WALL SHEAR-STRESS AND LAMINARISATION IN ACCELERATED TURBULENT COMPRESSIBLE BOUNDARY-LAYERS

by

JAMES LUDLOW NASH-WEBBER

April 1968

GAS TURBINE LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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Cambridge, Massachusetts
Per ardua ad astra

Eheu, fugaces !
ABSTRACT

Turbulent-laminar transition in compressible, steeply-accelerated, adiabatic, turbulent boundary layers on a smooth wall was investigated experimentally in the ranges of Mach and Reynold's numbers typical of nozzles used in propulsive devices. Correlation of the present and previously published data suggests that the transition of such a shear layer may be predicted by consideration of its trajectory on a plane having an acceleration parameter $K$ and a Reynold's number $R_{\delta_2}$ as coordinates. An ab initio design method has been developed, based on these findings, which will ensure laminar flow before and at the throat of a sufficiently small nozzle operating at sufficiently small total pressure. A new type of surface-pitot was developed and calibrated and used to measure wall shear stresses in both transitional and non-transitional flows. Decrease of wall shear-stress in laminarising flows was found.

General-purpose computer programs for data-reduction, surface-pitot calibration and interpretation and boundary layer development predictions were developed.
ACKNOWLEDGEMENTS

Being as it were the child of two of the Institute's families--the Aero & Astro Dept. Propulsion Group and the Gas Turbine Lab., I have many people to thank for their ever-available help in advancing my work. The pleasant working conditions within these groups provides a welcome degree of insulation from the insensate pressures of the Institute as a whole. Though it is perhaps invidious to single out anyone, I should like to thank most especially:

Professor Edward Taylor, whose outgoing personality and incisive insight into the problem made research a pleasure;

Professor Gordon Oates, who, as project supervisor always contrived to maintain a sense of humor and a degree of optimism despite all setbacks;

The other members of my Thesis Committee, for their time and trouble in setting me back on the straight and narrow at frequent intervals;

Thorwald Christensen, without whose skill and tireless efforts in the machine-shop and at a dozen other trades the experiment would have clanked to a halt;

John and Joan Moore for their valuable assistance with the machine calculations;

Lotti Gopalakrishnan, for typing and retyping the reports, often from my atrocious scrawl, and for feeding me coffee and cookies at frequent intervals; and

Bonnie, who saw the work through to the end, and was an ever-present help in time of trouble.
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NOTATION

\begin{itemize}
\item \(A, A_1\): Constants used in skin friction fence calibration relations
\item \(B, B_1\): Constants of skin friction law (See Appendix I)
\item \(C\): Skin friction coefficient \(\left(\frac{\tau_w}{\frac{1}{2} \rho \delta^2}\right)\)
\item \(a, b\): Walz' skin friction coefficient \(\left(\frac{\tau_w}{\rho \delta^2}\right)\)
\item \(c_f\): Walz' shear work integral
\item \(d\): Fence height
\item \(f_p, f_2, F\): Functions defined in Appendices I & II
\item \(F_R, F_L\): Functions defined in Appendices I & II
\item \(H_{12}\): Shape factor \(\equiv (\delta_1/u) / (\delta_2/u)\)
\item \(H_{12}^*\): Shape factor \(\equiv \delta_1 / \delta_2\)
\item \(H_{32}\): Shape factor \(\equiv (\delta_3/u) / (\delta_2/u)\)
\item \(H_{32}^*\): Shape factor \(\equiv \delta_3 / \delta_2\)
\item \(K\): Acceleration Parameter \(\equiv \frac{v_w}{\delta} \cdot \frac{d u_\delta}{d x}\)
\item \(M\): Mach Number
\item \(N, n\): Empirical exponents used in Walz' theory (See III.C.1)
\item \(P\): Pressure
\item \(\Delta P\): Pressure difference
\item \(R\): Reynold's Number (general); Radius
\item \(R_{\delta, 2}\): Reynold's Number based on momentum thickness \(\frac{\rho U_\delta \delta_2}{\nu_w}\)
\item \(r\): Recovery factor
\item \(T\): Temperature
\item \(T'\): Sommer & Short reference temperature:
\[T_\delta \left[ 1 + 0.035 \frac{M_\delta^2}{T_\delta} + 0.45 \left(\frac{T_w}{T_\delta} - 1\right)\right]\]
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<td>U, u</td>
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<td>v</td>
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**Greek letters**

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<td>$\Theta$</td>
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I. INTRODUCTION

A. Motivation and Background to the Problem

Fluid dynamicists of the more classical sort have spent much time and effort on trying to understand the behavior of boundary-layers in adverse pressure-gradients. Comparatively little effort has been devoted to the phenomena occurring in favorable pressure-gradient flows. Many propulsive devices, however, incorporate favorable pressure-gradient flows, and the continuing effort to push these to their limits of performance is currently rendering mandatory a much larger effort aimed at a better understanding of such flows. This study attempts to redress further the imbalance of research effort and, in particular, to advance our knowledge of the "relaminarisation" phenomenon.

The turbulent flow problem has been with us from the dawnings of fluid dynamics. Progress toward satisfactory enunciation of the problem, let alone elucidation, has been distressingly slow. In particular, when we come to examine the status of the turbulent shear flow problem, we find a wide divergence between the need of the designer of any sort of fluid-flow equipment for information and the ability of the theoretician to supply it. Since the problem will not go away, this has resulted in the formation of a large school of empiricists dedicated to the production of useable design data. The efforts of these workers are, nevertheless, greatly hampered by the inability of the available theory to point towards the correct design of experiments or greatly to help with the correlation of their results. It has accordingly been necessary to break the problem up into many smaller pieces, and these often indeed into several special cases before useful progress could be made.

On two such sub-problems, a great deal of work has been done. These are the laminar-turbulent transition problem and the equilibrium turbulent
boundary-layer problem. In these problems, for several special cases where the upstream history of the shear layer may be carefully controlled, "theory" and experiment have been notably at one. In particular, there have resulted the useful similarity laws of Clauser\textsuperscript{1} and of Coles\textsuperscript{2} for equilibrium and near-equilibrium shear flows, and several plausible transform theories to allow the extension of incompressible results to compressible flow have been proposed. Two major schools of thought on the subject of transforms are presided over by Coles\textsuperscript{3} and Crocco\textsuperscript{4}, but their differences have yet to be resolved. Efforts at a general treatment of the turbulent shear-flow problem have appeared at intervals, most recently and notably those of Hawthorne\textsuperscript{5} and Kutateladze and Leont'ev\textsuperscript{10}.

Several types of computational method for such flows have been developed, based on these ideas. These appear to enjoy success in direct ratio to the similarity between the problem to which they are applied and the data from which they were formulated. These methods will be treated in Section III of this report.

On the subject of turbulent shear flows which are very far from equilibrium, our ignorance is most profound. Nature has, however, decreed that such flows are to be found in many of our fluid-flow devices, which circumstance renders an attack on the problem highly desirable, though scarcely more tractable for all that. In particular, a new and important phenomenon has manifested itself in the special case of turbulent shear flows in a very strongly negative pressure gradient, such as can occur in certain duct flow devices, such as nozzles and turbine blades. This phenomenon is evidenced as a very considerable decrease in the macroscopic transport properties of the shear layer.

This has led to previously inexplicably low results in nozzle heat-transfer experiments such as those of Wilson and Pope\textsuperscript{5} in 1954, who were
apparently the first to encounter the phenomenon, at least in the Western
world: Papers by Deich et al. 29, 30 refer to a first discovery, in 1948, in
Russia, of the phenomenon, during study of trans- and supersonic flows past
a sphere, and later encounters, in steam ejectors, in 1954.

It was widely speculated 6, 7, 8 that this most desirable (in general)
phenomenon might be identified with the onset of a reverse transition from
turbulent to laminar shear flow. Conclusive evidence of reverse transition
was reported by Senoo 11 in 1957. He investigated the end-wall flow of a
turbine nozzle cascade, finding that for the conditions of his experiment,
the boundary layer, while demonstrably turbulent upstream of the throat,
exhibited a laminar profile in and downstream of the throat. Little insight
into the flow mechanisms involved could, however, be gained from this study.

During this period also, very large efforts in rocket-engine research
were getting under way. Countless tests to measure combustion chamber and
nozzle heat-transfer rates were run. Attempts to correlate and elucidate
these results proved less than satisfactory. Indeed, the best known of these,
the correlation of Bartz 12 could seldom be relied on to produce results for
the peak heat flux accurate within a factor of two either way, even in those
rare cases where the boundary-layer upstream of the nozzle was well determined.
References 13 through 16 are representative of the heat transfer measurements
reported in the literature.

To account for that residuum of grossly low heat transfer results
which could not be prodded into the same pen with the majority, invocation
of the notion of a turbulent-laminar reverse transition of the boundary-
layer was sometimes attempted, on frankly intuitive grounds. In 1954,
Preston 17 had suggested that there might be some minimum value of Reynolds
number, Re 2, say 320, based on momentum thickness, below which a turbulent
boundary layer could not exist. There might then perhaps be significant
effects if the sharply accelerated shear layer attained sufficiently small values of $\delta_2$. Qualitative arguments could be brought to bear on the effect of the stretching of vortex filaments in the shear layer by the acceleration of the free-stream flow: namely that the scale of the turbulence should be shifted towards the small wave lengths, thus promoting molecular-viscous dissipation. It was not clear whether the required deficit of turbulent energy production vis-à-vis dissipation would require, in addition, a mechanism for suppressing turbulence production. The details of such a mechanism were also wanting. Indeed, stability theory of turbulent shear flows was, and remains, in that limbo reserved for interesting but unsolved problems.

A common feature of these studies was the complexity of the flows studied. There was no way to separate out the effects of wall-cooling, wall curvature, chemical reaction, compressibility, multiphase flow, free-stream turbulence, wall roughness or free-stream acceleration, each of which could be expected to have some effect on the phenomenon.

This dilemma was apparent to several workers. Amongst the first to attempt more concisely defined experiments were: Back and his co-workers at JPL; the group at Stanford, separately reported by Kline and co-workers and by Moretti and Kays; the group at United Aircraft Research Labs; and several groups in Russia. References 18 through 20 contain essentially the same material from JPL; 21, 22 and 23 refer to Stanford works which themselves list their precursors; 24, 25 and 26 are complementary, and cover the UAC work; and 29 reviews the Russian work. Most significantly for a better understanding of the mechanisms involved, Launder$^{27, 28}$ conducted detailed experiments in which all effects bar that of free-stream acceleration were effectively absent. It is worthwhile to review briefly the important conclusions to be drawn from the above studies.
The JPL workers ran tests on a $45^\circ$-$15^\circ$ conical-conical convergent-divergent nozzle through which a mixture of air and methanol combustion products at some $1500^\circ$R was passed, with stagnation pressures from 30 to 250 psia. The nozzle was water-cooled in annular segments, and the mean heat flux per segment measured calorimetrically. Typical free-stream/wall temperature-ratios were about 2. Thickness of the inlet boundary layer, which was measured, could be varied by changing the length of the nozzle approach section.

Compared with the best current laminar and turbulent boundary-layer heat-transfer theories in use in that laboratory, it was found that although experimental values and the turbulent prediction were in fair agreement at the higher stagnation pressures tested, there was a tendency in the lower stagnation pressure tests — i.e. the lower Reynold's number tests — for the experimental data to fall away from the turbulent prediction and approach the laminar one. Although the detailed conclusions of this study were subsequently disputed by O'Brien, the general and expected tendency for preferential lammarisation at the lower Reynolds numbers appeared to be confirmed.

No detailed boundary-layer measurements were taken downstream of the nozzle entry, and there was no way to separate out conclusively the individual effects of wall-cooling and acceleration on the relaminarisation phenomenon.

The study at UAC Research Labs was similar in concept, but here a two-dimensional test geometry rather similar to the one reported in the present study was used. Cryogenic cooling of the test section was used, giving large free-stream/wall temperature-ratios and some detailed boundary-layer measurements were taken. It was again found that relaminarisation occurred for flow Reynolds Numbers below some upper limit for each of a
series of nozzles having different values of an acceleration parameter, and that this reverse transition was promoted by increased wall cooling. Since this study was definitely hardware-oriented, the data were presented in terms of quantities like rocket thrust level and throat Reynolds numbers, which inevitably makes comparison with other data difficult in any quantitative way. Again, the effects of wall cooling and acceleration of the free stream were not separately discoverable.

Moretti and Kays\textsuperscript{23} conducted an experiment in which both free-stream velocity and wall temperature could be varied continuously for a turbulent boundary layer having effectively constant fluid properties. They found clear evidence of relaminarisation for values of $K$, the free-stream acceleration parameter, greater than about $2.5 \times 10^{-6}$, with heat transfer reduced to levels anticipated for a laminar boundary layer. Significantly, there was no further reduction in heat transfer for values of $K$ greater than $3.5 \times 10^{-6}$, from which it was concluded that turbulence production was completely suppressed in this regime. Since the free-stream/wall temperature-ratio was near unity for this study, one might conclude that the results observed were due primarily to the acceleration effect.

In a massive study of flows in water channels, Schraub\textsuperscript{21} and Kline\textsuperscript{22} and their many co-workers found that turbulence production appears to occur in "bursts" of a complex flow structure within what has traditionally been called the "laminar sublayer", and that the rate of bursting could be correlated (inversely) with the value of $K$, with complete suppression for $K > 3.5 \times 10^{-6}$. These studies have advanced enormously our understanding of the mechanism of turbulence production.

Deich and Lazarev\textsuperscript{29} reported in 1964 on experiments run on three different test sections at the Moscow Power Engineering Institute: an axi-symmetric tunnel with a second throat, a two-dimensional tunnel with
a second throat and a tunnel with a skewed-axis nozzle and diffuser. For the first two, it was found that despite a high level of free-stream turbulence induced by a normal shock just downstream of the first throat, the acceleration imposed on the turbulent boundary layers approaching the second throat rendered them laminar, as detected both by total-head traverses and hot-wire measurements. More detailed shear layer measurements were made on the third test section, where relaminarisation was again found. It was found, moreover, that the onset of relaminarisation was marked always by a sharp decrease in momentum thickness not attributable to the effects of pressure gradient on a fully turbulent boundary layer.

This work is unfortunately reported in very sparse detail, making comparison with other work difficult. If one assumes, however, that the stagnation pressure was near one atmosphere for these tests, then $K$ was over $4 \times 10^{-6}$, i.e. well within the limits required for relaminarisation as found in the present study and earlier work.

In 1963 and 1964 Launder $^{27, 28}$ made very detailed boundary-layer measurements in highly accelerated flows of an adiabatic turbulent boundary layer at very low Mach and Reynolds's numbers ($M < 0.07$, $R_{\theta} < 1.2 \times 10^{3}$). He found that for $K > 2 \times 10^{-6}$ the boundary layer underwent a progressive reversion to a state almost indistinguishable from a laminar one in terms of mean velocity profile, shear stress, separation behavior, mean energy balance and re-transition (to turbulent) behavior, while nevertheless retaining a marked turbulence signal. He found also that the onset of relaminarisation was marked by a large and unmistakable rise in the value of the shape-factor $H_{12}$, and was preceded by large departures of the mean profile from the logarithmic "law of the wall" universally valid for near-equilibrium turbulent boundary layers. This work is a striking confirmation of the entirely independent Russian results mentioned above.
From the velocity-profile oriented point of view, it appeared that the relaminarisation process in Launder's flows proceeded through the growth of the "laminar sublayer" within the turbulent shear layer until the latter was supplanted entirely. This finding is in no way invalidated by the discovery by Kline et al.\textsuperscript{22} of the further details of the process, involving formation of an unsteady, bursting "streak" structure within the "laminar" flow. We might, however, in the light of these discoveries, do well to modify our conceptual model of a "laminar" sublayer, and simply note the existence of a region near the wall in which molecular viscosity dominates the flow.

There seems no reason to suppose a priori that the well-known Reynold's analogy between heat and momentum transfer should be valid for highly non-equilibrium turbulent flows like those treated above, and indeed, in 1963, Romanenko, Leont'ev and Oblivin\textsuperscript{39} reported on experiments in which both heat transfer and shear stresses in a highly accelerated flow near $M = 0.5$ were measured, leading them to conclude that the Reynold's analogy should indeed be abandoned for such flows. Their shear measurements were taken with hot-wire equipment, and wall shear stress was inferred from the conservation relations for the shear layer in integral form. No great accuracy is to be expected from this procedure (see ref. 35), and their alternate method of finding the wall shear stress, essentially that of Clauser\textsuperscript{1} seems equally suspect in view of the large departures from universality found by Launder and others for such shear layers.

B. Introduction to the Present Study

Given the above considerable body of knowledge about the relaminarisation phenomenon, it seemed desirable, when assessing the goals of the present study, to try to extend our knowledge into flow regimes typical of actual fluid machinery, as measured by such parameters as Mach and Reynold's numbers,
and to express the results in some form immediately useful for design purposes. It seemed clear that a necessary first step was the separation of the effects of free-stream acceleration and strong wall-cooling. This study treats the adiabatic-wall case. It also seemed desirable to measure as directly as possible wall shear stresses typical of highly non-equilibrium accelerating flows, this information being scanty or absent in all the reported work up to the time the project was initiated.

Recent work by Hopkins and Keener\textsuperscript{31} and by Patel\textsuperscript{32} gave reason to believe that some form of wall-pitot could be used for this purpose - see Section II.B.1. and Appendix I. The present study appears to incorporate the first published attempt to measure local wall shear-stress by a quasi-direct method in turbulent compressible boundary layers in extreme pressure gradients. The calibration procedure revealed a "universal" calibration for the essentially two-dimensional wall-pitot geometry chosen. This happy circumstance offers a new and useful technique for skin friction measurements in very thin boundary layers, with or without an imposed pressure gradient.

To facilitate both instrumentation and comparison with the major part of previously published data, a nominally two-dimensional experimental geometry was chosen. Provision was made for variation of the imposed pressure gradient, and for detailed shear-layer measurements. Unit Reynold's number could be varied through a 10:1 range. Section II.A treats the experimental facility.

Examination of velocity-profile development in a series of pressure gradients, taken in conjunction with the associated and confirmatory wall shear stress measurements allowed the delineation of the coupled effects of acceleration and characteristic Reynold's number. These results are presented in the form of a directly useful "laminarisation map" having
these parameters as coordinates.

Any given nozzle-wall shear-layer has a trajectory on such a diagram. Thus, if laminarisation is found to occur within some region of the diagram, entry of the trajectory corresponding to any actual or proposed device into such a region indicates the onset of laminarisation in that device. This process may, however, not be carried to completion if the boundary layer remains within the required ranges of $K$ and $R_{\infty}$ for an insufficient flow length. These matters are treated in Section IV.

A computation scheme suitable for general turbulent and laminar two-dimensional and axi-symmetric compressible boundary-layer calculations was developed, based essentially on that of Walz\textsuperscript{33, 34}. This scheme can be applied at the design stage of nozzle development, allowing rapid and reasonably accurate estimation of the onset and/or absence of any laminarising phenomena.

Further improvements in the prediction method were sought through incorporation of the recent, improved empirical relations of Fernholz\textsuperscript{35} and of Escudier\textsuperscript{36}. These various schemes were also extensively compared with data from the literature, covering both equilibrium and non-equilibrium cases in favorable and adverse pressure gradients.

A complete listing of the FORTRAN IV computer program associated with these schemes is given in Appendix V. Every effort has been made to facilitate the routine use of this program even by those totally unfamiliar with the details of the method.

Since the present study is for the adiabatic-wall case, and it has been found that wall cooling promotes reverse transition\textsuperscript{37, 38}, the suggested design procedure is essentially conservative except for cases where the fluid is cooler than the nozzle. It is qualitatively clear how
the "stability boundary" found in this study should move with positive or negative heat transfer, but quantitative information awaits much more experimental work.
C. The Method of Walz et al

1. A Brief Presentation of Theory

Building on earlier work, Walz\textsuperscript{33, 34} produced, in 1965, an explicit-integral theory for calculation of compressible, turbulent or laminar, two-dimensional or axisymmetric boundary layers, with or without heat transfer. The main features of this method are summarised below, but the reader is urged to consult the original references for a full presentation.

The equations for momentum- and energy-conservation and continuity take the forms:

\begin{align*}
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= - \frac{dP}{dx} + \frac{\partial \tau}{\partial y} \quad (18) \\
\frac{c_p}{\rho} \left( \rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} \right) &= u \frac{dP}{dx} + \tau \frac{\partial u}{\partial y} + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) \quad (19) \\
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} &= 0 \quad (20) \\
\text{where} \quad \tau &= \mu \frac{\partial u}{\partial y} \quad (21)
\end{align*}

Weighting (18) by $u^k$ and (20) by $(u^{k+1}/k+1)$ and integrating the sum in the interval $0 \leq y \leq \delta$ yields the relation

\begin{align*}
\frac{df_k}{dx} + f_k (2 + k + g_k) \left[ \frac{\partial u}{\partial \delta} \right] \frac{du}{dx} + e_k &= 0 \quad (22) \\
\text{where} \quad e_k &= (k + 1) \int_0^\delta \left( \frac{u}{u_\delta} \right)^k \frac{\partial \delta}{\partial y} \left( \frac{\tau}{\rho_\delta u_\delta^2} \right) dy \quad (23) \\
f_k &= \int_0^\delta \frac{\rho u}{\rho_\delta u_\delta} (1 - \frac{u}{u_\delta}^{k+1}) dy \quad (24) \\
g_k &= -(k + 1) \int_0^\delta \left[ \frac{\rho u}{\rho_\delta u_\delta} - \frac{(u}{u_\delta}^k \right] dy \quad (25)
\end{align*}
k can be any arbitrary dimensionless number, but the particular choices of $k = 0$ and $k = 1$ are particularly interesting physically, resulting in the usual momentum integral equation and the kinetic-energy integral equation, with

$$ g_0 \equiv \delta_1, \quad g_1 \equiv 2\delta_4, \quad f_0 \equiv \delta_2, \quad f_1 \equiv \delta_3 $$

(26)

and

$$ -e_0 \equiv \frac{w}{\rho_\delta u_\delta^2} = \bar{c}_f $$

(27)

$$ e_1 \equiv \bar{c}_D = \bar{c}_f \int_0^1 \frac{1}{\tau} \frac{d}{u_\delta} $$

(28)

These last two quantities, namely the skin friction coefficient $\bar{c}_f$ and the shear work (or "dissipation") integral $\bar{c}_D$ have been written with a tilde to emphasise that they are not the usual definitions, having exactly half the numerical value of the more conventional forms.

Walz also finds it useful to introduce the quantities $(\delta_1)_u, (\delta_2)_u, (\delta_3)_u, (\delta_4)_u$ where the subscript indicates that the integrals are to be performed with $\rho = \rho_\delta$.

Shape factors $H_{12}, H_{12}^*, H_{32}, H_{32}^*$ and $H_{43}^*$ can now be defined:

$$ H_{12} \equiv (\delta_1)_u/(\delta_2)_u \quad H_{12}^* \equiv \delta_1/\delta_2 $$

$$ H_{32} \equiv (\delta_3)_u/(\delta_2)_u \quad H_{32}^* \equiv \delta_3/\delta_2 $$

$$ H_{43}^* \equiv \delta_4/\delta_3 $$

(29)

A length parameter is also defined:

$$ z \equiv \delta_2 R \delta_2^n $$

(30)

Two auxiliary functions incorporating $\bar{c}_f$ and $\bar{c}_D$ may be introduced with advantage, viz.: 

$$ \bar{c}_f \equiv \frac{a}{R^n} \frac{\delta_2}{(\delta_2)_u} \quad \bar{c}_D \equiv \frac{8}{R^n} \frac{\delta_2}{(\delta_2)_u} $$

(31)
Finally, a heat-transfer parameter is required, viz.:

\[ \theta = \frac{T_e - T_w}{T_e - T_o} \]  

(32)

where \[ \frac{T_e}{T_o} = 1 + r \frac{Y - 1}{\gamma} M_o^2 \]  

(33)

Putting (23) - (33) into (22) results in the integral relations for momentum and kinetic-energy conservation for compressible laminar and turbulent boundary layers with heat transfer in the form:

\[ \frac{dz}{dx} + z \frac{d\mu_0/\mu}{u_0} (F_1 + n \frac{d\mu_0/\mu}{du_0/\mu}) = F_2 = 0 \]  

(34)

\[ \frac{dH_{32}^*}{dx} + H_{32}^* \frac{d\mu_0/\mu}{u_0} (F_3 - \frac{F_4}{z}) = 0 \]  

(35)

where \( F_1 \equiv (2 + n) + (1 + n) H_{12}^* - M_o^2 \)  

(36)

\( F_2 \equiv (1 + n) a (\delta_2/(\delta_2)_u) \)  

(37)

\( F_3 \equiv 1 - H_{12}^* + 2 H_{43}^* \)  

(38)

\( F_4 \equiv \frac{\delta_2}{(\delta_2)_u} (28 R_{62}^{H-N} - a H_{32}^*) \)  

(39)

It should also be noted that the shape factors are connected by the relations:

\[ H_{12}^* = \frac{H_{12}}{\delta_2/(\delta_2)_u} + r \frac{Y - 1}{2} M_o^2 (H_{32}^* - \theta) \]  

(40)

\[ H_{43}^* = r \frac{Y - 1}{2} M_o^2 \frac{H_{32} - \theta}{H_{32}^*} \]  

(41)

Up to this point, the development has involved no empirical approximations. For the solution of the coupled equations (34) and (35) we now require a hierarchy of empirical relations, summarised below. Only
chosen to impose the required pressure gradients on its boundary layer. These are shown in Figure 8.

