are

\[
\mathbf{K}_y^b = \mathbf{M} \times \hat{z} = \tilde{M}_zn e^{-jk_nz'} \hat{y} \quad (4.40)
\]
\[
\mathbf{K}_y^c = \mathbf{M} \times (-\hat{z}) = -\tilde{M}_zn e^{-jk_nz'} \hat{y}. \quad (4.41)
\]

So, the boundary conditions for magnetic flux density are

\[
-\tilde{B}_z^a + \tilde{B}_z^b = \mu_0 \tilde{M}_zn \quad (4.42)
\]
\[
-\tilde{B}_z^c + \tilde{B}_z^d = -\mu_0 \tilde{M}_zn. \quad (4.43)
\]

Solving the system of algebraic equations (4.35–4.39) and (4.42–4.43) yields

\[
\tilde{A}_y^a = \left( \frac{\mu_0}{2\gamma_n} \tilde{M}_zn - \frac{j\mu_0}{2k_n} \tilde{M}_znx \right) \left( 1 - e^{-\gamma_n\Delta} \right) \quad (4.44)
\]
\[
\tilde{A}_y^d = \left( -\frac{\mu_0}{2\gamma_n} \tilde{M}_zn - \frac{j\mu_0}{2k_n} \tilde{M}_znx \right) \left( 1 - e^{-\gamma_n\Delta} \right). \quad (4.45)
\]

From (4.32) and (4.37), and using (4.45), we find

\[
\tilde{B}_z^d = \left( -\frac{j\mu_0 k_n}{2\gamma_n} \tilde{M}_zn + \frac{\mu_0}{2} \tilde{M}_znx \right) \left( 1 - e^{-\gamma_n\Delta} \right) \quad (4.46)
\]
\[
\tilde{B}_z^d = \left( -\frac{\mu_0}{2} \tilde{M}_zn - \frac{j\mu_0 \gamma_n}{2k_n} \tilde{M}_znx \right) \left( 1 - e^{-\gamma_n\Delta} \right). \quad (4.47)
\]

Equations (4.45–4.47) are the potential and field solutions for the magnet at the boundary (d). With these in hand we turn our attention to the fields due to the stator currents. These fields can be considered separately because the problem as formulated is linear.
4.2.2 Field due to Stator Current

In the stator current region, the governing Poisson equation relating current density to vector potential is

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) A_{ynp} = -\mu_0 [J_f]_y = -\mu_0 \mathbf{J}_{yn} e^{-jk_nz}. \] (4.48)

Like the magnet case, a particular solution of the above Poisson equation is used to match the field driven by the volume current. That is,

\[ \tilde{A}_{ynp} = \frac{\mu_0}{k_n^2} \mathbf{J}_{yn}. \] (4.49)

Through similar steps as in the magnet case, we obtain the transfer relations for the stator current as\(^7\)

\[
\begin{bmatrix}
\tilde{B}_{zn}^f \\
\tilde{B}_{zn}^g
\end{bmatrix}
= k_n \begin{bmatrix}
\coth k_n \Gamma \\
\frac{1}{\sinh k_n \Gamma}
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_{yn}^f \\
\tilde{A}_{yn}^g
\end{bmatrix}
- \begin{bmatrix}
\frac{\cosh k_n \Gamma - 1}{\sinh k_n \Gamma} \\
\frac{\cosh k_n \Gamma - 1}{\sinh k_n \Gamma}
\end{bmatrix}
\frac{\mu_0}{k_n} \tilde{J}_{yn}. \] (4.50)

Transfer relations for the half-infinite regions above and below the stator are\(^8\)

\[ \tilde{B}_{zn}^e = -\gamma_n \tilde{A}_{yn}^e \] (4.51)

\[ \tilde{B}_{zn}^h = \gamma_n \tilde{A}_{yn}^h. \] (4.52)

The vector potential is continuous at the boundaries, and thus

\[ \tilde{A}_{yn}^e = \tilde{A}_{yn}^f \] (4.53)

\[ \tilde{A}_{yn}^g = \tilde{A}_{yn}^h. \] (4.54)

Since there is no surface current on the surface of the stator, the boundary conditions for magnetic flux density are

\[ -\tilde{B}_{zn}^e + \tilde{B}_{zn}^f = 0 \] (4.55)

\(^7\)Or, these transfer relations can be derived directly from (4.35) by comparing two Poisson equations' source terms, i.e., the right hand sides of (4.28) and (4.48).

\(^8\)Recall that we have assumed \(\mu_{magnet} = \mu_0\).
\[- \tilde{B}_z^n + \tilde{B}_z^h = 0. \quad (4.56)\]

Solving the system of algebraic equations (4.50–4.56) yields the vector potential on the upper and lower boundaries as

\[\tilde{A}_y^n = \frac{\mu_0}{2k_n^2} \tilde{J}_y^n \left( 1 - e^{-\gamma_n r} \right) \quad (4.57)\]

\[\tilde{A}_y^h = \frac{\mu_0}{2k_n^2} \tilde{J}_y^n \left( 1 - e^{-\gamma_n r} \right). \quad (4.58)\]

Applying (4.51) to (4.57) and applying (4.55–4.58) to (4.50) with \(k_n^2 = \gamma_n^2\) gives the fields on boundary \(e\) as

\[\tilde{B}_z^e = jk_n \tilde{A}_y^n = \frac{j \mu_0}{2k_n} \tilde{J}_y^n \left( 1 - e^{-\gamma_n r} \right) \quad (4.59)\]

\[\tilde{B}_z^e = -\frac{\mu_0}{2\gamma_n} \tilde{J}_y^n \left( 1 - e^{-\gamma_n r} \right). \quad (4.60)\]

### 4.2.3 Transfer Relations

This subsection summarizes the boundary conditions and the transfer relations derived in the previous subsections. Extending results developed in [Mel81], we derived the transfer relations which describe the model of Figure 4-1. As seen in the previous sections, the transfer relations give the normal flux density once the potential on the boundary surfaces is known, vice versa. So, in case of dealing with a region with uniform properties, we do not have to solve the Poisson equation everywhere in the region. The purpose of this subsection is to collect the results given earlier and tabulate them so that future workers can use these directly without going through the whole derivation over again.

- Vector potential

\[\tilde{A}_y^n = \tilde{A}_y^n \quad (4.61)\]

\[\tilde{A}_y^c = \tilde{A}_y^d \quad (4.62)\]

\[\tilde{A}_y^n = \tilde{A}_y^n \quad (4.63)\]

\[\tilde{A}_y^g = \tilde{A}_y^h \quad (4.64)\]

92
• Tangential Magnetic Flux Density

\[- \tilde{B}_{zn}^a + \tilde{B}_{zn}^b = \mu_0 \tilde{M}_{zn} \]  \hspace{1cm} (4.65)
\[- \tilde{B}_{zn}^c + \tilde{B}_{zn}^d = -\mu_0 \tilde{M}_{zn} \]  \hspace{1cm} (4.66)
\[- \tilde{B}_{zn}^e + \tilde{B}_{zn}^f = 0 \]  \hspace{1cm} (4.67)
\[- \tilde{B}_{zn}^g + \tilde{B}_{zn}^h = 0 \]  \hspace{1cm} (4.68)

• Transfer Relations

\[
\begin{bmatrix}
\tilde{B}_{zn}^b \\
\tilde{B}_{zn}^c
\end{bmatrix} = k_n \begin{bmatrix}
\coth k_n \Delta & \frac{-1}{\sinh k_n \Delta} \\
\frac{1}{\sinh k_n \Delta} & -\coth k_n \Delta
\end{bmatrix} \begin{bmatrix}
\tilde{A}_{yn}^b \\
\tilde{A}_{yn}^c
\end{bmatrix} + \begin{bmatrix}
\cosh k_n \Delta \frac{-1}{\sinh k_n \Delta} \\
\cosh k_n \Delta \frac{-1}{\sinh k_n \Delta}
\end{bmatrix} j \mu_0 \tilde{M}_{zn} \]  \hspace{1cm} (4.69)

\[
\begin{bmatrix}
\tilde{B}_{zn}^d \\
\tilde{B}_{zn}^e
\end{bmatrix} = k_n \begin{bmatrix}
\coth k_n z_0 & \frac{-1}{\sinh k_n z_0} \\
\frac{1}{\sinh k_n z_0} & -\coth k_n z_0
\end{bmatrix} \begin{bmatrix}
\tilde{A}_{yn}^d \\
\tilde{A}_{yn}^e
\end{bmatrix} \]  \hspace{1cm} (4.70)

\[
\begin{bmatrix}
\tilde{B}_{zn}^f \\
\tilde{B}_{zn}^g
\end{bmatrix} = k_n \begin{bmatrix}
\coth k_n \Gamma & \frac{-1}{\sinh k_n \Gamma} \\
\frac{1}{\sinh k_n \Gamma} & -\coth k_n \Gamma
\end{bmatrix} \begin{bmatrix}
\tilde{A}_{yn}^f \\
\tilde{A}_{yn}^g
\end{bmatrix} - \begin{bmatrix}
\cosh k_n \Gamma \frac{-1}{\sinh k_n \Gamma} \\
\cosh k_n \Gamma \frac{-1}{\sinh k_n \Gamma}
\end{bmatrix} \frac{\mu_0}{k_n} \tilde{J}_{yn} \]  \hspace{1cm} (4.71)

\[
\tilde{B}_{zn}^h = \gamma_n \tilde{A}_{yn}^h \]  \hspace{1cm} (4.72)

The terms containing \( \tilde{M}_{zn} \) and \( \tilde{J}_{yn} \) represent sources in the magnet and current regions, respectively. The other source term \( \tilde{M}_{zn} \) enters through boundary conditions and does not appear in the transfer relations. Since there is no source in the air gap between the stator and the magnet, the transfer relations (4.71) in this gap have a similar form without any source terms. These transfer relations express the constraints on field and potential quantities imposed by the MQS form of Maxwell’s equations. In our case, the transfer relations give eight equations in sixteen unknowns. In order to solve for the field quantities, eight more independent equations are required. These come from the boundary conditions on field and potential at each of the four boundaries.
We have already derived $\tilde{A}^a_{yn}$, $\tilde{A}^d_{yn}$, $\tilde{B}^a_{zn}$, and $\tilde{B}^d_{zn}$ due to the magnet, and $\tilde{A}^e_{yn}$, $\tilde{A}^h_{yn}$, $\tilde{B}^e_{zn}$, and $\tilde{B}^h_{zn}$ due to the stator current. To calculate the motor forces, it remains to express these quantities on a common boundary so as to evaluate the Maxwell stress tensor at that boundary.

### 4.2.4 Total Field

The field variables should decay exponentially in $x$ from the source in free space for the Cartesian geometry. From Figure 4-1, there is a transformation between the primed and the non-primed coordinates, $z = z_0 + z'$ especially for the lateral displacement. So, the flux density due to the current with respect to the primed frame fixed to the magnet is given by from (4.59–4.60)

\[
S\tilde{B}^d_{zn} = \frac{j}{2k_n} \mu_0 \tilde{J}_{yn} e^{-\gamma_n x_0} \left(1 - e^{-\gamma_n R}\right) e^{-jk_n z_0} 
\]

\[
S\tilde{D}^d_{zn} = -\frac{1}{2\gamma_n} \mu_0 \tilde{J}_{yn} e^{-\gamma_n x_0} \left(1 - e^{-\gamma_n R}\right) e^{-jk_n z_0}, 
\]

where we use a superscript $S$ to emphasize these fields originating from the stator current. Using these results and (4.46–4.47), the total fields due to the magnet and the stator current at (d) can be obtained as follows by superposition.

\[
\tilde{B}^d_{zn} = \frac{j}{2k_n} \mu_0 \tilde{J}_{yn} e^{-\gamma_n x_0} \left(1 - e^{-\gamma_n R}\right) e^{-jk_n z_0} + \left(-\frac{jk_n}{2\gamma_n} \mu_0 \tilde{M}_{zn} + \frac{1}{2} \mu_0 \tilde{M}_{zn}\right) \left(1 - e^{-\gamma_n \Delta}\right) 
\]

\[
\tilde{D}^d_{zn} = -\frac{1}{2\gamma_n} \mu_0 \tilde{J}_{yn} e^{-\gamma_n x_0} \left(1 - e^{-\gamma_n R}\right) e^{-jk_n z_0} + \left(-\frac{1}{2} \mu_0 \tilde{M}_{zn} - \frac{j\gamma_n}{2k_n} \mu_0 \tilde{M}_{zn}\right) \left(1 - e^{-\gamma_n \Delta}\right) 
\]

We choose to solve for the field at boundary (d) in order to calculate the force exerted on the magnet via the Maxwell stress tensor.

### 4.3 Force

Now that we have a field solution at the surface of magnet, we can calculate the interacting force between the magnet and the stator current. Recall the stress tensor
associated with the Korteweg-Helmholtz force density as introduced in Section 4.1.4,

\[ T_{ij} = \mu H_i H_j - \frac{\mu}{2} \delta_{ij} H_k H_k. \]  

(4.78)

Now, let us draw an imaginary box that encloses an integer number of magnet periods. The bottom of the box is at the bottom surface of the magnet \((d)\). We let the upper surface of the box be stretched to infinity. This is convenient because the fields are zero at infinity in the \(+x\)-direction. The stresses of the opposite side \((z\)-directional\) surfaces cancel with each other due to the periodicity of the geometry. We define the face area of the magnet enclosed by this surface as \(S\). This is the area of the \(-x\)-directed face of the enclosing box. Then, the vertical force on the enclosed section with area \(S\) is

\[ F_x = -S \langle T^{rd}_{zz} \rangle_z = -\frac{S \mu_0}{2} \langle H^{rd}_x H^d_x - H^d_z H^d_z \rangle_z. \]  

(4.79)

The lateral force is given by

\[ F_z = -S \langle T^{rd}_{zz} \rangle_z = -S \mu_0 \langle H^d_x H^d_z \rangle_z, \]  

(4.80)

where the angle bracket expression \(\langle \cdot \rangle_z\) represents spatial averaging operation with respect to \(z\) as discussed earlier (4.23). We use negative signs because the normal vector to the bottom surface of the magnet is pointed in the \(-x\)-direction. We use \(\mu_0\) in both the equations, because the surface \((d)\) is in free space. With (4.79) and (4.80) in hand, we are ready to calculate the forces on the motor once the Fourier coefficients of the magnetic field intensity are given.

Specific geometries for magnet array and current distribution will be discussed in Chapter 5. Fundamental and ripple force calculation for the linear motor in the magnetic levitator is one of the major contributions in that chapter. For the moment, we assume that the stator current is distributed sinusoidally with a fundamental period \(l\). This is a fictitious ideal case, since this cannot be readily achieved with real conductor windings. However, this assumption is reasonable because the fundamental component is primarily responsible for force generation. We let \(\tilde{J}_n\) be zero except for
the \( n = \pm 1 \) fundamentals. We represent the fundamental components with real and imaginary parts, \( \tilde{J}_{1} = J_{a} + jJ_{b} \), and \( \tilde{J}_{-1} = J_{a} - jJ_{b} \). That is, \( 2J_{a} \) and \( 2J_{b} \) are the peak current densities. Under this assumption, applying the spatial averaging theorem (4.23) to (4.79) and (4.80), and using the field solutions in the previous section, (4.77) and (4.76) yields, after some algebra, the forces acting on one spatial period of the magnet array as

\[
\begin{bmatrix}
J_{a} \\
J_{b}
\end{bmatrix} = \mu_{0}M_{0}Ge^{-\gamma_{1}z_{0}}
\begin{bmatrix}
-\sin \gamma_{1}z_{0} & \cos \gamma_{1}z_{0} \\
\cos \gamma_{1}z_{0} & \sin \gamma_{1}z_{0}
\end{bmatrix}
\begin{bmatrix}
J_{a} \\
J_{b}
\end{bmatrix},
\]

(4.81)

where \( f_{x\lambda} \) and \( f_{z\lambda} \) are the \( x \)-directed and \( z \)-directed forces per spatial wavelength, respectively. Here \( \mu_{0}M_{0} \) is the remanence of the permanent magnets. The constant

\[
G = \frac{\sqrt{2wl^{2}}}{\pi^{2}}(1 - e^{-\gamma_{1}^{\Gamma}})(1 - e^{-\gamma^{\Delta}})
\]

(4.82)

contains the effects of the motor geometry. The \( x_{0} \) and \( z_{0} \) are defined as in Figure 4-1 and they represent a relative displacement of the magnet array with respect to the stator. Equation (4.81) shows that our model represents Lorentz-force-type actuators. Comparing with the Lorentz force law, \( \mathbf{F} = \mathbf{J} \times \mathbf{B} \), we have \( J_{a} \) and \( J_{b} \) as \( \mathbf{J} \), \( \mu_{0}M_{0} \) as \( \mathbf{B} \), and other terms depending on the motor geometry and the relative displacement between its members. Since the motor geometric constant \( G \) has a unit in \( \text{m}^{3} \), the right hand side of (4.81) gives forces.

Sometimes it is desirable to have lifting magnets on the platen to compensate the gravity load of the platen itself. In this circumstance, we need a target for the lifting magnets with highly magnetic material such as laminated magnet iron. If we assume the permeability of the magnetic target to be infinity and the target is sufficiently thick, we can use the image method to calculate the lifting force, that is the attraction force between the magnets and the target [Jac75]. To do this requires only a straightforward extension of the transfer relations developed in the previous sections and is not undertaken herein.
Chapter 5

Analysis for Electromagnetic System

In Chapter 4, we derived field solutions and force equations for the general class of permanent-magnet machines. We discuss the electromagnetic analysis of permanent-magnet linear motors with a Halbach magnet array and a multi-phase winding in this chapter. We also discuss the lumped parameter model of the linear motor with its electrical terminal relation. We use a six-phase motor as a model for the analysis. However, motors with any number of phases can be analyzed in a very similar way. Eventually, I decided to build three-phase motors for the levitator after the force ripple computation for $q$-phase motors. Most materials in this chapter are based on the results derived in Chapter 4.

5.1 Single-Sided Field Sources

In magnet arrays used for conventional permanent-magnet linear motors, alternate magnet segments are magnetically oriented perpendicular to the air gap in directions which are $180^\circ$ rotated from each other about an axis perpendicular to the direction in which the array extends. Since the magnetization is anti-symmetrical with respect to
bo.ch sides in conventional magnet arrays, there exists an anti-symmetrical two-sided field if no iron backing is used. In our case with the winding on one side of the array, half of the field is wasted. So, magnet and winding structures which have stronger field on one side and weaker field on the other side are therefore desirable. There are also applications, such as maglev trains, in which magnetic field shielding is necessary for passengers’ health and thus single-sided field from a coil array may be desirable.

A type of magnet array which provides a magnetic field limited to one side of the array has been available for some years. The permanent magnet array used in my motor was first proposed by Halbach [Hal80, Hal85]. The Halbach magnet array has the remarkable property of primarily single-sided field pattern. Such an array is also studied by Marinescu, et al. [MM92], where it is represented by a two-dimensional multipole field expansion via complex variable theory. Abele, et al. [ARB92] represented the magnet array in spherical harmonics and calculated its coefficients. Leupold, et al. suggested a free-electron laser with a pair of permanent-magnet wigglers with a Halbach array [LIA90].

There is a natural question about the existence of a winding counterpart. I suggest a triangular winding pattern which has a single-sided field analogous to the Halbach magnet array. In this section, using the analytic methodology developed in the previous sections, I derived the vector potential, field, force, power, and commutation law. A discussion of the proposed winding pattern’s power efficiency follows.

5.1.1 Halbach Magnet Array

Halbach developed a single-sided rare-earth magnet array for use in undulator and particle accelerators [Hal85]. Figure 5-1 shows such a square Halbach magnet array. This type of magnet array differs from conventional arrays in that each adjacent magnet segment is rotated around an axis perpendicular to the direction in which the array extends by a predetermined angle, for instance 90° or 45°. As shown in Figure 5-1, one spatial pitch of such a Halbach array consists of four blocks of magnets with
magnetization rotated by 90° in each successive block. Such a linear Halbach array has $\sqrt{2}$ times stronger field than that of a conventional ironless magnet array with the same volume, thereby doubling the power efficiency of the linear motor or reducing magnet mass. The use of Halbach magnet arrays for motor design was presented in [TWN93] and [TKW96a, TKW94]. In order to utilize the Halbach arrays in planar position control application, I devised a two-dimensional Halbach magnet matrix. A more detailed description is in Appendix A.

5.1.2 Halbach Array Harmonics

The vertical and lateral magnetization components of the Halbach magnet array are represented by the complex Fourier coefficients via the analysis integrals

$$\tilde{M}_{zn} = \frac{1}{l} \int_{0}^{l} M_z e^{jkn z'} dz'$$

(5.1)

$$\tilde{M}_{zn} = \frac{1}{l} \int_{0}^{l} M_z e^{jkn z'} dz'$$

(5.2)

Using (5.1–5.2), the Fourier coefficients for a Halbach array whose magnet has a square cross-section (i.e., $\Delta = l/4$ as shown in Figure 5-2) with peak magnetization $M_0$ are
Figure 5-2: (a) Linear Halbach magnet array (b) vertical and (c) lateral magnetization components with respect to $z$

$$\vec{M}_{zn} = \begin{cases} \frac{\sqrt{2}M_0}{\pi|n|}, & n = \pm(8m + 1) \text{ or } \pm(8m + 3) \\ -\frac{\sqrt{2}M_0}{\pi|n|}, & n = \pm(8m + 5) \text{ or } \pm(8m + 7) \\ 0, & n : \text{even} \end{cases}$$  \hspace{1cm} (5.3)

and

$$\vec{M}_{zn} = j^n\vec{M}_{zn},$$  \hspace{1cm} (5.4)

where $m$ is a non-negative integer. Figure 5-3 shows Fourier coefficients of the Halbach magnet array flux density on the strong side. This result is obtained by substituting (5.3–5.4) into (4.46–4.47).

With the origin chosen as in Figure 5-2, there is no even-order field in Halbach arrays because the magnetization $\vec{M}$ is an odd function of $z$. Substitution of (5.3) and (5.4) into (4.46) and (4.47) yields an important result. The strong side (the $y$-$z$ plane in Figure 5-2 (a)) of the Halbach magnet array produces magnetic field of fundamental, 5th, 9th, $\cdots$, order; the 3rd, 7th, 11th, $\cdots$, order fields cancel out. This fact implies the Halbach array has a more purely sinusoidal field on its strong
side; this gives lower force ripple in the motor. In the next section, I will present the calculations for force ripple in a permanent-magnet motor with a Halbach array.

Another important advantage of the square Halbach array is that it has $\sqrt{2}$ times stronger field on its strong side than a conventional magnet array [TWN93]. The Halbach array thus gives double the power efficiency of a conventional motor of the same volume. A detailed analysis for Halbach array motor follows in the subsequent sections. Figure 5-4 shows the flux lines of a square Halbach magnet array.

### 5.1.3 Triangular Winding

Since we can define $\mathbf{J}_e = \nabla \times \mathbf{M}$, there should exist a winding pattern which has a single-sided field may have applications where it is desirable to build a single-sided electromagnet such as in maglev trains. In this application, superconducting coils mounted in the maglev train interact with driven coils in the track which levitate and propel the train. However, due to the strength of the superconducting magnet, it is
difficult to shield the passenger compartment from stray magnetic fields. Such fields may pose a health risk to passengers, especially those using field-sensitive devices such as pacemakers.

It is possible to reduce the shielding requirements if an electromagnet is used which has a predominantly single-sided field pattern. A possible winding pattern for such an electromagnet is illustrated in Figure 5-5 [TKW96b]. The figure shows the winding’s analogy with the Halbach magnet array through the relationship $K_c = n \times [M^a - M^b]$. Since it is not possible to implement true surface currents, we approximate this with a winding pattern that occupies the volume of the triangular regions shown.

**Current Distribution**

This single-sided winding pattern can be readily analyzed using the approach presented in previous chapter. The Fourier coefficient of the winding pattern in Figure 5-6 can be calculated by the analysis integral.

$$
\tilde{J}_{yn} = \frac{1}{l} \int_{-l/2}^{l/2} J_y(x)e^{jk_n x} \, dx
$$

(5.5)

Since the current distribution is not constant in $x$, the Fourier coefficient is a function
Figure 5-5: Electromagnetic dual of Halbach array (a) Halbach magnet array (b) Equivalent current model (c) triangular winding pattern

Figure 5-6: Triangular winding pattern with a phase filled with uniform current density
of \( x \). Introducing a new variable \( Z \),

\[
Z = \begin{cases} 
\frac{-\pi l}{4}, & -\frac{\Gamma}{2} \leq x < 0 \\
\frac{\pi l}{4}, & 0 \leq x \leq \frac{\Gamma}{2} 
\end{cases}.
\] (5.6)

Then, the fundamental Fourier coefficient for \(-\Gamma/2 \leq x \leq \Gamma/2\) is with analysis integral

\[
\tilde{J}_{y1}(x) = \frac{1}{l} \left[ \int_{-l/2+Z}^{-3l/8} \text{sgn}(x) \cdot \int_{-l/4+Z}^{-l/4+Z} + \int_{-l/8}^{l/8} - \int_{x}^{l/4-Z} \text{sgn}(x) \cdot \int_{l/4-Z}^{l/4+Z} - \int_{3l/8}^{l/2-Z} \right] \cdot J_0 e^{2\pi s l} dz \\
= \frac{j J_0}{\pi} \left( \sqrt{2} - 2 \cos \frac{\pi}{2\Gamma} x - 2 \sin \frac{\pi}{2\Gamma} x \right). \] (5.7)

We have used (5.6) to derive the above equation. Since \( J_y \) is a real function, its Fourier coefficients are conjugate symmetric. That is,

\[
\tilde{J}_{y,-1}(x) = -\frac{j J_0}{\pi} \left( \sqrt{2} - 2 \cos \frac{\pi}{2\Gamma} x - 2 \sin \frac{\pi}{2\Gamma} x \right) \] (5.8)

for \(-\Gamma/2 \leq X \leq \Gamma/2\).

**Triangular Winding Field**

Let us consider an imaginary boundary at \( x \) in Figure 5-6. We first derive the vector potential at the boundary \((e)\). The current density with infinitesimal thickness is denoted as \( \tilde{K}_{yn} \) \( \equiv \tilde{J}_{yn}(x)dx \). And the jump condition at the boundary \((e)\) for the flux density is \(-\tilde{B}_{zn}' + \tilde{B}_{zn}' = \mu_0 \tilde{K}_{yn} (\Gamma/2)\). The primes denote that the quantities are the flux densities and the potentials only from the infinitesimal current density. The transfer relations of the half-infinite regions are

\[
\tilde{B}_{zn}' = -\gamma_n \tilde{A}_{yn}' \] (5.9)

\[
\tilde{B}_{zn}' = \gamma_n \tilde{A}_{yn}'. \] (5.10)

---

\(^{1}\) Even if it happens to look like \( \Gamma = l/4 \) in Figure 5-6, we deal with more general triangular winding patterns with an arbitrary ratio between \( \Gamma \) and \( l \) in this section.
Using the jump condition, the transfer relations, and the continuity of the vector potential, \( \vec{A}'_y = \vec{A}'_y \),

\[
\vec{A}'_y = \frac{\mu_0}{2\gamma_n} \vec{K}_y(\Gamma/2) = \frac{\mu_0}{2\gamma_n} \vec{J}_y(\Gamma/2)dx. \tag{5.11}
\]

In order to calculate the vector potential due to the whole current distribution, we integrate across the current-carrying region. Recall that the vector potential decays exponentially in the distance from the source in Cartesian coordinates; thus

\[
\vec{A}'_y = \int_{-\Gamma/2}^{\Gamma/2} \frac{\mu_0}{2\gamma_n} \vec{J}_y(x)e^{-\gamma_n(\Gamma/2-z)}dx. \tag{5.12}
\]

For the fundamental component,

\[
\vec{A}_y = \frac{\mu_0}{2\gamma_1} e^{-\gamma_n\Gamma/2} \frac{j J_0}{\pi} \int_{-\Gamma/2}^{\Gamma/2} \left( \sqrt{2} - 2 \cos \frac{\pi}{2\Gamma} x - 2 \sin \frac{\pi}{2\Gamma} x \right) e^{\gamma_n x} dx. \tag{5.13}
\]

After carrying out the integration, the vector potential at the surface of the strong side is

\[
\vec{A}_y = \frac{j \mu_0 J_0}{\sqrt{2}\pi\gamma_1} \left[ \frac{1 - e^{-\gamma_n \Gamma}}{\gamma_1} - \frac{1}{\left( \frac{\pi}{2\Gamma} \right)^2 + \gamma_1^2} \left( 2\gamma_1 e^{-\gamma_n \Gamma} - \frac{\pi}{\Gamma} \right) \right]. \tag{5.14}
\]

Here, \( J_0 \) is magnitude of the current density within the triangular regions shown. Similarly, the vector potential at the surface of the weak side can be shown to be

\[
\vec{A}_y = \frac{j \mu_0 J_0}{\sqrt{2}\pi\gamma_1} \left[ \frac{1 - e^{-\gamma_n \Gamma}}{\gamma_1} + \frac{1}{\left( \frac{\pi}{2\Gamma} \right)^2 + \gamma_1^2} \left( 2\gamma_1 e^{-\gamma_n \Gamma} - \frac{\pi}{\Gamma} \right) \right]. \tag{5.15}
\]

By \( \vec{B}_y = jk_n \vec{A}_y \), the fundamental Fourier components of the normal magnetic flux density due to such a triangular stator current distribution are

\[
\vec{B}^e_{x1} = -\frac{\mu_0 J_0}{\sqrt{2}\pi} \left[ \frac{1 - e^{-\gamma_n \Gamma}}{\gamma_1} - \frac{1}{\left( \frac{\pi}{2\Gamma} \right)^2 + \gamma_1^2} \left( 2\gamma_1 e^{-\gamma_n \Gamma} - \frac{\pi}{\Gamma} \right) \right] \tag{5.16}
\]

\[
\vec{B}^h_{x1} = -\frac{\mu_0 J_0}{\sqrt{2}\pi} \left[ \frac{1 - e^{-\gamma_n \Gamma}}{\gamma_1} + \frac{1}{\left( \frac{\pi}{2\Gamma} \right)^2 + \gamma_1^2} \left( 2\gamma_1 e^{-\gamma_n \Gamma} - \frac{\pi}{\Gamma} \right) \right] \tag{5.17}
\]

at the boundaries (e) and (h), respectively.
We have been carrying along $\Gamma$ and $l$ (as in $\gamma_1 = 2\pi/l$) independently. The weak side ($h$) of the winding has no fundamental field ($n = \pm 1$) under the condition that $\Gamma = l/4$. If we substitute $l = 4\Gamma$ into (5.17), we get $\tilde{B}_x^h = 0$. In this case the fundamental of the tangential field $\tilde{B}_x^h$ is also zero on the weak side of the winding. As can be seen in Figure 5-6, $J_y$ is an odd function in $z$ (i.e., $J_y(z) = -J_y(-z)$) for all values of $x$. This implies $\tilde{J}_y$ vanishes for even numbers of $n$. So, there is no even-order harmonic field associated with the winding. We observed the same phenomena with a square ($\Delta = l/4$) Halbach magnet array.

In a similar manner, we can show the third harmonic for the strong side is

$$
\tilde{B}_x^e = \frac{\mu_0 J_0}{3\sqrt{2}\pi} \left[ \frac{1 - e^{-\gamma_3 \Gamma}}{\gamma_3} - \frac{1}{(\frac{3\pi}{2\Gamma})^2 + \gamma_3^2} \left(2\gamma_3 - \frac{3\pi}{\Gamma} e^{-\gamma_3 \Gamma}\right) \right].
$$

(5.18)

Under the same square geometry condition, the third harmonic for the strong side vanishes. Thus, the next harmonics on the strong side is the fifth one. This indicates that the strong-side field is highly sinusoidal. This would yield smoother operation if the motor were used in a linear motor, e.g., for a maglev train. In addition, since the fields fall off exponentially with spatial wave number, the absence of fundamental fields on the back side of the motor reduces the shielding requirements if the motor is used as the traveling excitation for such a maglev train. The single-sided winding pattern can also easily switch its strong and weak sides by changing the direction of excitations. I took vertical flux density components $\tilde{B}_{zn}$ for examples in this section. It can be shown that the same arguments are applied to the lateral flux density components, $\tilde{B}_{zn}$.

Discussions

To this point we have considered a one-phase coil array. The configuration of the winding for two-phase operation is shown in Figure 5-7. Here, phase 2 is electrically displaced by $90^\circ$ from phase 1 (i.e., $l/4$ spatial displacement). As compared with a conventional motor winding, the triangular winding pattern has the following merits:
Figure 5-7: Two-phase operation with superposition of two phases with 90° apart with each other

(1) has no fundamental field on the weak side, (2) has less distorted sinusoidal field on the strong side, and (3) saves magnetic field shielding material for certain applications, such as maglev vehicles.

However, that the major disadvantage is that the winding is less power-efficient. This limits its utility in general applications. Specifically, via steps similar to the above, we can derive the field of the conventional two-phase winding (like one in Figure 5-9 with two phases) as

$$\bar{B}_{x1} = -\frac{\mu_0 J_0}{\sqrt{2\pi}} \left[ \frac{1 - e^{-\gamma_1 R}}{\gamma_1} \right].$$

(5.19)

Comparing the field at (e) for the geometric data from the prototype motor described in the previous section, the magnitude of the field of the single-sided winding pattern is 53% of that of the conventional pattern with the same consumption of the electric power. Thus this single-sided winding is not likely to be used in power-sensitive applications in conventional machines. However, in superconducting machines this winding pattern, or others like it, may prove to be of interest.
5.2 Electrical Terminal Relation of Stator

We move now to consideration of the motor terminal characteristics. In order to derive the electrical terminal relation, we use the Faraday induction law

\[ \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a}. \]  

(5.20)

Here the integration is taken along a contour fixed in the non-moving frame (or the non-primed frame in Figure 4-1). The magnetic flux density \( \mathbf{B} \) inside the integral on the right-hand side in (5.20) consists of the flux densities due to the magnet \( ^M \mathbf{B} \) and those due to the stator \( ^S \mathbf{B} \). It can be shown that this integral can be expanded for two-dimensional geometries as

\[ -V_S + \oint_C \frac{\mathbf{J}_f}{\sigma} \cdot d\mathbf{l} + \oint_C \left( -v_x^M B_z + v_z^M B_x \right) i_y \cdot d\mathbf{l} = -\frac{d}{dt} \int_S ^S \mathbf{B} \cdot d\mathbf{a}. \]  

(5.21)

The term \( V_S \) is the driving voltage at the terminals of the winding. The second term is Ohmic drop denoted, i.e., \( R_S I_S \), where \( I_S \) is defined as positive into the positive terminal of \( V_S \), and \( I_S = J_f A_w \) where \( A_w \) is the winding wire cross-sectional area. The third term is the back electromotive force (emf), or speed voltage. Here, \( v_x \equiv \frac{d\alpha}{dt} \) and \( v_z \equiv \frac{d\alpha}{dt} \) are the velocities of the magnet array in the \( x \)- and \( z \)-directions, respectively. The right-hand-side term is the induced voltage due to self- and mutual-inductance of the stator coil. This terminal relation gives a complete electrical model for the linear motor. The rest of this section gives detailed derivations of the last two terms of (5.21).

5.2.1 Vector Potential inside Stator

We think of an imaginary boundary at \( x = X \) and calculate the vector potential there (Figure 5-8). Let the boundary upper and lower surfaces be \( (p) \) and \( (q) \), respectively. Vector potentials due to the current in the upper section \( X < x \leq \Gamma \) and the lower section \( 0 \leq x \leq X \) come directly from (4.57–4.58) as

\[ ^S \tilde{A}^p \quad \left( = ^S \tilde{A}^q \right) = \frac{\mu_0}{2k_n^2} \tilde{J}_m \left( 1 - e^{-\gamma (\Gamma - x)} \right) \]  

(5.22)
Figure 5-8: Imaginary boundary (dashed line at \( x = X \)) inside stator

\[
\frac{S \vec{A}_p}{L_y} (= \frac{S \vec{A}_q}{L_y}) = \frac{\mu_0}{2k_n^2} \vec{j}_{yn} (1 - e^{-\gamma n X}).
\]  
(5.23)

By superposition, the vector potential inside the stator at \( x = X \) is the sum of \( \frac{S \vec{A}_p}{L_y} \) and \( \frac{S \vec{A}_q}{L_y} \).

\[
\frac{S \vec{A}_p}{L_y} = \frac{\mu_0}{2k_n^2} \vec{j}_{yn} \left( 2 - e^{-\gamma n X} - e^{-\gamma n (\Gamma - X)} \right)
\]  
(5.24)

Thus, the total vector potential at \( p \) due to the magnet and the current is from (4.45) and (5.24)

\[
\vec{A}_p = \left( -\frac{\mu_0}{2\gamma_n} \vec{M}_z - \frac{j\mu_0}{2k_n} \vec{M}_z \right) e^{-\gamma n (x_0 + \Gamma - X)} e^{ikx_0} (1 - e^{-\gamma n \Delta})
\]  
(5.25)

This result is used to calculate flux linkage, self-inductance of the winding, and back emf in the following subsections.

### 5.2.2 Stator Flux Linkage

The vector potential simplifies the evaluation of the flux passing through a surface. The magnetic flux \( \Phi \) linked by a closed surface \( S \) is given by the integration of the magnetic flux density over \( S \). By the Stokes' theorem, this can be represented as the line integral of the vector potential \( \vec{A} \) around the contour \( C \) enclosing the surface, i.e.,

\[
\Phi = \int_S \vec{B} \cdot d\vec{a} = \int_S (\nabla \times \vec{A}) \cdot d\vec{a} = \oint_C \vec{A} \cdot d\ell.
\]  
(5.26)
Figure 5-9: Closed contour for magnetic flux, self-inductance, and back emf calculation

In the two-dimensional geometry of our model, the path of the line integral of interest is confined to a plane and is a rectangle. This situation is depicted in Figure 5-9. As seen in the figure, I take a six-phase machine for example where in the windings occupy rectangular windows of thickness \( \Gamma \) and width \( l/2q \), where \( l \) is the pitch and \( q \) is the number of phases of the machine. In the model, the vector potential \( \mathbf{A} \) has only a \( y \)-component which is constant in \( y \). This enables us to readily evaluate \( \Phi \) for every surface closed by the path \( C \) inside the stator current distribution. Since the winding and its return path are separated by half the pitch, the flux linked by one winding of negligible cross-section lying at \( x, z \) and \( x, z + l/2 \) is

\[
\Phi = \sum_{n=-\infty}^{\infty} \left[ \tilde{A}_{yn}(x)e^{-jknz} - \tilde{A}_{yn}(x)e^{-jkn(z+l/2)} \right] w, \tag{5.27}
\]

where \( w \) is the depth of the stator winding.

If the turn distribution is uniform with \( \eta_0 \) turns per unit cross-sectional area, then there are \( \eta_0 dx dz \) turns in an infinitesimal area. The total flux linked by the one phase winding for one pitch \( l \) is given by the integration of the linked infinitesimal flux over
one pitch of the winding (Figure 5-9) as
\[ \lambda_S = w \eta_0 \sum_{n=-\infty, odd}^{\infty} \int_0^{l/2q} 2e^{-jk_nz} dz \int_0^\Gamma A_{yn}(x) dx. \] (5.28)

### 5.2.3 Self-Inductance

Let us derive a formula for a six-phase square-wave winding as in Figure 5-9. From (5.28), with \( q = 6 \), and taking only the self-flux gives
\[ \lambda_S = w \eta_0 \sum_{n=-\infty, odd}^{\infty} \int_0^{l/12} 2e^{-jk_nz} dz \int_0^\Gamma S A_{yn}(x) dx. \] (5.29)

Using
\[ \int_0^{l/12} 2e^{-jk_nz} dz = \frac{jI}{\pi n} (e^{-j\pi n/6} - 1), \] (5.30)
and using (5.24)
\[ \int_0^\Gamma S A_{yn}(x) dx = \frac{\mu_0 j_{yn}}{n k_n^2} \left( \Gamma + \frac{e^{-\gamma_n \Gamma}}{\gamma_n} - 1 \right), \] (5.31)
the total flux linkage in terms of the current density may be expressed as
\[ \lambda_S = \frac{j \mu_0 w \eta_0 I_0}{\pi} \sum_{n=-\infty, odd}^{\infty} \frac{j_{yn}}{n k_n^2} (e^{-j\pi n/6} - 1) \left( \Gamma + \frac{e^{-\gamma_n \Gamma}}{\gamma_n} - 1 \right). \] (5.32)

The Fourier coefficients of the current density with the amplitude \( J_0 \) for the first phase out of six can be shown to be
\[ j_{yn} = \begin{cases} \frac{J_0}{j \pi n} (e^{j\pi n/6} - 1), & n \text{ : odd} \\ 0, & n \text{ : even} \end{cases} \] (5.33)

Here we are assuming that the currents in the other five phases are zero, and thus we will calculate the self flux linkage and thereby the self-inductance. Substituting this into (5.32) yields the self-flux linked by one phase in one pitch.
\[ \lambda_S = \frac{\mu_0 w \eta_0 I_0^3 J_0}{2\pi^4} \sum_{n=-\infty, odd}^{\infty} \frac{1}{n^4} \left( 1 - \cos \frac{\pi}{6} n \right) \left( \Gamma + \frac{e^{-\gamma_n \Gamma}}{\gamma_n} - 1 \right). \] (5.34)

Since \( J_0 = \eta_0 I_S \), where \( I_S \) is the stator winding current, the self-inductance of one phase winding per pitch is, via \( \lambda_S = L_S I_S \), given by
\[ L_S = \frac{\mu_0 w \eta_0^2 I_0^3}{2\pi^4} \sum_{n=-\infty, odd}^{\infty} \frac{1}{n^4} \left( 1 - \cos \frac{\pi}{6} n \right) \left( \Gamma + \frac{e^{-\gamma_n \Gamma}}{\gamma_n} - 1 \right). \] (5.35)
We use this formula to estimate the self-inductance of a prototype six-phase motor winding later in this chapter.

### 5.2.4 Back Electromotive Force

Now, let us work out the third term of (5.21). The Fourier components of the vector potential due to the magnet at \( x = X, z = Z \) are

\[
M A_n(X, Z) = M \tilde{A}_n(X) e^{-j k_n Z} \\
= \left( -\frac{\mu_0}{2\gamma_n} \tilde{M}_{zn} - \frac{j \mu_0}{2k_n} \tilde{M}_{zn} \right) \left( 1 - e^{-\gamma_n \Delta} \right) e^{-\gamma_n (x_0 + \Gamma - X)} e^{jk_n (z_0 - Z)}. \tag{5.36}
\]

One-half pitch later at \( x = X, z = Z + l/2 \), the potential is

\[
M A_n(X, Z + l/2) = M \tilde{A}_n(X) e^{-j k_n (Z + l/2)} \\
= \left( -\frac{\mu_0}{2\gamma_n} \tilde{M}_{zn} - \frac{j \mu_0}{2k_n} \tilde{M}_{zn} \right) \left( 1 - e^{-\gamma_n \Delta} \right) e^{-\gamma_n (x_0 + \Gamma - X)} e^{jk_n (z(t) - Z - l/2)}. \tag{5.37}
\]

Applying the relation \( \tilde{B}_zn = \frac{\partial}{\partial x} \tilde{A}_yn \) to (5.36) gives

\[
\tilde{B}_zn = \gamma_n \tilde{A}_yn. \tag{5.38}
\]

Using this and \( \tilde{B}_zn = j k_n \tilde{A}_yn \) gives the total flux linked by the one-phase winding denoted in Figure 5-9 due to the magnet as

\[
\int_S \frac{\mathbf{M} \cdot d\mathbf{a}}{\partial} = w \eta_0 \int_{0}^{l/2q} \int_{0}^{\Gamma} \sum_{n=-\infty}^{\infty} \left[ M \tilde{A}_yn(X, Z) - M \tilde{A}_yn(X, Z + l/2) \right] dXdZ \\
= w \eta_0 \int_{0}^{l/2q} \int_{0}^{\Gamma} \sum_{n=-\infty}^{\infty} \left[ M \tilde{A}_yn(X) - M \tilde{A}_yn(X) e^{-jk_n l/2} \right] e^{-jk_n Z} dXdZ. \tag{5.39}
\]

Recalling

\[
e^{-jk_n l/2} = e^{-j \pi n} = \begin{cases} 1, & n : \text{even} \\ -1, & n : \text{odd} \end{cases} \tag{5.40}
\]
and using (5.36),

$$\int S M \mathbf{B} \cdot d\mathbf{a} = 2w\eta_0 \sum_{n=-\infty,\text{odd}}^{\infty} \left( \frac{-\mu_0}{2\gamma_n} \tilde{M}_{zn} - \frac{j\mu_0}{2k_n} \tilde{M}_{zn} \right) \left( 1 - e^{-\gamma_n \Delta} \right)$$

$$\cdot e^{-\gamma_n(x(t)+\Gamma)} e^{jk_n z(t)} \int_0^{l/2q} e^{-jk_n z} dZ \int_0^\Gamma e^{\gamma_n x} dX. \quad (5.41)$$

The back emf for the one period terminal can be calculated as

$$V_{bemf} = \frac{d}{dt} \int S M \mathbf{B} \cdot d\mathbf{a} = 2w\eta_0 \sum_{n=-\infty,\text{odd}}^{\infty} \left( \frac{-\mu_0}{2\gamma_n} \tilde{M}_{zn} - \frac{j\mu_0}{2k_n} \tilde{M}_{zn} \right) \left( 1 - e^{-\gamma_n \Delta} \right)$$

$$\cdot ( -\gamma_n v_x(t) + j k_n v_z(t) ) e^{-\gamma_n(x(t)+\Gamma)} e^{jk_n z_0} \int_0^{l/2q} e^{-jk_n z} dZ \int_0^\Gamma e^{\gamma_n x} dX. \quad (5.42)$$

We can specifically derive the back emf for a six-phase motor ($q = 6$) with evaluations of the integrals. The back emf per pitch at the stator terminals of a one-phase winding denoted in Figure 5-9 then reduces to

$$V_{bemf} = w\eta_0 \sum_{n=-\infty,\text{odd}}^{\infty} \left( \frac{\mu_0}{\gamma_n} \tilde{M}_{zn} + \frac{j\mu_0}{k_n} \tilde{M}_{zn} \right) \left( \frac{j v_x}{k_n} + \frac{v_z}{\gamma_n} \right)$$

$$\cdot \left( 1 - e^{-\gamma_n \Delta} \right) \left( 1 - e^{-\gamma_n \Gamma} \right) \left( e^{-j\pi n/6} - 1 \right) e^{-\gamma_n x_0} e^{jk_n z_0}. \quad (5.43)$$

We can also derive the back emf directly from the field solutions. The Fourier components of the magnetic flux density due to the magnet at $x = X$, $z = Z$ are from the field solutions (4.46-4.47)

$$M\tilde{B}_{zn}(X, Z) = \left( -\frac{jk_n \mu_0}{2\gamma_n} \tilde{M}_{zn} + \frac{\mu_0}{2} \tilde{M}_{zn} \right) \left( 1 - e^{-\gamma_n \Delta} \right) e^{-\gamma_n(x(t)-X)} e^{jk_n(z(t)-Z)} \quad (5.44)$$

$$M\tilde{B}_{zn}(X, Z) = \left( -\frac{\mu_0}{2} \tilde{M}_{zn} - \frac{j\gamma_n \mu_0}{2k_n} \tilde{M}_{zn} \right) \left( 1 - e^{-\gamma_n \Delta} \right) e^{-\gamma_n(x(t)-X)} e^{jk_n(z(t)-Z)} \quad (5.45)$$

The Fourier components of the vector potential due to the magnet at $x = X$, $z = Z$ are by (4.45) with some intermediate steps

$$M\tilde{A}_{zn}(X, Z) = M\tilde{A}_{zn}(X) e^{-jk_n z}$$

$$= \left( -\frac{\mu_0}{2\gamma_n} \tilde{M}_{zn} - \frac{j\mu_0}{2k_n} \tilde{M}_{zn} \right) \left( 1 - e^{-\gamma_n \Delta} \right) e^{-\gamma_n(x(t)-X)} e^{jk_n z(t)} e^{-jk_n z}. \quad (5.46)$$
It can be shown that the back emf of the one-phase winding denoted in Figure 5-9 is represented as the third term of (5.21).

\[ V_{\text{bemf}} = \oint_C \left( -v_z M B_z + v_z^* M B_z^* \right) i_y \cdot dl = w \eta_0 \int_0^{l/2} \int_0^\Gamma N \sum_{n=0}^\infty \left( -\gamma_n v_z + j k_n v_z \right) \]  

where the integration path \( C \) includes the whole wire in one-phase winding of a pitch. With a similar argument in the flux linkage calculation case this becomes

\[ V_{\text{bemf}} = 2w \eta_0 \sum_{n=-\infty}^\infty \left\{ -v_z \left( -\frac{\mu_0}{2} \bar{M}_{zn} - \frac{j \gamma_n \mu_0}{2k_n} \bar{M}_{zn} \right) + v_z \left( -\frac{j k_n \mu_0}{2 \gamma_n} \bar{M}_{zn} + \frac{\mu_0}{2} \bar{M}_{zn} \right) \right\} \cdot \left( 1 - e^{-\gamma \Delta} \right) e^{-\gamma z(t) e^{j k_n z(t)}} \int_0^{l/2} e^{-j k_n z} dz \int_0^\Gamma e^{\gamma X} dX \]  

After simplification, The back emf expected at a stator terminal for a one-phase winding per pitch for a six-phase motor denoted in Figure 5-9 reduces to

\[ V_{\text{bemf}} = w \eta_0 \sum_{n=-\infty, \text{odd}}^\infty \left( \frac{\mu_0}{\gamma_n} \bar{M}_{zn} + \frac{j \mu_0}{k_n} \bar{M}_{zn} \right) \left( \frac{j v_z}{k_n} + \frac{v_z}{\gamma_n} \right) \cdot \left( 1 - e^{-\gamma \Delta} \right) \left( 1 - e^{\gamma R} \right) \left( 1 - e^{-j \pi n/6} \right) e^{-\gamma z(t) e^{j k_n z(t)}}. \]  

This is identical to (5.43) which was derived via the vector potential, thereby providing a confirmation of this earlier result.

