SOUND GENERATION FROM A
TRANSONIC COMPRESSOR STAGE

by

Joan Elsa Schaffner
K. Uno Ingard


GAS TURBINE & PLASMA DYNAMICS LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS
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ABSTRACT

Some aspects of the sound pressure wave characteristics produced by a transonic compressor stage are studied by the use of a simplified mathematical model involving flow parallel to a moving corrugated board. The critical parameters of the system are correlated with the various unknowns in the mathematical model resulting in a suitable representation of the physical system. The analysis is first performed for the rotor alone, calculating the sound pressure wave upstream and downstream from the rotor, spinning at a tip speed Mach number of 1.2. The model is then modified to account for the stator blade row, 3/4" axially displaced from the rotor. A study of the reflective characteristics of the stator blade row is performed with the result that the reflective characteristic due to straightening of the flow is much more significant than that due to the physical hardware. The complete description for the sound pressure wave in the three regions: upstream of the rotor, downstream of the stator, and between the rotor and the stator, is determined. The results of this analysis are presented in the form of graphs showing: 1) the reflection coefficients, RF and RS, as functions of tip speed, 2) the pressure wave magnitude as a function of rotor tip speed for specific values of flow speed, 3) the axial dependence of the pressure wave magnitude in the region between the rotor and stator, 4) the ratios PO/P1, P0/P2, and P1/P2, as functions of Mₜ. Experimental data obtained prior to this analysis at the M.I.T. Blowdown Facility is presented and shown to correlate quite favorably with the theoretical determination of the pressure ratios presented.
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NOMENCLATURE

A  Area
B  Stator interface
C  Speed of sound in medium
C_t  Speed of corrugated board
D  Axial chord of stator blade row
E  Distance measured between rotor and stator
f  Arbitrary function
K  Wavenumber
L  Wavelength
M  Mach number of fluid swirl velocity
M_t  Tangential tip Mach number of rotor
n  Distance measured along stator blade row
p  Total pressure field
J  Pressure amplitude
P  Normalized magnitude of total pressure field
R_F  Reflection coefficient characteristic of flow straightening
R_S  Reflection coefficient characteristic of stator hardware
S  Laboratory coordinate system
S'  Moving coordinate system
t  Time
T  Transmission coefficient
\( T_{m,n} \) Transmission matrix components

\( u \) Arbitrary velocity

\( V \) Fluid swirl velocity

\( x \) Axial coordinate

\( y \) Radial coordinate

\( z \) Impedance

\( \nabla \) Del operator, gradient

\( \theta \) Stagger angle of stator blade row

\( \xi \) Transverse displacement of boundary

\( \rho \) Density

\( \phi \) Angle of propagation

\( \omega \) Natural frequency of propagating wave
Aeroacoustics is concerned with sound generated by aerodynamic forces or motions originating in a flow rather than by the externally applied forces or motions of classical acoustics. Noise emanates from the unsteadiness of a flow field generally encountered in jet engines and turbomachinery. There are three main sources of this unsteadiness:

1) The motion of blades relative to an observer. If this motion is supersonic, propagating weak shocks are created which lead to the "buzz-saw" noise associated with high-pass turbofans.

2) The motion of blades relative to one another. This relative motion produces pure-tone sound, dominant on approach in early turbofans.

3) Turbulence and other fluid instabilities. This type is characterized by the radiation of sound from the interaction with turbomachine blading and other surfaces or from the fluid fluctuations themselves, as in jet noise.

For many years, the critical source of noise aircraft powered by pure jets and in the low bypass ratio fans was that created by the hot exhaust gas. This noise emanated from the turbulence and instability of the jet. With the introduction of the high bypass ratio turbofan engines, the jet velocities were reduced as was the overall noise level. Nevertheless a new set of noise problems have been encountered. The source of this noise is the blade motion. The so-called pure tones, generated in
fans and compressors at the blade passing frequency, are now of great concern to the engine manufacturers today. The problem is the more important because of the recent increased emphasis placed on noise control. A reduction of the pure tone noise requires a basic understanding of the noise-generation, transmission, and radiation mechanisms.

In general, there are two major kinds of fan-compressor, blade passing frequency, noise-generating mechanisms: 1) the first relating to the rotating blade row; 2) the second as a result of the interaction of moving blades with the stationary blade rows. The rotor noise arises from a rotating steady-pressure field that surrounds each blade as a consequence of its motion. The interaction-related noise results from: viscous wake interaction, potential field interaction, or wake vortex interaction.

Research has been performed on the study of turbomachinery parameters and their effect on sound generation (1). It has been found that specific parameters including, tip speed of the rotor, number of blades of the rotor, the ratio of vanes/blades, and blade row spacing, all have a significant effect on the sound generation properties of the turbomachinery.

It is the goal of this thesis to select an appropriate, simplified model which adequately describes a transonic rotor-stator compressor stage and which employs these basic parameters of the real system to obtain an understanding of the sound generating mechanisms associated with the compressor stage through the mathematical analysis of this model. The results of this analysis will provide relative pressure ratios between the various flow regions of the compressor stage which may then be compared to experimental data obtained earlier at the M.I.T. Blowdown Facility in order to check the validity of the analysis and model selected. It is not
possible to obtain the absolute pressure magnitudes with this model since it does not incorporate the appropriate forcing function of the rotor blades; however, the pressure ratios of upstream to downstream as well as upstream to rotor-stator pressure magnitudes may be compared with the experimental findings. These results are compared with the experimentally obtained ratios with quite favorable results.
CHAPTER II
LITERATURE REVIEW

Much research has been performed in the area of noise generated by turbomachinery, specifically high speed axial compressors and fans. Unfortunately, there has been a tremendous gap between theory and experimentation. The theoretical models can become terribly complicated when all effects are included; consequently, much simplification is applied to the models which, in turn, reduce their relevance to the experimental configuration. In addition, little finesse has been applied to the noise data retrieval and analysis from engines. These events have resulted in the general divergence of theoretical and experimental approaches in the search to understand noise generated by turbomachinery.

