REPORT No. 68

BOUNDARY LAYER SEPARATION

Preliminary Report

HAL L. MOSES

May, 1962

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Under the Sponsorship of:

Office of Naval Research

Gas Turbine Laboratory
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ABSTRACT

The phenomenon of incompressible boundary layer separation and the existing methods of predicting it are discussed. The failure of these theories in many cases clearly indicates a need for further investigation.

A program is proposed that, it is hoped, will improve the present situation. The study is directed primarily at turbulent flow, but the laminar case is treated as well.

A theoretical method is presented which involves the ability of the boundary layer to transfer momentum to the fluid near the wall by shear stress.

The apparatus, which has already been built for the experimental investigation, is described. Due to its flexibility, the apparatus should prove valuable in comparing and improving methods of predicting separation.

ACKNOWLEDGEMENTS

The author would like to acknowledge, with gratitude, the encouragement and advice of Professor Edward S. Taylor, Director of the Gas Turbine Laboratory, and Professor Philip G. Hill. He is also grateful to Mr. Dalton Baugh and staff, who built the experimental apparatus, and Mrs. Madelyn Euvrard, who typed the report.

TABLE OF CONTENTS

INTRODUCTION

The well known phenomenon that occurs when the flow near a solid boundary, such as an airfoil, ceases to follow the boundary is known as boundary layer separation, or stall. The usual two-dimensional model is quite clear -- fluid near the wall that has been retarded by viscous stresses is unable to continue into a region of increasing pressure. The separation point is defined as the point where $\left(\frac{\partial u}{\partial y}\right)_o = 0$, or where $\tau_w = 0$. However, in three-dimensional flow the picture is not quite so clear. Except in special cases (singular points) the velocity gradient and shear stress at the wall remain finite. The separation point is then defined as the point where the limiting streamlines leave the wall. A general description of separation in three-dimensional flow was given recently by Taylor(1). For turbulent boundary layers the concept is still more difficult, even for two-dimensional flow. As noted by Kline(2), there is no definite point of separation, but usually a somewhat unsteady region.

Undoubtably, boundary layer separation, or stall, is of great importance in fluid mechanics. The design of any device involving fluid flow is likely to be critically dependent upon the phenomenon of separation. Such devices include pumps, compressors, turbines, propellers, wings, vanes, hull shapes, etc. In general, without some knowledge of separation, very little can be said about the flow even outside the boundary layer.

Because of the importance, a great deal of work has already been done in this general field. However, there is still considerable doubt as to the value of the methods so far proposed for predicting stall. Several theories were compared recently by Stewart(3). Figure 1, which was taken from Stewart's paper, clearly demonstrates the unreliability of these theories. In most cases, somewhat conservative rules, such as the lift coefficient for airfoils and pressure rise for diffusers, are used in design.

After a critical discussion concerning the present theories, Clauser[4] concludes that "the field is still wide open for the advent of a reliable method for predicting the behavior of turbulent layers under the influence of pressure gradients".

TURBULENT BOUNDARY LAYER THEORY

The theory of turbulence dates back to O. Reynolds, who first introduced the velocity in terms of a mean and fluctuating component into the Navier-Stokes equations. Later Prandtl, in his famous boundary layer approximations, simplified the equations of motion for flow near a solid boundary. The resulting equation for turbulent boundary layers is usually written

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\partial \overline{u'v'}}{\partial y} \qquad (1)$$

where $-\rho\overline{u'v'}$ is the Reynolds shear stress.

However, the difficulties involved in solving equation (1) have led to the use of integral techniques. The von Kármán momentum integral equation, which can be derived by directly integrating equation (1) or by a control volume approach, is usually written in its two-dimensional form

$$\frac{d\theta}{dx} + (H + 2) \frac{\theta}{U} \frac{dU}{dx} = \frac{C_f}{2} \qquad (2)$$

Equation (2) contains three unknowns; the momentum thickness, the shape factor and the skin friction. The skin friction for pressure gradients is somewhat uncertain, but perhaps one of the most reliable equations so far proposed is that of Ludwieg and Tillmann[5]

$$C_f = 0.246 \left(\frac{U\theta}{\nu}\right)^{-.268} 10^{-.678H} \qquad (3)$$

which was based on heat transfer measurements. Since the skin friction term is relatively small in an adverse pressure gradient, a flat plate equation of the form

$$C_f = \text{const.} \times \left(\frac{U\theta}{\nu}\right)^{-1/n} \qquad (4)$$

is often used with equation (2).

Then, with a mean value of H, which might partly compensate for the error involved in using the flat plate skin friction, equation (2) can be integrated as outlined by Schlichting[6] or Thwaites[7].

$$\Theta \left(\frac{U\Theta}{\nu}\right)^{1/n} = \frac{1}{U^b}\left[C + a \int_{x_1}^{x} U^b \, dx\right] \tag{5}$$

where a, b and n are constants.

Due to poor agreement with some experimental data, especially near separation, several recent investigators have questioned the validity of equation (2) (Refs. 8-11). Most of these authors have suggested the addition of terms containing the variation of pressure across the boundary layer and Reynolds normal stress ($\rho\overline{u'^2}$). Clauser[4] suggests, however, that the discrepancies are due mainly to three-dimensional flow. The use of equation (2) might be questionable, but the integrated form (5) has been found to agree reasonably well with much of the experimental data for the momentum thickness.

In predicting separation, however, there has been no such general agreement with experiment. Several attempts (Refs. 12-14) have been made to develop a criterion for separation of the form

$$\frac{\Theta}{U}\frac{dU}{dx} R^{1/n} = \text{const.} \tag{6}$$

which corresponds to the Pohlhausen parameter for laminar flow. These attempts, which assume that the tendency of the boundary layer to separate is only a function of local conditions, have not been too successful in general. As pointed out by Prandtl, one would expect the rate of change in the velocity profile, not the profile itself, to be related to this local parameter.

Most recent methods make use of the assumption that the velocity profiles form a one-parameter family, H, which has been verified reasonably well by experiment. The usual assumption is that separation occurs when the shape factor, H, reaches a value of approximately 2.5. Since values of 1.8 to 2.8 have been quoted, this criterion has also received much criticism. However, one is mainly interested in predicting the pressure rise to separation, and H always increases sharply with pressure above a value of 2.0 -- above this value, the fluid near the wall has very little dynamic pressure. The uncertainties involved in determining experimentally the value of H at separation are probably due to the unsteadiness of the boundary layer itself and cannot be avoided.

The usual procedure, then, is to find an equation for the shape factor (Refs. 7, 11, 15-23 and 26) of the form

$$\theta \frac{dH}{dx} = f(R,H) \frac{\theta}{U} \frac{dU}{dx} + g(R, H) \tag{7}$$

Many of the investigators attempt to find this relation by purely empirical means, such as von Doenhoff and Tetervin[16]. Others use semi-empirical methods; for example, Spence[20] assumes a universal velocity profile holds for $y \le \theta$ and obtains an expression from the differential equation (1). The coefficients, f and g, in equation (7) are compared in Reference (26) for several methods.

Some of the most successful methods so far make use of an energy equation (Refs. 11, 18, 22 and 23), which is derived by multiplying equation (1) by u and integrating. After making the usual assumption of a one parameter family, the energy equation can be reduced to one of the form (7). One limitation to this approach is the appearance of a dissipation term;

$$\int_0^{\delta} \frac{\tau}{\rho U^2} \frac{\partial (u/U)}{\partial y} \, dy$$

which must be approximated from experiment.