### 5.2.5 Power Balance

The electrical power transduced into mechanical power is a summation of the product of the back emf and the current for all the filamentary winding loops in a phase. Since \( V_{\text{bemf}} \) derived in the last subsection is the back emf for a full one-phase winding, the electrical power per pitch is by symmetry of the magnet array and windings

\[ P_E = 2 \int_0^{l/2} \int_0^\Gamma \frac{V_{\text{bemf}} J_y}{2 \eta_0} dxdz. \]  

To make the discussion simple and to specialize to the sinusoidal stator to compare against the force formula derived earlier, let us assume the stator current is distributed
sinusoidally and has only fundamental \( \tilde{J}_1 \) and \( \tilde{J}_{-1} \). Then, the \( y \)-component of the current density can be denoted by

\[
J_y = (J_a + jJ_b)e^{-jk_1z} + (J_a - jJ_b)e^{-jk_{-1}z}.
\]  

(5.51)

Since there is no \( x \)-dependence in \( \tilde{J}_{yn} \), the total electrical power is from (5.42) and (5.51)

\[
P_E = 2\omega \eta_0 \sum_{n=-\infty, \text{odd}}^{\infty} \left( \frac{\mu_0}{2\gamma_n} \tilde{M}_{zn} - \frac{j\mu_0}{2k_n} \tilde{M}_{zn} \right) e^{-\gamma_n z(t)} e^{jk_n z(t)} (-\gamma_n v_x(t) + jk_n v_z(t))
\]

\[
\cdot (1 - e^{-\gamma_n \Delta}) \frac{1 - e^{-\gamma_1 l}}{\gamma_1} \int_0^{l/2} \frac{1}{\eta_0} \left\{ (J_a + jJ_b)e^{-jk_1z} + (J_a - jJ_b)e^{-jk_{-1}z} \right\} e^{-jk_n z} dz.
\]

(5.52)

By the following relation,

\[
\int_0^{l/2} e^{-jk_\pm z} e^{-jk_n z} dz = \begin{cases} 
0, & n \neq \mp 1, \text{odd} \\
\frac{l}{2}, & n = \mp 1 
\end{cases}
\]

(5.53)

the higher magnet harmonics will not contribute to the electrical power transduced into mechanical power anyhow under the assumption made above, the sinusoidal current distribution. The electric power formula reduces to (Recall here \( \gamma_{-1} = \gamma_1 = k_1 = -k_{-1} \).)

\[
P_E = w \left(1 - e^{-\gamma_1 \Delta}\right) \frac{1 - e^{-\gamma_1 l}}{\gamma_1} \frac{l}{2}
\]

\[
\cdot \left[ (-\gamma_1 v_x(t) + jk_1 v_z(t)) e^{jk_1 z(t)} \left( -\frac{\mu_0}{\gamma_1} \tilde{M}_{z1} - \frac{j\mu_0}{k_1} \tilde{M}_{z1} \right) (J_a - jJ_b)
\]

\[
+ (-\gamma_{-1} v_x(t) + jk_{-1} v_z(t)) e^{jk_{-1} z(t)} \left( -\frac{\mu_0}{\gamma_{-1}} \tilde{M}_{z,-1} - \frac{j\mu_0}{k_{-1}} \tilde{M}_{z,-1} \right) (J_a + jJ_b) \right].
\]

(5.54)

This is an electrical power formula for general permanent-magnet machines under sinusoidal excitation with fundamental frequency.

Using the fundamental Fourier coefficients of the Halbach magnet array given in (5.3–5.4) in Section 5.1, we can give the electrical power per pitch in the simplest
\[ P_E = \mu_0 M_0 G e^{-\gamma x_0} \]
\[ \cdot [v_z(t) \{ J_a \sin \gamma z_0 - J_b \cos \gamma z_0 \} - v_z(t) \{ J_a \cos \gamma z_0 + J_b \sin \gamma z_0 \}] \]

where the constant \( G \) is defined as (4.82). Let \( P_M \) be the mechanical power. Then,

\[ P_M = v_z(t) f_x + v_z(t) f_z. \]  

(5.56)

We can prove that the electrical power input \( P_E \) and the mechanical power output \( P_M \) coincide by using the force equations (4.81). Recall they use only fundamental harmonics of the fields due to magnet and current. This power balance provides a consistency check of the analytic results so far.

This completes the exposition of the electrical terminal relation given in (5.21). I will give experimental verifications to the above analytical results in the last section of this chapter.

### 5.3 Force Ripple with Respect to Phase Number

In this section, we discuss the prospective motor's force ripple with respect to the number of phases of the stator current, since the force ripple depends on the number of stator phases the details of the magnet array and the non-ideality of both of magnet array and stator current. They do not provide sinusoidal fields in reality. The ripples force can affect the motor positioning performance and is to be made as small as possible. In this section, we present analysis of the force ripple as a function of the number of phases in a linear ironless Halbach magnet array motor, and show that force ripples below 0.1% can be readily achieved with a reasonable number of phases.

#### 5.3.1 Fourier Harmonics of Stator Current

Now, to analyze the force ripple later in this section, we will derive Fourier coefficients of the stator current distribution the \( q \)-phase balanced operation. Let us assume that
Figure 5-10: Spatial current density distribution with six-phase winding as a function of spatial angle $\delta = 2\pi z/l$

the stator currents are in balanced operation, i.e.,

$$J_k(z, t) = J_0 \cos \left( \theta(t) + \frac{\pi k}{q} \right), \quad \frac{(2k - 1)l}{4q} + lp \leq z \leq \frac{(2k + 1)l}{4q} + lp,$$

(5.57)

where $q$: number of phases, $J_k$: instantaneous $y$-directed current density of slot $k$ ($0 \leq k \leq 2q - 1$) depending on the electrical angle $\theta(t)$, $p$: integer representing the successive winding cycles, and $J_0$: maximum current density. The electrical phase angle between two consecutive phases is $\pi/q$, which is $30^\circ$ in this six-phase case. Figure 5-10 shows a current distribution with $J_0 = 1$ and $\theta(t) = \pi / 4.17$, which are arbitrarily chosen. The spatial angle is defined by $\delta = 2\pi z/l$. The current distribution in the figure includes up to the 99th harmonics of the real staircase distribution. A fundamental period sinusoidal wave form is given for reference. Gibbs phenomena in the figure are an artifact of truncating the Fourier series, and are not present with the real windings. Since we approximate the ideal sinusoidal current distribution with a finite number of lumped phases, higher-order harmonics exist in the current
distribution. These stator harmonics interact with like-order magnet-array harmonics to cause force ripple. This force ripple can yield an error motion when driving the levitated stage.

We next calculate the complex Fourier coefficients of the current distribution. The current distribution is dependent only on \( z \) and \( t \).

\[
\tilde{J}_{yn} = \frac{1}{l} \int J_{sk}(z, t) e^{jknyz} dz = \frac{1}{l} \sum_{k=0}^{2q-1} \int_{(2k-1)/4q}^{(2k+1)/4q} J_0 \cos \left( \theta(t) + \frac{\pi k}{q} \right) e^{j\pi nz/l} dz, \tag{5.58}
\]

where \( \theta(t) \) is an arbitrary phase angle of the zeroth phase in the balanced operation. After the integration,

\[
\tilde{J}_{yn} = \frac{J_0}{j4\pi n} \sum_{k=0}^{2q-1} \left( e^{j\theta(t)} e^{j\pi k/q} + e^{-j\theta(t)} e^{-j\pi k/q} \right) \left( e^{j2\pi n(2k+1)/4q} - e^{j2\pi n(2k-1)/4q} \right). \tag{5.59}
\]

We find \( \tilde{J}_{yn} = 0 \) unless \( \pm \frac{j\pi k}{q} + \frac{j2\pi n 2k}{4q} = j2\pi km \), where \( m \) is an integer. The Fourier coefficients are thus:

\[
\tilde{J}_{yn} = \begin{cases} 
\frac{qJ_0}{4\pi n} \sin \left( \frac{n\pi}{2q} \right) e^{\mp j\theta(t)}, & n = 2mq \pm 1, \\
0, & \text{otherwise}
\end{cases} \tag{5.60}
\]

where the order of the double sign is significant.

### 5.3.2 Force Ripple

We have the complete field description due to the Halbach magnet array and stator current given as in (4.77) and (4.76). As we described in Subsection 4.1.4, the force can be more easily calculated with the Maxwell stress tensor. Recall the formulas for forces between the magnet and the stator current given in Section 4.3; using the Parseval’s theorem (4.23) given in Subsection 4.1.5 the vertical force on the enclosed section with area \( S \) is

\[
F_z = -S \langle T^d_{xx} \rangle_z = -\frac{S \mu_0}{2} \langle H_x^d H_x^d - H_z^d H_z^d \rangle_z = -\frac{S \mu_0}{2} \sum_{n=-\infty}^{\infty} \left( \bar{H}_x^d \bar{H}_x^d - \bar{H}_z^d \bar{H}_z^d \right). \tag{5.61}
\]
Table 5.1: Ripple force with various phases with a current density of $1.5 \times 10^6$ A/m²

<table>
<thead>
<tr>
<th>phases</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>force (N)</td>
<td>13.2</td>
<td>14.0</td>
<td>14.3</td>
<td>14.4</td>
<td>14.5</td>
</tr>
<tr>
<td>amplitude of force ripple (N)</td>
<td>0.11</td>
<td>0.11</td>
<td>0.013</td>
<td>0.013</td>
<td>0.0025</td>
</tr>
<tr>
<td>percentage error</td>
<td>0.83%</td>
<td>0.79%</td>
<td>0.091%</td>
<td>0.090%</td>
<td>0.017%</td>
</tr>
</tbody>
</table>

The lateral force is given by

$$F_z = -S\langle T_{zz}^d \rangle_z = -S\mu_0 \langle H_z^d H_z^d \rangle_z = -S\mu_0 \sum_{n=-\infty}^{\infty} \tilde{H}_{zn}^d \tilde{H}_{zn}^d.$$ \hspace{1cm} (5.62)

Recall the square Halbach array field in the air gap has only fundamental, 5th, 9th, 13th components, and so force, whereas a conventional magnet array field has all odd-order harmonics. With the result of Subsection 5.3.1, we know the winding field components are non-zero only for $n = 2mq \pm 1$, where $m$ is an integer. The $n$th order force ripple component exists by the previous force equations (5.61–5.62) only for such $n$ that these are non-zero like order components of the Halbach array field and the stator current field. Using the above analyses we construct, Table 5.1 which shows the numerically calculated amplitude of the ripple force and the percentage error for a prototype levitator. We assume a reasonable operating current density in the winding to be $1.5 \times 10^6$ A/m² for force calculation in Table 5.1. Up to 99th harmonics of the force components are added up. We observe an interesting fact in the table. For example, the force ripples in four-phase and five-phase cases are about the same. The square Halbach array field in the air gap was shown to have fundamental, 5th, 9th, ⋅⋅⋅, components, whereas a conventional magnet array field has all odd-order harmonics. We also showed that the winding field components are non-zero only for $n = 2mq \pm 1$, where $m$ is an integer. Therefore, the next higher-order force component above the fundamental is 9th order for both of the four-phase and five-phase cases.
The parameters of an example motor for the force ripple calculation are: number of phases, $q = 6$; pitch, $l = 25.4$ mm ($1''$); effective depth, $w = 91.44$ mm ($3.6''$); winding thickness, $\Gamma = l/5$; magnet-array thickness, $\Delta = l/4$; nominal air gap, $x_0 = 500$ $\mu$m. The stator has $5\frac{1}{2}$ pitches of winding (total length of 139.7 mm). The maximum allowable current density, $J_0 = 1.5 \times 10^6$ A/m² was arrived at via thermal considerations. The magnet array consists of $3\frac{3}{4}$ pitches (15 rows of $0.25 \times 0.25''$ magnets) of NdFeB material with remanence $\mu_0 M_0 = 1.23$ T. The total mass of platen is 6 kg. We neglect end effects in due to the finite magnet length and edge effect due to the finite width of the motor.

When this linear permanent-magnet levitator carries a quarter-$g$ load, where $g$ is the acceleration of gravity, 9.80 m/s², the maximum error acceleration of the six-phase levitator is thus $1.8 \times 10^{-3}$ m/s². However, in a real motor, manufacturing variations in magnet remanence and wire placement will likely yield force ripples higher than those calculated above. Thus, balancing against manufacturing challenges, I decided to realize the motors in three-phase. The discussion on the ripple force up to here is based on the sinusoidal balanced commutation. We have derived the force ripple for surface-wound permanent-magnet linear motors with Halbach array analytically in this section. Due to the characteristics of Halbach array field, nearest even-odd pairs $(2n-2n+1$, where $n$ is a positive integer) of phase number have similar force because of the alignments of harmonics. We calculated the maximum force ripple for our linear levitator. The force ripple is found to be less than 0.1% of applied force for motors using 4 or more phases if there is no manufacturing error.

5.4 Experimental Verifications

I use Mark Williams' motor (Figure 2-5) to verify my analytical work experimentally. The parameters of the motor are: number of phases, $q = 6$; number of turns per phase: 80 (with AWG#22 copper wire); turn density, $\eta_0 = 1.86 \times 10^6$ turns/m²;
pitch, \( l = 5.08 \text{ cm (2")} \); depth, \( w = 15 \text{ cm} \); winding thickness, \( \Gamma = l/5 \); magnet-array thickness, \( \Delta = l/4 \). A winding has 44-cm average turn length; its average end-turn length is 14 cm. The total winding resistance and self-inductance for one phase are 19.8 \( \Omega \) and 9.48 mH, respectively. The stator has 10 pitches of winding (total length of 50.8 cm). The magnet array consists of \( 5\frac{1}{4} \) pitches (21 rows of 0.5 \( \times \) 0.5” magnets) of NdFeB material with remanence \( \mu_0 M_0 = 1.1 \text{T} \).

With the motor geometry given above we can estimate the self-inductance of a stator winding directly with (5.35). Since (5.35) gives self-inductance per pitch, we need to multiply it by the number of pitches of winding which is 10 in this case. My analysis predicts a total self-inductance of 6.27 mH for a phase. The actual inductance measured by a dynamic signal analyzer is 9.48 mH. In spite of this 34% error, these figures are in good agreement considering that the analysis ignores the winding end turns (32% of the total length) which connect the successive phase sections. These end turns likely account for the larger measured inductance.

An experimental wave form for the back emf of one phase is shown in Figure 5-11. These data were acquired as follows. During the experiment the platen is suspended in five degrees of freedom. The driving circuits for the linear motor are disconnected. The platen is then pushed by hand while the voltage on one phase is recorded. The air gap between the magnet array and the stator winding is maintained at 400 \( \mu \text{m} \). The velocity along the \( z \)-axis is estimated graphically on the basis of the observed frequency as \( v_z = 160 \text{ mm/s} \). On the basis of the fundamental terms \( (n = \pm 1) \) of the square Halbach array’s Fourier coefficients, \( M_{z, \pm 1} = \frac{\sqrt{3}M_0}{\pi} \) and \( M_{z, \pm 1} = \pm j\frac{\sqrt{3}M_0}{\pi} \), we predict the peak magnitude of the back emf should be 1.6 V per phase via (5.43). Since the magnet array consists of \( 5\frac{1}{4} \) pitches, the total effective back emf is estimated as \( \pm 8.4 \text{ V peak} \). The actual data yield about \( \pm 8 \text{ V peak} \). The end and edge effects in the magnet and winding may be responsible for the decrease of the magnitude in the real experiment. That is, the effective flux density linked by the winding is less for the real motor than that predicted by the idealized theory.

121
5.5 Summary of Part II

In Part II, I have derived field solutions and force equations for linear permanent-magnet machines following the transfer relations approach. The analysis is very general so that it can be applied to iron- or ironless- machines and machines with multiple phases. The analysis has also been extended to tubular linear permanent-magnet machines as presented in Appendix B and [KBTL96].

A Halbach magnet array has stronger and weaker sides due to its magnetization pattern. It proves to be power efficient as the moving element in a linear motor if the stronger side is used for force production. A Halbach magnet array also causes smaller force ripple and helps save shielding material on its weaker side. Thus, I decided to use Halbach arrays for the levitator. A triangular winding pattern with a single-sided field has also been developed and analyzed.

Besides this continuum electromechanical analysis, the electrical terminal relation for multi-phase linear permanent-magnet machines are derived. This lumped model
analysis includes the stator flux linkage, self-inductance of the stator winding and back emf and could be extended to calculate mutual inductances. The analysis was verified theoretically by mechanical and electrical power balance, and experimentally with a prototype linear motor. The force ripple analysis shows that three phases are enough to fulfill the position stability specification of the levitator. A motor with fewer phases is more advantageous, since it is easier to manufacture. With the theoretical framework in place, we are now in a position to move on to the consideration of the system-level configuration and design choices for our levitated stage.
Part III

Design
In the three chapters to come, I present the design issues of the magnetic levitation stage. In Chapter 6, I discuss the conceptual design and parametric analysis. I generate several design concepts for a wafer stepper stage application, and discuss the candidate design selection. The selection criteria, such as precision, cost, control effort, and evolvability are applied. I suggest the best design concept with only one moving part for construction. Many design iterations and parametric analyses were done to fix dimensions of parts.

Since the performance of the stage is much affected by actuators (permanent-magnet linear motors in this thesis), detailed design of the motors is the major task of Chapter 7. I have developed analytic tools for this task in Part II. Chapter 7 includes designs and describes the fabrication of linear Halbach magnet arrays, stator windings, power amplifier circuit, instrumentation structure, and software. Solutions to several important mechanical design problems for the levitator are given in Chapter 8. The honeycomb sandwich structure for the platen, the stators, and packaging and assembly are described in detail. Associated fabrication issues are also discussed. At the end of this part, we have a prototype planar magnetic levitator to test.
Chapter 6

Conceptual Designs

This chapter develops conceptual designs for a magnetically levitated positioner for photolithography. We first set performance goals typical for a wafer stepper stage. Then, I generated several design concepts that may satisfy the performance goals. In this section, we discuss the advantages and disadvantages of each concept in terms of power dissipation, bulkiness and mass of the platen, thermal error, and so forth. We categorize selection criteria for prototyping, and apply a systematic method to choose the final candidate concept. On the basis of the selected design concept, a detailed design is developed for prototype construction in the two following chapters.

6.1 Design Considerations

Fundamental design considerations for the magnetic levitator are presented in this section. These include the performance goals, selection of actuators, mass of the platen, and power consumption. All conceptual designs in this chapter are on the basis of these design considerations.
6.1.1 Performance Goals Revisited

Here is the summary of the performance goals for the levitator, which were presented in Chapter 1. These goals are set to meet specifications for a next-generation wafer stepper.

- $x$ and $y$ travel range: $200 \times 200$ mm (300 $\times$ 300 mm in the near future)
- $z$ travel range: $\pm 150$ $\mu$m
- angular range: $\pm 400$ $\mu$rad
- linear positioning speed capability: 200 mm/sec
- 3-$\sigma$ position noise:
  - $x, y$: 20 nm
  - $z$: 100 nm\(^1\)
  - $\theta_x, \theta_y, \theta_z$: 1 $\mu$rad

The ultimate purpose of this thesis is to realize a magnetic levitator with only one moving part that achieves the above performance goals. I carried out conceptual studies and parametric analyses for $200 \times 200$ mm area coverage to accommodate 200-mm wafers in earlier my thesis work. Thus, all geometric parameters for design concepts in this section are given for $200 \times 200$ mm lateral travel range. Later, we decided to implement a prototype levitator with $50 \times 50$ mm travel range in order to limit experimental costs. In the following chapters (Chapters 7–8), I present electromagnetic and mechanical detailed design for a levitator with $50 \times 50$ mm lateral travel range on the basis of conceptual studies done in this section.

\(^1\)We know that the A/D converters limit us to this noise level. The stage itself and the capacitance gaging in our system are capable of noise on the 10-nm level with proper A/D conversion.
6.1.2 Selection of Actuators

The key elements of a magnetic levitator are the actuators which create forces for position stabilization and motion control. We have considered conventional variable-reluctance, induction, and permanent-magnet linear motors for the levitator in Section 3.1. As elaborated there, a variable-reluctance motor has inevitable cogging and attraction forces. It is thus hard to get high-precision position control. A linear induction motor is generally hard to control. Moreover, thermal expansion due to heat dissipation in the platen may deteriorate accuracy. Therefore, we ruled out variable-reluctance motors and induction motors for our application.

A permanent-magnet motor can generate suspension force as well as drive force by multi-phase operation. The motor structure preferably has Halbach permanent-magnet arrays on the moving platen with the driving stator coils in the fixed machine base. In this way, there is no umbilical cable to the platen. Unbalanced attraction force between the magnet arrays and the stator are eliminated if we use no ferromagnetic material. Then, it is also possible to eliminate the stator teeth and use all the stator surface for windings to increase efficiency. In other words, a prospective motor type is a surface-wound permanent-magnet machine with an air-gap armature winding. Such a machine takes advantage of the fact that modern permanent-magnet materials have very low permeability and that, therefore, the produced magnet field is relatively insensitive to the size of the air gap of the machine [Kir95]. Every design concept given in this section follows the above actuator guideline.

6.1.3 Mass of the Platen

The mass of the platen is a critical design parameter because the power consumption of the stage is proportional to it. The faces of the platen are made of aluminum with a mass density of 2.70 g/cm³. The material mass inside the platen will be minimized with an aluminum honeycomb structure. We presume in this section that aluminum fills 25% of the total platen volume, although the specific inner structure
will be determined in Section 8.1. The mass density of copper windings is 8.94 g/cm$^3$. The empty spaces among the wires are ignored in the winding mass calculation for simplicity's sake. We use 7.6 g/cm$^3$ for the mass density of NdFeB magnets [Del94]. The electromagnet mass is assumed 150 g each in case that any electromagnet is used in design concepts given in this chapter.

In the following section for design concepts, rough dimensions of platens and stators are given. With the dimensions and the mass densities of specific materials, I estimate approximate mass of a platen. To get a lighter platen, we would want to make it arbitrarily thin. However, a thin platen has a low resonant frequency and is susceptible to bending so that maintaining mechanical stability becomes harder. We assume the thickness of the platen to be at least one sixth of the width as a rule of thumb regardless of design concepts.

### 6.1.4 Power Consumption

Power efficiency is one of the most important design considerations. A small power consumption is recommended to alleviate cooling demands and thermal errors as well as to reduce the operational cost. To compare power efficiencies of design concepts, their power consumptions are calculated according to a previous work [TWN93]. I present only the final result in this section. The power per one spatial period was derived as

$$P_X = \frac{2wl\Gamma}{\sigma} \frac{e^{2\eta z_0}}{(\mu_0 M_0 G)^2} (f_{zd}^2 + f_{zd}^2),$$  \hspace{1cm} (6.1)

where $G$ is a motor geometric constant defined in Subsection 5.2.5. $f_{zd}$ and $f_{zd}$ are the $x$-directed and $z$-directed desired forces per spatial period, respectively. The conductivity $\sigma$ for copper wire is $5.65 \times 10^7$ S/m. We use 1.2 T for the remanence $\mu_0 M_0$ of NdFeB magnet. The electric power dissipation is calculated under the assumption that the platen moves in $z$-direction at a quarter-$g$. We assume no vertical force $f_{zd}$ in those calculations.
6.2 Design Concepts

In this section, I describe design concepts for the planar magnetic levitator and discuss pros and cons of each concept. I classify them into multiple-moving-part types and one-moving-part types. We concentrate on the one-moving-part design concepts, as they have great merits discussed in Section 3.4. I use mnemonics like ‘Four Two-Sided Motors Design’ for names of design concepts. In all the figures in this chapter, all dimensions are in millimeters.

6.2.1 Multiple-Moving-Part Types

Crossed-Axis Design

The crossed-axis design is one of the most common designs for $x$-$y$ stages. Detailed discussion of the traditional crossed-axis stages can be found in Section 3.3, and is not repeated in this section. Figure 6-1 shows one possible design with magnetic suspension. Each of the two long orthogonal axes has a permanent-magnet linear motor for driving in the corresponding direction. There are twelve electromagnets
per long axis for suspension. By controlling the current in the electromagnets, we can generate all rotations and vertical translations. This concept may be the simplest to achieve a large lateral travel range, but it includes a large number of actuators. Since all the focusing and alignment motions can be generated in the magnetically levitated stage, however, it needs no other fine motion control stages.

**Gantry Design**

A gantry, or H-type, design is one of the most common ways to realize planar motions. In a specific design concept in Figure 6-2, which is similar to a concept in a patent assigned to U.S. Philips Corporation [WB87], there is one tubular motor for $x$-directional motions beneath the platen and two tubular motors for $y$-directional motions. Since the $y$-motors should drive and support $x$-directional magnet array and steel target for electromagnets as well as the platen itself, they should have larger load capacity. Tubular motors reduce the power waste due to end turns, which are inevitable with linear motors. We can control all rotations and vertical translations with electromagnets in the platen. The gap sensors are on the bottom of the platen. A hybrid suspension with magnetic suspension and air bearings may be a good choice for frictionless suspension, if air bearings are allowed for the system.

Canon’s ceramic air-guide stage has conventional linear permanent-magnet motors in an H-shape [SON+91]. Since it has independent aerostatic bearings for each $x$- and $y$-stage, load change or vibration of an axis is not transferred to the other axis. Wafer stepper stages by ASM Lithography\(^2\) are also in an H-shape [EB92]. However, the gantry stage does not carry the wafer stage; it rides on an air puck on a granite slab. Although the many moving parts complicate the dynamics of the system, a multiple-moving-part design usually occupies a small footprint compared with the one-moving part designs which are presented later in this section. This small foot-

\(^2\)Philips led the formation of ASM Lithography and is its majority share holder at press time of this thesis.
Figure 6-2: Gantry design
print of multiple-moving-part designs is a big economical advantage in clean-room or vacuum environments.

### 6.2.2 One-Moving-Part Types

As we discussed in the previous subsection, multiple-moving-part designs are complicated in structure and dynamics. We discuss several one-moving-part design concepts in this subsection. Dimensions for platens and stators in figures, if any, are intended for only approximate comparison. In addition, all the end turns of the windings are excluded for simplicity. Figure 6-3 shows schematics of the parts used in drawings for design concepts. Lateral position feedback of the stage can be obtained by laser interferometry. For this purpose, we need two plane mirrors, which are omitted in some drawings.

There are a few design alternatives for one-moving-part design concepts.
• Does the moving part carry either magnets or windings?

• Does the moving part carry either gap sensors or their targets?

• Use electromagnets for fine motion control or not?

• Use preload magnets with their targets to cancel gravity or not?

We will explore various combinations of the above design alternatives. At the end of this section, we discuss advantages and disadvantages of them and suggest a best combination. In the rest of this section, we discuss several design concepts which satisfy all the design constraints given above and can achieve the design goals. I try to present various combinations in motor and sensor arrangements.

Figure 6-4 shows the platen of a one-moving part (so called, flying-puck) design, which will be the basis of the selected design concept. The platen has preload permanent magnets to compensate its gravity load. Figure 6-5 shows the cross-sectional view of the levitation system. In case of the design in Figures 6-4–6-5, magnet arrays are on the platen and windings are stationary. The working principle of this type was given in Section 1.2 and is not repeated here.

**Four Two-sided Motors Design**

As suggested earlier, there are alternatives whether to place magnets (and windings) on the platen or the stators. In another arrangement (moving winding–stationary magnet, Figures 6-6–6-7), the locations of magnet arrays and windings are exchanged; windings on the platen and magnet arrays on the stators. Figure 6-6 is the top view of the platen. Two windings are for \( x \)-directional motions, and the other two are for \( y \)-directional motions. Yaw motions can be generated with differential operations of the four motors. The bottom side (not shown here) also has four windings and four gap sensor targets. Figure 6-7 is the top view of the upper stator. There are eight magnet arrays on the stator including four on the lower stator (not shown here). In
Figure 6-4: Flying puck design (moving winding)
Figure 6-5: Cross-sectional view of the flying puck design

Figure 6-6: Four two-sided motors design (platen)
Figure 6-7: Four two-sided motors design (top-surface of stators)
other words, the whole stage thus has eight unbalanced motors altogether. As before, a moving magnet–stationary winding design is possible.

The vertical force (including the weight of the platen) can be balanced by simultaneous operations of the motors on top and bottom surfaces of the platen. The four two-sided linear motors enable us to control the vertical motion and all rotational motions by selective activation of the motors. Figure 6-8 shows the way the rotational motions are generated. Electromagnets are not necessary in this design to control small angular motions. Even though we do not require electromagnets and preload magnets, we must include the gap sensor target areas which are roughly equal to the area of the wafer.

The biggest disadvantage of this design is that it has a large footprint. Judging from Figure 6-6, the planar dimension of the stator is about $1.1 \times 1.1$ m. The estimated mass of the platen is as much as 65 kg. It is heavy because of the large amount of the copper winding in the platen. The total power consumption is about 4.4 W at a quarter-$g$ acceleration.
Three One-Sided Motors Design

Actually, four motors are redundant to generate all the required forces and torques in six axes. Figures 6-9 shows a positioner with three one-sided linear permanent-magnet motors. The magnet arrays are located only on the top side of the platen. The upper stator has three corresponding windings. If we put a steel target on top of each of the three stator windings, the gravity load can be canceled by the attraction forces between the magnet array and the steel plate. We may use the bottom side of the platen as the targets for gap sensors.

We can see that the sizes of the stators and the platen are significantly reduced compared with the four two-sided motors design.\(^3\) Since the platen has relatively small magnet volume, the mass of the platen is estimated to be as small as 8.3 kg. The windings are fixed over the platen. We may use the first scheme in Figure 6-10 for the primary \(x\) movements. We can direct \(\sqrt{3}/2\) of the total force of a motor to the effective direction in that case. We can also generate \(y\)-directional motions. Power is inevitably wasted more in this direction because of the ineffective directional forces which are to be canceled. The power consumption in the windings is estimated at 12 W altogether at a quarter-\(g\) acceleration.

A cross-sectional view through the motor is given in Figure 6-11. Controlling the current of the three stator windings independently, we may produce all the vertical motions including pitch and roll motions as in the four two-sided motors design. Therefore, we need no electromagnets for focusing and attitude adjustment. Yaw motions are generated by differential operation of the three motors; in a manner similar to that shown in Figure 6-8.

\(^3\)This is only true for a stage covering a circular area with 200-mm diameter. The concept in Figure 6-9 cannot cover a 200 × 200 mm square area. This square-area coverage is frequently required (even though a wafer is round) in real applications for alignment purposes. So, an actual machine with three one-sided motors design concept will be bigger than shown and does not have the advantage in platen mass and footprint.
Figure 6-9: Three one-sided motors design
(a) $x$-directional motion generation  (b) $y$-directional motion generation

Figure 6-10: Translational motion generation with three one-sided motors

---

Figure 6-11: Cross-sectional view of one motor in the three one-sided motors design
One-Sided Motors on the Bottom Design

Figure 6-12 is another possible configuration with one-sided motors and preload permanent magnets which compensate the gravity load of the platen. So, the upper stator, not shown in the figures, should be a steel target for the preload permanent magnets and electromagnets. The bottom side of the platen has one magnet array (one in the center) for \( x \)-directional motions and two for \( y \)-directional motions. It also has gap sensor targets.

Figure 6-13 shows the stators. Since the stators can be laid on a mounting table with low thermal resistance paths, heat dissipated in the windings can be removed easily. The motors can generate yaw motions and another rotational motions along the \( y \)-axis by selective activation of motors. Yet, they cannot generate the other rotational motions around the \( x \)-axis in itself. So, electromagnets should be used to control the roll motions. Fine focusing motions are also possible by electromagnets.

The gap sensors lie between the windings. As there are stator windings on the top surface of the lower stator, we cannot put electromagnets in push-pull pairs. A disadvantage of this design comparing with the previous design concept is that the platen's bottom surface needs a bigger total area of gap sensor targets. The estimated mass of the platen is 25 kg with 19-W power dissipation at a quarter-\( g \) acceleration.

Orthogonal Two-Sided Motors Design

Figure 6-14 shows the platen (top and bottom views) of a two-sided design with two orthogonal directional motors on each side. All the gap sensors can be located on the top surface of the lower stators and the bottom surface of the platen is used for gap sensor targets. The total volume of the platen is much smaller than the previous concept with merging all the gap sensor targets. The mass of the platen is about 12 kg. Figure 6-15 shows the upper stators which have two orthogonal directional windings. The lower stator, not shown here, looks similar except that it has gap sensors instead of the 50-mm aperture.
Figure 6-12: One-sided motors on the bottom design (platen)
Figure 6-13: One-sided motors on the bottom design (top view of stators)
Figure 6-14: Orthogonal two-sided motors design (platen)
Figure 6-15: Orthogonal two-sided motors design (bottom view of top stators)
Figure 6-16: Puck with hole design (perspective view)

We can generate $x$- and $y$-directional motions by activating corresponding $x$- and $y$-motors. Yaw motions are possible by differential operations of the two $y$-motors, one in the top and one in the bottom. Pitch motions are generated by similar methods as in the other design concepts. However, electromagnets are necessary to control the roll motions as in the previous concept.

**Puck with Hole Design**

Figure 6-16 shows a design concept in which the cross-shaped stator windings penetrate the platen. There are two magnet arrays for the two translational motions on the inner sides of the platen (Figure 6-17). The $x$-windings are only on top of the cross, and the $y$-windings are only on bottom of the cross. We can generate $x$- and $y$-directional motions by activating corresponding $x$- and $y$-motors. There are eight push-pull electromagnets; four of them are on the top surface of the platen and other four, not shown in the figure, are on the bottom surface. The upper stator is a steel target for electromagnets for lifting and other vertical rotational motions. The lower stator is also a steel target for electromagnets except for the gap sensors. The gap
Figure 6-17: Puck with hole design
sensors are on the top surface of the lower stator. The mass of the platen is about 31 kg.

Yaw motions are controlled by activating the two \( y \)-windings differentially. This is the reason there must be two sets of windings for \( y \)-directional motions. The other two rotational motions can be controlled by the push-pull pairs of the electromagnets. Vertical motions are also controlled by the electromagnets. One apparent disadvantage of this design concept is the force actuation points vary with the position of the platen. So, the stage needs more complicated coordinate transformation and commutation laws. We also expect the resonant frequency is low and the system is susceptible to vibration due to the hole structure. The power consumption is 12 W at a quarter-\( g \) acceleration.

### 6.3 Selection for Prototyping

In the previous section, I generated several design concepts for magnetically levitated stages with various combinations of actuators and sensors. Several suggestions for implementation are raised in the course of discussing performance goals to meet and features to have. I summarize them in this section. To select a candidate design concept for prototyping, we consider four important selection criteria—precision, cost, control effort, and evolvability.

#### 6.3.1 Selection Criteria

Four major selection criteria and sub-criteria for magnetically levitated stages are chosen. They are as follows:

- **precision**—positioning accuracy, thermal expansion, rigidity
- **cost**—labor to produce, components to buy, power consumption
- **control effort**—system complexity, number of I/O’s
• evolvability—larger sizes, production

Now, we discuss the selection criteria and sub-criteria.

**Precision**

The precision is no doubt the most important criterion for every high-precision machine. The platen should maintain its position in a small error boundary specified in the performance goals during the lithography exposition time to prevent blurs. So, a high positioning accuracy is very important. Unmodeled local thermal expansion deteriorates the precision. Moving magnet–stationary winding type design concepts have advantages in terms of the thermal expansion, since the platen is not much affected by the power dissipation by windings. The structural rigidity is also of importance. In the gantry design, the rails can bend and vibrate. Their sagging and bending can be noticeable with long travel range.

**Cost**

Since a prototype linear motor development requires much labor in machining, stator winding fabrication, and control hardware, with respect to this thesis the labor becomes the most important sub-criterion. The amount of rare-earth magnet material used in a design affects the cost of components to buy.

**Control Effort**

A magnetic levitation system is in itself a multi-input and multi-output system. So, the stabilization and control of this multivariable system is very challenging. If the system dynamics characteristics are complicated, much more control design effort will need to be exerted. A large number of sensor inputs and actuator outputs require more processing time.
Evolvability

The evolvability to production is also an important factor in selecting the best concept. The semiconductor industry is going to use bigger wafers to enhance throughput. Even if we decided to construct a stage to cover 50 × 50-mm planar area, we should consider if the current designs can be scaled up to dealing with 300-mm or larger wafers.

6.3.2 Suggestions for Implementation

First, we discuss advantages and disadvantages of two alternative arrangements of magnets and windings, a moving magnet–stationary winding type or a moving winding–stationary magnet type. The definite advantage of the moving magnet–stationary winding type is negligible thermal expansion error in the platen because there is no power dissipation due to resistive loss in the windings. So, the position of the platen is to be kept more accurate. Another advantage is that umbilical cables connecting from the power amplifiers to the platen can be eliminated. Since the magnet arrays occupy smaller area, we can save on relatively expensive permanent-magnet materials. However, this design wastes power, because the winding occupies a large area on the stator, i.e., only small portion of the winding area provides the motion generation. This power waste could be reduced by turning off those stator areas not in use.

On the other hand, the moving winding–stationary magnet type has significant disadvantages:

- We need two connecting wires to the platen per each phase per each motor. If we use four three-phase motors, altogether 24 wires, let alone those for electromagnets and gap sensors, are needed. Sagging umbilical cables may deteriorate the stability and performance of the stage.

- The platen itself may significantly expand due to heat dissipation in the winding. There is no efficient thermal path for this heat, since the puck is isolated
mechanically and thermally. This thermal error makes the position accuracy worse.

- Stator magnets occupy the larger area in this structure. This leads to higher manufacturing cost due to the relatively high price of rare-earth magnet material. Also assembling the magnets is challenging as the number of magnet pieces grows larger.

Considering the performance goals and specifications, the magnetically levitated stage should have the following features.

- One-moving part design is much preferable due to its easy fabrication and simple dynamics.

- There should be four reference surfaces; two for the motors in each orthogonal direction, one for the electromagnets which control vertical translational and rotational motions, and one for gap sensors. One or two of the surfaces can be eliminated to reduce the platen size, if we do without the electromagnets, or if we merge two surfaces for motors and gap sensors.

- Place gap sensors below the platen to clear a lens field of view above the wafer.

- To prevent thermal expansion and eliminate umbilical cables, the platen should not carry windings.

- There are two parts in a permanent-magnet planar motor; magnets and windings. We prefer the part which has smaller area to be put on the platen so that the action points of the generated forces should be fixed with respect to the platen regardless of the platen position. More complicated coordinate transformations are required if the action point varies with respect to the platen center of mass.
• Use four linear permanent-magnet motors orthogonally placed. To generate rotational motion, the actuators' line of force should not pass through the platen's center of mass. This configuration also leads to the most compact platen.

• A compact platen leads to a light platen. Magnet and winding materials can thus be saved.

• The laser interferometry beams should be on the same plane as the wafer to eliminate Abbe errors.

• Avoid using electromagnets in the platen if possible. They generate heat, and need targets, supporting means for the targets, umbilical cables, and tight clearance, which leads to a complicated mechanical structure.

• The electromagnets, if any, should be in push-pull pairs and operated at near-zero current operation. This makes the suspension system more robust from outside disturbances, avoiding magnetic saturation in the steel core of the electromagnets.

### 6.4 Selected Design Concept

In Part I, we discussed the current technology trends for precision positioners. Designs with one moving part and avoiding mechanical contacts are the most notable ones. I selected a moving magnet-stationary winding type with four permanent-magnet linear motors for prototyping. Figure 6-18 shows the overall perspective view of the selected design concept for prototyping. The advantages of this design are as follows.

• simple mechanical structure and compact arrangement

• no umbilical cables to the platen

• symmetry in the $x$- and $y$-directions
Figure 6-18: Selected design for prototyping
• no overhung steel targets for electromagnets and preload magnets—nothing above the platen

• no heat dissipation on the platen to minimize thermal expansion error

• heat easily removed to the mounting table

In two following chapters, we will reflect on detailed design issues, such as mechanical structure of the platen, linear motor design, power amplifier design, and packaging and assembly.
Chapter 7

Electromagnetic Design

Having settled on the candidate design concept in Chapter 6, we turn now to the electromagnetic design details of the levitator. This chapter is organized as follows. The development of linear motors which are compatible with the motion requirements stated in the specifications is the main focus here. Earlier in this thesis the magnet array part of the motor was determined to be a Halbach array. Specifications of magnetic material and dimension of the magnet array are specified in this chapter. On the basis of the ripple force estimation in Chapter 5, design parameters for the stator winding follow. Design of linear power amplifiers for stators is the next topic. I elaborate the instrumentation structure for control and monitoring the levitator's states. A brief summary of the control software and the user interface program ends the chapter.

156
7.1 Magnet Arrays

7.1.1 Permanent-Magnet Material

Lodestones were probably the first permanent magnetic material known in human history. In the first millennium, B.C., Chinese and Greek people not only knew the existence of the material but used it as a compass, or a ‘car pointing South.’ The lodestone is an iron oxide (dominantly Fe$_3$O$_4$ domains with Fe$_2$O$_3$ regions). It remained the principal permanent-magnet material before carbon steel magnets were manufactured in the early eighteenth century. However, wide usage of permanent magnets began with the production of iron-compound material, such as alnico in 1930’s. Unfortunately, the coercivity of alnico was not high enough for certain applications so that demagnetization of magnets was always problematic. Soon after, production of the ferrite magnet followed. Improvements in coercivity and maximum energy product in alnicos, ferrites, and steel magnets made it possible to use them in many applications.

A new era of permanent magnetism began in 1970, when a compound of samarium and cobalt (SmCo$_5$) was found by General Electric to have a strong coercivity and energy product [Liv96b]. This rare-earth magnet material made numerous devices, such as wrist watches, phone receivers and loudspeakers more compact and facilitated the design of such machines with high efficiency. This is because their high coercivity can resist well their own self-demagnetization field. In 1983, a new rare-earth iron (neodymium-iron-boron, NdFeB) magnet was discovered independently by Sumitomo Special Metals Company, Ltd. and General Motors [SFT+84]. Its discovery is important because iron is much cheaper than cobalt and neodymium is much more abundant than samarium. Delco Remy, a Division of General Motors announced a preliminary magnet with remanence as high as 1.37 T and energy product $(BH)_{max} = 3.6 \times 10^5 \text{J/m}^3 \ (\approx 45 \text{ MGOe})$ [Del94]. To generate 1.37-T magnetic flux density with

---

1Interesting applications and historical accounts for magnets can be found in [Liv96a].
an air-core coil, we should provide $1.09 \times 10^6$-A/m surface current density, which is virtually impossible in small applications. The permeability of neodymium magnet is practically the same as $\mu_0$, so this value is used throughout the thesis.

Neodymium magnets have some shortcomings. First, they have a low operating temperature range. The Curie temperature of neodymium magnets is typically 350°C, but they begin to lose magnetization at as low as 150°C. The samarium magnet has better thermal stability. However, since other mechanical properties and the price of rare-earth iron magnet are superior to those of rare-earth samarium magnet, NdFeB has largely replaced SmCo₅ in normal temperature applications. Second, neodymium magnets are susceptible to corrosion. So, they need surface coating. Adding some element to increase corrosion resistance is under study in the permanent-magnet material research field.

The remanence $B_r$ is the residual flux density of the magnetic material when there is no outside applied field. The remanence is one of the most important quantities, which tells how strong a filed the magnet can generate. The coercivity $H_c$ is the magnitude of the applied field that is required to drive the material's magnetic flux density to zero. The high coercivity of the recent rare-earth permanent magnets makes them resistant to self-demagnetization. So, a compact but strong permanent magnet can be manufactured. This results in the flexible design as in earphones in portable stereos. The maximum energy product of permanent magnetic material combines both the remanence and the coercivity. A material with higher maximum energy product can be smaller and more efficient for the same application.²

²Many of the literature and specification sheets still use the CGS units. Here are some conversion factors which may be useful: $1 \text{ G} = 10^{-4} \text{ T}$, $1 \text{ Oe} = 250/\pi \text{ A/m}$, So, with respect to maximum energy product, $1 \text{ MGOe} = 7.958 \text{ kJ/m}^3$. 

158
7.1.2 Magnet Specifications

With the discussions so far, we can readily conclude that NdFeB material is the best choice at present for the magnetic levitator. Alnico have low coercivity, ferrites have low remanence, and samarium magnets are still expensive. Since NdFeB material corrodes easily, proper coating is necessary for any application. One of coating methods, nickel coating, proved to be inappropriate for adhesion of epoxy in our previous experiences. So, phenolic resin coating was chosen for surface coating. The dimensions of the magnet given in Figure 7-1 include the thickness of this coating layer. I first used ferrite magnets with the same dimension to simulate the fabrication process which will be described later in this section. They are much cheaper than the neodymium magnets and have much lower remanence (about 0.4 T). They are thus also easier to handle. Table 7.1 contains the specifications of the neodymium magnets and ferrite magnets used in the experiments. The manufacturer\(^3\) guarantees 1.20 T as minimum remanence for these neodymium magnets.

Even at temperatures much below its Curie temperature, under conditions of reverse magnetic field intensity, permanent-magnet material can loose its remanence significantly and the loss can be irreversible. The maximum operation temperature of our neodymium magnets is 120°C. Also, NdFeB material is a moderate electric conductor; this can lead to heating problems if the magnets are exposed to an AC magnetic field.

\(^3\)Magnetic Component Engineering, Inc., 213 N. Cedar Avenue, Inglewood, California 90301
Table 7.1: Specifications of NdFeB and ferrite magnets

<table>
<thead>
<tr>
<th></th>
<th>NdFeB</th>
<th>ferrite</th>
</tr>
</thead>
<tbody>
<tr>
<td>remanence (T)</td>
<td>1.29</td>
<td>0.385</td>
</tr>
<tr>
<td>coercivity (kA/m)</td>
<td>990</td>
<td>230</td>
</tr>
<tr>
<td>maximum energy product (kJ/m³)</td>
<td>320</td>
<td>28</td>
</tr>
<tr>
<td>density (g/cm³)</td>
<td>7.49</td>
<td>4.97</td>
</tr>
<tr>
<td>Curie temperature (°C)</td>
<td>310</td>
<td>450</td>
</tr>
<tr>
<td>resistivity (µΩm)</td>
<td>1.5</td>
<td>-</td>
</tr>
</tbody>
</table>

7.1.3 Design of Magnet Arrays

We would be better off if the magnet array consists of many pitches, which decreases edge and end effects. Conversely, we definitely should not have too many numbers of pitches. Large number of pitches also increase the overall size of the magnet arrays and platen, which leads to a heavier platen. We would then consume more power to drive the stage. I chose to make the pitch of the array small in order to have a larger number of pitches. This leads to smaller force capacity, and requires small air gap. So, the fabrication should be of higher tolerance. Considering this trade-off, I decided the pitch of the motors in the levitator to be 25.4 mm (1").

For mechanical simplicity, I do not introduce any lifting magnet for this prototype design. We do not have to have any such lifting magnet because the power consumption in the stator windings proves to be acceptable even if the lifting force is obtained solely by the motor currents. This will be discussed fully in the next section. By finite element analysis simulations, I found the force capacity can be enhanced by some 10% by backing up the magnet array with a steel plate. However, the steel backing will add mass to the platen, which is not favorable. Moreover, using such a steel plate between the magnet array and the platen may make it harder to control
the parallelism of the magnet array surfaces.