Many key researcher have developed theories and models which have provided much insight into the study of turbomachinery noise. This subject is a rather complex one which has led to the separation of this research into numerous subdivisions depending upon the characteristics of the noise generated, how it propagates, and what conditions are present, both internally, within the compressor itself, and externally. Such divisions may be designated in the following ways: The type of noise may be classified under three headings: interaction tone noise, multiple pure tone noise, and broadband noise. The generation of such noise has a variety of sources:
the rotor alone, rotor-stator interaction, and interaction with atmospheric turbulence, to name a few. In addition, one may study the reflection and transmission of sound through the blades, the interaction of wakes from stationary blades with the rotor, as well as the characteristics or propagation through the duct with varying geometries and materials. Each of these areas of study are of great importance to the understanding of turbomachinery noise.

This thesis explores the problem of interaction tone noise generation specifically applied to a transonic rotor (23.25 inches in diameter) at a tip Mach number of 1.2 and a 3/4 inch axially spaced stator blade row. In the past, a number of researchers have studied interaction tone noise generation and the physical parameters of the compressor stage which most greatly affect this phenomenon. Among those researchers are: Tyler and Sofrin (2), Mani and Horvay (3), Kaji and Okazaki (4,5), Mani (6), Lowson (7), Koch (8), Hetherington (9), Amiet (10), and Morfey (11). Through the research that has been performed, particularly since the late 1960's, a number of interesting observations and conclusions have been attained. In particular, the interaction of the rotor and stator is considered to have two major components: a potential part which is most recognizable at small separation and a wake part which becomes dominant at larger separations since the wakes decay at a much slower rate than the potential field. The blades are typically assumed to be compact so that the fluctuating force may be characterized by an acoustic dipole source. With these assumptions, quite good agreement has been obtained between the measurements and the theoretically predicted tone power.

The models used to analyze the blade rows have varied with basic assumptions. As previously mentioned, many methods treat the blade rows
as compact sources, which has been leading towards the treatment of sound generation as an acoustic dipole source. Kaji and Okazaki use the semi-actuator disk approach for calculating sound transmission through a blade row, in which the blade now pitch becomes infinitely small. Mani and Horvay use a different approach in which the blade passages are assumed semi-infinite to facilitate the use of the Weiner-Hopf techniques. In some respects these two models are opposite extremes.

The article by Kaji and Okazaki is of particular interest for the account of sound generation presented in this thesis. Kaji and Okazaki have analyzed the propagation of sound through a blade row based on the semi-actuator disk theory and the acceleration potential method. They explored the effects of various physical parameters of the cascade on the propagation of sound. A few interesting conclusions which they obtained are summarized below:

1) Flow Mach numbers have a great influence on sound transmission and reflection. An increase in Mach number decreases the transmission coefficient.

2) There exists an angle of incidence, other than the chordwise direction, at which the sound wave has no reflection from the stator blade row.

3) The wavelength of the incident wave has a minor effect, from the phase difference associated with the cascade inlet and outlet.

4) An increase in stagger angle results in a decrease in the transmission coefficient.

The semi-actuator disk theory is a useful one if the condition of the sound field is not super-resonant, where many circumferential modes
propagate as the transmitted and reflected wave for one mode of incident waves.

Other methods have treated the blades as distributed sources, where the blade row is represented by a cascade of uncambered blades with zero mean lift. These analyses distribute the sources along the blades in order to match the incident upwash due to vorticity or pressure waves. Excellent quantitative results have been achieved in this area.

In summary, it has been found that the compact models are quite accurate in the analysis of pressure wave transmission and reflection by the blades while inaccurate for the prediction of the interaction of inlet distortion on the blade rows. The evidence suggests that the behavior of the blades is similar to that of a diffraction grating. The incidence of the blades is relatively significant while the extent of the blades is not (12).

With this background research as a foundation, this thesis attempts to analyze a specific set of pressure data obtained at M.I.T. During the testing of a transonic compressor stage at the Blowdown Facility on the M.I.T. campus, much data was gathered describing the pressure field upstream of the rotor, between the rotor and stator, and downstream of the stator. It is the objective of this thesis to analyze the acoustic characteristics of this compressor stage and compare the results with the experimental data to obtain relative orders of magnitude estimates of the acoustic pressure wave fields in these three regions and determine the acoustical significance of the pressure data accumulated from the compressor stage.
CHAPTER III

SOUND GENERATION BY THE ROTOR

Analysis of the acoustic pressure field representative of a transonic compressor stage consisting of a 23.25 inch O.D. rotor, with a tangential tip Mach number of 1.2, and 23 blades and a 3/4 inch, axially spaced, stator blade row of 48 blades, begins with the selection of the simplest model available which most accurately describes the actual configuration. Upon consideration of the critical characteristics of this system, rotor spinning in a flow field may be modeled simply by considering flow parallel to a moving corrugated board. The corrugations of the board represent the blades of the rotor while the parallel flow describes the swirling flow field produced between the rotor and stator. The stator may be represented by a plane boundary or a set of parallel planes perpendicular to the corrugated board, depending upon which reflective characteristics of the stator blade row is to be studied. For the present, the analysis of the moving corrugated board will be studied as a separate entity. The discussion of the reflective characteristics of the stator will be discussed in the following chapters.

Figure 1 is a schematic representation of the corrugated board and parallel flow. Region 0 is that region upstream of the rotor. Region 1 is the section downstream of the rotor in the real system.

The corrugated board symbolizes a wave of wavelength $L$ moving with a speed Mach number, $M_t$, corresponding to the tangential tip Mach number of the rotor. The wavelength is determined by the spacing of the blades in the rotor. The fluid velocity $V$, parallel to the flow,
is given the swirl velocity produced by the rotor. Note that there is no velocity of the fluid (V=0) in Region 0. The swirl is produced by the rotor; therefore, we have assumed there is no swirl velocity upstream of the rotor. The vector \( \hat{K} \) is perpendicular to the plane wavefronts in the medium where the magnitude \( K \) is called the wave-number. \( K \) is representative of the "angular frequency" of the wave and has the dimensions of inverse length. The natural frequency is denoted by \( \omega \)

\[
\omega = \frac{2\pi C_t}{L}
\]  

(1-1)

where \( C_t \) is the speed of the corrugated board. The angles of propagation of the plane wave are \( \phi_0 \) and \( \phi_1 \), the angles measured from the vertical to the vectors \( \hat{K}_0 \) and \( \hat{K}_1 \). The wavenumber describing the wave from the corrugated board is denoted, \( K_t \).

\[
K_t = \frac{\omega}{C_t}
\]  

(1-2)

These parameters describe the system completely and may now be used to calculate the pressure field in each region. This problem is treated in *Theoretical Acoustics* by Morse and Ingard (13); however, a fairly detailed derivation will be presented here as it is essential to the understanding of the acoustic pressure field produced by a rotor.

