REPORT NO. 56

CASCADE PERFORMANCE WITH ACCELERATED OR DECELERATED AXIAL VELOCITY

SHIGEO KUBOTA

September, 1959

GAS TURBINE LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE · 39 · MASSACHUSETTS

CASCADE PERFORMANCE WITH ACCELERATED OR DECELERATED AXIAL VELOCITY

by

SHIGEO KUBOTA

Under the sponsorship of:

General Electric Company
Allison Division of General Motors Corporation
Westinghouse Electric Corporation

Gas Turbine Laboratory
Report Number 56

September
1959

Massachusetts Institute of Technology

TABLE OF CONTENTS

ABSTRACT

A theoretical method to estimate the effect of axial velocity change through a cascade was investigated. The change of axial velocity was reproduced by distributing sinks and sources within the blade passages, and the conclusions are set forth in some simple formulae. Some graphs for the numerical evaluation of the performance of NACA 65 Series cascades were prepared, and several examples were compared with experimental data.

ACKNOWLEDGEMENTS

This report is the result of an investigation, which was performed during my stay at the Gas Turbine Laboratory for several months. During this period, I was treated quite the same as the other staff members and spent a most profitable and pleasant time. I should like to thank all the members of the Gas Turbine Laboratory, particularly Professors E. S. Taylor and Y. Senoo who always gave me the most useful advice and encouragement. Professor S. Otsuka of Nagoya University, Japan, sent me a useful report and valuable advice for which I am most thankful.

I must express many thanks to Mr. J. Brown who assisted me most effectively in the preparation of the manuscript. The skillful typewriting is due to the efforts of Mrs. N. Appleton.

My stay at this laboratory would not have been possible without the financial support of my company, Kawasaki Dockyard Co., Kobe, Japan; I take this opportunity to express many thanks to the company.

1. INTRODUCTION

The study of cascade performance, as a result of a changing axial velocity within the blade passages, is important in the design of axial flow turbomachinery. In most of the multi-stage axial flow compressors and turbines, the magnitude of axial velocity varies considerably from the first to the last stage, because of the design requirements. Small axial velocity changes accumulate from each stage. Therefore, it is somewhat inexact to use the experimental data of cascade tests which are obtained under the condition of constant axial velocity, without any correction.

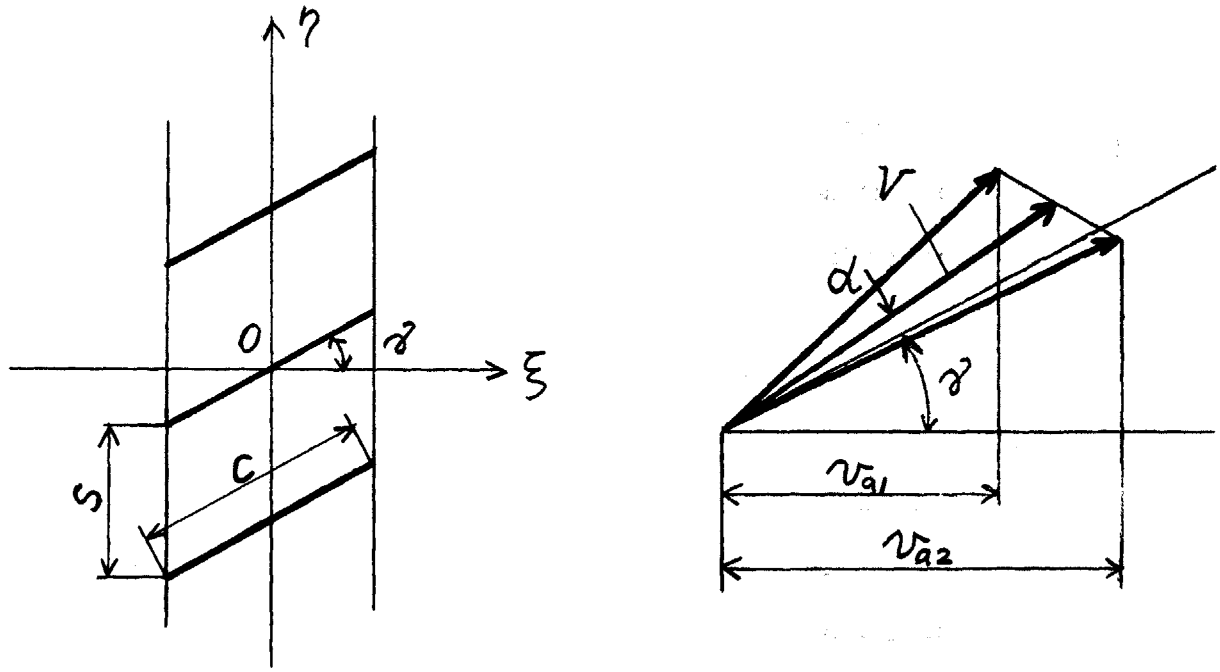
There is another reason why this problem is important. In the usual cascade wind tunnel test, owing to the development of a boundary layer on the surface of the side wall of the cascade, the axial velocity increases at the midspan through the cascade. In order to keep the axial velocity constant, the boundary layer must be sucked off in some way, making the experimental technique more time consuming. Therefore, if the experimental data without any boundary layer removal can be corrected to the case of constant axial velocity, there is a certain possibility of reducing the difficulty of cascade testing very much.

N. Scholz (1), S. Katzoff (2) and W. R. Hawthorne (3) wanted to solve this problem and devised some equations to correct such an effect. However, since their treatment was based on a simple one-dimensional consideration, it seems to be insufficient to explain such a complicated phenomenon. J. R. Erwin and J. C. Emery (4) proposed three empirical methods to correct the turning angle but, as the authors themselves said they didn't cover any of the experimental evidence. Later, T. Kawasaki (5) carried out a method based on the two-dimensional treatment. In his method, he put a concentrated source at the middle point of the blade passage corresponding to the increment of the axial velocity. However, there is some doubt whether or not it is proper to express the effect of the axial velocity increment by such a concentrated singularity. The method here developed is based on the idea that sources are distributed in the whole domain of the blade passage, corresponding to the increase or decrease of the axial velocity. This assumption seems to be a more accurate expression of the phenomenon.

Generally speaking, there are many factors which would be related to the change of axial velocity. They are; hub ratios at the inlet and exit of the stages, accumulation of boundary layer thickness through stages, effect of compressibility, effect of secondary flow and leakage or extraction at any stage. In most cases, one might consider that all of these effects appear separately. The method here described can cover all of the effects mentioned above, except the effect of compressibility and secondary flow. The correction regarding these two factors must be taken into account in a proper way, if it is necessary.

2. FLAT PLATE CASCADES

First consider a most simplified cascade, that is, the cascade composed of flat plates. In this theoretical consideration the following basic assumptions are presupposed; flow is steady, two-dimensional, nonviscous and incompressible.



The cascade to be considered is composed of flat plates, whose chord length is c, spacing is s, stagger angle is γ . At the inlet of the cascade flow is quite uniform, and its direction is changed by the cascade, flowing off to the right. In the cascade, from $\xi = -\frac{c}{2}\cos\gamma$ to $\xi = \frac{c}{2}\cos\gamma$, the axial velocity of the flow is accelerated (or inversely, decelerated) from v_{a1} to v_{a2}.

In such a case our problem is to determine the performance of the cascade. That is, given the values of mean velocity direction measured from the chord line α, the magnitude v, and the changing condition of axial velocity within the blade passage $\Delta v_a(\xi)$, calculate the value of the circulation around a blade Γ as a function of γ, $\lambda = s/c$, α, V and $\Delta v_a(\xi)$. In this paper, variations of axial velocity were treated as a function of ξ only, and independent of η.

To solve this problem, we divide the flow into three elementary parts:

(i) Flow around the given cascade with a given mean velocity vector, without any change of axial velocity.

(ii) Flow consisting of a source and sink distribution only, corresponding to the given axial velocity change (the boundary condition is not satisfied).

(iii) Flow to cancel the normal velocities on the blade surface which were induced by the flow (ii), and an additional circulation to satisfy the Kutta-Joukowski's condition at trailing edge.

By superimposing the three elementary flows above, we can obtain the flow which satisfies all the prescribed conditions.

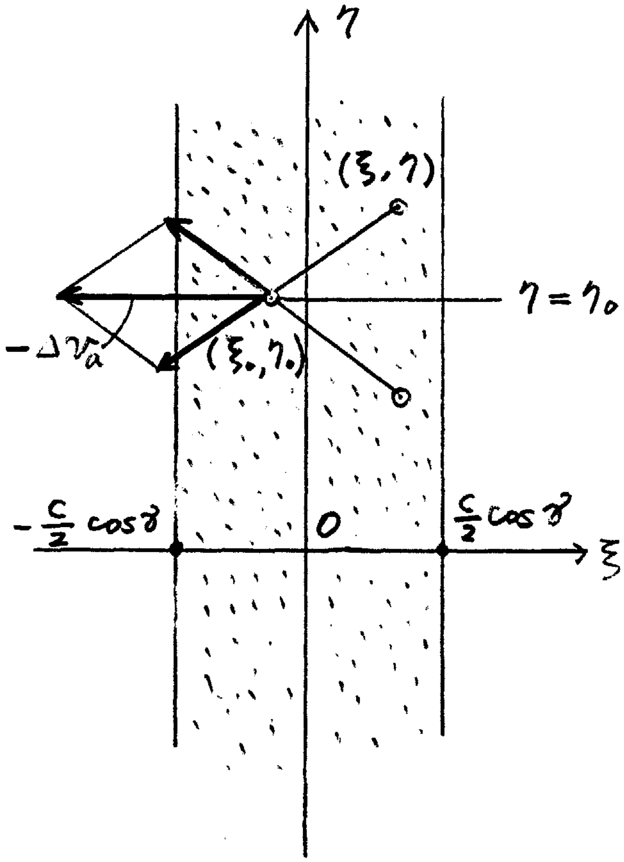
2 - 1. Flow (i)

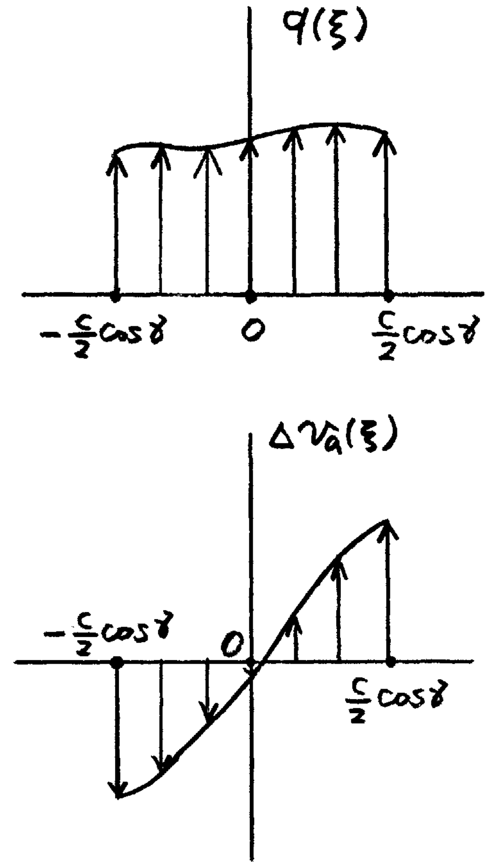
The solution of Flow (i) is already well known, and in this paper the conclusive equation only shall be given,

$$\Gamma = 4 s \kappa \sin\alpha \frac{V}{\sqrt{1 + 2\kappa^2 \cos 2\gamma + \kappa^4}} \quad \text{------} (1)$$

where, κ is a parameter depending upon the values of the blade pitch-chord ratio λ, and the stagger angle γ. Although the functional relation $\kappa = f(\lambda, \gamma)$ is rather complicated, we can easily find it in the numerical tables (6).

2 - 2. Flow (ii)





Next the relation between the source or sink distribution and the variation of axial velocity must be examined. Consider the source distribution spread over the belt-wise domain from $\xi = -\frac{c}{2}\cos\gamma$ to $\xi = \frac{c}{2}\cos\gamma$. The strength of source per unit area is expressed by q , and it is a function of ξ only.