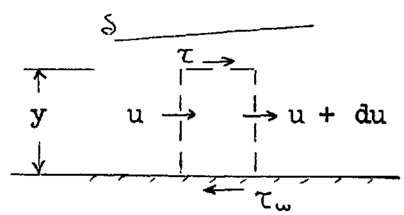
Truckenbrodt's method[22], which is outlined by Schlichting[6], uses the energy equation. The dissipation term is evaluated from the work of

Rotta[23]. One of the main advantages of this method is that the shape factor as well as the momentum thickness is obtained in integrated form.

PROPOSED METHOD

As the boundary layer proceeds in an adverse pressure gradient, all of the fluid experiences essentially the same pressure rise, and would suffer the same loss in dynamic pressure were it not for shear stress. Since the fluid near the wall has very little momentum, it must receive momentum from the outer layers in order to avoid separation. The shear stress is much greater in a turbulent boundary layer and it can experience a much greater pressure rise without separation than a laminar layer. It seems, then, that a method based on the distribution of turbulent shear stress, or ability to transfer momentum, would be a necessary approach.

By equating the forces on the partial control volume (similar to the method used by Rohsenow[24] for condensate boundary layers)



the shear stress can be expressed

$$\tau = \tau_w + y \frac{dp}{dx} + \frac{\partial}{\partial x}\int_0^y \rho u^2 dy - u \frac{\partial}{\partial x}\int_0^y \rho u dy \tag{8}$$

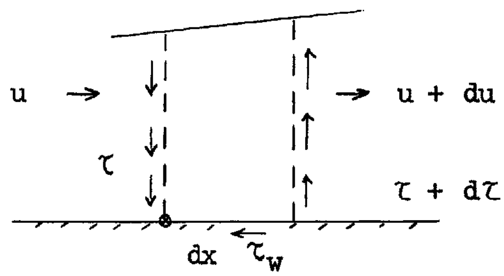
By using the power law velocity profile,

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/n} \qquad \text{where} \qquad n = \frac{2}{H-1}$$

equation (8) can be integrated across the boundary layer to obtain a shape factor equation

$$\frac{dH}{dx} = \left[-\frac{1}{U}\frac{dU}{dx} - \frac{H-1}{H^2(H+1)}\frac{1}{\theta^2}\int_0^\delta \frac{\tau}{\frac{1}{2}\rho U^2} dy + \frac{C_f}{\theta(H+1)} \right] \frac{H(H+1)^2(H-1)}{2} \tag{9}$$

An alternate approach consists of taking the moment of the momentum flux through the control volume



The resulting equation,

$$\frac{d}{dx}\int_0^{\delta} u^2 y\, dy - U\,\frac{d}{dx}\int_0^{\delta} u dy - \int_0^{\delta} uv dy = \frac{\delta^2}{2} U \frac{dU}{dx} - \frac{1}{\rho}\int_0^{\delta} \tau\, dy \qquad (10)$$

can be combined with the momentum equation (2), using the power law profiles, to obtain equation (9). The above expressions can also be derived by multiplying equation (1) by y and integrating. This approach was used by Tetervin and Lin[(25)] to obtain a general integral equation that contained both the energy and moment of momentum equations. Granville[(26)] used equation (9) as derived by Tetervin and Lin.

Although the velocity profiles can best be represented by a law such as that suggested by Coles[(27)], these laws involve the shear stress, τ_w. The simplicity and general experimental agreement of the power law tend to justify its use. As with the momentum equation, it might be expected that the error caused by using an approximate velocity profile would not be too great. In order to check this assumption, profiles of the form

$$\frac{u}{U} = 1 - m\left[\frac{y}{\delta} - 1\right]$$

were tried with very similar results.

Figure 2 shows the result of comparing equation (9) with the data of Schubauer and Klebanoff[(28)], where the turbulent shear stress was determined by hot-wire measurements. The agreement is very good when the shear stress is reduced by 30%, as suggested by Coles[(27)]. The dotted lines show the effect of a slight increase in pressure on the shape factor.

Equation (9) is very similar to that obtained from the energy method, as would be expected since the energy equation can be derived by multiplying equation (1) by $u = f(y)$ and the moment of momentum by y itself.

However, since the moment of momentum is directly connected to the shape and is definitely conserved, as opposed to the kinetic energy, the present analysis might have a sounder physical basis. Also, equation (9) contains the integral of the shear stress alone, which should be easier to determine than the dissipation function. With an assumption for the shear stress or dissipation term, both methods reduce to an equation of the form (7). However, only these terms need to be determined from experimental data.

It can be seen that equation (7) is made up of two terms; one a pressure term that tends to increase H towards separation and the other a shear stress term that decreases H. Several investigators have suggested that the shear stress term is relatively small (Refs. 29-30), which might be expected in sufficiently large adverse pressure gradients, and can be approximated by a mean value. However, this is not always the case -- for example, both terms are equal in an equilibrium boundary layer, where $dH = 0$. It might be noted that when $dH = 0$, equation (7) reduces to one of the form (6).

The statement that the first term in equation (7) is due to pressure rise alone is not exactly true, since some viscous action is necessary to hold the power law velocity profiles, especially near the wall. It appears that this viscous action is always present and important, except possibly at a shock wave or sharp bend.

In order to use the present method, some means of predicting the distribution of turbulent shear stress must be developed. One attempt is based on flat plate data, where the shear stress, and thus the turbulent viscosity, can be determined by equation (8). Approximately the same result is obtained using the data for pipe flow given by Schlichting[6] with the radius replaced by δ.

If it is assumed that the turbulent viscosity obtained in this way,

$$\frac{\epsilon}{U\delta} \cong 0.011 \left(\frac{U\theta}{\nu}\right)^{-\frac{1}{10}} \left[\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] \tag{11}$$

also applies to the boundary layer in an adverse pressure gradient, the turbulent shear stress can be expressed

$$\frac{\tau}{\frac{1}{2}\rho U^2} = 0.011 \left(\frac{U\theta}{\nu}\right)^{-\frac{1}{10}} (H-1) \left[\left(\frac{y}{\delta}\right)^{\frac{H-1}{2}} - \left(\frac{y}{\delta}\right)^{\frac{H+1}{2}}\right] \tag{12}$$

The shear stress from equation (12), as shown in Figure 3, agrees with the data of Schubauer and Klebanoff[28] much better than than proposed by Fediaevsky[32]:

$$\frac{\tau}{\frac{1}{2}\rho U^2} = \frac{\tau_w}{\frac{1}{2}\rho U^2}\left[1-4\left(\frac{y}{\delta}\right)^3 + 3\left(\frac{y}{\delta}\right)^4\right] - \frac{\delta}{U}\frac{dU}{dx}\left[\left(\frac{y}{\delta}\right) - 3\left(\frac{y}{\delta}\right)^3 + 2\left(\frac{y}{\delta}\right)^4\right] \tag{13}$$

Equation (12) can now be integrated for the shear stress term in equation (9).

$$\frac{1}{\theta}\int_0^\delta \frac{\tau}{\frac{1}{2}\rho U^2}\,dy = .044\,\frac{H}{H+3}\left(\frac{U\theta}{\nu}\right)^{-\frac{1}{10}} \tag{14}$$

This result has been plotted in Figure 3 for comparison with the data of Reference (28), which has been reduced by 30% to give more reasonable values at the wall.

Granville[26] assumed

$$\frac{1}{\delta^*}\int_0^\delta \frac{\tau}{\frac{1}{2}\rho U^2}\,dy$$

to be that obtained from flat plate data.