First, consider Region 0 where \( V = 0 \). The transverse displacement of the boundary, in lab coordinates, is given by:
The relative motion between the wave in the boundary and the wave in the medium produces a plane wave in the medium. The sound pressure wave will have the same frequency as that of the transverse wave, resulting in the following equation for the pressure wave:

\[ p_0 = J_0 e^{i(Kx_0x + Ky_0y - \omega t)} \]  \hspace{1cm} (1-4)

where:

\[ Kx_0 = K_0 \sin \phi_0 \]
\[ Ky_0 = K_0 \cos \phi_0 \]

To describe this pressure field, the amplitude \( J_0 \), the wavenumber \( K_0 \), and the angle of propagation \( \phi_0 \) must be solved. These are the goals of the following derivation.

Note that in order for the sound wave to fit the boundary displacement wave along the XZ-plane, the following condition must be satisfied:

\[ Ky_0 = Ky_1 = K_t \]  \hspace{1cm} (1-5)
Now, with $V = 0$, we have the following relationship:

$$K_0 = \omega/C$$  \hspace{1cm} (1-6)

With the use of Eqs. (1-2), (1-5), and (1-6):

$$\frac{(\omega/C) \cos \psi_0}{\cos \phi_0} = \omega/C_t$$  \hspace{1cm} (1-7)

$$\cos \phi_0 = \frac{C}{C_t} = 1/M_t$$  \hspace{1cm} (1-8)

This determines the angle of propagation of the second pressure wave in Region 0. Also note that if $(1/M_t) > 1$, in other words, $C > C_t$, $Kx_0$ is imaginary since $\sin^2 \phi_0 > 1$, and no true wave is propagated into the medium.

To relate the pressure amplitude to the boundary displacement, the wave equation states the following:

In physical terms, this equation says that an acceleration of the fluid is produced by a pressure gradient. Upon differentiating Eq. (1-3) twice with respect to time and Eq. (1-4) once with respect to displacement, the following equation is obtained for the pressure amplitude:
The pressure wave has been determined for Region 0 as $K_0$, $\phi_0$, and $J_0$ have been obtained.

The derivation of the sound pressure wave for Region 1 is similar to that of Region 0 with a slight modification to account for the swirl velocity, $V$. It may be advantageous to review this derivation.

Recall, the equations of mass and momentum balance in the moving coordinate system:

$$\frac{\partial \rho}{\partial t'} + \rho_0 \sum_i \frac{\partial u_i}{\partial x_i} = 0 \quad (1-11)$$

$$\rho_0 \frac{\partial u_i}{\partial t'} = -\frac{\partial p}{\partial x_i} \quad (1-12)$$

where $i$ refers to $x$, $y$, and $z$, respectively. The fluid has velocity components $V_i$, therefore the transformation to lab coordinates in the lab system $S$ become:

$$x_i = x'_i + V_i t', \quad t = t', \quad \tilde{V} = \text{constant}$$

Therefore, for any function $f$ of space and time, the following relations result:
Upon substitution into Eqs. (1-11) and (1-12), the following is obtained:

$$\frac{\partial f}{\partial x_i} = \frac{\partial f}{\partial x_i} \frac{\partial x_i}{\partial x_i} + \frac{\partial f}{\partial t} \frac{\partial x_i}{\partial t} = \frac{\partial f}{\partial x_i}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial t} + \sum_i \frac{\partial f}{\partial x_i} \frac{\partial x_i}{\partial t} = \frac{\partial f}{\partial t} + \sum_i V_i \frac{\partial f}{\partial x_i}$$

$$\therefore \frac{\partial f}{\partial t} = \frac{\partial f}{\partial t} + \sum_i V_i \frac{\partial f}{\partial x_i} = \frac{\partial f}{\partial t} + \vec{V} \cdot \vec{V} \quad (1-13)$$

Recall a basic relation for the sound pressure field:

$$\rho \left( \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right) p + \rho_0 \sum_i \frac{\partial u_i}{\partial x_i} = 0 \quad (1-14)$$

$$\rho_0 \left( \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right) u_i = -\frac{\partial p}{\partial x_i} \quad (1-15)$$

To obtain the wave equation, differentiate Eq. (1-14) with respect to time, take the divergence of Eq. (1-15), and combine the results.
The resulting wave equation in laboratory coordinates, accounting for the swirl velocity \( V \), is:

\[
\left[ \frac{\partial}{\partial t} + \mathbf{\hat{v}} \cdot \mathbf{\hat{v}} \right] p = c^2 \nabla^2 p \tag{1-16}
\]

In this example, consider only changes in the \( y \)-direction, therefore the equation becomes:

\[
\left[ \frac{\partial}{\partial t} + \mathbf{\hat{v}} \frac{\partial}{\partial y} \right] p = c^2 \nabla^2 p \tag{1-17}
\]

Upon differentiating Eq. (1-4) and substituting into Eq. (1-17) the following relationship is obtained:

\[
[-i\omega + \mathbf{\hat{v}}(iK_y)]^2 = c^2 k_1^2 \\
(\omega - \mathbf{\hat{v}}K_y)^2 = (cK_1)^2
\]

From this, solve for \( K_1 \) and \( \phi_1 \):

\[
\omega - \mathbf{\hat{v}}(\omega/C_t) = cK_1, \\
k_1 = (\omega/C)[1 - (\mathbf{\hat{v}}/C_t)] = (\omega/C)[1 - (VK_1\cos\phi_1/\omega)]
\]
\[ K_1 = \frac{\omega}{C} - \frac{VK_1 \cos \phi_1}{C} \]

\[ C = \frac{\omega}{K_1} - V \cos \phi_1 \]

\[ K_1 = \frac{\omega}{C + V \cos \phi_1} = \frac{\omega}{C} / (1 + M \cos \phi_1) \]  \hspace{1cm} (1-18)

where

\[ M = \frac{V}{C} \]

Consequently,

\[ Kx_1 = K_1 \sin \phi_1 = \frac{\omega \sin \phi_1}{C (1 + M \cos \phi_1)} \]

\[ \sin^2 \phi_1 = 1 - \cos^2 \phi_1 \]

Recall:

\[ Ky = \frac{\omega}{C_t} = K_1 \cos \phi_1 = \frac{\omega \cos \phi_1}{C (1 + M \cos \phi_1)} \]

\[ C (1 + M \cos \phi_1) = C_t \cos \phi_1 \]

\[ \cos \phi_1 (M - M_t) = -1 \]

\[ \cos \phi_1 = \frac{1}{M_t - M} \]  \hspace{1cm} (1-19)

Note the alterations to \( K \) and \( \phi \) due to the velocity \( V \). \( K_0 \) is reduced by a factor \( [1/(1+M \cos \phi_1)] \) and \( \cos \phi_0 \) is modified by the
the factor [1-(M/M_t)]. Observe that \(|\cos \phi_1| > 1\) when \(|M_t-M| < 1\), this results in an imaginary \(\sin \phi_1\) and consequently an imaginary \(Kx_1\). Therefore, no true wave is propagated into the medium if \(|M_t-M| < 1\).