For calibration of the skin friction fences, which required an accurately zero pressure gradient, a flexible 0.375" aluminum plate was opposed to the test plate. This was mounted on jackscrews to allow variation of its contour. The L.E. of this plate was again faired into the main contraction with a nose-piece. Considerable difficulty was initially experienced with highly non-uniform (in y) velocity distributions in the free-stream with the test section set up in this calibration configuration. This was a consequence of the 2:1 asymmetry of the second stage of the contraction. A first-order analysis of this potential flow problem due to Oates 52 indicated a method of determining that position of the leading-edge/nose-piece with respect to the test plate's nose-piece which would give minimum distortion of the free-stream velocity distribution near the entrance. This analysis is treated in Appendix III. Repositioning of the flexible plate L.E. yielded a distribution less than 1% from uniform at the first measuring station \((x = 12'')\).

Throughout this report, the origin of coordinates is the L.E. of the plastic test plate, which is also the location of the boundary layer trip. This was a 0.06" x 0.1" transverse slot in the surface. A more substantial roughness element or "spoiler" consisting of a 2" wide strip of 0.062" perforated steel plate, laid flat on the surface, could also be fitted at this point to provide an artificially thickened shear layer.

Nozzle-blocks A, B and C were chosen in their final forms with a view to obtaining interesting trajectories on a \(K - R_{62}\) diagram, discussion of which is the main burden of Section IV of this report. Nozzle-block A is in fact one of the two blocks used to form the uniform Mach 2 flow which is
the usual business of the tunnel. Table 1 lists the important characteristics of the free-stream flows produced by these three test sections. See also figure 9.

Thirty-nine static pressure orifices (0.016") were distributed along the centerline of the plastic plate, with a variable spacing so as to allow accurate determination of gradients in the converging and throat regions of the test sections. These were connected to a bank of mercury manometers. In addition, for the zero pressure gradient operation required for calibration, seven downstream orifices, distributed over the whole instrumented length of the test section, were monitored for differential static pressure with respect to the L.E. tap, using inclined water manometers. This enabled the jackscrews to be adjusted so as to obtain a maximum deviation from constant static pressure of less than 0.05%.

Stagnation pressure for the isentropic part of the flow was measured with an 0.059" pitot probe in the free-stream at \( x = 1\frac{4}{16} \). Total temperature was monitored with a bare thermocouple in the settling section.

3. Probes, skin-friction fences and associated equipment

Total-pressure distribution in the shear layers could be measured at twelve stations along the test plate. Conventional flattened, sharp-edged total head probes were used for this purpose. Some of these are shown in Figure 2. Tip sizes varied from 0.007" x 0.025" to 0.017" x 0.042", the smallest size being used wherever practicable. These probes were mounted in remotely controlled and/or driven traversing rigs, as illustrated in Figure 3. These had traversing ranges of 1.3" and 2.2", using micrometer heads as the rotating elements. A miniature 10-turn helipot was driven off each head by a step-down pulley system. Acting as voltage-dividers, these supplied the X-channel signals for a pair of Moseley X-Y recorders.
Pitot-static differential pressure was fed in each case to one of a number of differential-pressure transducers, having ranges between 0 ± 0.2 psid to 0 ± 15 psid. These were mostly of the Statham unbonded strain-gauge type except for an Eico 0 ± 0.5 psid variable reluctance type. This latter was found to be unreliable below 0.1 psid and was consequently not used in this range. Three well-filtered, smoothed solid-state DC power-supplies of nominal 6, 10 and 28 V ratings were used to supply the references for the various transducers and helipots. These were supplied, in turn, off a 115 V. AC magnetic type line-voltage stabilizer. A twenty-minute warmup period proved adequate to ensure that reference-voltage variations over the course of some hours remained smaller than the test-equipment resolution. In no case did ripple exceed 0.1%.

The signals from the transducers were fed to the Y-channels of the X-Y recorders. Grounded, shielded cable was used for all connections. The transducers themselves were isolated from rapid temperature fluctuations and mechanical shock in specially-constructed enclosures, which served to maintain their operating components in the same orientation with respect to the gravity vector. This last precaution was found to be an absolute necessity for accurate work. A valved bypass system provided for accurate zeroing of each transducer/recorder pair immediately before each traverse in the actual electrical and ambient-pressure environment then existing.

One of the probe-traversing rigs was motorised, the motor speed being variable from 3 to 15 rpm through variable input DC voltage, supplied by a nominal 28 V DC power supply connected to line through a variable transformer.

The skin-friction sublayer-fence configuration chosen was that of Figure 6. This provides for a square-edged step in the surface many times
wider than its height, preceded by a minimally-sized round orifice designed to measure the pressure of the field immediately in front of the step. This geometry is only one of several possible. Some of the others are shown in Figure 7. Although the pressure difference created between this type of fence and a local static orifice is only about a third that of a double-sided geometry, the former was chosen as being easiest to make in the very small sizes required (\(d < 0.002\)" in some cases). There was also the problem of dust and oil-film contamination. Other workers have had considerable difficulty detecting contamination in the double-sided configuration, due to the presence of two similarly-sized tiny orifices. In the present geometry, the local static tap has 16 times the area of the fence orifice and is effectively always clear. This allows an easier determination of the onset of contamination.

The fences of each nominal size (\(d = .010", .007", .004"\) and .002") were milled en masse and rigorously cleaned and degreased after the drilling and deburring of the nominal 0.0039" orifices. Deburring was accomplished under high magnification.

The fences were mounted in a \(4 \times 7\) array, with one fence of each of the four nominal sizes at each station. They were a light press-fit into reamed holes and were sealed on the underside with epoxy cement. After mounting, each fence-element L.E. was carefully blended into the plastic surface by hand scraping under magnification. Finally, the actual finished fence heights were measured to better than 0.0001" using a dial gauge and traverse. Figure 4 shows some of the fences mounted in the test plate. Precautions, not always entirely successful, were taken to exclude dust and oil from the test surface.

The fences, and the local static-pressure orifices at each of the seven \(x\) stations, were connected to a bank of 28 U-tube water manometers
except in cases where the large shear to be encountered necessitated the substitution of mercury instruments for certain fences. Since the total pressure differences to be measured were as little as 0.1% of the total dynamic head, and the tunnel ambient-pressure was usually considerably above or below atmospheric, great care had to be taken in sealing each of the several hundred tubing joints in the various pressure-measuring systems. "Glyptal"enamel was used for this purpose.

The theory of wall-pitots in general, and the single-sided skin-friction fence in particular is treated in Appendix I. The calibration and interpretation procedure used forms the burden of Section B.1 of this chapter.

It is believed that this constitutes the first reported use of this particular geometry for shear-stress measurements.

B. Calibration

1. Skin-Friction-Fence Calibration and Interpretation

The object of the calibration procedure used was to provide, for each individual fence, a unique relation between its readings, the local skin friction and any other pertinent parameters of the flow field. (See Appendix I)

A choice of "reference standard" for the calibration had to be made between three possibilities

a) skin-friction balance as used by Coles\(^2\) and others\(^{31,41}\),

b) Preston tube\(^{40}\),

c) calculation.

Choice a) was rejected on the grounds of complexity, difficulty and expense. Considerable doubt as to the calibration of the Preston tube is still evident in the literature, with various investigations\(^{40,41,42}\)
showing differences of up to 15% in their calibrations, whereas the recent work of Fernholz on a correlation of skin-friction data in zero-pressure-gradient flows gave a law which correlated the available data within 5% for some 80% of the data and within 10% for the remainder. It was, therefore, decided to calibrate with reference to this correlation law, which is set out in Appendix VII. The fairly elaborate logic of the calibration may best be understood through reference to the block diagram of Figure 29.

For an adiabatic flow, the Fernholz law requires knowledge of the local values of $R_{\delta_2}$ and $M_{\delta}$. The test section was operated in a zero-pressure-gradient flow, at fixed subsonic $M_{\delta}$, at a series of Reynolds numbers. Boundary-layer total-head traverses were taken at a series of streamwise-distributed stations for each nominal Reynolds number increment. Thus the momentum-thickness distributions on the test wall could be measured, and the local skin friction computed. At the same time, readings were taken of the skin-friction-fence manometers. Since the density corrections required for evaluation of the experimental velocity profiles were small at the Mach number chosen ($M_{\delta} = 0.45$), no total-temperature traverses were thought necessary. Reliance was placed on the well-known relation of Van Driest for the temperature profile of a turbulent boundary layer on an adiabatic flat plate, which gives $T = T(u_\delta, M_{\delta}, \bar{u}/u_\delta)$. This is set out in Appendix VIII.

A machine integration procedure was used to evaluate the experimental profiles for the integral thicknesses $\delta_1, \delta_2, \delta_3, (\delta_1)_{u}, (\delta_2)_{u}, (\delta_3)_{u}$ and for the shape factors $H_{12}, H_{32}, H^{*}_{12}, H^{*}_{32}$. This is set out in Appendix IX.

Finally, the measured results and the computed $c_f$ were plotted against each other in a suitable non-dimensional form for each fence. The method of non-dimensionalisation chosen was that of Hopkins and Keener, which
provides the necessary corrections for effects of compressibility. Since their procedure had proved to work well for both Preston tubes and Stanton tubes, there seemed little reason to doubt that it would prove equally reliable for the generically similar skin-friction fence. The details of this procedure are described in Appendix I.

Examination of the reduced calibration data, showed that for each individual fence, the data could be well fitted by a law of the form

\[ \log(F_R) = A + B \log(F_L) + C [\log(F_L)]^2 \]  (1)

where \( C \) is a very small coefficient. This was not unexpected, since the results for a Preston tube are similar, but the fidelity of the data to the above law was nevertheless comforting. Furthermore, it was found that all the data of all the fences could be fitted remarkably well by the linear part of the same law, viz.:

\[ \log(F_R) = A_1 + B_1 \log(F_L) \]  (2)

Figure 31 shows all 532 measured data points on the same plot, and the best-fit line for form (2) through them, while Figure 32 shows the results for a typical individual fence. All the curve-fitting was done by machine using standard techniques.

This whole data reduction procedure was combined into a single computer program which is listed in Appendix II, and whose logic is, in part, the subject of Figure 30.

Since curve-fitting routines are essentially "mindless" affairs, the precaution was taken of getting a machine plot (Calcomp) of the two- and three-term calibration curves, equation (1), for each fence, together with all the data points for that fence. This allowed an evaluation of which curves
could reasonably be extrapolated outside the ranges of shear covered in
the calibration runs. See figures 31 and 32. In those few cases where
fence readings showed marked scatter, it was decided to use the "universal"
calibration (equation (2)) instead. The "universal" calibration constants
found were:

\[
A_1 = -0.18668 \quad \text{and} \quad B_1 = 0.74325
\] (3)

To review, we now had a relation for each fence, and a "universal"
one for all fences, of the form:

\[
c_f = c_f (\Delta P, P_6, M_6, T_6)
\] (4)

or, since isentropic flow may be assumed,

\[
c_f = c_f (\Delta P, P_w, P_o, T_o)
\] (5)

in terms of experimentally measured quantities. These relations were
obtained at a fixed Mach number and in zero pressure gradient.

Thus, to interpret the readings of a given fence at some other Mach
number, only a simple inversion of the procedure of Hopkins and Keener was
required, but to complete the interpretation, a correction had to be made
for the effects on the readings of any pressure gradient present.

We have, up to now, no entirely successful analytical description of the
viscous flow over a step, with or without an axial pressure gradient,
despite several valiant attempts, most notably by Gadd\textsuperscript{44}, Thom\textsuperscript{45}, Trilling
and Hämkinen\textsuperscript{42}, Taylor\textsuperscript{51} and Patel\textsuperscript{32}.

The simple first-order correction procedure described by Patel did,
however, show that we might expect such corrections to be reasonably small -
less than 25\% - even in the most severe pressure gradients we proposed to
investigate. In contrast, Preston tube corrections can be over 60\%. With
this in mind, therefore, a somewhat extended form of Patel's analysis was completed, taking into account non-uniformity of fluid properties, and using a rather more realistic, but still inexact, prescription for the effect of the oncoming non-uniform (in $y$) momentum flux on the readings of the device. This is set out in Appendix I.

It should be recognised that the solution of this problem involves the solution of the full two-dimensional Navier-Stokes equations for a compressible fluid, with mixed boundary conditions, a task which is currently considerably beyond the state of the art. Even the attempt by Gadd $^{43}$ on the purely incompressible laminar Couette flow over a small step proved of limited applicability. There is certainly room for work on a better correction procedure - either theoretical or experimental - than that used here. Given such an improved procedure, the data of this study could easily be reworked to incorporate it.

The assumption is made that the effects of compressibility and of pressure gradient are only weakly coupled. This will be true for any boundary layer whose laminar sublayer has sensibly constant (in $y$) fluid properties, especially density. For flows up to at least Mach 2, this assumption will not be seriously in error to at least the order of accuracy implied by the pressure-gradient-effects correction-analysis.

It will be seen from Figure 30, which treats the logic of the interpretation procedure, that such a decoupling is in fact assumed, since in each cycle of the iteration procedure described in Appendix I, the raw reading is corrected before reference to the calibration equation.

The exponent in the relation

$$\Delta P = aT^b$$

required by the correction analysis to account for effects of unit Reynold's
number variation was obtained by a best-fit to all the calibration data. A value of 1.31 proved to fit all but 10% of the data to better than 5%.

The computer program for the iterative interpretation procedure described above, detailed in Appendix I and whose logic forms the subject of Figure 30 is listed in Appendix II. In addition to the RHS quantities of equations (1, 2), the program also requires a value, in each case for the relative pressure gradient, \( \frac{d(P/P_0)}{dx} \), obtained from some such diagram as Figure 8. The constants \( A, B \) and \( C \) for each fence and \( A_1, B_1 \) are punched onto cards as part of the output of the calibration program. These cards then form part of the data set of the interpretation program.

Examination of the output of the program for a given run reveals at once whether any of the four fences at each \( x \) station were partially blocked by dust or oil, through a clearly spurious low value of \( c_f \) by comparison with its neighbors. A choice can also be made between the calibration curves to be used in the event - such as at very low absolute values of shear - that extrapolation beyond the calibrated range is needed. In any event, the differences over which this somewhat subjective "weighting" had to be exercised seldom amounted to as much as 10%. Manometer-reading errors at very low absolute shears (\( \Delta P < 2 \) mm water) were the largest single source of random error, though the effects of dust and oil in putting certain fences completely out of commission for individual runs were certainly more prevalent at these low densities. It is thus not surprising to find that experimental scatter of \( c_f \) values is most marked at the lowest-density tunnel-operating conditions.

2. Linearity Checks of Transducers and Recorders

At several intervals during the course of the experiments, linearity checks were performed on the transducers and recorders used. A manometer
was used as the reference standard. Care was taken to ensure that the same transducer-recorder pairing was maintained for all subsequent tests.

It was found that each pair was commendably linear throughout its whole nominal range, except very close to zero pressure difference. The Statham transducers were superior to the Eico in this respect.

The linearity of response of the traverse-reporting potentiometers and recorder channels was also checked at intervals. Provided that due care was taken to allow sufficient warmup time for the line-voltage stabilisation device, power supplies and recorder D.C. amplifiers, no deviation from linearity, within the resolution capability of the recorder, was ever found.

Exclusive of the above caveat, the Moseley recorders were found to be astonishingly drift-free over a period of hours, and are to be highly recommended.

C. Experimental Procedure

For each nozzle profile to be tested, the tunnel was run at a series of standard conditions on $P_0$, $T_0$ and back-pressure, as nearly as possible. Local static pressures were measured, allowing establishment of local free-stream quantities, and, by numerical differentiation, all pressure-gradient-dependent quantities such as $K$. Assumption of a recovery factor of 0.88 (see e.g. Ref. 34) allowed determination of local fluid properties at the surface. The pressure-ratio data were smoothed before being put into the various computational schemes discussed in the next chapter.

Total-head traverses could be taken two at a time, using the two traversing rigs, for each standard operating condition, provided that the probes were not so placed as to cause mutual interference. In each case, the scale factors on the X-Y recorders were adjusted so as to yield the largest trace that would fit onto the 8-1/2" x 11" plotting sheets. The traversing
rigs were driven sufficiently slowly that there was negligible "lag" or "hysteresis" in the plotting procedure.

The data of these traces were then converted to digital form, using variable y spacing to improve accuracy in regions of large $u^2$ variation, and punched onto cards for machine computation of the various integral parameters required for this study. The program used is written-up and listed in Appendix IX. Since total-temperature measurements were not attempted in the shear layers, density corrections are supplied through the relation of Van Driest. (See Appendix VIII.) This applies, strictly speaking, only to the zero-pressure-gradient case, but, since the highest Mach number encountered for a traverse was only about 0.8, the density corrections are small, and the final effect on the integral parameters even further attenuated.

To further the general utility of the program, provision has been made for insertion of $u$ and $T$ data if required. The perennial problem of how to approximate the effect of the first element of such a digitised list is also treated in Appendix IX.

As a check, $\delta_2$ and $\delta_3$ were plotted versus $P_o$ for each $x$, for each nozzle as the data-taking proceeded. In this way faulty data could be culled and the required runs repeated. In cases where there appears to be no reason to favor or discard some data that conflicts with other data obtained under nominally the same conditions, all the conflicting data elements are listed or plotted elsewhere in this report. To resolve doubts, resort was sometimes had to "fill-in" runs at intermediate values of $P_o$.

The values chosen for the parameters required to initiate the prediction programs were the means of several sets of profile data at the particular upstream point chosen. The points chosen were at nearly zero pressure gradient for all runs.
The skin-friction-fence manometers were read at each standard operating condition, for each run. The values quoted in Tables 2–5 are weighted means of several runs, except when the data appeared to group around two or more values. This occurrence probably indicated an incipient problem with contamination, but in such cases, both values are listed, since there seems little valid means to choose between them. Interpretation of the readings required machine computation by the procedure listed in Appendix II.

The tunnel had to be shut down and the test-section doors removed for each change of probe stations. This procedure was unavoidably laborious and resulted in large accumulations of tunnel time for a relatively small amount of data.

In determining the set of "standard" conditions to be run for each nozzle, a quick check of the actual pressure field was made for each nozzle, and a starting profile measured. This enabled a prediction of the likely trajectories for the shear layer on a \( K - R_{\delta_2} \) plot, assuming fully turbulent behavior. In this way, the range of \( P_0 \) likely to bracket the appearance of laminarisation could be estimated, and much unnecessary data-gathering avoided.
III. THEORETICAL PROGRAM

A. Brief Survey of Computational Methods

There exists a vast multitude of computational methods for the incompressible turbulent two-dimensional boundary-layer. There exist also a few theories for compressible boundary layers, usually based on some adaptation of an incompressible theory. Fortunately, several excellent reviews of computation methods 36, 48, 49 have appeared recently, which offer some guidance in the maze. There is no point in paraphrasing these here except insofar as the special needs of this study dictate.

In his admirably clear scheme for classifying such theories, Spalding 49 distinguishes between:

(i) "Complete" theories, which aim to solve, by numerical means, more-or-less exactly the partial differential equations of conservation of mass, momentum, energy, etc., using as additional input empirical information about local quantities in the shear-layer such as effective viscosity, effective Prandtl number, etc. Patankar & Spalding 53 and Mellor & Gibson 55 have recently produced improved theories of this type.

(ii) "Parametric Integral" theories, in which profile relations containing several free parameters are assumed to hold good at each longitudinal station. The partial differential equations are then multiplied by each of a set of weighting functions of the dependent and/or independent variables and integrated across the layer, yielding a set of ordinary differential equations having the streamwise coordinate as the independent variable and the free parameters as the dependent variables, which appear linearly. These then require a matrix inversion for isolation, followed by numerical integration of the resultant set of first-
order equations, yielding finally the free paramenters as algebraic functions of the streamwise coordinate. From these, any desired property of the solution may be exhibited - to the accuracy to which that property may be described by the number and nature of the parameters involved.

(iii) "Explicit Integral" theories, in which the partial differential equations, weighted in some manner, are integrated over the shear layer as a first step, yielding ordinary differential equations in "integral parameters" such as $\delta_1, \delta_2, \delta_3$, etc. with the streamwise coordinate as the independent variable. These are then subject to relatively quick and simple numerical solution procedures. These methods differ chiefly in the weighting functions used, the number and nature of the conservation laws invoked and the auxiliary relations used to relate explicitly the various integral parameters one to another.

One has to agree with Spalding that the approach of type (i) offers much hope of eventual relief from the woes of trying to relate an ever larger hierarchy of empirical correlations as we attempt to compute ever more more complex flows. However, the empirical input required for use of any type (i) method in the present case of a compressible boundary layer which may or may not undergo laminarisation appears utterly lacking at this juncture. The inputs required for types (ii) and (iii) computation are more easily obtained from commonly measured quantities. To this end, a representative method of each of these latter types will be examined further.

The method of Moses $^{50}$ and Launder $^{28}$ represents an early attempt to apply a parametric integral method to the problem of an incompressible laminarising flow. Further work on this method is treated in Section B.

The method of Walz $^{33, 34}$ of type (iii) purports to compute all cases of laminar or turbulent, compressible or incompressible, two-
dimensional or axisymmetric boundary layers, with or without heat-transfer, and as such is probably unique in its field. It seemed worthwhile, for the ends of this study, to make a considerable effort to develop a readily useable and foolproof machine-computation procedure incorporating this overall method, and to explore some of its limitations, especially in highly non-equilibrium compressible shear-flows. The relatively transparent nature of the auxiliary relations used in type (iii) methods offers perhaps the best hope, at the present state of the art, of ascertaining the modifications required for prediction of such flows. Section C deals further with this phase of the study.

None of the above methods can be invoked without a more-or-less precise knowlege of the properties of the shear layer at the starting point. Given an error in the specification of the starting conditions, there is a uniform tendency for the prediction to diverge from experiment in adverse, and to approach it in favorable pressure-gradients (usually). Type (ii) methods appear particularly susceptible to this starting problem.

B. The Method of Moses and Launder

In 1964 Moses\textsuperscript{50} proposed a parametric integral theory for calculation of turbulent incompressible boundary layers. This corresponds precisely with the type (ii) scheme outlined above, where the momentum integral equation is the chief conservation relation invoked. A parametric description of the local velocity profile of the form

\[ \frac{u}{u_0} = 1 + \alpha \log \frac{Y}{\delta} + \beta (1 - 3 \left( \frac{Y}{\delta} \right)^2 + 2 \left( \frac{Y}{\delta} \right)^3) \]  

(7)

is used, where \( \alpha, \beta \) are the parameters, which are related to other variables of the problem by the relations
\[ \alpha = \frac{1}{\kappa} \left( \frac{c_f}{2} \right)^{1/2} \]  
(8)

\[ \beta = \alpha (\log (aR_\delta) + 1.1237) - 1 \]  
(9)

\( \kappa \) is the familiar constant from the Coles universal velocity profile:

\[ \frac{u}{u_t} = \frac{1}{\kappa} \log \left( \frac{\gamma u}{v} \right) + B + \frac{\pi}{\kappa} \omega (\gamma) \]  
(10)

Since two parameters are invoked, two conservation relations are required. This is accomplished by causing the momentum integral equation to be separately satisfied over two strips, each of height \( \delta/2 \). This requires empirical specification of the effective viscosity and turbulent normal stress at the join.

