A Photon-Photon Interface with an Atomic Ensemble in an Optical Cavity

by

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Abstract

Since the emergence of quantum computing as a field, interactions between two-level systems that can carry quantum information have been conceptualized in terms of quantum devices. Here, I demonstrate four such devices for optical photons traveling in two different modes: an all-optical switch and transistor, a nondestructive photon detector, a phase shifter, and a quantum state generator. These devices rely on a nonlinear optical interface comprised of a few thousand atoms inside of a high-finesse optical cavity. Signal light is sent through a mode transverse to the cavity and is stored or travels through the atoms as a collective excitation. The corresponding atomic population couples to the cavity mode's optical field. The strength of the resulting photon-photon interaction is governed by the optical depth of the atoms for the signal light, and the strong atom-photon coupling in the optical cavity.

I first demonstrate an all-optical switch and transistor using blocking interactions that reduce the cavity’s transmission by a factor of $11 \pm 1$ in the presence of a stored signal photon. This interaction creates anticorrelations between the output light in the signal and cavity paths. I show that these anticorrelations persist in time continuous operation when the photon signal and cavity arrive at the same time. Then, I turn to photon detection. I reconfigure the system and reconceptualize it as a non-destructive, cavity-based detector for signal light. I demonstrate strong correlations between this nondestructive detection and a subsequent destructive detection with non-destructive detection efficiency of $0.5\%$. Next, I show that a single cavity photon can shift the phase of stored signal light by up to $1.0 \pm 0.4$ rad and demonstrate entanglement between output cavity and signal photons. Finally, I present recent experiments where this entanglement is used to modify the phase and amplitude of the signal light by making a projective measurement on the cavity light.

Thesis Supervisor: Vladan Vuletić
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Chapter 1

Introduction

In 1981, in a keynote speech at Caltech, Richard Feynman explored the feasibility of simulating a quantum mechanical system. By combining ideas from computer science and physics, he not only showed that computing as it was could not efficiently simulate such a system but also proposed an answer: a universal quantum simulator. He emphasized that a new paradigm would come out of this sort of simulation, concluding:

Nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy. [25]

In the decades since Feynman’s speech, this melding of ideas between computing and quantum mechanics has given rise to the fields of quantum simulation and quantum computation. Quantum simulation uses one well-understood and controllable quantum mechanical system to model another system, while quantum computation uses quantum systems to address computer science problems and proposes road maps to achieving such computations in, for example, the circuit-based model [67]. In this model, the system’s state is understood as information; interactions within the system are the gates used for processing it. The experimental challenge is one of control: initializing the system’s state, tailoring the intrasystem dynamics and limiting interactions with the environment until a controlled read out of the result — the system’s state — at the end of the computation. Moreover, this quantum approach was shown
to make certain problems easier solve, from Feynman’s original suggestion of quantum mechanical simulations to the computer science problems of prime factorization (Shor’s algorithm, [84]) and reverse database lookup (Grover’s algorithm, [36]).

Many different platforms are currently being investigated to demonstrate such control for a single qubit, or two-level quantum system, and then to scale the system up to many qubits joined by controlled interactions. Already in 1998, a nuclear magnetic resonance system demonstrated quantum search with 7 qubits [16]. Three years later, a similar system was used to factor the number 15 into its prime factors [96]. An implementation of the quantum Fourier transform was reported using a trapped ion quantum computer with three qubits a few years later [14]. Ongoing and parallel efforts additionally study quantum computing with, for example, superconducting qubits [21, 104], nitrogen vacancy centers [15] and electrons levitated on the surface of liquid helium [7, 62]. These studies aim to demonstrate ever-more practical implementations of long-lived qubits, reliable quantum gates, and their combination in quantum algorithms as well as to overcome the engineering problems involved in scaling systems up to a workable multi-qubit quantum computer.

Among potential qubits, optical photons – single particles of light – are unique because they can carry quantum information over distances with low loss. However, photons barely interact with each other in vacuum, so using quantum information carried by photons to make calculations is difficult. While linear optics and post-selection on detection events enables the full suite of quantum computations with optical qubits [53, 70], the success probability of such a computation decreases exponentially with the number of photons and the number of operations, which makes linear optics an impractical scheme for large-scale optical computing [10]. Deterministic interactions are an alternate route for quantum information processing that is more scalable. Advances toward such interactions have been made in cavity QED with atoms [4, 54, 76, 89, 94, 95], in photonic crystals with artificial atoms [29, 41, 74], and

---

1Here and throughout, the photon is taken by its operational definition: a state of the electromagnetic field that produces at most one current pulse in a photodetector, or transfers at most one atom’s worth of population to another state in an ensemble of atoms. Loudon discusses photons beyond this functional definition in the introduction to his text The Quantum Theory of Light [60].
This thesis describes one journey to engineer photon-photon interactions using an ensemble of atoms in a high finesse cavity to mediate interactions between two modes of light. These interactions are applied to make proof-of-principle quantum devices that further elucidate the physics underlying these interactions. In this chapter, I give an overview of the basic interaction and describe how to quantify it through the photon statistics of light leaving the system. The key tools are the second-order correlation function, $g^{(2)}$, and conditional measurements.

In Chapter 2, I introduce atom-cavity interactions and electromagnetically induced transparency (EIT) and combine them to describe the particular photon-photon interactions I study in this thesis. In Chapter 3, I describe the physical apparatus, focusing on three technical challenges that I addressed in these experiments: imaging the atoms, controlling their position, and cooling them. (My approach to two additional challenges – data analysis and programmable frequency control – are presented in the appendices.)

I then turn to the quantum devices that I demonstrated with the apparatus. First, I study the change in photon statistics that arise when one photon blocks the transmission of subsequent photons through the system as an all-optical transistor and switch (Chapter 4) and as a cross-intensity modulator (Chapter 5). By adding an intensity reference, I demonstrate a partly nondestructive photon detector (Chapter 6). I then turn to the optical phase. I present a cross-phase modulator in which one photon changes the phase of a photon in the other mode (Chapter 7) as well as current work to produce phase and amplitude modulation with a blocking type interaction (Chapter 8).

### 1.1 A recipe for photon-photon interactions

To get two photons to interact, the first ingredient is an intermediary that has a nonlinear optical response so that the transmission of light through the material
Figure 1-1: **Strong coupling to a single atom.** The red circle represents the absorption cross-section of a single atom $\sigma = \lambda^2$. The gray area represents a focused beam with a diameter of $d$ at the atoms. In this illustration, $p \sim 0.05$, approximately the interaction probability for a single atom with the signal mode in my apparatus.

depends on the number of photons present. An atom is such an intermediary: it can absorb at most one photon within the relevant transition lifetime, about $30$ ns for the D2 line in the cesium atoms I use in my experiments.

However, getting a photon to couple strongly to an atom in order to realize this interaction is difficult. A two-level atom’s absorption cross-section is at most $\sigma \approx \lambda^2$. The interaction probability $p$ between one photon in a uniform beam of diameter $d$ is then the ratio of the areas, $p \approx \lambda^2/d^2$. In free space, the diffraction limit keeps $d > \lambda$, typically giving $p \ll 1$ as illustrated in Fig. 1-1. In our system (Fig. 1-2), we achieve $p \sim 0.05$ on the signal path and $p \sim 10^{-4}$ for a single pass of the cavity path.

To get strong coupling ($p \sim 1$), the second ingredient for photon-photon interactions, the atom-light coupling must be enhanced. This can be achieved by increasing the number of atoms, by sending the a photon through the atom many times or by confining light to $d \sim \lambda$ by, for example, using the evanescent field of an optical fiber [102] or a photonic waveguide with a structured bandgap [32].

In my experiments, I couple the signal and cavity light – two different spatial modes – to the same atoms. On the signal path, many atoms enhance the absorption probability. Atoms are prepared in one electronic state of the atoms; we map the photon onto an EIT *polariton*, a collective excitation to another electronic state with the same atomic population as the number of photons sent into the system. In the experiments presented in this thesis, the atoms absorb 50-99% of the resonant light. (Equivalently, our ensemble of atoms has optical densities, $OD$, between 0.7 and 3).

\footnote{This discussion follows the one presented by Chang et al. in [10].}
Figure 1-2: **General setup for photon-photon interactions.** The interaction probability between the atoms (red circle) and the signal beam (green) is enhanced by using many atoms. On the cavity path (yellow), the interaction probability is enhanced by multiple interactions with the same atom(s): photons make many passes through the atoms before leaving the cavity mode.

On the cavity path, photons pass through the atomic ensemble many times by using an optical resonator. Each pass a photon makes through the cavity increases the atom-light interaction, giving an enhancement of \( \frac{\mathcal{F}}{2\pi} = \frac{2L}{c} \frac{\kappa}{2\pi} \). Here, \( \mathcal{F} \) is the cavity finesse, \( L \) is the cavity length, \( \kappa \) is the cavity linewidth and \( c \) is the speed of light. Photons in our cavity pass through the atoms approximately \( 3 \times 10^4 \) times before being transmitted by or absorbed in one of the cavity mirrors, giving \( p > 1 \).

### 1.2 Correlation functions and quantum light

Fluctuations are a key signature of the quantum nature of light in a single, well-defined mode. (Such a mode has a single frequency, polarization and spatial profile.) The coherence describes fluctuations in the electric field. It is quantified by the first order correlation function, \( g^{(1)}(\tau) \), which compares the field at time \( t \) with itself at a
later time $t + \tau$:

$$
g^{(1)}(\tau) = \frac{\langle E^*(t)E(t + \tau) \rangle}{\langle E^*(t)E(t) \rangle} = \frac{\langle a^\dagger(t)a(t + \tau) \rangle}{\langle a^\dagger(t)a(t) \rangle} 
$$

(1.1)

Here, $a^\dagger$ ($a$) is the creation (destruction) operator for the field. When the light’s intensity envelope is independent of time, the assumption that the light field has just one frequency guarantees that the electric field is coherent and $g^{(1)}(\tau)$ evaluates to 1 for all $\tau$. This assumption is well justified for my experiments. The input sources – sub-kHz linewidth lasers – have coherence times longer than a millisecond, considerably longer than the atom-mediated interactions which have a timescale of a few microseconds.

Intensity correlations, unlike electric field correlations, are not determined by the single-mode assumption. These intensity fluctuations are described by the second order correlation function, $g^{(2)}(\tau)$:

$$
g^{(2)}(\tau) = \frac{\langle I(t)I(t + \tau) \rangle}{\langle I(t) \rangle^2} 
$$

(1.2)

The quantum form is not the same, which is easy to see in the photon number basis at $\tau = 0$:

$$
g_q^{(2)} = \frac{\langle a^\dagger(t)a^\dagger(t + \tau)a(t)a(t + \tau) \rangle}{\langle a^\dagger(t)a(t) \rangle^2} = \frac{\langle n^2 \rangle - \langle n \rangle}{\langle n \rangle^2} = 1 + \frac{(\Delta n)^2 - \langle n \rangle}{\langle n \rangle^2} 
$$

(1.3)

$$
g_q^{(2)}(0) = \frac{\langle a^\dagger(aa^\dagger - 1)a \rangle}{\langle a^\dagger a \rangle^2} = \frac{\langle n^2 \rangle - \langle n \rangle}{\langle n \rangle^2} = 1 + \frac{(\Delta n)^2 - \langle n \rangle}{\langle n \rangle^2} 
$$

(1.4)

where I have used the commutation relation $[a(t), a^\dagger(t)] = 1$ and the definitions $n = a^\dagger a$ and $(\Delta n)^2 = \langle n^2 \rangle - \langle n \rangle^2$.

The second order correlation function can be used to distinguish between different states of light. For a coherent state $|\alpha\rangle$, $g^{(2)} = 1$ as the variance is equal to the average photon number, $(\Delta n)^2 = \langle n \rangle = |\alpha|^2$. $g^{(2)}$ takes on different values for different states. For example, for a photon number state $|n\rangle$, $g^{(2)} = 1 - \frac{1}{n}$ as $\langle n^2 \rangle = \langle n \rangle^2 = n^2$.

Since the intensities $I(0)$ commute but the operators $a$ and $a^\dagger$ do not, the second
order correlation function can distinguish between classical and nonclassical states. Quantum mechanically, \(g^{(2)} \geq 0\). However, \(g^{(2)}_c \geq 1\), and \(g^{(2)} < 1\) is not allowed by classical correlations of intensities. The single photon state \(|1\rangle\) is nonclassical as it has \(g^{(2)} = 0 < 1\).

(Note that if \(g^{(2)} > 1\) for a given state, this does not mean that this state is classical. Instead, the test is agnostic: the \(g^{(2)}\) function does not tell us if the state is classical or not. For example, the nonclassical state \(\frac{1}{\sqrt{2}} (|0\rangle + |4\rangle)\) is has \(g^{(2)} = 1.5 > 1\).)

The photon number distribution in a mode provides some intuition for the meaning of this classicality. A coherent state has a Poisson distribution of photon numbers:

\[
|\alpha\rangle = e^{-|\alpha|^2} \sum_n \frac{|\alpha|^n}{\sqrt{n!}} |n\rangle
\]

\[
P(n) = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}
\]

Sub-Poissonian states such as photon number states have tighter photon-number distributions than the coherent state and have \(g^{(2)} < 1\). Phase-squeezed states have super-Poissonian statistics; low variance in phase is accompanied by high variance in the conjugate variable (photon number). Classically, it is possible to make states with more fluctuation in photon number, but not less, than classical (coherent) light sources.

For \(\tau \gg 0\) in real systems, the creation and annihilation operators commute and

\[
g^{(2)}(\tau) = \frac{\langle n(t) n(t+\tau) \rangle}{\langle n(t) \rangle^2}
\]

In our system, this drives measured correlations back toward their uncorrelated value at long separation times, \(g^{(2)}(\infty) = 1\).

The intensity correlation formalism can be extended to a two-mode system, like our signal-cavity system shown in Fig. 1-2.

\[
g^{(2)}_{ab}(\tau) = \frac{\langle a^\dagger_b(t) a^\dagger_b(t+\tau) a_a(t) a_b(t+\tau) \rangle}{\langle a^\dagger_a(t) a_a(t) \rangle \langle a^\dagger_b(t) a_b(t) \rangle} = \frac{\langle n_a(t) n_b(t+\tau) \rangle}{\langle n_a(t) \rangle \langle n_b(t) \rangle}
\]
Figure 1-3: **Classical cross-correlation calculations.** Different patterns of intensity modulation on two modes illustrate how the second-order correlation function distinguishes between (a) correlated $g_{ab}^{(2)} > 1$, (b) anticorrelated $g_{ab}^{(2)} < 1$ and (c) uncorrelated $g_{ab}^{(2)} = 1$ intensity modulation.

for different modes $a$ and $b$. This function can take on any positive value. In Fig. 1-3, I illustrate classical intensity patterns that give $g_{ab}^{(2)}(0)$ above, below, and equal to 1. If the light intensity in the two modes is correlated so that $n_b(t) = \beta n_a(t)$ (Fig. 1-3a), $g_{ab}^{(2)}(0) = \frac{n_b^2}{\langle n_a \rangle^2} \geq 1$ by application of the Cauchy-Schwarz inequality. If instead the light intensity is anticorrelated so that whenever $n_a(t)$ is nonzero, $n_b(t)$ is zero, $g_{ab}^{(2)} = 0$. $g_{ab}^{(2)} = 1$ if the intensity modulation between the two modes is uncorrelated.

As with single-mode correlations, $g_{ab}^{(2)}$ can be used to distinguish between different states of light. I use this later to compare the state of light sent into the experiment and the joint state of the output light, changed by interactions in the system. However, $g_{ab}^{(2)}$ does not say anything about quantum intensity correlations. Both the classical intensities and the quantum mechanical operators for different modes commute. If we want determine the classicality of a two-mode system, we need a new measure. One such measure is the normalized $g^{(2)}$ function, $G$ [18]:

$$G = \frac{\left(\frac{g_{ab}^{(2)}}{g_{aa}^{(2)} g_{bb}^{(2)}}\right)^2}{\text{classical}} \left(\frac{\langle I_a I_b \rangle}{\langle I_a \rangle \langle I_b \rangle}\right)^2 \frac{\langle I_a^2 \rangle}{\langle I_a^2 \rangle} \frac{\langle I_b^2 \rangle}{\langle I_b^2 \rangle}$$

(1.9)

$$= \frac{\langle I_a I_b \rangle^2}{\langle I_a^2 \rangle \langle I_b^2 \rangle}$$

(1.10)

Applying the Cauchy-Schwarz inequality, $\langle I_a I_b \rangle^2 \leq \langle I_a^2 \rangle \langle I_b^2 \rangle$, showing that (classi-
cally) $G \leq 1$. If we instead use the quantum mechanical definition of the single-mode correlators, we find that $G$ takes on any positive value. I will return to this function in Chapter 5.

A cross intensity modulator is an example of a device that exhibits a modified two-mode correlation function. In such a device, the power of one mode depends on the power of the other mode. In particular, if the system transmission with two photons is less than its transmission with one photon, then $g_{ab}^{(2)} < 1$. The change in correlations is caused by the atomic mediator; the timing of the resulting correlations as well as its strength allow us to understand the underlying interaction.

1.3 Conditional phase measurements

Interactions lead not only to intensity fluctuations but also to phase shifts. A linear system’s interaction with light is described by the susceptibility, $\chi$, a complex number with a real component that leads to attenuation or gain, and an imaginary component that leads to phase shifts or refraction. To access the phase, we perform conditional measurements. When a photon is present in mode $a$ – and only then – we reconstruct the phase in mode $b$. This measurement is similar to a lock-in detection scheme: by gating on the few photons in mode $a$, we can bring out a small signal in mode $b$ that otherwise might be buried in noise.

A cross-phase modulator is a device in which one photon shifts the phase of another optical mode. If the normal phase is $\phi_0 = 0$, the phase with one photon is changed to $\phi_1$. When we add the intensities of these two possible outcomes with probabilities $p_0$ and $p_1$, we can extract the average phase:

$$p_0 \sin(\omega t) + p_1 \sin(\omega t + \phi_1) = \sin(\omega t) \left\{ p_0 + p_1 \cos(\phi_1) \right\} + \cos(\omega t) \left\{ p_1 \sin(\phi_1) \right\}
= A \sin(\omega t + \theta)$$

(1.11)

where $A^2 = p_0^2 + p_1^2 + 2p_0p_1 \cos \phi_1$ and $\tan \theta = \sin \phi_1 / \left( p_0 / p_1 + \cos \phi_1 \right)$. The output has a reduced amplitude and a reduced phase between $\phi_0 = 0$ and $\phi_1$ (Fig. 1-4). The
Figure 1-4: **Conditional phase measurements.** The average phase (black) which consists of an unshifted component (blue) and shifted component (red). The dashed curves are the unshifted and shifted components normalized so that their maximum peak value is 1. The colored circles on the axis indicate the phase of the average, unshifted and shifted light, respectively.

full phase shift can be extracted from the signals by conditioning on detecting one photon, which gives a phase of $\phi_1$. 
Chapter 2

Atom-Photon-Cavity Interactions

Our ensemble of atoms mediates interactions between two optical modes by means of a cavity and EIT polaritons, the collective atomic excitations of slow or stored light. These interactions enable us to characterize the ensemble with average features such as modified transmission spectra, and to construct the quantum devices described in later chapters. This chapter begins by describing the interactions between a two-level atom and a cavity (Section 2.1) and generalizes this interaction to non-uniform coupling and collective excitations. I present an experiment that demonstrates the validity of this model for our system.

I then proceed to describe electromagnetically induced transparency (EIT) in a three level-atom (Section 2.2). I discuss the particular decoherence that arises from interactions between EIT polaritons and light in our optical cavity. I finally bring the cavity and free-space modes together to create a single model (Section 2.3) used in later chapters to describe a 4-level ($N$-type) atom simultaneously under EIT conditions and interacting with a cavity probe. I finally comment on how the two models are combined in the conceptually simpler experiments with stopped light where the fields are applied sequentially.
An optical cavity transmits light due to electric-field interference. Consider a standing-wave resonator of length $L$ with two lossless mirrors as depicted in Fig. 2-1. For mirrors with real transmission and reflection coefficients $q$ and $r$, the steady state cavity field, $\mathcal{E}_{\text{cav}}$, is given by [90]:

$$\mathcal{E}_{\text{cav}} = r^2 e^{i k L} \mathcal{E}_{\text{cav}} + q \mathcal{E}_{\text{in}} \approx \frac{\mathcal{E}_{\text{in}}}{q} \left( 1 - i \frac{2 \delta}{\kappa} \right)^{-1}$$

(2.1)

where $\mathcal{E}_{\text{in}}$ is the input electric field, $k = 2\pi/\lambda$ is the wavenumber of the light, $\kappa = q^2 c / L$ is the cavity linewidth ($1/\kappa$ is the $1/e$ lifetime of a photon in the cavity), $\delta = \omega - \omega_{\text{cav}}$ is the detuning of the input light from the cavity resonance at $\omega_{\text{cav}}$ ($\omega = c k$, with $c$ the speed of light), and $r^2 e^{i k L} \approx 1 - q^2 + 2i q^2 \delta / \kappa$. The highest cavity field – and thus the highest transmission $q \mathcal{E}_{\text{cav}}$ – occurs for $\omega = \omega_{\text{cav}}$. At this frequency, light enters the cavity with the same phase as the circulating light, interfering constructively with the cavity field. (These high transmission frequencies occur once per free spectral range, that is for light with frequencies separated by
Importantly, this interference depends on the round-trip phase $e^{ikL}$ and the loss $r^2$ from the cavity mode. The interference conditions change when an atom is present. Classically, this atom can be described as an oscillator with a complex polarizability [90], which results in additional single-pass phase shift and loss. The atom’s phase shift changes the effective cavity length (the new phase is $kL + \phi = kL'$) and thus changes the resonant frequency of the cavity. Additionally, close to atomic resonance (that is, the transition $|c\rangle \rightarrow |d\rangle$ in Fig. 2-1) where the polarizability changes quickly with the detuning $\Delta$ between the atomic resonance and cavity probe light, the cavity mode may split so that two different input frequencies in a single cavity free spectral range give the highest transmission. Loss induced by the atoms reduces the overall resonant cavity transmission.