Velocity induced at an arbitrary point (ξ_0, η_0) by this source distribution can be easily calculated by the law of Biot Savart, and is written as follows,

$$\Delta v_a = \frac{1}{\pi}\int_{-\frac{c}{2}\cos\gamma}^{\xi_0}\int_{\eta_0}^{\infty} q(\xi)\frac{\cos\theta}{r}\,d\xi\,d\eta - \frac{1}{\pi}\int_{\xi_0}^{\frac{c}{2}\cos\gamma}\int_{\eta_0}^{\infty} q(\xi)\frac{\cos\theta}{r}\,d\xi\,d\eta \qquad \text{-----}(2)$$

where
$$r = \sqrt{(\xi-\xi_0)^2 + (\eta-\eta_0)^2},$$
$$\cos\theta = \frac{\xi-\xi_0}{\sqrt{(\xi-\xi_0)^2+(\eta-\eta_0)^2}}.$$

Equation (2) can be integrated at once, and the result is as follows:

$$\Delta v_a(\xi) = \int_{-\frac{c}{2}\cos\gamma}^{\xi} q(\xi)\,d\xi - \frac{1}{2}\int_{-\frac{c}{2}\cos\gamma}^{\frac{c}{2}\cos\gamma} q(\xi)\,d\xi$$

The second term on the right hand side of the above equation is one half of the fluid volume welled out per unit length of η - direction, and therefore, it is identically equal to ($v_{a_2} - v_{a_1}$). So we have,

$$\Delta v_a(\xi) = \int_{-\frac{c}{2}\cos\gamma}^{\xi} q(\xi)\,d\xi - \frac{v_{a_2}-v_{a_1}}{2} \quad \text{-----}(3)$$

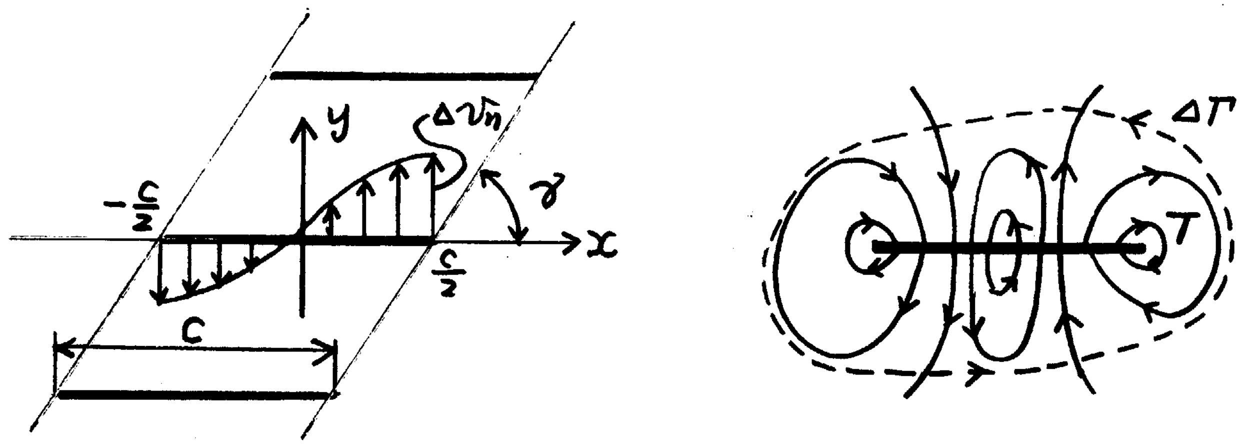
or, in differential form

$$\frac{d(\Delta v_a)}{d\xi} = q(\xi) \quad \text{------} \; (4)$$

Equations (3) and (4) show the conclusive relation between the source distribution and the axial velocity change. Whatever the change of axial velocity in ξ - direction may be we can now find the corresponding source distribution which satisfies the given axial velocity change, so long as the derivatives of Δv_a with respect to ξ have finite values.

2 - 3. Flow (iii)

Flow (i), of course, satisfies the boundary condition on the blade surface; however, Flow (ii) does not satisfy the boundary condition alone. Therefore, in order to cancel the normal velocity on the blade surface, Flow (iii) must now be considered.



The acceleration $\Delta v_a(\xi)$ of the axial velocity between the blade passages causes a normal velocity component on the blade surface, as follows:

$$\Delta v_n = \sin\gamma \cdot \Delta v_a$$

So, to cancel this normal velocity the following normal velocity shall be added on the blade surface,

$$\Delta v_n = -\sin\gamma \cdot \Delta v_a \qquad ----- (5)$$

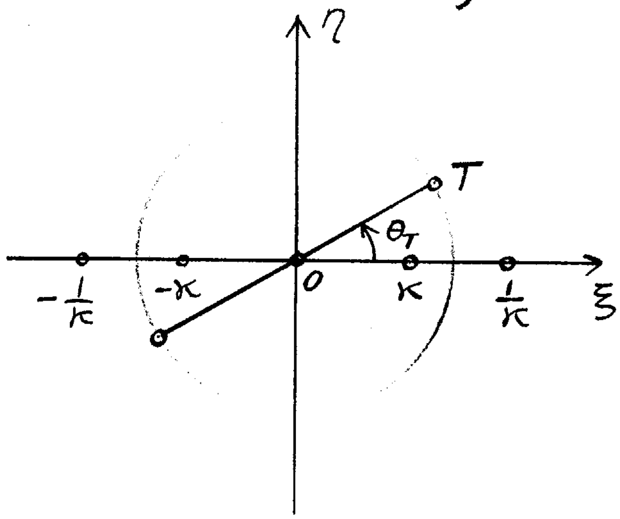
This means that the flow wells out on the upper (or lower) surface and sucks the same magnitude into the opposite surface. As a whole, the flow field can be considered as consisting of a doublet distribution as is schematically shown in the Figure. As can be seen easily from this figure, there occurs a flow from the upper surface to the lower surface surrounding the trailing edge T. This induces infinite velocity at T, and is inconsistent with the hypothesis of Kutta-Joukowski, so there occurs another circulation $\Delta\Gamma$ around the blade to cancel the infinite velocity at T. $\Delta\Gamma$ is nothing more than the change of circulation caused by axial velocity change, and what we want is to be able to calculate the value of $\Delta\Gamma$ as a function of the prescribed conditions.

Now we shall consider the present problem on the transformed imaginary plane of the complex number $\zeta = re^{i\theta}$.

The mapping function of flat plate cascade is already well known, and is written in the following equation.

$$z = \frac{s}{2\pi}\left[e^{-i\gamma}\log\frac{1+\kappa\zeta}{1-\kappa\zeta} + e^{i\gamma}\log\frac{\zeta+\kappa}{\zeta-\kappa}\right] \quad \text{------} \ (6)$$

where z = x + iy, $\quad \zeta = re^{i\theta} = \xi + i\eta$



By Eq. (6), the outer domain of cascade as shown on Page 7 can be transformed conformally to the outer domain of unit circle in ζ- plane. An illustration of the singular points on the ζ - plane is given in the Figure to the left. Substituting $\zeta = e^{i\theta}$ we get the following relation between the blade surface and the unit circle:

$$\left.\begin{aligned} x &= \frac{s}{\pi}\ \cos\gamma \tanh^{-1}\frac{2\kappa\cos\theta}{1+\kappa^2} + \sin\gamma \tan^{-1}\frac{2\kappa\sin\theta}{1-\kappa^2} \quad + ns\sin\gamma \\ y &= ns\cos\gamma\ , \quad n = 0, \pm 1, \pm 2, \ \text{---} \end{aligned}\right\} \text{-}(7)$$

The argument θ_T , which corresponds to the trailing edge of the blade is calculated by the next equation.

$$\tan\theta_T = \frac{1-\kappa^2}{1+\kappa^2}\tan\gamma \quad \text{-------} \ (8)$$

After the preparatory description given above, the characteristics of Flow (iii) must be analyzed in detail. By the law of continuity, the normal velocity component $q(\theta)$ on the unit circle is expressed as a function of the normal velocity component Δv_n on the blade surface as follows:

$$q(\theta) = \Delta v_n \frac{dx}{d\theta}$$

Substituting Eq. (5) into the above equation, we get,

$$q(\theta) = -\sin\gamma \, \Delta v_a(x) \frac{dx}{d\theta}$$

$dx/d\theta$ can be obtained by differentiating Eq. (7) with respect to θ, and the result is:

$$q(\theta) = \sin\gamma \, \Delta v_a(x) \frac{2SK}{\pi} \frac{(1+k^2)\cos\gamma\sin\theta - (1-k^2)\sin\gamma\cos\theta}{1-2k^2\cos 2\theta + k^4} \quad \text{-----} (9)$$

By the above equation the distribution of sources (or sinks) on the unit circle, corresponding to the normal velocity on a blade, can be calculated as a function of the argument θ.

On the other hand, let us consider the flow around the unit circle, which wells the fluid volume Q outside of the unit circle at an arbitrary point $\zeta = e^{i\theta}$. The complex velocity potential of this flow can be expressed as follows:

$$F_1 = \frac{Q}{\pi} \log(\zeta - e^{i\theta}) - \frac{Q}{4\pi} \log(\zeta^2 - k^2)(\zeta^2 - \frac{1}{k^2})$$

Hence the velocity of induced flow u at a point x_0 on the surface of the blade due to a source of strength Q placed at a point x is given by the following equation, assuming x and x_0 correspond to $\zeta = e^{i\theta}$ and $e^{i\theta_0}$ respectively.

$$u = \left(\frac{dF}{d\zeta}\frac{d\zeta}{dz}\right)_{\zeta=\zeta_0}$$

$$= \frac{-Q}{4SK} \frac{1-2k^2\cos 2\theta_0 + k^4}{\{(1+k^2)\cos\gamma\sin\theta_0 - (1-k^2)\sin\gamma\cos\theta_0} \left\{ \frac{\sin(\theta-\theta_0)}{1-\cos(\theta-\theta_0)} - \frac{2k^2\sin 2\theta_0}{1-2k^2\cos 2\theta_0 + k^4} \right\}$$

$$\text{-----} (10)$$

Replacing Q of Eq. (10) by $q(\theta)$ of Eq. (9), integrating over the unit circle and using the condition $\Sigma q(x)=0$, the induced velocity at the point x_o is given by the following relation,

$$u_q(\theta_o) = -\frac{\sin\gamma}{2\pi}\frac{1-2K^2\cos 2\theta_o + K^4}{\{(1+K^2)\cos\gamma\sin\theta_o - (1-K^2)\sin\gamma\cos\theta_o\}}\cdot$$

$$\int_{\theta_T}^{\theta_T+2\pi} \Delta v_a(x)\frac{\{(1+K^2)\cos\gamma\sin\theta - (1-K^2)\sin\gamma\cos\theta\}\sin(\theta_o-\theta)}{\{1-2K^2\cos 2\theta + K^4\}\{1-\cos(\theta_o-\theta)\}}\,d\theta \qquad \text{-----}(11)$$

In the neighborhood of the trailing edge i.e. $\theta = \theta_T + \epsilon$, where ϵ is a very small angle,

$$u_q(\theta_T+\epsilon) = \frac{(1-K^4)^2}{2\pi\epsilon K}\sin\gamma\int_{\theta_T}^{\theta_T+2\pi}\Delta v_a(x)\,\frac{1+\cos(\theta_T-\theta)}{K'}\,d\theta \quad \text{---}(12)$$

where, $K = 1+2K^2\cos 2\gamma + K^4$, $K' = 1-2K^2\cos 2\theta + K^4$. In the ζ - plane, the complex velocity potential of the flow due to the circulation of strength $\Delta\Gamma$ around each blade is given by the following relation.

$$F_z = \frac{i\Delta\Gamma}{4\pi}\left\{\log(\zeta^2 - K^2) - \log\left(\zeta^2 - \frac{1}{K^2}\right)\right\} \quad \text{-----}(13)$$

Hence the velocity $u_{\Delta\Gamma}$ at the point x_o due to this flow becomes

$$u_{\Delta\Gamma}(\theta_o) = \frac{\Delta\Gamma\,(1-K^4)}{4KS\{(1+K^2)\cos\gamma\sin\theta_o - (1-K^2)\sin\gamma\cos\theta_o\}} \quad \text{-----}(14)$$

and in the neighborhood of the trailing edge becomes,

$$u_{\Delta\Gamma}(\theta_T+\epsilon) = \frac{\Delta\Gamma\,(1-K^4)}{4SK\epsilon\sqrt{K}} \quad \text{-----}(15)$$

The circulation is determined from the above-mentioned condition i.e.

$$u_q(\theta_T) + u_{\Delta\Gamma}(\theta_T) = 0$$

and hence,

$$\Delta\Gamma = -\frac{2SK(1-K^4)}{\pi\sqrt{K}}\sin\gamma\int_{\theta_T}^{\theta_T+2\pi}\Delta V_a(x)\,\frac{1+\cos(\theta_T-\theta)}{K'}\,d\theta \quad --- \quad (16)$$

Eq. (16) is the conclusive equation, which gives the value to correct the circulation around the blade due to the prescribed axial velocity change.

