POWER FACTOR CORRECTION.

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A thesis submitted to the Faculty of the Massachusetts Institute of Technology in partial fulfillment of the requirements for the degree of Bachelor of Science in Electrical Engineering.

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Submitted by

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and

C.A. Williams
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Memorandum for Professor Jackson:

The thesis by L. Randall and C. A. Williams, Course VI, on "POWER FACTOR CORRECTION" is satisfactory.
FOREWORD

The purpose of this thesis, as originally conceived, was to cover in a general way all phases of the subject of "Power Factor Correction." This has been done, and in addition, some definite conclusions have been reached in the matter of reactive power rates. In the chapter on the "Industrial Application of Synchronous Motors" some rather fantastic schemes have been put forth, more to suggest the methods of the future than to establish a definite system.

The problem has been found large enough to make the study of a lifetime and we feel that, in what follows, we have only scratched the surface.
ACKNOWLEDGMENT

The authors are indebted to Professor T. H. Dillon for his suggestions in planning this work, and for his advice and criticism in its execution.
POWER FACTOR CORRECTION

POWER FACTOR

Power factor in a single phase circuit is always \[ \frac{\text{watts}}{\text{volt-amperes}} \], which reduces to \[ \frac{VI \cos \theta}{VI} \], where \( \theta \) is the phase angle between current and voltage.

Power factor in polyphase circuits has been the subject of a long controversy which has resulted in the following definitions:

1. Power factor in a polyphase circuit is the ratio of the total watts to the (arithmetical) sum of the volt-amperes in the several phases each measured to a non-inductive neutral point. This definition may be otherwise expressed as the weighted mean of the individual power factor in the phases (weighted according to the volt-amperes in each phase.)

\[ \frac{VI \cos \theta}{VI \text{ (arith. sum)}} \]

2. Power factor in a polyphase circuit is the ratio of the total watts to the vector sum of the volt-amperes in the several phases.

\[ \frac{VI \cos \theta}{VI \text{ (vector sum)}} \]

In the absence of a practical commercial incentive to a universally accepted understanding as to the purpose and use of the term power factor little progress has been made toward an agreement upon a single definition. However, for the purpose of discussion in what follows, the second definition will always be used.
POWER AND REACTIVE KVA

In order to use simple expressions for power relations in alternating current circuits it is necessary to assume sinusoidal voltages and currents, and to assume the phases balanced. Most commercial circuits so closely approximate these conditions that no great error is introduced. The expression for power then becomes $VI \cos \theta$. The quantity $VI \sin \theta$ represents the rate at which electrostatic or electromagnetic energy is being transferred from one part of the circuit to another without doing any work. This swashing back and forth through the circuit causes heating losses, so that the heating limitations of the circuit and machinery are reached long before they are fully loaded. $VI \sin \theta$ is called reactive KVA, and the minimizing of this factor is the problem of power factor correction.

CAUSES OF LOW POWER FACTOR

In general, on systems where the power factor is low the cause is almost entirely induction motors and transformers. Unreasonably low power factor will be usually found due to:
1. Induction motors running at less than full load.
2. The use of motors too large for the duty they perform.
3. Transformers operating at less than half load.
4. High voltage, causing excessive magnetizing currents in transformers and induction motors.
5. Special inductive apparatus, e.g. rectifiers, electric furnaces, arc lamps, etc.

**Effects of Low Power Factor**

The effects of low power factor are very numerous since they apply to practically every piece of apparatus connected to a transmission or distribution system. In general they are: -- lower efficiency due to increased copper losses, poor voltage regulation, and increased operating expense due to the necessity for oversize machinery. This last is clearly shown by the present method of rating turbo-alternators. A rating of 5000 kw. at 0.8 P. F. means that a 5000 kw. turbine is coupled to a 6250 kv.-a. alternator. The extra 1250 kv.-a. is the allowance made for low power factor.

Specifically as applied to alternators, the main effects are to decrease the capacity, lower the
efficiency, and impair the voltage regulation. This can more readily be seen by reference to the vector diagram. A reactive component of current which is lagging with respect to the induced voltage will have a demagnetizing effect on the generator field. The voltage, therefore, tends to fall off as the load comes on and this effect must be offset by an increase in exciting current through the generator field. Heating of the field system is thus increased and in some cases an additional margin of capacity is required in the field.

The regulation is also increased. Reg. \( \frac{E_a - V}{V} \). From the diagrams it can be seen that, for a given load, \( I \) increases with a decrease in power factor. This causes an increase in \( IR \) and \( IX \) which shortens \( V \). Hence \( \frac{E_a - V}{V} \) increases.

\( PF = 0.8 \) The basic cause of low power factor is magnetizing current. This current is always lagging, due to the flux relations in a magnetic circuit, and its value depends upon impressed voltage, degree of saturation of the iron, and counter effects such as
armature reaction, magnetic leakage, etc. If the impressed voltage is constant, all the other effects are comparatively small, and the magnetizing current is practically constant. Applied to induction motors and transformers, this means that the magnetizing or quadrature current is constant for any load from zero to full load, but the variation in the load component of current changes the power factor. This is illustrated in the diagram.

Thus considered from the standpoint of quadrature current, the problem is simply to introduce a certain leading reactive current which will counteract the lagging component.