The manufacturing process of neodymium magnets include sintering and cutting. Magnetization of the magnets follows after any machining process to avoid messiness with magnetized NdFeB chips, which could cause troubles for machine tools made of iron material. Finally, coating the magnets with phenolic resin follows. Figure 7-3 shows the dimension of the magnet unit which is used in the magnet array fabrication. The thickness to length ratio of the magnet is advised less than 1:5, because the NdFeB magnet material is very brittle. Magnetic Component Engineering, Inc. offered a good deal in price with the magnet size, $1.20 \times 0.25 \times 0.25''$, because they had a tooling for the specific size already. So, the magnet array shown in Figure 7-3 has 45 blocks of such magnets and is in $3.60 \times 3.75 \times 0.25''$ dimension, which is not exactly square. The magnet tolerance is $\pm 0.005''$. According to the manufacturer, it is very difficult to guarantee a better tolerance than that due to uncertainties in manufacturing processes. However, the actual tolerance of magnets is as good as $\pm 0.002$ in, which is better than specified. I confirmed it by measuring the dimensions of many magnets for myself with non-magnetic calipers.

7.1.4 Fabrication of Halbach Magnet Arrays

I tried several epoxy glues and mold releases to work with the phenolic resin coated magnet and the aluminum platen. By many trials and errors, PC-7 epoxy by Protective Coating\textsuperscript{4} was chosen for the glue material, which has long working time (2 hours) and curing time (18 hours). The epoxy glue provides a strong holding force against the magnets' repulsion forces against each other. I chose E408 Dry Film Mold Release by Stoner\textsuperscript{5} for the mold release. These materials work well with the ferrite magnets, too.

Figure 7-2 shows a photograph of the toolings used in magnet array fabrication.

\textsuperscript{4}Protective Coating Co., Allentown, Pennsylvania 18102

\textsuperscript{5}Stoner, 1070 Robert Fulton Highway, Quarryville, Pennsylvania 17566
The whole magnet array with 45 magnets consists of three rows of 15 magnets each as shown in Figure 7-3. My approach is to make these rows first and then glue three rows into the whole magnet array. Figures 7-4–7-5 show the tooling for such rows. These magnet-fixture toolings are necessary to make four identical magnet arrays with a tight tolerance. The magnets with epoxy glue on their sides are slid into the opening of a die one at a time. A small compass determines the orientation of the magnet magnetization. Due to the high repulsion force, the magnets would not be glued together nicely, but tend to align in zigzag pattern in the opening of the tooling as shown in Figure 7-6. This zig-zag pattern also confirms if the orientations of the magnet magnetization are correct. The tooling has two dies so that we can make two rows simultaneously. Inside the tooling there is a groove per die. These grooves are prepared as the space for squeezed-out epoxy. After inserting fifteen magnets, they are clamped down using 9 cap screws. The holding force of the 9 screws is strong enough to set the magnets into a planar configuration.

With three such cured magnet rows, a whole Halbach magnet array with 45 magnets is fabricated. Figure 7-7 shows the lid of the other tooling. Figures 7-8–7-9 show the bottom and top pieces to hold the three epoxied rows together. The purpose of
Figure 7-3: Magnet chip and magnet array consisting of forty-five such magnet chips.

Each drawing shows top and side views.
Figure 7-4: Tooling to make two magnet rows—bottom part
Figure 7-5: Tooling to make two magnet rows—top part
Figure 7-6: Magnet alignment in the tooling before cap screws are tightened

Figure 7-7: Tooling to make a whole magnet array—lid
Figure 7-8: Tooling to make a whole magnet array—bottom part
Figure 7-9: Tooling to make a whole magnet array—top part
Figure 7-10: Lettering for magnet arrays. The drawing shows the bottom view of the platen. The letters N and S indicate that the nearest poles from the edges are North and South, respectively.

The three grooves is the same as before. The overall dimensions of the magnet arrays are given in Table 7.2. Letters I to IV are assigned to the magnet arrays referring to Figure 7-10. The positions of the magnet arrays are identified with respect to the mirror (in dotted lines) on top of the platen. The fabrication was very successful so that the overall size tolerance is 0.001". The glue line absorbed the errors in the magnet. Now the pitch of the magnet array is 1.008". The added 0.008” is due to the four glue lines per pitch (i.e., 0.002” per epoxy layer).

Table 7.2: Magnet array dimensions

<table>
<thead>
<tr>
<th>magnet array</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
</table>
7.2 Stators

As determined in Chapter 6, the stationary part of the levitator is made up of four linear motor stators for suspension and lateral motions. The stators are surface-wound in a slotless structure with no iron for the following reasons.

First, a downward attraction force between the magnet arrays and the stators due to a single-sided, or unbalanced, motor structure is unavoidable if the stators have ferromagnetic material. We do not want this strong attraction force as this would overload the suspension capability of the motors. So, I designed the stators without any ferromagnetic material. The optical table purchased from TMC\(^6\) was specified with a non-magnetic top, but turns out to slightly attract the magnets. Thus, we also need a sufficient gap from the top of the optical table to the magnet arrays. The height of the stator should be at least 2.5 cm, which corresponds to one-pitch length. Since the magnetic field decays exponentially with distance, at one-pitch length, the fundamental field due to the magnet is \(e^{-\lambda t} = e^{-2\pi} \approx 0\) times that at the magnet surface.

Second, we should not use a slotted structure, since force ripple due to iron slots must be avoided. In this way, we reduce the cogging motion to the minimum as calculated in Chapter 5. Minimization of the cogging motion by reducing force ripples is necessary, since the stage is to be used in high-precision stepper or scanner stages. Thus, I decided to implement surface-wound stators with no iron, although they are somewhat less efficient in terms of force capacity. Even with ironless stator structure, we can have enough force to suspend the platen and drive it at a one-g acceleration, as presented in Chapter 10.

\(^6\)Technical Manufacturing Corp., 15 Centennial Drive, Peabody, Massachusetts 01960
7.2.1 Discussions on Winding Structure

There are at least four types of windings for linear motor stators: (1) single-layer winding, (2) double-layer winding, (3) triangular winding introduced in Chapter 5, and (4) wrap-around winding, which is termed a Gramme-type winding. The single-layer winding has moderately short end turns (about 30% of the total winding length). And it is appropriate for double-sided DC permanent-magnet linear motors as seen in [Cor95]. The biggest problem with this winding is that the end-turns must be stacked in a three-dimensional space. (We call any non-effective connecting portions of winding to be end-turns hereafter.)

To solve this problem, rotary machines of moderate size frequently employ the double-layer winding [FKU90]. The double-layer winding has slightly shorter end-turns and smoothly wraps around the perimeter of the stator in most cases. The phase windings can also be arranged to produce smoother current distribution. Unfortunately, it turns out to be inappropriate for slotless surface-mounted linear motors in the levitator. Each of the coils is hard to construct, and there still remains the three-dimensional end-turn problem.

The triangular winding introduced in Chapter 5 is the best in terms of the end-turns’ length. However, it has less power-efficiency as derived previously, and does not have the flexibility of a multi-phase winding. In other words, we can only control at most two phases for the triangular winding pattern.

The Gramme-type winding wraps around the stator core. It has much longer end-turns, since more than half of the winding does not generate any force. This results in higher power consumption at the same force capacity. However, it is much easier to fabricate in modules; and assembly is simple because the coils are just stacked side-by-side. Moreover, the straightness can be fairly good with heat bondable wires like Polybondex wire by MWS7 [MWS95].

Thus, I decided to build stators with Gramme-type windings. In this section,

7 MWS Wire Industries, 31200 Cedar Valley Drive, Westlake Village, CA 91362
every calculation concerning electrical parameters is based on the presumption to use the Gramme-type windings. More detailed the winding fabrication process is described in the next section.

7.2.2 Electrical Design Parameters

Figure 7-11 shows the shape and the dimension of the stator winding, in the Gramme pattern. Thirty-three windings are stacked side by side to form one linear motor. Each of the three phases consists of eleven such windings in series. One such winding has fifty-four turns with heavy-build AWG#23 resistance-bondable wire (diameter = 0.0249\textquotedbl). To maintain good flatness of the top surface, crossovers of the wires are placed in either of the short sides of the winding. Lead wires come out of the bottom side as indicated in the figure. These lead configuration makes it easy to solder the
Determinaton of the winding parameters, e.g., thickness of wire, number of layers, number of turns, together with the peak phase current and terminal voltage needed a number of design iterations. Here is the final decision for the winding specifications. According to the wire specifications, AWG#23 copper wire has resistivity of 0.0666 Ω/m. The total length of one phase over the length of the motor is calculated to be 217 m. So, the resistance of one phase winding is 14.4 Ω, which is exactly same as the real measurement value.

Assuming that each of the four motors support about a quarter of the platen weight (54.7 N)\(^8\), the peak current density for the suspension is \(1.2 \times 10^6\) A/m\(^2\) by the following relation with a 250-μm nominal air gap.

\[
J_p = \frac{2e^{\eta_0}}{\mu_0 M_0 G N_m} f xd
\]  
(7.1)

Now, the turn density is calculated as \(\eta_0 = 2.5 \times 10^6\) turns/m\(^2\) by the Figure 7-11. So, the corresponding peak terminal current is about 500 mA, which is confirmed with real experiments. The peak terminal voltage from Ohm’s law is \(V_p = I_p R = 7.2\) V. The power amplifier circuit should have at least three times higher current rating than this nominal rating to accommodate any sudden peak current for control.\(^9\) So, the power supply and amplifier current and voltage swing will be at least ±1.5 A and ±21.6 V. Details are described in the power amplifier and power supply section. Besides suspension force, the motor should be able to generate drive force. In the case of a half-\(g\) lateral acceleration, two responsible motors generate a quarter-\(g\) lateral acceleration. Then the resultant peak current for both a quarter-\(g\) lateral acceleration and a quarter-\(g\) vertical acceleration should be \(\sqrt{2} \times 500\) mA. So, 1.5 A-21.6 V swings are sufficient even in this case. The nominal steady-state power dissipation for the

---

\(^8\)In reality, each motor assumes slightly different weight, because the mass of the platen is unbalanced due to the mirror. This will be discussed in Chapter 10.

\(^9\)One might argue that the bigger the swings are, the better. However, bigger swings need bigger power supplies and power OP Amps, which leads more expensive power circuits.
suspension per motor is 5.4 W as calculated by $I^2R$, which is not too high for this prototype motor. So, we did not consider any active cooling to take heat from the stator cores.

7.2.3 Winding Fabrication

Figure 7-11 shows the overall dimension of the winding. It was shown that the power optimal thickness of the winding is 1/5 of pitch [TWN93]. Since we decided to make the pitch 1", the thickness is to be 0.2". The round corners are necessary to prevent cracking the insulation layer due to sharp bending. I designed the tooling shown in Figure 7-12 for winding fabrication used by WireWinders\textsuperscript{10}. Figure 7-13 shows dimensions of the parts of the tooling for winding fabrication. The AWG# 23 wire by MWS has two layers on the surface of the copper: a heat-sensitive bondable layer (epoxy) outside an insulation layer (polyester-amide-imide) [MWS95]. After 54 turns

\textsuperscript{10}WireWinders, Inc., 151 Mount Vernon Road, Milford, NH 02055
Figure 7-13: Dimensions of the winding tooling
are wound around the inner fixture, two side plates are attached to the two long side of the winding to maintain these surfaces flat. Then, the winding is baked in an oven at 130°C to allow the bonding layers of each wire to melt and the winding itself to set. To help dissolve the bonding material, a small amount of methyl-ethyl-ketone is used before placing the wire in the oven. After baking the wire the leads are stripped and tinned to prepare for soldering. I used methyl-ethyl-ketone to clean up any remaining epoxy on the winding surface.

### 7.3 Power Amplifier

Since there are four motors with three phases each, we need twelve power amplifiers to control the individual stator phase currents. The commutation command by the controller is transmitted through D/A converters (described in next section), which gives signals as voltages. I take an approach to control the phase currents directly with these signals instead of controlling the terminal voltages of the windings. Thus, the power amplifiers should be a transconductance type. They specify the stator current distributions and each motor thereby generates required suspension force and drive force by (4.81).

If the dynamics of the power amplifier circuit including the winding is fast enough compared with the mechanical dynamics of the system, the power amplifier dynamics can simply be ignored in the outer control loop. Then, the power amplifier can be seen as a controlled current source from the view point of the phase windings’ terminal.

#### 7.3.1 Power OP Amp

The voltage and current swings were determined in the previous section to be ±1.5 A and ±21.6 V. Among many power OP Amps, I choose the PA12A by Apex\(^{11}\) for the power amplifier circuit. The PA12A has proven to be a good linear power OP Amp in

\(^{11}\)Apex Microtechnology Corp., 5980 N. Shannon Road, Tucson, Arizona 85741
our research group's previous experience. It matches or exceeds the required current and voltage swings and has a decent voltage slew rate. The important maximum ratings and specifications of PA12A are as follows [Ape95].

- supply voltage, \( +V_S \) to \( -V_S = 100 \) V
- output current = 15 A
- power dissipation, internal = 125 W
- gain bandwidth product at 1 MHz = 4 MHz
- power bandwidth = 20 kHz
- voltage swing = \( \pm (V_S - 6) \) V
- slew rate = 4 V/\( \mu \)s

Heat sinks for the power OP Amps are provided by Thermalloy\(^{12}\). To limit the current at 2.0 A for safe operation purpose, 0.33 Ω resistors are used on the appropriate terminals on the OP Amp [Ape95].

### 7.3.2 Power Supplies

A switching power supply is in general less expensive, more efficient, and smaller than a linear power supply with the same power capacity. However, a linear power supply is more desirable for high-precision applications because it is much quieter than the other. The ripple errors of linear and switching power supplies are on the order of 5 mV versus 150 mV. Considering power capacity, we used four LNS-P28's (ratings: \( \pm 7.6 \) A-\( \pm 28 \) V at 40 °C) for the required current swings. So, two supplies serve two motors (six power amplifiers) as positive and negative biases. The current and voltage ratings of the power supply are determined by the power amplifier ratings,

\(^{12}\)Thermalloy Inc., 2021 W. Valley View Lane, Dallas, Texas 75234

177
±1.5 A±21.6 V. In the balanced three-phase operation, the maximum phase currents are like 1.5 A, 0.75 A, and 0.75 A, for example. So, total of 6.0 A is sufficient to drive two motors. Since the voltage swing of PA12A is ±(VS - 6) V, a ±28-V power supply rating is required for the ±21.6-V PA12A swing. We have one LND-P152 (ratings: 5.0 A±15 V at 40 °C) for bias for TL072ACP's, and one LND-X152 (ratings: 2.1 A-±15 V at 40 °C) for the capacitance probe electronics. All linear power supplies are manufactured by Lambda\(^{13}\).

### 7.3.3 Design Characteristics

Figure 7-14 is the power amplifier circuit. The circuit consists of three parts—differential amplifier, feedback amplifier, and power booster. The differential amplifier rejects common mode signals from the D/A converter. A feedback network is provided to stabilize the current control loop. The TL072ACP OP Amps by Texas Instruments\(^{14}\) are used for this differential and feedback amplifiers. A PA12A in the gain-of-two configuration serves as the power booster.

Almost all the phase current through the winding flows through the current sensing resistor (1 Ω-10 W), since the 1.5 kΩ loop gain resistor is much bigger than the current sensing resistor. The voltage across the current sensing resistor is fed back to the feedback amplifier. The low-pass network with a 0.01 μF capacitor and 27 kΩ resistor compensates the control loop. The element parameters are chosen to set the amplifier closed-loop bandwidth at 1.6 kHz. Considering that the mechanical dynamics and overall control bandwidth will not exceed 200 Hz, we can ignore the much faster power amplifier.

The printed circuit board artworks for the power amplifier circuit and the back plane were provided by Sandia National Laboratories. Each power amplifier circuit board accommodates two channels. Figure 7-15 is the power amplifier circuit board

\(^{13}\)Lambda Electronics, Inc., 3801 West Military Highway, McAllen, Texas 78503

\(^{14}\)Texas Instruments, Inc., P.O. Box 655303, Dallas, Texas 75265
Figure 7-14: Power amplifier circuit
with the power OP Amps and the heat sinks removed. The back plane consists of ±28 V and ±15 V rails and 96 pin Eurocard connectors which hold power amplifier circuit boards. The printed circuit boards were manufactured by Fineline Circuits & Technology.\textsuperscript{15}

### 7.4 Instrumentation Structure

Figure 7-16 shows the instrumentation structure for the planar magnetic levitation system. Control algorithms are implemented digitally in a Texas Instrument’s 320C40 digital signal processor (DSP) based Pentek\textsuperscript{16} 4284 board. A Radisys\textsuperscript{17} 80486-100 MHz VME PC takes care of user interface, such as monitoring levitator state variables and command interpretation. The two processing units, the PC and the DSP, communicate with each other using dual port shared RAM residing on the Pentek 4284 board over the VMEbus. On the VMEbus exist three channels of HP 10897A

\footnote{\textsuperscript{15}Fineline Circuits & Technology, Inc., 594 Apollo, Brea, California 92621}

\footnote{\textsuperscript{16}Pentek, Inc., 55 Walnut Street, Norwood, New Jersey 07648}

\footnote{\textsuperscript{17}Radisys Corp., 15025 S.W. Koll Parkway, Beaverton, Oregon 97006}
laser axis boards by Hewlett-Packard\textsuperscript{18} and a DATEL\textsuperscript{19} DVME-622 D/A converter board. There is a MIXbus local to the digital signal processor connected to a Pentek 4245 A/D converter board. More detailed descriptions of the instrumentation structure are given in the rest of this section.

7.4.1 VMEbus

The VMEbus (Versa Module Eurocard bus) is an asynchronous bus, which was defined in 1981 [Pet93]. It is a successor of the Versabus defined by Motorola for its 68000 microprocessor family in 1979. It is asynchronous since no clocks are used to synchronize data transfer and memory accessing. As data are passed by handshaking signals, the data transfer rate is determined by the slowest module on the bus. The VMEbus architecture went through several revisions. According to the VMEbus specifications’ revision C.3 (IEEE-1014-87), the maximum data transfer bandwidth is 40 Mbytes/s. We can choose either 16-, 24-, or 32-bit addressing modes dynamically. The data path is either 8-bit-, 16-bit- or 32-bit-wide, which can also be chosen dynamically. In the most recent revision, rev. D, 80 megabytes/s data transfer, 64-bit addressing and 64-bit data path width are allowed. The high bandwidth and wide addressing range and data path make the VMEbus attractive in complicated and demanding industrial applications.

A VMEbus accommodates up to 21 boards in a 6U Eurocard size. All of them can be bus masters, in other words, can generate bus request signals. The left-most slot (slot 1) should be occupied by the slot 1 system controller (in our case, the Radisys VME PC). It prioritizes bus request signals from the bus master modules and prevents bus contention. The VMEbus has four bus request levels. Each bus request signal line is daisy-chained so that the nearer module to the slot 1 system controller on each bus request chain has a higher priority to request the bus than farther boards on

\textsuperscript{18}Hewlett-Packard Co., 29 Burlington Mall Road, Burlington, Massachusetts 01803

\textsuperscript{19}DATEL, Inc., 11 Cabot Blvd., Mansfield, Massachusetts 02048
Figure 7-16: Instrumentation structure
Table 7.3: Modules in the VMEbus rack

<table>
<thead>
<tr>
<th>slot</th>
<th>module</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–4</td>
<td>Radisys 80486 VME PC with a floppy disc driver and a hard disc driver</td>
</tr>
<tr>
<td>5</td>
<td>Pentek 4284 320C40 digital signal processor board</td>
</tr>
<tr>
<td>6</td>
<td>Pentek 4245 32-channel, 16-bit A/D converter board</td>
</tr>
<tr>
<td>7</td>
<td>DATEL DVME-622 16-channel, 12-bit D/A converter board</td>
</tr>
<tr>
<td>8</td>
<td>anti-aliasing filters</td>
</tr>
<tr>
<td>16</td>
<td>HP 10897A laser axis board (for y)</td>
</tr>
<tr>
<td>17</td>
<td>HP 10897A laser axis board (for x1)</td>
</tr>
<tr>
<td>18</td>
<td>HP 10897A laser axis board (for x2)</td>
</tr>
</tbody>
</table>

the same daisy-chained bus request level. Among four bus request levels, the slot 1 controller (the bus arbiter) can give the bus request priority to level 3, then 2, then 1, and finally 0 (priority mode) or to levels 3, 2, 1, and 0 in sequence without priority (round robin mode). The VMEbus can handle a maximum of 7 interrupt levels.

A VMEbus programmer should be careful with byte ordering in case of using modules which follow the Intel byte ordering convention (little endian), such as IBM PC compatible computers together with modules which follow the Motorola byte ordering convention (big endian) [Hea89]. The Radisys VME PC as the slot 1 system controller has hardware byte swapping capability to take care of byte swapping between modules.

Table 7.3 shows the modules in the VMEbus rack with slot numbers. The high-speed HP laser axis boards which might generate high-frequency noise are intentionally separated from the other analog boards by Pentek and DATEL. This is recommended by DATEL.
Table 7.4: Selected memory map in Pentek 4284

<table>
<thead>
<tr>
<th>memory module</th>
<th>address</th>
</tr>
</thead>
<tbody>
<tr>
<td>internal local SRAM (2 kbyte)</td>
<td>0x002ff800–0x002ffff</td>
</tr>
<tr>
<td>external EEPROM (512 kbyte, 1 wait)</td>
<td>0x00b00000–0x00b7ffff</td>
</tr>
<tr>
<td>external local SRAM (1 Mbyte, 0 wait)</td>
<td>0x40000000–0x4003ffff</td>
</tr>
<tr>
<td>global dual DRAM (4 Mbyte, 3 wait)</td>
<td>0x80000000–0x800fffff</td>
</tr>
<tr>
<td>global SRAM (2 Mbyte, 0 wait)</td>
<td>0xC0000000–0xC07fffff</td>
</tr>
</tbody>
</table>

7.4.2 Digital Signal Processor

I chose a Pentek 4284 digital signal processor board with one TMS320C40 processor, since its predecessor TMS320C30 processor board was verified to work properly in our group. The Pentek 4284 has VMEbus master interface capability and could assume the slot 1 system controller functions, which are not used in the current VMEbus configuration. It is also a MIX baseboard and acts as a MIXbus master. The MIX interface was originally developed by Intel for its Multibus II. The 4284 board has memory resources given in Table 7.4. The instructions, data and stacks are stored in the fast global SRAM. The dual-port DRAM at 0x80000000–0x800fffff is used as shared memory with the Radisys VME PC for data exchange and semaphores.

The Texas Instruments' 50 MHz TMS320C40 floating-point digital signal processor has a 50-MFLOP (million floating-point operations) capability [Tex93]. It has a 40-ns instruction cycle times and does most floating-point operations in one cycle. It has 32-bit-wide data/address words, so its total memory space is 4 gigawords. Its minimum separable address is 32 bit away, so the 320C40's byte size is also 32 bits. So, careful translation of addresses on the VMEbus (which uses a byte of 8 bits) is required from the view-point of 320C40. The following formulae is used for the Pentek
4284 board [Pen94].

\[
\text{TMS320C40 address} = \frac{\text{VMEbus address}}{4} + 0xb0000000
\]  

(7.2)

The fact that a 32-bit portion of memory is the minimum unit of data storage makes all the data types in TMS320C40 C language are of 32-bit size. So, there is no difference among short, normal, and long data types. The TMS320C30/C40 family uses its unique floating-point format. Data conversion is needed when it communicates with modules following the IEEE floating-point format (IEEE-754), like IBM PC compatibles.

7.4.3 Sensors

In this subsection, a brief description for the measurement systems, capacitance probes, and laser interferometers is given. [Slo92] is a good reference for more detailed operational principles of these sensors.

Capacitance Probes

ADE\textsuperscript{20} provided the capacitance gaging systems. The ADE 2810 capacitance probe has 10-mm diameter active sensing area. Its outer diameter and height are 20 mm and 18 mm. A bigger gage generally shows lower noise due to its smoothing effect with large capacitance. However, we cannot go with a large capacitance gage because of packaging and the position range of the capacitance probe. The capacitance gaging system ADE 3800 converts the probe-to-target distance to electronic signal [ADE93]. Three gaging systems are daisy-chained to obtain three-channel measurements—vertical displacement and rotational angles around \(x\)- and \(y\)-axes. A drive clock synchronizes the measurements of three channels.

The output ranges of the ADE 3800 systems are modified to be \(\pm 7.5\) V to match the input voltage swing of Pentek 4245 A/D converter board. This maximizes the

\textsuperscript{20} ADE Corp., 77 Rowe Street, Newton, Massachusetts 12166
position sensitivity. The zero point of the gaging system is set at the nominal 450-μm air gap between the ADE 2810 capacitance probes and the targets on the bottom side of the platen. The scale factor (displacement-to-voltage ratio) is determined by the sensing range which is 200 μm–700 μm. The air gaps between the stator windings and the magnet arrays are designed 250 μm smaller than those over an air gap of between capacitance probes and targets. So the maximum sensing range of the platen is −50 μm–450 μm. Since negative travel range is physically not allowed due to contact between the magnet arrays and the stators, we thereby avoid sensor saturation. This prevents the problem of loosing position information at the initialization of the platen position.

Since we use 16-bit A/D converters, the scale factor is 34 μm/V and the resolution of this gaging system is 7.8 nm per least count. However, this does not imply that we can stabilize the vertical position at the nanometer level due to the noise in the Pentek A/D card. This noise corrupts the three least-significant bits of the 16-bit A/D converter. So, a typical noise amplitude in the gap information is 50 nm peak-to-peak or more. The bandwidth of the ADE 3800 systems is set at 1 kHz. ADE recommends to ground the target, i.e., the platen, to reduce noise; I did this with a wire attached to the platen.

**Laser Interferometers**

The HP 5517B is a HeNe laser source at a wavelength of 632.991 nm. The HP 10897A laser axis board gives position information with 0.6-nm resolution for a plane mirror system [Hew93b]. It gives 35-bit position data at a 10 MHz update rate. It also provides 24-bit velocity data up to 254 mm/s limited by our laser head slew rate specifications. This is a big asset since differentiating position data to get velocity is usually troublesome due to the noise component residing in the position data and due to timing jitter. There are three HP 10897A laser axis boards to measure translations and linear velocities along x- and y-axes and rotation and angular velocity around
z-axis.

Figure 7-17 shows the laser interferometer metrology arrangement. I use three of 10780A receivers, six of 10703A retroreflectors, three of 10722A plane mirror converters, two of 10707A beam benders, and two of 10701A 50% beam splitters, all manufactured by Hewlett-Packard. A square mirror (serial number 1394) and all the laser interferometric parts, except for the HP 10897A laser axis boards are taken from a Model 6300A wafer stepper donated by GCA\textsuperscript{21}.

The laser interferometric position sensing system gives only relative position data with respect to an initial position. It needs other means, such as micrometers for absolute position setting. Every laser interferometry system relies on the laser's wavelength as its scale of length. The wavelength depends on the air's index of refraction. We might need air sensors, material temperature sensors, humidity sensors, pressure sensors, and gas sensors to compensate errors due to the changing laser wavelength. We do not aim for achieving this high-level of accuracy for this prototype because of limited funding and resources. Also this is a problem in all such systems, and is not unique to our levitated stage. These issues are thus not addressed further in this thesis.

\subsection*{7.4.4 A/D and D/A Converter Boards}

\textbf{A/D Converter Board}

The Pentek 4245 is a 32-channel 16-bit resolution A/D converter Intel Multibus II MIX module [Pen90]. The maximum aggregate converting rate is 400 000 conversions per second. Its usable bandwidth is 40 kHz with a 15-V peak-to-peak input signal. According to the specifications, the input impedance is greater than 10 M\Omega and the common mode rejection is better than 80 dB. The integral linearity is \pm0.003\%. It

\textsuperscript{21}GCA Corp., Burlington Division, 209 Burlington Road, Bedford, Massachusetts 01730. GCA is out of business at press time of this thesis and has been superseded by Integrated Solutions, Inc., Tewksbury, Massachusetts.

187
Figure 7-17: Laser interferometry metrology with the platen
communicates with the Pentek 4284 TMS320C40 host via the M IXbus so that we can reduce the data traffic over the VMEbus.

This board can generate interrupts to the Pentek 4284 TMS320C40 host with various modes. We use the sample counter mode, which makes an Intel 82C54 clock timer counter chip generate interrupt signals at a programmed frequency no higher than 400 kHz. I programmed the sampling rate at 5 kHz (initially at 2.5 kHz), which is justified in Part IV of this thesis. If an interrupt occurs, the 320C40 host jumps to an interrupt service routine. The controller is implemented in this interrupt service routine (the C function, c_int01() in Appendix D). Once the interrupt has been serviced, the 320C40 host performs other jobs, such as bookkeeping the platen status, sending data requested by the Radisys VME PC, idling to wait for a command from the Radisys VME PC, interpreting the command, and preparing for the command execution.

The A/D converter board samples three channels of capacitance probe gap data transmitted by three ADE 3800 capacitance gaging systems. The Pentek 4245 board has an unfortunate bug in the sample counter mode. The first channel must be read twice to clear the input FIFO and to avoid occasional signal glitches.

The Pentek 4245 includes no analog anti-aliasing filter. So I built a filter bank between the three-channel capacitance probe output terminals and Pentek 4245 input terminals. The anti-aliasing filters are first-order RC low pass filters whose cut-off frequencies are selectable between 800 Hz, 1.0 kHz, 1.5 kHz, and 2.4 kHz. The 800-Hz anti-aliasing filter is used in experiments in Part IV, and the dynamics of this filter is ignored and is not modeled.

D/A Converter Board

The DATEL DVME-622 is a 16-channel, 12-bit resolution D/A converter board for VMEbus [DAT92]. The full scale output is programmed to ±10 V. According to the specifications, the linearity error is ±0.025% of the full scale range and the slew rate
Table 7.5: DATEL D/A converter board channel assignment

<table>
<thead>
<tr>
<th>channel</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>phase</td>
<td>IC</td>
<td>IB</td>
<td>IA</td>
<td>IIA</td>
<td>IIB</td>
<td>IIC</td>
<td>IIIA</td>
<td>IIIB</td>
<td>IIIC</td>
<td>IVC</td>
<td>IVB</td>
<td>IVA</td>
</tr>
</tbody>
</table>

is 10 V/μs. It also has three digital out lines, one of which is used to determine the execution time of the control routine. This is accomplished by turning the digital output on at the beginning of the control routine and off at the end. This output can be monitored with an oscilloscope to determine the execution time.

If the output registers of DATEL are updated with data and triggered, it outputs the data to all the channels simultaneously. Then, the D/A output signals are fed to the power amplifiers, which convert the voltage signals from the D/A converters to phase currents. Table 7.5 shows the channel assignment of the DATEL D/A converter board to the phase currents of the four linear motors. The sequences for motors I and IV are reversed to be consistent with the definition of the global coordinates.

I do not believe a 16-bit resolution D/A converting is really necessary. The manufacturing and alignment errors residing in the magnet array and stator windings can dominate the 16-bit resolution anyway.

7.4.5 Miscellaneies

A 76"-tall standard 19"-rack contains all chassis for the instrumentation and power supplies. The 21-slot VMEbus back plane and 6U Eurocard cage was manufactured by Elma\textsuperscript{22}. The Sandia National Laboratories provided the back plane for the power amplifiers manufactured by Electro Plate Circuitry\textsuperscript{23}. The cage for the power amplifier

\textsuperscript{22}Elma Electronics, Inc., 44350 Grimmer Boulevard, Fremont, California 94538

\textsuperscript{23}Electro Plate Circuitry, Inc., 1430 Centry, Carrollton, Texas 75006
cards came from Vector\textsuperscript{24}. There are altogether six fans to cool the power amplifiers and the power supplies by Schroff\textsuperscript{25}. All twisted shielded cables are manufactured by Weico\textsuperscript{26}.

7.5 Software

The software for the levitator consists of the real-time digital control code and other service routines realized in the Pentek 4284 320C40 processor board and user interface program implemented in the Radisys VME PC.

I did not see any necessity to develop the service routines and user interface routines from scratch for myself. The service routines in the digital signal processor was mainly developed by Sandia National Laboratories for 320C30 processors. However, I wrote all the machine-specific codes including communication software over the VME bus for the Pentek 4284 board. Development of the real-time digital control routine in Pentek 4284 320C40 board is wholly my contribution. I programmed the Intel 82C54 clock timer counter chip included in the Pentek 4245 board to generate an interrupt signal at 5 kHz.

The user interface routines in the Radisys VME PC were originally written by Tony Smith and others at Sandia National Laboratories. Later, Mark Williams contributed to development and expansion of the routines. I modified and ported the user interface program and wrote all the machine-dependable codes including communication software over the VME bus for the Radisys VME PC.

In the rest of this section, I describe functions of the control routine and user interface software.

\textsuperscript{24}Vector Electronics Co., 12460 Gladstone Avenue, Sylmar, California 91342

\textsuperscript{25}Schroff, Inc., 170 Commerce Drive, Warwick, Rhode Island 02886

\textsuperscript{26}Weico Wire & and Cable, Inc., 161 Rodeo Drive, Edgewood, New York 11717
7.5.1 Control Routine

I developed the real-time digital control routine (C.int01() in Appendix D). If we sample the position sensors at 5 kHz sampling rate, a 200-µs duration is available for a loop. At the beginning of each loop, the real-time control routine is called by a hardware interrupt. So, control updates happen exactly at the sampling rate, 5 kHz. The Pentek 4284 TMS320C40 host serves the crucial control routine in the first hundred microseconds (which depends on the length of the control code), and does other service routines in the remaining time after that. The following are important tasks the real-time control routine does.²⁷

1. Reset the FIFO in the Pentek 4245 A/D converter board.

2. Read the laser electronics registers for lateral positions and velocities.

3. Read the position registers in Pentek 4245 A/D converter board to get capacitance probe vertical position data²⁸.

4. coordinate transformation

5. command generation by controller

6. modal-decomposed force transformation

7. DQ-two-phase transformations

8. inverse Blondel-Park transformation

9. Output the motor current commands through the DATEL DVME-622 D/A converter board.

10. Save old command and error variables.

²⁷Detailed descriptions of the transformations are given in Chapter 9.
²⁸Postponing this routine after the previous one saves about 20-µs running time. The 4245 FIFO can gather data over the MIXbus while reading the laser boards is being done over the VMEbus.
11. If commanded, save state variables.

12. Clear the interrupt.

In the remaining 100 $\mu$s, the 320C40 processor performs the following non-real-time service routines.

1. Write state variable, force, current, and capacitance probe voltage data in the dual port memory data section.

2. Poll the command register to check if there is a command by the Radisys VME PC.

3. If not, keep checking.

4. If so, interpret the command and set appropriate semaphores, and fill the command register. Execute the command.

I revised the following software functions developed by Sandia National Laboratories or Mark Williams.

- command interpreter
- HP laser axis board initialization
- DATEL converter board driver
- transportation of 32-bit or 16-bit data between the Pentek 4284 TMS320C40 board and the Radisys 80486 VME PC over the VMEbus
- storage of state variables of the platen

The control routine and the user interface routine described in the next subsection set semaphores and interchange data through shared memory built in the global RAM of Pentek 4284 board.
7.5.2 User Interface Routine

The user interface routine is implemented in the Radisys 80486 VME PC in C language. The user interface program does the following tasks.

1. Update the screen information four times per second.

2. Poll any key pressed.

3. If not, keep checking.

4. If so, interpret the command and set appropriate semaphores, and fill the command register. Execute the command.

I designed the monitor program. The CRT screen shows the following.

• nine state variables (three lateral positions, three lateral velocities, and three vertical positions) of the platen

• commanded values for positions

• position and velocity errors, which represent the state variables subtracted by the commands

• eight force components (vertical and lateral force components per motor)

• three capacitance probe electronics output voltages

• twelve phase currents (three phase currents of four motors)

• status of the real-time control routine—alive or dead

The following are available commands. I modified most of the routines as developed by Sandia National Laboratories.

• F (Fly): Lift the platen by 250 μm.

• X (Execute): Execute the position commands.
• G (Gather): Gather position and velocity data of the platen.

• L (Reset Lasers): Reset the position registers in the HP10897A laser axis boards.

• R (Run): Run preset demonstration routines.

• Q (Quit): Quit the user interface routine.

• Esc (Escape): Emergency escape

• F1–F6: Accept position commands for $x$, $y$, $z$, $\theta_x$, $\theta_y$, and $\theta_z$, respectively.

• 4 (LMs): Turn on and off four linear motors.

• 5 (Get): Set the number of data to gather.
Chapter 8

Mechanical Design

In this chapter, I provide a detailed description of the mechanical design and fabrication issues. As we discussed in previous chapters, the one-moving-part design has many merits. One of them is that a high resonant frequency yields a fast response. Such high resonant frequency is achieved with a sandwich panel structure, which has a high bending stiffness-to-mass ratio. Along with a high resonant frequency, it is desirable that the structure is well-damped. This is achieved through passive shear damping. In the first section of this chapter, we go through design processes of the platen aluminum-honeycomb sandwich structure. In the next section, the levitator stator design is presented. I decided to use Gramme-type windings, since they are easy to fabricate and assemble. Metrology devices, mounting, packaging and assembly will be discussed as well. Dimensions of mechanical parts in this chapter are specified in English units since most machine tools available to me are scaled in inches.

8.1 Platen

The platen is the only moving part in our levitator system. It is supposed to carry a wafer in the real application, so devices to hold a wafer (such as vacuum or electrostatic chucks) and sensors for auxiliary tasks such as alignment are required. Such
supporting mechanism and sensors are neglected in this prototype levitator. Thus, the present platen only includes Halbach magnet arrays as parts of linear motors, a shear damper, and metrology devices, such as a square mirror and capacitance probe targets.

As determined in Chapter 6, we want to produce suspension forces as well as drive forces with four permanent-magnet linear motors below the platen. The bottom side of the platen needs space for four Halbach magnet arrays and capacitance probe targets as well. It is mechanically simple to grind the continuous bottom surface of the platen and use it for both magnets and targets. The top surface of the sandwich panel has mirror mounting pads which carry a standard square mirror. The raised pads and flat bottom surface are machined after the sandwich fabrication. The following subsection gives details of the fabrication process of the sandwich panel.

8.1.1 Optimal Design of Honeycomb Sandwich Panel

A sandwich structure is defined as a three-layer construction. It consists of two thin sheets of high-strength material between which a thicker layer of low average strength and density is sandwiched. The two thin sheets are called faces, and the intermediate layer is the core of the sandwich. Depending on applications, the material of the faces may be aluminum alloy, reinforced plastic, titanium, heat-resistant steel, and the like. The material and the geometric shape for the core vary widely. A very popular type of core is the 'honeycomb' core, which consists of thin foils in the form of hexagonal cells perpendicular to the faces. Other types of core are corrugated sheet with the corrugations running parallel to the faces, expanded materials, such as cellulose acetate, synthetic rubber, etc., and balsa wood. The material of honeycomb cores, corrugated cores, and the like can be similar to the material of the faces [Pla66].

Advantages of the honeycomb sandwich construction are as follows [Hex74].

- high strength-to-mass ratio
• high stiffness-to-mass ratio

• uniform crushing strength under compression

• high resistance to wrinkling

In the context of the current application high strength-to-mass ratio and high stiffness-to-mass ratio properties are most important. That is, a honeycomb sandwich can be a low deflection structure at minimum mass. From the structural standpoint, the function of the sandwich core is twofold. First, the core must keep the faces apart and stabilize them. It must, therefore, possess a certain rigidity against deformations perpendicular to the plane of the faces. Second, the core must enable the faces to act as the outer layers of a beam or plate, and to this end it must possess a certain shearing rigidity in planes perpendicular to the faces. Clearly the sandwich type of construction derives its strength and stiffness characteristics from the second property.

With geometric constraints for magnet arrays and stators described in Chapter 7, the minimal planar dimension of the platen must be $12.5 \times 12.5''$. Since there is a weight penalty\(^1\), we need to make the platen as light as possible. A small and light platen is also preferable as it has higher resonant frequency\(^2\). However, there is a nontrivial trade-off with regard to the thickness of the sandwich panel. As a rule, a thicker sandwich has a larger bending stiffness, so it has a higher resonant frequency. Unfortunately, this also leads to a heavier platen. A thicker platen also raises the center of mass relative to where the forces are applied at the face of the magnets. Thus, we need an optimization process for the honeycomb sandwich structure.

[Al69] presents three processes for the optimal design of sandwich panel.

• The core material and the facing material are specified as is the thickness of the faces. Determine the minimum thickness of the core material for the thinnest

---

\(^1\)We must consume more power to suspend and drive a heavier platen.

\(^2\)The natural frequency is proportional to the square root of the stiffness-to-mass ratio of the platen.
possible faces.

- The core material and the facing material are specified but the thickness of the faces and the core are to be determined. Find the optimal minimum-mass design.

- Choose the core density as well as the face and core thickness.

I reproduce below the optimization given in Section 11.3 of [All69]. The objective of the optimization is to maximize the bending stiffness of the sandwich panel with respect to its mass. We want to increase the bending stiffness which is directly related to the resonant frequency of the sandwich panel and the vibration amplitude of the platen. Let us denote:

- $D$: bending stiffness of a sandwich panel

- $t$: thickness of the face

- $d$: thickness of the core plus two halves of the thickness of the face ($t/2 + t/2$)

- $w$: total mass per unit area

- $\mu_c$, $\mu_f$: mass densities of the face and the core

There are three important assumptions involved in this optimization.

1. very thin faces (Definition of a 'thin' face is $100 > d/t > 5.77$.)

2. homogeneous core

3. negligible mass of the adhesive

Assumptions 1 and 3 are easily satisfied in our sandwich panel design. Even if the second assumption is not automatically true, it is more or less valid in case of the honeycomb core with a small cell size and a thin wall thickness as ours.
Now, we turn to the optimization procedure. The bending stiffness of a sandwich panel is specified as in [All69].

\[ D = \frac{E t d^2}{2} \]  

(8.1)

The core and face thicknesses \( d \) and \( t \) are to be adjusted to minimize the total mass of the structure (the second optimization process in the above).

\[ w = \mu_c d + 2 \mu_f t \]  

(8.2)

We can find the optimal relation omitting intermediate details as

\[ \frac{\mu_c d}{2 \mu_f t} = 2. \]  

(8.3)

Allen's conclusion is that the optimal sandwich in terms of maximum flexural rigidity with given mass is one where the core mass is twice the combined mass of the faces.

### 8.1.2 Sandwich Panel Fabrication

Various samples of honeycomb core were gathered within MIT laboratories. A carbon fiber reinforced polymer sandwich panel with corrugated Nomex honeycomb core is considered superior in the applications which need high compressive stress, which is not urgently needed in our application. Furthermore, the surface of carbon fiber reinforced polymer is difficult to machine. Thus, this material was excluded from considerations and classical aluminum honeycomb with aluminum faces was chosen for their easy machinability and low material price.

Dean Sheppard, a Bachelor's student in our lab, and I constructed and tested a few sandwich panels. FM123-2 high-shear film adhesive by Cytec\(^3\) was found superior to epoxy bonding due to its uniform thickness and superior damping property [Cyt94]. It gives 0.15-mm uniform thickness and is proven to work well with aluminum material from our experiments. The densest honeycomb core obtained from the Gas Turbine Laboratory machine shop at MIT and 3/16”-thick 6061-T6 aluminum alloy were

\(^3\)Cytec Engineered Materials, Inc., 1300 Revolution Street, Havre de Grace, Maryland 21078
chosen for the platen materials. With various experiments, I decided to make the thickness of the sandwich 2" for the 12.5 × 12.5" planar area.

Here are the specifications of the chosen aluminum honeycomb core:

- cell size: 1/16"
- wall thickness: 0.007"
- nominal density: 6.5 pcf (lb/ft³) = 0.0104 g/cm³
- thickness: 2" = 50.8 mm
- dimension: 12.5 × 12.5" = 317.5 × 317.5 mm

With the previous optimization process, we can estimate the optimal thickness of the faces with core specification given above. Substituting the data in (8.3) using 2.70 g/cm³ for the face density,

\[ t = \frac{\mu_c d}{4\mu_f} \approx \frac{0.0104 \times 2.54}{4 \times 2.70} \approx 0.05 \text{ cm} \]  

(8.4)

So, the optimization demands faces thinner than 1/32" plates. The assumption 1 is satisfied in this case since \( d/t \approx 50 \). The very thin film adhesive can satisfy the assumption 3. Because we use honeycomb core with very small cells, the assumption 2 can be considered approximately satisfied. However, there are a few reasons why we cannot meet this 0.05-cm thickness. (1) We need to machine the bottom surface flat because it is to be used for capacitance gage targets. (2) It is known that the sandwich panel may warp in the course of the curing process. So, we need some extra margin in thickness for machining. (3) Even if we can grind the surfaces, we cannot attempt to make the faces too thin. The honeycomb core may deform or even collapse due to the machine tool force. (4) We need to drill and tap on the bottom side for alignment jig fixture. So, a minimal thickness is necessary for that. Thus, 1/16" was chosen for the target thickness after all the machining processes. We used 3/16"-thick 6061-T6 aluminum plates for initial faces.
Since the faces may slide a little with respect to the core during the curing process, an oversized (14” × 14”) honeycomb core and faces are prepared to be glued together. The curing process for the sandwich panel with FM123-2 adhesive is as follows.

1. Scrape the honeycomb core and faces with sand paper. Clean the surfaces with methyl-ethyl-ketone.

2. Place film adhesives between the core and faces.

3. Place the sandwich panel on an aluminum bed. Put wood and steel plates around the sandwich to hold it during the curing process.

4. Place a glass breather and a nylon bagging material over the bed.

5. Place the whole assembly in the autoclave and apply 40-psig pressure.

6. Cure at 255°F for two hours.

7. Cool and depressurize slowly.

Two honeycomb sandwich panels were fabricated at the same time. One of them was intended for the back-up purpose in case of machining failures.

8.1.3 Machining Process

The sandwich panel was sent to the Laboratory of Nuclear Science machine shop to be machined. First, all the six 10-32 holes were drilled and tapped, and the top and bottom faces of the platen were milled off leaving the four raised pads. The bottom face was made flat with a Blanchard grinder, and then the four pads were ground to coplanar and parallel to the bottom surface. The coplanarity and flatness of the surfaces are better than 0.001” tolerance. Finally the excess material due to the oversized sandwich panel fabrication was cut to the desired dimension. Figures 8-1–8-2 show top and bottom views of the machined sandwich panel.
Figure 8-1: Top and side views of the sandwich panel
Figure 8-2: Side and bottom views of the sandwich panel
There are four mirror pads with 10-32 bolt taps on the top surface. We could machine the top face of the honeycomb plate and glue pads on top of that. However, we want to avoid any small relative movement between the mirror and platen, since maintaining the absolute position of the mirror is very important. These pads are prepared for the mirror mount and placed according to the positions of the three matched pads on the bottom side of the mirror. Another pad is provided for resonant frequency measurement experiment fixtures, which is described in detail in the following section. The raised height of the four pads is 1/8”; the gap between the mirror and the top surface of the platen is used for the damping layer. To protect the sides of the honeycomb core, four 1/16”-thick aluminum plates are epoxied to each side.

Figure 8-2 shows the bottom view of the sandwich panel. The surface is ground with a tolerance of 0.001” or better. This flatness is required because the surface is used for the capacitance probe targets. So, it serves as a reference surface in the vertical metrology. Since the magnet arrays will also directly be glued to the surface, we need an alignment jig for magnet array attachment. The bottom side has two tapped holes for alignment cross. Figure 8-3 is a photograph of the alignment cross for the magnet arrays, and Figure 8-4 shows its dimensions.

8.1.4 Damping

A honeycomb sandwich panel has high stiffness, since it has a continuous I-beam support. The adhesive between the faces and the honeycomb core gives a rigid joint. Thus, the total system shows high bending stiffness. In addition to stiffness, proper damping is necessary for system stability. There are two major methods to provide passive mechanical damping—tuned mass damping and shear plate damping [Slo92]. A tuned mass damper has a set of mass, spring, and damper. It gives a 180° out-of-phase vibration carefully matched to cancel the resonance vibration of the original system. Because we do not want to add any significant mass to the platen, however,
this method is discarded. It is also quite sensitive to knowing the resonant frequency accurately. A shear plate damper is a simple but efficient method to give mechanical damping. It consists of a constraint layer and a viscoelastic layer. Mechanical energy due to the relative motion of the constraint layer with respect to the original structure is dissipated as heat energy in the viscoelastic layer. The damping layer cannot be applied to the bottom face of the platen, since it is used as the reference for magnet arrays and capacitance probe targets. It is thus put in the space between the mirror and the top surface of the platen.

A 0.025"-thick non-magnetic stainless steel plate serves as the constraint layer. Between the constraint layer and the top face, a 0.060"-thick C1002 shear damping sheet by E-A-R\textsuperscript{4} is used as the viscoelastic layer. To bond the shear damping sheet and the constraint layer, 0.005"-thick ScotchVHB double-sided tape by 3M\textsuperscript{5} is used.

\textsuperscript{4}E-A-R Specialty Composites, 7911 Zionville Road, Indianapolis, Indiana 46268
\textsuperscript{5}3M, 3M Center, St. Paul, Minnesota 55144
Figure 8-4: Dimension of the alignment cross
The tape also adds shear damping. Figure 8-5 shows the overall view of the platen after fabrication and Figure 8-6 shows how they are assembled together.

