Technology and Labor Markets
by
Pascual Restrepo Mesa
B.A., Universidad de los Andes (2011)
Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June 2016
© Pascual Restrepo Mesa, MMXVI. All rights reserved.
The author hereby grants to MIT permission to reproduce and to distribute
publicly paper and electronic copies of this thesis document in whole or in
part in any medium now known or hereafter created.

Signature redacted

Author .................................................................
Department of Economics
May 15, 2016

Certified by..........................................................
Daron Acemoglu
Elizabeth and James Killian Professor of Economics
Thesis Supervisor

Certified by..........................................................
Ivan Werning
Robert M. Solow Professor of Economics
Thesis Supervisor

Accepted by..........................................................
Ricardo Caballero
Ford International Professor of Economics
Chairman, Departmental Committee on Graduate Studies
Abstract

This thesis is divided in three chapters. The first two chapters explore the role of structural changes and skill mismatches in generating unemployment. In the first chapter, I present a model in which technological change creates a mismatch between the skill requirements in novel jobs and workers’ current skills. Although the economy adjusts as workers are retrained and learn these skills, I argue that the adjustment may be sluggish and inefficient. Moreover, along this adjustment the economy becomes more responsive to aggregate shocks. The key mechanism behind these results is that, due to matching frictions and because skills are not yet standardized, firms face a high cost to recruit workers with the requisite skills for novel jobs and they respond by creating fewer novel jobs. In the second chapter, I document that the decline in routine-cognitive jobs outside manufacturing—a pervasive structural change that has affected U.S. labor markets since the mid 90s—accelerated during the Great Recession and contributed to the long-lasting increase in unemployment since 2007. I show that the local labor markets that were more exposed to this structural change experienced worst outcomes during the Great Recession. Moreover, at the local labor market, this structural change interacted with temporal shocks to the demand for goods and services. In the third chapter, which is joint work with Daron Acemoglu, we study how the automation of jobs performed by labor affects employment, wages and the share of labor in national income.
Acknowledgments

I thank Daron Acemoglu, Abhijit Banerjee and Ivan Werning for their thorough guidance with this project. They were the best advisors I could have asked for. It is a privilege to be your student.

Daron deserves a special acknowledgment. Since the day I started my PhD at MIT, he encouraged me to learn. He became my intellectual guide, my role model, and my friend. He is the best economist and the most generous person I know.

This project was enriched by the comments and suggestions from participants at the MIT Macro Seminar, and from Alex Bartik, Atif Mian, Emi Nakamura, Brendan Price, Mathew Rognlie and Dejanir Silva. Special thanks go to David Autor for his thoughtful advice on the empirical work.

MIT was an amazing and inspiring place, and I can certainly say that I lived the best years of my life here. My friends at MIT made my experience unique. For their friendship and support I thank Nicolas Caramp, David Colino, Diego Feijer, Mateo Montenegro and Roman D. Zarate.

To Ari and my Family: thanks for your love, for your support, and for always being by my side.
## Contents

1 A Model of Skill Mismatch and Structural Unemployment  
1.1 A Model of Structural Change  
1.1.1 Characterizing the Equilibrium  
1.1.2 Analysis of the model  
1.2 Quantitative exploration  
1.2.1 Numerical results  
1.3 Concluding remarks  
1.4 Theory Appendix  
1.4.1 Derivation of state variables and Bellman equations:  
1.4.2 Properties of the equilibrium and steady state behavior  
1.4.3 Details of the limit when $a, \lambda \to \infty$  
1.4.4 Comparative statics for the instantaneous equilibrium.  
1.4.5 Proofs of the propositions 2-6.  
1.4.6 Proof of Proposition 6  
1.4.7 Allowing workers to direct their search efforts.  
1.4.8 Behavior of wages when the number of open vacancies does not adjust immediately.  
1.4.9 Restructuring concentrates in recessions.  

2 Do Structural Changes Explain Part of the Current Employment Slump?  
2.1 Did the recession accelerate the decline in routine-cognitive jobs?  
2.2 Impacts at the commuting zone level  
2.2.1 How do local labor markets adjust?  
2.3 Spillovers at the Local Labor Market Level  
2.4 Concluding discussion  
2.5 Data Appendix  

3 The Race Between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment (joint with Daron Acemoglu)
3.1 Static Model .................................................. 123
   3.1.1 Environment ............................................. 123
   3.1.2 Equilibrium in the Static Model ....................... 126
3.2 Dynamics, Balanced Growth and the Productivity Effect .......... 133
   3.2.1 Balanced Growth ........................................ 133
   3.2.2 The Productivity Effect ................................ 138
3.3 Full Model: Tasks and Endogenous Technologies .................. 140
   3.3.1 Endogenous and Directed Technological Change .......... 141
   3.3.2 Equilibrium with Endogenous Technological Change ...... 142
3.4 Welfare ..................................................... 150
3.5 Extensions ................................................................
   3.5.1 Automation, New Tasks and Inequality .................... 152
   3.5.2 Creative Destruction of Profits .......................... 154
3.6 Conclusion ................................................................
3.7 Appendix A: Proofs .............................................. 159
   3.7.1 Proofs from Section 3.1 .................................... 159
   3.7.2 Proofs from Section 3.2 .................................... 163
   3.7.3 Proofs from Section 3.3 .................................... 166
3.8 Appendix B (Not-For-Publication): Omitted Proofs and Additional Results ........................................... 175
   3.8.1 Details of the Empirical Analysis ......................... 175
   3.8.2 Remaining Proofs from Section 3.1 ......................... 179
   3.8.3 Remaining Proofs from Section 3.2 ......................... 186
   3.8.4 Constrained Efficient Allocation and Proofs from Section 3.4 ... 188
   3.8.5 Proofs from Section 3.5 .................................... 198
   3.8.6 When New Tasks Also Use Capital ......................... 200
   3.8.7 Microfoundations for the Quasi-Labor Supply Function .... 202
List of Figures

1-1 U.S. employment rates for different occupational categories. Data from the BLS. .................. 16

1-2 Percent change in quarterly vacancies posted within broad occupational groups (relative to the first quarter of 2007). The light-blue bars plot 90% confidence intervals for the difference between both series in each quarter. Data for 22 occupations from Help Wanted Online, by the Conference Board. ................................................................. 19

1-3 Task space and a graphical representation of the effect of structural change on the productive structure. The top panel presents the status quo and the bottom panel the structural change. 21

1-4 Phase diagram for the equilibrium in terms of $x(t)$ and $\Omega(t)$................................. 29

1-5 Earnings, wages and employment for a displaced unskilled worker. Results for Scenario 1. 37

1-6 Equilibrium adjustment paths for different variables in my model in both scenarios. .... 38

1-7 Unemployment rate decomposition computed for Scenario 1 in the case with $x(0) = \gamma(0) = 1/3$. ................................................................................................................. 39

1-8 Representation of the equilibrium employment in stepping-stone jobs. ....................... 40

1-9 Adjustment paths for unemployment relative to its initial value at time 0. The gray line plots the counterfactual trends in an economy adjusting to a structural change that started at $t = -5$. The recession affects the economy from $t = 0$ to $t = 2.5$ (in years) .......... 42

1-10 Beveridge curves. Both figures center around the initial unemployment and the number of vacancies prior to the recession. Each point corresponds to a different year since the onset of the recession. ................................................................. 42

1-11 Quality locus (equation 1.18) and job-creation locus (equation 1.19). ......................... 58

1-12 Loci for the fixed points of the map in equation (9). ......................................................... 60

2-1 Left axis: Unemployment rate (in black) and U6 unemployment rate (in navy blue) for the U.S. Right axis: Employment to working age population ratio in the U.S. (in light blue and inverted). Data from the Bureau of Labor Statistics. ................................. 80

2-2 U.S. employment rates for different occupational categories. Data from the BLS. ........ 80
2-3 Percent change in quarterly vacancies posted within broad occupational groups (relative to the first quarter of 2007). The light-blue bars plot 90% confidence intervals for the difference between both series in each quarter. Data for 22 occupations from Help Wanted Online, by the Conference Board.

2-4 Percent change in the unemployment rate (short and long term) for workers who were last employed in routine-cognitive jobs. Data from the American Community Survey.

2-5 Share of current employed workers who were employed in routine-cognitive jobs at year $t-1$ and switched occupations at year $t$. Data from the Current Population Survey, Annual Social and Economic Supplement.

2-6 Geographical distribution of the exposure of U.S. commuting zones to the decline in routine-cognitive jobs, $R_{2000}^c$.

2-7 Estimated change in employment for commuting zones at the mean (in gray) and 90th percentile of exposure to structural change (in blue) relative to the year 2000. The light-blue bars plot 90% confidence intervals for the estimates in the highly exposed areas. Data from the County Business Patterns.

2-8 Estimated change in employment for commuting zones at the mean (in gray) and 90th percentile (in blue) of exposure to structural change. The light-blue bars plot 95% confidence intervals for a test of both series being different. Data from the County Business Patterns.

2-9 Estimated change in the share of workers who report service, routine, professional and managerial occupations and their 90% confidence intervals. The estimates compare commuting zones at the 95th percentile of exposure to structural change relative to commuting zones at the 5th percentile (2007 is the base year). Each panel indicates the occupational category.

2-10 Estimated change in the employment, unemployment and long term unemployment rate, and percent change in wages and their 90% confidence intervals. The estimates compare commuting zones at the 95th percentile of exposure to structural change relative to commuting zones at the 5th percentile (2007 is the base year).

2-11 Percent change in unemployed workers' job-finding rate (relative to the first quarter of 2007). The light-blue bars plot 90% confidence intervals for the difference between both series in each quarter. Data from the Longitudinal Employer-Household Dynamics.

2-12 Estimates for $\beta^c_0$ for commuting zones at 90th percentile of specialization in $R_{c}^S$ relative to the mean.

2-13 U.S. employment rates for different occupational categories. Data from the BLS.
3-1 Employment growth by decade plotted against the share of new job titles at the beginning of each decade for 330 occupations. Data from 1980 to 1990 (in dark blue), 1990 to 2000 (in blue) and 2000 to 2007 (in light blue, re-scaled to a 10-year change). Data source: See Appendix B. ................................................. 119

3-2 The task space and a representation of the effect of introducing new complex tasks (middle panel) and automating existing tasks (bottom panel). .......................................................... 126

3-3 Static equilibrium. The left panel depicts the case in which $I^* = I < \bar{I}$ so that the allocation of factors is constrained by technology. The right panel depicts the case in which $I^* = \bar{I} < I$ so that the allocation of factors is not constrained by technology. .................................................. 129

3-4 Balanced growth path and dynamic equilibrium when technology is exogenous and satisfies $n(t) \to n$ and $g(t) \to \Lambda \Delta$. ................................................................. 136

3-5 Dynamic behavior of wages following a permanent increase in automation. .................... 140

3-6 Determination of $n^D$ in steady state. ................................................................. 148

3-7 Left panel: Average years of schooling among employees against the share of new job titles at the beginning of each decade for 330 occupations. Right panel: Change in average years of schooling over the next 10 years (dark blue), next 20 years (blue) and next 30 years (in light blue) against the share of new job titles at the beginning of each decade. See Appendix B for data sources and detailed definitions. ................................................................. 153

3-8 Determination of $n^D$ when the structure of intellectual property rights features the creative destruction of rents. The model has an odd number of equilibria, which in the case depicted include a stable one at $n^D = 1$. ................................................................. 157

3-9 Construction of function $\omega(I^*, N, K)$. ................................................................. 161

3-10 Behavior of value functions in steady state with respect to changes in $n = N - I$. .......... 167

3-11 Phase diagram and global saddle path stability when $\theta = 0$. .................................. 169

3-12 Behavior of unit costs of labor with respect to changes in $n = N - I$ in steady state. ...... 187
List of Tables

1.1 Quarterly parametrization of the model. .................................................. 36

2.1 Changes in the non-manufacturing employment rate during the Great Recession among commuting zones with different exposure to the decline in routine-cognitive jobs. .................................................. 91

2.2 Changes in the employment rate during the Great Recession among commuting zones with different exposure to the decline in routine-cognitive jobs and experiencing different local demand shocks .................................. 96

2.3 Changes in the employment rate during the Great Recession among commuting zones and skill groups with different exposure to the decline in routine-cognitive jobs .................................................. 102

2.4 Changes in the employment rate during the Great Recession among commuting zones and skill groups with different exposure to the decline in routine-cognitive jobs .................................................. 105

2.5 Hiring and separation rates among commuting zones exposed, during and before the Great Recession. .................................................. 116

3.1 Differential employment growth in occupations with more new job titles ... 177

3.2 Reversal in skill content for occupations with more new job titles and in occupations that used to hire more educated workers. .................................................. 179
Chapter 1

A Model of Skill Mismatch and Structural Unemployment

Abstract

I present a model in which technological change creates a mismatch between the skill requirements in novel jobs and workers' current skills. Although the economy adjusts as workers are retrained and learn these skills, I argue that the adjustment may be sluggish and inefficient. Moreover, along this adjustment the economy becomes more responsive to aggregate shocks. The key mechanism behind these results is that, due to matching frictions and because skills are not yet standardized, firms face a high cost to recruit workers with the requisite skills for novel jobs and they respond by creating fewer novel jobs. The paucity of novel jobs increases unemployment for all workers, including those who already hold the requisite skills and discourages skill acquisition by other workers. As a consequence, when the skill mismatch is severe, labor markets go through a prolonged adjustment process wherein job creation and investments in skills are depressed and unemployment is high. A calibration of my model using reasonable parameters suggests that this mechanism changes the quantitative properties of business cycles and may generate long adjustment periods with high unemployment.

Keywords: Unemployment, Skill Mismatch, Structural Change, Business cycles.


Structural change leads to the obsolescence of old jobs and expands novel jobs that embody new technologies. But the expansion of novel jobs does not guarantee that workers reallocate at no cost. Old and novel jobs require different skills, and a large share of the workforce lacks the skills that novel jobs require. Structural change can create a skill mismatch.

For instance, since the mid 90s, cheaper computers have allowed firms to carry out tasks that previously had been performed by clerks, technicians, bookkeepers, salespersons, and other white-collar workers (see Autor, Dorn and Hanson, 2015). Figure 1-1 shows that, from 1996 to 2015, the share of the workforce employed in these jobs declined from 25.5% to 21%—a 4.5 percentage points decline, or 7 million jobs. The decline in routine-
cognitive jobs coincided with the expansion of a wide range of professional jobs such as audio-visual specialists, executive secretaries, data administrators and analysts, computer support specialists and engineering jobs. To avoid unemployment or low-pay service jobs, workers displaced from routine-cognitive jobs had to redeploy to professional jobs, but many of them lacked the analytical skills, training and formal education that are required in these jobs. By removing employment opportunities for middle-skill workers who specialized in routine tasks, the decline in routine-cognitive jobs created a skill mismatch.

![Employment rates by occupational category in the U.S.](image)

**Figure 1-1:** U.S. employment rates for different occupational categories. Data from the BLS.

In this paper I argue that when the skill mismatch is severe, labor markets go through a prolonged adjustment process in which job creation is low and unemployment is high for all workers. I also argue that the adjustment interacts with the business cycle, which causes a long-lasting increase in unemployment that concentrates in recessions; booms, on the other hand, mask the negative consequences of structural changes.

In Section 1.1 I develop my argument. I study a model in which an economy adjusts to a one-time structural change that leads to the gradual obsolescence of old jobs—routine-cognitive jobs, in my example—and expands novel jobs that require different skills—professional jobs, in my example. The economy adjusts as the unskilled workers who lack the skills required in novel jobs retrain by taking stepping-stone jobs.

The key assumption in my argument is that labor markets are frictional and are characterized by bargaining over the product of labor. Firms and workers face matching frictions when they form jobs; once matched the worker and the firm bargain over the surplus of
the relationship. Matching frictions are such that when firms post novel jobs they face the random arrival of workers and cannot guarantee to be matched only to workers with the requisite skills for these jobs. Thus, firms must incur some recruiting costs before learning if the match they obtained is proficient in the novel job. In such a world, firms cannot perfectly direct their search efforts and are less likely to obtain a skilled worker if the share of the unskilled among the unemployed is large. Thus, a surge in the share of unskilled workers that are searching for novel jobs—a severe skill mismatch—reduces the ability of the firm to locate workers with the requisite skills for novel jobs and crowds out matching opportunities for skilled workers (see Coate and Loury, 1993; Acemoglu 1996).

This description of the matching process seems plausible, especially when firms introduce new technologies or production methods. Firms may need to rely on search technologies that cannot be perfectly directed towards a particular type of worker. Moreover, the skills required in many novel jobs and occupations—including the professional jobs listed above—have not been standardized, and there are no credentials that reliably indicate proficiency in such jobs. In fact, many novel jobs require soft skills or skills that are learned on the job (e.g., team management, critical thinking, problem solving), and whose level an outsider cannot easily observe nor the worker communicate. The possibility that matching frictions feature such randomness is supported by the fact that new industries tend to agglomerate (see ). Intuitively, when recruiting is imperfect, a firm that wants to hire great programmers would locate in Silicon Valley, where workers with the required skills abound.

Structural change causes temporary but long-lasting unemployment. Because they lack the skills required in novel jobs, unskilled workers go through a prolonged period of unemployment and low wages until they retrain. I refer to this as the direct effect of structural change, which is the usual mechanism emphasized in the literature on job displacement and reallocation (see Kambourov and Manouskii, 2009; Alvarez and Shimer, 2011; and Jaimovic and Siu, 2014).

The direct effect is only a part of the story. Matching frictions introduce two amplification mechanisms that depend on the aggregate extent of skill mismatch—the share of unskilled workers among those who are searching for novel jobs. There is a job creation externality that amplifies unemployment. The skill mismatch lowers employer’s expectations about the probability of obtaining a skilled worker, and firms respond by creating less novel jobs. Thus, when the unskilled abound, the skill mismatch lowers the job-finding rate of both skilled and unskilled workers. This constitutes an externality because changes in the finding rate of workers have a first-order effect on their utility. Moreover, there is a complementarity effect, which dampens retraining and prolongs the adjustment. Since retraining is only useful in novel jobs, during periods of skill mismatch, the paucity of novel jobs reduces the incentives...

1Theoretical point, and alck of labor market institutions.
of workers to retrain. Firms respond by creating few stepping-stone jobs even if there are no contractual frictions or nominal rigidities involved (see Caballero and Hammour, 1996).

The distinctive implication of my model is that, due to the job creation externality and the complementarity effect, the finding rate of a worker not only depends on his skills, but also on the skill mismatch in his labor market. The skill mismatch reduces job opportunities for skilled workers and affects the redeployment and retraining of unskilled workers.

Although the decline in routine-cognitive jobs started in the mid 90s, Figure 2-2 shows that about two thirds of the decline in the last 20 years occurred during the Great Recession (see also the evidence by Jaimovic and Siu, 2014). Also, as I show in my empirical exercise, the effects of this structural change on unemployment concentrate in economic downturns. My framework underscores two potential mechanisms by which the structural change interacts with the business cycle, causing a long-lasting increase in unemployment that concentrates in recessions.

First, because unskilled workers produce a low surplus in the available jobs, their job-finding rate is more responsive to changes in productivity. Thus, during periods of low aggregate productivity, unskilled workers cannot find novel jobs easily; the share of unskilled workers among the unemployed rises, which exacerbates the skill mismatch and its negative externalities (see also Pries, 2008, who emphasizes the same mechanism).

Second, a literature going back to Schumpeter (1942) argues that, due to the low opportunity cost of adjustment during recessions, firms use crisis to replace old jobs with new technologies or restructure and close job positions that will soon become obsolete due to advances in technology. In the case of routine-cognitive jobs, the data supports the assumption that firms adjust their labor requirements during recessions, which caused a permanent shift in the demand for routine-cognitive labor during the Great Recession. Figure 1-2 shows that during the recession, job openings for routine-cognitive jobs—the analog of old jobs in my model—suffered a permanent decline of about 55% relative to other jobs.

I incorporate this possibility by assuming that recessions bring a temporary increase in the rate at which firms permanently close the available positions for old jobs and stop hiring labor to produce these tasks. During good times, old jobs are still plentiful; workers are not displaced nor forced to redeploy to novel jobs and the skill mismatch is modest. During recessions, firms adjust the type of jobs that they post and there is a permanent decline in openings for old jobs. The paucity of old jobs pushes unskilled workers into

---

2Hagedorn and Manouski (2008) and Ljungqvist and Sargent (2015), too, emphasize that when the net surplus is low, wages become endogenously rigid and the job-finding rate of workers becomes more cyclical.

3See also Davis and Haltiwanger (1990), Hall (1991), Caballero and Hammour (1994), Aghion and Saint Paul (1998), Koenders and Rogerson (2005), and Berger (2014).

4A recent report by Burning Glass Technologies (2014) shows that, even within the remaining job openings for middle-skill jobs, firms are demanding higher qualifications and workers are expected to perform different tasks than before.
unemployment and to redeploy to novel jobs, which exacerbates the skill mismatch and its negative externalities. I show that when training costs are high, the resulting increase in unemployment may outlast the recession and create a jobless recovery.

The theory section presents the formal statements and intuitions for these results. The model is dynamic and has several state variables, but a careful choice of assumptions allows me to characterize the adjustment path when gross flows are sufficiently large (as is the case in the data for the U.S.). Moreover, I characterize the inefficiencies that arise because of the job creation externality in terms of wedges between the private and social value of job searching for different workers (as in Shimer and Smith, 2001). The wedges indicate that the inefficiencies arise because unskilled workers deteriorate matching opportunities for all workers, while skilled workers improve them. Workers and firms do not internalize these first-order effects when forming matches or creating stepping-stone jobs. The inefficiencies arise even if the Hosios condition holds (Hosios, 1990).

Section 1.2 supplements my theoretical analysis with a quantitative exploration of the model. For plausible parameters, the external and complementarity effects explain about 40% of the increase in unemployment along the adjustment. The efficient allocation exhibits about 30% less unemployment than the decentralized one and features more stepping-stone jobs. Although firms and workers bargain over small quasi-rents (matching frictions are small in the parametrization I use), matching frictions can have a significant effect on skill acquisition and job creation. Moreover, the interaction between structural change and recessions is significant. I parametrize a recession as a decline of 5% in productivity and an increase in the rate at which firms close available positions for old jobs. This shock is calibrated to match the permanent decline in job openings for routine-cognitive jobs that I present below in Figure 2-3. Both shocks last for 10 quarters. When the recession affects an economy
that is adjusting to structural change, it increases unemployment by up to 3 percentage points. Five years after the recession ends, unemployment remains above its pre-recession trend for both types of workers. In the absence of structural change, the same recession would only increase unemployment by 1 percentage point, and unemployment would exhibit no propagation (see Shimer, 2005).

**Related literature:** The mechanism behind the job creation externality builds on the work of Acemoglu (1996, 1997), who presents models in which the ex-post bargaining of workers and firms over their joint surplus reduces investments in capital and training. My paper incorporates this mechanism into the canonical search model, which allows me to quantify the externalities that arise solely from the small matching frictions that are typically calibrated in the literature. Unlike previous studies, I examine if this mechanism affects how the economy adjusts to structural changes and if this mechanism interacts with the business cycle. I also provide evidence of the external amplification effects implied by this mechanism.

Finally, I contribute to the literature that examines the empirical performance of Mortensen and Pissarides's (1994) matching model. I show that through their interaction with structural change, recessions can generate a large and long-lasting increase in unemployment. As shown by Shimer (2005) the canonical search model by itself fails to generate these patterns. In keeping with the available evidence, due to their lack of requisite skills, unemployment spells for unskilled workers who have been displaced from old jobs may be more costly than in the canonical search model (see Davis and von Wachten, 2011).

### 1.1 A Model of Structural Change

I extend the matching model of Mortensen and Pissarides (1994) to include several types of jobs and workers. Time is continuous and I omit it whenever it causes no confusion. All individuals are risk neutral and discount the future at a constant rate \( r \).

In its status quo, the economy produces a final good \( Y \) (with price normalized to 1) by

---

5 In a related paper, Coates and Loury (1993) argue that imperfect learning by employers creates a negative spillover on all workers and reduces incentives for skill acquisition. Shimer and Smith (2001) also emphasize the role of externalities in a matching model with ex-ante heterogeneous agents. Beaudry, Green and Sand (2012) provide evidence that openings for high-paying jobs create a positive externality, as the job creation externality does in my model.

6 Shimer's paper sparked a whole literature that modified the canonical search model or the calibration used to improve the model's ability to match the data (see Hall, 2005; Hagedorn and Manouski, 2008; Hall and Milgrom, 2008; Pissarides, 2009; and Ljunquist and Sargent, 2015).

7 In my model, unemployment spells for unskilled workers are more costly during recessions and when markets face a severe skill mismatch. The reason is that the paucity of novel jobs affects the redeployment of unskilled workers. This provides an alternative perspective to recent scholarship (see Huckfeldt, 2014; Jarosch, 2014; and Krolikowski, 2014), which emphasizes how recurrent job losses create costly unemployment spells.
combining a mass of tasks \( y(i) \) with \( i \in [0, 1] \):

\[
Y = \int_0^1 y(i) di.
\]

A mass 1 of workers are employed in jobs, each of which produces a single task \( i \).

Structural change shifts the productive structure by an amount \( \Delta \) as shown in Figure 1-3.

![Diagram](image)

Figure 1-3: Task space and a graphical representation of the effect of structural change on the productive structure. The top panel presents the status quo and the bottom panel the structural change.

The shift in the productive structure partitions the task space in a variety of jobs:

- **Old jobs**—indexed by the superscript \( o \)—, which produce tasks in \([0, \Delta)\). These jobs are at risk of becoming obsolete due to the competition from technology or because these jobs embody old technologies. In the example in the Introduction, old jobs correspond to the routine-cognitive jobs.

- Old jobs in \([\Delta - I(t), \Delta)\) still hire labor. \( I(t) \) determines the number of available old jobs. These jobs become obsolete at an exogenous rate \( v(t) > 0 \), so that

  \[
  \dot{I} = -Iv(t), \text{ with } I(0) = \Delta.
  \]

Old jobs in \([0, \Delta - I(t))\) are obsolete and do not hire labor.

The rate \( v(t) \) may be constant during normal times, which reflects the secular advancement of structural change. Increases in \( v(t) \) reflect fast technological change or periods of adjustment in which firms adopt the existing technologies and permanently close positions for old jobs, such as during recessions.

- **Regular jobs**—indexed by the superscript \( r \)—, which perform tasks in \([\Delta, 1]\). Structural change does not affect these jobs. Regular jobs do not require new skills, and they provide
feasible employment alternatives for workers displaced from old jobs. In the example cited in the Introduction, regular jobs correspond to service jobs that do not require retraining.

**Novel jobs**—indexed by the superscript \( n \)—, which perform tasks in \((1,1+\Delta]\). Structural change expands these additional jobs. The defining characteristic of novel jobs is that they require additional skills which unskilled workers lack. In the example described in the Introduction, novel jobs correspond to professional jobs that rely on analytical and cognitive skills, and tend to require more training, job-related experience and formal education than routine-cognitive jobs.

Among novel jobs we have *stepping-stone jobs*, which provide retraining opportunities that allow unskilled workers to become skilled on the job. I think of stepping-stone jobs as created by firms that have enough time and resources to undertake costly investments to train new hires. For instance, a stepping-stone job for secretaries would allow them to learn over time the skills needed to become an executive secretary. Or firms could train technicians to perform some of the tasks reserved for professional engineers. Stepping-stone jobs become an attractive option for firms when unskilled workers abound and skilled workers are hard to find. Though the firm must incur training costs and wait for workers to become skilled, it can extract in the form of lower wages part of the gains of training, especially when unskilled workers highly value the acquisition of new skills (see Becker, 1964).

On the labor supply side, workers are of two types: skilled—indexed by the subscript \( s \)—or unskilled—indexed by the subscript \( u \). Skilled workers produce \( z(t) \) units of any task. Unskilled workers produce \( z(t) - q^n \) units of new tasks and \( z(t) \) units when employed in regular or old jobs. Here, \( z(t) \) is the marginal product of labor and \( q^n > 0 \) reflects unskilled workers' lack of expertise in novel jobs (it is in this sense that workers are unskilled). By taking stepping-stone jobs unskilled workers retrain and become skilled through on-the-job learning. When employed in these jobs, unskilled workers produce \( z(t) - q^n - q^l \) units of output and become skilled at a rate \( \alpha > 0 \). Here, \( q^l > 0 \) denotes the costs of training. Moreover, unskilled workers become skilled at a small exogenous rate \( \delta \approx 0 \), which captures other forces not modeled that can include the entry of new college cohorts or the standardization of new technologies. This exogenous rate guarantees that all the workforce eventually becomes skilled.

As old jobs obsolesce and close, firms stop hiring labor for these jobs and workers redeploy to novel jobs. The skill mismatch arises because unskilled workers are less productive at novel jobs. The economy adjusts as unskilled workers take stepping-stone jobs and become skilled, so that in the long run, all workers are employed in jobs that produce tasks in \([\Delta,1+\Delta]\).

To introduce my mechanism, I depart from competitive labor markets and assume that there are matching frictions. For each task, there is a separate *hiring market*. Unemployed workers populate these hiring markets in an undirected fashion, constantly churning across
markets, tinkering at job opportunities in different tasks until they are matched to a job opening. When unemployed, skilled workers spend a share \( \Delta \) of their time searching in the hiring markets for novel jobs, and they spend the remaining share \( 1 - \Delta \) searching in hiring markets for regular jobs. These frequencies reflect the share of regular and new tasks in the economy. Unskilled workers also take advantage of available old jobs. They spend a share \( \frac{\Delta}{1+\ell} \) of their time searching in the hiring markets for novel jobs; a share \( \frac{1-\Delta}{1+\ell} \) in the markets for regular jobs; and the remaining share \( \frac{1}{1+\ell} \) searching in the hiring markets for the available old jobs. Thus, at each point in time, the hiring market for old jobs is populated by unskilled workers, while the hiring markets for regular and novel jobs are populated by both types of workers.

To hire workers, firms post vacancies at a flow cost \( \kappa \), which are aimed at a particular type of worker. When hiring for novel jobs in \( i \in [1, 1+\Delta] \), firms may post stepping-stone jobs aimed at unskilled workers, \( v''_i \). As emphasized above, the creation of stepping-stone jobs is the main endogenous margin that drives the adjustment of the economy.

Alternatively, firms can attempt to hire workers who already hold the requisite skills by posting vacancies for jobs that do not offer any training, \( v''_i \), and that are aimed at skilled workers. The key assumption is that, when posting these jobs, firms cannot perfectly direct their search efforts, and so with some random probability they will be (mis)matched to an unskilled worker who is searching in the hiring market for task \( i \in [1, 1+\Delta] \). To model this possibility, I assume that with probability \( \pi > 0 \), unskilled workers fail to be screened out and end up in the queue for vacancies \( v''_i \). This allows some unskilled workers to obtain jobs faster, although these jobs do not offer training. In this case, firms only realize they were mismatched after meeting with the worker and having already incurred in search costs. With probability \( 1 - \pi \), unskilled workers reveal their true type and queue only for stepping-stone jobs. Skilled workers who search for novel jobs always queue for the vacancies \( v''_i \). Thus, firms that post vacancies \( v''_i \) are randomly matched to both skilled and unskilled workers at frequencies that depend on \( \pi \) and on the number of skilled and unskilled workers who are searching for novel jobs.

My model captures succinctly how random matching affects the probability with which firms expect to obtain a skilled match when posting a novel job. Let \( \gamma^n \) denote this probability, and \( \gamma \) be the share of skilled workers among the unemployed. Then

\[
\gamma^n = \frac{\gamma \Delta}{(1 - \gamma) \frac{\Delta}{1+\ell} + \gamma \Delta}.
\]  

When the unskilled abound among the unemployed (\( \gamma \) is low) and structural change makes more old jobs obsolete (\( I \) declines and pushes unskilled workers to redeploy to novel jobs), firms become pessimistic about obtaining a skilled match and about the expected profits
from job creation, captured here by a reduction in $\gamma^n$. Lower values of $\gamma^n$ reflect a severe skill mismatch.

The defining feature of random matching is not that firms cannot direct their search efforts (they could wait for a skilled match if they wanted to), but that an inflow of unskilled workers—the skill mismatch—crowds out matching opportunities for skilled workers and increases the risk for firms of being mismatched (see Shimer and Smith, 2001). Equation (1.1) captures this feature succinctly in a reduced form way. Here, $\pi$ represents the noise in the signals used by firms to screen candidates or the extent to which firms cannot perfectly direct their search efforts. Both of which reduce firms’ ability to locate skilled workers during periods of severe mismatch.\textsuperscript{8}

For tractability, and because they play no major role, the interactions in the hiring markets for regular and old jobs are simpler. In the market for regular jobs, firms post vacancies that are aimed at hiring skilled workers, $v^s(i)$, or unskilled ones $v^u(i)$, for all $i \in [\Delta, 1)$. Firms are able to separate workers by their type, and skilled workers in this market queue for vacancies $v^s(i)$; while unskilled workers queue for vacancies $v^u(i)$. Finally, in the hiring market for old jobs, firms post vacancies aimed at the unskilled workers that populate it, $v^o(i) \forall i \in [\Delta - I(t), \Delta)$. Hiring does not feature a random component in these markets.

When firms post a vacancy $v^s_j(i)$, they are matched to the workers in the queue for the job at a rate $q(\theta) = a\theta^{-\eta}$, with $\eta \in [0, 1]$. Here, $\theta$—the tightness—equals the ratio of vacancies to the number of workers who are searching for this particular job. Workers queuing for a vacancy are matched at a rate $f(\theta) = a\theta^{1-\eta}$. Thus, the matching process in each hiring market exhibits constant returns to scale. Once matched, the firm observes the worker type and decides whether to keep the match. If they do so, they split the surplus through Nash bargaining and the worker obtains a share $\beta > 0$. Ongoing matches separate at an exogenous rate $\lambda > 0$ and there are no endogenous separations. Finally, there is free entry of firms.

To complete the description of the environment, I now present the behavior of the state variables. I have to keep track of $x(t)$, the number of skilled workers; $u(t)$, the unemployment rate; $s(t)$, the number of stepping-stone jobs; $\gamma(t)$, the share of skilled workers among the unemployed; and $I(t)$, the remaining old jobs. The state variables evolve according to the

\textsuperscript{8}To think about the role of the random matching assumption, I find it useful to consider the case of a young firm that has just entered the market and is deciding whether to create a professional job. This firm has no large human-resources department, it does not receive hundreds of job applications from the best workers, and cannot go through long and costly processes to select its personnel. Such a firm must take chances, and will choose to expand depending on the expected skill level of workers who are searching for jobs. This example is relevant because the evidence suggests that young and new firms are responsible for the bulk of employment growth (see Haltiwanger, Jarmin and Miranda, 2011).
backward-looking differential equations:

\[
\begin{align*}
\dot{x} &= \alpha s + \delta u(1 - \gamma), \\
\dot{u} &= \lambda(1 - u) - u \gamma f_s - u(1 - \gamma)f_u, \\
\dot{s} &= u(1 - \gamma) \frac{\Delta}{1 + I} (1 - \pi) f(\theta_u) - (\alpha + \lambda)s, \\
\dot{\gamma} &= (1 - \gamma) \gamma (f_u - f_s) + \lambda \frac{x - \gamma}{u} + (1 - \gamma) \delta, \\
\dot{I} &= -I u(t).
\end{align*}
\] (1.2)

Here, \( \theta_u \) is the tightness in the queue for stepping-stone jobs. \( f_u \) and \( f_s \) are unskilled and skilled workers finding rates, respectively, which depend on the equilibrium tightness in all labor markets. The finding rates are given by (assuming all matches yield a positive surplus):

\[
\begin{align*}
\dot{f}_s &= \Delta f(\theta_s^e) + (1 - \Delta)f(\theta_s^e), \\
\dot{f}_u &= \frac{\Delta}{1 + I} [(1 - \pi) f(\theta_u^e) + \pi f(\theta_u^e)] + \frac{1 - \Delta}{1 + I} f(\theta_u^e) + \frac{I}{1 + I} f(\theta_u^e).
\end{align*}
\] (1.3)

Given a starting value for the state variables \( \{x(0), u(0), s(0), \gamma(0), I(0)\} \), an allocation consists of a path for tightness \( \{\theta_u^e(t), \theta_u^e(t), \theta_s^e(t), \theta_s^e(t), \theta_u^e(t)\} \), and a path for the state variables \( \{x(t), s(t), u(t), s(t), I(t)\} \) that solves the system of differential equations given by their initial condition and equation (1.2).

### 1.1.1 Characterizing the Equilibrium

An equilibrium is given by an allocation in which the tightness of all markets is determined by firm entry decisions, and firms enter the market motivated by the profits from job creation.

The surplus of different matches \( S_j^k - k \) indexes the type of job and \( j \) the type of worker—satisfy the Bellman equations:

\[
\begin{align*}
(r + \lambda)S_s^n &= z(t) - (rU_s - \hat{U}_s) + \hat{S}_s^n, \\
(r + \lambda)S_s^r &= z(t) - (rU_s - \hat{U}_s) + \hat{S}_s^r, \\
(r + \lambda)S_u^n &= z(t) - q^n - (rU_u - \hat{U}_u) + \hat{S}_u^n, \\
(r + \lambda)S_u^r &= z(t) - (rU_u - \hat{U}_u) + \hat{S}_u^r, \\
(r + \lambda)S_u^d &= z(t) - (rU_u - \hat{U}_u) + \hat{S}_u^d.
\end{align*}
\] (1.4)

The discounted surplus on the left equals the flow value of production, minus the opportunity cost of workers (their reservation wage, \( rU_j - \hat{U}_j \)), plus the appreciation of the match value. Free entry by firms implies that their opportunity cost of engaging in a match is zero.

The surplus of stepping-stone jobs has a different Bellman equation that is given by:

\[
(r + \lambda)S_u^d = z(t) - q^n + \max\{-q' + \alpha(U_s - \hat{U}_u) + \alpha(S_s^n - S_u^d), 0\} - (rU_u - \hat{U}_u) + \hat{S}_u^d. \] (1.5)

The terms \( \alpha(S_s^n - S_u^d) \) and \( \alpha(U_s - U_u) \) correspond to the gains, shared by the worker and the
firm, when the worker becomes skilled. The max operator on the right-hand side indicates that firms have the option value of not training workers if it is not profitable for the pair. The term $\alpha(U_s - U_u)$ underscores the fact that workers recognize the benefits of taking these jobs and they share these benefits with the employer through bargaining, who is then able to recover part of the training expenditures. No contractual problems affect stepping-stone jobs.\(^9\) However, as it will be clear in my analysis, when forming these jobs the firm and worker do not take into account the benefits that accrue to future employers, who benefit from the better chances of matching with a skilled worker.

Workers' reservation wages are given by the Bellman equations:

\[
\begin{align*}
W_s &= rU_s - \hat{U}_s = b + \Delta \beta f(\theta^s) \max\{S^s_0, 0\} + (1 - \Delta) \beta f(\theta^s) \max\{S^s_u, 0\}, \\
W_u &= rU_u - \hat{U}_u = b + \frac{\Delta}{1 + I} \beta f(\theta^u) \max\{S^u_0, 0\} + (1 - \beta) f(\theta^u) \max\{S^u_u, 0\}
\end{align*}
\]

\[
\begin{align*}
&+ \frac{1 - \Delta}{1 + I} \beta f(\theta^u) \max\{S^u_u, 0\} + \frac{I}{1 + I} \beta f(\theta^u) \max\{S^u_u, 0\} + \delta(U_s - U_u).
\end{align*}
\]

(1.6)

The reservation wage equals the value of leisure, $b$, plus a share $\beta$ of the expected surplus at different jobs multiplied by the rate at which the worker obtains these jobs.

The equilibrium tightness for each type of job is determined by free entry:

\[
\kappa \geq q(\theta^k) (1 - \beta) E_S[\max\{S, 0\} | k, j]
\]

(1.7)

with equality when $\theta^k > 0$. Here, $E_S[\max\{S, 0\} | k, j]$ denotes the expected surplus of a match that is obtained by posting a vacancy $V^k$. The max operator indicates that a firm rejects matches that yield a negative surplus. Given that vacancies in old, regular and stepping-stone jobs are matched to a single type of worker, the expected surplus is $\max\{S^k, 0\}$.

Because firms that post novel jobs, $V^k$, are matched to both skilled and unskilled workers, their free entry condition becomes:

\[
E_S[\max\{S, 0\} | n, j] = \gamma^n \max\{S^n_s, 0\} + (1 - \gamma^n) \max\{S^n_u, 0\},
\]

(1.8)

with $\gamma^n$ the probability that vacancies for novel jobs yield a match with a skilled worker (see equation 1.1). Because $S^n_s > S^n_u$ (see lemma 1 in the Theory Appendix), when the mismatch is severe and firms are pessimistic about finding skilled workers ($\gamma^n$ is low) they create less

---

\(^9\)One could incorporate these inefficiencies by assuming that with some probability $H > 0$, the worker captures the value of the increase in his outside option. This could also represent a lower bound on wages. I find that small values of $H$ reduce training, exacerbate the skill mismatch, and have a large effect on unemployment. See also Caballero and Hammour (1996) for models in which contractual problems slow down the adjustment of the economy.
novel jobs and reduce tightness. This response constitutes the job creation externality. The externality arises because firms earn quasi-rents in the form of a share of the surplus of the match, and so they care about obtaining workers who yield the largest surplus. If firms paid workers their full marginal product, wages would adjust to reflect the differences in productivity and this mechanism would not operate.

Given a starting value for the state variables \( \{x(0), u(0), s(0), \gamma(0), I(0)\} \), an equilibrium consists of an allocation in which the value functions \( \{U_u(t), U_a(t), S^a_u(t), S^a_s(t), S^a_u(t), \} \) satisfy the Bellman equations (1.4), (1.5) and (1.6); and the equilibrium tightnesses \( \{\theta^u_r(t), \theta^u_u(t), \theta^s_r(t), \theta^s_u(t)\} \) are determined implicitly by equation (1.7).

### 1.1.2 Analysis of the model

Throughout I assume that \( x(0) < 1 \) so that not all workers are skilled and the structural change induces a skill mismatch. The Theory Appendix contains the proofs of all the propositions.

I start by analyzing the long-run behavior of the equilibrium. Before the structural change, I assume that \( z(0) = 1 \) and that the economy is in steady state. By \( u^*, \theta^*, f^* \) and \( v^* \) I denote the equilibrium unemployment rate, tightness, finding rate and number of vacancies, respectively, in the status quo of this economy. These correspond to the equilibrium objects in the usual search and matching model with no heterogeneity.\(^{10}\) Proposition 1 shows that the effects of structural change are only temporary and the economy reverts to its status quo.

**Proposition 1 (Steady-state behavior)** *The economy converges to a unique steady state with* \( x(t), \gamma(t) \to 1, u(t) \to u^* \) *and* \( \theta(t) \to \theta^* \). *In this steady* \( f_\theta(t) \to f^* \) *and* \( f_a(t) \to f_a^* \).

The economy adjusts as \( x(t) \to 1 \) and \( I(t) \to 0 \), both because the economy creates stepping-stone jobs and because unskilled workers eventually become skilled at the rate \( \delta > 0 \). Because structural change does not affect the measure and productivity of jobs available to skilled workers, the economy reverts to its initial status quo over the long run.

To characterize the transitional dynamics, I focus in the case in which gross flows are large. In this case, all state variables but \( x(t) \)—the share of skilled workers—and \( I(t) \)—the number of available old jobs—adjust immediately and exhibit no propagation on their own, which simplifies the analysis and allows me to derive analytically a clean characterization of the adjustment. Because gross flows are so large in U.S. markets (see Davis and Haltiwanger, 1990) this case is also empirically relevant.

---

\(^{10}\) In particular, tightness and unemployment are implicitly defined by the equations \( (1 - \beta)(1 - b) = \frac{r + \lambda + \beta \theta^* q(\theta^*) \kappa}{q(\theta^*)} \) and \( u = \frac{\lambda}{\lambda + f(\theta^*)} \).
Let $a = \tilde{a} \xi$ and $\lambda = \tilde{\lambda} \xi$, and suppose $\xi \to \infty$, so that the gross flows between employment and unemployment are large. Because separation rates are large, future reservation wages or productivities do not affect the current surplus of jobs. The normalized surpluses, $\xi S_j^k$, in each job, the reservation wages, $w_s$ and $w_u$, the finding rates, $f_s$ and $f_u$, and the tightness $\theta_u$, are well defined in this limit and only depend on the current value of $z(t), x(t), I(t)$ and $\Omega(t) = U_s(t) - U_u(t)$—the incentives to acquire skills (see the Theory Appendix for details). 11

Moreover, the right hand sides of the equations for $u, u$ and $s$ must converge to zero, so that $u(t), u(t)$ and $s(t)$ are determined solely by the current value of $z(t), x(t), I(t)$ and $Q(t)$:

$$(1.9)$$

These equations implicitly define $\gamma(z, x, I, \Omega), u(z, x, I, \Omega)$ and $s(z, x, I, \Omega)$, which are independent of their past values and adjust immediately. Here, $1 - \gamma(t)$ tracks $1 - x(t)$, but takes into account the different finding rates of skilled and unskilled workers. Unemployment depends on the average finding rate $f = \gamma f_s + (1 - \gamma) f_u$. The variable $\Omega(t)$ summarizes the incentives to acquire skills, which determine employment in stepping-stone jobs.

To analyze the model, I maintain three assumptions. First, I assume that $q^l > \bar{q}$, with $\bar{q} = (\alpha + \tau) \Omega^* - q^a$ (here, $\Omega^*$ is the steady-state value for $\Omega(t)$). This restriction guarantees that $S_u^a \leq S_u^o < S_u^o$ in equilibrium, so that unskilled workers produce a lower surplus than skilled workers in novel jobs. Second, I assume that $\beta < \bar{\beta}$. This restriction guarantees that the job-finding rate of unskilled workers decreases when $I(t)$ is low and old jobs close. For large values of $\beta$, a decline in $I(t)$ lowers unskilled workers' reservation wages so much that firms could end up creating a large number of regular jobs and increasing unskilled workers' finding rates. Third, I assume that $\pi < \bar{\pi}$. This restriction guarantees that the equilibrium is unique and that the externalities do not introduce instabilities in the adjustment of the

11 Let $\tilde{f} = \tilde{a} \tilde{\theta}^{1-\eta}$ and $\tilde{q} = \tilde{a} \tilde{\theta}^{n-\eta}$ Formally, the normalized surpluses are well defined and given by:

$\xi S_s^a = \frac{z(t) - w_s}{\lambda}, \quad \xi S_s^o = \frac{z(t) - w_s}{\lambda}$

$\xi S_u^a = \frac{z(t) - w_u}{\lambda}, \quad \xi S_u^o = \frac{z(t) - q^a - w_u}{\lambda}$

The reservation wages are well defined in the limit, and are given by

$w_u = b + \Delta (f(\theta^*_a) \max \{\xi S_s^a, 0\} + (1 - \Delta) f(\theta^*_o) \max \{\xi S_s^o, 0\})$

$w_s = b + \frac{\Delta i}{1 + e} \beta f(\theta^*_a) \max \{\xi S_s^a, 0\} + (1 - \pi) f(\theta^*_u) \max \{\xi S_u^a, 0\} + \frac{1 - \Delta}{1 + e} \beta f(\theta^*_o) \max \{\xi S_u^o, 0\} + \beta \Omega$

Finally, the equilibrium tightnesses are given by $\kappa \geq \bar{\kappa}(\theta^*_k) (1 - \beta) \xi S \max \{\xi S, 0\} [k, j]$. These equations coincide with a steady state in which $z, x, I, \Omega$ are fixed over time.
economy. The thresholds $\bar{\beta}, \bar{\pi} > 0$ are derived in the Appendix. The conditions $\beta < \bar{\beta}$ and $\pi < \bar{\pi}$ are not demanding. For the parametrization of my model introduced in Section 1.2, any value of $\pi \in [0, 1)$ and values of $\beta$ as large as 0.9 satisfy these conditions.

The following proposition summarizes the properties of the transitional dynamics.

**Proposition 2 (Transitional Dynamics)** Let $a = \tilde{a} \xi$ and $\lambda = \tilde{\lambda} \xi$, and suppose $\xi \to \infty$.

1. The current values of $x, I,$ and $\Omega$ are a sufficient statistic for the equilibrium objects. The behavior of $x, I$ and $\Omega$ boils down to the system of differential equations:

\[
\begin{align*}
\dot{x} &= (1 - x) \left[ \frac{\Delta (1 - \pi) f(\theta^i_u)}{\lambda + f_u} + \delta \right], \\
\dot{\Omega} &= r \Omega + w_u - w_s \\
\dot{I} &= -v(t)I,
\end{align*}
\]

coupled with an initial condition for $x(0)$ and $I(0)$, and paths for $z(t)$ and $v(t)$.

2. The system is globally saddle-path stable and converges to $x(t) = 1, I(t) = 0, \Omega(t) = \Omega^*$. If $z(t) = 1$ for all $t$, the stable arm is described by a curve in which $x(t)$ and $\Omega(t)$ increase monotonically to their steady-state values and $I(t)$ declines at the exogenous rate $v(t)$.

Figure 1-4 shows the phase diagram for the equilibrium (holding $I(t)$ and $z(t)$ constant). The dotted lines are the loci for $\dot{U} = 0$ and $\dot{x} = 0$ (a vertical line through $x = 1$). Starting from any $x(0)$, the incentives to upgrade skills, $\Omega(0)$, jump to the stable arm and both $\Omega(t)$ and $x(t)$ converge monotonically to the steady state.

![Phase diagram](image)

**Figure 1-4**: Phase diagram for the equilibrium in terms of $x(t)$ and $\Omega(t)$.

When gross flows are large, as they are in the U.S. data, the bulk of the state dependence in my model and the labor market consequences of structural change are driven by the behavior of $x(t)$ and $I(t)$. The dynamics of the remaining state variables introduce minor
effects, as is the case in the usual parametrizations of the canonical search model (see Shimer, 2005).

The following proposition characterizes the adjustment when \( z(t) = 1, I(t) = 0 \) for all \( t \); there are no aggregate shocks and unemployment is driven by the endogenous behavior of \( x(t) \).

**Proposition 3 (Structural Unemployment)** Suppose \( z(t) = 1 \) and \( I(t) = 0 \) for all \( t \). The adjustment to structural change satisfies:

1. \( f_u(t) < f_s(t) \) for all \( t \geq 0 \).
2. Along the transition, we have that \( f_s(t) < f^* \) and \( f_u(t) < f_u^* \) for all \( t \geq 0 \). Moreover, both \( f_s(t) \) and \( f_u(t) \) increase over time for all \( t \geq 0 \).
3. A lower \( x(0) \) shifts down the entire equilibrium path for \( x(t) \), the average finding rate \( f(t) \) and the finding rates \( f_s(t), f_u(t) \).

The proposition shows that the skill mismatch induced by structural change—captured by the share of unskilled workers, \( 1 - x(0) \)—causes unemployment along the transition. Unemployment is driven by a decline in the average finding rate \( f = \gamma(t)f_s(t) + (1 - \gamma(t))f_u(t) \). The deviation of the average finding rate with respect to its initial level is given by:

\[
f - f^* = (1 - \gamma(t))[f_u(t) - f_s(t)] + [f_s(t) - f^*].
\]

The first term, \( (1 - \gamma(t))[f_u(t) - f_s(t)] < 0 \), captures the direct effect of structural change—as I labeled it in the Introduction. A lower \( x(0) \) increases the share of unskilled workers among the unemployed at all points in time, \( 1 - \gamma(t) \), and these workers have a lower finding rate.\(^{12}\) The second term, \( f_s(t) - f^* \leq 0 \) captures the effect of the job creation externality. When \( x(0) \) is small, firms anticipate that more unskilled workers will be searching for novel jobs. Firms respond to the skill mismatch by creating less novel jobs (per searcher), which reduces the finding rate of both skilled and unskilled workers below their steady-state levels: \( f_s(t) < f_s^*, f_u(t) < f_u^* \). In contrast, in the limit when \( \pi = 0 \) and there is no random matching we have \( f_s(t) = f^* \), and the skill mismatch only increases unemployment via the direct effect.

The proposition also clarifies the nature of unemployment in my model. Despite the fast flows, the skill mismatch—the interplay between a low \( x(t) \) and the lack of old jobs—creates unemployment by reducing the average finding rate. Contrary to models of reallocation that build on Lucas and Prescott (1976), the time it takes workers to move from searching for old to new jobs—search unemployment—plays no role in my framework (or at most a minor

\(^{12}\)Intuitively, this is the case because all workers are matched to novel and regular jobs at some rates, but because of training costs \( (q' > \overline{q}) \) and their lower productivity, they face lower finding rates for novel jobs than skilled workers.
role if my model is parametrized to match the large gross flows in the data).\textsuperscript{13} Matching frictions are important not because of the search unemployment they create but because of the way in which they affect job creation by firms.

Given the large inflow of unskilled workers searching for novel jobs, one would be tempted to conclude that firms could profit from creating a large number of stepping-stone jobs and that the skill mismatch would not last for long. However, through the complementarity effect outlined in the Introduction, the skill mismatch dampens the creation of stepping-stone jobs.

**Proposition 4 (Complementarities in Skill Upgrading)** Suppose $z(t) = 1$. Along the adjustment, we have that $\Omega(t)$—the incentive of unskilled workers to become skilled—increases over time. Moreover, a lower $x(0)$ shifts the entire equilibrium path for $\Omega(t)$ down.

The upward-sloping locus for the stable arm in Figure 1-4 depicts the complementarity effect: for small $x(t)$, the incentives to acquire skills, $\Omega(t)$ are lower, and these increase over time as more workers become skilled.

The complementarity effect results from the fact that skilled workers derive a larger increase in their utility from the novel jobs that are affected by the job creation externality than unskilled workers do. Thus, a worst skill mismatch hurts skilled workers more than it hurts the unskilled and reduces the value of becoming skilled, $\Omega(t)$. In my model, this feature follows from the fact that $S^a_s < S^a_u$—which reflects unskilled workers lower productivity in novel jobs—and the assumption that skilled workers exogenously search more often for these jobs than do unskilled workers. I find this assumption plausible and intuitive. If workers were able to direct their search efforts, and given that $S^a_s < S^a_u$, skilled workers would still search for novel jobs more often than will unskilled workers.\textsuperscript{14}

The main implication of 4 is that the complementarity effect reduces the creation of stepping-stone jobs, which further amplify unemployment and prolongs the skill mismatch. This occurs because stepping-stone jobs are profitable to the extent that workers are willing to take wage cuts to retrain (see equation 1.5). During periods of severe mismatch, workers perceive a lower value of acquiring skills. Thus, firms not only post few novel jobs; they do not take full advantage of the large inflow of unskilled workers whom they could retrain.

Propositions 3 and 4 combined imply that, due to the job creation externality and the complementarity effect, unemployment will be accompanied by a drop in tightness and va-

\textsuperscript{13} Pilossof (2014), too, argues that sectoral reallocation can create little unemployment when gross flows are large. The result for my limit case echoes her findings, and it shows that my theory of unemployment, which is based on the mismatch of skills, is not affected by this criticism.

\textsuperscript{14} To substantiate this point, in the Theory Appendix I present an extension of my model in which workers are able to partially direct their search efforts. In this extension, workers allocate their search efforts based on idiosyncratic shocks that garble their expected utility of searching for jobs in each particular task. I derive the equilibrium distribution of workers searching for each job and show that skilled workers allocate a greater share of their time to searching for novel jobs than unskilled workers. I also show that, even if allowed, skilled workers would search for old jobs less than unskilled workers do.

As mentioned in the Introduction, the negative effects of structural change may concentrate in recessions. To explore the interaction between structural change and the business cycle, I characterize the equilibrium of an economy adjusting to structural change which is hit by an unanticipated recession that lasts from time $T_i$ to $T_f$. I model recessions as bringing two aggregate shocks. First, the recession causes a temporary decline in productivity from $z(t) = 1$ to $z_L < 1$ for $t \in [T_i, T_f)$. In addition, the recession increases the rate at which firms close old jobs to $\bar{v}$ for $t \in [T_i, T_f]$, while $v(t) = v < \bar{v}$ otherwise. The high rate $\bar{v}$ reflects the possibility that firms use recessions to replace old jobs with new technologies, and restructure or close job positions that are at risk of becoming obsolete due to advances in technology, as discussed in the Introduction. The following proposition characterizes the effects of both shocks. To emphasize the business-cycle effects of the recession, I describe my results in terms of the deviations from the trend that would result if there were no recession.

**Proposition 5 (Interaction with a Recession)** Consider an economy that is adjusting to structural change in which $x(T_i) < 1$ and $I(T_i) > 0$. Then:

1. The decline in productivity reduces both $f_s(t)$ and $f_u(t)$ below their trend for $t \in [T_i, T_f)$. When $x(T_i)$ is small, the average finding rate, $f(t)$, and both finding rates $f_s(t)$ and $f_u(t)$ are more cyclical.

2. For any $T_p > T_f$, there exists a training cost $q(T_p) \in [\bar{q}, \infty)$, such that for $q' = q(T_p)$ we have that the increase in $v(t)$ reduces both $f_s(t)$ and $f_u(t)$ below their trend for $t \in [T_i, T_p]$. For a given $q'$, the reduction in the average finding rate, $f(t)$, and in both $f_s(t)$ and $f_u(t)$ is larger and more long-lasting when $x(T_i)$ is small.

Numeral 1 shows that the temporary fall in productivity reduces both workers’ finding rates. The interaction with a small $x(T_i)$ follows by noting that unskilled worker’s finding rate is more responsive to changes in productivity. Because the reservation wage of unskilled workers is close to their value of leisure, their wage does not adjust much in response to productivity shocks, but their finding rate does. Due to their low finding rate during recessions, unskilled workers become numerous among the unemployed, which reduces $\gamma(t)$ and exacerbates the job creation externality (see also Pries, 2008). This effect increases the cyclicality of both finding rates $f_s$ and $f_u$, as well as the average finding rate.

Although this mechanism explains why the finding rate of both workers is more cyclical, it does not create any significant source of propagation. When productivity recovers so does the finding rate of both workers. By itself, a temporary productivity shock causes no propagation because it does not affect the behavior of $x(t)$ nor $I(t)$. This observation extends
to an environment with heterogeneous agents, the result that productivity shocks create no internal propagation in the canonical search model (Shimer, 2005).

In contrast, Numeral 2 shows that the temporary increase in the rate at which firms close old jobs causes a long-lasting decline in both $f_s$ and $f_u$. This is so because a low $I(t)$ reduces employment opportunities for unskilled workers and pushes them to redeploy to novel jobs. Following a recession, the inflow of unskilled workers that are searching for novel jobs exacerbates the skill mismatch—an effect that becomes more severe when $x(T_t)$ is small. Firms respond by creating less novel jobs, which reduces both workers finding rates in a persistent manner.