With the procedure outlined in part A supra, the net result is a pair of simultaneous, first-order, ordinary differential equations in \( \alpha \) and \( \beta \) of the form

\[ \frac{d\delta}{dx} = F_1 (\alpha, \beta, \delta, u_\delta) \]  
(11)

\[ \frac{d\alpha}{dx} = F_2 (\alpha, \beta, \delta, u_\delta) \]  
(12)

which can be solved by the familiar Runge-Kutta numerical procedure, given starting values of \( \delta, \alpha \) and \( u_\delta \) at some \( x \). Further details of the procedure are given in the cited reference.

The author has strong objections to the use of the quantity \( \delta \) in either experimental or theoretical work of any sort. This quantity is extremely ill-defined, particularly for boundary layers having a large value of the shape factor \( H_{32} \). In particular, its use as a normalizing quantity either for presenting experimental data or for computational purposes is fundamentally unsound, though staggeringly widespread, nevertheless.
Launder subsequently modified this computational method in an effort to calculate the development of an incompressible turbulent boundary layer undergoing turbulent-laminar transition. The shear layer was again considered in two strips, characterised as a "viscous inner layer" and a "shear free outer layer", but the position of the boundary between the strips was allowed to vary. This boundary was at height $\delta_L$. A parametric representation of velocity profiles in the form

$$ \frac{u}{u_\delta} = \alpha \left[ (2 - \beta) \frac{y}{\delta_L} + (\beta - 1) \frac{y^2}{\delta_L} \right] \quad 0 \leq \frac{u}{u_\delta} \leq \alpha $$

$$ \frac{u}{u_\delta} = \left( \frac{y}{\delta_L} \right)^\beta \quad \alpha \leq \frac{u}{u_\delta} \leq 1 $$

was used, where $\alpha = (u/u_{\delta L})$. In addition, a Reynolds number was introduced as an explicit third parameter, defined as

$$ R_{\delta_L} = \frac{u_\delta \delta_L}{v} $$

This required another conservation equation, and conservation of mass within the shear layer was invoked. The net result was a set of three o.d.e.'s of form

$$ \frac{d\alpha}{dx} = F_1 (\alpha, \beta, R_{\delta_L}, u_\delta, \frac{du_\delta}{dx}) $$

$$ \frac{d\beta}{dx} = F_2 (\alpha, \beta, R_{\delta_L}, u_\delta, \frac{du_\delta}{dx}) $$

$$ \frac{dR_{\delta_L}}{dx} = F_3 (\alpha, \beta, R_{\delta_L}, u_\delta, \frac{du_\delta}{dx}) $$

These are again solvable by the Runge-Kutta technique, given starting values of $\alpha$, $\beta$ and $R_{\delta_L}$. It should be noticed that the parameter $\alpha$ is
connected to the wall shear-stress in the same way in the Launder and Moses formulations.

In order to obtain predictions of the data of his experiment, Launder required resort to a non-systematic specification of both the shear stress at the join between the strips and also the specification of that point (in x) from which the calculation should be initiated.

Additional criticisms of this calculation method also arise, from the point of view of ab initio calculation, as, e.g., in basic design: $a$, $b$ and $\delta_L$ are not easily estimable or empirically well-known. Also, the assumption of mass-conservation within the shear layer seems physically unrealistic and manifestly raises some question as to the identity of the outer part of the shear layer vis-à-vis the free stream.

During the early phase of the present study, an attempt was made to overcome these objections through a more systematic formulation, and specification of an entrainment condition, à la Head, in place of the mass-conservation relation. Attention was also given to a parametric representation of the density profile of this sort of shear layer in a compressible flow, with a view to obtaining a parametric integral prediction method for compressible relaminarising flows. This effort proved fruitless, due to the almost complete lack of experimental data and an inability to systemise the choice of initial conditions sufficiently well for an ab initio calculation, and was, therefore, abandoned.

Given a very considerable experimental effort over a period of some years, it may well be possible to generate sufficient data to permit the formulation of a sound parametric integral theory for the compressible case. This data would almost certainly, however, provide the required input for a "complete" theory of the type (i) discussed in section A. This latter approach is, in the end, likely to prove the most generally useful.
C. The Method of Walz et al

1. A Brief Presentation of Theory

Building on earlier work, Walz\textsuperscript{33, 34} produced, in 1965, an explicit-integral theory for calculation of compressible, turbulent or laminar, two-dimensional or axisymmetric boundary layers, with or without heat transfer. The main features of this method are summarised below, but the reader is urged to consult the original references for a full presentation.

The equations for momentum- and energy-conservation and continuity take the forms:

\begin{align}
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= - \frac{dP}{dx} + \frac{\partial \tau}{\partial y} \tag{18}
\end{align}

\begin{align}
c P \left( \rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} \right) &= u \frac{dP}{dx} + \frac{\tau}{\partial y} \frac{\partial u}{\partial y} + \frac{\partial}{\partial y} (\lambda \frac{\partial T}{\partial y}) \tag{19}
\end{align}

\begin{align}
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} &= 0 \tag{20}
\end{align}

where \( \tau = \mu \frac{\partial u}{\partial y} \) \tag{21}

Weighting (18) by \( u^k \) and (20) by \( u^{k+1}/k+1 \) and integrating the sum in the interval \( 0 \leq y \leq \delta \) yields the relation

\begin{align}
\frac{df_k}{dx} + f_k \left( 2 + k + \frac{\varepsilon_k}{r_k} - \frac{M_0^2}{\delta} \right) \frac{du}{dx} + e_k &= 0 \tag{22}
\end{align}

where \( e_k = (k + 1) \int_0^\delta \left( \frac{u}{u_\delta} \right)^k \frac{\partial}{\partial y} \left( \frac{\tau}{\rho_\delta u_\delta^2} \right) dy \tag{23}
\]

\[ f_k = \int_0^\delta \frac{\rho u}{\rho_\delta u_\delta} \left( 1 - \left( \frac{u}{u_\delta} \right)^{k+1} \right) dy \tag{24}
\]

\[ \varepsilon_k = - (k + 1) \int_0^\delta \left[ \frac{\rho u}{\rho_\delta u_\delta} - \left( \frac{u}{u_\delta} \right)^k \right] dy \tag{25} \]
k can be any arbitrary dimensionless number, but the particular choices of \( k = 0 \) and \( k = 1 \) are particularly interesting physically, resulting in the usual momentum integral equation and the kinetic-energy integral equation, with

\[
\begin{align*}
\varepsilon_0 &\equiv \delta_1, \quad \varepsilon_1 \equiv 2\delta_4, \quad f_0 \equiv \delta_2, \quad f_1 \equiv \delta_3 \\
\text{and} \quad -e_0 &\equiv \frac{\tau_w}{\rho \delta u_\delta^2} = \widetilde{\sigma}_f \\
e_1 &\equiv \widetilde{\sigma}_D = \widetilde{\sigma}_f \int_0^1 \frac{\tau_w}{\tau_w} \, d \left( \frac{u}{u_\delta} \right)
\end{align*}
\]  

(26)

(27)

(28)

These last two quantities, namely the skin friction coefficient \( \widetilde{\sigma}_f \) and the shear work (or "dissipation") integral \( \widetilde{\sigma}_D \) have been written with a tilde to emphasise that they are not the usual definitions, having exactly half the numerical value of the more conventional forms.

Walz also finds it useful to introduce the quantities \((\delta_1)_u\), \((\delta_2)_u\), \((\delta_3)_u\), \((\delta_4)_u\) where the subscript indicates that the integrals are to be performed with \( \rho = \rho_\delta \).

Shape factors \( H_{12}, H_{21}, H_{32}, H_{32}^* \) and \( H_{43}^* \) can now be defined:

\[
\begin{align*}
H_{12} &\equiv (\delta_1)_u / (\delta_2)_u \\
H_{21}^* &\equiv \delta_1 / \delta_2 \\
H_{32} &\equiv (\delta_3)_u / (\delta_2)_u \\
H_{32}^* &\equiv \delta_3 / \delta_2 \\
H_{43}^* &\equiv \delta_4 / \delta_3
\end{align*}
\]  

(29)

A length parameter is also defined:

\[
z \equiv \delta_2 R_{\delta_2}^n
\]  

(30)

Two auxiliary functions incorporating \( \widetilde{\sigma}_f \) and \( \widetilde{\sigma}_D \) may be introduced with advantage, viz.:

\[
\begin{align*}
\widetilde{\sigma}_f &\equiv \frac{a}{R^{\alpha/2}_\delta} \frac{\delta_2}{(\delta_2)_u} \\
\widetilde{\sigma}_D &\equiv \frac{\beta}{R^{\beta/2}_\delta} \frac{\delta_2}{(\delta_2)_u}
\end{align*}
\]  

(31)
Finally, a heat-transfer parameter is required, viz.:

$$\theta = \frac{T_e - T_w}{T_e - T_\delta} \quad (32)$$

where $$\frac{T_e}{T_\delta} = 1 + r \frac{Y - 1}{Y} M_\delta^2 \quad (33)$$

Putting (23) - (33) into (22) results in the integral relations for momentum and kinetic-energy conservation for compressible laminar and turbulent boundary layers with heat transfer in the form:

$$\frac{dz}{dx} + z \frac{du_\delta/dx}{u_\delta} (F_1 + n \frac{du_w/dx}{du_\delta/dx} \mu_\delta) - F_2 = 0 \quad (34)$$

$$\frac{dH_{32}}{dx} + H_{32} \frac{du_\delta/dx}{u_\delta} (F_3 - \frac{F_4}{z}) = 0 \quad (35)$$

where

$$F_1 \equiv (2 + n) + (1 + n) H_{12}^* - M_\delta^2 \quad (36)$$

$$F_2 \equiv (1 + n) \alpha (\delta_2/(\delta_2)_u) \quad (37)$$

$$F_3 \equiv 1 - H_{12}^* + 2 H_{43}^* \quad (38)$$

$$F_4 \equiv \frac{\delta_2}{(\delta_2)_u} (28 H_{62}^{n-N} - \alpha H_{32}^*) \quad (39)$$

It should also be noted that the shape factors are connected by the relations:

$$H_{12}^* = \frac{H_{12}}{\delta_2/(\delta_2)_u} + r \frac{Y-1}{2} M_\delta^2 (H_{32}^* - \theta) \quad (40)$$

$$H_{43}^* = r \frac{Y-1}{2} M_\delta^2 \frac{H_{32}}{H_{32}^*} \quad (41)$$

Up to this point, the development has involved no empirical approximations. For the solution of the coupled equations (34) and (35) we now require a hierarchy of empirical relations, summarised below. Only
those for a turbulent boundary layer are given, those for the laminar case being similar in nature:

\[
\frac{\delta_2}{(\delta_2)_u} = \left[1 + r \frac{y-1}{2} M^2 \delta \right] H^* (H^* - \theta) (2 - H^*) \rho
\]

(42)

\[
H^* = H \left[1 + (2 - H^*) \psi \right]
\]

(43)

where \( \psi = 1 - 0.0719 M^2 + 0.00419 M^2 \)

(44)

and

\( \psi = 0.0114 M^2 (2 - \theta)^0.8 \)

(45)

Also,

\[
H_{12} = 1 + 1.48 (2 - H^*) + 104 (2 - H^*)^{0.7}
\]

(46)

\[a = 0.0566 H^* - 0.0842 \]

(47)

\[b = 0.0056 \]

(48)

\[n = 0.268 \]

(49)

\[N = 0.168 \]

(50)

\[r = 0.88 \]

(51)

The functions \( \psi \) and \( \phi \) are seen to be correction functions for compressibility and heat-transfer effects, and are claimed to be useful up to \( M^2 = 5 \).

2. Computational Scheme

Mean values of \( F_{j} \) \( (j = 1, 4) \) may be defined over finite \( \Delta x \equiv |x_i - x_{i-1}| \):

\[
\overline{F_{j}} = \frac{1}{2} (F_{j,i-1} + F_{j,i})
\]

(52)

The coupled differential equations (34), (35) may be put into axisymmetric form by the Mangler \(^{58}\) coordinate transformation, and can then be written in the finite-difference form:
where
\[ A_Z = \left[ \frac{(u_\delta)_{i-1}}{(u_\delta)_i} \right] \quad A_H = \left[ \frac{(u_\delta)_i-1}{(u_\delta)_i} \right] \]

\[ B_Z = \frac{1 - A_Z (u_\delta)_{i-1}/(u_\delta)_i}{(1 + F_1)(1 - (u_\delta)_{i-1}/(u_\delta)_i)} \]

\[ B_H = \frac{1 - A_H (u_\delta)_{i-1}/(u_\delta)_i}{(1 + F_3)(1 - (u_\delta)_{i-1}/(u_\delta)_i)} \]

and \( R \) is the cross-sectional radius. For the two-dimensional case, \( R_i = 1 \) for all \( i \).

This development assumes that the "universal" functions \( F_j \) are in fact linearisable, an assumption which fails significantly only very near to a separation point.

There are several conceivable ways of "marching-out" the coupled equations (53) and (54) by computation involving iterative steps. Only one of these, to be described below, proves to be unconditionally stable for all cases short of separation.

The following quantities are necessary and sufficient input for a calculation:

(i) Initial conditions on: \( x, u_\delta, (\delta_2)_u, (\delta_3)_u \)

(ii) The quantities \( P_{i=1}, T_0, \Delta x \)
(iii) The velocity field, in the form of a table of $u_\delta$ vs. $x$ (or $(P_\delta/P_0)$ vs. $x$)
(iv) In the case of heat transfer, a table of $T_w$ vs. $x$
(v) In the axisymmetric case, a table of $R$ vs. $x$

This information, in conjunction with the hierarchy of exact and empirical relations (38) - (51) and their antecedents allows the following stepwise procedure:

(i) Find $(H_{32}^*)_{i=1}$
(ii) Estimate $(H_{32}^*)_{i=2}$
(iii) Thus find all coefficients of (53) and (54)
(iv) Hence obtain a first estimate of $z_{i=2}$
(v) Put these estimates of $(H_{32}^*)_{i=2}$ and $z_{i=2}$ into RHS of (54)
(vi) Hence obtain revised estimate of $(H_{32}^*)_{i=2}$
(vii) Compare the latest and penultimate estimated values of $(H_{32}^*)_{i=2}$.

If the difference exceeds some predetermined value, iterate, by returning to step (iii) with the latest estimate of $(H_{32}^*)_{i=2}$.

(viii) After sufficiently close convergence of successive estimates of $(H_{32}^*)_{i=2}$ the step is complete, and steps (ii) through (vii) are repeated, with iteration as required, to find $(H_{32}^*)_{i=3}$ and so forth.

For best accuracy, the largest allowable value of $\Delta x$ is that for which $\Delta u_\delta < 0.015 u_\delta$ in the worst case. Convergence to $|(H_{32}^*)_{i}^{(m)} - (H_{32}^*)_{i}^{(m-1)}| \leq 0.00001$ typically requires less than 4 iterations, except at very close approach to a separation point. ($m$ is the order of the iteration step.)

This accuracy is barely within the single-precision resolution capability of a computer, and oscillations of amplitude less than 0.0001 can occasionally arise. In such a case the convergence criterion may be relaxed an order of
magnitude without serious effect on the overall accuracy of the calculation.

Appendix V contains a FORTRAN IV program to carry out this calculation for all the cases for which it is useful, and specific instructions for use of the program. The program has been written in such a way as to facilitate its use even by those totally unfamiliar with the method. The step size is found automatically, and the convergence criterion is reset as required. Every effort has been made to ensure that numerical "disasters" are forecast by the program as the calculation proceeds, and the appropriate messages printed-out before the case is terminated and the next case started. However, the unexpected can still happen, especially if erroneous data is supplied, so that provision has also been made for a "debug" printout in which the major quantities of each subroutine are printed-out at each iteration or step, as well as much other information. This facility should not be invoked lightly, as the paper output is staggering.

Since $H_{32}$ is the chief auxiliary quantity of the calculation, it was decided to specify $(\delta_2)_u$ and $(\delta_3)_u$ as the input required at the start of the calculation rather than $\delta_2$ and $\delta_3$. The $(\ldots)_u$ quantities are easily obtained if the starting velocity profile is available, while $\delta_2$ and $\delta_3$ require also that the density profile be available. In the event that only the numerical values of $\delta_2$ and $\delta_3$ are known, the auxiliary relations (42) - (45) may be invoked for $(\delta_2)_u$ and $(\delta_3)_u$. In the event that only $(\delta_2)_u$ (or $\delta_2$) and $H_{12}$ (or $H_{12}^*$) are known, $H_{32}$ may be estimated by means of (40) and (46). The calculation method is naturally sensitive to poor estimates of $H_{32}$, but tends to be self-correcting after a number of steps, especially in zero or favorable pressure gradients.

It should be noted that the scheme adopted calculates the local values of $M_0$ through an isentropic-flow assumption. If the case in question
involves such a large heat addition to the flow that the isentropic assumption is seriously in error, a minor reprogramming, involving the use of $M_\delta$ as an additional input quantity will be required.

3. New Relations of Fernholz and Escudier

Fernholz\textsuperscript{35} has recently produced alternate empirical formulations of the skin-friction, dissipation and shape-factor relations for a compressible turbulent boundary layer on a flat plate. These may be used in place of the Walz relations in the following way:

Replace eqn. (43) with:

\[ H_{32}^* = 1.80 + 0.0072 M_\delta \] (58)

Replace (46) with

\[ H_{12} = (1 - \left(c_1 \frac{1}{2}\right)^{1/2}) (7.506 - 0.202 \log_{10} \left(R_\delta^* \sigma^* \right)) \] (59)

where

\[ c_1 = \frac{0.01015}{(R_\delta^* \sigma^*)^{0.15}} + \frac{0.786}{R_\delta^* \sigma^*} \] (60)

and

\[ \sigma^* = \left(\frac{\delta_2}{\delta_2}\right)^{0.7} \] (61)

where

\[ \frac{(\delta_2)}{\delta_2} = [H_{12}^* - r \frac{Y-1}{2} M_\delta^2 (H_{32}^* - \theta)]/ H_{12} \] (62)

where

\[ H_{12}^* = H_{12} = 0.4 M_\delta^2 f_2(\theta) \] (63)

The function $f_2(\theta)$ is unity for the adiabatic case.

We also obtain the skin friction coefficient

\[ c_f = r_1 \left[ 0.01015 \left(\frac{(\delta_2)}{\delta_2}\right)^{0.595} + \frac{0.786}{R_\delta^*} \right] \] (64)

where

\[ r_1 = 1 + r \frac{Y-1}{2} M_\delta^2 (1 - \theta) \] (65)
thus we can replace (47) by
\[
a = \frac{1}{2} c_f \frac{(\delta_2)u}{\delta_2} R_{0.268}^{0.268}
\]  
(66)

The dissipation coefficient \( \beta \) now becomes
\[
\beta = \frac{1}{4} c_f H_{32} \cdot R_{0.268}^{0.268} \frac{(\delta_2)u}{\delta_2}
\]  
(67)

in place of eqn. (48).

These relations have also been programmed into an alternate sub-
routine ("HANS") and are thus available for use as explained in Appendix V.

New relations, not restricted to zero pressure-gradient, are also
available from Escudier\(^{36}\), and take the following forms:

Eqn. (46) is to be replaced by:
\[
H_{12} = 1.55 \left[ 0.0971 + (0.009428 - 3.1 (1.431 - H_{32}^{1/2})^2) \right]^{-1}
\]  
(68)

Eqn. (47) is to be replaced by:
\[
a = S_s R_{0.268}^{0.268} \frac{(\delta_2)u}{\delta_2}
\]  
(69)

where
\[
S_s = \left[ 0.243 \zeta^2 + 0.0376 \zeta - 0.00106 + 0.0914 \zeta^2 (1 + \frac{65}{\zeta}) \right]/L^2
\]  
(70)

where
\[
\zeta = \frac{2}{3} H_{32} - 1 + \left[ \frac{2}{3} H_{32} \left( \frac{2}{3} H_{32} - 1 \right) \right]^{1/2}
\]  
(71)

and
\[
L = \ln \left[ \frac{3.389 R_{0.268} \zeta}{(1 - \zeta)(1 + 2 \zeta)} \right]
\]  
(72)

Eqn. (48) is to be replaced by:
\[
\beta = S \cdot R_{0.268}^{0.268} \frac{(\delta_2)u}{\delta_2}
\]  
(73)

where
\[
\overline{S} = (2 \zeta + 1) S_s/3 + T_q
\]  
(74)
where

\[ T_q = \begin{cases} 
0.00565 (1 - \zeta)^{2.715} & \text{for } \zeta < 1 \\
0.01 (\zeta - 1)^{3} & \text{for } \zeta > 1 
\end{cases} \quad (75) \]

These relations are also available in the alternate subroutine "MARCEL" and may be implemented according to the instructions of Appendix V.

It should be noted that these relations are also useful for cases where the velocity profile shows an internal maximum, including the extreme case of a wall jet. Since the formulation is for the incompressible case, the required compressibility and heat transfer corrections are put in by the usual Walz relations.

4. Comparisons with Existing Data

The predictions of the Walz method were compared with adverse pressure-gradient data of Moses \textsuperscript{50} and Goldberg \textsuperscript{62} for both separating and non-separating cases, with zero pressure-gradient data of Smith \& Walker \textsuperscript{63}, and with severe favorable pressure-gradient data of Launder \textsuperscript{28}, in addition to the data of the present study. The new relations of Escudier were also used for each test case, and the relations of Fernholz also used for a prediction of the zero pressure-gradient case. The results of the test cases based on previous data appear in Figures 11 - 13, while the data of the present study is treated in Figures 14 - 25.

Appendix VI details the sources of the particular data chosen, and lists any assumptions required to initiate the computations.

Looking first at the adverse pressure-gradient data of Figure 11 we note that both the Walz and Escudier relations predict early separation for the steep adverse, followed by relaxing, pressure-gradient of Moses # 5 while in the similar case of Goldberg # 3, the Walz relations result in an early
separation prediction. The Escudier relations show an under-prediction of shape-factor development. The Escudier relations give a more accurate prediction of $\delta_2$ development in each case. For the less severe, non-separating case of Moses # 6, there is little to choose between the methods, for $\delta_2$ prediction, but the Escudier relations for shape-factor again prove superior. The same findings hold good for the similar, but separating case of Goldberg # 6.

Figure 12 shows a conclusively laminarising case of Launder. Prediction of $R_{\delta_2}$, up to the turbulent-laminar transition, is good for both sets of relations, with those of Escudier slightly superior. Thereafter the data lies grossly below the prediction. The shape factor prediction is decidedly poor, especially after transition, with the Walz relations proving superior.

Precisely the same behavior in very steep negative pressure-gradients was found when the Walz method was applied to the prediction of the flows of the present study. The uniform tendency of laminarising flows to depart from predictions based on correlations of normal turbulent boundary layer behavior in these two ways is crucial to the further development of this report, and will be discussed at length in the next chapter.