In the course of testing, a mirror resonance mode at 400 Hz was found to be excited easily. The same C1002 shear damping sheets were thus applied between the mirror and the top surface of the platen. These shear damping sheets are greatly helpful in damping out this mirror resonance.

8.1.5 Resonant Frequency

It is generally difficult to have an exact mechanical model for a sandwich structure that has an inhomogeneous core. A finite element analysis needs a large number of variables to model it properly. Rather, we follow an approach to build several honeycomb sandwich structures with various thickness, core density and cell size, and test them with a dynamic signal analyzer to model and compare. I give test results for the sandwich platen with and without a shear damping layer in this subsection.

Figure 8-7 shows the test rig for the resonant frequency measurement. The suspension rack was designed and constructed by Dean Sheppard. The platen is suspended with four rubber straps to the rack. The straps are fastened to the four pads with
Figure 8-6: Exploded view of the platen
Figure 8-7: Test rig for resonant frequency and damping ratio

Figure 8-8: Accelerometer positions on the bottom side of the platen. The coordinates are relative to the lower left corner of the platen

10-32 bolts. A 353B15 accelerometer by PCB Piezotronics\(^6\) is attached to the platen at three different places as indicated in the Figure 8-8. The effective frequency range of the accelerometer is 1 Hz–10 kHz, the voltage sensitivity is 10.03 mV/g and the output excitation is 8.6 V. The signal from the accelerometer is fed to the HP 35565 Dynamic Signal Analyzer. The impulse response of the platen is obtained by hitting the platen vertically with a plastic hammer. The signal analyzer performs an FFT of the signal from the accelerometer and can also give time response of the vibration. Ten data sets are averaged for the power spectrum plot.

Figure 8-9 shows undamped (dashed line) and damped (solid line) power spectra of impulse responses. The responses were recorded by the accelerometer at the position

\(^6\)PCB Piezotronics, Inc., 3425 Walden Avenue, Depew, New York 14043
Figure 8-9: Power spectra of impulse responses with (solid) and without (dashed) damping

As in the Figure 8-8. The first resonance modes are as high as 1.35 kHz and 1.22 kHz, without and with the damping, respectively. The decreased resonant frequency for the damped case is expected since we add some mass for the constraint layer and the shear damping material. The resonance peak is significantly diminished in the damped case by as much as 50 dB near the first resonance mode. Figure 8-10 shows a time plot of the impulse response. In the figure we can see the effectiveness of the shear plate damping. The damping ratios calculated with the data in the figure are 0.004 and 0.05, respectively. So, the shear plate damping can improve the platen damping ratio by an order of magnitude. The test data in this subsection were made with a sandwich panel which does not include magnet arrays and the mirror.

\footnote{The vibration frequency in Figure 8-10 is apparently 2.5 kHz. This indicates that the fundamental resonant mode was not excited in the experiment for the impulse response.}
Figure 8-10: Time plot of impulse responses with (solid) and without (dashed) damping.
8.1.6 Magnet Placing

We want to obtain a good perpendicularity of four Halbach magnet arrays\(^8\) and coplanarity of four magnet-array surfaces to minimize errors in operations of linear motors. Even if small misalignment may be compensated by feedback control or model correction, we are better off with high accuracy. Obtaining a good parallelism of magnet-array surfaces with respect to the plane of the four mirror pads is also important as the overall coplanarity of magnet-array surfaces. To obtain a good coplanarity, I use two granite slabs with 0.0002" surface tolerance manufactured by Rock of Ages\(^9\). Figure 8-11 shows how the magnet arrays are placed using the alignment cross as in Figure 8-4 whose perpendicularity is better than 0.001". Of course, we cannot achieve this small tolerance for overall magnet surfaces anyway, since the surface of the magnet array itself depends on the tolerance of the magnet pieces, which is some ±0.002".

Here is the procedure how to put the magnet arrays to the platen.

- Scratch the bottom surfaces of the platen for the glue.

- Spray mold release on the cross and unnecessary surfaces of the platen.

- Bolt the alignment cross down on the bottom side of the platen.

- Put the epoxy on the weak sides of the magnet arrays, which face the platen.

- Place the epoxied magnet arrays on a granite slab.

- Place the platen on top of the magnet arrays and push the arrays to the cross.

- Put another granite slab on top of the platen.

- Cure for 18 hours.

---

\(^8\)The fabrication process of them was described in Chapter 7 in detail.

\(^9\)Rock of Ages, 560 Graniteville Road, Graniteville, Vermont 05654
Figure 8-11: Placing magnet arrays with the alignment cross
The same epoxy (PC-7) and mold release (E408) as in the magnet array fabrication are used in the process.

Coordinates of magnet arrays are measured by a coordinate measuring machine. The coordinate measuring machine gives a resolution down to 0.0001", but the measured data are sensitive to the speed of the probe tip just before taking the measurement. That is, the dynamics of the machine affects the measurement. I took more than five data for measurement and averaged them. Repeated data confirm the measurement accuracy on the order of 0.001". Figure 8-12 shows the measured heights of magnet array surfaces and capacitance probe target surfaces. The distance between the mirror pad plane and capacitance probe target plane is 2.250", which is exact as specified. So, we know that the thickness of the film adhesive layers between the honeycomb core and faces is really negligible, even if each has two FM123-2 sheets. This is because we pressurized the honeycomb up to 40 psig while we cured the sandwich platen. The numbers in Figure 8-12 represent the highest points of each surface. With the values, coplanarity of the four magnet array surfaces is ±0.003". It is quite good considering the specified tolerance of the magnet pieces, ±0.002".

Coplanarity and parallelism of the three capacitance probe target surfaces are better than ±0.001" as in Figure 8-12. This tolerance is expected because the target surfaces are ground within that tolerance. Parallelism of the three magnet arrays relative to the plane of the mirror pad surface is better than ±0.0015". The magnet array III has worse parallelism, ±0.004". A small stone or metal chip might be involved in the curing process, which might be thought as a cause for this worse parallelism. The error source for this is not clear and not investigated further herein. All x and y coordinates for the magnets are within 0.001" error bound of the specified values, which is bearable compared with the manufacturing error in windings described in the next section. Figure 8-13 shows the bottom view of the platen after the magnet arrays are placed.
Figure 8-12: Average heights of magnet-array and capacitance-probe-target surfaces on the bottom side of the platen. The reference plane is the one which encloses four top surfaces of mirror pads on the other side.
Figure 8-13: Bottom view of the platen
8.1.7 Mirror Mounting

The square mirror from a 1980's-vintage GCA stepper is mounted on top of three mounting pads on the top surface of the honeycomb sandwich. An aluminum block holds two plane mirrors for three lateral degree-of-freedom measurements\textsuperscript{10}. The mirror is kinematically mounted on the ground pads. To match the dimensions of the holes on the mirror, 10-32 bolt holes are tapped on the mounting pads accordingly (Figure 8-1). Thanks to the clearance holes on the mirror, a rotational adjustment on the order of a few degrees is possible. This adjustment is not necessary in our system since we have not attempted to image substrates.

8.2 Stators

I decided to make the four stator blocks identical. The linear motor stators are made from solid 6061-T6 aluminum so that there is no iron material in the stators. Winding modules are slid in to the stator core, which makes the stator fabrication easy. As we discussed in Chapter 5, we do not want to have a big downward attraction force or cogging force in the platen. The motor and sensor air gaps are decided to be 0.010" and 0.020", respectively to avoid sensor saturation and collision between the capacitance gages and the targets.

8.2.1 Capacitance Probe Mounting

We have three capacitance probes to gather three vertical degree-of-freedom position information. It is ideal to separate these metrology devices from actuators. In other words, it would be better to mount the capacitance probes mechanically isolated from the stator core. However, I decided to mount the probes directly to the side rails of the stator core. This also simplifies the machining process.

\textsuperscript{10}A sophisticated up-to-date metrology mirror uses Zerodur material (trademark of Schott glass) to minimize the thermal expansion of the mirror itself.
Figure 8-14: Side view of the stage indicating the nominal sensor gap of 500 μm, and the nominal actuator gap of 250 μm.

Figure 8-14 is a side view of the platen and a stator to show the air gaps. I set the representative height of the magnet arrays with respect to the capacitance probe targets to be 0.266". So, the thickness of the glue line of between the magnet arrays and the platen is 0.012". The nominal air gap between the magnet and the stator is suggested to be 0.010" (= 250 μm) and the nominal air gap between the capacitance probes and their targets is 0.020" (= 500 μm). So, the vertical range of the platen is 50–450 μm in terms of the motor air gap, and the minimal air gap between the capacitance probes and their targets are 250 μm. The capacitance probe electronics is set to generate 7.5 V at the maximum distance, 700 μm and −7.5 V at the minimum distance, 200 μm. This voltage range matches the input range of our system A/D converters.

### 8.2.2 Stator Core

Figure 8-15 shows the cross-sectional view of a stator. Neglecting the eddy current effect, I chose not to laminate the stator core for simplicity sake. So, the stator block is made of solid aluminum 6061-T6. The stator cores are machined by Eastern Tool\textsuperscript{11}.

\textsuperscript{11}Eastern Tool Corp., 35 Medford Street, Somerville, Massachusetts 02143

219
The width of the stator is 5.5" in Figure 8-16, so there are 2.5" wide empty spaces at both ends of the stator core. These empty spaces are provided room for the coil leads and connections of the leads. The thickness of the stator core is set 1" so that the height of the magnet from the optical table is more than one-pitch length as we discussed in Section 7.2. A thinner stator core would lead to windings with shorter end turns, which is good for power efficiency. However, it would also lead to a lower resonant frequency of the stator core. The 1" thickness is a reasonable trade-off with these considerations.

We need a little bigger (by 0.007" in diameter) clearance holes for the stator mounting to the base table. These clearance holes are to absorb thermal expansion and to adjust the rotational position of the stators in case of misalignments. No special alignment tool for the stators proves to be necessary.

In Figure 8-17, capacitance probe related heights are specified with high tolerance. Here (.001) means a ±0.0005" tolerance. The height of the rail for the capacitance probe is determined by the measurements done in the previous section. That is, the nominal height of the magnet arrays relative to the capacitance probe target surfaces is 0.266". The height of the capacitance probe is 0.709". The nominal air gaps are 0.010" and 0.020" for the magnet array and the capacitance probe respectively. So, the top surfaces of the capacitance probes are intact, even when the magnet arrays touch the surfaces of the stators. Considering the manufacturing error and tolerance, capacitance probes will not hit their targets in any case. This is important not to damage the sensor and target surfaces.

Figures 8-18–8-19 are the end view of the stator and the loose part. It is important to maintain the total height 1.500" is within ±0.0005" tolerance. I devised the specific stator core in Figure 8-20 to maintain the best parallelism. The bottom surfaces of two legs are milled at the same time to be on a reference plane, then the top surface is milled parallel to this plane. So, all four top surfaces of four stators on the optical table are coplanar.
Figure 8.17: Side view of the stator core

- clearance hole (.257") for 1/4-20 (x2) cap screws with counter bore this side
- round edges (corner radius about .001")
- tap 10-32 (x5)
- through hole for M3 (x3) cap screws with counter bore this side
- clearance hole for (.257")1/4-20 (x2) cap screws WITHOUT counter bore

1:1 scale
Figure 8.18: End view of the stator core

- Clearance hole (0.257") for 1/4-20 (x4) cap screws
- Round edges (corner radius about 1/16")
- Tap 10-32 (x5, 1.250" apart)
- Round edges (corner radius about 1/16")
- Reference sides

Dimensions:
- Width: 5.5"
- Height: 1.500" (.001)
- Height between holes: 0.750" (.001)

1:1 scale
Figure 8-19: Stator loose part
8.2.3 Winding Assembly

Figure 8-20 shows how the winding assembly is accomplished. Two 0.005"-thick Mylar sheets are on the top and the bottom surface of the stator core to provide another electrical insulation of the windings from the stator core and to prevent scratching the winding surface. To prevent scratch to the windings, sharp edges are also avoided in the stator. A Mylar ring is placed between the left wall of the stator core and the left side of the first winding. Then, as the order indicated in the figure: (1) Thirty-three windings are slid in from the right side of the stator core. (2) Another Mylar ring is placed to the right side of the last winding. We then clamp the windings with the end block. (3) A 1/32"-thick aluminum spacer is placed between the Mylar ring and the bottom-left surface of the stator core's right end. Were it not for this spacer, the last (thirty-third) winding could not be slid and rotated to its position. Heat generated
Figure 8-21: Top view of a motor stator with a capacitance probe mounted on its rail from the winding is dissipated through low thermal resistive paths from the legs to the optical table.

Soldering the leads of the same phase is done between every third winding. One pair of leads can easily soldered outside the winding on the bottom. The other pair of leads should be soldered and placed in the area between the inner surface of the winding and the right side of the stator core. Heat shrinkable tubing with 1/32" diameter are used to protect and insulate soldered bare wire. Phase A of each motor has blue tubing; phase B, yellow; and phase C, green. Finally, an electrical terminal block is epoxied to the end of the stator to serve as an attachment point for the motor leads. Figure 8-21 is the top view of a stator with a capacitance probe after the winding assembly.
8.3 Packaging and Assembly

In this section, we discuss packaging and assembly issues, especially for stationary parts—stators, metrology devices, and the like. Figure 8-22 shows the overall stator assembly. For the metrology device mounting, refer to Figure 7-17.

8.3.1 Mounting Table

We need a large flat mounting table on which all the stationary parts are laid. The mounting table should also have good mechanical properties in terms of resonance peak and external vibration isolation. There are a few well-known choices for the table top—granite, cast iron, and honeycomb sandwich. (1) A granite slab shows a good stiffness but high resonance peak. It has low thermal expansion. It is possible to grind the surface to get a very flat plane down to 0.0001”, even though the machining cost is expensive at this surface flatness level. One of its disadvantages is that it is hard to make threaded attachment points. (2) We can easily drill bolt tabs in cast iron which is comparatively inexpensive. However, it has low resonant frequency in general. Since we do not want to have a strong downward attraction force, a cast iron table top is not used in this project. (3) A honeycombed optical table is a good choice for stiffness and resonance properties. Advantages of the honeycomb structure given earlier in this chapter are also applicable to the table top.

We bought a 4 × 4 ft optical table from TMC. It uses non-magnetic stainless steel for its top plate. It has 1/4-20 bolt taps which make it easy to mount the stators and bumpers. These taps are located at the cross-sections of dotted 1” grid in Figure 8-22. The table top’s surface flatness is ±0.002” in any 1-m² area. It has 188 Hz mode to reject by its specifications. The table top needs four air legs to isolate the floor vibration effectively. A 80-psi compressed air is supplied by a can of 2200-psi dry air provided by BOC\textsuperscript{12} and two-stage regulators are used for the air supply. Three air

\textsuperscript{12}BOC Gases, 575 Mountain Avenue, Murray Hill, New Jersey 07974
Figure 8-22: Top and side views of stationary parts on the mounting table except for the metrology devices
legs have leveling valves to make the optical table leveled.

8.3.2 Stators

As in Figure 8-22, four stators are arranged orthogonally. The stators are bolted directly to the optical table with four 1/4-20 cap screws for this prototype. The 1/4-20 and 10-32 cap screws are made from non-magnetic stainless steel. If a high orthogonality is necessary for the stators, another alignment cross as in Figure 8-4 is required. In this prototype, the system works well without any fine alignment. Magnetic levitation does not need high mechanical tolerance as mentioned earlier in this thesis.

8.3.3 Metrology Devices

As I described in the previous section, a capacitance probe is mounted directly to each stator block on a side rail. The laser head, beam splitters, and beam benders are attached to the optical table. The old GCA stepper had small holders of a retroreflector, a plane mirror converter, and a receiver. I use three of them with height adjustment by cutting and milling its legs by 0.250”. Then, they are fixed to the optical table with 1/4-20 screws. Refer to Figure 7-17 for their arrangement in a proportional scale.

8.3.4 Bumpers

There are eight VPH-6 post holders by Newport\textsuperscript{13} as bumpers. They are originally supposed to be used as holders of lenses, mirrors, and other optical devices on the optical table. They have 1/4-20 bolts implemented on their bottom sides to screw directly to the optical table. They are made of aluminum and so are non-magnetic. Foam sheets are wrapped around the posts for protection. They are required to stop

\textsuperscript{13}Newport Corp., 1791 Deere Avenue, Irvine, California 92714

230
any erroneous motion of the platen. I also used them as posts with Mylar tape to constrain degrees of freedom other than the vertical motion during the initial testing phase.

8.3.5 Mechanical Tolerances

Here is a summary of mechanical tolerance of parts.

- mounting table: ±0.002" in any 1-m² area
- magnet pieces: ±0.002" (better than ±0.005" in manufacture's specifications)
- flatness of the surface of the platen: ±0.002"
- parallelism of the capacitance probe targets: ±0.0005"
- flatness of surfaces of magnet arrays: 0.0025" (except for III: 0.004")
- coplanarity of four top surfaces of stators: ±0.001"

8.4 Summary of Part III

In Part III, we came up with a candidate conceptual design for the planar magnetic levitator after generating and comparing several design concepts and parametric analyses. The selected candidate consists of only one moving part, and includes four permanent-magnet linear motors that are two-degree-of-freedom actuators. In other words, they can generate suspension force as well as drive force. The magnetic levitator is structurally simple and has no umbilical cables. It generates all six-degree-of-freedom motions for small adjustments and large two-dimensional motions with only one moving part, the platen.

In the latter two chapters in this part, I presented detailed electromagnetic and mechanical designs for the levitator. As the Halbach magnet-array linear motors are the key actuators of the levitator, we concentrated on the motor design. The linear
motors are three-phase surface-wound surface-magnet type actuators. The stators have Gramme-type windings for easy assembly. We also discussed power amplifier design, instrumentation structure, and software in the Electromagnetic Design chapter.

The platen in the planar magnetic levitator stage is in aluminum honeycomb structure for high bending stiffness, and has a shear damper to add mechanical damping to the system. Finally, I specified all electrical and mechanical design parameters and elaborated the fabrication processes. Now the levitator is ready to test. Its modeling and control will be covered in Part IV.
Part IV

Dynamics and Control
In this part, we present dynamics and control of the levitator. For modeling purposes, we estimate mechanical parameters of the platen, such as mass, center of mass, rotational inertia, and measure electrical parameters of the stators, such as winding resistance and self-inductance, and the like. The dynamic equations are derived from Newton's second law and the force equations given in Section 4.3 with a linearization around an operation point. To decouple the vertical and lateral dynamics, we use Blondel-Park transformations and DQ transformations (two reaction theory) [Blo13, Par29]. With these electromechanical parameters in hand, we derive decoupled and state-space dynamic models of the stage.

Chapter 10 elaborates the testing processes. For initial debugging, we begin with the simplest model for the plant—decoupled dynamics and ignore the product of inertia terms. Since ours is a redundant actuator system, there exist non-unique solutions to force allocation, i.e., modal-decomposed force transformation. Testing includes vertical and lateral modal controls and step responses. Utilizing the full state feedback provided by the laser interferometry electronics, a multivariable linear quadratic control of lateral dynamics of the platen is implemented. A discussion of the time-optimal control, which is directly related to the machine throughput, is given. Finally, a description of the experimental demonstrations follows.
Chapter 9

Modeling and Dynamic Behavior

To get a first-hand dynamic model of the levitator, we need to measure or calculate
electromechanical parameters. These include the mass and inertia tensor of the
platen, resistance and inductance of the phase winding, and the like. In this chapter,
we summarize these parameters and the specifications of the whole levitator system.
To derive lateral and vertical dynamic models of the system, we decompose force
and stator current into the lateral and vertical components by applying a modified
DQ decomposition of the classical electric machine theory. We eventually derive a
linearized state-space model of the platen. The state space model is used in the
multivariable linear quadratic control in Chapter 10.

9.1 Electromechanical Parameters

9.1.1 Mass and Inertia Tensor of the Platen

An electronic scale with 0.1-g resolution is used to measure the mass of the parts.
The square mirror (serial number 1394 from the GCA stepper) and one magnet array
weigh 1.5150 kg and 0.411 kg, respectively. The mass of the honeycomb sandwich

\footnote{From now on, the positive z-axis directs opposite to gravity and the large motion plane is the
x-y plane. The axes are different from those in Part II, where they follow Melcher's convention.}
with the shear damper and the screws for the mirror is 2.4195 kg. The total mass of the platen, which includes the honeycomb core, top and bottom skins, four side plates, the stainless-steel constraint layer, the shear damping layer, four magnet arrays, the square mirror, three screws for mirror mounting, epoxy, and film adhesive, is \( M = 5.5785 \text{ kg} \). So, the platen weight is 54.7 N.

The center of mass of the total platen from the geometric center of the platen honeycomb core is calculated to be

\[
CM = \begin{bmatrix}
CM_x & CM_y & CM_z
\end{bmatrix}^T = \begin{bmatrix} 11.2 \quad 11.2 \quad 2.8 \end{bmatrix}^T
\]  

(9.1)
in millimeters. Thus, the offset of the platen center of mass is significant in the \( x - y \) plane due to the mirror position as seen in Figure 8-6. This mirror position was intended to place the wafer at the center of the platen top surface.

Now, we calculate the moment of inertia of the platen. The inertia tensor is represented by a \( 3 \times 3 \) matrix

\[
I = \begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{bmatrix}
\]  

(9.2)
where the quantities \( I_{xx}, I_{yy} \) and \( I_{zz} \) are known as the moments of inertia of a body about the respective axes, and \( I_{xy}, I_{yz}, I_{xz}, I_{yx}, I_{yz}, I_{zx} \) and \( I_{zz} \) are known as products of inertia. By the definition of products of inertia, it is observed that \( I_{xy} = I_{yx}, I_{yz} = I_{zy} \) and \( I_{zz} = I_z \). [Wil96]

The moment of inertia of a rectangular prism in Figure 9-1 is derived as [Wil96].

\[
I_{xx} = \frac{1}{12}M(l_y^2 + l_z^2)
\]  

(9.3)
\[
I_{yy} = \frac{1}{12}M(l_z^2 + l_x^2)
\]  

(9.4)
\[
I_{zz} = \frac{1}{12}M(l_x^2 + l_y^2)
\]  

(9.5)
\[
I_{xy} = I_{yx} = 0
\]  

(9.6)
\[
I_{yz} = I_{zy} = 0
\]  

(9.7)
\[
I_{zx} = I_{xz} = 0
\]  

(9.8)
Figure 9-1: A rectangular prism with indication of its center of mass

where, \( M \) is the mass of the body assumed uniformly distributed, and \( l_x, l_y \) and \( l_z \) are the lengths of the edges of the rectangular prism. The origin of the \( xyz \)-coordinate system is located at the center of mass of the rectangular prism.

We have a useful theorem to derive the inertia tensor of the same body about different coordinate axes in case corresponding coordinate axes are parallel to each other. Here is a statement of the parallel-axes theorem [Wil96].

\[
I_A = I_{CM} + M \begin{bmatrix}
  b^2 + c^2 & -ab & -ac \\
  -ba & c^2 + a^2 & -bc \\
  -ca & -cb & a^2 + b^2 \\
\end{bmatrix}
\]

(9.9)

As depicted in Figure 9-2, we want to find the inertia tensor \([I]_A\) with respect to the coordinate axes \( x'y'z' \) with the origin at \( A \). The inertia tensor \([I]_{CM}\) for the center of mass of a rectangular prism is calculated with (9.4–9.8). \([ a \; b \; c ]^T\) is the displacement vector from the center of mass \( CM \) to the origin \( A \).

Each part of the platen except the mirror can be represented as a set of rectangular prisms. We think of the mirror as consisting of two rectangular prisms. Using (9.4–9.8) and the parallel-axes theorem the inertia tensor of the whole platen about the
Figure 9-2: Two parallel axes attached on a rigid body with origins at its center of mass and at an arbitrary point \( A \)

Platen center of mass is calculated as

\[
I_{CM} = \begin{bmatrix}
0.0542 & 0.00276 & -0.00253 \\
0.00276 & 0.0541 & -0.00261 \\
-0.00253 & -0.00261 & 0.0981 \\
\end{bmatrix}
\] (9.10)

in \( \text{kg-m}^2 \).

### 9.1.2 Winding Resistance and Self-Inductance

By direct measurement, the resistance of one winding is found to be 1.3 \( \Omega \). Since there are 11 windings per phase per motor, the total resistance of the windings of a phase is 14.3 \( \Omega \). The measured total resistance turns out to be 14.4 \( \Omega \) after stator fabrication.

The self-inductance of a winding is found by using the HP 35665 dynamic analyzer's swept sine mode, where it automatically generates sinusoidal waves with various frequencies through its source terminal. Figure 9-3 is the schematic diagram for the self-inductance measurement. We get the Bode plot of the transfer function
from the Channels 1 and 2 inputs. The winding is modeled with an inductor series-connected with a winding resistor. A 1-Ω resistor is used for a reference resistor in this experiment. The transfer function from $V_{in}$ to $V_{out}$ turns out to be as follows.

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1.00}{(j\omega L + 1.3) + 1.00} \quad (9.11)$$

So, the DC gain of the circuit is $-7.24$ dB. From the Bode plot from by the dynamic analyzer, the 3-dB frequency is read as 665 Hz. At the 3-dB frequency,

$$20 \log_{10} \left| \frac{V_{out}(j \cdot 2\pi \cdot 665)}{V_{in}(j \cdot 2\pi \cdot 665)} \right| = 20 \log_{10} \left| \frac{1}{(j2\pi 665L + 1.3) + 1.00} \right| = -10.24. \quad (9.12)$$

So, the self-inductance of a winding is calculated as 0.54 mH. The total self-inductance of a phase is calculated as 5.94 mH since there are eleven such windings in a phase.

This inductance calculation is for air-core windings. Since the actual windings have a solid aluminum core, the inductance measurements for the total phase windings are done after the stator assembly is complete. Table 9.1 shows the inductance measurements for the aluminum core windings with the same methodology given above. The average inductance 3.44 mH turns out to be smaller than the estimation for air core windings, 5.94 mH. The error is believed to be from the opposing magnetic flux generated by eddy current in the solid aluminum core.

To simulate the real situation where the winding currents react with the magnet arrays, the phase inductances with the magnet arrays resting on the top stator surfaces
Table 9.1: Self-inductance of the phase windings without the effect of the magnet arrays

<table>
<thead>
<tr>
<th>stator</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>phase A (mH)</td>
<td>3.44</td>
<td>3.44</td>
<td>3.43</td>
<td>3.44</td>
</tr>
<tr>
<td>phase B (mH)</td>
<td>3.44</td>
<td>3.44</td>
<td>3.44</td>
<td>3.45</td>
</tr>
<tr>
<td>phase C (mH)</td>
<td>3.45</td>
<td>3.44</td>
<td>3.45</td>
<td>3.44</td>
</tr>
</tbody>
</table>

Table 9.2: Self-inductance of the phase windings with the effect of the magnet arrays

<table>
<thead>
<tr>
<th>stator</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>phase A (mH)</td>
<td>3.47</td>
<td>3.48</td>
<td>3.44</td>
<td>3.48</td>
</tr>
<tr>
<td>phase B (mH)</td>
<td>3.48</td>
<td>3.47</td>
<td>3.44</td>
<td>3.52</td>
</tr>
<tr>
<td>phase C (mH)</td>
<td>3.48</td>
<td>3.47</td>
<td>3.47</td>
<td>3.49</td>
</tr>
</tbody>
</table>

with two 0.005” Mylar sheets (about 250-μm gap) in between are also measured as in Table 9.2. The inductances measured with the magnet arrays are not far from those measured with magnets. This fact also confirms the validity of our approximation of the magnet permeability as the free-space permeability $\mu_0$.

### 9.1.3 Specifications of the Levitator

This subsection summarizes geometric parameters and ratings of the levitator determined in Part III.

Each of four linear motors are surface-wound ironless permanent-magnet machines. Its ratings are as follows.

- number of phases, $q = 3$
- phase inductance = 3.44 mH,
- phase resistance = 14.4 Ω
- nominal phase current = ±0.5 A
- maximum phase current = ±1.5 A
- nominal phase voltage = ±7.2 V
- maximum phase voltage = ±22 V
- rail voltages for power supplies = ±28 V
- rail voltage for TL072ACP OP Amp = ±15 V
- nominal power dissipation per motor = 5.4 W
- suspension efficiency = 7.2 mW/N².

Here are the parameters of the magnet arrays and windings:
- turn density, \( \eta_0 = 2.491 \times 10^6 \) turns/m²
- pitch, \( l = 25.6 \) mm
- magnet array width, \( w = 92.0 \) mm
- magnet array length = 96.1 mm
- number of magnet pitches, \( N_m = 3.75 \)
- number of stator pitches = 5.5
- magnet thickness, \( \Delta = l/4 \)
- winding thickness, \( \Gamma = l/5 \)
- magnet remanence, \( B_r = \mu_0 M_0 = 1.29 \) T
- motor geometric constant, \( G = 4.89 \times 10^{-6} \, \text{m}^3 \)

- nominal peak current density, \( J_p = 1.23 \times 10^6 \, \text{A/m}^2 \)

The following are motion capabilities of the levitator:

- planar travel range = 50 \times 50 \, \text{mm}

- nominal motor air gap = 250 \, \mu\text{m}

- vertical range = \pm 200 \, \mu\text{m}

- angular ranges = \pm 600 \, \mu\text{rad}

- maximum velocity = 254 \, \text{mm/s} \text{ limited by the laser head slew rate}

- maximum acceleration = 10 \, \text{m/s}^2 (1 \, g)

### 9.2 Decoupled Equations of Motion

We derive the linearized force equations and vertical and lateral linear equations of motions in this section. They prove to be a second-order dynamic equations with mass and (positive or negative) magnetic spring.

#### 9.2.1 DQ Decomposition

The DQ decomposition in conventional rotary machines was introduced to separate the stator current component that generates torque [FKU90]. The direct-axis \((D\text{-axis})\) and quadrature-axis \((Q\text{-axis})\) are attached to the rotor frame, and rotate with the rotor. Then, force equations and commutation described in the DQ frame do not contain the position dependence with respect to the stator. So, nonlinearity due to the trigonometric function dependence in the model can be eliminated, which is a better choice for control. Conventionally, the \(D\)-axis is aligned to the rotor magnetic axis. The \(Q\)-axis is orthogonal to the \(D\)-axis and leads it by 90°. The \(Q\)-component of
the stator current at a given time generates useful torque. The $D$-component usually does not do effective work because the rotor is pinned around its symmetry axis.

The linear motor in our levitator is designed to generate suspension force as well as drive force; it is a two-degree-of-freedom actuator. Decoupling the two orthogonal force components should be performed to control the two degrees of freedom independently. As in Figure 9-4, we defined the $D$-axis as the $z'$-axis in the platen frame. The $Q$-axis leads $D$-axis by $l/4$, which is $90^\circ$ in electrical angle, in the $+y$-direction.

Now, we have the transformation from $[J_a \ J_b]^T$ to $[J_Q \ J_D]^T$.

$$
\begin{bmatrix}
J_Q \\
J_D
\end{bmatrix} = e^{\gamma_1 y_0 J} \begin{bmatrix}
J_a \\
J_b
\end{bmatrix}
$$

(9.13)

where $e^{\gamma_1 y_0 J}$ is a transformation matrix given by

$$
e^{\gamma_1 y_0 J} = e^{\begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix}} = \begin{bmatrix}
\cos \gamma_1 y_0 & \sin \gamma_1 y_0 \\
-\sin \gamma_1 y_0 & \cos \gamma_1 y_0
\end{bmatrix}.
$$

(9.14)

In conventional machine literature, $i_\alpha = J_a/\eta_0$ is defined as the magnetic flux axis of the first phase current $i_\alpha$. The other component, $i_\beta = J_\beta/\eta_0$ leads $i_\alpha$ by $90^\circ$ (where $\eta_0$ represents the turn density of windings). We need to point out again that $[J_a \ J_b]^T$
is differently defined from $[J_\alpha \ J_\beta]^T$ in the above transformation. In Section 4.3, the $J_\alpha$ component was defined as the real part of the $\tilde{J}_1$ and $\tilde{J}_{-1}$. So, the $a$-axis matches with the $x$-axis in the continuum model in Figure 4-1. Thus, in our framework, the $\alpha\beta$-axis leads $ab$-axis by $l/4$ (again, by $90^\circ$ in electrical angles). We stick to the $ab$-frame, rather than the $\alpha\beta$-frame to be consistent with the analysis done in previous work [KT96].

### 9.2.2 Linearized Force Equations

We derive linearized force equations in steady-state suspension in a dynamic equilibrium. At a fixed time, if we generate the current sinusoidally distributed in the $y$-direction as in Figure 9-5, there is vertical repulsive force between the same magnetic poles of the magnet array and the current distribution (i.e., corresponding North to North and South to South). This vertical repulsive force is also responsible to lift the platen against gravity. Since this equilibrium is unstable in the lateral direction (in the $y$-direction in the figure), however, we need active feedback control to stabilize the motion of the platen around this dynamic equilibrium. Conceptually, we can
control the magnitude of the vertical force by changing the magnitude of the current, and control the lateral force by commutation.

Using the relationships, \( f_{y,z} = N_m f_{(y,z)\lambda} \) and \( i_{a,b} = 2\eta_0 J_{a,b} \), we rewrite the relationship between the total vertical and lateral forces \( f_y \) and \( f_z \) and the peak current components \( i_a \) and \( i_b \).

\[
\begin{bmatrix}
  f_y \\
  f_z
\end{bmatrix} = \frac{1}{2} \mu_0 M_0 \eta_0 N_m Ge^{-\gamma_1 z_0} \begin{bmatrix}
  \cos \gamma_1 y_0 & \sin \gamma_1 y_0 \\
  -\sin \gamma_1 y_0 & \cos \gamma_1 y_0
\end{bmatrix} \begin{bmatrix}
  i_a \\
  i_b
\end{bmatrix}
\]  

(9.15)

From (9.15) and the following relationship

\[
\begin{bmatrix}
  i_Q \\
  i_D
\end{bmatrix} = e^{\gamma_1 y_0 J} \begin{bmatrix}
  i_a \\
  i_b
\end{bmatrix},
\]

(9.16)

we can decouple the lateral and vertical force components in \( i_Q \) and \( i_D \).

\[
f_{k_y} = \frac{1}{2} \mu_0 M_0 \eta_0 N_m Ge^{-\gamma_1 z_0} i_Q \]  

(9.17)

\[
f_{k_z} = \frac{1}{2} \mu_0 M_0 \eta_0 N_m Ge^{-\gamma_1 z_0} i_D, \]

(9.18)

where \( k \) can be a number representing a stator. The above equations have multiplicative terms of position \( z_0 \) and current \( i_Q \) or \( i_D \). To linearize these force equations, we set

\[
z_0 = \tilde{z}_0 + \hat{z}_0 \]

(9.19)

\[
i_D = \tilde{i}_D + \hat{i}_D \]

(9.20)

\[
i_Q = \tilde{i}_Q + \hat{i}_Q, \]

(9.21)

where \( \tilde{z}_0 \) is the nominal levitation height, which is 250 \( \mu \text{m} \). The nominal direct current component \( \tilde{i}_D \) is calculated 500 mA to levitate the platen (which generates 14 N, a quarter of the weight of the platen). The nominal quadrature current component \( \tilde{i}_Q \) is zero, because the platen is in a dynamic equilibrium then \( f_y \) is zero.
9.2.3 Vertical Equations of Motion

Retaining zeroth and first order terms, the equilibrium condition for the vertical direction becomes from (9.18)

\[ f_z - Mg = 4 \cdot \frac{1}{2} \mu_0 M_0 \eta_0 N_m G \left( e^{-\gamma_l z_0 i_D} - \gamma_1 e^{-\gamma_l z_0 i_D} \tilde{z}_0 + e^{-\gamma_l z_0 i_D} \right). \tag{9.22} \]

There is a factor of four, since four linear motors are equally responsible to suspend the platen neglecting the mass unbalance due to the square mirror. When the platen is in dynamic equilibrium, the weight \( Mg \) of the platen should be matched with \( 4 \cdot \frac{1}{2} \mu_0 M_0 \eta_0 N_m G e^{-\gamma_l z_0 i_D} \). Now, the force equation becomes,

\[ f_z = 2 \mu_0 M_0 \eta_0 N_m G e^{-\gamma_l z_0 i_D} - 2 \mu_0 M_0 \eta_0 N_m G \gamma_1 e^{-\gamma_l z_0 i_D} \tilde{z}_0. \tag{9.23} \]

So, the incremental equation of motion in the vertical direction is

\[ M \frac{d^2 \tilde{z}_0}{dt^2} + 2 \mu_0 M_0 \eta_0 N_m G \gamma_1 e^{-\gamma_l z_0 i_D} \tilde{z}_0 = 2 \mu_0 M_0 \eta_0 N_m G e^{-\gamma_l z_0 i_D}. \tag{9.24} \]

When there is no control, \( \tilde{i}_D = 0 \), the vertical dynamics is marginally stable as expected. The uncompensated resonant frequency of this platen mass-magnetic spring system is calculated as 7.85 Hz, which is very close to the measured natural frequency 8 Hz during testing.

9.2.4 Lateral Equations of Motion

The equilibrium condition for the lateral direction becomes from (9.15)

\[ f_y = 2 \cdot \frac{1}{2} \mu_0 M_0 \eta_0 N_m G e^{-\gamma_l z_0} (\cos(\gamma_1 y_0) i_a + \sin(\gamma_1 y_0) i_b). \tag{9.25} \]

Here is a factor of two, since two diagonally opposite linear motors are equally responsible to drive the platen in one direction neglecting the mass unbalance due to the square mirror. To linearize this force equations, we set

\[ z_0 = \tilde{z}_0 \tag{9.26} \]
\[ y_0 = \bar{y}_0 + \bar{y}_0 \]  
(9.27)

\[ i_a = \bar{i}_a + \bar{i}_a \]  
(9.28)

\[ i_b = \bar{i}_b + \bar{i}_b. \]  
(9.29)

Then,

\[ f_y = \mu_0 M_0 \eta_0 N_m G e^{-\eta_0 \bar{z}_0} \left\{ \cos \gamma_1 (\bar{y}_0 + \bar{y}_0) (\bar{i}_a + \bar{i}_a) + \sin \gamma_1 (\bar{y}_0 + \bar{y}_0) (\bar{i}_b + \bar{i}_b) \right\} \]

\[ = \mu_0 M_0 \eta_0 N_m G e^{-\eta_0 \bar{z}_0} \left\{ \cos \gamma_1 \bar{y}_0 (\bar{i}_a + \bar{i}_a) - \bar{y}_0 \sin \gamma_1 \bar{y}_0 (\bar{i}_a + \bar{i}_a) \right. \]

\[ + \sin \gamma_1 \bar{y}_0 (\bar{i}_b + \bar{i}_b) + \bar{y}_0 \cos \gamma_1 \bar{y}_0 (\bar{i}_b + \bar{i}_b) \right\} \]

\[ = \mu_0 M_0 \eta_0 N_m G e^{-\eta_0 \bar{z}_0} \left( \bar{i}_Q + \bar{i}_Q + \bar{y}_0 \bar{i}_D + \bar{y}_0 \bar{i}_D \right). \]  
(9.30)

We want \( \bar{i}_Q = 0 \) in equilibrium. The last term can be neglected as it is a second-order term. So, the incremental equation of motion in lateral direction is

\[ M \frac{d^2 \bar{y}_0}{dt^2} - \mu_0 M_0 \eta_0 N_m G e^{-\eta_0 \bar{z}_0} \bar{i}_D \bar{y}_0 = \mu_0 M_0 \eta_0 N_m G e^{-\eta_0 \bar{z}_0} \bar{i}_Q. \]  
(9.31)

When there is no control, \( \bar{i}_Q = 0 \), the lateral dynamics shows its unstable nature as expected.

### 9.3 State-Space Equations of Motion

In this section, we derive linearized state-space equations of motion of the platen. First, we derive full nonlinear equations of motion with no approximations and then linearize those equations about an operating point that is a dynamic equilibrium described earlier in this chapter. The full equations of motion are nonlinear not only because the vertical force \( f_x \) depends on the air gap exponentially, but because the motion depends in trigonometric functions on the electrical angles of the platen with respect to the inertial frame. So, first of all, let us define the angles (so-called, Euler angles) between coordinate frames.

247
9.3.1 Euler Angles

We need a transformation between the two coordinate systems (the inertial frame and the body frame). In other words, we want to know of the orientation of the body frame $x'\ y'\ z'$ with respect to the inertial frame $xyz$. There are a few conventions to define Euler angles. I follow the $xyz$ convention, which is commonly used in engineering applications [Gol80]. We should be careful to define the angles and the order of rotations, since the finite rotations are not commutative.

The complete transformation matrix $A$ can be obtained by multiplying following three transformations, $D$, $C$ and $B$. First, Define the angle $\phi$ as a counterclockwise rotation around the inertial $z$-axis. And then, define the new intermediate coordinates as $\xi\eta\zeta$. Thus, the initial rotation can be represented by a matrix $D$.

$$\xi = D\vec{x} \tag{9.32}$$

Then, define the angle $\theta$ as a counterclockwise rotation around the intermediate $\eta$-axis. And then, define the other new intermediate coordinates as $\xi'\eta'\zeta'$. The second rotation can be represented by a matrix $C$.

$$\xi' = C\xi \tag{9.33}$$

Finally, Define the angle $\psi$ as a counterclockwise rotation around the intermediate $\xi'$-axis. Then, the new vector representation are matched with the vector $x'\ y'\ z'$ represented in the body frame. The final rotation can be represented by a matrix $B$.

$$x' = B\xi' \tag{9.34}$$

So, the complete transformation matrix $A$

$$x' = Ax \tag{9.35}$$

is the product of the successive transformation matrices.

$$A = BCD \tag{9.36}$$
For clarity, the transformation matrix $\mathbf{A}$ operates on the component of a vector in the unprimed frame and yields the components of the same vector in the primed frame. Incidentally, the rotational transformations in matrix form are

$$
\mathbf{D} = \begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix} \quad (9.37)
$$

$$
\mathbf{C} = \begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix} \quad (9.38)
$$

$$
\mathbf{B} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \psi & \sin \psi \\
0 & -\sin \psi & \cos \psi
\end{bmatrix} \quad (9.39)
$$

Thus, the complete transformation matrix $\mathbf{A}$ is

$$
\mathbf{A} = \begin{bmatrix}
\cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\
\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \cos \theta \sin \psi \\
\cos \psi \sin \theta \cos \phi + \sin \phi \sin \psi & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \theta \cos \psi
\end{bmatrix} \quad (9.40)
$$

The inverse transformation from the body frame to the inertial frame is represented by the inverse of the matrix $\mathbf{A}$.

$$
\mathbf{A}^{-1} = \mathbf{A}^T
$$

$$
= \begin{bmatrix}
\cos \theta \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi & \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\
\cos \theta \sin \phi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi \\
-\sin \theta & \cos \theta \sin \psi & \cos \theta \cos \psi
\end{bmatrix} \quad (9.41)
$$
9.3.2 Forces and Torques

Let the inertial frame $xyz$ be attached to the stators on the optical table. The origin of the inertial frame $xyz$ is located on the plane parallel to the top surface of the optical table (See Figure 8-22). This plane is parallel and 500-$\mu$m above to the surface which contains the top surfaces of four stators. The origin is in the middle of the stators on the plane. Thus, with the nominal sensor gap of 500 $\mu$m, the bottom surface of the platen, which also serves as the capacitance probe targets, is at $z = 0$. The body frame $x'y'z'$ is attached to the platen and its origin is at the center of mass calculated in Section 9.1.

Let $f'_1$, $f'_2$, $f'_3$ and $f'_4$ be the force vectors on four magnet arrays I, II, III and IV in the body frame (Figure 1-4). Since each magnet-stator pair generates both suspension and drive forces

\begin{align*}
  f'_1 &= f'_{1x}i'_1 + f'_{1z}i'_z \\  f'_2 &= f'_{2y}i'_y + f'_{2z}i'_z \\  f'_3 &= f'_{3x}i'_x + f'_{3z}i'_z \\  f'_4 &= f'_{4y}i'_y + f'_{4z}i'_z,
\end{align*}

where $i'_x$, $i'_y$ and $i'_z$ are the unit vectors in the body frame.