The finding rates $f_s$ and $f_u$ will be depressed until workers retrain and the skill mismatch abates. Numeral 2 of the proposition emphasizes this point and shows that when training costs are high, the effect of the decline in $I(t)$ on job-finding rates outlasts the recession. This mechanism creates a jobless recovery in which the finding rate of both workers remains depressed, relative to their trends, even though productivity has already recovered.$^{15}$

The effect of a decline in $I(t)$ on labor markets hinges on the assumption that it affects the finding rate of unskilled workers more than it affects the finding rate of skilled workers. In my model this feature follows from the fact that only unskilled workers search for old jobs. Though clearly a stark simplification, the general idea that unskilled workers will search more frequently for old jobs than skilled ones seems plausible. After all, unskilled workers lack the skills that are required in other jobs (see also footnote 14). Moreover, both results in Proposition 5 hinge on the assumption that the worsening composition of the unemployment pool affects firm hiring efforts. This would still apply if, while on the job, workers also searched for jobs so long as they do so less frequently than unemployed workers. All the same, my analysis applies to firm’s hiring efforts directed at workers who are currently unemployed, and implies a reduction in the rate at which unemployed workers find jobs.

I derived the results in Proposition 5 under the assumption that $I(t)$ declines exogenously during recessions. Although my empirical findings and Figure 2-3 support this assumption, it is worth discussing it more thoroughly. The purpose of the assumption is to show what could happen if firms restructured their demand for different types of labor during a recession, without explaining why that could be the case. My results here indicate that it is important to understand when and why do recessions prompt such behavior by firms. In the Theory Appendix I show that one possibility is that, due to the competition from technology, the production of old tasks using labor becomes unprofitable. Firms do not close this vacancies

$^{15}$As this discussion clarifies, the closure of old job positions during recessions is different from an increase in the separation rate (as emphasized by Jaimovic and Siu, 2014). An increase in separations contributes to unemployment but it does not affect the state variables $x(t)$ and $I(t)$, and therefore cannot generate propagation.
because they made irreversible investments which they have to liquidate or redeploy to the production of other tasks. But liquidating or redeploying investments disrupts current production (see Aghion and Saint Paul, 1998). Thus, firms would endogenously concentrate their liquidation and restructuring efforts during recessions, when the opportunity cost of the foregone production is small. When the recession is over, firms do not create new openings for old jobs because these are unprofitable. This extension generates the same pattern as an exogenous increase in v(t) during recessions.

Proposition 5 has two key implications. First, it suggests that the incidence of skill mismatch rises persistently and lowers job creation both during the recession and the recovery. This feature is consistent with the evidence by Sahin et al. (2014), who show that the incidence of occupational mismatch rose at the onset of the Great Recession. However, using indices of occupational mismatch that are based on the Jackman-Roper condition (see Jackman and Roper, 1987), the literature finds a fast recovery of occupational mismatch after the recession. My model suggests that these indices decline faster than the underlying skill mismatch because, although unskilled workers redeploy to novel jobs—as required by the Jackman-Roper condition—they lack the requisite skills in these jobs. In my model, the required redeployment of unskilled workers exacerbates the skill mismatch and continues to dampen job creation during the recovery.

Second, the proposition shows that a recession that takes place during periods of structural change produces a different business cycle, which exhibits a larger and more long-lasting increase in unemployment.

I complete my theoretical exploration of the model by characterizing the inefficiencies in the decentralized allocation. This characterization holds for the general case in which a and A take any positive values.

**PROPOSITION 6 (WELFARE ANALYSIS)** Suppose that $\beta = \eta$ and the Hosios condition holds. The constrained efficient allocation has the same structure as the decentralized equilibrium, but the planner values the opportunity cost of workers at $\mu_s$ and $\mu_u$ given by

$$
\mu_s - w_s = (1 - \eta)(1 - \gamma^n)f(\theta^n)(\max\{S^n_s, 0\} - \max\{S^n_u, 0\}) > 0,
$$

$$
\mu_u - w_u = - (1 - \eta)\frac{\Delta}{1 + \pi\gamma^n f(\theta^n)(\max\{S^n_s, 0\} - \max\{S^n_u, 0\})} < 0. \quad (1.10)
$$

Thus, the adjustment of the economy is inefficient. However, the decentralized allocation is constrained efficient in steady state or in the limit case in which $\pi = 0$.

The intuition behind the inefficiency is that, because workers earn quasi rents when they are employed, a reduction in their finding rate has a first-order effect on their utility. Thus, the job creation externality renders the adjustment inefficient.
The Hosios condition internalizes some but not all of the failures in the market. For instance, when \( \pi = 0 \) and there is no job creation externality, the economy is constrained efficient. In this case, workers who acquire skills are held up by future employers, but this is offset by the congestion these workers create on other skilled workers, as shown by Acemoglu and Shimer (1999). When workers are heterogeneous this reasoning breaks down. Under these circumstances, when a worker becomes skilled he improves matching opportunities for firms that post novel jobs. These firms will be able to extract part of the higher surplus and avoid unskilled matches. This additional external benefit, which translates into more job creation, is not internalized by the Hosios condition (see Shimer and Smith, 2001).

The proposition shows that the planner allocation could be decentralized by taxing search efforts by unskilled workers and subsidizing search efforts by skilled ones (see Shimer and Smith, 2001). The proposition also implies that the returns to training are compressed relative to their social value. When workers become skilled, they reduce the incidence of the job creation externality, but firms and workers do not internalize this social benefit. Subsidizing training increases welfare, as the following corollary shows.

**Corollary 1** The social value of skill upgrading exceeds its private counterpart:

\[
\int_{t}^{\infty} e^{-\tau(t-t)}(\mu_s - \mu_u) d\tau > \Omega(t).
\]

A temporary subsidy to stepping-stone jobs reduces unemployment and increases welfare.

### 1.2 Quantitative exploration

This section explores quantitatively the mechanisms in my model. My numerical exercises also show that the insights derived analytically in the previous section continue to apply when I calibrate gross flows (\( \alpha \) and \( \lambda \)) to match the U.S. data.

Table 1.1 describes a quarterly parametrization of my model. The top panel summarizes standard parameters from the matching literature. The bottom panel presents the parameters that quantify the structural change. For these parameters, I define two scenarios: one

---

16 This resembles what unemployment insurance and other welfare programs achieve when, as critics argue, they reduce search effort by unskilled takers. Subsidizing old jobs affected by structural change to keep them from becoming obsolete would produce a similar result because it would keep unskilled workers from searching for novel jobs.

17 I set the elasticity of the matching function, \( \eta_c \) to 0.5 following Pissarides (2009) and the evidence in Mortensen and Petrongolo (2000). I also impose the Hosios condition \( \beta = \eta_c \). In this case with random matching the usual argument that justifies this assumption does not apply (see Shimer 2005). Instead, I assume the Hosios condition to isolate the role of the job creation externality from the other well-known inefficiencies present in matching models. I target quarterly data and set \( z = 1, \alpha = 1.3, \kappa = 0.235, b = 0.7, \lambda = 0.1 \); which guarantee in steady state \( \theta^* = 1 \)—a normalization—, a quarterly finding rate of 1.3 and a unemployment rate of 7% in steady state. Finally, I set the quarterly discount rate to \( r = 0.012 \).
calibration with $\Delta = 1$ and another with $\Delta = 0.8$. The small values for $1 - \Delta$ reflect the fact that workers displaced by structural change may have few employment alternatives that do not require retraining. In the case of workers displaced from routine-cognitive jobs, service jobs correspond to the main alternative that does not require retraining (see Autor and Dorn, 2013). Despite their growth since 1980, service jobs only employ 13% of workers, and these jobs involve lower wages, which makes them an inviable alternative for many displaced workers. Moreover, during the last 30 years, new professional jobs that are intensive in analytical tasks—reminiscent of novel jobs in my model—account for the bulk of employment growth (see Acemoglu and Restrepo, 2015), which supports my choice of a large value for $\Delta$.

Table 1.1: Quarterly parametrization of the model.

<table>
<thead>
<tr>
<th>Search model Parameters:</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Source or target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-state productivity, $z(t)$</td>
<td>1</td>
<td>1</td>
<td>Normalization.</td>
</tr>
<tr>
<td>Discount rate, $r$</td>
<td>0.012</td>
<td>0.012</td>
<td>From Pissarides (2009).</td>
</tr>
<tr>
<td>Matching function elasticity, $\eta$</td>
<td>0.5</td>
<td>0.5</td>
<td>Mortensen and Petrongolo (2000).</td>
</tr>
<tr>
<td>Workers’ value of unemployment, $b$</td>
<td>0.7</td>
<td>0.7</td>
<td>From Pissarides (2009).</td>
</tr>
<tr>
<td>Matching function scale, $\alpha$</td>
<td>1.3</td>
<td>1.3</td>
<td>Quarterly rate from Shimer (2005).</td>
</tr>
<tr>
<td>Flow cost of vacancies, $\kappa$</td>
<td>0.235</td>
<td>0.235</td>
<td>Normalization $\theta^* = 1$.</td>
</tr>
<tr>
<td>Separation rate $\lambda$</td>
<td>0.1</td>
<td>0.1</td>
<td>Quarterly rate from Shimer (2005).</td>
</tr>
<tr>
<td>Workers’ bargaining power, $\beta$</td>
<td>0.5</td>
<td>0.5</td>
<td>Hosios condition.</td>
</tr>
</tbody>
</table>

| Structural change parameters: | | |
| Share of novel jobs, $\Delta$ | 1 | 0.8 | Employment growth in high-skill jobs. |
| Learning rate in stepping-stone jobs, $\alpha$ | 1/6 | 1/6 | Half the average time required to master occupations in $O \ast NET$. |
| Exogenous replacement rate $\delta$ | 1/16 | 1/16 | Replacement by new college cohort. |
| Lower productivity in novel jobs, $q^u$ | 0.05 | 0.05 | Wage losses for displaced workers (Davis and von Wachten, 2011). |
| Training costs, $q^1$ | 0.45 | 0.3 | Earning losses for displaced workers (Davis and von Wachten, 2011). |
| Random matching, $\pi$ | 0.5 | 0.5 | Assumed. |

Notes: The table presents the value of the parameters used in my numerical exercises. The columns labeled Scenario 1 and Scenario 2 present the two alternative scenarios I explore.

My choice for $\alpha$ is supported by data from $O \ast NET$, which shows that it takes on average 3 years of training and experience to master a particular occupation or job.\(^{18}\) I set $1/\alpha$ to half this value (6 quarters) to account for the possibility that workers productivity increases throughout this period. I also set $1/\delta = 16$, so that on average workers exogenously upgrade their skills every four years. This choice is motivated by the entry of new college cohorts, and the specific value I use plays no role in my results so long as it is small and positive.

In the first scenario, I calibrate values of $q^u = 0.05$ and $q^1 = 0.45$ to match estimates for the wage and earning losses for an unskilled worker displaced from an old job (see Davis and von Wachten, 2011).\(^{18}\) Among 729 occupational groups in the $O \ast NET$ data, workers require on average 2.98 years (standard deviation=2.45) of vocational training, plant training or job-related experience to master each occupation.

\(^{18}\) Among 729 occupational groups in the $O \ast NET$ data, workers require on average 2.98 years (standard deviation=2.45) of vocational training, plant training or job-related experience to master each occupation.
von Wachten, 2011). The literature estimates that 15 years after losing a job, a displaced worker's earnings are 10% lower than his previous income—which informs my choice for $q^n$—, and the present discounted value of the losses amounts to a full year of his income—which informs my choice of $q^l$. For these parameters, Figure 1-5 presents the paths for earnings, wages and employment (relative to their pre-displacement level) for an unskilled worker displaced from an old job at time 0. Expected earnings are 10% lower 15 years after and the present discounted value of the earning losses amounts to 1.06 times the worker’s yearly earnings.

![Figure 1-5: Earnings, wages and employment for a displaced unskilled worker. Results for Scenario 1.](image)

For the second scenario with $\Delta = 0.8$, I scale $q^l$ down to 0.3, which keeps the surplus of stepping-stone jobs at a level that is similar to that of the first scenario—roughly 0.32 in steady state. The purpose of this scenario is to investigate how the availability of regular jobs that require no skills affects the adjustment of the economy.

Finally, I assume $\pi = 0.5$, so that there is an intermediate but large degree of random matching. Larger values of $\pi$ exacerbate the externalities in my model.

### 1.2.1 Numerical results

I start by computing the equilibrium adjustment to structural change when $I(0) = 0$, so that no old jobs are available. Figure 1-6 depicts the equilibrium, which presents the results for the first scenario with $\Delta = 1$ in the top panel and for the scenario with $\Delta = 0.8$ in the

---

19 This coincides with the cost of unemployment spells estimated by Davis and von Wachten (2011) during periods with low aggregate unemployment. To match this setting in my model, I estimate the earning losses for a single unskilled worker assuming that the tightness of all labor markets is at its steady state level. The large value of $q^l$ implies unskilled workers upgrade their skills at a low rate, matching the persistent losses in earnings and wages in the data. The small positive value of $q^n$ implies unskilled workers may be able to obtain novel jobs without upgrading their skills for several years, but at a slightly lower wage than what they earned before. A larger value of $q^n$ implies a counterfactual sharp drop in earnings followed by a rapid recovery.

---

20 The values of $\pi$ and $\beta$ used satisfy the restrictions derived for the particular case in which $\alpha, \lambda \to \infty$. Moreover, the condition $q^l > \bar{q}$ is satisfied. In the first numerical scenario, I have $\bar{q} = 0.34$, which is smaller than $q^l = 0.45$. In the second numerical scenario, I have $\bar{q} = 0.19$, which is smaller than $q^l = 0.3$. 

37
bottom panel. In both cases, the blue lines present the equilibrium paths for an economy with $x(0) = \gamma(0) = 1/3$, so that a third of the workers are skilled, and the black lines present the paths for an economy with $x(0) = \gamma(0) = 2/3$. In addition, I set $u(0) = u^*$ and $s(0) = 0$.

**SCENARIO 1, $\Delta = 1$**

- Unemployment
- Skilled workers
- Tightness
- Vacancies
- Finding rate skilled
- Finding rate unskilled
- Stepping-stone jobs
- Skill upgrading returns

Paths for $x(0) = \gamma(0) = 1/3$.

---

Efficient allocation for $x(0) = \gamma(0) = 1/3$.

Paths for $x(0) = \gamma(0) = 2/3$.

**SCENARIO 2, $\Delta = 0.8$**

- Unemployment
- Skilled workers
- Tightness
- Vacancies
- Finding rate skilled
- Finding rate unskilled
- Stepping-stone jobs
- Skill upgrading returns

Paths for $x(0) = \gamma(0) = 1/3$.

---

Efficient allocation for $x(0) = \gamma(0) = 1/3$.

Paths for $x(0) = \gamma(0) = 2/3$.

Figure 1-6: Equilibrium adjustment paths for different variables in my model in both scenarios.

When old jobs close immediately, structural change creates a large and long-lasting increase in unemployment. In the first scenario with $\gamma(0) = 1/3$, structural change raises the unemployment rate by 3.2 percentage points in the short run and 1.5 percentage points 10 years thereafter. Unemployment is accompanied by low tightness and few vacancies, which shows that my model overcomes the Abraham and Katz' (1986) critique; during periods of structural change, vacancies and unemployment trace a downward sloping Beveridge curve.

The increase in unemployment is driven by the 30% lower finding rate among unskilled workers—the direct effect—and by the 17% decline in skilled workers' finding rate (relative to its steady state level $f^* = 1.3$) that is caused by the job creation externality. Figure 1-7
decomposes the unemployment rate for the first scenario and for \( x(0) = \gamma(0) = 1/3 \). The solid line depicts the unemployment rate. The dashed line shows the unemployment rate that would prevail if both workers’ finding rates were set at their steady-state levels, thus removing the job creation externality. The dotted line shows the additional reduction in unemployment that would result if the incentives to acquire skills along the transition were given by \( \Omega^* \) instead of \( \Omega(t) \), which removes the complementarity effect. Although this is one of several possible decompositions, it shows that the job creation externality and the complementarity effect may explain up to 40% of the increase in unemployment.

Figure 1-7: Unemployment rate decomposition computed for Scenario 1 in the case with \( x(0) = \gamma(0) = 1/3 \).

The market failure is quantitatively relevant. This can be seen from a comparison of the market equilibrium with the paths for the constrained efficient allocation for \( x(0) = \gamma(0) = 1/3 \) in the dotted blue lines in Figure 1-6. The constrained efficient allocation involves about 30% less unemployment along the transition and a faster adjustment that is driven by the creation of 50% more stepping-stone jobs in the first years of the adjustment. The figures also show that the private value of acquiring skills is about 10 to 15% smaller than its social value.

This is surprising given that in the calibration used search frictions create only a small wedge between wages and the marginal product of labor. In particular, workers earn a share \( \frac{r + \lambda + \eta (1 - \gamma)}{r + \lambda + \eta} \in [90\%, 93\%] \) of the gross value of a match, which implies that they are effectively bargaining with firms over small rents. The inefficiencies are large despite the small matching frictions for two reasons. First, rents determine job creation decisions. Even if these rents are small, changes in the frequency at which firms that post novel jobs match
with skilled or unskilled workers cause large changes in the creation of novel jobs. Thus, the job creation externality is large (as my decomposition in Figure 1-7 confirms), and this introduces a wedge between the private and social value of retraining of about 10-15%. Second, when the surplus of stepping-stone jobs is small—as in my calibrations—, a small change in the value of retraining can have a large effect on the number of stepping-stone jobs that are created. In this case, the wage paid to unskilled workers in stepping-stone jobs is close to their outside option and becomes endogenously rigid. A decline in the gross value of stepping-stone jobs—driven by workers’ willingness to acquire skills—results in large changes in quantities instead of wages.

A complementary intuition is that, due to the large training costs, the quasi-supply of unskilled labor in stepping-stone jobs is very elastic, as is shown in Figure 1-8. In addition, because of the complementarity effect, the demand curve for stepping-stone jobs (i.e., their flow value) is upward sloping in equilibrium. Both forces imply that a small change in the gross value of these jobs creates a large increase in quantities. If the surplus of stepping-stone jobs were large, there would still be an externality. But because the planner would face a much inelastic quasi-supply of unskilled labor, it would not create many additional stepping-stone jobs in response. The inefficiencies would be reflected in prices and not in quantities, and the welfare cost (shaded in gray in the figure) would be smaller than in my calibration.

![Figure 1-8: Representation of the equilibrium employment in stepping-stone jobs.](image)

Finally, I ask whether my model generates a large interaction between an underlying structural change and recessions. For both scenarios in Table 1.1, I consider an economy that is adjusting to structural change and I compute its response to an unanticipated recession that takes place 5 years into the adjustment (so that \( T_i = 20 \)). I assume initially \( \gamma(0) = 1/5 \) and \( I(0) = 1 \) so that the recession hits the economy when \( \gamma(T_i) \approx 1/3 \) and \( I(T_i) \) is still large. The recession lasts for 10 quarters and reduces labor productivity by 5%, which matches the
available estimates for factor productivity during the Great Recession. I set \( v = 0.01 \) so that old jobs become obsolete at a small secular rate, and I calibrate \( \bar{v} = 0.09 \) to match the permanent decline in old job openings of roughly 55% depicted in Figure 2-3.

Figure 1-9 presents the deviations in unemployment from its level at \( T \), for both scenarios, as well as the equilibrium path for \( \gamma^n \). For simplicity, I normalize the starting time of the recession to zero in the figures so that productivity fully recovers by 2.5 years. The gray dotted line presents the (almost negligible) trend in unemployment in the absence of a recession. The slow decline in \( I(t) \) guarantees the absence of a trend. The red line presents the behavior of unemployment in the recession. As stated in Proposition 5, when the economy is adjusting to structural change, the recession creates a large and long-lasting increase in unemployment. In the first scenario, unemployment increases by 2.75 percentage points above its trend during the crisis and it remains 1 percentage point above its initial level (and trend) 5 years after the recession ends. In both scenarios, the share of skilled workers among those who are searching for novel jobs, \( \gamma^n \), falls in a persistent manner during the recession, which shows how the crisis exacerbates the skill mismatch.

As emphasized in my theoretical analysis, the permanent decline in old jobs has a long-lasting effect because it exacerbates the skill mismatch. The skill mismatch lasts because firms and workers do not take full advantage of the opportunity to retrain workers. For comparison, Figure 1-9 presents the constrained efficient allocation in green. In this allocation, firms and workers engage in the efficient amount of retraining, the increase in the skill mismatch abates shortly after the recession ends, and there is little propagation of unemployment. However, in the efficient allocation, unemployment may be larger during the onset of the crisis. This occurs because the planner keeps skilled workers searching for jobs to compensate for the more volatile finding rate of unskilled workers. By doing so, the planner maintains a more favorable composition of the unemployment pool, which reduces the job creation externality.

The black line in Figure 1-9 presents the response of unemployment in an economy that is not experiencing structural change. In line with Shimer's (2005) findings, unemployment only increases slightly (by less than a percentage point) and the finding rate recovers fully by the end of the recession; there is no propagation. The blue line presents the response of unemployment to only the decline in productivity in an economy that is adjusting to structural change. In this case, unemployment is amplified during the crisis, and it becomes about two times more volatile than in an economy that is not affected by structural change. But as anticipated, in this case, too, there is no significant propagation beyond \( T_f \).

Finally, my model matches two salient facts of recessions. First, as Figure 1-10 shows,  

\[ \text{Footnote: The unemployment rate goes below its initial trend because the reduction in old jobs causes workers to upgrade their skills at a faster rate than what they would otherwise do.} \]
**Scenario 1, \( \Delta = 1 \)**

- **Business-cycle unemployment**
- **Business-cycle mismatch \( \gamma^o \)**

**Scenario 2, \( \Delta = 0.8 \)**

- **Business-cycle unemployment**
- **Business-cycle mismatch \( \gamma^o \)**

---

**Figure 1-9:** Adjustment paths for unemployment relative to its initial value at time 0. The gray line plots the counterfactual trends in an economy adjusting to a structural change that started at \( t = -5 \). The recession affects the economy from \( t = 0 \) to \( t = 2.5 \) (in years).

**Figure 1-10:** Beveridge curves. Both figures center around the initial unemployment and the number of vacancies prior to the recession. Each point corresponds to a different year since the onset of the recession.

When the recession interacts with a structural change the economy recovers through a more pronounced and sluggish counter-clockwise trajectory in the vacancy-unemployment space, as was observed in recent recessions (see Barlevy, 2011 and Veracierto, 2011). The black
line shows that the adjustment is less pronounced and faster for an economy not undergoing any structural change. Second, in keeping with the evidence, my model predicts that unemployment spells are more costly during a recession (see Davis and von Wachten, 2011). Due to the lack of old jobs and the skill mismatch, displacement costs unskilled workers an additional 18% loss in earnings when it occurs during a recession.

1.3 Concluding remarks

This paper argues that economies fail to adjust properly when they are affected by a severe skill mismatch. Plausible matching frictions that limit the ability of firms to direct their search efforts can have large aggregate effects when the mismatch is severe. In a companion paper using U.S. data and through a study of the decline of routine-cognitive jobs, I find support for the aggregate implications of these externalities, which operate at the commuting-zone level.

The inefficiencies in my model open room for a wide range of temporary policies during periods of skill mismatch, especially during recessions, when the effects of structural change are likely to concentrate. Though unemployment is structural, policies aimed at increasing demand or government expenditure during the initial stages of a recession can increase welfare by raising the returns to skill upgrading during the recovery and by avoiding the fast closure of old jobs during the crisis. Temporary subsidies to retraining workers can ease the adjustment of the economy, increase welfare and reduce the excess unemployment.

Behind the job creation externality is a key assumption: firms cannot perfectly direct their search efforts. If this was not the case, the search behavior of unskilled workers would not affect matching opportunities for skilled workers, which seems like a restrictive requirement. I believe that the random matching assumption is plausible, especially in the context of new technologies for which there is no standardized set of skills or credentials that are useful signals of proficiency. But whether this assumption holds remains an empirical question. Micro-evidence is needed.

For instance, I plan to follow a micro approach that is designed to understand how firms change their hiring patterns when they face a skill mismatch, whether their response varies during recessions, and whether it is consistent with the predictions of models of random matching. An extension of my model that allows firms to direct their search efforts based on multiple signals suggests that when a skill mismatch occurs, firms focus most of their recruiting efforts on candidates who project the best (but scarce) signals. At the same time they devote few resources to hiring and training new candidates who have little experience or qualifications. This shift in hiring practices can affect the rate at which workers find jobs and firms fill their vacancies. In future research I will study these issues in more detail through
analyses of proprietary data on job openings, which include detailed job characteristics, requirements and a description of the tasks performed by employees.

In recent years some of these phenomena have affected routine-cognitive jobs. These jobs, which formerly demanded few requirements, are professionalizing, and firms now routinely ask for additional requirements and credentials. The increase in requirements partly reflects changes in supply (see Modestino, Shoag and Ballance, 2015) and a shift away from routine-cognitive tasks. That said, a report by Burning Glass Technologies indicates that although in recent years 65% of the current job openings for executive secretaries call for a bachelor's degree, only 19% of those currently employed in these jobs satisfy this requirement. The report also argues that, in some cases, the formal requirements do not correspond to observed changes in tasks performed by workers. For some firms the requirements might serve as restrictive recruiting filters that exclude suitable matches and reduce the rate at which firms fill vacancies. My model views these hiring practices unfavorably, seeing them as inefficient bottlenecks for the reallocation of displaced workers and the creation and discovery of new talent.

At the macro level, my model suggests that there is value to a perspective that regards recessions as times of adjustment and reorganization. Once we adopt this view, the usual distinction between structural and business-cycle phenomena blurs and the two become intricately related. The interaction between structural factors and business cycles can be a useful addition to models of search unemployment. My calibrations suggest that the interactions can be quantitatively significant and go a long way towards explaining the propagation of otherwise short-lived shocks. The possibility of an interaction raises questions about policy and crisis management during periods of structural change as well as the timing of adjustments. I intend to address these questions in future work.

References


Schumpeter, Joseph A. (1942) *Capitalism, Socialism, and Democracy*.


### 1.4 Theory Appendix

The Theory Appendix has the following structure. First, I describe the details of the behavior for the state variables and the Bellman equations. Second, I discuss some special conditions that guarantee the existence and uniqueness of the equilibrium for the general
case in which \( a, \lambda \) take any positive values. In this subsection I also characterize the asymptotic behavior of the economy and provide general lemmas that I will use throughout the appendix. Third, I provide the details of the limit in which \( a, \lambda \to \infty \), and provide conditions for the uniqueness of an equilibrium. In this subsection I also provide the proof of Proposition 2, and comparative statics results for the effects of \( z, x, I, \Omega \) on the finding rates and reservation wages. Fourth, I provide the details of the proof for Proposition 3, 4 and 5. Finally, I derive the constraint efficient allocation in the general case in which \( a, \lambda \) take any positive values.

1.4.1 Derivation of state variables and Bellman equations:

**Derivation of the state variables behavior.** The state variables of the model include the share of skilled workers \( x \), the number of skilled unemployment workers \( u_s \), the number of skilled unemployment workers \( u_u \), and the number of workers of each type employed in different jobs \( e_{ij} \), where \( k \) indexes the task performed and \( j \) the type of worker.

The behavior of unemployment for both groups is given by:

\[
\dot{u}_s = \lambda x - (\Delta f(\theta_s^u) + (1 - \Delta) f(\theta_s^l))u_s + \delta u_s
\]

\[
\dot{u}_u = \lambda(1 - x) - \left( \frac{\Delta}{1 + I} f(\theta_s^u) + \frac{\Delta}{1 + I} (1 - \pi) f(\theta_s^l) + \frac{(1 - \Delta)}{1 + I} f(\theta_u^l) + \frac{I}{1 + I} f(\theta_u^s) \right) u_u - \delta u_u
\]

Using these expressions, I can calculate the behavior of the unemployment rate, \( u \), and the share of skilled workers among the unemployed as:

\[
\dot{u} = \lambda(1 - u) - u\gamma f_u - u(1 - \gamma) f_u,
\]

\[
\dot{\gamma} = (1 - \gamma) \gamma(f_u - f_u) + \lambda \frac{x - \gamma}{u} + (1 - \gamma) \delta,
\]

with \( f_u \) and \( f_u \) defined in equation (1.3) in the main text. To save on notation, these expressions assume all matches produce a positive surplus and are always formed. In the general case, when the surplus of a job is negative or zero, firms and workers reject these matches.

The behavior employment counts in each job is given by

\[
\dot{e}_s^n = \Delta f(\theta_s^n) u\gamma - \lambda e_s^n
\]

\[
\dot{e}_u^l = \frac{\Delta}{1 + I} (1 - \pi) f(\theta_u^l) u(1 - \gamma) - (\lambda + \alpha) e_u^l
\]

\[
\dot{e}_u^r = \frac{1 - \Delta}{1 + I} f(\theta_u^r) u(1 - \gamma) - \lambda e_u^r
\]

\[
\dot{e}_s^n = \frac{\Delta}{1 + I} f(\theta_u^s) u(1 - \gamma) - \lambda e_s^n
\]

(1.11)
Derivation of the state variables behavior. I now derive the Bellman equation for the surplus of a match. Denote by $J^k_j$ the firm surplus and $E^k_j$ the worker surplus. These surpluses are given by the Bellman equations:

\[ rJ^k_j - J^k_j = z^k_j - w^k_j - \lambda J^k_j \]
\[ rE^k_j - E^k_j = w^k_j + \lambda(U_j - E^k_j). \]  

(1.12)

Here, $z^k_j$ is the flow value of the match production (equal to $z(t)$ and adjusted by workers' productivity if needed). Also $-\lambda J^k_j$ and $\lambda(U_j - E^k_j)$ are the losses incurred by the firm and worker in the event that the match separates (recall that the firm outside option is set to zero by free entry).

Nash bargaining implies that $\beta J^k_j = (1 - \beta)(E^k_j - U_j)$, and $\beta J^k_j = (1 - \beta)(E^k_j - U_j)$. Multiplying the first expression in equation (1.12) by $\beta$, the second by $(1-\beta)$, and subtracting them yields the following formula for the wage rate:

\[ w^k_j = \beta z^k_j + (1 - \beta)(rU_j - U_j). \]

Plugging the wage in the equation for $J^k_j$ yields:

\[ (r + \lambda)J^k_j - J^k_j = (1 - \beta) \left[ z^k_j - (rU_j - U_j) \right]. \]

Nash bargaining implies that $J^k_j = (1 - \beta)S^k_j$. Therefore, we obtain:

\[ (r + \lambda)S^k_j - S^k_j = z^k_j - (rU_j - U_j), \]

which is the expression used in the main text.

In the case of stepping-stone jobs, the derivation is different since I have to take into account the gains from training. In this case:

\[ rJ^l_u - J^l_u = z^l_u - w^l_u - \lambda J^l_u + \alpha(J^n_u - J^l_u) \]
\[ rE^l_u - E^l_u = w^l_u + \lambda(U_j - E^l_u) + \alpha(E^n_u - E^l_u). \]  

(1.13)

Here, $z^l_u$ is the flow value of the match production, $z(t) - q^n - q^l$, adjusted by workers' productivity and training costs. $-\lambda J^l_u$ and $\lambda(U_j - E^l_u)$ are the losses on the firm and worker, respectively, in the event that the match is exogenously separated. $\alpha(J^n_u - J^l_u)$ and $\alpha(E^n_u - E^l_u)$ are the gains on the firm and worker, respectively, in the event that the worker becomes skilled.

Nash bargaining implies that $\beta J^l_u = (1 - \beta)(E^l_u - U_u)$, and $\beta J^l_u = (1 - \beta)(E^l_u - U_u)$. Multiplying the first expression in equation (1.13) by $\beta$, the second by $(1-\beta)$, and subtracting
them yields the wage:

\[ w^T_u = \beta z^T_u + (1 - \beta)(rU_j - \dot{U}_j) - (1 - \beta)\alpha(U_s - U_u) \]

This equation shows that, as mentioned in the text, workers willingness to acquire skills reflects in lower wages at stepping-stone jobs (see Becker, 1964).

Plugging the wage in the equation for \( J^T_u \) yields:

\[ (r + \lambda)J^T_u - J^T_u = (1 - \beta) \left[ z^T_u - (rU_j - \dot{U}_j) + \alpha(U_s - U_t) \right] + \alpha(J^n_s - J^T_u). \]

Nash bargaining implies that \( J^T_u = (1 - \beta)S^T_u \). Therefore, we obtain:

\[ (r + \lambda)S^T_u - \dot{S}^T_u = z^T_u - (rU_j - \dot{U}_j) + \alpha(S^n_s - S^T_u) + \alpha(U_s - U_t). \]

The expression presented in the main text incorporates the slight variation that the worker and the firm have the option value of not incurring in training costs if it is not profitable:

\[ (r + \lambda)S^T_u = z(t) - q^n + \max\{-q^T + \alpha(U_s - U_u) + \alpha(S^n_s - S^T_u), 0\} - (rU_u - \dot{U}_u) + \dot{S}^T_u. \]

Minimal set of state variables required to compute equilibrium. In the main text, I define the equilibrium based on fewer state variables than the set introduced above. The reason for doing that is that the equilibrium admits a recursive structure in which surpluses and \( \gamma \)—the share of skilled workers among the unemployed—determine tightness, tightness determines workers reservation wages and this feeds back into the surplus. Unlike the traditional search model, the fact that \( \gamma \) affects tightness—see equation (1.8) in the main text—implies that to determine the path for surplus and tightness we have to keep track of their joint behavior with \( \gamma \).

To characterize the behavior of \( \gamma \), I need to keep track of \( x, u, I \) and \( s \). As equation (1.2) shows, the behavior of these variables only depend on tightness and their current values, so I can determine the equilibrium by focusing on this subset of the state variables. This is the minimal set of state variables required to characterize labor market tightness.

1.4.2 Properties of the equilibrium and steady state behavior

Steady-state behavior of the economy.

PROOF OF PROPOSITION 1. The equation for \( \dot{x} \) shows that \( x \) converges monotonically to 1. Thus, in any steady state we have \( x(t) \to 1 \). Moreover, the exogenous behavior of \( I(t) \) implies \( I(t) \to 0 \) by assumption.
For $x(t) = 1$ and $I(t) = 0$, the equilibrium conditions for the steady state are given by:

$$
S^n_s^* = \frac{z - w^*_s}{r + \lambda}, \quad S^n_u^* = \frac{z - q^n - w^*_u}{r + \lambda},
$$

$$
S^n_s^* = \frac{z - w^*_s}{r + \lambda}, \quad S^n_u^* = \frac{z - q^n - w^*_u + \max\{-q' + \alpha\Omega^*\}}{r + \lambda},
$$

with $w^*_s, w^*_u$ the reservation wage of skilled and unskilled workers respectively, and $\Omega^* = W^*_s - W^*_u$ the incentives to acquire skills. Moreover, the reservation wages satisfy:

$$
w^*_s = rU^*_s + b + \Delta\beta f(\theta^*_s) \max\{S^n_s^*, 0\} + (1 - \Delta)\beta f(\theta^*_s) \max\{S^r_s^*, 0\},
$$

$$
w^*_u = rU^*_u + b + \Delta\beta [\pi f(\theta^*_u) \max\{S^n_u^*, 0\} + (1 - \pi) f(\theta^*_u)] \max\{S^r_u^*, 0\} + (1 - \Delta)\beta f(\theta^*_u) \max\{S^l_u^*, 0\} + \delta\Omega^*.
$$

These formulas imply that $S^n_s^* = S^r_s^* = S^*_s$. Moreover, since $x(t) = 1$, we have that in steady state $\gamma^n = 1$ and $\theta^n_s^* = \theta^n_s^* = \theta^*_s$. The equilibrium surplus $S^*_s$ and $\theta^*_s$ are therefore equal to what one would obtain in the traditional search and matching model with homogeneous workers and jobs, and given by:

$$
(1 - \beta)(1 - b) = \frac{r + \lambda + \beta\theta^* q(\theta^*)}{q(\theta^*)} \kappa, \quad u = \frac{\lambda}{\lambda + f(\theta^*)}.
$$

Although there are no unskilled workers in steady state, we can compute $\theta^*_u$ and $\theta^*_u$ for completeness. We have that $S^l_u^* = S^l_u(w^*_u), \theta^*_u = \theta^*_u(w^*_u)$ are implicit and decreasing functions of $w^*_u$. The same holds for $S^r_u^* = S^r_u(w^*_u), \theta^*_u = \theta^*_u(w^*_u)$ and $S^n_u^* = S^n_u(w^*_u)$. Plugging these expressions in the equation for $w^*_u$ we obtain

$$
w^*_u = rU^*_u + b + \Delta\beta [\pi f(\theta^*_u) \max\{S^n_u(w^*_u), 0\} + (1 - \pi) f(\theta^*(u^*_u))] \max\{S^l_u(w^*_u), 0\}.
$$

Since the left-hand side is increasing in $w^*_u$ while the right hand side decreases, this equation defines a unique steady-state value for $w^*_u$. This reservation wage determines the steady-state tightness and surplus for jobs employing unskilled workers. ■

**Dynamic equilibrium in the general case in which $a, \lambda$ take any value.**

**Proposition 7** There exists a threshold $\beta \in [0, 1]$ such that, for $\beta \leq \beta$, the equilibrium exists and is unique.

**Proof.** Consider the limit case in which $\beta \to 0$. In this case, we have that $w_s, w_u \to b$.

Since $w_s, w_u$ are pinned down by the value of leisure, the surpluses of different jobs and
the tightness become jump variables, which do not depend on the path of future wages and only depend on the current value of $z(t)$.

In particular, the surpluses are given by

$$
S^n_s(t) = \frac{z(t) - b}{r + \lambda}, \\
S^n_u(t) = \frac{z(t) - q^n - b}{r + \lambda}, \\
S^G_u(t) = \frac{z(t) - b}{r + \lambda}, \\
S^G(t) = \frac{z(t) - b}{r + \lambda},
$$

And the tightness for different jobs is given by the free entry-conditions, which only depend on $z(t)$ and $\gamma(t)$.

That this is the unique solution for surpluses and tightness follows from the same argument presented in Pissarides (1985). Other values imply an explosive behavior for tightness and surpluses. Thus, there is a unique equilibrium path for finding rates, tightness, value functions and reservation wages. Since all equations and maps determining surpluses and tightness are continuous in $\beta$, these results extent to $\beta \in [0, \bar{\beta}]$.

The tightness at different jobs and the exogenous decline in $I(t)$ imply a unique and deterministic path for the finding rates, $f_s(t)$ and $f_u(t)$. Thus, the steady state behavior is pinned down by the solution to the boundary problem:

$$
\dot{x} = \delta u(1 - \gamma(t)), \\
\dot{u} = \lambda (1 - u(t)) - u(t) \gamma(t) f_s(t) - u(t)(1 - \gamma(t)) f_u(t), \\
\dot{\gamma} = (1 - \gamma(t)) \gamma(t) (f_u(t) - f_s(t)) + \lambda \frac{x(t) - \gamma(t)}{u(t)} + (1 - \gamma(t)) \delta,
$$

coupled with an initial condition for $x(0), u(0), \gamma(0)$.

**Remark**: Though the proof of the first numeral relies on the limit case $\beta \to 0$, and does not provide any intuition of what values of $\beta$ lead to a unique equilibrium, the following subsection provides a tighter characterization of this threshold in the empirical relevant case in which gross flows are large.

**Remark 2**: The key simplification in the proposition is that $w_s, w_u$ do not change over time and converge rapidly to their steady state values. In the general case, the change over time of both reservation wages introduces additional complications and in some cases multiplicities.

**Additional properties of the transition in the general case.**

LEMMA 1 *In any equilibrium, we have $S^n_s > S^n_u$ at all points in time.*

**Proof.** I first prove this is the case in steady state. Suppose by way of contradiction that
S_u^* \leq S_u^*$. Since $S_u^* \geq S_u^*$ and $S_u^* \geq S_u^*$, we have that $S_u^* \geq S_u^*$, $S_u^*$, $S_u^* \geq S_u^* = S_u^*$.

Since unskilled workers would produce a higher surplus at all jobs, we have $w_u^* \geq w_u^*$. But then $S_u^* = \frac{z-w_u^*}{r+\lambda} > \frac{z-w_u^*}{r+\lambda} = S_u^*$, which yields a contradiction.

Now, I prove the same holds along the transition. In particular, I prove that if $S_s^*(T) \leq S_u^*(T)$ for some $T$, we have $S_s^*(t) < S_u^*(t)$ for all $t > T$. However, this contradicts the fact that in steady state we have $S_u^* > S_u^*$.

Suppose $S_s^*(T) \leq S_u^*(T)$. We have that $S_u^*(T) \geq S_u^*(T)$ since stepping-stone jobs have the additional option value of actually training workers. We also have that $S_s^*(T) = S_u^*(T) > S_u^*(T)$, since the opportunity cost of workers is the same in all these jobs, but unskilled workers are more productive at regular and old jobs. Therefore, if $S_s^*(T) \leq S_u^*(T)$, we have $S_u^*(T), S_u^*(T), S_u^*(T) > S_u^*(T) = S_s^*(T)$.

The equation for $w_u(T)$ and $w_u(T)$ imply that $w_u(T) \geq w_u(T)$. Plugging this inequality in the Bellman equation for $S_s^*(t)$ and $S_u^*(t)$ at time $T$ shows that the inequality $S_s^*(T) \geq S_s^*(T)$ can only hold if $S_u^*(T) > S_u^*(T)$. However, this implies that $S_s^*(t)$ appreciates more than $S_s^*(t)$ over time, and since $S_u^*(T) \geq S_u^*(T)$, we have $S_u^*(t) < S_u^*(t)$ for all $t > T$, which yields the desired contradiction.

1.4.3 Details of the limit when $a, \lambda \to \infty$.

Formal description of the limit. I start by characterizing the equilibrium conditions in this limit case.

Define $\tilde{f}_s = f_s/\xi$, $\tilde{f}_u = f_u/\xi$ and $\tilde{f}(\theta) = f(\theta)/\xi$, $\tilde{q}(\theta) = q(\theta)/\xi$.

Taking the limit $\xi \to \infty$, we obtain that the behavior of the state variables converges to:

\[
\begin{align*}
\dot{s} &= \alpha s + \delta u(1 - \gamma), \\
0 &= \tilde{\lambda}(1 - u) - u\gamma \tilde{f}_s - u(1 - \gamma) \tilde{f}_u, \\
0 &= u(1 - \gamma) \frac{A}{1 + I(1 - \pi)} \tilde{f}(\theta_i) - \tilde{\lambda}s, \\
0 &= (1 - \gamma) \gamma (\tilde{f}_u - \tilde{f}_s) + \tilde{\lambda} \frac{\tau - \gamma}{u}, \\
\dot{t} &= -I v(t).
\end{align*}
\]

This follows by noting that the right-hand side of the equations for $\dot{u}, \dot{\gamma}$ and $\dot{s}$ explode otherwise. A complementary intuition for this limit is that these stock variables converge to the stocks determined by current finding and separation rates at a rate of the order of $(a + \lambda)$ over time.

Rearranging the equations in (1.14) yields the system in equation (1.9) that determines the behavior of $\gamma, s, u$ in terms of the finding rates and the share of skilled workers in the economy.

To compute the equilibrium tightness, define the normalized surplus as $\lim_{\xi \to \infty} \xi S^k_j = \tilde{S}$. 54
The normalized surpluses are given by

\[
\tilde{S}_j^a(t) = \lim_{\xi \to \infty} \xi \int_t^\infty e^{-(r+\lambda\xi)(t-\tau)} h_j^k(\tau) d\tau
\]

\[
= \lim_{\xi \to \infty} \frac{\xi}{r + \lambda\xi} h_j^k(t) + \frac{\xi}{r + \xi} \int_t^\infty e^{-(r+\lambda\xi)(t-\tau)} \frac{dh_j^k}{dt}(\tau) d\tau
\]

\[
= \frac{h_j^k(t)}{\lambda}.
\]

with \(h_j^k(t)\) given by the right-hand side of the flow value of a match. The second line uses integration by parts. This line does not require \(h_j^k(t)\) to be differentiable, but simply to have a representation of the form \(h_j^k(t) = \int_0^t \frac{dh_j^k}{dt}(\tau) d\tau\) for some integrable function \(\frac{dh_j^k}{dt}\).

Thus, normalized surpluses are well defined (without the normalization, the surplus converges to zero) and given by the solution to the static system:

\[
\tilde{S}_s = \frac{z(t) - w_s}{\lambda}, \quad \tilde{S}_r = \frac{z(t) - w_s}{\lambda},
\]

\[
\tilde{S}_u = \frac{z(t) - q^n - w_u}{\lambda}, \quad \tilde{S}_r = \frac{z(t) - w_u}{\lambda},
\]

\[
\tilde{S}_o = \frac{z(t) - w_u}{\lambda}, \quad \tilde{S}_o = \frac{z(t) - q^n - w_u + \max\{-q' + \alpha\Omega\}}{\lambda},
\]

with \(w_s, w_u\) the reservation wage of skilled and unskilled workers respectively, and \(\Omega = U_s - U_u\) the incentives to acquire skills.

The reservation wages are well defined in the limit, and are given by

\[
w_s = b + \Delta \beta \tilde{f}(\theta_s^o) \max\{\tilde{S}_s, 0\} + (1 - \Delta) \beta \tilde{f}(\theta_s^e) \max\{\tilde{S}_e, 0\},
\]

\[
w_u = b + \Delta \beta \tilde{f}(\theta_u^o) \max\{\tilde{S}_u, 0\} + (1 - \pi) \tilde{f}(\theta_u^i) \max\{\tilde{S}_i, 0\}
\]

\[+ \frac{1 - \Delta}{1 + I} \beta \tilde{f}(\theta_u^o) \max\{\tilde{S}_o, 0\} + \frac{I}{1 + I} \beta \tilde{f}(\theta_u^i) \max\{\tilde{S}_i, 0\} + \delta\Omega.\]

Finally, the normalized surpluses and \(\gamma^n\) determine the equilibrium tightness as:

\[
k \geq \tilde{\gamma}(\theta_j^k)(1 - \beta) E_s[\max\{\tilde{S}, 0\}] [k, j].
\]

These equations mimic those obtained for the steady state, with the difference that \(x\) (or more precisely \(\gamma^n\)) and \(I\) are moving in the background and shifting all the equilibrium variables. In particular, \(z, x, I, \Omega\) implicitly determine all equilibrium objects in the model as the solution to the system of equations given by (1.14, 1.15, 1.16, 1.17). I refer to the corresponding values of surpluses, tightness and reservation wages as the instantaneous equilibrium.
Properties of the instantaneous equilibrium.

I start by proving the existence of an instantaneous equilibrium.

PROPOSITION 8 For a set of given values of $z, x, I, \Omega \geq 0$, and $q^l > \bar{q}$, for a positive threshold $\bar{q}$, there exists at least one instantaneous equilibrium.

PROOF. For a fixed set of values $z, x, I, \Omega \geq 0$, I define a mapping $T$ from $\theta^n_s, w_s, w_u$ as follows:

1. $\theta^n_s, w_s, w_u$ determine $f(\theta^n_s)$ and the surplus of all jobs $\max\{\bar{S}_j^k, 0\}$.
2. The surpluses determine the finding rates at jobs other than novel ones: $\tilde{f}_j^k$.
3. The finding rates $\tilde{f}_j^k$ and $f(\theta^n_s)$ determine the total finding rates of workers: $\tilde{f}_s$ and $\tilde{f}_u$.
4. The finding rates determine the share of unskilled workers among those that are searching for novel jobs:

   $\gamma^n = \frac{\Delta \gamma}{\Delta \gamma + \frac{\Delta \pi I}{1 + I}(1 - \gamma)},$

   $(1 - \gamma) = \frac{\lambda + \gamma \tilde{f}_s + (1 - \gamma)\tilde{f}_u}{\lambda + \tilde{f}_u}.$

5. $T_{\theta^n_s}(\theta^n_s, w_s, w_u)$ is defined as the implied job creation in novel jobs, given the expected quality of matches determined by $\gamma^n$ and the surpluses $\max\{\bar{S}_n^0, 0\} > \max\{\bar{S}_u^0, 0\}$.

   That is:

   $\kappa = \bar{q}(T_{\theta^n_s}(\theta^n_s, w_s, w_u)) \left[ \gamma^n \frac{z - w_s}{\lambda} + (1 - \gamma^n) \frac{z - q^n - w_u}{\lambda} \right],$

   (1.19)

6. The mapping for reservation wages is defined as the implied value of searching, which is given by:

   $T_{\omega_s}(\theta^n_s, w_s, w_u) = b + \Delta \beta f(\theta^n_s) \frac{z - w_s}{\lambda} + (1 - \Delta) \beta M(z - w_s),$

   $T_{\omega_u}(\theta^n_s, w_s, w_u) = b + \frac{\Delta}{1 + I} \pi \beta f(\theta^n_s) \max\{z - q^n - w_u, 0\} + \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u)$

   $+ \frac{\Delta}{1 + I} (1 - \pi) \beta M(z - q^n + \max\{q^l + \alpha \Omega, 0\} - w_u) + \delta \Omega.$

   (1.20)

Here, for each job performed by worker $j \in \{s, u\}$ at task of type $k \in \{n, r, l, o\}$, I denote by $M(z^k_j - w_j)$ the expected surplus obtained in these jobs, which includes
the probability of finding the job. This expected surplus is given by \( \tilde{f}(\theta^*_{I}) \max\{S^k_I, 0\} \), which depends solely on \( z^k_j - w_j \).

An instantaneous equilibrium corresponds to a fixed point of this map: \( T_w(\theta^*_{I}, w_s, w_u) = w_s \), \( T_w(\theta^*_{I}, w_s, w_u) = w_u \), and \( T_w(\theta^*_{I}, w_s, w_u) = \theta^*_{I} \). I now prove such a fixed point exists. The mapping \( T \) is continuous. Since surpluses are bounded and non-negative, it maps the compact set \([0, \bar{\theta}] \times [b, M(z) + \delta \Omega]^2 \) into itself. Thus, Brouwer’s fixed-point theorem implies that there exists a fixed point of this mapping, as wanted.

I now provide conditions for the unicity of an equilibrium. To simplify my notation, let \( \Phi \) be the vector of current values of \( z, x, I, \Omega \). I also assume \( \delta \approx 0 \) as in the main text. I present an equivalent construction of the equilibrium mapping that allows me to prove the unicity of a fixed point under certain conditions. Taking \( w_s, w_u, \Phi \) as given, I define the tightness in novel jobs \( \theta^*_{I}(w_s, w_u; \Phi) \) implicitly as the interception of equation (1.18)—which determines the average quality of the matching pool in novel jobs—and the equation (1.19)—which determines the creation of novel jobs for a given expected quality and surpluses.

The following lemma provides a condition which I use to prove that \( \theta^*_{I}(w_s, w_u; \Phi) \) is uniquely defined.

**Lemma 2** For all values of \( \Phi, w_s, w_u \), such that \( w_s \geq w_u \) and \( z - w_s - q^n > 0 \), we have that:

\[
\frac{1 - \gamma}{\gamma} \frac{\pi}{1 + I} < \frac{1 - x}{x}.
\]

**Proof.** Because \( w_s \geq w_u \) we have that \( \tilde{f}^u > \tilde{f}^s > \tilde{f}^n \). This implies that \( \tilde{f}^u > \frac{\pi}{1 + I} \tilde{f}^s \), in the worst case in which \( \tilde{f}^u = 0 \). That is, unskilled workers can guarantee to find jobs at least at the rate at which they mimic skilled workers.

Therefore:

\[
(1 - \gamma) = (1 - x) \frac{\tilde{\lambda} + \gamma \tilde{f}_s + (1 - \gamma) \tilde{f}_u}{\tilde{\lambda} + \tilde{f}_u} < (1 - x) \frac{\tilde{\lambda} + \gamma \tilde{f}_s + (1 - \gamma) \frac{\pi}{1 + I} \tilde{f}_s}{\tilde{\lambda} + \frac{\pi}{1 + I} \tilde{f}_s}
\]

\[
< (1 - x) \frac{\gamma \tilde{f}_s + (1 - \gamma) \frac{\pi}{1 + I} \tilde{f}_s}{\frac{\pi}{1 + I} \tilde{f}_s}
\]

\[
= (1 - x) \frac{\gamma + (1 - \gamma) \frac{\pi}{1 + I}}{\frac{\pi}{1 + I}}
\]

The first inequality follows from the fact that \( \tilde{f}^u > \frac{\pi}{1 + I} \tilde{f}^s \). The second one follows by noting that \( \gamma \tilde{f}_s + (1 - \gamma) \frac{\pi}{1 + I} \tilde{f}_s > \frac{\pi}{1 + I} \tilde{f}_u \). The last line follows after canceling the term \( \tilde{f}_u \) in the numerator and denominator.

Rearranging the last expression yields \( \frac{1 - \gamma}{\gamma} \frac{\pi}{1 + I} < \frac{1 - x}{x} \), as wanted.

Using this lemma, I now prove that \( \theta^*_{I}(w_s, w_u; \Phi) \) is uniquely defined.
LEMMA 3 For \( w_s \geq w_u \), the tightness \( \theta_s^n(w_s, w_u; \Phi) \) is unique. Moreover, \( \theta_s^n(w_s, w_u; \Phi) \) and \( f(\theta_s^n(w_s, w_u; \Phi)) \) increase with \( x, I, \Omega \).

PROOF. Equation (1.19) defines an increasing locus between \( \theta_s^n \) and \( \gamma^n \) in the \((\gamma^n, \theta_s^n)\) space, which I refer to as the job-creation locus. This follows from Lemma 1, which implies that \( \frac{z-w_s}{\lambda} > \frac{z-q^n-w_u}{\lambda} \).

Equations (1.18) determine a decreasing locus between \( \gamma^n \) and \( \theta_s^n \)—which reflects the intuitive fact that when tightness in novel jobs increases, skilled workers find jobs faster and there are fewer skilled workers among the unemployed searching for novel jobs. I refer to this curve as the quality locus.

To show that this locus is decreasing, is equivalent to the claim that an increase in \( f_s^n \)—the finding rate of workers in novel jobs—reduces \( \gamma \) for fixed values of \( w_s, w_u, z, x, I, U \). If \( z-w_s-q^n < 0 \), the increase in \( f_s^n \) raises \( f_s \) and does not affect \( f_u \). Therefore, \( \gamma \) and \( \gamma^n \) increase in the last two equations of (1.18) as wanted. If \( z-w_s-q^n > 0 \), lemma 2 applies (recall that \( w_s \geq w_u \)). The increase in \( f_s^n \) raises \( f_s \) by \( \Delta \) and \( f_u \) by \( \frac{\Delta z}{1+I} \). Therefore, in the last two equations of (1.18), we have that \( \gamma \) falls by \( d\gamma = -\left(\frac{1-x}{x} - \frac{1-\gamma}{\gamma} \right) \Delta d_s^n < 0 \) and \( \gamma^n \) falls as well, which proofs these equations describe a downward sloping locus, as depicted in Figure 1-11.

Figure 1-11: Quality locus (equation 1.18) and job-creation locus (equation 1.19).

To finalize, notice that for \( \gamma^n = 0 \) we have \( f_s^n \geq 0 \) in the job-creation locus. The quality locus crosses \( f_s^n = 0 \) at \( \gamma^n \in (0, 1) \). Therefore, at \( f_s^n = 0 \), the job-creation locus is above the quality locus. As \( f_s^n \to \infty \), the quality locus converges to \( \gamma^n = 0 \). Thus, both loci cross at a unique point that determines unique values for \( \theta_s^n(w_s, w_u; \Phi) \) and \( \gamma^n(w_s, w_u; \Phi) \).

Moreover, an increase in \( x, \Omega \) and \( I \) shift the quality locus upwards, which implies that both \( \gamma^n(w_s, w_u; \Phi) \) and \( \theta_s^n(w_s, w_u; \Phi) \) increase in \( x, \Omega \) and \( I \) for fixed values of \( w_s, w_u \).
same comparative statics applies for $\tilde{f}(\theta^n_s(w_s, w_u; \Phi))$.  

I denote the resulting finding rate in novel jobs as $\tilde{f^n_s}(w_s, w_u; \Phi) = \tilde{f}(\theta^n_s(w_s, w_u; \Phi))$, which increases in $x, I, \Omega$. The effect of $x, I, \Omega$ through $\Phi$ capture the indirect effect of these variables through changes of the composition of workers that are searching for jobs—shifts in the quality locus. In my model, these indirect effects correspond to the extent to which different variables exacerbate or attenuate the skill mismatch.

The next proposition provides conditions for the uniqueness of the instantaneous equilibrium as a function of $(z, x, I, \Omega)$.

**Proposition 9** There exists a positive threshold $q'$ such that, if $q' > -q$ and $\beta \Delta \frac{\partial f^n_s(w_s, w_u; \Phi)}{\partial w_s}$ is uniformly bounded above by 1, the instantaneous equilibrium is unique.

**Proof.** I set $q'$ as the threshold for which $q' \geq \alpha \Omega - q^n + w_s - w_u$ if $q' > -q$ (this threshold exists since $w_s - w_u$ is bounded across all possible instantaneous equilibria). Thus, the condition $q' > -q$ guarantees $S'_s \leq S^n_s$, and therefore $w_s \geq w_u$ in any instantaneous equilibria.

Therefore, to find an instantaneous equilibrium, I may restrict to pairs $(w_s, w_u)$ such that $w_s \geq w_u$. For any such pair, the function $\tilde{f^n_s}(w_s, w_u; \Phi)$ is well defined. Thus, the instantaneous equilibrium is fully determined by a pair of reservation wages $w_s \geq w_u$, which solve the fixed point problem:

\[
T_s(w_s, w_u) = b + \Delta \beta f^n_s(w_s, w_u; \Phi) \frac{z - w_s}{\lambda} + (1 - \Delta) \beta M(z - w_s),
\]

\[
T_u(w_s, w_u) = b + \frac{\Delta}{1 + I} \pi \beta f^n_s(w_s, w_u; \Phi) \max\{z - q^n - w_u, 0\} + \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u)
\]

\[
+ \frac{\Delta}{1 + I} (1 - \pi) \beta M(z - q^n + \max\{-q' + \alpha \Omega, 0\} - w_u),
\]

with $T_s(w_s, w_u)$ and $T_u(w_s, w_u)$ a continuous mapping which has at least one fixed point, and such that all fixed points satisfy $w_s \geq w_u$.

I now prove that a solution to the previous system is unique when $\beta \Delta \frac{\partial f^n_s(w_s, w_u; \Phi)}{\partial w_s}$ is uniformly bounded above by 1.