2 - 4. Analogy with Thin Aerofoil Theory

The above mentioned theoretical considerations are quite similar in mathematical procedure to those of "thin aerofoil theory" (7), developed by Hirose and Hudimoto about ten years ago. In their work, they expressed the effect of the shape of camber lines by doublets distributed on the chord line. Their result was the following formula:

$$\Gamma = \frac{2SK(1-K^4)}{\pi\sqrt{K}}\int_{\theta_T}^{\theta_T+2\pi}\left(V\sin\alpha - V\cos\alpha\frac{dy}{dx}\right)\frac{1+\cos(\theta_T-\theta)}{K'}\,d\theta \quad --- \quad (17)$$

In this equation, denotes the circulation around a blade, and y is the ordinate of a given camber line, while the other symbols are the same as they appear elsewhere in this paper. From Eq. (17), the effect of only the camber line is obtained by setting $\alpha = 0°$. Therefore, in this case,

$$\Gamma = -\frac{2SK(1-K^4)V}{\pi\sqrt{K}}\int_{\theta_T}^{\theta_T+2\pi}\frac{dy}{dx}\,\frac{1+\cos(\theta_T-\theta)}{K'}\,d\theta \quad -- \quad (17')$$

Comparing Eqs. (16) and (17)', it can be seen that the following analogy exists between the flow changing axial velocity through a flat plate cascade and the flow through a cambered cascade.

$$\Delta V_a \sin\gamma = V\frac{dy}{dx} \quad ---- \quad (18)$$

Therefore, if the changing state of axial velocity through a flat plate cascade is given by the formula,

$$\frac{\Delta V_a}{V} = \sum_{n=0}^{\infty} a_n \left(\frac{x}{c}\right)^n \quad ---- \quad (19 \cdot A)$$

then, the camber lines of a corresponding cascade in a constant axial velocity field with a zero attack angle, can be calculated by the following equation.

$$\frac{y}{c} = \sin\gamma \sum_{n=1}^{\infty} \frac{a_n}{n+1} \left(\frac{x}{c}\right)^{n+1} + \text{Const.} \quad --- \quad (19 \cdot B)$$

From the above considerations, it can now be proved, that the flow around a cascade, which has a changing axial velocity, has a direct, close correlation with the flow through a slightly cambered wing lattice. This conclusion is also seen intuitively, since the change of axial velocity between blade passages means the deformation of blade shape itself. Here, the most important matter to be recognized is the limitation of the above analogy.

Although Eqs. (16) and (17) have quite the same form, their meanings are substanially different. Eq. (16) is the exact solution of the flow through a flat plate cascade with an axial velocity change, however, Eq. (17), unlike Eq. (16), is an approximate solution and is applicable only for a very slightly cambered wing lattice. Because of this it is not quite correct to use Eq. 17 for the case of a finite, cambered blade. Therefore, the analogy given by Eq. (18) is correct only for the small cambered profile, or inversely only for a small rate of axial velocity change. On the other hand, if the change rate of axial velocity becomes large, Eq. (16) gives the exact solution of cascade performance.

Therefore, speaking purely mathematically, the treatment of Hirose and Hudimoto, gave not only the approximate solution through cascades of slightly cambered blade profile with constant axial velocity, but also offered, ten years ago, the exact solution of changing axial velocity through flat plate cascades. Unfortunately, they were not aware of such a physical meaning at that time.

2 - 5. Linear Axial Velocity Change

In the case of a linear axial velocity change in the x - direction, Eq. (19-A) can be expressed as follows:

$$\frac{\Delta V_a}{V} = \frac{x}{c}\,\frac{V_{a2} - V_{a1}}{V} \qquad (20)$$

where, V_{a1} is the axial velocity at the inlet, V_{a2} is the axial velocity at the exit of the cascade. By the above analogy, this corresponds to a cascade composed of parabolic cambered blades with constant axial velocity and zero angle of attack. The corresponding cambered profile is expressed by the following equation.

$$\frac{y}{c} = \sin\gamma\,\frac{V_{a2} - V_{a1}}{2V}\left(\frac{x}{c}\right)^2$$

Hence, the relation between the rate of change of axial velocity and the magnitude of camber f/c of a corresponding parabolic camber line can be written as follows.

$$\frac{f}{c} = -\sin\gamma\,\frac{V_{a2} - V_{a1}}{8V} \qquad ---- (21)$$

Now substituting Eq. (20) into (16), we obtain,

$$\Delta\Gamma = -\frac{2SK(1-k^4)}{\pi\sqrt{K}}\,\frac{V_{a2} - V_{a1}}{c}\,\sin\gamma\int_{\theta_T}^{\theta_T+2\pi} X\cdot\frac{1+\cos(\theta_T-\theta)}{K'}\,d\theta$$

Substituting Eq. (7) into the above equation, we get finally,

$$\frac{\Delta\Gamma}{Vc} = -\frac{2\lambda^2K(1-k^4)}{\pi^2\sqrt{K}}\,\frac{V_{a2} - V_{a1}}{V}\,\sin\gamma\,\times$$

$$\int_{\theta_T}^{\theta_T+2\pi}\left[\cos\gamma\tanh^{-1}\frac{2k\cos\theta}{1+k^2} + \sin\gamma\tan^{-1}\frac{2k\sin\theta}{1-k^2}\right]\frac{1+\cos(\theta_T-\theta)}{K'}\,d\theta$$

$$---(22)$$

Although this definite integration has already been solved by Hirose and Hudimoto, their mathematical procedures are somewhat interesting, so they shall be reproduced:

We put $$\varphi(\theta) = \cos\gamma \tanh^{-1}\frac{2k\cos\theta}{1+k^2} + \sin\gamma \tan^{-1}\frac{2k\sin\theta}{1-k^2}$$

and expand $\varphi(\theta)$ in a Fourier Series such as:

$$\varphi(\theta) = \sum_{n=0}^{\infty} C_n \cos n\theta + \sum_{n=1}^{\infty} d_n \sin n\theta$$

then, $$\int_0^{2\pi} \frac{\cos n\theta}{1-2K^2\cos 2\theta + k^4} d\theta = \frac{2\pi K^n}{1-k^4}, \quad \text{when } n \text{ is even,}$$

$$0 \quad , \quad \text{when } n \text{ is odd.}$$

hence, $$C_{2m+1} = \frac{2K^{2m+1}\cos\gamma}{2m+1}, \quad d_{2m+1} = \frac{2K^{2m+1}\sin\gamma}{2m+1},$$

$$m = 0, 1, 2, ----$$

and $$\int_{\theta_T}^{\theta_T+2\pi} \varphi(\theta) \frac{1+\cos(\theta_T-\theta)}{K'} d\theta = 2\pi\left(\frac{\cos\gamma\cos\theta_T}{1-k^2} + \frac{\sin\gamma\sin\theta_T}{1+k^2}\right)\sum_{n=0}^{\infty}\frac{K^{4m+1}}{2m+1}$$

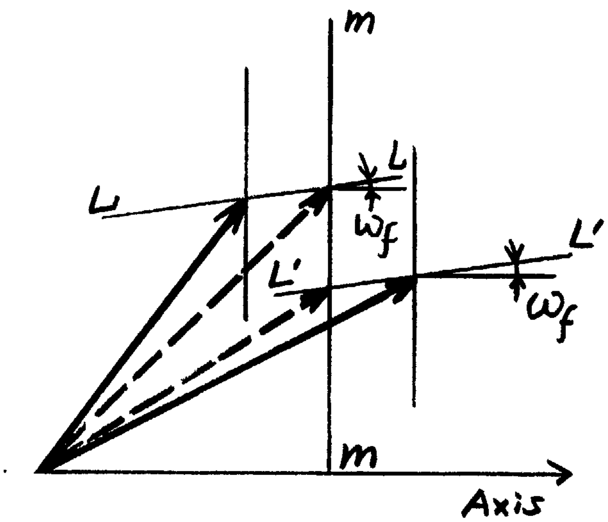
$$= \frac{\pi\sqrt{K}}{k(1-k^4)} \log\frac{1+k^2}{1-k^2}.$$

Substituting the above equation into Eq. (20), we get the following conclusive equation.

$$\frac{\Delta T}{(v_{a1} - v_{a2}) S} = \frac{2\lambda\sin\gamma}{\pi} \log\frac{1+k^2}{1-k^2} \quad --- \quad (23)$$

Fig. 1 shows the result of a numerical calculation of the above equation, which gives the correction values for the circulation due to the change of axial velocity as a function of pitch-chord ratio λ and stagger angle γ.

Now, from the following figure, a concrete method can be suggested



for correcting the velocity triangles in the case of accelerated or decelerated axial velocity to that of constant axial velocity. The practical drawing method is as seen at the left. Suppose the prescribed velocity triangle is as shown by arrows (solid lines) of the Figure. First, we draw the mean axial velocity line m - m. Second, from the tops of both arrows, draw two parallel straight lines L - L and L' - L', which are inclined to the axial direction by an angle ω_f. ω_f has the following physical meaning.

$$\tan \omega_f = \frac{\Delta \Gamma}{(v_{a_1} - v_{a_2})\, s}$$

The values of ω_f can easily be found in Fig. 1, if the values of pitch-chord ratio λ and stagger angle γ are known. Then, the two points of intersection of the lines m - m and L - L, m - m and L' - L' correspond respectively to the velocity vectors at the inlet and exit of the cascade with a constant axial velocity. In Fig. 1, the most useful domain for practical purposes is shown by the shaded area. In this domain, tan ω_f seems to vary from 0.3 to 0.9, and for the most practical purposes it will be enough to put tan ω_f = 0.5 for the usual axial flow compressor design.

2 - 6. Correction of the Turning Angle

In the procedures of the foregoing considerations, the mean of the inlet and exit velocity vectors is taken as the standard velocity, for the sake of mathematical simplicity. On the other hand, what we usually want to get is the turning angle through the cascade keeping the inlet velocity vector constant. Therefore, it would be necessary to refer to the method of turning angle correction in order to keep the inlet velocity vector constant.