Now, we define the displacement vectors from the center of mass of the platen (the origin of the primed frame) to the center of mass of each of four magnet arrays as $R'_1 = \begin{bmatrix} X'_1 & Y'_1 & Z'_1 \end{bmatrix}^T$, $R'_2 = \begin{bmatrix} X'_2 & Y'_2 & Z'_2 \end{bmatrix}^T$, $R'_3 = \begin{bmatrix} X'_3 & Y'_3 & Z'_3 \end{bmatrix}^T$, and $R'_4 = \begin{bmatrix} X'_4 & Y'_4 & Z'_4 \end{bmatrix}^T$. The value of the geometric parameters $X'_1 = X'_4 = Y'_3 = Y'_4 = -112.8$ mm, $X'_2 = X'_3 = Y'_1 = Y'_2 = 96.4$ mm, and $Z'_1 = Z'_2 = Z'_3 = Z'_4 = -33.4$ mm in our levitator. Let $\tau'_1$, $\tau'_2$, $\tau'_3$ and $\tau'_4$ be the force vectors on four magnet arrays I, II, III and IV in the body frame. Then, we can model the torques on the center of

\footnote{In reality, forces are distributed on the whole region of the magnet arrays. However, we can show that the resultant forces act on the center of the mass of the magnet array with a good approximation.}
the mass of the platen as follows.

\[ \tau'_1 = \mathbf{R}'_1 \times \mathbf{f}'_1 = Y'_1 f'_1 x'_x + (Z'_1 f'_1 x' - X'_1 f'_1 y' x'_y - Y'_1 f'_1 x'_z \]  
\[ \tau'_2 = \mathbf{R}'_2 \times \mathbf{f}'_2 = (Y'_2 f'_2 x' - Z'_2 f'_2 y') x'_x - X'_2 f'_2 x'_y + X'_2 f'_2 x'_z \]  
\[ \tau'_3 = \mathbf{R}'_3 \times \mathbf{f}'_3 = Y'_3 f'_3 x'_x + (Z'_3 f'_3 x' - X'_3 f'_3 y' x'_y - Y'_3 f'_3 x'_z) \]  
\[ \tau'_4 = \mathbf{R}'_4 \times \mathbf{f}'_4 = (Y'_4 f'_4 x' - Z'_4 f'_4 y') x'_x - X'_4 f'_4 x'_y + X'_4 f'_4 x'_z. \]  

9.3.3 Linearized Equations of Motion

We need to define twelve state variables to describe general motions of the platen. They are

\[ \begin{bmatrix} x & y & z & u & v & w & \psi & \theta & \phi & p' & q' & r' \end{bmatrix}^T. \]  

The first six states are the position (in m) and velocity (in m/s) components of the center of the mass of the platen with respect to the origin of the inertial frame described in the inertial frame. The seventh, eighth and ninth states are Euler angles (in rad) defined earlier. The last three states are the angular velocity (in rad/s) components of the platen described in the body frame. The angular velocity components described in the inertial frame are approximately the same as those described in the body frame in case of a small signal linearized equations of motion. However, we retain the primes on the last three states for the moment. Actually, the following relations are hold for the rate of change of the Euler angles and the angular velocity components in the inertial frame and in the body frame [Gol80].

\[ \omega' = \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} = \begin{bmatrix} \dot{\psi} - \dot{\phi} \sin \theta \\ \dot{\theta} \cos \psi + \dot{\phi} \cos \theta \sin \psi \\ -\dot{\theta} \sin \psi + \dot{\phi} \cos \theta \cos \psi \end{bmatrix} \]  
\[ \omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \psi \cos \theta \cos \phi - \dot{\theta} \sin \phi \\ \psi \cos \theta \sin \phi + \dot{\theta} \cos \phi \\ \dot{\phi} - \psi \sin \theta \end{bmatrix} \]  

251
We have six kinematic relationships among the state variables.

\[
\dot{x} = u \quad \text{(9.53)}
\]
\[
\dot{y} = v \quad \text{(9.54)}
\]
\[
\dot{z} = w \quad \text{(9.55)}
\]
\[
\dot{\psi} = p' + \frac{\sin \theta \sin \psi}{\cos \theta} q' + \frac{\sin \theta \cos \psi}{\cos \theta} r' \quad \text{(9.56)}
\]
\[
\dot{\theta} = \cos \psi q' - \sin \psi r' \quad \text{(9.57)}
\]
\[
\dot{\phi} = \frac{\sin \psi}{\cos \theta} q' + \frac{\cos \psi}{\cos \theta} r' \quad \text{(9.58)}
\]

Using the conservation of linear momentum in the inertial frame, \( \sum \mathbf{f} = \mathbf{p} = M \mathbf{\dot{v}} \),

\[
M \mathbf{\dot{u}} = \sum f_x = \sum A^{-1} f'_x
\]
\[
= (\cos \theta \cos \phi) (f'_{1x} + f'_{3x})
\]
\[
+ (\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) (f'_{2y} + f'_{4y})
\]
\[
+ (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) (f'_{1z} + f'_{2z} + f'_{3z} + f'_{4z}) \quad \text{(9.59)}
\]

\[
M \mathbf{\dot{v}} = \sum f_y = \sum A^{-1} f'_y
\]
\[
= (\cos \theta \sin \phi) (f'_{1x} + f'_{3x})
\]
\[
+ (\sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi) (f'_{2y} + f'_{4y})
\]
\[
+ (\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi) (f'_{1z} + f'_{2z} + f'_{3z} + f'_{4z}) \quad \text{(9.60)}
\]

\[
M \mathbf{\dot{w}} = \sum f_z = \sum A^{-1} f'_z
\]
\[
= - \sin \theta (f'_{1x} + f'_{3x})
\]
\[
+ \cos \theta \sin \psi (f'_{2y} + f'_{4y})
\]
\[
+ \cos \theta \cos \psi (f'_{1z} + f'_{2z} + f'_{3z} + f'_{4z}) - Mg, \quad \text{(9.61)}
\]

where \( g \) is the gravitational constant.
From Euler's equation, \( \sum \tau = \left( \frac{\partial L'}{\partial t} \right)_{\text{body}} + \omega' \times L' \). The angular momentum represented in the body frame is

\[
L' = I \omega' = (I_{x'x'}' + I_{xy}' + I_{zz}'') \dot{i}_x' + (I_{yz}' + I_{y'y} + I_{y'z}) \dot{i}_y' + (I_{x'x'} + I_{y'y} + I_{zz}'') \dot{i}_z'.
\]

(9.62)

So, the component-wise Euler’s equations are

\[
\sum \tau_x = I_{x'x'}' \dot{x}' + I_{xy}' \dot{y}' + I_{zz}' \dot{z}' - I_{yz}' \dot{y}' - I_{y'z} - I_{yy} \dot{y}' - I_{y'z} \dot{z}' \tag{9.63}
\]

\[
\sum \tau_y = I_{yz}' \dot{x}' + I_{y'y} \dot{y}' + I_{x'y}' \dot{z}' + I_{xy} \dot{y}' - I_{yy} \dot{x}' - I_{y'z} \dot{x}' \tag{9.64}
\]

\[
\sum \tau_z = I_{x'x'} \dot{x}' + I_{y'z} \dot{y}' - I_{yy} \dot{x}' - I_{y'z} \dot{y}' - I_{zz} \dot{z}' - I_{xz}' \dot{z}' \tag{9.65}
\]

From \( \sum \tau = A^{-1} \sum \tau' \),

\[
\sum \tau_x = (\cos \theta \cos \phi) (Y_1 f_{1x}' + Y_2 f_{2x}' - Z_2 f_{2y}' + Y_3 f_{3x}' + Y_4 f_{4x}' - Z_4 f_{4y}') \\
\quad + (\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) (Z_1 f_{1x}' - X_1 f_{1x}' - X_2 f_{2x}' + Z_3 f_{3x}' - X_3 f_{3x}' - X_4 f_{4x}') \\
\quad + (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) (-Y_1 f_{1y}' + X_2 f_{2x}' - Y_3 f_{3y}' + X_4 f_{4x}') \tag{9.66}
\]

\[
\sum \tau_y = (\cos \theta \sin \phi) (Y_1 f_{1x}' + Y_2 f_{2x}' - Z_2 f_{2y}' + Y_3 f_{3x}' + Y_4 f_{4x}' - Z_4 f_{4y}') \\
\quad + (\sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi) (Z_1 f_{1x}' - X_1 f_{1x}' - X_2 f_{2x}' + Z_3 f_{3x}' - X_3 f_{3x}' - X_4 f_{4x}') \\
\quad + (\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi) (-Y_1 f_{1y}' + X_2 f_{2x}' - Y_3 f_{3y}' + X_4 f_{4x}') \tag{9.67}
\]

\[
\sum \tau_z = -\sin \theta (Y_1 f_{1x}' + Y_2 f_{2x}' - Z_2 f_{2y}' + Y_3 f_{3x}' + Y_4 f_{4x}' - Z_4 f_{4y}') \\
\quad + \cos \theta \sin \psi (X_1 f_{1z}' - X_2 f_{2z}' + Z_3 f_{3z}' - X_3 f_{3z}' - X_4 f_{4z}') \\
\quad + \cos \theta \cos \psi (-Y_1 f_{1y}' + X_2 f_{2x}' - Y_3 f_{3y}' + X_4 f_{4z}'). \tag{9.68}
\]
Let us define \( k_D = \frac{1}{2} \mu_0 M_0 \gamma_0 N_m G e^{-\gamma_0} \). Using the linearization equations for forces

\[
\ddot{f}_{kx} = k_D \ddot{i}_{kD} \ddot{x} + k_D \ddot{i}_{kQ} \\
\ddot{f}_{ky} = k_D \ddot{i}_{kD} \ddot{y} + k_D \ddot{i}_{kQ} \\
\ddot{f}_{kz} = -k_D \gamma_1 \ddot{i}_{kD} \ddot{z} + k_D \ddot{i}_{kD},
\]

and the perturbation formulae, the full state equations for small signals around the equilibrium point for the levitator become

\[
\dot{x} = \ddot{u} \\
\dot{y} = \ddot{v} \\
\dot{z} = \ddot{w} \\
\dot{u} = \frac{1}{M} k_D (\ddot{i}_{1D} + \ddot{i}_{3D}) \ddot{x} + \frac{1}{M} k_D (\ddot{i}_{1Q} + \ddot{i}_{3Q}) \\
\dot{v} = \frac{1}{M} k_D (\ddot{i}_{2D} + \ddot{i}_{4D}) \ddot{y} + \frac{1}{M} k_D (\ddot{i}_{2Q} + \ddot{i}_{4Q}) \\
\dot{w} = -\frac{1}{M} k_D \gamma_1 (\ddot{i}_{1D} + \ddot{i}_{2D} + \ddot{i}_{3D} + \ddot{i}_{4D}) \ddot{z} + \frac{1}{M} k_D (\ddot{i}_{1D} + \ddot{i}_{2D} + \ddot{i}_{3D} + \ddot{i}_{4D}) \\
\dot{\psi} = \ddot{p} \\
\dot{\theta} = \ddot{q} \\
\dot{\phi} = \ddot{r}
\]

\[
\ddot{p} = I_{xx}^{-1} k_D \left(-Y_1^2 \gamma_1 \ddot{i}_{1D} - Y_2^2 \gamma_1 \ddot{i}_{2D} + Z_2^2 \ddot{i}_{3D} - Y_3^2 \gamma_1 \ddot{i}_{4D} - Y_4^2 \gamma_1 \ddot{i}_{4D} + Z_4^2 \ddot{i}_{4D} \right) \ddot{\psi} \\
+ I_{xy}^{-1} k_D \left(Z_1^2 \ddot{i}_{1D} - X_1^2 \gamma_1 \ddot{i}_{2D} + X_2^2 \gamma_1 \ddot{i}_{3D} - Z_3^2 \gamma_1 \ddot{i}_{4D} - X_4^2 \gamma_1 \ddot{i}_{4D} \right) \ddot{\theta} \\
+ I_{xx}^{-1} k_D \left(Y_1^2 \ddot{i}_{1D} + X_2^2 \ddot{i}_{2D} + Y_3^2 \ddot{i}_{3D} + X_4^2 \ddot{i}_{4D} \right) \ddot{\phi} \\
+ k_D \left(I_{xy}^{-1} Z_1 - I_{xx}^{-1} Y_1 \right) \ddot{i}_{1Q} + k_D \left(-I_{xx}^{-1} Z_2 + I_{xx}^{-1} X_2 \right) \ddot{i}_{2Q} \\
+ k_D \left(I_{xy}^{-1} Z_3 - I_{xx}^{-1} Y_3 \right) \ddot{i}_{3Q} + k_D \left(-I_{xx}^{-1} Z_4 + I_{xx}^{-1} Y_4 \right) \ddot{i}_{4Q} \\
+ k_D \left(I_{xx}^{-1} Y_1 - I_{xy}^{-1} X_1 \right) \ddot{i}_{1D} + k_D \left(I_{xx}^{-1} Y_2 - I_{xy}^{-1} X_2 \right) \ddot{i}_{2D} \\
+ k_D \left(I_{xx}^{-1} Y_3 - I_{xy}^{-1} X_3 \right) \ddot{i}_{3D} + k_D \left(I_{xx}^{-1} Y_4 - I_{xy}^{-1} X_4 \right) \ddot{i}_{4D}
\]
\[ \ddot{q} = I_{yz}^{-1} k_D \left( -Y_1^2 \gamma_1 \dot{i}_{1D} - Y_2^2 \gamma_1 \dot{i}_{2D} + Z_2^2 \dot{i}_{2D} - Y_3^2 \gamma_1 \dot{i}_{3D} - Y_4^2 \gamma_1 \dot{i}_{4D} + Z_4^2 \dot{i}_{4D} \right) \dot{\psi} \\
+ I_{yy}^{-1} k_D \left( Z_1^2 \dot{i}_{1D} - X_1^2 \gamma_1 \dot{i}_{1D} - X_2^2 \gamma_1 \dot{i}_{2D} + Z_3^2 \dot{i}_{3D} - X_3^2 \gamma_1 \dot{i}_{3D} - X_4^2 \gamma_1 \dot{i}_{4D} \right) \dot{\theta} \\
+ I_{yz}^{-1} k_D \left( Y_1^2 \dot{i}_{1D} + X_2^2 \dot{i}_{2D} + Y_3^2 \dot{i}_{3D} + X_4^2 \dot{i}_{4D} \right) \dot{\phi} \\
+ k_D \left( I_{yy}^{-1} Z_1 - I_{y2}^{-1} Y_1 \right) \dot{i}_{1Q} + k_D \left( -I_{yx}^{-1} Z_2 + I_{yx}^{-1} X_2 \right) \dot{i}_{2Q} \\
+ k_D \left( I_{yy}^{-1} Z_3 - I_{y2}^{-1} Y_3 \right) \dot{i}_{3Q} + k_D \left( -I_{yx}^{-1} Z_4 + I_{yx}^{-1} Y_4 \right) \dot{i}_{4Q} \\
+ k_D \left( I_{yx}^{-1} Y_1 - I_{yy}^{-1} X_1 \right) \dot{i}_{1D} + k_D \left( I_{yx}^{-1} Y_2 - I_{yy}^{-1} X_2 \right) \dot{i}_{2D} \\
+ k_D \left( I_{yx}^{-1} Y_3 - I_{yy}^{-1} X_3 \right) \dot{i}_{3D} + k_D \left( I_{yx}^{-1} Y_4 - I_{yy}^{-1} X_4 \right) \dot{i}_{4D} \] (9.82)

\[ \ddot{r} = I_{xx}^{-1} k_D \left( -Y_1^2 \gamma_1 \dot{i}_{1D} - Y_2^2 \gamma_1 \dot{i}_{2D} + Z_2^2 \dot{i}_{2D} - Y_3^2 \gamma_1 \dot{i}_{3D} - Y_4^2 \gamma_1 \dot{i}_{4D} + Z_4^2 \dot{i}_{4D} \right) \dot{\psi} \\
+ I_{xy}^{-1} k_D \left( Z_1^2 \dot{i}_{1D} - X_1^2 \gamma_1 \dot{i}_{1D} - X_2^2 \gamma_1 \dot{i}_{2D} + Z_3^2 \dot{i}_{3D} - X_3^2 \gamma_1 \dot{i}_{3D} - X_4^2 \gamma_1 \dot{i}_{4D} \right) \dot{\theta} \\
+ I_{xx}^{-1} k_D \left( Y_1^2 \dot{i}_{1D} + X_2^2 \dot{i}_{2D} + Y_3^2 \dot{i}_{3D} + X_4^2 \dot{i}_{4D} \right) \dot{\phi} \\
+ k_D \left( I_{xy}^{-1} Z_1 - I_{xx}^{-1} Y_1 \right) \dot{i}_{1Q} + k_D \left( -I_{xx}^{-1} Z_2 + I_{xx}^{-1} X_2 \right) \dot{i}_{2Q} \\
+ k_D \left( I_{xy}^{-1} Z_3 - I_{xx}^{-1} Y_3 \right) \dot{i}_{3Q} + k_D \left( -I_{xx}^{-1} Z_4 + I_{xx}^{-1} Y_4 \right) \dot{i}_{4Q} \\
+ k_D \left( I_{xx}^{-1} Y_1 - I_{xy}^{-1} X_1 \right) \dot{i}_{1D} + k_D \left( I_{xx}^{-1} Y_2 - I_{xy}^{-1} X_2 \right) \dot{i}_{2D} \\
+ k_D \left( I_{xx}^{-1} Y_3 - I_{xy}^{-1} X_3 \right) \dot{i}_{3D} + k_D \left( I_{xx}^{-1} Y_4 - I_{xy}^{-1} X_4 \right) \dot{i}_{4D}. \] (9.83)

In the above equations, we omitted the primes for simplicity’s sake.

Using given parameters, we get the following state space model for the levitator.
The states are rearranged to be grouped as vertical modes and lateral modes.

\[
\begin{bmatrix}
\dot{x}
\dot{y}
\dot{\phi}
\dot{u}
\dot{v}
\dot{\tau}
\dot{\psi}
\dot{\theta}
\dot{z}
\dot{p}
\dot{q}
\dot{w}
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
4.9672 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 4.9672 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 5.8410 & 0 & 0 & 0 & -63.6820 & -65.8997 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
-26.0325 & 26.0092 & 31.4955 & -31.5107 & 2.6040 & -0.0396 & -2.5942 & 0.0494
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
-0.2805 & 18.3069 & 2.2722 & 15.6741 & 43.5212 & 48.7261 & -55.5949 & -60.7998
0 & 0 & 0 & 0 & 0 & 4.9672 & 4.9672 & 4.9672 & 4.9672
\end{bmatrix}
\begin{bmatrix}
\ddot{x}
\ddot{y}
\ddot{\phi}
\ddot{u}
\ddot{v}
\ddot{\tau}
\ddot{\psi}
\ddot{\theta}
\ddot{z}
\ddot{p}
\ddot{q}
\ddot{w}
\end{bmatrix}
\]

(9.84)
9.3.4 Sensor Equations

Let us define the following measurement variables,

- \( \tilde{x}_{t1} \): position \( x \) measured by the \( X_1 \) laser interferometer (in m)
- \( \tilde{x}_{t2} \): position \( x \) measured by the \( X_2 \) laser interferometer (in m)
- \( \tilde{y}_t \): position \( y \) measured by the \( Y \) laser interferometer (in m)
- \( \tilde{u}_{t1} \): velocity \( u \) measured by the \( X_1 \) laser interferometer (in m/s)
- \( \tilde{u}_{t2} \): velocity \( u \) measured by the \( X_2 \) laser interferometer (in m/s)
- \( \tilde{v}_t \): velocity \( v \) measured by the \( Y \) laser interferometer (in m/s)
- \( \tilde{z}_{z2} \): position \( z \) measured by the \( Z_2 \) capacitance probe (in m)
- \( \tilde{z}_{z3} \): position \( z \) measured by the \( Z_3 \) capacitance probe (in m)
- \( \tilde{z}_{z4} \): position \( z \) measured by the \( Z_4 \) capacitance probe (in m)

and the following geometric parameters in metrology.

- \( d_x = 114.2 \) mm: distance in \( x \) of the mirror surface for \( x \) measurement from the center of mass
- \( d_y = 114.2 \) mm: distance in \( y \) of the mirror surface for \( y \) measurement from the center of mass
- \( d_z = 29.8 \) mm: distance in \( z \) of the capacitance targets for \( z \) measurement from the center of mass
- \( X_i = -44 \) mm: displacement in \( x \) of the \( Y \) laser beam from the \( y \)-axis
- \( Y_{t1} = 69 \) mm: displacement in \( y \) of the \( X_1 \) laser beam from the \( x \)-axis
- \( Y_{t2} = -30 \) mm: displacement in \( y \) of the \( X_1 \) laser beam from the \( x \)-axis
• $Y_{c1} = 101.6$ mm: displacement in $y$ of the $Z_1$ capacitance probe from the inertial origin

• $X_{c2} = 101.6$ mm: displacement in $x$ of the $Z_2$ capacitance probe from the inertial origin

• $Y_{c3} = -101.6$ mm: displacement in $y$ of the $Z_3$ capacitance probe from the inertial origin

• $X_{c4} = -101.6$ mm: displacement in $x$ of the $Z_4$ capacitance probe from the inertial origin

• $Z_l = 83.3$ mm: height in $z$ of the $Y$ laser beam from the inertial origin

• $Z_{l1} = 82.6$ mm: height in $z$ of the $X_1$ laser beam from the inertial origin

• $Z_{l2} = 82.1$ mm: height in $z$ of the $X_2$ laser beam from the inertial origin

Then, the following relationships hold.

$$x_{l1} = x + \frac{d_x}{\cos \phi} + (y - Y_{l1}) \tan \phi$$ \hspace{1cm} (9.85)

$$x_{l2} = x + \frac{d_x}{\cos \phi} + (y - Y_{l2}) \tan \phi$$ \hspace{1cm} (9.86)

$$y_l = y + \frac{d_y}{\cos \phi} - (x - X_l) \tan \phi$$ \hspace{1cm} (9.87)

For small $\tilde{\phi}$ and ignoring second order terms,

$$\tilde{x}_{l1} \equiv x_{l1} - \bar{x} - \bar{d}_x \cong \tilde{x} + (\bar{y} - Y_{l1}) \tilde{\phi}$$ \hspace{1cm} (9.88)

$$\tilde{x}_{l2} \equiv x_{l2} - \bar{x} - \bar{d}_x \cong \tilde{x} + (\bar{y} - Y_{l2}) \tilde{\phi}$$ \hspace{1cm} (9.89)

$$\tilde{y}_l \equiv y_l - \bar{y} - \bar{d}_y \cong \tilde{y} - (\bar{x} - X_l) \tilde{\phi}.$$ \hspace{1cm} (9.90)

Since $\ddot{u}_{l1} = \frac{d\ddot{x}_{l1}}{dt} = \frac{d\ddot{x}_{l1}}{dt}$, and likewise,

$$\ddot{u}_{l1} = \ddot{u} + (\ddot{y} - Y_{l1}) \ddot{r}$$ \hspace{1cm} (9.91)

$$\ddot{u}_{l2} = \ddot{u} + (\ddot{y} - Y_{l2}) \ddot{r}$$ \hspace{1cm} (9.92)

$$\ddot{v}_l = \ddot{v} - (\ddot{x} - X_l) \ddot{r}.$$ \hspace{1cm} (9.93)
For the capacitance gage gaps,
\[ z_{c1} = z - \frac{d_z}{\cos \psi} - (y - Y_{c1}) \tan \psi \]  
(9.94)
\[ z_{c2} = z - \frac{d_z}{\cos \theta} + (x - X_{c2}) \tan \theta \]  
(9.95)
\[ z_{c3} = z - \frac{d_z}{\cos \psi} - (y - Y_{c3}) \tan \psi \]  
(9.96)
\[ z_{c4} = z - \frac{d_z}{\cos \theta} + (x - X_{c4}) \tan \theta. \]  
(9.97)

Recall that we use only three capacitance probes (# 2, 3 and 4) in the system. For small \( \tilde{\psi} \) and \( \tilde{\theta} \), and ignoring second order terms,
\[ \tilde{z}_c1 \equiv z_{c1} - \tilde{z} + d_z \approx \tilde{z} - (\tilde{y} - Y_{c1}) \tilde{\psi} \]  
(9.98)
\[ \tilde{z}_c2 \equiv z_{c2} - \tilde{z} + d_z \approx \tilde{z} + (\bar{x} - X_{c2}) \tilde{\theta} \]  
(9.99)
\[ \tilde{z}_c3 \equiv z_{c3} - \tilde{z} + d_z \approx \tilde{z} - (\tilde{y} - Y_{c3}) \tilde{\psi} \]  
(9.100)
\[ \tilde{z}_c4 \equiv z_{c4} - \tilde{z} + d_z \approx \tilde{z} + (\bar{x} - X_{c4}) \tilde{\theta}. \]  
(9.101)

Thus, here is the \( C \) matrix of the state space model. This \( C \) matrix depends on the large-signal variable \( \bar{x} \) and \( \bar{y} \), so it is time-varying.

\[
\begin{bmatrix}
\bar{x}_{l1} \\
\bar{x}_{l2} \\
\bar{y}_l \\
\bar{u}_{l1} \\
\bar{u}_{l2} \\
\bar{\nu}_l \\
\bar{z}_{c2} \\
\bar{z}_{c3} \\
\bar{z}_{c4}
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 & \bar{y} - Y_{l1} & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & \bar{y} - Y_{l2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & - (\bar{x} - X_l) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \bar{y} - Y_{l1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \bar{y} - Y_{l2} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & - (\bar{x} - X_l) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \bar{x} - X_{c2} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \bar{y} - Y_{c3} & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \bar{x} - X_{c4} & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\bar{x} \\
\bar{y} \\
\bar{\phi} \\
\bar{u} \\
\bar{v} \\
\bar{\tilde{\psi}} \\
\bar{\tilde{\theta}} \\
\bar{z} \\
\bar{\tilde{p}} \\
\bar{\tilde{q}} \\
\bar{\tilde{w}}
\end{bmatrix}
\]  
(9.102)
With geometric parameters in metrology system, we obtain the following $C$ matrix. Large signal variables $\tilde{x}$ and $\tilde{y}$ are set to zero here.

$$y = \begin{bmatrix} \ddot{x}_{l1} \\ \ddot{x}_{l2} \\ \ddot{y}_t \\ \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{v}_t \\ \ddot{z}_c2 \\ \ddot{z}_c3 \\ \ddot{z}_c4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.0690 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0.0300 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -0.0440 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -0.0690 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0.0300 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.0440 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.1016 & 1 \\ 0 & 0 & 0 & 0 & 0 & -0.1016 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1016 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\phi} \\ \ddot{u} \\ \ddot{v} \\ \ddot{r} \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix}$$  

(9.103)

This completes the derivation of linearized state-space equations of motion of the platen.
Chapter 10

Control

In Chapter 9, we derived decoupled and state-space dynamic models of the levitator. The dynamic models are linearized at an operating point. At the operating point, the platen is in stable equilibrium in vertical modes and in unstable equilibrium in lateral modes. In this chapter, we test and control the levitator using these dynamic models. Figure 10-1 shows the overall control loop of the stage. More specifically, the purpose of this chapter is to design the compensators block in the figure. We have two important control objectives for the levitator as a prototype semiconductor manufacturing equipment. (1) Maintain the position of the platen with smallest possible position noise during the die site exposure time. (2) Move and settle the platen as fast as possible in the transition from one die site to another.

The testing process begins with only the z-direction motion control with other five degrees of freedom confined with Mylar tapes. Eventually, all six-axis motions are stabilized and tested at the same time. Classical decoupled lead-lag control and multivariable linear quadratic control are applied to stabilize the platen motion. I also provide testing results on position error and step responses in this chapter.
Figure 10-1: Control loop of the planar magnetic levitator system
10.1 Force Allocation

To implement decoupled digital controllers for six-degree-of-freedom stabilization of the platen, we simplify the model and decouple the platen dynamics and force equations in this section.

10.1.1 Commutation Law

We derive the commutation law for the physical three-phase current (density), $J_A$, $J_B$ and $J_C$ in the context of the DQ theory. Recall the definitions of the current density components $J_a$ and $J_b$ in the previous chapter.

\[ \tilde{J}_1 = J_a + j J_b \]  
\[ \tilde{J}_{-1} = J_a - j J_b \]

The stator current density is a function of the lateral position $y$ on the stator frame and may be approximated with its fundamental components.

\[ J_x(y) = \sum_{n=-\infty}^{\infty} \tilde{J}_{xn} e^{-j k_n y} \]

\[ \cong (J_a + j J_b) e^{-j k_1 y} + (J_a - j J_b) e^{-j k_{-1} y} \]

\[ = 2 J_a \cos \gamma_1 y + 2 J_b \sin \gamma_1 y \]

The physical phase current densities $J_A$, $J_B$, $J_C$, $J'_A$, $J'_B$, $J'_C$ are located on the stator whose boundaries are at $z = 0, l/6, l/3, l/2, 2l/3, 5l/6$, and so forth, and

\[ J_A = -J'_A \]  
\[ J_B = -J'_B \]  
\[ J_C = -J'_C. \]
Then, the inverse Blondel-Park transformation holds between \([J_a \ J_b]^T\) and \([J_A \ J_B \ J_C]^T\) in the balanced three-phase operation [Blo13, Par29].

\[
\begin{bmatrix}
  J_A \\
  J_B \\
  J_C
\end{bmatrix} = \begin{bmatrix}
  2 & 0 \\
  2 \cos \frac{\pi}{3} & 2 \sin \frac{\pi}{3} \\
  2 \cos \frac{2\pi}{3} & 2 \sin \frac{2\pi}{3}
\end{bmatrix}
\begin{bmatrix}
  J_a \\
  J_b
\end{bmatrix} = \sqrt{6} T_{32}
\begin{bmatrix}
  J_a \\
  J_b
\end{bmatrix}.
\]  

(10.7)

The factor of \(\sqrt{6}\) is introduced to match up with the definition of \([J_a \ J_b]^T\). Actually, \(J_a\) is one half of the peak current density. The inverse Blondel-Park transformation, \(T_{32}\) is defined by

\[
T_{32} = \begin{bmatrix}
  \frac{\sqrt{2}}{\sqrt{3}} & 0 \\
  \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\
  -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}}
\end{bmatrix}.
\]  

(10.8)

Then, the commutation law for the physical three-phase current \([i_A \ i_B \ i_C]^T\) and the desired force \([f_x \ f_z]^T\) becomes

\[
\begin{bmatrix}
  i_A \\
  i_B \\
  i_C
\end{bmatrix} = \frac{2e^{\gamma \alpha_0}}{\mu_0 M_0 \eta_0 G N m}
\begin{bmatrix}
  1 & 0 \\
  1/2 & \sqrt{3}/2 \\
  -1/2 & \sqrt{3}/2
\end{bmatrix} e^{-\gamma \alpha_0 J}
\begin{bmatrix}
  f_x \\
  f_y
\end{bmatrix}, \quad \text{where } J = \begin{bmatrix}
  0 & -1 \\
  1 & 0
\end{bmatrix}.
\]  

(10.9)

The inverse Blondel-Park transformation matrix in the above equation has different elements from the usual inverse Blondel-Park transformation [Blo13, Par29]. The difference originates from the phase current convention we are using, and the above representation is consistent with our previous work.

We now have a relationship between the desired forces and the phase currents. Since the current levitation stage is a redundant actuator system\(^1\), we need to allocate the modal forces and torques to the vertical and lateral decomposed forces. In the following subsections, we discuss this force allocation.

\(^1\)We have eight force components, instead of six which suffice to generate all six-degree-of-freedom motions.
10.1.2 Vertical Force Allocation

The vertical force allocation can be derived by a free body diagram (Figure 10-2) and by a dynamic equilibrium. The vertical force components should generate three-degree-of-freedom focusing and alignment motions as well as cancel the weight of the platen (54.7 N). So, the nominal vertical forces should satisfy the following relation.

$$\bar{f}_{1z} + \bar{f}_{2z} + \bar{f}_{3z} + \bar{f}_{4z} = M g = 54.7 \text{ N}$$ \hspace{1cm} (10.10)

By symmetry, two large signal vertical force components $\bar{f}_{1z}$ are $\bar{f}_{3z}$ should be equal for the suspension purpose. They altogether take care of half the weight, which is arbitrarily chosen. So,

$$\bar{f}_{1z} = \bar{f}_{3z} = M g / 4.$$ \hspace{1cm} (10.11)
Then, \( \ddot{f}_{2z} \) are \( \ddot{f}_{4z} \) take the rest of the platen weight.

\[
\ddot{f}_{2z} + \ddot{f}_{4z} = \frac{Mg}{2}
\]  

(10.12)

Let us apply another equilibrium condition for torques around the \( Y \)-axis in Figure 10-2. Since the moment arms of \( \ddot{f}_{1z} \), \( \ddot{f}_{2z} \), \( \ddot{f}_{3z} \), and \( \ddot{f}_{4z} \) are \( \sqrt{2}(l_t - l_s) \), \( \sqrt{2}l_s \), \( \sqrt{2}(l_t - l_s) \), \( \sqrt{2}l_t \), respectively,

\[
(\ddot{f}_{1z} + \ddot{f}_{3z})(l_t - l_s) + \ddot{f}_{4z}l_t = \ddot{f}_{2z}l_s.
\]  

(10.13)

If we solve the above equations (10.10–10.13),

\[
\begin{bmatrix}
\ddot{f}_{1z} \\
\ddot{f}_{2z} \\
\ddot{f}_{3z} \\
\ddot{f}_{4z}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{4} \\
\frac{2l_s - l_t}{2(l_s + l_t)} \\
\frac{1}{4} \\
\frac{2l_t - l_s}{2(l_s + l_t)}
\end{bmatrix} Mg = \begin{bmatrix}
0.250 \\
0.333 \\
0.250 \\
0.167
\end{bmatrix} Mg,
\]  

(10.14)

where \( l_s + l_t = 203 \text{ mm (8")} \) is the distance between the centers of two adjacent magnet arrays. Thus, with the force-current relationship, the nominal direct components of the current are

\[
\begin{bmatrix}
\ddot{i}_{1D} \\
\ddot{i}_{2D} \\
\ddot{i}_{3D} \\
\ddot{i}_{4D}
\end{bmatrix} = \begin{bmatrix}
0.494 \\
0.658 \\
0.494 \\
0.331
\end{bmatrix}.
\]  

(10.15)

in A. The equilibrium conditions are also valid for small-signal force relations. Thus, we can use the same relations for small-signal vertical force allocation.

\[
\begin{bmatrix}
\ddot{f}_{1z} \\
\ddot{f}_{2z} \\
\ddot{f}_{3z} \\
\ddot{f}_{4z}
\end{bmatrix} = \begin{bmatrix}
0.250 \\
0.333 \\
0.250 \\
0.167
\end{bmatrix} \ddot{f}_z
\]  

(10.16)

Now let us think about the two other rotational degrees of freedom in the vertical dynamics. Since we do not want any net vertical force disturbance,

\[
\ddot{f}_{1z} + \ddot{f}_{2z} + \ddot{f}_{3z} + \ddot{f}_{4z} = 0.
\]  

(10.17)

266
Now the modal torques have the following relationships with decomposed vertical force components.

\[
\ddot{\tau}_x = (\ddot{f}_{1z} + \ddot{f}_{2z})l_s - (\ddot{f}_{3z} + \ddot{f}_{4z})l_l \tag{10.18}
\]
\[
\ddot{\tau}_y = (\ddot{f}_{4z} + \ddot{f}_{1z})l_l - (\ddot{f}_{2z} + \ddot{f}_{3z})l_s \tag{10.19}
\]

in N·m. By the symmetry of the problem for the rotations around x- and y-axes.

\[
\ddot{f}_{3z} = -\ddot{f}_{1z} \tag{10.20}
\]

Solving (10.17–10.20) yields

\[
\begin{bmatrix}
\ddot{f}_{1z} \\
\ddot{f}_{2z} \\
\ddot{f}_{3z} \\
\ddot{f}_{4z}
\end{bmatrix}
= \frac{1}{2(l_s + l_l)}
\begin{bmatrix}
\ddot{\tau}_x + \ddot{\tau}_y \\
\ddot{\tau}_x - \ddot{\tau}_y \\
-\ddot{\tau}_x - \ddot{\tau}_y \\
-\ddot{\tau}_x + \ddot{\tau}_y
\end{bmatrix} \tag{10.21}
\]

in N. So, the allocation of direct components of current is

\[
\begin{bmatrix}
\ddot{i}_{1D} \\
\ddot{i}_{2D} \\
\ddot{i}_{3D} \\
\ddot{i}_{4D}
\end{bmatrix}
= 0.0889
\begin{bmatrix}
\ddot{\tau}_x + \ddot{\tau}_y \\
\ddot{\tau}_x - \ddot{\tau}_y \\
-\ddot{\tau}_x - \ddot{\tau}_y \\
-\ddot{\tau}_x + \ddot{\tau}_y
\end{bmatrix} \tag{10.22}
\]

in A. This completes the vertical force and current allocation with the modal vertical force and torques.

### 10.1.3 Lateral Force Allocation

The following are two relationships for the lateral force components.

\[
\ddot{f}_x = \ddot{f}_{1z} + \ddot{f}_{3z} \tag{10.23}
\]
\[
\ddot{f}_y = \ddot{f}_{2y} + \ddot{f}_{4y} \tag{10.24}
\]
We need to generate the modal torque around z-axis with the four lateral force components. We have some freedom to achieve this, and the following are one of the choices.

\[ \tilde{\tau}_z = \tilde{f}_{2y} l_s - \tilde{f}_{4y} l_l \]  \hspace{1cm} (10.25)

\[ 0 = \tilde{f}_{1x} l_s + \tilde{f}_{3x} l_l \]  \hspace{1cm} (10.26)

in N-m. So, we generate \( \tilde{\tau}_z \) with \( \tilde{f}_{2y} \) and \( \tilde{f}_{4y} \) only in the above specific force allocation. The force components, \( \tilde{f}_{1x} \) and \( \tilde{f}_{3x} \) do nothing about the torque generation. Thus, the motors in y-direction will assume an additional task for small-angle adjustment around the z-axis, which is also arbitrarily chosen.

Solving (10.23–10.26) yields,

\[
\begin{bmatrix}
\tilde{f}_{1x} \\
\tilde{f}_{2y} \\
\tilde{f}_{3x} \\
\tilde{f}_{4y}
\end{bmatrix} = \frac{1}{l_s + l_l} \begin{bmatrix}
\tilde{f}_x l_l \\
\tilde{f}_y l_l + \tilde{\tau}_z \\
\tilde{f}_x l_s \\
\tilde{f}_y l_s - \tilde{\tau}_z
\end{bmatrix}
\]  \hspace{1cm} (10.27)

in N. So, the allocation of quadrature components of current is

\[
\begin{bmatrix}
\tilde{i}_{1Q} \\
\tilde{i}_{2Q} \\
\tilde{i}_{3Q} \\
\tilde{i}_{4Q}
\end{bmatrix} = \begin{bmatrix}
0.0201 \tilde{f}_x \\
0.0201 \tilde{f}_y + 0.178 \tilde{\tau}_z \\
0.0161 \tilde{f}_x \\
0.0161 \tilde{f}_y - 0.178 \tilde{\tau}_z
\end{bmatrix}
\]  \hspace{1cm} (10.28)

in A. This completes the lateral force and current allocation with the modal lateral forces and torque.
10.1.4 Modal-Decomposed Force Transformation

We present the summary of the transformation from the decomposed forces to modal forces obtained in previous subsections.

\[
\begin{bmatrix}
\tilde{f}_x \\
\tilde{f}_y \\
\tilde{r}_z \\
\tilde{r}_x \\
\tilde{r}_y \\
\tilde{f}_z
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 \\
-l_s & l_s & l_t & -l_t & 0 & 0 & 0 \\
0 & l_x & 0 & l_z & l_s & l_s & -l_t & -l_t \\
-l_z & 0 & -l_z & 0 & l_t & -l_s & -l_s & l_t \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
\tilde{f}_{1x} \\
\tilde{f}_{2y} \\
\tilde{f}_{3x} \\
\tilde{f}_{4y} \\
\tilde{f}_{1z} \\
\tilde{f}_{2z} \\
\tilde{f}_{3z} \\
\tilde{f}_{4z}
\end{bmatrix}, \tag{10.29}
\]

where the platen center-of-mass offsets are \(l_t = 112.8\) mm, \(l_s = 90.4\) mm, \(l_z = 30\) mm,\(^2\) and \(l_s + l_t = 203\) mm (8”). The above transformation matrix has a similar form to a subset of the \(B\) matrix in the state space model (9.84) except for ignorance of unimportant coupling terms and coefficients for force-current conversions.

Since the above transformation matrix from the decomposed forces to modal forces is not square, the inversion of the transformation is not unique. We discussed this freedom to allocate the decomposed forces in two previous subsections. The transfor-

\(^2\)Here, \(l_z\) is \(CM_z\) in (9.1) plus the vertical displacement between the geometric centers of the honeycomb core and the magnet array. Four vertical force components have slightly different \(l_s\)'s depending on the magnet surface heights in Figure 8-12. We use the average value \(l_z = 30\) mm.
mation from the modal forces to decomposed forces determined therein is

\[
\begin{bmatrix}
\tilde{f}_{1x} \\
\tilde{f}_{2y} \\
\tilde{f}_{3x} \\
\tilde{f}_{4y} \\
\tilde{f}_{1z} \\
\tilde{f}_{2z} \\
\tilde{f}_{3z} \\
\tilde{f}_{4z}
\end{bmatrix}
= \frac{1}{l_s + l_t}
\begin{bmatrix}
l_t & 0 & 0 & 1/2 & 1/2 & (l_s + l_t)/4 \\
0 & l_t & 1 & 0 & 0 & 0 \\
l_s & 0 & 0 & 1/2 & -1/2 & (2l_s - l_t)/2 \\
0 & l_s & -1 & -1/2 & -1/2 & (l_s + l_t)/4 \\
0 & 0 & 0 & -1/2 & 1/2 & (2l_t - l_s)/2
\end{bmatrix}
\begin{bmatrix}
\tilde{f}_x \\
\tilde{f}_y \\
\tilde{t}_z \\
\tilde{r}_x \\
\tilde{r}_y \\
\tilde{r}_z
\end{bmatrix}.
\] (10.30)

By comparing two transformations, we see there does exist a coupling from the modal vertical forces to the decomposed lateral forces. This coupling originates from the 30-mm vertical offset of the platen center of mass from the plane the lateral forces act on. For instance, a \( y \)-directional force also generates a rotational motion around the \( x \)-axis, which is actually a vertical mode motion. This coupling in the transformation is denoted by an asterisk, and we will not take this coupling into consideration for the decoupled control purpose. We ignore this coupling terms as in the previous force allocation and let it be a \( 4 \times 3 \) zero matrix. This lack of contributions of the modal vertical forces in the decomposed lateral forces would turn out to be perturbations as unmodeled dynamics. Later in the testing procedures, we confirm that this ignorance, if any, does not make the platen dynamics unstable.

\section{10.2 Decoupled Control}

In this section, we design digital controllers for six-degree-of-freedom stabilization. We use the decoupled model developed in Chapter 9. The decoupled model is second-order one representing the (negative or positive) magnetic spring and mass system. We do not model any damping for the platen dynamics in this section as the mechanical damping is comparatively much lower than what can be achieved by feedback
control. The equations of motion are represented with modal forces and torques. The relationships of the modal forces and decomposed forces were derived in the previous section.

As derived in the previous section, the platen dynamics are coupled in all six degrees of freedom. However, we neglect the coupling between the vertical and lateral modes. We thus decouple them in two three-degree-of-freedom subsystems. The current inputs for the lateral modes (\(i_0\)'s) also generate stray motions in vertical modes. So, the \(B\) matrix (the input matrix) also contains coupling terms. Interestingly enough, the current inputs for vertical modes (\(i_D\)'s) do not affect the lateral dynamics much. This is because the platen lateral dynamics except for the small perturbed rotation around \(z\)-axis have nothing to do with the vertical force components. Therefore, we decompose vertical and lateral modes simply by neglecting the coupling effects and by considering them as perturbation. Perturbed motions also come from other unmodeled dynamics of the system.

The issue of the position initialization of the platen is important because the laser interferometers give only relative displacements from initial set-points. If maintaining the position repeatability is very crucial for a certain application, then initial position setting means with high repeatability, such as high-precision differential micrometers, are required. For this prototype stage, we do not have such devices. The platen is initially placed at the center of the stators by visual means and aligned for the three channels of laser interferometers with checking the alignment indicator lamps on the laser receivers. The controllers for testing turn out to be robust enough to endure sub-millimeter-order initial position errors (on the order of ten degrees in electrical angle) which can be achieved by a careful platen placement by eye measure. However, this position error yields unrepeatable force offsets, which can be checked in the four-degree-of-freedom suspension experiment given at the end of this chapter.
10.2.1 Sampling Rate

Classical texts as [FPW90, HL85, Kuo80] give discussions on the determination of the sampling rate. The very minimum bound for the sampling rate is governed by the sampling theorem by Shannon.\(^3\) In reality, we need to oversample for the following reasons.

- Reduce phase lag due to the time-delay effect of the zero order hold. The averaged command is lagged by \(T_s/2\), half of the sampling period.

- If the sample rate is fast enough, we can emulate the dynamics of the discrete-time system as the continuous-time counterpart.

So, sampling faster than twenty times higher than the system bandwidth is advisable, if we have enough computational power.

Sampling rate determination with regard to the system resonant frequency \(\omega_r\) is discussed in [PK75]. If we sample at an integer fraction of \(2\omega_r\), the digital controller cannot see the resonance at all, and thus this produces unobservability. In the levitation system the significant resonance modes come from the mirror dynamics at 400 Hz. So, sampling at much higher than 800 Hz is desirable to avoid this sensitive resonance problem.

The sampling rate for the initial testing is set at 2.5 kHz (i.e., the sampling period \(T_s = 400 \mu s\). A control routine spends typically 120 \(\mu s\) for its completion. By optimization of the source code for the control routine, I was able to reduce the control-routine time down to 96 \(\mu s\). The optimization techniques for this reduction are as follows.

- Any overhead routines such as for display, user interface, and trajectory generation are out of the control routine.

\(^3\) The Nyquist frequency is \(\omega_s = 2\omega_b\) (where \(\omega_b\) is the system bandwidth) to restore the original signal.
• Since the Pentek 4245 A/D converter board samples the air gap information at
400 kHz at the fastest, it needs 2.5 \( \mu s \) to fill a data in the FIFO. So, instead of
idling for the next available data, the laser interferometry position data can be
read in the mean time.

• For the communication through the VMEbus, the addressing mode should be
setup properly. Minimize the number of these setup changes.

• Use 320C40 registers (instead of the memory) for frequently used variables. The
320C40 C compiler can designate even floating-point variables in registers.

Sampling up to at 5 kHz proves to be successful, and this 5-kHz sampling rate is
believed to be the maximum for this hardware setup. The 200-\( \mu s \) sampling period
consists of the real-time control routine, overhead routine, and preparations for inter-
rupts. Sophisticated control systems for fast high-precision motion control increase
the sampling rate by employing multiprocessing units individually dedicated to sub-
procedures as processing the sensor data, control law calculation, actuating signal
generation, and so forth. Higher sampling rate is generally beneficial, since the phase
lag and the quantization error decrease thanks to more samples.

With decoupled models of the levitator and the sampling rate in hand, we develop
decoupled controllers in the rest of this section.

10.2.2 Vertical Mode Control

Since vertical modes are stable at the operation point we set, it is a proper approach
to attempt to close the vertical control loop first. We confine other three-degree-of-
freedom unstable lateral motions for initial vertical motion testing. This confinement
could be achieved with foam blocks placed between the sides of the platen and the
bumper pillars. I rather choose to use Mylar tapes by 3M to fix four corners of the
top surface of the platen with eight bumper posts. By trial-and-errors, I figured out
an appropriate attachment scheme. Since the nominal lateral forces at the operation point (a dynamic equilibrium) is zero, the tapes have only to withstand small perturbation forces and lateral forces due to misplacement of the platen from the equilibrium point resulting from the tape compliance. The three-degree-of-freedom vertical modes are free from constraint with this scheme. The dynamics due to Mylar tapes, if any, are very slow and negligible.

As derived in the previous chapter, the decoupled vertical translational dynamics can be presented as follows.

$$M \frac{d^2 \ddot{z}}{dt^2} + 2 \mu_0 M_0 \eta_0 N_m G \gamma_1 e^{-\eta z_iD} \ddot{z} = 2 \mu_0 M_0 \eta_0 N_m G e^{-\eta z_iD}$$

(10.31)

By plugging nominal and geometric parameters,

$$5.58 \frac{d^2 \ddot{z}}{dt^2} + 13600 \ddot{z} = \ddot{f}_z.$$  (10.32)

The modal force $\ddot{f}_z$ is a sum of the decomposed vertical force components as in the previous section.

There are a few methods to design digital controllers [FPW90]. One of them is a $z$-plane design which uses similar classical design methodology as root locus, Nyquist plot, and Bode plot directly in the $z$-domain. We need to convert the continuous time model to a discrete-time model via the zero-order-hold equivalence to apply this method. For example, the zero-order-hold equivalence of a pure mass system $G(s) = \frac{1}{Ms^2}$ is $G(z) = \frac{T_s^2(z+1)}{2M(z-1)^2}$, where $T_s$ is the sampling period. We can also deal with the inevitable latency in digital design in this methodology. There is no choice but to use this methodology in case the sampling rate is not much faster than the system bandwidth.

Another method is design using emulation. We design a continuous-time controller for a continuous-time plant and then transfer the controller poles and zeros through the following pole-zero mapping technique.

$$z = e^{sT_s}$$  (10.33)
Classical control design methodologies such as the graphical Bode plot is still applicable. For an accurate controller design, however, the sampling rate should be much higher than the closed-loop control bandwidth. And we cannot study the delay effect in this method. It proved that the sampling rate for testing can be as high as 5 kHz thanks to the 320C40 processor’s high speed. It is much faster than the target closed-loop control bandwidth for testing, which is set 50 Hz. Thus, we can use the emulation methodology without much trouble.

The lead zero of the continuous time controller is set at 30 Hz. The lead pole is a decade higher than the lead zero. The lag zero is a decade lower than the lead zero and the lag pole is a decade lower than the lag zero. This pole-zero placement is a rule of thumb for a lead-lag compensator design. The controller is designed by using MATLAB root locus routines and the gain is determined to have the damping ratio $\zeta = 0.5$. The following lead-lag compensator is to give damping and stiffness to the system.

$$G_z(s) = 2.6966 \times 10^6 \left( \frac{s + 188.5}{s + 1885} \right) \left( \frac{s + 18.85}{s + 1.885} \right)$$  \hspace{1cm} (10.34)

Now, the dominant poles are at $-120 \pm j210$ rad/s. The closed-loop natural frequency is 39 Hz. Then, via the pole-zero mapping technique with the sampling period $T = 200 \mu s$ (sampled at 5 kHz), the corresponding digital lead-lag compensator is shown to be

$$G_z(z) = 2.3134 \times 10^6 \left( \frac{z - 0.96300}{z - 0.68592} \right) \left( \frac{z - 0.99624}{z - 0.99962} \right).$$  \hspace{1cm} (10.35)

The gain of the digital controller is determined by the DC gain matching with the corresponding continuous-time controller. Figure 10-3 shows the block diagram of this vertical motion controller with $A = 0.96300$, $B = 0.68592$, $A = 0.99624$, $A = 0.99962$, and $K = 2.3134 \times 10^6$. Figure 10-4 shows the root locus for dominant poles. The chosen closed-loop poles are denoted with a plus sign (+).

Figure 10-5 shows very close response expected by MATLAB simulations represented by the dotted line. This step response was obtained with Mylar tape confinement that prevented instability in other axes. In other words, there is no active
Figure 10-3: Decoupled lead-lag controller for $z$

Figure 10-4: Root locus for $z$
Figure 10-5: 3-μm vertical step responses by real measurement (solid) and by MATLAB simulation (dashed)

control for other five degrees of freedom except for z in this experiment. The vertical position was set at 50 μm and a 3-μm step was generated. The figure shows about 1.2-μm steady-state position error. This error comes from the fact that there is no integrator in this specific controller scheme, i.e., we place a lag pole not at s = 0. This can be fixed with integral control by placing a pole at the origin. The real response turns out to be slightly less damped than expected by the MATLAB simulation. Our second-order model implemented in MATLAB does not represent any resonance of the platen. The 400 Hz mirror resonant frequency could be one of the error sources. The error can be diminished by system identification of the platen and by using an improved model for the controller design. The other possible source of this error is the eddy current effect in the solid aluminum core. The motor force decrease due to the eddy current results in the decrease of the magnitude of the system

277
transfer function, and further the loss of the phase margin by the Bode gain-phase theorem [FPEN86]. This eddy current loss is not considered in the plant model at present. However, the two responses are in reasonably good agreement, verifying our model in this degree-of-freedom.

The vertical control-loop was tried to be closed at a higher lead zero frequency at 60 Hz, with the following compensator.

\[
G_z(z) = 0.77916 \times 10^7 \left( \frac{z - 0.92719}{z - 0.46955} \right) \left( \frac{z - 0.99247}{z - 0.99924} \right)
\]  

(10.36)

This compensator causes the platen to vibrate badly, even though the platen does not go unstable. It is believed that the mirror resonance mode is excited by significant control power around the mirror resonance.