Although the present analysis needs further development and simplification, it seems quite promising thus far. The method may be summarized as follows:

1) momentum integral equation (2)
2) assumption of power law velocity profiles
3) shape factor equation (9)
4) skin friction equation (3) or (4)

5) assumption that equation (12) applies for adverse pressure gradients, which results in equation (14)

The present method has been developed for two-dimensional flows, but it can easily be extended for axisymmetric and three-dimensional cases. However, when the boundary layer is skewed, some further assumptions must be made regarding the cross-flow velocity profiles. The analysis will then provide a theoretical method of determining the importance of three-dimensionality on separation.

Laminar Boundary Layers

The difficulties involved with series solutions have also led to integral techniques for laminar boundary layers. The best known of these methods is that of Pohlhausen, where the shape factor is related to the pressure gradient by the local parameter

$$\Lambda = \frac{\delta^2}{\nu}\frac{dU}{dx}$$

However, according to this method separation occurs when $\Lambda = -12$, while the exact solutions of Hartree and Howarth, as well as the experiment of Schubauer[33] show separation at $\Lambda \cong -5$. (The Pohlhausen method and the exact solutions are outlined by Schlichting[6]).

As with the energy method, the present method should also apply for laminar boundary layers. However, a different assumption must be made for the velocity profiles. The profiles that have been used here (Fig. 4)

$$\frac{u}{U} = (2\eta - \eta^2) + \beta(\eta - 2\eta^2 + \eta^3)$$

satisfy the boundary conditions; $\qquad \eta = \frac{y}{\delta}$

$$\eta = 0, \quad u = 0 \qquad \eta = 1, \quad \frac{u}{U} = 1$$

$$\frac{\partial u}{\partial y} = \tau_w/\mu \qquad \frac{\partial u}{\partial y} = 0$$

The resulting thickness and shape equations can be written

$$\frac{d\theta}{dx} + M(\beta)\,\frac{\theta}{U}\frac{dU}{dx} = \frac{\nu}{U\theta}\,N(\beta) \qquad (15)$$

$$P(\beta)\ \frac{d\beta}{dx} = \frac{1}{U}\frac{dU}{dx} + Q(\beta)\ \frac{\nu}{U\theta^2} \qquad (16)$$

where $M(\beta)$, $N(\beta)$, $P(\beta)$ and $Q(\beta)$ are given in Figure 5. Separation occurs at a value of β of -2.0.

For the case of zero pressure gradient, $\beta = -.360$. Equation (15) then yields

$$\tau_w = 0.335\ \rho U^2 \sqrt{\frac{\nu}{Ux}} \qquad (17)$$

where the constant corresponds to the value of .332 obtained from the exact Blasius solution.

In the case of wedge flow $(U = U_o x^m)$ the present analysis gives similar profiles with the shape factor related to m by

$$m = \frac{Q}{(1-2M)Q-2N}$$

For separation, m = -.0927 compared to the exact Hartree solution of -.091.

Experimental Apparatus

An apparatus, which was suggested by Professor P. G. Hill, has already been built in the Gas Turbine Laboratory for an experimental study of turbulent boundary layer separation. The test section (Fig. 6) consists of an annulus between a 5 1/2 inch plastic cylinder and a 15 inch porous metal cylinder. The boundary layer to be studied grows along the plastic cylinder, and can be measured at any position. Velocity profiles are taken by a total pressure probe inserted through the porous metal. The adverse pressure gradient is obtained by closing the end of the annulus with an adjustable plate, which causes the flow to diffuse through the porous cylinder. Static pressures are measured by a row of static taps along the plastic cylinder, which can be rotated or moved axially to any position. The pressure distribution can be controlled by simply varying the amount of open area along the porous cylinder.

An apparatus of this particular type has several distinct advantages that may be summarized as follows:

1) the factors affecting the boundary layer; pressure distribution, thickness and length (or Reynolds number) can easily be controlled

2) measurements can be made at any position

3) the flow is quite steady, even with separation

4) undesired corner and secondary flow effects are eliminated

5) the boundary layer can be made skewed or collateral

Since the boundary layer is thin compared to the radius of the plastic cylinder, the effect of transverse curvature is neglected as a first approximation. If necessary, the above equations can be transformed to include this effect for comparison with the experiments. In case of zero pressure gradient the effect of transverse curvature has been calculated (similar to the method used by Eckert[31]) assuming a skin friction equation of the form (4). The result for the momentum thickness can be written

$$\theta_{cyl.} \cong \theta_{f.p.}\left[1 + \frac{\delta}{3r}\right]^{-\frac{4}{5}} \tag{18}$$

This result has been experimentally verified on the above apparatus.

Preliminary Results

Some preliminary tests have been made with the above apparatus in order to demonstrate the effect of different factors upon the maximum pressure rise before separation. Figure 7 shows the effect of pressure distribution. It can be seen that the pressure coefficient $\left(C_p \equiv \frac{\Delta p}{\frac{1}{2}\rho U_1^2}\right)$ can be nearly doubled by simply changing the pressure distribution, with all other factors remaining constant. The advantage of early deceleration may be explained by the greater ability of a thin boundary layer to transfer momentum to the retarded flow by turbulent shear stress.

Figure 8 shows the effect of three-dimensionality on separation. The three-dimensional flow was obtained by closing the open area on top of the porous cylinder. The pressure coefficient was increased to approximately 0.85 on the top of the cylinder, where the retarded flow is removed from the boundary layer, and decreased to 0.5 on the bottom, where it collects. The corresponding C_p for two-dimensional flow was 0.6.

An example of an experiment designed to isolate the effect of turbulent shear stress in avoiding separation (the function g in equation (7)) is shown in Figure 9. The pressure was increased very rapidly and then held constant over a considerable length. It can be seen that the shear stress causes the shape factor to return to the original value fairly quickly.

Summary of Program

1. Evaluation of Existing Contributions to the Problem

Due to the great importance of the problem, a vast amount of work has been done in this and related fields. However, as comparison with the experimental data clearly shows, there is still no universally satisfactory method for predicting boundary layer separation. Since most of the theories are ultimately based on experiment, they do correlate some of the data, but there is no means of knowing when they are reliable. Almost all of the methods so far proposed consist of the following elements:

1) the assumption that the velocity profiles can be described by one parameter, the shape factor
2) the momentum integral equation
3) an empirical skin friction equation
4) an expression for the shape factor, H

It is in the determination of the shape factor, by which separation is to be predicted, that the methods differ and often fail.

The results of some of the existing theories have been compared (Refs. 3, 6, 7, 26), but there is still a definite need for an ordered evaluation of the state of the field. The evaluation should attempt to identify the essential elements of the methods -- their relative strength and weakness and an explanation for when and why they fail.

Most of the work that has been done so far is for two-dimensional flow, while in reality, this is seldom the case. It is usually assumed that the effect of three-dimensionality is negligible, but a quantitative examination is definitely needed.

2. Proposed Theoretical Method

The proposed method (page 5) for the determination of the shape factor is based on the fundamental law of motion. Since the boundary layer is of finite thickness, two equations may be used to improve on existing integral methods -- one force and one moment. An assumption must be made for the velocity profiles, but this is apparently not critical. The resulting expression for turbulent flow agrees with the result of Tetervin and Lin[25], who used a mathematical rather than a physical approach. A preliminary check of the fundamental equation (9) is shown in Figure 2.

As pointed out by Granville[26], the most critical part of the method for turbulent flow is the determination of the shear stress across the boundary layer. The proposed method is based on well-established flat plate and pipe flow data, and is compared with experiment in Figure 3. For laminar flow the shear stress term can be integrated directly.

The theory has been discussed for two-dimensional flow, but it can be extended for three-dimensional cases.