The transmitted ($P_{tr}$), reflected ($P_r$), and scattered ($P_{4\pi}$) power out of the cavity mode due to the interaction with one atom are expressed as fractions of the input power, $P_{in}$, as [90]:

$$\frac{P_{tr}}{P_{in}} = \frac{1}{(1 + \frac{\eta}{1+\Delta^2})^2 + (\bar{\delta} - \frac{\Delta\eta}{1+\Delta^2})^2}$$  \hspace{1cm} (2.2)

$$\frac{P_{4\pi}}{P_{in}} = \frac{2\eta}{(1 + \bar{\Delta}^2)}$$  \hspace{1cm} (2.3)

$$\frac{P_r}{P_{in}} = 1 - \frac{P_{tr}}{P_{in}} - \frac{P_{4\pi}}{P_{in}}$$ \hspace{1cm} (2.4)

where $\bar{\Delta} = 2\Delta/\Gamma$ is the atom-light detuning normalized to the atomic linewidth $\Gamma$, $\bar{\delta} = 2\delta/\kappa$ is the cavity-light detuning normalized to the cavity linewidth, and $\eta$ is the single-atom cooperativity which describes the strength of the atom-cavity interaction ($\eta$ is the same as the interaction probability $p$ described in Chapter 1). On resonance, where $\Delta = \bar{\delta} = 0$, the cooperativity entirely describes the system’s behavior:

$$\frac{P_{tr}}{P_{in}} = \frac{1}{(1 + \eta)^2}$$ \hspace{1cm} (2.5)

$$\frac{P_{4\pi}}{P_{in}} = \frac{2\eta}{(1 + \eta)^2}$$ \hspace{1cm} (2.6)
\[
\frac{P_r}{P_{\text{in}}} = \frac{\eta^2}{(1 + \eta)^2} \quad (2.7)
\]

The resonant single-atom cooperativity is the single-pass absorption probability \( \frac{\eta}{A} \) (for a cavity mode with effective area \( A \)) multiplied by the average number of times a photon cycles through the cavity \( \mathcal{F}/2\pi \) and a factor of 4 that accounts for the difference in intensities between a uniform field and the antinode of the standing wave field in the optical cavity:

\[
\eta = 4 \times \frac{6\pi/k_0^2}{\pi w^2/2} \times \frac{\mathcal{F}}{2\pi} = \frac{24\mathcal{F}/\pi}{k_0^2 w^2} \quad (2.8)
\]

where \( w \) is the waist of the TEM00 cavity mode. This is equivalently the ratio of coherent processes in the cavity (energy exchange between the atom and the cavity) and loss processes:

\[
\eta = \frac{4g^2}{\kappa L} \quad (2.9)
\]

where \( 2g \) is the single photon Rabi frequency for a maximally coupled atom. In our system, the largest achievable cooperativity is \( \eta = 8.6 \pm 0.2 \).

### 2.1.1 Inhomogeneous coupling

The single-photon Rabi frequency depends on the electric field strength and therefore the atomic position. To account for this, the single photon Rabi frequency \( 2g \) needs to be replaced by a position-dependent value \( 2g(r, z) \) where

\[
4g^2(r, z) = 4g^2 \cos^2(kz) e^{-\frac{r^2}{w^2}}.
\]

Here, \( z \) is the atom’s position along the cavity axis and \( r \) is the radial distance from the cavity axis. The variation along \( z \) describes the standing-wave cavity mode whereas the Gaussian factor (dependent on \( r \)) describes the mode profile’s decay away from the cavity axis.

An additional factor that can reduce the coupling is the atomic dipole’s coupling strength, quantified by the Clebsch-Gordon coefficient. For cycling transitions such as the \( |S_{1/2}; F = 4, m_F = r\rangle \rightarrow |P_{3/2}; 5, 5\rangle \) transition used in the experiments I present in this thesis, the Clebsch-Gordon coefficient takes on its maximal value of 1.
2.1.2 Collective excitations and ensemble-cavity interactions

When we have more than one atom in state $|c\rangle$, each atom individually couples to the cavity field with a coupling strength that depends on the local strength of the cavity field at that position, providing a phase shift and loss for the local cavity field. These atoms collectively couple to the field with an effective coupling strength $NH\eta$ where the $N$ is the number of atoms and $H$ is a factor between 0 and 1 that describes the coupling [90].

With the exception of the atom number detection scheme presented in Section 4.4, the experiments in this thesis have just with one collective excitation ($N = 1$). (Since our inputs are coherent states, the results for higher atom numbers are important for predictions of cavity transmission properties for the full system, but not to understand the underlying interactions we want to study.) These collective excitations, described in more detail below, can be thought of as many atoms sharing one atom’s worth of population in state $|c\rangle$ with an effective coupling strength $H\eta$.

The most appropriate distribution of coupling strengths $H$ depends on the experiment. In the all-optical transistor experiment (Chapter 4), we model the polaritons as single atoms that have a single position in each experimental attempt. This one atom has an effective coupling strength $H\eta = \eta \cos^2(kz)e^{-\frac{z^2}{w^2}}$. We then predict the cavity’s transmission and free-space scattering by averaging the transmission and free-space scattering for an atom at position $(z, r)$ over the atomic density distribution weighted by the signal beam profile. This model is justified by the strong cavity field in this experiment, which decoheres collective excitations through scattering out of the cavity mode, leaving an ensemble with one atom in state $|c\rangle$. In other experiments where we do not decohere the polariton, the effective cooperativity is reduced because the polariton’s population is distributed over different positions during in each measurement.
Figure 2-2: Experimental illustration of atom-cavity interactions with detuning. The cavity transmission is plotted in red (black) with atoms (without atoms) in the state near-resonant with the optical cavity for different values of the atom-cavity detuning $\Delta$. The data agree well with theory for $N\eta \simeq 10$.
2.1.3 An experimental illustration

In Fig. 2-2, I show a set of illustrative cavity spectra measured with a small atomic population sent into the atomic ensemble as EIT polaritons. The black (red) lines indicate cavity spectra without (with) atomic population in the state resonant with the cavity. The cavity spectrum on atom-cavity resonance is a low-amplitude Lorentzian. As we detune from $\Delta = 0$, the transmission changes its center frequency. This frequency cares about the sign of the detuning. At large detunings, the transmission spectra are similar again, with the detuning of the atom-cavity peak returning close to 0 and the transmission increasing to a large value (close to full transmission).

2.2 Electromagnetically Induced Transparency

Electromagnetically Induced Transparency (EIT) is a mechanism that is used to control the propagation of light through an ensemble of atoms using a secondary light field [28, 39, 40, 47, 81]. In Fig. 2-3, I show the energy level diagram for the system. Atoms in state $|a\rangle$ absorbs light on the $|a\rangle \rightarrow |b\rangle$ transition. However, in the presence of a second optical field (typically called the control field) resonant with $|b\rangle \rightarrow |c\rangle$, 

Figure 2-3: Electromagnetically Induced Transparency (EIT) (a) cartoon and (b) energy level diagram. State $|b\rangle$ has linewidth $\Gamma$. 

29
the ensemble instead transmits light. This transparency arises due to quantum interference; under EIT conditions, the driving lasers ensure that the population in state $|b\rangle$ is zero because the amplitude from direct excitation $|a\rangle \rightarrow |b\rangle$ and from interaction with the control beam $|a\rangle \rightarrow |b\rangle \rightarrow |c\rangle \rightarrow |b\rangle$ cancel. This interference leads to a strong modification of the atoms' susceptibility, which not only changes the transmission but also changes the group velocity of the light.

The transmission of the ensemble is given by

$$T = \exp\left(-\mathcal{OD}\frac{\Gamma}{2} \frac{8\Gamma(\delta - \Delta)^2 + 2\gamma(\Omega^2 + \gamma\Gamma)}{(\Omega^2 + \gamma\Gamma - 4\delta(\delta - \Delta))^2 + (2\Gamma(\delta - \Delta) + 2\gamma\delta)^2}\right)$$

(2.10)

where $\mathcal{OD}$ is the ensemble's optical density, $\Gamma$ is the linewidth of state $|b\rangle$, $\gamma$ is the decoherence of state $|c\rangle$ (relative to state $|a\rangle$), $\Delta$ is the control detuning $\omega_c - \omega_{bc}$, $\delta$ is the signal (probe) detuning $\omega_s - \omega_{ab}$ and $\Omega$ is the Rabi frequency of the control light.

Light propagating through an atomic ensemble under EIT conditions travels as a polariton, a quantum excitation that is a mixture of a photonic component and some atomic population in state $|c\rangle$. The relative amplitudes in these different components are given by the control field's Rabi frequency; lower Rabi frequencies correspond to more population in state $|c\rangle$. The group velocity of the propagating light is proportional to by $\Gamma/\Omega^2$.

Transparency is limited by how much the polariton decoheres before it exits the ensemble, as can be seen in Eq. 2.10. Typically decoherence is limited by atomic temperature, which scrambles the phase of different atoms in the ensemble by relative motion, or magnetic-field noise which changes the energy levels.

### 2.2.1 Stored (Stopped) light

The polariton's relative proportion of photonic and atomic components can take on any value. In particular, a fully photonic polariton is the limit of infinite Rabi frequency. The opposite limit, however, is quite different; a photon sent into an EIT medium can exist as a purely atomic excitation.

Of course, a photon cannot enter the ensemble as an EIT polariton without a
control field. However, if a polariton is sent into the ensemble with a large control field which is then adiabatically reduced, the polariton can be ‘stopped’ or ‘stored’ in the atoms [27]. The conversion efficiency between an input photon and a stored polariton depends on the optical density of the atomic ensemble, the atomic linewidth and other decoherence processes, and the adiabaticity of temporal changes [33].

2.2.2 Cavity-induced decoherence

Interactions with cavity light also can decohere the EIT polariton. These can be generically described by the way that the interaction spatially changes the spatial structure of the polariton which records the input photon’s electric field. To retrieve the photon into its original mode, the coherence and relative phase between different parts of the polariton must be preserved.

First, if light in the cavity mode scattered off of the polariton into free space, a detector outside of the cavity can in principle determine from the trajectory of the light which photon scattered the atom. (Alternatively, this can be seen as an entanglement between the photon and the atomic ensemble, which is decohered by a measurement by the environment.) This decoherence scales with the number of cavity photons at a rate of\(2\eta/(1 + \gamma)^2\) per cavity photon.

Even if light is not scattered off of the polariton, interactions with light in the cavity change the weight of different components of the spin wave. If we detect that there is population in state \(|c\rangle\), as we do in the nondestructive detection experiment (Chapter 6), we change the relative weighting of different parts of the spin wave. In particular, our knowledge about the atomic population is weighted toward the cavity axis and the antinodes of the cavity standing wave. This localization makes the spin wave couple poorly to the control beam, reducing the efficiency with which the polariton transverses the atoms into its original optical mode.

While the majority of the experiments I present in this thesis use cavity light that is resonant with the atomic transition, the phase shift measurements in Chapter 7 use off-resonant light. This off resonant light imparts non-uniform phase shifts on the stored light that depend on the local field intensity. This leads to mode distortions, as
well as to a frequency chirp across the retrieved light that corresponds to the cavity’s geometric mode.

2.3 A photon-photon interface

EIT and atom-cavity interactions can be combined into an $N$ type system as described in the introduction (Fig. 1-2) to induce interactions between photons traveling in the signal and the cavity modes. The system Hamiltonian is the combination of the Hamiltonians for the two paths:

$$ H_{\text{eff}} = \sum_k c|a_k\rangle\langle a_k| + \sum_j (\omega_j - i\kappa_j/2)|b_j\rangle\langle b_j| + \sum_j (\omega_{j\text{d}} - i\Delta/2)|d_j\rangle\langle d_j| + \sum_j (\omega_{j\text{c}} - i\gamma_j/2)|c_j\rangle\langle c_j| + \sum_x (\Omega_2 e^{i\omega x t}|c_x\rangle\langle b_x| + g_2 a_d^\dagger(x)|a_d\rangle\langle b_d| + g_s b_d^\dagger|b_d\rangle + h.c.) $$

This Hamiltonian includes decay and a multimode signal (free-space) beam and is used in the theoretical predictions for cross-intensity modulation ([2] and Chapter 5).

Here, the electric field operators for the signal (free-space) and cavity fields can be written as $\hat{E}_s(x) = \sqrt{\hbar c k_0/V} a(x)$ and $\hat{E}_c = \sqrt{\hbar c k_0/V} b$, where $a(x) = N^{-1/2} \sum_k e^{ikx} a_k$ and $b$ are bosonic annihilation operators, $ck_0$ is the center frequency of the signal field, and $V$ is the quantization volume. Additionally, $\mathcal{E}$ is the amplitude of the cavity input field, $\omega_{\mu\nu}$ is the atomic transition energy between states $\mu$ and $\nu$, $\Omega$ is the classical Rabi frequency for the coupling field, $\Gamma$ is the linewidth of the excited states $b$ and $d$, $\gamma$ is decoherence rate of two stable ground states $a$ and $c$, and $g_s$, $g_c$ are the bare couplings of the atomic transition to the two fields. We take the gate and signal fields to be resonant with the atoms so that $ck_0 = \omega_{ab}$ and $\omega_c = \omega_{cd}$.

2.3.1 Photon-photon interactions with stopped light

For experiments with stored light, the photon-photon interaction can be decomposed into three separate light-atom interactions. The typical sequence consists of storing
signal light in the atoms, probing the cavity's transmission, and retrieving the signal light. The storage is characterized the storage efficiency, the probability of converting an input signal to atomic population in state $|c\rangle$. The cavity interaction is described by the interaction between the cavity and a collective excitation, giving the reduced transmission probability and the scattering rate of light out of the cavity mode. Light is retrieved with the same efficiency as it is stored, reduced by the additional decoherence that comes from the cavity interaction. The light-cavity interaction, then, can be well described with loss rates added by hand to the light-atom interactions for each of the signal and the cavity modes individually.
Chapter 3

Experimental Setup

In this chapter, I describe the apparatus used to perform the experiments described in this thesis. In Sec. 3.1, I describe how we trap, cool and prepare atoms from the background gas in the chamber. In Sec. 3.2, I describe how we calibrate and quantify ensemble properties through absorption imaging and atomic interactions with our optical cavity. In Sec. 3.3, I turn to our optical cavity and describe calibration methods for optimizing the atom-cavity interaction. Sec. 3.4 focuses on properties of the detection system. Additional details on data analysis and dynamic frequency control are provided in Appendices A and B, respectively.

3.1 Atomic preparation

3.1.1 Magneto Optical Trap (MOT)

We collect our cesium atoms from the small background pressure of cesium atoms in our vacuum chamber into a 6-beam magneto-optical trap (MOT) in a 10 G/cm gradient field. The MOT operates on the D2 line in cesium, that is the transitions between the $S_{1/2}$ and $P_{3/2}$ electronic states with a wavelength of 852 nm. The cooling laser is slightly red detuned from the $|S_{1/2}; F = 4\rangle \rightarrow |P_{3/2}; 5\rangle$ transition. The repumping laser is resonant with the $|S_{1/2}; 3\rangle \rightarrow |P_{3/2}; 4\rangle$ transition and ensures atoms in state $|S_{1/2}; 3\rangle$ can be returned to the cooling transition (see Fig. 3-1). A pair of in-
Figure 3-1: **Magneto-optical trap.** (a) **Geometry** and (B) **energy level diagram** for the magneto-optical trap (MOT) used to collect atoms for the experiments described in this thesis. Circularly-polarized laser beams are retroreflected on three paths (indicated). Atoms are slowed and trapped in the quadrupole magnetic field produced by a pair of in-vacuum anti-Helmholtz coils indicated in (a). Three pairs of Helmholtz coils (not shown) are used to trim the position of the field zero where the atoms collect. Each beam contains light at both the cooling and repumper transitions indicated in (b), where \( F \) indicates the hyperfine quantum number.

Vacuum electromagnetic coils produce the gradient field. The initial MOT collection time varies between 250ms and 1s depending on the desired atomic density and the desired heat load on the chamber from the resistive heating of these coils. Adam Black details of the vacuum chamber in his thesis [5]. Jon Simon’s thesis discusses our current in-vacuum structure, including the quadrupole coils [85].

After about 500ms of loading, the MOT reaches steady state. The density of this cloud is set by the competition between the scattering of photons directly from the MOT beams and the photons that are reabsorbed after emission by another atom. In order to collect higher atomic densities, we compress the MOT [20]. We increase the gradient field to 22 G/cm for 27 ms. This changes the potential the atoms see in the presence of the light and are able to win over the increased photon pressure from the cloud at the center for short times. The trim coils are used to adjust the position of the compressed MOT to optimize loading into the optical trap.
The position and shape of the MOT evolves over the course of the day and between different loadings, resulting in large shot-to-shot variation in our atom number. We conjecture that this loading instability is due to interference fringes from light scattered by the cavity and other structures in the vacuum chamber combined with slow drifts in the position of our beams and the position of our magnetic field zero. We have tried to improve the MOT stability in several ways, addressing both the magnetic and optical fields. Early in my Ph.D., we added MOT compression (described above) to our experimental sequence. Since compression uses a higher gradient field, the field zero is less sensitive to drifts in ambient field. Additionally, we cut one of the aluminum optical breadboards close to the vacuum chamber to reduce Eddy currents, which reduced the settling time for the field and likely reduced ambient field drifts as well.

On the optical paths, we realign the retro-reflected beams in the MOT about once a week, and occasionally rebalance the power in the different optical paths. To improve polarization purity, I 3D-printed waveplate holders that attach to our vacuum flanges. We use these to mount the quarter waveplates that make our circularly polarized light after the dielectric mirrors in the beam path. Dielectric mirrors only preserve polarization for linearly polarized light in or normal to the plane made by the incident and reflected light. We also installed piezoelectric stacks into two of the three optical paths so that we could modulate the path lengths independently to wash out interference fringes during the MOT loading time. None of these efforts significantly improved the MOT stability. However, some optical trap configurations (Sec. 3.1.3) mitigate the effect of atom number loading fluctuations.

3.1.2 Optical cooling

The atoms’ temperature is set by evaporative cooling in the optical dipole trap leading to a temperature of about 17% of the trap depth. The temperature of atoms increases the linewidth of EIT through the Doppler effect of the atoms as well as through atomic collisions. These increased collision rates reduce the lifetime of atoms in the trap, reducing total data rate because the amount of time that we can use for data
collection is reduced in each experimental cycle.

In some of our experiments, we used additional cooling to improve the system performance and to reach higher atomic densities. Polarization gradient cooling uses optical interference to set up a sinusoidal energy landscape introduced by the variation in local polarization. By careful choice of the detuning of the laser beams and by setting the magnetic field to zero, this energy structure can mediate additional cooling. Photons absorbed where the energy difference is lowest can only lose additional energy—when the atom transitions back to the lower level, it will release additional energy if the transition happens anywhere in the energy structure other than this lowest energy difference point.

We also used Raman cooling. This also needs particular magnetic field values—here, we want the trap’s phonon energy to match the Zeeman energy between two adjacent magnetic states. We do this in two ways: both by changing the magnetic field and by changing the power of the optical trap. Changing the power of the optical trap tends to be more controllable and continuous than changing the magnetic field.

3.1.3 Optical trapping and atomic placement

We gather the atoms into a far-off-resonant dipole trap where we hold the atoms during subsequent preparation and the experiment. The primary trapping light is at 937 nm, close to the magic wavelength for the cesium D2 line. For the experiments I will describe in this thesis, it is only the atoms that are in the geometric overlap between our probe beam and the cavity mode that mediate interactions between the photons. Atoms elsewhere in the cavity mode dispersively shift the cavity frequency and contribute to backgrounds. We have used two different configurations of this trap in our experiments to differently optimize the atom-cavity mode overlap and the optical density.

The trapping light is produced by a 100 mW DFB laser from Eagleyard (EY-DFB-0937) which delivers up to 15 mW of power to the cavity. This laser’s linewidth is narrowed by external feedback from a 0.7 m fiber [58]. This light is used to injection lock a 300 mW laser from Axcel Photonics (M9-940-0300-DSP).
- **Cavity trap** The optical cavity has a finesse of 368 ± 20 at 937 nm, which enhances the average power of the intracavity trap by a factor of $F/\pi \approx 100$. Atoms in this trap are well coupled to the cavity; all of them are held in the same TEM00 cavity mode that we use for the 852 nm coupling which ensures that the radial profile of the atomic cloud and 852 nm cavity mode are well matched.