Correction is then effective at all loads and no other adjustment is necessary. It is more convenient in dealing with electrical machinery to use the product of volts x amperes, or better, kilovoltamperes, (KVA), for the unit of rating, and since the voltage is constant, this does not change the calculations.

**ELECTRICAL PRINCIPLES OF POWER FACTOR CORRECTION**

The problem of power factor correction, from the electrical standpoint, is perfectly straightforward
and does not involve complicated mathematics. There are numerous charts, tables and short-cut methods which may be used, but the principles are the same in each, and it will usually be found easier to apply the theory directly to each case. The results are easier to check and more reliable.

Typical problems will be solved showing the application of a synchronous motor in three ways:

1. As a unity power factor motor, i.e., all the input being used for energy.

2. As a synchronous condenser, i.e., all the input being used to supply "wattless leading" current to the line.

3. As a motor-condenser, i.e., part of the input being used for energy and part for furnishing "wattless leading" current to the line.

Assume a load of 600 kw. at 60% power factor.

1. As a unity power factor motor: The apparent load is originally: \( \frac{600}{0.6} = 1000 \text{ kVA} \). The reactive component is:

\[ \sqrt{(1000)^2 - (600)^2} = 800 \text{ kVA}. \]

The addition of 600 kw. at unity p. f. does not increase the reactive component, but increases the power com-
ponent to 1200 kW. The apparent power is then:
\[ \sqrt{(1200)^2 + (800)^2} = 1442 \text{ kVA} \]
The resultant power factor is:
\[ \cos \theta = \frac{1200}{1442} = 83.2\% \]

The result is that a 44.2% increase in kVA capacity doubles the kW. output and raises the power factor from 60% to 83.2%.

2. As a synchronous condenser. It is desired to raise the power factor to 90%.

As before, the apparent load is 1000 kVA; the reactive component is 800 kVA.

The apparent load at 90% p. f. is
\[ \frac{600}{0.9} = 667 \text{ kVA} \text{ and the reactive component is } \sqrt{(667)^2 - (600)^2} = 291 \text{ kVA}. \]
The capacity of the condenser required is then 800 - 291 = 509 kVA.

The generator capacity is reduced 333 kVA by adding a 509 kVA condenser. But if the original 1000 kVA were available for additional load, the total load that could be carried, using this condenser would be:
\[ \sqrt{(1000)^2 - (291)^2} = 957 \text{ kilowatts}. \]
Increasing the kVA capacity 50.9% increases the output 59.6%.
3. As a motor condenser. It is desired to add a load which will require 300 kW. to drive, without exceeding the capacity (1000 KVA) of the apparatus available for this service. As before, we start with an apparent load of 1000 KVA, the reactive component being 800 KVA. Adding 300 kW., the load becomes 900 kW. Since the apparent load is to remain constant, the reactive component must change to:

$$\sqrt{(1000)^2 - (900)^2} = 436 \text{ KVA.}$$

Therefore, the reactive component to be supplied is 800 - 436 = 364 KVA. The synchronous motor must be good for 300 Kw. and also deliver 364 KVA for power factor correction. Its rating should be

$$\sqrt{(300)^2 + (364)^2} = 472 \text{ KVA.}$$

Increasing the apparatus 47.2% increases the output 50%.

Static condenser calculations are the same as for "2" of synchronous motor calculations. In fact, all power factor correction calculations consist in breaking up each load into active and reactive components, adding or subtracting, and finding the resultant.

**ECONOMIC PRINCIPLES OF POWER FACTOR CORRECTION**

The economic principles of power factor correction are much more complex than the electrical
principles. The solution of an engineering problem ultimately depends upon costs, and it is particularly true in this case. Bad power factor is a condition which represents material waste and financial loss, and it is inevitable that economy should be the prime factor. The underlying principle is Kelvin's Law, which states that for the most economical installation, the sum of the annual charges for interest, depreciation, etc., plus the total annual cost of energy losses shall be a minimum. This cannot be applied strictly to power factor correction, because the additional factor of annual saving in power cost complicates the solution. It is better to analyze all charges separately, and plot a curve of the total annual cost. The minimum point of the curve will indicate the result.

The various items of expense connected with any machinery installations may be listed as follows:

A. Initial Expense
   1. Price
   2. Transportation
   3. Auxiliary Equipment
   4. Installation Expense
   5. Special Housing
   6. Other Special Charges.

B. Annual Percentage Charges. (Capital Charges)
   1. Depreciation or Amortization
   2. Interest on Investment
   3. Insurance

C. Annual Power Costs.
C. Annual Power Costs
   1. Losses dependent on Efficiency

D. Annual Fixed Charges
   1. Maintenance and Supplies
   2. Attendance
   3. Housing Rental

Total Annual Cost = B + C + D.

B = A \times R_{cc}, where R_{cc} is the percent capital charge rate.

C = (KWH loss) \times R_p, where R_p is the power rate in $/KW yr.

Annual Power Cost = (KVA \times P. F. \times R_p) + D.

To apply to the specific case of power factor corrective apparatus used by a central station company the following relations can be developed.