To reduce the steady-state error in the previous experiment, the following lead-lag controller with a pole at the origin is implemented.

\[
G_z(z) = 2.3134 \times 10^6 \left( \frac{z - 0.96300}{z - 0.68592} \right) \left( \frac{z - 0.99624}{z - 1} \right).
\]  

(10.37)

Figures 10-6 and 10-7 show the loop transmission and the closed-loop Bode plot for the \( z \) motion by MATLAB simulations, respectively. The loop transmission shows a resonance peak at 8 Hz, which comes from the magnetic spring and mass resonance of the platen. As seen in Figure 10-6, the control bandwidth is 50 Hz for this controller. Figure 13-8 shows a 5-μm step response in \( z \) with this lead-lag compensator. All the six-axis controllers are operational in this experiment. Because of the coupling nature of the platen, there are perturbed motions in other five axes. The step response shows that the system loses about 20° phase margin and shows more ringings compared with the previous step response with Mylar tape confinement without other axis control. Unmodeled coupling effect is likely to be responsible for this phenomenon. The cause for this decrease in the phase margin and damping is under investigation.

After the closing the loop for the vertical translational motion, the other two control loops for vertical degree-of-freedom motions \( \psi \) and \( \theta \) are closed. Because of the geometrical symmetry in \( \psi \) and \( \theta \) of the levitation system, they have identical
Figure 10-6: Loop transmission for $z$
Figure 10-7: Closed-loop Bode plot for $z$
Figure 10-8: 5-μm step response in z with perturbed motions in the other five axes
controllers. Their gain must be different from that in the $G_i(z)$ since the numerical values of the moments of inertia ($I_{xx} = I_{yy} = 0.0541$ kg-m$^2$) are different from that of the mass. We neglect any product of inertia terms or any cross-coupling terms between the motions as alluded to previously. The desired modal forces are resolved into the decomposed forces by (10.30). Here is the controllers for $\psi$ and $\theta$.

$$G_{\psi,\theta}(z) = 2.2504 \times 10^4 \left( \frac{z - 0.96300}{z - 0.68592} \right) \left( \frac{z - 0.99624}{z - 1} \right)$$ (10.38)

Figure 10-9 shows the loop transmissions for the $\psi$ and $\theta$ motion. Figures 10-10 and 10-11 are 50-\(\mu\)rad step responses in $\psi$ and $\theta$. Again, there are significant perturbed motions in the other axes.
Figure 10-10: 50-μrad step response in $\psi$ with perturbed motions in the other five axes
Figure 10-11: 50-μrad step response in $\theta$ with perturbed motions in the other five axes
10.2.3 Lateral Mode Control

As derived in the previous chapter, the decoupled lateral translational dynamics can be presented as follows.\(^4\)

\[
M \frac{d^2 \bar{x}}{dt^2} - \mu_0 M_0 \eta_0 N_m G e^{-\gamma \bar{x}_d} \bar{x} = \mu_0 M_0 \eta_0 N_m G e^{-\gamma \bar{x}_Q} \tag{10.39}
\]

By plugging nominal and geometric parameters,

\[
5.58 \frac{d^2 \bar{x}}{dt^2} - 27.71 \bar{x} = \tilde{f}_x. \tag{10.40}
\]

The modal force \(\tilde{f}_x\) is a sum of the decomposed lateral force components as in the previous section. We design similar lead-lag compensators with MATLAB tools and use the following digital controllers for \(x\) and \(y\).

\[
G_{x,y}(z) = 2.2261 \times 10^6 \left( \frac{z - 0.96300}{z - 0.68592} \right) \left( \frac{z - 0.99624}{z - 1} \right) \tag{10.41}
\]

Similarly, the controller for \(\phi\) is

\[
G_\phi(z) = 3.9804 \times 10^4 \left( \frac{z - 0.96300}{z - 0.68592} \right) \left( \frac{z - 0.99624}{z - 1} \right). \tag{10.42}
\]

Figures 10-12 and 10-13 show the loop transmissions for \(x\), \(y\) and \(\phi\). These loop transmissions have no resonance peak as in the loop transmissions for vertical motions. Figures 10-14 through 10-16 are 5-\(\mu\)m step responses in \(x\) and \(y\) and a 50-\(\mu\)rad step response in \(\phi\), respectively. This completes the decoupled controller design for six-degree-of-freedom stabilization of the levitator.

10.2.4 Position Noise

Figure 10-17 shows the position regulation results by the decoupled lead-lag compensators designed above. The platen's position is held at the origin in the global coordinate frame. The position noises in \(x\)- and \(y\)-axes are on the order of 5 nm in the

\(^4\)We need to replace \(x\) with \(y\) for motors II and IV.
Figure 10-12: Loop transmission for $x$ and $y$
Figure 10-13: Loop transmission for $\phi$
Figure 10-14: 5-μm step response in x with perturbed motions in the other five axes
Figure 10-15: 5-μm step response in $y$ with perturbed motions in the other five axes
Figure 10-16: 50-μrad step response in φ with perturbed motions in the other five axes
Figure 10-17: Position noises in six-degree-of-freedom position regulation with the six decoupled lead-lag compensators rms sense. The position noise in $\phi$ is 0.05 $\mu$rad. For reference, it also shows the vertical displacement $z$, and the velocity $u$ in the $x$-direction. The vertical displacement shows a 70-nm-order position error envelope. The A/D electronics noise is primarily responsible for the worse position noise in $z$.

The testing result in Figure 10-17 has a dominant 120-Hz noise. Our lab floor vibration due to a HVAC equipment located in the next room is believed to generate this noise. To trace this mechanical noise more closely, I measured the position noise without levitating the platen. Figures 10-18 and 10-19 are the noise measurements with the optical table turned on and off, respectively. As seen in Figure 10-18, the
lateral modes measured by laser interferometers have a 120-Hz noise.\textsuperscript{5} The vertical modes measured by capacitance probes contain no significant trace of this noise. When the optical table is turned off as in Figure 10-19, we can see the position noise for the lateral modes are larger. It is consistent with our expectation, since the system is now more vulnerable to the floor vibration without the vibration isolation from the optical table. In the six-degree-of-freedom position regulation testing data given in Figure 10-17, therefore, the 120-Hz noise component in the vertical modes is possibly introduced through couplings from the lateral modes. The floor vibration of our lab includes other lower frequency components at 7.5 Hz, 8 Hz and probably at 15 Hz and 28 Hz. A more detailed trace of this floor vibration can be found in [Lud96].

Figures 10-18 and 10-19 show drifting tendencies of positions in $x$, $y$ and $\phi$. Figure 10-20 shows the trend in longer time frame. This tens-of-nanometer-level position fluctuation is believed to originate from the laser wavelength change due to air movement, local air density and humidity change. In the current experiment setup, no compensation for this error is provided. In a commercial high-precision positioner system, the error compensation is required to maintain the necessary nanometer-order position accuracy.

\section{10.3 Multivariable Control}

As mentioned earlier, one of the most important control objectives is to maintain the position noise of the platen as small as possible while a die site on the wafer is exposed by the lithography beam. Since only the platen generates all required motions, the dynamics is coupled in six degrees of freedom. So, a multivariable linear quadratic control is a natural choice to control the system. At present, I implemented a multivariable linear quadratic regulator for the lateral modes ($x$, $y$ and $\phi$) of the stage. A full state feedback is provided by the laser interferometer electronics (HP

\textsuperscript{5}Incidentally, a 0.6-nm position resolution by the laser interferometry is noticeable in Figure 10-17.
Figure 10-18: Position noise with the optical table air legs pressurized. Platen is resting on stators with control turned off.
Figure 10-19: Position noise with the optical table air legs not pressurized. Flaten is resting on stators with control turned off.
Figure 10-20: Position fluctuation due to the laser wavelength change induced by air movement.
10897A) for the lateral mode control; the laser interferometer also provides 24-bit velocity data over the VMEbus. Thus, we do not need to build a state estimator for velocity feedback or differentiate position data, which is susceptible to noise. We use the state-space model of the platen dynamics developed in Chapter 9 to apply the multivariable control theory.

### 10.3.1 Multivariable Linear Quadratic Control

In this subsection, we design a multivariable linear quadratic regulator for the lateral motions of the platen. If we neglect coupling terms between the vertical and lateral modes, the linearized small-signal lateral-mode dynamics can be represented as follows from (9.84).

\[
\begin{bmatrix}
\dot{\ddot{x}} \\
\dot{\ddot{y}} \\
\dot{\ddot{\phi}} \\
\dot{\ddot{u}} \\
\dot{\ddot{v}} \\
\dot{\ddot{r}}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
4.9672 & 0 & 0 & 0 & 0 & 0 \\
0 & 4.9672 & 0 & 0 & 0 & 0 \\
0 & 0 & 5.8440 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{\phi} \\
\ddot{u} \\
\ddot{v} \\
\ddot{r}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
4.9672 & 0 & 4.9672 & 0 & 0 & 0 \\
0 & 4.9672 & 0 & 4.9672 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dddot{i}_{1Q} \\
\dddot{i}_{2Q} \\
\dddot{i}_{3Q} \\
\dddot{i}_{4Q}
\end{bmatrix}
\]

\[(10.43)\]

The open-loop poles are at $\pm2.2287$, $\pm2.2287$ and $\pm2.4174$; the lateral modes are unstable. The instability comes from the negative springs from the electromagnetic origin.
Now represent the above system as follows.

$$\dot{x}(t) = Ax(t) + Bu(t)$$  \hspace{1cm} (10.44)

where $x$ is the state vector and $u$ is the input vector. Define the performance index

$$V(x(t_0), u(\cdot), t_0) = \int_{t_0}^{\infty} \left( u^T(t)Ru(t) + x^T(t)Qx(t) \right) dt.$$  \hspace{1cm} (10.45)

The time-invariant infinite-time regulator problem is the minimization problem to find an optimal control $u^*$ to minimize $V$. The solution of this problem is well-known and can be found texts on optimal control as [AM71].

$$u^*(t) = -R^{-1}B^TPx(t),$$  \hspace{1cm} (10.46)

where $P$ is the solution of an algebraic Riccati equation,

$$PA + A^TP - PBR^{-1}B^TP + Q = 0.$$  \hspace{1cm} (10.47)

Since regulating the position state variables as small as possible is one of the important control goals, we allocate heavier weight to the position states. So, the $Q$ matrix is set

$$Q = \text{diag}(10000 \ 10000 \ 10000 \ 10 \ 10 \ 10).$$  \hspace{1cm} (10.48)

The $R$ matrix is set as follows.

$$R = \text{diag}(1 \ 1 \ 1)$$  \hspace{1cm} (10.49)

We find the optimal controller associated with the performance index defined above as follows.

$$\begin{bmatrix} \tilde{i}_{1Q} \\ \tilde{i}_{2Q} \\ \tilde{i}_{3Q} \\ \tilde{i}_{4Q} \end{bmatrix}^* = \begin{bmatrix} 74.0904 & -2.8995 & -45.6450 & 4.5886 & -0.1925 & -1.6894 \\ -2.8833 & 74.1054 & 45.6230 & -0.1914 & 4.5897 & 1.6887 \\ 68.0698 & 3.1536 & 54.0928 & 4.1955 & 0.2028 & 1.9696 \\ 3.1374 & 68.0522 & -54.1164 & 0.2017 & 4.1944 & -1.9704 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\phi} \\ \tilde{\theta} \end{bmatrix}$$  \hspace{1cm} (10.50)
The corresponding closed-loop system \( \dot{x}(t) = (A - BR^{-1}B^TP)x(t) \) is

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\phi} \\
\dot{\bar{u}} \\
\dot{\bar{v}} \\
\dot{\bar{r}}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-701.23 & -1.2623 & -41.961 & -43.633 & -0.051255 & -1.3918 \\
-1.2623 & -701.22 & 42.188 & -0.051255 & -43.632 & 1.3993 \\
-41.948 & 42.175 & -5777.8 & -1.3905 & 1.3980 & -212.01
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\phi \\
\bar{u} \\
\bar{v} \\
\bar{r}
\end{bmatrix}
\] (10.51)

The closed-loop poles are at \(-179.92, -32.12, -21.84 \pm j15.01, \) and \(-21.78 \pm j15.00\). The closed-loop system with this controller is stable, which is guaranteed by the optimal control theory [AM71]. This multivariable linear quadratic regulator does stabilize the platen lateral dynamics. Since the controller is implemented digitally in a 320C40 digital signal processor, we need the control gain in the discrete-time domain. The discrete feedback gain is calculated with the 'lqrd' function in MATLAB. The 'lqrd' function discretizes the continuous plant and continuous cost function weighting matrices using the sampling period and the zero order hold approximation [Mat92]. The sampling rate of the controller is 5 kHz as in the decoupled lead-lag control.

Figure 10-21 shows the lateral position regulation result by the linear quadratic controller designed above. It shows about the same performance as that of the decoupled controller case in terms of the position noise. It is understandable since the linear quadratic regulator is optimal in the sense that it minimizes a specific performance index not the noise. The test results indicate some steady-state errors, because we have no integral control in the current design of the linear quadratic regulator.

### 10.3.2 Time-Optimal Control

The time-optimal control is an important motion control objective where a fast response saves operational cost and enhances throughput. There are many researches done on the time-optimal control especially in path planning in robotics [SM85, SY89].
Figure 10-21: Position noises in six-degree-of-freedom position regulation with the linear quadratic regulator for the lateral modes
A similar economical consideration is also applicable for wafer handling applications in the semiconductor manufacturing, since a faster movement and settling increases the machine throughput. The time-optimal control problem with actuator saturation was solved and can be found in optimal control texts as [AM71]. We adapt this result and demonstrate that the stage can generate very fast movements.

We can formulate a time-optimal control problem as a position regulation problem without penalty in the control efforts. That is, we set the performance index for the time-optimal position control in the linear quadratic control problem context as follows.

\[ V = \int_{t_0}^{\infty} x'Qx\,dt \]  

(10.52)

The above is a special case of the general form of the performance index,

\[ V = \int_{t_0}^{\infty} (x'Qx + u'Ru)\,dt \]  

(10.53)

with a constraint of the maximum available input command, \(|u| \leq u_{max}\). This singular optimal control problem was solved in the 1960’s and well-described in [AM71]. The solution is called a dual-mode control, which consists of a bang-bang control in a saturation mode and a sliding-mode control in a linear mode. In a practical system under the influence of disturbance and noise, chattering is a significant problem especially at the end of the trajectory. Workman, et al. adapted the dual-mode control solution to devise the proximate time-optimal control to fix this chattering problem [WKF87]. They use a non-zero width region for the linear mode.

Many practical systems frequently have limitations on the availability of control. Taking our levitation system for example, we designed the maximum phase current of the linear motors as 1.5 A. This serves as the above constraint of the maximum available input command, \(|i| \leq |i_{max}| = 1.5\) A. In high-precision motion control system as ours employing laser interferometer system, the laser head slew rate constraint proves to be a much more significant constraint. The laser head (HP 5517B) has the maximum velocity rating of 254 mm/s for a plane mirror measurement system. If we
accelerate the platen at 12.5 m/s², it takes only 20 ms to reach the 250-mm/s velocity and the platen displacement is 5 mm at t = 20 ms. If we use pure a bang-bang control for 20-mm steps, the velocity of the platen will be out of range because of the slew rate constraint of the laser head.

For the fastest motion control, we accelerate the platen at the highest acceleration possible until the velocity reaches the maximum slew rate, hold this velocity for a time, then decelerate the platen at the same maximum acceleration to brake the platen motion. This is called a trapezoidal velocity profile. The platen follows a parabolic, a linear, and another parabolic reference trajectory. Figure 10-22 shows test results with repetitive 20-mm steps in the y-direction. The acceleration given to the platen is 10 m/s² (about 1 g) and the platen maintains 200-mm/s velocity in the middle sections of the contour. The platen follows a 120-ms command for a 20-mm step. This fast response confirms the utility of the magnetic levitation technology in fast high-precision active motion control applications.

Testing results also show perturbed motions in z and φ, and all the other axes. These perturbed motions result from the coupling of the platen dynamics as there is only one moving part in the levitation system for all six-degree-of-freedom motion generation. These motions can be significantly reduced by feedforwarding the forces against the perturbed motions. It typically takes 300 ms for these perturbed motions to settle to the specified position noises. It apparently takes some more time for the y-movement settling.

We observe the optical table pitching while the platen makes high-acceleration movements. These pitching motions came from the action-reaction principle. These pitching motions of the mounting table deteriorate settling of the platen motions. Commercial systems usually have an active vibration cancellation mechanism that feedfowards opposite forces to reduce these motions. We did not pursue this vibration cancellation in this thesis. The magnitude of the perturbed motions in z is small
Figure 10-22: 20-mm repetitive steps in $y$ with its velocity profile ($v$) and perturbed motions in $z$ and $x$

enough that there is no collision between the platen and the stator at any time.\textsuperscript{6} We could not operate the platen at the maximum velocity rating (254 mm/s) of the laser head. Any velocity ringings which exceed 254-$\mu$m at the beginning and the end of step motions make the platen unstable, since the laser interferometers lose counts.

10.4 Demonstrations

Several routines are developed to demonstrate the platen’s motion capabilities. These are described below.

\textsuperscript{6}Recall the zero point for $z$ of the platen is set at 250-$\mu$m air gap.
20-mm Steps

The platen makes two 20-mm steps to the positive \( y \)-direction, two to the negative \( y \)-direction and repeats them. During the development of this demonstration, the acceleration was increased from a minimum of 0.1-\( g \). The platen follows the trapezoidal velocity profile with 120-ms step command, which is pseudo-time optimal described in the previous section. The maximum acceleration is 10 m/s\(^2\) (about 1 \( g \)).

30-mm Diameter Circles

The platen makes a circle with 30-mm diameter per second. The main purpose of this demonstration is to show the levitation system can generate any unified two-dimensional motions. The maximum speed achieved for the demonstration is completion of a circle in \( \frac{2}{3} \) s (The velocity in this case is 140 mm/s.). At this speed, the platen is found to noticeably vibrate. However, the cause of this vibration has not yet been determined.

Four-Degree-of-Freedom Suspension

We turn off controls for the two long axes, i.e., the \( x \)- and \( y \)-axes in this demonstration. The platen is movable freely in a plane with 250-\( \mu \)m air gap, but other four-degree-of-freedom motions are regulated by the individual modal controllers. The platen levitates smoothly as if it were on air bearings, and can be pushed by hand in any direction in the \( x-y \) plane.

Steps-and-Settles

The system simulates motions of a wafer stepper; the platen makes step-and-settle motions described in the 20-mm steps demonstration in the positive \( y \)-direction, steps over in the positive \( x \)-direction, makes step-and-settle motions in the negative \( y \)-direction, and so on. It also demonstrates that the platen can generate diagonal motions as it returns to the start of the pattern.
10.5 Summary of Part IV

In this part, we developed decoupled and state-space dynamic equations of motion for the levitator. For this purpose, electromechanical parameters, such as the mass and the inertia tensor of the platen, and the resistance and the self-inductance of the stator windings were measured and calculated. The adaptation of the DQ-decomposition theory plays a crucial role for the decoupled dynamics. A linearized state-space model was derived for a multivariable linear quadratic control purpose.

A force allocation, i.e., modal-decomposed force transformation is required for the decoupled control. With the transformation and the decoupled system model, classical lead-lag controllers are designed and tested. A multivariable linear quadratic control is also tried to stabilize the lateral dynamics of the platen. Important achievements include 5-nm position noise, 1-g acceleration, following 120-ms command for 20-mm steps. Finally, we developed demonstrations for the platen's motion capabilities—20-mm steps, 30-mm diameter circles, four-degree-of-freedom suspension, and steps-and-settles.
Chapter 11

Conclusions and Suggestions of Future Work

In this final chapter, we discuss the contributions and achievements of this thesis. I suggest incremental improvements of the current design and discuss the scaling issues of the prototype levitator for a wafer stepper in the next-generation photolithography.

11.1 Conclusions

In this thesis, I designed and implemented a world’s first high-precision six-degree-of-freedom magnetic levitator with large two-dimensional motion capability for photolithography in semiconductor manufacturing. The magnetically levitated platen generates all the required small motions for focusing and alignments as well as large planar motions for wafer positioning. This magnetic levitation stage can be readily used in a clean room, since there is no wear particle generation and no lubrication is required, since there is no mechanical contact between the moving part and the stators. This design is also highly suitable for vacuum environments, since the heat generated in the motors is in the fixed frame and thus can be readily removed by material conduction. Furthermore, there is no backlash because the levitation system
uses no intermediate power transmission device like ball screws. Thanks to the lack of friction or backlash, the position accuracy depends primarily on sensors. Thus, the stage precision depends primarily on the fundamental limits of metrology and control. Another advantage is that no fine finishing of surfaces of mechanical parts, such as for bearings, is necessary. This simplifies the production process and reduces the manufacturing cost.

We find in Part I that the one-moving part design is a technical trend and has many advantages for the high-precision position control application area. The one-moving part has a simple mechanical structure that yields fast dynamics. This fast dynamics is directly related to high throughput, which is an important design specification for manufacturing equipment. One difficulty with for the one-moving-part design is that it has a comparatively large footprint, which is economically disadvantageous in vacuum-chamber or clean-room environments. Improved designs are being sought at press time of this thesis [TK97].

I decided to implement and designed the permanent-magnet linear motors for the levitator. Strong cogging force and heat generation in the platen exclude variable-reluctance motors and induction motors, respectively. I derived a general design and analysis framework for linear permanent-magnet machines, using for this magnetic levitator as a case study. Additionally, this general framework is applied to Halbach magnet-array motors. This analytical work was extended to analyze permanent-magnet tubular linear machines. The Halbach array is chosen for the magnet arrays in the levitator as it has a stronger field and low field distortion. The force capacity, power consumption, self-inductance, back emf, and force ripple calculations resulting from this analysis serve as design tools for the development of the high-precision maglev positioning stage. I also provide experimental results with a six-degree-of-freedom magnetic levitator. These results are in good agreement with analytical estimations.

Among several design concepts, a concept with four permanent-magnet linear mo-
tors was selected for prototyping. The platen has the magnet arrays on its bottom side so that it consumes no power except for the negligible eddy current loss. In this way, thermal expansion error is minimized, and we are free from the umbilical wires. The single moving part, the platen, can be designed to have a high natural frequency and thus can be moved rapidly vis-a-vis multi-element stages which have more complex dynamics. This allows higher bandwidth and thus increases machine throughput. A shear plate damper adds significant damping to the platen. The honeycomb sandwich platen shows a high first resonant frequency of 1.22 kHz by experiments. Its damped high first resonance mode enables fast control laws to be implemented. The actuators for the levitator are four surface-wound slotless permanent-magnet linear motors with Gramme-type windings to maintain winding straightness and easy fabrication, and to avoid cogging force. They have the capability to generate magnetic suspension force as well as drive force, i.e., they are two-degree-of-freedom actuators. Thus, with the motor configuration given in this thesis, I demonstrated the generation of all six-degree-of-freedom motions for focusing and alignment, and large two-dimensional motions for positioning with only one moving part.

The magnetically levitated stage has been tested successfully. I implemented classical decoupled lead-lag as well as multivariable linear quadratic real-time digital controllers in a 320C40 digital signal processor. The sampling rate of the system is 5 kHz. The control loop for the levitator is able to be closed at a 50-Hz bandwidth. A mirror resonance at 400 Hz limits the ability to achieve high bandwidth. Several demonstrations including six-degree-of-freedom stabilization and step-and-settle motions are presented. Important experimental achievements include 5-nm rms position error in $x$ and $y$, 30-nm rms position error in $z$, 20-mm steps following 120-ms references, and 1-$g$ acceleration. I thus have demonstrated that the planar magnetic levitator is a promising candidate as the positioning stage in next-generation semiconductor manufacturing equipment.
11.2 Suggestions for Future Work

This section discusses incremental improvements to the current design and its adaptation for the future-generation wafer steppers.

11.2.1 Incremental Improvements of the Current Design

- We would rather have small grooves on the inner surfaces of the tooling for the magnet array fabrication. These grooves will prevent the epoxy from smearing in between the magnets and the tooling.

- The laser interferometers can give only relative position information with respect to the initial set position of the platen. For better repeatability, we need an initialization fixture. Some devices that can set the absolute position like differential micrometers will do. We can place the micrometers on the side of the stage opposite the laser interferometers.

- To protect the top surfaces of the stator, we may place epoxy on the surface and grind them after the epoxy cures.

- Although we can easily ignore eddy current, stator core lamination is recommended to eliminate parasitic effects, such as heat generation due to eddy currents and to prevent force reduction.

- The mirror obtained from the GCA stepper is much bigger than necessary for the current system. To get better dynamic properties for the whole platen, we would be better off with a carefully designed mirror with a proper size. To minimize the thermal expansion of the mirror, a square mirror on the Zerodur material is highly recommended.

- We should use vacuum-compatible epoxy and wire with vacuum-compatible insulation in case that the system is intended to be used in a vacuum chamber.
Insulation material and epoxy can evaporate in a low pressure in vacuum environments. We did not care much about this requirement in this prototype design.

- We could use five channels of laser interferometry, i.e., all six degrees of freedom except for the vertical translational degrees of freedom, to enhance rotational position resolution around $x$- and $y$-axes. Then, only one capacitance probe is required below the platen to measure $z$-directional motions.

- Ceramic materials can be better for the platen in terms of the structural stiffness, which yields high resonant frequency.

### 11.2.2 Accommodating Larger Wafers—Scaling Issues

It is a strong trend to use larger wafers to enhance the productivity in semiconductor manufacturing. A larger wafer contains more dies per wafer and yields better throughput. Conversion to 300-mm diameter wafers is expected beginning in 1998–2000. The year 2010 is considered to be a reasonable time frame for 450-mm wafers [And97]. Along with decreased feature size, the transition to larger wafers throws new challenges to semiconductor equipment manufacturers. For instance, the ratio of the position accuracy versus the total travel range decreases. To accommodate larger wafers, the future equipment must occupy a bigger footprint and the wafer handler must also be bigger. Since a larger structure generally has a lower resonant frequency, control to generate swift motions for high throughput becomes more difficult. To drive a larger and heavier platen, the system must have bigger actuators that consume more power. This larger power consumption causes other problems including aggravated thermal expansion error.

The area of the platen is quadrupled as the diameter of the wafer is doubled. The basic limitation of the one-moving part design is that the coil sets must have dimensions of at least travel plus magnet width. I discuss below how the current
prototype design can be adapted in this situation.

- In the current design, each stator consumes only several watts to levitate the platen. So, this small amount of heat is easily removed through the low thermal resistance path to the optical table. To drive a heavier platen, we need bigger rated motors. It is believed that the motor current can be raised to as much as 15 A which is ten times bigger than the current maximum current level. In this case, however, active cooling of the motor stator is required perhaps with liquid flowing through the stator core. Of course, we will also need to modify the power amplifier circuit for this bigger power rating.

- If the thermal expansion of the stator should be significant, we should mount the stators kinematically to prevent the stators from bending and for better position repeatability of them.

- Compensation of the gravity of the platen can be achieved preload magnets and steel targets above the platen. Since the preload magnets can take the gravity load, the driving magnet-array size can be reduced. In this case the lens field of view should still be unobstructed. Careful design for the additional structural dynamics of the steel targets is required.

- Using a steel plate behind the Halbach magnet array provides a better magnetic circuit and enhances the force capacity of the motor by some 10%. Development of high-remanence permanent-magnet material will also help to build high-power motors.

- To achieve a high accuracy, we need to compensate errors due to change in ambient temperature, humidity, and air movement with proper sensors. However, this additional compensation equipment was not used in the present efforts since we wanted to first demonstrate the principles of the stage levitation and control before concerning ourselves with accuracy issues.
In conclusion, this thesis has demonstrated a new class of precision positioning stages in which novel permanent-magnet linear motors are used to both suspend and drive a platen. I have developed all the supporting electromechanical analyses to predict the performance of this high-precision planar magnetic levitator. The fabricated prototype stage operates successfully at the precision required for photolithography and verifies the analyses developed in this thesis. This thesis provides a basis for the development of this new class of machines and thereby provides one of the key enabling technologies for next-generation photolithography.
Appendices
Appendix A

Planar Levitator with Superimposed Halbach Magnet Matrix

As mentioned in Chapter 3, significant efforts have been done to realize permanent-magnet motors which have unified large two-dimensional planar motion capabilities. In this appendix, I summarize the existing class of permanent-magnet matrices for planar motors and suggest a two-dimensional superimposed Halbach magnet matrix.

A.1 Magnet Matrices—Prior Art

Asakawa [Asa85], Hinds [Hin87], and Ebihara et al. [EW89] have studied such planar permanent-magnet motors. Their magnet matrices are constructed as shown in Figure A-1. In the figure, non-magnetic material is shown with blank spaces. Among them, Hinds’ magnet matrix is the direct superposition of two orthogonal conventional one-dimensional magnet arrays.\(^1\) Since the maximum residual flux density is limited

---

\(^1\)Cancellation of opposite poles from two orthogonal conventional linear magnet arrays with 180° magnetization leads to the blank spaces in the Hinds’ magnet matrix.
Figure A-1: Conventional planar motor magnet matrices (a) Asakawa (b) Hinds (c) Ebihara

by the magnetic material property, the maximum remanence of the one-dimensional magnet arrays which the magnet matrix is presumed to consist of cannot go beyond one half of that of conventional magnet arrays. This fact results in a smaller thrust force relative to one-degree-of-freedom motors.

A.2 Conception of Superimposed Halbach Magnet Matrix

An alternative is to consider the superposition of two orthogonal Halbach arrays. The realization of this idea is given in Figure A-2 [TKW96b]. In the figure, magnet blocks with an arrow have $1/\sqrt{2}$ remanence of the magnets noted with North (N) and South (S) poles. Blocks with solid arrows tip up at 45°, whereas blocks with hollow arrows tip down at 45°. As shown in the previous work [TWN93], the Halbach array has a stronger fundamental field by the factor $\sqrt{2}$. This will also be the case with the two-dimensional magnet matrices, since the magnetic field obeys linear superposition. All the previous analytic results can be applied directly to this magnet matrix via superposition. Such an array would allow the construction of planar motors with higher power efficiency than those which utilize more conventional magnetization patterns.
Figure A-2: Magnet matrix for planar motors
A.3 Analysis of Superimposed Halbach Matrix

A.3.1 Transfer Relations

Let us assume the platen is magnetized identically along y- and z-axis.

\[ M = \sum_{n=-\infty}^{\infty} \left[ \tilde{M}_{zn} \hat{i}_x + \tilde{M}_{zn} \hat{i}_z \right] e^{-j kn z'} + \left[ \tilde{M}_{zn} \hat{i}_x + \tilde{M}_{yn} \hat{i}_y \right] e^{-j kn y'} \]  \hspace{1cm} (A.1)

\[ J = \sum_{n=-\infty}^{\infty} \tilde{J}_{yn} \hat{i}_y e^{-j kn z} + \tilde{J}_{zn} \hat{i}_z e^{-j kn y} \]  \hspace{1cm} (A.2)

\[ A = \sum_{n=-\infty}^{\infty} \tilde{A}_{yn} \hat{i}_y e^{-j kn z} + \tilde{A}_{zn} \hat{i}_z e^{-j kn y} \]  \hspace{1cm} (A.3)

\[ B = \sum_{n=-\infty}^{\infty} \left[ \tilde{B}_{zn} \hat{i}_x + \tilde{B}_{zn} \hat{i}_z \right] e^{-j kn z'} + \left[ \tilde{B}_{zn} \hat{i}_x + \tilde{B}_{yn} \hat{i}_y \right] e^{-j kn y'} \]  \hspace{1cm} (A.4)

Equations (4.61–4.73) are the complete boundary conditions and transfer relations for the case that the magnet array and the winding are aligned to the y-axis. For the magnet array and the winding with z-axis alignments,

- Vector Potential

\[ \tilde{A}^{a}_{zn} = \tilde{A}^{b}_{zn} \]  \hspace{1cm} (A.5)

\[ \tilde{A}^{c}_{zn} = \tilde{A}^{d}_{zn} \]  \hspace{1cm} (A.6)

\[ \tilde{A}^{e}_{zn} = \tilde{A}^{f}_{zn} \]  \hspace{1cm} (A.7)

\[ \tilde{A}^{g}_{zn} = \tilde{A}^{h}_{zn} \]  \hspace{1cm} (A.8)

- Tangential Magnetic Flux Density

\[ \tilde{B}^{a}_{yn} - \tilde{B}^{b}_{yn} = -\mu_0 \tilde{M}_{yn} \]  \hspace{1cm} (A.9)

\[ \tilde{B}^{c}_{yn} - \tilde{B}^{d}_{yn} = \mu_0 \tilde{M}_{yn} \]  \hspace{1cm} (A.10)

\[ \tilde{B}^{e}_{yn} - \tilde{B}^{f}_{yn} = 0 \]  \hspace{1cm} (A.11)

\[ \tilde{B}^{g}_{yn} - \tilde{B}^{h}_{yn} = 0 \]  \hspace{1cm} (A.12)
A.4 Force and Commutation Law

Let the current density have only fundamental component, that is, as in (5.51), then the two-dimensional superimposed current density can be represented as

\[
J = [(J_{ya} + j J_{yb}) e^{-j k_1 z} + (J_{ya} - j J_{yb}) e^{-j k_{-1} z}] i_y \\
+ [(J_{za} + j J_{zb}) e^{-j k_1 y} + (J_{za} - j J_{zb}) e^{-j k_{-1} y}] i_z. \tag{A.18}
\]

Then, the overall force and commutation law for the two lateral degrees of freedom is a direct extension of the result in Section 5.2.

\[
\begin{bmatrix}
J_{za} \\
J_{zb} \\
J_{ya} \\
J_{yb}
\end{bmatrix} = \begin{bmatrix}
\sin \gamma_1 y_0 & -\cos \gamma_1 y_0 & -\sin \gamma_1 z_0 & \cos \gamma_1 z_0 \\
-\cos \gamma_1 y_0 & -\sin \gamma_1 y_0 & 0 & 0 \\
0 & 0 & \cos \gamma_1 z_0 & \sin \gamma_1 z_0 \\
0 & 0 & \sin \gamma_1 z_0 & \cos \gamma_1 z_0
\end{bmatrix}
\begin{bmatrix}
f_{z\lambda} \\
f_{y\lambda} \\
f_{z\lambda} \\
f_{y\lambda}
\end{bmatrix}, \tag{A.19}
\]

\[
\begin{bmatrix}
J_{zas} \\
J_{zbs} \\
J_{yas} \\
J_{ybs}
\end{bmatrix} = \begin{bmatrix}
\sin \gamma_1 y_0 & -\cos \gamma_1 y_0 & 0 \\
-\cos \gamma_1 y_0 & -\sin \gamma_1 y_0 & 0 \\
-\sin \gamma_1 z_0 & 0 & \cos \gamma_1 z_0 \\
\cos \gamma_1 z_0 & 0 & \sin \gamma_1 z_0
\end{bmatrix}
\begin{bmatrix}
f_{z\lambda d} \\
f_{y\lambda d} \\
f_{z\lambda d} \\
f_{y\lambda d}
\end{bmatrix}, \tag{A.20}
\]
where $f_x$, $f_y$, and $f_z$ are $x$, $y$, and $z$-directed forces per spatial wavelength, respectively. $f_x$, $f_y$, and $f_z$ are the desired forces for the controller. $J_{zas}$, $J_{zbs}$, $J_{yas}$, and $J_{ybs}$ are current densities calculated to achieve the desired forces. Since two orthogonal Halbach magnet arrays are superimposed, the magnitude of the magnetization $M_0$ in the above equations is a half of the actual magnetization of magnet pieces with vertical or lateral magnetization in Figure A-2.

### A.5 Planar Levitator Concept with Superimposed Halbach Magnetic Matrix

Figure A-3 shows one possible configuration for a superimposed Halbach magnet matrix permanent-magnet planar levitator. The bottom side of the platen (Figure A-4) has the two-dimensional superimposed Halbach magnet matrix in Figure A-2. The superimposed Halbach magnet matrix levitator yields the most compact platen. However, a complex stator winding pattern (stacking up or weaving) is necessary in order to make the platen generate both $x$ and $y$ motions. The platen has preload permanent magnets to compensate its gravity load. Targets for the top-surface electromagnets and for the preload magnets are omitted in the figure. The pitch and roll motions can be controlled with four electromagnets. Controlling yaw motions is possible by selective activation of winding sets, which needs switches and more complicated commutation laws. As in [Asa85, Hin87], a stationary magnet-moving winding configuration is also possible.
Figure A-3: Flying puck design with superimposed Halbach magnet matrix with interweaved windings
Figure A-4: Superimposed Halbach magnet matrix on bottom of the platen
Appendix B

Tubular Linear Motor

Chapter 4 presents the modeling and analysis of linear permanent-magnet machines. In this appendix, we extend the analysis to permanent-magnet tubular linear machines. Brief descriptions on the modified Bessel functions and the variation of the parameters method are given as mathematical preliminaries. The transfer relations between the magnetic flux density and the vector potential for an axisymmetric geometry are derived. A 50-lb thrust tubular linear motor was constructed by Michael Berhan, a Master’s student, on the basis of these analyses [Ber96].

B.1 Tubular Motor Model

Figure B-1 is the basic arrangement for radially magnetizing a cylindrical magnet [Mos76], and Figure B-2 is a model with a tubular Halbach array. The magnet array consists of alternating radially- and axially-magnetized rings as shown in the figure. For analytical simplicity, we assume that the total equivalent charges on the inner and outer surfaces are the same. From magnetic surface charge density, $\sigma_m = n \times \mu_0 M$ [HM89], the total magnetic charge on the inner surface is $q_m = -2\pi \mu_0 \gamma DM(\gamma)$. On the outer surface, $q_m = 2\pi \mu_0 \delta DM(\delta)$.

In order that the total charges are the same, $M$ must depend on the inverse of
Figure B-1: Fabrication of radially magnetized tubular magnet

Figure B-2: Tubular linear motor model with a tubular Halbach array and three-phase windings
the radius $r$. So, in the tubular case, the magnetization vector should be represented as follows.

$$M = \sum_{n=-\infty}^{\infty} [M_{rn} \mathbf{i}_r + M_{zn} \mathbf{i}_z] = \sum_{n=-\infty}^{\infty} \left[ \frac{\tilde{M}_{rn}}{r} e^{-jknz} \mathbf{i}_r + \tilde{M}_{zn} e^{-jknz} \mathbf{i}_z \right], \quad (B.1)$$

where $\tilde{M}_{rn}$ and $\tilde{M}_{zn}$ are the complex Fourier coefficients of the $n$th order radial and axial magnetization components, respectively. We see $M_{rn}$ is not a constant in $r$.

### B.2 Mathematical Preliminaries

#### B.2.1 Modified Bessel Functions [Arf85]

In problems with axisymmetric geometry, we encounter the diffusion version of the Bessel equation,

$$r^2 \frac{d^2}{dr^2} Y_\nu(kr) + r \frac{d}{dr} Y_\nu(kr) - (k^2 r^2 + \nu^2) Y_\nu(kr) = 0. \quad (B.2)$$

The solutions of (B.2) are Bessel functions of imaginary argument. The modified Bessel function of the first kind is defined as $I_\nu(x) \equiv j^{-\nu} J_\nu(jx)$. For integral $n$, it can be shown that $I_n(x) = I_{-n}(x)$. The recurrence relations satisfied by $I_\nu(x)$ are

$$I_{\nu-1}(x) + I_{\nu+1}(x) = 2I'_\nu(x). \quad (B.3)$$

Especially, $I'_0(x) = I_1(x)$.

The second set of independent solutions of (B.2) can be defined in terms of the Hankel function, $H^{(1)}_\nu(x)$ by $K_\nu(x) \equiv \frac{\pi}{2} j^{\nu+1} H^{(1)}_\nu(jx)$. For integral $n$, it can be shown that $K_n(x) = K_{-n}(x)$. The recurrence relations for $K_\nu(x)$ are

$$K_{\nu-1}(x) + K_{\nu+1}(x) = -2K'_\nu(x). \quad (B.4)$$

Especially, $K'_0(x) = -K_1(x)$.

$I_\nu(x)$ and $K_\nu(x)$ satisfy the Wronskian relation

$$W[I_\nu, K_\nu] = I_\nu(x)K'_\nu(x) - I'_\nu(x)K_\nu(x) = -\frac{1}{x}. \quad (B.5)$$

323
The modified Bessel functions with negative argument can be obtained with the following relations (44–45 in p. 80 of [Erd53])

\[ I_{\nu}(ze^{im\pi}) = e^{im\pi\nu}I_{\nu}(z) \]  \hspace{1cm} (B.6)

\[ K_{\nu}(ze^{im\pi}) = e^{-im\pi\nu}K_{\nu}(z) - j\pi\frac{\sin(m\pi\nu)}{\sin(\pi\nu)}I_{\nu}(z). \]  \hspace{1cm} (B.7)

In case \( \nu \) is an integer equal to \( n \), then

\[ \lim_{\nu \rightarrow n} \frac{\sin(l\pi\nu)}{\sin(\pi\nu)} = (-1)^{n(l+1)}, \]  \hspace{1cm} (B.8)

where \( l \) is equal to \( m - 1, m \) or \( m + 1 \) respectively. So, for real positive \( x \),

\[ I_{n}(-x) = (-1)^{n}I_{n}(x) \]  \hspace{1cm} (B.9)

\[ K_{n}(-x) = (-1)^{n}K_{n}(x) \pm j\pi I_{n}(x). \]  \hspace{1cm} (B.10)

The plus and minus signs depend on which branch of \( K_{n}(ze^{im\pi}) \) is chosen. The imaginary part of \( K_{n}(-x) \) seems troublesome, but it will eventually cancel out in the following analysis. Now we can represent the general solution of (B.2) with a linear combination of \( I_{n}(\cdot) \) and \( K_{n}(\cdot) \).

### B.2.2 Variation of Parameters [RiR68]

The method of variation of parameters enables us to find a particular solution of an inhomogeneous equation whenever two linearly independent solutions of the homogeneous equation are known. We consider the equation

\[ a(x)y'' + b(x)y' + c(x)y = f(x). \]  \hspace{1cm} (B.11)

We assume that \( y_{1}(x) \) and \( y_{2}(x) \) are linearly independent solutions of the homogeneous equation. It can be shown that the Wronskian of \( y_{1} \) and \( y_{2} \) is always different from zero if \( y_{1} \) and \( y_{2} \) are linearly independent. That is,

\[ W[y_{1}, y_{2}] = y_{1}y_{2}' - y_{1}'y_{2} \neq 0. \]  \hspace{1cm} (B.12)
Then, a solution of the inhomogeneous equation can be written as

\[ y_p = v_1(x)y_1(x) + v_2(x)y_2(x), \quad (B.13) \]

where

\[ v_1 = -\int \frac{y_2(x)f(x)}{a(x)W[y_1, y_2]} dx \quad (B.14) \]
\[ v_2 = \int \frac{y_1(x)f(x)}{a(x)W[y_1, y_2]} dx. \quad (B.15) \]

### B.3 Field Solutions

#### B.3.1 Solution to Poisson Equation

By geometry, the vector potential \( \mathbf{A} \) has only \( \theta \)-component and \( r \)- and \( z \)-dependences only. And the Fourier coefficient of the vector potential \( \tilde{A}_{\theta n} \) satisfies a scalar Poisson equation. By setting the Coulomb gauge as in the Cartesian case,

\[ \nabla^2 \mathbf{A}_n = -\nabla \times \nabla \times \mathbf{A}_n. \quad (B.16) \]

In cylindrical coordinates,

\[ \mathbf{B}_n \equiv \nabla \times \mathbf{A}_n = jk_n \tilde{A}_{\theta n} e^{-jk_n z i_z} + \left( \frac{\tilde{A}_{\theta n}}{r} + \frac{d}{dr} \tilde{A}_{\theta n} \right) e^{-jk_n z i_z}. \quad (B.17) \]

So,

\[ \tilde{B}_{rn} = jk_n \tilde{A}_{\theta n} \quad (B.18) \]
\[ \tilde{B}_{zn} = \frac{\tilde{A}_{\theta n}}{r} + \frac{d}{dr} \tilde{A}_{\theta n}. \quad (B.19) \]

And,

\[ \nabla \times \nabla \times \mathbf{A}_n = \left( -\frac{d^2 \tilde{A}_{\theta n}}{dr^2} - \frac{1}{r} \frac{d\tilde{A}_{\theta n}}{dr} + \left( k_n^2 + \frac{1}{r^2} \right) \tilde{A}_{\theta n} \right) e^{-jk_n z i_\theta}. \quad (B.20) \]

So, the \( \theta \)-component Poisson equation for the stator current is

\[ \frac{d^2 \tilde{A}_{\theta n}}{dr^2} + \frac{1}{r} \frac{d\tilde{A}_{\theta n}}{dr} - \left( k_n^2 + \frac{1}{r^2} \right) \tilde{A}_{\theta n} = -\mu_0 \tilde{J}_{\theta n}. \quad (B.21) \]
The homogeneous part of this has the form of modified Bessel equation (B.2) with $k = k_n$ and $\nu = 1$.

Let us find out a particular solution of the Poisson equation (B.21). It will be necessary in the course of deriving transfer relations in cylindrical geometry. The modified Bessel functions, $I_1(k_n r)$ and $K_1(k_n r)$ are two independent homogeneous solutions. And from the Wronskian relation (B.12),

$$I_1(k_n r)K'_1(k_n r) - I'_1(k_n r)K_1(k_n r) = -\frac{1}{k_n r}.$$  \hspace{1cm} (B.22)

The ‘prime’ operator represents the derivative with respect to the whole argument. For example, $K'_1(k_n r) = \frac{d}{d(k_n r)} K_1(k_n r)$. We look for a particular solution of (B.21) in the form $\tilde{A}_{\theta np} = v_1(r)I_1(k_n r) + v_2(r)K_1(k_n r)$. The method of variation of parameters gives a particular solution of the Poisson equation with integration terms of Bessel functions.

$$\tilde{A}_{\theta np} = \frac{\mu_0 j_{\theta n}}{k_n^2} \left( 1 - k_n r I_1(k_n r) \int_0^r K_0(k_n r) dr - k_n r K_1(k_n r) \int_0^r I_0(k_n r) dr \right)$$

$$\equiv \frac{\mu_0 j_{\theta n}}{k_n^2} A(r).$$ \hspace{1cm} (B.23)

### B.3.2 Field due to Stator Current

Using the result of the previous section, the homogeneous solution in the stator current region can be written as follows.

$$\tilde{A}_{\theta nh} = \left( \tilde{A}^b_{\theta n} - \tilde{A}_{\theta np}(\alpha) \right) \frac{K_1(k_n \beta) I_1(k_n r) - I_1(k_n \beta) K_1(k_n r)}{I_1(k_n \alpha) K_1(k_n \beta) - K_1(k_n \alpha) I_1(k_n \beta)}$$

$$+ \left( \tilde{A}^c_{\theta n} - \tilde{A}_{\theta np}(\beta) \right) \frac{I_1(k_n \alpha) K_1(k_n r) - K_1(k_n \alpha) I_1(k_n r)}{I_1(k_n \alpha) K_1(k_n \beta) - K_1(k_n \alpha) I_1(k_n \beta)}.$$ \hspace{1cm} (B.24)

From (B.19),

$$\tilde{B}_{zn} = \frac{1}{r} \frac{d}{dr} \left( r \tilde{A}_{\theta n} \right) = \frac{1}{r} \frac{d}{dr} \left( r \tilde{A}_{\theta nh} \right) + \frac{1}{r} \frac{d}{dr} \left( r \tilde{A}_{\theta np} \right).$$ \hspace{1cm} (B.25)

Here,

$$\frac{1}{r} \frac{d}{dr} \left( r \tilde{A}_{\theta np} \right) = -\mu_0 j_{\theta n} \left( I_0(k_n r) \int_0^r K_0(k_n r) dr - K_0(k_n r) \int_0^r I_0(k_n r) dr \right)$$

$$\equiv -\mu_0 j_{\theta n} B(r).$$ \hspace{1cm} (B.26)
The transfer relations for the stator current is

\[
\begin{bmatrix}
\tilde{B}_{zn}^b \\
\tilde{B}_{zn}^a
\end{bmatrix}
= -k_n^2
\begin{bmatrix}
F_0(\beta, \alpha) & G_0(\alpha, \beta) \\
G_0(\beta, \alpha) & F_0(\alpha, \beta)
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_{bn}^b \\
\tilde{A}_{bn}^a
\end{bmatrix}
- \mu_0 J_{bn}
\begin{bmatrix}
F_0(\beta, \alpha) A(\alpha) + G_0(\alpha, \beta) A(\beta) + B(\alpha) \\
G_0(\beta, \alpha) A(\alpha) + F_0(\alpha, \beta) A(\beta) + B(\beta)
\end{bmatrix},
\]  

(B.27)

where for positive wave numbers \(k_n = 2\pi n/l\),

\[
F_0(x, y) = \frac{I_0'(k_n x) K_0(k_n y) - K_0'(k_n x) I_0(k_n y)}{k_n \{I_0'(k_n y) K_0'(k_n x) - K_0'(k_n y) I_0'(k_n x)\}}
- \frac{I_1(k_n x) K_0(k_n y) + K_1(k_n x) I_0(k_n y)}{k_n \{I_1(k_n y) K_1(k_n x) - K_1(k_n y) I_1(k_n x)\}}
\]  

(B.28)

\[
G_0(x, y) = \frac{1}{k_n^2 x \{I_0'(k_n y) K_0'(k_n x) - K_0'(k_n y) I_0'(k_n x)\}}
- \frac{1}{1 - \frac{k_n^2 x \{I_1(k_n y) K_1(k_n x) - K_1(k_n y) I_1(k_n x)\}}}{1 - \frac{k_n^2 x \{I_1(-k_n y) K_1(-k_n x) - K_1(-k_n y) I_1(-k_n x)\}}{}}.
\]  

(B.29)

The wave number \(k_n\) can take a negative number. We can use the same formula for \(F_0\) and \(G_0\). Or, it can be shown that for negative arguments,

\[
F_0(x, y) = -\frac{\{-I_1(-k_n x)\} K_0(-k_n y) + \{-K_1(-k_n x)\} I_0(-k_n y)}{k_n \{-I_1(-k_n y)\} \{-K_1(-k_n x)\} - \{-K_1(-k_n y)\} \{-I_1(-k_n x)\}}
\]  

(B.30)

\[
G_0(x, y) = -\frac{1}{k_n^2 x \{-I_1(-k_n y)\} \{-K_1(-k_n x)\} - \{-K_1(-k_n y)\} \{-I_1(-k_n x)\}}.
\]  

(B.31)

So, we can use the modified Bessel functions with positive arguments to calculate \(F_0\) and \(G_0\).