However, along the cavity axis ($z$), the trap is a standing wave so its intensity depends sinusoidally on its position, with a factor of 4 intensity increase at antinodes and 0 trap intensity at the nodes. This means that the trap this laser creates gives us a series of atomic ‘pancakes’ that are separated by $\frac{1}{2} \times 937$ nm = 413.5 nm. Importantly, this is incommensurate with the sinusoidal variation of the cavity coupling light on the D2 transition (which has 425 nm periodicity). We have attempted to use this as a feature in our system (see Sec. 7.6), but in our experiments so far it has been an often annoying technical detail.

- **Shelving atoms** In the cavity trap, we hold many atoms along the cavity axis. Our signal beam only interacts with a small fraction of these atoms. The sample loaded is approximately 100 μm-long, and our signal beam has a waist of 2.2 μm at the location of the atoms. By reducing the extent of the cloud, we reduce cavity frequency fluctuations and atomic backgrounds without greatly reducing the ensemble’s optical depth.

To do this, we shelve atoms. We pumped the atoms into the $|S_{1/2}; F = 4\rangle$ ground state manifold and used a beam slightly bigger than the signal beam at the atoms to pump just the atoms in that portion of the loaded sample to the $|S_{1/2}; 3\rangle$ ground state. Then, we applied a single beam on the $|S_{1/2}; 4\rangle \rightarrow |F_{3/2}; 5\rangle$ cycling transition (tuned close to resonance this time) to push atoms out of the trap. This beam off-resonantly excited some of the atoms in the held region; the best performance we achieved was when we repeated the prepare-optically pump-push sequence three times. In the all-optical transistor experiments, we used shelving to reduce the total atom number to 3% ($5 \times 10^5$).
to $1.6 \times 10^4$ atoms) while reducing the optical density in the signal path by less than 10%.

- **Transverse ‘sidebeam’ trap** Since some of our experiments were starved for optical density for high mode overlap, we used the intracavity lenses to trapping the atoms in a transverse trap. Using a similar focusing system to the one that used for the signal probe path (see Haruka Tanji-Suzuki’s thesis for details [88]), we focus light at the atoms down to a waist of about 2 $\mu$m. While the intensity at the atoms is similar to the intensity maximum in the cavity trap; the waist of the cavity trap is 35 $\mu$m so the ratio of the areas is about 300, comparable to the intensity enhancement in the cavity mode due to the finesse and standing wave mode structure. Images of the atoms in the trap showed that the atoms extended beyond the cavity mode; we estimate that between half and a third of the atoms are in the cavity mode.

There was a new source of heating in this trap. Polarization variation along the highly focused beam leads to an effective magnetic field at the atoms. When we mapped out the decoherence of the atoms as a function of the polarization, the lowest trap-induced decoherence occurred for vertically polarized light. In practice, we do not use our maximal trap power in this trap due to this noise; we instead increase it to the point that we have the best retrieval efficiency for stored light at our desired retrieval time.

- **1064 nm trap**

An alternate way of improving the atomic density in the overlap region is to use a pair of crossed traps. We put in a trap from the side of the cavity with a 5 W 1064 nm fiber laser. By using a beam telescope, we were able to focus this laser to 20 $\mu$m at the atoms. The trap did not result in a smaller size of the atoms along the direction of the transverse trap, as we had hoped, but it did help combat atom loss by providing a reservoir of atoms that could enter the other trap during the experimental time.
3.1.4 Optical pumping

Once we have trapped the atoms, we prepare the atomic sample in the desired hyperfine level by applying elliptically polarized light to the atoms, which in the atomic basis has no $\sigma^-$ component. As I show in Appendix C, in any geometric configuration, you can choose to eliminate one polarization of light in the atomic basis. The state we want to prepare ($|S_{1/2}, F = 3, m_F = 3\rangle$) is a dark state for this system, that is that it has no resonant allowed transitions. (It can be excited to $|P_{3/2}, F = 3, m_F = 3\rangle$, $|P_{3/2}, F = 4, m_F = 3\rangle$ and $|P_{3/2}, F = 4, m_F = 4\rangle$ off resonantly). Each of the excited states can decay to states with $\Delta m_F = \{-1, 0, 1\}$, although each of these decays does not occur with equal probability, so that atoms eventually end up in the desired state. During optical pumping, we also apply depumping light on the atoms on the $|S_{1/2}, F = 4\rangle \to |P_{3/2}, F = 3\rangle$ transition to clear atoms out of the long-lived $|S_{1/2}, F = 4\rangle$ state.

When our quantization axis is along the $z$-axis, we use light at angle of about 22 deg to the horizontal to do this optical pumping. The beam is a collimated elliptical beam with an aspect ratio of 1:3. This ratio was chosen to couple well to the atoms in the cavity trap while not wasting power by illuminating areas where atoms are not held in that trap.

3.2 Detecting and calibrating the atomic ensemble

To understand the experiment, we need to understand the atomic ensemble. We quantify the atomic ensemble through a variety of methods to determine the number of atoms present in the system, their density distribution, their temperature, and the atomic state distribution, all of which are important to understand the way the atoms mediate interactions between photons in the two paths.
Figure 3-2: **Energy level diagram for optical pumping.** In a small magnetic field, the energy of different $m_F$ levels are split by the Zeeman effect. This diagram shows a partial energy-level diagram for the D2 line in our optical pumping configuration. In the atomic basis, the optical pumping beam has $\pi$- and $\sigma^+$-polarized components, but no $\sigma^-$. This asymmetry makes the $|S_{1/2}, F = 3, m_F = 3\rangle$ state a dark state; atoms in this state scatter light at a much lower rate than the other $m_F$ levels (the only scatter is through off-resonant transitions) and so atoms end up collecting in this state. Not indicated in this figure is light that removes atoms out of the $|S_{1/2}, F = 4\rangle$ manifold.
Figure 3-3: Calibrating absorption imaging with gravity. Images of atoms that have been dropping for $t\ \mu$s. The lower graph shows the location (in pixels in the measured image) of the peak atomic density fit to the function $at^2 + c$. $a$ fits to $-1.63 \pm 0.05\ \text{pixels}/(\mu\text{s})^2$. These data were taken with 1/4 resolution.
3.2.1 Absorption imaging

The absorption imaging system was installed in 2012 and designed by visiting student Robert Bücker. Light illuminates the atoms from a collimated beam typically chosen to be resonant with the $|S_{1/2}; F = 4, m_F = 4\rangle \rightarrow |P_{3/2}; 5, 5\rangle$ transition. Atoms resonant with the beam absorb photons and scatter them out of the beam. Atoms appear in images light that is blocked from the beam; each image is calibrated to a second image taken in the same experimental cycle without atoms present.

The system is set up as a telescope to image atoms. Light from the atoms is focused by a pair of lenses and detected by the sensor of a camera (PointGrey Grasshopper Express GX-FW-28S5M) which has a pixel size of 4.54 μm and an active area of 1932 by 1452 pixels. It needs a delay time of approximately 15 ms between images. The optics are arranged so that one pixel is 0.7 μm at the atoms. When we take lower resolution images, the calibration needs to be adjusted accordingly.

3.2.2 Cavity shift

The dispersive interaction between the atomic ensemble and the optical cavity can be used to measure the number of atoms interacting with the cavity mode. At large frequencies, the coupled atom number is calculated as

$$N \eta = \frac{4\Delta s}{(\Gamma \kappa) \sum_i C_{g_i}^2 \delta_i}$$ (3.1)

where $\Delta s$ is the measured cavity center shift, $C_{g_i}$ is the Clebsch-Gordon coefficient that describes the dipole coupling strength, and $\delta_i$ is the detuning of transition $i$ from the cavity frequency.

3.2.3 Thermometry

The temperature of the ensemble is a useful parameter for understanding not only our atomic preparation but also the Doppler decoherence in the system. We have used two methods to calibrate it. When we have many atoms,
This can be measured by time of flight spectroscopy, in which the initial cloud of atoms is quickly released from its trap and falls under gravity as shown in Fig. 3-3. The atomic cloud expands with the momentum of the atoms mapping onto position. Several measurements as a function of time are used to fit size of the expanding cloud to derive the average speed of the atoms, and thus the initial temperature in the cloud.

More recently, we have been measuring temperature through the correlation function of light scattered into the cavity mode from a beampath transverse to the cavity. The $g^{(2)}$ of the light is 2 at short times and 1 at long times. Atoms' relative positions are unchanged at times less than the Doppler decoherence time, maintains the higher correlation for a timescale related to the temperature described in [35].

### 3.2.4 Microwave spectroscopy

To measure the quality of the optical pumping of our ensemble, we use microwave spectroscopy. For this measurement, atoms are prepared in the $F = 3$ groundstate manifold in a low magnetic field. A long microwave pulse is applied to the sample through a microwave horn outside of the vacuum chamber at a frequency close to 9.81 GHz, the transition frequency between the $F = 3$ and $F = 4$ groundstate manifolds. The microwave frequency is swept. At each frequency, the number of atoms is measured either by cavity shift or by absorption imaging (Fig. 3-4).

This method can also be used to measure the magnetic field strength and fluctuations. The separation between hyperfine levels is determined by the magnetic field; magnetic field (and/or effective magnetic field) fluctuations are responsible for the difference in linewidth between transitions that are first-order magnetic field insensitive ($|S_{1/2}; F = 3, m_F = 0\rangle \rightarrow |S_{1/2}; 4, 0\rangle$) and transitions that couple strongly to the magnetic field, such as the $|S_{1/2}; F = 3, m_F = 3\rangle \rightarrow |S_{1/2}; 4, 4\rangle$ transition, whose frequency changes 350 kHz/G.
Figure 3-4: Microwave spectroscopy of the $F = 3$ ground state manifold. The atom number transferred to $F = 4$ can be measured by (a) the dispersive frequency shift of the optical cavity or (b) by integrating the number of atoms in absorption imaging. $y$-axis in each graph extracts a number that increases with increasing atom number. In (a), we see all 15 possible transitions (partially indicated on the upper axis) indicating that there are atoms in all $m_F$ levels in the $F=3$ groundstate manifold. In (b), we only see 3 transitions, identified to be those from $\pi$-polarized microwave fields for $m_F = \{-3, 0, 3\}$, after optical pumping with $\pi$-polarized light on the $|S_{1/2}, F = 3\rangle \rightarrow |P_{3/2}, F = 3\rangle$ transition. (C) zooms in on the central peak (black circles) which is insensitive to magnetic fields and one of the magnetic field sensitive transitions (red circles).
3.3 Optical cavity

The optical cavity is formed by a pair of highly polished mirrors mounted on piezoelectric stacks in the vacuum chamber. The curved mirrors have a radius of curvature of $10 \pm 0.2 \text{ mm}$ and are separated by $13.7 \text{ mm}$ to give a free spectral range $\nu_{\text{FSR}} = 10.9 \text{ GHz}$. The cavity linewidth is directly measured by ring-down spectroscopy giving a linewidth of $\kappa/2\pi = 142 \pm 1 \text{ kHz}$. The linewidth is confirmed by measuring the cavity transmission spectrum and fitting it with a Lorentzian curve, which gives a linewidth of $\kappa/2\pi = 158 \pm 7 \text{ kHz}$, in good agreement with the ring-down measurement once the additional noise from our cavity lock ($\sim20 \text{ kHz}$) is taken into account.

The cavity mode waist at the location of the atoms is $35.5 \pm 0.2 \mu\text{m}$, determined by the geometry of the cavity. Taken together with the cavity linewidth (which determines the cavity finesse $F = (7.71 \pm 0.05) \times 10^4$), this geometry determines the maximum cavity cooperativity $\eta = 8.6 \pm 0.3$.

We have used a variety of methods to make sure that the atoms are highly coupled to both the signal and the cavity in every trap setup. There are four primary geometric considerations that need to be dealt with once at a time to make sure that the atoms in the trap are coupled the best to the signal and cavity.

- **Signal focus** The focus of the signal beam needs to be centered on the atoms so that the optical density is highest at the atoms on the cavity axis. To optimize this, trap atoms in the cavity trap and measure the rate at which a resonant beam saturates the atoms so that they no longer absorb. This rate depends on the local intensity; the beam’s position is optimized when this rate takes on its maximal value.

- **Signal placement** The position of the signal beam relative to the cavity axis determines number of atoms that are interacting with both the signal and the cavity mode. With atoms trapped in the cavity mode, measure the optical density as a function of the beam displacement perpendicular to the cavity mode. The optimal position is where the $\text{OD}$ is maximum.
• **Sidetrap placement** With the signal beam placed relative to the cavity, the sidetrap needs to be aligned. First, the position of the sidetrap is changed to optimize the optical depth of atoms held in the sidetrap as measured by the signal beam.

• **Sidetrap focus** Finally, the focus of the sidetrap needs to be centered on the atomic cavity. To measure this, use the cavity birefringence. Send a linearly-polarized probe through the cavity and detect it on a polarizing beamsplitter where the output polarization basis has been adjusted with waveplates so that only one of the detectors measures light in the absence of trapped atoms. The amount of light measured in the otherwise dark port depends on the atom number coupled to the cavity mode. The focus is optimized when the dark port signal is highest.

### 3.4 Detectors

Our detectors are single photon counting modules that we get from Excelitas.\(^1\) These are gallium-arsenide detectors run in Geiger mode, so that one single photon triggers an avalanche of electrons. Electronics within the detector convert this pulse to a voltage spike, which we then detect with a 4-channel multichannel sequencer (P7888 from Fast Com'Tec) with 2 ns resolution.

These detectors have two known detector artifacts. The detectors are not able to detect photons for about 40 ns after detecting one photon, a time known as the detector ‘dark’ time. This is due to the reset process for the detector. This effect reduces the average rates detected at high rates. Measured average rates need to be corrected for this effect using calibrations provided for particular detector from the company. These calibrations can be confirmed by using a calibrated set of neutral density attenuators before the detector. Since the correction is small at low rates, the low rate measurement can be taken as the true value.

\(^1\)The detectors are the same, but the division was been acquired by Excelitas while I was in graduate school. Some of ours were purchased when they were still part of Perkin-Elmer.
Table 3.1: **Efficiency and dark count data for our single photon detectors.**
From our measurements, the dark count rate increases with the age of the detectors.

Secondarily, the detectors are known to ‘afterpulse.’ When a photon is detected, the detector can emit a broadband optical pulse. When this light is reflected back to the detector, it may register as an additional count. To limit the detection of these photons, we take advantage of the fact that they are broadband and thus many will not travel in the guided mode of the fiber. By coiling the fiber, we can increase the attenuation for photons traveling in the fiber cladding. There will be some photons detected from this effect, either by reflection off of the backside of interference filters or photons that are close to 852 nm that travel through our interference filters and reflect back to the detectors off of other optics in the optical path. These photons show up in single-channel correlation measurements like the example shown in Fig. 3-5 as one or more peaks in coincidences for times less than about 100 ns. These events can be removed from data by removing photons that arrive on a single channel more closely spaced than the measured afterpulsing time as described in Sec. A.1.1.

We use a stack of interference filters before each fiber-coupled detector. On the cavity path, we use a pair of 10 nm Semrock laser line filters (Semrock LL01-852) centered at 850 nm that have about 98% transmission. In between these filters, we have a long-pass filter from Chroma. This spectral filtering, combined with the spatial filtering from the single-mode fiber we couple to the detector and additional spectral filtering we get by attenuating light traveling in the optical cladding by blacking out and coiling the fiber preceding the detector, allows us to achieve dark count rates during the experimental acquisition time that are close to separately measured values.
Figure 3-5: **Detector artifacts.** Single-channel correlation on the cavity path as a function of the time difference, \( \tau = |t_1 - t_2| \). The two primary detector artifacts that we take into account in our analysis are depicted in this figure. The detector registers only a few counts that are separated by less than the detector deadtime of 60 \( \mu s \). The peak in coincidences from 60-80 \( \mu s \) is caused by afterpulsing, where the detector is fully blocked.

To record our photon counts, we use a fast multi-channel acquisition board from Fast ComTech (P7888). This detector records the times when voltages go over (or fall below) software-selected thresholds with 2 ns resolution. The data rate recorded by this card is limited by the on-card buffers and the PCI bus in its host computer. When the data rate exceeds the buffer and/or PCI bus capacity, data are lost. Typically this is seen as a loss of data from the later portion of an experimental run. Average detected rates show a decrease in detected rates as a function of time. We have run up against this limitation when using a combination of high rates and/or long acquisition times. Data are written to the file in 1024 kB chunks, which is noticeable when measuring at very low rates (for example, when calibrating detector dark counts). The last part of data will be saved to the file after acquisition is stopped. The card can acquire for longer than 100 ms. However, at these longer acquisition times, the proprietary software runs slowly.
Chapter 4

A Photon Transistor, Switch, and Photon-Number Detector

In 1947, John Bardeen, Walter Brattain and William Shockley invented the transistor, for which they were awarded the Nobel Prize in 1956 [78]. This scientific discovery provided a way for a small electrical signal to control a larger electrical signal. In the appropriate configuration, the transistor can also be used as a switch or as an amplifier for the original signal. Transistors replaced vacuum tubes as elements in electronic logic circuits and catalyzed the electronics revolution to give us ever-smaller personal computers and powerful supercomputers.

Optical transistors, similarly, aim to control a large optical signal with a small one. In an era where information is routinely carried by light carried on fibers, optical transistors would enable to make logic circuits with light. To compete with electronic transistors in classical computing, these optical transistors need to operate at a similar speed and energy to electronic transistors. Alternatively, in the weak signal limit, an optical transistor might provide a feature that cannot be easily replicated using electronic transistor – quantum coherence. Such coherence is a step towards a deterministic quantum gate for optical qubits.

Current electronic transistors operate on a picosecond timescale with total energy consumption equivalent to a few hundred optical photons [65]. Comparable speeds and powers have been demonstrated individually, but not together. A single-mode
optical modulator was demonstrated with silicon microtoroids that operates with picowatt-strength light fields and switching speeds of up to 300 MHz [103], and, in later work, as a transistor with 10 GHz switching speeds [23]. Single-mode optical switching has been achieved with lower powers by strongly coupling a single optical mode to atoms [1, 59, 73, 98]. The atomic lifetime limited the switching speed to a few megahertz.

Two-mode photon switching experiments in $N$-type energy level structures were proposed in the theoretical literature in both free-space and cavity-mediated setups [9, 47]. Previous work showed that not only could a single resonant atom block the transmission of a highly coupled cavity [92], but also that collective excitations blocked transmission much like atoms [90].

In this chapter, we demonstrate a two-mode optical transistor gated by one stored photon. Weak stored light (the transistor ‘gate’) induces a bimodal cavity transmission distribution which can switch over 400 photons when operated as a transistor. The cavity transmission is analogous to the source-drain current. With fewer cavity photons, I demonstrate that these interactions do not always result in the loss of stored light, which I quantify through the light’s retrieval probability and through the cross correlation function $g^{(2)}_{ac} = 0.29^{+0.09}_{-0.08}$ of the output beams. With even more light, we can distinguish between 0, 1, 2 and more absorbed photons.

This work is summarized in the publication:


and in Sec. 3.5 and Chapter 4 of Wenlan Chen’s thesis [11].

### 4.1 Experimental setup

Our experimental setup is shown in Fig. 4-1. An ensemble of laser-cooled atoms is prepared in a far-off-resonant standing-wave optical trap in the cavity mode and
Figure 4-1: (a) **Setup** and (b) **energy level diagram** for the photon switch and transistor experiments. An ensemble of laser-cooled atoms is prepared in the geometric overlap between the $\sigma^+$-polarized, free-space signal (green) and linearly-polarized cavity (yellow) modes and held in a far-off-resonant standing-wave trap in the TEM00 mode of the optical cavity. Atoms are prepared in state $|a\rangle$. Signal photons resonant with the $|a\rangle \rightarrow |b\rangle$ transition are stored in the ensemble by adiabatically turning off the $\pi$-polarized EIT control beam (purple), which is resonant with the $|b\rangle \rightarrow |c\rangle$ transition. Photons resonant with the $|c\rangle \rightarrow |d\rangle$ transition probe the cavity. The atomic states used in this experiment are $|a\rangle = |6S_{1/2}; F = 3, m_F = 3\rangle$, $|b\rangle = |6P_{3/2}; 4, 4\rangle$, $|c\rangle = |6S_{1/2}; 4, 4\rangle$ and $|d\rangle = |6P_{3/2}; 5, 5\rangle$, where $F$ and $m_F$ denote the hyperfine and magnetic sublevels.

optically pumped to state $|a\rangle$. By choosing the polarizations and frequencies of the light that interrogates the atoms, we pick out a four-state energy level structure $|a\rangle \leftrightarrow |b\rangle \leftrightarrow |c\rangle \leftrightarrow |d\rangle$ in the cesium D2 line. The atoms mediate interactions between the signal photons sent into the atomic ensemble from a focused beam and the cavity photons that travel in the mode of our high finesse optical cavity.