Let:  R_p = power cost $/ kw hr.

\[ R_EP = \text{part of power cost chargeable to electric plant} \]

\[ R_{SP} = \text{part of power cost chargeable to steam plant} \]

\[ KW = \text{Maximum load} \]

\[ LF = \text{Annual Load Factor} \]

\[ PF = \text{P. F. of system} \]

\[ CKVA = \text{Corrective KVA} \]

\[ C = \$/KVA \text{ for } CKVA \]

p.f. = P.F. of corrective apparatus
\[ B_c = \text{Capital charge against corrective apparatus} \]

\[ RKVA = \text{Reactive KVA before correction} \]

\[ NRKVA = \text{Net Reactive KVA (RKVA - CKVA)} \]

\[ Hr/yr = \text{Number of hours per year plant or machinery is in operation.} \]

Then: Total Cost of Power = \( KW \times LF \times \left[ R_{SP} + \frac{R_{FP}}{P.F.} \right] \times Hr/yr \)

Power chargeable to the electric plant is assumed to vary inversely as the power factor. If we assume that the operating costs and fixed charges increase directly as the generator capacity is increased, the above assumption will be correct, and the error introduced is so small that it will not affect the results.

\[ \text{(2) Annual Cost of Corrective Apparatus} = \text{Capital Charges + losses} = (CKVA \times B_c \times D \times 100) + (CKVA \times p.f. \times R_p \times Hr/yr). \]

The sum of equations (1) and (2) will be the total annual cost including corrective apparatus. In order to analyze the equation, p.f. must be expressed in terms of other quantities.

\[ RKVA = \frac{KW \sin \theta}{\cos \theta} = KW \tan \theta \quad (\cos \theta = \text{original P.F.}) \]

\[ NRKVA = KW \tan \theta - CKVA \]

Resultant P.F. = \( \cos \left( \tan^{-1} \frac{NRKVA}{KW} \right) \)
Substituting these values and clearing terms, we get:

\[
\text{Total Annual Cost} = KW \times \frac{hr/yr \times LF \times \frac{REP}{cos \tan^{-1} (\tan \theta - \frac{CKVA}{KW})}}{hr/yr \times LF \times \frac{REP}{cos \tan^{-1} (\tan \theta - \frac{CKVA}{KW})}} + CKVA \left[ (Bc \times D \times 100) + (p.f. \times Rp \times hr/yr) \right]
\]

The result is in cents per year. To convert to cents per kilowatt hour it should be multiplied by: \( \frac{1}{KW \times LF \times hr/yr} \)

For a given KW load, the first term is practically constant. The increase in losses in the generating equipment caused by low power factor does not appreciably affect \( R_{sp} \), according to the consensus of opinion among engineers. The reason for this is that by far the greatest part of the loss in a distribution system is made up of the core losses in the transformers, and an increase in copper losses has a relatively slight effect upon the total.

The term \( \cos \tan^{-1} (\tan \theta - \frac{CKVA}{KW}) \) is the power factor of the system with corrective apparatus. Therefore the second term of the general equation varies inversely as the final power factor.

The third term resolves into a constant times the CKVA.
The following data was secured from the Edison Electric Illuminating Company of Boston and other sources, and is outlined and discussed in Appendix A.

\[ R_p = 4.809 \ (\$/KW \ Hr) = \text{power cost} \]

\[ R_{EP} = 1.221 \ (\$/KW \ Hr) = \text{part of power cost chargeable to electric plant} \]

\[ R_{SP} = 3.588 \ (\$/KW \ Hr) = \text{part of power cost chargeable to steam plant} \]

\[ KW = 30,000 = \text{maximum load} \]

\[ LF = 40\% = \text{load factor} \]

\[ PF = 60\% = \text{power factor of system} \]

\[ D = $11.25/KVA = \$/KVA \text{ for CKVA} \]

\[ p.f. = 4.5\% = \text{power factor of corrective apparatus} \]

\[ B_c = 20\% = \text{Capital charges against corrective apparatus} \]

\[ Hr/yr = 8760 \text{ for load} \]

\[ Hr/yr = 7000 \text{ for corrective apparatus} \]

Total Annual Cost = 30,000 x 8760 x 0.40 x 3.588

\[ + 30,000 \times 8760 \times 0.40 \times \frac{1.221}{\cos \tan^{-1} \left( \frac{0.8 - \text{CKVA}}{0.6} \right) / 30000} \]

\[ + \text{CKVA} \ (0.20 \times 11.25 \times 100) + (0.045 \times 4.809 \times 7000) \]

Total Annual Cost (T.A.C.) = 377 x 10^6 + \frac{128 \times 10^6}{\cos \tan^{-1} \left( \frac{40000 - \text{CKVA}}{30000} \right)} + 1740 \times \text{CKVA}
The cost, of course, will depend upon the amount of corrective apparatus (KVA) used, which is determined by the resultant power factor desired. Comparative costs will be figured for resultant power factors of 100%, 90% and 80%, the initial p.f. being 60%.

Reactive kv.-a = VI sin θ

\[ VA = \frac{\text{Power}}{\cos θ} \]

Therefore KVA = Power x \( \frac{\sin θ}{\cos θ} \)

To correct to 100% power factor it is necessary to use corrective kv.-a equal to the reactive kv.-a of the load

Reactive kv.-a of load = 30000 x \( \frac{0.80}{0.60} \) = 40,000 kv.-a

To 100% p.f. Corrective kv.-a = 40,000 kv.-a.