If one wishes to decide on a "best" method between the three relations offered by Walz, Fernholz and Escudier, the flat-plate data represented by Figure 13 proves singularly frustrating: For $\delta_2$ calculation, there is little to choose between Walz and Fernholz, while the prediction of Escudier is definitely somewhat off. The same is true of the $c_f$ prediction, with the Fernholz relations showing a slight initial superiority. For the shape-factor calculation, however, the Escudier relation corrects itself most rapidly, while the other two methods both predict high values of $H_{12}$. 
The general conclusions, based on a comparison of all the predictions with their corresponding data, seem to be that $\delta_2$ is well predicted, with the relations of Walz proving slightly better, most especially in favorable pressure-gradients. $H_{12}^*$ is, however, relatively poorly predicted, with the Escudier relations clearly superior in zero and adverse pressure-gradients, and the Walz relations superior in favorable pressure-gradients. A high prediction of $\delta_2$ development, and a low prediction of $H_{12}^*$ development are consequent upon the appearance of turbulent-laminar transition in the physical flow.
IV. PRESENTATION AND DISCUSSION OF RESULTS

A. Location of Turbulent-Laminar Transition Points

Previous work on laminarisation\textsuperscript{27, 28, 29} indicated that the onset of laminarisation in a turbulent boundary layer is marked by a sharp decrease of the momentum thickness below that to be expected for a "normal" turbulent boundary layer. This showed clearly in the work of Launder\textsuperscript{27, 28}, who presented his results in the form of $H_{12}$ vs. $x$ diagrams. At the low velocities with which that study was concerned, $H_{12}$ and $H_{12}^*$ (see definition in Chapter III, Section C) are virtually identical. The Russian work\textsuperscript{29} was, by contrast, carried out in the high-subsonic regime, in the converging section of a two-throat tunnel, downstream of a normal shock standing at the exit of the first converging-diverging nozzle. This identification of the turbulent-laminar transition points was confirmed by hot-wire measurements in each of the studies cited.

A difficulty arises, however, if one wishes to apply a shape-factor criterion to the high-subsonic or transonic free-stream velocity regimes, where the fluid properties change rapidly. In particular, the rapidly decreasing density in that part of the shear layer closest to the wall causes a strong rise in the value of $H_{12}^*$ for $M_6 > 0.7$, as is evident from examination of the definition of this parameter, viz:

$$H_{12}^* = \frac{\int_0^\delta \left(1 - \frac{\rho u}{\rho_0 u_0}\right) dy}{\int_0^\delta \frac{\rho u}{\rho_0 u_0} \left(1 - \frac{u}{u_0}\right) dy}$$

Thus, if one proposes to use a change in the value of $H_{12}^*$ as an indicator of the onset of laminarisation in this regime, it is necessary to observe a
rise superimposed on another rise. This is likely to degrade further the inherently poor precision of identification due to the finite spacing of measuring stations. Following, however, a suggestion of Fernholz\textsuperscript{56}, we might seek to observe the behavior of the parameter $H_{12}$, which depends only on the geometrical shape of the velocity profiles, and disregards the weighting effect of the density ratios. A sharp rise from the reasonably constant values exhibited by this quantity for non-laminarising boundary layers would signal with greater precision the point of onset of an unusual phenomenon. As will be detailed below, it was in fact found that any sharp rises in $H_{12}$ were coincident both with marked increases in the rate of change of $H_{12}^*$ and also with marked decreases in the measured skin-friction. It is thus not unreasonable to infer that a sharp rise in the value of $H_{12}$ for a turbulent compressible boundary layer is also indicative of turbulent-laminar transition.

Three nozzle profiles (Figure 8) were used in the present study, producing the free-stream flows detailed in Figure 9 and Table 1. In addition, a fourth case was created by placing a spoiler just upstream of the trip for a series of runs with nozzle C. This was in an effort to increase the range of $R_{\delta_2}$ studied with this nozzle. The spoiler was the largest that would still allow the relaxation of the shear layer to a state indistinguishable from a normal flat-plate boundary layer before the onset of acceleration.

Tables 2 - 5 and Figures 14 - 25 present the measured values of $c_f$, $H_{12}^*$, $H_{12}$, $H_{32}$ and $R_{\delta_2}$ as functions of longitudinal coordinate $x$ and stagnation pressure $P_o$. Since the test sections were in each case operated in the supersonic throat mode, changes in stagnation conditions should simply be regarded as giving rise to different characteristic densities. The
spread of stagnation temperatures was only some $15^\circ R$, thus changes in $\mu$ between the various "standard conditions" are small.

The qualitative behavior of $H_{12}^*$ and $H_{12}$ as functions of $x$ and $P_o$, as illustrated in Figures 18 - 21 was rather similar for each of the four cases. Only one case need, therefore, be discussed in detail, e.g., nozzle A:

Beyond $x \geq 30''$, the $H_{12}^*$ data appears to divide into two groups, that for $P_o = 5$ and $10''Hg. \ abs.$ diverging from that of $P_o = 15$ and 20 and from the prediction. Beyond $x = 32.8''$, the data for $P_o = 15$ also rises sharply. The behavior for $H_{12}$ is similar, but more marked, especially since the majority of the data and the predictions show an almost constant value of $H_{12}$ with both $x$ and $P_o$ except for the behavior of the errant few.

Turning now to the behavior of $\delta_2$ itself, we note from Figure 22 that while for $P_o = 20$, $\delta_2$ has risen, at $x = 35''$, slightly above the minimum value encountered, (following the sense of the prediction) the trend in the other three cases is for a continued strong decrease. As has been discussed in Chapter III, the general agreement between measurement and calculation is comparatively good, except in this particular range of $x$ and $P_o$ values.

The skin-friction measurements for nozzle A (Figure 14) appear to confirm the trends noted above: At the most upstream measuring station ($x = 19''$), the trend of $c_f$ values with increase of unit Reynolds number (i.e. increase of $P_o$) is in line with the conventional experience with zero-pressure-gradient boundary layers, i.e. $c_f$ decreases with increased Reynolds number. However, as the expansion proceeds, the behavior changes in a surprising way, viz: By $x = 41''$, $c_f$ for $P_o = 20$ has risen steadily to roughly double the initial value; for $P_o = 15$, $c_f$ reaches a plateau at about $x = 35''$, and is less than that of $P_o = 20$ at $x = 41''$, the final
measuring station; for $P_o = 10$, there is a maximum around $x = 30^\circ$, followed by a steady fall, while the $P_o = 5$ data set follows the same trend, but peaks earlier and decreases very much more rapidly thereafter. The end result is that by the final measuring station, the order of $c_f$ values with Reynold's number has been inverted, $c_f$ now increasing with unit Reynold's number.

Given the above behavior of $H_{12}^*$, $H_{12}$, $\delta_2$ and $c_f$ with $x$ and $P_o$, and remembering that $K$ is an almost linear function of $P_o$ at any $x$, it seems not unreasonable to interpret the data in the following way:

(i) the $P_o = 5$ boundary layer entered turbulent-laminar transition at $28^\circ < x < 30^\circ$, with immediate anomalous reduction of $\delta_2$ and $c_f$,

(ii) the $P_o = 10$ layer similarly entered transition at $29^\circ < x < 32^\circ$,

(iii) the $P_o = 15$ layer entered transition for $x > 32^\circ$, with little change in $c_f$ from the entering value,

(iv) the $P_o = 20$ layer did not enter transition at all.

In an entirely similar way, transition points can be located for the other three cases. Nozzle B, for example, would be declared to have the $P_o = 5$ data showing transition at about $x = 37^\circ$, while the $P_o = 10$ data shows a rather "hesitant" transition at about $x = 39^\circ$. The $P_o = 20$ data behaves in an unexceptionally turbulent manner.

B. Shear-Layer Trajectories in the $K - R_{\delta_2}$ Plane

At this stage in our knowledge of laminarising turbulent boundary layers, it would be distinctly unwise to assert that we could list all the important parameters of the problem and thus perform a definitive dimensional analysis, vide Taylor$^{57}$. The following quantities, however, have proven experimentally to influence the onset and course of the turbulent-
laminar transition phenomenon in the adiabatic-wall case: \( u_\delta, \delta, u_w, p_w, \rho_\delta, du_\delta/dx \).

Clearly this list is neither exhaustive nor unique, and the reference conditions are arbitrary. We can, however, construct the following two dimensionless groups:

\[
K \equiv \left( \frac{u_w}{\rho_w u_\delta} \right) \frac{du_\delta}{dx} \quad \text{and} \quad R_{\delta 2} \equiv \left( \frac{\rho_\delta u_\delta}{u_w} \right)
\]

The significance of the group \( K \) has been remarked upon by Rotta, Mellor and Launder and several others in their consideration of incompressible flows. Since it is the ratio of pressure and shear forces acting on that limiting region of the shear layer at the wall, the only additional specification that has to be made for the compressible case is that fluid properties be evaluated at wall conditions, as has been indicated.

Examination of the literature reveals a surprising lack of uniformity in the definition of momentum thickness Reynolds number for compressible boundary layers, even on an adiabatic wall. Some authors use \( \rho_w \) in the numerator or \( u_\delta \) in the denominator. The form cited above seems, however, to be the most logical for the needs of this problem when considered thus: We wish to express the ratio of the inertial forces to the viscous forces which exist in that part of the shear layer nearest to the wall. Since we do not know with precision any other characteristic momentum, we choose that of the free stream, and so, for consistency, the density term is to be evaluated at free-stream conditions. The viscosity term should be evaluated at those well-defined conditions most closely approximating those of the near-wall layer. This indicates the choice of \( T_w \) as the suitable temperature for evaluation of \( \mu \).
An important point to notice about $K$ and $R_{\delta_2}$ is that while they share several variables, $K$ contains only variables pertaining to the free-stream, the only boundary-layer parameter, $\delta_2$, being contained in $R_{\delta_2}$. With an eye to achieving rational design procedures for nozzle flows, we might, therefore, seek to understand the laminarising phenomenon in terms of the behavior of these two quantities. It will now be shown that this approach is largely successful, rendering more intelligible the data of this investigation and the other detailed laminarisation data published up to now.

We come first to the concept of a "trajectory" of the shear-layer in the $K - R_{\delta_2}$ plane. This is simply obtained by a cross-plot of the data of, for example, Figures 9 and 22, for each discrete value of $P_o$ and with $x$ as the cursory polar coordinate. This way of presenting the data appears to be particularly useful in describing the "history" of a compressible shear layer in a rapidly expanding flow. The local density appears in both the force-ratios $K$ and $R_{\delta_2}$, but in an opposite sense. The mere fact of its rapid change makes separate evaluation of acceleration and Reynold's number effects extremely difficult at best. The cross-plotting technique offers, however, a way of assessing the coupled effects.

Figure 26 shows some typical shear-layer trajectories encountered in this and former studies. Since there exist no shockless flows having a discontinuous first spatial derivative of velocity, all trajectories must enter the diagram from the abscissa if considered far enough upstream. (It might be noted, in passing, that the negative $K$ part of the plane is not unpopulated, and that the negative $R_{\delta_2}$ part of the plane contains the interesting wall-jets treated by Escudier\textsuperscript{36.}.) It is an interesting exercise to try to visualize the flow required to produce an arbitrary trajectory. It quickly becomes clear that the upper right part of the positive-positive
quadrant is difficult of approach, requiring ever larger flow devices
operating at ever lower characteristic densities, once the practical limits
of \( \frac{du}{dx} \) have been realised. These latter are set by the tendency of a
converging flow to accommodate a rapid change in area ratio through
formation of an extended separation zone:

Nozzle B of this study exhibited clearly such a separated zone.

Since \( x \) is a cursory parameter for each trajectory, we can readily
mark on each trajectory that point at which turbulent-laminar transition,
if any, is detected. Figure 27 shows the result of such an operation for
each of the four cases of this study. It will be observed that the
trajectories for each case form a clearly evolutionary "family", though
the fundamental shapes involved are hardly intuitively obvious. Open
symbols mark transition points as determined by the methods described in
Section A. Solid symbols denote that last transition point encountered in
each hierarchy of increasing values of \( P_0 \) (or equivalently, unit Reynold's
number). The solid symbols may thus be regarded as defining the least
favorable condition for each case that will in fact lead to transition in
the converging portion of the duct. Such points will be called "boundary-
points" for purposes of further discussion.

Taken as a group, the trajectories of Figure 27 strongly suggest the
existence of a region of the \( K - R_6 \) plane which is uniquely susceptible to
the relaminarisation phenomenon. Figure 28 shows the happy result of
combining all the available data, including also that other published data
which can be reworked into the required form, namely that of Launder and
Wilson. The observations of Kline et al. and the tentative recommendation of Launder have also been indicated for their respective ranges of $R_{\delta_2}$.

One observes that the data populates a region to the upper left of a line of boundary points. A line has been fitted through the available points and christened the "transition boundary". A suggested design procedure invoking the properties of this boundary and its associated regions is treated next.

C. Recommended Design Procedure for Laminar-Throat Supersonic Nozzles

1. Adiabatic Wall

The presentation and discussion up to this point admits of the following statements:

(i) There exists a parameter $K$ which is a function of flow-device geometry and upstream conditions alone.

(ii) The development of the quantity $R_{\delta_2}$ may be adequately calculated up to the point of turbulent-laminar transition by several methods, one of which is that due to Walz.

(iii) Beyond such a transition point, the value predicted for $R_{\delta_2}$ is too large.

(iv) There exists a region of the $K - R_{\delta_2}$ plane outside of which turbulent-laminar transition has not been observed.

(v) Sustained entry of a shear-layer trajectory into this region results in turbulent-laminar transition.

On the basis of the available evidence, it is difficult to be more precise about the units in which "sustained entry" is to be measured. As a very tentative number, however, it would seem that residence over the boundary for $\Delta x > 20 \delta_2$ is likely to ensure laminarisation of the shear layer. It should be noted in this regard that the effect is not a discontinuity in any sense. The effect of $K$ on the supression of turbulence
production is monotonic in \( 0 \leq K \leq 3.5 \times 10^{-6} \) (Kline\textsuperscript{22}), and the effects on departure of the velocity profiles from the logarithmic law of the wall are similarly continuous, with effects being clear long before the jump in \( H_{12} \) (Launder\textsuperscript{27, 28}). The position of the recommended transition-boundary is simply a consequence of the transition marker chosen. No difficulty need, however, arise providing due care is taken to maintain a consistent approach to the question of prediction.

In the light of these observations, therefore, the following steps taken at the basic design stage of an adiabatic-wall nozzle seem likely to ensure an effectively laminar flow in the shear layer at the throat or before:

(a) Using conventional potential-flow theory, compute the near-wall free-stream flow, and obtain, in particular \( K = K(x) \) for each nozzle profile of interest.

(b) Estimate as accurately as possible, from the known flow conditions upstream of the nozzle, those properties of the boundary layer necessary to initiate a computation from a point near to the entrance of the nozzle. Any calculation scheme capable of reasonably accurate prediction of \( R_{\delta_2} = R_{\delta_2}(x) \) may be used. The method of Walz discussed in Chapter III is one such method.

(c) Plot \( K \) vs. \( R_{\delta_2} \) trajectories for each nozzle and shear-layer combination of interest and observe the relation of these to the transition boundary of Figure 28. At this point it should be clear which of the design parameters need be changed to ensure a sustained entry of a trajectory into the laminarising region. Ensure that the value of \( x \) at which this occurs is as far upstream of the throat as possible. Note that the predicted value of \( \delta_2 \) is likely to be too high after the boundary has been crossed.

NB. It may well be impossible to secure a turbulent-laminar transition in subsonic flow for large, high-density devices.
2. More General Cases

Our detailed, quantitative information is essentially limited to the case treated above, viz. the flow of non-reacting, semi-perfect gases in the shear layer on smooth adiabatic walls, with reasonably small freestream turbulence. One might, however, expect the following trends to apply, judging from the body of literature surveyed in Chapter I:

(i) In the case of wall-cooling, the transition boundary of Figure 28 will be displaced to lower $K$ and higher $R_2^6$. Thus the design procedure of part 1 of this section is clearly conservative for this case.

(ii) The reverse is true for hot-wall boundary layers or for boundary layers having internal energy addition by any means, including chemical reaction.

(iii) The rough-wall case is less favorable for laminarisation.

(iv) The effect of wall curvature on the laminarisation phenomenon is probably additive, i.e. the convex-wall case is more favorable than the concave-wall case.

D. General Discussion

The use of the shape-factor change as an indicator for turbulent-laminar transition appears to be reliable and reproducible. Its use in further research into the phenomenon is to be recommended.

In view of the reservations expressed in Appendix I about the procedure used for correction of sublayer-fence readings for severe pressure-gradient effects, it is felt that the skin-friction values quoted should be treated with some caution for exact quantitative purposes, especially since these appear to be the first determinations of high-pressure-gradient transitional skin-friction by a quasi-direct method. Since the same correction
procedure has been consistently applied to all cases of this study, it
does, however, seem valid to compare trends in the way discussed in part 1
of this section.

There is no apparent reason to doubt the usefulness of the single-
sided fence geometry for measurements in very thin, compressible boundary
layers in a small pressure-gradient. The universal calibration factors
obtained in this study may be considered definitive for want of any other
data. In the light of experience with Preston tubes, the calibration is
likely to be invalid for fully laminar flows. The use of single-sided
fences in this case will, therefore, require a new calibration. Non-
adiabatic flows are also excluded from the turbulent calibration obtained
in these studies.

It is probable that the profile measurements made in this study were
in no case carried out sufficiently far downstream to be in a fully
laminarised shear-layer. This was a dual consequence of the extremely
thin shear-layers encountered and the appearance of bow-shocks on the
probes, rendering further measurements of highly questionable accuracy.
We have, however, the previous work of Launder\textsuperscript{27,28} as a guide to the
characteristics of a fully laminarised shear-layer.
V. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

(i) It has been confirmed that an adiabatic turbulent boundary layer can be caused to enter turbulent-laminar transition for sufficiently low values of momentum thickness Reynold's number \( R_{\delta} \) and sufficiently high values of a free-stream acceleration parameter \( K \).

(ii) Previously published incompressible results have been extended into the ranges of Mach and Reynold's numbers typical of actual engineering devices, viz: \( M \approx 1, R_{\delta} \approx 10^4 \).

(iii) It has been found that the behavior of a boundary layer in respect of turbulent-laminar transition may be described by consideration of its trajectory in the \( K - R_{\delta} \) plane.

(iv) There exists a region of this plane, the boundary of which has been experimentally determined, inside of which turbulent-laminar transition occurs. Turbulent-laminar transition has not been observed outside this region.

(v) An ab initio design method has been developed which incorporates these findings and is capable of ensuring laminar flow upstream of, and at the throat of, a sufficiently small nozzle. As a by-product, there has resulted a computer program suitable for computation of turbulent or laminar, two-dimensional or axisymmetric, compressible boundary layers, with and without heat-transfer.

(vi) A new instrument of extremely small characteristic height, the single-sided sublayer fence, has been developed for measurements of wall shear-stress in thin, turbulent, adiabatic, compressible boundary layers with or without an imposed axial pressure-gradient.

(vii) A universal calibration function for this class of instrument has been developed.
(viii) Wall shear-stress measurements taken in regions of turbulent-laminar transition show a progressive reduction with streamwise distance when compared to measurements in non-laminarising boundary layers. These are believed to be the first measurements of wall shear-stress in a steep pressure-gradient by a quasi-direct method.

(ix) It is recommended that similar studies be performed in a larger facility to enable delineation of the turbulent-laminar transition boundary for still larger values of $R_s^2$.

(x) The effects of heat-transfer on turbulent-laminar transition should be further investigated.

(xi) There is a need for more work on shear flows over imbedded small steps, both with and without an imposed pressure-gradient, and in the presence of heat-transfer to or from the wall.
VI. REFERENCES


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APPENDIX I - THEORY OF THE SKIN-FRICTION FENCE

In 1953, Preston \(^{40}\) showed that for incompressible turbulent boundary layers, if some form of universal "law of the wall" of form:

\[
\frac{u}{u^*} = f \left( \frac{u_{\tau} y}{v_\delta} \right)
\]

(I-1)

was assumed, and if no displacement effects were taken into account, then, from a dynamical similarity analysis for wall-pitots:

\[
\frac{\Delta P d^2}{\frac{1}{4} \rho_\delta v_\delta^2} = f_p \left( \frac{\tau_w d^2}{\frac{1}{4} \rho_\delta v_\delta^2} \right)
\]

(I-2)

He obtained an empirical form for \(f_p\), and was followed by several other workers \(^{32}, 41, 42\) who also obtained formulations for the incompressible calibration function \(f_p\) differing between themselves by up to 15%. In 1966 Hopkins and Keener \(^{31}\) proposed a correlation method based on (I-2) which dealt highly successfully with flows up to \(M = 3.4\). This requires non-dimensionalisation in the form:

\[
f_2(T') R_d^2(M_s/M_\delta) = F[f_2(T') R_d c_f]
\]

(I-3)

where

\[
f_2(T') \equiv \left( \frac{u_\delta}{u^*} \right) \frac{\rho^*}{\rho_\delta}
\]

(I-4)

the prime denoting evaluation of fluid properties at the Sommer and Short \(^{43}\) reference temperature \(T'\) as defined in the notation. The remaining functions requiring definition are:

\[
R_d \equiv (\rho_\delta u_\delta d)/u_\delta
\]

(I-5)

\[
M_s \equiv \left( \frac{2 \Delta P}{\frac{\gamma}{\rho} \frac{P_s}{\gamma}} \right)^{1/2}
\]

(I-6)
It was found that this correlation collapsed all their supersonic Preston and Stanton tube data onto the original Preston incompressible calibration curve with quite astonishing accuracy, leading to substantial identification of the functions \( f_p \) and \( F \) above. Since the considerations leading to (I-3) are by no means specialised to Preston and Stanton tube geometries, this correlation method was adopted in calibrating the single-sided fences used in this study so as to account adequately for the effects of compressibility.

(I-4) was specialised for air by use of the Sutherland viscosity formula, yielding, with the assumption of an adiabatic wall:

\[
T' = T_0 \left(1 + 0.1142 M_0^2\right) \quad (I-7)
\]

\[
f_2(T') = \left(\frac{T'}{T_0 + 198.6}\right)^2 \left(1 + 0.1142 M_0^2\right)^{-n} \quad (I-8)
\]

In 1965 Patel\(^{32}\) discussed the application of wall pitots to shear measurements in pressure-gradients. He indicated that the sublayer fence was far less subject to error than Preston or Stanton tubes and presented a first-order analysis for the effect of pressure-gradient on fence readings. This may be rederived and further elaborated in a way relevant to the needs of the present study:

\[
r = \rho_w v_w \frac{\partial u}{\partial y} \quad (I-9)
\]

which, with the boundary condition at the wall:

\[
\frac{\partial r}{\partial y} = \frac{dP}{dx} + h.o.t. \quad (I-10)
\]

implies, after differentiating with respect to \( y \):

\[
\frac{d^2 u}{dy^2} = \frac{1}{u_w} \frac{dP}{dx} \quad (I-11)
\]
This has the solution
\[ u = \frac{\tau_w}{\mu_w} y + \frac{1}{2\mu_w} \left( \frac{dP}{dx} \right) y^2 \]  
(I-12)

This gives an estimate of the sign and magnitude of the curvature induced in the sublayer velocity profile by the imposed pressure gradient.

If we now assume, not unreasonably, that \( \Delta P \) depends essentially on the momentum flux entering the region of influence of the fence, we can define a mean:
\[ \bar{u}^2 = \frac{1}{d} \int_0^d u^2 \, dy \]  
(I-13)

and, substituting from (I-12) supra,
\[ \bar{u}^2 = \frac{1}{d} \int_0^d \left( \frac{\tau_w}{\mu_w} y + \frac{1}{2\mu_w} \frac{dP}{dx} y^2 \right) \, dy \]  
(I-14)

Now, for zero pressure gradient, the second RHS term of (I-12) falls away, and thus we get the comparison:
\[
\frac{\Delta P}{dP/dx} \neq 0 \quad \frac{\Delta P}{dP/dx} = 0
\]
\[
= \left[ 1 + 3 \frac{d}{\tau_w} \frac{dP}{dx} + \frac{3}{4} \frac{d}{\tau_w} \left( \frac{dP}{dx} \right)^2 \right]
\]  
(I-15)

Comparing surface-pitot data of many workers, Patel found that

Reynold's number variation could be accounted for by writing
\[ P = a \tau_w^b \]  
(I-16)

where \( b = b(R_d) \)  
(I-17)
and  
\[ a = a(p, v, d, \ldots) \]  
(I-18)

Experimental values of \( b \) for various geometries varied between 1 and 1.67.

The exponent \( b \) was determined as 1.31 for the data of the present study as described in Chapter II Section B.