From the asymptotic properties, \(\lim_{x \to \infty} I_n(x) = \infty\) and \(\lim_{x \to 0} K_n(x) = \infty\), we know

\[
F_0(\infty, \beta) = \frac{1}{\gamma_n K_1(\gamma_n \beta)}
\]  

(B.32)

\[
F_0(0, \alpha) = -\frac{1}{\gamma_n I_0(\gamma_n \alpha)}.
\]  

(B.33)

So, we can tell the transfer relations for half-infinite free region.

\[
\tilde{B}_{zn}^a = -\gamma_n \frac{K_0(\gamma_n \alpha)}{K_1(\gamma_n \alpha)} \tilde{A}_{bn}^a
\]  

(B.34)
\[ \tilde{B}_z^d = \gamma_n \frac{I_0(\gamma_n \beta)}{I_1(\gamma_n \beta)} \tilde{A}_\theta^d \]  

(B.35)

Since there is no impulse of field everywhere, vector potential is continuous at the boundaries

\[ \tilde{A}_\theta^a = \tilde{A}_\theta^b \]  

(B.36)

\[ \tilde{A}_\theta^c = \tilde{A}_\theta^d. \]  

(B.37)

There is no surface current on the surfaces of the stator. So, the magnetic flux density is also continuous.

\[ -\tilde{B}_z^a + \tilde{B}_z^b = 0 \]  

(B.38)

\[ -\tilde{B}_z^c + \tilde{B}_z^d = 0. \]  

(B.39)

The axial component of flux density \( \tilde{B}_z^a \) is obtained by solving (B.27) and (B.34–B.39).

\[
\begin{bmatrix}
\tilde{B}_z^a \\
\tilde{B}_z^d 
\end{bmatrix} = 
\begin{bmatrix}
1 - \gamma_n \frac{K_1(\gamma_n \alpha)}{K_0(\gamma_n \alpha)} F_0(\beta, \alpha) & \gamma_n \frac{I_1(\gamma_n \beta)}{I_0(\gamma_n \beta)} G_0(\alpha, \beta) \\
-\gamma_n \frac{K_1(\gamma_n \alpha)}{K_0(\gamma_n \alpha)} G_0(\beta, \alpha) & 1 + \gamma_n \frac{I_1(\gamma_n \beta)}{I_0(\gamma_n \beta)} F_0(\alpha, \beta)
\end{bmatrix}^{-1} 
\begin{bmatrix}
F_0(\beta, \alpha) \mathcal{A}(\alpha) + G_0(\alpha, \beta) \mathcal{A}(\beta) + \mathcal{B}(\alpha) \\
G_0(\beta, \alpha) \mathcal{A}(\alpha) + F_0(\alpha, \beta) \mathcal{A}(\beta) + \mathcal{B}(\beta)
\end{bmatrix}
\cdot (-\mu_0 \tilde{j}_\theta^a).
\]  

(B.40)

By the transfer relations in the half infinite region,

\[
\begin{bmatrix}
\tilde{A}_\theta^a \\
\tilde{A}_\theta^d
\end{bmatrix} = 
\begin{bmatrix}
-\frac{1}{\gamma_n} \frac{K_1(\gamma_n \alpha)}{K_0(\gamma_n \alpha)} \tilde{B}_z^a \\
\frac{1}{\gamma_n} \frac{I_1(\gamma_n \beta)}{I_0(\gamma_n \beta)} \tilde{B}_z^d
\end{bmatrix}.
\]  

(B.41)

And by (B.18),

\[
\begin{bmatrix}
\tilde{B}_r^a \\
\tilde{B}_r^d
\end{bmatrix} = 
\begin{bmatrix}
jk_n \tilde{A}_\theta^a \\
jk_n \tilde{A}_\theta^d
\end{bmatrix}.
\]  

(B.42)

Equations (B.40) and (B.42) specify the field on the outer surface of the stator current region.
B.3.3 Field due to Magnet

The curl of the $n$th order term of the magnetization in axisymmetric coordinates is calculated as

$$\nabla \times M_n = -j k_n \frac{\tilde{M}_{rn}}{r} e^{-jk_n z} \hat{z}_\theta.$$  \hspace{1cm} (B.43)

The Ampere's law with magnetic material is $\nabla \times B = \mu_0 (J_f + \nabla \times M)$. With absence of free current density $J_f$ in this problem, the vector potential satisfies the following scalar Poisson equation for $\theta$-component in the axisymmetric coordinate.

$$\frac{d^2 \tilde{A}_\theta}{dr^2} + \frac{1}{r} \frac{d \tilde{A}_\theta}{dr} - \left( k_n^2 + \frac{1}{r^2} \right) \tilde{A}_\theta = \frac{j \mu_0 k_n}{r} \tilde{M}_{rn}$$  \hspace{1cm} (B.44)

By inspection, a particular solution could be as follows

$$\tilde{A}_{\theta np} = \begin{cases} \frac{-j \mu_0}{k_n r} \tilde{M}_{rn} & (n \neq 0) \\ \frac{j \mu_0 k_n}{2 r} \ln r \tilde{M}_{rn} & (n = 0) \end{cases}.$$  \hspace{1cm} (B.45)

For motor applications, the case $n = 0$ is not interesting; a non-zero $n$ is assumed after on. With the particular solution (B.45), $\frac{d}{dr} (r \tilde{A}_{\theta np}) = 0$, so, $\tilde{B}_{zn} = \frac{d}{dr} (r \tilde{A}_{\theta n}) = \frac{1}{r} \frac{d}{dr} (r \tilde{A}_{\theta nh})$. The transfer relations for the magnet are

$$\begin{bmatrix} \tilde{B}_{zn}^f \\ \tilde{B}_{zn}^g \end{bmatrix} = -k_n^2 \begin{bmatrix} F_0(\delta, \gamma) & G_0(\gamma, \delta) \\ G_0(\delta, \gamma) & F_0(\gamma, \delta) \end{bmatrix} \begin{bmatrix} \tilde{A}_{\theta n}^f \\ \tilde{A}_{\theta n}^g \end{bmatrix} + 0.$$  \hspace{1cm} (B.46)

Using the particular solution (B.45) we chose,

$$\begin{bmatrix} \tilde{B}_{zn}^f \\ \tilde{B}_{zn}^g \end{bmatrix} = -k_n^2 \begin{bmatrix} F_0(\delta, \gamma) & G_0(\gamma, \delta) \\ G_0(\delta, \gamma) & F_0(\gamma, \delta) \end{bmatrix} \begin{bmatrix} \tilde{A}_{\theta n}^f \\ \tilde{A}_{\theta n}^g \end{bmatrix}$$

$$-j \mu_0 k_n \tilde{M}_{rn} \begin{bmatrix} F_0(\delta, \gamma)/\gamma + G_0(\gamma, \delta)/\delta \\ G_0(\delta, \gamma)/\gamma + F_0(\gamma, \delta)/\delta \end{bmatrix}.$$  \hspace{1cm} (B.47)

Equivalent surface current densities at the boundary $(f)$ and $(g)$ are

$$K_{nf}^f = M_n \times i_r = \tilde{M}_{zn} e^{-jk_n z} \hat{i}_\theta$$  \hspace{1cm} (B.48)

$$K_{nf}^g = M_n \times (-i_r) = -\tilde{M}_{zn} e^{-jk_n z} \hat{i}_\theta.$$  \hspace{1cm} (B.49)

329
So, the boundary conditions for magnetic flux density are

\[ -\tilde{B}^e_{zn} + \tilde{B}^h_{zn} = \mu_0 \tilde{M}_{zn} \]  \hspace{1cm} (B.50)

\[ -\tilde{B}^g_{zn} + \tilde{B}^h_{zn} = -\mu_0 \tilde{M}_{zn}. \]  \hspace{1cm} (B.51)

Solving (B.47) and (B.50–B.51) with the continuity of the vector potential, we obtain the axial component of flux density.

\[
\begin{bmatrix}
\tilde{B}^e_{zn} \\
\tilde{B}^h_{zn}
\end{bmatrix}
= \begin{bmatrix}
1 - \gamma_n K_1(\gamma_n \gamma) F_0(\delta, \gamma) & \gamma_n \frac{I_1(\gamma_n \delta)}{I_0(\gamma_n \delta)} G_0(\gamma, \delta) \\
-\gamma_n K_1(\gamma_n \gamma) G_0(\delta, \gamma) & 1 + \gamma_n \frac{I_1(\gamma_n \delta)}{I_0(\gamma_n \delta)} F_0(\gamma, \delta)
\end{bmatrix}^{-1}
\begin{bmatrix}
-\mu_0 \tilde{M}_{zn} - j \mu_0 k_n \tilde{M}_{rn} \{F_0(\delta, \gamma)/\gamma + G_0(\gamma, \delta)/\delta\} \\
-\mu_0 \tilde{M}_{zn} - j \mu_0 k_n \tilde{M}_{rn} \{G_0(\delta, \gamma)/\gamma + F_0(\gamma, \delta)/\delta\}
\end{bmatrix}
\hspace{1cm} (B.52)
\]

By the transfer relations outside and inside the magnet region,

\[
\begin{bmatrix}
\tilde{A}^e_{\theta n} \\
\tilde{A}^h_{\theta n}
\end{bmatrix}
= \begin{bmatrix}
-\frac{1}{\gamma_n} K_1(\gamma_n \gamma) \tilde{B}^e_{zn} \\
\frac{1}{\gamma_n} \frac{I_1(\gamma_n \delta)}{I_0(\gamma_n \delta)} \tilde{B}^h_{zn}
\end{bmatrix}
\hspace{1cm} (B.53)
\]

And by (B.18),

\[
\begin{bmatrix}
\tilde{B}^e_{rn} \\
\tilde{B}^h_{rn}
\end{bmatrix}
= \begin{bmatrix}
j k_n \tilde{A}^e_{\theta n} \\
j k_n \tilde{A}^h_{\theta n}
\end{bmatrix}
\hspace{1cm} (B.54)
\]

Equations (B.52) and (B.54) specify the field on the outer surface of the magnetized region.

**B.3.4 Approximate Stator Field Solution**

The exact solution (B.40) assumes a uniform current distribution. It contains definite integrals of Bessel functions which cause tremendous computational time. In this section, a current distribution proportional to inverse of the radius is proposed. As seen in the previous section, a Poisson equation with a source term proportional to the inverse of the radius has solutions in simpler form. It is expected that the approximate solution gives almost the same result if the radii of the tubes are sufficiently large. So,
let us consider the following simplified version of the Poisson equation. (Cf. (B.21) and (B.44)).
\[
\frac{d^2 \tilde{A}_{\theta n}}{dr^2} + \frac{1}{r} \frac{d \tilde{A}_{\theta n}}{dr} - \left( k^2_n + \frac{1}{r^2} \right) \tilde{A}_{\theta n} = -\mu_0 \frac{1}{r} \tilde{J}_{\theta n} \tag{B.55}
\]
As in (B.45), a particular solution for \( n \neq 0 \) could be \( \tilde{A}_{\theta mp} = \frac{\mu_0}{k^2_n} \tilde{J}_{\theta n} \). Now the derivation of the transfer relations is technically the same as in the magnet array case. So, the transfer relations for the simplified current distribution are
\[
\begin{bmatrix}
\tilde{B}^a_{2n} \\
\tilde{B}^d_{2n}
\end{bmatrix}
= -k^2_n
\begin{bmatrix}
F_0(\beta, \alpha) & G_0(\alpha, \beta) \\
G_0(\beta, \alpha) & F_0(\alpha, \beta)
\end{bmatrix}
\begin{bmatrix}
\tilde{A}^a_{\theta n} \\
\tilde{A}^d_{\theta n}
\end{bmatrix}
+ \mu_0 \tilde{J}_{\theta n}
\begin{bmatrix}
F_0(\beta, \alpha)/\alpha + G_0(\alpha, \beta)/\beta \\
G_0(\beta, \alpha)/\alpha + F_0(\alpha, \beta)/\beta
\end{bmatrix}. \tag{B.56}
\]
There is no equivalent surface current density at the boundary (b) and (c). So, the tangential component of the magnetic flux density should be continuous.
\[
\begin{align*}
- \tilde{B}^a_{2n} + \tilde{B}^b_{2n} &= 0 \tag{B.57} \\
- \tilde{B}^c_{2n} + \tilde{B}^d_{2n} &= 0 \tag{B.58}
\end{align*}
\]
Solving (B.57–B.58) with the continuity of the vector potential, we obtain the axial component of flux density.
\[
\begin{bmatrix}
\tilde{B}^a_{2n} \\
\tilde{B}^d_{2n}
\end{bmatrix}
= \begin{bmatrix}
1 - \gamma_n \frac{K_1(\gamma_n \alpha)}{K_0(\gamma_n \alpha)} F_0(\beta, \alpha) & \gamma_n \frac{I_1(\gamma_n \beta)}{I_0(\gamma_n \beta)} G_0(\alpha, \beta) \\
-\gamma_n \frac{K_1(\gamma_n \alpha)}{K_0(\gamma_n \alpha)} G_0(\beta, \alpha) & 1 + \gamma_n \frac{I_1(\gamma_n \beta)}{I_0(\gamma_n \beta)} F_0(\alpha, \beta)
\end{bmatrix}^{-1}
\left[
\begin{array}{c}
F_0(\beta, \alpha)/\alpha + G_0(\alpha, \beta)/\beta \\
G_0(\beta, \alpha)/\alpha + F_0(\alpha, \beta)/\beta
\end{array}
\right]
\cdot \mu_0 \tilde{J}_{\theta n}. \tag{B.59}
\]
By the transfer relations outside and inside the magnet region,
\[
\begin{bmatrix}
\tilde{A}^a_{\theta n} \\
\tilde{A}^d_{\theta n}
\end{bmatrix}
= \begin{bmatrix}
- \frac{1}{\gamma_n} \frac{K_1(\gamma_n \alpha)}{K_0(\gamma_n \alpha)} \tilde{B}^a_{2n} \\
\frac{1}{\gamma_n} \frac{I_1(\gamma_n \beta)}{I_0(\gamma_n \beta)} \tilde{B}^d_{2n}
\end{bmatrix} \tag{B.60}
\]
and
\[
\begin{bmatrix}
\tilde{B}^a_{r_n} \\
\tilde{B}^d_{r_n}
\end{bmatrix}
= \begin{bmatrix}
j k_n \tilde{A}^a_{\theta n} \\
j k_n \tilde{A}^d_{\theta n}
\end{bmatrix}. \tag{B.61}
\]
331
Equations (B.59) and (B.61) specify the field on the outer surface of the stator current region.

### B.3.5 Transfer Relations in the Air Gap

Since there is no source term in the air gap between stator and the magnet, the transfer relations are as follows.

\[
\begin{bmatrix}
\tilde{B}_zn^h \\
\tilde{B}_zn^a
\end{bmatrix}
= -k_n^2
\begin{bmatrix}
F_0(\alpha, \delta) & G_0(\delta, \alpha) \\
G_0(\alpha, \delta) & F_0(\delta, \alpha)
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_\theta^h \\
\tilde{A}_\theta^a
\end{bmatrix}
\]  

(B.62)

Solving for \( \tilde{A}_\theta^a \),

\[
\tilde{A}_\theta^a = -\frac{\tilde{B}_zn^h + k_n^2 F_0(\alpha, \delta) \tilde{A}_\theta^h}{k_n^2 G_0(\delta, \alpha)}.
\]  

(B.63)

The axial and radial flux densities at the boundary (a) are

\[
\tilde{B}_zn^a = -k_n^2 \left( G_0(\alpha, \delta) \tilde{A}_\theta^h + F_0(\delta, \alpha) \tilde{A}_\theta^a \right)
\]  

(B.64)

\[
\tilde{B}_r a = jk_n \tilde{A}_\theta a.
\]  

(B.65)

### B.4 Boundary Conditions and Transfer Relations

This section summarizes boundary conditions and transfer relations for tubular motors.

- **Vector Potential**

  \[
  \tilde{A}_\theta c^e = \tilde{A}_\theta c^f
  \]  

  (B.66)

  \[
  \tilde{A}_\theta a^c = \tilde{A}_\theta b^c
  \]  

  (B.67)

  \[
  \tilde{A}_\theta c^a = \tilde{A}_\theta c^b
  \]  

  (B.68)

  \[
  \tilde{A}_\theta a^c = \tilde{A}_\theta d^c
  \]  

  (B.69)

- **Tangential Magnetic Flux Density**
\[ -\bar{B}_{zn}^e + \bar{B}_{zn}^f = \mu_0 \bar{M}_{zn} \] (B.70)
\[ -\bar{B}_{zn}^g + \bar{B}_{zn}^h = -\mu_0 \bar{M}_{zn} \] (B.71)
\[ -\bar{B}_{zn}^a + \bar{B}_{zn}^b = 0 \] (B.72)
\[ -\bar{B}_{zn}^c + \bar{B}_{zn}^d = 0 \] (B.73)

- Transfer Relations

\[ \bar{B}_{zn}^e = -\gamma_n \frac{K_0(\gamma_n \gamma)}{K_1(\gamma_n \gamma)} \tilde{A}_{\theta n}^c \] (B.74)

\[
\begin{bmatrix}
\bar{B}_{zn}^f \\
\bar{B}_{zn}^g
\end{bmatrix} = -k_n^2
\begin{bmatrix}
F_0(\delta, \gamma) & G_0(\gamma, \delta) \\
G_0(\delta, \gamma) & F_0(\gamma, \delta)
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_{\theta n}^f \\
\tilde{A}_{\theta n}^g
\end{bmatrix}
- j\mu_0 k_n \bar{M}_{rn}
\begin{bmatrix}
F_0(\delta, \gamma)/\gamma + G_0(\gamma, \delta)/\delta \\
G_0(\delta, \gamma)/\gamma + F_0(\gamma, \delta)/\delta
\end{bmatrix}
\] (B.75)

\[
\begin{bmatrix}
\bar{B}_{zn}^h \\
\bar{B}_{zn}^a
\end{bmatrix} = -k_n^2
\begin{bmatrix}
F_0(\alpha, \delta) & G_0(\delta, \alpha) \\
G_0(\alpha, \delta) & F_0(\delta, \alpha)
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_{\theta n}^h \\
\tilde{A}_{\theta n}^a
\end{bmatrix}
\] (B.76)

\[
\begin{bmatrix}
\bar{B}_{zn}^b \\
\bar{B}_{zn}^c
\end{bmatrix} = -k_n^2
\begin{bmatrix}
F_0(\beta, \alpha) & G_0(\alpha, \beta) \\
G_0(\beta, \alpha) & F_0(\alpha, \beta)
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_{\theta n}^b \\
\tilde{A}_{\theta n}^c
\end{bmatrix}
+ \mu_0 \tilde{j}_{\theta n}
\begin{bmatrix}
F_0(\beta, \alpha)/\alpha + G_0(\alpha, \beta)/\beta \\
G_0(\beta, \alpha)/\alpha + F_0(\alpha, \beta)/\beta
\end{bmatrix}
\] (B.77)

\[ \bar{B}_{zn}^d = \gamma_n \frac{I_0(\gamma_n \beta)}{I_1(\gamma_n \beta)} \tilde{A}_{\theta n}^d \] (B.78)

B.5 Parametric Analysis of a Tubular Motor

I present a parametric analysis for a tubular motor in this section. The geometric parameters are \(\alpha = 0.05\) m, \(\beta = 0.04\) m, \(\gamma = 0.0635\) m, and \(\delta = 0.051\) m so that \(\Gamma = 0.01\) m, \(\Delta = 0.025\) m, and \(x_0 = 0.001\) m. The pitch is \(l = 0.05\) m. The remanence of permanent magnet is assumed 1.2 T.
Table B.1: Forces of tubular and linear motors (N/cell)

<table>
<thead>
<tr>
<th>alpha (m)</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
<th>0.35</th>
<th>0.40</th>
<th>0.45</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>tubular motor</td>
<td>67.4</td>
<td>135</td>
<td>203</td>
<td>271</td>
<td>338</td>
<td>406</td>
<td>473</td>
<td>541</td>
<td>608</td>
<td>676</td>
</tr>
<tr>
<td>linear motor</td>
<td>67.4</td>
<td>135</td>
<td>202</td>
<td>270</td>
<td>337</td>
<td>405</td>
<td>472</td>
<td>540</td>
<td>607</td>
<td>674</td>
</tr>
</tbody>
</table>

For force calculation, let us set the surface for Maxwell stress tensor just outside the stator (at \(a\), or at \(r = \alpha\)). The average force per spatial period in the axial direction is

\[
f_z = -2\pi \alpha l (\vec{H}_r \cdot \vec{H}_z)_z
\]

\[
= -\frac{2\pi \alpha l}{\mu_0} \sum_{n=-\infty}^{\infty} \vec{B}_{rn}^a \vec{B}_{zn}^a
\]

\[
= -\frac{2\pi \alpha l}{\mu_0} \sum_{n=-\infty}^{\infty} (M \vec{B}_{rn}^a + S \vec{B}_{rn}^a) (M \vec{B}_{zn}^a + S \vec{B}_{zn}^a).
\]

(B.79)

Since the geometric parameters \((\alpha, \beta, \gamma, \text{and} \delta)\) are arguments of Bessel functions, it is cumbersome to carry the parameters algebraically to the end. It is even more harder with the exact stator current model which contains integrals of Bessel functions. So, for simplicity’s sake, we assume that the current distribution is purely sinusoidal. That is, the current model is as in (5.51), \(J_0 = (J_a + j J_b) e^{-jkz} + (J_a - j J_b) e^{-jk-1z}\). The force due to the fundamental field is calculated with the MAPLE symbolic mathematics software. Table B.1 shows the forces per spatial period of the simplified tubular motor model and of the linear motor with the current density \(J_a = 1 \times 10^6 \text{ A/m}^2\) and \(J_b = 0\). The forces are computed for various sizes. However, the thickness of the winding and magnetic layers, and the air gap remain unchanged. The two motors have approximately the same force capacities. We can use the result for design of a linear machine tool axis.
B.6 Implementation of a Tubular Motor

Michael Berhan, a master's student in our group, implemented a permanent-magnet linear motor in a tubular geometry with a Halbach magnet array on the basis of the developed analytic tools in this thesis [KBTL96, Ber96]. The work is motivated by the desire to develop a direct-drive linear actuator for machine tool applications. The applications for such a motor range from material handling devices to semiconductor wafer stepper applications, diamond turning machines and other precision applications. This tubular motor is intended to demonstrate the utility for such a drive unit in mid- to high-thrust applications. Many commercial linear synchronous motors use unipolar or linear brushless DC type actuators with a linear encoder. The references [CTN86, AE90] are examples of tubular linear synchronous motors. Tubular linear induction motors in various applications can be found in the literature, such as [dGH90]. A tubular linear permanent-magnet synchronous motor was chosen for our study in light of its advantages: (1) higher maximum speeds and acceleration limits, (2) minimization of power loss due to end-turn effects, (3) higher position accuracy without anti-backlash devices, (4) no direct physical constraint in the axial direction of propulsion, (5) no power loss in rotary-linear power conversion, (6) no friction except in the ball bearings that support the platen weight, and (7) normal force canceled out due to symmetry of the motor.

Figure B-3 shows the tubular motor stage and its test setup built by Michael Berhan. The motor has a three-phase and 52.32-mm pitch with 300-mm axial travel. This motor is sized via our analyses to produce 125 N at a peak current density of $3 \times 10^6 \text{ A/m}^2$, with a power dissipation of 97 W over the 9-pitch length (471 mm) which supports the windings. The three pitches of four magnet orientation directions apiece are made from twelve aluminum plates, each holding eight NdFeB magnets arranged in an octagonal ring, circumscribing a circle 67.44 mm in diameter. The rectangular magnets are characterized by a remanence $B_r = 1.08 \text{ T}$, and are $12.7 \times 12.7 \times 25.4 \text{ mm}$ in size. The Halbach magnet array consists of three pitches with four
octagonal rings per pitch. So there are a total of 96 such rectangular magnets on the platen.

This compact tubular configuration of the motor is designed to address limitations of planar permanent-magnet motor designs. When the entire stator circumference is surrounded by the magnet array, end turns are eliminated, thereby increasing the motor efficiency. This configuration also avoids edge effects because the magnet ring sees a continuous stator circumference. The cross-section of the magnet ring is an octagon rather than a circle because a radially magnetized ring magnet of the proper dimensions was difficult to obtain. The air gap between the magnets and the windings is 2.54 mm at the closest point; this was chosen to make it easy to build and align the stator in this first prototype. The plates are located in position by long rods which extend through each plate at the corners. The plate assembly is mounted to a steel carriage platen (total mass: 31 kg) riding atop four precision ball-bearing linear guides. The guide rails are set into a precision ground cast iron bed. The magnet chassis together with the carriage plate form the moving platen.
The stator uses 3 phases over 9 pitches. The motor's fifty-four individual coils (six coils per pitch) are wound from 76 turns of AWG #18 copper wire. Each coil is wound on the outside of an aluminum pipe and the coils have an outer diameter of 62.36 mm. The force output per peak ampere of the motor is calculated to be 34.4 N/A, and is taken as a constant value. Our experimental results show an average output of 26.9 N/A. For example, a 6.04-A peak phase current for a maximum current density in the winding region of $5 \times 10^6$ A/m$^2$ produces a measured thrust value of 168 N. Our results return an average experimental power to force ratio of $1.02 \times 10^{-2}$ W/N$^2$. The expected value is $6.22 \times 10^{-3}$ W/N$^2$.

The actual axial thrust measurements are about 80% of the predicted value from the tubular motor analysis. We suspect the main cause of the offset is the friction in the guide rail for the platen. The friction range as measured with a spring gage is on the order of 20 N. Also, even if we use an air gap value that averages the spacing from the minimum air gap to the corners between the magnets, it does not account for the missing spaces between the magnets. Thus, our predicted thrust values are higher than physically realizable by the system. The 20% averaged error between the predicted and measured thrust values seems reasonable considering these error sources. This discrepancy bears further study, however, the experimental results with this motor support the accuracy of the analyses developed in this appendix.
Appendix C

Superconducting Levitation—A Case Study

Since its discovery in 1986, high-temperature superconductivity opens new domains in many applications. In this appendix, I provide a case study of current high-temperature superconducting levitation technology. We review history and characteristics of superconductors to interpret designs and experimental results given in the selected papers. Even if the superconducting levitation is out of the main focus of this thesis, it is an important enabling technology that has a big impact. On the ground of reviewed materials, a new hybrid superconducting bearing is proposed. The original form of this appendix was submitted to the Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology as my Area Exam report. In is included here because our levitator topology can be used for suspension with superconducting materials. This option may be of increasing interest as higher-temperature superconductors are developed.
C.1 Introduction

Superconductivity is a physical phenomenon usually represented as lossless electric conduction. Another important, yet sometimes overlooked property of it is diamagnetism or expulsion of magnetic fields. In fact, a number of pure metals with modest resistivity at room temperature turn into superconductors when they are refrigerated near zero Kelvin. So, we call a certain material a superconductor if the material is to be superconducting under certain circumstances. More precisely, it can be a superconductor in low temperature, low current density and low magnetic field.\(^{1}\) Beyond the critical temperature \((T_c)\), the superconductor simply loses superconductivity. It ceases superconducting when magnetic field increases to the critical field \((H_c)\). Superconductivity can also be defeated by excessive current through the superconductor. The biggest amount of current density that can be flown without resistance is called the critical current density \((J_c)\). These critical circumstances vary by superconducting materials and processes to produce them. Figure C-1 shows a critical surface of a generic superconductor. The material is superconducting inside the critical surface in \(HJT\)-coordinate.

Then, what is a *high-temperature* superconductor and what does *high* temperature mean? A high-temperature superconductor (HTS) is described as a material that acts as a superconductor even at higher temperature than the boiling point of liquid nitrogen (77 K). Sometimes, it is called more specifically a high-critical-temperature (high-\(T_c\)) superconductor. The field of high-temperature superconductivity is coming out of a state of a laboratory curiosity now. We discuss advantages of high-temperature superconductors to normal metal conductors and low-temperature superconductors.

There are vast opportunities of applications of superconductivity including high-temperature superconductivity. First of all, we can imagine immediate applications with the perfect conducting property of superconductors. They compete with normal

\(^{1}\)Figure 1.2 in [BB94] is a periodic table of the elements that indicates superconducting metals with critical temperatures and critical fields.
metal conductors in many fields, such as power transmission lines, generators and motors [CKHU79, CL91, TBB93], and electromagnets [Iwa94, Wil83]. Among them, producing high magnetic field with superconducting electromagnets is a proven application. Superconducting magnets can be used in health care, for instances, magnetic resonance imaging (MRI) and nuclear magnetic resonance (NMR), or in transportation as in a superconducting maglev. However, conventional low-temperature superconductors (LTS’s) lose superconductivity under relatively low magnetic field. Metal alloy superconductors, such as niobium tin (Nb$_3$Sn), can maintain superconductivity under stronger field.

Second, one of the most important devices with superconductivity is a Josephson junction, which can be used as a fast switch. Its switching time is in the order of pico seconds, which is roughly a thousand times faster than a silicon transistor. An extremely sensitive detector with Josephson junction is called the superconducting quantum interference device (SQUID). They can detect flux densities below $10^{-14}$ T [She94]. A 30-pV resolution voltmeter using SQUIDs was announced by University of California at Berkeley [Bro95].
A third promising application field of superconductivity relies on the diamagnetic property of superconductors. A passive stable suspension without external control input can be achieved. Superconducting bearings and superconducting flywheel energy storage devices are in this category [CMM+93, NHT+95, WLH90]. In this appendix, we deal with the high-temperature superconductivity technology especially on suspension and bearing applications.

There are two ways to achieve stable suspension with high-temperature superconductors: (1) by using diamagnetism and flux pinning of bulk superconductors, and (2) by using electromagnets whose coil is made with a superconducting tape. This appendix is based on recent researches presented in three papers [MMLC93, DGM+92, GMEA+95]. They represent prominent research groups in applied superconductivity in the U.S. and Europe. The authors suggested design concepts, built prototypes, and provided experimental results. The first two papers discuss diamagnetic passive suspension [MMLC93, DGM+92] and the last one a high-temperature superconducting magnet [GMEA+95]. The papers generally aim at presenting the authors' most recent experimental results and do not deal with sufficient historical and physical background which leads to their designs. They are far from self-containing, but give only parts of on-going researches. Thus, it is necessary to review extensive references by the authors and others in order to get 'big pictures' of the whole projects.

Solid understanding of high-temperature superconducting phenomena and characteristics is also mandatory to interpret experimental results given in the selected papers. In this appendix, we investigate related physics that attempts to explain physical behaviors of the devices, although the theory for high-temperature superconductors is in a rudimentary stage at present. A brief history of superconductivity precedes the phenomena and characteristics section. It is essential to appreciate the impact of macroscopic application of high-temperature superconductivity including those suggested in the selected papers. Discussions on the three selected papers follow on the ground of these materials.
C.2 Brief History of Superconductivity

Heike Kamerlingh Onnes, a Dutch scientist, is accredited to discover superconductivity phenomenon in 1911.² His research was to confirm a tendency that resistance increases steadily as temperature decreases. He did experiments on metal properties at a very low temperature. A newly invented (in 1908) cryogenic refrigerator that could attain liquid helium at 4.2 K was available then. He discovered that mercury allowed electricity to flow without resistance at that low temperature. He even found that a conductor ring could keep current of the original strength for a year. He named this phenomenon superconductivity. The physical mechanism of superconductivity could not be explained then.

Since it was very expensive and difficult to operate cryogenic apparatus at 4.2 K, a significant amount of efforts had been made to discover superconducting material with higher critical temperature. Some metal alloys were found superconducting at 10 K in 1933. In the 1960s, alloy of niobium (Nb₃Sn) was found superconducting at a little higher temperature, and the highest critical temperature was jumped to 23 K in 1973. Superconducting electromagnets were probably the most important application by then.³ They became more popular in application when some medical equipment, such as Magnetic Resonance Imaging, became a standard device in hospitals in the 1970s. No further progress with metal alloys was made for thirteen years after 1973.

The first breakthrough towards high-temperature superconductivity was achieved by Karl Alex Müller and J. George Bednorz at IBM in Zurich [MB87] in 1986. They

²There are several good survey materials. [She94] gives a relevant overview and history of superconductivity. Fascinating pictures and informal stories of the discovery of high-temperature superconductivity can be found in [Bil91]. Refer to [Bra89] for a broad survey of levitation in conjunction with superconducting levitation. An early outlook of the application of the high-temperature superconductivity can be found in [GH88]. [Chu95] presents the most recent status of high-temperature superconductivity.

³A hybrid magnet system (resistive coils residing inside superconducting coils) has been proposed recently to achieve very high magnetic field (about 45 T) at low temperature around 2 K [MBB⁺94].
paid attention to a new ceramic oxide material with barium, lanthanum and copper, and succeeded in reaching a critical temperature of 38 K. This was a notable achievement against a widespread belief that no way could raise critical temperature above 30 K. However, another big breakthrough was achieved by Paul C. W. Chu and Maw-Kuen Wu. They reached a 92-K critical temperature with an oxide of metals, yttrium, barium and copper. The discovery of oxide ceramic superconductors was remarkable, since nitrogen is liquefied at 77 K. Liquid nitrogen costs only about a percent of liquid helium, and it has much greater cooling capacity. Those discoveries opened a new horizon in applications; they became more feasible economically. Figure C-2 shows history of superconducting critical temperatures for metal, metal-alloy and ceramic superconductors.

Subsequent discoveries followed to create a new class of high-temperature cuprate (CuO) ceramics superconductors. In their conventional notation they are YBCO (yttrium barium copper oxide, YBa$_2$Cu$_3$O$_7$; $T_c = 92$ K), BSCCO (bisnuth strontium calcium copper oxide, (Bi,Pb)$_2$Sr$_2$Ca$_2$Cu$_3$O$_{x}$; $T_c = 105$ K), TBCCO (thallium barium calcium copper oxide, TlBa$_2$Ca$_2$Cu$_3$O$_{y}$; $T_c = 115$ K), and HBCCO (mercury barium calcium copper oxide, HgBa$_2$Ca$_2$Cu$_3$O$_{y}$; $T_c = 135$ K). Under high pressure HBCCO can be superconducting at 164 K, which is attainable with technology used in household air-conditioning [Chu95]. Further, no theory yet prohibits a room-temperature superconductor. If found, it will surely be a great impact on everyday life.

C.3 Phenomena and Characteristics

In this section, we concentrate on diamagnetism, hysteresis, flux pinning, and other related phenomena and characteristics of superconductors. These properties make it

---

$^4$Heat of vaporization of nitrogen (161 J/cm$^3$) is much greater than that of helium (2.6 J/cm$^3$) [Iwa94].

$^5$Sometimes phase of compounds is specified. For example, Y-123 or YBCO-123 represents YBa$_2$Cu$_3$O$_7$. 

343
Figure C-2: History of superconducting critical temperatures (after [BB94])
possible for a rare-earth permanent magnet to overcome gravity and to levitate.

C.3.1 Earnshaw’s Theorem

A nineteenth century English minister and natural philosopher Samuel Earnshaw (1805–1888, [Sco59] for a brief biography) stated a fundamental proposition on passive electromagnetic stability [Ear42]. A modern statement of the Earnshaw’s theorem can be found in [Str41] and other texts. Here is a magnetic version of the statement.

*A magnetized body placed in an magnetostatic field cannot be maintained in stable equilibrium in systems under the influence of the magnetic force alone.*

The magnetic potential satisfies the Laplace equation, so it is a harmonic function. It has no local minima or maxima by the maximum principle [Ahl79]. Therefore, there is no static equilibrium for stable suspension of the magnetized body. However, the Earnshaw’s theorem has nothing to do with the dynamic equilibrium. This is exactly how conventional magnetic bearings work, but they need external energy for control. Another way to evade the Earnshaw’s theorem is to use diamagnetic materials. Diamagnetism is a macroscopic phenomenon of Faraday’s law and Lenz’s law acting on the atomic level. The induced current opposing field change is dispersive in normal resistive conductors. Hence, they have very weak diamagnetism, if any. If a superconductor is merely a lossless conductor, we could not take advantage of it for levitation. Yet, a superconductor is more than a lossless conductor; it is also a diamagnet. This fact was what confused people in the early stage of superconductivity. A superconductor expels existing field even when the cooling process is applied under the external field (Meissner effect—zero relative permeability or zero flux density inside superconductor). On the contrary, a perfect conductor is expected to freeze not to expel the existing field. Figure C-3 depicts this situation. A quotation from [OD91]

---

6Historical debates and experimental verification can be found in [Dah92].
Figure C-3: Low-temperature behaviors of superconductor and perfect conductor (after [OD91])

is adequate to distinguish a superconductor from a perfect conductor.

*A perfect conductor is a flux conserving medium; a superconductor is a flux repelling medium.*

### C.3.2 Meissner Effect

Discovered in 1933, the Meissner effect is related with the perfect diamagnetism of superconductors. In other words, a superconductor can keep out magnetic field. The image method can be an effective way to visualize this phenomenon. Let us suppose a permanent magnet and superconductor system, where the permanent magnet is placed above the superconducting plate (Figure C-4). We can imagine the image magnet inside the superconducting plate to keep the normal component of the flux density zero at the surface of the superconductor. Therefore, a passive levitation in
the vertical direction is possible with this permanent magnet-superconductor system. Here *passive* means there is no power input to the system to stabilize or control the attitude of the magnet.

The Meissner effect cannot be explained with conventional conduction models, like Ohm's law \( \mathbf{J} = \sigma \mathbf{E} \). Rather we postulate the current density is proportional to the vector potential [Kit86].

\[
\mathbf{J} = -\frac{1}{\mu_0 \lambda_L^2} \mathbf{A}
\]  
(C.1)

This is the London equation and \( \lambda_L \) is called the London penetration depth. Here we set the London gauge, \( \nabla \cdot \mathbf{A} = 0 \), and \( \mathbf{A}_n = 0 \), where the subscript \( n \) denotes the normal component. Thus, \( \nabla \cdot \mathbf{J} = 0 \), and \( J_n = 0 \). These are the actual boundary conditions of current at the surface of a superconductor. By a Ampere's law without time-varying electric field, \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \),

\[
\nabla^2 \mathbf{B} = -\nabla \times \nabla \times \mathbf{B} = -\mu_0 \nabla \times \mathbf{J}.
\]  
(C.2)

Thus, by the definition of vector potential, \( \nabla \times \mathbf{A} = \mathbf{B} \),

\[
\nabla^2 \mathbf{B} = \mathbf{B}/\lambda_L^2
\]  
(C.3)

This equation accounts for the Meissner effect, because the only constant solution is \( \mathbf{B} = 0 \). In other words, the only field allowed inside a pure superconductor decays
exponentially as

$$B(x) = B_0 \exp(-x/\lambda_L).$$  \hspace{1cm} (C.4)

A typical value of $\lambda_L$ of ceramic high-temperature superconductors is 200 nm [BB94].

Earlier metallic superconductors, such as mercury, tin and lead, have only one critical field. They are classified as type I. Namely, type I material expels all magnetic field below $H_c(T)$. Recall the definition of the magnetization $M$,

$$B = \mu_0(H + M).$$ \hspace{1cm} (C.5)

Under this perfect diamagnetism condition, the magnetic flux density $B$ should be zero inside the superconductor. This leads to $M = -H$ below $H_c$. Magnetic field above $H_c(T)$ can penetrate type I material freely. The magnetization $M$ is zero above $H_c$. This situation is depicted in Figure C-5 (a).

Type II superconductors including metal alloys or rare-earth metal cuprates show more complexity. Type II superconductors have two critical fields. Up to the lower critical field $H_{c1}$, they expel all the fields like type I superconductors under $H_c$. Between the lower and upper critical field some field can penetrate the superconductor, although the superconductor can conduct electricity without resistance. In other words, diamagnetism and lossless conductivity can coexist in type II superconduc-
tors. Above the upper critical field $H_{c2}$, they stop being superconductors. This situation is depicted in Figure C-5 (b). It should note that cuprate high-temperature superconductors generally allow some flux to penetrate themselves even $H < H_{c1}$ due to material imperfection.

Unfortunately, the Meissner effect alone cannot explain the stable levitation of a permanent magnet over a high-temperature superconductor. The estimated critical field for type I regime of cuprate high-temperature superconductors is typically as low as around 10 mT (= $\mu_0 H_{c1}$ at 77 K) [BB94]. It is believed that a much higher field is involved in the levitation. So, the complete diamagnetic model may not be quite useful for high-temperature superconducting levitation. Another important point is that the Meissner effect gives no account for lateral force. The Meissner model is invariant with respect to lateral translations. How can the lateral movement be stabilized? Some attempts to explain the lateral stabilization are given in the next section.

### C.3.3 Flux Pinning

It can be shown by quantum mechanics that magnetic flux is quantized [Kit86]. The flux quantum, or fluxoid is derived as

$$\Phi_0 = \frac{h}{2e} = 2.0678 \times 10^{-15} \text{ Wb},$$  

(C.6)

where $h$ is Planck’s constant and $e$ is the electron’s charge. By Abrikosov’s vortex model [Abr88], a type II superconductor comes into a vortex state after it goes over the lower critical field $H_{c1}$. The vortex state is a mixed state of the diamagnetic state with flux penetration. The superconductor changes its tiny region to normal state for a flux line to penetrate. In the superconducting region around the normal region, a circular current should exist to shield the flux line. The observed damping phenomenon with the suspended specimen is a counterexample against perfect Meissner state. If all flux is expelled from the superconductor, there is no mechanism to dissipate
energy inside the superconductor. So, we know that some flux has penetrated the superconductor in the vortex state. Of course, there is weak residual damping due to eddy current inside the permanent magnet induced by the relative movement to the superconductor, which is negligible in most cases.

When current is flowing inside the superconductor, flux lines tend to move by the Lorentz-like force. A flux line motion requires a voltage to sustain the current constant. In other words, by the Faraday's law \( \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \) electric field is induced by the motion of the fluxoid. So, there is a dissipation of energy, although there is no mechanism like Ohmic loss inside superconductors. If the flux lines can move around freely inside the superconductor, the lossless conduction could not be possible. However, the motions of flux lines are stopped at grain boundaries, structural imperfection and chemical impurities in the crystalline structure of superconductor. The flux line is said to be pinned, and the phenomenon is called flux pinning. A perfect pinning force acts as an elastic restoring force. Actually, there is a small resistance due to imperfect pinning, although the resistance is negligible at low temperature. This resistance is advantageous from the viewpoint of rejection of unwanted perturbed motions. An increased pinning force due to proper process can the lateral restoration force be much larger.

\section{C.3.4 Bean's Critical State Model}

Now we know that magnetic field can penetrate in a type II superconductor in its vortex state. Then, how does the current distribution look like inside the superconductor? Charles P. Bean was one of the researchers who tried to set a model for the current distribution and its interaction with the external field. His model is based on rather simple hypotheses: (1) The magnitude of the current density inside a superconductor induced by external field, however small, can take only three values \( +J_c \), \( -J_c \) or zero. (2) In the case of the flux removal, the last flux in should be the first out [Bea64].
In Bean's critical state model, a critical current density \( J_c \) flows parallel to the surface reacting the external field \( H \). Depending on the field history, the direction of current may be different as a function of depth \( z \). One of the most important points here is the magnitude of the current is always \( J_c \) or zero. This model is not reversible and hysteresis is involved in it. The critical current is established inside the superconductor as a negative gradient of the field \( (J_c = -\frac{dH}{dz}) \) in the region between the surface and depth \( \delta_0 = H_0/J_c \), where \( H_0 \) is the magnetic field intensity at the surface. If \( H_0 \) reaches a maximum \( H_{max} \) and decreased monotonically, the last flux coming into the superconductor comes out first by Hypothesis (2). Due to pinning, the flux line deep inside remains in place. According to Hypothesis (1), a reverse-directional current with magnitude of \( J_c \) should be induced by the decrement of the surface field. The region of negative current extends from the surface to \( \delta = (H_{max} - H_0)/2J_c \). This distribution of current is in contrast to the idea that \( \delta_0 \) would decrease as \( H_0 \) decreases without introducing the current reversal near the surface. Figure C-6 summarizes the current density distributions reacting external field based on Bean's model. Small figures from 0 to 6 in the current density diagrams match the subscripts for the field \( H \). Experimental confirmation of Bean's model is first done by H. T. Coffey [Cof67].

### C.3.5 Hysteresis in Type II Superconductors

The hysteresis in superconductors comes out from the flux pinning. Moving flux lines have to be depinned and exert a frictional force on the atomic lattice; they behave like plucked strings in a viscous medium [Bra90]. Figure C-7 represents hysteresis curves for high-temperature superconductors in the \( HM \)-coordinate for (a) weak and (b) strong pinning of flux lines. The hysteresis curve does not follow the ideal trajectory given in Figure C-5 (b) even below \( H_{c1} \), since there are some imperfections inside the superconductor. Even after the external field returns to zero, the superconductor may have magnetization exactly like a permanent magnet. This is due to the flux trapped
Figure C-6: Bean's critical state model
in the superconductor. The magnetization disappears when temperature goes higher than the critical temperature. Recent researches show a sample of YBCO can trap bigger magnetic flux than rare-earth permanent magnets [CL91, WCL+90]. This kind of samples can increase the levitation force and magnetic stiffness significantly and even levitate a small chunk of permanent magnet below itself [SHM+89].

We can expect that there is a hysteresis loss when the curve makes a cycle. This sort of loss exists in either AC applications or in the involvement of movement. Therefore, DC applications have no loss. Due to the hysteresis phenomenon, type II superconductors are sometimes classified as hard superconductors. The hysteretic phenomenon in type II superconductors adds stability to the magnet-superconductor system. For type I superconductors following perfect diamagnetism in Figure C-5 (a), there is no damping due to hysteretic energy loss. A small perturbation causes indefinite oscillation to the motion. There are experimental evidences for velocity-independent damping in type II superconductors; superconducting friction seems like
Figure C-8: Flux lines causing levitation and lateral stability of a magnet over type II superconductor (after [HGJ+88])

a velocity-independent Coulomb friction with high fields [DLS88, Moo94, Moo90]. Thus this is not a kind of damping due to eddy current. The friction force due to eddy current is proportional to velocity and vanishes when the sample stands still [Bra88]. In the following section, we discuss important empirical results on levitation and lateral friction force in type II superconductors.

**C.3.6 Levitation and Lateral Force**

As described in the previous sections, the complete Meissner effect cannot explain the stable levitation in the lateral mode. Hellman and others explain the lateral stability in levitation of a permanent magnet above a YBCO superconductor disk [HGJ+88]. They point out that the penetration and pinning of the flux in the superconductor cause the position of the magnet to be stable over a flat disk; a complete Meissner effect would make the position unstable. Figure C-8 is a proposed mechanism how flux is trapped and lateral stability is obtained. Most of the flux lines are repelled out. This gives the levitation force which can overcome the gravity load of the magnet as in the Meissner model. Some flux lines penetrate the superconductor. Because the
thickness of the superconductor is not thin enough, some flux lines may close inside the superconductor as in Figure C-8. However, the vortices trapped and pinned inside the superconductor prevents the lateral instability. Attempts to move the permanent magnet laterally should overcome the pinning forces. This additional required energy produces the stabilization force.

The magnetic friction force which causes damping is calculated with Bean’s critical current model in Davis and others’ paper [DLS88]. They also show a diamagnetic planar disk with finite size will not provide lateral stability. Later, Davis derives the lateral restoring force for a permanent magnet and a superconductor slab pair with the same model [Dav90]. Johnson and others refine the analysis and verify with experiments [JYB+91]. They claim that Davis’ model does not give a satisfactory quantitative force prediction for materials with low $J_c$.