To correct to 90% power factor the corrective kv.-a must be:

\[ \text{Reactive kv.-a before correction} - \text{reactive kv.-a after correction} \]

Reactive kv.-a at 90% power factor = 30000 x \( \frac{0.436}{0.90} \) = 14,550 kv.-a

Reactive kv.-a of load = 40,000 KVA

To 90% p.f. Corrective kv.-a = 40,000 - 14,550 = 25,450 kv.-a
To correct to 80% power factor the corrective kv.-a must be:

Reactive kv.-a at 60% minus reactive kv.-a at 80%
Reactive kv.-a at 80% power factor = \(30000 \times \frac{0.60}{0.80} = 22,500\) kv.-a

Reactive KVA of load = 40,000
To 80% p.f. Corrective kv.-a = 40,000 - 22,500 = 17,500 kv.-a

Total annual cost = \(377 \times 10^6 + \frac{128 \times 10^6}{\cos \tan^{-1} 0} + 1740 \times 40000\)
To 100% p.f. = \(377 \times 10^6 + 128 \times 10^6 + 69.6 \times 10^6 = 574.5 \times 10^6 \) \(\phi/yr.\)

Total Annual Cost = \(377 \times 10^6 + \frac{128 \times 10^6}{\cos \tan^{-1} \frac{14,550}{30,000}} + 1740 \times 25,450\)
To 90% p.f. = \(377 \times 10^6 + 142 \times 10^6 + 44.3 \times 10^6 = 563.3 \times 10^6 \) \(\phi/yr.\)

Total Annual Cost = \(377 \times 10^6 + \frac{128 \times 10^6}{\cos \tan^{-1} \frac{22,500}{30,000}} + 1740 \times 17,500\)
To 80% p.f. = \(377 \times 10^6 + 160 \times 10^6 + 30.5 \times 10^6 = 567.5 \times 10^6 \) \(\phi/yr.\)
From these results it can be seen that the economical power factor to which correction should be made is close to 90%, and if a curve be plotted cost vs. power factor, the minimum point will show this economical power factor. Such curves are plotted on page 23 showing cost in $/kw-h against final power factor, for initial power factors of 90%, 80%, 70% and 60%. The minimum cost in all cases is for a power factor of 90%, showing that for the conditions given it is good economy to correct from any lower power factor to 90%, but no higher. Different conditions of capital charges, number of hours corrective apparatus is used, etc., will affect this value slightly, and the foregoing is to be considered a method of analysis to be followed for a specific problem, and not the solution of a general case.

In the substitution in the general equation on page 16 it is seen that the third term is

$$CKVA \times 225 + 1615 = 1740 \times CKVA$$

The smaller factor, 225, is affected by a charge in initial cost or annual charges, while the larger factor, 1515, is affected by power factor of corrective apparatus, cost of power, and number
of hours corrective apparatus is used. A 100% increase in annual charges or first cost produces only 1% increase in total annual cost and lowers the economical power factor about 2%. The same effect is produced by a 15% increase in cost of power, number of hours used, or power factor of corrective apparatus, the last term representing the power losses in the apparatus.

The two variable terms of the general equation are plotted separately for 60% original power factor on page 24 to show how the component factors vary as the corrected power factor is raised. The total curve is similar to the curve on page 25 and has already been discussed. It can be seen from the component curves that the total will be a minimum when the positive slope of one component equals the negative slope of the other. Since the data for these curves are based upon assumptions and exact costs are never definitely known, changing the assumptions will change the slopes and will have a direct effect upon the results, except that no change will cause the cost to decrease as the power factor is raised above 90%. Then we may say that it is never economical to correct above 90%, but
there are conditions which will make it good economy to stop at a value as low as 80%.

In view of this conclusion, it probably would not be worth while to attempt to correct a system having an inherent power factor of 80% as an erroneous assumption might make the correction an added expense, and at best the amount to be gained would hardly justify a large installation. This value of 80% has been generally recognized by large central station companies, and they have been, on the whole, satisfied if their power factor could be kept up this high.

This in no way affects the upper limit of correction when the initial power factor is 60% or 70%, it should be corrected to the economic value previously shown. The conclusion is that a central station company should correct its power factor, if it is below 80%, up to a value determined by this method of analysis, and which generally will be about 90%, but the company should not correct its power factor if it is already above 80% unless other considerations overrule the economic principles.
### Calculated Data from Which Curves are Plotted

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<th>Corrective KVA</th>
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<th>2nd Term $R_{EP}$</th>
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COST OF POWER AT CENTRAL STATION
SHOWING SAVING EFFECTED BY
USING CORRECTIVE APPARATUS

TOTAL COST OF POWER AND CORRECTIVE APPARATUS IN CENTS PER KWH.

POWER COST AT UNITY POWER FACTOR - NO CORRECTIVE APPARATUS

FINAL POWER FACTOR, %

50 60 70 80 90 100

5.8
5.6
5.4
5.2
5.0
4.8

ORIGINAL POWER FACTOR, %

60
70
80
90
VARIABLE FACTORS OF THE GENERAL EQUATION SHOWING THEIR EFFECT UPON THE SHAPE OF THE TOTAL COST CURVE.
Conditions may exist, and frequently do, which make it desirable for the consumer to correct his own power factor. Central station companies are fast realizing the tremendous disadvantages of low power factor on their lines, and a great many now have "power factor clauses" in their rates which increase the charge for power used at low power factor. This is the most effective way of getting results, from the standpoint of the company, and is entirely equitable because it does not make the company pay for a condition brought about, in nine cases out of ten, by poor engineering on the part of the consumer.