As an example, if \(C_t = 0\) a wave will propagate only if \(V > C\); \(V\) is supersonic. The relative speed of the board with respect to the fluid must be supersonic for a wave to propagate in the medium.

Finally, the pressure amplitude must be derived. Equation (1-9) must be modified to take \(V\) into account. Upon substitution of the transformation relations derived previously, the following relation results:

\[
\rho \left[ \frac{\partial}{\partial t} + V \frac{\partial}{\partial y} \right] \xi = -\frac{\partial p}{\partial x} \tag{1-20}
\]

Again, performing the appropriate differentiations of Eqs. (1-3) and (1-4), and substituting into Eq. (1-20):

\[
\rho [K_t^2(C_t-V)^2 \xi_0 ] = i J_1 \sin \phi_1 [(K_t/C)(C_t-V)]
\]

\[
J_1 = -i\rho C \xi_0 [K_t(C_t-V)/\sin \phi_1]
\]

\[
J_1 = -i\rho C \xi_0 (\omega/C_t)(C_t-V)/\sin \phi_1
\]

Recall:
\[ C_t = \omega / K \cos \phi_1 \quad ; \quad \cos \phi_1 = C / (C_t - V) \]

\[ K_1 = (\omega / C) / (1 + M \cos \phi_1) \]

Therefore:

\[ J_1 = -i\rho C \varepsilon_0 (KC / \cos \phi_1)(\cos \phi_1 / \sin \phi_1) \]

\[ J_1 = -i\rho C \varepsilon_0 (C / \sin \phi_1)(\omega / C) / (1 + M \cos \phi_1) \]

\[ J_1 = -i\rho C \varepsilon_0 \{ 1 / [(1 + M \cos \phi_1) \sin \phi_1] \} \quad (1-20) \]

Again note that \( J_0 \) is modified by the factor \( 1 / (1 + M \cos \phi_1) \). To see the results more clearly, substitute:

\[ \sin^2 \phi_1 = 1 - (\omega / K_t C_t)^2 \]

\[ \sin^2 \phi_1 = 1 - [\omega / C_t \{ 1 / [(\omega / C) (1 - V / C_t)] \}]^2 \]

\[ \sin^2 \phi_1 = 1 - [C / (C_t - V)]^2 \]

\[ \sin^2 \phi_1 = [(C_t - V)^2 - C^2] / (C_t - V)^2 \]
Now: if \( \sin \phi_1 \) is zero, \( J_1 \) approaches infinity. This occurs when \( C_t - V = C \). This corresponds to the cut-off limit obtained previously that a wave will propagate only when the relative speed of the board with respect to the fluid is supersonic. Also, when \( \sin \phi_1 \) approaches infinity or, in other words, when \( C_t = V \), \( J_1 \) will approach zero. To determine the behavior of \( Kx_1 \) when \( C_t = V \), one may write \( Kx \) in the following manner:

\[
Kx_1 = \left(\frac{\omega}{C}\right) \left[1 - \frac{V}{C_t}\right] \left\{ \left(\frac{C_t - V}{C_t}\right)^2 - \frac{1}{2}\right\}^{1/2} / \left(C_t - V\right)
\]

\[
= \left(\frac{\omega}{C_t}\right) \left(\frac{C_t - V}{C_t}\right)^2 - \frac{1}{2}^{1/2}
\]

Now, as \( C_t \) approaches \( V \), \( Kx_1 \) becomes:

\[
Kx_1 = i(\omega/C_t) = iKy = iK_t
\]

In other words, \( Kx_1 \) is imaginary and approaches the value of the wavenumber determined by the corrugated board parameters.
CHAPTER IV
REFLECTION COEFFICIENTS OF THE STATOR

The pressure field derived in the preceding chapter accounts only for the rotor motion. The rotor is the wave source in this system; however, the stator blade row has certain reflective characteristics which alter the flow field and sound pressure wave.

4.1 Reflective Characteristic of Flow Straightening

At a distance 3/4 inch downstream from the rotor, the stator blade row performs the function of removing the swirl produced by the rotor from the flow. To represent this function of the stator in the model introduced in Chapter 1, a plane boundary, B, is erected at a distance E, (E is 3/4 inch in this system) from the moving corrugated board. This is illustrated in Figure 2.

This boundary will have no effect on the pressure wave in region 0; however, region 1 is now transformed into two regions labelled region 1 and region 2. Region 1 is the section of the compressor stage between the rotor and the stator while region 2 is that region downstream of the stator in the actual system. Note that the swirl velocity, V, is zero in region 2 as in region 0 due to the stator blade row.

In order to calculate the sound pressure wave in regions 1 and 2, the reflection and transmission characteristics of the stator must be analyzed in order to compute the respective coefficients. These
coefficients are crucial in the determination of the complete sound pressure wave in Regions 1 and 2. The derivation of these coefficients along with the resulting sound pressure wave field in Regions 1 and 2 are the goals of this section.