It has already been proven elsewhere (8) that the following general relationship holds for all kinds of cascades.

$$\tan\beta_2 = \sigma_0 + \sigma_1 \tan\beta_1 \quad \text{-----} \quad (24)$$

Where, β_1 is the inlet velocity direction measured from the axis, β_2 is the same one at exit, and σ_0 , σ_1 are the constants peculiar to the prescribed cascade profiles and their physical arrangements. σ_0 and σ_1 are expressed by the following equations, if the conformal mapping of their boundaries into the unit circle is possible (9).

$$\left.\begin{aligned} \sigma_1 &= \frac{\frac{1}{k} + k - 2\cos\theta_T}{\frac{1}{k} + k + 2\cos\theta_T} \\ \sigma_0 &= \frac{4\left(\frac{1}{k} + k\right)\sin\theta_T}{\left(\frac{1}{k} - k\right)\left(\frac{1}{k} + k + 2\cos\theta_T\right)} \end{aligned}\right\} \quad \text{---} \quad (25)$$

Now, let the values of β_1, β_2 which are obtained by the drawing procedure and shown in the foregoing paragraph to be β_{10} and β_{20} respectively, and expand both sides of Eq. (24) in a Taylor Series, that is;

$$\tan\beta_{20} + \sec^2\beta_{20}(\beta_2 - \beta_{20}) + \cdots = \sigma_0 + \sigma_1 \tan\beta_{10} + \sigma_1 \sec^2\beta_{10}(\beta_1 - \beta_{10}) + \cdots$$

Thus, in the vicinity of the origin of expansion, there exists the following approximating relation

$$\sec^2\beta_{20}(\beta_2-\beta_{20}) = \sigma_1 \sec^2\beta_{10}(\beta_1-\beta_{10})$$

Putting,

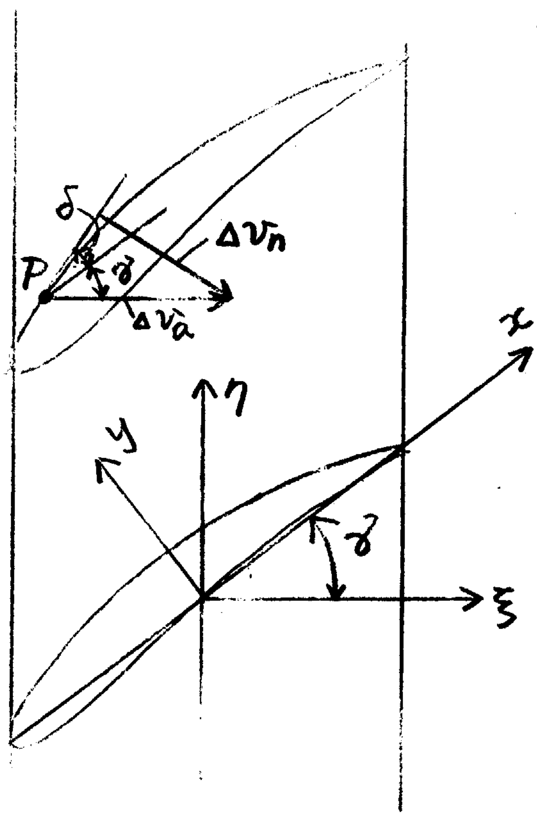
$$\Delta\beta_2 = \beta_2 - \beta_{20} \quad , \quad \Delta\beta_1 = \beta_1 - \beta_{10} \; ,$$

we obtain this relation:

$$\Delta\beta_2 = \sigma_1 \left(\frac{\cos\beta_{20}}{\cos\beta_{10}}\right)^2 \Delta\beta_1 \quad \text{-----} \quad (26)$$

The above equation is valid for all kinds of profiles, and in the case of a flat plate cascade, we can deduce Eq. (26) from Eq. (1) and (8). Using the above result, we can obtain the correction values $\Delta\beta_2$ of the exit angle β_2, which correspond to the required change $\Delta\beta_1$ of the inlet angle β_1. Fig. 2 shows the values of σ_1 for flat plate cascades as a function of pitch-chord ratio λ and stagger angle γ. It can be seen from this figure, that in the vicinity of a pitch-chord ratio $\lambda = 1$, all of the values of σ_1 are very near 0.05. This means, for normal values of turning angle, the correction value $\Delta\beta_2$ of the exit angle is nearly 0.1° per degree of inlet angle adjustment. Therefore, in most cases, it does not seem necessary to correct the exit angle. However, in the case of pitch-chord ratio larger than 1.2 and large stagger angle such as $\gamma = 60°$, the values of exit angle correction might exceed 0.5° or more per degree of inlet angle adjustment. In such cases, correction of exit angle should be taken into account.

3. CASCADE COMPOSED OF ARBITRARILY SHAPED BLADES



To approach the more practical solution, let's consider the flow through cascades composed of arbitrarily shaped blades. There are several methods to solve such a problem involving a small perturbation based on the solution of flat plate cascades. In this paper, one of the simplest approximation theories similar to the Hudimoto method (6) shall be adopted. Since we require very small quantities in the final result, we shall consider only the first approximation as sufficient for our purposes.

3 - 1. General Equations

Consider a cascade composed of an arbitrary blade shape such as the figure above. The whole blade shape is supposed to be prescribed by the following equation

$$y = f(x) \quad ----- \quad (27)$$

Now let's consider the source distribution corresponding to the axial velocity change from v_{a1} to v_{a2}. Restricting our problem to the case of uniformly distributed sources, the strength of source per unit area q must first be determined. In this case, defining the cross sectional area of the blade as A, the whole quantity of fluid Q welled out from the one passage between two adjacent blades is,

$$Q = q \cdot (cS\cos\gamma - A)$$

and this must coincide with the change of axial velocity before and after the cascade, therefore,

$$(v_{a2} - v_{a1})S = q(Sc\cos\gamma - A)$$

hence,

$$q = \frac{(v_{a2} - v_{a1})S}{cS\cos\gamma - A} \qquad \text{------ (28)}$$

Finally, using Eq. (4), the apparent change of axial velocity is given by the following equation.

$$\Delta v_a = \frac{v_{a2} - v_{a1}}{1 - B}\frac{x}{c} \qquad \text{------ (29)}$$

where ,

$$B = \frac{A}{cS\cos\gamma}$$

B is the "blocking ratio" of the passage by the blade, and in the usual case of the axial flow compressor cascade, it has always small value around 0.1.

Then, the local apparent axial velocity change Δv_a at any point P on the blade surface, brings a normal velocity component Δv_n to the blade surface at this point.

The relation between Δv_n and Δv_a is as follows,

$$\Delta v_n = -\sin(\delta + \gamma)\cdot \Delta v_a \qquad \text{------ (30)}$$

where ,

$$\tan\delta = \frac{dy}{dx}$$

and δ is given numerically, since the blade shape is prescribed by Eq. (27). Assuming $\delta \ll 1$, so that $\delta \fallingdotseq dy/dx$, Δv_n becomes as follows.

$$\Delta v_n = -\frac{dy}{dx}\cos\gamma\,\Delta v_a - \sin\gamma\,\Delta v_a \qquad \text{------ (31)}$$

In the above equation, the second term on the right hand side is the same as Eq. (5). Therefore, all we have to do, is to solve for the first term only, and superimpose it onto the solution of a flat plate cascade. Now, considering the first term only, the radial velocity distribution around the unit circle becomes,

$$q(\theta) = \frac{dx}{d\theta}\,\Delta v_{\bar n} = -\frac{dy}{d\theta}\cos\gamma\,\Delta v_{\hat a}$$

$$= -\frac{d\left(\frac{y}{c}\right)}{d\theta}\,\frac{\frac{x}{c}\cos\gamma}{1-B}\,C\,(v_{\hat a_2} - v_{\hat a_1}) \quad \text{-----} \; (32)$$

On the other hand, let's consider the flow which cancels the normal velocity around the unit circle. First, to satisfy the boundary condition, a quantity of fluid $q \cdot A$ must be sucked into the unit circle. Furthermore, considering that the quantity of fluid $q \cdot A$ must be sucked into the blade from an infinite distance in both directions perpendicular to the cascade axis, the following complex velocity potential function is given.

$$W = \frac{qA}{4\pi}\left\{\log\frac{\zeta - \frac{1}{k}}{\zeta - k} + \log\frac{\zeta + \frac{1}{k}}{\zeta + k}\right\} + \sum_{n=1}^{\infty}\frac{E_n' + iF_n'}{\zeta'^n}$$

where , $\zeta' = \zeta e^{-i\theta_T}$

Using Eq. (28) the above equation can be rewritten in the following form.

$$W = \frac{C(v_{\hat a_2} - v_{\hat a_1})}{4\pi}\,\frac{\lambda B}{1-B}\left\{\log\frac{\zeta - \frac{1}{k}}{\zeta - k} + \log\frac{\zeta + \frac{1}{k}}{\zeta + k}\right\}$$

$$+ C(v_{\hat a_2} - v_{\hat a_1})\sum_{n=1}^{\infty}\frac{E_n + iF_n}{\zeta'^n} \quad \text{----} \; (33)$$

where $\zeta' = \zeta e^{-i\theta_T}$

From the above equation, the radial velocity v_r and the tangential velocity v_θ on the unit circle become:

$$v_r = -\frac{c(v_{a2}-v_{a1})}{2\pi}\frac{\lambda B}{1-B} - \frac{c(v_{a2}-v_{a1})}{\pi}\frac{\lambda B}{1-B}\sum_{n=1}^{\infty}K^{2n}\cos 2n\theta$$

$$- c(v_{a2}-v_{a1})\sum_{n=1}^{\infty} n(E_n\cos n\theta' + F_n\sin n\theta') \text{ ----(34-A)}$$

$$v_\theta = -c(v_{a2}-v_{a1})\sum_{n=1}^{\infty} n(E_n\sin n\theta' - F_n\cos n\theta') \text{----(34-B)}$$

Therefore, setting Eq. (32) and Eq. (34-A) equal, we can determine the values of the Fourier Series E_n and F_n as follows.

$$\sum_{n=1}^{\infty} n(E_n\cos n\theta' + F_n\sin n\theta') = \frac{d\frac{y}{c}}{d\theta}\frac{\frac{x}{c}\cos\gamma}{1-B} - \frac{\lambda B}{1-B}\frac{1}{2\pi}$$
$$- \frac{\lambda B}{\pi(1-B)}\sum_{n=1}^{\infty}K^{2n}\cos 2n\theta \text{ ----(35)}$$

Now, we can calculate the tangential velocity on the unit circle using Eq. (34-B) and the determined values of E_n and F_n.

At the trailing edge,

$$(v_\theta)_{\theta'=0} = c(v_{a2}-v_{a1})\sum_{n=1}^{\infty} nF_n \text{ --- (36)}$$

However, the tangential velocity at the trailing edge, due to the complex velocity potential given by Eq. (13) is;

$$(v_{\theta\Gamma})_{\theta'=0} = -\frac{\Delta\Gamma}{2\pi}\frac{K}{1-K^4} \text{ ----- (37)}$$

From Eqs. (36) and (37), can be determined as follows.

$$\frac{\Delta\Gamma}{(v_{a2}-v_{a1})S} = \frac{2\pi}{\lambda}\frac{1-K^4}{K}\sum_{n=1}^{\infty} nF_n \text{ ---- (38)}$$

Now we can calculate the value of the circulation, which corresponds to the first term of Eq. (31). Therefore, superimposing this onto the solution of the flat plate cascade, we can calculate the correction value for the circulation in the case of an arbitrarily prescribed blade profile.

3 - 2. Calculation for NACA 65 - Series Compressor Cascades

Since the calculation for each case is a rather laborious one, for convenience it is preferable to make some numerical graphs for certain blade profile series. In this paper, numerical graphs for the rapid characteristic estimation of NACA 65 - Series Compressor Cascades were prepared. NACA 65 - Series blade profiles consist of a camber line and thickness distribution, and both values change proportionally to some "designation" with the distribution remaining constant. The designation for camber line is C_{lo}, which expresses the lift coefficient in the case of an isolated wing. The designation for thickness is the "value of the maximum thickness", and in this paper, let us designate it temporarily t.

Now, as it can be seen from the above description, taking the chord line as the abscissa x, the ordinate y on the blade surface can be expressed by the following equation:

$$y = y_c \pm y_t$$

where, y_c represents the ordinate of the camber line, y_t represents the ordinate of thickness only, the + sign is taken for the upper surface and the - sign for the lower surface. Furthermore, we can write:

$$y = C_{lo} \times (y_c)_{C_{lo}=1.0} \pm \frac{t}{t_o} \times (y_t)_{t_o}$$

Hence,
$$\frac{dy}{dx} = C_{l0} \times \left(\frac{dy_c}{dx}\right)_{C_{l0}=1.0} \pm \frac{B}{B_0}\left(\frac{dy_t}{dx}\right)_{t=t_0} \quad ---- (39)$$

where B_o is a blocking ratio corresponding to t_o and t_o is some standard thickness ratio.