The difficult question is how to adjust the rate, or, what the power factor clause should be. Should a charge be made which discriminates against consumers whose power factor is low, or should a charge be made for all reactive power used? Should a certain amount of reactive power be allowed free? Should exception be made to power users whose power factor is normally above some fixed value, say 80%, or should the charge be applied alike to all power consumers? If a charge is made for reactive power should it be fixed or should it
vary with the power factor of the load?

There is such a wide divergence of opinion on these questions that the general trend cannot easily be estimated, and each company has its own particular method of making the charge.

In general there are three ways in which the charge for low power factor may be made:

1. Charge at reduced rate for reactive KVA added to KW charge.
   A. Fixed rate for all power factors.
   B. Rate increasing as power factor decreases.
2. Charge on the basis of KVA instead of KW.
3. Charge with penalty and bonus for power factors less or greater than some fixed value.

There is a very small tendency toward the last two methods, due partly to difficulty in metering. The first method is greatly favored, and most of the progress, in the United States and abroad, has been made with a reduced charge for reactive KVA added to the KW charge, the advantages being extreme simplicity in determining reactive KVA and applying the charge. Any of the methods is logical and can be analyzed without much difficulty, and all should arrive at similar results in the end, for, from an engineering analysis, the costs must be analyzed and the excess cost of re-
active power charged to the consumer. It should not make any difference how we arrive at the cost, or what terms it is expressed in. If all the methods of reasoning are on the same basis they must lead to the same results. The first method, 1-A, above, will therefore be used as a basis for investigation, since it seems to be the most reasonable and direct.

The method of determining the amount of the fixed charge will be applied to the data previously used in Appendix A. The following is based upon 1 KW at a power rate of 1¢ per KWH, so the results are in reality percentages:

<table>
<thead>
<tr>
<th>POWER FACTOR</th>
<th>(2) KVA</th>
<th>(3) KVA</th>
<th>(4) d/KWH</th>
<th>(5) d/KVAH</th>
<th>(6) TOTAL COST</th>
<th>(7) d/KVA</th>
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<td>0.02</td>
<td>49.99</td>
<td>50.00</td>
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<td>0.658</td>
<td>1.254</td>
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The first three columns are self-explanatory.

R_sp, column 4, represents the percent of total cost of power chargeable to the steam equipment. REP, column 5,
represents the percent of total cost chargeable to electric equipment multiplied by the total kVA, column 3. \( R_P \), column 6, represents the sum of \( R_{SP} + R_{EP} \) or the total cost of power generation at various power factors. Column 7 is the increase in cost of production due to low power factor divided by the reactive kVA, for example, at 10% p.f.

\[
\frac{3.286 - 1.0}{9.95} = 0.230 \frac{\$}{\text{RKVA}}
\]

The results are plotted on page 28, and it is evident that the charge should vary with the power factor, increasing to a limiting value of 25.4% of the kW charge at zero power factor. It will be shown that this value is the percent cost of power chargeable to the electric equipment.

\[
\text{RKVA Rate} = \frac{(\text{total cost at low p.f.}) - (\text{total cost at unity p.f.})}{\text{RKVA}}
\]

but Total Cost = \( R_{SP} + R_{EP}(\text{kVA}) \)

and \( (\text{kVA} = \text{kW sec } \theta) \) where \( \theta \) is the power factor angle, \( (\text{RKVA} = \text{kW tan } \theta) \)

therefore:

\[
\text{Rate} = \frac{R_{SP} + R_{EP}(\text{kW sec } \theta)}{\text{KW tan } \theta} - \frac{(R_{SP} + R_{EP})}{\text{KW tan } \theta}
\]

\[
\text{RKVA Rate} = R_{EP} \left( 1 + \frac{\text{kW sec } \theta}{\text{KW tan } \theta} \right)
\]
Now as the power factor approaches zero, \( \tan \theta \) and \( \sec \theta \) approach infinity, and the ratio within the parenthesis approaches unity, therefore, the rate approaches \( R_{FP} \).

This means that the \( RKVA \) charge, in percent of the \( kW \) charge, should never exceed the percent cost of power chargeable to the electric equipment. This is perfectly reasonable because we have assumed that the reactive power represents no actual energy output, hence does not require any excess steam.

If the cost of one kilowatt of active power is divided 75% to steam plant and 25% to electric plant, then the cost of one kilovoltampere of reactive power which should all be charged to electric plant, because it does not require any steam to produce it, will be 25% of the total cost of one kilowatt.

The average of all the values from 30% to 100% power factor is 0.110, but use of this charge would not cover the cost below 65% power factor, and the income would be very small from loads having higher power factors. It would probably be better to average the rates between 30% and 80%, as this covers the usual range of power factors which cause
trouble. The result is then 0.147, or say 15\%, and the analysis will be made on this basis.