Thus, finally, combining (I-16) and (I-15),

\[
\frac{\Delta P}{\Delta \rho} = \frac{\Delta P}{\Delta \rho} = 0 \left[ 1 + \frac{3}{4} \left( \frac{\delta}{\delta x} \right) \frac{\Delta P}{\Delta x} + \frac{3}{4} \left( \frac{\delta}{\delta x} \right)^2 \frac{\Delta P}{\Delta x^2} \right]^{1.31} \]  
(I-19)

(I-19) thus allows the relation of a measurement made in a pressure gradient to a calibration made in a zero pressure gradient flow, although an iterative process is required, since the calibration formula and (I-19) both involve \( \tau_w \). Convergence is, however, rapid. Thus (I-19) taken in conjunction with the existence of a pre-determined \( F \) in (I-3) allows the interpretation of a sublayer fence measurement at arbitrary Mach number and pressure gradient. The machine realization of this procedure is treated in the following Appendix. See also Figure 30.

It must be realized that all of the above is merely a highly simplified model of an exceedingly complex flow-field involving separation and re-attachment with unsteady, eddying flow. The calibration procedure is to be defended in the last analysis by empiricism alone. The interpretation procedure rests on no more than a plausible foundation especially for fences extending substantially outside the "sublayer", and, this being so, must share the limbo of all hypotheses, standing until disproven or superceded.
APPENDIX II - FORTRAN IV PROGRAMS TO CALIBRATE AND INTERPRET SKIN FRICTION FENCE READINGS.

Program CALIBRATE is used to reduce the calibration data to a particular form of (I-3), viz.:

\[ F_R = a_1 + a_2 \log_{10} F_L + a_3 (\log_{10} F_L)^2 \]

or

\[ F_R = b_1 + b_2 \log_{10} F_L \] (II-1)

where \( F_R \) is the RHS and \( F_L \) the LHS of (I-3).

The notation used in the program is consistent with this report.

Input quantities are:

- \( AA, DD(J) \) = \( d \), the actual fence height (in inches)
- \( NOR \) = integer label of the experiment run
- \( PZERO \) = \( P_o \) (in inches of mercury)
- \( P \) = static pressure (in inches of mercury)
- \( TZERO \) = \( T_o \) (in \(^\circ\) R)
- \( FACT \) = correction factor -
  \[ \frac{(T_o/T_o \text{ std})}{(P_o \text{ std}/P_o)} \]
- \( NFENCE \) = integer fence label
- \( DELP \) = fence differential pressure reading \( \Delta P \)
  (in mm. water)
- \( DEL2 \) = \( \delta_2 \) (in inches)

The program will print out and punch onto cards the coefficients of (II-1), and produce machine plots of the data points and curve-fits. Curve fitting is done by subroutine LSFIT using standard numerical techniques, and the relations of Fernholz which are treated in Appendix VIII. Note that the coefficients associated with label numbers 29 and 30 are the...
"universal" calibration, obtained by two and three term curve-fits to all the data of all the fences. See also Figure 29.

Program INTERPRET requires as input the deck of coefficients produced by CALIBRATE and some additional information about the readings to be interpreted:

- **A, B** array of coefficients for each fence as implied by (II-1)
- **H** fence heights (in inches)
- **NR** integer label of experiment run
- **NDATA** number of input values from that run which are to be interpreted
- **PZERO, TZERO** $P_o, T_o$, the actual values of the run (in "Hg and $^\circ R$"
- **PSTD, TSTD** = "standard values" of experiment run (in "Hg and $^\circ R$"
- **DELP** = fence reading (in mm. water)
- **N** = integer fence label
- **PR** local free-stream pressure ratio ($P_{\text{static}}/P_o$)
- **GRAD** = local pressure gradient, $\left| \frac{d(P/P_o)}{dx} \right|$ (in inches$^{-1}$)

The program reverses the procedure of (II-1) and invokes the iterative pressure-gradient correction procedure described in Appendix I. Uncorrected and corrected $c_f$ values are printed out, for two- and three-term calibration relations both of each individual fence, and the "universal" calibration. The notation is again consistent with this report. See also Figure 30.
C  PROGRAM CALIBRATE

DIMENSION DD(28), XX(532), YY(532), X(19), Y(19), XA(532), YA(532)
1 *XAL(532), YAL(532), XL(19), YL(19), C(11), E(2), Q(2), R(100), S(100)
2 *SL(100)
DIMENSION XLABEL(2), YLABEL(2)
DATA XLABEL, YLABEL, LOG, FR, 'LOG FL /
CALL NEWPLT ('M5155', '4862', 'WHITE', 'BLACK')
DO 39 J = 1, 28
READ 40, AA
DD(J) = AA
39 CONTINUE
K = 1
DO 200 JJ = 1, 19
44 CONTINUE
READ 11, NOR
READ 22, PZERO, P, TZERO, FACT
DO 200 II = 1, 28
NFENCE = II
D = DD(II)
DELP = FACT*DELP
READ 22, DELP, DEL2
D = D/12.0
DEL2 = DEL2/12.0
ACHM = (5.00*((PZERO/P)**0.2857 - 1.e0)**0.5
ACHMS = (5.00(((P+DELP/345.18)/P)**0.2857 - 1.e0)**0.5
ROZERO = 1.32595 PZERO/TZERO
RO = ROZERO/(1.0 + 0.2*ACHM*ACHM)**2.5
T = TZERO/(1.0 + 0.2*ACHM*ACHM)
VIS = 0.00001248*{(T/540.0)**1.5)*738.0/(T+198.0)
U = (12013.9*(TZERO-T))**0.5
RD = RO*U*D/VIS
TP = TZERO*(1.0 + 0.1142*ACHM*ACHM)
F2 = ((TP+198.6)/(TZERO+198.6))**2.0*(1.0 + 0.1142*ACHM*ACHM)**
1 (-4.0)
C NOW INVOKE FERNHOLZ CF PROCEDURE
NN = 0
R1 = (1.0 + 0.176*ACHM*ACHM)**(-1.0)
H32S = 1.80 + 0.0072*ACHM
I = 0
RDEL2 = RO*U*DEL2/VIS
IF (NN) 1, 2
1 SIGS = 1.001
NN = NN + 1
2 CC = 0.01015/(RDEL2*SIGS)**0.15 + 0.786/(RDEL2*SIGS)
H12 = (1.0 - ((0.5*CC)**0.5)*(7.506 - 0.08773*ALOG(RDEL2*SIGS)))
1 **(-1.0)
H12S = H12 + 0.4*ACHM*ACHM
RAT = (H12S - 0.176*ACHM*ACHM*H32S)/H12
SIGS = RAT**0.725
I = I + 1
IF (I = 3) 3, 4.4
GO TO 2
4 CF = R1*SIGS*CC
FL = F2*RD*RD*(ACHMS/ACHM)**2.0
FR = F2*RD*RD*CF
YY(K) = FL
XX(K) = FR
K = K + 1
200 CONTINUE
JK = 0
PRINT 302
DO 300 KN=1,28
NN = KN-1
DO 3002 KM = 1,19
JK = JK+1
NJ = 1+(KM-1)*28 + NN
X(KM) =XX(NJ)
Y(KM) = YY(NJ)
XL(KM) = ALOG10(XX(NJ))
YL(KM) = ALOG10(YY(NJ))
XA(JK) = XX(NJ)
YA(JK) = YY(NJ)
XAL(JK) = ALOG10(XX(NJ))
YAL(JK) = ALOG10(YY(NJ))
3002 CONTINUE
PRINT 301, KN
PRINT 304, (XL(KM), KM=1,19)
PRINT 304, (YL(KM), KM=1,19)
IF (KN=28) 55,55,56
55 E(1) = 1.0
Q(1) = 2.0
E(2) = 6.0
Q(2) = 7.0
56 CONTINUE
C VALUE OF M DETERMINES TYPE OF FIT
M = 1
DO 54 J=1,11
C(J) = 0.0
CALL LSFIT(YL*XL*C,19,2,M)
PRINT 304, (C(I), I=1,2)
C1 = C(1)
C2 = C(2)
PUNCH 306, C1,C2,KN
CALL LSFIT(YL*XL*C,19,3,M)
PRINT 304, (C(I), I=1,3)
PUNCH 305, C(1),C(2),C(3),KN
DO 890 K=1,100
R(K) = 0.0
SL(K) = 0.0
890 S(K) = 0.0
DO 880 J=1,70
R(J) = 2.1 + 0.1*J
SL(J) = C1 + C2*R(J)
880 S(J) = C(1)+C(2)*R(J)+C(3)*R(J)*R(J)
CALL PICTUR(10,5,XLABEL,-8,YLABEL,-8,EQ,-2.0,1,-1,
1 XLYL,-19,0.1,-1,SLR,45,0,0,0,SR,45,0,0,0)
300 CONTINUE
PRINT 304, (XAL(JK), JK=1,532)
PRINT 304, (YAL(JK), JK=1,532)
DO 17 L=2,5
CALL LSFIT(YAL*XAL*C,532,L,M)
PRINT 304, (C(I), I=1,L)
IF (L-2) 70,70,17
70 C1 = C(1)
C2 = C(2)
KN = 29
PUNCH 306, C1,C2,KN
CONTINUE
CALL LSFIT(YAL,XAL,C,532,3,M)
KN = 30
PUNCH 305, C(1),C(2)*C(3),KN
DO 89 K=1,100
R(K) = 0.0
SL(K) = 0.0
89 S(K) = 0.0
DO 88 J=1,70
R(J) = 2.1 + 0.1*J
SL(J) = C1 + C2*R(J)
88 S(J) = C(1)+C(2)*R(J)+C(3)*R(J)*R(J)
PRINT 304, R
PRINT 304, S
PRINT 304, SL
PRINT 307, M
C N.B. R IS THE 'Y AXIS' AND S THE 'X AXIS'
C N.B. Q IS THE 'Y AXIS' AND E THE 'X AXIS'
CALL PICTUR(10C,5,XLABEL,-8,YLABEL,-8,E,Q,-2,0.06,-1,
1 XAL,YAL,-532,0.06,-1,SL,R,45,0,0)
CALL ENDPLT
40 FORMAT (F10.5)
151 FORMAT (25H FENCE HEIGHTS (INCHES)///)
12 FORMAT (36H SUBLAYER FENCE CALIBRATION DATA / )
11 FORMAT (I3)
111 FORMAT (11H RUN NO. K=,I3/ )
14 FORMAT (30H FENCE FL FR / )
22 FORMAT (F12.6)
13 FORMAT (17X,2F12.0 )
15 FORMAT (9F12.5)
302 FORMAT (9H WEED-OUT//)
47 FORMAT (10F10.0)
301 FORMAT (///15H FENCE NUMBER ,I3/ )
303 FORMAT (10F10.0)
9 FORMAT (2F12.4)
304 FORMAT (10F10.5)
305 FORMAT (3F20.10,9X,I3)
306 FORMAT (2F20.10,29X,I3)
307 FORMAT (///22H ***NOTA BENE*** M IS ,I1///)
CALL END
STOP
EXIT
SUBROUTINE LSFIT(X, Y, C, N, L, M)

C IF M IS 1, SUBROUTINE GIVES (L-1)TH ORDER CURVE-FIT TO MINIMISE THE
C SUM OF THE SQUARES OF THE ABSOLUTE ERRORS.
C IF M IS 0, SUBROUTINE GIVES (L-1)TH ORDER CURVE-FIT TO MINIMISE THE
C SUM OF SQUARES OF THE ERRORS RELATIVE TO THE MAGNITUDE OF THE DATA
C MAXIMUM VALUE OF L IS 10
C N IS THE NO. OF DATA PTS. TO BE FITTED, ARR. IN ARRAYS X AND Y
C C IS THE ARRAY OF COEFFICIENTS OF THE RESULTING POLYNOMIAL.
C THAT IS  Y = C(1) + C(2)*X + C(3)*X**2 + C(4)*X**3 + ........

DIMENSION X(600), Y(600), C(11), A(11, 11), B(11), R(11)
DO 1 I = 1, 11
   B(I) = 0.
   DO 1 J = 1, 11
1         A(I, J) = 0.0
   L1 = L+1
   DO 3 I = L1, 11
3         A(I, I) = 1.
   DO 2 ID = 1, N
   IF (M) 5, 9, 10
5            WT = Y(ID)
   GO TO 11
   WT = 1.0
10         DO 8 I = 1, L
8            R(I) = X(ID)**(I-1)
   R(I) ARE THE FUNCTIONS AS REQD. FOR FIT, IF NOT A POLYNOMIAL.*
   DO 2 I = 1, L
   B(I) = B(I) + R(I)*Y(ID)/WT
   DO 2 J = 1, L
2            A(I, J) = A(I, J) + R(I)*R(J)/WT
   CALL SIMO(A, B, 11, KS)
   IF (KS) 6, 6, 5
5            PRINT 7
7            FORMAT (18H NO SOLN IN LSFIT )
   RETURN
6         DO 4 I = 1, L
4            C(I) = B(I)
   RETURN
END
PROGRAM INTERPRET

DIMENSION A(3,30), B(2,28), H(28)
READ 13, A
READ 14, B
READ 4, H
DO 5 K=1,28
H(K) = H(K)/12.0
5
READ 3, NR*NDATA
READ 2, PZERO, TZERO, PSTD, TSTD
PRINT 9, NR
DO 6 L=1,NDATA
READ 7, DELP, P*PR*GRAD
GRAD = -GRAD*PZERO*848.72
P = PZERO*PR
DELP = DELP*(PSTD/PZERO)*TZERO/TSTD
CONST = DELP
M = 0
KL = 0
D = H(N)
ACHM = (5.0*{(PZERO/P)**0.2857 - 1.0})**0.5
ACHMS = (5.0*{(P+DELP/345.18)/P)**0.2857 - 1.0})**0.5
ROZERO = 1.0*32595*PZERO/TZERO
RO = ROZERO/(1.0+0.5*ACHM*ACHM)**2.5
T = TZERO/(1.0+0.5*ACHM*ACHM)
TE = T*(1.0+0.5*ACHM*ACHM)
VISW = 0.00001248*((TE/540.0)**1.5)*738.0/(T+198.0)
ROW = RO*TE/T
VIS = 0.00001248*((T/540.0)**1.5)*738.0/(T+198.0)
U = (12013.9*(TZERO-T))**0.5
RD = RO*U*D/VIS
TP = TZERO*(1.0+0.1142*ACHM*ACHM)
F2 = ((TP+198.6)/(TZERO+198.6)**2.0*(1.0+0.1142*ACHM*ACHM)**
1 (-4.0)
FL = F2*R*D*(ACHMS/ACHM)**2.0
FLL = ALOG10(FL)
DELP = ALOG10(DELP)
FRL1 = A(1,N)+A(2,N)*ALOG10(FL)+A(3,N)*ALOG10(FL)*ALOG10(FL)
FRL2 = B(1,N)+B(2,N)*ALOG10(FL)
FRL3 = A(1,30)+A(2,30)*ALOG10(FL)+A(3,30)*ALOG10(FL)*ALOG10(FL)
FRL4 = A(1,29)+A(2,29)*ALOG10(FL)
FR1 = 10.0**FRL1
FR2 = 10.0**FRL2
FR3 = 10.0**FRL3
FR4 = 10.0**FRL4
IF (M) 11 12
CF5 = FR1/(F2*R*D)
CF6 = FR2/(F2*R*D)
CF7 = FR3/(F2*R*D)
CF8 = FR4/(F2*R*D)
GO TO 15
11
CF1 = FR1/(F2*R*D)
CF2 = FR2/(F2*R*D)
CF3 = FR3/(F2*R*D)
CF4 = FR4/(F2*R*D)
TW = CF4*0.5*RD*U*U/32.17
GO TO 18
15 TW = CF8*0.5*RO*U*U/32.17
18 TERM = D/TW*GRAD
TWL = ALOG10(TW)
IF (KL) 24, 24, 16
24 UTAU = (TW*32.17/ROW)**0.5
DELTA = VISW/(ROW*ROW*UTAU**3.)*GRAD*32.17
TERD = (DELTA*UTAU*D*ROW/VISW)
CT = (1.0+0.75*TERM+0.75*TERM*TERM)**1.31
DELP = CONST/CT
IF (CT-1.0) 22, 23, 23
23 KL = 1
M = M+1
GO TO 1
22 M = M+1
21 IF (M-6) 1, 1, 16
16 PRINT 8, N, CF1, CF2, CF3, CF4, FLL, DELPL, TWL
PRINT 17, CF5, CF6, CF7, CF8, DELTA, CT
6 CONTINUE
GO TO 10
2 FORMAT (4F10.3)
3 FORMAT (2I3)
4 FORMAT (4F10.7)
7 FORMAT (F18.1, I2, 2F20.5)
8 FORMAT (16*F16.6, 3F15.6, 3X, 3F10.2)
9 FORMAT (14H1***RESULTS***, 30X, 15H RUN NUMBER D-912///76H FENCE NO
1. 3 TERM IND.  2 TERM IND.  3 TERM UNIV.  2TERM UNIV.
2 //)
13 FORMAT (3F20.10)
14 FORMAT (2F20.10)
17 FORMAT (7H CORR., 4F15.6, 3X, 2F10.6/)
19 FORMAT (7F15.5)
END
A first-order small-perturbation analysis, due to Oates\textsuperscript{52} of the potential flow through an asymmetrically convergent channel is briefly summarised below. The solution provided by this analysis is exhibited parametrically in \( x \) and \( y \) for each of the contraction sections required to form the zero pressure-gradient calibration flows. Since the analysis is linear, the results may be subtracted from each other at each \( y \) for any constant \( x \) - graphically for the purpose at hand. This allows the determination of that relative position of the two nose-pieces which minimises the distortion of the free-stream flow downstream of the contraction.

Following Marble\textsuperscript{54}, with the assumption of incompressible, irrotational flow, with the usual stream function \( \phi \) notation:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{III-1}
\]

and, differentiating with respect to \( y \):

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \tag{III-2}
\]

Now if we are given \( r_o = f_o(x) \), the linearised boundary condition then becomes

\[
v = 0 \quad \text{at} \quad r = r_h \tag{III-3}
\]

and introducing the axial velocity \( U \),
\[
\frac{v}{U} = \frac{dr}{dx} \quad \text{at} \quad r = r_o
\]  \hspace{1cm} (\text{III}-4)

and taking an integral transform by writing

\[
\tilde{v}(y, k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} v(y, k) e^{-ikx} \, dx
\]  \hspace{1cm} (\text{III}-5)

we obtain the transformed equation

\[
\frac{d^2\tilde{v}}{dy^2} - (ik)^2 \tilde{v} = 0
\]  \hspace{1cm} (\text{III}-6)

having boundary conditions

\[
\tilde{v}(r_h, k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} v(r_h, x) e^{-ikx} \, dx
\]  \hspace{1cm} (\text{III}-7)

\[
\tilde{v}(r_o, k) = \frac{U}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{dr}{dx} e^{-ikx} \, dx \quad U \quad F_o(k)
\]  \hspace{1cm} (\text{III}-8)

where \( F_o \) is the transformed shape function.

Now (III-6) has a solution of the form

\[
\tilde{v} = A \sin (iky) + B \cos (iky)
\]  \hspace{1cm} (III-9)

Application of (III-7) and (III-8) and taking \( r_o = 0 \), we obtain

\[
\tilde{v} = U \quad F_o(k) \quad \frac{\sin [ik (r_h - y)]}{\sin ik r_h}
\]  \hspace{1cm} (III-10)

and, formally inverting:

\[
\frac{v}{U} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F_o(k) \frac{\sin [ik (r_h - y)]}{\sin ik r_h} e^{ikx} \, dk
\]  \hspace{1cm} (III-11)

where

\[
F_o(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{dr}{dx} e^{-ikx} \, dx
\]  \hspace{1cm} (III-12)
Now, the shape of the nose-pieces of the channel could be approximated by writing, with the notation implied by the sketch:

\[ r_0 = A \sin \left( \frac{\pi x}{2L} \right), \quad -L \leq x \leq L \]  

\[ \text{hence} \quad \frac{dr_0}{dx} = \frac{\pi A}{2L} \cos \left( \frac{\pi x}{2L} \right), \quad -L \leq x \leq L \]  

\[ = 0, \quad |L| < x \]  

this yields from (III-12)

\[ F_o(k) = \frac{A}{2\sqrt{2\pi}} \left( \frac{\pi}{L} \right)^2 \frac{\cos kL}{(\frac{\pi}{2L})^2 - k^2} \]  

This, with (III-11) and after a contour integration procedure, leads to:

\[ \frac{v}{U} = \frac{2A}{r_h} \sum_{n=1}^{\infty} \frac{\cosh \frac{n\pi L}{r_h}}{1 + \left( \frac{2n\pi}{r_h} \right)^2} \sin \left( \frac{n\pi y}{r_h} \right) e^{-\frac{n\pi x}{r_h}} \]  

for \( x > L \)

Now, from continuity, the axial velocity follows:

\[ u = v - \int \frac{3v}{2y} \, dx \]  

and, since from the irrotationality, at \( x = \infty \), \( u/U = \text{constant} \), putting (III-17) into (III-18) yields for the shape of the axial velocity
distribution at exit from the contraction:

\[
\frac{\Delta u}{U} = \left(\frac{2A}{r_h}\right) \sum_{n=1}^{\infty} \frac{\cosh \left(\frac{n\pi L}{r_h}\right)}{1 + \left(\frac{2n\pi}{r_h}\right)^2} \cos \left(\frac{n\pi y}{r_h}\right) e^{-\frac{n\pi x}{r_h}} \tag{III-19}
\]

and, since only the fundamental will predominate after one characteristic height or so, this is conveniently simplified to the approximate form:

\[
\frac{\Delta u}{U} = A \frac{\cos \frac{\pi y}{r_h}}{1 + \left(\frac{2\pi}{r_h}\right)^2} e^{-\pi \frac{x-L}{r_h}} \tag{III-20}
\]

i.e., an axial velocity profile of the form:

We can now superpose two solutions of the form (III-20) for the situation sketched below:

Furthermore, since \(S = (x_1 - x_2)\), we can exhibit the quantity

\[
\left|\frac{\Delta u}{U}^{(1)} - \frac{\Delta u}{U}^{(2)}\right| \text{ at any } x \text{ with } S \text{ as parameter, and thus determine}
\]
that value of $S$ which minimises this quantity.