In Figure 2, the swirl velocity, \( V \), is shown parallel to the boundary in Region 1. As stated previously, \( V = 0 \) in Region 2. The sound pressure wave from the corrugated board is shown approaching at an angle of incidence of \( \phi_1 \) in Region 1. As the wave contacts the boundary, a portion is reflected back into Region 1 and the remaining wave is transmitted at an angle \( \phi_2 \) in Region 2. The general equations which describe the total sound pressure wave in Regions 1 and 2 are the following:

\[
P_1 = p_I + p_R = J_1 \left[ e^{i(Kx_1X + Ky_1Y - \omega t)} + R_F e^{i(-Kx_1X + Ky_1Y - \omega t)} \right] \quad (2-1)
\]

\[
P_2 = p_T = J_1 T_F e^{i(Kx_2X + Ky_2Y - \omega t)} \quad (2-2)
\]

Note, in these two equations no distinction is made between \( Ky_1 \) and \( Ky_2 \). As shown previously, \( Ky_1 = Ky_2 = Ky = K_t \) in order for the incident, reflected, and transmitted waves to "match" at the interface, \( B \). The unknowns in these equations are the two coefficients, \( R_F \) and \( T_F \), the wavenumber \( K_2 \) in Region 2, and the angle of refraction, \( \phi_2 \).
Consider the boundary conditions at B. First note that the velocity in the $y$-direction must be equal at the interface in Regions 1 and 2. Mathematically stated:

\[
\frac{C}{\cos \phi_1} + V = \frac{C}{\cos \phi_2}
\]

\[
\cos \phi_2 + \left( V \cos \phi_1 \cos \phi_2 / C \right) = \cos \phi_1
\]

\[
\cos \phi_2 [1 + (V \cos \phi_1 / C)] = \cos \phi_1
\]

\[
\cos \phi_2 = \cos \phi_1 / (1 + M \cos \phi_1)
\] (2-3)

The remaining boundary conditions are: continuity of pressure at the interface, $X = 0$, and continuity of displacement, $\xi_x$, at the interface, $X = 0$. The continuity of pressure is described quite simply by the following relation:

\[
J_1 (1 + R_F) = J_1 T_F
\]

\[
T = 1 + R_F
\] (2-4)

The continuity of displacement relation requires a more detailed derivation.
From Chapter 1, the following relation was found:

\[ \rho \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial y} \right)^2 \xi_x = - \frac{\partial \rho}{\partial x} \]  \hspace{1cm} (1-20)

This relation may be solved with the use of Eqs. (1-3), (2-1), and (2-2). Upon performing the appropriate differentiations of each equation and substituting into (1-20), the following relations are obtained:

\[ \xi_{x_1} = (iJ_1 \sin \phi_1 / \rho K_1 C^2) e^{i(Kx_1 X + Ky Y - \omega t)} - R_F e^{i(-Kx_1 X + Ky Y - \omega t)} \]

\[ \xi_{x_2} = (iJ_1 \sin \phi_2 / \rho K_2 C^2) e^{i(Kx_2 X + Ky Y - \omega t)} \]

Equating these two relations:

\[ \xi_{x_1} = \xi_{x_2} \]

\[ K_2 \sin \phi_1 (1 - R_F) = K_1 \sin \phi_2 T_F \]  \hspace{1cm} (2-5)

As noted previously, \( Ky_1 = Ky_2 \), therefore:

\[ K_1 \cos \phi_1 = K_2 \cos \phi_2 \]

\[ K_2 = K_1 (\cos \phi_1 / \cos \phi_2) \]  \hspace{1cm} (2-6)
With the use of these relations, an expression for $R_F$ may be obtained. Upon substitution of Eq. (2-4) into Eq. (2-5):

\[(K_1 \sin \phi_2)(1 + R_F) = (K_2 \sin \phi_1)(1 - R_F)\]

\[R_F = \frac{K_2 \sin \phi_1 - K_1 \sin \phi_2}{K_2 \sin \phi_1 + K_1 \sin \phi_2}\]

Substitute expression for $K_2$ from Eq. (2-6) and find:

\[R_F = \frac{\sin \phi_1 \cos \phi_1 - \sin \phi_2 \cos \phi_2}{\sin \phi_1 \cos \phi_1 + \sin \phi_2 \cos \phi_2}\]  \hspace{1cm} (2-7)

\[R_F = \frac{\sin(2\phi_1) - \sin(2\phi_2)}{\sin(2\phi_1) + \sin(2\phi_2)}\]  \hspace{1cm} (2-8)

Finally, from Eq. (2-4):

\[T_F = 1 + \left[ \frac{\sin \phi_1 \cos \phi_1 - \sin \phi_2 \cos \phi_2}{\sin \phi_1 \cos \phi_1 + \sin \phi_2 \cos \phi_2} \right]\]

\[T_F = \frac{2 \sin \phi_1 \cos \phi_1}{\sin \phi_1 \cos \phi_1 + \sin \phi_2 \cos \phi_2}\]  \hspace{1cm} (2-9)

\[T_F = \frac{2 \sin(2\phi_1)}{\sin(2\phi_1) + \sin(2\phi_2)}\]  \hspace{1cm} (2-10)
The two coefficients have been derived. The remaining task is to determine the wavenumber, $K_2$, for Region 2. Since $V = 0$ in this region, it is the same as in Region 0:

$$K_2 = \frac{\omega}{C}$$

Now:

$$K_2^2 = K_Y^2 + K_X^2$$

$$K_Y = K_Y = K_t = (\omega/C_t)$$

$$K_X = K_{x_2} = (\omega/C)^2 - (\omega/C_t)^2 = (\omega/C)^2[1 - (1/M_t)^2]$$

$$K_{x_2} = K_2 \sin\phi_2$$

A complete description of the sound pressure field for the model of a moving corrugated board with an axially displaced plane boundary has been obtained. The results for each region are summarized here for easy reference.

**Region 0**

$$P_0 = J_0 e^{i(K_X X + K_Y Y - \omega t)}$$

where

$$J_0 = -i \rho C \omega \xi_0 / \sin\phi_0$$
\[ K_0 = \omega / c \]

\[ K_y = K_0 \cos \phi_0 = \omega / c_t \]

\[ K_{x0} = K_0 \sin \phi_0 \]

\[ \cos \phi_0 = \frac{1}{M_t} \]

\[ \sin^2 \phi_0 = 1 - \left(\frac{1}{M_t}\right)^2 \]

\[ p_0 = |p_0| / \rho c \varepsilon_0 \omega \]