The symbols used below are the same as in the previous case with the following additions:

\[ k = \% \text{ KW charge for reactive KVA} \]

\[ m = \% \text{ CKVA actually used with respect to the amount necessary to correct to 100\% power factor.} \]

Applying the economic principles as before:

Total Annual Cost of Power =

\[ (\text{KW} \times \text{Hr/yr} \times \text{Rp}) + (\text{RKVA} \times \text{Hr/yr} \times k\text{Rp}) \]

\[ \text{RKVA} = \text{KW} \tan \theta, \text{ where } \theta = \cos^{-1} \text{PF}. \]

When:

Total Annual Cost of Power =

\[ (\text{KW} \times \text{Hr/yr} \times \text{Rp}) (1 + k \tan \theta) \]

If corrective apparatus is added, RKVA becomes \[ \text{NRKVA} = \text{RKVA} - \text{CKVA} \] and:

Total Annual Cost of Power =

\[ (\text{KW} \times \text{Hr/yr} \times \text{Rp}) \left[ 1 + k(\tan \theta - \frac{\text{CKVA}}{\text{KW}}) \right] \]

Also:

Total Annual Cost of CKVA = CKVA \[ (B_c \times D \times 100) + \]

\[ (\text{p.f.} \times \text{Rp} \times \text{Hr/yr}) \]

The sum of these two is the total annual cost. The equations can be simplified and the
solution facilitated by reducing to units and expressing CKVA in terms of the total CKVA necessary to correct to 100% power factor.

Thus let: \( \text{CKVA} = m \times \text{KW tan } \theta \)

where \( m = \text{percent of CKVA used with respect to amount necessary to correct to unity p.f.} \)

Substituting and collecting terms:

\[
\text{Total Annual Cost} = \frac{Hr/yr \times R_p}{m} \left[ 1 + k(1-m) + (p.f. \times m) \tan \theta \right]
\]

\[
m \times B_c \times D \times 100 \tan \theta
\]

which may be written:

\[
\text{Total Annual Cost} = (1 + k) \frac{Hr/yr \times R_p}{m} + \left[ (B_c \times D \times 100 \tan \theta) - (k \times \text{p.f.}) \tan \theta \right] m
\]

which is of the form:

\[
\text{Total Annual Cost} = Q + (R-S)m, \text{ where } Q, R \text{ & } S \text{ are constants.}
\]

This means that the total annual cost will either increase or decrease continuously, as corrective apparatus is added, depending upon whether \( R \) is greater or less than \( S \). At first sight this does not seem reasonable as it implies that correction should be made to 100% p.f., which is not in accord with previous results or with engineering opinion in general. The relation seems to be correct, however, if a fixed charge is made for reactive kVA,
and if the unit cost of CKVA remains constant. Even
if the unit cost of CKVA changes, it will decrease
as the CKVA increases, tending to accentuate the
above facts. Curves and data follow which illustrate
the relations.

Assumed values:

\[ \text{Hr/yr} = 3500 \quad \text{KW} = 1500 \]

\[ R_P = \$0.05 \text{ per KWH, which is the Boston Edison} \]
\[ \text{rate for 1500 KW.} \]

\[ k = 0.15 \]

\[ \text{p.f.} = 0.045 \quad m \text{ and } \theta \text{ are variables} \]

\[ B_0 = 0.20 \]

\[ D = \$11.25 \text{ per KVA as used before} \]

Total Annual Cost per KW =

\[ 3500 \times 0.05 \times 1 + 0.15(1-m) + 0.045 \times m \tan \theta + 2.25 \times m \tan \theta \]

(Data are also computed for \( k \) variable as shown on page 27.)
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<th>P.F. tan ( \theta )</th>
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<th>k variable</th>
<th>Resultant P.F.</th>
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COST OF POWER TO CONSUMERS
SHOWING SAVING EFFECTED BY
USING CORRECTIVE APPARATUS

POWER RATE 6 CENTS PER KWH

REACTIVE KVAR RATES
BLUE-0.75 CENTS PER KVAR
BLACK-VARIABLE RATE

TOTAL COST OF POWER AND CORRECTIVE APPARATUS IN DOLLARS PER KW PER YEAR

FINAL POWER FACTOR %

ORIGINAL POWER FACTOR %
Using a constant charge for reactive KVA, the consumer pays a constant amount for a certain KW load, plus a value which increases directly with the reactive KVA. Thus the total power cost increases directly and linearly with the reactive KVA. The addition of corrective apparatus decreases the net reactive KVA, and so decreases the value of the excess charge based upon this item, but at the same time the corrective apparatus adds to the cost of operation of the plant. The corrective apparatus costs have been assumed to vary directly with the corrective KVA, so the operating costs increase linearly with the amount of correction. The net effect will be linear and its slope will depend upon the relation between the power saving and the increase in operating expense. The result is that correction should be made all the way to unity power factor or not at all.

Plotted against final power factor instead of CKVA added, the cost curves will not be linear, but will curve downward as unity power factor is approached, as shown by the blue curves on page 36. This further emphasizes the fact that correction, if made at all, should be made to 100% power factor.
Using the constant 15% charge, at 80% p.f. the gain in correcting to unity is $12.00 per kW per year or about 6% of the original power cost. Below this the percent saving increases, at 30% p.f. the gain being over 20%, although at this value the reactive KVA costs the company more than at 80% as was shown above.