For the purposes of this study, (III-20) was machine-calculated for cases (1) and (2) for several $x$, and the optimum value of $S$ found by graphical superposition of the solutions.
APPENDIX IV - EXTRACT FROM THE EXPERIMENTAL DATA

TABLES
TABLE 1A - TEST SECTION PARAMETERS - NOZZLE A

<table>
<thead>
<tr>
<th>x inches</th>
<th>$U_0$, ft/sec ($P_o=10''$Hg.a.) ($T_o=515^\circ$R)</th>
<th>M₀</th>
<th>Channel depth inches</th>
<th>Kx10⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$P_o=5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$T_o=515$</td>
</tr>
<tr>
<td>20</td>
<td>162</td>
<td>.145</td>
<td>10.00</td>
<td>1.52</td>
</tr>
<tr>
<td>22</td>
<td>173</td>
<td>.155</td>
<td>10.00</td>
<td>2.88</td>
</tr>
<tr>
<td>24</td>
<td>193</td>
<td>.174</td>
<td>10.00</td>
<td>3.96</td>
</tr>
<tr>
<td>26</td>
<td>222</td>
<td>.200</td>
<td>8.74</td>
<td>4.83</td>
</tr>
<tr>
<td>28</td>
<td>281</td>
<td>.253</td>
<td>6.34</td>
<td>5.88</td>
</tr>
<tr>
<td>30</td>
<td>380</td>
<td>.344</td>
<td>4.43</td>
<td>5.06</td>
</tr>
<tr>
<td>32</td>
<td>521</td>
<td>.477</td>
<td>3.19</td>
<td>4.12</td>
</tr>
<tr>
<td>34</td>
<td>735</td>
<td>.688</td>
<td>2.42</td>
<td>3.12</td>
</tr>
<tr>
<td>35</td>
<td>843</td>
<td>.801</td>
<td>2.34</td>
<td>2.29</td>
</tr>
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<td>36</td>
<td>928</td>
<td>.894</td>
<td>2.28</td>
<td>1.50</td>
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<td>37</td>
<td>983</td>
<td>.957</td>
<td>2.26</td>
<td>1.24</td>
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<tr>
<td>38</td>
<td>1057</td>
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<td>2.33</td>
<td>2.08</td>
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<tr>
<td>39</td>
<td>1221</td>
<td>1.252</td>
<td>2.58</td>
<td>3.94</td>
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<tr>
<td>40</td>
<td>1421</td>
<td>1.546</td>
<td>2.82</td>
<td>3.22</td>
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<tr>
<td>42</td>
<td>1652</td>
<td>1.970</td>
<td>1.98</td>
<td>1.01</td>
</tr>
<tr>
<td>44</td>
<td>1686</td>
<td>2.045</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sharp-edged geom. throat is at $x = 37.75$
TABLE 1B - TEST SECTION PARAMETERS - NOZZLE B

<table>
<thead>
<tr>
<th>x</th>
<th>$U_0$ ft/sec</th>
<th>$M_0$</th>
<th>Channel depth</th>
<th>Kx10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches</td>
<td>(P_o=10&quot;Hg.a.)</td>
<td>(T_o=525°R)</td>
<td>inches</td>
<td>$P_o=5$</td>
</tr>
<tr>
<td>26</td>
<td>441</td>
<td>.399</td>
<td>10.00</td>
<td>.08</td>
</tr>
<tr>
<td>28</td>
<td>443</td>
<td>.401</td>
<td>10.00</td>
<td>.10</td>
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<tr>
<td>30</td>
<td>448</td>
<td>.405</td>
<td>10.00</td>
<td>.17</td>
</tr>
<tr>
<td>32</td>
<td>458</td>
<td>.415</td>
<td>10.00</td>
<td>.44</td>
</tr>
<tr>
<td>34</td>
<td>475</td>
<td>.431</td>
<td>10.00</td>
<td>.62</td>
</tr>
<tr>
<td>35</td>
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<td>.440</td>
<td>10.00</td>
<td>.75</td>
</tr>
<tr>
<td>36</td>
<td>508</td>
<td>.462</td>
<td>10.00</td>
<td>1.51</td>
</tr>
<tr>
<td>37</td>
<td>544</td>
<td>.496</td>
<td>9.90</td>
<td>2.03</td>
</tr>
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<td>600</td>
<td>.550</td>
<td>9.05</td>
<td>2.66</td>
</tr>
<tr>
<td>39</td>
<td>685</td>
<td>.634</td>
<td>6.44</td>
<td>3.19</td>
</tr>
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<td>812</td>
<td>.764</td>
<td>3.49</td>
<td>3.60</td>
</tr>
<tr>
<td>41</td>
<td>979</td>
<td>.947</td>
<td>3.02</td>
<td>3.92</td>
</tr>
<tr>
<td>42</td>
<td>1187</td>
<td>1.200</td>
<td>4.00</td>
<td>3.94</td>
</tr>
<tr>
<td>43</td>
<td>1388</td>
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Sharp-edged geometric throat is at $x = 40.90"$
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Sharp-edged geom. throat is at \( x = 40.00 \)
TABLE 2 - SHEAR LAYER MEASUREMENTS - NOZZLE A

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| 28.0     | 0.0030 | 9522     | 1.341    | 1.410      | 1.784      |
| 30.0     | 0.0032 | 9260     | 1.326    | 1.406      | 1.793      |
| 32.0     | 0.0032 | 8779     | 1.302    | 1.379      | 1.805      |
| 32.8     | 0.0037 |          |          |            |            |
| 35.0     | 0.0036 |          |          |            |            |
| 36.0     | 0.0047 | 7093     | 1.234    | 1.336      | 1.841      |
| 38.0     | 0.0047 | 4797     | 1.204    | 1.296      | 1.876      |
| 40.1     | 0.0051 | 2581     | 1.239    | 1.373      | 1.908      |
| 41.0     | 0.0051 |          |          |            |            |

| $P_o = 20$ |
| 24.0     | 0.0028 |          |          |            |            |
| 28.0     | 0.0031 | 16210    | 1.257    | 1.323      | 1.820      |
| 30.0     | 0.0032 | 15800    | 1.249    | 1.315      | 1.818      |
| 32.0     | 0.0032 | 14840    | 1.229    | 1.287      | 1.833      |
| 32.8     | 0.0038 |          |          |            |            |
| 35.0     | 0.0038 |          |          |            |            |
| 36.0     | 0.0042 | 11120    | 1.182    | 1.281      | 1.857      |
| 38.0     | 0.0042 | 7525     | 1.184    | 1.307      | 1.896      |
| 40.1     | 0.0047 | 4003     | 1.159    | 1.361      | 1.901      |
| 41.0     | 0.0047 |          |          |            |            |

| $P_o = 30$ |
| 28.0     | 0.0027 |          |          |            |            |
| 32.8     | 0.0027 | 21160    | 1.235    | 1.302      | 1.830      |
| 36.0     | 0.0028 | 19870    | 1.232    | 1.297      | 1.831      |
| 38.0     | 0.0028 | 16900    | 1.148    | 1.161      | 1.891      |
| 41.0     | 0.0047 |          |          |            |            |
### TABLE 4 - SHEAR LAYER MEASUREMENTS - NOZZLE C

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88.
### TABLE 5 - SHEAR LAYER MEASUREMENTS - NOZZLE C WITH SPOILER

| x inches |  |  |  |  |  |
|----------|-------|-------|-------|-------|
|          | $c_F$ | $R_{\delta_2}$ | $H_{12}$ | $H_{12}^*$ | $H_{32}^*$ |
| $P_o = 10"$Hg.abs. |       |       |       |       |       |
| 19.0     | .0051 | 4252  | 1.260 | 1.276  | 1.840  |
| 24.0     | .0049 | 3850  | 1.229 | 1.247  | 1.842  |
| 28.0     | .0049 | 2363  | 1.179 | 1.197  | 1.851  |
| 30.0     | .0052 | 1598  | 1.202 | 1.221  | 1.879  |
| 32.0     | .0053 | 1227  | 1.262 | 1.293  | 1.873  |
| 32.8     |       | 590   | 1.558 | 1.645  | 1.905  |
| 35.0     | .0058 |       |       |       |       |
| 36.0     | .0056 | 910   | 1.571 | 1.702  | 1.887  |
| 38.0     |       |       |       |       |       |
| 41.0     | .0042 |       |       |       |       |

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<td>35.0</td>
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<th>$P_o = 20$</th>
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APPENDIX V - FORTRAN IV PROGRAM TO COMPUTE BOUNDARY LAYER DEVELOPMENT ACCORDING TO THE MODIFIED METHOD OF WALZ

This program has been written in such a way as to promote great flexibility of use, while nevertheless taking care of most routine calculations with a minimum of effort. The required input deck takes the following form:

First card: Operations card having 6 integer fields of 5 columns.

ID
An integer \( > 1 \) identifying the particular computation to the user.

NUMBER
An integer \( 0 < n \leq 9 \) informing the program what type of boundary layer is under consideration. Table 6 lists the possibilities.

NIM
An integer \( > 1 \) giving the number of \( x \) stations at which information about the free-stream is given.

NBUG
In the event that a detailed output is required for debugging purposes, this should be 1. For normal use, it should be left blank.

NHW
Can have values \(-1, 0 \) or 1. If \(-1\), the Escudier auxiliary relations are used. If 0 (or left blank), the Walz auxiliary relations are used. If 1, the Fernholz relations are used.

ND
Can have values \(-2, -1, 0 \) or 1. If \(-2\), free-stream information may be given in terms of \((P/P_o)\) vs. \(x\) (see below), and output will be both printed out and punched onto cards. If \(-1\), \(u_\delta\) vs. \(x\) is required as input, and both printed and punched output results. If 0 (or blank)
u₀ vs. x input is required and only printed output results. If 1, \( \frac{P}{P_0} \) vs. x input is required, and only printed output results.

**TABLE 6**

<table>
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<tr>
<th>Value of NUMBER</th>
<th>Type of Boundary Layer</th>
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<tr>
<td>1</td>
<td>Turb., 2-D, Adiabatic, Zero p.g., Fernholz incompressible aux. relations</td>
</tr>
<tr>
<td>2</td>
<td>Lam., Axisymmetric, Spec. Wall Temps.</td>
</tr>
<tr>
<td>3</td>
<td>Turb., 2-D, Adiabatic Wall.</td>
</tr>
<tr>
<td>4</td>
<td>Lam., 2-D, Adiabatic Wall.</td>
</tr>
<tr>
<td>5</td>
<td>Turb., 2-D, Spec. Wall Temps.</td>
</tr>
<tr>
<td>6</td>
<td>Lam., 2-D, Spec. Wall Temps.</td>
</tr>
<tr>
<td>7</td>
<td>Lam., Axisymmetric, Adiabatic Wall</td>
</tr>
<tr>
<td>8</td>
<td>Turb., Axisymmetric, Spec. Wall Temps.</td>
</tr>
<tr>
<td>9</td>
<td>Turb., Axisymmetric, Adiabatic Wall</td>
</tr>
</tbody>
</table>

N.B. if NUMBER = 1, NHW must also be 1.

**Second Card:** Contains the parameters for the particular gas involved. The usual values for air are in parentheses. It has 4 decimal fields of 10 columns.

- **GAMMA** Ratio of specific heats (1.4)
- **R** Recovery factor (0.88 for turb., 0.86 for lam. b.l.)
- **RGAS** Gas constant, (\text{ft-lb/lb}^°\text{R}), (53.3)
- **CP** Specific heat at constant pressure, (\text{ft-lb/lb}), (186)

**Third Card:** Contains initial values of boundary layer parameters, having 5 decimal fields of 10 columns.
DEL2UL
Value of $(\delta_2)_u$ at first $x$ station, inches

DEL3UL
Value of $(\delta_3)_u$ at first $x$ station, inches

Pl
Value of static pressure at first $x$ station, inches of mercury absolute.

TZERO
Stagnation temperature of free-stream, $^\circ$R (absolute).

SPACE
Spacing, in inches, of the $x$ stations at which the output is to be printed. The spacing of the input must be some even multiple or sub-multiple of this length.

Fourth and Successive Cards: These contain the free-stream information, one card for each $x$ station. Each card has 4 decimal fields of 10 columns.

XIM(I)
The $i^{th}$ ($1 \leq i \leq NIM$) value of $x$, in inches, to be given as input. Col. 1-10.

UIM(I)
The value of $u_0$, in ft/sec, at $x_i$. In the event that $ND$ is -2 or 1, the value of $(P_{\text{static}}/P_0)$ at $x_i$. Col. 11-20.

TIM(I)
The value of $T_w$, in $^\circ$R, at $x_i$. Col. 21-30. In the case of an adiabatic wall, these columns should be left blank.

RIM(I)
The value of $R$, the radius of the flow passage, in inches, at $x_i$. Col. 31-40. In the case of a two-dimensional boundary layer, these columns should be left blank.

No further cards are required for non-zero pressure-gradient calculations. Thus the input data set for each separate computation has $(NIM + 3)$ cards.

In the case of a zero pressure-gradient calculation, $NIM = 1$, and a final card, having 1 field of 10 columns, is required at the end of each data set, viz:
PZERO Free-stream total pressure at $x_i$, in inches of mercury absolute.

When deciding on the value of NIM required for any given computation, it should be borne in mind that the machine will extrapolate the input between $x_i$ and $x_i + 1$ by considering it to lie on that unique parabola which may be drawn through the input values of the required quantity at $x_i$, $x_i + 1$, and $x_i + 2$. Thus, for accuracy, closer spacing of the input is needed in regions of rapid free-stream changes. This variable spacing must, however, be an even multiple or sub-multiple of SPACE.

Any number of input data decks may be submitted simultaneously. Printout for each case will cover the range $x_i$ to $x_j$, where $j = NIM - 2$. The program will usually catch the impending numerical disaster associated with the prediction of a separation, and print out an appropriate series of messages. Not all types of disaster can be foreseen, however, and unexplained stoppages should be further investigated by invoking the NBUG = 1 option on Card 1.

Typical output, with NBUG = 0 and 1, is shown at the end of the listing.

Exclusive of compilation time, a typical boundary layer calculation takes about 4 seconds on an IBM O/S 360, Mod. 65. The following library functions/subroutines are used (From the IBM Scientific Subroutines Package, Version II):

IFIX ALOG
ABS ALOG10
AMAX1
WALZ METHOD MOD 5 MAIN PROGRAM

DIMENSION H(1000),HST(1000),U(1000),Z(1000),XA(1000),RR(1000),
1 UIM(100),XIM(100),RIM(100),TIM(100),TA(1000),D(100)
2 *RN(1000),*H12S(1000)

COMMON P1,U1,PZERO,TZERO,UDEL,VISVAL,ACHM,RODEL,THETA,RGAS,
1 GAMMA,NUMBER,ALPHA,BETA,H12,HSTAR,DELRAT,EN,BIGN,NN,NBUG
2 *MKS,RDEL2U,RDEL2,DEL12,DEL2,DELTAX,X,NHW,CP,TW

C INITIALISE ARRAYS, INDICES AND TAGS

DO 44 I=1,100
UIM(I) = 0.0
XIM(I) = 0.0
TIM(I) = 1.0
RIM(I) = 1.0
NBUG = 0
NHW = 0
ND = 0
NN = 0
MKS = 0
MKR = 0
ID = 0
J = 1
MOD = 5
E = 0.000001

C READ INITIAL DATA
READ 1, ID, NUMBER, NIM, NBUG, NHW, ND
READ 11, GAMMA, RGAS, CP
READ 2, RDEL2U, RDEL2, DEL12, DEL2, DELTAX, X, NHW, CP, TW

C INITIALISE DATA
READ 3, XIM(I), UIM(I), TIM(I), RIM(I)
IF (ND) 46, 451, 647
IF (ND+1) 647, 932, 15, 16
DO 461 I=1,NIM
TI = TZERO*UIM(I)**((GAMMA-1.)/GAMMA)
UIM(I) = (64.3*GAMMA*RGAS*(TZERO-TI)/(GAMMA-1.))**0.5
461 DO 461 I=1,NIM

PRINT 17
PRINT 601, (XIM(I), I=1,NIM)
PRINT 601, (UIM(I), I=1,NIM)

GO TO (7, 45, 7, 8, 9, 45, 9), NUMBER
7 DO 47 I=1,NIM
TIM(I) = 1.0
RIM(I) = 1.0
GO TO 45
8 DO 48 I=1,NIM
RIM(I) = 1.0
GO TO 45
9 DO 49 I=1,NIM
TIM(I) = 1.0
45 C ASSIGN THE STEP SIZE
45 IF (NIM-1) 932, 15, 16
74 READ 640, K
GO TO 76
K = IFIX((XIM(NIM-1)-XIM(1))/SPACE)

IF (NIM-1) 932,467,468

D1 = ABS((UIM(2)-UIM(1))/UIM(1))
DO 462 I=2,NIM
D(I) = ABS((UIM(I)-UIM(I-1))/UIM(I-1))

D1 = AMAX1(D(I)*D1)
DO 463 I=2,NIM
D(I) = ABS((UIM(I)-UIM(I-1))/UIM(I-1))+.0001
IF (D(I)-D1) 463,464,464

L = I
GO TO 465

CONTINUE

KKB = 2*IFIX((UIM(L)-UIM(L-1))/(UIM(L-1)*0.03))
IF (KKB-2) 71,72,72

KKB = 2
GO TO 469

DELTAX = SPACE/120.0
KK = 10
KKB = 10
GO TO 73

DELTAX = (XIM(L)-XIM(L-1))/(12*KKB)
IF (DELTAX-SPACE/12.0) 648,747,747

DELTAX = SPACE /12.

KK = IFIX(SPACE/(12.0*DELTAX) + .001)
IF (NDEBUG) 932,18,19
PRINT 10, KD1,L,KKB,KK,DELTAX

DO 466 I=1,NIM
XIM(I) = XIM(I)/12.0
RIM(I) = RIM(I)/12.0
X = XIM(1)
C CREATE DETAILED DATA ARRAYS
IF (NIM-1) 932,470,471

XY = XIM(1) - DELTAX
MKK = K*KK+1
DO 472 I=1,MKK
XY = XY+DELTAX
U(I) = UIM(1)
RR(I) = RIM(1)
TA(I) = TIM(1)

XA(I) = XY
GO TO 473

NIMM = NIM-1
U(1) = UIM(1)
XA(1) = XIM(1)
TA(1) = TIM(1)
RR(1) = RIM(1)

MM = 2
DO 600 IM = 2,NIMM
MK = IFIX((XIM(IM)-XIM(IM-1))/DELTAX + .001)
IF (IM-2) 932,503,504

MKK = MK + 1
GO TO 505

MKK = MM + MK - 1
IF (MKK-1000) 505,505,506
PRINT 507, ID
GO TO 932
96.

DO 500 I = MM, MKK
    X = X + DELTAX
    XA(I) = X
    Y1 = UIM(IM-1)
    Y2 = UIM(IM)
    Y3 = UIM(IM+1)
    XX1 = XIM(IM-1)
    XX2 = XIM(IM)
    XX3 = XIM(IM+1)
    S1 = RIM(IM-1)
    S2 = RIM(IM)
    S3 = RIM(IM+1)
    T1 = TIM(IM-1)
    T2 = TIM(IM)
    T3 = TIM(IM+1)
    U(I) = Y3*(X-XX1)*(X-XX2)/(XX3-XX1)/(XX3-XX2) + Y2*(X-XX3)*(X-XX1)
         1/(XX2-XX3)/(XX2-XX1) + Y1*(X-XX2)*(X-XX3)/(XX1-XX2)/(XX1-XX3)
    TA(I) = T3*(X-XX1)*(X-XX2)/(XX3-XX1)/(XX3-XX2) + T2*(X-XX3)*(X-XX1)
         1/(XX2-XX3)/(XX2-XX1) + T1*(X-XX2)*(X-XX3)/(XX1-XX2)/(XX1-XX3)
    RR(I) = S3*(X-XX1)*(X-XX2)/(XX3-XX1)/(XX3-XX2) + S2*(X-XX3)*(X-XX1)
         1/(XX2-XX3)/(XX2-XX1) + S1*(X-XX2)*(X-XX3)/(XX1-XX2)/(XX1-XX3)
500 CONTINUE

MM = MKK + 1
600 CONTINUE

473 IF (NBUG) 932, 475, 474
474 PRINT 476, MKK, XA(MKK), XA(MKK-1), U(MKK), U(MKK-1), TA(MKK),
       1 TA(MKK-1), RR(MKK), RR(MKK-1)
       PRINT 601, (U(I), I=1, MKK)
475 PZERO = P1*(1.0-((GAMMA-1.0)*UIM(1)*UIM(1))/(2.0*GAMMA*RGAS
       1 *TZERO*32.17))**(-GAMMA/(GAMMA-1.0))
       X = XIM(1)
       DEL2U1 = DEL2U1/12.0
       DEL3U1 = DEL3U1/12.0
       P1 = 70.72622 * P1
       PZERO = 70.72622*PZERO
C CALCULATE INITIAL VALUES FOR PRINTOUT
121 UU = U(J)
       TW = TA(1)
       U1 = U(1)
       CALL PROPS (J, UU)
       HH = DEL3U1/DEL2U1
       RDEL2U = RODEL*UDEL*DEL2U1/VISWAL
C GUESS INITIAL VALUE OF 'DELRAT' THEN ITERATE 3 TIMES
       RDEL2 = RDEL2U*0.998
       DO 783 IJK=1,3
       GO TO (78, 13, 78, 13, 78, 13, 78, 78, 78)*NUMBER
13 CALL BLAM (HH)
       GO TO 782
78 CALL TURB (HH)
782 DEL2 = DEL2U1*DELRAT
       RDEL2 = RODEL*UDEL*DEL2/VISWAL
       RN(1) = RDEL2
       H12S(1) = DEL12
       IF (MKS) 783, 783, 932
783 CONTINUE
Z(1) = DEL2*RDEL2**EN
HST(1) = HSTAR
H(1) = MH
CALL HEAD (NUMBER MOD ID): IF (NBUG) 932,884,883
883 PRINT 70, J
884 CALL PRINT (U1,HSTAR,HH)
LL = KK + 1
L = 2
DO 200 N = 1,K
DO 100 I = L,LL
IF (MII = I) 927,927,22
22 JJ = 0
C GUESS NEXT VALUE OF HSTAR THEN ITERATE UNTIL
C DIFFERENCE BETWEEN SUCCESSIVE VALUES IS LESS THAN E:
IF (I = 2) 20,20,21
20 HST(2) = HST(1) - 0.001
GO TO 210
21 HST(I) = HST(I-1) + 0.5*(HST(I-1)-HST(I-2))
210 PSI = 0.0114*ACHM*(2.0-THETA)**0.8
H(I) = (1.0+2.0*PSI-((1.0+2.0*PSI)**2.0-4.0*PSI*HST(I))**0.5)
1 /(2.0*PSI)
HBAR = (H(I) + H(I-1))**0.5
GO TO (23,14,23,14,23,14,23,14,23,14)
14 CALL BLAM (HBAR)
GO TO 891
23 CALL TURB (HBAR)
891 JJ = JJ + 1
IF (MKS) 116,116,932
116 IF (JJ = 20) 24,24,114
24 CALL FUNCT (F1,F2,F3,F4,HBAR)
UU = U(I)
TW = TA(I)
CALL PROPS (I,UU)
U(I) = UDEL
M = I - 1
UA = U(M)
UB = U(I)
ZA = Z(M)
HSTA = HST(M)
RA = RR(M)
RB = RR(I)
CALL SOLVE(F1,F2,F3,F4,ZA,ZB,HSTA,HSTB,UA,UB,DELTAX,NBUG,RA,RB,EN)
VAL = ABS(HST(I) - HSTB)
IF (VAL = E) 32,32,31
31 HST(I) = HSTB
GO TO 210
32 Z(I) = ZB
HST(I) = HSTB
RDEL2 = (RODEL*UB*ZB/VISWAL)**(1.0/(1.0+EN))
RDEL2U = RDEL2/DELRAT
RN(I) = RDEL2
H12S(I) = DEL12
II = I
HHH = H(I)
X = X + DELTAX
IF (NBUG) 932,100,638
638 PRINT 639, JJ
100 CONTINUE
C
AFTER KK STEPS, PRINT OUT VALUES
GO TO (5,6,5,6,5,6,5,6,5,5), NUMBER
6 CALL BLAM (HHH)
GO TO 915
5 CALL TURB (HHH)
IF (NBUG) 932,918,915
915 PRINT 70, II
918 RDEL2 = (RODEL*UB*ZB/VISWAL)**(1.0/(1.0+EN))
RDEL2U = RDEL2/DELRAT
DEL2 = ZB/(RDEL2**EN)
CALL PRINT (UB,HSTB,HHH)
L = LL + 1
LL = (N+1)*KK + 1
200 CONTINUE
927 DEL2U1 = DEL2U1 * 12.0
DEL3U1 = DEL3U1 * 12.0
P1 = P1/70.72622
PZERO = PZERO/70.72622
PRINT 303, DEL2U1,DEL3U1,P1,U1,TZERO,PZERO
GO TO 932
114 MKR = MKR+1
IF (MKR-1) 120,120,118
120 PRINT 117
E = E*10
NN = 0
J = 1
X = XIM(1)
GO TO 121
118 PRINT 115
932 PRINT 311
IF (ND) 641,642,642
641 MKKN = MKK - 1
PUNCH 640, ID
DO 643 I=1,MKKN
643 PUNCH 644, XA(I),U(I),RN(I),H12S(I),HST(I)
642 GO TO 4
1 FORMAT (6I5)
2 FORMAT (2F10.6,3F10.4)
3 FORMAT (4F10.4)
10 FORMAT ('' ***ARRAY PARAMS***,I5,F10.4,316,F12.5//)
11 FORMAT (4F10.5)
17 FORMAT (''1 *** INPUT DEBUG ***'')
70 FORMAT (3H I=,I5)
101 FORMAT (/I3)
115 FORMAT (/69H *****NO CONVERGENCE AFTER OVER 20 ITERATIONS. CAS
1E ABANDONED.*****/) 
117 FORMAT(/1F10.5) 
118 FORMAT(///69H *****NO CONVERGENCE. NEW ATTEMPT WITH WEAKER CONVERGENCE
1E CRITERION FOLLOWS*****)
303 FORMAT (/46H ***END OF COMPUTATION HAVING STARTING VALUES ,
1 /8H DEL2U1=,F9.5 ,9H DEL3U1=,F9.5 ,5H P1=,F7.3 ,
2 5H U1=,F8.2,8H TZERO=,F5.0,8H PZERO=,F7.3)
311 FORMAT (/33H ***QUIT***VRYSTAAT INTEENDEEL***)
476 FORMAT (12H ***INPUT***,16,8F11.5/)
601 FORMAT (2X,10F10.4)
507 FORMAT ('1***COMPUTATION NUMBER',I5,' REQUIRES TOO MUCH STORAGE.
1BREAK IT UP INTO SHORTER PARTS***')
639 FORMAT (19H **NEXT STEP AFTER 12,13H ITERATIONS**) 
640 FORMAT (I5) 
644 FORMAT (5F13.5)
END
SUBROUTINE PROPS (I,UU)