Fig. 4-2 shows the sequence timing. Signal photons enter the ensemble from free-space. They are stored as EIT polarizations in the atomic ensemble by adiabatically ramping down the power of the control beam [39, 40, 47, 81]. Then, we probe the
cavity transmission by sending a pulse of light resonant with the $|c\rangle \rightarrow |d\rangle$ transition onto the cavity. We use a long (50 $\mu$s) pulse to measure the system’s performance as a transistor (Fig. 4-2c), or a short (1 $\mu$s) cavity pulse (Fig. 4-2a-b) to measure the system performance with signal photon retrieval. We retrieve the stored light after the cavity transmission measurement by ramping up the control beam’s power.

A combination of optical depth, atomic temperature, and suboptimal waveform control limit the signal photon storage efficiency to 10-20%, within a factor of two of the best expected storage efficiency for this system [33]. Since the storage and retrieval efficiency are similar, the input-output storage and retrieval efficiency, $R$, is $1 - 4\%$, as seen in the example time traces in Fig. 4-2b. In Fig. 4-3, I plot $R$ as a function of storage time. The trend fits to an exponential curve with a time-constant of $2.0 \pm 0.3$ $\mu$s. The better fit of the exponential decay (compared to the Gaussian fit) indicates that the storage time is limited by non-thermal decoherence. Thermal decoherence results in Gaussian-shaped decay, while random decoherence processes that happen at a fixed rate give exponential decays.

The spatial distribution of atoms was controlled by shelving atoms in the desired region by using a beam copropagating with the free-space beam with a larger beam waist, as described in Sec. 3.1.3. It was important to limit the total number of atoms. Off-resonant excitations transfer atoms from state $|a\rangle$ to state $|c\rangle$, a process that is proportional to the total atom number. The cavity interaction that determines the cavity transmission does not discriminate between the population in $|c\rangle$ due to off-resonant excitations and the EIT polaritons. By removing most of the atoms outside of the geometric overlap between the free-space beam and the cavity mode, we maintained optical depth while reducing our backgrounds. Reducing the total atom number also made the cavity center frequency more stable.

4.2 An all-optical transistor

To quantify system operation, we first measured the average cavity transmission using a long probe pulse with many photons. Fig. 4-4a shows transmission spectra for
Figure 4-2: **Timing sequence showing (a) input and (b) output pulses.** Signal light is stored in the atomic ensemble by adiabatically ramping down the control beam at $t = 0$ ("storage"). The cavity transmission is then measured by sending in a 250 ns pulse onto the cavity and detecting its transmission. Finally, the control beam is adiabatically ramped up to retrieve the signal photon. For the data set shown here, the combined storage and retrieval probability is 2.8%. The shaded regions are the analysis regions used to extract $g_{ac}^{(2)} = 0.29(9)$ reported in the text and in [12]. (c) shows the output pulse timing for measuring the system performance as a transistor. The cavity transmission is interrogated for 50 μs and there is no signal retrieval.
different mean stored signal photon numbers \( \langle n_s \rangle \). The average cavity transmission reduced to about a third at \( \langle n_s \rangle = 1.4 \), considerably less than the expected single-atom cavity blocking of \( 1/(1 + \eta)^2 \simeq 4\% \). (The maximal cooperativity \( \eta \) is \( 4.2 \pm 0.2 \), reduced by a factor of two because we probe the cavity with linear, instead of \( \sigma^+ \)-polarized, light.)

This higher average transmission arises because the stored signal light is a weak coherent state with average photon number \( \langle n_s \rangle \), and the number of stored photons – and thus the population in state \( \ket{c} \) – has a distribution different photon numbers \( n \). The fractional cavity transmission is then a weighted sum of the expected transmission spectra for different numbers of stored photons:

\[
T = \sum_n T_n P(n) = e^{-\langle n_s \rangle} \sum_n T_n \frac{\langle n_s \rangle^n}{n!} \geq e^{-\langle n_s \rangle}
\]

where \( T_n = 1/(1 + N\eta)^2 \) and the lower bound comes from the observation that for \( P(0) = e^{-\langle n_s \rangle} \) of the cases, there are no stored photons and the cavity fully transmits light.

This limit translates to an upper limit of \( 1 - e^{-\langle n_s \rangle} \) on the switching contrast, the difference between the full cavity transmission and the transmission with stored signal light. I plot this in Fig. 4-4b. The grayed out area represents contrast values that are not allowed by the Poisson statistics of the stored light. The measured data points use the resonant contrast extracted from the average curves and lie close to
Figure 4-4: **Average cavity transmission.** (a) **Transmission spectra** for the cavity and (b) **contrast** between the resonant spectra with and without stored signal photons. Transmission spectra were measured with average stored signal photon numbers \( \langle n_s \rangle = 0, 0.4, 1.4 \) and 2.9 (top to bottom). The solid lines are Lorentzian fits to the data.

the maximal switching contrast.

Stored photon number quantization is evident in the statistics of the cavity’s transmitted photon number. In Fig. 4-5, I plot histograms of the detected photon number for average stored signal photon numbers \( \langle n_s \rangle = 0 \) and \( \langle n_s \rangle = 0.4 \). The black data points show the cavity transmission in the absence of stored photons (\( \langle n_s \rangle = 0 \)). This transmission is well described by a Poisson function (black curve) with a mean photon number of \( 33.5 \pm 0.1 \), which describes the combined projection efficiency of coherent state used to probe the cavity’s transmission and the detection efficiency of the detector. A lower-transmission pedestal (not included in the theory) comes from atoms excited into state \( |c\rangle \) by off-resonant light during the measurement time. The cavity transmission then depends on the time these background atoms are deposited.

The red data were measured with \( \langle n_s \rangle = 0.4 \). The histogram shows two clearly separated components. The high transmission component corresponds to \( n_s = 0 \). The low-transmission peak corresponds to \( n_g \geq 1 \). The predicted transmission is a sum of the expected transmission with one atom and full transmission weighted by the coherent state probabilities.

The high-to-low peak transmission gives an extinction factor for one stored gate photon of \( T^{-1} = 11 \pm 1 \). The optical gain per signal photon, \( G \), is the difference
Figure 4-5: **Histogrammed cavity transmission.** In black (red) is a histogram of the detected cavity photons measured for no stored signal photons (0.4 stored signal photons). The black solid line is a Poisson fit to the empty cavity transmission histogram. The red solid line is a prediction (see text) for the transmission of the cavity with \( \langle n_s \rangle = 0.4 \), a weighted sum of the empty cavity transmission (black) and the cavity transmission with one photon (shaded, gray).

In transmitted cavity photon number with \( n_s = 0 \) and \( n_s = 1 \), quantified as the difference between the average of the 0- and 1- photon transmission. For this data, \( G \) is 25 detected cavity photons. Correcting for the cavity outcoupling efficiency, path loss, and detector efficiency, the intrinsic gain in this measurement is 170 photons, 110 of which are available outside of the cavity. The outcoupling efficiency is \( \frac{T}{T+L} = 0.66 \). (Here, \( T = 27 \) ppm is the cavity mirror transmission, and \( L = 14 \) ppm is the mirror loss.)

The detection path efficiency combines the 45% detector efficiency and additional measured path loss of 50%.

We measured similar histograms for five different cavity powers and two different integration times (25 \( \mu s \) and 50 \( \mu s \)). In Fig. 4-6, I plot the gain after correcting for the detection efficiency. The gain saturation occurs around 1000 transmitted cavity photons, likely due to optically pumping the atom into other magnetic sublevels with weaker cavity coupling. One stored signal photon can block more than 600 photons, of which about 400 are available outside of the cavity.

While the predicted cavity transmission is a simple bimodal distribution when
atoms have a single coupling to the cavity, the atomic distribution relative to the cavity standing wave complicates the picture. Atoms at different positions along the cavity axis experience different cavity coupling strengths, and thus have a different effect on the cavity transmission. In Fig 4-7a, I show a cartoon of the relative placement of atoms, the cavity coupling, and the signal beam spatial profile along the cavity axis \( z \). After scattering several photons out of the cavity, the stored signal polariton decoheres into an atom at a single position \( z \). Fig. 4-7b shows a series of shot-noise-broadened transmission histograms for the different atomic positions. The spectra depend on the relative position \( D \) between the center of the Gaussian signal beam along the cavity axis and the next-nearest atomic pancake and are weighted by the local power of the signal beam. In fact, since the input beam drifts with movement in the room, we are more likely averaging over multiple graphs. The data we measure correspond well to the prediction for a Gaussian beam with \( D = 140 \text{ nm} \) and a reduced atom-cavity coupling of \( \eta = 2.6 \).
Figure 4-7: Cavity transmission with atom position. (a) Cartoon of atom placement for transmission. Atoms are held in the cavity trap with a spacing of $\frac{1}{2} \times 937$ nm (red) and illuminated from the side with the Gaussian signal beam, indicated in green. The relative placement between an atom and the cavity standing wave (yellow, spacing $\frac{1}{2} \times 852$ nm) determines the cavity transmission. The spacing between the most likely atom position and the closest cavity antinode is indicated as $D$, 140 nm in this illustration. (b) Predicted cavity transmission. The black curves are the shot-noise broadened prediction for cavity transmission for atoms in each position indicated in (a) with an empty cavity transmission that matches the measured transmission for the histograms measured in Fig. 4-5. Their amplitude is weighted by the Gaussian signal beam profile, and sum to the predicted transmission indicated in gray.
4.3 Photon switching by stored light

In order to operate the device with gate retrieval and measure the switching of one photon by another, we reduced the number of photons sent through the cavity as well as the cavity integration time, as shown in Fig. 4-2a-b. As in the higher power measurements, one stored free-space photon should reduce the cavity transmission by a factor $1/(1 + \eta)^2$, introducing correlations between the cavity transmission and the number of stored signal photons, and thus the number of retrieved signal photons. To measure this switching, we measure the correlations $g_{s,c}^{(2)} = 0.29^{+0.09}_{-0.08}$ between the cavity transmission and the retrieved signal light on cavity resonance using weak coherent states with 0.2 average retrieved signal photons and 0.1 transmitted cavity photons. Correcting for independently measured backgrounds (see below), the anticorrelation is $g_{s,c}^{(2)} = 0.17^{+0.08}_{-0.06}$. Importantly, the measured $g^{(2)}$ is less than one, indicating that the output light is anticorrelated and that a single stored photon switches switch a photon in the cavity mode. These correlations are extracted from the raw data using the method described in Sec. A.2.3.

We can make a simple truth table for the expected output depending on the number of stored signal photons:

<table>
<thead>
<tr>
<th>stored signal cavity free-space photon number transmission transmission</th>
<th>cavity transmission</th>
<th>free-space transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$T$</td>
<td>$R'$</td>
</tr>
</tbody>
</table>

where $R'$ is the probability that a stored photon will be retrieved after interacting with a cavity photon ($R' \leq R$, the probability the stored photon will be retrieved without interacting with a cavity photon). If our input states are both weak coherent states, $|\epsilon\rangle \approx |0\rangle + \epsilon|1\rangle$, the output state is

$$
|0_s0_c\rangle + \epsilon_c|0_s1_c\rangle + \epsilon_s R|1_s0\rangle + \epsilon_s \epsilon_c R'T|1_s1_c\rangle
$$

(4.2)

and the expected second order correlation function is

$$
g_{sc}^{(2)} = \frac{\langle n_s n_c \rangle}{\langle n_s \rangle \langle n_c \rangle} = \frac{\epsilon_s \epsilon_c R'T}{(\epsilon_s R + \epsilon_s \epsilon_c R')(\epsilon_c + \epsilon_s \epsilon_c T')} \approx \frac{R'}{R} T
$$

(4.3)

61
Uncorrelated background counts on our detectors bring the measured correlation function closer to the uncorrelated value of 1.

\[
g_{sc}^{(2)} = \frac{\langle (n_s + b_s)(n_c + b_c) \rangle}{\langle n_s + b_s \rangle \langle n_c + b_s \rangle} = \frac{\tilde{g}_{fc}^{(2)} + B}{1 + B} \tag{4.4}
\]

where the background-to-signal ratios on each path have been combined into the variable \( B = \frac{\langle b_f \rangle}{\langle n_f \rangle} + \frac{\langle b_c \rangle}{\langle n_c \rangle} + \frac{\langle b_f \rangle \langle b_c \rangle}{\langle n_f \rangle \langle n_c \rangle} \).

The measured cavity transmission at high power gives an expected value of \( T \) between 0.09 and 0.24 (extracted from the mode and mean of the 0- and 1- photon peaks, respectively), which agrees well with our measured value after correcting for backgrounds.

The second order correlation function \( g^{(2)} \) demonstrates that the atomic interaction mediates photon switching, but does not tell us how much gain we might have while still retrieving the signal photon. To do this, we measured the fractional retrieval efficiency \( \frac{R'}{R} \), as a function of the average cavity photon number \( \langle n_c \rangle \mid_{n_s = 0} \). As in the average cavity transmission where the transmission depended on the statistics of the weak stored signal light, the fractional retrieval efficiency depends on the photon statistics of the cavity light. A weak coherent state has at least one photon in a fraction \( 1 - e^{-\langle n_c \rangle} \) of cases. If one cavity photon entirely decoheres stored light, the recovery should still be \( e^{-\langle n_c \rangle} \) as there are no cavity photons to decohere the stored light in this fraction of measurements.

In Fig. 4-8, I plot the fractional recovery of stored light with increasing cavity power. The red curve is an exponential fit to the decay and fits to a \( 1/e \) cavity photon number of \( 1.9 \pm 0.1 \) (\( 2.8 \pm 0.2 \) without cavity outcoupling losses). This agrees well with the simple model of decoherence from cavity photons scattering out of the cavity; the probability of scattering a cavity input photon is \( 2\eta/(1 + \eta)^2 \), which gives an anticipated \( 1/e \) photon number of 3.3 without outcoupling losses, in good agreement with the measured value.

The total amount of gain available at the \( 1/e \) retrieval photon number is outside the cavity is \( 1.4 \pm 0.1 \) (\( 2.2 \pm 0.2 \) without cavity outcoupling losses) using the linear
Figure 4-8: **Fractional recovery of the stored light in the transistor.** The average fractional recovery decays exponentially (solid red curve) with the cavity photon number. The $1/e$ recovery point is at $\langle n_c \rangle|_{n_s=0} = 1.9 \pm 0.1$ ($2.8 \pm 0.2$ without cavity outcoupling losses).

This gain estimate depends critically on the calibrations that we have made in the detection paths. The measurement itself, however, gave us a check on the storage efficiency in the signal path. The histogrammed transmission curves allow us to make a good estimate of the atomic population of the cavity by using the weight in the high-transmission peak ($n_s = 0$) to estimate the stored photon number. We directly compared this to the stored photon number predicted from the photon storage efficiency and the measured detection path efficiency (detector efficiency of 45% and path efficiency of 30%). The results agreed within 10%.

### 4.4 Detecting 0 to 3 absorbed photons

In the transistor experiment, we limited the total cavity power to about 1 photon per cavity lifetime to avoid saturating the atoms or the single photon detectors\(^1\) and

\(^1\)The single photon detectors we use have an efficiency that starts deviating from its nominal value at a count rate of $10^6$/s, which was about the rate that we used for the transistor measurements. The detectors should not be operated with rates exceeding $10 \times 10^6$/s. In these experiments, we use smaller detector safety margins and/or neutral density filters to measure transmission with higher
integrated for 50 μs. By increasing the average intracavity power, we could saturate the $|c\rangle$ state to make the cavity transmission multi-modal, so that we could distinguish between 0, 1, 2 and more atoms in state $|c\rangle$. Such a device is an atom-number resolving detector [105] or a photon-number resolving detector [46, 48].

Saturation increases cavity transmission by reducing the effective coupling between an atom and the cavity by a factor of $1/(1 + s)$ where $s$ is the intracavity saturation factor. The transmission is then modified from Eq. 4.1 to take into account the atom-number dependent cavity power and its effect on the cavity coupling, replacing $T_n$ by $T_{n,s}$.

The experimental setup for these experiments is shown in Fig. 4-9. There are a few critical changes. First, the quantization axis is now along the cavity axis, increasing the maximal coupling to an atom on the $|c\rangle \rightarrow |d\rangle$ transition because input light can have the right polarization to drive this transition directly. We also incoupled signal photons by absorption instead of as stored light, reducing the probability of converting a signal photon into an atom in state $|c\rangle$. We then increased the input photon number until we had significant population in each transmission peak.

In Fig. 4-10, I show histograms of cavity transmission for different cavity powers, which are plotted on the $x$ axis. The fractional cavity transmission is $T = 1/(1+n_s\eta)^2$ until the cavity photon number is enough to saturate the atoms, at which point the fractional cavity transmission increases. This allows us to discriminate between different atom numbers by choosing the input power of the cavity.

The curves depict the predicted transmission by modeling the transmission of the cavity with atomic saturation with the intracavity saturation parameter $s$ found by solving the equation:

$$s = s_{in} \left(1 + \frac{\eta}{1+s}\right)^{-2}$$

where $s_{in}$ is the intracavity saturation parameter in the absence of atoms. The modified cavity transmission is $T = (1 + \eta/(1 + s))^{-2}$ For small $s \ll 1$, $s = s_{in}/(1 + \eta)^2$, as we would expect for the reduced cavity power in the presence of the atomic average rates.
Figure 4-9: **Experimental Setup for photon number counting by absorption.** Photons are absorbed on the $|a\rangle = |S_{1/2}; F = 3, m_F = 3 \rangle \rightarrow |b\rangle = |P_{3/2}; 4, 4 \rangle$ transition. Atoms decay with 46% probability into the state $|c\rangle = |S_{1/2}; 4, 4 \rangle$. These atoms are then detected by measuring the cavity transmission on the $|c\rangle \rightarrow |d\rangle = |P_{3/2}; 5, 5 \rangle$ transition. We measured the cavity transmission twice per experimental cycle, once with absorbed photons (population in state $|c\rangle$) and once with atoms in state $|a\rangle$. Cavity transmission was integrated for 400 μs.
Figure 4-10: **Cavity transmission as a function of input power.** The figure shows the transmission of a cavity with a few atoms as the amount of incident power onto the cavity is increased (see text).

blocking. For large $s$, $s = s_{\text{in}} - 2\eta$. In this plot, the curves show transmission for 0, 1 and 2 atoms. The low transmission peak at high cavity input rate is the transmission for 3 or more atoms.

This detection scheme can also be used to illustrate how atoms are excited into the detection state by trap and cavity light. In Fig. 4-11, I show histograms of the cavity transmission as a function of integration time. Each row is a histogram measured for the photons detected in 50 $\mu$s starting at times between 0 and 300 $\mu$s in the 400 $\mu$s detection window. The top panel shows histograms of the cavity transmission (color) as a function the integration time (vertical axis) with $n_s = 0$. The histograms show discrete transmission ranges, with the weight of lower transmissions increasing as the integration time increases. This indicates that more atoms are in state $|c\rangle$ at later times. The lower panel shows transmission histograms for $\langle n_s \rangle > 1$. The same transmission ranges are visible, with additional weight on the lower transmission ranges that correspond to 2 or more atoms.
Figure 4-11: **Optical pumping during multiphoton detection.** The color shows the number of occurrences of that number of photons exiting the cavity during each subsequent 50 μs measurement time. The top panel shows the optical pumping for a sample with no absorbed signal photons; the bottom shows the pumping of a sample that has absorbed a few photons on so it starts out with several different blocking factors.

### 4.5 Summary

In this chapter, I demonstrated that we are able to make blocking type interactions using signal photons in the form of slow light polaritons to block the cavity. These measurements demonstrate not only that single photons can block others with a probability of about 75% but also show that these interactions can occur without destroying the stored signal light, a coherent regime of interactions that suggest the use of similar interactions for coherent manipulation, preparation, and measurement of optical states.

In a related experiment, atomic saturation was used to distinguish between 0, 1, 2 and 3 or more atoms resonant with the cavity.
Chapter 5

Cross-intensity modulation

In the last chapter, an atom-cavity system induced strong anticorrelations in initially uncorrelated light. However, this output light itself was still classical; the pulses were recovered at different times. A system operating with slow light instead of stopped light should exhibit the same blocking interactions, while enabling asynchronous switching behavior. The anticorrelations induced by interaction may enable continuous entanglement of two light beams for measurements below the standard quantum limit [19, 51] and continuous, nondestructive detection of optical photons [77].

In this chapter, I demonstrate cross-intensity modulation at the individual photon level between two continuous beams of light with different frequencies. The originally uncorrelated beams become anti-correlated through their interactions, which I quantify through average signals and the equal-time correlation function $g^{(2)}(0) = 0.41 \pm 0.07$, which is the hallmark of continuous photon switching. The cross-correlation function is asymmetric with time separation $\tau$, revealing that either a signal photon or a cavity photon can switch the other within a timescale that describes the information decay of that mode. Moreover, the measured function underscores the observation made in the previous chapter that signal photons are not always destroyed in the interaction.

This work is summarized in the publication:
and discussed in Chap. 5 of Wenlan Chen's thesis [11].

5.1 Experimental setup

The essential features of the experimental setup (Fig. 4-1) are the same as the all-optical switch experiment presented in Chapter 4. Atoms prepared in the geometric overlap of the cavity and signal beams are optically pumped into state $|a\rangle = |S_{1/2}; F = 3, m_F = 3\rangle$ along the quantization axis introduced by a 12 G field. The atoms are interrogated by two independent lasers that drive the signal ($|a\rangle \rightarrow |b\rangle = |P_{3/2}; 4, 4\rangle$) transition and the cavity ($|c\rangle = |S_{1/2}; 4, 4\rangle \rightarrow |d\rangle = |P_{3/2}; 5, 5\rangle$) transitions, transitions that are joined by a classical control laser that couples $|c\rangle \rightarrow |b\rangle$.