The function $f^n_s(w_s, w_u; \Phi)$ declines in $w_u$ for two reasons: first, $w_u$ reduces the surplus $S^n_u$. Second, $w_u$ reduces $f_u$, which increases $\gamma^n$ (see equation 1.9). Instead, $f^n_s(w_s, w_u; \Phi)$ may increase or decrease in $w_s$ depending on how much do changes in $w_s$ shift the quality and job-creation locus. Given my assumption on $\beta \Delta \frac{\partial f^n_s(w_s, w_u; \Phi)}{\partial w_s}$, the curve for $w_s$ in equation (1.21) describes a decreasing locus on the $(w_s, w_u)$ space, as shown in Figure 1-12, while the curve for $w_u$ describes another locus on the $(w_s, w_u)$ space (which may be decreasing or increasing).

Suppose there are two equilibria $(w^1_s, w^1_u)$ and $(w^2_s, w^2_u)$, with $w^1_s < w^2_s$ without lost of generality. Since the locus determined by the equation for $w_s$ is decreasing, we must have
First, assume that $z > q^n + w_u^1$, so that all matches are formed in both equilibria. We have that:

$$w_i^1 - (1 - \Delta)\beta M(z - w_u^i) \frac{z - w_i^1}{\lambda} = f_s^n(w_i^1, w_u^i, \Phi)$$

$$= w_i^1 - \frac{\Delta(1 - \pi)}{1 + I} \beta M(z - q^n + \max\{-q^i + \alpha \Omega, 0\} - w_u^1) - \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u^i) - \delta \Omega - \frac{\Delta}{1 + I} \pi \beta \frac{z - q^n - w_u^i}{\lambda}$$

(1.22)

with the left-hand side and the right-hand side, two increasing functions of $w_s^i$ and $w_u^i$, respectively. Since $w_s^1 < w_s^2$, but $w_u^1 > w_u^2$, both equalities cannot hold simultaneously for $i = 1, 2$. This contradiction shows that there is a unique equilibrium.

Now, suppose that $z < q^n + w_u^1$. This implies that

$$w_u^1 = b + \frac{\Delta}{1 + I} (1 - \pi) \beta M(z - q^n + \max\{-q^i + \alpha \Omega - w_u^1, 0\}) + \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u^1) + \delta \Omega$$

$$< b + \frac{\Delta}{1 + I} (1 - \pi) \beta M(z - q^n + \max\{-q^i + \alpha \Omega - w_u^2, 0\}) + \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u^2) + \delta \Omega$$

$$< b + \frac{\Delta}{1 + I} \pi \beta f_s^n(w_s^1, w_u^2, \Phi) \frac{\max\{z - q^n - w_u^2, 0\}}{\lambda}$$

$$+ \frac{\Delta}{1 + I} (1 - \pi) \beta M(z - q^n + \max\{-q^i + \alpha \Omega - w_u^2, 0\}) + \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u^2) + \delta \Omega$$

$$= w_u^2.$$

This contradicts the fact that $w_u^1 > w_u^2$ and proves that there is only one equilibrium. □

Remark: The bound on $\beta \Delta \frac{\partial f^n_s(w_s, w_u; z, x, I, \Omega)}{\partial w_u} < 1$ is akin to assuming that complementarities are weak. One can guarantee it in a number of ways. In particular, there are thresholds
such that for $\beta < \overline{\beta}$, or for $\pi < \overline{\pi}$, the condition holds. Numerically, this condition is not too demanding, and for the parameters used in the calibration of my model the condition holds when $\beta = 0.5$ for all values of $\pi$. In the following propositions I provide a sharper characterization for $\overline{\varphi}$, which is the one I used in the main text.

1.4.4 Comparative statics for the instantaneous equilibrium.

I now provide comparative statics for the behavior of the instantaneous equilibrium. I concentrate in the case in which $q_1 > \overline{q}$, and $\beta \Delta \frac{\partial f^\alpha_s(w_s,w_u;\Phi)}{\partial w_s}$ is uniformly bounded above by 1, so that the equilibrium is unique.

Let $w_s(\Phi, z, x, I, \Omega)$ and $w_u(\Phi, z, x, I, \Omega)$ be the reservation wages in the instantaneous equilibrium which solve the fixed point problem given by equation (1.21). Here, I introduce the argument $\Phi$ to separate the effect of all variables through $f^\alpha_s(w_s, w_u; \Phi)$—the effects through the quality of matches and the job creation externality—from the direct effects on the surpluses and matching rates. Likewise, denote by $f_s(\Phi, z, x, I, \Omega)$ and $f_u(\Phi, z, x, I, \Omega)$ the finding rates for both workers.

To save on notation I assume that the unique equilibrium is such that all surpluses are positive and all matches are formed, as is the case in my numerical exercise. All the results generalize to the case in which some matches are not formed with small modifications of the arguments that I present below.

**Proposition 10** Suppose $q_1 > \overline{q}$ and $\beta \Delta \frac{\partial f^\alpha_s(w_s,w_u;\Phi)}{\partial w_s}$ is uniformly bounded above by 1, so that the equilibrium is unique. There exist thresholds $\overline{\pi}$ and $\overline{\beta}$ such that the instantaneous equilibrium satisfies the following properties:

1. Consider a change in $\Phi$ from $\Phi_1$ to $\Phi_2$ such that $f^\alpha_s(w_s^1, w_u^1; \Phi_2) > f^\alpha_s(w_s^1, w_u^1; \Phi_1)$. Then, $w_s, w_u, f_s, f_u$ and $w_u - w_s$ increase. As a corollary, it follows that the external effect of $x, I, \Omega$ through $\gamma^\alpha$ is to increase $w_s, w_u, f_s, f_u$ and $w_u - w_s$.

2. $w_u$ increases in $I$ and $\Omega$. Moreover, if $\pi < \overline{\pi}$, we have that $w_u - w_s$ increases in $I$ and $\Omega$.

3. If $\beta < \overline{\beta}$ and $\pi < \overline{\pi}$ we have that both finding rates $f_s, f_u$ increase in $I$ and $\Omega$. Moreover, $\gamma^\alpha, f^\alpha_s$ and $w_s$ also increase in $I$ and $\Omega$.

**Proof of Numeral 1:** Notice that the increase in $f^\alpha_s(w_s, w_u; \Phi)$ shifts the loci for $T_s(w_s, w_u) = w_s$ and $T_u(w_s, w_u) = w_u$ upwards and to the right in Figure 1-12. Since the locus for $T_s(w_s, w_u) = w_s$ is a decreasing curve, the new equilibrium must involve an increase in $w_s$ or an increase in $w_u$, or both. However, equation (1.22) implies that both reservation
wages must move in the same direction, which implies that both $w_s$ and $w_u$ increase, and so does $f^n_s(w_s, w_u; \Phi)$.

In addition, since the left hand side of equation (1.22) is less steep than the right hand side—which reflects the different frequencies with which workers match with novel jobs and the wage they obtain—$w_s$ increases more than $w_u$, as wanted.

I now prove that both finding rates increase. As claimed above, $f^n_s(w_s, w_u; \Phi)$ increases. Moreover, due to the increase in reservation wages, the surplus obtained by skilled workers, which is given by $\tilde{S}^n_s = \frac{z - w_s}{\lambda}$, decreases.

Suppose by way of contradiction that $f_s$ decreases. If this were the case and since $w_s = b + \beta f_s \frac{z - w_s}{\lambda}$, $w_s$ would decline, which contradicts the fact that $w_s$ increases. This contradiction implies $f_s$ increases.

For unskilled workers we have that the surpluses $\tilde{S}^n_u = \frac{z - w_u - q^n}{\lambda}$, $\tilde{S}^l_u = \frac{z - w_u - q^n + \max\{-q^l + \alpha \Omega\}}{\lambda}$, and $\tilde{S}^n_u = \frac{z - w_u}{\lambda}$ decrease due to the increase in the reservation wage, $w_u$.

Let $w_u$ increase from $w_u^1$ to $w_u^2$, and denote by $\tilde{f}_j^n(\Phi)$ the resulting finding rates. We have:

$$w_u^2 = \frac{\Delta}{1 + I} \beta \left[ \frac{\pi \tilde{f}_n^s(\Phi_2) z - q^n - w_u^2}{\lambda} + (1 - \pi) \tilde{f}_l^n(\Phi_2) z - q^n - w_u^2 + \max\{-q^l + \alpha \Omega\} \right]$$

$$> \frac{\Delta}{1 + I} \beta \left[ \frac{\pi \tilde{f}_n^s(\Phi_1) z - q^n - w_u^1}{\lambda} + (1 - \pi) \tilde{f}_l^n(\Phi_1) z - q^n - w_u^1 + \max\{-q^l + \alpha \Omega\} \right]$$

The first inequality uses the fact that $w_u^2 > w_u^1$, and the second uses the decline in surpluses as we move from $\Phi_1$ to $\Phi_2$.

The last inequality implies that:

$$(\tilde{f}_n^s(\Phi_2) - \tilde{f}_n^s(\Phi_1)) \frac{\Delta \pi}{1 + I} > (\tilde{f}_l^n(\Phi_1) - \tilde{f}_l^n(\Phi_2)) \frac{\Delta (1 - \pi)}{1 + I} \frac{\tilde{S}_u^l}{\tilde{S}_u^n} (\tilde{f}_n^s(\Phi_1) - \tilde{f}_n^s(\Phi_2)) \frac{1 - \Delta + I \frac{\tilde{S}_u^r}{\tilde{S}_u^n}}{1 + I}$$

The first line follows from rearranging the previous inequalities. The second line follows from the fact that $\tilde{S}_u^n$ is smaller than the surplus obtained by unskilled workers in all other jobs.

This inequality implies that $f_u$ increases after rearranging it.
PROOF OF NUMERAL 2: Since $q > \bar{q}$, we have that $\tilde{S}_u^m \leq \tilde{S}_u^l < \tilde{S}_u^r = \tilde{S}_u^o$.

Therefore, for fixed values of $w_s, w_u$, an increase in $I$ shifts the locus for $w_u = T_s(w_s, w_u)$ to the right. Through its effect on $\Phi$, it also shifts the locus for $w_u = T_s(w_s, w_u)$ upwards. This implies that at least one among $w_s, w_u$ increases.

Suppose $w_s$ increases but $w_u$ declines. The equation 1.22 implies that $f_u^s(w_s, w_u, \Phi)$ increases. Since $w_u$ is given by

$$w_u = b + \frac{\Delta}{1 + I} \pi \beta f_u s \max\{z - q^n - w_u, 0\} + \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u)$$

$$+ \frac{\Delta}{1 + I} (1 - \pi) \beta M(z - q^n + \max\{-q^l + \alpha \Omega, 0\} - w_u) + \delta \Omega,$$

and the increase in $I$ shifts the right-hand side upwards (including $f_u^s$), $w_u$ increases as well. This implies that $w_u$ always increases.

Likewise, for fixed values of $w_s, w_u$, an increase in $\Omega$ shifts the locus for $w_u = T_s(w_s, w_u)$ to the right. Through its effect on $\Phi$, it also shifts the locus for $w_u = T_s(w_s, w_u)$ upwards. This implies that at least one among $w_s, w_u$ increases. Using the same argument as above, I can discard a situation in which $w_s$ increases but $w_u$ declines, which implies that $w_u$ always increases.

The threshold $\bar{\pi} > 0$ is defined by noting that, as $\pi \to 0$, the effect of both $I$ and $\Omega$ on $w_s$ converges to zero, while the effect of both $I$ and $\Omega$ on $w_u$ remains positive and bounded away from zero. Therefore, there exists a threshold $\bar{\pi}$ that guarantees that the direct effects of $I$ and $\Omega$ on $w_u$ dominate and $w_u - w_s$ increases in $I, \Omega □$

PROOF OF NUMERAL 3: The finding rate for unskilled workers is given by

$$f_u = \frac{\Delta}{1 + I} \pi f_u^s + \frac{\Delta}{1 + I} (1 - \pi) f_u^l + \frac{1 - \Delta + I}{1 + I} f_u^r,$$

which already takes into account the fact that $f_u^r = f_u^o$.

The effect of a change in $I$ on the finding rate can be written as:

$$df_u = \frac{\Delta}{(1 + I)^2} \pi (f_u^r - f_u^s) dI + \frac{\Delta}{(1 + I)^2} (1 - \pi) (f_u^r - f_u^l) dI + \frac{\Delta}{1 + I} \pi df_u^s + O(\beta).$$

Here, the term $O(\beta)$ stands for the change in $w_s$ and $w_u$, which are of the same order of magnitude as $\beta$.

The condition $q' > \bar{q}$ guarantees that $\tilde{S}_u^m \leq \tilde{S}_u^l < \tilde{S}_s^m \leq \tilde{S}_s^r = \tilde{S}_u^o$. This implies that $f_u^r > f_u^s$ and $f_u^r > f_u^l$.

These inequalities show that the term $\frac{\Delta}{(1 + I)^2} \pi (f_u^r - f_u^s) dI + \frac{\Delta}{(1 + I)^2} (1 - \pi) (f_u^r - f_u^l) dI$ in the expression for $df_u$ above is positive.
Turning to $d\gamma$, we have that
\[
d\gamma \propto \frac{1 - \gamma}{\gamma} df_u - \frac{1 - x}{x} df_s
= \left( \frac{1 - \gamma}{\gamma} \frac{\pi}{1 + I} - \frac{1 - x}{x} \right) \Delta df^n_s
+ \frac{1 - \gamma}{\gamma} \left( \frac{\Delta}{(1 + I)^2} f'_u - f'_s \right) + \frac{\Delta}{(1 + I)^2} (1 - \pi)(f'_u - f'_s) \right) dI + O(\beta).
\]
Thus, if $df^n_s < 0$, we would have that $d\gamma > 0$. But this implies $d\gamma^n > 0$ and we can write $df^n_s = A\gamma^n + O(\beta)$, and $df_s = A\Delta d\gamma^n + O(\beta)$ with $A > 0$ given by the slope of the job-creation locus. If, on the other hand, we had $df^n_s > 0$, we would still have $df_s = \Delta df^n_s + O(\beta)$.

In either case, the expressions for $df_u$, $df^n_s$ and $df_s$ imply that there exists a threshold $\beta > 0$ such that, for $\beta < \beta$, a rise in $I$ increases $f_u$, $f_s$ and $f^n_s$. Finally, since in this case the rise in $I$ increases $f^n_s$, equation (1.22) implies that $w_s$ increases as well. The only way in which $f^n_s$ may increase while both reservation wages, $w_s$ and $w_u$, are larger, is if the composition $\gamma^n_s$ improves.

Turning to the effect of an increase in $\Omega$, we can write:
\[
df_u = \Delta \frac{1}{1 + I} \pi df^n_s + \Delta \frac{1}{1 + I} (1 - \pi) df^l_u + O(\beta).
\]
Here, $df^l_u > 0$ due to the increase in $\Omega$, while $O(\beta)$ takes into account the decline in $f'_u$ due to the increase in reservation wages.

Turning to $d\gamma$, we have that
\[
d\gamma \propto \frac{1 - \gamma}{\gamma} df_u - \frac{1 - x}{x} df_s
= \left( \frac{1 - \gamma}{\gamma} \frac{\pi}{1 + I} - \frac{1 - x}{x} \right) \Delta df^n_s
+ \frac{1 - \gamma}{\gamma} \left( \frac{\Delta}{1 + I} (1 - \pi) df^l_u + O(\beta). \right.
\]
Thus, if $df^n_s < 0$ we would have that $d\gamma > 0$. But this implies $d\gamma^n > 0$ and we can write $df^n_s = A\gamma^n + O(\beta)$, and $df_s = A\Delta d\gamma^n + O(\beta)$ with $A > 0$ given by the slope of the job-creation locus. If, on the other hand, we had $df^n_s > 0$, we would still have $df_s = \Delta df^n_s + O(\beta)$.

In either case, the expressions for $df_u$, $df^n_s$ and $df_s$ imply that there exists a threshold $\beta > 0$ such that, for $\beta < \beta$, a rise in $\Omega$ increases $f_u$, $f_s$ and $f^n_s$. Finally, since in this case the rise in $\Omega$ increases $f^n_s$, equation (1.22) implies that $w_s$ increases as well. The only way in which $f^n_s$ may increase while both reservation wages, $w_s$ and $w_u$, are larger, is if the composition $\gamma^n_s$ improves, as wanted.

**Remarks:** The threshold $\overline{\pi}$ bounds the job creation externality. It guarantees that shocks affecting the surplus of unskilled workers (reductions in $I$ or $\Omega$) increase their incen-
tives to become skilled. This regularity condition guarantees the saddle path stability of the system as I show below. In the main text, this condition is used to guarantee the uniqueness of the instantaneous equilibrium and this regularity condition that leads to the saddle path stability.

The condition \( \beta < \bar{\beta} \) guarantees two things. First, it guarantees that increases in the surplus of one particular type of job do not reduce workers' finding rates. This counter-intuitive effect would result if, due to the increase in unskilled workers' reservation wage, firms created less of other jobs and this resulted in a net decline in workers' finding rates. Second, the condition guarantees that, when unskilled workers are displaced from old jobs or few stepping-stone jobs are created, they reduce the expected surplus of job creation for firms that are posting novel jobs. This could fail to be the case if, due to the decline in their reservation wage, the abundant number of unskilled workers became more profitable matches in novel jobs than before. The condition \( \beta < \bar{\beta} \) keeps unskilled workers' reservation wage from falling that much, so that the net effect of an inflow of displaced workers that are searching for novel jobs causes a reduction in job creation. I used this condition to guarantee that an improvement in the quality of potential matches, \( d\gamma^a > 0 \), is associated with the creation of more novel jobs.

The thresholds are not restrictive in my numerical exercise. In my calibration of the model, the instantaneous equilibrium is unique and \( w_u - w_s \) is increasing in \( \Omega \) and \( I \) for any value of \( \pi \in [0, 1] \). Moreover, in my baseline calibration, one can have values for \( \beta \) as high as \( \beta = 0.9 \), for which \( I \) and \( \Omega \) increase both workers finding rates and reservation wages. Thus, these conditions are not demanding numerically, but are required conceptually.

1.4.5 Proofs of the propositions 2-6.

The previous results allow me to prove the following generalization of Proposition 2 presented in the main text:

PROPOSITION 11 Suppose we are in the limit case in which \( a, \lambda \rightarrow \infty \). Moreover, assume \( \pi < \bar{\pi} \) and \( q_1 > \bar{q} \). We have that:

1. The current values of \( z, x, I, \Omega \) uniquely determine all equilibrium objects.

2. The equilibrium behavior of \( x, I, \Omega \) boils down to the system of equations:

\[
\dot{x} = (1 - x) \left[ \frac{\Delta (1 - \pi) f(\theta_u)}{\lambda + \hat{f}_u} + \delta \right], \quad \dot{\Omega} = r \Omega + w_u - w_s, \quad \dot{I} = -v(t) I,
\]

coupled with an initial condition for \( x(0), I(0) \).
3. Near the unique steady state, the equilibrium is saddle-path stable.

4. The system is globally saddle path stable, with $\Omega(t)$ increasing monotonically to $\Omega^*$.

5. The threshold $\bar{q}$ is defined explicitly as the one that guarantees $\bar{q} = (r + \alpha)\Omega^* - q^a$.

PROOF. Numerals 1 and 2 follow from the results presented above on the unicity and existence of the instantaneous equilibrium. The equilibrium behavior of $x, I, \Omega$ follows from rearranging equation (1.14).

For numeral 3, we have that the behavior of the system can be approximated linearly around the steady state as (variables with an asterisk denote their steady state levels):

$$
\dot{x} = -\left[\alpha \frac{\Delta(1-\pi)f(\theta^m_u)}{\lambda + f^*_u} + \delta\right](x - x^*)
$$

$$
\dot{I} = -v(I - I^*)
$$

$$
\dot{\Omega} = \left(\frac{\partial w_u}{\partial x} - \frac{\partial w_s}{\partial x}\right)(x - x^*) + \frac{\partial w_u}{\partial I}(I - I^*) + \left[r + \frac{\partial w_u}{\partial \Omega}\right](\Omega - \Omega^*)
$$

The reason why there is no effect of $I, \Omega$ on $x$ is because around the steady we have $x^* = 1$ and these effects are second order. Likewise, $I$ has no effect on $w_s$ near the steady state.

Locally, the system is recursive, with $x$ and $I$ converging monotonically to their steady state values at fixed rates, and $\Omega$ uniquely determined by forward integration over the resulting paths. To confirm this, notice that the linear system has two negative eigenvalues given by $-\left[\alpha \frac{\Delta(1-\pi)f(\theta^m_u)}{\lambda + f^*_u} + \delta\right]$ and $-v$, and a positive one given by $r + \frac{\partial w_u}{\partial \Omega}$. Since the system has two state variables and a forward looking variable, it is locally saddle path stable.

To extend this result outside a neighborhood of the steady state, notice that $I \to 0$ and $x \to 1$ always, which implies the global stability of the steady state.

I now turn to an analysis of the transitional dynamics. The equation for $\dot{x}$ implies that $x(t)$ increases monotonically and converges to 1. The equation for $\dot{I}$ implies that $I(t)$ decreases monotonically and converges to zero.

Consider the equilibrium when $z(t) = 1\forall t$ and $I(t) = 0$. The equation for $\dot{\Omega}$ implies that as time goes by, $x$ increases and $w_u - w_s$ declines (see Proposition 10). Also, as $\Omega$ increases, and since $\pi < \bar{\pi}$, we have that $w_u - w_s$ increases. Therefore, I can write the differential equation for $\Omega$ as $\dot{\Omega} = h(\Omega, t)$, with $h_\Omega > 0$ and $h_t < 0$.

The same holds when $z(t) = 1\forall t$ but we also have $I(t) > 0$. In this case, and since $\pi < \bar{\pi}$ and $q' > \bar{q}$, we have that the decline in $I(t)$ over time reduces $w_u - w_s$. Thus, in this case we also have that $\dot{\Omega} = h(\Omega, t)$, with $h_\Omega > 0$ and $h_t < 0$.

The steady state value for $\Omega(t)$ satisfies $\lim_{t \to \infty} h(\Omega^*, t) = 0$. Suppose that along the transition $\Omega(T) \geq \Omega^*$. Then $h(\Omega(T), T) > h(\Omega^*, t) = 0$, which implies $\dot{\Omega}(T) > 0$. Thus, for
all \( t > T \) we have \( \Omega(t) > \Omega^* \) and \( \dot{\Omega}(t) > 0 \) so that \( \Omega(t) \) is increasing, which contradicts the fact that in the unique steady state we always have \( \lim_{t \to \infty} \Omega(t) = \Omega^* \). This contradiction implies that \( \Omega(t) < \Omega^* \) along the transition.

Now, suppose that \( h(\Omega(T), T) < 0 \). Then \( \Omega(t) < \Omega(T) \) for \( t \in (T, T + \epsilon) \), for some \( \epsilon > 0 \). I claim that in this case \( \Omega(t) \) declines for \( t \geq T \). To prove it, suppose that it does not. Then there is a time \( T' > T \) in which \( h(\Omega(T'), T') = 0 \) but \( h(\Omega(t), t) < 0 \) for \( t \in [T, T') \). We have that \( \Omega(t) > \Omega(T') \) for \( t \in [T, T') \) by the election of \( T' \). But this implies \( 0 > h(\Omega(t), t) > h(\Omega(T'), T') = 0 \), a contradiction. This implies that \( \Omega(t) \) declines for \( t \geq T \), but then \( \lim_{t \to \infty} \Omega(t) \neq \Omega^* \). This contradiction implies my initial supposition is false, and we have \( h(\Omega(t), t) > 0 \) for all \( t \), which implies that \( \Omega(t) \) increases monotonically until it reaches its steady state level.

I now prove the final numeral of the proposition. If \( q' > (r + \alpha)\Omega^* - q^n \), we have that \( q' > \alpha \Omega(t) + w_s - w_u - q^n \). This inequality uses the fact that \( \Omega^* \geq \Omega(t) \) and \( r \Omega \geq w_s - w_u \), since \( \Omega \geq 0 \).

Therefore, \( q = (r + \alpha)\Omega^* - q^n \) is enough to guarantee the uniqueness of the equilibrium and the comparative statics results established above.

**Remark:** In the main text I state all the propositions using the tighter condition \( q' > \bar{q} \), with \( \bar{q} = (\alpha + r)\Omega^* - q^n \), which offers a sufficient characterization of \( \bar{q} \) that applies for all propositions proved here. This threshold is reasonable for several reasons. First, the condition \( q' > \bar{q} \) guarantees that \( S_u^l < S_u^n \), \( \Omega \geq 0 \) and \( w_s \geq w_u \). Thus, \( q' > \bar{q} \) is the right assumption required to obtain the intuitive feature that skilled workers have larger reservation wages and produce a larger surplus in novel jobs than unskilled workers. I used these regularity conditions throughout the text when giving several intuitions, or describing the adjustment of the economy.

The condition is also satisfied in my numerical simulations. In the first numerical scenario, I have \( \bar{q} = 0.34 \), which is smaller than \( q' = 0.45 \). In the second numerical scenario, I have \( \bar{q} = 0.19 \), which is smaller than \( q' = 0.3 \).

Moreover, this condition is not restrictive. We have that \( \bar{q} = (\alpha + r)\Omega^* - q^n = \alpha \Omega^* + w_s^* - w_u^* - q^n \). Lemma 1 implies that \( w_s^* - w_u^* - q^n < 0 \), which implies \( \alpha \Omega^* > \bar{q} \). Thus, for \( q' \approx \bar{q} \) firms and workers face positive incentives to retrain.

Using the conditions \( q' > \bar{q}, \beta < \bar{\beta} \) and \( \pi < \bar{\pi} \), I am now in a position to prove Propositions 3-5.

**Proof of Proposition 3:** Since \( q^n + q' > (\alpha + r)\Omega \), we have that \( q^n + q' > \alpha \Omega + w_s - w_u \). Here, I used the fact that \( \dot{\Omega} = r \Omega + w_u - w_s > 0 \) along the adjustment (see Proposition 11). The inequality \( q^n + q' > \alpha \Omega + w_s - w_u \) implies that \( \tilde{S}_u^l > \tilde{S}_u^l \).

Thus we have \( \tilde{S}_u^l > \tilde{S}_u^l \) (by lemma 1) and \( \tilde{S}_u^l > \tilde{S}_u^l \). These observations imply that the finding rate for unskilled workers can only be larger than that of skilled workers if
$(1 - \Delta)(f_u^* - f_s^*)$ is large enough, or equivalently, if $w_s - w_u$ is large enough. The condition $\beta < \bar{\beta}$ guarantees this is not the case, and the increase in hiring in regular jobs does not fully compensate for the depressed finding rates of unskilled workers elsewhere.

The statement in the second numeral of the proposition follows from the observation that, along the transition we have that $\Omega(t)$ and $x(t)$ are increasing, as established in Proposition 10. Since $f_s, f_u$ increase with both $\Omega(t)$ and $x(t)$, we have that $f_s(t)$ and $f_u(t)$ increase over time and approach their corresponding steady-state values.

The statement in the third numeral of the proposition follows from the observation that the stable arm for $\Omega$ is given by an increasing curve between $\Omega$ and $x$. Thus, a fall in $x(0)$ shifts the entire path for $x(t)$ and $\Omega(t)$ downwards. The comparative statics in Proposition 10 imply that $f_s$ and $f_u$ increase with both $x$ and $\Omega$. Therefore, a fall in $x(0)$ shifts down the entire path for $f_s(t)$ and $f_u(t)$.

**Proof of Proposition 4:** The first part of the proposition follows as a corollary of Numeral 5 in Proposition 11. The second part follows from the fact that a low $x(0)$ shifts $\Omega(0)$ downwards along the stable arm of the system, which implies that it shifts down the entire equilibrium path for $\Omega(t)$.

**Remark:** Note that this result does not require any condition on $\beta$. This is because this proposition does not deal with the behavior of finding rates, but only of reservation wages.

I finalize this section with a proof of Proposition 5. As before, I assume we have $\pi < \bar{\pi}$, $\beta < \bar{\beta}$ and $q^d > \bar{q}$ so that the instantaneous equilibrium is unique and the comparative statics developed in Proposition 10 apply.

Before presenting the proof, I introduce some notation that I will use.

In order to separate the direct effects from those that operate through $\gamma^n$ — that is, the job creation externality —, I define the functions $f^n_s(\gamma^n, z, I, \Omega)$ and $f^n_u(\gamma^n, z, I, \Omega)$, as the finding rates one would obtain for a fixed $\gamma^n$, which leave the quality of potential matches fixed.

The finding rates obtained once the change in $\gamma^n$ is taken into account, are given by $f_s(z, x, I, \Omega)$ and $f_u(z, x, I, \Omega)$. These are defined by the unique solution to the system:

$$f_s = f^n_s(\gamma^n(f_s, f_u, x, I), z, I, \Omega) \quad f_u = f^n_u(\gamma^n(f_s, f_u, x, I), z, I, \Omega).$$

The function $\gamma^n_s(f_s, f_u, x, I)$ is defined implicitly by:

$$\gamma^n_s(f_s, f_u, x, I) = \frac{\gamma(f_s, f_u, x)}{(1 - \gamma(f_s, f_u, x))} = \frac{1}{1 + f_u} + \gamma(f_s, f_u, x),$$

$$(1 - \gamma(f_s, f_u, x)) = (1 - x) + \gamma(f_s, f_u, x) f_u + (1 - \gamma(f_s, f_u, x)) f_u.$$
The comparative static results in Proposition 10, imply that both \( f_s \) and \( f_u \) increase with \( x \). Moreover, we have that:

\[
\frac{df_s}{dx} = \frac{\frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial x}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}} dx,
\]

\[
\frac{df_u}{dx} = \frac{\frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial x}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}} dx.
\]

Therefore, \( \frac{\partial f_s^p}{\partial \gamma^n}, \frac{\partial f_u^p}{\partial \gamma^n} > 0 \)—this relies on Numeral 1 of the comparative statistics, and \( 1 > \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u} \)—this condition guarantees that compositional effects are not so strong as to have an improvement in match quality end up reducing finding rates. In fact, this condition follows from the fact that \( \beta \Delta \frac{\partial f_s^p(u, w, x, z, \Delta)}{\partial w_s} < 1 \).

Using these functions, I am now in a position to prove Proposition 5.

**PROOF OF PROPOSITION 5:** We can write:

\[
\frac{df_s}{dz} = \frac{\frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} + \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}} dz,
\]

\[
\frac{df_u}{dz} = \frac{\frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} + \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}} dz.
\]

This expression shows the effect of the decline in \( z \) can be decomposed in a direct effect—holding \( \gamma^n \) constant—and an indirect effect through the decline in \( \gamma^n \)—the job creation externality.

The comparative static results presented above imply that \( \frac{\partial f_s^p}{\partial \gamma^n}, \frac{\partial f_u^p}{\partial \gamma^n} > 0 \). Moreover, holding \( \gamma^n \) constant we have \( \frac{\partial f_s^p}{\partial z} > \frac{\partial f_u^p}{\partial z} > 0 \), which is the standard effect of a change in productivity on finding rates, and is stronger on workers with lower productivity (see Shimer, 2005).

The solution to this system is given by:

\[
\frac{d}{dz} = \frac{\frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} + \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}} dz
\]

\[
\frac{d}{dz} = \frac{\frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} + \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma^n}{\partial f_u}} dz.
\]

Here, the first term captures the direct effect of the recession, and the second term captures the way the recession exacerbates the skill mismatch by reducing the share of skilled workers who are searching for jobs.
This second term is positive (so that this mechanism reduces the finding rate for both workers) because:

\[
\frac{\partial \gamma_p}{\partial z} + \frac{\partial \gamma_u}{\partial z} \propto \left( \frac{1 - \gamma}{\gamma} \frac{\partial \gamma_p}{\partial x} - \frac{1 - \gamma}{x} \frac{\partial f_s}{\partial z} \right) > 0. \tag{1.23}
\]

The inequality follows from noting that \( \frac{1 - \gamma}{\gamma} > \frac{1 - \gamma}{x} \) (recall that this is the case because \( f_s > f_u \)), and \( \frac{\partial f_s}{\partial z} > \frac{\partial f_s}{\partial z} \) because unskilled workers finding rate is more cyclical.

For small \( x \), \( \gamma \) is small, which increases the cyclicity of the finding rate because \( f_s > f_u \) is more cyclical. In addition, a smaller \( x \) increases the response of \( \gamma^n \) to changes in the finding rates, which makes the term \( d\gamma^n = \left( \frac{\partial \gamma_p}{\partial f_s} + \frac{\partial \gamma_u}{\partial f_u} \right) dI \) larger.

Now consider a change in \( I \), given by \( dI \). We can write

\[
\begin{align*}
dfs &= \frac{\partial f_s}{\partial I} dI + \left( \frac{\partial \gamma_p}{\partial f_s} df_s + \frac{\partial \gamma_u}{\partial f_s} df_u \right) \frac{\partial f_s}{\partial \gamma^n}, \\
\du &= \frac{\partial f_u}{\partial I} dI + \left( \frac{\partial \gamma_p}{\partial f_u} df_s + \frac{\partial \gamma_u}{\partial f_u} df_u \right) \frac{\partial f_u}{\partial \gamma^n}.
\end{align*}
\]

The resulting change in finding rates is given by:

\[
\begin{align*}
dfs &= \frac{\partial f_s}{\partial I} dI + \left( \frac{\partial \gamma_p}{\partial f_s} + \frac{\partial \gamma_u}{\partial f_s} \frac{df_s}{\partial \gamma^n} \right) dI \\
\du &= \frac{\partial f_u}{\partial I} dI + \left( \frac{\partial \gamma_p}{\partial f_u} + \frac{\partial \gamma_u}{\partial f_u} \frac{df_u}{\partial \gamma^n} \right) dI.
\end{align*}
\]

Moreover, we have that \( d\gamma^n = \left( \frac{\partial \gamma_p}{\partial f_s} + \frac{\partial \gamma_u}{\partial f_u} \right) dI \). The comparative statics results imply that \( \gamma^n \) increases with \( I \), which implies that \( \frac{\partial \gamma_p}{\partial f_s} + \frac{\partial \gamma_u}{\partial f_u} > 0 \). Moreover, the condition \( \beta < \beta \) implies that, although \( \frac{\partial \gamma_p}{\partial I} < 0 \)—since \( I \) reduces the profitability of matches with unskilled workers—this effect is dominated by \( \frac{\partial \gamma_p}{\partial \gamma^n} > 0 \).

As above, a when \( x \) is smaller the finding rate is more responsive both because there are more unskilled workers directly affected by \( I \), and because \( d\gamma^n = \left( \frac{\partial \gamma_p}{\partial f_s} + \frac{\partial \gamma_u}{\partial f_u} \right) dI \) is more responsive to changes in \( I \).

To finalize the proof I need to specify how does \( \Omega \) respond to changes in \( I \) and \( z \). For a decline in \( z \), \( \Omega \) actually declines temporarily because, during the crisis, \( w_s \) falls more than \( w_z \). Thus, this effect creates a force in the direction of increasing unemployment too. However, this effect is not quantitatively relevant since \( \Omega \) is forward looking and the decline in productivity is only temporary.

In the case of a decline in \( I \), \( \Omega \) increases as a response. However, we have that \( \Omega < \Omega^* \). Therefore, for large values of \( q' \), the response of job creation and the subsequent increase in
x are not strong enough to compensate for the decline in I. Over time, x increases and the average finding rate and \( f_s \) return to their pretrend.

To prove the propagation can take as much time as wanted, consider the case in which \( q' > \alpha \Omega^* \), so that there is no training along equilibrium. In this case, the effect of I on \( f_s \) and \( f_u \) is permanent and x does not adjust to compensate for it. The effect of a decline in I only disappears asymptotically, when due to the exogenous acquisition of skills by unskilled workers, we have that \( x = 1 \) □

1.4.6 Proof of Proposition 6

PROOF OF THE CHARACTERIZATION OF THE CONSTRAINED EFFICIENT ALLOCATION: In the general case, the planner’s problem is to maximize

\[
\max \mathcal{W} = \int_0^\infty e^{-rt} \left( u_s b + u_u b + \sum_{k,j} e_j^k(t) z^k - \kappa \theta_s^n \left[ u_s \Delta + u_u \frac{\Delta}{1 + I} \right] - \kappa \theta_u^n u_u \frac{\Delta}{1 + I} (1 - \pi) - \kappa \theta_u^n u_u \left( 1 - \Delta \right) - \kappa \theta_u^n u_u \frac{1 - \Delta}{1 + I} - \kappa \theta_u^n u_u \frac{I}{1 + I} \right) dt,
\]

subject to the behavior of the state variables described in equation (1.11). Here, \( z^k_j \) is the product of a match, \( z(t) \), net of training costs \( q^n \) and \( q' \).

The co-state for \( u_s \), which I label \( \Gamma_s \)—and determines the social value of unemployment for skilled workers is given by

\[
r \Gamma_s - \dot{\Gamma}_s = b + \Delta \left( f(\theta^n_s)SV^n_s - \kappa \theta_s^n \right) + (1 - \Delta) \left( f(\theta^n_s)SV^n_s - \kappa \theta^n_s \right).
\]

Here, \( SV^k_j \) denotes the value of a match \( e_j^k(t) \), which is equal to the co-state for the state variable \( e_j^k(t) \).

The co-state for \( u_u \), which I label \( \Gamma_u \)—and determines the social value of unemployment for skilled workers is given by

\[
r \Gamma_u - \dot{\Gamma}_u = b + \Delta \left( f(\theta^n_u)SV^n_u - \kappa \theta_u^n \right) + \Delta \left( 1 - \pi \right) \left( f(\theta^n_u)SV^n_u - \kappa \theta_u^n \right) + \frac{1 - \Delta}{1 + I} \left( f(\theta^n_u)SV^n_u - \kappa \theta_u^n \right) + \frac{I}{1 + I} \left( f(\theta^n_u)SV^n_u - \kappa \theta_u^n \right).
\]

The planner chooses tightness as to maximize the current value Hamiltonian. Thus, tightness is given by:

\[
\kappa = f'(\theta^n_j) \mathbb{E}_{SV}[\max\{SV, 0\}|k, j].
\]
This equation implies that
\[ \kappa \theta_j^k = \theta_j^k f'(\theta_j^k)E_{SV} \{ \max \{ SV, 0 \} | k, j \} = (1 - \eta) f(\theta_j^k)E_{SV} \{ \max \{ SV, 0 \} | k, j \}. \]

Replacing these terms in the reservation wages yields
\[ r \Gamma_s - \hat{\Gamma}_s = b + \Delta f(\theta_s^n) (SV_s^n - (1 - \eta)(\gamma^n SV_s^n + (1 - \gamma^n)SV_u^n)) + \eta(1 - \Delta)f(\theta_s^u)SV_s^u, \]
and
\[ r \Gamma_u - \hat{\Gamma}_u = b + \Delta f(\theta_u^n) (SV_u^n - (1 - \eta)(\gamma^n SV_s^n + (1 - \gamma^n)SV_u^n)) + \Delta f(\theta_u^u)SV_u^u \]
\[ + \frac{(1 - \Delta)}{1 + I} f(\theta_u^u)SV_u^u + \frac{I}{1 + I} f(\theta_u^u)SV_u^u. \]

Equation 1.10 in the main text follows from these expressions after imposing the Hosios condition \( \beta = \eta \) and noting that \( \mu_j = r \Gamma_j - \hat{\Gamma}_j \). In addition, the values of employment \( SV_j^k \) satisfy the same Bellman equations derived for the decentralized economy, with \( \Gamma_j \) playing the role of \( U_j \).  

**PROOF OF THE COROLLARY TO PROPOSITION 6 IN THE MAIN TEXT:** I prove that, in the limit case with large gross flows, subsidizing training at the margin increases welfare.

We have that along any given allocation:
\[ \Gamma_s - \Gamma_u = \int_0^\infty e^{-rt}(\mu_s - \mu_u)dt > \int_0^\infty e^{-rt}(w_s - w_u)dt = \Omega. \]

Therefore, along the decentralized allocation \( \overline{SV}_u^j > \overline{S}_u^j \).

I now prove that there is a path of non-negative subsidies for training that increase welfare in the decentralized allocation.

Suppose that firms creating stepping-stone jobs earn a surplus \( S_u^t(t) + \sigma \), with \( \sigma > 0 \) for \( t \in [0, T] \) denoting the subsidy.

For small values of \( \sigma \), the subsidy increases tightness in stepping-stone jobs by \( d_\theta(t) > 0 \), and the effect on welfare is given by:
\[ d\mathcal{W} = \int_0^T e^{-rt} \left[ (1 - \eta) \overline{q}(\theta_u^t) \overline{SV}_u^t - \kappa \right] d_\theta(t)dt. \]

However, we have that along the decentralized allocation (and using the Hosios condition),
Therefore, the effect of a small subsidy to training is given by

$$d\mathbb{W} = \int_0^T e^{-rt}(1-\eta)\mathcal{v}(\theta_u^t) \left[ SV_u^p - S_u^{\theta_u^t} \right] d\phi(t) dt > 0,$$

as wanted.

### 1.4.7 Allowing workers to direct their search efforts.

In this sub-section I describe an extension of my model in which I allow unemployed workers to direct their search efforts.

Each period unskilled workers draw an idiosyncratic shock $\varepsilon(i)$ determining their search efficiency when looking for jobs in task $i$. Moreover:

$$\varepsilon(i) \sim F(\phi, (1+I)^{-1/\phi} \mu),$$

with $F(\phi, \mu)$ the Frechet distribution with shape $\phi > 1$ and scale $\mu$. I normalize $\mu$ so that $\Gamma(1-1/\phi)\mu = 1$, and on average unskilled workers have one unit of search efficiency in the task they choose to search. After they obtain their draws, workers decide in which task they search for jobs. When their search efficiency is $\varepsilon(i)$ and they search for jobs in this task, their finding rate is $\varepsilon(i)f(\theta(i))$, with $\theta(i)$ the tightness at the task.

Workers search for jobs in the task that maximizes their expected utility from searching. With this distributional assumptions, the probability that the worker searches for a novel job is given by $\frac{\Delta}{1+I} \lambda_u^{n\phi-1}$, with

$$\lambda_u^n = \frac{\pi f(\theta_u^n)S_u^n + (1-\pi)f(\theta_u^t)S_u^t}{\left(\frac{\Delta}{1+I} [\pi f(\theta_u^n)S_u^n + (1-\pi)f(\theta_u^t)S_u^t]^\phi + \frac{1-\phi}{1+I} [f(\theta_u^n)S_u^n]^\phi + \frac{I}{1+I} [f(\theta_u^t)S_u^t]^\phi\right)^{1/\phi}},$$

The probability that the worker searches for a regular job is given by $\frac{1-\Delta}{1+I} \lambda_u^{r\phi-1}$, with

$$\lambda_u^r = \frac{f(\theta_u^r)S_u^r}{\left(\frac{\Delta}{1+I} [\pi f(\theta_u^n)S_u^n + (1-\pi)f(\theta_u^t)S_u^t]^\phi + \frac{1-\phi}{1+I} [f(\theta_u^n)S_u^n]^\phi + \frac{I}{1+I} [f(\theta_u^t)S_u^t]^\phi\right)^{1/\phi}},$$

Finally, the probability that the worker searches for an old job is given by $\frac{I}{1+I} \lambda_u^{o\phi-1}$, with

$$\lambda_u^o = \frac{f(\theta_u^o)S_u^o}{\left(\frac{\Delta}{1+I} [\pi f(\theta_u^n)S_u^n + (1-\pi)f(\theta_u^t)S_u^t]^\phi + \frac{1-\phi}{1+I} [f(\theta_u^n)S_u^n]^\phi + \frac{I}{1+I} [f(\theta_u^t)S_u^t]^\phi\right)^{1/\phi}},$$

We have that these probabilities collapse to $\frac{\Delta}{1+I}, \frac{1-\Delta}{1+I}$ and $\frac{I}{1+I}$ for $\phi \to 1$. Thus, the general-
The derivation for skilled workers is similar so I do not present it here.

For $\phi > 1$ workers direct their search efforts to tasks yielding a higher surplus, but only respond to differences in surpluses with an elasticity $\phi - 1$—captured by the terms $\lambda_j^{\phi-1}$—garbling the initial probabilities. Thus $\phi$ determines the extent to which workers may direct their search efforts. As $\phi \to \infty$, we converge to the case in which workers only search in their preferred task.

For intermediate values of $\phi$, we have that as $I$ declines, the probability that workers search for novel jobs, $\Delta \lambda_u^{n \phi-1}$, increases (both terms increase). Which generalizes to this environment the main force exacerbating the skill mismatch when $I(t)$ declines.

Likewise, if skilled workers had the chance to search for old jobs, we would have $\lambda_u^o > \lambda_u^n$ so long as $S_n^u > S_u^n$, which is always the case in equilibrium.

This exercise confirms that, if given the chance to direct their search efforts—albeit imperfectly—a reduction in old jobs $I(t)$ would still exacerbate the mismatch, since these jobs would hire more unskilled than skilled workers, and the unskilled would respond by moving to novel jobs.

1.4.8 Behavior of wages when the number of open vacancies does not adjust immediately.

In this subsection I outline an extension of my model in which the number of open job vacancies does not adjust immediately, but does so gradually.

As mentioned in the main text, the gradual adjustment implies that, along the transition, the opportunity cost of a firm with an empty vacancy may depart from zero temporarily. This introduces an additional force that determines wages, which now depend on the change in both the worker's and the firm's outside options.

Formally, let $V_j^k(t)$ be the value of entering the market by opening a vacancy, $v_j^k$, at time $t$ (to simplify the notation, I omit the task index, since equal tasks have an equal number of vacancies this is not needed). When $V_j^k(t) > 0$, a mass $1$ of potential entrants are able to enter the market at a rate $\phi^\text{in} \in (0, \infty)$. When $V_j^k(t) < 0$, firms that hold an open vacancy are able to exit the market at a rate $\phi^\text{out} \in (0, \infty)$. When $V_j^k(t) = 0$ firms are indifferent between entering or exiting the market, and so the number of open vacancies does not change.

The model in the main text corresponds to the limit case in which $\phi^\text{in} = \phi^\text{out} = \infty$, which implies that the number of vacancies adjusts immediately and $V_j^k(t) = 0$ in equilibrium (if firms are entering the market). The assumption of a gradual adjustment of vacancies may be though as a reduced form to capture irreversible investments (at least in the short run).
made by firms to create jobs.

The value function \( V_k^j \) satisfies the Bellman equation:

\[
\tau V_k^j - \dot{V}_k^j = -\kappa + \beta q(t^n) (\gamma^n \max\{S^n_s, 0\} + (1 - \gamma^n) \max\{S^n_u, 0\}) + \phi^{out} \max\{0, -V_k^j\}.
\]

with \( \tau V_k^j - \dot{V}_k^j = \rho_k^j \) the opportunity cost of the firm of engaging in the match. This equation shows that, when a firm enters the market, it must always pay the flow cost \( \kappa \). I think of this assumption as a reduced form that incorporates the opportunity cost of the resources and capital allocated to a particular job opening. The rate \( \phi^{out} \) determines the speed at which the firm can redeploy these resources to other uses and close the vacancy.

The behavior of the number of open vacancies, \( v_k^j \), satisfies:

\[
\dot{v}_k^j = \phi^{in} \mathbb{1}\{v_k^j > 0\} - v_k^j \phi^{out} \mathbb{1}\{v_k^j < 0\}.
\]

Nash bargaining implies that wages in different jobs are given by:

\[
w_k^j = \beta z_j^k + (1 - \beta)(rU_j - \bar{U}_j) - \beta(rV_k^j - \dot{V}_k^j).
\]

Unlike the case in which firms enter and exit the market immediately, now the wage also reflects the opportunity cost of the firm. Wages increase when \( \tau V_k^j - \dot{V}_k^j \) is low because workers shield firms from having to stay with an open and unprofitable vacancy. Acemoglu (1997) discusses a similar effect in a model in which firms have to decide if they stay with their current match or they search for a new match.

Consider again the limit in which \( \alpha, \lambda \to \infty \). The following proposition shows that, when \( x(0) \) declines, the wage of skilled workers at novel jobs increases temporarily.

**Proposition 12** For any values of \( \phi^{in}, \phi^{out} < \infty \), an unanticipated decline in \( \gamma^n \)—a worst skill mismatch—causes a temporary increase in the wage of skilled workers.

**Proof.** Since \( \phi^{in}, \phi^{out} < \infty \), the number of vacancies for all jobs remain fixed at time 0.

This implies that skilled workers’ job-finding rates and outside options remain fixed too, so that \( w_s(0) \) does not change.

Lemma 1, implies that a decline in \( \gamma^n \) reduces \( \rho_s^j(0) \) and \( V_s^n(0) \). Importantly, the decline in \( x(0) \) also reduces \( \Omega(0) \), which creates a further decline in \( \gamma^n \).

The equation for wages at time 0, implies that:

\[
w_s^n(0) = \beta z(0) + (1 - \beta)w_s(0) - \beta \rho_s^n(0), w_s^n(0) = \beta z(0) + (1 - \beta)w_s(0) - \beta \rho_s^n(0).
\]

Therefore, at time 0, \( w_s^n(0) \) increases and \( w_s^n(0) \) remains unchanged. Thus, the wage of
skilled workers increases at time 0 and for a positive amount of time until vacancies adjust.

The key implication of Proposition 12 is that, through its indirect effect on $\gamma$, a worst skill mismatch causes a temporary increase in the wage of skilled workers. We know from Proposition 10, that when vacancies adjust to keep $V^k_j = 0$ the worst skill mismatch reduces $w_s$ and hence reduces the average wage of skilled workers. Thus, both results combined imply that, while vacancies adjust, we may have a temporary increase in the wage of skilled workers followed by a decline below its initial level.

This behavior of wages may explain the results found in Table ?? in the main text.

1.4.9 Restructuring concentrates in recessions.

In the main text, I assumed that firms restructure their labor demand more during recessions. This assumption is a reduced-form way of capturing the idea that firms restructure during recessions.

In this subsection I discuss an extension of my model that endogenizes this feature.

Suppose that to produce an old task, firms need to purchase one unit of a capital good $m(i)$ produced by a monopolist for each $i \in [0, A)$.

The monopolist produces the good at a marginal cost $0$, but also faces a fixed cost of production $C$. Moreover, the monopolist prices the good at $p^m > 0$, which is exogenously determined by a fringe of competitive firms that could otherwise supply the capital good. For firms that post old jobs, $p^m$ is a part of the recruiting cost $\kappa$, so that $\kappa > p^m$.

The monopolist is removed from the market and replaced by new technologies at a rate $v > 0$, which denotes the secular advancement of technology.

While the monopolist operates in the market, it may restructure its operation or liquidate its firm. Doing so allows the monopolist to loose the least value from its failed investment in the production of old tasks.

Restructuring (or liquidation) costs the monopolists $R$ units of labor. This cost represents resources that are diverted away from production, and which are valued at an opportunity cost of $z(t)$. The assumption that the opportunity cost of restructuring is lower during recessions builds on the work of Hall (1991), and Aghion and Saint Paul (1998).

When the monopolist starts a restructuring process, it succeeds with Poisson probability $\overline{v} - v > 0$, in which case it pays the cost of restructuring and stops providing the capital good to firms producing the old task with labor.

Let $V$ be the value of the monopoly. We have that

$$(r + v)V - \dot{V} = u(1 - \gamma)\theta_u^e \frac{1}{1 + \pi} p^m - C + (\overline{v} - v) \max\{-Rz(t) - V, 0\}. $$
Structural change lowers the value of the monopoly. Because of competition from technology and the fact that workers become skilled and stop searching for old jobs, we have that at some point $u(1 - \gamma)\theta_u \frac{1}{1 + \gamma} p^m < C$ and the monopolist starts making negative profits.

Suppose that $z(t) = 1$ and $\frac{C}{r+\psi} > R$. Thus, there is a time $T$ at which the firm decides to restructure. At this point, we have $V(T) = -R$, and $V(t) > -R$ for $t < T$.

In this case, a decline in productivity may prompt monopolists to restructure before time $T$. In particular, suppose that at time $T' < T$, productivity declines below $\frac{V(T')}{R} \in (0, 1)$, with $V(t)$ the value of the monopolist when the path for productivity is fixed and equal to $z(t) = 1\forall t$. Due to the decline in productivity, the monopolist finds it profitable to liquidate the firm at time $T'$ for two reasons. First, because the opportunity cost of assigning labor to liquidate the firm is lower. Second, because recessions cause a large drop in vacancies, which make the monopoly less profitable. Thus, the value of retaining the monopoly falls.

Now, suppose that $z(t) = 1$ but $\frac{C}{r+\psi} > R$, so that firms would not liquidate along the adjustment and $V(t) \in (-R, 0)$ from some point onwards. In this case, a large productivity shock could also prompt restructuring efforts, which would not have happened otherwise.

Thus, recessions may cause an increase in the rate at which firms stop hiring labor for old jobs because firms front load the liquidation of old jobs to take advantage of the low opportunity cost to do so during recessions.

**Remark:** Instead of the fixed cost $C$, one could have that the monopolist pays a liquidation cost $L$ if it is replaced by technology (this could be equal to $Rz(t)$ for the current value of $z(t)$ that determines productivity when the firm is replaced). Here, the difference in liquidation costs that the monopolist could save by restructuring during the recession plays the same role as $C$ in the previous analysis. Moreover, a large $L$ guarantees that the monopoly profits are negative along the whole transition because of the competition from technology, which is embedded in the liquidation cost.

When $L = Rz(t)$, the firm never restructures if $z(t)$ is constant and equal to 1, but it will do so if $z(t)$ declines temporarily below $\frac{V}{r+\psi}$.
Chapter 2

Do Structural Changes Explain Part of the Current Employment Slump?

Abstract

I document that the decline in routine-cognitive jobs outside manufacturing—a pervasive structural change that has affected U.S. labor markets since the mid 90s—accelerated during the Great Recession and contributed to the long-lasting increase in unemployment since 2007. Exploiting differences in the extent of routine-cognitive jobs across U.S. commuting zones, I show that the local labor markets that were more exposed to this structural change experienced worst outcomes during the Great Recession. Moreover, at the local labor market, this structural change interacted with temporal shocks to the demand for goods and services. I provide evidence that the adjustment to the decline in routine-cognitive jobs created strong negative spillovers at the local labor market level, which explain about 70% of the negative effects created by this structural change since 2007. The spillovers affect all middle-skill workers, including workers with specialized and non-routine skills. These spillovers are consistent with the existence of important human capital externalities that operate at the local labor market level, but not with theories that emphasize the role of demand externalities.

Keywords: Unemployment, Skill Mismatch, Structural Change, Polarization, Great Recession.


What explains the large and long-lasting increase in joblessness experienced in the U.S. during the Great Recession and depicted in Figure 2-1?

Since the mid 90s, cheaper computers have allowed firms to carry out tasks that previously had been performed by clerks, technicians, bookkeepers, salespersons, and other white-collar workers. This technological change led to a major restructuring of the labor market, characterized by a decline in these routine-cognitive jobs that can be easily computerized because they follow precise procedures (see Autor, Dorn and Hanson, 2015). Figure 2-2 shows that, from 1996 to 2015, the share of the workforce employed in these white-collar
jobs declined from 25.5% to 21%—a 4.5 percentage points decline, or 7 million jobs.\(^1\) (The more publicized decline in manufacturing during the same period removed 9 million jobs.)

---

1 The figure uses employment counts by broad occupational category, which are provided by the Bureau of Labor Statistics (BLS). In this figure, I define routine-cognitive jobs as the sum of the employment count in clerical and sales jobs. The Data Appendix shows additional decompositions of these employment patterns.
Figure 2-2 also shows that about two thirds of the decline in routine-cognitive jobs over the last 20 years occurred during the Great Recession (see also the evidence by Jaimovic and Siu, 2014). The decline in routine-cognitive jobs coincided with the expansion of service jobs and a wide range of professional jobs (e.g., audio-visual specialists, executive secretaries, data administrators and analysts, computer support specialists and engineering jobs). Although the employment rate in these jobs expanded during the recovery of the Great Recession, their growth was insufficient to keep overall employment from contracting and unemployment from increasing in a long-lasting manner in recent years.

Motivated by these patterns, in this paper I ask if the secular decline in routine-cognitive jobs explains part of the long-lasting increase in joblessness during the recent Great Recession. My hypothesis is that the Great Recession accelerated the rate at which firms stopped hiring workers to perform routine-cognitive jobs. By removing employment opportunities for middle-skill workers who specialized in routine tasks, the decline in routine-cognitive jobs forced workers to redeploy to professional jobs (or low-wage services), but many of them lacked the analytical skills, training and formal education that are required in these jobs. As I show in a companion paper (see Restrepo, 2016), this skill mismatch may result in a long-lasting increase in joblessness driven by a depressed job creation and a low job-finding rate for workers. Using data for the U.S., I provide evidence consistent with this view.

In Section 2.1 I review the evidence that indicates that the Great Recession accelerated the decline in routine-cognitive jobs. In line with Figure 2-2 and recent papers by Jaimovic and Siu (2014), Cortes, Jaimovic, Nekarda and Siu (2014) and Hershbein and Kahn (2016), I find that during the onset of the Great Recession, there was a large and persistent decline in the number of job openings in routine-cognitive jobs. Since 2007, routine-cognitive job openings declined by about 50% more than openings for other jobs. This gap persisted over time, and even by 2015 had not recovered.

In Section 2.2 I estimate the local labor market effects of the accelerated decline in routine-cognitive jobs during the Great Recession. I leverage the fact that, due to historical reasons, some commuting zones—or local labor markets—were more specialized than others in routine-cognitive jobs, and hence were differentially exposed to this structural change. During the Great Recession, the most exposed commuting zones experienced a large shift in their occupational structure, lower employment rates, higher unemployment rates and lower wages than other labor markets. My estimates imply that, from 2007 to 2013, an additional 10 percentage points of exposure to routine-cognitive jobs—the difference between commuting zones at the 95th and 5th percentiles of exposure—reduced the employment rate by about 2 percentage points. The rise in joblessness is entirely accounted for by the incidence

---

2This approach builds on the work of Autor, Dorn and Hanson (2013, 2015), who explore the aggregate consequences of trade and technology on local labor markets.
of long-term unemployment and a large and long-lasting decline in workers’ job-finding rate.

The timing of these effects suggests that the secular decline in routine-cognitive jobs interacted with the Great Recession. Although the local labor market effects of the decline in routine-cognitive jobs were small or negligible before the Great Recession, during the recession and its recovery the effects became large and significant. The same occurred during the 2001 recession, but not during the 1990 recession, which preceded the decline in routine-cognitive jobs. Moreover, I find that reductions in local economic activity—proxied by a decline in household net worth in a commuting zone (see Mian and Sufi, 2014) or by deleveraging (see Mian, Rao and Sufi, 2013)—interacted with the underlying decline in routine-cognitive jobs. In the commuting zones most exposed to this structural change, the decline in local demand had a large and persistent effect on employment that lasted up to 2013. In the least exposed commuting zones, the decline in local demand had a modest and short-lived impact on employment, which fully vanished by 2013.