C MOD 5

C COMMON P1,U1,PZERO,TZERO,UDEL,VISWAL,ACHM,RODEL,THETA,R,RGAS,
1 GAMMA,NUMBER,ALPHA,BETA,H12,HSTAR,DELRA,EN,BIGNN,NN,NBUG
2 MKS,RDEL2U,RDEL2,DEL12,DEL2,DELTA*X,NHW,CP,TW

C IF (NUMBER-1) 1,1,10
1 IF (I - 1) 6,6,7
6 ACHM1 = ((2.0/(GAMMA - 1.0))**((PZERO/P1)**((GAMMA-1.0)/GAMMA)
1 - 1.0))**0.5
ROZERO = PZERO/(RGAS*TZERO)
TDEL1 = TZERO/(1.0 + ((GAMMA-1.0)/2.0)*ACHM1*ACHM1)
TE1 = TDEL1*(1.0 + R*( (GAMMA-1.0)/2.0)*ACHM1*ACHM1)
RODEL1 = ROZERO*(P1/PZERO)**((1.0/GAMMA))
VIS1 = 0.00001248*((TE1/540.0)**1.5)*738.0/(TE1 + 198.0)

7 ACHM = ACHM1
RODEL = RODEL1
VISWAL = VIS1
GO TO (8,9,8,8,9,9,8,9,8),NUMBER
8 THETA = 0.0
GO TO 11
9 THETA = 64.34*CP*(TE-TW)/(U1*U1)
11 TDEL = TDEL1
TE = TE1
UDEL = U1
GO TO 2
10 UDEL = UU
ACHM = (2.0/(GAMMA-1.0))*((2.0*GAMMA*32.174*TZERO*RGAS)
1 /((2.0*GAMMA*RGAS*32.174*TZERO) - (GAMMA-1.0)*UU*UU) - 1.0)
2 )**0.5
ROZERO = PZERO/(RGAS*TZERO)
TDEL = TZERO/(1.0 + ((GAMMA-1.0)/2.0)*ACHM*ACHM)
TE = TDEL *(1.0 + R*( (GAMMA-1.0)/2.0)*ACHM*ACHM)
RODEL =ROZERO/(1.0+(GAMMA-1.0)*5*ACHM*ACHM)**(1.0/(GAMMA-1.0))
VISWAL=0.00001248*((TE /540.0)**1.5)*738.0/(TE + 198.0)
GO TO (12,13,12,12,12,13,12,12,13,12,13,12,12),NUMBER
12 THETA = 0.0
GO TO 2
13 THETA = 64.34*CP*(TE-TW)/(UU*UU)
2 IF (NBUG) 4*4,5
5 PRINT 3, UDEL,ACHM,ROZERO,TDEL,TE,RODEL,VISWAL,THETA
4 RETURN
3 FORMAT (9H **PROPS **8F13.8)
END
SUBROUTINE TURB (H)

C
C MOD 5
C
C THIS SUBROUTINE CALLS 'HANS' OR 'WALZ' OR 'MARCEL' TO COMPUTE
C VALUES OF ALPHA BETA H12 DEL12 PHI PSI HSTAR AND DELRAT DEPENDING
C ON THE VALUE GIVEN TO PARAMETER 'NHW'
C NHW= 1 GETS ALL THE ABOVE FROM 'HANS'
C NHW= 0 GETS ALL THE ABOVE FROM 'WALZ'
C NHW=-1 GETS ALL THE ABOVE FROM 'MARCEL'
C
COMMON P1,U1,PZERO,TZERO,UDEL,VISWAL,ACHM,RODEL,THETA,R,RGAS,
1 GAMMA,NUMBER,ALPHA,BETA,H12,HSTAR,DELRAT,EN,BIGN,NN,NUBUG,
2 MKS,DEL2U,DEL2L,DEL12,DEL2,DELTAX,X,NHW,CP,TW
C
C TEST VALUE OF SHAPE FACTOR
NMJ = NN
2 IF (H - 1.57) 2*2*1
19 PRINT 4
NN = NN+1
19 IF (H-1.50) 20,20*1
20 IF (NN-4) 23,23*1
23 PRINT 21
NN = NN+1
5 PRINT 6
MKS = 1
GO TO 18
1 IF (NHW) 8,12,11
8 CALL MARCEL (H)
GO TO 9
12 CALL WALZ (H)
GO TO 9
11 CALL HANS (H)
9 IF (ALPHA) 14,14*10
14 IF (NN-6) 16,16*10
16 PRINT 15
MKS = 1
GO TO 18
10 EN = 0.268
BIGN = 0.168
IF (NUBUG) 17,17,18
18 PRINT 13, ALPHA,BETA,H12,DEL12,HSTAR,DELRAT,H,NMJ
17 CONTINUE
7 RETURN
4 FORMAT ('0**SEPARATION IMMINENT** H32 L.T. 1.57 */)
6 FORMAT ('0**CASE ABANDONED** H32 L.T. 1.49 */)
13 FORMAT (8H **TURB 9F12.6,13)
15 FORMAT ('0**NEGATIVE WALL SHEAR STRESS COMPUTED**/)
21 FORMAT ('0**SEPARATION REACHED BY WALZ CRITERION H32 L.T. 1.50 */)
END
SUBROUTINE BLAM (H)

MOD 5

COMMON P1,U1,PZERO,TZERO,UDEL,VISWAL,ACHM,RODEL,THETA*R,RGAS,
GAMMA,NUMBER,ALPHA,BETA,H12,HSTAR,DELRAT,EN,BIGN,NN,NBUG

C
C TEST H
C IF (H - 1.5151) 16,16,3
1
ALPHA = 1.441*(H-1.515)**0.66
BETA = (0.1573 + 1.691*(H-1.515)**1.637)*((1.0+0.6667*R*((GAMMA-
1.0)/2.0)*ACHM*ACHM*(1.0-0.75*THETA.))**0.65)
H12 = 4.03-4.183*(H-1.515)**0.3945
PHI = 0.936-0.0572*ACHM
PSI = 0.0114*ACHM*(2.0-THETA)**0.8
IF (ALPHA - 0.001) 14,14,100
IF (NN-2) 16,16,100

PRINT 15

NN = NN + 1
MKS = NN
GO TO 7

HSTAR = H *(1.0 + (2.0 - H)*PSI)
DELRAT = (1.0 + R*((GAMMA-1.0)/2.0)*ACHM*ACHM*(HSTAR-THETA)*)
(2.0 - H)*PHI)**(-1.0)

DEL12 = H12/DELRAT+R*((GAMMA-1.0)/2.0)*ACHM*ACHM*(HSTAR-THETA)
EN = 1.0
BIGN = 1.0
IF (NBUG) 17,17,18

PRINT 13, ALPHA,BETA,H12,DEL12,PHI,PSI,HSTAR,DELRAT,H

CONTINUE

RETURN

FORMAT (///49H ****BOUNDARY LAYER HAS NOW REACHED SEPARATION***//)

FORMAT (8H **BLAM ,9F12.6)

END

SUBROUTINE FUNCT (F1,F2,F3,F4,HBAR)

MOD 5

COMMON P1,U1,PZERO,TZERO,UDEL,VISWAL,ACHM,RODEL,THETA*R,RGAS,
GAMMA,NUMBER,ALPHA,BETA,H12,HSTAR,DELRAT,EN,BIGN,NN,NBUG

DEL43 = R*((GAMMA-1.0)/2.0)*ACHM*ACHM*(HSTAR-THETA)/HSTAR
F1 = 2.0 + EN + (1.0+EN)*DEL12 - ACHM*ACHM
F2 = (1.0+EN)*DELRAT*ALPHA
F3 = 1.0 - DEL12 + 2.0*DEL43
F4 = DELRAT*(2.0*BETA*RDEL2**2(EN-BIGN)) - ALPHA*HSTAR

IF (NBUG) 3,3,1

PRINT 2, HBAR,DEL43,F1,F2,F3,F4

RETURN

FORMAT (9H **FUNCT ,6F13.7)

END
SUBROUTINE HEAD (NUMBER, MOD, ID)

C
C
C
MOD 5
C

PRINT 10
PRINT 20, NUMBER, MOD, ID
GO TO (1,2,3,4,5,6,7,8,9), NUMBER
1 PRINT 11
GO TO 69
2 PRINT 21
GO TO 69
3 PRINT 31
GO TO 69
4 PRINT 41
GO TO 69
5 PRINT 51
GO TO 69
6 PRINT 61
GO TO 69
7 PRINT 71
GO TO 69
8 PRINT 81
GO TO 69
9 PRINT 91
69 PRINT 70
PRINT 80
RETURN
10 FORMAT ( '1 COMPRESSIBLE BOUNDARY LAYER CALCULATION BY THE METHOD
1 OF WALZ ET AL.' )
20 FORMAT ( '0 CASE NUMBER' , I3 , ' PROGRAM MOD. ' , I12 , '9X,' COMPUTATION NUMBER ' , I5 )
11 FORMAT ( '0 TURBULENT, ZERO PRESSURE GRADIENT, ADIABATIC WALL,
1 TWO DIMENSIONAL, FERNHOLZ (1967) INCOMPRESSIBLE RELATIONS')
21 FORMAT ( '0 LAMINAR, AXISYMMETRIC, SPECIFIED WALL TEMPERATURE')
31 FORMAT ( '0 TURBULENT, TWO DIMENSIONAL, ADIABATIC WALL')
41 FORMAT ( '0 LAMINAR, TWO DIMENSIONAL, ADIABATIC WALL')
51 FORMAT ( '0 TURBULENT, TWO DIMENSIONAL, SPECIFIED WALL TEMPS.')
61 FORMAT ( '0 LAMINAR, TWO DIMENSIONAL, SPECIFIED WALL TEMPS.')
71 FORMAT ( '0 LAMINAR, AXISYMMETRIC, ADIABATIC WALL')
81 FORMAT ( '0 TURBULENT, AXISYMMETRIC, SPECIFIED WALL TEMPERATURE')
91 FORMAT ( '0 TURBULENT, AXISYMMETRIC, ADIABATIC WALL')
70 FORMAT ( '1FCT. MOM. THK. M0M. THK. DISP. SHP. FCT. INCOMP. ')
80 FORMAT ( '1(S) INCHES REN. NO. THK. H32 (S) H12 ' )
END
SUBROUTINE HANS (H)

MOD 5

THIS SUBROUTINE COMPUTES VALUES OF ALPHABETAH12,DEL12,PHIPSISTAR AND DELRAT ACCORDING TO RELATIONS OF FERNHOLZ (1967) FOR THE TURBULENT BOUNDARY LAYER ON A FLAT PLATE

COMMON P1,U1,PZERO,TZERO,UDEL,VISWAL,ACHM,RODEL,THETA,R,RGAS,1 GAMMA,NUMBER,ALPHA,BETA,H12,HSTAR,DELSTAR,EN,BIGN,NN,NBUG 2 MKS,RDEL2U,RDEL2,DEL2,DEL2,DEL2,DEL2,TW  
R1 = (1.0+R*(GAMMA-1.0)/2.0*ACHM*ACHM*(1.0-THETA))**(1.0) 
H32S = 1.80 + 0.0072*ACHM 
F2THET = 1.* 
I = 0 
SIGS = 1.002 
CC = 0.01015/(RDEL2*SIGS)**0.15 + 0.786/(RDEL2*SIGS) 
H12 = (1.0 - ((0.5*CC)**0.5)*(7.506-0.202*ALOG10(RDEL2*SIGS)))/((1.0-CC)**0.5) 
H12S = H12 + 4*ACHM*ACHM*F2THET 
RAT = (H12S - R*(GAMMA-1.0)/2.0*ACHM*ACHM*(H32S-THETA))/H12 
SIGS = RAT**0.7 
I = I + 1 
IF (I<5) 3,4,4 
GO TO 2 
4 CF=R1*(0.01015*(RAT**0.595)/(RDEL2**0.15)+0.786/RDEL2) 
ALPHA1 = 0.5*CF*RAT*(RDEL2**0.268) 
CFU = 0.01015/(RDEL2U**0.15)+0.786/RDEL2U 
ALPHA2 = CFU*0.5*(RDEL2U**0.268) 
IF (NUMBER<1) 7,6,5 
6 ALPHA = ALPHA2 
GO TO 17 
5 ALPHA = ALPHA1 
17 IF (NUMBER<1) 7,13,9 
13 BETA = 0.25*CFU*(H + 0.017)*RDEL2U**0.168 
GO TO 11 
9 BETA = 0.25*CF*H32S*(RDEL2**0.168)*RAT 
11 DEL12 = H12S 
DELSTAR = 1.0/RAT 
PSI = 0.0114*ACHM*(2.0 - THETA)**0.8 
HSTAR = H *(1.0 + (2.0 - H)*PSI) 
IF (NBUG<7) 102,7 
103 PRINT 20, H,PSI,R1,SIGS,RAT,CF,CFU,NN,NHW,MKS 
102 GO TO 12 
7 MKS = 1 
12 RETURN 
20 FORMAT (8H **HANS 7F12.8,3I5) 
END
SUBROUTINE MARCEL (H)

MOD 5

THIS SUBROUTINE COMPUTES VALUES OF ALPHA, BETA AND H12 ACCORDING TO
RELATIONS OF ESCUDIER ET AL (1966). VALUES OF THE COMPRESSIBILITY-
CORRECTED QUANTITIES DEL12 AND HSTAR ARE ACCORDING TO WALZ (1965)
CORRECTION RELATIONS

COMMON P1, U1, PZERO, TZERO, UDEL, VISWAL, ACHM, RODEL, THETA, R, RGAS,
1 GAMMA, NUMBER, ALPHA, BETA, H12, HSTAR, DELRAT, EN, BIGN, NN, NBUG,
2 * MKS, RDEL2U, RDEL2, DEL12, DEL2, DELTAX, X, NHW, CP, TW

H12 = 1.55/(0.0971 + (0.009428 - 3.1*(1.431 - H))**0.5)
PHI = 1.0 - 0.0719 * ACHM + 0.00419 * ACHM * ACHM
PSI = 0.0114 * ACHM * (2.0 - THETA)**0.8
HSTAR = H * (1.0 + (2.0 - H) * PSI)
DELRAT = (1.0 + R*(((GAMMA-1.0)/2.0)*ACHM*ACHM*(HSTAR-THETA))
1 (2.0-H)*PHI)**(-1.0)

DEL12 = H12/DELRAT + R*((GAMMA-1.0)/2.0)*ACHM*ACHM*(HSTAR-THETA)
HT = 2.0 * H/3.0
ZETA = HT-1.0 + (HT*(HT-1.0))**0.5
IF (ZETA-1.0) 1, 1, 2
1 TERM = 0.00565*(1.0-ZETA)**2.715
GO TO 3
2 TERM = 0.01*(ZETA-1.0)**3.0
3 EL = ALOG((3.389*RDEL2*ZETA)/((1.0-ZETA)*((1.0+2.0*ZETA))))
SS = (0.243*ZETA*ZETA+0.0376*ZETA-0.00106+0.0914*ZETA*ZETA/
1 (1.0+65./ZETA))/((EL*EL)

BY USUAL DEFINITION, CF = 2.0*SS
ALPHA = SS*(RDEL2**2.68)/DELRAT
SBAR = (2.0*ZETA+1.0)*SS/3.0 + TERM
BETA = SBAR*(RDEL2**1.68)/DELRAT
IF (NBUG) 5, 5, 6
6 PRINT 4, H*PHI, PSI, HT, ZETA, EL, SS, SBAR
5 RETURN
4 FORMAT (1, ' **MARCEL ', 8F12.8)
END

SUBROUTINE PRINT (U*HST*H)

MOD 5

COMMON P1, U1, PZERO, TZERO, UDEL, VISWAL, ACHM, RODEL, THETA, R, RGAS,
1 GAMMA, NUMBER, ALPHA, BETA, H12, HSTAR, DELRAT, EN, BIGN, NN, NBUG,
2 * MKS, RDEL2U, RDEL2, DEL12, DEL2, DELTAX, X, NHW, CP, TW

CF = ALPHA*DELRAT/(RDEL2**EN)*2.0
3 QX = 12.0 * X
QDEL2 = 12.0 * DEL2
QDXTAX = 12.0 * DELTAX
DELI = QDEL2**DELI2
PRINT 13, QX, U, CF, ACHM, THETA, DEL12, QDEL2, RDEL2, DEL1, HST, H12
RETURN
13 FORMAT (1X, F6.2, 3X, F7.2, 3X, F8.6, 2X, F5.3, 2X, F6.3, 3X, F8.4, 3X, F6.4,
1 X, F10.0, 3X, F6.4, 3X, F7.4, F9.4/)
END
SUBROUTINE SOLVE (F1,F2,F3,F4,ZA,ZB,HSTA,HSTB,UA,UB,DELTAX,1
NBUG,RA,RB,EN )

C
C MOD 5
C
UR = (UA/UB)
IF (UR - 1.0) 2,3,2

AZ = 1.0
BZ = 1.0
AH = 1.0
BH = 1.0
GO TO 4

2 AZ = UR**F1
BZ = (1.0 - AZ*UR)/((1.0 + F1)*(1.0 - UR))
AH = UR**F3
BH = (1.0 - AH*UR)/((1.0 + F3)*(1.0 - UR))

4 ZB = ZA*(RA/RB)**(1.0+EN)*(AZ+(BZ*F2*DELTAX/ZA)*(1.0+(RB/RA)**(1.0+EN))/2.0)
HSTB = HSTA * (AH + BH*F4*DELTAX*2.0/(HSTA*(ZB + ZA)))
IF (NBUG) 13,13,17

17 PRINT 5, UR,ZA,ZB,HSTA,HSTB,AZ,BZ,AH,BH,RA,RB
13 CONTINUE
RETURN
5 FORMAT (9H **SOLVE ,11F10.6)
END

SUBROUTINE WALZ (H)

C
C MOD 5
C
THIS SUBROUTINE COMPUTES VALUES OF ALPHA,BETA,H12,DEL12,PHI,PSI,
HSTAR AND DELRAT ACCORDING TO RELATIONS OF WALZ (1965) FOR A
TURBULENT BOUNDARY LAYER.
C
COMMON P1,U1,PZERO,TZERO,UDEL,VISWAL,ACHM,RODEL,THETA,R,RGAS,
1 GAMMA,NUMBER,ALPHA,BETA,H12,HSTAR,DELRAT,EN,BIGN,NN,NBUG
2 ,MKS,RDEL2U,RDEL2,DEL12,DELTAX,X,NHW,CP,TW

C
ALPHA = 0.00566*H - 0.0842
BETA = 0.0056
H12 = 1.0 + 1.48 *(2.0-H) + 104.0 * (2.0-H)**6.7
PHI = 1.0 - 0.0719*ACHM + 0.00419*ACHM*ACHM
PSI = 0.0114*ACHM*(2.0 - THETA)**0.8
HSTAR = H *(1.0 + (2.0 - H)*PSI)
DELRAT = (1.0 + R*((GAMMA-1.0)/2.0)*ACHM*ACHM*(HSTAR-THETA)*
1 (2.0 - H)*PHI)**((-1.0)
DEL12 = H12/DELRAT+R*((GAMMA-1.0)/2.0)*ACHM*ACHM*(HSTAR-THETA)
IF (NBUG) 3,3,2
2 PRINT 1, H,PHI,PSI,NN,NHW,MKS
3 RETURN
1 FORMAT (8H **WALZ ,3F12.8,48X,315)
END
COMPRESSIBLE BOUNDARY LAYER CALCULATION BY THE METHOD OF WALZ ET AL.

CASE NUMBER 3  PROGRAM MOD. 5  COMPUTATION NUMBER 4

TURBULENT, TWO DIMENSIONAL, ADIABATIC WALL

<table>
<thead>
<tr>
<th>POSN. X IN.</th>
<th>VELOCITY 85.00</th>
<th>SKIN FRIC. 0.003101</th>
<th>MACH 0.075</th>
<th>HI.THR. 1.4529</th>
<th>SHP.FCT. 0.0570</th>
<th>MGM.THK. 2387.</th>
<th>MUM.THK. 0.0828</th>
<th>DISP. 1.7130</th>
<th>SHP.FCT. 1.4505</th>
<th>INCOMP. 1.50 L.T. 1.57</th>
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<td>1.4340</td>
<td>0.0602</td>
<td>2521.</td>
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<td>0.074</td>
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<td>0.0726</td>
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<td>1.9401</td>
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**SEPARATION IMMINENT. H32 L.T. 1.57**

**SEPARATION IMMINENT. H32 L.T. 1.57**

**SEPARATION REACHED BY WALZ CRITERION. H32 L.T. 1.50**
**COMPRESSIBLE BOUNDARY LAYER CALCULATION** BY THE METHOD OF WALZ ET AL.

**CASE NUMBER** 1  
**PROGRAM MOD.** 5  
**COMPUTATION NUMBER** 24

TURBULENT, ZERO PRESSURE GRADIENT, ADIABATIC WALL, TWO DIMENSIONAL, FERNHOLZ (1967) INCOMPRESSION

**POSN. VELOCITY** SKIN FRIC. MACH HT. TFR. SHP. FCT. MOM. THK. MUM. THK. DISP. SHP. FCT. INCOMP.
**XI** IN. FT/SEC. CF LOCAL NO. PARAM. H12 (S) INCHES  
REN. NO. THK. H32 (S) INCHES

<table>
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<th>i</th>
<th>15.75</th>
<th>350.22</th>
<th>0.002705</th>
<th>C.309</th>
<th>0.0</th>
<th>1.3671</th>
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<th>8250.</th>
<th>0.0336</th>
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<td>1.00416279</td>
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<tr>
<td><strong>TURB</strong></td>
<td>0.015251</td>
<td>0.005448</td>
<td>1.328911</td>
<td>1.367126</td>
<td>1.747163</td>
<td>0.994083</td>
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<td>534.78198242</td>
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<td>0.14444309</td>
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<tr>
<td><strong>SOLVE</strong></td>
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<td>0.023258</td>
<td>1.747667</td>
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<tr>
<td><strong>NEXT STEP AFTER 2 ITERATIONS</strong></td>
<td></td>
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**HANS** | 1.74511051 | 0.006135C7 | 0.98346382 | 1.00595474 | 0.00010069 | 0.00271127 |
| **TURB** | 0.015251 | 0.005448 | 1.328911 | 1.366335 | 1.747838 | 0.994061 | 1.745111 |
| **FUNCT** | 1.7451105 | 0.0168149 | 3.9053526 | 0.0192421 | -0.3330055 | 0.0002007 |
| **PROPS** | 350.21655273 | 0.309C9413 | 0.15144247 | 534.78198242 | 543.77392578 | 0.14444309 | 0.0001255 |
| **SOLVE** | 1.000000 | 0.023258 | 1.328911 | 1.366335 | 1.747872 | 0.994083 | 1.745145 |
| **NEXT STEP AFTER 2 ITERATIONS** | |

**HANS** | 1.74525166 | 0.006135C7 | 0.98346382 | 1.00595665 | 0.00010069 | 0.00271127 |
| **TURB** | 0.015280 | 0.005449 | 1.328920 | 1.366635 | 1.747838 | 0.994060 | 1.745252 |
| **FUNCT** | 1.7452517 | 0.0168149 | 3.9047375 | 0.0192655 | -0.3332511 | 0.0002009 |
| **PROPS** | 350.21655273 | 0.309C9413 | 0.15144247 | 534.78198242 | 543.77392578 | 0.14444309 | 0.0001255 |
| **SOLVE** | 1.000000 | 0.023258 | 0.023578 | 1.747952 | 1.748091 | 1.000000 | 1.000000 |
| **NEXT STEP AFTER 2 ITERATIONS** | |

**HANS** | 1.7453928C | 0.006135C7 | 0.98346382 | 1.00595856 | 0.00010069 | 0.00271087 |
| **TURB** | 0.015280 | 0.005449 | 1.328920 | 1.366635 | 1.747838 | 0.994061 | 1.745252 |
| **FUNCT** | 1.7453928 | 0.0168149 | 3.9047375 | 0.0192655 | -0.3332511 | 0.0002009 |
| **PROPS** | 350.21655273 | 0.309C9413 | 0.15144247 | 534.78198242 | 543.77392578 | 0.14444309 | 0.0001255 |
| **SOLVE** | 1.000000 | 0.023258 | 0.023578 | 1.747952 | 1.748091 | 1.000000 | 1.000000 |
| **NEXT STEP AFTER 2 ITERATIONS** | |
APPENDIX VI - NOTES ON EXPERIMENTAL DATA USED FOR TESTING THE WALZ METHOD

Moses 50 #5.