The validity of the approximate method for laminar flow can be checked with the known exact solutions of Hartree and Howarth as well as experimental data, such as the Schubauser ellipse. The agreement for zero pressure gradient is shown in Figure 4. However, for turbulent boundary layers the test must rely entirely on experiment.

3. Experimental Methods

An apparatus (Fig. 6) has been built for the investigation of turbulent separation. The boundary layer to be studied is located on the inner cylinder. The static pressure is measured by wall taps spaced 2 inches apart. Velocity profiles can be taken at any position by a total pressure probe with a micrometer traversing mechanism. If necessary, other facilities are available, including the possibility of hot-wire turbulence measurements.

The apparatus has several distinct advantages over previous types. The boundary layer can be made skewed or definitely collateral, as desired; the factors affecting the boundary layer, such as pressure distribution and Reynolds number, can be easily controlled; and the flow is steady. By this means experimental data over a wide range of conditions can be obtained to compare and improve the theoretical methods.

Some preliminary results, as discussed on page 11, are shown in Figures 7 - 9.

LIST OF SYMBOLS

H	Turbulent shape factor, δ^*/θ
p	Static Pressure
q	Dynamic Pressure, $\frac{1}{2}\rho U^2$
R	Reynolds Number, $U\theta/\nu$
u	Mean velocity parallel to the wall
u'	Fluctuating velocity parallel to the wall
v	Mean velocity perpendicular to the wall
v'	Fluctuating velocity perpendicular to the wall
U	Free stream velocity
x	Coordinate parallel to the wall
y	Coordinate perpendicular to the wall
C_p	Pressure coefficient, $\Delta p/\frac{1}{2}\rho U_1^2$
C_f	Skin friction coefficient, $\tau_w/\frac{1}{2}\rho U^2$
β	Laminar shape factor
δ	Boundary layer thickness
δ^*	Displacement thickness
θ	Momentum thickness
ϵ	Turbulent viscosity
ρ	Density
τ	Shear stress
τ_w	Shear stress at the wall
ν	Kinematic viscosity

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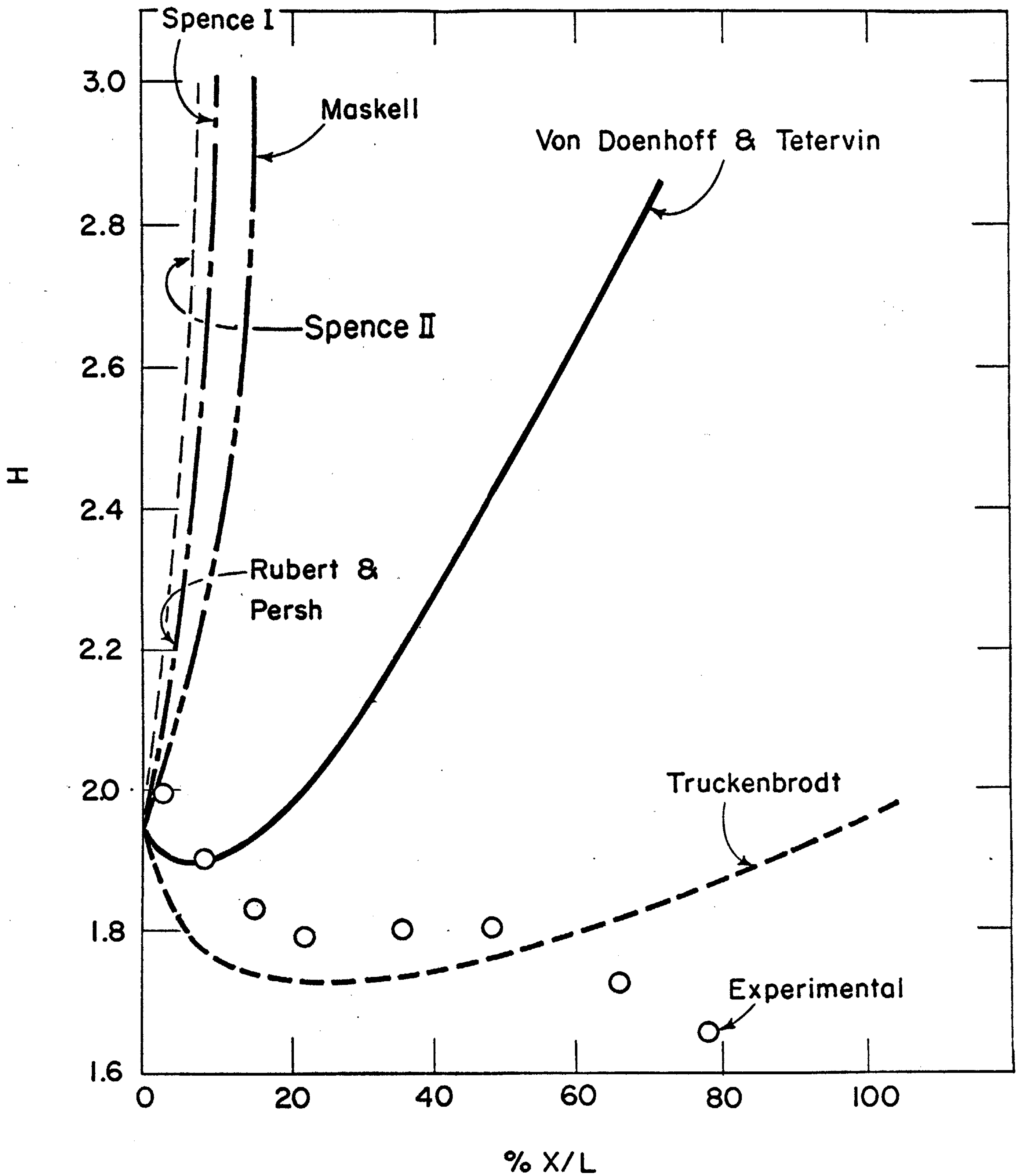


SHAPE FACTOR vs PERCENT OF TEST SECTION LENGTH - CLAUSER'S NO. 2 PRESSURE DISTRIBUTION

FIGURE I

(TAKEN FROM REFERENCE 3)