A growing literature documents that the impact of shocks that require the reallocation of workers are more visible not in national-level outcomes, but locally, in the most exposed labor markets (see Acemoglu et al. 2014; and Autor, Dorn and Hanson, 2015, Beaudry, Galizia and Portier, 2015). This pattern would emerge if there are important spillovers (e.g., demand externalities, increases in taxation to support disability payments) at the local labor market level. In Section 2.3, I investigate if the large local labor market impact of the decline in routine-cognitive jobs is driven by spillovers that operate at the local labor market level.

To test for the presence of spillovers, I partition workers in the American Community Survey into 200 skill groups (defined by age, education, sex and region of residence), and measure their exposure to routine-cognitive jobs using their year 2000 share of employment in these jobs. My estimates imply that, from 2007 to 2013, an additional 10 percentage points of exposure to routine-cognitive jobs reduced the employment of workers in a skill group by about 0.6 percentage points. In line with these estimates, I consistently find that the effects on exposed skill groups are smaller than the effects obtained at the commuting zone level. This pattern suggests that the decline in routine-cognitive jobs generated negative spillovers at the local labor market level.

To formally disentangle the importance of spillovers and the direct effect on affected workers, I compare the employment rate of workers with similar skills—and hence a comparable reliance on routine-cognitive jobs—but who reside in commuting zones with different levels of overall exposure to routine-cognitive jobs. I find that, for workers in a given skill group and commuting zone, not only their skills and exposure to routine-cognitive jobs explain their labor market outcomes—a direct effect on affected workers—, but also the average exposure

---

3This builds on work by Acemoglu and Autor (2011) and Foote and Ryan (2014), who follow the same procedure to study the effects of technology on workers in different skill groups.
of other workers in their local labor market—a spillover effect. My estimates suggest that the direct impact on affected workers explains 30% of the effect of the decline in routine-cognitive jobs in a local labor market; while the spillovers explain the remaining 70% of the effect. Thus, a national-level comparison of workers with different skills misses the bulk of the effect of this structural change.

Several mechanisms could explain the existence of these spillovers. For instance, they could be driven by local demand externalities (see Beaudry, Galizia and Portier, 2014), they could result from the decreasing marginal value of jobs receiving displaced workers, or they could result from higher taxes needed to support transfer payments—a fiscal externality. However, none of these alternatives explain my findings. The estimated negative spillovers are present among workers who specialize in the tradable sector and are not present on workers who specialize in the non-tradable service jobs, which rules out an explanation based on local demand or fiscal externalities. In addition, the estimated negative external effects are robust when I control for changes in employment by occupation, which deals with potential changes in task prices that stem from decreasing returns to scale.

Instead, as I argue in the conclusions, these spillovers are consistent with models in which, due to human capital externalities, professional jobs become less profitable when the share of workers with routine and no specialized skills that need to be redeployed to these jobs raises. Because of these human capital externalities, it is hard for commuting zones in which many workers are used to routine jobs to develop new jobs and occupations that rely on specialized and professional skills. For instance, in models with matching frictions and random matching, having more routine-cognitive workers displaced from their jobs and searching for new jobs may end up reducing job creation in professional occupations that require specialized skills, and that were supposed to pick-up the slack in the labor market (see Acemoglu, 1996; and Restrepo, 2016). In line with this explanation, I find that the spillovers only affect the labor market outcomes of workers who specialize in routine jobs or workers with specialized skills in professional occupations. In contrast to middle-skill workers, the local exposure to the decline in routine-cognitive jobs does not affect workers in skill groups that specialize in services—which require no specialized skills—, nor workers in skill groups that specialize in managerial occupations—which did not receive an inflow of unskilled workers.

**Related literature:** My paper contributes to the literature on job polarization (Autor, Levy and Murnane, 2003; Goos and Manning, 2007). In a recent paper, Autor, Dorn and Hanson (2015) explore the aggregate consequences of polarization on employment from 1990 to 2007, when the decline in routine-cognitive jobs did not have a major impact. Closest to my paper is a study by Jaimovic and Siu (2014), who argue that employment polarization interacted with the last three recessions and generated jobless recoveries. Their findings and
mine are complementary, but our studies differ in several respects. Jaimovic and Siu present evidence that is based on employment counts at the national level, and they focus on the decline of all middle-skill jobs, which includes manufacturing jobs.\footnote{Their findings were criticized by Foote and Ryan (2014) on the grounds that their time series patterns could be explained by differences in cyclicality among manufacturing industries. By focusing on the reduction of routine-cognitive jobs outside manufacturing and controlling for differences in industry cyclicality, my empirical approach overcomes this criticism.}

My paper also contributes to the literature that examines how sectoral or occupational shocks, as opposed to aggregate shocks, drive unemployment fluctuations. This literature goes back to Lillien (1982) and re-emerged with the debate over whether unemployment during the Great Recession reflected a sectoral or occupational mismatch between available jobs and unemployed workers (see Kocherlakota, 2010). Using U.S. data, Chodorow-Reich and Wieland (2015) construct a measure of sectoral reallocation at the local labor market and show that this reallocation contributes to worst employment outcomes, especially during recessions (see also Garin, Pries and Sims, 2013; and Mehrota and Sergeyev, 2013). Using a decomposition based on aggregate data, Sahin et al. (2014) find a smaller role for industry mismatch or sectoral shocks in explaining unemployment during the Great Recession, though they find some role for occupational mismatch. However, the role of occupational or skill mismatches remains a matter of debate (see Lazear and Spletzer, 2012; and Wiczer, 2013). I contribute to this literature by showing that, during the Great Recession, the large reduction in routine-cognitive jobs—a source of occupational or skill mismatch—contributed to the large and prolonged increase in joblessness.\footnote{Kroft et al. (2014) and Barnichon and Figura (2015) emphasize the role of pure duration dependence as opposed to worker ex-ante heterogeneity or amplification effects of the sort that I propose. However, these approaches assume there is no unobserved heterogeneity or externalities that affect job creation.}

Finally, A literature going back to Schumpeter (1942) argues that, due to the low opportunity cost of adjustment during recessions, firms use crisis to replace old jobs with new technologies or restructure and close job positions that will soon become obsolete due to advances in technology (see also Davis and Haltinwanger, 1990; Hall, 1991; Caballero and Hammour, 1994; Aghion and Saint Paul, 1998; Koenders and Rogerson, 2005; and Berger, 2014). I contribute to this literature by showing that, in the case of routine-cognitive jobs, the data supports the view that firms adjust their labor requirements during recessions, which caused a permanent shift in the demand for routine-cognitive labor during the Great Recession (see also Hershbein and Kahn, 2016).
2.1 Did the recession accelerate the decline in routine-cognitive jobs?

In this section I document that the Great Recession accelerated the decline in routine-cognitive jobs. In particular, I provide evidence that the decline occurred through a permanent reduction in job openings for these jobs. I use several data sources which I describe in detail in the appendix.

Figure 2-2 shows that, in line with this view, about two thirds of the decline in routine-cognitive jobs over the last 20 years occurred during the Great Recession. This coincides with the evidence presented by Jaimovic and Siu (2014), who argue that, during the last 30 years, most of the employment losses in middle-skill occupations (including routine-cognitive jobs) occurred during economic downturns.

Did the decline occur through a reduction in job openings for routine-cognitive jobs or an increase in the rate at which firms fire workers? Several pieces of evidence suggest a key role for the former margin; recent recessions created a permanent reduction in job openings for routine-cognitive jobs.

Figure 2-3: Percent change in quarterly vacancies posted within broad occupational groups (relative to the first quarter of 2007). The light-blue bars plot 90% confidence intervals for the difference between both series in each quarter. Data for 22 occupations from Help Wanted Online, by the Conference Board.

Figure 2-3 uses data from Help Wanted Online (HWOL) to show that, during the Great Recession, job openings for routine-cognitive jobs suffered a permanent decline of about 55% relative to other jobs. This decline probably understates the real reduction in openings.

---

6 The figure uses employment counts by broad occupational category, which are provided by the Conference Board; the repository of the Help Wanted Online Data. The figure is constructed from a regression in which I explain the change from the first quarter of 2007 in the log of job posting as a function of the share of
for jobs that require routine skills; a recent report by Burning Glass Technologies (2014) shows that, even within the remaining job openings for routine jobs, firms recently started to demand higher qualifications and workers are expected to perform different tasks than before. In a related paper that uses more detailed job openings data, Hershbein and Kahn (2016) find that firms responded to the recent crisis by demanding more skilled workers, especially in routine-cognitive occupations. This increase was driven by an observable shift away from routine tasks and toward cognitive skills, the set of tasks that workers were supposed to perform. According to their estimates, the Great Recession could have prompted a 12 percentage points increase in the share of job openings that require skill credentials (from an initial mean of 53%). This shift in the demand by firms became permanent, did not reverse during the recovery, and seems unrelated to changes in the composition of the labor supply.

Likewise, the Current Population Survey (CPS) data used by Cortes et al. (2014) also suggests that the decline in middle-wage routine jobs was driven by a secular decline in the rate at which workers find such jobs, which accelerated and became particularly relevant during the Great Recession. Since 2007, even prime-age workers saw a steeper decline in the rate at which they could find routine-cognitive jobs than in the rate at which they could find other jobs.

Data from the American Community Survey (ACS) also supports the role of a lower finding rate for routine-cognitive jobs. Figure 2-4 shows that, since 2007, and compared to an average worker, the probability that a worker who was last employed in a routine-cognitive job was long-term unemployed increased by 17% in 2010 and 8% in 2012. However, during the same period and compared to an average worker, the probability that a worker who was last employed in a routine-cognitive job was unemployed for less than a year declined by about 10%. This pattern suggests that, workers who specialize in routine-cognitive jobs were less likely than others to find a new job—and hence the relative increase in their long-term unemployment rate—, but not more likely to be fired from their current jobs—and hence the decline in their short-term unemployment rate.

A decline in the rate at which middle-skill workers find routine-cognitive jobs should prompt their redeployment to other jobs. Using data from the Current Population Survey, Annual Social and Economic Supplement, Figure 2-5 plots the share of currently employed prime-age workers who, starting from a routine-cognitive job at year $t - 1$, switched occupations at year $t$. This exercise uses a consistent aggregation of jobs into the 330 occupational

routine jobs in each occupational category. Using the estimates from this regression, I plotted the predicted job openings for an occupational category with no routine jobs compared to one in which all jobs are routine.

The definition of routine-cognitive jobs is introduced in Section 2.2. The figure plots yearly estimates of the likelihood of unemployment (both short and long term) as a function of whether a worker was last employed in a routine-cognitive job or not. All the estimates are relative to the year 2007, which is the base year. These results also control for workers' socio-demographic characteristics.
categories that were proposed by Autor and Dorn (2013). Both the raw numbers and those adjusted by changes in demographic characteristics reveal that, although the rate at which workers switch occupations has been on a long secular decline, it experienced a significant increase during the recovery from the Great Recession. In order to remain employed, workers who reported to be employed in routine-cognitive jobs had to switch occupations. The slow labor market recovery in recent years was characterized by a large number of previous routine-cognitive workers attempting to switch occupations, rather than returning to their previous jobs.

The evidence in this section supports the view that, because in a recession firms face a low opportunity cost of restructuring, they tend to adjust their labor requirements during downturns. Thus, a structural change that lowers the surplus that routine-cognitive jobs generate will manifest during recessions, when firms adjustments induce a large and permanent decline in the creation of routine-cognitive jobs and a permanent shift in the composition of available jobs. Through this channel, the Great Recession could have accelerated the decline in routine-cognitive jobs and permanently affected the finding rate of middle-skill workers (despite their efforts to reallocate).8

8Notwithstanding this evidence, there is an alternative view that recessions are sullying and do not accelerate reallocation and structural change. For instance, Barlevy (2002) shows that in a model with search frictions and on-the-job search, the value of switching jobs declines with recessions, and this may slow down reallocation. Carrillo-Tudela and Visschers (2014) provide evidence showing that, during the initial years of a recession, there is a decline in the gross rate at which workers change occupations. However, their data does not cover subsequent recovery years, or takes into account the possibility that this may be driven by a decline in the rate at which workers obtain routine-cognitive jobs. Relatedly, Mueller (2015) shows that, contrary to what one would expect if recessions had a cleansing effect, recessions tend to increase the unemployment rate of high-wage workers more than that of low-wage workers.
2.2 Impacts at the commuting zone level

The previous section established that the Great Recession accelerated the decline in routine-cognitive jobs. If this structural change creates unemployment, one would expect that local labor markets that were more exposed to routine-cognitive jobs had worst labor market outcomes during the Great Recession. In this section I test this hypothesis.

To define routine-cognitive jobs, I use the set of 330 occupations proposed by Autor and Dorn (2013), which are consistently defined over time and include all non-military jobs in the U.S. Census. I follow the literature on employment polarization and label as routine the jobs in the top tercile of occupations with the highest routine content. In turn, the routine content of an occupation is measured from the Dictionary of Occupational Titles (1977) and described in detail in Autor, Dorn and Hanson (2014). I refer to routine jobs outside of manufacturing as routine-cognitive jobs. These jobs comprise precise and repetitive tasks, which are typical of white-collar jobs performed by middle-skilled workers. The occupations with the highest routine content outside of manufacturing are telephone operators, payroll, postal and time-keeping clerks, and bank tellers. However, routine jobs also include manual jobs in the manufacturing sector. Because routine-manual jobs have been on a secular decline, and the manufacturing sector is more sensitive to the business cycle (see Abraham and Katz, 1986, and Foote and Ryan, 2014), I focus only on the more recent decline in routine-cognitive jobs—the routine jobs outside of manufacturing.

I define local labor markets using 722 commuting zones which cover the entire continental
U.S. territory (excluding Alaska and Hawaii). For each commuting zone, I use the 2000 Census to compute the share of workers outside of manufacturing who were employed in routine jobs. For each commuting zone, this procedure yields a measure of exposure to routine-cognitive jobs $R_{000}^e$, which I depict in Figure 2-6. There is large variation in this measure of exposure. The difference between the 95th percentile and the 5th percentile of exposure to routine-cognitive jobs is of about 10 percentage points. The variation in $R_{000}^e$ stems from historical patterns of specialization as I show in detail below. Because commuting zones that have high values of $R_{000}^e$ have more workers specialized in routine-cognitive jobs, they are the most exposed to the decline in routine-cognitive jobs and experience the largest change in productive structure. As Autor and Dorn (2013) have shown, in recent decades these commuting zones saw a fast adoption of computers and information technologies, as cheaper computers allowed firms to replace labor in many of the repetitive tasks that comprise routine-cognitive jobs.

![Share of employment in routine-cognitive jobs from the 2000 Census](image)

Figure 2-6: Geographical distribution of the exposure of U.S. commuting zones to the decline in routine-cognitive jobs, $R_{000}^e$.

To test whether during the Great Recession exposed commuting zones diverged in terms of labor market outcomes, I estimate the model

$$\Delta Y_{ct} = \delta d(c) + \beta_t R_{000}^e + \theta_t X_t + \epsilon_{ct},$$

(2.1)
for different years, \( t \). Here, \( \Delta Y_{ct} \) is the change in a commuting zone \( c \) between 2007—a year before the Great Recession—and year \( t \) in the labor market outcome \( Y_{ct} \). The vector \( \delta_{d(c)t} \) contains a full set of Census × year division dummies, whose effects vary by year \( (d(c) \) denotes the division that contains commuting zone \( c \)). The vector \( X_c \) includes additional covariates, which I describe below. When estimating this equation, I allow the error term \( \varepsilon_{ct} \) to be correlated within States and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight commuting zones by their year 2000 working-age population.

Provided that \( \mathbb{E}[R_{c2000}\varepsilon_{ct}] = 0 \), the coefficient \( \beta_t \) captures the differential change in employment in commuting zones that were more exposed to the decline in routine-cognitive jobs. One might be worried that mean reversion or measurement error in my measure of exposure to routine-cognitive jobs could lead to violations of the assumption \( \mathbb{E}[R_{c2000}\varepsilon_{ct}] = 0 \). To deal with these concerns, I also use the 1990 Census to compute an exposure measure \( R_{c1990} \), which I use as an instrument for \( R_{c2000} \) in some exercises. In this case, the identifying assumption becomes \( \mathbb{E}[R_{c1990}\varepsilon_{ct}] = 0 \). I discuss these identifying assumptions in detail below.

Table 2.1 presents my estimates of equation 2.1 using the non-manufacturing employment rate in each commuting zone as the outcome of interest. This measure comes from the County Business Patterns (CBP), and I aggregate it at the commuting zone level. I focus on the non-manufacturing sector because this is where the effects of the decline in routine-cognitive jobs should be present. Moreover, this choice allows me to abstract from the secular decline in manufacturing and the greater volatility of manufacturing employment during crisis. All the same, the results using the overall CBP employment rate as outcome are very similar.

In Columns 1 to 3 I focus on the change in the employment rate from 2007 to 2009. These models explore if, during the onset of the Great Recession, commuting zones that were more exposed to routine-cognitive jobs experienced a greater employment decline than other areas. The OLS estimate in Column 1 of the top panel suggests that, for every additional 10 percentage points of exposure to routine-cognitive jobs—the difference between the most and the least exposed areas—, the employment rate in a commuting zone declined an additional 2.58 percentage points (standard error=0.29). Thus, my estimates suggest that the most exposed commuting zones experienced an additional 2.6 decline in their employment rate when compared to the least exposed zones. A back of the envelope calculation shows that, if all commuting zones with above-average exposure had experienced the same employment rate as the average, the 2007 to 2009 employment decline in the U.S. would have been a third of what it was.

Columns 2 to 3 include additional controls. Column 2 presents estimates that control for the year 2000 share of employment in manufacturing, durable, tradable and construction industries, which take into account differences in industry cyclicality and other sectoral shocks.
Table 2.1: Changes in the non-manufacturing employment rate during the Great Recession among commuting zones with different exposure to the decline in routine-cognitive jobs.

<table>
<thead>
<tr>
<th>Dep. var: non-manufacturing employment rate</th>
<th>From 2007 to 2009</th>
<th>From 2007 to 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-0.258***</td>
<td>-0.246***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.49</td>
<td>0.51</td>
</tr>
<tr>
<td>Observations</td>
<td>722</td>
<td>722</td>
</tr>
</tbody>
</table>

**I. OLS estimates**

| CZ exposure to routine-cognitive jobs        | -0.270***        | -0.261***        |
|                                             | (0.034)          | (0.036)          |
| R-squared                                   | 0.49             | 0.51             |
| Observations                                | 722              | 722              |
| First-stage F statistic                      | 1295.3           | 725.1            |

**II. 2SLS estimates**

| CZ exposure to routine-cognitive jobs        | -0.270***        | -0.261***        |
|                                             | (0.034)          | (0.036)          |
| R-squared                                   | 0.49             | 0.51             |
| Observations                                | 722              | 722              |
| First-stage F statistic                      | 1295.3           | 725.1            |

**Notes:** Panel I presents OLS estimates of the differential change in the non-manufacturing employment rate from 2007 onward among commuting zones more exposed to structural change. Panel II presents 2SLS estimates of the differential change in the non-manufacturing employment rate from 2007 onward among commuting zones more exposed to structural change. In these models, I instrument the commuting zone exposure to routine-cognitive jobs, $R_{200}$, using its measurement from the 1990 Census, $R_{1990}$. The bottom rows indicate the set of covariates included in each model. In all models, I allow the error term to be correlated within States and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight each commuting zone by its size in 2000.

Column 3 present estimates that control for observable characteristics of the workforce, as measured in the 2000 Census, and which determine changes in the labor supply. These characteristics include the population density, the share of the population of different races, the share of the population that is older than 65 years, the share of foreign workers, the share of workers who have different levels of schooling, the female labor force participation and the share of workers who earn the minimum wage. The estimates in Columns 2 and 3 remain in the ballpark of those in Column 1, which suggests that my measure of exposure to routine-cognitive jobs is not capturing differences in industry or labor-force composition across local labor markets.

In Columns 4 to 6 I focus on the change in the employment rate from 2007 to 2013. These models explore if, during the recovery from the Great Recession, commuting zones that were exposed to routine-cognitive jobs continued to diverge from other areas in terms of employment, or if instead they quickly converged back to the national employment level. The OLS estimate in Column 4 of the top panel suggests that, from 2007 to 2013, for every
additional 10 percentage points of exposure to routine-cognitive jobs, the employment rate in a commuting zone declined by an additional 2.15 percentage points (standard error=0.44). This estimate implies that, in exposed labor markets, the initial effects of the decline in routine-cognitive jobs persisted and remained of roughly the same size during both the Great Recession and its subsequent recovery. As before, Columns 5 and 6 present estimates that control for the industry and labor-force composition of exposed commuting zones. If anything, when I include these controls the estimates become slightly more negative.

Panel II of Table 2.1 presents my 2SLS estimates. As mentioned above, I use the measured exposure in 1990 to instrument the measured exposure in 2000 ($R_{1990}^{2000}$ as an instrument for $R_{e}^{2000}$). All estimates are in the ballpark of their corresponding OLS values but are slightly more negative, which may be explained by a small amount of measurement error in my measure of exposure in 2000. Importantly, the reported $F$ statistics for the first-stage relation, which are all above 500, show that there is a strong persistence in the exposure of different local labor markets to routine-cognitive jobs. These patterns suggest that, the variation in exposure captures the historical and persistent specialization of some commuting zones in routine-cognitive jobs. Thus, any violation of the identifying assumptions $E[R_{e}^{2000} \varepsilon_{ct}] = 0$ or $E[R_{e}^{1990} \varepsilon_{ct}] = 0$ must stem from unobserved and persistent differences across commuting zones that rendered exposed areas historically more responsive to business cycles.

To explore in more detail the timing of these effects, and the historical responsiveness of exposed commuting zones to business cycles, I estimate equation 2.1 for every year from 1988 to 2013. In this exercise, I focus on the 2SLS specification introduced above, and use the change in the non-manufacturing employment rate relative to the year 2000 as the dependent variable. Figure 2.2 plots the over-time effects of exposure to routine-cognitive jobs. To ease the interpretation, the figure plots the average non-manufacturing employment rate in gray, and the predicted employment rate in a commuting zone at the 95th percentile of exposure in blue (which is obtained from my estimates for $\beta_{t}$). Because my estimates are relative to the year 2000, both series coincide at zero in this year. The 1990, 2001 and 2007 recessions and their respective recovery years are shaded in gray. Likewise, Figure 2-8 separately examines the behavior of employment during these three recessions relative to the year that preceded the start of each recession.

The figures reveal a clear pattern: the effects of the decline in routine-cognitive jobs concentrate during the 2001 and 2007 recessions. Exposed labor markets did not experience divergent employment paths before 2000 or during the booming years of 2004 to 2007. For each additional 5 percentage points of exposure—the difference between the mean and the 95th percentile of exposure—, employment from 2000 to 2004 declined by 2 percentage points (standard error=0.56). As described above, from 2007 to 2010, the corresponding decline in employment was 1.3 percentage points (standard error=0.15). From 2004 to 2007—years
Employment rate from County Business Patterns

Figure 2-7: Estimated change in employment for commuting zones at the mean (in gray) and 90th percentile of exposure to structural change (in blue) relative to the year 2000. The light-blue bars plot 90% confidence intervals for the estimates in the highly exposed areas. Data from the County Business Patterns.

that had a strong economy and that fell between recessions—the corresponding decline in employment in exposed zones was not statistically significant and only of 0.25 percentage points (standard error=0.2).

For the 1990 recession I do not find any divergent path for employment in the most exposed commuting zones. This serves as an useful placebo test because this recession preceded the decline in routine-cognitive jobs that started in the late 90s. The pattern that surrounds the 1990 recession confirms that highly exposed commuting zones only became more responsive to the business cycle when the decline in routine-cognitive jobs started. As required by the identifying assumption $E[R_{1990}^{e} \varepsilon_{ret}] = 0$, exposed commuting zones were not historically more responsive to business cycles. The fact that exposed commuting zones were not on a divergent employment trend before the 2001 recession or immediately before the Great Recession also supports the assumption that these areas do not systematically differ in important unobserved characteristics.

Although the timing of the effects provides evidence of an interaction between the secular decline in routine-cognitive jobs and recessions, I also exploit cross-sectional variation in the incidence of the Great Recession to test directly for an interaction between temporary demand shocks and structural changes. I explore whether the decline in routine-cognitive jobs had a larger effect in commuting zones that, because of factors orthogonal to this structural change, suffered during the Great Recession from a greater reduction in demand.
Employment rate during the 2007 recession

Employment rate during the 2001 recession

Employment rate during the 1990 recession

Figure 2-8: Estimated change in employment for commuting zones at the mean (in gray) and 90th percentile (in blue) of exposure to structural change. The light-blue bars plot 95% confidence intervals for a test of both series being different. Data from the County Business Patterns.

and economic activity. To do this I estimate the model

$$
\Delta Y_{ct} = \delta_{d(c)} + \beta_t R_c^{2000} + \theta_t DSC + \alpha_t R_c^{2000} \times DSC + \theta_t X_t + \varepsilon_{ct}. \tag{2.2}
$$
Here, $DS_c$ is a local demand shock that caused a decline in local economic activity at commuting zone $c$ during the Great Recession. I explore two different proxies for $DS_c$: the decline in household net worth during the recession (see Mian and Sufi, 2014), and the increase in household leverage from 2000 to 2006, which created a subsequent deleveraging during the crisis (see Mian, Rao and Sufi, 2013). Both measures capture demand-driven declines in local economic activity that are temporary and orthogonal to the decline in routine-cognitive jobs.\footnote{These measures are available for a subset of counties that I aggregate to the commuting-zone level. Overall, I find that both $DS_c$ and $R_{c2000}^{2000}$ are weakly correlated across commuting zones. Although not reported to save space, I also include quadratic terms for $DS_c$ and $R_{c2000}^{2000}$, which guarantee that I am not capturing non linearities.}

In equation (2.2), $\beta_t$ captures the effect of the decline in routine-cognitive jobs; $\theta_t$ captures the direct effect of a temporary decline in demand on economic activity (either through household balance sheet effects or deleveraging); and the coefficient of the interaction term $\alpha_t$ captures the interaction in a local labor market between the demand-driven decline in economic activity and the underlying decline in routine-cognitive jobs.

Table 2.2 reports my estimates of equation 2.2.\footnote{In this table and what follows I focus on OLS estimates in which I treat $R_{c2000}^{2000}$ as exogenous. I do not report 2SLS results using $R_{c1990}^{1990}$ as an instrument to save space, but in all cases these are very similar to their OLS counterparts.} The top panel presents results using the decline in net worth as the proxy for the decline in local economic activity. This measure is available for 365 commuting zones, which comprise my sample. Column 1 shows that during the onset of the Great Recession (from 2007 to 2009), for every additional reduction of 10 percentage points in household net worth non-manufacturing employment decreased by 0.45 percentage points (standard error=0.18).\footnote{My estimates are smaller than Mian and Sufi's estimate of 1.9 percentage points, which they obtained at the county level. Presumably, the differences result from my level of aggregation and the fact that I focus on all non-manufacturing employment while they concentrate on employment in service jobs.} Although my sample includes about half of all 722 commuting zones, my estimates of the effect of exposure to the decline in routine-cognitive jobs are remarkably similar to those in Table 2.1.

Column 2 present estimates of the full model in equation 2.2. The estimates for $\alpha_t$ indicate that local demand shocks amplified the effects of the decline in routine-cognitive jobs during the onset of the Great Recession (from 2007 to 2009). Columns 3 and 4 present results for the recovery period (from 2007 to 2013). Column 4 shows that the interaction with the underlying decline in routine-cognitive jobs propagated demand shocks over time, so that their effects lasted until the recovery. The interactions are quantitatively relevant. In the average commuting zone, a 10 percentage point loss in net worth caused a .45 percentage point reduction in employment. By 2013, the effect had already vanished, suggesting that on average the effects of temporary demand shocks are short lived. In contrast, in a commuting zone that is at the 95th percentile of exposure, a 10 percentage point loss in net worth caused
Table 2.2: Changes in the employment rate during the Great Recession among commuting zones with different exposure to the decline in routine-cognitive jobs and experiencing different local demand shocks

<table>
<thead>
<tr>
<th></th>
<th>DEP. VAR: NON-MANUFACTURING EMPLOYMENT RATE</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From 2007 to 2009</td>
<td>From 2007 to 2013</td>
<td>From 2007 to 2009</td>
<td>From 2007 to 2013</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td><strong>I. Decline in households’ net worth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-0.209***</td>
<td>-0.210***</td>
<td>-0.245***</td>
<td>-0.211**</td>
</tr>
<tr>
<td></td>
<td>(0.069)</td>
<td>(0.075)</td>
<td>(0.074)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>Demand shock</td>
<td>-0.045**</td>
<td>-0.073***</td>
<td>-0.028</td>
<td>-0.051*</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.020)</td>
<td>(0.026)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs x Demand shock</td>
<td>-0.805***</td>
<td>-1.344***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.272)</td>
<td>(0.471)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.63</td>
<td>0.65</td>
<td>0.57</td>
<td>0.59</td>
</tr>
<tr>
<td>Observations</td>
<td>365</td>
<td>365</td>
<td>365</td>
<td>365</td>
</tr>
<tr>
<td><strong>II. Pre-crisis increase in leverage</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-0.206***</td>
<td>-0.230***</td>
<td>-0.232***</td>
<td>-0.248***</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.069)</td>
<td>(0.061)</td>
<td>(0.079)</td>
</tr>
<tr>
<td>Demand shock</td>
<td>-0.014***</td>
<td>-0.023***</td>
<td>-0.011</td>
<td>-0.025***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs x Demand shock</td>
<td>-0.319***</td>
<td>-0.478***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.133)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.61</td>
<td>0.63</td>
<td>0.49</td>
<td>0.51</td>
</tr>
<tr>
<td>Observations</td>
<td>606</td>
<td>606</td>
<td>606</td>
<td>606</td>
</tr>
</tbody>
</table>

Notes: Panel I presents OLS estimates of the differential change in the non-manufacturing employment rate from 2007 onward among commuting zones more exposed to structural change, and its interaction with the decline in households’ net worth. Panel II presents OLS estimates of the differential change in the non-manufacturing employment rate from 2007 onward among commuting zones more exposed to structural change, and its interaction with the increase in leverage from 2002 to 2006. All models include a full set of covariates, which includes census division dummies, and the industry and labor supply composition. In all models, I allow the error term to be correlated within States and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight each commuting zone by its size in 2000.

a .85 percentage point reduction in employment. By 2013, the employment rate was still down by .95 percentage points. In contrast to the average local labor market, in markets exposed to the secular decline in routine-cognitive jobs the effects of demand shocks are large and long lived, presumably because demand shocks accelerate this structural change and worsen its consequences.

The bottom panel presents my estimates when I use the increase in leverage from 2000 to 2006 as a proxy for the subsequent decline in economic activity during the Great Recession. In this case, I have a bigger sample of 606 commuting zones, but I obtain similar results.

This results are consistent with the evidence presented in Section 2.1, which suggests that firms respond to large declines in demand—even if temporary—by restructuring their labor requirements and production processes. By doing so, the recession exacerbated the decline in routine-cognitive jobs, especially in markets that experienced the greatest declines in demand. These results also suggest that, on the other hand, the economic boom during
the years that preceded the Great Recession could have masked the effects of the decline in routine-cognitive jobs.

2.2.1 How do local labor markets adjust?

This subsection explores how local labor markets adjust to the decline in routine-cognitive jobs. I use data from the American Community Survey (ACS), which can be consistently aggregated at the commuting zone level and provides several labor market outcomes that cover the 2005-2013 period.

I start by estimating the reallocation patterns in exposed commuting zones. To do so, I estimate equation 2.1 using as dependent variable the change between 2007—before the start of the Great Recession—and year $t$ in the share of workers who report some given occupational categories. Workers who report an occupation include both those who are currently employed and unemployed workers who were last employed in each occupation. The occupational categories include service, routine, professional, and managerial jobs. Service, professional and managerial jobs have the lowest routine content, and they are not directly affected by the computerization of routine tasks.

Figure 2-9 shows that, during the Great Recession and its recovery, exposed commuting zones experienced an accelerated redeployment of workers away from routine jobs and into service and professional jobs. The figure plots year-by-year estimates of equation (2.1) that are scaled so that they reflect the difference in reallocation between commuting zones at the 95th percentile of exposure to structural change relative to commuting zones at the 5th percentile. From 2007 to 2013, among commuting zones that were exposed the most to
routine-cognitive jobs, employment in routine jobs fell by 1.8 percentage points relative to the least exposed commuting zones. The decline was matched by a redeployment to professional jobs (1 percentage points increase) and service jobs (1 percentage points increase). However, displaced workers did not go as far as to become managers, presumably because they lacked the qualifications required in these occupations. The reallocation of workers concentrated during the Great Recession and its recovery; it did not occur before, from 2005 to 2007, as is shown by the estimates for these years.

These findings do not result from a mechanical decline in the share of workers currently employed in routine jobs or from mean reversion. If workers did not reallocate, they would still report they were last employed in routine-cognitive jobs, and the patterns in the figures would not emerge. Contrary to what I find in the data, mean reversion would show up strongly before the Great Recession. Moreover, I obtain similar results when I use the historical exposure $R_c^{1990}$ as an instrument for the contemporary exposure, even though mean reversion should not play any significant role in this specification which only exploits persistent differences in exposure.

Although Figure 2-9 suggests that workers attempted to reallocate in response to fewer available routine-cognitive jobs, as documented above, this structural change still had aggregate effects and reduced overall employment. I now explore if other local labor market outcomes are also affected by estimating equation 2.1 using the change in unemployment, long-term unemployment and wages as dependent variables. Figure 2-10 is constructed in an analogous way as Figure 2-9 and presents these additional results.

My estimates show that, following the accelerated decline in routine-cognitive jobs during the onset of the Great Recession, exposed commuting zones experienced a wide range of worsening labor market outcomes. From 2007 to 2013, among commuting zones that were exposed the most to routine-cognitive jobs, the employment rate—measured using the ACS this time—fell by 1.5 percentage points relative to the least exposed commuting zones. The decline was matched by a similar increase in unemployment. Interestingly, the increase in unemployment and reduction in employment coincides with a sharp decline in wages of nearly 5%. This large decline in wages was not enough to ease the reallocation of workers to non-routine jobs.

\[ \text{The ACS reports the occupation held by non-employed workers in their last job, provided that they had a job in the last 5 years. Thus, it is unlikely that these facts are explained by differences in attrition.} \]

\[ \text{Moreover, for the period from 1990 to 2000, I estimate an annual convergence rate of 0.0429 percentage points per year (standard error=0.0275) in the share of workers employed in routine-cognitive jobs among areas with a 10 percentage points additional share in 1980. For the period from 1980 to 1990, I estimate an annual convergence rate of 0.01 percentage points per year (standard error=0.003) in the share of workers employed in routine-cognitive jobs among areas with a 10 percentage points additional share in 1980. Both estimates imply a level of mean reversion that could explain at most a tenth of the documented decline in routine-cognitive jobs from 2007 to 2013 in highly exposed commuting zones.} \]
The increase in unemployment is entirely accounted for by the incidence of long-term unemployment, which I compute as the share of workers who currently are unemployed and report that they did not have a job during the last year. This observation suggests that, in exposed commuting zones the decline in employment and the rise of unemployment is driven by a persistent decline in the job-finding rate.

To test this possibility formally, I use quarterly job-finding rates data from the Longitudinal Employer-Household Dynamics, which can be consistently aggregated at the commuting zone level. The resulting data covers 698 commuting zones for the period from 2005 to 2015. Figure 2-11 plots the over-time estimates of equation 2.1 using the log of the quarterly job-finding rate for unemployed workers as the outcome. To ease the interpretation, the figure plots the average job-finding rate in gray and the predicted job-finding rate in a commuting zone at the 95th percentile of exposure in blue. Because my estimates are relative to the first quarter of the year 2007, both series coincide at zero at this time. On average, all workers experienced a sharp decline in their job-finding rate, which by 2010 was 60 log points below its pre-recession level. However, commuting zones at the 95th percentile of exposure suffered a persistent 15 log points additional decline in their job-finding rate from 2007 to 2014. The lower job-finding rate fully explains the 1.5 percentage points decline in the employment rate of workers in exposed commuting zones.

In the Data Appendix, I present additional exercises using these data and show that, during the Great Recession, highly exposed commuting zones not only experienced a decline in their job-finding rates but also a slight decline in the separation rate of employed workers, especially during the recovery years. Both effects resulted in a significant reduction in worker
Figure 2-11: Percent change in unemployed workers' job-finding rate (relative to the first quarter of 2007). The light-blue bars plot 90% confidence intervals for the difference between both series in each quarter. Data from the Longitudinal Employer-Household Dynamics.

turnover in exposed markets. Contrary to models that emphasize how recurrent job losses or an increase in separations affect displaced workers (see Lillien, 1982; and Jarosch, 2014), my evidence shows that, during periods of structural change, unemployment coincides with lower turnover, and both are driven by a decline in workers' job-finding rates.

Overall, my findings are consistent with the view that, because workers face high reallocation costs or lack the requisite skills for other jobs, the workers affected by the decline in routine-cognitive jobs face lower job-finding rates when they attempt to redeploy. For instance, the lack of skills prevents the swift redeployment of routine-cognitive workers to professional jobs. Several pieces of evidence support this possibility. Data from O*NET shows that professional jobs are among the most intensive in analytical tasks, while workers in these jobs perform few routine tasks. Workers who specialized in routine-cognitive jobs may lack these analytical skills. Data from O*NET also suggests that professional jobs have stringent training requirements: they require on average 2.5 years of training and experience, unlike routine-cognitive jobs, which require only 1.5 years. In line with these figures, in additional results not reported here I find that, in exposed commuting zones, the adjustment to the decline in routine-cognitive jobs required the reallocation of workers to the top tercile of jobs with the most stringent training requirements. Professional jobs also differ in other dimensions. In 2000, 90% of the workers in professional jobs had some college education, but only 64% of the workers in routine-cognitive jobs did. This, too, points to a mismatch in educational requirements. Finally, data examined by Lin (2013) show that, according to the 2000 Census, 12% of professional jobs corresponded to new job titles. The novelty of these jobs suggests that, particularly among those specialized in routine-cognitive
jobs, many of the requisite skills were not commonly held by workers.

2.3 Spillovers at the Local Labor Market Level

In this section I investigate which workers were affected by the decline in routine-cognitive jobs and test for the existence of spillovers that operate at the local labor market.

To do so, I partition the working-age population in the American Community Survey and the Census into skill groups that are defined by their demographic characteristics. This yields 200 groups that are defined by sex (male, female), age (16-24, 25-34, 35-44, 45-54 and 55-64 years), education (less than high school, high school, some college, completed college and more than college), and region of residence (Midwest, North, South and West). For each skill group \( g \), I use the 2000 Census to compute the share of workers outside manufacturing who were employed in routine-cognitive jobs. This procedure yields a measure of the exposure of each skill group to routine-cognitive jobs, \( R_{2000}^g \). Underlying my approach are two assumptions: that employment shares in 2000 reflect each skill group’s inherent abilities for different jobs; and that these abilities persist over time.

To investigate if workers in the skill groups exposed to routine-cognitive jobs were the most affected by the decline of such jobs, I use the ACS to estimate the following variant of equation 2.1:

\[
\Delta Y_{gt} = \mu_t R_{2000}^g + \gamma_t X_g + \varepsilon_{gt}.
\]

This model explains changes in the labor market outcomes of workers in skill group \( g \) as a function of their exposure to the decline in routine-cognitive jobs \( R_{2000}^g \). In this equation I control for a set of skill group characteristics, \( X_g \), measured using the 2000 Census. These include the share of employment in manufacturing, durable, tradable and construction industries, which takes into account differences in industry cyclicality and other sectoral shocks. I also include a full set of gender\( \times \)age, education, and region of residence dummies so that my estimates do not capture differences in formal education, female participation in the labor force or life-cycle dynamics (which are also allowed to vary by gender). When estimating this equation, I allow the error term \( \varepsilon_{gt} \) to be correlated within skill groups and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight skill groups by their size in 2000.

Table 2.3 presents estimates of this equation using the change in the employment rate (Panel I), the change in the unemployment rate (Panel II), and the change in the long-term unemployment rate (Panel III) as dependent variables. Column 1 focuses on the change in these outcomes from 2007 to 2009; while Column 3 presents results for the change in these outcomes from 2007 to 2013. The estimates in Column 1 suggest that, for every additional
10 percentage points of exposure to routine-cognitive jobs, workers in a given skill group face a 0.49 percentage points lower employment rate. The reduction in employment leads to a 0.22 percentage points increase in the unemployment rate, which is entirely driven by the incidence of long-term unemployment. The estimates in Column 3 reveal a similar pattern, though I do not find any effects on the unemployment rate.

Table 2.3: Changes in the employment rate during the Great Recession among commuting zones and skill groups with different exposure to the decline in routine-cognitive jobs

<table>
<thead>
<tr>
<th></th>
<th>FROM 2007 TO 2009</th>
<th>FROM 2007 TO 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>I. Employment rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group exposure to routine-cognitive jobs</td>
<td>-0.049**</td>
<td>-0.063**</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-0.060</td>
<td>-0.137**</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.04</td>
<td>0.31</td>
</tr>
<tr>
<td>Observations</td>
<td>200</td>
<td>722</td>
</tr>
<tr>
<td>II. Unemployment rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group exposure to routine-cognitive jobs</td>
<td>0.022**</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>0.079**</td>
<td>0.076**</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.04</td>
<td>0.38</td>
</tr>
<tr>
<td>Observations</td>
<td>200</td>
<td>722</td>
</tr>
<tr>
<td>III. Long-term unemployment rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group exposure to routine-cognitive jobs</td>
<td>0.022***</td>
<td>0.019***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>0.038**</td>
<td>0.117***</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.08</td>
<td>0.37</td>
</tr>
<tr>
<td>Observations</td>
<td>200</td>
<td>722</td>
</tr>
</tbody>
</table>

Notes: Panel I presents OLS estimates of the differential change in the employment rate from 2007 onward among skill groups more exposed to structural change (Columns 1 and 3) and commuting zones more exposed to structural change (Columns 2 and 4). In Panel II the dependent variable is the unemployment rate, and in Panel III the dependent variable is the long-term unemployment rate. All models include a full set of covariates. The skill group models in Columns 1 and 3 control for the baseline employment in different industries and skill group characteristics described in the main text. The commuting zone models in Columns 2 and 4 control for census division dummies, and the industry and labor supply composition. I allow the error term to be correlated within States and over time (Columns 2 and 4), and within skill groups and over time (Columns 1 and 3), and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight each commuting zone and skill group cell by its size in 2000.

For comparison, Columns 2 and 4 of Table 2.3 present estimates of equation 2.1 at the commuting zone level (these are the same estimates plotted in Figure 2-10). The clear pattern that emerges is that the exposure of a commuting zone to the decline in routine-cognitive jobs has a bigger impact on labor market outcomes than exposure at the skill group-level. This finding is not driven by measurement error; instrumenting $R_{g}^{2000}$ with a measure of exposure computed using the 1990 Census ($R_{c}^{1990}$) yields similar estimates of the decline of
routine cognitive jobs on workers in exposed skill groups.

The fact that the commuting zone impacts of this structural change are significantly larger than the impacts on exposed skill groups suggests that there are important spillovers or general equilibrium effects that amplify the effects of a structural change at the commuting zone level. While the commuting zone estimates capture these spillovers and the direct effect on affected workers, the skill group estimates only capture the direct effect on affected workers because workers in a given skill group do not reside in the same labor market.

To illustrate this point and motivate the rest of my analysis, consider the following model. Suppose that there are only two types of workers: those directly affected by the lack of routine-cognitive jobs—or routine-cognitive workers—and others, and these types are invariant. Suppose also that the change in the employment rate of a worker $i$ in skill group $g$ who resides at commuting zone $c$, which I denote by $\Delta e_{igt}$, can be written as

$$E[\Delta e_{igt}] = \begin{cases} 
\delta_t + \mu_t + \rho_t R_c^{2000} & \text{if } i \text{ is a routine-cognitive worker} \\
\delta_t + \rho_t R_c^{2000} & \text{otherwise.}
\end{cases} \quad (2.4)$$

Here, $\mu_t \leq 0$ is the direct effect of the decline in routine-cognitive jobs on affected workers, and $\rho_t R_c^{2000}$ are the spillover effects generated by overall exposure in a given local labor market.

Aggregating the above equation across commuting zones yields

$$E[\Delta e_{gt}|R_c^{2000}] = \delta_t + (\mu_t + \rho_t) R_c^{2000}.$$ 

This implies that the effect of the decline in routine-cognitive jobs on exposed commuting zones is given by $\beta_t = \mu_t^r - \mu_t^n + \rho_t$. On the other hand, aggregating the above equation across skill groups yields

$$E[\Delta e_{gt}|R_g^{2000}] = \delta_t + \mu_t R_g^{2000} + \rho_t E[R_c^{2000}|R_g^{2000}].$$

This implies that the impact of the exposure of a skill group is given by $\mu_t + \rho_t \frac{\text{cov}(R_c^{2000}, R_g^{2000})}{\text{var}(R_g^{2000})}$. Because skill groups are close to evenly distributed across local labor markets, we have that $0 \approx \frac{\text{cov}(R_c^{2000}, R_g^{2000})}{\text{var}(R_g^{2000})}$ (indeed, this coefficient can be estimated as 0.0003 in my data). Thus, the presence of spillovers at the local labor market implies that, in line with my findings, $\beta_t$—the commuting zone impact of exposure in equation 2.1—will be more negative than $\mu_t$—the skill group impact of exposure in equation 2.3.

To formally decompose the impact of the decline in routine-cognitive jobs in the direct effect on affected workers and spillovers that operate at the commuting zone level, I aggregate equation 2.4 at the skill group $\times$ commuting zone cell. In a given cell, the expected change
in the employment rate is given by

$$E[\Delta \epsilon_{gc}] = \delta_t + \mu_t R_{gc}^{2000} + \rho_t R_{c}^{2000}.$$ 

This suggests estimating the model

$$\Delta Y_{gc} = \mu_t R_{gc}^{2000} + \rho_t R_{c}^{2000} + \delta_{d(c)t} + \theta_t X_c + \gamma_t X_g + \epsilon_{gc}.$$ (2.5)

Here, $\Delta Y_{gc}$ is the average change in a labor market outcome in a skill group $x$ commuting zone cell. Also, $R_{gc}^{2000}$ is the 2000 Census share of workers in the skill group $x$ commuting zone cell $(c, g)$ employed in routine-cognitive jobs. Besides the other terms, which I already introduced above, $\epsilon_{gc}$ is an error term. When estimating this equation, I allow the error term $\epsilon_{gc}$ to be correlated within States and within skill groups, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight each cell by its year 2000 working-age population.

Provided that outcomes can be linearly decomposed in the direct effect on affected workers and spillovers that operate at the local labor market level, consistent estimates of $\mu_t$ and $\rho_t$ in equation 2.5 separately identify these effects, respectively.

Table 2.4 presents estimates of equation 2.5 focusing on the employment rate, which I compute for each cell from the American Community Survey. The top panel considers the change in the employment rate from 2007 to 2009, the middle panel from 2007 to 2013, and the bottom panel pools all estimates from 2007 to 2009, 2010, 2011, 2012 and 2013 in a single average coefficient. Column 1 includes only the commuting zone exposure to routine-cognitive jobs, in addition to all the controls that capture differences in the industry and labor-supply composition. Column 2 includes the cell exposure to routine-cognitive jobs, $R_{gc}^{2000}$, as an explanatory variable. In all panels, my estimates for both the direct effect on affected workers—the coefficient on the cell exposure term, $\mu_t$—and the amplification effects at the commuting zone level—the coefficient on the commuting zone exposure, $\rho_t$—are negative and significant. However, once I include the cell exposure in Column 2, the estimate of the commuting zone exposure falls by about 30%, which suggests that 70% of the commuting zone impact was driven by spillovers that operate at the local labor market. This evidence suggests that the decline in routine-cognitive jobs not only reduced the employment rate of directly affected workers—those specialized in routine-cognitive jobs—but also created negative spillovers that amplified these effects in exposed labor markets.

One concern with the estimates in Column 2 is that the cell exposure might be measured with error. For instance, $R_{cg}^{2000}$ might be contaminated by temporal shocks or because there are few observations in a given cell. Because the commuting zone exposure $R_{c}^{2000}$ is the
Table 2.4: Changes in the employment rate during the Great Recession among commuting zones and skill groups with different exposure to the decline in routine-cognitive jobs

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I. Estimates from 2007 to 2009</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-0.126**</td>
<td>-0.095*</td>
<td>-0.085</td>
<td>-0.072</td>
<td>-0.075</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.057)</td>
<td>(0.059)</td>
<td>(0.056)</td>
<td>(0.056)</td>
</tr>
<tr>
<td>Cell exposure to routine-cognitive jobs</td>
<td>-0.032**</td>
<td>-0.042**</td>
<td>-0.056***</td>
<td>-0.052***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.020)</td>
<td>(0.019)</td>
<td>(0.019)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Observations</td>
<td>35237</td>
<td>35233</td>
<td>35195</td>
<td>35233</td>
<td>35195</td>
</tr>
<tr>
<td>First stage F-statistic</td>
<td>609.1</td>
<td>22763.0</td>
<td>18243.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overid test p-value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>II. Estimates from 2007 to 2013</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-0.187***</td>
<td>-0.137**</td>
<td>-0.111*</td>
<td>-0.122*</td>
<td>-0.119*</td>
</tr>
<tr>
<td></td>
<td>(0.069)</td>
<td>(0.064)</td>
<td>(0.067)</td>
<td>(0.066)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>Cell exposure to routine-cognitive jobs</td>
<td>-0.051***</td>
<td>-0.078***</td>
<td>-0.067**</td>
<td>-0.070**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.025)</td>
<td>(0.028)</td>
<td>(0.027)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Observations</td>
<td>35472</td>
<td>35466</td>
<td>35401</td>
<td>35466</td>
<td>35401</td>
</tr>
<tr>
<td>First stage F-statistic</td>
<td>599.8</td>
<td>22850.6</td>
<td>18288.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overid test p-value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>III. Pooling 2009 to 2013</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-0.174***</td>
<td>-0.134**</td>
<td>-0.118*</td>
<td>-0.125**</td>
<td>-0.123**</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.057)</td>
<td>(0.059)</td>
<td>(0.058)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>Cell exposure to routine-cognitive jobs</td>
<td>-0.041***</td>
<td>-0.058***</td>
<td>-0.051**</td>
<td>-0.052***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.020)</td>
<td>(0.021)</td>
<td>(0.020)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Observations</td>
<td>176793</td>
<td>176762</td>
<td>176498</td>
<td>176762</td>
<td>176498</td>
</tr>
<tr>
<td>First stage F-statistic</td>
<td>605.3</td>
<td>22813.1</td>
<td>18230.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overid test p-value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Instruments:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group exposure in 1990</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>National-level group exposure</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table presents estimates of the direct effect of the decline in routine-cognitive jobs on affected workers and the spillovers at the commuting zone level. Panel I presents results focusing on the employment rate change from 2007 to 2009. Panel II presents results focusing on the employment rate change from 2007 to 2013. Panel III presents results focusing on the average employment rate change from 2007 to 2009, 2010, 2011, 2012 and 2013. In different models, I instrument the cell exposure to routine-cognitive jobs using the instruments indicated in the bottom rows. In all models, I allow the error term  \( \varepsilon_{ct} \) to be correlated within States and over time, and within skill groups and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight each commuting zone x skill group cell by its size in 2000.

average of \( R_{cg}^{2000} \), any measurement error in the later variable would load up into the coefficient of the former. Thus, even if there are no important spillovers, the positive and significant estimates for the spillovers, \( \rho_t \), may simply reflect measurement error in the cell exposure to routine-cognitive jobs (see Acemoglu and Angrist, 2000).

I pursue several strategies to address this concern. In Column 3 I instrument the cell exposure using its historical level from the 1990 Census, \( R_{cg}^{1990} \). This specification has the benefit of removing temporary sources of measurement error in my cell measure of exposure, \( R_{cg}^{2000} \). This strategy makes the estimated partial equilibrium effects more negative and reduces the estimated impact of the spillovers that operate at the commuting zone level (the
estimates in the first panel are not significant at conventional level, but remain sizable). In Column 4 I follow an additional instrumental variables strategy, and instrument the cell exposure using the national-level skill group exposure, \( R_{g}^{2000} \). This specification has the advantage of removing any measurement error introduced shocks that are specific to a given labor market. The case for using \( R_{g}^{2000} \) as an instrument for \( R_{cg}^{2000} \) rests on the exclusion restriction that there are no spillovers across workers in the same skill group but who reside in different labor markets. This requires enough segmentation and little mobility across local labor markets, two features that seem plausible in my application. Finally, in Column 5 I include both instruments simultaneously. For each panel, the results in Columns 3 to 5 are remarkably similar. This is reassuring, given that these estimates exploit different sources of variation. In line with this pattern, an overidentification test of the models in Column 5 cannot reject the null that all instruments are exogenous. In the 2SLS models in Columns 3 to 5, my estimates of the direct effect of the decline in routine-cognitive jobs on affected workers are slightly larger than in Column 2, and the estimated spillovers are slightly smaller than in Column 2.

The estimates in Column 5 of the bottom panel suggest that, during the Great Recession and its recovery, for every additional 10 percentage points of exposure to the decline in routine-cognitive jobs, the direct effect of the decline in routine-cognitive jobs on affected workers created a reduction in the employment rate of 0.52 percentage points. However, through spillovers on exposed labor markets, this structural change reduced the employment rate by an additional 1.23 percentage points. Thus, in exposed labor markets, about 70% of the joblessness caused by the decline in routine-cognitive was driven by spillovers that amplify the effects at the local labor market.

So far I have estimated the average spillovers that operate at the commuting zone level. However, to disentangle between some potential theories that predict the existence of these spillovers, it is useful to decompose their incidence. To do so, I allow the spillovers in equation 2.4 to vary depending on the type of jobs typically held by workers in a skill group, and estimate the model:

\[
\Delta Y_{gst} = \mu_{t} R_{gc}^{2000} + \lambda_{gt} + \sum_{o} \rho_{t}^{o} R_{c}^{2000} \times S_{og} + \delta_{d(c)t} + \theta_{t} X_{c} + \varepsilon_{gst}. \tag{2.6}
\]

Here, \( S_{og} \) is the share of workers in a given skill group \( g \) who, in the 2000 Census, were employed in a broad occupational category \( o \). These broad occupational categories include service jobs, routine-manual jobs, routine-cognitive jobs, professional jobs and managerial jobs. Because the shares of workers employed in these broad categories add up to 1, the coefficients \( \rho_{t}^{o} \) decompose the incidence of the spillovers on workers specialized in different jobs. In equation 2.6, I also include a full set of skill group \( \times \) year dummies \( \lambda_{gt} \), which fully
take into account the role of unobserved differences across skill groups that are common nationwide.

Figure 2-12 plots my OLS estimates of equation 2.6 (the results instrumenting my different measures of exposure are very similar). The figure plots in dark blue the average spillover effect on all skill groups, $\rho_t$, followed by the spillover on workers specialized in different jobs, $\rho_t^j$, separately by type of job. The estimates indicate that there are no negative spillovers on the employment rate of workers specialized in service and managerial jobs. However, there are large and negative spillovers from the decline in routine-cognitive jobs on workers specialized in all middle-skill jobs, including professional jobs, which do not comprise routine tasks and are not directly affected by the computerization of such tasks.

![Figure 2-12: Estimates for $\beta_0^\tau$ for commuting zones at 90th percentile of specialization in $RS_e$ relative to the mean.](image)

The bottom panel of Figure 2-12 presents my estimates using the change in (the log of) wages as the dependent variable. Although in commuting zones exposed to routine-cognitive jobs, the employment rate of workers specialized in professional jobs declines, their wage actually increases. The behavior of wages in the data weighs against theories that emphasize nominal rigidities or contractual frictions as the main limit to the redeployment of displaced workers. Unlike models with wage rigidities, here, unemployment is associated with a large decline in average wages for routine-cognitive workers and an increase in wage.
dispersion.

These findings are not explained by a mechanical decline in the marginal product of labor employed in professional or service jobs. To make this point, the estimates in light blue also control for the percent change in employment in professional and service jobs—the two occupational categories absorbing displaced workers. Because the change in employment in professional and service jobs is endogenous, I instrument it using its pre-recession level, which exploits the existing mean reversion on employment levels. As can be seen from the figure, the increase in employment in professional and service jobs per se does not explain these spillovers. It is only when the composition of displaced workers shifts towards more unqualified ones that I observe a negative effect on the employment rate of middle-skill workers.

2.4 Concluding discussion

This paper documented the role of the decline in routine-cognitive jobs—a pervasive structural change affecting U.S. labor markets since the mid 90s—in contributing to the long-lasting increase in joblessness observed during the recent Great Recession.

My evidence supports three main conclusions.

First, together with the evidence in Jaimovic and Siu (2014), Cortes, Jaimovic, Nekarda and Siu (2014) and Hershbein and Kahn (2016), my evidence supports the view that the Great Recession accelerated the decline in routine-cognitive jobs. This feature is consistent with theories going back to Schumpeter (1942), and that argue that because the opportunity cost of a firm depends on its current demand, even temporary demand shocks may lead to a deep restructuring of firms, their labor requirements, their use of technologies, and the labor market. This restructuring manifests during the onset of a recession and its recovery as a permanent decline in the creation of additional routine-cognitive jobs. Notwithstanding the evidence in this paper, there are alternative accounts that suggest that, because the gains from reallocation are low when demand or productivity is depressed, recessions may be sullying.

Second, I documented a new fact: during the Great Recession, commuting zones that were more exposed to routine-cognitive jobs fared worst in terms of employment and wages than other local labor markets; while these effects were masked during the booming years that preceded the recession. In the most exposed local labor markets, the joblessness resulted from the imperfect and costly reallocation of workers away from routine-cognitive jobs. This evidence suggests that, during the Great Recession, the accelerated decline in routine-cognitive jobs had aggregate effects that contributed to the observed joblessness.

Third, my evidence is consistent with the decline in routine-cognitive jobs creating large
spillovers that operate at the local labor market level. These spillovers explain why the effects of this structural change are most visible not in national-level statistics, but in the dynamics of adjustment of exposed labor markets. A linear decomposition of my estimates suggests that nearly 70% of the overall effect of this structural change occurred through spillovers at the local labor market. These spillovers may explain why studies that use nationwide comparisons of the labor market outcomes of different workers tend to find a limited role for structural factors in explaining unemployment during the Great Recession (see Lazear and Spletzer, 2012; Kroft et al., 2014; and Barnichon and Figura, 2015).

Given the large estimated role for spillovers, this paper suggest that theories of structural unemployment and reallocation should not only focus on the direct effect of structural changes on affected workers, but also on the possibility that the adjustment process may disrupt the overall labor market.