According to Davis’ analysis [Dav90], the restoring force per unit length for small displacement $\Delta x$ can be derived as

$$\frac{F_x}{L_y} = -\frac{\mu_0 \Delta x}{J_c} \int_{-\infty}^{0} dx H_0(x) \left( \frac{dH_0(x)}{dx} \right)^2$$

(C.7)

where $L_y$ is the length in the $y$-direction and $H_0(x)$ is the magnetic field at the surface of the superconductor. He assumes an infinite size superconductor slab and the following field profile.

$$H_0(x) = H_0(0) \frac{h^2}{x^2 + h^2}$$

(C.8)

where $h$ is the height of the levitation. We can find the force to be

$$\frac{F_x}{L_y} = -\frac{5\pi \mu_0 H_0^3(0) \Delta x}{64 J_c h}.$$  

(C.9)

We will refer to this formula later in the next section.

We surveyed phenomenological explanations and models for behaviors of superconductors. Although there has been a significant amount of efforts, no theory seems to predict exactly levitation and lateral force for high-temperature superconductors. This fact is due to our lack of knowledge about high-temperature superconductiv-
ity itself. The papers discussed in the next section deal with empirical results and qualitative justifications but not much on theory behind them.

C.4 Current Researches on High-temperature Superconducting Suspension

C.4.1 Ma, et al.—Superconducting Bearing, Torque Coupler, and Damper

The authors implement mechanical devices, such as a bearing, a torque coupler (clutch), and a damper with a permanent-magnet assembly and a superconductor slab. YBCO (YBa$_2$Cu$_3$O$_7$) ceramic high-temperature superconductor is bathed in liquid nitrogen. The devices are based on pinning force and hysteresis that are intrinsic in high-temperature superconductors. Highly hysteretic $H_M$ behavior as in Figure C-7 accounts for the hysteresis in force between the permanent magnet and the superconductor. The hysteretic force is generated to oppose motions of the magnet when there is a field change inside the superconductor from the relative motion of the magnet. This hysteretic loss of mechanical energy leads to vibration absorption in a superconducting damper. The authors give experimental results as proofs of principle. This paper is rather a short summary of the research trends in Texas Center for Superconductivity at University of Houston. More detailed description of their separate systems can be found in the Proceedings of the 1992 TCSUH Workshop, HTS Materials, Bulk Processing and Bulk Application, edited by C. W. Chu, et al. [LMM+92, MMC92, MMLC92, MGC+92]. Other researches on superconducting bearings include [CCC95, MMTK89, TMK+92].
Figure C-9: No hysteretic loss due to axial symmetry

Bearing

The main purpose of magnetic bearings, either conventional bearings or superconducting bearings, is to eliminate frictional force between mechanical members. Superconducting bearings attract attention because they can be stabilized passively due to the Meissner effect and flux pinning. Usual active magnetic bearings need continuous power consumption to control the attitude of the body being lifted since the whole system is inherently unstable. They generally need a complicated multivariable feedback system comprising position sensors and force actuators. Moreover, a fast computing unit is mandatory for multiple-degree-of-freedom stabilization.

We studied in previous sections that friction force is one of the inherent properties in high-temperature superconductivity. The friction force could impede intended motion of magnet. However, we want to have a bearing that has drag as small as possible, which seems to be contradictory. Here is a point: The friction and drag forces are generated only when the magnetic field penetrated in the superconductor changes. So, if the bearing magnet has a rotational symmetry in magnetic field, there is no drag between the rotating magnet and the superconductor. Let us take a thrust bearing in Figure C-9 for example. A perfect cylindrical magnet generates no field change for the rotational degree of freedom around its symmetry axis. Any other five-degree-of-freedom motions cause field change, and the magnet experiences an opposing force due to flux pinning. So, unwanted lateral and vertical motions are
Figure C-10: Hybrid superconducting magnetic bearing

damped out without dragging the rotation of the magnet.

There are important concerns related to the bearing application of high-temperature superconductors. In the case of thrust bearings, one concern is how big the vertical thrust that the bearing can handle is. Unfortunately, a simple pair of permanent-magnet and a superconductor is proved to be insufficient to support a usual weight of the bearing shaft. The magnetic pressure is typically on the order of 10 mN/mm² [MCH+90]. Another concern is the lateral stiffness of the bearing, which is important to overcome lateral instability.

To solve the first concern, the authors' group suggests a hybrid structure of a thrust bearing with magnet-superconductor-magnet structure [MMC92, MML+92].

Figure C-10 shows its structure schematically. The whole system is supposed to be

7Their system is also introduced in other texts such as [Moo94, She94].

358
used in a turbine generator. The main thrust is provided by strong repulsion force of two NdFeB magnets with the same poles facing with each other. The YBCO slab in between is used to stabilize the magnet-magnet system laterally. To calculate lateral force we may use Davis’ analysis given in Section C.3.6. As in [JYB+91], we can make an approximation $H_0(0) \approx J_c d$, and using $B = \mu_0 H_0(0)$, (C.9) becomes

$$\frac{F_x}{L_y} \approx -\frac{5\pi B J_c d^2 \Delta x}{64 h}.$$ (C.10)

Because this formula is valid for the interaction of a magnet and a superconductor without the lower magnet slab, it needs some modification to fit for Ma and others’ case. They are believed to derive the force relation as in (2) in [MMLC93] directly from the Lorentz-like force. Let us assume that the induced current is the critical current $J_c$ (This is a consequence of Bean’s critical model with high external field due to the permanent magnets.) and the magnet is perturbed by $\Delta x$. Then the magnetic flux density $B$ due to the magnet sweeps the effective volume in the superconductor, which is $2Rd\Delta x$. Then the force is

$$F_x = B(2R)J_c d\Delta x,$$ (C.11)

where the diameter of the rotor magnet $2R = 0.6$ cm, the flux density $B = 0.2$ T at the superconductor, which has a thickness $d = 0.4$ cm and a critical current density $J_c = 10^4$ A/cm$^2$. The stiffness is calculated as $F_x/\Delta x = 0.5$ N/mm and they claim this agrees well with the experimental data given in Figure 4 in [MMLC93]. We should note that this force formula can only give a very rough estimation. The magnetic field cannot be uniform inside a superconductor and the end effect is significant. Comparing the force equations (C.11) by Ma and others and (C.9) by Davis, there is a $5\pi d/64 h$ factor difference. The information on the magnet height $h$ is not given in the paper. If $h$ is about four times smaller than $d$, then Davis’ formula is also compatible with the experimental results given in this paper.

---

8All dimensions in figures are in millimeters throughout this appendix.
Figure C-11: Magnet-superconductor torque coupler

Figure 4 in [MMLC93] shows the hysteretic lateral force versus displacement. It indicates that even a negative force is possible in some interval due to trapped flux in the superconductors. We can see that there are more than one equilibrium lateral position. That is, the magnet may not return to the original position after it completes a hysteretic cycle. The authors conclude the hybrid bearing makes practical application possible due to big enough thrust force but it does not have a high stiffness. So, its main advantage is its capacity to support a high, but steady load, as in a flywheel energy storage device.

**Torque Coupler**

The strong flux pinning property of a high-temperature superconductor suggests a possibility of a torque coupler or clutch. Ma and others have constructed a prototype magnetic clutch in Figure C-11. A magnetic clutch can be realized using normal conductors. Within a finite-resistance material like a copper disk, eddy current is induced due to the change of magnetic field. The interaction between the induced current and the external field generates axial force and magnetic drag force. A torque can be transmitted only when there is a slip between the magnet and the copper disk. However, the mechanism of transmitting angular momenta in superconducting clutch is totally different. The pinning force enables the superconducting clutch to transmit a torque without slip. The torque is independent of rotating speed and there is no dissipation of energy in a superconducting clutch. In addition, it can damp out
unintended motion as in the superconducting bearing case.

The authors give a torque estimation as

$$\Gamma = 2\pi a^3 djB,$$  \hspace{1cm} (C.12)

where $a$ is the radius of the disk. They overestimate the current path length and the moment arm as $a$. Assuming the current occupies the whole cross section $2\pi ad$, the effective current is $i = 2\pi adj$. From $F = iBl$, where $l$ is the current path length, $F = 2\pi a^2 djB$. So, the (over)estimated torque is $\Gamma = Fa = 2\pi a^3 djB$. Equation (C.12) gives a very rough estimation, but it serves well to get the order of the torque that the torque coupler can generate. The torque due to the pinning force is found to be independent of the rotating speed as expected (Figure 2 in [MMLC93]). The torque decreases as the air gap between the magnet disk and the superconductor slab increases. Therefore, we can control delivered torque by adjusting the air gap.

For fair comparison, they repeated the experiment with a copper disk in liquid nitrogen at 77 K. Interestingly, the torque due to the magnetic induction is larger (about by 20%) in the range of rotating speed given in the paper. However, the copper clutch cannot deliver any torque when the magnetic disk at stationary. So, we can expect the torque from the copper clutch would vanish at the limit of zero speed whereas the superconducting clutch can deliver a torque at a very low speed. For recapitulation, the advantage of superconducting magnetic clutches is their capability as a wearless, lossless and frequency-independent torque transmitter.

**Damper**

A superconducting damper is based on two energy loss mechanisms—hysteretic loss by pinning and depinning of flux and viscous loss by flux motion. If the frequency of the vibration increases, damping due to eddy current loss becomes significant. Figure C-12 is a schematic diagram of a vibration absorber with stripe magnets and a high-temperature superconducting slab. The superconducting slab is fixed and bathed in liquid nitrogen while the magnet is attached to a cantilever beam. Cyclic
motions of magnets relative to the superconductor slab cause a hysteretic loss of mechanical energy, so the motions are damped. The level of damping can be adjusted by changing the distance between magnets and the superconducting slab. Comparing with an eddy current damper with a copper slab, the superconductor damper has 20 times faster settling time and 20% of steady-state response peak. The quality factor of the damper is as small as 3. A superconducting damper has a superior damping properties.

C.4.2 Delprete, et al.—High-Speed Induction Motor with Superconducting Bearings

A simple high-speed asynchronous induction machine with superconducting diamagnetic bearings is the theme of this paper. Static and dynamic characterization is presented with experiments. This paper gives experimental works on radial and axial force hysteresis and stiffness with respect the distant in which the magnet and the superconductor are separated at the time of cooling. The dynamic behavior of the rotor, which is beyond the scope of this appendix, is estimated with the finite element code developed by the authors. The paper evaluates the bearing drag to find it is negligible.

Damping comes from the resistance of a superconductor against flux change that can cause the rearrangement of vortices. A cylindrical permanent magnet levitating with its axis parallel to the high-temperature superconductor surface has a restoring force or damping force against any translational motion but a rotational motion
Figure C-13: High-speed induction motor using high-temperature superconductor bearings around its axis. Because there is no flux change in this rotational motion, the rotation is frictionless theoretically. In a real measurement the drag torque is as low as $10^{-9}$ N-m for a new melt-textured YBCO [MOF+91], and this torque essentially comes from inhomogeneities of the magnetic filed due to the permanent magnet. The melt-texturing-type processing eliminates the weak grain boundary link and inadequate flux pinning. In bulk YBCO, $J_c$ can be as large as $3 \times 10^8$ A/m$^2$ in a 1-T field at 77 K. It is also reported that the melt-quenched samples produce forces from 96–170% higher than sintered specimens [MCH+90]. Although this processing is not easily applicable to long wires, monolithic samples good for magnetic bearings, high speed machinery, flywheels and gyroscopes can be made.

Figure C-13 shows a cross-sectional view and a top view of the rotor and the superconducting bearings of the motor. The bearings are conceptually the same with ones suggested in [MC90]. (See also [Moo94].) At both ends of the rotor, two NdFeB axially magnetized magnets with 0.32-T surface flux density sit inside housings. Their
magnetic axis is collinear of the rotor axis. Because the bearings cannot carry high loads with magnet-superconductor interaction only, the rotor's mass is kept minimum (18.77 g). The driving torque is originated from the induced eddy current on the bell-shaped part of the rotor. The stator has three phases (120° apart). To avoid the thermal deformation of the rotor by the heat from the stator, the air gap between the rotor and the stator is kept relatively large (about 2 mm). It leads to a low air-gap flux density (0.1 T). The motor can be driven up to as fast as 120 000 rpm, and the tip speed is 140 m/s.

When a superconductor cooled under $T_c$ expels an existing field due to Meissner effect, it is important to distinguish the zero-field cooled (ZFC) condition and the field cooled (FC) condition. Under the ZFC condition, the superconductor is cooled with experiencing no outer field. On the contrary, the superconductor is cooled in some outer field under the FC condition. It was reported that the ZFC condition can provide more levitation force and the FC condition gives much higher magnetic stiffness [MML+92]. This results from partial flux repulsion of type II superconductors. In other words, a specimen under FC condition has smaller room for additional flux penetration and pinning. This phenomenon is related to the irreversibility of type II superconductors, and a perfect diamagnet would not have this property. Ma and others' experiment on their bearing in the last section is believed to be done under FC condition. They might not worry about the loss of the levitation force because the magnet-superconductor-magnet interaction can provide sufficient suspension force.

The force characteristic depends on the hysteretic material property as well as the initial relative position between the magnets and the superconductor. The authors confirm this fact with their static characterization of the bearings. They did experiments for the force hysteresis in large scale (15-mm vertical travel range in the rotor axis) and in small scale (about 1-mm vertical travel range in the rotor axis). In Figure C-14, the ZFC case shows the highest equilibrium position in levitation, that is the ZFC case can provide larger levitation force at the same height as expected.
Figure C-14: Levitation force as a function of vertical position

The ZFC case has the least steep tangent in the $F_z$-$z$ graph, which means it has the smallest stiffness (Table I in [DGM+92]). So, there is a tradeoff between the equilibrium clearance that should be big enough to guarantee a safe operation without collision between members and the stiffness that is responsible for a stable operation. In other words, we must choose the optimal initial cooling distance to have large levitation force and high stiffness.

The authors report that the initial levitation force-height relationship under zero-field condition can be accurately fitted globally with an exponential function, $F_z = F_0 e^{-\alpha z}$, where $F_0 = 1.31$ N and $\alpha = 307$ m$^{-1}$ [CMHM90]. The diamagnetism due to the perfect Meissner effect rather predicts that the force should be a function of a single power of $z$. We can check here again that there is other mechanism involved in high-temperature superconducting levitation than Meissner effect. They also give hysteretic behaviors for radial and axial force like in [MMLC93].

The authors give discussions on dynamic behavior of the rotor with a finite element code developed by their institution. They use a simpler model for the rotor (Figure 6 a) in [DGM+92]) and ignore any rotor-stator magnetic interaction. So,
there is no superconducting hysteretic damping or drag involved in the model. They also give a Campbell diagram [Gen93], which gives the natural frequency profile with respect to the rotor speed (Figure 6 d) in [DGM+92]). The unbalance response of a damped model with the static unbalance \( m\epsilon = 0.0597 \text{ g-mm} \) says the first four natural frequencies (at 830, 1200, 1400 and 1700 rpm) can be easily passed. The maximum operating speed without instability due to deformation is estimated to be 140 000 rpm (Figure 7 in [DGM+92]). Incidentally, the real natural frequencies are observed as 2200 and 2800 rpm, which are higher than the values 1400 and 1700 rpm given in the static simulation. This result confirms that the dynamic stiffness of the superconducting bearings is higher than the static stiffness.\(^9\)

The authors report that some magnetic flux creep is observed in experiments. Magnetic flux creep is thermally activated by motion of flux vortices due to incomplete pinning. The field decays logarithmically as a function of time [CE72]. Decaying field decreases the levitation force of the rotor. It leads to lower equilibrium position and a smaller clearance between the rotor and stator. So, magnetic flux creep can cause a stability problem for a larger magnetic bearing prototype. It is reported that one sample lost 13% of its field in one week [Moo94]. However the next 13% would be lost in 13 years due to the logarithmic relation. The thermally activated flux motion at 77 K is always exists, and thus this effect never vanishes.

The overall drag torque to the rotor is calculated by differentiating spin-down test results in Figure 8 in [DGM+92] computed as \( M = Jd\omega/dt \). The drag torque includes aerodynamic drag, drag due to non-rotating damping and magnetic bearing drag. The authors were not successful in extracting the magnetic bearing drag because of uncertainties of the other two drag components. However, the total drag is already as small as tens of mN-mm. This fact is consistent with Ma and others’ result. There is no change in flux pattern due to axial symmetry of the rotor magnet while the rotor is rotating. So, if we can neglect any asymmetry due to manufacturing error

\(^9\)Recall \( k \sim \omega_n^2 \) for a second-order system.
in the magnets, we do not expect any drag that could come from the hysteresis. In reality, commercial rare-earth magnets have a little inhomogeneity in the field, which leads to depinning of the flux lines. Then, the flux lines dissipate energy by friction inside the superconductor. This friction prevents the rotor from rotating indefinitely in a spin-down test even in vacuum [Moo90]. The friction depends also on how deep the potential wells that hold the pinned flux lines are. This depends on the material property, which can be enhanced. The losses in superconducting bearings would be much smaller than those in any other type of mechanical or magnetic bearing systems.

The contribution of this paper is to realize a high-speed induction motor with high-temperature superconducting bearings. The motor was built using melt-textured YBCO for passive suspension. Because of a very small drag, the rotor can be operated up to 120 000 rpm. They reassure prominent properties of high-temperature superconductivity, such as hysteresis, damping and force creep with more realistic experiments. However, there is hardly anything new in the theoretical discussions or experimental methodology. The authors also give dynamic behaviors of the rotor, which is a necessary analysis to build a real motor with superconducting bearings.

C.4.3 Goodall, et al.—Superconducting Magnet

Previous papers deal with applications of high-temperature superconductors on bearing and suspension. Besides passive suspension with bulk high-temperature superconductors, active suspension with electromagnets with superconducting wires takes good portion in application. The purpose of this paper is to understand issues relating to the controllability of superconducting magnets. The authors investigate the effects of both varying excitation in the superconducting coil and varying reluctance in the magnetic circuit. They discuss what should be modified in a suspension system model with conventional electromagnets and give a state-space model for a superconducting suspension system.

Low-temperature superconducting magnets operated at 4.2 K are not suitable for
controlling magnets because of their low specific heat [Iwa94] and low temperature margin between the operation temperature and the critical temperature. On the other hand, YBCO can carry 400 000 A/cm² at 9 T at 77 K and has much bigger heat capacity [Chu95]. It was believed that high-temperature ceramic superconductors were not suitable for magnet wires primarily due to their high brittleness. The brittleness makes it very difficult to form ceramic superconductors in fine filaments. Thanks to the development of material and process, however, BSCCO can be made into magnet wire [SSM+93]. Nevertheless, existing high-temperature superconducting winding needs an iron core to generate sufficient fields at 77 K according to the authors. So, it is necessary to investigate the response of an electric magnet with high-temperature superconducting wire in an iron circuit to both excitation and reluctance changes.

This paper [GMEA+95] comes from a collaborative research between Loughborough University, Cambridge University, and University of Oxford in the U.K. Fabrication of the high-temperature superconducting magnet was done in Clarendon Laboratory at University of Oxford. A detailed fabrication process of winding with BSCCO-2212 superconducting tape can be found in [JJY+95].

To characterize the superconducting magnet, we need to know about the terminal voltage. First, let us think about the effect of the current change with flux fixed. The terminal voltage consists of the inductive voltage, which is the same that as in an identical copper solenoid, and the resistive voltage due to losses in the circuit. Although there is no Ohmic voltage drop in the superconducting wire under a proper operation condition, there is nonlinear resistive voltage due to losses in the system. Second, we should consider the effect due to the air gap change with current fixed. In this case, the induced voltage at the terminal is the time derivative of the iron core flux. Thus, the linearized terminal voltage can be modeled as

\[ V = L \frac{di}{dt} + n \frac{d\Phi}{dt}, \]  

(C.13)

where \( L \) the inductance of the aircore coil, \( n \) is the number of turns, and \( \Phi \) the flux in the iron core. Both \( \Phi \) and \( L \) can be complex to take account of the losses.

368
In some circumstances with no reluctance change the current terminal can be removed after the magnet has been charged to full current. Before the current is disconnected, the magnet terminals should be shorted with a superconducting connector. Then, the magnet is called in persistent mode and the current in the magnet will flow indefinitely [Wil83]. So, flux can be kept constant without any active control. In real mechanical situations, there are always small perturbation motions, flux leakage and fringing effects. So, an active control is necessary to stabilize the system. Loss due to these stray effects is usually small. As we can see in (C.13), using superconducting magnets leads to a large $VI$-power reduction by removing the voltage due to the Ohmic drop.

The authors begin with the linearized model for a conventional magnetic suspension as in [Goo85]. Figure 5 in [GMEA+95] is a linearized block diagram around a nominal operating point. The four important variables in the suspension system are force $F$, flux density $B$, air gap $G$, and coil current $I$. If we expand the variables around nominal values, we can rewrite, for instance, $F = F_0 + f$, where $F_0$ is the force at the steady-state suspension and $f$ is a small variation. Because the flux density is proportional to the coil current and inversely proportional to the air gap, we can deduce that

$$b = (B_0/I_0)i - (B_0/G_0)g.$$  \hspace{1cm} (C.14)

Since the force is proportional to the square of the flux density,

$$f = 2(F_0/B_0)b.$$  \hspace{1cm} (C.15)

The linearized terminal relation is

$$v = Ri + nA(db/dt),$$  \hspace{1cm} (C.16)

where $A$ is the pole face area, $n$ the number of turns, and $R$ the coil resistance. Since $F_0$ is equal to the weight of the suspended mass $M$, the vertical acceleration is given by $\ddot{z} = f/M$. Then, we have a block diagram model for a conventional
suspension system as in Figure 5 in [GMEA+95] with $K_t = B_0/I_0$, $K_g = B_0/G_0$, and $K_b = 2F_0/B_0$. The situation for the superconducting suspension is almost the same except that the resistance of the coil $R$, which appears a divisor in the diagram (Figure 5 in [GMEA+95], is nominally zero. The authors could fix this problem with a state space model (Figure C-15). Both of the model can be shown to represent the same third-order plant,

$$\frac{d^3z}{dt^3} + \frac{R}{NAK_t} \frac{d^2z}{dt^2} + \frac{RK_gK_b}{NAMK_t}(z_a - z) = \frac{K_b}{NAM}v. \quad (C.17)$$

Figure C-16 shows the testing fixture for experiment. Four of BSCCO coils are connected in series and immersed in liquid nitrogen. At the room temperature side,
Figure C-17: Impedance plots with respect to frequency variation

there is a moving ferromagnetic material in a 5-mm air gap. The effective impedance measurements were done covering 0.1 to 60 Hz frequency range both for fixed air gap with varying excitation and for varying air gap reluctance. The leakage flux turns out to be large due to the long vertical and lateral arms of the magnetic circuit, which are necessary to fulfill the cryogenic requirements [JYY+95]. The large leakage flux reduces the impact of changing reluctance of the coil. To make things worse, their switched mode power amplifier is a significant noise source. So, the experiments with varying reluctance are not satisfactory yet. They give experimental results only with variable excitation in [GMEA+95].

They acquire trans-impedance data for the superconducting magnet through a frequency response analyzer and convert the total impedance into the in-phase (resistive) component and the quadrature (inductive) component. Two testing data sets are given in the paper: (1) different levels of current variation (i) with the same steady current (I₀) and (2) the same level of variation with different steady current. Figure C-17 (a) gives the first set of the data. The total impedance is dominated by the inductive component as expected. However, the resistive component is still
significant and cannot be ignored. It becomes larger as the frequency goes higher. This is because the loss mechanism represents the losses both in the superconductor and the iron core. All impedances increase with the larger AC variation, although the effect is not large. Figure C-17 (b) shows a contrast with different steady currents. Judging from the DC resistance in the 2-A steady current case, the magnet is being operated beyond its critical current level. The lower level of the inductive component comparing with that of the 1-A steady current case at the higher frequencies is explained as the effect of magnetic saturation.

The contribution of this paper is the authors' modeling effort to determine resistive and inductive impedances in a new type of the high-temperature superconducting magnet. As expected, the inductive impedance is not dominated by the resistive impedance because wires are superconducting. However, the authors have shown experimentally that the resistive loss is not insignificant. The resistive loss containing iron loss has a tendency to increase as the excitation frequency increases.

C.5 Discussions

In this appendix, we discussed phenomena and characteristics of superconductors putting emphasis on high-temperature superconductors. The ultimate goal in high-temperature superconductivity is to discover and utilize a superconducting material at room temperature. It might be a real revolution in industry and in everyday life. No theory as yet prohibits the possibility of room-temperature superconductivity.

Passive stable levitation with superconductor-magnet interaction is possible due to the Meissner effect and flux pinning. Levitation and bearing application of high-temperature superconductors is very attractive. As a matter of fact, high-temperature passive bearings including magnet-superconductor-magnet hybrid bearings have great merits. Basically they need no complex control electronics to stabilize the rotor in six degrees of freedom, so the overall system is simple except for the cryogenics. The
operational cost for cryogenics for high-temperature superconductivity applications is much more affordable than that for low-temperature superconductivity applications.

Passive superconducting bearings can compete with mechanical bearings and usual magnetic bearings. Superconducting bearings require no lubricant and do not generate wear particles; a clean operation is possible. They are suitable for high-speed devices since drag force is minimal. The drag in superconducting bearings is 1000 and 25 times less than those in mechanical rolling bearings and conventional magnetic bearings, respectively. There is little loss or friction in rotational motion. However, no-loss condition can only be achieved with perfect axial symmetric field. The energy dissipation due to imperfect axial symmetry is proportional to the cube of the azimuthal fluctuation in the magnetic field [Bea64]. So, manufacturing magnets with tight tolerance in symmetric axial magnetic field is one of the most important issues. Furthermore, superconducting bearings have inherent hysteretic damping properties to damp out unwanted perturbed motions.

However, there are still non-obvious practical obstacles to overcome. One of the most important drawbacks is the low stiffness of superconducting bearings. The stiffness needed for a real machine is the order of $10^6$ N/m [MCH+90]. No superconducting bearing has a comparable stiffness of this amount yet. At present, a superconducting bearing can only be used under steady load as in flywheel energy storage devices. Suspension height and orientation are not unique due to hysteresis. Moreover, we cannot avoid drift of the stabilized position of the rotor due to flux creep, which is not avoidable at high temperature. Hence, a superconducting bearing may not be suitable for precision applications.

Apart from the application of bulk high-temperature superconductors, many efforts have been made to use high-temperature superconducting magnets. Because high-temperature superconductors are ceramic materials, they are very brittle. Although we can make magnet wires out of those, the price for those is relatively high at present comparing with low-temperature superconductor wire, such as $\text{Nb}_3\text{Sn}$.
There is another issue related to stability of high-temperature superconducting magnets [Iwa94]. They are at the very elementary stage for application. In this appendix, we discussed impedance characteristics of them with varying current. It is hard to predict whether high-temperature superconducting magnets can replace low-temperature counterparts in application areas of NMR and MRI.

In order to utilize high-temperature superconductors more widely, there is much to be improved in material and process for stronger pinning force and smaller flux creep. It is difficult to expect how far the development can reach, but there could always be a breakthrough. No exact theory can predict levitation force or drag force yet. Theoretical development is necessary at the same time. An adequate model and theory could give a clue to material and process development. Application of high-temperature superconductors in magnetic suspension has a limited success as now. However, they have great potential and their future is bright.
Appendix D

Code

In this appendix, I provide the real-time control code in C and selections of MATLAB code for simulation.

D.1 Real-Time Control Code

The real-time control code is implemented in the 320C40 digital signal processor. The function c.int01() is called every 200 $\mu$s by interrupt signals from an Intel 82C54 Clock Timer Circuit chip built in the Pentek 4245 A/D Converter board. It contains machine-dependent support functions such as tr_lcw(). These functions, user interface, and command interpretation routines are not presented in this appendix.

D.1.1 Decoupled Lead-Lag Compensators

This real-time control routine consists of several parts—position measurements, coordinate transformation, compensators, transformations for commutation, current outputs, and save old data.

```c
void c_int01()
{
    register unsigned int i;
```
register float f2i;
register signed long test;
register float gain;
float cos_x, sin_x, cos_y, sin_y;
unsigned long address;
float it[12];

tr_low();
*(unsigned long int *)&LIFE = 1;

/* POSITION MEASUREMENTS */

/* Read capacitance probe voltages. */
address = 0x8000001c;
ResetFifo();

/* Read laser interferometers. */

*(unsigned long int *)&0xb0310003 = 0x0041;
raw_x1_pos = (*(long int *)&0xb0310048 << 16) & 0xffff0000;
raw_x1_vel = (*(long int *)&0xb031004e << 16) & 0xffff0000;
*(unsigned long int *)&0xb0320003 = 0x0041;
raw_x2_pos = (*(long int *)&0xb0320048 << 16) & 0xffff0000;
raw_x2_vel = (*(long int *)&0xb032004e << 16) & 0xffff0000;
*(unsigned long int *)&0xb0300003 = 0x0041;
raw_y_pos = (*(long int *)&0xb0300048 << 16) & 0xffff0000;
raw_y_vel = (*(long int *)&0xb030004e << 16) & 0xffff0000;

tr_high();
raw_x1_pos |= (*(long int *)&0xb0310048 >> 16) & 0x0000ffff;
raw_x1_vel |= (*(long int *)&0xb031004e >> 16) & 0x0000ffff;
raw_x2_pos |= (*(long int *)&0xb0320048 >> 16) & 0x0000ffff;
raw_x2_vel |= (*(long int *)&0xb032004e >> 16) & 0x0000ffff;
raw_y_pos |= (*(long int *)&0xb0300048 >> 16) & 0x0000ffff;
raw_y_vel |= (*(long int *)0xb030004e >> 16) & 0x0000ffff;

/* Translate in m/s for plane mirrors. See p. 4-29 in the 10897A manual. */

pos_x = ((float)(raw_x1_pos)) * 6.1815119987e-10;
ur = ((float)(raw_x1_vel)) * 3.77292037e-7;

pos_x1 = ((float)(raw_x2_pos)) * 6.1815119987e-10;
vel_x1 = ((float)(raw_x2_vel)) * 3.77292037e-7;

pos_y = ((float)(raw_y_pos)) * 6.1815119987e-10;
vr = ((float)(raw_y_vel)) * 3.77292037e-7;

for(i=0;i<3;i++)
{
    test = *InData; /* dummy read to fix 4245 bug */
    if(test & 0xffff)
    {
        test = *InData & 0xffff;
        if(test & 0x8000)
        {
            test = test | 0xffff0000; /* sign bit extension */
        }
    }
}

/* Conversion factor: 7.809667693e-9 = (15/2**16) * (3.41209588 m/V) */

raw_z_pos[i] = 7.809667693e-9 * test - 50e-6; /* zero set at 50e-6 m */

/* COORDINATE TRANSFORMATION */

/* 0.099 m is the distance between two x-lasers */

hr = (pos_x1 - pos_x) / 0.099;
rr = (vel_x1 - ur) / 0.099;

zs = (raw_z_pos[1] + raw_z_pos[2]) / 2;
sr = (zs - raw_z_pos[0]) / 0.1616; /* 4-inch apart */

tr = (raw_z_pos[1] - raw_z_pos[2]) / 0.2032; /* 8-inch apart */

/* Skip all control routine if fly = 0 */

if(ifly) goto skip;
ze = zc - zr;
se = sc - sr;
te = tc - tr;

/* COMPENSATORS */

/* lead-lag compensators */
/* lead zero at 30 Hz and zeta = 0.5 */
/* leadlag_z */
gain = 2.3134e6;
zu = 1.68592*zu1 - 0.68592*zu2 + gain * (ze - 1.95924*ze1 + 0.95938*ze2);

/* leadlag_psi */
gain = 2.2504e4;
su = 1.68592*su1 - 0.68592*su2 + gain * (se - 1.95924*se1 + 0.95938*se2);

/* leadlag_theta */
gain = 2.2504e4;
tu = 1.68592*tu1 - 0.68592*tu2 + gain * (te - 1.95924*te1 + 0.95938*te2);

he = hc - hr;

he = hc - hr;

he = hc - hr;

he = hc - hr;

// /* leadlag_x */
gain = 2.2261e6;
xu = 1.68592*xu1 - 0.68592*xu2 + gain * (xe - 1.95924*xe1 + 0.95938*xe2);

/* leadlag_y */
gain = 2.2261e6;
\[ y_u = 1.68592*y_u1 - 0.68592*y_u2 + \text{gain} \times (y_e - 1.95924*y_e1 + 0.95938*y_e2); \]

\[
/* \text{leadlag\_phi} */
\]

\[ \text{gain} = 3.9809e4; \]

\[ h_u = 1.68592*h_u1 - 0.68592*h_u2 + \text{gain} \times (h_e - 1.95924*h_e1 + 0.95938*h_e2); \]

\[
/* \text{force to current conversion} */
\]

\[
/* \text{At 250-micrometer air gap, } f_2i = 0.03612 \text{ N/A} */
\]

\[ f_2i = 0.033974 \times \exp(245.4063 \times (x_r+250e-6)); \]

\[
/* \text{TRANSFORMATIONS FOR COMMUTATION} */
\]

\*[modal-decomposed force transform] /*

/* compensation with torques for psi, theta, psi */

\*[direct current components] /*

\[ i_{1d} = 0.9882 + 0.25*f_2i*z_u + 2.4606*f_2i*(s_u+t_u); \]

\[ i_{2d} = 0.332677165*f_2i*z_u + 2.4606*f_2i*(s_u-t_u); \]

\[ i_{3d} = 0.9882 + 0.25*f_2i*z_u - 2.4606*f_2i*(s_u+t_u); \]

\[ i_{4d} = 0.167322835*f_2i*z_u - 2.4606*f_2i*(s_u-t_u); \]

\*[quadrature current components] /*

\[ i_{1q} = 0.55509*f_2i*x_u; \]

\[ i_{2q} = 0.55509*f_2i*y_u + 4.9225*f_2i*h_u; \]

\[ i_{3q} = 0.44491*f_2i*x_u; \]

\[ i_{4q} = 0.44491*f_2i*y_u - 4.9225*f_2i*h_u; \]

\*[for 4-DOF suspension demo only] /*

\*[ ]

\[ i_{1q} = 0.0; \]

\[ i_{2q} = 4.9225*f_2i*h_u; \]

\[ i_{3q} = 0.0; \]

\[ i_{4q} = -4.9225*f_2i*h_u; \]

379
\[
cos_x = \cos(245.4063*\text{xr}); \quad /* \text{gamma}_1 = 245.4063 \text{ m}^{-1} */
\]
\[
cos_y = \cos(245.4063*\text{yr});
\sin_x = \sin(245.4063*\text{xr});
\sin_y = \sin(245.4063*\text{yr});
\]

\[
/* \text{DQ—2—phase transformation} */
\]
\[
i_{1a} = \cos_x i_{1q} - \sin_x i_{1d};
i_{2a} = \cos_y i_{2q} - \sin_y i_{2d};
i_{3a} = \cos_x i_{3q} - \sin_x i_{3d};
i_{4a} = \cos_y i_{4q} - \sin_y i_{4d};
i_{1b} = \sin_x i_{1q} + \cos_x i_{1d};
i_{2b} = \sin_y i_{2q} + \cos_y i_{2d};
i_{3b} = \sin_x i_{3q} + \cos_x i_{3d};
i_{4b} = \sin_y i_{4q} + \cos_y i_{4d};
\]

\[
/* \text{inverse Blondel—Park transformation} */
\]
\[
it[0] = i_{1a};
it[1] = 0.5*i_{1a} + \text{SQR}T3.2*i_{1b};
it[2] = -0.5*i_{1a} + \text{SQR}T3.2*i_{1b};
it[3] = i_{2a};
it[4] = 0.5*i_{2a} + \text{SQR}T3.2*i_{2b};
it[5] = -0.5*i_{2a} + \text{SQR}T3.2*i_{2b};
it[6] = i_{3a};
it[7] = 0.5*i_{3a} + \text{SQR}T3.2*i_{3b};
it[8] = -0.5*i_{3a} + \text{SQR}T3.2*i_{3b};
it[9] = i_{4a};
it[10] = 0.5*i_{4a} + \text{SQR}T3.2*i_{4b};
it[11] = -0.5*i_{4a} + \text{SQR}T3.2*i_{4b};
\]

\[
/* \text{CURRENT OUTPUTS} */
\]
for(i=0;i<12;i++)
    d2a[i] = DATEL(it[i]);
d2awrt();

force2current = f2i; /* force2current is a global variable. */

/* SAVE OLD DATA */

ze2 = ze1; zu2 = zu1;
ze1 = ze; zu1 = zu;
se2 = se1; su2 = su1;
se1 = se; su1 = su;
te2 = te1; tu2 = tu1;
te1 = te; tu1 = tu;
he2 = he1; hu2 = hu1;
he1 = he; hu1 = hu;
xe2 = xe1; xu2 = xu1;
xe1 = xe; xu1 = xu;
ye2 = ye1; yu2 = yu1;
ye1 = ye; yu1 = yu;

/* DEMONSTRATION ROUTINES */

if(lm_scan_enable == 1){
    /*
    time_optimal();
circle();
    */
    step_and_settle();
}

skip:
D.1.2 Multivariable Linear Quadratic Regulator

For the multivariable linear quadratic regulator for the lateral motions of the platen, replace ll. 129–132 in the above real-time control code with the following code.

D.1.3 Demonstration Routines

The following code includes functions, time_optimal(), circle(), step_and_settle() for demonstrations described in Section 10.4. For the four-degree-of-freedom suspension demonstration, see the comment in l. 134 and the following 4 lines in the real-time control code.
/* time-optimal control */
/* 20mm steps in 120 ms, acceleration = 10 m/s^2 */

void time_optimal()
{
    const float acc = 5.0;
    t += T;
    if (0.0 <= t && t < 2.0) yc += (2e-6);
    else if (2.0 <= t && t < 3.0) yc = 0.02;

    else if (3.0 <= t && t < 3.02) yc = -1*acc*(t-3.0)*(t-3.0) + 0.02;
    else if (3.02 <= t && t < 3.1) yc = -0.2*(t-3.02) + 0.018;
    else if (3.1 <= t && t < 3.12) yc = acc*(t-3.12)*(t-3.12);
    else if (3.12 <= t && t < 3.4) yc = 0.0;

    else if (3.4 <= t && t < 3.42) yc = -1*acc*(t-3.4)*(t-3.4);
    else if (3.42 <= t && t < 3.5) yc = -0.2*(t-3.42) - 0.002;
    else if (3.5 <= t && t < 3.52) yc = acc*(t-3.52)*(t-3.52) - 0.02;
    else if (3.52 <= t && t < 3.8) yc = -0.02;

    else if (3.8 <= t && t < 3.82) yc = acc*(t-3.8)*(t-3.8) - 0.02;
    else if (3.82 <= t && t < 3.9) yc = 0.2*(t-3.82) - 0.018;
    else if (3.9 <= t && t < 3.92) yc = -1*acc*(t-3.92)*(t-3.92);
    else if (3.92 <= t && t < 4.2) yc = 0.0;

    else if (4.2 <= t && t < 4.22) yc = acc*(t-4.2)*(t-4.2);
    else if (4.22 <= t && t < 4.3) yc = 0.2*(t-4.22) + 0.002;
    else if (4.3 <= t && t < 4.32) yc = -1*acc*(t-4.32)*(t-4.32) + 0.02;
    else if (4.32 <= t && t < 4.6) yc = 0.02;

    else if (4.6 <= t) t = 3.0;
}

383
/ * circles with 30-mm diameter in 1 second */
void circle()
{
  const float omega = 2.0*PI;
  t += Ts;
  if (0.0 <= t && t < 1.5) xc = 0.01*t;
  else if (1.5 <= t && t < 2.0) xc = 0.015;
  else if (2.0 <= t){
    xc = 0.015 * cos(omega*(t-2.0));
    yc = 0.015 * sin(omega*(t-2.0));
  }
}

/ * step-and-settle motions */
void step_and_settle()
{
  const float acc = 5.0; /* acc for y = 10 m/s^2 */
  const float accx = 0.25; /* acc for x = 2 m/s^2 */
  t += Ts;
  if (0.0 <= t && t < 2.0){
    xc -= (2e-6);
    yc -= (2e-6);
  }
  else if (2.0 <= t && t < 3.0) xc = yc = -0.02;

  / * step in y */
  else if (3.0 <= t && t < 3.02) yc = acc*(t-3.0)*(t-3.0) - 0.02;
  else if (3.02 <= t && t < 3.1) yc = 0.2*(t-3.02) - 0.018;
  else if (3.1 <= t && t < 3.12) yc = -acc*(t-3.12)*(t-3.12);
  else if (3.12 <= t && t < 3.4) yc = 0.0;

  else if (3.4 <= t && t < 3.42) yc = acc*(t-3.4)*(t-3.4);
  else if (3.42 <= t && t < 3.5) yc = 0.2*(t-3.42) + 0.002;
else if (3.5 <= t && t < 3.52) yc = -acc*(t-3.52)*(t-3.52) + 0.02;
else if (3.52 <= t && t < 3.8) yc = 0.02;
else if (3.8 <= t && t < 4.0) yc = 0.02;

/* step in x 20mm step in 200 ms, acceleration = 2 m/s^2 */
else if (4.0 <= t && t < 4.2) xc = accx*(t-4.0)*(t-4.0) - 0.02;
else if (4.2 <= t && t < 4.4) xc = -accx*(t-4.4)*(t-4.4);
else if (4.4 <= t && t < 4.8) xc = 0.0;

/* step in -y */
else if (4.8 <= t && t < 4.82) yc = -acc*(t-4.8)*(t-4.8) + 0.02;
else if (4.82 <= t && t < 4.9) yc = -0.2*(t-4.82) + 0.018;
else if (4.9 <= t && t < 4.92) yc = acc*(t-4.92)*(t-4.92);
else if (4.92 <= t && t < 5.2) yc = 0.0;
else if (5.2 <= t && t < 5.22) yc = -acc*(t-5.2)*(t-5.2);
else if (5.22 <= t && t < 5.3) yc = -0.2*(t-5.22) - 0.002;
else if (5.3 <= t && t < 5.32) yc = acc*(t-5.32)*(t-5.32) - 0.02;
else if (5.32 <= t && t < 5.6) yc = -0.02;
else if (5.6 <= t && t < 5.8) yc = -0.02;

/* step in x */
else if (5.8 <= t && t < 6.0) xc = accx*(t-5.8)*(t-5.8);
else if (6.0 <= t && t < 6.2) xc = -accx*(t-6.2)*(t-6.2) + 0.02;
else if (6.2 <= t && t < 6.6) xc = 0.02;

/* step in y */
else if (6.6 <= t && t < 6.62) yc = acc*(t-6.6)*(t-6.6) - 0.02;
else if (6.62 <= t && t < 6.7) yc = 0.2*(t-6.62) - 0.018;
else if (6.7 <= t && t < 6.72) yc = -acc*(t-6.72)*(t-6.72);
else if (6.72 <= t && t < 7.0) yc = 0.0;
else if (7.0 <= t && t < 7.02) yc = acc*(t-7.0)*(t-7.0);
else if (7.02 <= t && t < 7.1) yc = 0.2*(t-7.02) + 0.002;
else if (7.1 <= t && t < 7.12) yc = -acc*(t-7.12)*(t-7.12) + 0.02;
else if (7.12 <= t && t < 7.4) yc = 0.02;
else if (7.4 <= t && t < 7.6) yc = 0.02;

else if (7.6 <= t && t < 9.6) {
    xc -= (2e-6);
    yc -= (2e-6);
}
else if (t >= 9.6) t = 0.0;

---

D.2 MATLAB Simulation Code

D.2.1 Decoupled Lead-Lag Compensators

This m-file simulates the decoupled lead-lag controllers. It generates root locus, Bode plots, and step responses.

% leadlag4.m (1/16/97) (11/25/96)

Ts = 200e-6; % sampling period (sampling at 5 kHz)

% plant model
num=[0.17926]; % 1/M for z, x, y
%num=[18.54]; % I_xx_inv for psi, theta
%num=[10.2153]; % I_ww_inv for phi
% platen natural frequency at 7.86Hz
den=[1 0 2409.2]; % z
%den=[1 0 2511.8]; % psi, theta
%den=[1 0 -4.9086]; % x, y
%den=[1 0 -5.6345]; % phi

% lead compensator with lead zero at 30 Hz
numlead=[1 188.50]; denlead=[1 1885.0];

% lag compensator with lag pole at the origin
numlag=[1 18.850]; denlag=[1 0];
[numc,denc] = series(num,den,numlead,denlead);
[numc,denc] = series(numc,denc,numlag,denlag);

rlocus(numc,denc);
axis([-500,0,-250,250]);
sgrid;
[k,poles] = RLOCFIND(numc,denc)
pause;

numc=k.*numc;
bode(numc,denc) % for loop transmission
pause;

[numf,denf] = feedback(numc,denc,1,1);
bode(numf,denf) % for closed-loop system
pause
step(numf,denf);

end

D.2.2 Self-Inductance

This m-file calculates the self-inductance of one-phase winding per spatial period for the linear motor stator.

% self_L4.m (12/6/96) (8/4/95) (7/7/95) (5/3/94)

inch_to_meter = 0.02540; % conversion factor

mu_0 = pi*4e-7; % permeability of free space (H/m)
pitch = 1.008 * inch_to_meter; % pitch (in)
Gamma = pitch/5; % stator winding depth (m)
w = 3.624*inch_to_meter; % stator winding length (m)
eta_0 = 2.491e6; % turn density (turns/m^2)

n_modes = 101;
n_max = 2*n_modes-1; % highest mode to be included in calculation

coeff = 0.5 * mu_0 * w * eta_0^2 * pitch^3 / pi^4;
L = 0;
nn=[-n_max:2:n_max];

for kk = [1:n_max+1]
n=nn(kk);

388
k_n = 2*pi*n/pitch; % wave number

gamma_n = abs(k_n);

L_n = coeff * (1/n^4) * (1-cos(pi*n/6)) ...

(Gamma + (exp(-1*gamma_n*Gamma)-1)/gamma_n);

L = L + L_n;

end

D.2.3 Force and Force Ripple

This m-file calculates force and force ripples from the interaction between the three-phase windings and the Halbach magnet array with fixed air gap.

% ph3_ha15.m (2/19/96) (7/7/95) (6/21/95)

q = 3; % number of phases

pitch = 0.0256; % 2.56 centimeter pitch (m)

Delta = pitch/4; % thickness of the magnet array

Gamma = pitch/5; % thickness of the winding

x_0 = 250e-6; % air gap (m)

w = 0.09144; % width of the motor (m)

B_r = 1.29; % permanent magnet remanence (T)

mu_0 = pi*4e-7; % permeability of free space (H/m)

M_0 = B_r/mu_0; % magnet magnetization (A/m)

n_max = 49; % highest mode to be included in calculation

steps = 60; % view steps = 2*steps+1
d = [-pitch/2 : (pitch/2)/steps : pitch/2];
z.0 = [-pitch/2 : (pitch/2)/steps : pitch/2];
theta = 2*pi*d/pitch; % phase angle
Ia = zeros(1,2*steps+1);
I = zeros(1,n_max+1);

% Halbach flux density coefficients as in (5.3--5.4).
mB.x = zeros(1,n_max+1);
mB.z = zeros(1,n_max+1);
for n = [1:4:n_max+1]
mB.x(n) = B_r*(-1)^((n-1)/4)*sqrt(2)/(n*pi);
mB.z(n) = (j^n) *mB.x(n);
end;

% Calculate force.
f.x = zeros(1,2*steps+1);
f.xz = zeros(1,2*steps+1);

for disp = 1:2*steps+1,
for n = 1:2:n_max+1,
k.n = 2*pi*n/pitch;
gam.n = abs(k.n);

% Fourier coefficients of current

if rem(n,(2*q)) == 1,
I(n) = q/(n*pi)*sin(n*pi/(2*q))*exp(j*theta(disp));
elseif rem(n,(2*q)) == 2*q-1,
I(n) = q/(n*pi)*sin(n*pi/(2*q))*exp(-j*theta(disp));
.end;

% vertical and horizontal flux densities

B_xd = j*mu_0*(1.5*10^6)*I(n)/(2*k_n)...
exp(-gam_n*x_0)*(1-exp(-gam_n*Gamma)).*exp(-j*k_n*z_0(disp))...
+(mB_x(n)-j*mB_z(n))*(1-exp(-gam_n*Delta))/2;
B_zd = -mu_0*(1.5*10^6)*I(n)/(2*gam_n)...
exp(-gam_n*x_0)*(1-exp(-gam_n*Gamma)).*exp(-j*k_n*z_0(disp))...
+(-j*mB_x(n)-mB_z(n))*(1-exp(-gam_n*Delta))/2;

% forces by Maxwell stress tensor

f_x(disp) = f_x(disp) - 2*(w*(15/4)*pitch/(2*mu_0))...
(conj(B_xd).*B_xd - conj(B_zd).*B_zd);
f_xz(disp) = f_xz(disp) - (w*(15/4)*pitch/mu_0)...
(conj(B_zd).*B_xd + conj(B_xd).*B_zd);

end;
end;

plot(z_0, f_xz),title('lateral force'),
xlabel('z(m)'), ylabel('force(N)');
pause
plot(z_0, f_x),title('vertical force'),
xlabel('z(m)'), ylabel('force(N)');
end
Bibliography


399


400