Substituting Eq. (39) into Eq. (35) we can write,

$$\sum_{n=1}^{\infty} n(E_n \cos n\theta' + F_n \sin n\theta') = C_{l0} \frac{d\left(\frac{y_c}{c}\right)_{C_{l0}=1.0}}{d\theta} \frac{\frac{x}{c}\cos\gamma}{1-B}$$
$$+ \frac{B}{1-B}\left\{\frac{1}{B_0}\frac{d\left(\frac{y_t}{c}\right)_{t=t_0}}{d\theta}\frac{x}{c}\cos\gamma - \frac{\lambda}{2\pi} - \frac{\lambda}{2\pi}\sum_{n=1}^{\infty} K^{2n}\cos 2n\theta\right\}$$

Then, we divide the above equation into two parts as follows:

$$\sum_{n=1}^{\infty} n({}_cE_n \cos n\theta' + {}_cF_n \sin n\theta') = \frac{C_{l0}}{1-B} \frac{d\left(\frac{y_c}{c}\right)_{C_{l0}=1.0}}{d\theta} \frac{x}{c}\cos\gamma$$

$$\sum_{n=1}^{\infty} n({}_tE_n \cos n\theta' + {}_tF_n \sin n\theta') = \frac{B}{1-B}\left\{\frac{1}{B_0}\frac{d\left(\frac{y_t}{c}\right)_{t=t_0}}{d\theta}\frac{x}{c}\cos\gamma - \frac{\lambda}{2\pi} - \frac{\lambda}{\pi}\sum_{n=1}^{\infty} K^{2n}\cos 2n\theta\right\}$$

Finally, putting

$$\tan \omega_c = \frac{2\pi}{\lambda}\frac{1-K^4}{K}\sum_{n=1}^{\infty} n_c F_n$$
$$\tan \omega_t = \frac{2\pi}{\lambda}\frac{1-K^4}{K}\sum_{n=1}^{\infty} n_t F_n$$

We get the following conclusive equation

$$\frac{\Delta T}{(v_{a2} - v_{a1})S} = \tan \omega_c + \tan \omega_t$$

In Fig. 3, tan ω_c and tan ω_t are shown for the case of $C_{l0} = 1.0$, t = 0, and C_{10} = 0, t = 0.1 respectively. It can be seen from the previous description that the solutions for all other combinations of C_{10} and t can be easily found from this numerical graph.

4. CORRECTION FOR SECONDARY FLOW

As was briefly mentioned in the introduction, secondary flow occuring in the passages of a cascade due to boundary layer causes the deviation of turning angle. Usually, this factor can be considered almost independent of the phenomena considered in previous sections.

Therefore, to reach a more accurate solution, we must add the correction value for secondary flow to our calculation. For the correction of secondary flow, it is considered appropriate to use the Otsuka's method (10). Otsuka developed the secondary flow theories built by Squire-Winter (11), Preston (12), and Smith (13), and devised a convenient formula to estimate the deviation of turning angle due to the secondary flow in the passages of cascades.

According to his theory, deviation of the turning angle δ owing to the secondary flow can be calculated by the next equation.

$$\delta = \frac{\Delta w_1}{w_{1m}} \frac{\cos\beta_2}{\cos\beta_1}\left(\frac{\sin\beta_1}{\cos\beta_2} - \frac{\sin\beta_2}{\sin\beta_1}\right) \qquad (40)$$

Where, w_{1m} is the magnitude of average inlet velocity along the span, Δw_1 is the locally accelerated inlet velocity caused by the existence of a boundary layer on the inelt side wall.

Using Eq. (40) and the experimental value of $\Delta w_1 / w_{1m}$, we can estimate the value of δ, which must be superimposed into our calculations. Fig. 4 shows the values of $\delta \Big/ \frac{\Delta w_1}{w_{1m}}$ for various inlet angles β_1 and exit angles β_2.

5. NUMERICAL EXAMPLES

To confirm the validity of the theory, comparisons were made with several sets of experimental data. All of the experimental data here used (14), (4), (15) were collected from the publications on hand. All of the data were for NACA 65 Series Compressor Cascades using low speed wind tunnels. In them, turning angles are plotted against attack angles for the cases of both solid wall and porous wall. In the latter case, two-dimensional flow is supposed to be maintained at the middle of the span. For both cases, all data gave the rate of axial velocity change v_{a2}/v_{a1}, before and after the cascades. Now starting with the value for solid wall, what we want to do is to correct the effect of secondary flow and the effect of axial velocity change on them, and to compare the final curve to that of the porous wall.

In the first group of data (14) (from Fig. 5 to Fig. 10), the value of $\Delta w_1/w_{1m}$ was estimated to be 0.03, according to the velocity distribution shown in the report. In all figures, thin solid lines express the value for the porous wall, thin dotted lines show the value for the solid wall, while thick solid lines show the results of calculations started from solid wall data.

Since the NASA porous wall data (4) (Fig. 11) uses a somewhat different value compared with the other report (15), in this paper the value of the latter was adopted as the value of the porous wall experiment. The last data (Fig. 12) came from M.I.T. (16) and in this case, as the data of a decelerated case was also presented, it was used in comparison, and the result is given by the dot-dash line in the figures. In the decelerated case, since the secondary flow theory no longer has its meaning, the correction for the secondary flow was omitted. In both the NASA and MIT data, because of the lack of description of inlet velocity distribution, it was assumed that $\Delta w_1 / w_{1m}$ = 0.1 temporary.

6. CONCLUSIONS

In this paper, a general method was developed for the evaluation of cascade performance with accelerated or decelerated axial velocity. For the series of NASA 65 Series Compressor Cascades especially, numerical graphs for practical use were presented. From the comparison of the theory with the data from several experiments, we see that good agreement exists between them. However, there still remains some descrepancy. On the one hand, it seems to depend upon the accuracy of the cascade testing technique itself: on the other hand, the more complicated factors such as the effect of viscosity might affect the performance, which is beyond the scope of the present treatment. Restricting the discussion to the procedure treated here, the accuracy of the calculation would be doubtful in the more highly cambered blade or in a narrower spacing than the present example, because of the invalidity of the simple conformal transformation function used. Fortunately, such a difficult case would not appear frequently in the practical design of axial flow compressors.

BIBLIOGRAPHY

1. Scholz, N., "Two Dimensional Correction of the Outlet Angle in Cascades Flow", Journal of Aero. Sciences, Vol. 20, 1953.

2. Katzoff, S., Bogdonoff, E. H., Boyet, H., "Comparison of Theoretical and Experimental Lift and Pressure Distribution on Airfoils in Cascades", NACA TN 1388, 1947.

3. Hawthorne, W. R., "Induced Deflection Angle in Cascades", Reader's Forum, Journal of Aero. Sciences, Vol. 16, No. 4, 1949.

4. Erwin, J. R., Emery, J. C., "Effect of Tunnel Configuration and Testing Technique on Cascade Performance", NACA TR 1016, 1951.

5. Kawasaki, T., "Some Tests on Cascades of a Compressor Blade", Journal of the Japan Society of Mechanical Engineers, Vol. 22, No. 113, 1956.

6. Hudimoto, B., Kamimoto, G., Hirose, K., "Theory of Wing Lattice", Technical Reports of the Engineering Research Institute, Kyoto University, 1952.

7. Hirose, K., Hudimoto, B., Trans. of the Japan Soc. of Mech. Engineers, Vol. 4, No. 27.

8. Traupel, W., "On the Potential Theory of Blade Cascades", Sulzer Review, 1948.

9. Kubota, S., "Aerodynamic Researches on Turbine Cascades", Technical Report, Kawasaki Dockyard Co., 1958.

10. Otsuka, S., "A Theory about the Secondary Flow in Cascades", Proceedings of the 6th Japan National Congress for Applied Mechanics, 1956.

11. Squire, H. B., Winter, K. G., "The Secondary Flow in a Cascade of Airfoils in a Non-uniform Stream", Journal of Aero. Sciences, Vol. 18, 1951.

12. Preston, J. H., "A Simple Approach to the Theory of Secondary Flows", The Aero. Quarterly, 1954.

13. Smith, L. H., "Secondary Flow in Axial-Flow Turbomachinery", Trans. of ASME, 1955.

14. Takeya, K., "On the Wind Tunnel Tests of Compressor Cascades with Boundary Layer Suction", Presented at the Conference of Turbo Jet Engines, Japan Society of Aero. Engineering, June 1956.

15. Herrig, L. J., Emery, J. C., Erwin, J. R., "Systematic Two-Dimensional Cascade Tests of NACA 65-Series Compressor Blade at Low Speeds", NACA TN 3916, 1957.