**Region 1**

\[ p_1 = J_1 [e^{i(K_{x1}X + KyY - \omega t)} + R_F e^{i(-K_{x1}X + KyY - \omega t)}] \]

where

\[ J_1 = \frac{-i \rho c \varepsilon_0}{(1 + M \cos \phi_1) \sin \phi_1} \]

\[ K_1 = \frac{\omega / c}{[1 / (1 + M \cos \phi_1)]} \]

\[ K_{x1} = K_1 \sin \phi_1 \]
\[ \cos \phi_1 = \frac{1}{M_t - M} \]

\[ \sin^2 \phi_1 = 1 - \left[ \frac{1}{(M_t - M)^2} \right] \]

\[ R_F = \frac{\sin(2\phi_1) - \sin(2\phi_2)}{\sin(2\phi_1) + \sin(2\phi_2)} \]

\[ P_1 = |p_1|/\rho C \xi_0 \omega \]

Region 2

\[ p_2 = j_1 T_F e^{i(Kx_2X + KyY - \omega t)} \]

where:

\[ T_F = \frac{2\sin(2\phi_1)}{\sin(2\phi_1) + \sin(2\phi_2)} \]

\[ K_2 = \omega/C \]

\[ Kx_2 = K_2 \sin \phi_2 \]

\[ \cos \phi_2 = \frac{1}{M_t} \]
$$\sin^2 \phi_2 = 1 - (1/M_t)^2$$

$$P_2 = |p_2|/\rho C_0 \omega$$

4.2 Reflective Characteristic of Hardware

As stated previously in Chapter 1, the stator blade row has essentially two types of reflective characteristics. The first type, studied in the preceding section, is the reflection due to the "straightening" of the flow. In this section, the reflection of the sound pressure wave by the actual hardware of the stator blades will be studied.

This characteristic of the reflection by the stator blade row may be modelled by the addition of a row of parallel planes aligned parallel to the corrugated board in place of the plane boundary from Chapter 2. This configuration is shown in Figure 3. The distance D, or length of the parallel planes, corresponds to the axial chord of the stator blade row. The angle \( \theta \) describes the angular incidence of the stator blade at the exit. From geometry:

$$\frac{D}{n} = \cos \theta \quad \therefore \quad n = \frac{D}{\cos \theta} \quad (2-11)$$

The transmission matrix for such a configuration is the following:
From the flow diagram insert of Figure 3 the following relationships are determined:

\[ u_1 = u'_1 \cos \theta \quad \therefore \quad u'_1 = u_1 / \cos \theta \quad (2-13) \]
\[ u_2 = u'_2 \cos \theta \quad \therefore \quad u'_2 = u_2 / \cos \theta \quad (2-14) \]

Also:

\[ p_2 \cos \theta' = \rho C u_2 \quad (2-15) \]

From this information one may write the following relation:

\[
\begin{bmatrix}
  p_1 \\
  \rho C u_1'
\end{bmatrix}
= \begin{bmatrix}
  T_{11} & T_{12} \\
  T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
  p_2 \\
  \rho C u_2'
\end{bmatrix}
\]

Upon the substitution of \( u'_1 \), \( u'_2 \), and \( \rho C u_2 / p_2 \), the following description for \( Z \), the impedance, is obtained:
\[ Z = \frac{p_1}{\rho Cu_1} = \left[ \frac{T_{11} + T_{12}(\cos \phi' / \cos \theta)}{T_{21} + T_{22}(\cos \phi' / \cos \theta)} \right] (1/\cos \theta) \]

Substitute the values for the transmission matrix and reduce. The resulting relation for \( Z \) is:

\[ Z = \frac{\cos \phi' (1 + \tan^2 \theta) + i(\tan \theta \cos \phi) (\cos^2 \theta - \cos^2 \phi')}{\cos^2 \phi' + (\tan \theta \cos \phi)^2} \]

(2-17)

Now, recall the following equations for \( p_1 \) and \( \rho Cu_1 \):

\[ p_1 = J_1 e^{i(Kx_1 X + Ky_1 Y - \omega t)} + R_s e^{i(-Kx_1 X + Ky_1 Y - \omega t)} \]

\[ \rho Cu_1 = J_1 \cos \phi' [e^{i(Kx_1 X + Ky_1 Y - \omega t)} - R_s e^{i(-Kx_1 X + Ky_1 Y - \omega t)}] \]

\[ Z = \frac{p_1}{\rho Cu_1} = (1/\cos \phi') \left[ \frac{1 + R_s}{1 - R_s} \right] \]

\[ Z \cos \phi' (1 - R_s) = 1 + R_s \]
$$R_s = \frac{Z \cos \phi' - 1}{Z \cos \phi' + 1} \quad (2-18)$$

Hence, the reflection coefficient is derived for the reflection by the stator blade hardware.

For this derivation, $V$ is taken to be zero. It will be shown in the following section that the magnitude of $R_s$ is negligible in comparison to the magnitude of $R_F$; consequently, a more detailed analysis of the derivation of $R_s$ accounting for $V$ was not made.

The values of the parameters to be used are the following:

$$\cos \phi' = \frac{1}{M_t}$$

$$K = \frac{\omega}{C} = 2\pi \frac{M_t}{L}$$

$$\omega = 2\pi C \frac{t}{L}$$

These values were determined from the previous analysis.
CHAPTER 5
RESULTS

It is now necessary to evaluate the results of the analysis from the preceding two chapters and compare this evaluation with experimental data. First, a comparison of the magnitudes of the two reflection coefficients, $R_F$ and $R_S$, will be presented. With the knowledge of the appropriate reflection and transmission coefficients, the magnitude of the sound pressure field is calculated in each region as a function of $M_t$, the Mach number of the corrugated board, and $M$, the swirl Mach number. The ratios of upstream (Region 0) to downstream (Region 2) pressure amplitudes and upstream (Region 0) to rotor-stator (Region 1) pressure amplitudes are evaluated and compared with the experimental data obtained for a transonic compressor stage, run at the M.I.T. Blowdown Facility.

5.1 Theoretical Analysis

Figure 4A is a graph of the complex reflection coefficient, $R_S$, obtained for the reflective characteristic of the stator hardware as a function of $M_t$. Figure 4B is a plot of the amplitude $|R_S|$ of this complex reflection coefficient as a function of $M_t$. As may be noted from the graphs in Figures 4A and 4B, there exists specific values for $M_t$ at which $R_S$ approaches zero, quite abruptly. Since the value of $M_t$ and the value of $\phi_1$, the angle of incidence of the sound pressure wave, are dependent, a physical explanation for
this phenomenon may be postulated. One explanation may be that at specific angles of incidence the geometry of the system is such that the wave is essentially "trapped" in the blade row resulting in very small reflection of the wave by the stator blade row. As one might recall, Kaji and Okazaki noted this phenomenon as well in their study of sound propagation through a blade row.