Since no tabulated values were available, $\delta_2 (\equiv \delta_2 u)$ and $H_{12}$ were read off the plots for $x = 0^\circ$ as 0.02" and 1.35 respectively. This value of $H_{12}$ was used with eqn. (46) to compute a value of $H_{32}$, and hence to obtain a value of $\delta_3$ at $x = 0$. The actual value of $u_\delta$ at $x = 0$ was not given, but was assumed to be 100 ft/sec. This enabled a computation of $u_\delta$ at each succeeding data point. $P_{x=0}$ was assumed to be 30.0" Hg.abs. and $T_0$ to be 540 °R.

Moses #6.

The identical starting values were used for this case.

Goldberg 62 #3.

Values of $\delta_2 (\equiv \delta_2 u)$ and $\delta_3 (\equiv \delta_3 u)$ were read off the published plots at $x = 4^\circ$ as 0.033" and 0.0575" respectively. The report stated that the free-stream velocity in the test-section was "about 85 ft/sec" and this number was used for $u_\delta$ at $x = 4^\circ$ to initiate the calculations. $P_{x=4}$ was assumed to be 30.0 °Hg.abs. and $T_0$ to be 540 °R.

Goldberg #6

The same procedure yielded $\delta_2 = 0.057$, $\delta_3 = 0.0976$, the other starting quantities being unchanged for a computation starting at $x = 16^\circ$.

Launder 28 case (a), (i)

Tabulated values of $u_\delta$ and $R_{\delta 2}$ were available for $x = 11^\circ$. With the assumption of $v = 0.00017$ ft$^2$/sec, this yielded $(\delta_2 u) = 0.04321"$. $H_{12}$ was given as 1.68 and equation (46) was used
to estimate $H_{32}$ as $1.673$, giving $(\delta_3)_u = 0.07229"$. The tabulated experimental values of $u_\delta$ vs. $x$ were plotted, and values of $u_\delta$ read off a faired curve through the data at one inch intervals up to $x = 31"$. $P_{x=11}$ was assumed to be 30.0 "Hg.abs. and $T_0$ to be 540 °R.

Smith and Walker

Data was extracted from the complete tabulations of this report for cases where $M_\delta \approx 0.309$ and $P_0 \approx 30.55$ psia. The tabulated values of energy and momentum thickness at $x = 15.75"$ were assumed to correspond to the $\delta_2$ and $\delta_3$ definitions of the present report, although it is not clear whether Smith and Walker made any density corrections in evaluating either their velocity-profiles or integral parameters. Consequently, the auxiliary relations of the present report were used to produce values of $(\delta_2)_u$ and $(\delta_3)_u$ consequent upon $M_\delta = 0.309$. (It might be remarked that this is rather too large a Mach no. to allow the accurate evaluation of shape-factors without density corrections being applied both to the derivation of velocity-profiles from $\Delta P$ data and to the evaluation of integral parameters.)

The consequent values of $(\delta_2)_u$ and $(\delta_3)_u$ were 0.0247" and 0.0431". Mean values of $P_0$ and $T_0$ over the data were calculated, which, with adoption of $M_\delta = 0.3095$ yielded values of $P_{x=15.75} = 58.213 "$Hg.abs. and $T_0 = 545 °R.$

Data of the present study.

In each case, the experimental data tabulated in Tables 1-5 were used, in conjunction with the associated experimental values of $(\delta_2)_u$ and $(\delta_3)_u$, to initiate computations.
APPENDIX VII – THE SKIN FRICTION LAW OF FERNHOLZ

In 1967 Fernholz\textsuperscript{35} produced a new form of correlation for the wall shear stress in the turbulent compressible flat plate boundary layer with or without heat transfer. When specialised to the adiabatic wall case, this may be presented in the following form:

\[ c_f = \frac{\rho_w}{\rho_\delta} \sigma^* (\overline{c_f}) u = f \left( M_\delta, R_{\delta 2} \right) \quad (VII-1) \]

where \[ \frac{\rho_w}{\rho_\delta} = [1 + 0.176 M_\delta^2]^{-1} \quad (VII-2) \]

and \[ (\overline{c_f}) u = \frac{0.01015}{(R_{\delta 2} \sigma^*)^{0.15}} + \frac{0.786}{(R_{\delta 2} \sigma^*)} \quad (VII-3) \]

and \( \sigma^* \), the Coles\textsuperscript{3} transform function takes the form

\[ \sigma^* = \left[ \frac{(\delta_2) u}{\delta_2} \right]^{0.7} \quad (VII-4) \]

Now the correlation for \( \left[ (\delta_2) u/\delta_2 \right] \) takes the form

\[ (\delta_2) u/\delta_2 = \left[ H_{12}^* - 0.176 M_\delta^2 H_{32}^* \right] H_{12}^{-1} \quad (VII-5) \]

where \[ H_{12} = \left[ 1 - \left( \frac{1}{2} (\overline{c_f}) u \right)^{1/2} \left( 7.506 - 0.202 \log_{10}(R_{\delta 2} \sigma^*) \right) \right]^{-1} \quad (VII-6) \]

and \[ H_{12}^* = H_{12} + 0.14 M_\delta^* \]

and \[ H_{32}^* = 1.80 + 0.0072 M_\delta \]

This cascaded series of relations clearly requires iteration on \( \sigma^* \) for final evaluation, given \( R_{\delta 2} \) and \( M_\delta \), but this converges very quickly. This procedure is realised in Appendix II.
APPENDIX VIII - THE TEMPERATURE PROFILE RELATION OF VAN DRIEST

Solution of the energy equation for the turbulent compressible flow in the shear layer of a flat plate with a negligible longitudinal enthalpy gradient, unity Prandtl number and constant \( c_p \) yields, for the time-averaged quantities:

\[
\frac{T}{T_\delta} = \frac{T_w}{T_\delta} - \left( \frac{T_w}{T_\delta} - 1 \right) \frac{u}{u_\delta} - \frac{Y - 1}{2} \frac{M_\delta^2}{u_\delta} \left( 1 - \frac{u}{u_\delta} \right) \quad (VIII-1)
\]

This, together with the usual relation for the recovery temperature of an adiabatic wall, viz.

\[
T_w = T \left( 1 + r \frac{Y - 1}{2} M_\delta^2 \right) \quad (VIII-2)
\]

allows determination of the temperature and hence density profile of a shear layer given only the mean velocity profile, Mach number and local free-stream properties. The LHS of (VIII-1) is close to unity for all the cases treated in this report.
APPENDIX IX - FORTRAN IV PROGRAM TO COMPUTE BOUNDARY LAYER INTEGRAL PARAMETERS FROM RAW DATA

This program provides for the computation of $\delta_1$, $\delta_2$, $\delta_3$, $(\delta_1'u, (\delta_2'u, (\delta_3'u, H_{12}, H_{32}, H_{32}^*, H_{32}^*, \delta_2' / (\delta_2' u, u_\delta, R_{\delta_2}$ and $M_0$, given either values of $\Delta P$ vs. $y$ only or values of $(u/u_\delta)$ and $(T/T_\delta)$ vs. $y$. In the former case, density corrections are put in according to the relation of Van Driest set out in Appendix VIII.

The required input deck is:

- **NSER**: An integer tag identifying the computation to the user.
- **STN**: The value of $x$ at which the profile was measured.
- **MOD**: If this is 1, $(u/u_\delta)$ and $(T/T_\delta)$ must be given vs. $y$.
  If it is 3, $\Delta P$ (in any units) must be given vs. $y$ (in any units).
- **PZERO**: Free-stream stagnation pressure in inches of mercury absolute.
- **P**: Local static pressure, inches of mercury absolute.
- **TZERO**: Free-stream stagnation temperature, in °R.
- **DISP**: The effective displacement, if any, of the probe tip from the $y$ coordinates quoted below. (e.g. in the present study, this was half the $y$ O.D. dimension, in inches, of the probe tips used, since $y = 0$ was declared to be the case of the probe touching the wall.)
- **YSCALE**: A scale factor relating the numbers given for $y$ coordinates below to the actual physical lengths. This is useful if data is read off X-Y plots of varying scales.
There are now two possibilities:

(i) If MOD = 3, a table of values of ΔP vs. y follows, a pair of values to a card [USQ(I) and Y(I)]. This must terminate with a card having only the quantity "-1.0" punched in columns 11-14.

(ii) If MOD = 1, the following card contains the value of T at the first y station, and is followed by a table of y, (T/T₀) and (u/u₀), a triplet of values to a card, [Y(I), TS(I), and U(I)] which must terminate with "-1.0" punched in columns 1-4.

Any number of such data sets may be submitted simultaneously. A trapezoidal integration rule is followed, except for the first element. To attenuate the inaccuracy introduced by the finite size of the first experimental y value, the profile is automatically assigned a spurious value of (u/u₀) at y = 0 by extrapolating to y = 0 that unique parabola which may be drawn through the data of the first, second and third given y stations.

A sample printout is given at the end of the listing.
COMPRESSIBLE BOUNDARY LAYER PROFILES --- INTEGRATED QUANTITIES

THE VALUE OF 'MOD' DETERMINES THE FORMAT OF THE DATA
MOD = 1 REQUIRES BOTH VELOCITY RATIO AND STATIC TEMP. RATIO TO BE
GIVEN IN TERMS OF COORDINATE 'Y'
MOD = 3 REQUIRES ONLY VALUES OF DELTAP FOR EACH Y. DENSITY
CORRECTIONS ARE PUT IN THROUGH RELATION OF VAN DRIEST.
ARRAY 'A' REFERS TO AXISYMMETRIC CASE NOT IMPLEMENTED HERE

DIMENSIONS(6,50),T(6),Y(50),U(50),USQ(50),TS(50),XI(50),A(50)

111 PRINT 211
NCOM = 0
NCOM2 = 0
READ 75, NSER
READ 82, STN
READ 80, MOD
READ 82, PZERO,P,TZERO,DISP,YScale
ACHM = (5.0*(((PZERO/P)**0.2857-1.0))**0.5
TFS = TZERO/(1. + 2*ACHM*ACHM)
TWALL = TFS*(1. + 176*ACHM*ACHM)
VISC = 0.00001248*((TWALL/540.)**1.5)*738./(TWALL+198.)
RODEL = P*70.72622/(53.3*TFS)
VEL = ((1.4*53.3*32.17*TFS)**0.5)*ACHM
TR = 1.0 + 0.176*ACHM*ACHM
PRINT 10
I = 1
IF (MOD-3) 98,44,21
44 I = I + 1
READ 13, USQ(I),Y(I)
Y(I) = Y(I) + DISP*YScale
IF (Y(I)) 33,44,44
33 IMAX = I - 1
UMAX = (USQ(IMAX))**0.5
DO 15 J=2,IMAX
U(J) = (USQ(J)**0.5)/UMAX
IF (U(J)-1.00001) 71,71,72
72 NCOM2 = 1
71 TS(J) = TR-(TR-1.)*U(J)+2*ACHM*ACHM*U(J)*(1.-U(J))
U(J) = U(J)*TS(J)**0.5
15 Y(J) = Y(J)/YScale
GO TO 66
98 READ 82, TS(1)
4 I = I + 1
READ 2, Y(I),TS(I),U(I)
IF (Y(I)) 3,4,4
3 IMAX = I - 1
C SET ARRAY 'A' TO ZERO FOR 2-DIM. CASE
66 DO 58 IJKL = 1,IMAX
58 A(IJKL) = 0.0
Y(1) = 0.0
U(1) = U(4)*((Y(1)-Y(2))*(Y(1)-Y(3))/(Y(4)-Y(2))/(Y(4)-Y(3))
   + U(3)*((Y(1)-Y(4))*(Y(1)-Y(2))/(Y(3)-Y(4))/(Y(3)-Y(2))
   + U(2)*((Y(1)-Y(3))*(Y(1)-Y(4))/(Y(2)-Y(3))/(Y(2)-Y(4))
   IF (U(1)) 14,14,151
14 U(1) = 0.0
NCOM = 1

151  TS(I) = TR-(TR-1)*U(1)*2*ACHM*ACHM*U(1)*(1-1-U(1))
DO 6 I=1,IMAX
XI(I) = 100/TS(I)
X=XI(I)
F(I) = (10-X*U(I))*(10+A(I))
F(2) = U(I)*X*(10-U(I))*(10+A(I))
F(3) = U(I)*X*(10-U(I))*U(I) *(10+A(I))
F(4) = (10-U(I))*(10+A(I))
F(5) = U(I)*U(I)*(10-U(I))*(10+A(I))
F(6) = U(I)*(10-U(I))*U(I) *(10+A(I))
DO 6 I=1,IMAX
XI(I) = 100/TS(I)
X=XI(I)
F(1) = (10-X*U(I))*(10+A(I))
F(2) = A(I)-U(I))*(10+A(I))
F(3) = U(I)*U(I)*(10-U(I))*(10+A(I))
F(4) = (10-U(I))*(10+A(I))
F(5) = U(I)*U(I)*(10-U(I))*(10+A(I))
F(6) = U(I)*(10-U(I))*U(I) *(10+A(I))
PRINT 7, Y(I),U(I),XI(I)
DO 8 L=1,6
T(L) = 0.0
DO 8 I=2,IMAX
DY = Y(I) - Y(I-1)
8  T(L) = T(L) + 0.5*(F(L)+F(L-1))*DY
PRINT 9, (T(L),L=1,6)
H12 = T(4)/T(5)
H32 = T(6)/T(5)
H12S = T(1)/T(2)
H32S = T(3)/T(2)
RATD2 = T(2)/T(5)
RENO = RODEL*VEL*T(2)/(VISC*12)
PRINT 11, H12S,H32S
PRINT 12, H12,H32
PRINT 756, PZERO,P,TZERO,RATD2,ACHM,VEL,RENO
PRINT 74, STN,NSER
IF (NCOM) 16,16,17
16  IF (NCOM) 25,25,26
26  PRINT 27
25  PRINT 20
20  GO TO 111
11  CALL EXIT
21  FORMAT (3F10.5)
7  FORMAT (2X*3F9.4)
9  FORMAT (14H DELTA 1,2,3 =3X,3F13.5/)
117H (DELTA 1,2,3)U = ,3F13.5/)
10  FORMAT (1, Y U/UD RHO/RHOD/)1
11  FORMAT (1, COMP, H12S,H32S =,2F9.4/)
12  FORMAT (1, INCOMP, H12,H32 =,2F9.4/)
13  FORMAT (2F10.5)
18  FORMAT (1, **** SUSPECT DATA NEAR WALL ****/)
20  FORMAT (1, ****COMPRRESSIBLE BOUNDARY LAYER** MK III ****/)
27  FORMAT (1, ****SPURIOUS DATA MAXIMUM ****/)
74  FORMAT (1, STATION X =,F6.2,10X, SERIAL NO.9,14)
75  FORMAT (I2)
80  FORMAT (I1)
82  FORMAT (F10.5)
211 FORMAT (1, NEXT PROFILE****/)
756 FORMAT (1, PZERO,P,TZERO,D2/D2U =,2F9.3,F7.1,F7.4/)
1' MACH NO.,FREESTREAM VEL., REN. NO. = ,F6.3,F6.1,F8.0)
END
***NEXT PROFILE***

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\[
\text{DELTA } 1, 2, 3 = 0.13222, 0.10152, 0.18577
\]

\[
(\text{DELTA } 1, 2, 3)U = 0.12651, 0.10245, 0.18740
\]

COMP. H12S, H32S = 1.3023 1.8298

INCOMP. H12, H32 = 1.2349 1.8292

P_{\text{ELEC}}/P_{\text{ZERO}} = 26.925 534.0 0.9910

MACH No., FREESTREAK VEL., REN. NO. = 0.401 446.9 21164.

STATION X = 28.00

SERIAL NO. 81

*COMPRESSIBLE BOUNDARY LAYER** MK III **
FIG. 2 TEST EQUIPMENT AND TRAVERSING RIGS
FIG. 3 MINIATURE TOTAL-HEAD PROBES.
FIG. 4   SKIN-FRICTION FENCES
FIG. 5 SCHEMATIC LAYOUT OF TEST SECTION
FIG. 6 CONSTRUCTION OF SKIN-FRICTION FENCES

PLASTIC TEST PLATE

MATERIALS
1/4" D BRASS STOCK
.059" D STAINLESS STEEL TUBING
SILVER SOLDER

JOINT BLENDED BY HAND SCRAPING

IN. NOMINAL
.002
.004
.007
.010

DRILL .0039" D

0.25" D

0.75"

0.042" D

1.00"
FIG. 7 VARIOUS WALL-PITOT GEOMETRIES
FIG 8. TEST-SECTION NOZZLE PROFILES
FIG. 9 (CONTINUED) TEST SECTION PRESSURE GRADIENT DISTRIBUTIONS
FIG. 9 VELOCITY AND ACCELERATION PARAMETER DISTRIBUTIONS — NOZZLE A

$P_0 = 5$ IN. Hg. ABS.
$P_0 = 5 \text{ in. Hg. abs.}$

**FIG. 9 (CONTINUED) VELOCITY AND ACCELERATION PARAMETER DISTRIBUTIONS NOZZLE B**
$P_0 = 5 \text{ in. Hg. Abs.}$

**FIG. 9 (CONT) VELOCITY AND ACCELERATION PARAMETER DISTRIBUTIONS**

NOZZLE C
FIG. 10 TYPICAL X-Y RECORDER TRACE

Run D = 81
X = 25°C

ΔP

Y

1.80

FIG. 10 TYPICAL X-Y RECORDER TRACE
FIG. 11 COMPARISON OF $H_{12}$ AND $\delta_2$ CALCULATIONS WITH DATA OF MOSES\(^{(50)}\) AND GOLDBERG\(^{(62)}\) IN ADVERSE PRESSURE GRADIENTS.
FIG. II (CONT.) COMPARISON OF $H_{12}^*$ AND $\delta_2$ CALCULATIONS WITH DATA OF MOSES$^{(50)}$ AND GOLDBERG$^{(62)}$ IN ADVERSE PRESSURE GRADIENTS
FIG. 12 COMPARISON OF $H_{12}^*$ AND $R_{\delta_2}$ CALCULATIONS WITH DATA OF LAUNER\(^{(28)}\) IN A FAVORABLE PRESSURE GRADIENT
FIG. 13 COMPARISON OF $H^{*}_{12}$, $C_f$ AND $\delta_2$ CALCULATIONS WITH DATA OF SMITH AND WALKER$^{(63)}$ IN ZERO PRESSURE GRADIENT
FIG. 14 COMPARISON OF MEASURED AND PREDICTED SKIN FRICITION NOZZLE A
FIG. 15 COMPARISON OF MEASURED AND PREDICTED SKIN FRICTION—NOZZLE B
FIG. 16 COMPARISON OF MEASURED AND PREDICTED SKIN FRICTION
NOZZLE C
Fig. 17 Comparison of Measured and Predicted Skin Friction
Nozzle C with Spoiler

\[ P_0 = 20 \text{ in. Hg. abs.} \]

WALZ METHOD PREDICTION

\[ P_0 = 20 \text{ in. Hg. abs.} \]

DATA

\[ P_0 = 20 \text{ in. Hg. abs.} \]

\[ 15 \]

\[ 10 \]
FIG. 18 SHAPE FACTORS DEVELOPMENT NOZZLE A

DATA

\[ H_{12} \]

\[ x \text{ INCHES} \]
FIG. 19 SHAPE FACTORS DEVELOPMENT — NOZZLE B
FIG. 20 SHAPE FACTORS DEVELOPMENT
NOZZLE C
FIG. 21 SHAPE FACTORS DEVELOPMENT
NOZZLE C WITH SPOILER
FIG. 22 COMPARISON OF MEASURED AND COMPUTED $R_{d_2}$
NOZZLE A

$P_0 = 20$ IN. Hg. ABS.
FIG. 23 COMPARISON OF MEASURED AND COMPUTED \( R_{\theta_2} \)

\( P_0 = 20 \) IN. HG. ABS.

X INCHES

\( R_{\theta_2} \times 10^{-3} \)

28 30 32 34 36 38 40 42
FIG. 24 COMPARISON OF MEASURED AND COMPUTED $R_{\delta_2}$
NOZZLE C

$P_0 = \text{50 IN. HG. ABS.}$
FIG. 25 COMPARISON OF MEASURED AND COMPUTED $R_{\delta_2}$
NOZZLE C WITH SPOILER
FIG. 26 TYPICAL TRAJECTORIES ON $K - R_{\delta_2}$ PLOT

1. NOZZLE C, $P_0 = 50$
2. NOZZLE B, $P_0 = 5$
3. NOZZLE A, $P_0 = 5$
4. LAUNDER$^{(26)}$ PROFILE $a(ii)$
5. LAUNDER$^{(26)}$ PROFILE $c(i)$
**Fig. 27** Construction of boundary points for the laminarising region

**NOTATION:**
- A-10
- A-5
- A-15
- A-20

**NOZZLE A**

**NOZZLE B**

---

**Expected trajectory past final measurement**
FIG. 27 (CONTINUED) CONSTRUCTION OF BOUNDARY POINTS FOR THE LAMINARISING REGION
FIG. 28 TURBULENT–LAMINAR TRANSITION BOUNDARY FOR AN ADIABATIC WALL SHEAR LAYER
ZERO PRESSURE GRADIENT TEST
SECTION OPERATION AT FIXED SUBSONIC MACH NUMBER AND SEVERAL REYNOLD'S NUMBERS

TAKE READINGS OF ALL 28 PRESSURE DIFFERENTIALS ($\Delta P$) FOR EACH RUN

FIRST ORDER CORRECTIONS OF READINGS FOR SMALL DEVIATIONS FROM NOM. TUNNEL OPERATING PARAMETERS

PLOT $\Delta P$ VS. $c_f$ IN DIMENSIONLESS FORM USING PROCEDURE OF HOPKINS & KEENER TO REDUCE TO "INCOMPRESSIBLE" VALUES

VAN DRIEST RELATION FOR TEMP. PROFILE

MEAS. OF TOTAL HEAD B.L. PROFILES BY PROBE TRANVERSES AT EACH NOM. REN. NO. & SEVERAL $x$

FIRST ORDER CORR. OF RDGS. FOR SMALL DEV. FROM NOM. TUNNEL OPERATING PARAMETERS

OBTAIN EXPONENT IN EMP. RELATION
$\Delta P = q^b$ FOR EACH FENCE

PLOT $\delta_\gamma$ VS. $x$ FOR EACH NOM. REN. NO. USE FAIRED CURVES TO GET $\delta_\gamma$ AT 7 FENCE STATIONS

MACHINE INTEGRATION TO GIVE VEL. PROFILES AND MOMENTUM THICKNESS VALUES

PLT $c_f$ AT EACH FENCE STN. AT EACH REN. NO. USING ITERATIVE PROCEDURE OF FERNHOLZ

FIG. 29 SUBLAYER FENCE CALIBRATION PROCEDURE
FIG. 30 INTERPRETATION OF SUBLAYER FENCE READINGS
FOR FLOW WITH ARBITRARY MACH NUMBER
AND PRESSURE GRADIENT
FIG. 31 COMBINED CALIBRATION DATA POINTS FOR ALL FENCES AND RUNS AND BEST-FIT LINE THROUGH 532 POINTS
FIG. 32 MACHINE PLOT OF CALIBRATION DATA POINTS AND BEST-FIT LINES FOR FENCE 28