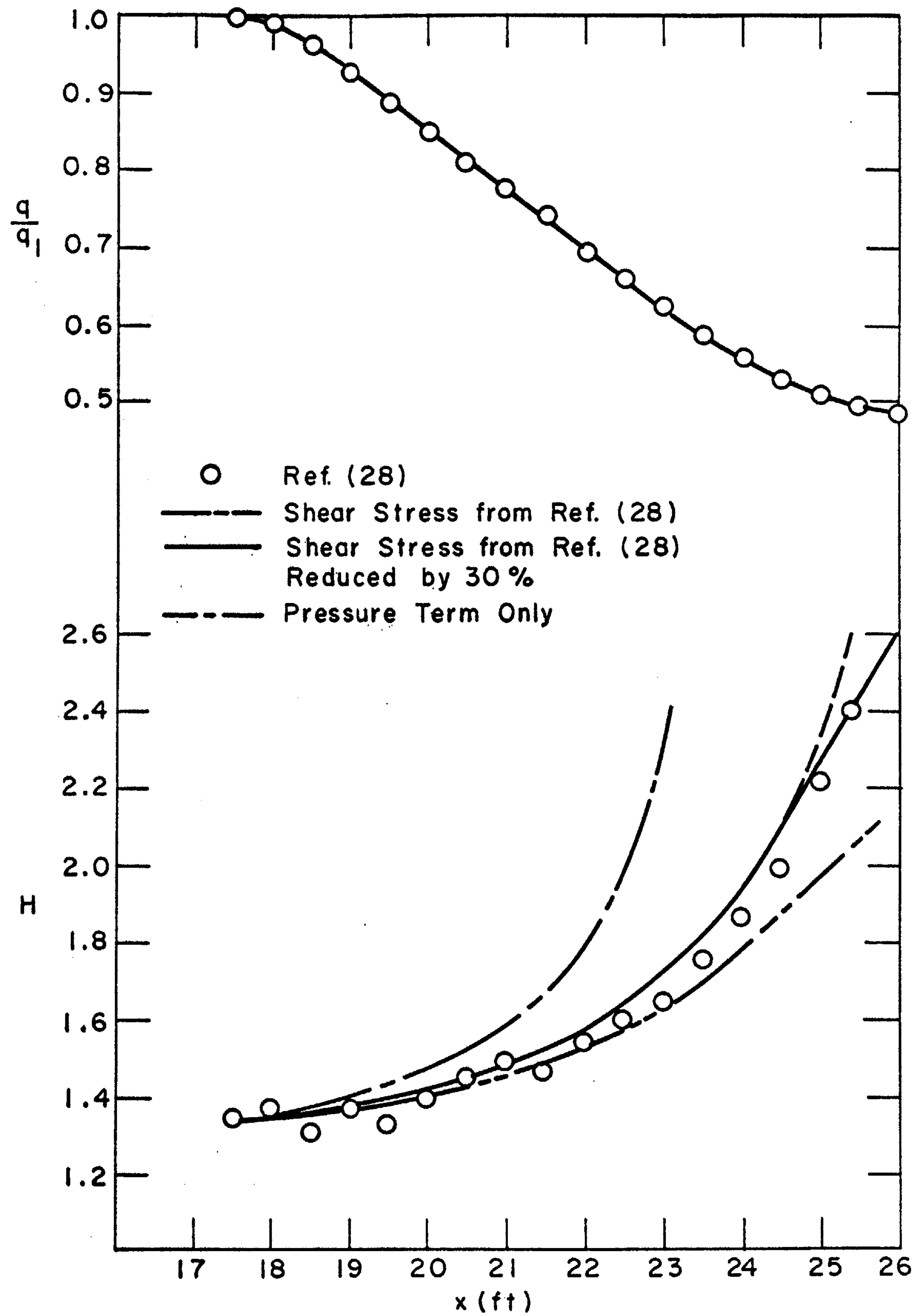


FIGURE 2 SHAPE FACTOR EQUATION (9) COMPARED WITH DATA FROM REF (28)

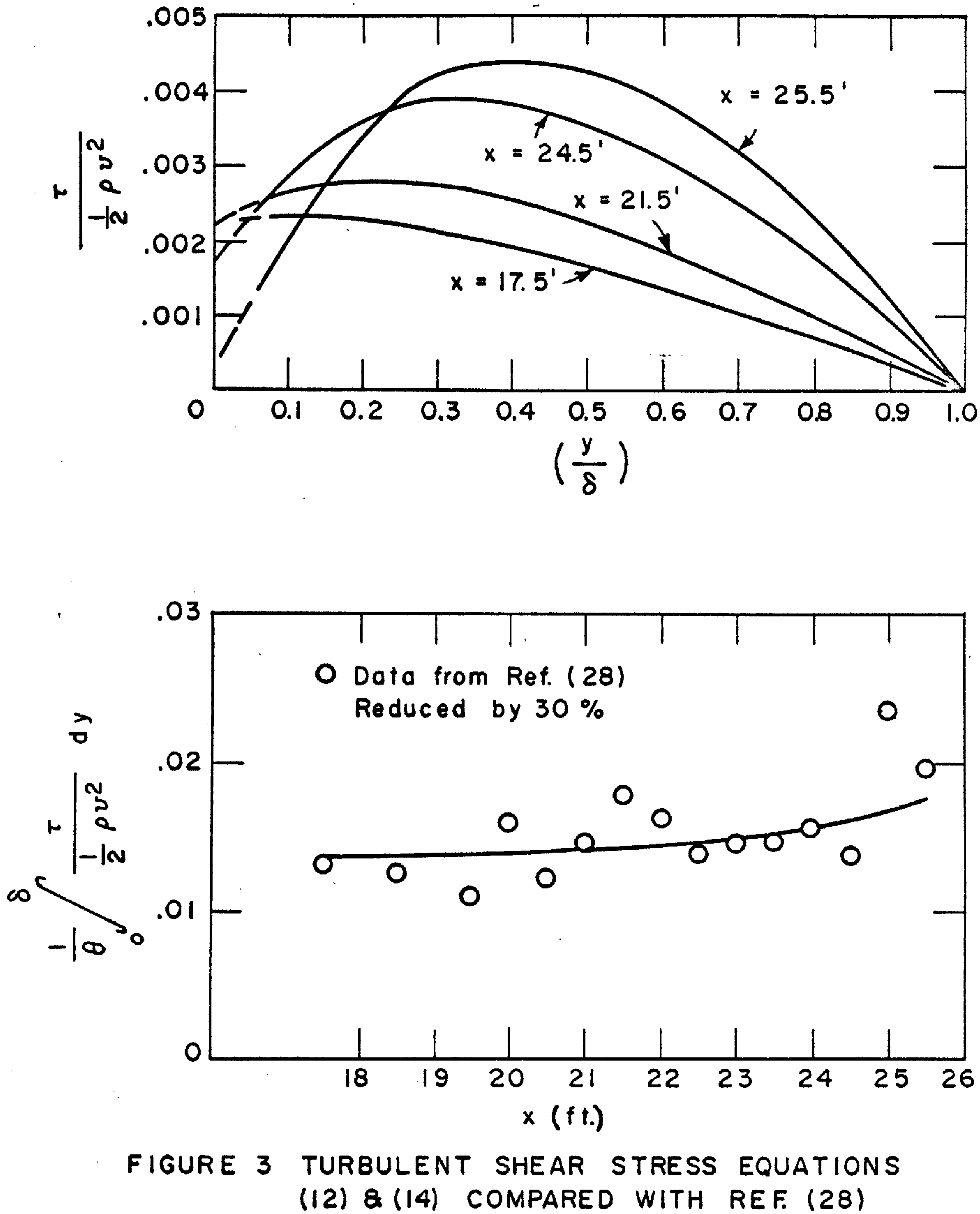


FIGURE 3 TURBULENT SHEAR STRESS EQUATIONS (12) & (14) COMPARED WITH REF. (28)