16. Montgomery, S. R., "Three-Dimensional Flow in Compressor Cascades", MIT Gas Turbine Laboratory Report No. 48, 1958.

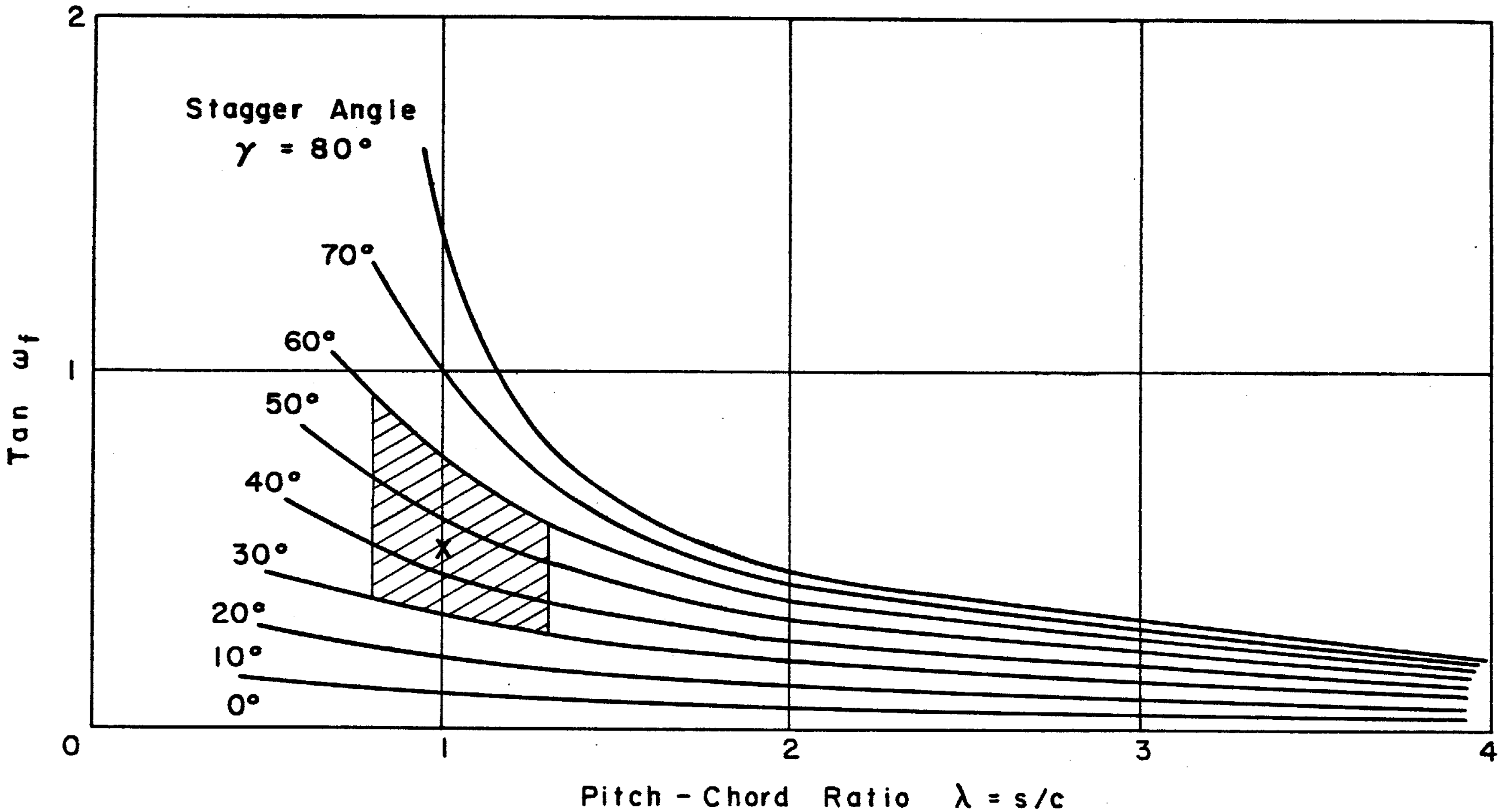


FIG. I CORRECTION OF CIRCULATION AROUND BLADES
FOR AXIAL VELOCITY CHANGE

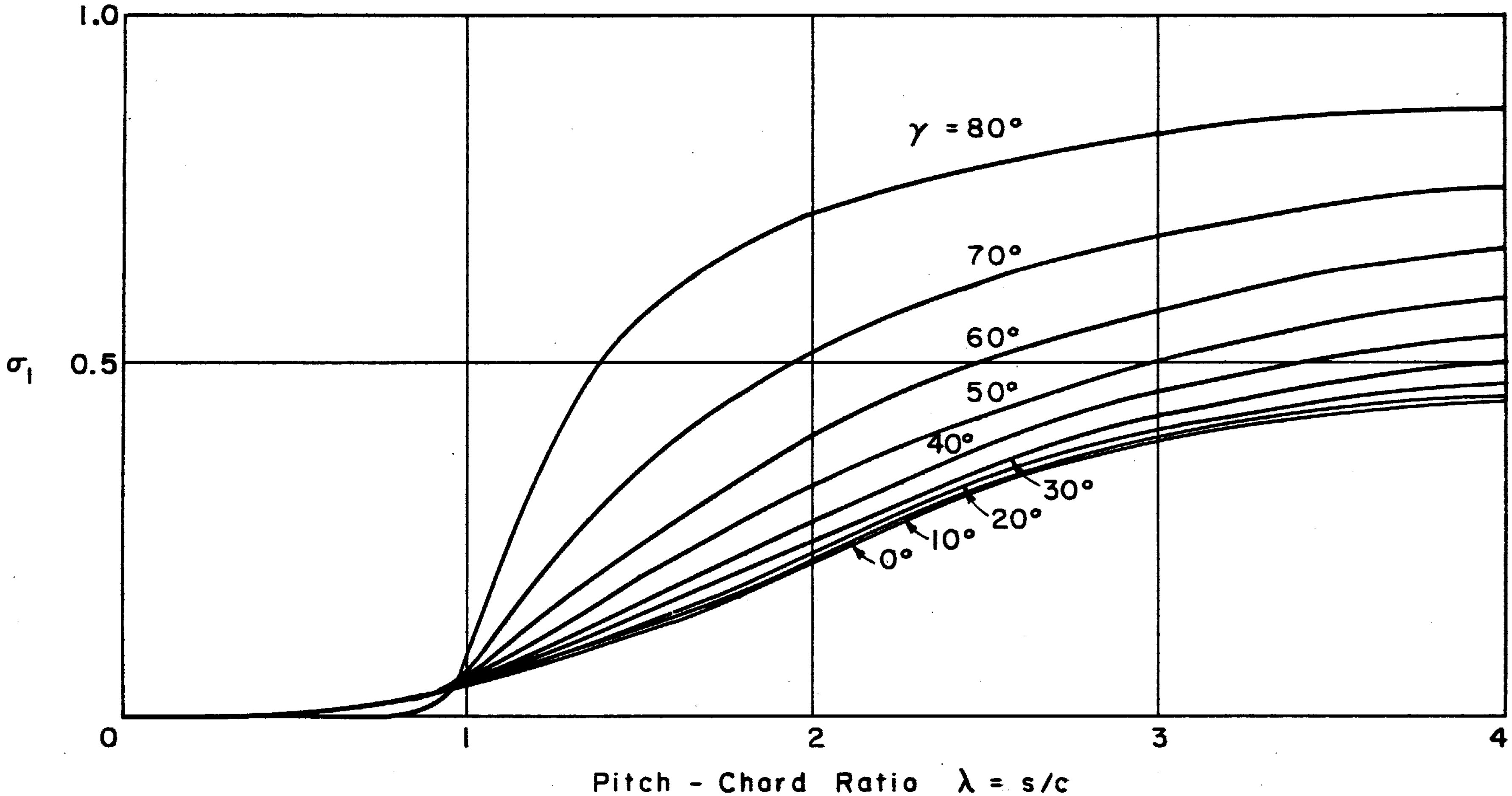


FIG. 2 CORRECTION OF TURNING ANGLE

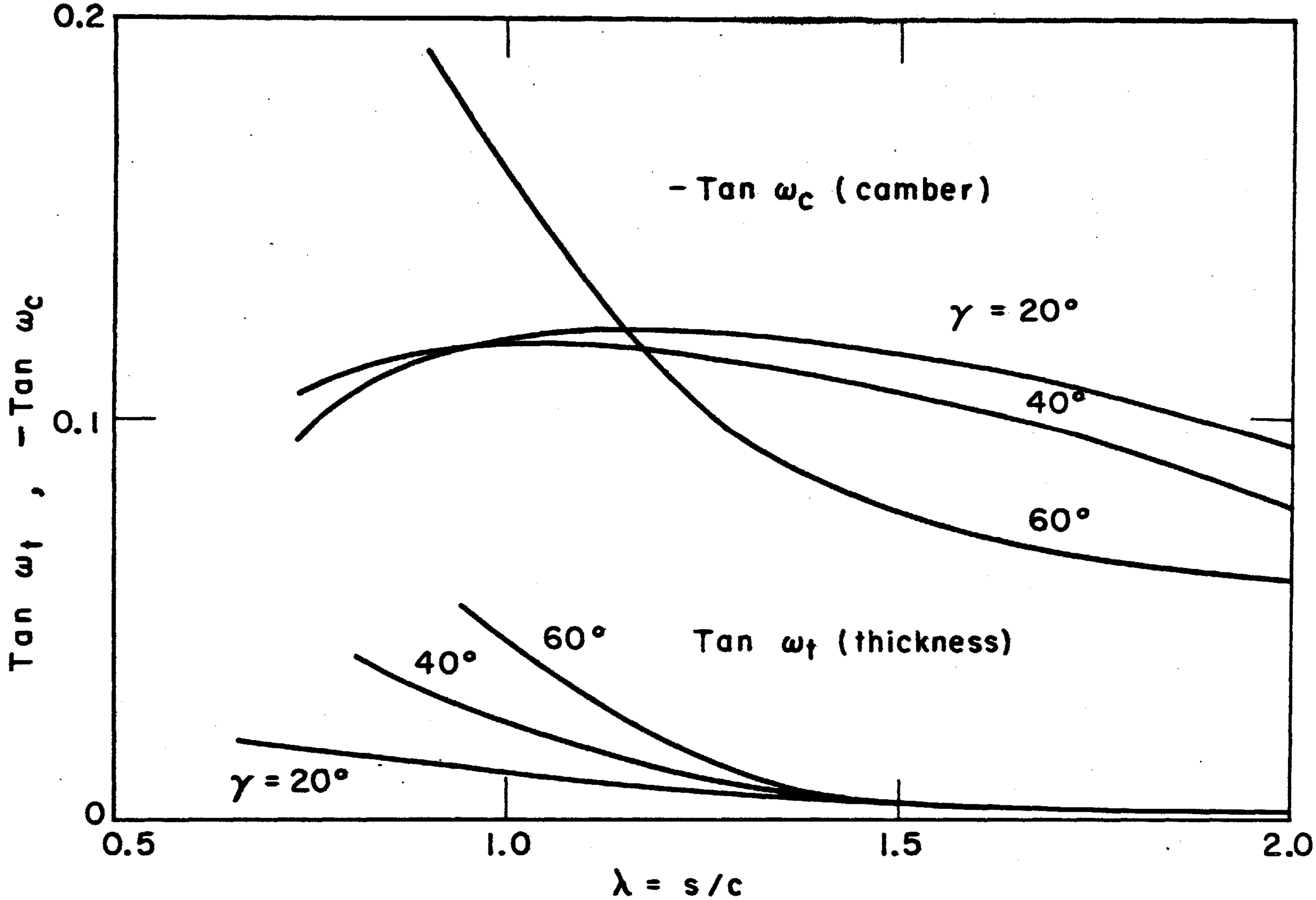


FIG. 3 EFFECT OF CAMBER AND THICKNESS

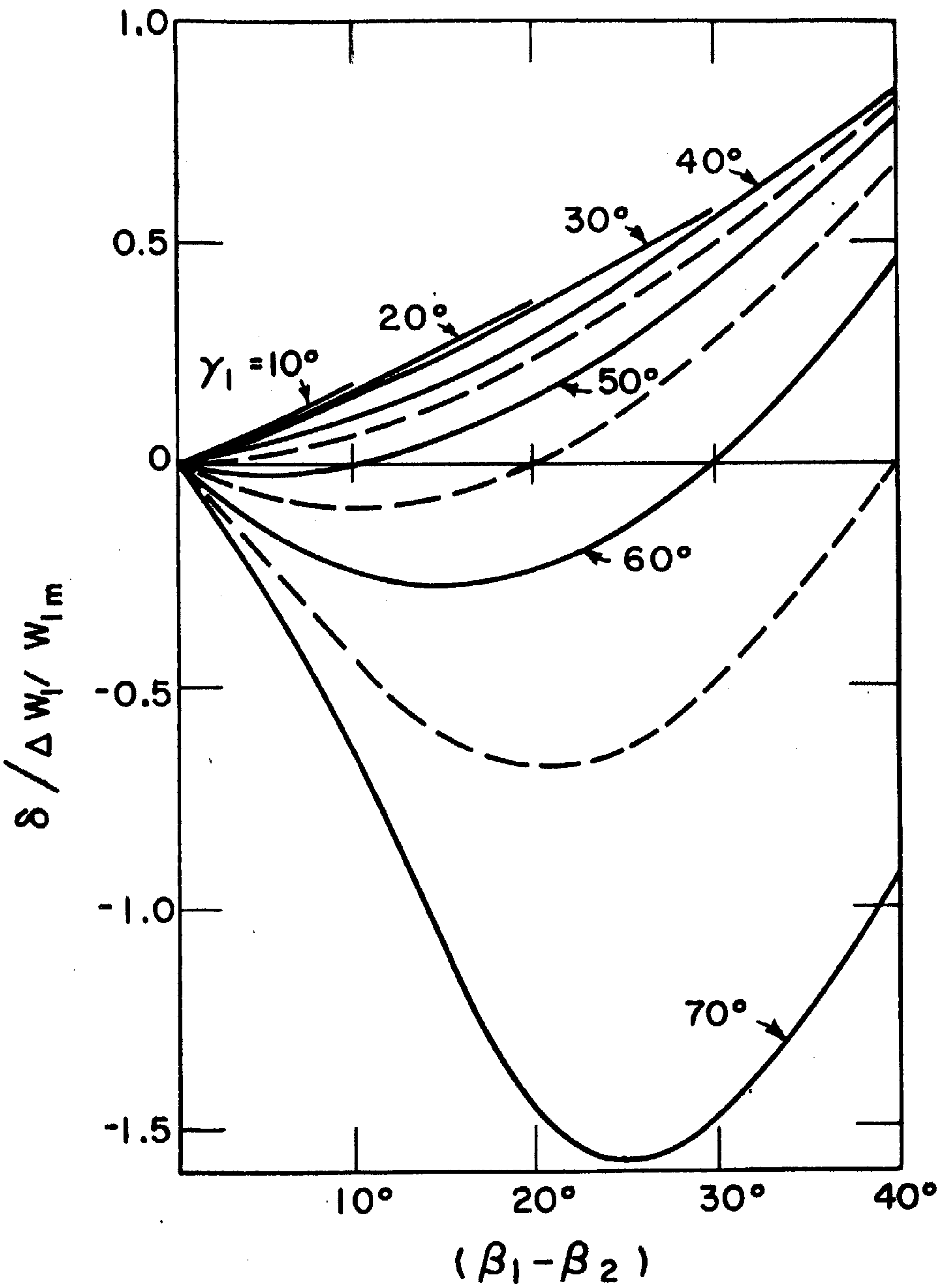