The black curves on page 36 show that using a variable charge there is very little gain for the consumer in correcting his power factor if it is already above 70%, while the blue curves show a substantial saving at all power factors. Now the average power factor of a feeder is determined by the ratio of kilowatts to total KVA, and it is not fair that one consumer should pay a lower rate for reactive KVA than another simply because he uses more kilowatts. The cost of production of reactive KVA is determined by the average power factor of the feeder. One reactive KVA is the same as another, no matter what part of the load causes it, and if the power factor is to be improved, it is reactive KVA that determines the size of the corrective apparatus, not any one consumer's power factor. Therefore, the constant
charge, if the value is properly determined, is more equitable to all: consumers and company alike, and there is no more reason why a certain number of reactive kVAs should be allowed free than a certain number of kilowatts. This makes the customer feel that he is getting something for nothing, when he is probably paying a higher power rate to make up for it. Each and every reactive kVA represents the same amount of CKVA, hence the same cost, for its neutralization, and all should be charged alike.

These results and the foregoing discussion show that the 15% charge for reactive kVA has the following points in its favor:

a. It is fixed charge and is easy to apply.

b. Its application rests upon metering the reactive kVA, which can be done easily and accurately.

c. It is based upon actual costs, and as long as the net power factor of the system is above 50%, it will compensate the company for the added expense of production.

d. It does not overcharge the consumer.
e. It makes it economical for the consumer to install and operate corrective apparatus no matter what his power factor may be, and correct to unity power factor, which is the result the central station companies desire.

The object of a power factor rate clause is to better power factor conditions, not to produce additional revenue. Successful rate making embodies not only the cost of production and the value of the service, but also considerations of public policy and other elements which are not subject to exact measurement and weight. It is necessary to show the consumer why his rate must be adjusted, and in this connection the word "penalty" should not be used, even by central station men themselves, because of its psychological effect upon the consumer. Failure of a customer to meet a given requirement may call for a correction or adjustment, but never a penalty.

This digression on the subject of rates has been necessary to show that correction of power factor cannot be made intelligently unless all details of the rates are known, and also to show that the problem of power factor correction can be solved
for the central station company by using a charge which is based upon actual costs. In the past a power factor clause has been determined by following what another company has done, or by averaging the rates used by several companies. This is mere guesswork, and one guess is as bad as another. What the situation needs is a sound engineering analysis in every case. If the power companies will deal fairly and squarely with the consumers in adjusting the rates, the matter of correction will be left up to the consumer, and it will be so clearly to his advantage to improve his power factor, if it is low, that it will not even require moral suasion on the part of the company.

CONSIDERATION OF SOME ASSUMED CASES

Assume a constant induction motor load of 500 kW operating at an average power factor of 60% for 3500 hours per year. Other assumed data will be the same as that used for computing the curves on page 36, with the constant 15% charge for reactive KVA.

The addition of a 500 KVA condenser would correct the power factor to 95.2% and reduce the annual cost as follows:
Cost without correction = 500 x $210 = $105,000
Cost with correction = 522.5 x $194 = 101,300
Saving per year = $ 3,700

At $11.25/kVA the condenser would cost $5725.

There are three ways of looking at the investment in corrective apparatus. It is undoubtedly desirable if it saves money, but it must save enough so that the money could not be invested to better advantage elsewhere. A particular appeal to customers is found in showing how long it will take the apparatus to pay for itself. The three viewpoints are, therefore:

1. Saving per year in total cost.
2. Interest upon the investment.
3. Time in which apparatus will pay for itself in saving.

As the capital charge of 20% used in computing the curves was made large purposely to allow for earnings in other investments, any saving at all will indicate that the corrective apparatus is paying good dividends; but this will be disregarded in computing the saving.

1. Saving per year is above example: $3700
2. Interest on investment of $5725: 65.8%
3. Time to pay for itself: 18.25 months
If only 200 KVA were added, the corrected power factor would be 73.7%, annual cost $103,200, Condenser cost $2250.

1. Saving per year: $1800
2. Interest on investment: 80%
3. Time to pay for itself: 15 months.

The savings indicated are surprisingly large, yet no error can be found in the theory or processes used, and the assumptions made are all conservative. Much more unfavorable conditions would still yield favorable results; and, as mentioned above, any saving at all represents a profitable investment, due to the large capital charge used.

The general conclusion from the economic standpoint is that the consumer should correct his own power factor. This must be brought about by the central station company through the use of an added charge for reactive power. However, small companies, or companies which have only one or two low power factor loads will probably find it easier to install corrective apparatus themselves than go to the trouble of having a power factor clause approved by the Public Service Commission. There are other questions here which prevent a fixed set of rules being laid down,
as a matter of fact the officials of the company will
use their best judgment anyhow, so the foregoing
should be looked upon as an outline of procedure,
not a statement of rigid policy.
CORRECTIVE APPARATUS & EQUIPMENT

Power factor corrective apparatus is limited to those devices which will supply comparatively large reactive currents. Under this classification are included synchronous motors, condensers, converters, static condensers and phase advancers. Each of the above devices possesses certain inherent advantages for particular applications which can, perhaps, be better understood after a general survey of their individual characteristics.