Although some theories emphasize the importance of spillovers, they are not consisting with some of the patterns documented here. For instance, one theory is that labor may have a decreasing marginal value in jobs receiving displaced workers (professional and service jobs). In this case, the reallocation of workers could directly affect the employment opportunities for workers specialized in professional jobs. This explanation does not fit several features of the data. First, if I control for the change in employment in professional and service jobs—the two employment categories that are absorbing workers—, I obtain very similar results. These estimates are plotted in light blue in Figure 2-12. Second, this mechanism cannot explain why there are no spillovers on workers specialized in the service sector, which would also be subject to decreasing returns, or why do wages for skilled workers increase.

Another theory emphasizes the role played by local demand spillovers. For instance, when workers are displaced from routine-cognitive jobs or learn that their skills became obsolete they might consume less. This mechanism reduces local demand and could affect employment for other workers through demand externalities (see Beaudry, Portier and Green, 2014; and Acemoglu et al., 2014). However, demand externalities would have the strongest effect on workers specialized in the highly non-tradable service jobs and would lower the wage of workers who specialized in professional jobs, which are not the case in the data.

Instead, I believe the spillovers documented here point towards theories of agglomeration, human capital or skill externalities. The estimated spillovers and their incidence are consistent with models in which, due to human capital externalities or agglomeration effects, professional jobs become less profitable when the share of workers with routine and no specialized skills that need to be redeployed to these jobs raises. Because of these human capital externalities, it is hard for commuting zones in which many workers only have routine skills to develop an industry for new jobs and occupations that rely on specialized and professional skills. This slows down the reallocation of workers and affects the labor market outcomes
of professional workers who depend on the development of these industry. For instance, in models with matching frictions and random matching, having more routine-cognitive workers displaced from their jobs and searching for new jobs may end up reducing job creation in professional occupations that require specialized skills, and that were supposed to pick-up the slack in the labor market (see Acemoglu, 1996; and Restrepo, 2016).

In line with this explanation, I find that the spillovers only affect the labor market outcomes of workers who specialize in routine jobs or workers with specialized skills in professional occupations. In contrast to middle-skill workers, the local exposure to the decline in routine-cognitive jobs does not affect workers in skill groups that specialize in services—which require no specialized skills—, nor workers in skill groups that specialize in managerial occupations—which did not receive an inflow of unskilled workers.

If indeed human capital externalities or agglomeration effects are behind the spillovers in the data, one would expect a long-lasting an inefficient adjustment to the decline in routine-cognitive jobs. The spillovers may be so large as to prevent some local labor markets to adjust at all and develop a new economy based on professional jobs that require specialized skills.

References


Schumpeter, Joseph A. (1942) *Capitalism, Socialism, and Democracy*.


### 2.5 Data Appendix

**Occupational groups and routine jobs:** To define routine-cognitive jobs, I use the 330 occupational groups proposed by Dorn (2009). These partition into consistently aggregated groups the occupations reported in the 1980, 1990 and 2000 Census and the American Community Survey. Military occupations are not included, and I exclude military personnel when using Census and ACS data.

These 330 occupational groups can be also consistently aggregated to match the broad occupational categories that are available for other of the datasets I use.

For each occupational group I use the routine-content index computed by Autor, David, and Murnane (2003), who in turn use data from the Dictionary of Occupational Titles (1977). Following the literature on job polarization, I define routine jobs as those in the occupational categories in the top three deciles of routine content.

**Commuting zones:** I use 722 commuting zones that cover the entire continental U.S. but do not include Alaska and Hawai. David Dorn’s crosswalks (available at his webpage http://www.ddorn.net/data.htm) are used to aggregate Census and ACS geographic units at the commuting zone level. Recent waves of the American Community Survey use a new coding for Public Use Microdata Areas (PUMAS) to report geography. Using the available Census maps for the new PUMAs I do a geographic match to Counties in 1990. Then using David Dorn’s crosswalks I match Counties to commuting zones.

**BLS data on employment counts by occupation:** The employment counts used to construct Figure 2-2 are from the Bureau of Labor Statistics, which in turn construct these estimates suing the Current Population Survey. These series include the total population aged 16 years or older employed in each broad occupational category. The data is available online with the following series identifiers:

- **Professional jobs:** Employment Level - Professional and Related Occupations. Series LNU02032203.
- **Service jobs:** Employment Level - Service Occupations. Series LNU02032204
• *Routine-cognitive jobs*: Employment Level - Sales and Office Occupations. Series LNU02032205.

Using these series, I construct the employment rate by dividing the employment counts by the U.S. working-age population (people between 16 and 64 years in the U.S.).

![Employment rates by occupational category in the U.S.](image)

**Employment rate in sales jobs (left axis)**

**Employment rate in clerical jobs (right axis)**

*Figure 2-13: U.S. employment rates for different occupational categories. Data from the BLS.*

The employment counts for sales and office occupations can be separated in sales jobs and office jobs (series LNU02032206 and LNU02032207, respectively). These are presented separately in the following figure, which shows that, during the Great Recession, both sales and clerical jobs declined sharply.

**Help Wanted Online (HWOL) data**: The job openings data is from the Conference Board, Help Wanted Online Statistics. These data includes monthly job-opening counts for 22 broad occupational categories starting in January, 2006, and ending in December, 2015. To reduce measurement error, I aggregate the data to the quarterly level and adjust for seasonality.

After aggregating the 330 occupational categories to the 22 broad ones included in the HWOL data, I am able to compute the average share of routine jobs in each broad occupational category. Some broad occupations, like “education and training,” have no routine jobs. Others, like “office and administrative support,” have a share of routine jobs of 93%.

Figure 2-3 plots the predicted change in the log of job openings (with respect to the first quarter of 2007) among occupations with no routine jobs, and among occupations in which...
all jobs are routine. These predicted values are obtained from a regression of the change in the log of job openings (with respect to the first quarter of 2007) against the share of routine jobs in each occupation.

**Current Population Survey (CPS) data:** To construct Figure 2-5, I use data from the Annual Social and Economic Supplement. These data reports, for each worker, her employment status and occupation both for the current and last year. I use the set of all prime-age workers who are currently employed and were employed in a routine job last year. In this sample, I compute the share of workers that changed occupations for every year from 1990 to 2015.

To adjust the data by demographics, I regress the share of workers that switched occupations on a full set of age, education and gender dummies. The adjusted share is computed for male workers of 30 years (the average age in the sample) with some college.

**Census and American Community Survey data (ACS):** I use the IPUMS extracts of the Census and American Community Survey. I match the occupations and areas of residence to the 330 occupational groups and 722 commuting zones described above. The details of how this match is done are described above.

Moreover, as explained in the text, I define skill groups by sex (2 categories), age (5 categories), educational attainment (5 categories), and region (4 regions), as reported in the Census and American Community Survey. This procedure yields a partition of the civilian workforce into 200 skill groups that I use in my analysis.

**Longitudinal Employer-Household Dynamics data and additional results:** Finally, I use the public-use data from the Longitudinal Employer-Household Dynamics. These data include figures on quarterly hirings and turnover for several (but not all) U.S. Counties. I aggregate these data to the commuting zone level and use them to construct Figure 2-11 as well as the complementary analysis that I present in the Appendix.

I aggregate the data by year at the commuting zone level and compute the annual hiring rate as total hirings per unemployed workers. Likewise, I compute the average turnover rate by averaging the turnover over all the quarters in a given year.

Table 2.5 presents my estimates of equation 2.1 using the annual hiring rate (Panel I) and turnover rate (Panel II) as dependent variables. Columns 1 to 3 focus on all industries; while Columns 4 to 6 focus on non-manufacturing industries. Different columns present models that investigate the change in these outcomes over different time periods, which I indicate in the top rows.

Overall, I find that the increase and persistence in joblessness documented in the paper is driven by a significant decline in the hiring rate. The estimates in Column 1 of this table imply that a 10 percentage point increase in exposure to routine-cognitive jobs is associated with a 15 percentage point reduction in the annual hiring rate during the onset of the Great
Table 2.5: Hiring and separation rates among commuting zones exposed, during and before the Great Recession.

<table>
<thead>
<tr>
<th></th>
<th>All industries</th>
<th>Non-manufacturing industries</th>
</tr>
</thead>
<tbody>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-1.649*** (0.555)</td>
<td>-1.528** (0.576)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.59</td>
<td>0.46</td>
</tr>
<tr>
<td>Observations</td>
<td>698</td>
<td>698</td>
</tr>
</tbody>
</table>

Panel I: Dependent variable: annual hiring rate from LEHD.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CZ exposure to routine-cognitive jobs</td>
<td>-0.195** (0.075)</td>
<td>-0.244** (0.112)</td>
<td>0.017 (0.049)</td>
<td>-0.377*** (0.109)</td>
<td>-0.322*** (0.120)</td>
<td>0.084 (0.050)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.50</td>
<td>0.53</td>
<td>0.20</td>
<td>0.18</td>
<td>0.17</td>
<td>0.07</td>
</tr>
<tr>
<td>Observations</td>
<td>698</td>
<td>698</td>
<td>698</td>
<td>698</td>
<td>698</td>
<td>698</td>
</tr>
</tbody>
</table>

Panel II: Dependent variable: annual turnover rate from LEHD.

Notes: The table presents OLS estimates of the differential change in the hiring and turnover rates from 2007 onward among commuting zones more exposed to structural change. The dependent variable is the change in the annual hiring rate (Panel I), and the turnover rate (Panel II). The change in the dependent variable is computed over the years indicated on top of each column. Columns 1 to 3 use data for all industries, while Columns 4 to 6 use only data for non-manufacturing industries. When estimating this equation, I allow the error term to be correlated within States and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight commuting zones by the size of their workforce in 2000.

Recession (standard error=4.5). Columns 4 to 6 show that this effect is driven entirely by a reduction of hires within non-manufacturing industries. This effect implies that, relative to the least exposed areas, the most exposed areas to the decline in routine-cognitive jobs experienced a 16% decline in the average job-finding rate, and this in turn translated into an additional 1.5 percentage point reduction in employment during the Great Recession—a figure that matches my previous estimates. Finally, Columns 3 and 6 show that exposed commuting zones were not experiencing divergent trends before the Great Recession.

---

15 The LEHD data yields an annual hiring rate of 1. This is considerably smaller than the finding rate reported by Shimer (2005). The difference arises because I compute the rate per worker without a job rather than per unemployed worker. Using the rate per unemployed worker yields similar but less precise estimates.
Chapter 3

The Race Between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment (joint with Daron Acemoglu)

Abstract

The advent of automation and the simultaneous decline in the labor share and employment among advanced economies raise concerns that labor will be marginalized and made redundant by new technologies. We examine this proposition using a task-based framework in which tasks previously performed by labor can be automated and more complex versions of existing tasks, in which labor has a comparative advantage, can be created. We characterize the equilibrium in this model and establish how the available technologies and the choices of firms between producing with capital or labor determine factor prices and the allocation of factors to tasks. In a static version of our model where capital is fixed and technology is exogenous, automation reduces employment and the share of labor in national income and may even reduce wages, while the creation of more complex tasks has the opposite effects.

Our full model endogenizes capital accumulation and the direction of research towards automation and the creation of new complex tasks. Under reasonable conditions, there exists a stable balanced growth path in which the two types of innovations go hand-in-hand. An increase in automation reduces the cost of producing using labor, and thus discourages further automation and encourages the faster creation of new complex tasks. The endogenous response of technology restores the labor share and employment back to their initial level. Although the economy contains powerful self-correcting forces, the equilibrium generates too much automation. Finally, we extend the model to include workers of different skills. We find that inequality increases during transitions, but the self-correcting forces in our model also limit the increase in inequality over the long-run.

Keywords: Automation, Directed Technological Change, Economic Growth, Endogenous Growth, Factor Shares, Productivity, Tasks, Technological Change.

JEL Classification: O33, O14, O31, J23, J24.
The accelerated automation of tasks performed by labor raises concerns that new technologies will make labor redundant (e.g., Brynjolfsson and McAfee, 2012, Akst, 2014, Autor, 2015). The recent declines in the share of labor in national income and the employment to population ratio in the U.S. (e.g., Karabarbounis and Neiman, 2014, and Oberfield and Raval, 2014) are often interpreted to support the claims that, as digital technologies, robotics and artificial intelligence penetrate the economy, workers will find it increasingly difficult to compete against machines, and their compensation will experience a relative or even absolute decline. Yet, we lack a comprehensive framework incorporating such effects, as well as potential countervailing forces.

The need for such a framework stems not only from the importance of understanding how and when automation will transform the labor market, but also from the fact that similar claims have been made, but have not always come true, about previous waves of new technologies. Keynes famously foresaw the steady increase in per capita income during the 20th century from the introduction of new technologies, but incorrectly predicted that this would create widespread technological unemployment as machines replaced human labor (Keynes, 1930). In 1965, economic historian Robert Heilbroner confidently stated that “as machines continue to invade society, duplicating greater and greater numbers of social tasks, it is human labor itself—at least, as we now think of ‘labor’—that is gradually rendered redundant” (quoted in Akst, 2014). Wassily Leontief was equally pessimistic about the implications of new machines. By drawing an analogy with the technologies of the early 20th century that made horses redundant, he speculated that “Labor will become less and less important... More and more workers will be replaced by machines. I do not see that new industries can employ everybody who wants a job” (Leontief, 1952).

This paper is a first step in developing a conceptual framework to study how machines replace human labor and why this might (or might not) lead to lower employment and stagnant wages. Our main conceptual innovation is to propose a unified framework in which tasks previously performed by labor are automated, while at the same time more complex versions of existing tasks in which labor has a comparative advantage are created.¹ The importance of these new complex tasks is well illustrated by the technological and organizational changes during the Second Industrial Revolution, which not only involved the replacement of the stagecoach by the railroad, sailboats by steamboats, and of manual dock workers by cranes, but also the creation of new labor-intensive tasks. These new tasks generated jobs for a new class of engineers, machinists, repairmen, and conductors as well as of modern managers and financiers involved with the introduction and operation of new technologies (e.g., Landes, 1969, Chandler, 1977, and Mokyr, 1990).

¹Herein lies our answer to Leontief’s analogy: the difference between human labor and horses is that humans have a comparative advantage in new and more complex tasks. Horses did not.
Today, while industrial robots, digital technologies and computer-controlled machines replace labor, we are once again simultaneously witnessing the emergence of new tasks ranging from engineering and programming functions to those performed by audio-visual specialists, executive assistants, data administrators and analysts, meeting planners or computer support specialists. Indeed, during the last 30 years, new tasks and new job titles account for a large fraction of U.S. employment growth. To document this fact, we use data from Lin (2011) that measures the share of new job titles—in which workers perform newer tasks than those employed in more traditional jobs—within each occupation. In 2000, about 70% of the workers employed as computer software developers (an occupation employing one million people at the time) held new job titles. Similarly, in 1990 a radiology technician and in 1980 a management analyst were new job titles. Figure 3-1 shows that for each decade since 1980, employment growth has been greater in occupations with more new job titles. The regression line shows that occupations with 10 percentage points more new job titles at the beginning of each decade grow 5.05% faster over the next 10 years (standard error=1.3%). From 1980 to 2007, total employment in the U.S. grew by 17.5%. About half (8.84%) of this growth is explained by the additional employment growth in occupations with new job titles relative to a benchmark category with no new job titles.2

Figure 3-1: Employment growth by decade plotted against the share of new job titles at the beginning of each decade for 330 occupations. Data from 1980 to 1990 (in dark blue), 1990 to 2000 (in blue) and 2000 to 2007 (in light blue, re-scaled to a 10-year change). Data source: See Appendix B.

We start with a static model in which capital is fixed and technology is exogenous. There

---

2The data for 1980, 1990 and 2000 are from the U.S. Census. The data for 2007 are from the American Community Survey. Additional information on the data and our sample is provided in Appendix B, where we also document in detail the robustness of the relationship depicted in Figure 3-1.
are two types of technological changes: the automation of existing tasks and new complex
tasks in which labor has a comparative advantage. Our static model provides a rich but
tractable framework to study how automation and the creation of new complex tasks impact
factor prices, factor shares in national income and employment. Automation allows firms
to produce tasks previously performed by labor with capital, while the creation of new
complex tasks allow firms to replace old tasks by new variants in which labor has a higher
productivity. In contrast to the more commonly-used models featuring factor-augmenting
technologies, here automation always reduces the share of labor in national income and
employment, and may even reduce wages. Conversely, the creation of new complex tasks
always increases wages, employment and the share of labor, and may even reduce the rate of
return to capital. These comparative statics follow because factor prices are determined by
the range of tasks performed by capital and labor, and exogenous shifts in technology alter
the range of tasks performed by each factor (see also Acemoglu and Autor, 2011).

We then embed this framework in a dynamic economy in which capital accumulation
is endogenous, and we characterize restrictions under which the model delivers balanced
growth—which we take to be a good approximation to economic growth in the United States
and the United Kingdom over the last two centuries. The key restrictions are that there is
exponential productivity growth from the creation of new tasks and that the two types of
technological changes—automation and the creation of new complex tasks—advance at equal
paces. A critical difference from our static model is that capital accumulation responds to
permanent shifts in technology in order to keep the interest rate constant. Thus, the dynamic
effects of technology on factor prices depend on the response of capital accumulation as
well. We show that the response of capital ensures that the productivity gains from both
automation and the introduction of new complex tasks fully accrue to labor (the relatively
inelastic factor) and increase overall wages in the long run—a feature we refer to as the
productivity effect. Although real wages increase due to the productivity effect, automation
always reduces the labor share.

Our full model endogenizes the rates of improvement of these two types of technologies
by marrying our task-based framework with a directed technological change setup. This full
version of the model remains tractable, and under natural assumptions, generates asymp-
totically stable balanced growth with equal advancement of the two types of technologies. If
automation runs ahead of the creation of new complex tasks, market forces induce a slow-
down of subsequent automation and countervailing advances in the creation of new complex
tasks. As a result, in the long run, the share of labor in national income and employment
return to their initial levels. The economics of these self-correcting forces are instructive
and highlight a crucial new force: a wave of automation pushes down the effective cost of
producing with labor. When technology is endogenous, this discourages further efforts to
automate additional tasks and pushes the economy to redirect its research efforts towards the creation of new (labor-intensive) tasks.\(^3\)

In our model, the stability of the balanced growth path implies that periods in which automation runs ahead of the creation of new complex tasks tend to self-correct. Contrary to the increasingly widespread concerns discussed above, our model raises the (theoretical) possibility that rapid automation need not signal the demise of labor, but might simply be a prelude to a phase of new technologies favoring labor. In addition, our analysis clarifies the long-run implications of different types of technological shocks. For example, if a wave of automation is triggered by a change in the innovation possibilities frontier (that is, in the technology for creating new technologies) that makes it easier to automate tasks, the economy will settle in a new balanced growth path with a greater share of tasks performed by capital, lower employment and lower labor share.

The final implication of our full model concerns the efficiency of the market equilibrium. In addition to the standard inefficiencies due to monopoly markups and appropriability problems in endogenous technological change models, our analysis identifies a new source of inefficiency that pushes towards too much automation and too few new tasks being created. This inefficiency arises because automation, which enables firms to economize on wage payments, responds to high wages; when some of the wage payments accruing to workers are rents (e.g., efficiency wages or quasi-rents created by labor market frictions), there will be more automation than what the social planner would desire, and technology becomes inefficiently biased towards replacing labor.

We also consider two extensions of our model. First, we introduce heterogeneity in skills, and assume that skilled labor has a comparative advantage in new complex tasks, which we view as a natural assumption.\(^4\) Because of the comparative advantage of skilled workers relative to the unskilled in higher-index tasks, automation directly takes jobs from unskilled labor and thus increases inequality. Similarly, because skilled workers have a comparative advantage in new complex tasks, the creation of such tasks at first increase inequality as well. However, inequality increases tend to reverse themselves over longer periods as new tasks are standardized and can employ unskilled labor more productively. This extension formalizes the intuitive idea that both automation and the creation of new complex tasks increase inequality in the short run, but also points out that, over the long run, the self-correcting forces in our economy limit the increase in inequality. Our second extension modifies our baseline patent structure and reintroduces the creative destruction of the profits of previous innovators, which was absent in our main model but is often assumed in the endogenous

\(^3\)Stability in addition requires that the productivity effect is not too strong; otherwise, automation would have a large impact on wages and could potentially discourage the creation of new tasks.

growth literature. The results in this case are similar, but the conditions for uniqueness and stability of the balanced growth path are more demanding.

Our paper relates to several literatures. It can be viewed as a combination of task-based models of the labor market with directed technological change models. Task-based models have been developed both in the economic growth and labor literatures, dating back at least to Roy’s seminal work (1955). The first important recent contribution is Zeira (1998), which proposed a model of economic growth based on capital-labor substitution and constitutes a special case of our model. Acemoglu and Zilibotti (2000) developed a simple task-based model with endogenous technology and applied it to the study of productivity differences across countries, illustrating the potential mismatch between new technologies and the skills of developing economies (see also Zeira, 2006, Acemoglu, 2010). Autor, Levy and Murnane (2003) suggested that the increase in inequality in the U.S. labor market reflects the automation and computerization of routine, labor-intensive tasks. Our static model is most similar to Acemoglu and Autor (2011). Our full framework extends this model not only because of the dynamic equilibrium incorporating directed technological change, but also because tasks are combined with a general elasticity of substitution, and because the equilibrium allocation of tasks critically depends both on factor prices and the state of technology.

Three papers from the economic growth literature that are particularly related to our work are Acemoglu (2003), Jones (2005), and Hemous and Olson (2015). The first two papers develop growth models in which the aggregate production function is endogenous and, in the long run, adapts to make balanced growth possible. In Jones (2005), this occurs because of endogenous choices about different combinations of activities/technologies being used. In Acemoglu (2003), this long-run behavior is a consequence of directed technological change in a model of factor-augmenting technologies. Our task-based framework here is a significant departure from this model, especially since it enables us to address questions related to automation, its impact on factor prices and its endogenous evolution. In addition, our framework provides a more robust economic force ensuring the stability of the balanced growth path: while in models with factor-augmenting technologies stability requires an elasticity of

---


6 Acemoglu and Autor (2011), Autor and Dorn (2013), Jaimovich and Siu (2014), Foote and Ryan (2014), Burstein and Vogel (2012), and Burstein, Morales and Vogel (2014) provide various pieces of empirical evidence and quantitative evaluations on the importance of the endogenous allocation of tasks to factors in recent labor market dynamics.

7 Acemoglu and Autor’s model, like ours, is one in which a discrete number of labor types are allocated to a continuum of tasks. Costinot and Vogel (2010) develop a complementary model in which there is a continuum of skills and a continuum of tasks. See also the recent paper by Hawkins, Ryan and Oh (2015), which shows how a task-based model is more successful than standard models in matching the co-movement of investment and employment at the firm level.
substitution between capital and labor that is less than 1 (so that the more abundant factor commands a lower share of national income), we do not need such a condition in this framework. Finally, Hemous and Olson (2015) develop a model of automation and horizontal innovation with endogenous technology, and use it to study consequences of different types of technologies on inequality. High wages (in their model for low-skill workers) encourage automation. But unlike our model, the unbalanced dynamics that this generates are not countered by other types of innovations in the long run.

The rest of the paper is organized as follows. Section 3.1 presents our task-based framework in the context of a static economy. Section 3.2 introduces capital accumulation and clarifies the conditions for balanced growth in this economy. Section 3.3 presents our full model with endogenous technology and establishes, under some plausible conditions, the existence, uniqueness and stability of a balanced growth path with two types of technologies advancing simultaneously. Section 3.4 examines the efficiency of equilibrium composition of new technologies. Section 3.5 considers the two extensions mentioned above. Section 3.6 concludes. Appendix A contains the proofs of some of the proofs omitted in the text, while Appendix B, which is not for publication, contains the remaining proofs, some additional results and the details of the empirical analysis presented above.

3.1 Static Model

We start with a static version of our model with exogenous technology, which will enable us to introduce our main setup in the simplest fashion and characterize the impact of different types of technological change on factor prices.

3.1.1 Environment

The economy contains a unique final good $Y$, produced by combining a continuum of tasks, $y(i)$, of unit measure with an elasticity of substitution $\sigma \in (0, \infty)$. Namely,

$$Y = \left( \int_{N-1}^{N} y(i)^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}. \quad (3.1)$$

The role of technologies replacing tasks in this result can be seen by noting that with factor-augmenting technological changes, the impact on relative factor prices is ambiguous and the direction of innovation may be dominated by a strong market size effect (e.g., Acemoglu, 2002). Instead, in our model, the difference between factor prices regulates the future path of technological change and thus generates a powerful force that ensures stability.

9Kotlikoff and Sachs (2012) develop an overlapping generation model in which automation may have long-lasting effects because it reduces the wages of current workers, and via this channel, also depresses their savings and capital accumulation.
All tasks and the final good are produced competitively. This formulation will enable us to model the upgrading of the quality (productivity) of the unit measure of tasks as an increase in \( N \). The fact that the limits of integration run between \( N - 1 \) and \( N \) also imposes that the measure of tasks used in production always remains at 1, and a new more complex task replaces or upgrades the lowest-index task.\(^{10}\)

Each task is produced combining labor or capital with a task-specific intermediate \( q(i) \), which embodies the technology used both for production and for the possible automation of tasks. In preparation for our full model in Section 3.3, we assume that property rights to each intermediate are held by a technology monopolist which can produce it at the marginal cost \( \mu \psi \) in terms of the final good, where \( \mu \in (0,1) \) and \( \psi > 0 \). The technology for each intermediate can be copied by a fringe of competitive firms, which can replicate the intermediate at a higher marginal cost of \( \psi \). We assume that \( \mu \) is such that the unconstrained monopoly price of an intermediate is greater than \( \psi \), ensuring that the unique equilibrium price for all types of intermediates in the presence of the competitive fringe will be a limit price at \( \psi \).

All tasks can be produced by labor. On the other hand, we model the technological constraints on automation by assuming that there exists \( I \in [N - 1, N] \) such that tasks \( i \leq I \) are technologically automated in the sense that they can feasibly be produced by capital as well. Though tasks \( i \leq I \) are technologically automated, in equilibrium they do not need to be produced with capital. Whether they will or not depends on relative factor prices as we describe below. Conversely, tasks \( i > I \) are not technologically automated. Independently of factor prices, they cannot be produced by capital and must be produced with labor.

Tasks \( i > I \) can only be produced using labor, and their production function takes the form

\[
y(i) = B \left[ \eta q(i) \frac{\zeta - 1}{\zeta} + (1 - \eta) (\gamma(i)l(i)) \frac{\zeta - 1}{\zeta} \right] \frac{\zeta - 1}{\zeta}, \tag{3.2}
\]

where \( \gamma(i) \) denotes the productivity of labor in task \( i \), \( \zeta \in (0, \infty) \) is the elasticity of substitution between intermediates and labor, \( \eta \in (0,1) \) is the distribution parameter of this constant elasticity of substitution production function, and finally, \( B \) is a normalizing constant set as \( B \equiv (1 - \eta)^{\zeta/(1-\zeta)} \) to simplify the algebra.

Tasks \( i \leq I \) can be produced using labor or capital, and their production function takes

---

\(^{10}\)This formulation imposes that once a new task is created at \( N \), it will automatically be utilized and as a consequence, will replace the lowest available task located at \( N - 1 \). In Section 3.2, we provide conditions under which firms will indeed prefer to utilize such tasks immediately (see footnote 13). The reason why, once adopted, they replace older tasks is technological: as already noted, there is a unit measure of tasks, so newly created tasks are simply higher productivity versions of already existing tasks; moreover, we are also assuming that task \( i \) is not compatible and will not be used together with tasks \( i' < i - 1 \) (see also footnote 18).
the form
\[ y(i) = B \left[ \eta q(i) \frac{\xi - 1}{\xi} + (1 - \eta) \left( k(i) + \gamma(i) l(i) \right) \frac{\xi - 1}{\xi} \right] ^{\frac{\xi - 1}{\xi}}. \] (3.3)

All of the parameters are thus common between the production function of tasks above and below the threshold \( I \), with the only difference being that those with \( i \leq I \) can be produced by capital as well as labor. This feature is embedded in (3.3) via the assumption that capital and labor are perfect substitutes in the production of these tasks.\(^{11}\)

Throughout, we assume that \( \gamma(i) \) is strictly increasing, so that labor has strict comparative advantage in tasks with a high index. In the next section, we strengthen this assumption by imposing a parametric form for \( \gamma(i) \), which will ensure that the productivity gains from the creation of new tasks generate balanced growth (see in particular, equation (3.12)). The key implication of the strict comparative advantage of labor in high-index tasks is that, in equilibrium, there will exist some threshold task \( I^* \leq I \) such that all tasks \( i \leq I^* \) are produced with capital, while all tasks \( i > I^* \) are produced with labor (see below).\(^{12}\)

Figure 1 depicts the resulting allocation of tasks to factors and also shows how, as already noted, the creation of new complex tasks replaces existing tasks from the bottom of the distribution.

In the static model, we take the capital stock to be fixed at \( K \) (we will endogenize it via household saving decisions in Section 3.2). In addition, since we wish to study the impact of new technologies not just on factor prices but also on employment, we assume that the employment level is given by a quasi-labor supply taken to be an increasing function of the wage rate \( W \) relative to capital payments \( RK \), i.e., \( L^* \left( \frac{W}{RK} \right) \). This quasi-labor supply curve thus implies that as the wage rate increases relative to payments to capital, the employment level increases as well. As we show in Section 3.2, the assumption that the level of employment depends on the ratio \( \frac{W}{RK} \) and not simply on wages ensures that this quasi-labor supply will be consistent with balanced growth (see Acemoglu 2003). Though we impose this as a reduced-form in the text, it is straightforward to derive it from various micro foundations as we do in Appendix B.

With this specification of the supply of factors, capital and labor market clearing imply the conditions
\[ \int_{N-1}^{N} k(i) di = K, \quad \text{and} \quad \int_{N-1}^{N} l(i) di = L^* \left( \frac{W}{RK} \right). \]

\(^{11}\)A simplifying feature of the technology described in equation (3.3) is that capital has the same productivity in all tasks, while labor has a different productivity. This assumption could be relaxed at the cost of additional complexity in our notation.

Another, perhaps more important simplifying assumption is that high-index tasks can be produced with just labor. Having these tasks combine labor and capital would have no impact on our main results, and in Appendix B, we show how adding an additional layer of capital-labor substitution in (3.2) and (3.3) has no substantive impact on our results.

\(^{12}\)Without loss of generality, we impose that when indifferent firms use capital. This explains our convention of writing that all tasks \( i \leq I^* \) (rather than \( i < I^* \)) are produced using capital.
3.1.2 Equilibrium in the Static Model

We now characterize the equilibrium in the static model, which can be summarized by the wage rate, $W$, the rental rate of capital (rental rate for short), $R$, and the equilibrium threshold $I^*$. 

We proceed by characterizing the unit cost of producing each task as a function of factor prices and the automation possibilities represented by $I$. Because tasks are produced competitively, their price, $p(i)$, will be equal to the minimum unit cost of production:

$$
p(i) = \begin{cases} 
  c^u \left( \min \left\{ R, \frac{W}{\gamma(i)} \right\} \right) = \left[ \left( \frac{\eta}{1-\eta} \right)^{\zeta} \psi^{1-\zeta} + \min \left\{ R, \frac{W}{\gamma(i)} \right\} \right]^{1-\zeta} & \text{if } i \leq I, \\
  c^u \left( \frac{W}{\gamma(i)} \right) = \left[ \left( \frac{\eta}{1-\eta} \right)^{\zeta} \psi^{1-\zeta} + \left( \frac{W}{\gamma(i)} \right)^{1-\zeta} \right]^{1-\zeta} & \text{if } i > I.
\end{cases} \tag{3.4}$$

Here $c^u(\cdot)$ is the unit cost of production for task $i$, derived from the task production functions, (3.2) and (3.3). This unit cost also depends on the price of intermediates, $\psi$, but we suppress this dependence to simplify the notation. The unit cost for tasks $i \leq I$ is written as a function of $\min \left\{ R, \frac{W}{\gamma(i)} \right\}$ to reflect the fact that capital and labor are perfect substitutes in the production of automated tasks. In these tasks, firms will choose whichever factor has a lower effective cost—where the effective cost for labor is $W/\gamma(i)$ in view of the fact that the productivity of labor in task $i$ is $\gamma(i)$.

The expression for $p(i)$ immediately implies that, given strict comparative advantage, there is a threshold $\tilde{I}$ such that tasks below $I^* = \min\{I, \tilde{I}\}$ will be produced using capital.
and the remaining more complex tasks will be produced using labor. Specifically, whenever \( R > W/\gamma(i) \) and \( i \leq I \), the relevant task is produced using capital, and otherwise it is produced using labor.\(^{13}\) Since \( \gamma(i) \) is strictly increasing, this implies that there exists a threshold \( \tilde{I} \) at which, if technologically feasible, firms would be indifferent between using capital and labor. Namely, at task \( \tilde{I} \), we have that \( R = W/\gamma(\tilde{I}) \), or

\[
\frac{W}{R} = \gamma(\tilde{I}).
\]  

(3.5)

This threshold represents the task for which the costs of producing with capital or labor are equal. Without any other constraints, the cost-minimizing allocation of factors is to produce all tasks \( i < \tilde{I} \) with capital. However, if \( \tilde{I} > I \), firms will not be able to use capital all the way up to task \( \tilde{I} \) because of the constraint imposed by the available automation technology. For this reason, the equilibrium threshold below which tasks are produced using capital is given by

\[
I^* = \text{min}\{I, \tilde{I}\}.
\]

To fully characterize the static equilibrium, we next derive the demand for factors in terms of the (endogenous) threshold \( I^* \). We choose the final good as the numeraire, which from (3.1) gives the demand for task \( i \) as

\[
y(i) = Yp(i)^{-\sigma}.
\]

(3.6)

From equations (3.4) and (3.6), equilibrium levels of task production can be written as

\[
y(i) = \begin{cases} 
Yc^u \left( \min \left\{ R, \frac{W}{\gamma(i)} \right\} \right)^{-\sigma} & \text{if } i \leq I, \\
Yc^u \left( \frac{W}{\gamma(i)} \right)^{-\sigma} & \text{if } i > I.
\end{cases}
\]

From the technologies in equations (3.2) and (3.3), it follows that the demand for capital and labor in each task is given by:

\[
k(i) = \begin{cases} 
Yc^u(R)^{1-\sigma}R^{-\zeta} & \text{if } i \leq I^*, \\
0 & \text{if } i > I^*.
\end{cases}
\]

\(^{13}\)This discussion reveals an asymmetry in our treatment of automation and new labor-intensive tasks. As already noted in footnote 10, we have assumed that the latter type of technology is always used when it is created (and hence we have not distinguished \( N, N^* \) and \( \tilde{N} \)). This is because, as we show in Proposition 15, in the interesting part of the parameter space, where the interest rate is not too small (which in turn results from the discount rate in our full model, \( \rho \), being above some threshold \( \tilde{\rho} \)), all new labor-intensive technologies will be used immediately, whereas all new automation technologies may or may not be utilized immediately depending on the relative state of the two types of technologies.
and

\[
l(i) = \begin{cases} 
0 & \text{if } i \leq I^*, \\
Y \gamma(i)^{\zeta-1} c^u \left( \frac{W}{\gamma(i)} \right)^{-\sigma} W^{-\zeta} & \text{if } i > I^*. 
\end{cases}
\]

Thus, capital and labor market clearing yield the following equilibrium conditions:

\[
Y(I^* - N + 1)c^u(R)^{\zeta-\sigma}R^{-\zeta} = K, 
\]

(3.7)

and

\[
Y \int_{I^*}^{N} \gamma(i)^{\zeta-1} c^u \left( \frac{W}{\gamma(i)} \right)^{-\sigma} W^{-\zeta} di = L^* \left( \frac{W}{RK} \right). 
\]

(3.8)

Though the set of equations characterizing an equilibrium are relatively straightforward, the substitution between factors (capital or labor) and intermediates (the \(q(i)\)'s) makes the relative demands for factors non-homothetic, opening the way to counterintuitive results. For instance, automation, which increases the productivity of capital, may end up raising the demand for labor more than the demand for capital. In what follows, we focus on the economically relevant case in which, at given factor proportions, automation reduces the relative demand for labor. The next assumption ensures this by limiting the extent of departures from homotheticity.

**Assumption 1:** One of the following three conditions holds:

1. \( \left( \frac{\gamma(N-1)}{\gamma(N)} \right)^{2+\sigma+\eta} > |\sigma - \zeta|, \) or
2. \( \zeta \to 1, \) or
3. \( \eta \to 0. \)

When \( \sigma = \zeta, \) the elasticities of substitution between tasks and between factors in the production of tasks are equal, ensuring homotheticity. Thus the first option in Assumption 1 requires that this departure from homotheticity is small relative to the inverse of the productivity gains from new tasks (where \( \gamma(N)/\gamma(N-1) \) measures these productivity gains). The second option corresponds to the case where the task production function becomes Cobb-Douglas, which implies that intermediates account for a constant share of costs and also ensures homotheticity. Finally, the third option guarantees homotheticity as well because it makes the share of intermediates in the task production function small. The next proposition completes the characterization of the equilibrium under Assumption 1.

**Proposition 13 (Equilibrium in the Static Model):** Suppose that Assumption 1 holds. Then, given a range of tasks \([N - 1, N] \), automation technology \(I \in (N - 1, N) \), and capital stock \(K \), there exists a unique equilibrium characterized by factor prices, \(W\) and \(R\), and
threshold tasks, $\tilde{I}$ and $I^*$, such that: (i) $\tilde{I}$ is determined by equation (3.5) and $I^* = \min\{I, \tilde{I}\}$; (ii) all tasks $i \leq I^*$ are produced using capital and all tasks $i > I^*$ are produced using labor; (iii) the capital and labor market clearing conditions, equations (3.7) and (3.8), are satisfied; and (iv) factor prices satisfy the ideal price index condition:

$$(I^* - N + 1)c^\sigma(R)^{1-\sigma} + \int_{I^*}^{N} c^\sigma \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} di = 1. \quad (3.9)$$

**Proof.** All of the expressions in this proposition follow from the preceding derivations. Existence and uniqueness are proved in Appendix A.

Figure 3-3 illustrates the unique equilibrium described in Proposition 13. The equilibrium is represented by the intersection of an upward and downward-sloping curve in the $(\omega, I)$ space, which determines $\omega = \frac{W}{ NK}$ and $I^*$. The downward-sloping curve, $\omega(I^*, N, K)$, corresponds to the relative demand for labor, which we obtain by combining the market clearing conditions for capital and labor, (3.7) and (3.8), together with the ideal price index condition, given in equation (3.9). Assumption 1 ensures that the relative demand curve $\omega(I^*, N, K)$ is decreasing in $I^*$—that is, automation reduces the relative demand for labor. The upward-sloping curve represents the cost-minimizing allocation of capital and labor to tasks represented by equation (3.5), with the constraint that the equilibrium level of automation can never exceed $I$.

![Figure 3-3: Static equilibrium. The left panel depicts the case in which $I^* = I < \tilde{I}$ so that the allocation of factors is constrained by technology. The right panel depicts the case in which $I^* = \tilde{I} < I$ so that the allocation of factors is not constrained by technology.](image-url)

The figure distinguishes between the two cases highlighted above. In the left panel, we have $I^* = I < \tilde{I}$ and the allocation of factors is constrained by technology, while the right panel plots the case in which $I^* = \tilde{I} < I$ and firms choose the cost-minimizing allocation given factor prices.

The next proposition gives a complete characterization of comparative statics. In what follows, $\frac{\partial \omega}{\partial I^*}$, $\frac{\partial \omega}{\partial N}$ and $\frac{\partial \omega}{\partial K}$ denote the partial derivatives of $\omega(I^*, N, K)$.
PROPOSITION 14 (COMPARATIVE STATICS) Suppose that Assumption 1 holds.

- If $I^* = I < \bar{I}$—so that the allocation of tasks to factors is constrained by technology—then:
  
  - the impact of technological change on relative factor prices is given by
    
    $$\frac{d\ln(W/R)}{dI} = \frac{d\ln w}{dI} = \frac{1}{\omega} \frac{\partial \omega}{\partial I^*} < 0, \quad \frac{d\ln(W/R)}{dN} = \frac{d\ln w}{dN} = \frac{1}{\omega} \frac{\partial \omega}{\partial N} > 0$$

  - the impact of capital on relative factor prices is given by
    
    $$\frac{d\ln(W/R)}{d\ln K} = \frac{d\ln w}{d\ln K} + 1 = \frac{1 + \varepsilon_L}{\sigma_{SR} + \varepsilon_L} > 0,$$
    
    where $\sigma_{SR} \in (0, \infty)$ is the short-run elasticity of substitution between labor and capital holding the allocation of factors to tasks fixed, and is given by a weighted average of $\sigma$ and $\zeta$.

  - Moreover, if $\sigma$ is sufficiently large, $\frac{d\ln W}{dI}, \frac{d\ln R}{dN} < 0$. Otherwise, $\frac{d\ln W}{dN}, \frac{d\ln R}{dN} \geq 0$.

- If $I^* = \bar{I} < I$—so that tasks are allocated to factors in the unconstrained cost minimizing fashion—then

  - the impact of technological change on relative factor prices is given by
    
    $$\frac{d\ln(W/R)}{dI} = \frac{d\ln w}{dI} = 0, \quad \frac{d\ln(W/R)}{dN} = \frac{d\ln w}{dN} = \frac{1}{\omega} \frac{\partial \omega}{\partial I^* \varepsilon} > 0 \text{ and}$$

  - the impact of capital on relative factor prices is given by
    
    $$\frac{d\ln(W/R)}{d\ln K} = \frac{d\ln w}{d\ln K} + 1 = \left(\frac{1 + \varepsilon_L}{\sigma_{SR} + \varepsilon_L}\right) \frac{1}{1 - \frac{1}{\omega} \frac{\partial \omega}{\partial I^* \varepsilon}} > 0,$$
    
    where $\varepsilon = \frac{d\ln r(I)}{dI} > 0$ is the semi-elasticity of the comparative advantage schedule.

    Here, the medium run elasticity of substitution $\sigma_{MR} \in (0, \infty)$ is

    $$\sigma_{MR} = (\sigma_{SR} + \varepsilon_L) \left(1 - \frac{1}{\omega} \frac{\partial \omega}{\partial I^* \varepsilon}\right) - \varepsilon_L > \sigma_{SR}.$$

  - Moreover, if $\sigma$ is sufficiently large, $\frac{d\ln R}{dN} < 0$. Otherwise, $\frac{d\ln R}{dN} \geq 0$.

---

14In this proposition, we do not explicitly treat the case in which $I^* = I = \bar{I}$ in order to save on space and notation, since in this case left and right derivatives with respect to $I$ are different.
Finally, in both parts of the proposition, the labor share and employment move in the same direction as \( \omega \).

PROOF. See Appendix A.

The main implication of Proposition 14 is that the two types of technological changes—automation and the creation of new complex tasks—have polar implications. Automation, corresponding to an increase in \( I \), reduces \( W/R \), the labor share and employment (unless \( I^* = \bar{I} < I \) and firms are not constrained by technology in their automation choice), while the creation of new tasks, corresponding to an increase in \( N \), raises \( W/R \), the labor share and employment.

These comparative static results are illustrated in Figure 3-3: automation moves us along the relative labor demand curve in the technology-constrained case shown in the left panel (and has no impact in the right panel), while the creation of new tasks shifts out the relative labor demand curve in both cases.

Another important implication of Proposition 14 is that when \( I^* = I \), automation—an increase in \( I \)—can reduce real wages. This happens because automation expands the range of tasks performed by capital and contracts the set of tasks performed by labor. This last feature, combined with the diminishing returns to the quantity of a task in the aggregate production function, (3.1), puts downward pressure on the wage, but is counteracted by a positive effect coming from the fact that tasks are \( q \)-complements in (3.1). This positive effect is weaker when \( \sigma \) is greater, explaining why the overall impact of automation on the wage rate is negative when \( \sigma \) is large.15 Likewise, when \( \sigma \) is large the creation of new complex tasks—that is, an increase in \( N \)—can reduce the rental rate.

These results are major consequences of the task-based framework developed here. With factor-augmenting technologies, technological improvements always increase the price of both capital and labor, but this is no longer the case when technological change alters the range of tasks performed by both factors (see also Acemoglu and Autor, 2011).16 Furthermore, as it is well known, with factor-augmenting technologies, whether different types of technological improvements are biased towards one factor or the other depends on the elasticity of substitution. But this too is different in our task-based framework, where automation is always capital-biased (that is, it reduces \( W/R \)), while the creation of new complex tasks is always labor-biased (that is, it increases \( W/R \)).

A final implication of Proposition 14 is that the short-run elasticity of substitution be-

---

15 We could have a negative impact of automation on wages even for moderate values of \( \sigma \). For example, if \( \sigma = \zeta = 1 \) and \( K/Y < 2.72 \), automation reduces the marginal product of labor.

16 For instance, with a constant returns to scale production function and two factors, capital and labor are \( q \)-complements. Thus, capital-augmenting technologies always increases the marginal product of labor. To see this, let \( F(A_K, A_L, L) \) be such a production function. Then \( W = F_L \), and \( \frac{\partial W}{\partial A_K} = KF_{LK} = -LF_{LL} > 0 \) (because of constant returns to scale).
tween capital and labor differs from the “medium-run” elasticity. The short-run elasticity, \(\sigma_{SR}\), applies when the range of tasks allocated to capital and labor is held fixed (as in the case where \(I^* = I\)). The medium-run elasticity, \(\sigma_{MR}\), applies when the allocation of factors to different tasks responds to changes in factor prices (as in the case where \(I^* = \tilde{I}\)).

Though Proposition 14 provides a complete characterization of the responses of relative factor prices, factor shares and employment to automation and the creation of new complex tasks, the results are qualitative and the explicit expressions for the partial derivatives are complicated. This problem arises because of the aforementioned non-homotheticity in the demand for capital and labor. In two of the special cases mentioned above, when \(\eta \to 0\) (the share of intermediates goes to zero), and when \(\zeta \to 1\) (intermediates represent a constant share of the production of tasks), we can provide an explicit characterization of the equilibrium and the comparative statics. We further simplify the presentation of these results by taking \(L(\omega) = L\) (and \(\varepsilon_L = 0\)), so that the quasi-labor supply coincides with the inelastic labor supply in the economy.

In both of these special cases we obtain a revealing expression for aggregate output:

\[
Y = \left[ (I^* - N + 1) \frac{1}{\hat{\sigma}} K^{\frac{\hat{\sigma} - 1}{\hat{\sigma}}} + \left( \int_{I^*}^{N} \gamma(i)^{\hat{\sigma} - 1} di \right) \frac{1}{\hat{\sigma}} L^{\frac{\hat{\sigma} - 1}{\hat{\sigma}}} \right]^{\frac{\hat{\sigma}}{\hat{\sigma} - 1}},
\]

(3.10)

where \(\hat{\sigma} \equiv \eta + (1 - \eta)\sigma\) (which also implies that when \(\eta \to 0\), we have \(\hat{\sigma} = \sigma\)).

This expression emphasizes that, in these special cases, output is a constant elasticity of substitution aggregate of capital and labor (with the short-run elasticity of substitution between capital and labor, \(\sigma_{SR}\), simply being equal to \(\hat{\sigma}\)). Critically, the distribution parameters are endogenous and depend on the state of the two types of technologies in the economy. Indeed, automation increases the share of capital and reduces the share of labor in this aggregate production function, while the creation of new complex tasks does the opposite.

In these cases, the relative demand for labor can be obtained directly by differentiating (3.10):

\[
\ln \omega = \left( \frac{1}{\hat{\sigma}} - 1 \right) \ln K + \frac{1}{\hat{\sigma}} \ln \left( \int_{I^*}^{N} \gamma(i)^{\hat{\sigma} - 1} di \right).
\]

(3.11)

\(^{17}\)Another noteworthy corollary of this proposition is that a long-run negative association between capital accumulation and the labor share is not sufficient to conclude that \(\sigma\)—the elasticity of substitution between labor and capital—is above 1 (e.g., as argued by Karabarbounis and Neiman, 2014). This reasoning is valid only when technology is factor-augmenting, but not when the allocation of tasks to factors is endogenous. In particular, when automation responds to an increase in the capital stock, we could have a situation in which capital accumulation reduces the labor share, regardless of whether \(\sigma \lessgtr 1\). For example, when \(\sigma = \zeta = 1\) and \(I^* = I\), as we show in Corollary 2, factor shares in our model are entirely independent of \(\sigma\), and just depend on the extent of automation and the creation of new tasks.
In these two special cases, equation (3.11) provides a more explicit characterization of the comparative statics derived in Proposition 14.

**Corollary 2** Suppose $\eta \to 0$ or $\zeta \to 1$. Then, there exists a unique equilibrium, and

- If $I^* = I < \bar{I}$:

$$d \ln \omega = \left( \frac{1}{\bar{\sigma}} - 1 \right) d \ln K + \frac{1}{\bar{\sigma}} \left[ \frac{\gamma(N)^{\bar{\theta}-1}}{\int_{I^*}^{\bar{N}} \gamma(i)^{\bar{\theta}-1} di} + \frac{1}{I^* - N + 1} \right] dN - \frac{1}{\bar{\sigma}} \left[ \frac{\gamma(I^*)^{\bar{\theta}-1}}{\int_{I^*}^{\bar{N}} \gamma(i)^{\bar{\theta}-1} di} + \frac{1}{I^* - N + 1} \right] dI.$$

- If $I^* = \bar{I} < I$:

$$d \ln \omega = \left( \frac{1}{\bar{\sigma} + \Lambda/\varepsilon \gamma} - 1 \right) d \ln K + \frac{1}{\bar{\sigma} + \Lambda/\varepsilon \gamma} \left[ \frac{\gamma(N)^{\bar{\theta}-1}}{\int_{I^*}^{\bar{N}} \gamma(i)^{\bar{\theta}-1} di} + \frac{1}{I^* - N + 1} \right] dN,$$

where

$$\Lambda \equiv \frac{\gamma(I^*)^{\bar{\theta}-1}}{\int_{I^*}^{\bar{N}} \gamma(i)^{\bar{\theta}-1} di} + \frac{1}{I^* - N + 1} > 0,$$

and $\bar{\sigma} \equiv \eta + (1 - \eta)\sigma$.

In this corollary, the difference between the short-run and the medium-run elasticity of substitution can be seen quite clearly: $\sigma_{SR} = \bar{\sigma}$, and $\sigma_{MR} = \bar{\sigma} + \Lambda/\varepsilon \gamma$. Moreover, the effect of shifts in technology are the same as in Proposition 14.

### 3.2 Dynamics, Balanced Growth and the Productivity Effect

In this section, we extend our model to a dynamic economy in which the evolution of the capital stock is determined by the saving decisions of a representative household. We then investigate the conditions under which the economy admits a balanced growth path (BGP), where output, the capital stock and wages grow at a constant rate. We conclude by discussing the long-run effects of automation on wages, the labor share and employment, and by highlighting an important “productivity effect,” which stems from capital accumulation and creates a force from automation towards higher wages.

#### 3.2.1 Balanced Growth

In order to ensure balanced growth, we need to put more structure on the comparative advantage schedule. Specifically, because balanced growth is driven by technology, and in
this model sustained technological change comes from the creation of new complex tasks, constant growth requires the productivity gains from new tasks to be exponential:

\[ \gamma(i) = e^{Ai} \text{ with } A > 0. \]  

(3.12)

We impose this functional form in the remainder of the paper. We also impose a simplified version of Assumption 1 under this functional form:

**Assumption 1':** One of the following three conditions are satisfied:

1. \( e^{-\left(2+2\sigma+\eta\right)A} > |\sigma - \zeta| \), or
2. \( \zeta \to 1 \), or
3. \( \eta \to 0 \).

We start by assuming exogenous paths for technological change, given by \( \{I(t), N(t)\} \), and we define

\[ n(t) \equiv N(t) - I(t) \]

as a summary measure of the state of technology. Automation reduces \( n(t) \), and conversely, as more new complex tasks are created, \( n(t) \) increases. We simplify the discussion and notation by assuming that \( I^*(t) = I(t) \), so that newly automated tasks are immediately produced with capital. We discuss the conditions that ensure \( I^*(t) = I(t) \) in Proposition 15 below.

Let \( \{I(t), N(t), K(t)\} \) denote the path of the state variables: technology and capital. Also, let \( \{R(t), W(t), Y(t), C(t)\} \) denote the path of factor prices, the equilibrium output, and the representative household’s consumption at each period. We assume that the representative household’s preferences over consumption paths, \( \{C(t)\} \), are given by

\[ \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1 - \theta} dt. \]  

(3.13)

The resource constraint of the economy takes the form

\[ \dot{K}(t) = Y(t) - C(t) - \delta K(t) - \psi \mu \int_{N-1}^N q(i,t) \, di, \]

\[ \delta = \frac{\psi \mu}{\int_{N-1}^N q(i,t) \, di}. \]

18 As usual we could have imposed this functional form only asymptotically, but we simplify the analysis and exposition by imposing it at all times.

Notice also that in this dynamic economy, as in our static model, the productivity of capital is the same in all automated tasks. This does not, however, imply that any of the previously automated tasks can be used regardless of \( N \). As \( N \) increases, as emphasized by equation (3.1) and in footnote 10, the set of feasible tasks shifts to the right, and only tasks above \( N - 1 \) remain compatible with and can be combined with those currently in use.
where $Y(t)$ continues to be given by (3.1), and $\delta$ is the depreciation rate of capital. Recall also that $\psi \mu$, with $\mu \in [0,1]$, parametrizes the marginal cost of producing intermediates (we maintain an exogenous markup of $1 - \mu > 0$ for intermediate goods, which plays no important role until the next section).

We characterize the equilibrium in terms of the normalized variables $y(t) \equiv Y(t)/\gamma(I(t))$, $k(t) \equiv K(t)/\gamma(I(t))$, and $c(t) \equiv C(t)/\gamma(I(t))$. We also define two relevant normalizations for wages: $w_I(t) \equiv W(t)/\gamma(I(t))$, which is the effective wage at the least complex task that has not yet been automated, and $w_N(t) \equiv W(t)/\gamma(N(t))$, which is the effective wage at the newest, most complex task. Finally, as in our static model, $R(t)$ denotes the rental rate, and the interest rate on savings is $r(t) = R(t) - \delta$.

At each point in time, technology and capital, $n(t)$ and $k(t)$, fully determine output and factor prices as in the static equilibrium. Specifically, the market clearing conditions for capital and labor, (3.7) and (3.8), and the ideal price index condition, (3.9), give the following equilibrium conditions:

$$y(t)(1 - n(t))e^u(R(t))^{\zeta - \sigma} R(t)^{-\zeta} = k(t),$$

$$y(t) \int_0^{n(t)} \gamma(i)^{\zeta - 1} c^u \left(\frac{w_I(t)}{\gamma(i)}\right)^{\zeta - \sigma} w_I(t)^{-\zeta} di = L^\sigma \left(\frac{w_I(t)}{R(t)k(t)}\right),$$

$$(1 - n(t))e^u(R(t))^{1 - \sigma} + \int_0^{n(t)} c^u \left(\frac{w_I(t)}{\gamma(i)}\right)^{1 - \sigma} di = 1$$

Proposition 13 coupled with Assumption I' guarantees that the equilibrium rental rate can be uniquely written as $R(t) = R(n(t), k(t))$. We also denote the normalized output net of intermediate costs by $f^E(n(t), k(t))$ and the growth rate of $\gamma(I(t))$ by $g(t)$.

Using this notation, we can describe the dynamic equilibrium of our model as paths for $c(t)$ and $k(t)$ that satisfy the Euler equation,

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta} (R^E(n(t), k(t)) - \delta - \rho) - g(t), \quad (3.14)$$

the resource constraint,

$$\dot{k}(t) = f^E(n(t), k(t)) - c(t) - (\delta + g(t))k(t), \quad (3.15)$$

and the household’s transversality condition,

$$\lim_{t \to \infty} (k(t) + \Pi(t))e^{-\int_0^t (\sigma - (1 - \theta)g(s))da} = 0, \quad (3.16)$$

135
where $\Pi(t)$ denotes the (normalized) net present value of corporate profits, which results from the presence of the monopoly markup. The dynamic equilibrium is defined for any exogenous path for technology, which is fully summarized by \{n, g\}.

Consider an exogenous path for technology in which $\dot{I} = \dot{N} = \Delta$ so that both automation and the creation of new complex tasks advance in tandem. Along such a path $n(t) \to n$ and $g(t) \to A\Delta$. Figure 3-4 presents the phase diagram for the system of differential equations (3.14) and (3.15) in this case, which fully determines the structure of the dynamic equilibrium. The structure of the above system resembles the standard neoclassical growth model, except that we use a different normalization and the exogenous markups on intermediate goods introduce a wedge between $R$ and $\partial f^E/\partial k$.

![Phase diagram](image)

**Figure 3-4**: Balanced growth path and dynamic equilibrium when technology is exogenous and satisfies $n(t) \to n$ and $g(t) \to A\Delta$.

We define a BGP as an allocation in which $Y(t), C(t), K(t)$ and $W(t)$ grow at a constant rate and $R(t)$ is constant. The next proposition characterizes the conditions under which the asymptotic behavior of this economy converges to a BGP.

**Proposition 15 (Dynamic equilibrium with exogenous technological change)**

Suppose that Assumption 1' holds. There exists a threshold $\bar{\rho}$ such that for $\rho > \bar{\rho}$, we have:

1. There exists $\bar{n}$ such that for $n(t) < \bar{n}$, we have $I^*(t) < I(t)$, while for $n(t) \geq \bar{n}$, $I^*(t) = I(t)$.

2. Suppose that there exists $T$ such that for $t \geq T$, $n(t) \in [\bar{n}, 1]$. Then a BGP exists (and is unique) if and only if asymptotically $\dot{N} = \dot{I} = \Delta$, so that $\lim_{t \to \infty} n(t) = n$. In this BGP, $I^*(t) = I(t); Y(t), C(t), K(t)$ and $W(t)$ grow at a constant rate $g = A\Delta$; and $R$ is constant.
3. Suppose instead that there exists T such that for \( t \geq T \), \( n(t) < \bar{n} \). Then a BGP exists (and is unique) if and only if asymptotically \( N(t) = \Delta \). In this BGP \( I^*(t) < I(t) \) and \( N(t) - I^*(t) \rightarrow \bar{n} \); \( Y(t), C(t), K(t) \) and \( W(t) \) grow at a constant rate \( g = A\Delta \); and \( R \) is constant.

4. Moreover, given such a path of technological change (with \( n(t) \in [\bar{n}, 1] \) or \( n(t) \leq \bar{n} \) for all \( t \geq T \)), the dynamic equilibrium is unique. Starting from any level of capital the economy converges to the unique BGP.

PROOF. See Appendix A. ■

The key implication of Proposition 15 is that balanced growth results from the simultaneous process of automation and the creation of new complex tasks. But the proposition also highlights that this process needs to be “balanced” itself: the two types of technologies need to advance at the same rate so that \( \lim_{t \to \infty} n(t) = n > \bar{n} \) (in the more interesting case where automated tasks are immediately produced with capital). Concerns about automation we cited in the Introduction notwithstanding, the proposition shows that a process of continuous automation is compatible with balanced growth.