The main difference in the experimental setup is that the signal photons propagate through the atoms as EIT polaritons during the experiment. The simplified picture of atom-cavity interactions used to understand the cavity transmission spectra needs to be modified to take into account not only for how cavity light is changed by atoms, but also how cavity light affects the propagation of signal light and the timescale of these interactions. In a typical experiment, 20 $\mu$s interrogation periods were interleaved with 10 $\mu$s repumping periods to ensure that the atomic preparation was maintained during the experiment. These interrogation periods were about an order of magnitude longer than the longest interaction timescales in the experiments.

We use approximately $2 \times 10^4$ atoms in these experiments with a typical resonant optical depth of $\mathcal{O}D = 0.9$. The atoms are spatially selected by shelving and then held in a far-off resonant dipole trap in the cavity mode.
5.2 Average modulation

When there are no signal photons, the state $|c\rangle$ is empty and the cavity is unblocked. Cavity photons sent onto the cavity are transmitted. (Our cavity has a resonant transmission of 43%, of which we detect 49%.) Similarly, when there are no cavity photons, signal photons are transmitted through the atoms as slow-light polaritons. However, when both a signal and a cavity photon is present in the system, their transmission is reduced. A signal polariton's atomic population in state $|c\rangle$ blocks cavity transmission and broadens the cavity line by introducing a new decay path: scattering photons out of the cavity mode on the $|c\rangle \rightarrow |d\rangle$ transition.

When signal photons travel through the system as EIT polaritons, there is an average population in state $|c\rangle$ that reduces the cavity's transmission by a factor of $(1+n_s\eta \cos \left(\frac{2\pi}{\lambda}\right))^2$. The transmission depends on the number of signal atoms $n_s$ and the local coupling strength $\eta \cos \left(\frac{2\pi}{\lambda}\right)^2$. The average transmission of the cavity is a weighted average over atom number and coupling strength. The atom number reflects the Poisson fluctuations of the input signal light, with an average photon number of $\langle n_s \rangle \approx R_s \left(1 - e^{-\mathcal{OD}}\right) \Gamma/\Omega^2$ where $R_s$ is the incident signal rate, $\Gamma/\Omega^2$ is the EIT lifetime, and $\mathcal{OD}$ is the optical depth. If all atomic positions (relative to the 852 nm standing wave) are equally likely, the average transmission is $\frac{1}{2\pi} \int_0^{2\pi} dx \left(1 + \eta \cos x\right)^2$. Combining these two averages, the expected transmission as a function of the average atom number is:

$$T = \sum_n T(n) P(n) = \sum_n \frac{2 + n\eta}{2(n\eta + 1)^{3/2}} e^{-(n\eta)} \langle n_c \rangle^n \frac{(n_s)^n}{n!}$$

In Fig. 5-1a, I show the transmission of the cavity as a function of the average number of photons sent into the system per EIT lifetime.

The same scattering adds loss for the polariton, as well, reducing the transmission of the signal path. The transmission on EIT resonance is $T_s = \exp \left[-\mathcal{OD}/(1 + \Omega^2/(\Gamma \gamma))\right]$ where $\gamma$ is the decoherence between the two ground states $|a\rangle$ and $|c\rangle$. This decoherence consists of two components: the bare system decoherence, which is dominated by the thermal Doppler broadening, and the decoherence induced by cavity light cou-
Figure 5-1: **Average cavity transmission with signal photon number.** (a) The fractional cavity transmission versus the average signal photon number per EIT lifetime $\langle n_s \rangle = R_s \Gamma / \Omega^2$. The expected transmission (solid curve) takes into account the calculated cavity blocking with the coherent state's Poisson photon number distribution. The transmission was measured with an average cavity photon number of $\langle n_c \rangle|_{n_s=0} = 2.7$ and an EIT lifetime $\Omega^2 / \Gamma = 1.4(1) \mu s^{-1}$. (b) Cavity transmission reduction (red curve) due to the presence of signal photons as seen in a scan of the cavity transmission spectra. Cavity transmission without signal photons (black curve) is shown for comparison.

I characterize the thermal broadening by a Gaussian profile of full width at half maximum (FWHM) $2\gamma_d$. The cavity-induced decoherence for $n_c$ cavity photons is approximately $\gamma_c = 4n_c g^2 / \Gamma = n_c \eta \kappa$. The number of cavity photons is reduced by the number of signal atoms $n_s$ by cavity blocking, so that the cavity photon number is $\langle n_c \rangle|_{n_s=0} / (1 + \bar{n})^2$ with $\bar{n} = n / (1 + \Omega^2 / (\kappa \Gamma))$ in the limit of $\mathcal{OD} > 1$. The resulting total decoherence is then the convolution of the thermal (Gaussian) and cavity-induced decoherence (Lorentzian) which gives a Voigt profile, or approximately $\gamma = 0.535\gamma_c + 0.217\gamma_c^2 + \gamma_d^2$. The extracted decoherence from EIT spectra are depicted in Fig. 5-2b and fit to this function (solid line). The fit agrees well with the data and gives $\gamma_d / (2\pi) = 110 \pm 10$ kHz for the Doppler broadening and a cavity-induced decoherence per photon of $\gamma_c / (2\pi n_c) = 70 \pm 25$ kHz. This value agrees with the expected value of $\gamma_c / (2\pi n_c) = 71 \pm 8$ kHz. As for the cavity transmission, this
5.3 Cross modulation between slow light and cavity light

Unlike the transistor experiment in the last chapter where signal light is stopped, the dynamics of the system are time dependent because of fluctuations in the photon number in the cavity and the number of polaritons in the system. The average measurements (above) show that both the signal and cavity light attenuate each other in a way that we can predict with cavity QED calculations. With fewer photons \((\langle n_c \rangle, \langle n_s \rangle < 1)\), the \(N\)-type atomic structure mediates an effective interaction between the signal and cavity modes, and photon number fluctuations change the transmission properties of the system. I measure this by the second-order cross-correlation function \(g^{(2)}_{sc}(\tau)\) where \(\tau = t_s - t_c\) is the time separation between the signal and cavity photons. At \(\tau = 0\), the correlation function can be interpreted as the modified...
transmission of one mode conditioned on the presence of a photon in the other mode: \( \langle n_c n_s \rangle / \langle n_c \rangle \) normalized to the average photon number in the signal \( \langle n_s \rangle \).

The expected functional\(^1\) form is:

\[
g^{(2)}(\tau) = 1 - \left( 1 - e^{-\sigma_{\text{D}}/(2\chi)} \frac{\eta}{1 + \eta} e^{-\kappa_< |\tau|/2} \right)^2
\]

(5.2)

For \( \tau < 0 \), \( \kappa_<= \kappa \) is the cavity linewidth. For \( \tau > 0 \), \( \kappa_> = \Omega^2 / \Gamma + \gamma_d \) is the EIT linewidth including the thermal decoherence \( \gamma_d \). The correction factor

\[
\chi = \frac{\kappa_>}{\kappa_> - \gamma} \left[ 1 + \frac{\kappa_>}{\kappa_< (1 + \eta)} \right]
\]

(5.3)

consists of factor that arises from imperfect EIT and one that accounts for the difference between the full (broadened) cavity lifetime in the presence of an atom in state \( |c \rangle \) and the polariton lifetime: when the polariton lifetime is shorter than the cavity lifetime \( (\kappa_> > \kappa_<) \), the interaction time and the cross modulation is reduced.

This correlation function is asymmetric in the time separation \( \tau \) between photon arrivals. The detection of a cavity photon at time \( t = 0 \) implies that the EIT transmission must have been reduced for times \( t < 0 \) on a time scale on the order of the cavity lifetime \( \kappa^{-1} \) and will approach its uncorrelated steady state value \( g^{(2)}_{bc}(\infty) = 1 \) for times \( t > 0 \) with a time constant determined by the polariton time lifetime \( \kappa_>^{-1} \). A similar argument can be made if we assume, instead, that a signal photon is detected at \( t = 0 \).

Fig. 5-3 shows measured cross-correlation functions \( g^{(2)}_{bc}(\tau) \) that match the theoretical prediction. The signal and cavity output light are uncorrelated at large time separations but have a marked anticorrelation dip near \( \tau = 0 \). The data in Fig. 5-3a-c are taken in the experimental setup described in this chapter; Fig. 5-3d was measured in the setup used for photon detection in the next chapter.

The data are well described by the theoretical prediction. Each correlation function is fit by a double exponential with an offset at \( \tau = 0 \). The time constants from

\(^1\)Note that this is different than the published version, which squares the entire left hand side instead. The difference comes from how we average the amplitudes to extract \( g^{(2)}_{bc} \) with decoherence.
Figure 5-3: **Cross-intensity modulation.** The second-order cross-correlation function $g^{(2)}(\tau)$ versus the time separation $\tau = t_s - t_c$ for different coupling beam powers. $\Omega^2/\Gamma$ = (a) $0.5 \pm 0.1 \mu s^{-1}$, (b) $1.4 \pm 0.1 \mu s^{-1}$, (c) $4.0 \pm 0.1 \mu s^{-1}$, and (d) $1.3 \pm 0.1 \mu s^{-1}$. The solid lines are fits of the data to the model presented in the text. (a)-(c) were measured with the experimental setup and level diagram described in this chapter. (d) was measured when I revisited this cross-modulation with the optical trap, quantization axis, and intermediate level $|b\rangle$ used for photon detection (Chapter 6). The measurements were performed at photon numbers $\langle n_c \rangle \simeq 0.2$ and $\langle n_a \rangle \simeq 0.2$. 
Figure 5-4: **Decay constants with EIT group velocity.** Fitted rate constants $\kappa_>$ from the cross-correlation functions plotted in Fig. 5-3 versus the EIT linewidth $\Omega^2/\Gamma$. The solid lines are linear fits to the extracted constants. The positive rate constant $\kappa_>$ has a slope of $0.7 \pm 0.1$ and a $y$-intercept of $1.1 \pm 0.2 \, \mu s^{-1}$. The negative rate constant is independent of EIT linewidth and fits to an average value of $1.6 \pm 0.1 \, \mu s^{-1}$. The blue circles highlight the rate constants measured under different conditions (Fig. 5-3d) and are not included in the fits.

The fit confirm the asymmetric shape of the cross-correlation function as shown in Fig. 5-4.

### 5.4 Summary

In this chapter, I have shown that we can introduce anticorrelations into two originally uncorrelated modes of light by studying the output correlations of the light. These output correlations show that there is a single photon interaction induced by the energy level diagram in our system.
Chapter 6

A (mostly) nondestructive photon detector

In 1999, Nogues, et al. reported that they could measure 'a single photon without destroying it' [69]. This experiment used the dispersive interaction between a microwave photon’s electric field and the large dipole moment of a Rydberg atom to measure a microwave photon bound to a cavity without destroying it. This nondestructive measurement enables the detection of the microwave photon with sufficiently little measurement-induced loss so that subsequent measurements give the same result. This particular measurement is actually a quantum non-demolition measurement, a measurement that not only gives the same value on subsequent measurements but also does not increase the variance in the variable being measured – in this case, the photon number [34].

The microwave measurement relied on a strong interaction between the intracavity field and the probe atom, as well as a small loss process introduced by the interaction that allowed for multiple measurements before the measurement-induced decoherence caused the field to decay. In the optical domain, finding a strong interaction is more difficult. Atomic cross sections scale with the transition wavelength $\propto \lambda^2$ so that the longer wavelengths of microwaves benefit from an enhancement of $(\frac{\lambda_m}{\lambda_{opt}})^2 \approx 10^7$. This weaker interaction means that, for the same amount of information gained, there is more measurement-induced loss. Additionally, while possible, high efficiency state
transfer in EIT is difficult to achieve, with the highest probabilities reaching about 90% [42]. Nonetheless, in the last two chapters, we observed that photons were not entirely destroyed in switching interactions which we quantified by the $1/e$ photon retrieval in Chapter 4. These interactions should enable nondestructive photon detection.

That said, neither the photon switch nor the cross modulation experiments constitute a good photon detector. A detector’s detection output correlates strongly with its input. In Chapter 5, I demonstrated intensity modulation with weak coherent states. In the typical interaction time (2.5 μs), the probability of having a cavity photon is $e^{-\langle n_c \rangle} = 25\%$. ‘Detecting a photon’ in this scheme is a reduction of the cavity transmission probability by our maximum modulation depth (41 ± 7%), resulting in a reduced transmission probability of 10%. Even if we collected every photon transmitted through the cavity, this small modulation is not detectable on a single shot basis. Moreover, the weak cavity probe means that the detector is ‘on’ at most a quarter of the time.

In this chapter, I present a modified scheme to detect photons using the same blocking interaction and demonstrate a partially nondestructive detector with observed conditional quantum efficiencies $QE$ between 13% and 1% for signal transmission $T$ between 2% and 35%. The maximal device nondestructive efficiency $Q = QE \cdot T = 0.5\%$.

This work is summarized in the publication:


### 6.1 Nondestructive detection

Quantum state detection is at the heart of quantum mechanics and has profound implications for quantum information technologies. Photodetectors detect photons by converting a photon’s energy into a measurable signal, like a photoelectron in
an avalanche photodiode or increased resistance in a superconductor brought over
threshold. Nondestructive detection is possible through nonlinear interactions. Non-
destructive measurements have been realized for microwave photons with Rydberg
atoms [31, 37, 69], for microwave photons in a circuit cavity QED system using a su-
perconducting qubit [50] and for optical photons with a limited detection time using
a single atom in an optical cavity [77].

To quantify how well any detector works, we need to correlate the input to be
measured with the detection output. In describing quantum nondemolition measure-
ments, Walls and Milburn outline three criteria that can equally well be applied to
characterize nondestructive detection [100]: (1) quality, (2) state preservation (also
called the back-action evasion criterion) and (3) state preparation. While often these
are expressed in terms of continuous variables, our detection scheme requires discrete
variables, i.e. correlations between photon clicks [75].

The detection quality is described by the measured correlation $g_{ac}^{(2)}$ between the
transmitted light (measured by a second detector) and the detection signal combined
with the transmission probability for signal light. The conditional signal autocorre-
lation function $g_{ss|c}^{(2)}$, that is the autocorrelation of the signal when detection photon
is measured, describes the state preparation. A detection will reduce the 0-photon
component of the stored signal light, so we expect $g_{ss|c}^{(2)} < 1$.

One criticism that has been raised about nondestructive (and particularly quan-
tum nondemolition) measurement is that this measurement could be replicated
by a destructive detector and a gated source [66]. However, this does not account for
measurements that are intentionally agnostic to a non-conjugate variable, such as
measuring the photon number and delaying the measurement of the polarization.

6.2 Experimental setup

The experimental setup is shown in cartoon form in Fig. 6-1. An ensemble of cesium
atoms is prepared in a tightly-focused 937 nm dipole trap transverse to the cavity
mode so that the atoms partially overlap with the cavity mode. Atoms are prepared
Figure 6-1: **Experimental setup.** (a) Cartoon of experimental setup and (b) energy level diagram. \(\pi\)-polarized signal light resonant with the \(|a\rangle = |6S_{1/2}; F = 3, m_F = 3\rangle \rightarrow |b\rangle = |6P_{3/2}; 3, 3\rangle\) travels through the ensemble with a slow group velocity due to EIT induced by the linearly-polarized control beam resonant with the \(|b\rangle \rightarrow |c\rangle = |6S_{1/2}; 4, 4\rangle\) transition. (The \(\sigma^+\) only couples off-resonantly to the atoms on the \(|c\rangle \rightarrow |d\rangle = |6P_{3/2}; 5, 5\rangle\), and does not affect the EIT.) Linearly-polarized cavity light is sent into the system resonant with the optical cavity and the \(|c\rangle \rightarrow |d\rangle\) transition. The \(\sigma^+\) component interacts with the atoms, while the \(\sigma^-\) serves as a frequency reference. The cavity output light is detected on a polarizing beamsplitter. The polarization analysis basis is tweaked by waveplates that precede the beamsplitter (not shown) to correct for birefringence induced by atoms and the cavity.
in state $|a\rangle$. Signal light is sent in on the $|a\rangle \rightarrow |b\rangle$ transition under EIT conditions
induced by a control beam on the $|c\rangle \rightarrow |b\rangle$ transition that enters the atoms at
a 45 deg angle to the cavity axis. Polaritons have an atomic component in state
$|c\rangle$ that strongly couples to the optical cavity with $\sigma^+$-polarized light on the $|c\rangle \rightarrow
|d\rangle$ transition. To generate a useful detection signal in transmission, we add $\sigma^-$
reference light and probe the cavity continuously with horizontally polarized light.
The reference light weakly interacts with the atoms: the atomic coupling strength
on the $|c\rangle \rightarrow |6P_{3/2};5,3\rangle$ transition is 45 times smaller than the strength of the $\sigma^+$
transition and is also detuned from resonance by $\Delta/(2\pi) = 6$ MHz by a 5.2 G magnetic
field along the cavity axis. Light is transmitted through the cavity and analyzed by
a polarizing beam splitter in the horizontal/vertical basis.

Using the small but optically dense ensemble of atoms in the sidetrap was essential
for this experiment. Since the atomic ensemble is polarized in state $|a\rangle$, the cavity
transmission for $\sigma^+$ and $\sigma^-$ light is shifted by different amounts by the dispersive
interaction with the ensemble 9.2 GHz away in frequency space. With the side trap,
all of the atoms that couple to the cavity also couple to the signal probe, giving a
much smaller number of atoms in the cavity mode than in a cavity trap. The total
shift is less than a cavity linewidth (about 80 kHz), and the differential shift between
the $\sigma^+$ and $\sigma^-$ light is small enough to get close to full transmission on both.

6.3 Nondestructive detection

In Fig. 6-1a, I show how we can conceptualize our system as two different detection
systems for the signal light: a nondestructive detection by the cavity followed by
a destructive single photon detector. The cross correlation function can then be
understood as the likelihood of measuring the signal photon twice. In Fig. 6-2, I plot
one such cross-correlation function with zero-time value $g^{(2)}_{sc}(0) = 4.0 \pm 0.3$. This
demonstrates that simultaneous nondestructive and destructive measurements of the
signal photon occur four times more often than by random chance, which would give
$g^{(2)} = 1$. (This agrees with the theoretical expectation which comes from a careful
Figure 6-2: Correlation function for nondestructive photon detection. Example signal-detection correlation functions plotted as a function of separation time $\tau = t_s - t_c$. (a) Single-pass correlation function. The decay time constant for negative (positive) times is $\tau_- = 1.2(2) \mu$s ($\tau_+ = 1.3(2) \mu$s) is consistent with the cavity (EIT) lifetime. This measurement is done with mean input cavity photon number $3.7$. (b) Interpretation of correlation function with detection and background areas marked.

consideration of rates, cooperativity $\eta = 4.3$, the optical depth in the overlap region $OD = 3$ and the measured anticorrelation in Fig. 5-3d [43]).

This system can also be understood in terms of its average response. In Fig. 6-3, I plot the cavity output rate in the detection port normalized to the total output power without signal photons as a function of the average input signal photon number per EIT lifetime $\langle n_{in}^{in} \rangle = R_{in}^{in} \tau_{EIT}/q_s$ where the input rate corrected for detection efficiencies is $\langle \rangle = R_{in}^{in}/q_s$. The system’s output is described by the truth table:

<table>
<thead>
<tr>
<th>stored signal photon number</th>
<th>$\sigma^+$ cavity transmission</th>
<th>normalized detection port</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$1/(1 + \eta)^2 \eta^\rightarrow\infty$ 0</td>
<td>$1/4$</td>
</tr>
</tbody>
</table>

I plot this ideal normalized rate in Fig. 6-3 as a dashed blue line. This ideal value is achieved for infinite cooperativity $\eta$ and optical depth. Even with our finite system parameters, we measure a slope of $0.20 \pm 0.01$, which represents the detection probability per input cavity photon and includes both nondestructive and destructive detection of the signal photon.

The efficiency of detecting a photon twice – once nondestructively and once destructively – is given by the area under the $g^{(2)}$ curve (Fig. 6-2b) times the cavity
Figure 6-3: **Average detection signal.** The detection rate is plotted against the input signal photon number. The detection rate is normalized to the input cavity power, so that the expected normalized rate is 0.25 with one signal polariton. The dashed line represents the maximum possible rate assuming that all $\sigma^+$ photons are blocked. The data (black circles) show the measured normalized rates. The data fit to a line with a slope of $0.20 \pm 0.01$ and an offset of $0.004 \pm 0.001$ that agrees with the measured background.

... probe rate (a rate of how likely these measurements are attempted). Call this figure of merit, $Q = R_p \int [g^{(2)}(\tau) - 1] \, d\tau$, the conditional nondestructive detection efficiency. I plot this in Fig. 6-4a as a function of the cavity photon number. $Q$ is linear with input photon rate and also increases as the lifetime of the polariton is increased (smaller $\Omega$), increasing the interaction probability with the cavity as the polariton has a larger atomic component.