Figures 5, 6, and 7 are plots of the reflection coefficient, $R_F$, due to the removal of swirl from the flow. This reflection coefficient may be a real or complex number depending upon the relationship between $M_t$, $M$, and $C$. $R_F$ is shown as a function of $M_t$ for different values of $M$, ranging from $M = 0.3$ to $M = 0.7$. Note that for $|M_t - M| = 0$ to $M_t = 1$, $R_F$ is a real number approaching the value of unity at either end. Recall that the reflection coefficient is the ratio between the incident and reflected pressure waves; therefore, a value of unity for $R_F$ means that the reflected wave differs only in phase angle from the incident wave, with the reflected intensity equal to the incident intensity. $R_F$ is also real for values of $|M_t - M| \geq 1$. For values of $M_t = 1$ to $|M_t - M| = 1$, $R_F$ is an imaginary number.

These specific values for the quantities of $M_t$ and $(M_t-M)$ represent critical values for the angles of incidence and refraction. Recall that the definition for $\cos \theta_1$ and $\cos \theta_2$ from the preceding derivations are:
\[
\cos \phi_1 = \frac{1}{(M_t - M)}
\]

\[
\cos \phi_2 = \cos \phi_1/(1 + M \cos \phi_1) = \frac{1}{M_t}
\]

Now, for real angles \( \phi_1 \) and \( \phi_2 \), \(|\cos \phi_1| \leq 1\) and \(|\cos \phi_2| \leq 1\); therefore, for \(|M_t-M| < 1\) and/or \(|M_t| < 1\), either \( \phi_1 \), \( \phi_2 \), or both \( \phi_1 \) and \( \phi_2 \) are imaginary.

In physical terms, if \(|M_t-M| = 1\), \(|\cos \phi_1| = 1\) and \(\phi_1 = 0^\circ\) or \(180^\circ\). This represents the smallest possible value of \(\phi_2\). The region \( \phi_2 < \phi_2 \), min represents a shadow section in Region 2. This is reflected in the value of \(R_F\) approaching \(-1\).

If \(|M_t| = 1\), \(|\cos \phi_2| = 1\) and \(\phi_2 = 0^\circ\) or \(180^\circ\). This represents the critical value for \(\phi_1\). For values of \(\phi_1 > \phi_{1c}\), total reflection will occur. This is demonstrated by the fact that \(R_F = 1\) at \(M_t = 1\). See Figure 8 for a sketch of this phenomenon.

The value for \(|M_t-M| = 0\) and/or \(M_t = 0\) represents a singularity of the mathematical equations. In physical terms, \(|M_t-M| = 0\) represents a situation in which the corrugated board velocity and the swirl velocity are equal. There is no relative motion between the wave in the boundary and the wave in the medium; therefore, no acoustic wave is generated, \(P\) approaches \(0\). At \(|M_t| = 0\), the boundary is stationary; the rotor is not spinning. In these two instances, the model no longer represents the physical system.
The range of values for $M_t$ and $M$ which are of importance in this instance for a transonic rotor-stator configuration lies within those values for which $R_F$ is imaginary, specifically $M_t = 1.2$ and $M = .5$. From the graphs of $R_F$ and $R_S$ the values obtained are $(.818 + .575i)$ and $(-.147 - .052i)$ respectively. These correspond to magnitudes of $|R_F| \approx 1.0$ and $|R_S| \approx .156$. From this, it may be seen that $|R_F/R_S| \approx 6.4$; consequently, the reflective characteristic due to the removal of swirl is the dominant reflective effect of the stator blade row. For our purposes, the reflection due to the stator hardware will be considered to be negligible in the following analysis.

The next step is to calculate the magnitude of the sound pressure wave in each region. To accomplish this a normalized value of the pressure, $p/pC_0$, is taken as the new value of the sound pressure wave and will be represented by the value $P$. The graphs of $P_0$, $P_1$, and $P_2$ vs. $M_t$ respectively for different values of $M$, from $M = .4$ to $M = .6$, are shown in Figures 9-11. Recall that $M=0$ in Regions 0 and 2; however, in Region 2 there are curves corresponding to differing values of $M$. These values for $M$ are those associated with the swirl velocity in Region 1 which affects the sound pressure wave transmitted to Region 2. Note that the general shape of the curves are all similar with a peak in the magnitude, $P$ approaches infinity, occurring at $|M_t-M| = 1$. Also note that $P$ decreases with an increase in $M$ given a specific value for $M_t$. Again, recall that the value of $|M_t-M| = 1$ corresponds to $\cos \phi_1 = 1$ or $\phi_1 = 0^\circ$. 
This is in support of a maximum occurring at $|M_t-M| = 1$ in the magnitude of the pressure field. Mathematically, note that

$$J_1 = -i\rho C\omega\zeta_0 /[(1+M \cos \phi_1)\sin \phi_1] .$$

Now since $\sin^2 \phi_1 = (1-\cos^2 \phi_1)$, $\sin \phi_1 = 0$ when $\cos \phi_1 = 1$. This will result in $J_1$ approaching infinity. In this case, note that it is the value of $J_1$ which dominates rather than the value of $R$ or $T$.

For the values $M_t = 1.2$ and $M = .5$ in Region 1, the equation for the pressure magnitude is dependent upon $X$, the axial location between the rotor and stator. Specifically, the relationship is one of exponential decay. In regions 0 and 2, the pressure amplitude is independent of $X$. Figure 12 shows the dependence upon $X$ in Region 1.