The additional requirement in Proposition 15, that \( \rho > \bar{\rho} \), ensures that the long-run equilibrium interest rate is not too close to 0. Provided that this is case, newly created complex tasks will be immediately utilized and produced with labor. If, instead, \( \rho \) were close to zero, capital would be so cheap that new complex tasks might remain unutilized (at least for a while) in a BGP. Thus, the condition \( \rho > \bar{\rho} \) defines the relevant range of parameters for our investigation (and it is in this range that \( \bar{n} \) is well-defined). Appendix A also characterizes the equilibrium when \( \rho < \bar{\rho} \).

Proposition 15 can be further illustrated and strengthened in the two special cases considered in the previous section; \( \eta \to 0 \) or \( \zeta \to 1 \). Suppose that, as required in part 2 of the proposition, we have \( \bar{N} = \bar{I} = \Delta \). The aggregate production function in equation (3.10) can be simplified to \( Y(t) = f(K(t), B(t)L) \), with

\[
B(t) = \left( \int_{I(t)}^{N(t)} \gamma(i)^{\delta-1} di \right)^{\frac{1}{\delta-1}} = e^{A\bar{I}(t)} \left( \frac{e^{A(\delta-1)n(t)} - 1}{A \delta - 1} \right)^{\frac{1}{\delta-1}},
\]

so that \( B(t) \) grows at a rate \( A\Delta \). This example shows that the net effect of balanced automation and the creation of new technologies is to augment labor. Intuitively, technology becomes purely labor augmenting on net because labor and capital perform a fixed share of

\[19\] The conditions \( \rho > \bar{\rho} \) and \( \lim_{t \to \infty} n(t) \geq \bar{n} \) are not restrictive. Focusing on a standard annual parametrization of our model with \( \theta = 1, \delta = 0.06, g = 0.016, \sigma = 0.5, \zeta = 0.2 \) (so that the elasticity of substitution between capital and labor lies between 0.5 and 0.2), \( A = 2, \eta = 0.5 \) and \( \psi = 0.9 \), we obtain \( \bar{\rho} = 0.012 \), so that the standard value of the discount rate, \( \rho = 0.05 \), is comfortably above this threshold. These parameters also imply \( \bar{n} = 0.96 \).
tasks, and the creation of new tasks directly increases the productivity of labor. This special case of our model also provides a direct connection between Proposition 15 and Uzawa’s Theorem, which implies that balanced growth requires a representation of the production function with purely labor-augmenting technological change (e.g., Acemoglu, 2009, or Grossman, Helpman and Oberfield, 2016).

### 3.2.2 The Productivity Effect

We now study the dynamic implications of a permanent decline in \( n(t) \), which in this dynamic setup corresponds to automation running ahead of the creation of new tasks. Because in the short run capital is fixed, the short-run implications of this change in technology are the same as in our static analysis in the previous section. However, over the long run, capital accumulation responds to the shift in technology in such a way as to keep the interest rate constant at its initial level, and in consequence, all the productivity gains from automation will accrue to the relatively inelastic factor, labor. Thus, the dynamic economy underscores the role of another economic force, which we call the productivity effect: automation, by enabling the substitution of the cheaper capital for labor, increases productivity and thus the demand for labor and wages.\(^{20}\) The productivity effect has been present in our analysis so far (in particular, it is because of this effect that automation need not reduce real wages in the short run), but as we show next, due to the induced capital accumulation, it becomes more powerful in the long run.

The next proposition characterizes the long-run impact of automation on factor prices and shares.

**Proposition 16 (Long-run Comparative Statics)** Suppose that technology evolves exogenously and satisfies \( \lim_{t \to \infty} n(t) = n \), Assumption 1’ holds, and \( \rho > \bar{\rho} \) (where \( \bar{\rho} \) was introduced in Proposition 15). Also, let \( \bar{n} \) be the threshold introduced in Proposition 15. Then we have:

1. For \( n < \bar{n} \), small changes in \( n \) do not affect the asymptotic behavior of the economy.
2. For \( n > \bar{n} \), we have that:
   
   - The long-run rental rate \( R = \lim_{t \to \infty} R(t) \) is equal to \( \rho + \delta + \theta g \), and is thus independent of the extent of automation given by \( n \). The capital stock adjusts in the long run so that \( R^E(n, k) = \rho + \delta + \theta g \).

\(^{20}\)This is similar to the productivity or efficiency effect in models of offshoring such as Grossman and Rossi-Hansberg (2008), Rodriguez-Clare (2010) and Acemoglu, Gancia and Zilibotti (2015), which results from the substitution of cheaper foreign labor for domestic labor in certain tasks.
• Long-run effective wages, \( w_I(n) = \lim_{t \to \infty} w_I \) and \( w_N(n) = \lim_{t \to \infty} w_N \), depend only on \( n \). Moreover, \( w_I(n) \) is increasing and \( w_N(n) \) is decreasing in \( n \).

• The long-run labor share, \( \omega \), and the employment rate are increasing in \( n \).

PROOF. See Appendix A.

This proposition illustrates the role of the productivity effect, and why this effect becomes more important in the long run. In the BGP, capital adjusts to keep the rental rate fixed at \( R = \rho + \delta + \theta g \), and thus independent of the extent of automation summarized by \( n \). In consequence, although in the short run the productivity gains from both technologies accrue to both factors, in the long run they only accrue to the inelastic factor—labor.

More specifically, the long-run behavior of wages is fully determined by the ideal price index condition, (3.9), which we can rewrite as

\[ 1 = (1 - n)c'(\rho + \delta + \theta g) + \int_0^n c'(w_I(i))^{1-\sigma} di = (1 - n)c'(\rho + \delta + \theta g) + \int_0^n c'(w_N(i))^{1-\sigma} di. \]

Taking derivatives, we obtain

\[ w_I'(n) = A w_I(n) \frac{c'(\rho + \delta + \theta g)^{1-\sigma} - c'(w_N(n))^{1-\sigma}}{c'(w_I(n))^{1-\sigma} - c'(w_N(n))^{1-\sigma}} > 0, \]  

\[ w_N'(n) = A w_N(n) \frac{c'(\rho + \delta + \theta g)^{1-\sigma} - c'(w_I(n))^{1-\sigma}}{c'(w_I(n))^{1-\sigma} - c'(w_N(n))^{1-\sigma}} < 0. \]

The signs of \( w_I'(n) \) and \( w_N'(n) \) can be derived from the following observations: since we are in the region in which \( \rho > \bar{\rho} \) and \( n > \bar{n} \), we have \( w_I(n) > \rho + \delta + \theta g > w_N(n) \) (which is equivalent to both types of technologies being immediately utilized). These inequalities then imply that \( w_I'(n) > 0 \) and \( w_N'(n) < 0 \).

The behavior of the effective wages also implies that, following a permanent increase in \( I(t) \) (to \( I(t) + \nu \) with \( \nu > 0 \) for all \( t \geq T \)), \( W(t) \) will eventually rise above its initial trajectory. Likewise, the creation of new complex tasks increases wages in the sense that a permanent increase in \( N(t) \) (to \( N(t) + \nu \) with \( \nu > 0 \) for all \( t \geq T \)) will increase \( W(t) \) above its initial path.\(^{21}\)

In summary, although the results from the static model continue to apply in the short run when capital does not adjust, because of the productivity effect, the potential negative impact of automation on the equilibrium wage level disappears in the long run. This is illustrated in Figure 3-5, which plots the dynamic behavior of the equilibrium wage, \( W \), following a permanent decline in \( n \). Even though the equilibrium wage may fall in the short

\(^{21}\)The first claim follows because \( W(t) \to w_N(n) \gamma(N(t)) \), and due to the productivity effect, automation raises \( w_N(n) \). The second claim follows because \( W(t) \to w_I(n) \gamma(I(t)) \), and again due to the productivity effect, the creation of new complex tasks raises \( w_I(n) \).
run following a surge in automation (if $\sigma$ is sufficiently large, as established in Proposition 14), it necessarily increases in the long run because of the induced capital accumulation and the productivity effect.\textsuperscript{22} The duration of the period with stagnant or depressed wages depends on $\theta$, which determines the speed of capital adjustment.\textsuperscript{23}

Importantly, Proposition 16 also establishes that, while the wage rate increases in the long run with automation (a decrease in $n$), the long-run labor share and employment decrease with automation. In fact, Appendix B shows that, following a wave of automation, if $\sigma_{SR} < 1$, capital accumulation mitigates the short-run response of the labor share but does not fully offset it; while, if $\sigma_{SR} > 1$, capital accumulation further depresses the labor share. In light of these results, the recent decline in the labor share and the employment to population ratio can be interpreted as a consequence of automation outpacing the creation of new labor-intensive tasks over the last two decades. This phenomenon could be accompanied by stagnant or lower wages in the short-run while capital adjusts to its new BGP level.

\section*{3.3 Full Model: Tasks and Endogenous Technologies}

The previous section established the existence of a BGP under the assumption that $\dot{N} = \dot{I} = \Delta$. But why should these two types of technologies advance at the same rate? This is the

\footnote{The expression for $w_I(n)$ also shows that when $w_I(n) \approx R$—which corresponds the case where productivity gains from automation are modest—an increase in automation always reduces wages in the short run and leaves them approximately unchanged in the long run. This is because, in this case, productivity gains from automation are limited, and thus the long-run benefit of the inelastic factor, labor, is also limited.}

\footnote{This comparative static serves as an alternative explanation for what historian Robert C. Allen termed the “Engel’s pause;” the period covering the first half of the 19th century in Britain (see Allen, 2009). During this period, real wages stagnated while output per worker and capital profits increased. Wages started increasing in the second half of the 19th century. These patterns are consistent with the dynamic predictions of our model following a wave of automation.}
question at the center of our paper, and to answer it, we now develop our full model, which endogenizes the pace at which automation and the creation of new complex tasks proceeds.

3.3.1 Endogenous and Directed Technological Change

We endogenize technological change by allowing new intermediates, which embody the technologies that automate existing tasks or create new complex tasks, to be introduced by technology monopolists. We assume that successful innovations always achieve automation or the creation of new tasks in the order of the intermediate indices, \( i \in [0, \infty) \). As a consequence, automation and the creation of new complex tasks correspond, respectively, to an increase in \( I \) and to an increase in \( N \). We continue to assume that all intermediates, including those that have just been invented, can be produced at the fixed marginal cost of \( \mu \psi \), and that the fringe of competitive firms forces the technology monopolists to price intermediates at \( \psi \), which implies an exogenous per-unit profit of \( (1 - \mu)\psi \).

In this section, we adopt a structure of intellectual property rights whereby automating or replacing an existing task is viewed as an infringement of the patent of the technology previously used to produce that task. Consequently, a firm automating a task must license or buy the relevant patent from the technology monopolist that supplied the intermediate good used in combination with labor to produce this task. Similarly, a firm introducing a new complex task, which is creating a new more complex version of an existing task, will have to obtain the relevant patent from the previous technology monopolist of that task. We finally assume that both purchases take place with the new inventors making a take-it-or-leave-it offer to the holder of the existing patent.

This game form ensures that each technology monopolist will receive the same flow of revenues regardless of whether its own intermediate is replaced or not—either as profits when he is operating or as payments for its patent when he is replaced. Moreover, because new entrants must compensate the incumbent technology monopolists that they replace, our patent structure removes the creative destruction of profits, which is present in other models of quality improvements under the alternative assumption that new firms do not have to respect the intellectual property rights of the technology on which they are building (e.g., Aghion and Howitt, 1992; Grossman and Helpman, 1991). In Section 3.5, we explore how our main results change when the intellectual property rights regime allows for the creative destruction of profits.

We assume that innovation of both types requires scientists.\(^{24}\) There is a fixed supply of

\(^{24}\)Focusing on an innovation possibilities frontier using just scientists, rather than variable factors such as in the lab-equipment specifications (see Acemoglu 2009), is convenient because it enables us to focus on the direction of technological change—and not on the overall amount of technological change—especially when we turn to the welfare analysis in the next section.
scientists, which will be allocated to automation \( (S_I(t) \geq 0) \) or the creation of new tasks \( (S_N(t) \geq 0) \),

\[
S_I(t) + S_N(t) \leq S.
\]

When a scientist is employed in automation, she automates \( \kappa_I \) tasks per unit of time, and receives a wage \( W_I^S \). When she is employed in the creation of new tasks, she creates \( \kappa_N \) new tasks per unit of time, and receives a wage \( W_N^S \). However, there is also a comparative advantage structure to the allocation of scientists in that the cost of effort for scientists depends on their exact skills and the type of technology on which they are working. More specifically, we assume that when working in automation, scientist \( i \) incurs a cost of \( \chi_I^i Y(t)/\lambda \), and when working in the creation of new tasks, she incurs a cost of \( \chi_N^i Y(t)/\lambda \). These costs are multiplied with aggregate output in the economy to ensure balanced growth,\(^{25}\) while \( \lambda > 0 \) parameterizes the importance of wage income for scientists relative to the effort cost. We assume that the distribution of \( \chi_N^i - \chi_I^i \) among scientists is given by a continuous and strictly increasing distribution function \( G \) over a support containing 0 as an interior point. For notational convenience, we also adopt the normalization \( G(0) = \frac{\kappa_N}{\kappa_I + \kappa_N} \). This formulation ensures that the allocation of scientists responds to the relative profitability of innovation in the two sectors in a "smooth" fashion, avoiding discontinuous shifts which otherwise complicate the analysis of dynamics. As \( \lambda \to \infty \), we recover these discontinuous shifts, while in the case where \( \lambda = 0 \), a constant fraction \( \frac{\kappa_N}{\kappa_I + \kappa_N} \) of scientists will work on automation regardless of the profitabilities of the two types of innovation (thus making technological change "undirected").

The productivity of scientists into two types of innovations, together with their comparative advantages, determine their wages in these two sectors, \( W_I^S \) and \( W_N^S \). Since comparative advantage only affects the costs of effort and all scientists have the same productivity in the two sectors, the innovation possibility frontier of the economy can be summarized as

\[
\hat{I}(t) = \kappa_I S_I(t), \quad \hat{N}(t) = \kappa_N S_N(t).
\]

### 3.3.2 Equilibrium with Endogenous Technological Change

To characterize the equilibrium with endogenous technological change, we need to compute the value functions that determine the net present discounted value accruing to monopolists from automation and the creation of new complex tasks. We denote these value functions by \( V_I(t) \) and \( V_N(t) \), respectively. More specifically, \( V_I(t) \) is the value of automating the task at \( i = I(t)^+ \) (i.e., the highest-indexed task that has not yet been automated, or more formally

\(^{25}\)Alternatively, these costs could be in terms of the scientists' wage with very similar implications.
Throughout we also assume that the costs of effort are sufficiently low that all scientists will work either in automation or the creation of new tasks.
\( i = I(t) + \varepsilon \text{ for } \varepsilon \text{ arbitrarily small and positive} \). Likewise, \( V_N(t) \) is the value of a new technology creating a more complex task at \( i = N(t)^+ \).

An equilibrium with endogenous technology is given by paths \( \{K(t), N(t), I(t)\} \) for capital and technology (starting from initial values \( K(0), N(0), I(0) \)) paths \( \{R(t), W(t), W^S(t), W^N(t)\} \) for factor prices, paths \( \{V_N(t), V_I(t)\} \) for the value functions of technology monopolists, and paths \( \{S_N(t), S_I(t)\} \) for the allocation of scientists such that all markets clear, all firms, including prospective technology monopolists, maximize profits, the representative household maximizes its utility, and \( N(t) \) and \( I(t) \) evolve endogenously according to equation (3.19).

We start by characterizing the value functions for technology monopolists. Suppose that in this equilibrium \( n(t) > \bar{n} \), so that \( I^*(t) = I(t) \) and new automated tasks start being produced with capital immediately. The flow profits accruing to a technology monopolists selling an intermediate good \( q(i) \) for an automated task \( i \) are

\[
\pi_I(t, i) = (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^{\zeta} \psi^{1 - \zeta} Y(t) c^u(R(t))^{\zeta - \sigma}. \tag{3.20}
\]

Intuitively, these profits come from the ability of firms to produce task \( i \) using capital, which is embodied in the intermediate good provided by the technology monopolist. Similarly, the flow profits accruing to a technology monopolist that sells an intermediate good \( q(i) \) for a non-automated technology (for labor with productivity \( \gamma(i) \) in the corresponding task) are

\[
\pi_N(t, i) = (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^{\zeta} \psi^{1 - \zeta} Y(t) c^u \left( \frac{W(t)}{\gamma(i)} \right)^{\zeta - \sigma}. \tag{3.21}
\]

It is then straightforward to compute the offer that a monopolist with a new technology (embodied in its intermediate good) automating task \( I^+ \) at time \( t \) needs to make to the firm currently holding the patent for the (labor-intensive) technology of that intermediate. This offer will be given by the net present discounted value of the profit streams, discounted using the path of future interest rates, that the existing patent-holder would obtain

\[
(1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^{\zeta} \psi^{1 - \zeta} \int_t^\infty e^{-\int_t^\tau (R(s) - \delta) ds} Y(\tau) c^u \left( \frac{W(\tau)}{\gamma(I)} \right)^{\zeta - \sigma} d\tau.
\]

Similarly, the offer that a monopolist with a new technology (embodied in its intermediate good) allowing the creation of a new complex task \( N^+ \) at time \( t \) needs to make to the firm currently holding the patent for the old technology for task \( N - 1 \) (which is necessarily being produced with capital) is

\[
(1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^{\zeta} \psi^{1 - \zeta} \int_t^\infty e^{-\int_t^\tau (R(s) - \delta) ds} Y(\tau) c^u (R(\tau))^{\zeta - \sigma} d\tau.
\]

143
Since these are take-it-or-leave-it offers, the best response of the patent-holders is to accept them. Thus, we can compute the values of innovating and becoming a technology monopolist as:

\[
V_0(t) = MY(t) \int_t^\infty e^{-\int_t^\tau (R(s) - \delta - g(s))ds} \left( c^u (R(\tau))^{\xi - \sigma} - c^u \left( w_I(\tau)e^{\int_0^\tau g(s)ds} \right)^{\xi - \sigma} \right) d\tau,
\]

(3.22)

and

\[
V_N(t) = MY(t) \int_t^\infty e^{-\int_t^\tau (R(s) - \delta - g(s))ds} \left( c^u \left( w_N(\tau) \frac{\gamma(n(\tau))}{\gamma(n(t))} e^{\int_0^\tau g(s)ds} \right)^{\xi - \sigma} - c^u (R(\tau))^{\xi - \sigma} \right) d\tau,
\]

(3.23)

with \( M = (1 - \mu) \left( \frac{n}{1-n} \right)^\zeta \psi^1 - \zeta \). In what follows, define the normalized value functions \( \nu_I = V_I(t)/Y(t) \) and \( \nu_N = V_N(t)/Y(t) \), which will be convenient to work with, especially since as \( t \to \infty \) they will only depend on \( n \).

The expressions for the value functions, \( V_I(t) \) and \( V_N(t) \), share a common form: they subtract the lower cost of producing a task with the factor for which the new technology is designed from the higher cost of producing the same task with the older technology. Because our structure of intellectual property rights removes the creative destruction effects, the profits from introducing new intermediates always depend on the alternative cost of producing a task with the older technology.

Competition among prospective technology monopolists to hire scientists implies that, when employed in automating existing tasks, a scientist earns a wage of \( W_I^S(t) = \kappa_I V_I(t) \). Likewise, when employed in creating new complex tasks, a scientist earns a wage of \( W_N^S(t) = \kappa_N V_N(t) \). These combined with the costs of effort specified above imply that the numbers of scientists working in automation and the creation of new tasks are

\[
S_I(t) = SG \left( \frac{\kappa_I \nu_I - \kappa_N \nu_N}{\lambda} \right) \in [0, S], \quad S_N(t) = S \left[ 1 - G \left( \frac{\kappa_I \nu_I - \kappa_N \nu_N}{\lambda} \right) \right] \in [0, S].
\]

(3.24)

Intuitively, whenever one of the two types of innovation is more profitable, more scientists will be allocated to that activity. Our formulation further implies that the allocation of scientists to the two different types of innovation is independent of the level of aggregate output, and guarantees that \( \dot{n}(t) > 0 \) if and only if \( \kappa_N \nu_N > \kappa_I \nu_I \), and \( \dot{n}(t) < 0 \) if and only if \( \kappa_N \nu_N < \kappa_I \nu_I \).

---

26 This expression is written by assuming that the patent-holder will also turn down subsequent less generous offers in the future. Writing it using dynamic programming and the one-step ahead deviation principle leads to the same conclusion.

27 To avoid confusion, and with a slight abuse of notation, we always write \( \nu_I \) and \( \nu_N \) as functions of \( n \) (and never explicitly as functions of time).
Using the same normalizations as in the previous section, we can represent the equilibrium with endogenous technology by a time path of the tuple \( \{c, k, n, S_I, v_I, v_N\} \) such that:

- Consumption satisfies the Euler equation (3.14) coupled with the transversality condition in equation (3.16) (where in addition we can note that the net present value of corporate profits in equation (3.16) is simply \( \Pi(t) = I v_I + N v_N \)).
- Capital satisfies the resource constraint in equation (3.15).
- The gap between automation and the creation of new tasks, \( n(t) = N(t) - I(t) \), satisfies:
  \[
  h(t) = K N S - (\kappa_I + \kappa_N) G \left( \frac{\kappa_I v_I - \kappa_N v_N}{\lambda} \right) S.
  \]
  This implies that \( n = 0 \) if and only if \( \kappa_I v_I = \kappa_N v_N \).
- The allocation of scientists satisfies the allocation rule in equation (3.24).
- The growth rate of \( \gamma(I(t)) \) and of aggregate variables is \( g(t) = A \kappa_I S_I(t) \).
- The value functions that determine the allocation of scientists, \( v_I(t), v_N(t) \), are given by (3.22) and (3.23).

A BGP is defined as in Proposition 15, as an allocation in which the normalized variables \( c(t), k(t), n(t) \) and the rental rate \( R(t) \) are constant—except that now \( n \) will be determined endogenously. The next proposition gives another one of the main results of the paper. It establishes conditions for the existence, uniqueness and asymptotic stability of a BGP in which there are both types of technological changes.

**PROPOSITION 17 (EQUILIBRIUM WITH ENDOGENOUS TECHNOLOGICAL CHANGE)** Suppose that \( \sigma > \zeta \) and Assumption 1' holds. Then there exist \( \bar{\rho} \) and \( \bar{A} \) such that for \( \rho > \bar{\rho} \) and \( A < \bar{A} \) (where \( \bar{\rho} \) is as defined in Proposition 15), the following are true:

1. There exists \( \bar{\kappa} \) such that for \( \frac{\gamma I}{\kappa_N} > \bar{\kappa} \), there is a BGP, where \( \check{I} = \frac{\kappa_{1\kappa N}}{\kappa_I + \kappa_N} S \), and \( Y, C, K \) and \( W \) grow at the constant rate \( g = A \frac{\kappa_{1\kappa N}}{\kappa_I + \kappa_N} S \), and the rental rate, \( R \), the labor share and employment are constant. Along this path, we have \( N(t) - I(t) = n^D \), with \( n^D \) determined endogenously from the condition \( \kappa_N v_N = \kappa_I v_I \), and satisfying \( n^D \in (\bar{n}, 1) \), where \( \bar{n} \) is as defined in Proposition 15. In addition, there exists \( \bar{\rho} > \rho \) such that if \( \rho > \bar{\rho} \), the BGP is unique.

2. Suppose that \( \frac{\gamma I}{\kappa_N} > \bar{\kappa} \) and \( \rho > \bar{\rho} \) so that the BGP is unique. When \( \theta = 0 \), the dynamic equilibrium is globally (saddle-path) stable. Moreover, there exists \( \bar{A} \leq \bar{\bar{A}} \) such that when \( A < \bar{\bar{A}} \), for any value of \( \theta \), the dynamic equilibrium is unique in the neighborhood of the BGP and is asymptotically (saddle-path) stable.
3. If, on the other hand, $\frac{\kappa_L}{N} < \bar{\kappa}$, we have $\kappa_{NVN} > \kappa_{IIV}$ and there exists a globally stable BGP where all scientists are allocated to create new labor-intensive tasks. Therefore, asymptotically $n \to 1$, and the economy converges to a BGP in which all tasks are produced by labor and long-run growth is driven by the creation of new tasks.

4. For $\rho < \bar{\rho}$, there exists an asymptotically stable BGP where $\kappa_{NVN} < \kappa_{IIV}$, all tasks are produced with capital, and long-run growth is propelled by capital accumulation.

**Proof.** See Appendix A. □

The first important result contained in this proposition (part 1) is the existence of a BGP and its uniqueness (when $\rho > \bar{\rho}$). The second critical result, established in part 2 of the proposition, is that this BGP is asymptotically stable and also globally stable when $\theta = 0$ (so that preferences have an infinite elasticity of intertemporal substitution). This result implies that there are powerful market forces pushing the economy towards the BGP.

These results are established under several conditions. First, we require $\sigma > \zeta$. This condition ensures that innovations are directed towards technologies that allow firms to produce tasks by using the cheaper (or more productive) factors. The profitability of introducing an intermediate that embodies a new technology will depend on the demand for the intermediate good. As the factor that needs to be combined with the intermediate to produce a given task (labor or capital) becomes cheaper, two effects come into play. First, the decline in costs allows firms to scale up their production, which increases the demand for the intermediate good. The extent of this positive scale effect is regulated by the elasticity of substitution $\sigma$: the greater is $\sigma$, the more powerful is this effect directing innovation towards technologies that work with cheaper factors. However, there is a countervailing force as well: as a factor becomes cheaper, it is substituted for the intermediate it is combined with, reducing the demand for the intermediate good that embodies the new technology. This countervailing substitution effect is regulated by the elasticity of substitution $\zeta$: the greater is $\zeta$, the more powerful is this effect discouraging innovations towards technologies that work with cheaper factors. The condition $\sigma > \zeta$ guarantees that the former, positive effect dominates, so that prospective technology monopolists have an incentive to introduce technologies that allow firms to produce tasks more cheaply. When the opposite holds and $\zeta > \sigma$, we could have a situation in which technologies that work with more costly factors are more profitable. Pathologically, in this case the net present discounted values from innovation would be negative.$^{28}$ The condition $\sigma > \zeta$ is not only theoretically necessary in our model but is also empirically plausible. Because intermediates embody the technology that allows firms to

$^{28}$Take, for example, (3.22). Because a task performed by labor is being automated, $c^u(R(\tau)) < c^u\left(\frac{u(\tau)}{\gamma(I(\tau))}\right)$, and hence if we had $\zeta > \sigma$, the profit stream would be negative. Thus, in order to ensure positive incentives for innovation, the condition $\sigma > \zeta$ is imposed even for the existence result in part 1 of the proposition.
produce with certain factors, we expect the elasticity of substitution between factors and intermediates, $\zeta$, to be very low, so that they are highly complementary. The condition $\sigma > \zeta$ thus ensures that, quite intuitively, intermediates are gross complements to factors they work with.

Second, we require that $A < \bar{A}$ to guarantee that the growth rate of the economy is not too high. If the growth rate is above the threshold implied by $\bar{A}$, the creation of new tasks is discouraged (even if current wages are low) because firms anticipate that wages will grow very rapidly, which will reduce the future profitability of these labor-intensive tasks. This requirement is strengthened to $A < \bar{A}$ in the second part of the proposition when we consider local stability, which allows us to use Taylor approximations of the value functions.\(^{29}\)

Third, as in Proposition 15, parts 1-3 of the proposition require $\rho > \bar{\rho}$. As discussed in that context, this assumption ensures that the interest rate is not too close to 0, which in turn guarantees that newly created complex tasks will immediately start being used with labor. The proposition further shows that when $\rho > \bar{\rho}$, and the other conditions in the proposition are satisfied, the long-run equilibrium endogenously involves $n > \bar{n}$. In this region there are productivity gains from automating tasks, and therefore a demand for automation. Instead, when $n < \bar{n}$, prospective monopolists have no incentives to automate tasks.

The economic forces ensuring the stability of the BGP in parts 1 and 2 of the proposition are intuitive. In a BGP, the normalized value of different types of innovations converges to $v_I(n) = \lim_{t \to \infty} \frac{V_I(t)}{Y(t)}$ and $v_N(n) = \lim_{t \to \infty} \frac{V_N(t)}{Y(t)}$. These limiting functions, $v_I(n)$ and $v_N(n)$, are fully determined by the technology gap $n$, as can be seen from equations (3.22) and (3.23). Moreover, in a BGP we must have $\kappa_I v_I(n) = \kappa_N v_N(n)$ so that $\bar{n} = 0$, which implies that both technologies should advance in parallel. When $\kappa_I v_I(n) > \kappa_N v_N(n)$, we instead have $\bar{n} < 0$, and when $\kappa_I v_I(n) < \kappa_N v_N(n)$, we have $\bar{n} > 0$. Neither of these possibilities is consistent with balanced growth.

Figure 3-6 draws the net present discounted value (normalized by output) of allocating scientists to creating new complex tasks or to automation when $\sigma > \zeta$, $\rho > \bar{\rho}$ and $A < \bar{A}$. The figure shows that, in the region where $n > \bar{n}$, as more tasks are automated (as $n$ decreases), the value of additional automation, $v_I(n)$, decreases. This is the key economic force that generates stability in our model: greater automation reduces the effective wage in the next task to be automated, $w_I(n)$, relative to the interest rate (which is fixed and independent of technology). Consequently, the (normalized) long-run value of automating additional tasks declines with automation.

Nevertheless, this property is not sufficient for the stability or uniqueness of a BGP. Figure

\(^{29}\)The restriction that $A < \bar{A}$ also ensures that the net present discounted value of the representative household is finite and thus the transversality condition is satisfied. Moreover, this restriction also guarantees that the condition from Assumption 1', $e^{-(2+2\sigma+\zeta)A} > |\sigma - \zeta|$, holds.
Figure 3-6: Determination of $n^D$ in steady state.

3-6 also shows that, asymptotically, an increase in automation (a decrease in $n$) reduces the value of creating new complex tasks, $v_N(n)$: because of the productivity effect, the effective wage paid in the most complex tasks, $w_N(n)$, also increases with automation. As a result, the (normalized) value of creating new complex tasks declines with automation, which generates a force towards instability and multiplicity.\(^{30}\) The condition $\rho > \bar{\rho}$ guarantees that the productivity effect of automation on the wage $w_N(n)$ is small, or equivalently, that the induced capital accumulation does not reverse the direction in which subsequent technological improvements respond to a wave of automation. In contrast, when this condition fails and $\rho \leq \bar{\rho}$, we could have situations in which, asymptotically, an increase in automation raises the wage $w_N(n)$ so much that it discourages the introduction of new more complex tasks, paving the way to multiple steady states in our baseline model.

Observe also that the conditions we have discussed so far are not sufficient to guarantee that the curves for $\kappa_Iv_I(n)$ and $\kappa_Nv_N(n)$ intersect. The former always starts below the latter as shown in Figure 3-6, but may always remain below it. The last condition in Proposition 17, that $\kappa_I/\kappa_N$ is sufficiently large, ensures that such an intersection takes place and thus there exists a unique "interior" BGP. This discussion then immediately leads to part 3 of the proposition, which establishes that when this condition does not hold, the long-run equilibrium will be one in which only new tasks are developed, and there is no automation. BGP is now achieved by continuous creation of more productive tasks (replacing older tasks), and hence, our economy in this case looks very similar to a standard "Schumpeterian" economy.

\(^{30}\)The intuition for the productivity effect can be alternatively viewed as follows: automation induces the accumulation of capital, which by raising wages in the long run makes the creation of new labor-intensive tasks less profitable. Thus, the induced capital accumulation crowds out some of the self-correcting forces that stem from the response of technology to changes in factor prices. The condition $\rho > \bar{\rho}$ ensures that this indirect capital accumulation effect does not dominate the direct, stabilizing effect described in the previous paragraph.
with growth propelled by quality improvements (e.g., Aghion and Howitt, 1992; Grossman and Helpman, 1991).

Finally, part 4 of the proposition shows that when \( \rho < \bar{\rho} \), we have a balanced growth path in which all (except possibly a measure zero subset modem) of tasks are produced with capital, and the economy grows by accumulating capital.\(^{31}\) This equilibrium can therefore be likened to Leontief’s “horse equilibrium,” because it makes labor redundant. Notably, however, such an equilibrium is possible only when the discount rate, and thus the long-run interest rate, is very low (recall from footnote 19 that under standard values of the other parameters, \( \rho < \bar{\rho} \) would require the annual discount rate to be less than 0.012).

In summary, the critical economic force highlighted by Proposition 17 is that, differently from models with factor-augmenting technologies, it is factor prices—not primarily the market sizes—that guide the direction of technological change.\(^{32}\) Consequently, there are stronger incentives to undertake the type of innovation that will work with the factor that has a lower effective cost.

Proposition 17 shows that shifts in technology, for example in the form of a series of new automation technologies (corresponding to an unanticipated decline in \( n \)), will set in motion self-correcting forces. Following such a change, there will be an adjustment process bringing the level of employment and the labor share back to their initial BGP values. This does not, however, imply that all shocks will leave the long-run prospects of labor unchanged. The next corollary shows that if there is a change in the innovation possibilities frontier that makes automation easier than before, the economy will move to a new BGP with lower employment and a lower share of labor in national income.

**Corollary 3** Suppose that all the conditions in Proposition 17 are satisfied and there is a one-time permanent increase in \( \kappa_I/\kappa_N \). Then the economy converges to a new BGP with lower \( n^D \), lower employment and a lower share of labor in national income.

This corollary follows by noting that an increase in \( \kappa_I/\kappa_N \) shifts the intersection of the curves \( \kappa_1 v_1(n) \) and \( \kappa_N v(n) \) in Figure 3-6 to the left, leading to a lower value of \( n^D \) in the BGP. This will trigger an adjustment process in which the labor share and employment decline over time, but ultimately settle to their new steady-state values.

\(^{31}\)The qualifier “except possibly a measure zero subset” is introduced, since when \( L(0) > 0 \), there will still be positive labor supply even with very low wages, but asymptotically all of this labor will be employed in the highest-indexed task; along the BGP, output of this task will grow by allocating additional capital there as well.

Note also that this BGP is not necessarily globally stable, because the two curves in the right panel of Figure 3-10 in Appendix A may cross, in which case the interior BGP will be unstable, and there will be another locally stable BGP in which the economy behaves as in a Schumpeterian growth model as described in part 3.

\(^{32}\)We should also note that this does not overturn the “weak bias” results in Acemoglu (2007), since these were derived in a setting that is general enough to nest the current environment.
Together this corollary and Proposition 17 delineate the types of changes in technology that will trigger self-correcting dynamics: those driven by faster than usual arrival of automation technologies will do so, while those which alter the ability of the society to create new automation technologies will not (and thus will result in worse prospects for labor in the future).

3.4 Welfare

In this section we turn to an analysis of the efficiency of the equilibrium described in Proposition 17. Our main finding is that the presence of rents for workers, as captured by our quasi-labor supply, biases the composition of equilibrium technology towards too much automation and the creation of too few new complex tasks (and this is in addition to other distortions that exist in models of endogenous technology). Two complementary results illustrate this inefficiency. First, in Appendix B we characterize the constrained efficient allocation chosen by a social planner who is subject to the same innovation possibilities frontier, the same quasi-labor supply schedule and the constraint that wages equal the marginal product of labor. We then show how this constrained efficient allocation can be decentralized by taxes and subsidies. In addition to the usual wedges (taxes/subsidies) between the social planner’s allocation and the decentralized equilibrium, workers’ rents create an additional reason to tax automation relative to the creation of new complex tasks. Second, we show how welfare in the decentralized equilibrium can be improved by altering the composition of R&D in the direction of creating more new complex tasks and automating fewer of the existing ones.

Let $F^P(I, N, K, L)$ denote the aggregate output net of the costs of producing intermediates when the level of employment is $L$, the capital stock is $K$, the state of technologies is represented by $N$ and $I$, and intermediates are priced at their marginal cost (which is the relevant net aggregate output expression for the social planner, since she would always price all intermediates at marginal cost). Because the level of employment is given by the quasi-labor supply $L = L^*(\omega)$, we have that in the constrained efficient allocation the resulting marginal products of labor and capital are fully determined by technology and capital, and we can write the resulting value of $\omega$ as $\omega^P(I, N, K)$.

The constrained efficient allocation maximizes the representative household’s utility given by (3.13) subject to the endogenous evolution of the state variables:

$$\dot{K}(t) = F^P(N(t), I(t), K(t), L(t)) - C(t) - \delta K(t), \dot{N}(t) = \kappa_N S_N(t), \quad \dot{I}(t) = \kappa_I S_I(t),$$

and because the planner faces the same quasi-labor supply schedule and labor demand rela-
tions, we also have:

\[ L(t) \leq L^*(\omega^P(I, N, K)). \]

From the fact that the social planner maximizes (3.13) we can see that there is no utility loss or opportunity cost of supplying labor; the higher wages needed to employ additional workers are "quasi rents" (see Appendix B). Notice also that we have focused on the case in which the planner chooses \( I^*(t) = I(t) \) (in the terminology of our previous section), so that all automated tasks are produced immediately with capital. As in the previous section, this will be the case when \( \rho > \bar{\rho} \), since in this case the planner will always choose to operate in the region where \( n > \bar{n} \).

We show in Appendix B that the solution to this maximization problem gives the constrained efficient allocation and can also be used to characterize the taxes and subsidies that need to be imposed on the decentralized equilibrium to achieve this constrained efficient allocation. In addition to the usual taxes and subsidies to internalize monopoly markups and technological externalities, the social planner will need to impose a tax on automation and a subsidy on the creation of new tasks in order to combat the tendency of the decentralized equilibrium to automate excessively. Intuitively, while the planner recognizes that automating tasks reduces employment (or creating new complex tasks increases it) and this has a first-order effect on workers (recall that there is no utility loss or opportunity cost of supplying labor), innovators do not internalize this externality.

Here, we demonstrate the presence of excessive automation in the decentralized equilibrium more directly. We show that starting from a decentralized allocation, the social planner can improve the allocation of resources by discouraging automation. The next proposition establishes this result when \( \zeta \geq 1 \) (which includes the tractable special case of our model in which \( \zeta \to 1 \)). We also assume that intermediates are subsidized at the rate \( 1 - \mu \), which removes the main effect of monopoly markups, or equivalently that \( \mu \to 1 \).

**Proposition 18 (Excessive Automation)** Suppose that \( \rho > \bar{\rho} \) as in Proposition 17, Assumption 1' holds, and \( \zeta \geq 1 \). Moreover, suppose that intermediate goods are subsidized and can be purchased at their marginal cost (or equivalently \( \mu \to 1 \)). Consider the decentralized equilibrium path described in Proposition 17, and which converges to a BGP with \( N(t) - I(t) \to n^D \). Then there exists a feasible allocation with \( n^P(t) \geq n(t) \) and \( \lim_{t \to \infty} n^P(t) > n^D \) that achieves strictly greater welfare than the decentralized equilibrium.

**Proof.** See Appendix B. \( \blacksquare \)

This proposition establishes that departing from the equilibrium in the direction of discouraging automation and encouraging the creation of new complex tasks improves welfare. Intuitively, because of the gap between the equilibrium wage and the opportunity cost of
labor, redirecting research towards the creation of new complex tasks instead of automation has a positive first-order effect on workers, while it only has a second-order impact on the profits of prospective technology monopolists.

3.5 Extensions

In this section, we discuss two extensions. First we introduce heterogeneous skills, which allow us to analyze the impact of technological changes on inequality. Second, we study a different structure of intellectual property rights that introduces the creative destruction of profits.

3.5.1 Automation, New Tasks and Inequality

In this subsection, we introduce heterogeneous skills and study how automation and the creation of new tasks impact inequality. This extension is motivated by the observation that, because new tasks are more complex, their creation may favor high-skill workers. The natural assumption that high-skill workers have a comparative advantage in new complex tasks receives support from the data. For instance, the left panel of Figure 3-7 shows that in each decade since 1980, occupations with more new job titles had higher skill requirements in terms of the average years of schooling among employees at the start of each decade (relative to the rest of the economy). However, the right panel of the same figure also shows a pattern of “mean reversion” whereby average years of schooling in these occupations decline in each subsequent decade, most likely, reflecting the fact that new job titles became more open to lower-skilled workers over time.

We incorporate these features into our model by assuming that there are two types of workers: low-skill and high-skill. We introduce a pattern of comparative advantage that reflects our interpretation of the patterns in the data: the productivity of high-skill labor is the same as before,

\[ \gamma_H(i) = e^{A_H i}, \]

while the productivity of low-skill labor improves over time as new tasks become more “standardized”.\(^{33}\) In particular, for tasks \( i \leq N(t) \), the productivity of low-skill workers is

\[ \gamma_L(i, t) = \gamma_H(i) \cdot \Gamma(N(t) - i), \]

\(^{33}\)This formulation captures the feature that new technologies and tasks are standardized over time (e.g., Acemoglu, Gancia and Zilibotti, 2010) or that low-skill workers require more time to adapt to new technologies (e.g., Schultz, 1965, Nelson and Phelps, 1966, Greenwood and Yorukoglu, 1997, Caselli, 1999, Galor and Moav, 2000, Beaudry, Green and Sand, 2013, and Goldin and Katz, 2008).
where $\Gamma(\cdot)$ is increasing, bounded above by 1, and satisfies $\Gamma(0) < 1$.

This structure implies that the productivity of low-skill labor in a new task (which means in task $i$ at time $t$ such that $N(t) = i$) starts at $\gamma_L(i, t) = \gamma_H(i) \Gamma(0) < \gamma_H(0)$. This productivity then increases as time passes. Since $\gamma_L(i, t)/\gamma_H(i) = \Gamma(N(t) - i)$ is decreasing in $i$, high-skill labor has a comparative advantage at high-index tasks. This structure of comparative advantage ensures that there exists a threshold task $M$ such that high-skill labor performs tasks in $[M, N]$, low-skill labor performs tasks in $(I, M)$, and capital performs tasks in $[N - 1, A]$.

We denote the wages of high and low-skill labor by $W_H$ and $W_L$, respectively. As before, we assume that there is a quasi-labor supply of high-skill labor given by $H^s(W_H)$, and a quasi-labor supply of low-skill labor given by $L^s(W_L)$, both of which have a constant and equal elasticity $\nu_L \geq 0$.

**Proposition 19 (Automation, New Tasks and Inequality)** Suppose technology evolves exogenously and either one of $\sigma - \zeta$, $\zeta - 1$, or $\eta$ is sufficiently close to 0.

1. Suppose that $\dot{N} = \dot{I} = \Delta$ and $I^*(t) = I(t)$ (and $A_H(1 - \theta)\Delta < \rho$ so that net present discounted value of household income is finite). Then, the economy admits a unique BGP. In this BGP $W_H$ and $W_L$ grow at the same rate as the economy and the wage gap, $w_H/w_L$, remains constant. Moreover, both low-skill and high-skill workers perform a constant share of the tasks.

2. Given such a path of technological change, the dynamic equilibrium is unique starting from any initial condition and converges to the BGP.
3. The immediate effect of increases in both $I$ and $N$ is to raise the wage gap $W_H/W_L$.

But the medium-run impact of an increase in $N$ is to reduce inequality.

PROOF. See Appendix B. ■

A number of features are worth noting. First, this extended model generates not only an endogenous distribution of income between capital and labor, but also inequality between high-skill and low-skill workers. Here, inequality reflects the assumed structure of comparative advantage for workers of different skill levels in different tasks. The short-run comparative statics in the proposition imply that automation, by squeezing out tasks previously performed by low-skill labor, increases inequality between the two types of skills. Interestingly, because it is high-skill labor that has a comparative advantage in high-index tasks, the creation of new complex tasks also tends to increase inequality at first. However, because tasks become standardized over time, which raises the productivity of low-skill workers, the medium-term implications of automation and the creation of new tasks are very different. The former increases inequality both in the short and the medium run. In contrast, the creation of new tasks increases inequality in the short run, but not in the medium run. In fact, low-skill workers gain relative to capital in the medium run from the creation of new tasks. Interestingly, inequality may be particularly high following a period of adjustment in which the labor share first declines—due to increases in automation—and then recovers—due to the introduction of new complex tasks. Inequality may remain high for a while, and only start declining after recently-introduced new tasks become sufficiently standardized.

3.5.2 Creative Destruction of Profits

In this subsection, we modify our baseline assumption on intellectual property rights, reverting to the classical setup in the literature in which new technologies do not infringe the patents of the products that they replace (Aghion and Howitt, 1992, and Grossman and Helpman, 1991). This assumption introduces creative destruction effects—the destruction of profits of previous inventors by new innovators. We will see that this alternative structure has little effect on the nature of a BGP in our model, but we require more demanding conditions to guarantee its uniqueness and stability.

Let us first define $V_N(t, i)$ and $V_I(t, i)$ as the values at time $t$ of having introduced different technologies for the production of task $i$ (respectively, new complex tasks and automation). The value functions satisfy the following Bellman equations:

$$r(t)V_N(t, i) - \dot{V}_N(t, i) = \pi_N(t, i)$$
$$r(t)V_I(t, i) - \dot{V}_I(t, i) = \pi_I(t, i).$$

Here $\pi_I(t, i)$ and $\pi_N(t, i)$ denote the flow profits from automating and creating new complex
tasks, respectively, which are given by the formulas in equations (3.20) and (3.21).

For a firm creating a new complex task $i$, let $T^N(i)$ denote the time at which it will be replaced by a technology allowing the automation of this task. Likewise, for a firm automating task $i$ at time $t$, let $T^I(i)$ denote the time at which it will be replaced by a more complex technology using labor. Since firms anticipate these deterministic replacement dates, their value functions also satisfy the boundary conditions $V_N(T^N(i), t) = 0$ and $V_I(T^I(i), t) = 0$.

Using the Bellman equations together with the boundary conditions derived above, we obtain:

$$
V_N(t) = V_N(N(t), t) = (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^{\zeta} \psi_{1-\zeta} \int_t^{T^N(N(t))} e^{\int_t^\tau (R(s) - \delta)ds} Y(\tau) e^{\frac{W(\tau)}{\gamma(N(t))}} \zeta^{-\sigma} d\tau,
$$

$$
V_I(t) = V_I(I(t), t) = (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^{\zeta} \psi_{1-\zeta} \int_t^{T^I(I(t))} e^{\int_t^\tau (R(s) - \delta)ds} Y(\tau) e^{\left( \min \left\{ R(\tau), \frac{w(\tau)}{\gamma(I(t))} \right\} \right) \zeta^{-\sigma}} d\tau.
$$

In addition, for reasons that will become readily clear, we modify the evolution of the technology frontier and assume that advances in automation take the form

$$
\dot{I}(t) = \kappa_I \phi(n(t)) S_I(t), \quad \text{and} \quad \dot{N}(t) = \kappa_N S_N(t)
$$

(3.25)

Here, the function $\phi(n(t))$ is included and assumed to be weakly increasing to capture the possibility that automating tasks closer to the frontier (defined as the highest available task) may be more difficult.

The characterization of the BGP is similar to that in Proposition 17 with (3.25) replacing (3.19). But there may exist multiple BGPs and additional assumptions on the function $\phi(n)$ need to be imposed to guarantee stability.

PROPOSITION 20 (EQUILIBRIUM WITH CREATIVE DESTRUCTION) Suppose that $\sigma > \zeta$, Assumption 1' holds, $\rho > \overline{\rho}$, $A < \overline{A}$ (where $\overline{\rho}$ and $\overline{A}$ are defined as in Proposition 17), and there is creative destruction of profits. Then:

1. There exist $\overline{\phi}$ and $\rho < \phi < \overline{\phi}$ such that if $\phi(0) < \phi$ and $\phi(1) > \overline{\phi}$, then there exists at least one (interior) stable BGP in which there is research in both automation and the creation of new complex tasks. In this BGP, we have $N(t) - I(t) = n^D$, $\kappa_N v_N(n^D) = \kappa_I \phi(n^D) v_I(n^D)$ and $\dot{N} = \dot{I} = \frac{\kappa_I \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S$. Also, $Y, C, K$ and $w$ grow at the constant rate based on $g = \frac{\kappa_I \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S$, $R$ is constant, and the labor share and employment are constant.

2. If $\phi(n)$ is constant, there is no asymptotically stable BGP with $n^D \in (0, 1)$. Any asymptotically stable equilibrium involves $n(t) \to 0$ or $n(t) \to 1$. 155
PROOF. See Appendix B. ■

The first part of the proposition follows from an analogous argument to that in the proof of Proposition 17, with the only difference being that, because of the presence of the function \( \phi(n) \) in equation (3.25), the key condition determining a BGP becomes \( \kappa_I \phi(n) v_I(n) = \kappa_N v_N(n) \). In addition, in a BGP, newly created tasks are automated after a period of length \( T^N(N(t)) - t = \frac{n^D}{\Delta} \), and newly automated tasks are replaced by new complex ones after a period of length \( T^I(I(t)) - t = \frac{1-n^D}{\Delta} \). Here, \( \Delta = \frac{\kappa_I \phi(n)}{\kappa_I \phi(n) + \kappa_N} \) is the endogenous rate at which \( N \) and \( I \) increase. Thus, both types of innovations are replaced after a fixed length of time, which ensures that the creative destruction of profits does not change the balance of the incentives for innovation.

The major difference with our previous analysis is that, in the presence of the creative destruction of profits, \( v_I(n) \) decreases with \( n \)—that is, \( v_I(n) \) now increases when more tasks are automated and \( n \) declines—generating a new force towards instability. The reason is that, when \( n \) declines, automated tasks are replaced by new complex ones less frequently, because newly automated tasks are replaced after \( \frac{1-n^D}{\Delta} \) units of time. This then increases the net present discounted value of profits for automating tasks. This effect is not compensated by changes in the flow profits from automation: because of the response in capital accumulation, the long-run interest rate, and hence the flow profits from automation, remains unchanged. Likewise, \( v_N(n) \) increases with \( n \)—that is, \( v_N(n) \) decreases when more tasks are automated—also contributing to instability. Intuitively, when \( n \) decreases, newly created tasks are automated more often, because newly created tasks are automated after \( \frac{n^D}{\Delta} \) units of time, and this reduces the net present discounted value of profits from new tasks. The productivity effect contributes another force towards instability: when \( n \) decreases (there is greater automation) the effective wage in the newest tasks, \( w_N(n) \), increases, further reducing the value from new tasks.

These observations imply that, if \( \phi(n) \) were constant, the intersection between the curves \( \kappa_N v_N(n) \) and \( \kappa_I \phi(n) v_I(n) \) would give an unstable BGP. This result is in stark contrast to the asymptotic stability of the BGP characterized in Proposition 17. Economically, this difference is a consequence of the fact that, in contrast to our baseline model (and the socially planned economy), the creative destruction of profits implies that the incentives for prospective monopolists to innovate depend on the total revenue that a technology generates and not on the incremental value created by their innovation (which is the difference between these revenues and the revenues that the replaced technology generated). In our baseline model and in the constrained efficient allocation, the fact that automation reduced the incremental value of automating additional tasks was the key force generating stability, but this force is absent when innovators destroy the profits of previous inventors.

As shown in Figure 3-8, with the creative destruction of profits, stability imposes some
restrictions on the \( \phi(n) \) function, and specifically, stability requires the conditions that \( \phi(0) < \frac{\phi}{\phi} \) and \( \phi(1) > \frac{\phi}{\phi} \). This ensures that the first intersection of these two curves takes place in the region in which \( \kappa_1 \phi_1(n) v_1(n) \) is steeper than \( \kappa_2 v_2(n) \), and their interception yields an asymptotically stable BGP. As the figure also shows, even in this case there might be additional BGPs, each having different stability properties. Notice also that, as shown in Figure 3-8, the two cars can intersect more than once. If so, the second intersection is unstable, and there may also exist a corner equilibrium in which, as in Proposition 17 part 3, there is a BGP in which all tasks are produced with labor.

![Figure 3-8: Determination of \( n^D \) when the structure of intellectual property rights features the creative destruction of rents. The model has an odd number of equilibria, which in the case depicted include a stable one at \( n^D = 1 \).](image)

### 3.6 Conclusion

As more tasks performed by labor are being automated, concerns that these new technologies will make labor redundant have intensified. This paper developed a comprehensive framework in which these forces can be analyzed and contrasted with countervailing effects. At the center of our model is a task-based framework. Automation is modeled as the (endogenous) expansion of the set of tasks that can be performed by capital, thus replacing labor in tasks that it previously produced. The main new feature of our framework is that, in addition to automation, there is another type of technological change enabling the creation of new, more complex versions of existing tasks, and it is labor that tends to have a comparative advantage in these new tasks. We characterize the structure of equilibrium in such a model, showing how, given factor prices, the allocation of tasks between capital and labor is determined both by available technology and the endogenous choices of firms between producing with capital or labor.
One attractive feature of task-based models is that they highlight the link between factor prices and the range of tasks allocated to factors: when the equilibrium range of tasks allocated to capital increases (for example, as a result of automation), the wage relative to the rental rate and the share of labor in national income decline, and the equilibrium wage rate may also fall. Conversely, as the equilibrium range of tasks allocated to labor increases, the opposite result obtains. In our model, because the supply of labor is elastic, automation tends to reduce employment, while the creation of new tasks increases employment. These results highlight that, while both types of technological changes undergird economic growth, they have very different implications for the factor distribution of income and also for employment.

Our full model endogenizes the direction of research towards automation and the creation of new complex tasks, showing how this framework generates a BGP in which both types of innovations go hand-in-hand. Moreover, under reasonable assumptions, the dynamic equilibrium is unique and locally converges to the BGP. Underpinning this stability result is the impact of relative factor prices on the direction of technological change. The task-based framework—differently from the standard models of directed technological change which are based on factor-augmenting technologies—implies that as a factor becomes cheaper, this not only influences the range of tasks allocated to it, but also generates incentives for prospective technology monopolists to introduce technologies that allow firms to utilize this factor more intensively. These economic incentives then imply that by reducing the effective cost of labor in the least complex tasks, automation discourages further automation and generates a powerful self-correcting force towards stability.

Though market forces ensure the stability of the BGP, they do not necessarily generate the efficient composition of technology. If the elastic labor supply relationship results from rents (so that there is a wedge between the wage and the opportunity cost of labor), then there is an important and new distortion in the direction of technological change. Because firms value reducing the rents earned by workers, there is a natural bias towards excessive automation. On the other hand, because the planner recognizes that these rents are just transfers, she has weaker incentives to automate and replace labor with capital in additional tasks.

In addition to claims about automation leading to the demise of labor, several commentators are concerned about the inequality implications of automation and related new technologies. In one of our extensions, we have studied this question by introducing a distinction between low-skill and high-skill labor, with the latter having a comparative advantage in producing the new complex tasks. In this extension, both automation (which squeezes out tasks previously performed by low-skill labor) and the creation of new tasks (which directly benefits high-skill labor) will increase inequality. Nevertheless, the medium-term implications of the creation of new tasks could be very different, because new tasks are later
standardized and used by low-skill labor. As a result of this effect, there exists a unique BGP in which not only the factor distribution of income (between capital and labor) but also inequality between the two skill types is endogenous but constant.

We consider our paper to be a first step towards a systematic investigation of different types of technological changes that impact capital and labor differentially. Several areas of research appear fruitful based on this first step. First, we introduced labor market distortions in this model in the form of a reduced-form quasi-labor supply curve. Going beyond this, an important set of issues center around how the process of automation and replacement of workers by capital interplays with the costly and potentially slow reallocation of workers across tasks and firms. We take some steps in this direction in our companion paper, Acemoglu and Restrepo (2016). Second, our model implies that it is always to tasks at the bottom that are automated; in reality, it may be those in the middle (e.g., Acemoglu and Autor, 2001). Ensuring a pattern of productivity growth consistent with balanced growth is more challenging in this case, though incorporating the possibility of such “middling tasks” being automated is an important generalization. Third, there may be major differences in the ability of technology to automate and also to create new tasks across industries (e.g., Polanyi, 1966, Autor, Levy and Murnane, 2003). An interesting step is to construct realistic models in which the sectoral composition of tasks performed by capital and labor as well as technology evolves endogenously and is subject to industry-level technological constraints (e.g., on the feasibility or speed of automation). Finally, and perhaps most importantly, our model highlights the need for additional empirical evidence on how automation takes place and how the incentives for automation and the creation of new tasks respond to policies and changes in the environment. One interesting direction would be to construct measures of automation and the creation of new tasks, potentially at the industry level, and then explore the impact on technology choices and innovation of industry-level variation in wages and institutional restrictions on capital-labor substitution.

3.7 Appendix A: Proofs

3.7.1 Proofs from Section 3.1

Proof of Proposition 1: We proceed in three steps. First, we show that \( I^*, N \) and \( K \), determine unique equilibrium values for \( R, W \) and \( Y \), thus allowing us to define the function \( \omega(I^*, N, K) \) representing the relative demand for labor, which was introduced in the text. Second, we provide a lemma which ensures that \( \omega(I^*, N, K) \) is decreasing in \( I^* \) (and increasing in \( N \)). Third, we show that \( \min\{I, \tilde{I}\} \) is nondecreasing in \( \omega \) and conclude that there is a unique pair \( \{\omega^*, I^*\} \) such that \( I^* = \min\{I, \tilde{I}\} \) and \( \omega^* = \omega(I^*, N, K) \). This
pair uniquely determines the equilibrium relative factor prices and range of tasks that get effectively automated.

**Step 1:** Consider I*, N and K such that I* ∈ (N − 1, N). Then, R, W and Y satisfy the system of equations given by capital and labor market clearing, equations (3.7) and (3.8), and the ideal price index, equation (3.9).

Taking the ratio of (3.7) and (3.8), we obtain

\[
\frac{\int_{I^*}^N \gamma(i)^{\xi-1} c^a \left( \frac{W}{\gamma(0)} \right)^{\xi-\sigma} W^{-\xi} di}{L^a \left( \frac{W}{R^K} \right) (I^* - N + 1)c^u(R)^{\xi-\sigma} R^{-\xi} = \frac{1}{K}}
\]

\[\text{(3.26)}\]

In view of the fact that L^a is nondecreasing and the function c^u(x)^{\xi-\sigma}x^{-\xi} is decreasing everywhere in x (as it can be verified directly by differentiation), it follows that the left-hand side is decreasing in W and increasing in R. Therefore, (3.26) defines an upward-sloping relationship between W and R, which we refer to as the relative demand curve.

On the other hand, inspection of equation (3.9) readily shows that this equation gives a downward-sloping locus between R and W as shown in Figure 3-9, which we refer to as the ideal price curve.

For a given I*, N and K, the intersection point between the relative demand and the ideal price curves determines the equilibrium factor prices (if it exists and is unique).