However, the signal transmission decreases with increasing cavity power and increased polariton lifetime, plotted in Fig. 6-4b. There are two features of note here. The first is that the conditional transmission even at low cavity rate is considerably reduced. This is due in part to the increased losses the polariton experiences when it interacts with the cavity. Scattering off the atoms give the environment information about the location of the atoms, changing the polariton into a single decohered atom. A second, more subtle, reason for this loss is something we have called polariton localization. The cavity measurement of a polariton tells us not only that an atom was in the cavity, but that that atom was likely highly coupled to the cavity. This additional information about the atom's position in the cavity changes the spatial distribution of the polariton.
Clearly, there is a tradeoff between high signal transmission (and thus more non-destructive detection) and high conditional detection probability \( Q \). When we plot \( Q \) versus \( T \) (Fig. 6-4c), the data collapse onto a single curve independent of the polariton lifetime.

The signal transmission and conditional nondestructive detection efficiency combine to give the device quantum efficiency \( Q = QE \times T \). I plot this along with the probabilities of different detection outcomes in Fig. 6-5 where \( P_{sc} \) are the probabilities to detect 0,1 signal and 0,1 cavity photons when there is one signal input photon. These probabilities can be obtained from measured quantities in the weak signal input limit:

\[
\begin{align*}
QE &= P_{11} / (P_{11} + P_{10}) \\
T &= P_{11} + P_{10} \\
\langle n_c \rangle &= P_{01} + P_{11} \\
\sum P_{sc} &= 1
\end{align*}
\]  

(6.1) (6.2) (6.3) (6.4)

Note that \( P_{11} = Q \).

### 6.4 Nonclassical state production

All the experiments to this point have studied the cross correlations between two different modes of light. It is straightforward to show that classical switching can lead to anticorrelations (or correlations) like the ones we observe without resorting to quantum effects. The figure of merit for quantum correlations in two modes is instead the normalized cross correlation \( G \), which compares the correlation function between the two modes to the correlation functions of the single modes (see Chapter 1). If \( G > 1 \), the light is nonclassical. When we run the experiment in a slightly modified way, sending our light through the atoms twice (Fig. 6-6) to improve our interaction probability, we measure a nonclassical \( G = 2.7 \pm 0.5 > 1 \) which comes from zero-time correlation measurements of \( g^{(2)}_{sc}(0) = 4.4 \pm 0.5 \), \( g^{(2)}_{ss}(0) = 1.6 \pm 0.3 \) and \( g^{(2)}_{cc}(0) = \)
Figure 6-4: **Quantifying conditional nondestructive detection.** (a) The observed conditional nondestructive quantum efficiency $Q$ is plotted against mean cavity photon number $\langle n_c \rangle_{n_s=0}$. The slope of the fitted curves (solid lines) is $dQ/d\langle n_c \rangle_{n_s=0} = \{10(2), 5(1), 1.9(5)\}$% for $\Omega/(2\pi) = \{1.8, 2.9, 3.5\}$ MHz (top to bottom) and represents the observed detection efficiency per input cavity photon. (b) Signal transmission $T_s$ for the same data presented in (a). Exponential fits give $1/e$ transmission at cavity photon numbers of $\{1.2(1), 1.9(1), 2.1(1)\}$ (bottom to top). (c) Redisplays the data shown in (a) and (b) together to plot the nondestructive quantum efficiency $Q$ as a function of signal transmission.
Figure 6-5: Device nondestructive detection and error probability. (a) The four characterizing probabilities $P_d$ with $s, p = \{0,1\}$. (b) zooms in on the device nondestructive detection ($P_{11} = Q = QE \times T_s$) and the error probability $P_{err}$ for joint detection of signal and probe outputs. $P_{err}$ represents the false detection of probe photons in the absence of signal photons.
Figure 6-6: **Setup to determine classicality of nondestructive detector.** To determine the classicality of the output states of the nondestructive detector, we changed the setup so that we could measure the correlation function on each path. Input signal light was sent into the system through the weakly coupled port of a 90-10 fiber beam splitter. The signal light was sent through the atoms twice by reflecting them off a mirror to enhance the effective optical depth. Retro-reflected light was coupled into the original fiber and measured after a 50-50 fiber beam splitter in the 90% coupled port of the input beam splitter.

This confirms the system’s output state is nonclassical.

Our attempt to directly measure the production of 1-photon number states in this setup, which would test the state preparation criterion for nondestructive measurement, failed due to insufficient statistics.

### 6.5 Summary

I demonstrated that we can measure photons without destroying them. There is a trade-off between the how well we detect the photons and how likely it is for the light to be transmitted through the system.
Chapter 7

Single-photon cross-phase modulation

Deterministic quantum logic requires nonlinear interactions that modify the phase of an optical state by \( \pi \) by interacting with one photon. This large single-photon phase shift is an important step towards constructing a quantum gate [17]. Phase shifts naturally arise in off-resonant interactions. Additionally, these interactions are less lossy than on-resonant interactions. As we increase the atom-light detuning, scattering losses \( (\propto 1/\Delta^2) \) decrease more quickly than the phase shift \( (\propto 1/\Delta) \). These interactions will lead to less transmission reduction of the signal state than we had in the blocking interactions used in the experiments I have described thus far which occur on atomic resonance.

Cavity QED first was used to look at this sort of interaction in a seminal paper by Turchette et al. [95] in which the authors show that an atom in a cavity produces a phase shift. Their measured average phase shift is well explained by a single-photon cross phase modulation of 0.28 rad. A recent paper reporting an experiment in a free space EIT system [24] measured an 18 \( \mu \)rad single-photon phase shift by reconstructing the phase after postselecting just the measurements in which two photons were measured leaving the system, instead of relying on calibration to correct for cases when there was only one photon present, and thus no expected phase shift. A preprint for a similar experiment using Rydberg EIT posted shortly after we posted our manuscript reporting a phase shift of \( \pi \) [93].

Despite these advances, locality and causality limit the fidelity of large cross phase
modulation interactions for continuous beams [83, 30]. In this experiment, we overcome this theoretical limitation by storing light as stopped-light polaritons in the atoms during the cavity interrogation time.

In this chapter, I demonstrate a large conditional cross-phase modulation of $\pi/6$ by one photon on another and entanglement between the output modes demonstrated by a positive concurrence, $C = 0.08 \pm 0.02$. By postselecting on photons that interact for longer times than average, the measured single-photon cross-phase modulation increases to $\pi/3$. Our work on cross-phase modulation is summarized in the preprint:


### 7.1 Experimental setup

The setup for the experiment is shown in Fig. 7-1. Similar to the transistor experiment, signal light is stored in the atoms by changing the amplitude of the control and signal beam (Fig. 7-2a). The signal beam is sent into the atoms perpendicular to the cavity axis. The control beam copropagates with the signal beam which reduces Doppler decoherence for the stored light. In each experimental attempt, the cavity light was on during both the storage and subsequent retrieval to keep the bandwidth of the photons well matched to the cavity's linewidth.

To measure the phase of the retrieved light, a reference pulse was sent into the signal detector along with the retrieved light. This reference light was detuned by 30 MHz from the signal and shaped to match the amplitude of the retrieved light. (The reference light was also pulsed on during light storage to enable checks on the phase.) This detuning was chosen to have minimal absorption by the atoms while simultaneously making a beatnote signal within the detection system's 2 ns resolution. Control light was blocked from the signal detector by a 620 MHz linewidth temperature-tuned etalon and slight beam misalignment. In each experimental cycle, the conditional phase, phase reference (without cavity light) and signal recovery (without reference light) were each measured 410 times by repeating these three mea-
Figure 7-1: **Setup for cross-phase modulation experiments** \( \pi \)-polarized signal photons on the \(|a\rangle = |S_{1/2}; F = 3, m_F = 3\rangle \rightarrow |b\rangle = |P_{3/2}; 3, 3\rangle\) transition are stored in the atomic ensemble by adiabatically turning off the quasi-copropagating control beam on the \(|b\rangle \rightarrow |c\rangle = |S_{1/2}; 4, 4\rangle\) transition. Light is continuously applied to the cavity during the storage and retrieval time on the \(|c\rangle \rightarrow |d\rangle = |P_{3/2}; 5, 5\rangle\) transition.

The signal and reference beams copropagated for all but approximately 20 cm of path length. There, the beams were sent through separate AOMs driven at 80 MHz and 110 MHz, respectively, derived from two phase-locked channels of a single DDS frequency generator board (EVAL-AD9959, a four-channel DDS board from Analog Devices). These AOMs not only provide the relative frequency shift of 30 MHz but also allowed for independent control of the power of each beam. The beams were then recombined on a non-polarizing beamsplitter with the same polarization and sent through a single fiber to the experimental setup. These path lengths were phase stable over the experimental acquisition time (about 50 ms). However, they were not stable over the 1 s atom preparation cycle time due to timing stabilization in the sequence generator and thermal drifts. The phase between the signal and reference was measured each cycle and corrected for in the analysis.

To reconstruct the phase, the photon arrival times in the recovery window are binned by their phase in the 33 ns beatnote period and aggregated into a single
Figure 7-2: **Timing for cross-phase modulation experiments.** (a) Timing for continuous measurements. (b) In each experimental cycle, we measure the phase without the cavity, the signal recovery, and the conditional phase. This 30 μs set of measurements is repeated 410 times per experimental cycle. (c) Timing for measuring phase as a function of cavity interaction time.
Figure 7-3: Typical reconstructed beatnote curve. Histograms of signal photon arrival times as a function of the signal-reference phase. Reference (without cavity photons, black) and conditional (conditioned on detecting on cavity photon, red) histograms are fit with sine waves to extract the phase difference. These curves were measured for a detuning $\Delta/2\pi = 8$ MHz. The conditional curve was reconstructed from 885 photons.

histogram to fit the phase. (Since the beatnote period was is not evenly divided by our detection system’s 2 ns resolution, we were actually able to slightly oversample taking advantage of the aliasing between our detection system clock and the phase of the heterodyne signal.) A typical reconstructed beatnote curve is shown in Fig. 7-3.

7.2 Cross-phase modulation

Fig. 7-4a shows the measured conditional phase shift as a function of the detuning $\Delta$ between the $|c\rangle \rightarrow |d\rangle$ transition in the atoms and the input cavity light. A photon in the cavity imparts an AC stark shift of $\delta = \eta \frac{\kappa_0}{2} \frac{2\Delta/\Gamma}{1+(2\Delta/\Gamma)^2}$ on the atom, which changes the energy different between the atomic states $|a\rangle$ and $|c\rangle$ in the polariton. The phase shift is then $\phi = \delta/\kappa$, where $\kappa = \kappa_0 \left[ 1 + \eta/(1 + (2\Delta/\Gamma)^2) \right]$ is the cavity lifetime in the presence of one polariton (one atom’s worth of population in state $|c\rangle$). We measure a conditional single-photon phase shift of $|\phi| = 0.4 \pm 0.1$ rad at $|\Delta/2\pi = 8$ MHz$^1$.

The interaction with light must take into account the multimode nature of the

---

$^1$This result agrees well with 18 $\mu$rad. shift Feizpour et al. [24] measure in free space and the simple model that the cavity-light interaction is increased by the number of times the cavity photon interacts with the signal photon, $F/\pi$, which would give us a predicted phase shift of 0.4 rad. If we reduce the number of round trips to account for the increased cavity linewidth in the presence of the signal polariton, this estimate is reduced to 0.08 rad.
Figure 7-4: **Conditional phase shift of signal light.** (a) The phase shift of the stored signal light conditioned on the detection of a cavity photon is plotted versus the atom-cavity detuning $\Delta$. The dashed line is the prediction for one photon; the solid line accounts for the effect of multiple photons in the cavity’s coherent state. (b) The phase shift of the cavity photon conditioned on the detection of a recovered signal photon. The dashed line assumes $\delta_c = 0$; the difference between the data and the prediction are well explained by including a light-cavity detuning of $\delta_c = 25$ kHz.

atom-cavity interaction. When we do this (as described in the supplemental to [3]), the resulting phase of the output light is given by

$$\phi = \arctan \left( \frac{2\delta_c}{\kappa} + \frac{\eta}{2} + \frac{2\Delta/\Gamma}{2 + (2\Delta/\Gamma)^2} \right) - \arctan \left( \frac{2\delta_c}{\kappa_0} \right)$$

(7.1)

for interaction with one photon. If the detuning of the input light from the cavity light is $\delta_c = 0$, this reduces to $\phi = \arctan \left( \frac{\eta}{2} \frac{2\Delta/\Gamma}{2 + (2\Delta/\Gamma)^2} \right)$ and is well approximated by $\phi = \frac{\eta}{2} \frac{2\Delta/\Gamma}{2 + (2\Delta/\Gamma)^2}$ for small $\eta$ and/or large $\Delta$, as we see in the intuitive description via the AC stark shift.

The dashed line in Fig. 7-4a is the above prediction, which is calculated for one cavity photon. The solid line sums over the higher photon number components in the cavity’s coherent state.

The cavity photon’s phase is also changed by its interaction with the system. On the cavity path, we measure phase shift by polarization rotation\(^2\). Linearly polarized

\(^2\)The $\sigma^-$-polarized phase reference was used after trying to use a detuned reference as we had for the signal path. Since the detuned light was blocked by the cavity, we had to send the detuned light onto the cavity detector on the other side of the vacuum chamber. The larger path length difference resulted in phase drifts that were faster than our acquisition time, and stabilizing them would have required active stabilization.
light is sent into the system. To good approximation (as described in Chapter 6), the \( \sigma^- \)-polarized component does not interact and serves as a reference. Phase shifts on the \( \sigma^+ \)-polarized component changes the ellipticity of the output light. We measure this change on a polarizing beamsplitter.

The cavity phase is plotted as a function of detuning in Fig. 7-4. The dashed line shows the prediction for all photon number components; the solid line additionally includes an average cavity-light detuning of \( \delta_c = 25 \text{ kHz} \). This detuning is reasonable as we maximized the cavity transmission at the end of our experimental cycle. The atom number decreases during the cycle, resulting in a change in the cavity resonant frequency.

The effect of multiple photons is explored in Fig. 7-5, which shows the conditional and average phase shifts as a function of the average cavity photon number at a \( \Delta/2\pi = -8 \text{ MHz} \). The average phase shift is linear in cavity photon number, the mean number of input cavity photons in the 2 \( \mu \text{s} \) conditioning window. It fits to a line with a slope of 0.43(1) rad/photon. This fit slope is used to calculate the zero-parameter prediction for the conditional phase. The slope of this prediction is lower than the slope fitted for the average phase because conditioning removes events where more than one cavity photon is detected. The prediction does not include backgrounds, which reduce the measured conditional phase at low cavity photon number.

### 7.3 Entanglement production

The cross-phase modulation introduced by our atomic mediator should give rise to an entangled state:

\[
|\Psi\rangle = p_{00}|0_s0_c\rangle + p_{10}|1_s0_c\rangle + p_{01}|0_s1_c\rangle + p_{11}e^{i\theta}|1_s1_c\rangle
\]  

(7.2)

where \( 0_s \) (\( 1_s \)) represents zero (one) signal photons and \( 0_c \) (\( 1_c \)) represents one \( \sigma^- \) (\( \sigma^+ \)) cavity photon, \( p_{ij} \) is the probability amplitude, and \( \theta \) is the nonlinear interaction
Figure 7-5: **Conditional phase shift of signal light with different cavity photon numbers.** The black data points are the average phase; in red is the conditional phase shift. The average phase is fit to a line (black line), whose fit is used as input to the zero-parameter prediction for the conditional phase shift (red line).

phase, which ideally should be equal to the conditional phase shift measured for the signal ($\phi$) and for the cavity ($\Psi$).

We did state tomography measuring the output state in 16 separate measurements that compared the phase and amplitude of the signal and cavity states (see supplemental to [3]) and then used maximal likelihood reconstruction [49] to extract the density matrix for the joint state. We used this reconstructed density matrix to the concurrence. The concurrence is a measure of entanglement [101]; when the concurrence is positive, the state has entanglement. For our state, we calculate a concurrence of $0.08(2) > 0$ and a nonlinear phase shift of $\theta = 0.45(2)$ rad after correcting for detection efficiencies and propagation losses.

### 7.4 Conditional decoherence

Accompanying the phase shift are changes in the system decoherence. In Fig. 7-6a, I show the signal transmission as a function of the atom-light detuning $\Delta$. The signal transmission is reduced by photon scattering into free space, shown as the solid line. Additional decoherence (not incorporated into this curve) arises due to mode changes from polariton localization which are responsible for the additional loss between the measured conditional transmission and the prediction.
Figure 7-6: (a) **Conditional signal transmission.** The signal transmission conditioned on detecting a cavity photon (red circles), normalized to the transmission without cavity light, is shown as a function of atom-light detuning $\Delta$. The solid line is the predicted signal transmission reduction due to scattering photons out of the cavity mode. (b) **Conditional cavity transmission.** Measured cavity blocking of $\sigma^+$ light normalized to the transmission of $\sigma^-$ light.

The cavity transmission is also reduced. Population in state $|c\rangle$ reduces cavity transmission for $\sigma^+$-polarized light close to atomic resonance. In Fig. 7-6b, I show the blocking of the $\sigma^+$ component normalized to the output $\sigma^-$ light. This normalization accounts for power level drifts and reduced transmission for small variations in light-cavity detuning.

The fidelity of cross-phase modulation is limited for traveling photons [83, 30]. To investigate these effects in our system, we measured the contrast of the output phase (that is, the contrast of the reconstructed histograms) and normalized out the contrast reduction due to the separately-measured transmission, which is a separate loss mechanism than the localization-related decoherence discussed in these papers. (This also does not correct for uncertainty in the interaction time for our cavity photons due to our limited storage lifetime.) In Fig. 7-7, I plot the visibility as a function of atom-light detuning $\Delta$. The visibility has an average value of $0.9 \pm 0.1$ and appears to be independent of $\Delta$.

The lifetime of the cavity photon, $1/\kappa_0$, decreases in the presence of an atomic excitation to $1/\kappa$ because the atomic excitation opens up a new decay path [86]. In Fig. 7-8, I show the cavity decay time conditioned on detecting a stored signal photon.
Figure 7-7: **Signal visibility.** The fringe visibility of the recovered signal light after correction for the transmission loss plotted in Fig. 7-6a. The black line shows perfect fringe visibility (1) for comparison.

![Signal visibility graph]

Figure 7-8: **Conditional cavity linewidth** The cavity linewidth conditioned on detecting a signal photon (red and average cavity linewidth (black), normalized to the bare linewidth $\kappa_0 = 2\pi \times 150 \text{ kHz}$, measured for $(n_s) = 0.2$. The solid line is the cavity linewidth increase by the presence of one atom in state $|c\rangle$. The dashed line represents no increase in cavity linewidth.

![Conditional cavity linewidth graph]

We excite the cavity with a short (200 ns) pulse (Fig. 7-2c) and fit the conditional decay as a function of the light-cavity detuning. A single atom in state $|c\rangle$ increases the cavity linewidth by $\kappa/\kappa_0 = 1 + \frac{n}{1+(2\Delta/\Gamma)^2}$. This is plotted as the theoretical curve in Fig. 7-8. Surprisingly, the decoherence shortens the cavity lifetime even when stored light is recovered.
Figure 7-9: **Conditional phase shift conditioned on cavity photon dwell time.**
The conditional phase shift measured as a function of the cavity photon dwell time. The solid lines model the phase as linear in interaction time; the dashed lines include the effect of multiple cavity photons and background counts.

### 7.5 Enhanced phase shift

In our model, phase shift increases with interaction time. In the short pulse excitation scheme used to measure the cavity decay, photons are sent onto the cavity at one time. The time that photons exit the cavity tells us how long they interact with the polariton. While many photons exit the cavity by the cavity lifetime $1/\kappa$, some photons stay in the cavity longer than average. In Fig. 7-9, I plot the reconstructed phase conditioned on different cavity output times for $\Delta/2\pi = \pm 8$ MHz. The observed conditional phase shift increases with the interaction time, although the increase is not linear (as shown in the dashed lines) due to false conditioning on background counts and the multiple photons present in the cavity coherent state. The largest observed phase is $1.0 \pm 0.4$ rad.

### 7.6 Single-photon beam deflection

This phase shift measurement – and for that matter all of the experiments I have talked about up to this point – are hindered by the distribution of effective cooperativities across the atoms by reducing blocking or decreasing signal transmission through polariton localization. However, in the detuned limit, this distribution can be a boon. The different phase shifts experienced by the stored light can deflect the
beam$^3$, changing the $k$-vector of the output light [52, 82].

If we hold the atoms in the cavity trap, each pancake of atoms experiences roughly the same coupling to the cavity and thus the same phase shift from a single photon. If the atoms in a range of pancakes each experience a different phase shift, the recovered light’s mode is distorted. However, when phases are arranged so that they increase as a function of position, the beam is instead deflected. The different amounts of accumulated phase turn into an overall shift of the wavevector of the outgoing light.

In our system, we might be able to see a weak effect. Our sidebeam is focused to a waist of 2 $\mu$m, and so stored light will have significant population over about 4 $\mu$m. The beatnote length between the 937 nm trap and 852 nm probe light is 4.7 $\mu$m, which we have seen in measurements of the atom-cavity coupling (see Sec. 6.4 of [11]). While we do not expect to see uniform deflection, the different parts of the stored light should experience different phases and we should see mode distortion due to the cavity interaction. As an initial test, we sent light through the atoms as EIT and continuously probed the cavity. We focused the output beam onto a camera with a cylindrical lens so that the light from the mode was focuses onto 1-2 pixels in the horizontal direction to give us the best signal to noise we could hope for in this direction. The images with and without cavity light were indistinguishable.