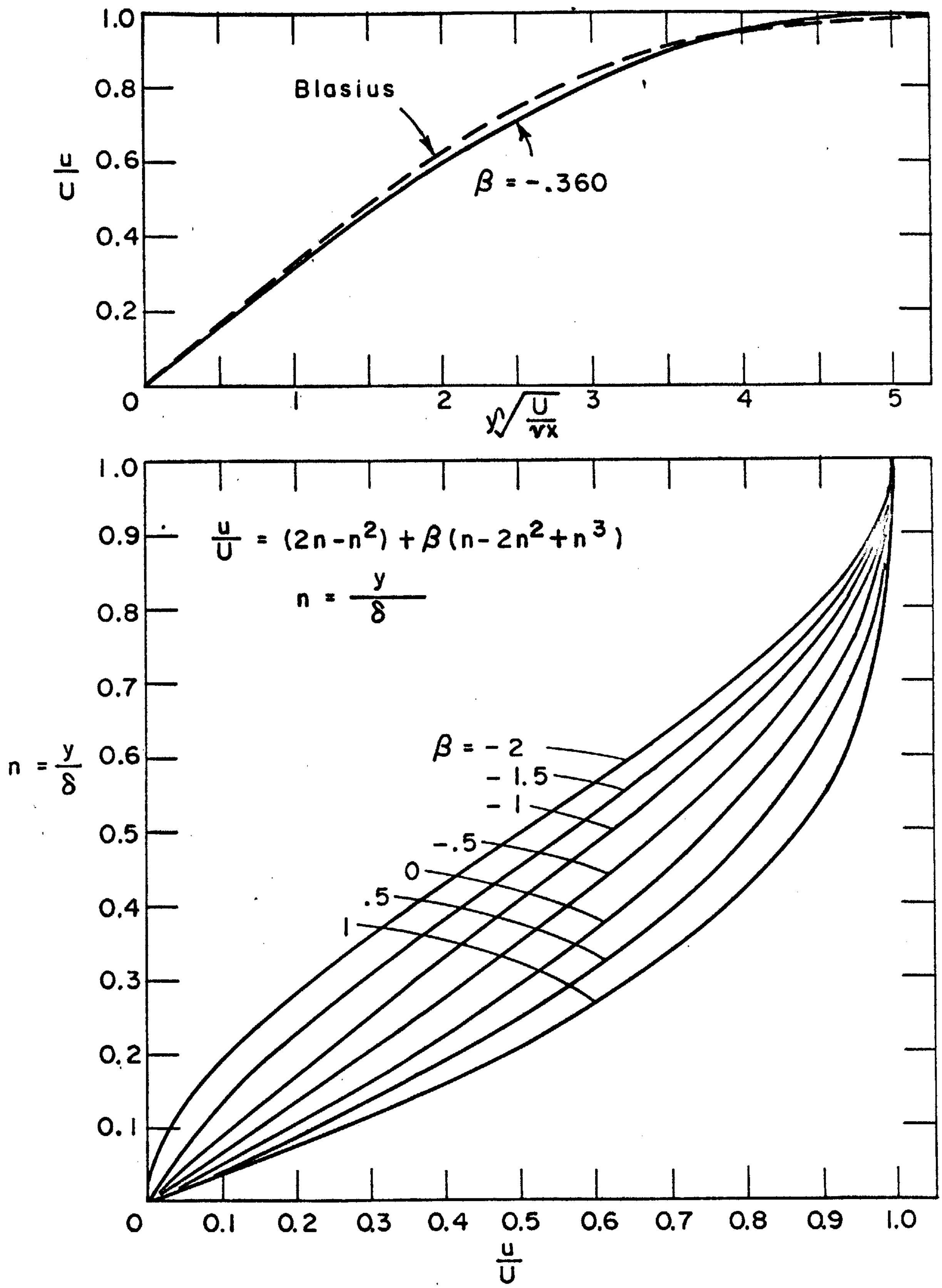


FIGURE 4 LAMINAR PROFILES

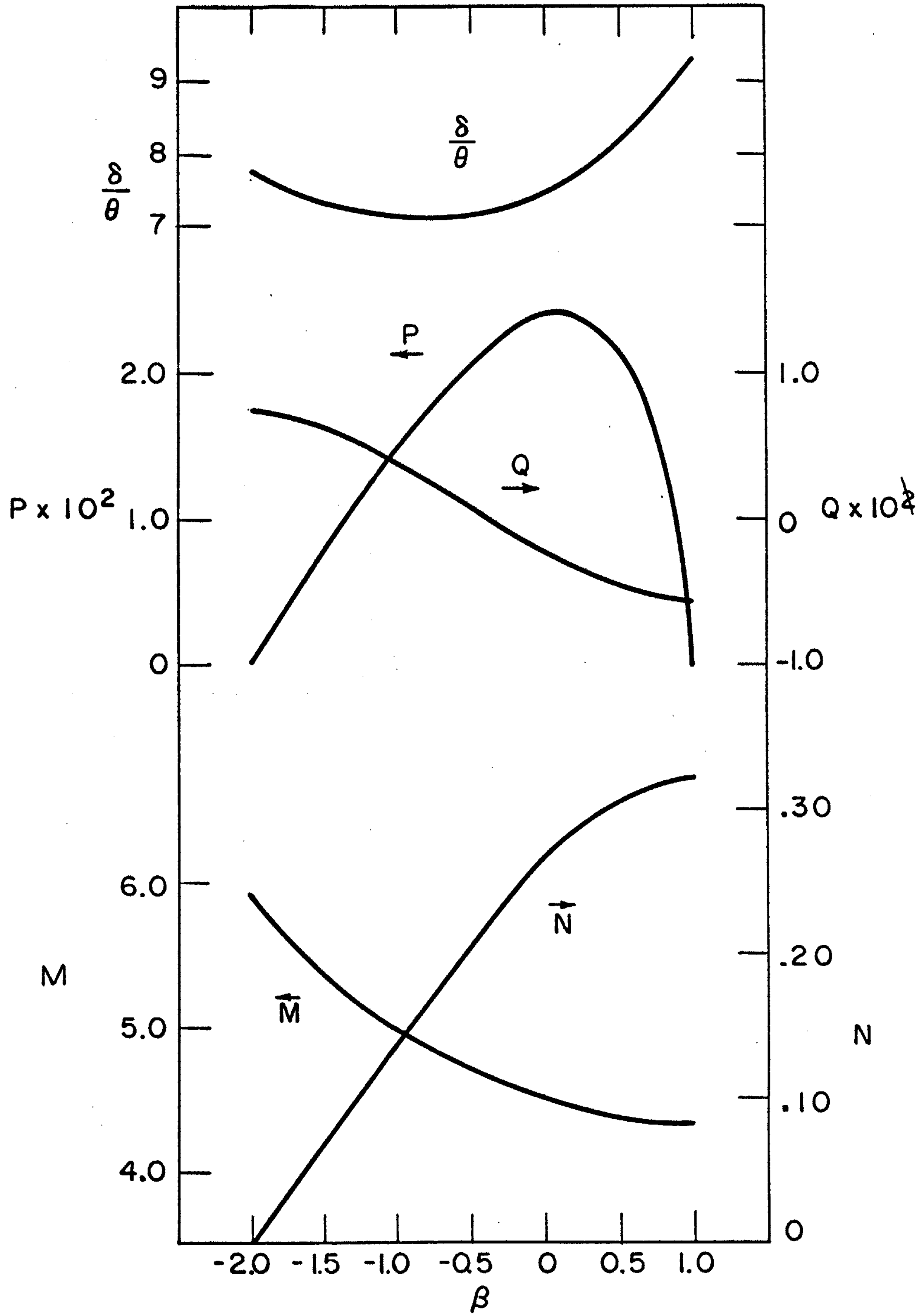
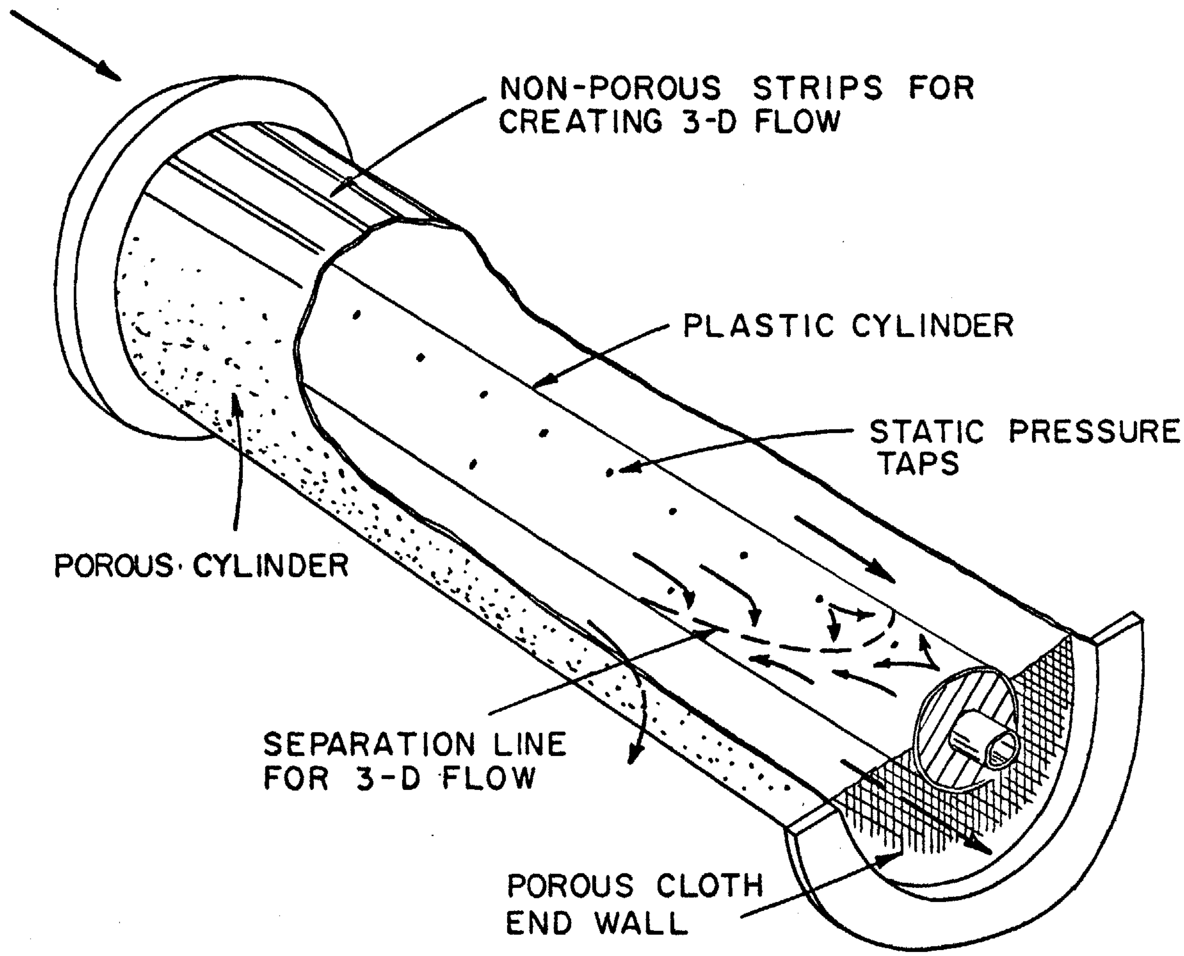