Because the relative demand curve is upward sloping and the ideal price index curve is downward sloping, there can be at most one intersection. To prove that there always exists an intersection, observe that \( \lim_{x \to 0} c^u(x)^{\xi-\sigma}x^{-\xi} = \infty \), and that \( \lim_{x \to \infty} c^u(x)^{\xi-\sigma}x^{-\xi} = 0 \). These observations imply that as W → 0, the numerator of (3.26) limits to infinity, and hence, so must the denominator, i.e., R → 0. This proves that the relative demand curve starts from the origin. Similarly, as W → ∞, the numerator of (3.26) limits to zero, and so must the denominator (i.e., R → ∞). This then implies that the relative demand curve goes to infinity as R → ∞. Thus, the upward-sloping relative demand curve necessarily starts below and ends above the ideal price curve, which ensures that there always exists an intersection between these curves. The unique intersection defines the equilibrium values of W and R, and therefore the function \( \omega(I^*, N, K) = \frac{W}{R^K} \).

**Step 2:** Step 2 follows directly from the following lemma, which we prove in Appendix B.

**Lemma 4** Suppose Assumption 1 holds. Then \( \omega(I^*, N, K) \) is decreasing in I* and is increasing in N.

**Step 3:** We now establish that \( I^* = \min\{I, \bar{I}\} \) is uniquely defined. Since \( \gamma(\bar{I}) = \omega K \), \( \bar{I} \) is increasing in \( \omega \), and thus \( I^* = \min\{I, \bar{I}\} \) is nondecreasing in \( \omega \). Consider the pair
of equations \(\omega = \omega(I^*, N, K)\) and \(I^* = \min\{I, \bar{I}\}\) plotted in Figure 3-3. Because \(\omega = \omega(I^*, N, K)\) is decreasing in \(I^*\) and \(I^* = \min\{I, \bar{I}\}\) is increasing in \(\omega\), there exists at most a single pair \((\omega, I^*)\) satisfying these two equations (or a single intersection in the figure).

To prove existence, we again verify the appropriate boundary conditions. Suppose that \(I^* \rightarrow N - 1\). Then from (3.7), \(R \rightarrow 0\), while \(W > 0\), and thus \(\omega \rightarrow \infty\). This ensures that the curve \(\omega(I^*, N, K)\) starts above \(I^* = \min\{I, \bar{I}\}\) in Figure 3-3. Since \(I^* = \min\{I, \bar{I}\}\), it is bounded above by \(I\), and cannot be below \(\omega = \omega(I^*, N, K)\) at \(I^* = I\), ensuring that there must exist a unique intersection between the two curves over the interval \(I^* \in (N - 1, I]\), which completes the proof of Proposition 13.

**Proof of Proposition 14:** We first establish the comparative statics of \(\omega\) with respect to \(I, N\) and \(K\) when both \(I^* = I < \bar{I}\) and \(I^* = \bar{I} < I\), and then turn to their effects on the level of factor prices.

**Comparative statics with respect to \(I\):** The relative demand locus \(\omega = \omega(I^*, N, K)\) does not directly depend on \(I\). Thus, the comparative statics are entirely determined by the effect of changes in \(I\) on the \(I^* = \min\{I, \bar{I}\}\) schedule in Figure 3-3. When \(I^* = \bar{I} < I\), small changes in \(I\) have no effect as claimed in the proposition. Suppose next that \(I^* = I < \bar{I}\). In this case, an increase in \(I\) shifts the curve \(I^* = \min\{I, \bar{I}\}\) to the right in Figure 3-3. From Lemma 4, we have that \(\omega(I^*, N, K)\) is decreasing in \(I^*\). This shift the shift in \(I\) increases \(I^*\) and reduces \(\omega\)—as stated in the proposition. Moreover, because \(I^* = I\), we have

\[
\frac{d\ln(W/R)}{dI} = \frac{d\ln \omega}{dI^*} = \frac{1}{\omega} \frac{\partial \omega}{\partial I^*} < 0,
\]

where \(\frac{\partial \omega}{\partial I^*}\) denotes the partial derivative of \(\omega(I^*, N, K)\) with respect to \(I^*\).

**Comparative statics for \(N\):** From Lemma 4, changes in \(N\) only shift the relative demand curve up in Figure 3-3. Hence, when \(I^* = I < \bar{I}\), we have

\[
\frac{d\ln(W/R)}{dN} = \frac{d\ln \omega}{dN} = \frac{1}{\omega} \frac{\partial \omega}{\partial N} > 0,
\]
where \( \frac{\partial \omega}{\partial N} \) denotes the partial derivative of \( \omega(I^*, N, K) \) with respect to \( N \).

Turning next to the case where \( I^* = \tilde{I} < I \), note that the threshold task is given by \( \gamma(I^*) = \omega K \). Therefore, \( dI^* = \frac{1}{\varepsilon_\gamma} d\ln \omega \) (where recall that \( \varepsilon_\gamma \) is the semi-elasticity of the \( \gamma \) function as defined in the proposition). Therefore, \( \frac{d\ln(W/R)}{dN} = \frac{d\ln \omega}{dN} \), and we can compute this total derivative as claimed in proposition:

\[
\frac{d \ln \omega}{dN} = \frac{1}{\omega} \frac{\partial \omega}{\partial N} + \frac{1}{\omega} \frac{\partial \omega}{\partial I^*} \varepsilon_\gamma \frac{d \ln \omega}{dN} = \frac{1}{\omega} \frac{\partial \omega}{\partial N} \left( 1 - \frac{1}{\omega} \frac{\partial \omega}{\partial I^*} \varepsilon_\gamma \right).
\]

**Comparative statics for \( K \):** The curve \( I^* = \min\{I, \tilde{I}\} \) does not depend on \( K \), all comparative statics are entirely determined by the effect of capital on \( \omega(I^*, N, K) \). An increase in \( K \) shifts up the relative demand locus in Figure 3-9 (this does not affect the ideal price index condition, which simplifies the analysis in this case), and thus increases \( W \) and reduces \( R \). The impact on \( \omega = \frac{W}{R} \) depends on whether the initial effect on \( W/R \) has elasticity greater than one (since \( K \) is in the denominator).

Notice that the function \( \omega(I^*, N, K) \) already incorporates the equilibrium labor supply response. To distinguish this supply response from the elasticity of substitution determined by factor demands, we define \( \omega^L(I^*, N, K, L) \) as the static equilibrium for a fixed level of the labor supply \( L \).

Once again using the notation \( \frac{\partial \omega^L}{\partial K} \omega^L = \frac{1}{\sigma_{SR}} - 1 \), and \( \frac{\partial \omega^L}{\partial L} \omega^L = \frac{1}{\sigma_{SR}} \), with \( \sigma_{SR} \) denoting the short-run elasticity of substitution between labor and capital, in the case where \( I^* = I < \tilde{I} \), we have

\[
\frac{d \ln \omega}{dN} = \left( \frac{1}{\sigma_{SR}} - 1 \right) \frac{d \ln K}{\frac{1}{\sigma_{SR}} + \varepsilon_L} + \frac{1}{\sigma_{SR}} \varepsilon_L \frac{d \ln K}{d \ln \omega} = \frac{1}{\sigma_{SR}} \varepsilon_L \frac{d \ln K}{d \ln \omega} = \frac{1 - \sigma_{SR}}{\sigma_{SR} + \varepsilon_L} \frac{d \ln K}{d \ln \omega},
\]

where we have used the fact that \( \omega(I^*, N, K) = \omega^L(I^*, N, K, L^*(\omega)) \). This establishes the claims about the comparative statics with respect to \( K \) when \( I^* = I < \tilde{I} \).

For the case where \( I^* = \tilde{I} < I \), we have

\[
\frac{d \ln \omega}{dN} = \frac{1 - \sigma_{SR}}{\sigma_{SR} + \varepsilon_L} \frac{d \ln K}{d \ln \omega} + \frac{1}{\omega} \frac{\partial \omega}{\partial I^*} \frac{1}{\varepsilon_\gamma} \frac{d \ln \omega}{dN} = \frac{1 - \sigma_{SR}}{\sigma_{SR} + \varepsilon_L} \frac{d \ln K}{d \ln \omega}.
\]

This expression implies the formula in the proposition.

**Effects on factor price levels:** Consider an increase in \( I \) in the case in which \( I^* = I < \tilde{I} \). In Figure 3-9, this increase in automation rotates the relative demand curve clockwise around the origin, but also shifts up the downward-sloping ideal price curve — which reflects the gains in productivity accruing to both factors.

Although in general the effects on the wage level are ambiguous, when \( \sigma \) is large, the
ideal price curve shifts by little and the effect through the relative demand curve dominates. In the limit in which \( \sigma \to \infty \), the ideal price curve does not shift, which implies that when automation increases the wage level declines. The proofs of the results for the effects of \( N \) on the rental rate are analogous. □

### 3.7.2 Proofs from Section 3.2

As in the main text, define \( w_N(n) = \lim_{t \to \infty} W(t)/\gamma(N(t)) \) and \( w_I(n) = \lim_{t \to \infty} W(t)/\gamma(I^*(t)) \). Notice that now we explicitly take into account the possibility that \( I^*(t) \) might be different from \( I(t) \).

Because in a BGP \( R = \rho + \delta + \theta g \), these effective wages will just be functions of \( n = \lim_{t \to \infty} N(t) - I(t) \). The next lemma characterizes their behavior in the BGP.

**Lemma 5 (Behavior of Effective Wages \( w_N(n) \) and \( w_I(n) \))** There exists \( \bar{\rho} > 0 \) such that:

1. For \( \rho > \bar{\rho} \), there exists \( \bar{n} \in (0,1) \) such that:
   - for \( n \geq \bar{n} \), we have \( I^* = I \), and for \( n < \bar{n} \), we have \( I^* < I \);
   - for \( n \geq \bar{n} \), \( w_N(n) \) is increasing and \( w_I(n) \) is decreasing in \( n \). Both wages are constant for \( n \leq \bar{n} \);

2. For \( \rho \leq \bar{\rho} \), there exists a different threshold \( \bar{n} \in [0,1) \) such that:
   - for \( n \geq \bar{n} \), both technologies are used, while for \( n < \bar{n} \), firms do not create or use new tasks (because labor is not productive or cheap enough compared to capital);
   - for \( n \geq \bar{n} \), \( w_N(n) \) is increasing and \( w_I(n) \) decreasing in \( n \). Both wages are decreasing in \( n \) for \( n < \bar{n} \).

**Proof:** See Appendix B.

Before proceeding, we will now state a result claimed in the text as a corollary of Lemma 5:

**Corollary 4** Suppose \( \rho > \bar{\rho} \). Then in the BGP, all new tasks will be produced with labor immediately.

This corollary follows immediately by noting that, in this case, \( \rho + \delta + \theta g > w_N(n) \) for all \( n \). Note however that this conclusion does not hold for \( \rho < \bar{\rho} \). In this case, for \( n \leq \bar{n} \), we have \( w_N(n) > \rho + \delta + \theta g \), which implies that new tasks will not be immediately produced with labor, which is more costly. This corollary justifies our focus in the main text on the case
where $\rho > \bar{\rho}$, and hence the fact that we did not introduce a separate notation to account for the possibility that, when created, new complex tasks might not be immediately produced using labor.

We now return to the rest of the proof of Proposition 15.

**Proof of Proposition 15:** We prove each part of the proposition separately.

**Part 1:** Since $\rho > \bar{\rho}$, this part follows directly from Lemma 5.

**Part 2:** We start by proving the “if” part. Suppose that $\hat{N} = \hat{I} = \Delta$ and that $\lim_{t \to \infty} n(t) = n \geq \bar{n}$. Then the normalized variables converge to values that solely depend on $n$. Given the functional form of $\gamma$ in (3.12), it follows that $Y, C, K, W$ grow at the same rate $g = A\Delta$, while the rental rate, $R$, remains constant. This shows that balanced growth emerges when both technologies advance at the same speed.

For the “only if” part, note that in any BGP, $Y, C, K$ and $W$ must grow at some constant rate $g$, and thus $y, c, k$ and $w_I$ must also grow at some constant rate $\tilde{g}$ while $R$ remains constant at $\rho + \delta + \theta g$. If $\tilde{g} = 0$, we have that $y, c, k$ and $w_I$ must converge. Because the behavior of these normalized variables only depends on $n(t)$, we must also have that $\lim_{t \to \infty} n(t) = n \in [\bar{n}, 1]$, which then implies that $\hat{N} = \hat{I}$. Moreover, $\hat{N} = \hat{I} = \Delta$ because $Y, C, K$ and $W$ grow at a constant rate. These observations show that to establish the “only if” part it is enough to show that $\tilde{g} = 0$.

Suppose to obtain a contradiction that $\tilde{g} < 0$. Then, for $t$ large enough, we will have $w_I(t) < R(t)$. This implies that, at this point, newly automated tasks are not immediately produced with capital, which contradicts Lemma 5 (recall that $\rho > \bar{\rho}$ and $n(t) \in [\bar{n}, 1]$).

Next, suppose once again, to obtain a contradiction, that $\tilde{g} > 0$. This implies that for $t$ large enough, we will have $w_I(t)/\gamma(1) > R(t)$, which implies that $w_N(t) > R(t)$. At this point in time, newly created tasks are not immediately produced with labor, which contradicts Lemma 3-12 (since we have $\rho > \bar{\rho}$). This establishes the “only if” direction of the proof.

**Part 3:** We start by proving the “if” part. Suppose that $n(t) < \bar{n}$ for all $t > T$, and that $\dot{N} = \Delta$. Lemma 5 implies that $I^*(t) = I(t) < I(t)$, and therefore, $w_I(t) = R(t)$.

Let us also define $n^*(t) \equiv N(t) - I^*(t)$. Then, from the same steps that we used in the main text, it follows that $y, c, k$, and $w_I$ converge to values that depend only on $n^*(t)$. Because asymptotically, $R(t)$ is constant and $w_I(t) = R(t)$, we must have that $n^*(t)$ converges to $\bar{n}$. Moreover, because the normalized variables converge to values that only depend on $n^*$, the economy achieves a BGP in which $Y, C, K$ and $W$ grow at the rate $A\Delta$—the same rate as $\gamma(N(t))$ and $\gamma(I^*(t))$—while the rental rate, $R$, remains constant.

For the “only if” part, note that in any BGP with $n(t) < \bar{n}$, we have that $n^*(t) = \bar{n}$. This implies that $w_I, w_N, y, c, k$ converge to constant values. Because $Y, C, K$ and $W$ grow at some constant rate $g$, we must have that $\gamma(I^*(t))$ and $\gamma(N(t))$ also grow at this constant.
rate, which implies that \( \dot{N} = \Delta \) and \( g = A\Delta \), as claimed.

**Part 4:** Starting with any initial value of \( k(0) \) and \( n(0) \), the equilibrium behavior is given by equations (3.14), (3.16) and (3.15). This is identical to the equations characterizing dynamics in the canonical neoclassical growth model (see, for example, Proposition 8.5 and 8.6 in Acemoglu (2009)). Moreover, the condition that \( \rho > \bar{\rho} \) guarantees that \( \rho > A(1-\theta)\Delta \), which ensures that the transversality condition holds, establishing part 4.

**Proof of Proposition 16:** That there are no effects when \( n \leq \bar{n} \) follows from Lemma 5. Next consider the case \( n > \bar{n} \), in which technology changes the asymptotic behavior of the economy. The expressions and comparative statics for the wages follow from equation (3.18), which we derived in the proof of Lemma 5.

The asymptotic behavior of the labor share can be established by considering two separate cases. First, suppose that \( \sigma_{SR} \leq 1 \). Let \( k_{I}(n) \) denote the steady-state value for \( K/\gamma(I) \). We have \( \omega(0, n, k_{I}(n)) \equiv \frac{w_{I}(n)}{(\rho+\delta+\theta_{2})k_{I}(n)} \). Differentiating this expression, we obtain

\[
k'_{I}(n) = \frac{w'_{I}(n)\frac{1}{Rk} - \frac{\partial \omega}{\partial N}}{\omega \frac{1+\varepsilon_{L}}{k \sigma_{SR}+\varepsilon_{L}}}.\]

Using this expression, we compute the total effect of technology on \( \omega \) as

\[
\frac{d\omega}{dn} = \frac{\partial \omega}{\partial N} \left( \sigma_{SR} + \varepsilon_{L} \right) + \frac{w'_{I}(n)}{Rk} \left( 1 - \sigma_{SR} \right).
\]

Because \( \frac{\partial \omega}{\partial N} > 0 \) and \( w'_{I}(n) > 0 \), we have that, whenever \( \sigma_{SR} \leq 1 \), \( \omega \) increases with \( n \).

Next suppose that \( \sigma_{SR} > 1 \). Let \( k_{N}(n) \) denote the steady-state value for \( K/\gamma(N) \). We have that \( \omega(0, n, k_{N}(n)) \equiv \frac{w_{N}(n)}{(\rho+\delta+\theta_{2})k_{N}(n)} \). Differentiating this expression, we have

\[
k'_{N}(n) = \frac{w'_{N}(n)\frac{1}{Rk} + \frac{\partial \omega}{\partial I^{*}}}{\omega \frac{1+\varepsilon_{L}}{k \sigma_{SR}+\varepsilon_{L}}} < 0.
\]

We can then compute the total effect of technology on \( \omega \) as

\[
\frac{d\omega}{dn} = -\frac{\partial \omega}{\partial I^{*}} \left( \sigma_{SR} + \varepsilon_{L} \right) + \frac{w'_{N}(n)}{Rk} \left( 1 - \sigma_{SR} \right).
\]

Because \( \frac{\partial \omega}{\partial I^{*}} < 0 \) and \( w'_{N}(n) < 0 \), we have that, whenever \( \sigma_{SR} \geq 1 \), \( \omega \) increases with \( n \). Since \( \omega \equiv \omega(-n, 0, k_{N}(n)) \equiv \omega(0, n, k_{I}(n)) \), We again conclude that, in the long run, \( \omega \) always increases with \( n \). ■
3.7.3 Proofs from Section 3.3

Let \( v_N(n) \equiv \lim_{t \to \infty} V_N(t)/Y(t) \) and \( v_I(n) \equiv \lim_{t \to \infty} V_I(t)/Y(t) \) be the normalized value functions, which in the BGP only depend on \( n \).

**Lemma 6 (Asymptotic Behavior of the Normalized Value Functions)** Suppose that \( \sigma > \zeta \), and that the conditions required in Lemma 5 hold. Let \( \overline{\rho} \) be as defined in Proposition 15. Then there exist thresholds \( \overline{A} \) and \( \overline{\rho} \) such that:

1. For \( \rho > \overline{\rho} \) and \( A < \overline{A} \):
   - if \( n < \overline{n} \), we have \( \kappa_N v_N(n) > \kappa_I v_I(n) = O(g) \);
   - if \( n \geq \overline{n} \), \( v_N(n) \) and \( v_I(n) \) are strictly increasing in \( n \);
   - if in addition \( \rho > \overline{\rho} \), then for \( n \geq \overline{n} \), \( v_I(n)/v_N(n) \) is increasing in \( n \).

2. For \( \rho \leq \overline{\rho} \):
   - for \( n < \overline{n} \), we have \( v_I(n) > 0 \) and \( v_N(n) \leq 0 \);
   - for \( n \geq \overline{n} \), both \( v_N(n) \) and \( v_I(n) \) are positive and strictly increasing.

**Proof.** Let \( g \equiv \frac{A_{\kappa_N}}{\kappa_I + \kappa_N} \) be the growth rate of the economy in the BGP. Suppose \( \rho > \overline{\rho} \). Then for \( n \geq \overline{n} \), we can write the value functions in the BGP as:

\[
\begin{align*}
v_N(n) &= M \int_0^\infty e^{-(\rho + (1 - \theta)g)\tau} \left[ c^u (w_N(n)e^{\theta\tau})^{\zeta-\sigma} - c^u (w_I(n)e^{\theta\tau})^{\zeta-\sigma} + O(g) \right] d\tau, \\
v_I(n) &= M \int_0^\infty e^{-(\rho + (1 - \theta)g)\tau} \left[ c^u (\rho + \delta + \theta g)^{\zeta-\sigma} - c^u (w_I(n)e^{\theta\tau})^{\zeta-\sigma} + O(g) \right] d\tau,
\end{align*}
\]

where we have defined \( M \equiv (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^{\zeta} \psi^{1-\zeta} \). Thus, the value functions only depend on the unit cost of labor \( w_N(n) \) and \( w_I(n) \), and on the rental rate, which is equal to \( \rho + \delta + \theta g \) in the BGP.

Now consider Taylor expansions of both of these expressions (which are continuously differentiable) around \( g = 0 \):

\[
\begin{align*}
v_N(n) &= \frac{M}{\rho} \left[ c^u (w_N(n))^{\zeta-\sigma} - c^u (\rho + \delta)^{\zeta-\sigma} + O(g) \right], \\
v_I(n) &= \frac{M}{\rho} \left[ c^u (\rho + \delta)^{\zeta-\sigma} - c^u (w_I(n))^{\zeta-\sigma} + O(g) \right],
\end{align*}
\]

where \( O(g) \) denotes terms that vanish when the growth rate, \( g \), is small, which enables us to approximate the integrals (solving them out explicitly when \( g = 0 \)). Moreover, when
$n < \bar{n}$, automated tasks are not immediately produced with capital. Instead, capital is used when wages have grown enough. Because future wage growth provides the only incentive for automation, we have $v_I(n) = O(g)$. On the other hand, the expression for $v_N(n)$ still applies and remains bounded away from zero. Thus, there exists $\bar{A} > 0$ such that for $A < \bar{A}$ (which guarantees that $g$ is small), $\kappa_N v_N(n) > \kappa_I v_I(n) > 0$ as claimed.

Differentiating the value functions in (3.27) when $n \geq \bar{n}$ immediately establishes that they are both strictly increasing (since $w_I(n) > \rho + \delta + \theta g > w_N(n)$ from Lemma 5). The implied behavior of the normalized value functions is depicted in Figure 3-10.

![Figure 3-10: Behavior of value functions in steady state with respect to changes in $n = N - I$.](image)

We now prove the existence of a threshold $\bar{\rho}$, which guarantees that the curves $\kappa_N v_N(n)$ and $\kappa_I v_I(n)$ cross at most once. To prove this result, note that $\frac{v'_I(n)}{v'_N(n)} > \frac{v'_N(n)}{v'_N(n)}$ if and only if

$$c^u(w_I(n))^{1-\sigma} s(I) c^u(\rho + \delta)^{1-\sigma} c^u(w_N(n))^{1-\sigma} (c^u(w_N(n))^{1-\sigma} - c^u(\rho + \delta))^{1-\sigma} >
\frac{c^u(w_N(n))^{1-\sigma} s(I) c^u(w_I(n))^{1-\sigma} - c^u(\rho + \delta)^{1-\sigma}}{1 - \sigma} (c^u(\rho + \delta)^{1-\sigma} - c^u(w_I(n))^{1-\sigma}),$$

(3.28)

where recall that $s(i)$ is the labor share in the production of task $i$.

For $\rho$ sufficiently large, we have that $\rho + \delta \to w_I(n)^+$, and thus the inequality (3.28) necessarily holds (notice that, while the right-hand side is strictly positive, the left-hand side always converges to zero). Thus, there exists a threshold $\bar{\rho}$ such that for $\rho > \bar{\rho}$ and $A < \bar{A}$, the single crossing condition (3.28), which ensures that $v_I/v_N$ is increasing in $n$, holds for all $n \geq \bar{n}$. (If the value of $\bar{\rho}$ that ensures this property is strictly less than $\bar{\rho}$, then we simply set $\bar{\rho} = \bar{\rho}$.)

The second part of the lemma has an analogous proof, with the behavior of normalized values given as in the right panel of Figure 3-10, and the details are omitted.

Proof of Proposition 17:
**Part 1:** A BGP emerges if and only if \( \dot{\hat{r}} = \hat{N}, \) and \( n(t) = n^D. \) To ensure that asymptotically \( \dot{n} = 0, \) in a BGP we must have

\[
\kappa_I v_I(n) = \kappa_N v_N(n).
\]

Thus, a BGP exists if and only if there exists a solution \( n^D \) to this equation.

Given \( \rho > \bar{\rho} \) and \( \bar{A} > A, \) Lemma 6 implies that \( \kappa_N v_N(\bar{n}) > \kappa_I v_I(\bar{n}). \) This shows that at \( \bar{n}, \) the curve \( \kappa_N v_N(\bar{n}) \) is above \( \kappa_I v_I(\bar{n}). \) However, as the ratio \( \frac{\kappa_I}{\kappa_N} \) increases—starting from zero—the curves \( \kappa_I v_I(n) \) and \( \kappa_N v_N(n) \) eventually cross at some point. This proves that for \( \kappa > \bar{\kappa}, \) there exists a BGP represented by the first intersection at \( n^D \in (\bar{n}, 1) \) between these two curves. Figure 3-6 in the main text illustrates the determination of \( n^D \) in this case. Finally, if \( \rho > \bar{\rho}, \) from Lemma 6 \( v_I/v_N \) is increasing in \( n, \) so \( n^D \) defined by \( \kappa_I v_I(n^D) = \kappa_N v_N(n^D) \) is unique.

**Part 2:** We will first prove the global stability claim for \( \theta = 0, \) and then turn to local stability when \( \theta > 0. \)

**Proof of stability of the unique BGP when \( \theta = 0: \)** Suppose that \( \theta = 0. \) In this case, capital adjusts immediately and its equilibrium stock only depends on \( n, \) which becomes the unique state variable of the model. The rental rate is fixed at \( R = \rho + \delta \) (or the interest rate is \( r = \rho), \) and the effective wages are given by \( w_I(n) \) and \( w_N(n) \).

Define next \( v \equiv \kappa_I v_I - \kappa_N v_N. \) Now starting from any \( n(0), \) an equilibrium with endogenous technology is given by the path of \( (n, v) \) such that the evolution of the state variable is given by

\[
\dot{n} = \kappa_N S - (\kappa_N + \kappa_I)G \left( \frac{v}{\lambda} \right) S,
\]

and the difference of the normalized value functions \( v \) satisfies the forward looking differential equation:

\[
\rho v - \dot{v} = M\kappa_I \left( c^u (\rho + \delta)^{\xi-\sigma} - c^u (w_I)^{\xi-\sigma} \right) - M\kappa_N \left( c^u (w_N)^{\xi-\sigma} - c^u (\rho + \delta)^{\xi-\sigma} \right) + O(g),
\]

and in addition the transversality condition (3.16) holds.

Let \( n^D \) denote the BGP value for \( n(t) \) in the unique BGP. Since \( \kappa_I v_I(n) - \kappa_N v_N(n) \) crosses zero only once at \( n^D \) in this case, equilibrium dynamics can be analyzed using the behavior of the value difference, \( v, \) as in Figure 3-6. We now prove that this BGP is globally stable. Figure 3-11 presents the phase diagram of the system in \( (v, n). \) Importantly, the locus for \( \dot{v} = 0 \) crosses \( v = 0 \) at \( n^D \) from below only once. This follows from the fact that \( \kappa_I v_I(n^D) > \kappa_N v_N(n^D) \) (that is, \( \kappa_I v_I(n) \) cuts \( \kappa_N v_N(n) \) from below at \( n^D \) as shown in Figure 3-6).
The system of differential equations determining the behavior of \( n, v \) near this BGP can be linearized as:
\[
\dot{n} = \frac{(\kappa_N + \kappa_I)}{\lambda} G'(0) S v, \quad \text{and} \quad \dot{v} = \rho v - Q,
\]
where \( Q > 0 \) denotes the derivative of \(-M_{\kappa I} c^u(w_I)^{\xi-\sigma} + M_{\kappa N} c^u(w_N)^{\xi-\sigma}\) with respect to \( n \) (this derivative is positive because \( \kappa_I v_I(n) < \kappa_N v_N(n) \) from below at \( n^D \)). Thus, the eigenvalues of the characteristic polynomial of this system add up to \( \rho > 0 \), and their product is given by \(-Q (\kappa_N + \kappa_I) G'(0) S < 0\). This implies that there is one positive and one negative eigenvalue, ensuring asymptotic (saddle-path) stability. It also follows from the same argument that for each \( n(0) \), there is a unique \( v(0) \) in the stable arm of the system, and thus guarantees uniqueness in the neighborhood of the BGP.

In order to show that globally all equilibria must be along the stable arm, we need to rule out other potential equilibrium paths. From the figure it is clear that if the equilibrium does not settle at \( n^D \), it must reach the region with \( \dot{v} > 0 \) and \( \dot{n} < 0 \), or it must reach the region with \( \dot{v} < 0 \) and \( \dot{n} > 0 \). In the first case, \( v \) is strictly increasing and \( n \) is strictly decreasing, and hence there are no interior limit points. This implies \( v \rightarrow \infty \) along any such path, and thus \( u_I \rightarrow \infty \), which violates the transversality condition (3.16). In the second case, \( v \rightarrow -\infty \) and \( n \rightarrow 1 \), which again analogously violates the transversality condition.

**Proof of local stability of the unique BGP when \( \theta > 0 \):** Let us next turn to the case in which \( \theta > 0 \). An equilibrium is given by a solution to the following system of differential equations: Starting from any \( n(0), k(0) \) the equilibrium path with endogenous technology is given by a tuple \( \{n, k, c, v\} \) such that:

- The evolution of the state variables is given by
\[
\dot{n}(t) = \kappa_N S - (\kappa_N + \kappa_I) G\left(\frac{v}{\lambda}\right) S,
\]
• The paths for \(c(t)\) and \(k(t)\) satisfy the Euler equation,

\[
\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(R^E(n(t), k(t)) - \delta - \rho) - \mathcal{O}(g)
\]

coupled with the transversality condition in equation (3.16), and the resource constraint,

\[
\dot{k}(t) = f^E(n(t), k(t)) - c(t) - \delta k(t).
\]

• The value function \(v\) satisfies the forward looking differential equation:

\[
\rho v - \dot{v} = \kappa_1 \pi_1(n, k) - \kappa_N \pi_N(n, k) + \mathcal{O}(g),
\]

with

\[
\pi_N(n, k) = M \left( c^a \left( w^E_N(n, k) \right)^{\xi - \sigma} - c^a \left( R^E(n, k) \right)^{\xi - \sigma} \right) \pi_1(n, k) = M \left( c^a \left( R^E(n, k) \right)^{\xi - \sigma} - c^a \left( w^F(n, k) \right)^{\xi - \sigma} \right)
\]

denoting the flow profits for innovators. Here, \(w^E_N = \frac{W}{\gamma(n)}\) and \(w^F_I = \frac{W}{\gamma(I)}\), which also depend on the stock of capital outside of the BGP.

By continuity, there exists a threshold \(\overline{A} \leq \overline{A}\) such that, for \(A < \overline{A}\), the local behavior of the above system matches that of the system in which we take the limit \(g \to 0\) in the above system of equations.

To simplify the notation, define the partial derivatives

\[
Q_k = \kappa_1 \frac{\partial \pi_1}{\partial k} - \kappa_N \frac{\partial \pi_N}{\partial k} > 0, Q_n = \kappa_1 \frac{\partial \pi_1}{\partial n} - \kappa_N \frac{\partial \pi_N}{\partial n} > 0, Q_v = \frac{\kappa_1 + \kappa_N}{\lambda} G'(0) S > 0.
\]

evaluated at their BGP values. Then local equilibrium dynamics can be linearly approximated around the BGP as follows (where \(n^D, v^D(= 0), k^D\) and \(c^D\) designate the BGP values):

\[
\dot{n} = -Q_v v,
\]

\[
\dot{v} = \rho v - Q_k [k(t) - k^D] - Q_n [n(t) - n^D],
\]

\[
\dot{c} = \frac{c^D}{\theta} R_n^E [n(t) - n^D] + \frac{c^D}{\theta} R_k^E [k(t) - k^D]
\]

\[
\dot{k} = f_n^E [n(t) - n^D] + f_k^E [k(t) - k^D] - [c - c^D].
\]

Here, the \(f_n^E, R_n^E\) and \(f_k^E, R_k^E\) denote the partial derivatives of \(f\) and \(R\) with respect to \(n\) and \(k\). The characteristic polynomial of the linearized system of differential equations (with
all derivatives still evaluated at their BGP values) can be written as

\[
P(\lambda) = \begin{pmatrix}
-\lambda & -Q_v & 0 & 0 \\
-Q_n & \rho - \lambda & 0 & -Q_k \\
c^D \theta R^E_n & 0 & -\lambda & c^D \theta R^E_k \\
f^E_n & 0 & -1 & f^E_k - \delta - \lambda
\end{pmatrix},
\]

or expanding it:

\[
P(z) = z^4 - 3(f^E_k - \delta + \rho) + z^2 \left(-Q_v Q_n + \frac{c^D}{\theta} R^E_k + \rho(f^E_k - \delta)\right)
- z \left(Q_v(f^E_n Q_k - (f^E_k - \delta) Q_n) + \rho \frac{c^D}{\theta} R^E_k\right) + Q_v(R^E_n Q_k - R^E_k Q_n) \frac{c^D}{\theta}.
\]

We now show that this polynomial has exactly two eigenvalues with positive real parts and two eigenvalues with negative real parts. First, note that \(R^E_n Q_k - R^E_k Q_n > 0\), which is also the condition for the curve \(\kappa_{I^j}(n)\) cutting \(\kappa_{N^j}(n)\) from below. Indeed, the term \(R^E_n Q_k - R^E_k Q_n > 0\) corresponds to the change in flow profits, \(\kappa_{I^j} - \kappa_{N^j}\) that results from an increase in \(n\) when capital adjusts to keep the interest rate constant. Next, let \(z_1, z_2, z_3\) and \(z_4\) be the eigenvalues of the above system. Then \(z_1 z_2 z_3 z_4 = Q_v(R^E_n Q_k - R^E_k Q_n) \frac{c^D}{\theta} > 0\). Moreover, \(z_1 + z_2 + z_3 + z_4 = f^E_k - \delta + \rho > 0\) (which is the trace of the matrix \(P(z)\)). This implies that either there are exactly two eigenvalues with positive real parts and two eigenvalues with negative real parts, or all eigenvalues have positive real parts. We rule out the latter possibility by showing that the system has at least one negative real root.

To do so, we prove instead that the polynomial \(P(-z)\) has at least one positive real root. Descartes’ rule of signs, applied to the polynomial \(P(-z)\), implies that this will be the case provided that at least one of the coefficients \(Q_v(f^E_n Q_k - (f^E_k - \delta) Q_n) + \rho \frac{c^D}{\theta} R^E_k\) and \(-Q_v Q_n + c^D \frac{c^D}{\theta} R^E_k + \rho(f^E_k - \delta)\) is negative. This is indeed the case as we can see by separately considering two cases. First, suppose that \(f^E_k - \delta < 0\). Then

\[-Q_v Q_n + \frac{c^D}{\theta} R^E_k + \rho(f^E_k - \delta) < 0.\]

This follows from Proposition 14, which establishes that \(Q_n > 0\) (i.e., as \(n\) increases—holding capital constant—the incentives to do automation increase). Suppose, alternatively, that \(f^E_k - \delta > 0\). Then \(f^E_n Q_k - (f^E_k - \delta) Q_n < 0\), this also follows from Proposition 14, which ensures that an increase in \(n\), holding output constant, raises the wage relative to the rental rate, or equivalently

\[Q_n - Q_k \frac{f^E_n}{f^E_k - \delta} > 0,\]
which implies that, when $f_k^E - \delta > 0$, we must also have $f_k^E Q_n - (f_k^E - \delta)Q_n < 0$, and thus

$$Q_v(f_k^E Q_n - (f_k^E - \delta)Q_n) + \rho \frac{c^D}{\theta} R_k^E < 0.$$ 

Because the model has two state variables and there are exactly two roots with negative real parts, equilibrium dynamics are asymptotically (saddle-path) stable.

**Part 3:** When $\kappa < \bar{\kappa}$, we have $\kappa IV_1 < \kappa N vN$ throughout, so $\dot{n} > 0$. Thus, asymptotically we have $n(t) = 1$ and the BGP is identical to that of an endogenous growth model with purely labor-augmenting technological change.

**Part 4:** As the right panel of Figure 3-10 shows, when $\rho \leq \bar{\rho}$, Lemma 6 implies that for $n \leq \bar{n} \kappa IV_1(n) > \kappa N vN(n)$. This observation implies that there is an asymptotically stable BGP with $n(t) = 0$. ■

**References**


172


3.8 Appendix B (Not-For-Publication): Omitted Proofs and Additional Results

3.8.1 Details of the Empirical Analysis

Here we provide information about the data used in constructing Figures 3-1 and 3-7. We also provide a regression analysis documenting the robustness of the patterns illustrated in these figures.

Data: We use data on the demographic characteristics of workers and employment counts in each of the 330 consistently defined occupations proposed by David Dorn (see http://www.ddorn.net/data.htm). Our sources of data are the U.S. Censuses for 1980, 1990 and 2000, and the American Community Survey for 2007. We focus on the set of workers between 16 and 64 years of age.


Detailed Analysis for Figure 3-1: To document the role of new job titles in employment growth, we estimate the regression

$$\ln E_{it+10} - \ln E_{it} = \beta N_{it} + \delta_t + \Gamma_t X_{it} + \varepsilon_{it}. \quad (3.29)$$

Here, the dependent variable is the percent change in employment from year $t$ to $t + 10$ in each occupation $i$. We stack the data for $t = 1980, 1990, 2000$. For $t = 2000$, we use the change from 2000 to 2007 as the dependent variable and re-scale it to a 10-year change. In all regressions we include a full set of decadal effects $\delta_t$, and in some models we also
control for differential decadal trends that vary depending on observable characteristics of each occupation, $\Gamma_t X_{it}$. These characteristics include the share of workers in different 5-year age brackets and from different races (Black, Hispanic), and the share of foreign and female workers. These covariates flexibly control for demographic changes that may affect the labor supply that is relevant for each occupation. Finally, $\varepsilon_{it}$ is an error term. Throughout, all standard errors are robust against arbitrary heteroskedasticity and serial correlation of the error term within occupations.

The coefficient of interest is $\beta$, which represents the additional employment growth in occupations with a large share of new job titles, $N_{it}$.

Panel A in Table 3.1 presents estimates of equation (3.29). Column 1 contains no additional covariates (the number of observations in this column is 989. We miss one observation because Lin's measure only covers 329 occupations in 1980). Our estimates indicate that occupations with 10 percentage points more new job titles at the beginning of each decade grew 5.05% faster over the decade (standard error = 1.29%). If occupations with more new job titles did not grow any faster than the benchmark category with no novel jobs, employment growth from 1980 to 2007 would have been, on average, 8.66% instead of the actual 17.5%, implying that approximately 8.84% of the 17.5% growth is accounted for by new job titles as reported at the bottom rows of the panel.

In column 2 we control for the log of employment at the beginning of the decade (year $t$). The coefficient of interest increases slightly to 0.560 and continues to be precisely estimated. The log of employment at year $t$ appears with a negative coefficient, which indicates that smaller occupations tend to grow more over time. The quantitative contribution of new tasks and job titles remains very similar to column 1, increasing slightly to 9.8%.

In column 3 we control for the trends that depend on the demographic covariates described above, which have little effect on the quantitative results. In column 4, we also control for the average years of schooling among workers in each occupation at the beginning of the decade. Although this covariate reduces the magnitude of the coefficient of the share of new job titles, our estimate for $\beta$ remains highly significant. The contribution of new job titles is now estimated at 6.62% out of the 17.5% growth between 1980 and 2007.

Column 5 repeats the specification of column 4, but this time we reweight the data by the 1980 share of employment in each occupation. This weakens the relationship of interest, and the share of novel tasks and jobs is no longer statistically significant. However, this lack of significance is driven by a few large occupations that are outliers in the estimated relationship. (In contrast, there are no major outliers in the unweighted regressions reported in columns 1-4). These outliers include office supervisors, office clerks, and production supervisors; three occupations that had combined employment of about 4 million workers in 1980 and have been contracting since then. Though these occupations introduced a significant number
Table 3.1: Differential employment growth in occupations with more new job titles

<table>
<thead>
<tr>
<th></th>
<th>Dep. var: Percent change in employment growth by decade.</th>
<th>Weighted by size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>Panel A: Stacked differences over decades.</td>
<td></td>
</tr>
<tr>
<td>Share of new job titles at the start of decade</td>
<td>0.505***</td>
<td>0.560***</td>
</tr>
<tr>
<td></td>
<td>(0.129)</td>
<td>(0.127)</td>
</tr>
<tr>
<td>log of employment at start of decade</td>
<td>-0.031**</td>
<td>-0.045***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Average years of schooling at start of decade</td>
<td></td>
<td>9.602***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.864)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>Observations</td>
<td>989</td>
<td>989</td>
</tr>
<tr>
<td>Occupations</td>
<td>330</td>
<td>330</td>
</tr>
<tr>
<td>Employment growth from 1980-2007 in p.p.</td>
<td>17.5</td>
<td>17.5</td>
</tr>
<tr>
<td>Contribution of novel tasks and jobs</td>
<td>8.84</td>
<td>9.8</td>
</tr>
<tr>
<td>Share of new job titles in 1980</td>
<td>1.247***</td>
<td>1.398***</td>
</tr>
<tr>
<td></td>
<td>(0.392)</td>
<td>(0.348)</td>
</tr>
<tr>
<td>log of employment in 1980</td>
<td>-0.150***</td>
<td>-0.183***</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Average years of schooling in 1980</td>
<td>21.779***</td>
<td>15.978***</td>
</tr>
<tr>
<td></td>
<td>(4.162)</td>
<td>(4.204)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>Observations</td>
<td>329</td>
<td>329</td>
</tr>
<tr>
<td>Occupations</td>
<td>329</td>
<td>329</td>
</tr>
<tr>
<td>Employment growth from 1980-2007 in p.p.</td>
<td>17.5</td>
<td>17.5</td>
</tr>
<tr>
<td>Contribution of novel tasks and jobs</td>
<td>7.27</td>
<td>8.155</td>
</tr>
</tbody>
</table>

**Covariates:**
- Decade fixed effects
- Demographics \times decade effects

**Notes:** The table presents 10-years stacked-differences estimates (Panel A) and long-differences estimates (Panel B) of the share of new job titles in an occupation on subsequent employment growth. The bottom row in each panel reports the observed growth and the share explained by growth in occupations with more new job titles. The bottom rows indicate additional covariates included in each model. In column 5 we reweight the data using the baseline share of employment in each occupation in 1980, and in column 6 we exclude three large employment categories that are outliers in the model of column 5. These include office supervisors, office clerks, and production supervisors. Standard errors robust against heteroskedasticity and serial correlation within occupations are presented in parentheses.

of new job titles in 1980, they shed a large amount of workers in the subsequent years. In column 6, we exclude these three occupations from our analysis, and obtain a similar pattern to that of column 4.

Finally, in Panel B, we present a set of regressions, which are analogous to those in the top panel, but that focus on a long difference specification between 1980 and 2007. The overall patterns are very similar, and now the contribution of novel tasks and new job titles
to the 17.5% growth in employment between 1980 and 2007 is between 7.27 and 8.5%.

**Detailed Analysis for Figure 3-7:** To document the presence of some amount of "standardization" of occupations with greater new job titles,

\[ \Delta Y_{it} = \beta N_{it} + \delta_t + \Gamma_t X_{it} + \varepsilon_{it}. \]  

(3.30)

Here, the dependent variable is the change in the average years of schooling among workers employed in occupation \( i \) and measured over different time horizons (10 years, 20 years or 30 years). We stack the data for \( t = 1980, 1990, 2000 \), but our sample becomes smaller as we measure the change in average years of schooling over longer periods of time. We again use the ACS dayear 2007. The covariates and independent variable are the same ones that we used in equation (3.29).

Panel A in Table 3.2 presents estimates of equation (3.30). Columns 1 and 2 present models in which the dependent variable is the change in average years of education over a 10-year period (hence, we get 989 observations for 330 occupations). Columns 3 and 4 focus on the change in average years of education over a 20-year period. Columns 5 and 6 focus on the change in average years of education over a 30-year period. In the models presented in the even columns we include a full set of trends that are allowed to vary depending on the composition of employment in each occupational category.

Our estimates indicate that, although occupations with more new job titles tend to hire more skilled workers initially, this pattern slowly reverts over time. Figure 3-7 shows that, at the time of their introduction, occupations with 10 percentage points more new job titles hire workers with 0.35 more years of schooling. But our estimates in Column 6 of Table 3.2 show that this initial difference in the skill requirements of workers slowly vanishes over time. 30 years after their introduction, occupations with 10 percentage points more new job titles hire workers with 0.0411 fewer years of education than the workers hired initially (standard error= 0.0176).

Relatedly, Panel B of the same table shows a similar pattern when we look at occupations that start with greater average years of schooling at the beginning of a decade. In particular, for each decade since 1980, employment growth has been faster in occupations with greater skill requirements—as measured by the average years of education among employees at the start of each decade (see Table 3.1). But estimating a version of (3.30) with average years of schooling at the beginning of the decade on the right-hand side, we find significant mean reversion. For example, Column 6 shows that occupations that used to hire workers with one additional year of schooling workers reduce their average years of schooling by 0.149 years relative to baseline after 30 years (standard error=0.021).
Table 3.2: Reversal in skill content for occupations with more new job titles and in occupations that used to hire more educated workers.

<table>
<thead>
<tr>
<th></th>
<th>Dep. var: Change in average years of schooling.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Over 10 years</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Share of new job titles at the start of decade</td>
<td>-0.085</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.32</td>
</tr>
<tr>
<td>Observations</td>
<td>989</td>
</tr>
<tr>
<td>Occupations</td>
<td>989</td>
</tr>
<tr>
<td>Average years of education at the start of decade</td>
<td>-0.030***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.36</td>
</tr>
<tr>
<td>Observations</td>
<td>990</td>
</tr>
<tr>
<td>Occupations</td>
<td>990</td>
</tr>
<tr>
<td>Covariates:</td>
<td></td>
</tr>
<tr>
<td>Decade fixed effects</td>
<td>✓</td>
</tr>
<tr>
<td>Demographics × decade effects</td>
<td>✓</td>
</tr>
</tbody>
</table>

Notes: The table presents OLS estimates that explain the change in average years of schooling among workers employed in a given occupation. These changes are computed over 10 years (Columns 1 and 2), 20 years (Columns 3 and 4) and 30 years (Columns 5 and 6). In Panel A we explain the subsequent change in years of schooling as a function of share of new job titles in each occupation at the start of the decade. In Panel B we explain the subsequent change in years of schooling as a function of the years of schooling in each occupation at the start of the decade. The bottom rows indicate additional covariates included in each model. Standard errors robust against heteroskedasticity and serial correlation within occupations are presented in parentheses.

3.8.2 Remaining Proofs from Section 3.1

We start by providing the proof of Lemma 4.

Proof of Lemma 4. As we have just seen, the equilibrium conditions that uniquely determine \( W(I^*, N, K) \) and \( R(I^*, N, K) \), are:

\[
L^S \left( \frac{W}{RK} \right) (I^* - N + 1) e^u(R) \gamma^{-\sigma} R^{-\zeta} - K \int_{I^*}^{N} \gamma(i)^{\zeta-1} e^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} di = 0
\]

\[
(i^* - N + 1) e^u(R)^{1-\sigma} \int_{I^*}^{N} e^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} di = 1.
\]
Taking total derivatives with respect to $I^*$, $N$, the first equation yields
\[
dN \left( L^*e^u(r)\gamma^{-\sigma}r^{-\zeta} + K\gamma(N)\gamma^{-1}c^u \left( \frac{W}{\gamma(N)} \right)^{\zeta-\sigma} W^{-\zeta} \right)
\]
\[-dI^* \left( L^*e^u(r)\gamma^{-\sigma}r^{-\zeta} + K\gamma(I^*)\gamma^{-1}c^u \left( \frac{W}{\gamma(I^*)} \right)^{\zeta-\sigma} W^{-\zeta} \right) \]
\[
=(d\ln W - d\ln r) \left( K \int_{I^*}^{N} \gamma(i)^{\zeta-1}c^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} [s(i)(\sigma - \zeta) + \zeta] di + \varepsilon_L L^2(I^* - N + 1)c^u(r)^{\zeta-\sigma}r^{-\zeta} \right)
\]
\[+d\ln r \left( K \int_{I^*}^{N} \gamma(i)^{\zeta-1}c^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} [(s(i) - s_k)(\sigma - \zeta)] di \right).
\]

Here, $s(i)$ is the share of labor in the production of task $i$ and $s_k$ is the share of capital in tasks produced with capital.

The last term in the above equation captures the non-homotheticity introduced by the presence of intermediate goods. In all the special cases in which $(s(i) - s_k)(\sigma - \zeta) = 0$, the demand system is homothetic and the results outlined in Lemma 4 follow easily. These cases include the limits when $\zeta \to 1$ (and $s(i) = s_k$ are constant) or $\eta \to 0$ (and $s(i) = s_k = 1$ are constant). Intuitively, in these cases the relative demand curve consists of a ray passing through the origin. However, when $(s(i) - s_k)(\sigma - \zeta) \neq 0$, the non-homotheticity becomes important and we need to take into account the movements in the ideal price curve to determine the behavior of relative factor prices.

Differentiation of the ideal price index condition gives us the equation:
\[
dN \frac{1}{1-\sigma} \left( c^u(r)^{1-\sigma} - c^u \left( \frac{W}{\gamma(N)} \right)^{1-\sigma} \right) + dI \frac{1}{1-\sigma} \left( c^u \left( \frac{W}{\gamma(I^*)} \right)^{1-\sigma} - c^u(r)^{1-\sigma} \right) = (d\ln W - d\ln R) A
\]
\[
=(d\ln W - d\ln r) \left( \int_{I^*}^{N} s(i)c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} di \right) + d\ln r \left( (I^* - N + 1)s_k c^u(r)^{1-\sigma} + \int_{I^*}^{N} s(i)c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} di \right).
\]

Combining both expressions, we can solve for $d\ln W - d\ln R$ as
\[
(d\ln W - d\ln R) A = dNP - dI P_I,
\]
where:
\[
A = \left( K \int_{I^*}^{N} \gamma(i)^{\zeta-1}c^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} [s(i)(\sigma - \zeta) + \zeta] di + \varepsilon_L L^2(I^* - N + 1)c^u(r)^{\zeta-\sigma}r^{-\zeta} \right)
\]
\[
\times \left( (I^* - N + 1)s_k c^u(r)^{1-\sigma} + \int_{I^*}^{N} s(i)c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} di \right)
\]
\[-\left( \int_{I^*}^{N} s(i)c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} di \right) \times \left( K \int_{I^*}^{N} \gamma(i)^{\zeta-1}c^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} [(s(i) - s_k)(\sigma - \zeta)] di \right),
\]
We now show that, under the conditions of Lemma 4, we have that $A > 0$. A sufficient condition for $A > 0$ is that:

\[
\begin{align*}
K \int_{I^*} \gamma(i)^{\zeta-1} e^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} \cdot [s(i)(\sigma - \zeta) + \zeta \cdot di] 
& \geq \int_{I^*} s(i)^{\zeta-1} e^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} \cdot [di - (s(i) - s_k)(\sigma - \zeta)] 
\end{align*}
\]

After canceling common terms on both sides of this inequality, it boils down to:

\[
\int_{I^*} \gamma(i)^{\zeta-1} e^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} [s(i)(\sigma - \zeta) + \zeta \cdot di] 
\geq \int_{I^*} s(i)^{\zeta-1} e^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} [(s(i) - s_k)(\sigma - \zeta)] 
\]

This can be rewritten as:

\[
\int_{I^*} \gamma(i)^{\zeta-1} e^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} [s(i)(\sigma - \zeta) + \zeta - (s(i) - s_k)(\sigma - \zeta)] 
\geq 0.
\]

The last inequality always holds because $s(i)(\sigma - \zeta) + \zeta - (s(i) - s_k)(\sigma - \zeta) = \sigma s_k + \zeta(1 - s_k) > 0$.

To determine the signs of $P_N$ and $P_I$, we regroup terms as follows. First, we group the terms that are multiplied by $s_k$ in the expression for $P_I$. To guarantee that these terms add up to a positive number, a sufficient condition is given by:

\[
\begin{align*}
\left( L^* e^u(r)^{\zeta-\sigma} r^{-\zeta} \right) \times (I^* - N + 1) s_k e^u(r)^{1-\sigma} 
& \geq \left| \sigma - \zeta \right| e^u \left( \frac{W}{\gamma(I^*)} \right)^{1-\sigma} - e^u(r)^{1-\sigma} \right| \times K \int_{I^*} s_k \gamma(i)^{\zeta-1} e^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} \cdot di 
\end{align*}
\]
The relative demand for factors implies that

$$(I^* - N + 1) s_k c^u (R) ^{1 - \sigma} = s_k c^u (R) ^{1 - \sigma} K \int_{I^*}^N \gamma(i) \xi - 1 c^u \left( \frac{W}{\gamma(i)} \right) ^{\zeta - \sigma} W ^{-\zeta} d \ell. $$

Therefore, we can rewrite the sufficient condition as:

$$s_k c^u (r) ^{1 - \sigma} K \int_{I^*}^N \gamma(i) \xi - 1 c^u \left( \frac{W}{\gamma(i)} \right) ^{\zeta - \sigma} W ^{-\zeta} d \ell \geq s_k \begin{vmatrix} \sigma - \xi \\ 1 - \sigma \end{vmatrix} ^{1 - \sigma} \left( c^u \left( \frac{W}{\gamma(I^* \xi)} \right) ^{1 - \sigma} - c^u (r) ^{1 - \sigma} \right) \times K \int_{I^*}^N \gamma(i) \xi - 1 c^u \left( \frac{W}{\gamma(i)} \right) ^{\zeta - \sigma} W ^{-\zeta} d \ell. $$

After removing common terms on both sides, this condition becomes:

$$c^u (R) ^{1 - \sigma} \geq \begin{vmatrix} \sigma - \xi \\ 1 - \sigma \end{vmatrix} ^{1 - \sigma} \left( c^u \left( \frac{W}{\gamma(I^* \xi)} \right) ^{1 - \sigma} - c^u (R) ^{1 - \sigma} \right) . \quad (3.31)$$

Second, we group the terms that are multiplied by $s(i)$ in the expression for $P_1$. To guarantee that these terms add up to a positive number, a sufficient condition is given by:

$$\begin{vmatrix} \sigma - \xi \\ 1 - \sigma \end{vmatrix} ^{1 - \sigma} \left( c^u \left( \frac{W}{\gamma(I^* \xi)} \right) ^{\zeta - \sigma} W ^{-\zeta} \xi - 1 c^u \left( \frac{W}{\gamma(I^* \xi)} \right) ^{\zeta - \sigma} \right) \times \left( \int_{I^*}^N s(i) c^u \left( \frac{W}{\gamma(i)} \right) ^{1 - \sigma} W ^{-\zeta} d \ell \right) \geq 0. \quad \text{(3.32)}$$

After removing common terms and re-grouping, this condition becomes

$$\int_{I^*}^N s(i) W ^{-\zeta} \left( \gamma(i) \xi - 1 c^u \left( \frac{W}{\gamma(I^* \xi)} \right) ^{\zeta - \sigma} \left( c^u \left( \frac{W}{\gamma(I^* \xi)} \right) ^{1 - \sigma} - c^u (r) ^{1 - \sigma} \right) \right) \geq 0. \quad (3.32)$$

We now show that, for this condition to hold, it suffices that

$$\begin{vmatrix} \sigma - \xi \\ 1 - \sigma \end{vmatrix} ^{1 - \sigma} c^u \left( \frac{W}{\gamma(i)} \right) ^{1 - \sigma} \geq \begin{vmatrix} \sigma - \xi \\ 1 - \sigma \end{vmatrix} ^{1 - \sigma} \left( c^u \left( \frac{W}{\gamma(I^* \xi)} \right) ^{1 - \sigma} - c^u (R) ^{1 - \sigma} \right). \quad (3.33)$$

We first show this in the case in which $\sigma > \xi$. In this case, we have that

$$c^u \left( \frac{W}{\gamma(I^* \xi)} \right) ^{\zeta - \sigma} \geq c^u \left( \frac{W}{\gamma(i)} \right) ^{\zeta - \sigma} \left( \frac{\gamma(i)}{\gamma(I^* \xi)} \right) ^{\zeta - \sigma} .$$
This follows from the fact that
\[
\frac{c^u \left( \frac{W}{\gamma(0)} \right)}{c^u \left( \frac{W}{\gamma(I^*)} \right)} \geq \gamma(I^*) \gamma(i),
\]
an inequality which can be proven by straightforward differentiation of the function \( c^u(x)/x \), which is weakly decreasing.

Plugging this in the inequality in equation (3.32), we obtain the sufficient condition
\[
\int_{I^*}^N s(i) W^{-\xi} \gamma(i)^{1-\sigma} \left( \gamma(I^*)^{\sigma-1} c^u \left( \frac{W}{\gamma(i)} \right) \right)^\xi c^u \left( \frac{W}{\gamma(0)} \right)^{1-\sigma} - \gamma(i)^{\sigma-1} c^u \left( \frac{W}{\gamma(0)} \right)^{1-\sigma} \left| \sigma - \xi \left( \frac{W}{\gamma(I^*)} \right)^{1-\sigma} - c^u(R)^{1-\sigma} \right|, \]
which holds provided that
\[
\left( \frac{\gamma(I^*)}{\gamma(i)} \right)^{\xi-1} c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} \geq \left| \sigma - \xi \left( \frac{W}{\gamma(I^*)} \right)^{1-\sigma} - c^u(R)^{1-\sigma} \right|.
\]
This inequality coincides with the condition in equation (3.33), completing this step of the proof.

Turning next to the case where \( \xi \geq \sigma \), because the cost function is increasing we have that:
\[
c^u \left( \frac{W}{\gamma(I^*)} \right)^{\xi-\sigma} \geq c^u \left( \frac{W}{\gamma(i)} \right)^{\xi-\sigma}\]

Plugging this in the sufficient condition in equation (3.32) yields:
\[
\int_{I^*}^N s(i) W^{-\xi} \gamma(i)^{1-\sigma} \left( \gamma(I^*)^{\sigma-1} c^u \left( \frac{W}{\gamma(i)} \right) \right)^\xi c^u \left( \frac{W}{\gamma(0)} \right)^{1-\sigma} - \gamma(i)^{\sigma-1} c^u \left( \frac{W}{\gamma(0)} \right)^{1-\sigma} \left| \sigma - \xi \left( \frac{W}{\gamma(I^*)} \right)^{1-\sigma} - c^u(R)^{1-\sigma} \right|, \]
which holds provided that
\[
\left( \frac{\gamma(I^*)}{\gamma(i)} \right)^{\xi-1} c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} \geq \left| \sigma - \xi \left( \frac{W}{\gamma(I^*)} \right)^{1-\sigma} - c^u(R)^{1-\sigma} \right|,
\]
and once again establishing (3.33).