FIG. 4 CORRECTION OF TURNING ANGLE FOR SECONDARY FLOW

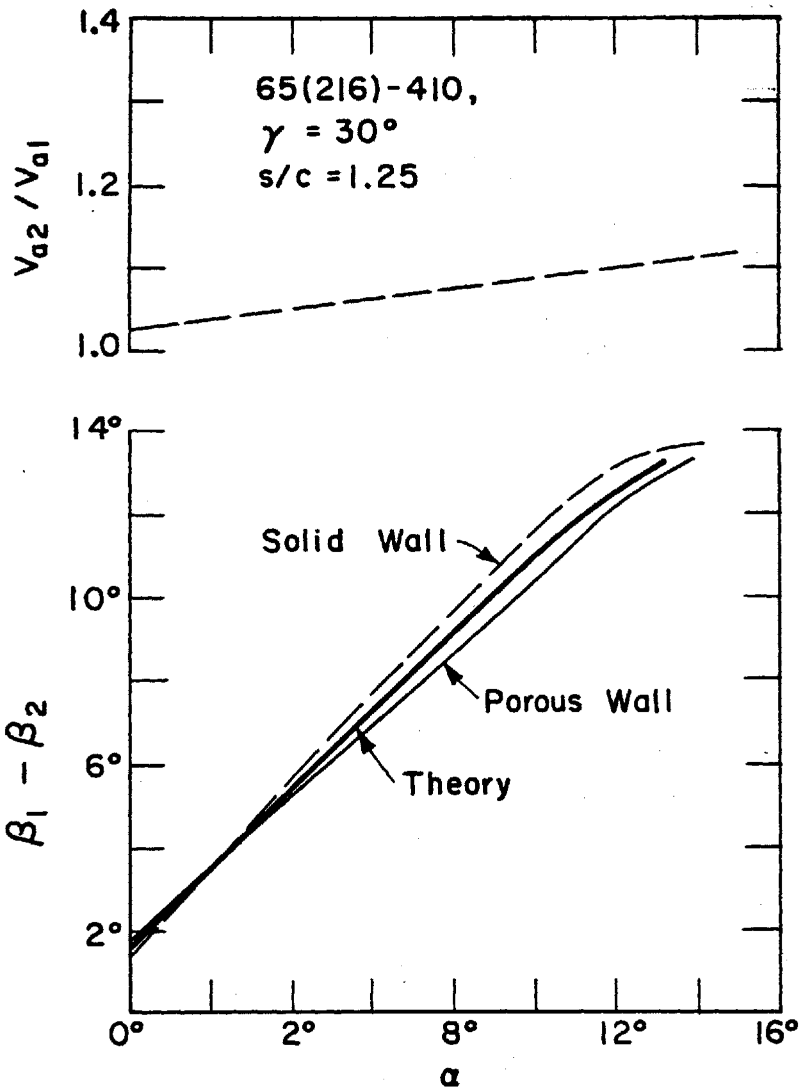


FIG. 5 CORRECTION OF TURNING ANGLE BY THEORY. EXAMPLE 1

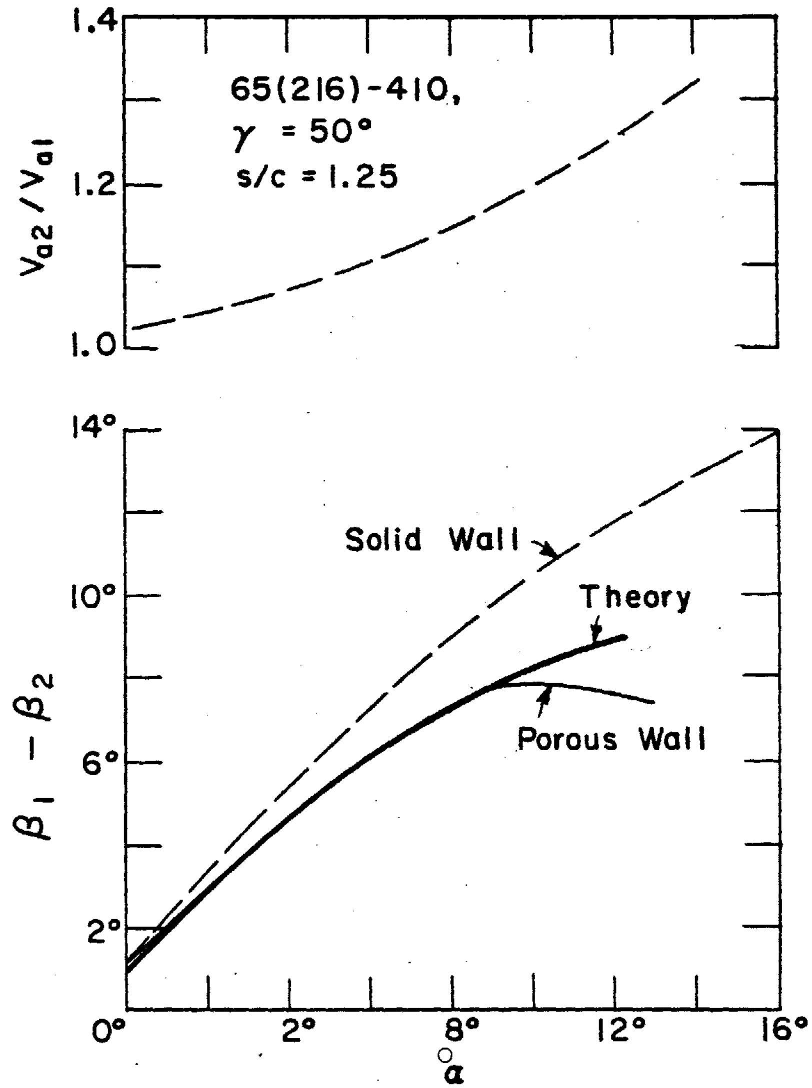


FIG. 6 CORRECTION OF TURNING ANGLE BY THEORY. EXAMPLE 2

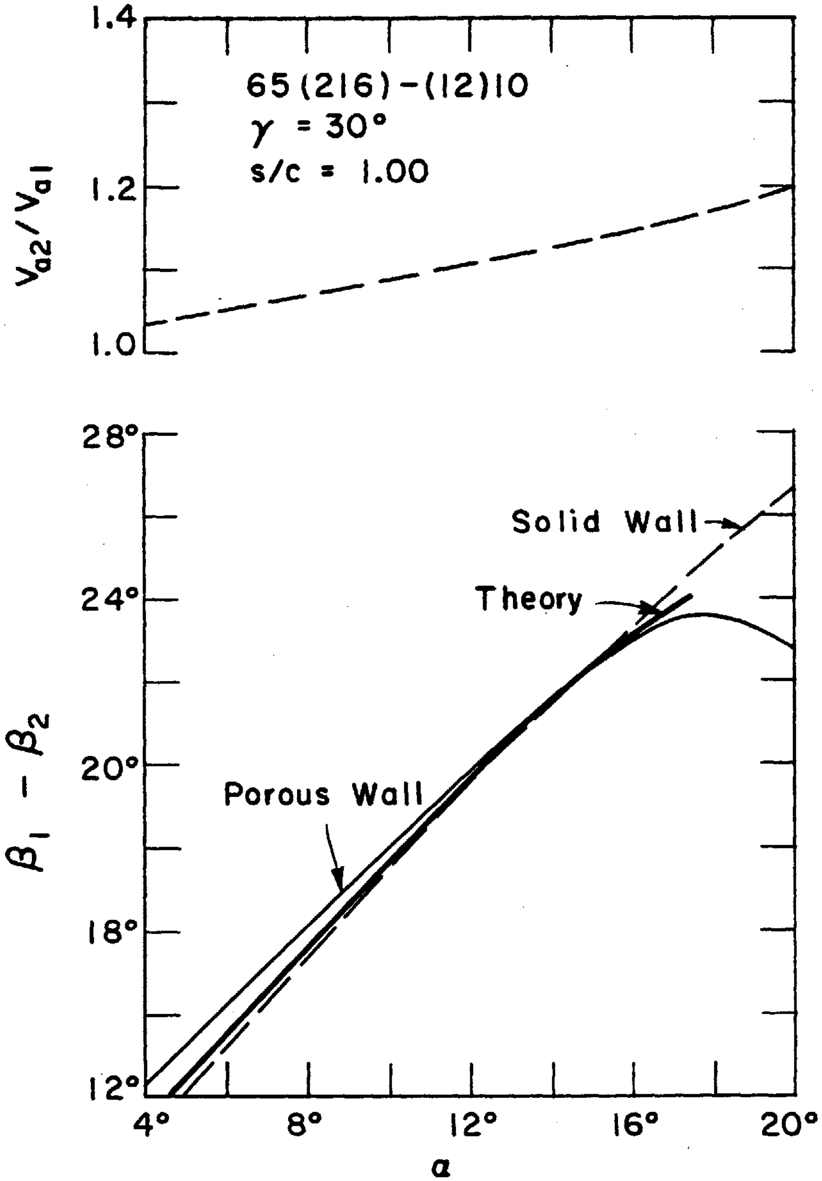


FIG. 7 CORRECTION OF TURNING ANGLE BY THEORY. EXAMPLE 3

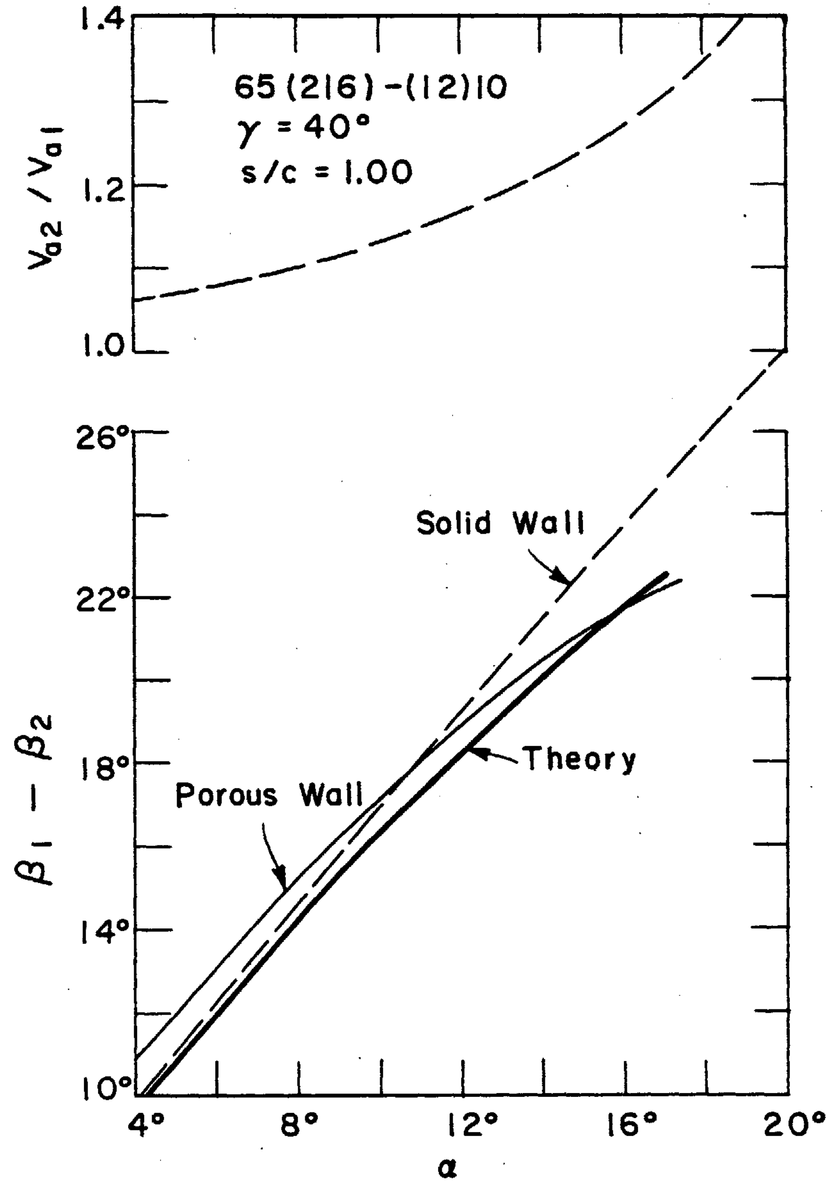