FIGURE 5 COEFFICIENTS FOR LAMINAR BOUNDARY LAYER EQUATIONS (15) & (16)



TEST SECTION

FIGURE 6

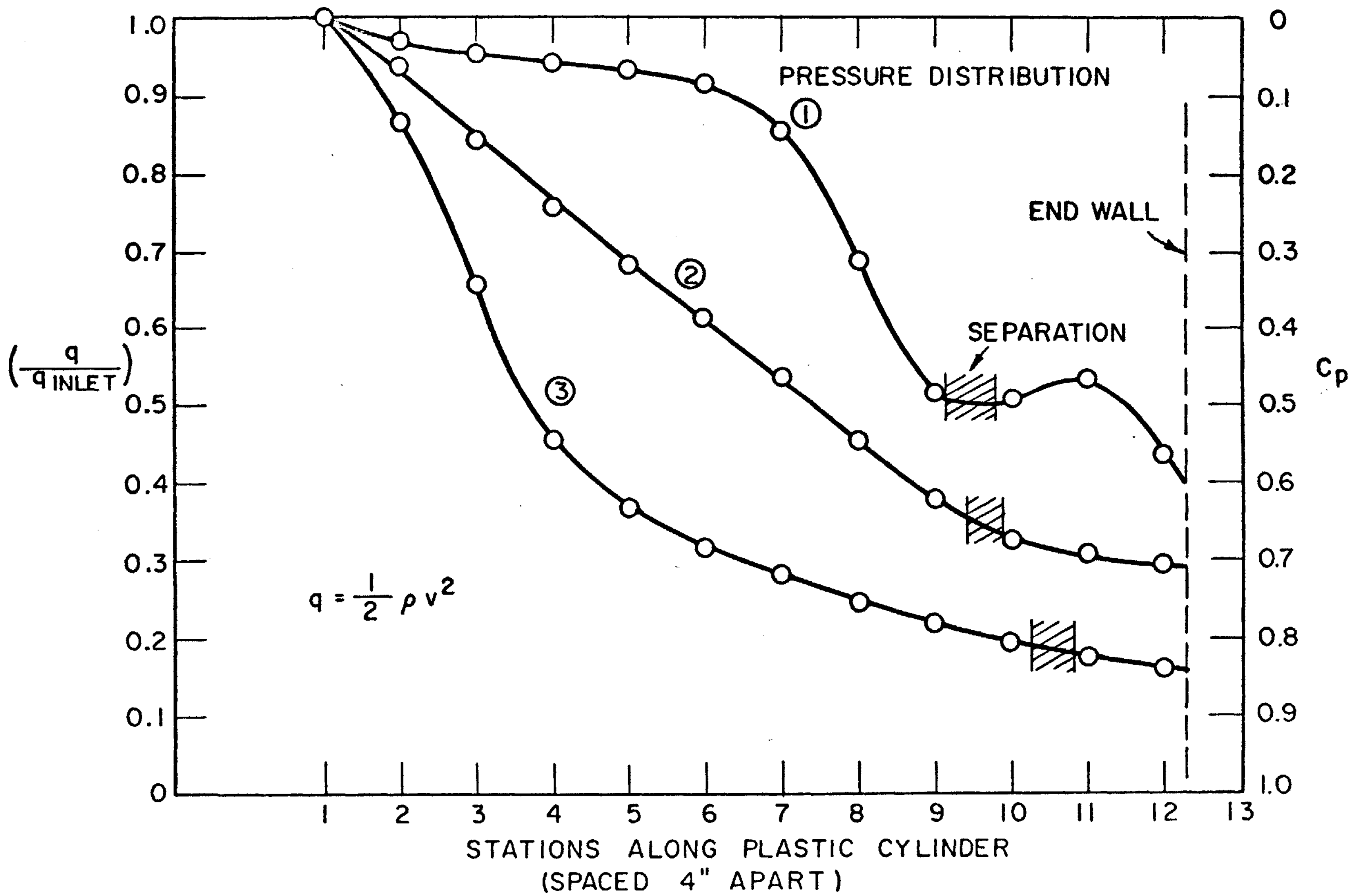


FIGURE 7 EFFECT OF PRESSURE DISTRIBUTION ON SEPARATION IN TWO-DIMENSIONAL FLOW.