This deflection should show up as additional loss in the transmission. The difference between the measured loss in conditional signal transmission and the expected loss from photon scattering (Fig. 7-6) provides an upper bound for the beam deflection. The light is retrieved into a single mode fiber; any (spatially separable) deflected light would not be detected in this path.

While these tests did not give promising results, we should be able to realize photon deflection with cavity light based on the phase shift measurement and our knowledge of the atom-cavity coupling. To measure it, I would repeat the experiment with the atoms held in a different trap that has a longer beatnote with our 852nm resonant light and measure transmission loss as a function of detuning.$^4$ To calibrate

$^3$Our interest in this topic came out of a conversation with Arno Rauschenbeutel during when he visited our lab in 2014.

$^4$Before we did the phase measurements, we built up a 923 nm trapping laser hoping to take
the effect after mode distortion has been observed, send in many photons into the cavity one or two free spectral ranges away to cancel the phase. The power needed to cancel the shift would quantify the phase shift.

7.7 Summary

In this chapter, I demonstrated a cross phase modulation of up to $0.4 \pm 0.1$ rad in a possibly deterministic protocol, which resulted in two mode entangled states of light between the phase of output signal light and the polarization of output cavity light. The entanglement was demonstrated by tomographically reconstructing the output state using the maximum likelihood method and calculating the concurrence of the resulting state. I also showed that the phase shift could be increased to $1.0 \pm 0.4$ rad by postselecting on photons that happened to interact for longer than average.
Chapter 8

Arbitrary optical state preparation by measurement

The entanglement produced by our photon-photon interface can be used not only to measure light but also to manipulate its state. Producing quantum states has become a cottage industry in atomic physics, producing atomic squeezed states [44, 57] and non-Gaussian entangled states [64, 87] for enhanced metrology with atomic sensors. By storing light as EIT polaritions, these manipulations that have been done with atoms should enable similar manipulations of optical states. Interactions with microwave qubits in circuit QED have enabled the production of non-Gaussian states [79, 97] of microwave photons.

There are several schemes to prepare optical states with atoms. The most direct engineers the photon emission process to make the states through, for example, a single atom that can at most emit single photons [61] or a cavity QED system used to produce photon pairs [91]. The second method manipulates atoms while they are storing light [6]. The modified atomic state is mapped onto retrieved light. In fact, the measurement that I demonstrated with nondestructive photon detection (Chapter 6 and [3]) is a special case of state preparation: we measure the presence of an atom in the atomic state resonant with the optical cavity and therefore expect that the conditionally detected light is closer to a 1-photon state than the weak coherent state we start with.
In this chapter, I demonstrate a third method to produce an optical state with near-arbitrary phase and amplitude by measurement on another mode. The atoms are used to produce a two-mode entangled state. A projective measurement of one of the modes prepares the output state of the other mode. This method illustrates a possible way to create a large family of optical states with a single experimental knob. Peters et al. demonstrated this projective preparation using polarization-entangled photon pairs to produce polarization states that span the Poincaré sphere [72].

In contrast to a previous experiment on the same system (Chapter 7 and [43]), this phase shift comes about due to entanglement generated by the atoms, instead of a differential light shift between levels in an optical quantum memory. As such, the phase shift is probabilistic; the phase shift depends on the measurement result on the other mode. Such an interaction illustrates a new way that single photons can lead to phase shifts.

Using this method, I present measurements of an anomalously high phase shift of 1.5 rad by a single photon on a weak coherent state, and demonstrate that the phase shift can be tuned between -1.5 and 1.5 rad by changing the projective measurement. The state amplitude changes by a factor between 0.4 and 4.5.

8.1 Experimental setup

In each iteration of this experiment, we use an ensemble atoms to create a two mode entangled state of light. A weak coherent state is stored in the atoms between states $|a\rangle$ and $|c\rangle$. A linearly-polarized pulse of light is then sent onto the cavity resonant with the $|c\rangle \rightarrow |d\rangle$ transition. As in the nondestructive detector experiment (Chapter 6 and [3]), a stored signal photon partially blocks $\sigma^+$ photons, while transmitting the $\sigma^-$ light. Finally, we retrieve the stored signal light. When there is an output photon the cavity mode, we produce a two-mode entangled state of light, $|\Psi\rangle$.

Having created this two-mode entangled state, we project the cavity mode of the resulting state onto a particular polarization, which we experimentally change by tuning the angles of the half and quarter waveplates located before a polarizing
Figure 8-1: Setup for entanglement-based state preparation π-polarized signal photons on the $|a\rangle = |S_{1/2}; F = 3, m_F = 3\rangle \rightarrow |b\rangle = |P_{3/2}; 3, 3\rangle$ transition are stored in the atomic ensemble held in a far-off-resonant dipole trap transverse to the cavity axis by adiabatically turning off the quasi-copropagating control beam on the $|b\rangle \rightarrow |c\rangle = |S_{1/2}; 4, 4\rangle$ transition. Linearly-polarized light is pulsed onto the cavity during the storage time resonant with the $|c\rangle \rightarrow |d\rangle = |P_{3/2}; 5, 5\rangle$ producing a two mode entangled state in the cavity output light's polarization and the amplitude and phase of weak coherent signal state stored in the atoms. (This experimental setup is the same as the one used in Chapter. 7.)
beamsplitter after the experimental cavity. When this projection succeeds, we measure a photon click on the detector $D1$ and reconstruct the phase of the second mode by heterodyning it with a strong reference beam detuned from the signal by 30 MHz, using the same method we used for phase reconstruction in Chapter 7.

### 8.2 Theory

For sufficiently weak states, the (unnormalized) joint state of the retrieved signal light $(s)$ and transmitted cavity light $(c)$ is

$$|\Psi\rangle = |0\rangle_s \left(|\sigma^-\rangle_c + |\sigma^+\rangle_c\right) + \epsilon|1\rangle_s \left(|\sigma^-\rangle_c + T|\sigma^+\rangle_c\right) \quad (8.1)$$

where $I$ have dropped terms of order $\epsilon^2$. $T$ is the transmission probability for $\sigma^+$-polarized light in the presence of a stored signal photon.

To illustrate how the reconstructed signal light changes, consider projecting the cavity part of our two-mode entangled state $|\Psi\rangle$ onto the polarization $\frac{1}{N} (A|\sigma^-| + B|\sigma^+|)$, where $A$ and $B$ are the (possibly complex) amplitudes of the two different circular components of the cavity light and $N = \sqrt{|A|^2 + |B|^2}$ is a normalization factor for the cavity projection:

$$\frac{1}{N} (A|\sigma^-| + B|\sigma^+|) |\Psi\rangle = \frac{1}{N} (A + B)|0\rangle_s + \epsilon (A + TB)|1\rangle_s \quad (8.2)$$

$$= \frac{1}{N} (A + B) \left(|0\rangle_s + \epsilon \frac{A + TB}{A + B}|1\rangle_s\right) \quad (8.3)$$

The retrieved state has a different amplitude and phase than the original state. To see this, choose $A$ and $T$ to be real and set $B = 1$. Since $T^2$ is a probability, $0 \leq T \leq 1$. The retrieved signal state's amplitude is decreased if $A < 0$ and increased if $A > 0$. If $A < -T$, $\frac{A + T}{A + 1} < 0$ and the phase of the retrieved state is changed by $\pi$.

The highest amplitude we can measure quantified by the cross-correlation of the output light in the special case where it is operating as a nondestructive detector. The probability of detecting the state scales with the state overlap between the original
Figure 8-2: (a,b) Reconstructed phase and (c-d) amplitude of signal light post-selected on polarization projection of cavity light shown as a function of the angle of the quarter waveplate from the axis, $\theta_Q$. Curves were measured with $\theta_H$ set at (a,c) 98 deg and (b,d) 108 deg.

input state and the target state, so that states with higher phase shift and larger amplitude factors are the most difficult to produce.

### 8.3 Arbitrary state preparation

The phase we detect is shown in Fig. 8-2a-b as a function of $\theta_Q$, the angle of the quarter wave plate, for two different angles $\theta_H$ of the half wave plate. Importantly, the reconstructed phase depends on the projected polarization. Fig. 8-2b shows the relative amplitude of the retrieved light with conditioning to the unconditioned retrieved light.

Despite operating on atomic resonance where the assumption that $T$ is real should
hold, the phases we measure in Fig. 8-2a-b are smaller than what we predict from the simple model presented above. There are two primary reasons for this. First, the expected high phase region is fine-tuned in several parameters. Shot-to-shot variation in the number of atoms we load in state $|a\rangle$ changes the transmitted light’s polarization as well as changing the laser-cavity resonance frequency, which changes the effective waveplate angle between loadings, as well as during the measurement time due to the loss of atoms from the trap. Second, we do not herald the production of our entangled states, so we average in an incoherent mixture of signal with 0 phase with the signal we create by conditioning the entangled state. If the phase shift is small, this averaging primarily reduces the maximal reconstructed phase. (At close to $\pi$ phase, it instead reduces the contrast of the reconstructed sine wave, see Sec. 1.3.)

The inhomogeneous cavity coupling also reduces the maximal phase production. Averaging over different atom-cavity couplings changes the positions of the maximal phase and amplitude, which results in a beam with different phases and amplitudes as a function of the local atom-cavity coupling where the light was stored.

### 8.4 Summary

In this chapter, I demonstrated that entanglement produced between two optical modes can be used to probabilistically create optical states with amplitude and phase that can be dialed in by choosing the projection basis for one of the modes. This was used to create states with phases between -1.5 and 1.5 rad and amplitude changes by factors between 0.4 and 4.5. Using the same interaction in a system with uniform coupling [56] and better atom number stabilization would enable the production of states with near arbitrary phase and amplitude by such projective measurements.

This type of interaction can be used to produce different sorts of states. For example, if we could make use of the atom-cavity coupling in order to displace the optical state as I explored briefly in Sec. 7.7, the output polarization of the cavity could be used to determine the displacement of the output state. Here, the displacement of the beam is proportional to the cavity interactions, which to good approximation scales
with the amplitude of the $\sigma^+$ component of the light. Maximal displacement occurs for a $\sigma^+$ photon; if we instead measure a $\sigma^-$ photon, there should be no displacement. (In practice, there would be a small shift due to finite detection efficiency.)
Chapter 9

Summary and Outlook

In this thesis, I demonstrated strong photon-photon interactions mediated by an ensemble of atoms in a high finesse cavity. I explored these interactions in a series of experiments that used the interactions to realize proof-of-principle quantum devices: an all-optical switch and transistor operating with one stored photon (Chapter 4 and [12]), a cross-intensity modulator (Chapter 5 and [2]), a partially nondestructive photon detector (Chapter 6 and [43]), a cross-phase modulator (Chapter 7 and [3]), and a quantum state manipulator (Chapter 8). These experiments show that this engineered interaction enables both time-stopped and time-continuous interactions which can change photon transmission statistics or control the optical phase.

However, our current system’s fidelities are limited by the out coupling of stored light to a level where further explorations of these interactions will at the most be proof-of-principle experiments. If we were to realize a photon-photon gate in the current system, its fidelity would be limited to below the threshold necessary for quantum computation, and moreover its loss-limited performance would at best reach the same level as existing probabilistic quantum gates. This loss also kept us from realizing squeezed optical states by measurement when we tried to replicate the nondestructive detection measurement off-resonant to count the number of atoms in a larger polariton as was done with atoms in [105].

With this in mind, the most interesting experiments that the system is poised to do are either experiments that make use of photon correlations, which are not effected
by losses, or that use the cavity to prepare atomic states which do not need to be retrieved and therefore are not limited by the storage and retrieval efficiency that we have achieved in the current system.

A previously explored direction – vacuum induced transparency – used the cavity’s atom-light coupling like an EIT control field [89]. Frequency-dependent interactions with the cavity mode changed the ensemble’s transmission for a probe beam sent into the atoms transverse to the cavity. Photon-number states travel through this system with different group velocities [55, 68]. However, this was not realized experimentally due to limited optical depth and atom-cavity coupling. While the atom-cavity coupling is unchanged, the higher optical depths that we are now able to achieve may allow us to see signatures of this photon-number dependence. James Douglas and Darrick Chang have simulated a model system that suggests that if we probe this system with Gaussian pulses of light, the correlation function $g^{(2)}(0)$ will depend on the $t$ in the pulse. Additionally, since our lab initially realized vacuum induced transparency, I realized that quantum effects in the light scattered into the cavity (and detected as leakage light out of the cavity mirrors) should be more pronounced because this scattered light self-selects for strong atom-light interaction, so it is less sensitive to limited optical density and biases the interaction toward the highest coupled regions in the standing wave cavity interaction. This additional experimental trick might enable us to see this previously unrealized and counterintuitive feature of the vacuum induced transparency system.

Our experiment is also poised to realize a proposal Chen et al. made to make highly entangled atomic states using a multi-frequency cavity probe [13]. In this proposal, different frequency components couple to different collective atomic spin eigenstates (Dicke states). When a photon is detected in transmission, it heralds the production of an entangled state. McConnell et al. produced the first atomic Dicke state with a similar method [64]. However, with in the new proposal, the experimental requirements are more stringent. In particular, the input state must have a very small intensity as the input power directly limits the quality of state preparation. Doing this state preparation will be a tour de force experiment that will push the limitations
of our current experiment and force us to make new experimental improvements.

Finally, I propose two system upgrades to increase the tools available in the current experiment. First, we are limited in our ability to distinguish the polariton number by the nonuniform cavity coupling. By using a trap at twice the wavelength of the 852 nm cavity probe light (1704 nm), we could realize uniform coupling along the cavity axis as was realized by Hosten, et al. for rubidium atoms [56]. (A feasibility test would be to measure the reflectivity of the cavity mirrors at long wavelengths; they have only been characterized up to 1100 nm.) Second, in experiments that probe cavity transmission, we should also measure cavity reflection. With recent changes to the cavity probe filtering made to enable ongoing cavity cooling measurements [8, 80, 99], we could feasibly simultaneously measure the cavity’s transmission and reflection to study interesting predictions for the cavity-induced phase shift that explain the high phase shift observed by Reiserer et al. [76].
Appendix A

MATLAB analysis

I wrote a set of functions to enable customized data analysis in MATLAB. This started by a need to do analysis of only part of the data saved in a data file when we were having stability issues with our experimental cavity during my first year in the lab. In practice, the added flexibility means these functions are not as easy to use as the Labview GUI interface for Jon's compiled C scripts that we still use for data monitoring during acquisition, but they have enabled different data analyses, monitoring of correlation functions during data acquisition, postselection and, more generally, a hands-on approach to analyzing the raw data.

In this appendix, I document the core codes and approaches in my data analyses to aid reading existing code and building new code based on the same utility functions.

A.1 Utility functions and code snippets

These functions are used to load data from the photon arrival time data our Fast ComTec P7888 photon counting card, to save it into a MATLAB structure and to select the right data for subsequent analysis.
**A.1.1 Loading data**

We acquire data using proprietary software provided by Fast ComTec. The software saves data in *.lst files, which consist of a plain text header with a description of the card settings during the acquisition followed by a list of arrival time and channel data saved as 32 bit unsigned integers. I import it into MATLAB in two structure arrays using the script `readLST`: desc formats the information in the header and data is a collection of lists that describe each pulse events. This script processes the raw 32 bit numerical descriptors to get the channel number, trigger (true/false, called a start event in code to mirror Fast ComTec’s documentation), arrival time after the last trigger in nanoseconds and the number of accumulated triggers, which I call the run number (described below). It should be used in the format `[desc data]=readLST('filename')`.

```matlab
%an excerpt from readLST showing how we process the raw 32 bit numerical
%descriptions
data.raw=fread(mFile,inf,'*uint32');
data.channel=bitshift(data.raw,-31+desc.fourchannelmode);
data.evtime=bitand(data.raw, hex2dec('3fffffff'))*(1+desc.fourchannelmode);
if desc.fourchannelmode
    data.startev=(data.raw==hex2dec('C0000000'));
else
    data.startev=(data.raw==hex2dec('80000000'));
end
```

This data structure can be used directly for analysis (Sec. A.2), although we typically perform some additional cuts on the data before analysis.

**Multiple files**

To import multiple files, import the files separately and then combine the two using the function `joinLST`. This simple utility function was written to join two datasets that were read in using the `readLST` function or were originally many such data sets joined using the `joinLST` function. It takes two pairs of description and data structure arrays as input: `[desc data]=joinLST(desc1, data1, desc2, data2)`. The descriptor arrays are joined to create an n-by-1 structure array which is output as the description array desc. Each structure entry in the data array is separately concatenated and
output together as a new 1-by-1 structure array (\texttt{data}) that includes the entries of both of the original two data arrays. Note that none of the structure entries are changed in this import, which means data run numbers will need to be renumbered (for example by including the line \texttt{data2.runnum=data2.runnum+max(data1.runnum)+1} before \texttt{joinLST}) if they are used in subsequent analysis.

This function will also work with any pair of data structures that have the same fields (it does not presuppose the set \{\texttt{raw'}, \texttt{channel'}, \texttt{evtime'}, \texttt{startev'}, \texttt{runnum'}\} that are produced by the \texttt{readLST} function).

Masking dark-time detections and single-channel afterpulsing

There are two types of detector artifacts that I occasionally clean up when we read in the data. The first is detections during the detector dark time. The detectors we use need to reset after detecting a photon before they can detect another one, a process that takes about 40 ns. Since the detectors cannot detect photons during this reset time, it is called the ‘dark time.’ Any photons detected within 40 ns of the last detected photon on one detector cannot be physical.

Afterpulsing is a type of detector artifact in which results in another photon click after one photon is detected. The timing of these events depends on the length of fiber and the optics before the detector. Typical timescales in our data are about 100 ns (see Sec. 3.4).

We directly remove these unphysical detection ‘events’ when we read in the data. I first make a vector, \texttt{ap}, which is the time difference between the current photon and the previous photon detected on the same detector, and use it to select only those photons that arrive more than the detector reset time after the previous photons:

\begin{verbatim}
%excerpt from readLST that removes dark-time detections from our data
ap(data.channel==0) = single(data.evtime(data.channel==0)) - ...
    single(circshift(data.evtime(data.channel==0),1));
ap(data.channel==1) = single(data.evtime(data.channel==1)) - ...
    single(circshift(data.evtime(data.channel==1),1));
ap(data.channel==2) = single(data.evtime(data.channel==2)) - ...
    single(circshift(data.evtime(data.channel==2),1));
ap(data.channel==3) = single(data.evtime(data.channel==3)) - ...
    single(circshift(data.evtime(data.channel==3),1));
\end{verbatim}
data = masktrialsLST(data, ap'>40 | data.startev==1);

To remove afterpulsing events as well, this timing cutoff should be increased. Since afterpulsing timing depends on the optical elements before each detector, the timing for each channel will be different and should be checked when the beampaths are changed. This can be implemented by substituting the last line of the preceding code snippet with:

\[
apcutoff = (\text{data.channel} == 0) \times t_0 + (\text{data.channel} == 1) \times t_1 + \ldots
\]
\[
+ (\text{data.channel} == 2) \times t_2 + (\text{data.channel} == 3) \times t_3;
\]
\[
data = \text{masktrialsLST(data, ap'>apcutoff | data.startev==1);
\]

where variables \(t_0\)-\(t_3\) represent the separately measured afterpulsing times on each channel.

**Memory and speed considerations**

When reading in large data files, you may run into memory issues in MATLAB. If MATLAB is running slow, a first check is to restart the program; MATLAB has known memory leaks that are most noticeable when working with large data sets. With especially large sets, we have imported data in 4 MB parts using the `memmapfile` function and selected only the parts of the data we want for subsequent analysis during the import. This is done in the scripts named `readAndMaskForMemLST`. Reading in parts of data is accomplished by the following code:

```matlab
currentPlace=772;  %end of header
toRead=1e6; toReadByte=4*toRead;
while endOfFile>currentPlace
  %read in toReadByte of data at a time
  if (endOfFile-currentPlace)>toReadByte
    raw=memmapfile(filename,  
        'Offset',currentPlace,'Format','uint32','Repeat',toRead);
  else
    raw=memmapfile(filename,  
        'Offset',currentPlace,'Format','uint32','Repeat',Inf);
  end
  %insert code here to process raw data and select photon clicks
  currentPlace=currentPlace+toReadByte;
end
```
I usually find that these special data reading routines are not necessary. The system is rarely stable for long enough to get truly onerous file sizes. I achieve reasonable performance by removing data fields that are not used in subsequent analysis (typically 'raw' and 'startev') after reading in data, as well as removing any data outside of the analysis window such as system checks and repumping light during repeated measurements (trials).

A.1.2 Logical groups of data

In typical data, we collect photon counts for 5-40 ms once every 0.5-2 s. This collection starts after a trigger, which appears in our data structure as a start event. I index each collection by a run number, which is a monotonically increasing number that counts the number of collections in the data set. When data are loaded, this is just the number of accumulated triggers:

```plaintext
% the portion of readLST that numbers runs
data.runnum=cumsum(single(data.startev))-1;
```

After removing part of the data by, for example, selecting only those runs that meet a check like sufficient cavity transmission, it is important to recalculate the run numbers for the data. Unphysical empty runs will result in incorrect normalization for rate and correlation analyses.