Third, we group the terms that are multiplied by \( s(i) \) in the expression for \( P_N \). To guarantee that these terms add up to a positive number, a sufficient condition is given by:
\[
\left( K \gamma(N)^{\xi-1} c^u \left( \frac{W}{\gamma(N)} \right)^{\xi-\sigma} W^{-\xi} \right) \times \left( \int_{I^*}^N s(i) c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} di \right)
\geq \left| \sigma - \xi \left( c^u \left( \frac{W}{\gamma(N)} \right)^{1-\sigma} - c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} \right) \right| \times \left( K \int_{I^*}^N s(i) \gamma(i)^{\xi-1} c^u \left( \frac{W}{\gamma(i)} \right)^{\xi-\sigma} W^{-\xi} di \right).
\]
This condition can be rewritten as:

$$\int_{1}^{N} s(i) W^{-\zeta} \left[ c^{u} \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} \gamma(N)^{\zeta-1} c^{u} \left( \frac{W}{\gamma(N)} \right)^{\zeta-\sigma} - \left| \frac{\sigma - \zeta}{1 - \sigma} \left( c^{u}(R)^{1-\sigma} - c^{u} \left( \frac{W}{\gamma(N)} \right)^{1-\sigma} \right) \right| \right] d\bar{i} \geq 0.$$ 

In Step 1 of the proof of this proposition, we showed that $x^{-\zeta} c^{u}(x)^{\sigma-\zeta}$ is decreasing in $x$. This implies that $\gamma(N)^{\zeta} c^{u} \left( \frac{W}{\gamma(N)} \right)^{\zeta-\sigma} > \gamma(i)^{\zeta} c^{u} \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma}$.

Therefore, a sufficient condition for the terms that are multiplied by $s(i)$ in the expression for $P_N$ to add up to a positive number is

$$\int_{1}^{N} s(i) \gamma(i)^{\zeta-1} c^{u} \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} \times \left[ \frac{\gamma(i)}{\gamma(N)} c^{u} \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} - \left| \frac{\sigma - \zeta}{1 - \sigma} \left( c^{u}(R)^{1-\sigma} - c^{u} \left( \frac{W}{\gamma(N)} \right)^{1-\sigma} \right) \right| \right] d\bar{i} \geq 0.$$ 

This expression implies that, to guarantee that these terms are positive, the following relationship would be sufficient

$$\frac{\gamma(I)}{\gamma(N)} c^{u} \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} \geq \left| \frac{\sigma - \zeta}{1 - \sigma} \left( c^{u}(R)^{1-\sigma} - c^{u} \left( \frac{W}{\gamma(N)} \right)^{1-\sigma} \right) \right|. \quad (3.34)$$

Fourth, and lastly, we group the terms that are multiplied by $s_K$ in the expression for $P_N$. To guarantee that these terms add up to a positive number, a sufficient condition is given by:

$$L^{\zeta} c^{u}(r)^{\zeta-\sigma} (I^{*} - N + 1) s_k c^{u}(r)^{1-\sigma} \geq \left| \frac{\sigma - \zeta}{1 - \sigma} \left( c^{u}(r)^{1-\sigma} - c^{u} \left( \frac{W}{\gamma(N)} \right)^{1-\sigma} \right) \right| \times \left( K \int_{1}^{N} s_k \gamma(i)^{\zeta-1} c^{u} \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} d\bar{i} \right).$$

The relative demand for factors implies that

$$L^{\zeta} c^{u}(R)^{\zeta-\sigma} (I^{*} - N + 1) s_k c^{u}(R)^{1-\sigma} = s_k c^{u}(R)^{1-\sigma} K \int_{1}^{N} \gamma(i)^{\zeta-1} c^{u} \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} d\bar{i}.$$ 

Therefore, we can rewrite the sufficient condition as:

$$s_k c^{u}(r)^{1-\sigma} K \int_{1}^{N} \gamma(i)^{\zeta-1} c^{u} \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} d\bar{i} \geq s_k \left| \frac{\sigma - \zeta}{1 - \sigma} \left( c^{u} \left( \frac{W}{\gamma(N)} \right)^{1-\sigma} - c^{u}(r)^{1-\sigma} \right) \right| \times K \int_{1}^{N} \gamma(i)^{\zeta-1} c^{u} \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} d\bar{i}.$$ 

After removing common terms on both sides, this condition becomes

$$c^{u}(R)^{1-\sigma} \geq \left| \frac{\sigma - \zeta}{1 - \sigma} \left( c^{u} \left( \frac{W}{\gamma(N)} \right)^{1-\sigma} - c^{u}(R)^{1-\sigma} \right) \right|. \quad (3.35)$$
To finalize the proof of the lemma, we use the fact that for any effective factor prices \( p_1, p_2 \) (the price per effective unit of labor or capital at any given task), we have

\[
\left| \frac{\sigma - \zeta}{1 - \sigma} \left( c^u(p_1)^{1 - \sigma} - c^u(p_2)^{1 - \sigma} \right) \right| \leq |\sigma - \zeta| c^u \left( \frac{W}{\gamma(N-1)} \right) c^u \left( \frac{W}{\gamma(N)} \right)^{-\sigma}.
\]  

(3.36)

This inequality follows because \( f(x) = \frac{1}{1-\sigma} x^{1-\sigma} \) is a concave function and the effective factor prices satisfy \( p_1, p_2 \in \left[ \frac{W}{\gamma(N)}, \frac{W}{\gamma(N-1)} \right] \).

Inequality (3.36) then implies that to guarantee (3.31), (3.33), (3.34) and (3.35), the following would suffice

\[
\frac{c^u(R)}{\gamma(N)} \leq |\sigma - \zeta| c^u \left( \frac{W}{\gamma(N-1)} \right) c^u \left( \frac{W}{\gamma(N)} \right)^{-\sigma}.
\]

These inequalities can be, in turn, rewritten as

\[
\begin{align*}
\left( \frac{c^u(R)}{c^u \left( \frac{W}{\gamma(N-1)} \right)} \right) \left( \frac{c^u \left( \frac{W}{\gamma(N-1)} \right)}{c^u \left( \frac{W}{\gamma(N)} \right)} \right)^\sigma & \geq |\sigma - \zeta|.
\end{align*}
\]

\[
\begin{align*}
\left( \frac{c^u \left( \frac{W}{\gamma(N-1)} \right)}{c^u \left( \frac{W}{\gamma(N)} \right)} \right) \left( \frac{c^u \left( \frac{W}{\gamma(N)} \right)}{c^u \left( \frac{W}{\gamma(N-1)} \right)} \right)^\sigma & \geq |\sigma - \zeta|.
\end{align*}
\]

From the properties of unit cost functions, it follows that for all \( p \in \left[ \frac{W}{\gamma(N)}, \frac{W}{\gamma(N-1)} \right] \) we have that \( c^u(p) \geq c^u \left( \frac{W}{\gamma(N-1)} \right) \frac{\gamma(N-1)}{\gamma(N)} \) and \( c^u \left( \frac{W}{\gamma(N)} \right) \geq c^u(p) \frac{\gamma(N)}{\gamma(N-1)} \). Using these properties, it follows that the sufficient conditions above hold whenever

\[
\begin{align*}
\left( \frac{\gamma(N-1)}{\gamma(N)} \right)^{1+\sigma} & \geq |\sigma - \zeta| \\
\left( \frac{\gamma(N-1)}{\gamma(N)} \right)^{\max\{\sigma, \zeta\}+\sigma} & \geq |\sigma - \zeta| \\
\left( \frac{\gamma(N-1)}{\gamma(N)} \right)^{2+\sigma} & \geq |\sigma - \zeta|.
\end{align*}
\]

Thus a sufficient condition for all three inequalities to hold is

\[
\left( \frac{\gamma(N-1)}{\gamma(N)} \right)^{2+2\sigma+\zeta} \geq |\sigma - \zeta|.
\]

185
establishing the desired result. ■

3.8.3 Remaining Proofs from Section 3.2

Proof of Lemma 5. Let us first suppose that there exists \( x \in [0, 1] \) for which

\[
w_I(x) > \rho + \delta + \theta g > w_N(x).
\]

This condition states that in the neighborhood of \( n(t) = x \), both automation and the creation of new complex tasks are profitable, and thus newly created tasks will be immediately produced using labor, and newly automated tasks will be immediately produced using capital. We discuss the cases in which this inequality does not hold for any \( x \) below.

We proceed in several steps.

Step 1: We show that \( w_I(n) \) is increasing and \( w_N(n) \) decreasing in \( n \) for all \( n \geq x \).

Rewrite the ideal price index condition (3.9), as in Section 3.2.2, substituting for the BGP value of the rental rate, \( R = \rho + \delta + \theta g \), which yields (3.17). Differentiating the expressions for effective wages using this BGP value of the interest rate yields equation (3.18) in the main text.

Suppose first that \( \sigma < 1 \). Then, for \( n \geq x \), the numerator of the first expression is positive, and the numerator of the second expression is negative, and their denominators are positive, and thus \( w'_I(n) > 0 > w'_N(n) \). Suppose next that \( \sigma > 1 \). In this case, the signs of the numerators are flipped, but the denominators are negative, so we reach the same conclusion. From the Fundamental Theorem of Calculus, we can conclude that

\[
w_I(n) > \rho + \delta + \theta g > w_N(n)
\]

for all \( n \geq x \).

Step 2: We now show that the inequality (3.38) cannot hold for all \( n \in [0, x) \).

Using the same argument as in Step 1, we have that for \( n < x \) and provided that (3.38) holds, \( w_I(n) \) continues to be increasing and \( w_N(n) \) decreasing in \( n \). To obtain a contradiction, suppose that equation (3.38) holds for all \( n \in [0, x) \). Then,

\[
w_I(0) > \rho + \delta + \theta g > w_N(0),
\]

but this is impossible, since at \( n = 0 \), \( w_I(0) = w_N(0) \), thus yielding a contradiction.

Step 3: We next show that either there exists \( \bar{n} \) or \( \tilde{n} \) as in the lemma, but both thresholds cannot exist simultaneously. Moreover, which threshold exists and is relevant depends on whether \( \rho \leq \bar{\rho} \). Since (3.38) holds at \( n = x \), but not at \( n = 0 \), and both \( w_I(n) \) and
$w_N(n)$ are continuous, there exists either $\overline{n}$ such that $w_I(\overline{n}) = \rho + \delta + \theta g$, or $\overline{n}$ such that $w_N(\overline{n}) = \rho + \delta + \theta g$, or both. We now show that only one of these cases may occur, and that $\rho$ determines which case it is.

First, suppose that as we move from $n = x$ to the left, we reach $\overline{n}$ first. Because $R > w_I(n)$ for $n \leq \overline{n}$, in this region there are no incentives to use capital in automated tasks, which implies $I^* < I$. Further increases in $I$—or reductions in $n$—do not change the equilibrium allocation. Thus, for $n \leq \overline{n}$, $w_I(\overline{n})$ and $w_N(n)$ are constant, as shown in the left panel of Figure 3-12. This establishes that, in this case, $\rho + \delta + \theta g > w_N(n)$ for all $n \in [0, 1]$, and there is no threshold $\overline{n}$. In this case, for $n > \overline{n}$ we have that inequality (3.38) holds, which implies that automated tasks are immediately produced with capital and newly created tasks are immediately produced with labor.

Case 1: $\rho > \overline{\rho}$

Case 2: $\rho \leq \overline{\rho}$

![Figure 3-12: Behavior of unit costs of labor with respect to changes in $n = N - I$ in steady state.](image)

Suppose next that as we move from $x$ to the left, we reach $\overline{n}$ first. For $n < \overline{n}$, we have that $w_N(n) > \rho + \delta + \theta g$, and thus newly created tasks that use labor are less productive than their old automated. Moreover, equations (3.18) and (3.38) imply that both $w_I(n)$ and $w_N(n)$ are decreasing to the left of $\overline{n}$ as shown in the right panel of Figure 3-12. This establishes that, in this case, $w_I(n) > \rho + \delta + \theta g$ for all $n \in [0, 1]$, and there is no threshold $\overline{n}$.

Consequently, one and only one of $\overline{n}$ and $\overline{n}$ will be reached. We now show that which one of these two thresholds is reached first is determined by the discount rate, $\rho$. For $\rho$ sufficiently small, we necessarily reach the threshold $\overline{n}$ first. This is the case depicted in the right panel of Figure 3-12. Moreover, because

$$\frac{\partial w_N(n)}{\partial \rho} = -\frac{1}{\gamma(n)} \int_0^n \frac{1}{\gamma(i)} c'(\rho + \delta + \theta g) c^\alpha(\rho + \delta + \theta g)^{-\alpha} c^\omega(w_I/\gamma(i))^{-\omega} c^\mu(w_I/\gamma(i)) \alpha i \, \gamma(i) < 0,$$
as \( \rho \) increases the curve for \( w_N(n) \) shifts down, while the curve for \( \rho + \delta + \theta g \) shifts upwards in Figure 3-12. This implies that, as \( \rho \) increases, the interception between these curves, \( \tilde{n} \), shifts to the left and for some \( \rho \) we will have that the curves \( w_N(n) \) and \( \rho + \delta + \theta g \) will intercept exactly at \( \tilde{n} = 0 \). These observations imply that there exists a value \( \bar{\rho} \) such that

\[
w_N(0) = w_I(0) = \bar{\rho} + \delta + \theta g.
\]

Let \( \bar{\rho} \) denote the smallest \( \rho \) for which this is the case. The definition of \( \bar{\rho} \) implies that, for \( \rho < \bar{\rho} \) the threshold \( \tilde{n} \) is reached and we have the case depicted in the right panel of Figure 3-12. On the other hand, for \( \rho > \bar{\rho} \) we necessarily have that the threshold \( \tilde{n} \) is reached (the curve \( w_N(n) \) is below \( \rho + \delta + \theta g \) for these values of \( \rho \)) and we have the case depicted in the left panel of Figure 3-12.

**Step 4:** We finalize the proof by dealing with the cases in which the inequality (3.37) does not hold. Suppose first that the right inequality does not hold. Then we can simply define \( \tilde{n} = 1 \), and the lemma applies as is. Suppose next that the left inequality does not hold. Then we define \( \tilde{n} = 1 \), and the lemma applies as is. This concludes the proof of the lemma. ■

### 3.8.4 Constrained Efficient Allocation and Proofs from Section 3.4

In this part of the Appendix, we complete the characterization of the constrained efficient allocation. The constrained efficient allocation solves the maximization problem introduced in the main text. To simplify the notation, we denote the marginal product of labor and capital in the planner’s allocation by \( W^P(I, N, K) = \frac{\partial P^P}{\partial L} \) and \( R^P(I, N, K) = \frac{\partial P^P}{\partial K} \), respectively.

The current value Hamiltonian for the planner’s problem is given by:

\[
H \equiv \frac{C^{1-\theta}}{1-\theta} + \mu_k \left( F^P(I, N, K, L) - \delta K - C \right) + \mu_L (L^P(I, N, K)) - L) + \mu_1 \kappa_1 S_1 + \mu_N \kappa_N S_N.
\]

Here, \( \mu_N \) and \( \mu_L \) denote the shadow values of the two types of technology, respectively, and \( \mu_L \) and \( \mu_K \) the shadow values of labor and capital. The maximum principle implies that these objects satisfy the necessary conditions:

\[
\begin{align*}
\rho \mu_N - \dot{\mu}_N &= \mu_K \frac{\partial F^P}{\partial N} + \mu_L \frac{\partial L^P}{\partial \omega} \frac{\partial \omega^P}{\partial N}, \\
\rho \mu_L - \dot{\mu}_L &= \mu_k \left( R^P - \delta \right) + \mu_L \frac{\partial L^P}{\partial \omega} \frac{\partial \omega^P}{\partial K}, \\
\rho \mu_K - \dot{\mu}_K &= \mu_K W^P.
\end{align*}
\]

All the functions in the above differential equations are evaluated at their corresponding arguments at time \( t \).
Moreover, the current value Hamiltonian associated with the planner’s problem is concave, so these conditions (plus the Euler equation for consumption and the transversality condition) are sufficient for characterizing the constrained efficient allocation.

We start by providing formulas for $\frac{\partial F^p}{\partial N}$ and $\frac{\partial F^p}{\partial I}$. To derive these formulas, we rewrite the function $F^p(I, N, K, L)$ as:

$$F^p(I, N, K, L) = \max_{k(i), l(i), qK(i), qL(i)} \left[ \int_{N-1}^{I} y^p(qK(i), k(i))^\frac{\sigma-1}{\sigma} + \int_{I}^{N} y^p(qL(i), \gamma(i)l(i))^\frac{\sigma-1}{\sigma} \right] \frac{\sigma}{\sigma-1}$$

subject to: $\int_{N-1}^{I} qK(i)di = K$, and $\int_{I}^{N} l(i)di = L$.

Here, $y^p(qK(i), k(i))$ and $y^p(qL(i), \gamma(i)l(i))$ denote the production of a task when using $qK(i)$ and $qL(i)$ units of the intermediate mixed with $k(i)$ units of capital or $\gamma(i)l(i)$ units of effective labor, respectively (see equations (3.2) and (3.3)). Notice that we have written this problem assuming that the planner chooses $I^* = I$ at all times, as we remarked in the proposition. 21. Also, notice that the multipliers for the restriction on total capital is $R^p$—the shadow rental rate—, and the multiplier for the restriction on total labor is $W^p$—the shadow wage.

Denote by $c^p(\cdot)$—rather than $c^m(\cdot)$—the unit cost of producing a task when intermediates are priced at their marginal cost. An application of the envelope theorem to the above problem yields

$$\frac{\partial F^p}{\partial I} = \frac{\sigma}{\sigma-1} Y^\frac{1}{\sigma} \left[ y^p(qK(I), k(I))^\frac{\sigma-1}{\sigma} - y^p(qL(I), \gamma(I)l(I))^\frac{\sigma-1}{\sigma} \right]$$

$$- \mu \psi (qK(I) - qL(I)) - (R^p k(I) - W^p l(I))$$

$$= \frac{\sigma}{\sigma-1} Y \left[ c^p (R^p)^{1-\sigma} - c^p \left( \frac{W^p}{\gamma(I)} \right)^{1-\sigma} \right] - Y \left[ c^p (R^p)^{1-\sigma} - c^p \left( \frac{W^p}{\gamma(I)} \right)^{1-\sigma} \right]$$

$$= \frac{1}{1-\sigma} Y \left[ c^p \left( \frac{W^p}{\gamma(I)} \right)^{1-\sigma} - c^p (R^p)^{1-\sigma} \right].$$

Here, we have used the fact that the planner sets the level of production of a task to $c^p(\cdot)$ $Y^{-\sigma}$, with $c^p(\cdot)$ its unitary cost.
Likewise,

\[
\frac{\partial F_p}{\partial N} = \frac{\sigma}{\sigma - 1} Y^{\frac{1}{\sigma}} \left[ y_p^\prime(q_{L}(N), \gamma(N)l(N))^\frac{\sigma - 1}{\sigma} - y_p^\prime(q_{K}(N - 1), k(N - 1))^\frac{\sigma - 1}{\sigma} \right] \\
- \frac{\mu}{\gamma(N) - q_{K}(N - 1)} - (W_p l(N) - R_p k(N - 1)) \\
= \frac{\sigma}{\sigma - 1} Y \left[ c^p \left( \frac{W_p}{\gamma(N)} \right)^{1-\sigma} - c^p \left( R_p \right)^{1-\sigma} \right] \\
- Y \left[ c^\prime \left( \frac{W_p}{\gamma(N)} \right)^{1-\sigma} - c^\prime \left( R_p \right)^{1-\sigma} \right] \\
= \frac{1}{1 - \sigma} Y \left[ c^p \left( R_p \right)^{1-\sigma} - c^p \left( \frac{W_p}{\gamma(N)} \right)^{1-\sigma} \right].
\]

Let \( \Psi_N(t) \equiv \frac{\mu_{N(t)}}{\mu_{K(t)}Y(t)} \) and \( \Psi_I \equiv \frac{\mu_{l(t)}}{\mu_{K(t)}Y(t)} \) be the shadow discounted net present values of new technologies (in terms of additional net output they create). These values are then given by:

\[
\Psi_I(t) = \int_t^\infty e^{-\int_t^\tau \left(R_p(1+\omega_p L^\epsilon L \frac{\partial \ln \omega_p}{\partial l_k}) - \delta - g(s)\right) \partial \ln \omega_p} \left( c^p(w^P_p(\tau))^{1-\sigma} - c^p(R^P(\tau))^{1-\sigma} \right) \frac{1}{1 - \sigma} + s_L L^\epsilon L \frac{\partial \ln \omega_p}{\partial I} d\tau \\
\Psi_N(t) = \int_t^\infty e^{-\int_t^\tau \left(R_p(1+\omega_p L^\epsilon L \frac{\partial \ln \omega_p}{\partial l_k}) - \delta - g(s)\right) \partial \ln \omega_p} \left( c^p(R^P(\tau))^{1-\sigma} - c^p(w^P_N(\tau))^{1-\sigma} \right) \frac{1}{1 - \sigma} + s_L L^\epsilon L \frac{\partial \ln \omega_p}{\partial N} d\tau.
\]

(3.40)

Here, \( s_L \) is the economy-wide labor share, \( w^P_I \) is the normalized wage \( W_p / \gamma(I) \), and \( w^P_N \) is the normalized wage \( W_p / \gamma(N) \).

These equations are analogous to the expressions for \( v_I \) and \( v_N \) in the decentralized equilibrium given by equations (3.22) and (3.23). In particular, notice that like prospective technology monopolists, the planner also values the automation of existing tasks or the introduction of new more complex tasks depending on the difference in production costs. Thus, our structure of intellectual property rights produces private incentives for innovation that share this common feature with the social value of such innovations. However, because technology monopolists only capture a non-constant share of the surplus that new tasks generate, the exact expressions for \( \Psi_I \) and \( \Psi_N \) differ from those of \( v_I \) and \( v_N \). Besides these differences the expressions for the social value of introducing different technologies show that the planner also take into account their effect on employment, captured by the terms \( s_L L^\epsilon L \frac{\partial \ln \omega_p}{\partial I} \) and \( s_L L^\epsilon L \frac{\partial \ln \omega_p}{\partial N} \) in equation (3.40). The social planner cares about this margin because changes in employment have a first-order positive effect on workers owing to the gap between wage and the opportunity cost of work.

Given the current values for \( \Psi_I(t) \) and \( \Psi_N(t) \), the optimal allocation of scientists to the
two different types of research then satisfies
\[ S_I(t) = S G \left( \frac{\kappa_I \Psi_I - \kappa_N \Psi_N}{\lambda} \right) \in [0, S] \quad S_N(t) = S \left[ 1 - G \left( \frac{\kappa_I \Psi_I - \kappa_N \Psi_N}{\lambda} \right) \right] \in [0, S]. \] (3.41)

Following our characterization in Section 3.3, we denote the normalized output in the socially planned economy by \( P^P(k,n) = F^P(I,N,K,L(\omega^P(I,N,K))/\gamma(I) \), and the rental rate, \( R^P(n,k) \) as functions of technology and capital. The constrained efficient allocation can be represented as a time path for the variables \( \{c(t), k(t), n(t), S_I(t), \Psi_I(t), \Psi_N(t)\}_{t=0}^\infty \) such that:

- Consumption satisfies the Euler equation
  \[ \frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta} \left( R^P(n(t),k(t)) \left( 1 + \omega^P L^\epsilon_k \frac{\partial \ln \omega^P}{\partial \ln K} \right) - \delta - \rho \right) - A \kappa_I S_I(t), \]
  together with the transversality conditions
  \[ \lim_{t \to \infty} \mu_k K e^{-\rho t} = 0, \quad \lim_{t \to \infty} I \mu_k e^{-\rho t} = 0, \quad \lim_{t \to \infty} N \mu_k e^{-\rho t} = 0. \]

- Capital satisfies the resource constraint in equation
  \[ \dot{k}(t) = f^P(n(t),k(t)) - c(t) - (\delta + A \kappa_I S_I(t)) k(t) \]

- The gap between automation and the creation of new tasks, \( n(t) = N(t) - I(t) \), satisfies:
  \[ \dot{n}(t) = \kappa_N S - (\kappa_I + \kappa_N) G \left( \frac{\kappa_I \Psi_I - \kappa_N \Psi_N}{\lambda} \right) S \]

- The allocation of scientists satisfies the allocation rule in equation (3.41).

- The social values of allocating scientists to develop different technologies, \( \Psi_I(t) \) and \( \Psi_N(t) \), satisfy equation (3.40).

The following proposition summarizes the properties of the constrained efficient allocation and provides a set of taxes and subsidies that can be used to decentralize it.

**Proposition 21 (Constrained Efficient Allocation and Decentralization)** Under the same conditions as in Proposition 17, the constrained efficient allocation admits a unique BGP.
Moreover, the constrained efficient allocation locally converges to this BGP, and if $\theta \to 0$, it globally converges to this BGP.

Finally, the constrained efficient allocation can be decentralized by using the following sets of taxes and subsidies:

1. A proportional subsidy at the rate $1 - \mu$ on intermediate prices to remove the monopoly markups.

2. A proportional subsidy/tax of $\omega^P I^s L^s \varepsilon_L \frac{\partial \ln \omega^P}{\partial \ln K}$ on gross interests on savings. This subsidy/tax corrects for the impact of capital on employment (this expression yields a positive subsidy when $\sigma_{SR} < 1$ and capital raises the labor share, and a tax in the opposite case).

3. Proportional subsidies/taxes on the profits of successful innovators who entered the market at time $t' \leq t$. These subsidies/taxes correct for the technological externality generated by the two different types of innovation and appropriability problems.

4. A proportional subsidy on the profits of successful innovators who create new complex tasks. This subsidy is proportional to $s_L \varepsilon_L \frac{\partial \ln \omega^P}{\partial N}$ $\geq 0$, and corrects for the fact that these technology monopolists do not take into account the positive effect of new complex tasks on the level of equilibrium employment.

5. A proportional tax on the profits of successful innovators who automate existing tasks. This tax is proportional to $-s_L \varepsilon_L \frac{\partial \ln \omega^P}{\partial I}$ $\geq 0$, and corrects for the fact that these technology monopolists do not take into account the negative effect of automation on the level of equilibrium employment.

PROOF. Using the formula in equation (3.40), we now establish the decentralization result by construction. First, assume the planner subsidizes a fraction $1 - \mu$ to the price of intermediate goods, and sets a subsidy to interests on capital savings of $s_k = \omega^P I^s L^s \varepsilon_L \frac{\partial \ln \omega^P}{\partial \ln K}$. This guarantees households discount future income at the socially optimal rate.

Now, we can decentralize the planner’s allocation by subsidizing/taxing the period $\tau$ profits of a firm that automates task $I(t)$ at time $t$ at the rate $s_I(t, \tau)$, and the period $\tau$ profits of a firm that introduces a new complex task $N(t)$ at time $t$ at the rate $s_N(t, \tau)$.

With these subsidies, the value of automating jobs or creating new tasks is given by a small modification of equations (3.22) and (3.23), which takes into account that firms sell intermediates at a price $\psi$, but buyers perceive a price $\mu \psi$ because of the subsidy. These values also discount future profits as the same rate the planner does because of the subsidies/taxes to capital accumulation and the subsidies to the two different types of innovations, $s_I(t, \tau)$.
and \( s_N(t, \tau) \). Thus:

\[
V_I(t) = Y(t) \int_t^{\infty} e^{-\int_0^t \gamma(RP(\tau)) \frac{\partial \ln w_P}{\partial \tau} d\tau} dM \cdot \left( c^p \left( R_P(\tau) \right)^{\zeta - \sigma} - c^p \left( w^p_P(\tau) \frac{\gamma(I(\tau))}{\gamma(I(\tau))} \right)^{\zeta - \sigma} \right) \cdot (1 + s_I(\tau)) d\tau,
\]

\[
V_N(t) = Y(t) \int_t^{\infty} e^{-\int_0^t \gamma(RP(\tau)) \frac{\partial \ln w_N}{\partial \tau} d\tau} dM \cdot \left( c^p \left( w^p_N(\tau) \frac{\gamma(N(\tau))}{\gamma(N(\tau))} \right)^{\zeta - \sigma} - c^p \left( R_P(\tau) \right)^{\zeta - \sigma} \right) \cdot (1 + s_N(\tau)) d\tau,
\]

where recall that \( M \equiv (1 - \mu) \psi \left( \frac{\eta}{1 - \eta} \right)^\varsigma (\mu \psi)^{-\varsigma} \). Here we used the cost function \( c^p(\cdot) \) which takes into account that intermediates are already priced at their marginal cost.

The following subsidy-tax policy ensures that the incentives of successful innovators are aligned with the social value of their innovations and provide a way to decentralize the planner’s allocation:

\[
1 + s_I(t, \tau) = \left( \frac{1}{(1 - \mu) \psi \left( \frac{\eta}{1 - \eta} \right)^\varsigma (\mu \psi)^{-\varsigma}} \right) \times \frac{c^p \left( R_P(\tau) \right)^{\zeta - \sigma} - c^p \left( w^p_P(\tau) \right)^{\zeta - \sigma}}{c^p \left( R_P(\tau) \right)^{\zeta - \sigma} - c^p \left( w^p_P(\tau) \frac{\gamma(I(\tau))}{\gamma(I(\tau))} \right)^{\zeta - \sigma}} + s_L \varepsilon L \frac{\partial \ln w_P}{\partial \tau},
\]

and

\[
1 + s_N(t, \tau) = \left( \frac{1}{(1 - \mu) \psi \left( \frac{\eta}{1 - \eta} \right)^\varsigma (\mu \psi)^{-\varsigma}} \right) \times \frac{c^p \left( w^p_N(\tau) \frac{\gamma(N(\tau))}{\gamma(N(\tau))} \right)^{\zeta - \sigma} - c^p \left( R_P(\tau) \right)^{\zeta - \sigma}}{c^p \left( w^p_N(\tau) \frac{\gamma(N(\tau))}{\gamma(N(\tau))} \right)^{\zeta - \sigma} - c^p \left( R_P(\tau) \right)^{\zeta - \sigma}} + s_L \varepsilon L \frac{\partial \ln w_N}{\partial \tau},
\]

These subsidies/taxes can be separated into several components as illustrated by the way in which we have written them. First, profits from both types of innovations get a gross subsidy of

\[
\left( \frac{1}{(1 - \mu) \psi \left( \frac{\eta}{1 - \eta} \right)^\varsigma (\mu \psi)^{-\varsigma}} \right) > 1.
\]

This term captures the known fact that innovators only manage to extract a fraction of the surplus they generate.

Second, profits from automation get taxed at a gross rate

\[
\frac{c^p \left( R_P(\tau) \right)^{\zeta - \sigma} - c^p \left( w^p_P(\tau) \right)^{\zeta - \sigma}}{c^p \left( R_P(\tau) \right)^{\zeta - \sigma} - c^p \left( w^p_P(\tau) \frac{\gamma(I(\tau))}{\gamma(I(\tau))} \right)^{\zeta - \sigma}} < 1;
\]

193
while profits from creating new complex tasks get subsidized at the rate
\[
\frac{c^p \left( w_N^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}}{c^p \left( w_N^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}} > 1.
\]

These taxes/subsidies correct for a technological externality: by inventing new tasks and increasing \(N\), monopolists improve the quality of intermediates that future entrants are able to develop, which creates a positive externality on subsequent innovators. The opposite occurs for automation: by automating task \(I\), new entrants will be forced to automate more complex tasks, in which they will obtain fewer profits. These taxes/subsidies depend on the time at which a task was introduced \(t\) — since they are a compensation (or charge) for all technologies built on top of them. This is why the subsidies that are needed to decentralize the planner allocation, \(s_I(t, \tau)\) and \(s_N(t, \tau)\), not only depend on the current time period, \(\tau\), but also on the time that the innovation took place, \(t\).

Third, profits from automation get taxed/subsidized at a gross rate:
\[
\frac{c^p \left( w^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}}{c^p \left( w^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}} > 1.
\]

while profits from creating new complex task get subsidized at a gross rate:
\[
\frac{c^p \left( w_N^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}}{c^p \left( w_N^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}} < 1.
\]

These subsidies account for the fact that, as the technology for producing tasks is a constant elasticity of substitution function, the monopolists who supply intermediate goods will charge a constant markup. When \(\zeta = 1\) (and \(\sigma > 1\)), so that monopolists earn a constant fraction of the value of the task, these taxes/subsidies collapse to \(\frac{1}{\sigma-1} \leq 1\).

Finally, profits from automation get taxed at a gross rate:
\[
\frac{c^p \left( w^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}}{c^p \left( w^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}} + s_L \frac{\ln w^F}{\partial t} < 1;
\]

while profits from creating new complex task get subsidized at a gross rate:
\[
\frac{c^p \left( w_N^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}}{c^p \left( w_N^F(\tau) \right)^{1-\sigma} - c^p \left( R^F(\tau) \right)^{1-\sigma}} + s_L \frac{\ln w^F}{\partial N} > 1.
\]

these taxes/subsidies correct for the fact that technology monopolists do not take into ac-
count the effect of technologies on the quasi-supply of labor. While, all else equal, automation decreases the supply of labor, the creation of new complex tasks increases it. Because at the margin increasing the level of employment improves welfare (given that it has no opportunity cost), the planner taxes profits from automation and subsidizes profits from the creation of new complex tasks.

Notice that the scientist allocation can be decentralized in many ways. In particular, since there is a fixed supply of scientists, we only need to get the relative expected profits from each type of innovation right. The particular decentralization outlined here guarantees the level of innovators' profits also matches the social value of innovation. Even if both types of technology end up being subsidized in equilibrium, this does not matter because the money can be recovered by taxing scientists.

Remarks: Note first that in contrast to neoclassical models of capital taxation (e.g., Chamley, 1986 and Judd, 1985, but also see Straub and Werning, 2014), the decentralization of the constrained efficient allocation requires taxing or subsidizing capital accumulation. This is because the capital stock affects wages and thus the level of employment through the quasi-labor supply schedule. For instance, if $\sigma_{SR} < 1$, capital increases the labor share and employment in the short run (see Proposition 14) which is, as noted above, beneficial (recall that workers strictly prefer to work than not). Thus in this case, the social planner would subsidize capital accumulation. When $\sigma_{SR} > 1$, and capital reduces the labor share and employment, the opposite applies.

Second, the quality ladder structure in the creation of new complex tasks introduces a technological externality. By undertaking this type of innovation and thus increasing $N$, a technology monopolist also allows future innovators to create more productive new tasks (because $\gamma(N)$ is increasing). Automation creates an opposite and somewhat more subtle externality. Because capital has the same productivity in all automated tasks, there is no direct technological externality. But automation today forces future innovators to automate higher-indexed tasks, which are the ones in which labor has a comparative advantage (because $\gamma(I)$ is increasing), and this reduces the incremental profits of future innovators.

Finally and most importantly, the quasi-labor supply schedule creates an additional and novel distortion in the equilibrium relative to the constrained efficient allocation. Because firms do not internalize that the quasi-rents received by workers are transfers, they automate tasks taking into account the wage rate. In contrast, the social planner understand that these quasi-rents are transfers, and thus at the margin bases her automation decisions on the opportunity cost of labor rather than the market wage. Equivalently, because the planner recognizes that wages are above the opportunity cost of labor, she prefers to create more employment. In the market allocation, the resulting greater incentives of firms to automate tasks than what is socially optimal translate into too much R&D directed towards automation.

195
and too little R&D directed towards the creation of new complex tasks. For this reason, the social planner would like to encourage (subsidize) the creation of new complex tasks and discourage (tax) automation, as outlined in parts 4 and 5 of the proposition.

**Proof of Proposition 18:** Let \( S_I(t) \) and \( S_N(t) \) denote the allocation of scientists, and consider the allocation obtained by a small deviation \( S_{IP}(t) = \min\{S_N(t) + \nu, 1\} \) and \( S_{IN}(t) = \max\{S_I(t) - \nu, 0\} \) if \( S_I(t) < 1 \), and \( S_{IP}(t) = S_N(t), S_{IN}(t) = S_I(t) \) otherwise.

Clearly, the new allocation satisfies \( n^P(t) \geq n(t) \). Furthermore, we have that asymptotically \( n^P(t) > n(t) \). We prove that for a small \( \nu > 0 \), such deviation increases welfare and reduces the extent of automation.

For \( \nu \) small enough, we have that the above allocation changes welfare by \( \nu(\kappa_N \mu_N(t) - \kappa_I \mu_I(t)) \), whenever \( S_I(t), S_N(t) \in (0, 1) \). Moreover, whenever the allocation of scientists is an interior one, we have \( \kappa_N V_N(t) = \kappa_I V_I(t) \).

Thus, to prove that the reallocation of scientists away from automation increases welfare, it is enough to verify that, whenever \( \kappa_N V_N(t) = \kappa_I V_I(t) \), we have \( \kappa_N V_N(t) - \kappa_I V_I(t) > 0 \). This is equivalent to proving the inequality:

\[
\frac{\Psi_N(t)}{\Psi_I(t)} > \frac{V_N(t)}{V_I(t)},
\]

with all value functions evaluated at the decentralized equilibrium path.

Because we assumed that in the decentralized allocation the intermediate goods are subsidized at the rate \( 1 - \mu \) (or equivalently, that \( \mu \to 1 \)), unit costs are given by \( c^P(\cdot) \), and factor prices are given by \( w^P_N, w^P_I \) and \( R^P \).

Thus, we can compute \( \Psi_N \) and \( \Psi_I \) as:

\[
\Psi_I(t) = Y(t) \int_t^\infty e^{-\int_s^t (R^P - \delta - g(s)) ds} \left( \frac{\partial c^P(w^P_I(\tau))}{1 - \sigma} - \frac{\partial c^P(R^P(\tau))}{1 - \sigma} \right) d\tau + s_L \xi L \frac{\partial \ln \omega^P}{\partial T},
\]

\[
\Psi_N(t) = Y(t) \int_t^\infty e^{-\int_s^t (R^P - \delta - g(s)) ds} \left( \frac{\partial c^P(R^P(\tau))}{1 - \sigma} - \frac{\partial c^P(w^P_N(\tau))}{1 - \sigma} \right) d\tau + s_L \xi L \frac{\partial \ln \omega^P}{\partial N}.
\]

These are variants of the formula provided in equation (3.40) in the main text, in which we now discount future welfare gains from technology at the household discount rate.
However, this implies the inequalities:

\[
\Psi_N(t) = \int_t^\infty e^{-\int_0^\tau (R_P - \delta - g(s))ds} \left( \frac{\theta P(R_P) \left( 1 - \sigma \right) - \theta P(w_L) \left( 1 - \sigma \right)}{1 - \sigma} + s L \varepsilon L \frac{\partial \ln P}{\partial N} \right) d\tau \\
\Psi_I(t) = \int_t^\infty e^{-\int_0^\tau (R_P - \delta - g(s))ds} \left( \frac{\theta P(R_P) \left( 1 - \sigma \right) - \theta P(w_L) \left( 1 - \sigma \right)}{1 - \sigma} + s L \varepsilon L \frac{\partial \ln P}{\partial I} \right) d\tau \\
> \int_t^\infty e^{-\int_0^\tau (R_P - \delta - g(s))ds} \frac{\theta P(R_P) \left( 1 - \sigma \right) - \theta P(w_L) \left( 1 - \sigma \right)}{1 - \sigma} d\tau \\
> \int_t^\infty e^{-\int_0^\tau (R_P - \delta - g(s))ds} \frac{\theta P(w_L) \left( 1 - \sigma \right) - \theta P(R_P) \left( 1 - \sigma \right)}{1 - \sigma} d\tau \\
> \int_t^\infty e^{-\int_0^\tau (R_P - \delta - g(s))ds} \left( \frac{\theta P(w_L) \left( 1 - \sigma \right) - \theta P(R_P) \left( 1 - \sigma \right)}{1 - \sigma} \right) d\tau \\
= \frac{V_N(t)}{V_I(t)},
\]

as we set out to prove.

The first inequality follows from the novel inefficiency introduced in this paper: the fact that labor gets rents in equilibrium pushes towards the underprovision of new tasks and excessive automation. The second inequality follows from the technological externality; which as explained above pushes towards the underprovision of new tasks. The last inequality follows by noting that:

\[
\frac{\theta P(R_P) \left( 1 - \sigma \right) - \theta P(w_L) \left( 1 - \sigma \right)}{1 - \sigma} \geq \frac{\theta P(w_L) \left( 1 - \sigma \right) - \theta P(R_P) \left( 1 - \sigma \right)}{1 - \sigma} \geq \frac{\theta P(w_L) \left( 1 - \sigma \right) - \theta P(R_P) \left( 1 - \sigma \right)}{1 - \sigma}.
\]

To prove this inequality, denote by \( A_N = \theta P \left( \frac{w_n}{N(t)} \right) \left( 1 - \sigma \right), A_I = \theta P \left( \frac{w_n}{I(t)} \right) \left( 1 - \sigma \right), \) \( A_K = \theta P \left( R^P \left( \theta P \right) \right) \left( 1 - \sigma \right). \)

The last inequality is then equivalent to

\[
\frac{h(A_K) - h(A_I)}{A_K - A_I} \geq \frac{h(A_N) - h(A_K)}{A_N - A_K},
\]

where \( h(x) = \frac{1 - x}{1 - \sigma} x^{\frac{1 - \sigma}{1 - \sigma}}. \) When \( \zeta \geq 1 \) this function is (weakly) convex, and the above inequality follows from the convexity of \( h(\cdot) \) and the observation that \( A_N > A_K > A_I. \)

When \( \zeta < 1 \) the last inequality is reversed. Thus, in this case, the inequality \( \frac{\Psi_N(t)}{\Psi_I(t)} > \frac{V_N(t)}{V_I(t)} \)
holds only if \( \varepsilon_L \) is sufficiently large.\]
3.8.5 Proofs from Section 3.5

**Proof of Proposition 19:** Consider an exogenous path for technology in which $\dot{N} = \dot{I} = \Delta$ and $I^*(t) = I(t)$ so that $N(t) - I^*(t) = n$. We show that the economy admits a BGP in which $\dot{M} = \Delta$, and therefore $N(t) - M(t) = n - m$.

Denote the normalized wages of low-skill and high-skill labor by $w_L = W_L/\gamma(I^*)$ and $w_H = W_H/\gamma(I^*)$, respectively. Balanced growth requires that these two normalized wages converge to constant values, so that both grow at the same rate as the aggregate economy. A BGP must satisfy two additional conditions. First, because at time $t$ firms are indifferent between producing task $M(t)$ with low-skill or high-skill workers, we must have

$$\Gamma(N(t) - M(t)) = \Gamma(n - m) = \frac{w_L}{w_H}. \quad (3.42)$$

Moreover, because $\Gamma(N(t) - i)$ is decreasing in $i$, low-skill workers have comparative advantage in low-index tasks, and thus will produce the tasks in $(I^*(t), M(t))$, while high-skill workers will produce the tasks in $[M(t), N(t)]$.

The second condition is that the wage gap must also be consistent with market clearing. The market clearing conditions are given by:

$$K = Y(I^* - N + 1)e(R)\zeta^{-\sigma}R^{-\zeta},$$

$$L^s\left(\frac{W_L}{RK}\right) = Y \int_{I^*}^M \gamma_L(i, t)\zeta^{-1}c^u\left(\frac{W_L}{\gamma_L(i, t)}\right)^{\zeta-\sigma}W_L^{-\zeta}di,$$

$$H^s\left(\frac{W_H}{RK}\right) = Y \int_{I^*}^N \gamma_H(i)\zeta^{-1}c^u\left(\frac{W_H}{\gamma_H(i)}\right)^{\zeta-\sigma}W_H^{-\zeta}di.$$ 

Using the normalized variables $k \equiv K/\gamma(I^*)$ and $y \equiv Y/\gamma(I^*)$, these conditions can be rewritten as

$$k = y(1 - n)c^u(R)\zeta^{-\sigma}R^{-\zeta},$$

$$L^s\left(\frac{w_L}{Rk}\right) = y \int_0^m \gamma_H(i)\Gamma(n - i))\zeta^{-1}c^u\left(\frac{w_L}{\gamma_H(i)\Gamma(n - i)}\right)^{\zeta-\sigma}w_L^{-\zeta}di,$$

$$H^s\left(\frac{w_H}{Rk}\right) = y \int_0^m \gamma_H(i)\zeta^{-1}c^u\left(\frac{w_H}{\gamma_H(i)}\right)^{\zeta-\sigma}w_H^{-\zeta}di.$$ 

We can also observe that, under our maintained assumptions, the demand for low-skill and high-skill labor is isoelastic with a negative elasticity of $\varepsilon_D$. In particular, this elasticity is equal to $\sigma$ if $\sigma \rightarrow \zeta$ or $\eta \rightarrow 0$, or to $\sigma s + (1 - s)$ if $\zeta \rightarrow 1$, with $s$ the constant share of labor or capital in the production of a task.

Moreover, because we assumed that the supply of labor is isoelastic, we can write the
relative demand for low-skill and high-skill labor as:

$$\frac{L^*(1) w_L}{H^*(1) w_H} = \frac{\int_0^m (\gamma_H(i) \Gamma(n-i))^{\zeta-1} c^{\delta} \left( \frac{1}{\gamma_H(i) \Gamma(n-i)} \right)^{\zeta-\sigma} di}{\int_m^n \gamma_H(i)^{\zeta-1} c^{\delta} \left( \frac{1}{\gamma_H(i)} \right)^{\zeta-\sigma} di} \left( \frac{w_L}{w_H} \right)^{-\varepsilon_D} \tag{3.43}$$

Combining equations (3.42) and (3.43), we conclude that there is a BGP if and only if there exists a threshold $m$ such that:

$$\frac{L^*(1) \Gamma(n-m) \nu_L}{H^*(1) \Gamma(n-m+\varepsilon_D) \nu_L} = \frac{\int_0^m (\gamma_H(i) \Gamma(n-i))^{\zeta-1} c^{\delta} \left( \frac{1}{\gamma_H(i) \Gamma(n-i)} \right)^{\zeta-\sigma} di}{\int_m^n \gamma_H(i)^{\zeta-1} c^{\delta} \left( \frac{1}{\gamma_H(i)} \right)^{\zeta-\sigma} di}.$$  

To show that this threshold always exists, notice that the left-hand side of the above equation decreases with $m$, while the right-hand side increases with $m$. Furthermore, when $m \to 0$ the right-hand side converges to zero, and when $m \to n$ the right-hand side converges to infinity. Thus, there is a unique value of $m \in (0, n)$ for which the above equality holds. This establishes our claim that there is a unique BGP.

We omit the proof of part 3 of the proposition, which establishes the effect of changes in technology on inequality. The required steps are similar to those in the proof of Lemma 4. Specifically, similar steps established that automation reduces the relative demand for low-skill labor, but at the same time the productivity gains it generates increase the demand for high-skilled labor, thus increasing inequality. On the other hand, an increase in $N$ raises the demand for high-skill labor relative to the demand for low-skill workers, also increasing inequality.

Proof of Proposition 20: We provide formulas for the asymptotic behavior of the value functions in this case. From the Bellman equations provided in the main text, it follows that along a BGP we have

$$v_N(n) = M \int_0^\Delta e^{-(\rho-(1-\theta)g)\tau} c^n (w_N(n)e^{\delta \tau})^{\zeta-\sigma} d\tau,$$

$$v_I(n) = M \int_0^{1-\Delta} e^{-(\rho-(1-\theta)g)\tau} c^n (\rho + \delta + \theta g)^{\zeta-\sigma} d\tau.$$

Here $\Delta = \kappa_{I\phi}(n_D)/\kappa_{I\phi}(n_D)+\kappa_N$ is the endogenous rate at which both technologies grow in a BGP.

As before, a BGP requires that $n_D$ satisfies

$$\kappa_I\phi(n_D)v_I(n_D) = \kappa_nv_N(n_D).$$

Using these formulas, the proof of the proposition follows the same steps as in the proof of
Proposition 17. Using the same steps, we also obtain that the equilibrium in this case is locally stable whenever $\kappa_I \phi(n) \nu_I(n)$ cuts $\kappa_N \nu_N(n)$ from below.

### 3.8.6 When New Tasks Also Use Capital

In our baseline model, new tasks use only labor. This simplifying assumption facilitated our analysis, but is not crucial or even important for our results. Here we outline a version of the model where new tasks also use capital and show that all of our results continue to hold in this case. Suppose, in particular, that the production function for non-automated tasks is

$$y(i) = B \left[ \eta q(i)^{\frac{\xi-1}{\xi}} + (1 - \eta) \left( B_\nu (\gamma(i) l(i))^\nu k(i)^{1-\nu} \right)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}},$$

(3.44)

where $k(i)$ is the capital used in the production of the task, $\nu \in (0, 1)$, and $B_\nu \equiv \nu^{-\nu}(1 - \nu)^{-(1-\nu)}$ is a constant that is re-scaled to simplify the algebra.

Automated tasks $i \leq I$ can be produced using labor or capital, and their production function takes the form

$$y(i) = B \left[ \eta q(i)^{\frac{\xi-1}{\xi}} + (1 - \eta) \left( k(i) + B_\nu (\gamma(i) l(i))^\nu k(i)^{1-\nu} \right)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}.$$  

(3.45)

Comparing these production functions to those in our baseline model ((3.2) and (3.3)), we readily see that the only difference is the requirement that labor has to be combined with capital in all tasks (while automated tasks continue not to use any labor). Note also that when $\nu \to 1$, we recover the model in the main text as a special case. It can be shown using a very similar analysis to that in our main model that most of the results continue to hold with minimal modifications. For example, there will exist a threshold $\tilde{I}$ such that tasks below $I^* \equiv \min\{I, \tilde{I}\}$ will be produced using capital and the remaining more complex tasks will be produced using labor. Specifically, whenever $R \in \arg \min \left\{ R, R^{1-\nu} \left( \frac{W}{\gamma(i)} \right)^\nu \right\}$ and $i \leq I$, the relevant task is produced using capital, and otherwise it is produced using labor. Since $\gamma(i)$ is strictly increasing, this implies that there exists a threshold $\tilde{I}$ at which, if technologically feasible, firms would be indifferent between using capital and labor. Namely, at task $\tilde{I}$, we have that $R = W/\gamma(\tilde{I})$, or

$$\frac{W}{R} = \gamma(\tilde{I}).$$

This threshold represents the index up to which using capital to produce a task yields the cost-minimizing allocation of factors. However, if $\tilde{I} > I$, firms will not be able to use capital all the way up to task $\tilde{I}$ because of the constraint imposed by the available automation technology. For this reason, the equilibrium threshold below which tasks are produced using...
capital is given by

\[ I^* = \min\{I, \tilde{I}\}, \]

meaning that \( I^* = \tilde{I} < I \) when it is technologically feasible to produce task \( \tilde{I} \) with capital, and \( I^* = I < \tilde{I} \) otherwise.

The demand curves for capital and labor are similar, with the only modification that the demand for capital also comes from non-automated tasks. Following the same steps as in the text, we can then establish analogous results. This requires the more demanding Assumption 1", which guarantees that the demand for factors is homothetic:

**Assumption 1":** One of the following three conditions holds:

1. \( \sigma - \zeta \to 0 \), or
2. \( \zeta \to 1 \), or
3. \( \eta \to 0 \).

The following proposition is very similar to Proposition 13, with the only difference being in the ideal price condition.

**Proposition 22 (Equilibrium in the static model when \( \epsilon \in (0, 1) \))** Suppose that Assumption 1" holds. Then, for any range of tasks \([N - 1, N]\), automation technology \( I \in (N - 1, N] \), and capital stock \( K \), there exists a unique equilibrium characterized by factor prices, \( W \) and \( R \), and threshold tasks, \( \tilde{I} \) and \( I^* \), such that: (i) \( \tilde{I} \) is determined by equation (3.5) and \( I^* = \min\{I, \tilde{I}\} \); (ii) all tasks \( i \leq I^* \) are produced using capital and all tasks \( i > I^* \) are produced using labor; (iii) the capital and labor market clearing conditions, equations (3.7) and (3.8), are satisfied; and (iv) factor prices satisfy the ideal price index condition:

\[
(I^* - N + 1)c^u(R)^{1-\sigma} + \int_{I^*}^{N} c^u\left(R^{1-\nu}\left(\frac{W}{\gamma(i)}\right)\right)^{1-\sigma} di = 1. \tag{3.46}
\]

**Proof.** The proof follows the same steps as Proposition 13. □

Comparative statics in this case are also identical to those in the baseline model (as summarized in Proposition 14) and we omit them to avoid repetition. The dynamic extension of this more general model is also very similar, and in fact, Proposition 15 applies identically, and is also omitted. To highlight the parallels, we just present the equivalent of Proposition 17.

**Proposition 23 (Equilibrium with endogenous technology when \( \nu \in (0, 1) \))** Suppose that \( \sigma > \zeta \) and Assumption 1" holds. Then there exist \( \bar{p} \) and \( \bar{A} \) such that for \( \rho > \bar{p} \) and \( A < \bar{A} \), the following are true:
1. There exists \( \bar{\kappa} \) such that for \( \frac{2\kappa_1}{\kappa_N} > \bar{\kappa} \), there is a BGP, where \( \dot{N} = \dot{I} = \frac{\kappa_1 S}{\kappa_I + \kappa_N} \), and \( Y, C, K \) and \( W \) grow at the constant rate \( g = \frac{\kappa_1 + \kappa_N}{\kappa_I + \kappa_N} S \), and the rental rate, \( R \), the labor share and employment are constant. Along this path, we have \( N(t) - I(t) = n^D \), with \( n^D \) determined endogenously from the condition \( \kappa_N V_N = \kappa_I V_I \), and satisfying \( n^D \in (\bar{n}, 1) \). In addition, there exists \( \bar{\rho} > \rho \) such that if \( \rho > \bar{\rho} \), the balance growth path is unique.

2. Suppose that \( \rho > \bar{\rho} \) so that the balance growth path is unique. Then, when \( \theta = 0 \), the dynamic equilibrium is globally (saddle-path) stable. Moreover, there exists \( \bar{A} \leq \bar{A} \) such that provided that \( A < \bar{A} \), and for any value of \( \theta \), the dynamic equilibrium is unique in the neighborhood of the BGP and is locally saddle-path stable.

**Proof.** The proof of this result follows closely that of Proposition 17, especially exploiting the fact that the behavior of profits of automation and the creation of new tasks behave identically to those in the baseline model, and thus the value functions behave identically also.

### 3.8.7 Microfoundations for the Quasi-Labor Supply Function

We provide various micro-foundations for the quasi-labor supply expression used in the main text, \( L^s (\frac{W}{R K}) \).

**Efficiency wages:** Our first micro-foundation relies on an efficiency wage story. Suppose that, when a firm hires a worker to perform a task, the worker could shirk and, instead of working, use her time and effort to divert resources away from the firm.

Each firm monitors its employees, but it is only able to detect those who shirk with probability \( q \) at each instant of time. If the worker is caught shirking, the firm does not pay wages and retains its resources. Otherwise, the worker earns her wage and a fraction of the resources that she diverted away from the firm.

In particular, assume that each firm holds a sum \( R K \) of liquid assets that the worker could divert, and that if uncaught, a worker who shirks earns a fraction \( b(i) \) of this income. We assume that the sum of money that the worker may be able to divert is \( R K \) to simplify the algebra. In general, we obtain a similar quasi-supply curve for labor so long as these funds are proportional to total income \( Y = R K + W L \).

In this formulation, \( b(i) \) measures how untrustworthy worker \( i \) is, and we assume that this information is observed by firms. \( b(i) \) is distributed with support \([0, \infty)\) and has a cumulative density function \( G \). Moreover, we assume there is a mass \( L \) of workers. A worker
of type $b(i)$ does not shirk if and only if:

$$W \geq (1 - q)[W + b(i)RK] \rightarrow \frac{W}{RK} \frac{q}{1-q} \geq b(i).$$

Thus, when the market wage is $W$, firms can only afford to hire workers who are sufficiently trustworthy. The employment level is therefore given by:

$$L^s = G \left( \frac{W}{RK} \frac{q}{1-q} \right) L.$$

When $q = 1$—so that there is no monitoring problem,— we have that $G(\frac{W}{RK} \frac{q}{1-q}) = 1$, and the supply of labor is fixed at $L$ for all wages $W \geq 0$. However, when $q < 1$—so that there is a monitoring problem,— we have that $L^s < L$. Even though all workers would rather work than stay unemployed, the monitoring problem implies that not all of them can be hired at the market wage. Notice that, though it is privately too costly to hire workers with $b(i) > \frac{W}{RK} \frac{q}{1-q}$, these workers strictly prefer employment unemployment.

Alternatively, one could also have a case in which firms do not observe $b(i)$, which is private information. This also requires that firms do not learn about workers. To achieve that, we assume that workers draw a new value of $b(i)$ at each point in time.

When the marginal value of labor is $W$, firms are willing to hire workers so long as the market wage $\tilde{W}$ satisfies:

$$(W - \tilde{W})G \left( \frac{\tilde{W}}{RK} \frac{q}{1-q} \right) - (1 - q) \left( \tilde{W} + RK \int_{\frac{\tilde{W}}{RK} \frac{q}{1-q}}^{\infty} bG(b) \right) \geq 0.$$

This condition guarantees that the firm makes positive profits from hiring an additional worker, whose type is not known.

Competition among firms implies that the equilibrium wage at each point in time satisfies:

$$(W - \tilde{W})G \left( \frac{\tilde{W}}{RK} \frac{q}{1-q} \right) - (1 - q) \left( \tilde{W} + RK \int_{\frac{\tilde{W}}{RK} \frac{q}{1-q}}^{\infty} bG(b) \right) = 0.$$

This curve yields an increasing mapping from $\frac{W}{RK}$ to $\frac{\tilde{W}}{RK}$, which we denote by

$$\frac{\tilde{W}}{RK} = h \left( \frac{W}{RK} \right).$$

Therefore, the effective labor supply in this economy, or the quasi-supply of labor, is given
by

\[ L^* = G \left( \frac{\bar{W}}{RK} \frac{q}{1 - q} \right) = G \left( h \left( \frac{W}{RK} \right) \frac{q}{1 - q} \right) L. \]

As in the previous model, even though the opportunity cost of labor is zero, the economy only manages to use a fraction of its total labor.

**Minimum wages:** Following Acemoglu (2003), another way in which we could obtain a quasi-labor supply curve is if there is a binding minimum wage. Suppose that the government imposes a (binding) minimum wage \( \bar{W} \) and indexes it to the income level and current employment, \( L \). In particular, assume that

\[ \bar{W} = M^{-1}(L)(RK + WL). \]

Here, \( RK + WL \) represents the total income in this economy (net of intermediate goods' costs), and \( M^{-1}(\cdot) \) is an increasing function with inverse \( M(\cdot) \), which determines the indexation. Rearranging this expression we obtain:

\[ L^* = M \left( \frac{\bar{W}}{RK + \bar{WL}^*} \right). \]

This functional equation defines the quasi-labor supply in this economy as the solution to the above functional equation. This quasi-labor supply is an increasing function of \( \frac{\bar{W}}{RK} = \frac{W}{RK} \) (because the minimum wage is binding).

**Additional References for Appendix B**