Finally, the ratios $P_0/P_1$, $P_0/P_2$, and $P_1/P_2$ are calculated for values of $M_t = 1.2$ and $M = .5$. These values correspond with experimental values. The magnitudes of these ratios are found to be: 1.49, 1.66, and 1.11, respectively. These values will be compared with those obtained experimentally, in the following section. Graphs of $P_0/P_1$, $P_0/P_2$, and $P_1/P_2$ as functions of $M_t$ are shown in Figures 13, 14, and 15 respectively. It is interesting to observe in Figures 13 and 14 that the values of $P_0/P_1$ and $P_0/P_2$ decrease abruptly as $M_t$ increases until $M_t-M = 1$ at which point the $P$'s approach infinity. In Figure 15 however, $P_1/P_2$ varies linearly with $M_t$. 
5.2 Experimental Analysis and Comparison with Theory

The data on the magnitude of the pressure field in a transonic compressor stage used to compare with the theoretical analysis presented was performed by Mohammed Durali in the Blowdown Facility at M.I.T.'s Gas Turbine Laboratory as part of his Ph.D. thesis work. The M.I.T. Blowdown Facility is quite a unique experimental facility which enables the experimentalist to test full-scale compressor stage rotors and stators with modest expenditures of money and power. This is accomplished by performing the test in a pulsed mode instead of the typical steady-state mode. This is possible since most of the aerodynamic data of importance have time scales of at most, one-rotor rotation period. A detailed description of the facility may be found in references (14, 15) while an in-depth description of the specific experimental work accomplished by Mohammed Durali may be found in reference (16).

The pressure field data in Regions 0 and 2 were obtained by the use of conventional strain-gauge transducers having frequency response flat to 1 KHz. The time resolved measurements of the fluctuating pressures behind the rotor, in Region 1, were determined using a spherical probe described in reference (17). The probe was traversed across the annulus using a pneumatic driver during the test run. The probe data was then analyzed to obtain the total and static pressures in this region.
Due to the configuration of the compressor stage, the area of the duct in Region 0 is greater than the area of the duct in Regions 1 and 2. This difference in cross-sectional area of the duct must be accounted for when comparing the magnitudes of the sound pressure waves in each region. This compensation, or correction factor, may be based on the principle of the conservation of energy in a sound pressure wave which, stated mathematically, says:

\[ P_0^2 A_0 = P_0^2 A_0 \tag{3-1} \]

where \( P_0 \) and \( A_0 \) are the pressure amplitude and area respectively, measured in Region 0 while \( P_0 \) and \( A_0 \) are the corrected values compensated to correspond to the area in Regions 1 and 2. In other words, \( A_0 = A_1 = A_2 \) and \( P_0 \) would be the pressure measured in Region 0 if the area were the same as that in Regions 1 and 2. From relation (3-1), it is determined that:

\[ P_0 = P_0 \sqrt{A_0 / A_0} \tag{3-2} \]

When calculating the pressure ratios \( P_0/P_1 \) and \( P_0/P_2 \), the value of \( P_0 \) corresponds to \( P_0 \) as stated above in relation (3-2). Upon analyzing the data obtained experimentally, resulting in the retrieval of \( P_0 , P_1 \), and \( P_2 \), and then evaluating the pressure ratios
\( P_0/P_2 \) and \( P_0/P_1 \), the following values are obtained: 1.59 and 1.22, respectively.

In comparing the values obtained analytically with those obtained experimentally, one may note that the values correspond quite well. The percent difference between \( (P_0/P_1)_{\text{analytical}} \) and \( (P_0/P_1)_{\text{experimental}} \) is 18% while the same comparison for \( P_0/P_2 \) is 4%. This comparison leads to the conclusion that the critical component of the pressure wave in a compressor stage is acoustic in nature.
CHAPTER VI
CONCLUSIONS

The following is a summary of the conclusions obtained from this study of sound generation from a transonic compressor stage.

1) A moving corrugated board model for a rotor is a simple, adequate method to obtain a basic understanding of the sound generation characteristics of a compressor stage.

2) The stator blade row has essentially two reflective characteristics: a) reflection by straightening the flow, $R_F$; b) reflection by the actual stator hardware, $R_S$.

3) Upon comparison of the magnitudes of the respective reflection coefficients, $R_F$ and $R_S$, the reflection due to straightening of the flow dominates, specifically in the flow regime studied with $M_t = 1.2$ and $M = .5$.

4) There are specific values of $M_t$ and, correspondingly, $f_1$, for which there is essentially no reflection due to $R_S$.

5) At particular values of $M_t$ and $|M_t-M|$, $R_F$ approaches $|1|$. These values represent critical values of $f_1$ and minimum values of $f_2$ for which
transmission will occur, where \( \phi_1 \) is the angle of incidence and \( \phi_2 \), the angle of transmission.

6) The magnitude of the sound pressure field "blows up" when \( |M_t-M| = 1 \), corresponding to the pressure amplitude, \( J \), approaching infinity.

7) The magnitude of the sound pressure field is dependent upon \( X \), axial displacement, only in Region 1, the region between the rotor and stator. Here, the relationship is one of exponential decay with increasing distance from the rotor.

8) The ratios of the specific magnitudes in the various regions, specifically: \( P_0/P_1 \), \( P_0/P_2 \), \( P_1/P_2 \), calculated analytically, compare favorably with those observed experimentally. This result leads to the conclusion that a critical component of the pressure wave in a compressor stage is acoustic in nature.
Figure 1. Model of Rotor.
Figure 2. Model of Stator as Plane Boundary.
Figure 3. Model of Stator as Set of Parallel Planes.
Figure 4A.  GRAPH OF COMPLEX $R_S$ vs. VALUES FOR $M_t$
Figure 4B. GRAPH OF $|R_s|$ vs. $M_t$

$(M = 0, \ \theta = 10^\circ)$
Figure 5. GRAPH OF $R_F$ vs. $M_t$
Figure 6. GRAPH OF COMPLEX $R_F$ WITH VALUES OF $M_t$ FOR $M = .5$
Figure 7. GRAPH OF $R_F$ vs. $M_t$
Figure 8. Illustration of Critical Angles of Propagation.
Figure 9. GRAPH OF PRESSURE MAGNITUDE vs. $M_t$ FOR REGION 0
Figure 10. GRAPH OF PRESSURE MAGNITUDE vs $M_i$ REGION 1
Figure 11. GRAPH OF PRESSURE MAGNITUDE vs. $M_t$ REGION 2
Figure 12. GRAPH OF PRESSURE MAGNITUDE vs. \(X (M_t = 1.2)\) REGION 1
Figure 13. GRAPH OF $P_0/P_1$ vs. $M_t$
Figure 14. GRAPH OF $P_0/P_2$ vs. $M_t$

KEY:
- $\Delta M = .4$
- $\bullet M = .5$
- $\blacksquare M = .6$
- $\blacktriangle M = .7$
Figure 15. GRAPH OF $\frac{P_1}{P_2}$ vs. $M_t$
REFERENCES