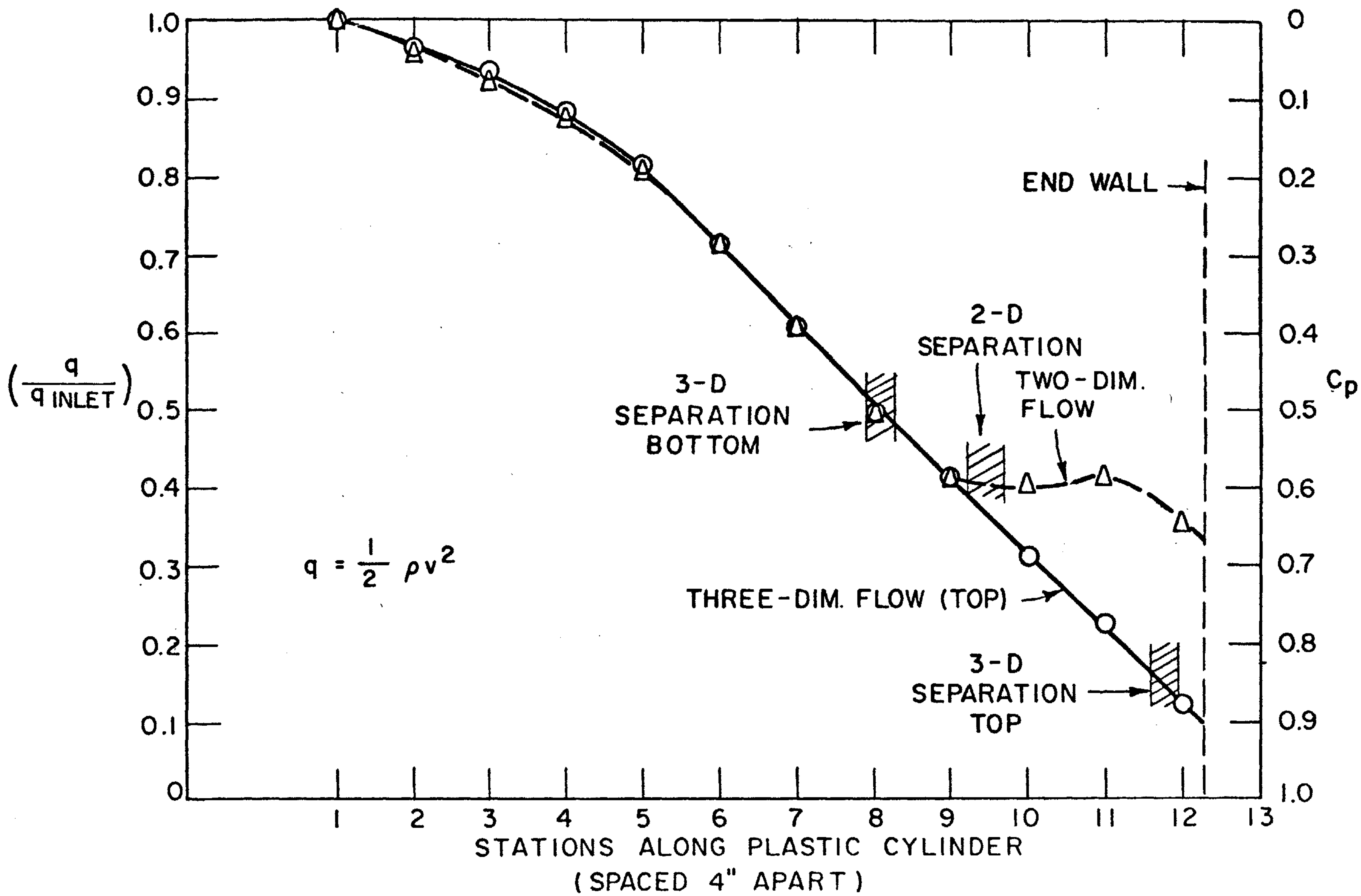


FIGURE 8 EFFECT OF THREE-DIMENSIONALITY ON TURBULENT BOUNDARY LAYER SEPARATION

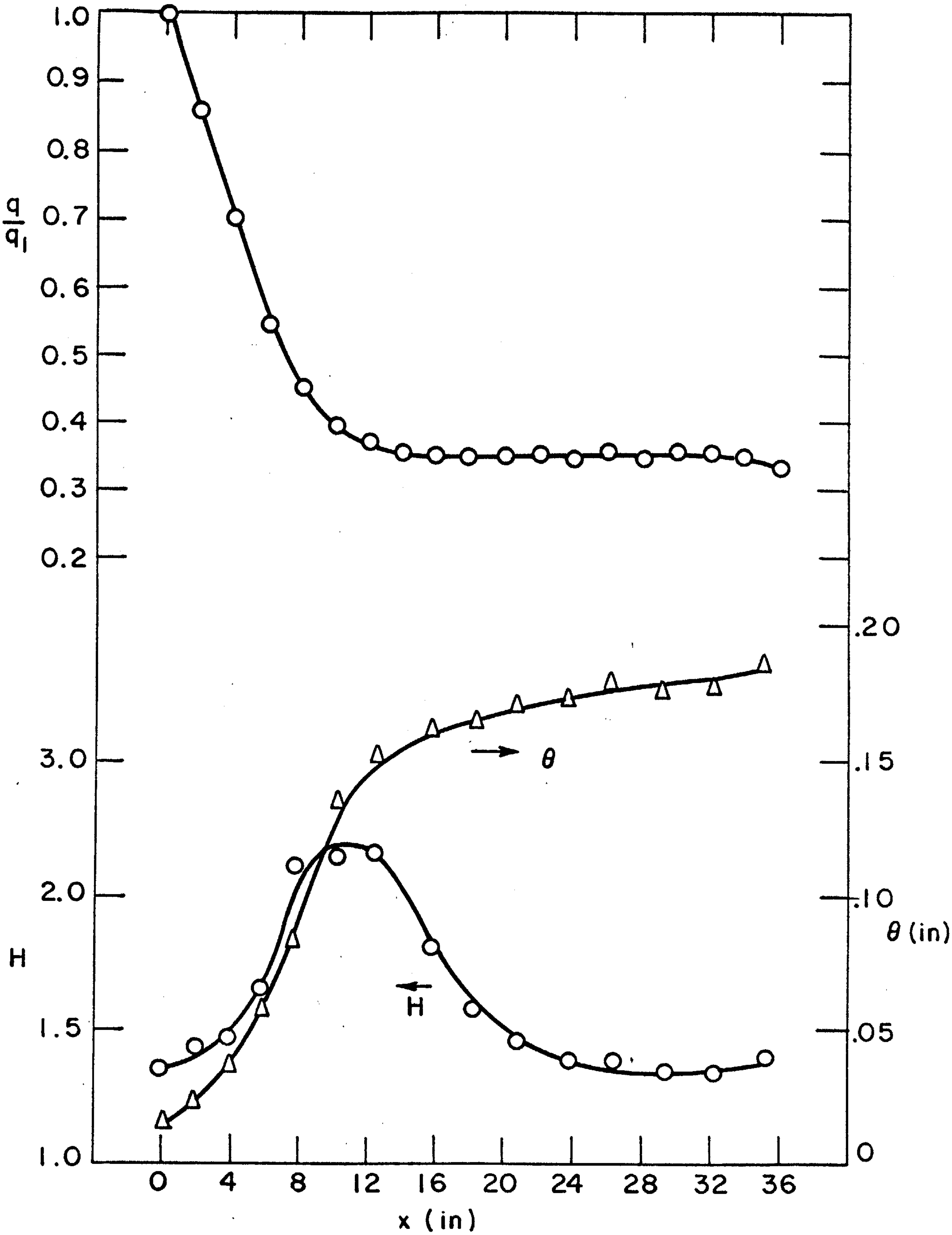


FIGURE 9 EFFECT OF SHEAR STRESS ON H IN ZERO PRESSURE GRADIENT