FIG. 8 CORRECTION OF TURNING ANGLE BY THEORY. EXAMPLE 4

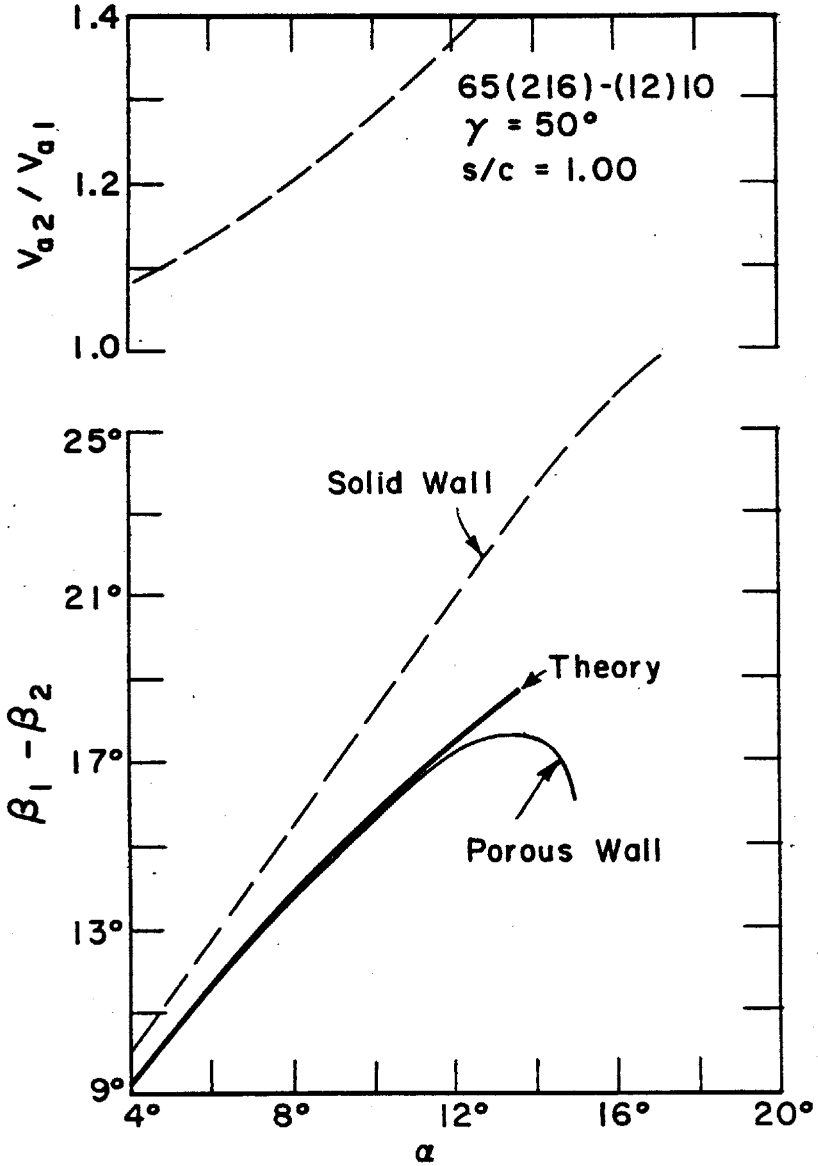


FIG. 9 CORRECTION OF TURNING ANGLE BY THEORY. EXAMPLE 5

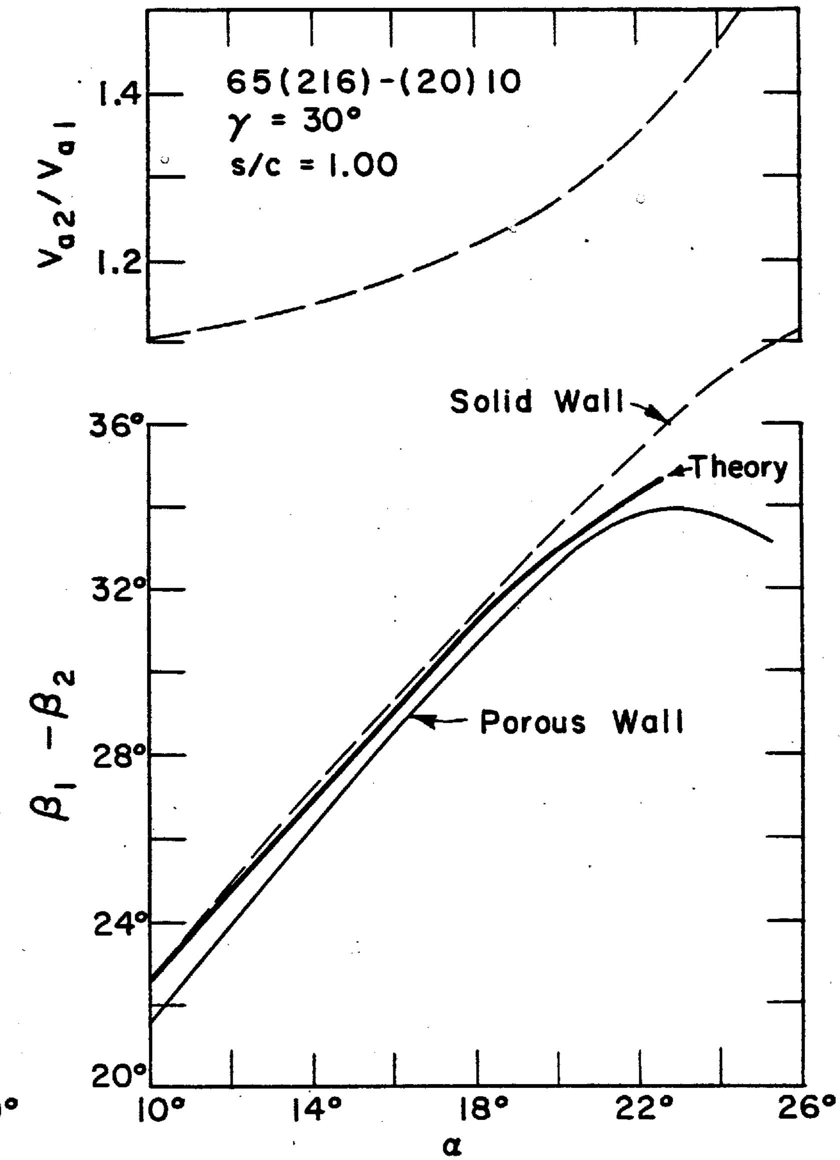


FIG. 10 CORRECTION OF TURNING ANGLE BY THEORY. EXAMPLE 6

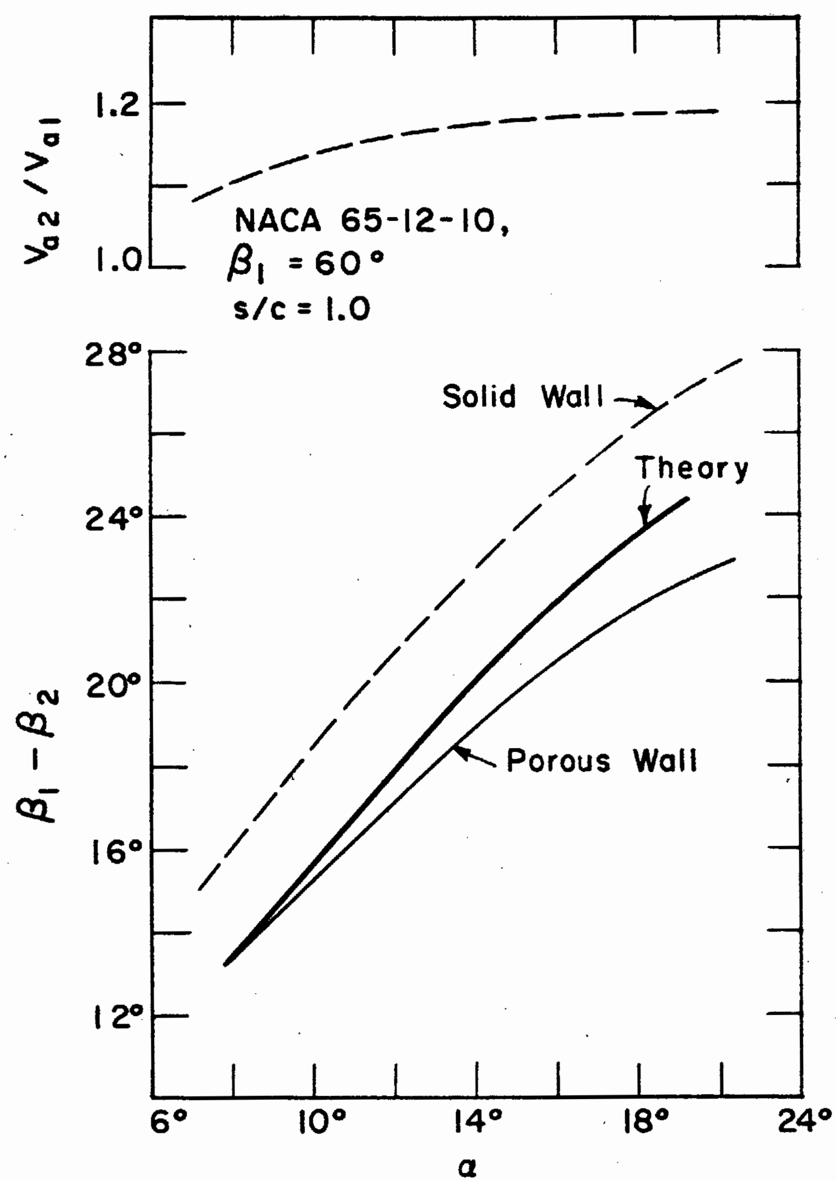


FIG. 11 CORRECTION OF TURNING ANGLE BY THEORY. EXAMPLE 7

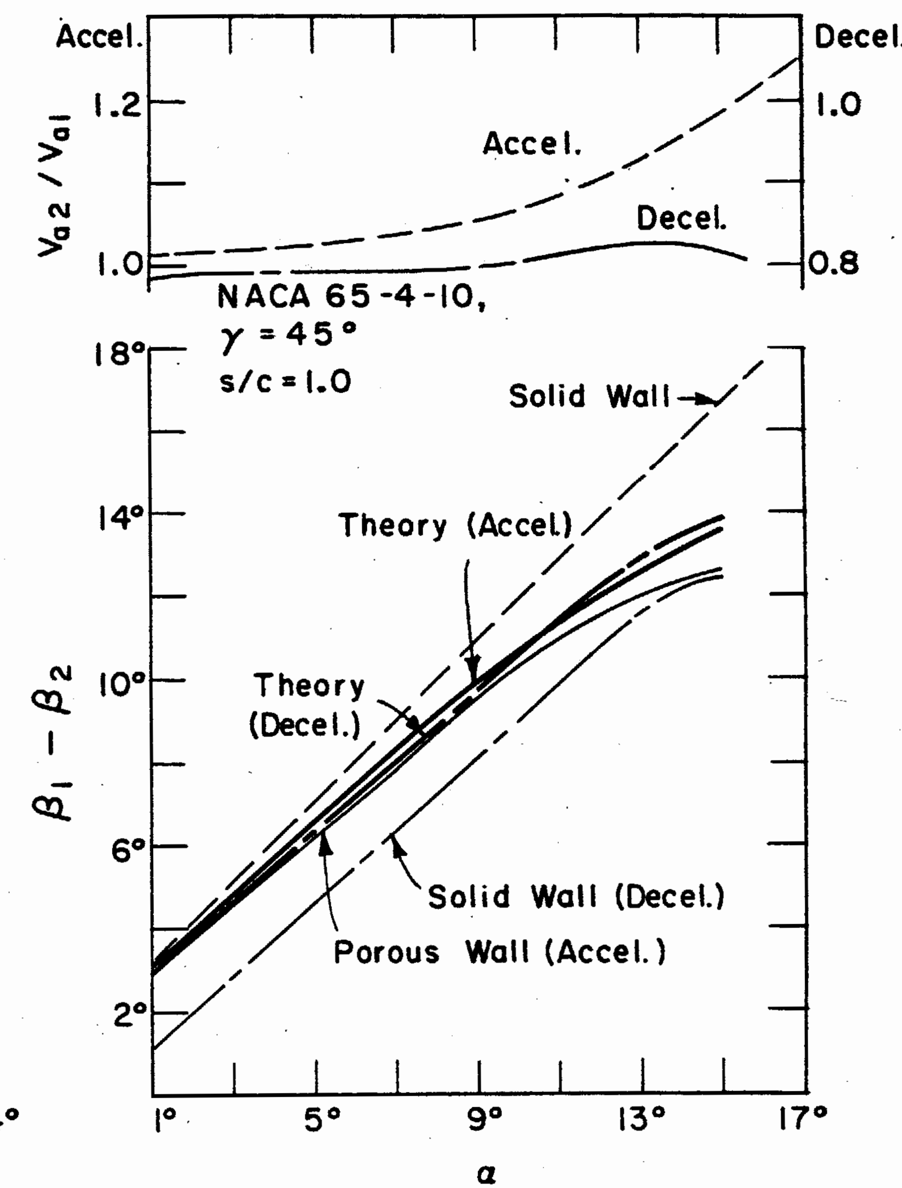


FIG. 12 CORRECTION OF TURNING ANGLE BY THEORY. EXAMPLE 8