To renumber runs, use the function `renumberrunsLST`. Instead of calculating runs from start events as we originally did, we instead calculate runs by a count of unique run numbers. (Start events are often removed from the data when we select data by time or channel; they show up on channel 3 at 0 ns.) The core of the code is:

```plaintext
% core code of renumberrunsLST
runind=cumsum(histc(unique(data.runnum),0:max(data.runnum)))-1;
data.runnum=runind(data.runnum+1);
```

`runind` should be thought of as a lookup table of length `max(data.runnum)+1` which gives the new run number in the old run number’s position. For example, if we have runs `{0,3,4,7}` (each with one event) represented in the data, `runind` would be `{0,1,2,3}` and the new run numbers would be `{0,1,2,3}`. To use this function, the
format is \texttt{data = renumberrunsLST(data)}. (Note that this function assumes that each run has a unique number, and so will give expected results for joined data structures whose run numbers have not been adjusted before joining them.)

While the particulars of the experiment determine what we do during each run, we can logically divide the acquisition time into measurements and system performance checks. The measurements are repeated under the same conditions each run.\footnote{The most common exception to this is when we do microwave spectroscopy, in which we change the frequency of the microwaves for each run.} When measurements are repeated multiple times in a given run, such as when we store and retrieve light in the transistor measurements (Chapter 4), we add an additional number to index the measurement number, which I call the trial number. To calculate the trial number for events occurring between times \texttt{pulsemin} and \texttt{pulsemax}, use this code:

\begin{verbatim}
data.trialnum=reshape(floor(double((data.evtime-pulsemin))/trialtime) +... ... (floor((pulsemax-pulsemin)/trialtime)+1)*data.runnum,[],1);
\end{verbatim}

Note that the trial number is meaningless (and often confusing, as it overlaps with trial numbers in other runs) outside of the range \texttt{pulsemin} to \texttt{pulsemax}. In practice, I calculate the trial number after processing system checks, postselecting data (and recalculating run numbers, if necessary) and selecting only those events in this time window.

\subsection{Choosing part of the data}

Often, we want to select part of the data for analysis. Usually, it’s the data from one detector, the data between certain times, or the data for the experimental runs when the system is working. Because we’re working with a data structure, I wrote the function \texttt{masktrialsLST} to make sure that all of the lists in the data structure are updated. To use it, the format is \texttt{data = masktrialsLST(data, mask)} where \texttt{mask} is a boolean vector with the same length as the vector in each field of the structure. For example, a mask for all the data on channel 1 is \texttt{data.channel==1}. To get the data that arrive between 0.5 \(\mu\text{s}\) and 2 \(\mu\text{s}\) during a 10 \(\mu\text{s}\) repeated measurement, use
\[ \text{mod(data.channel,10e3)}>0.5e3 \& \text{mod(data.channel,10e3)}<2e3. \]

This function goes through each field and keeps only those elements where the mask is true:

```matlab
core code of masktrialsLST
s=fieldnames(data);
for j=1:length(s)
    data.(s{j})=data.(s{j})(mask);
end
```

**Memory considerations while masking**

A data structure with more fields in it takes more memory to manipulate. When masking data, also consider removing fields from the structure that are no longer useful in analysis. For example, `raw` and `startev` are rarely used after data are imported. To do this, use this code snippet:

```matlab
removing fields raw and startev from data structure
data = rmfield(data, {'raw','startev'})
```

Alternatively, if you only want to keep a few fields (say `channel`, `evtime` and `runnum`), use code similar to the core of `masktrialsLST` (above) with the first line substituted for `s={'channel','evtime','runnum'}`.

**Postselection**

System checks within a data set allow us to select only those runs for which the system is performing well. The simplest example of this is when the cavity falls out of lock. In this case, we want to remove all data for subsequent runs (in which the cavity is out of lock): `data=masktrialsLST(data,data.runnum>123)`.

This can also be done systematically. To check if the cavity is in lock, record how many cavity arrive on each run (we typically use channels 0 and 1 for cavity transmission). Then, remove data for runs where there are very few photons detected, which indicates that the cavity transmission was low and the system was most likely out of lock:

```matlab
temp=masktrialsLST(data,data.channel==0 | data.channel==1);
```
postselCav = histc(temp.runnum, 0:max(data.runnum+1));
data = masktrialsLST(data, postselCav(data.runnum+1) > 20);

Alternatively, postselCav can be calculated without making temp, although the resulting code is harder to read: postselCav = histc(data.runnum(data.channel==0 | data.channel==1), 0:max(data.runnum+1)). Occasionally it also makes sense to add the postselection values to the data structure: data.psCav = postselCav(data.runnum+1) (The +1 accounts for the MATLAB convention that indexes start at 1, while data.runnum starts at 0).

To find the appropriate threshold, I use a percentage of maximum transmission (0.1*max(postselCav)) or plot the distribution of values in postselCav and hand code a threshold: plot(histc(postselCav, 0:max(postselCav))).

Common postselections in my analyses select for cavity transmission to check for cavity stability, cavity frequency and/or input power, free-space detection rate to check for input power drifts, and ratios of free-space detection rates measured near and far from atomic resonance to check the optical density.

A.2 Common analyses

A.2.1 Rates

The most straightforward analysis is that of the data rate on different channels. These are used to reconstruct absorption spectra or to monitor power levels. In its simplest form, this is just a normalized histogram of photon counts:

\[
\text{binsize}=10^3; \text{bins}=0: \text{binsize}:40e6; \%\text{analyzing data from 0-40 ms in 10 us increments}
\text{spectrum}=\text{histc}(\text{data.evtime(data.channel==2), bins})/\text{max(data.runnum+1)}*...\text{binsize}*1e-6); \%\text{in 10}^-3 \text{ counts/sec}
\]

This normalization factor is also calculated by the function countsToHz(data, binsize_in_ns).

To get rates in trials, instead calculate the histogram of the time in the trial and normalize by total trials:

\[
\text{trialtime}=10^3; \text{binsize}=100; \text{bins}=0: \text{binsize}:\text{trialtime}; \%\text{10 us trials in 100 ns increments}
\]
spectrum = \textbf{histc} (mod (data.evtime (data.channel == 1), bins), trialtime) / ... \\
\hspace{1em} (max (data.trialnum + 1) * binsize * 1e-6); \% in $10^{-3}$ counts/sec

Conditional rates are the measured rates given that something else has also occurred. For example, we measure the cavity decay when we measure a retrieved photon in the phase shift experiment (Chapter 7) to check how storing a photon changes the cavity decay rate.

\texttt{sigPhotInTrial} = \textbf{histc} (data.trialnum (data.channel == 2), 0:max (data.trialnum + 1)); \\
\texttt{spectrum} = \textbf{histc} (mod (data.evtime (data.channel == 1 \& sigPhotInTrial(data.trialnum+1)==1), bins), trialtime) / sum (sigPhotInTrial==1) / (max (data.trialnum + 1) * binsize * 1e-6); \\
\% in $10^{-3}$ counts/sec

This analysis adds a new conditioning vector (\texttt{sigPhotInTrials}) and a new normalization factor—the number of times conditioning succeeds.

A.2.2 Count statistics

We used count statistics to construct histograms of cavity transmission for our transistor measurements and atom-number detection (Chapter 4).

\texttt{bins} = 0:100; \% calculating a distribution with 1 photon binning up to 100 detected photons \\
\texttt{dist} = \textbf{histc} (\textbf{histc} (data.trialnum (data.channel == 1), 0:max (data.trialnum + 1)), bins);

This analysis is particularly sensitive to trial numbering. Incorrect numbering (or failing to renumber after postselection) may show up as an excess of trials in the first bin, i.e. trials with no recorded counts.

A.2.3 Correlation functions

Time-continuous data

We have used two approaches to calculate correlation functions of time-continuous data: calculating the arrival time correlation matrix and a vectorized approach that calculates time differences between photons. Each method calculates coincidences for individual runs and then sums total coincidences to calculate the correlation function. The second approach is at least an order of magnitude faster than the first. Moreover,
its calculation time is mostly independent of the desired resolution. I document the second approach here.²

Correlation functions are calculated using the script \texttt{calc\_g2\_timediffs}. Before using the script, you need to define the maximum separation time \texttt{taumax} and binning \texttt{binsize} in nanoseconds for the output correlation. The correlation function will calculated from the structure \texttt{data} between channels defined in \texttt{probevec} and \texttt{cavvec}. Data edges are dealt with by using a gating function, which assumes repeated measurements of time \texttt{gatetimewith} with data to be analyzed between times \texttt{gateLow} and \texttt{gateHigh} to calculate the singles vectors. (Note that the script does not mask data to ensure that this structure is used, so you need to enforce it by masking the data before analysis.) Data should occur between \texttt{maxAnaT} and \texttt{startT} in the data file; these numbers are only used to calculate the total number of trials analyzed. The script overwrites vectors \texttt{npnc}, \texttt{npnp} and \texttt{ncnc}, which record the number of coincidences as a function of separation time \texttt{bins}, the normalization counts (singles) \texttt{np} and \texttt{nc}, and the total number of trials \texttt{T}.

After defining vectors \texttt{npsel} and \texttt{ncsel} to select data on the probe and cavity channels, respectively, this script calculates arrival time differences for the \texttt{i}th run and sums over all runs:

\begin{verbatim}
%excerpt from calc\_g2\_timediffs
nct=dataC.evtime(ncesel & dataC.startev==0 & dataC.runnum==i);
np=dataC.evtime(npsel & dataC.startev==0 & dataC.runnum==i);

temp=ones(length(nct),1)*double(npt')-double(nct)*ones(1,length(npt));
temp=reshape(temp,1,1);
npnc=npnc+histc(temp,bins);
\end{verbatim}

and similarly for \texttt{npnp} and \texttt{ncnc}. Here, \texttt{temp} is a vector of all time differences between photons that arrive in the probe channel(s) and the cavity channel(s).

Then, it calculates singles by summing over all photons that could contribute to coincidences using a sliding gating function:

\begin{verbatim}
np(i)=np(i)+sum(npsel & dataC.startev==0 &...)
\end{verbatim}

²Analyses prior to January 2015 use the historical functions function \texttt{g2sub\_func} and script \texttt{calc\_g2\_kmb} and their predecessors \texttt{func\_CavPhoLater\_Gated2} and \texttt{func\_ProbPhoLater\_Gated2}.  

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\[
\text{mod}(\text{dataC.evtime}, \text{gatetime}) \leq \min(\text{gatehigh}+\text{bins}(i)+\text{binsize}, \text{gatehigh}) \quad \& \quad \text{mod}(\text{dataC.evtime}, \text{gatetime}) > \max(\text{gatelow}+\text{bins}(i), \text{gatelow})
\]

and similarly for \( \text{nc} \). This treats edge effects in the analysis for non-zero binning. For example, a probe photon that arrives at the minimum time analyzed (\( \text{gatelow} \)) could not contribute to a coincidences in which the probe photon arrives after another photon.

The code then calculates the trial number \( T \). To get the correlation function, calculate \( g_2 = \text{npnc} \cdot T / \text{nc} / \text{np} \).

**Pulsed data**

For pulsed data, calculating the correlation function is much more straightforward. I calculate vectors of the number of photons that occur on each channel in each trial. Their product gives coincidences:

\[
\begin{align*}
\text{np} &= \text{histc}(\text{data.trialnum(\text{data.channel}==2)}, 0: \text{max}(\text{data.trialnum})); \\
\text{nc} &= \text{histc}(\text{data.trialnum(\text{data.channel}==1)}, 0: \text{max}(\text{data.trialnum})); \\
\text{npnc} &= \text{np} \cdot \text{nc}; \\
g_2 &= \text{sum}(\text{npnc}) \cdot \text{length}(\text{npnc}) / \text{sum}(\text{np}) / \text{sum}(\text{nc});
\end{align*}
\]

A useful check here is to count the number of trials where more than one photon is detected: \( \text{sum(np}>1 | \text{nc}>1) \). When this is large, the second order correlation function does not give a full description of the data.

**A.2.4 Phase**

Reconstructing a sinusoidal heterodyne signal is the same as calculating a histogram where the trial time is \( 1/(2\pi f) \). However, since we reconstruct these phases in pulsed data, and the relative phase between the reference and signal light for the heterodyne can drift between runs, reconstructing the phase is slightly more complicated. Here, I outline the general procedure I use in my analyses as a guide to understanding existing scripts.

First, I calculate the heterodyne signal from a 1 ms pulse where both the signal and reference light are on at relatively high power (the average combined detected rate is about \( 10^6 \) counts/sec) as \( \text{histc} \text{(mod(data.evtime,bntime),0:bntime)} \) and fit
its phase. We typically use a period of 33 ns, a compromise between having enough resolution in our reconstructed sine wave and minimizing absorption of the reference light, which travels though the atoms.\(^3\) This gives the reference phase (as a time offset) for this run which I save as the vector \texttt{sigOff} in the data structure and use to calculate \texttt{data.sigtime=mod(double(data.evtime)-data.sigOff,sigtime)}, the time within the period of the heterodyne signal relative to the 0-phase point. After data cleaning and/or selecting coincidence data, I reconstruct the heterodyne signal as: \texttt{histc(data.sigtime,0:bntime)} and fit the phase.

Note that this reconstructed phase is not yet the physical phase from our interaction; we need to compare to a phase we fit from stored light because the storage and retrieval process also imparts phase.

\(^3\)The 33 ns period allows us to oversample slightly: our data are recorded with a time resolution of 2 ns, but the detection system has better accuracy than 2 ns. This allows us to sample at 1 ns as long as we acquire data over many heterodyne periods.
Appendix B

Frequency control

The experiments presented here depend not only on the ability to stabilize our lasers (and the optical cavity) to atomic frequencies, but also the ability to change their frequencies during an experimental cycle. Combining frequency control with control of the intensity of light in different beam paths during our experimental cycle allows us to use the same laser for different purposes, as well as to repeatably measure single-shot spectra.

In this appendix, I give a brief overview of the experimental frequency control which is described in several theses from our lab ([11, 85, 88]) before discussing our programmable frequency sources. During my graduate career, I developed tools to use these sources to change the frequencies of our signal and cavity probe beams quickly and reproducibly, and to use system outputs to do simple feedback to the frequency of these sources.

B.1 System overview

Our experimental system has nine lasers, five of which are 852 nm sources near resonant with the cesium D2 line. These lasers are actively stabilized to cesium spectroscopy through a frequency stabilization lock chain graphically depicted in Fig. B-1.

The reference laser is stabilized to atomic spectroscopy of a gas cell using the DAVLL (Dichroic Atomic Vapor Laser Lock) technique. The stabilized reference light
Figure B-1: **Frequency stabilization lock chain** This cartoon shows the lock dependencies of our frequency stabilization as well as the computerized controls. Each tier is directly stabilized through an analog frequency lock indicated by the lines to the tier above it. The stability of higher tiers is transferred through the locks to the lower levels. The line type indicates the type of experimental frequency control. Black lines indicate a fixed frequency difference. Dashed lines indicate control by voltages changed during the experimental cycle. The blue line is controlled by a potentiometer that is changed between experimental runs, but not during them. The red lines indicate frequencies controlled by a fast-updating programmable frequency source.
is combined on fast photodiodes with light from the MOT, Repumper, Gamma and Eta lasers producing an RF beatnote signal. For the MOT, Repumper and Gamma locks, this beatnote is mixed with an RF tone that is partially derived from a voltage controlled oscillator (VCO) with a voltage source that is actively controlled during the experimental cycle, allowing us to dial in a frequency for the laser at different times during the experimental cycle. The settling time of the frequency step depends on the lock parameters for that laser.

The Eta laser is locked to the transfer cavity by a Pound-Drever-Hall lock. The beatnote signal between that laser and the atomic reference is used to stabilize the transfer cavity. The beatnote can be tuned using a potentiometer-controlled VCO. As such, this frequency is not changed during the experimental run, but can be tuned in, for example, our measurements of phase shift in Chapter 7. The 817 transfer laser is stabilized to the transfer cavity, as well, and is used to stabilize the experimental cavity. This locking scheme stabilizes the experimental cavity relative to the Eta laser without probing it with light near the cesium D2 line.

If we are using an intracavity dipole trap, the 937 nm light is stabilized to the experimental cavity. This has been done either by using a PDH lock to the TEM00 mode or by locking to the TEM01 mode with a small amount of light. This secondary lock scheme allows us to change the power of the intracavity trap without changing the lock gain.

Not indicated in this lock chain is the 1064 nm dipole trap laser, which is not stabilized.

Each laser is split into multiple beam paths before interrogating the atoms, which may have different frequencies. I've indicated these for the Gamma and Eta lasers only, as these lasers have subsequent frequency control on one of the beam paths. (These beam paths are used to probe the signal and cavity paths, respectively.) Each of these lasers have a beam path that travels through an electro-optic modulator (EOM) modulated at a frequency derived from a programmable frequency source that is digitally controlled. I describe this control in more detail in the following section.
B.2 Programmable frequency source setup

The programmable frequency source setup consists of a PTS3200 (Precision Test Source 3200 series) frequency synthesizer and an FPGA to control its digital interface. Each PTS3200 provides one programmable frequency output that can be controlled between 1 and 3200 MHz controlled by approximately 30 TTL inputs. The Cs Lab controls these digital channels with an FPGA (in particular, Xilinx Spartan-3 XC32400-4PQ208 on a dev board from Opal Kelly, XEM3001). We program the FPGA asynchronously using a pre-compiled program (written in Verilog) and USB inputs (controlled through either an OpalKelly FrontPanel or LabView interface to provided C .dll libraries). If we are using the FPGA to change frequencies during our sequence, we trigger it on our master clock using a digital channel from one of our NI cards. The FPGA catches these asynchronous triggers on a separate FPGA output pin.

In some implementations of the code, the FPGA suffers from timing jitter and/or missed triggers. While this timing jitter should be considered in serious data analysis, the primary hardware limitation in day-to-day operation is the PTS update and stabilization rate, which can be as much as 20 μs when updating digits 10 MHz and higher. (As a general rule, for best operation, avoid changing the 10 MHz digit (or higher) during parts of the sequence where you care about the timing or ignore the switching period in data analysis.)

The code provides the ability to dial in numbers for the frequency four times per trigger and to change the frequency by a user-defined step after one of the frequency steps. Additionally, a new feature implemented in late 2015 provides feed forward to the cavity probe frequency to correct for the center-frequency drift due to atom number loss during the experimental cycle.
Appendix C

Polarization projection

In order to achieve EIT in a quasi-copropagating configuration, we typically use light with two different polarizations of light in order to connect the $|S_{1/2}, F = 3, m_F = 3\rangle$ state, which is straightforward to prepare with standard optical pumping techniques, and the $|S_{1/2}, 4, 4\rangle$ state. Typically, we choose circularly- and $\pi$- polarized light for the two EIT transitions, which cannot both have pure polarization in the atomic basis with the same $k$-vector. In this appendix, I describe the polarization projection in the atomic basis.

When light interacts with atoms, the $k$-vector of the light relative to the magnetic quantization axis determines the possible transitions that the atoms can make. One way to represent these transitions is to represent the light in terms of the polarization decomposition ($\pi$, $\sigma^+$ and $\sigma^-$) in the atomic basis. If the $k$-vector is not perpendicular or parallel to the quantization axis, we can get some light in any polarization at the expense of mixing in some amount of another polarization.

To make this more concrete, say our beam with wavevector $\mathbf{k}$ makes a small angle $\theta$ the magnetic field $\mathbf{B}$ that defines our quantization axis. Call the plane perpendicular to the one defined by $\mathbf{k}$ and $\mathbf{B}$ the vertical plane. Then, write the polarization of the incoming light in the vertical and horizontal basis (along $\mathbf{k}$): $Xh + Yv$

Transforming into the analogous (primed) coordinate frame of our quantization
axis (and thus the atoms), write

\[ h = (\cos \theta h' + \sin \theta \pi') \]  \hspace{1cm} (C.1)
\[ v = v' \]  \hspace{1cm} (C.2)

In the primed frame,

\[ h' = \frac{1}{\sqrt{2}} (\sigma^+ + \sigma^-) \]  \hspace{1cm} (C.3)
\[ v' = \frac{i}{\sqrt{2}} (\sigma^+ - \sigma^-) \]  \hspace{1cm} (C.4)

such that

\[
\frac{X \cos \theta - i Y}{\sqrt{2}} \sigma^+ + \frac{X \cos \theta + i Y}{\sqrt{2}} \sigma^- + X \sin \theta \pi
\]  \hspace{1cm} (C.5)

For a fixed angle \( \theta \) given by the system geometry, we can choose to eliminate one of the three polarizations of the control beam by carefully adjusting the amplitude and phase of \( X \) and \( Y \). If we take purely vertical light \( X = 0 \), we eliminate the \( \pi \) component. Alternatively, we can choose \( |X \cos \theta| = |Y| \) and the relative phase between these components to either cancel the coefficient of \( \sigma^+ \) or \( \sigma^- \), so long as \( \cos \theta \neq 0 \). (Under the same condition, the converse argument shows that there can be some amount of light in any polarization.)
Bibliography