Synchronous motors and condensers have the ability to draw either leading or lagging currents depending on the manipulation of their direct current fields. This can best be seen by reference to the vector diagrams. Figure No. 1 shows the condition
for normal field excitation which is that field strength giving unity power factor. If the excitation is increased $F$ will be lengthened. This in turn will increase $E$ (the generated E.M.F.) and by so doing will also tend to increase $V$. The terminal voltage, $V$, is in synchronism with the voltage of the supply system and hence is constant in magnitude. The increase in the difference between $E$ and $V$ means a greater current, and since $I = \frac{V - E_a}{Z}$ I must increase. With these various changes of voltage and current there is no accompanying change of power as the external load remains the same. If the power and voltage are constant and the current increases there is only one thing which can change, and that is the power factor. Hence it can be seen that by increasing the field the current vector will swing downward in a leading position, and by decreasing the field, the current vector will swing upward in a lagging position its tip at all times touching on a line perpendicular to the voltage vector ($-V$), With a leading current the effect of armature reaction on the field is demagnetizing and tends to decrease the leading current, but it has a stabilizing effect which cuts down hunting; with a lagging
current the reverse effect is true. The desirable electrical features of a synchronous motor or condenser for power factor correction are:

1. A large field worked at low saturation with a wide range of variation.
2. High current carrying capacity and
3. Large armature reaction.

The first two are obtained by putting in more iron and copper and the third by cutting down the length of the air gap.

When a synchronous motor is used to correct power factor it can also be used to carry a load. To perform these two functions it must not only possess the aforementioned electrical features, but must also be built rugged enough to satisfy load requirements. Starting under load is the chief difficulty which limits the use of synchronous motors. The best way to equip a motor used to carry a load and for power factor correction is to provide a heavy damping winding. With this the motor can be started as an induction motor with practically any reasonable load. The damping winding is also useful in preventing hunting. High speed is also desirable as this cuts down the amount of material
and hence the cost of the motor. Of course the matter of speed is largely dictated by the speed of the load to be carried, and the motor has to be chosen accordingly.

The maximum usefulness of a motor occurs when the power factor is 70.7% leading. With this condition the load carried will be 70.7% of the KVA rating and the reactive KVA for corrective purposes will also be 70.7% of the KVA rating. The efficiency of the motor will be somewhat lower for this condition than for unity power factor, but this effect is practically negligible compared to the saving in power produced by the reactive KVA. However, it is undesirable to operate at power factors below 50% because the low efficiency of the motor brings down the net economy.

Synchronous condensers is the name applied to synchronous motors which are designed to run at no load for purposes of power factor correction and voltage regulation. The difference between the two types lies in the size of the field structure, and the mechanical strength. Condensers are designed with very large fields in order to give a
wide range of variation of leading and lagging current. As they do not carry load, they are much less rugged than synchronous motors of the same rating. Polyphase condensers are usually equipped with amortisseur windings and are started as induction motors by means of compensators. Single phase condensers are not self-starting, and always have a small induction motor mounted on the shaft for bringing them up to synchronous speed.

Synchronous converters operate in much the same way as synchronous motors in regard to their ability to take leading or lagging currents, but they possess certain characteristics which confine their field of application within narrow limits. In the first place a converter drawing a large reactive current will overheat at the points where the different phases are tapped into the armature winding. Another undesirable feature is the large decrease in output due to low power factor. For this latter reason a converter should never operate at a power factor less than 0.9.

The use of static condensers has recently been practically stopped by fire insurance underwriters because of the inflammability of the paper
insulation. As a result of this the General Electric Co., who in the past have been the sole makers, have suspended manufacture. However, the field of application of static condensers is so wide, and the demand so insistent that it will undoubtedly be only a short time before a fireproof substitute for paper will be found and manufacture resumed. In view of this condition of affairs, it is thought best to include a complete analysis of the use of static condensers as applied to power factor correction.

The electrical characteristics of static condensers are somewhat different from those of synchronous condensers. The former will take a leading current at a fixed power factor, and cannot be adjusted to suit more than one condition. However, their efficiency is very high at all loads, and they operate at lower power factors than is possible with synchronous apparatus. Up to the present time they have not been made for very large capacities due to their bulk. The volume varies inversely with the voltage so that for large capacity and low voltage the size is prohibitive. At present the largest condensers manufactured have a rating of 1000 KVA. In order
to avoid excessive size for large units operating at low voltage, it is customary to provide auto transformers along with the condenser units. The transformers step the voltage up to 1200 volts which reduces the amount of active material and cuts down the cost. For potentials higher than 2300 volts step down transformers are used to eliminate the danger of puncturing or breaking down the insulation. Series reactors are also used to dampen out higher harmonics of the voltage waves.

Static condensers require practically no attention outside of occasional inspections. The absence of rotating parts makes it possible to place a condenser almost anywhere without special foundations or reinforcements. The yearly maintenance cost is practically negligible as there are no parts subject to wear and tear.

PHASE ADVANCERS

There are several devices available for advancing the phase of induction motors, foremost of which, in development and application, is a modification of the Scherbius system of control as manufactured by the General Electric Co. The prime object is to control the speed of a large induction motor, but the characteristics of the system permit
improvement of power factor without modification of auxiliary apparatus and without sacrifice of efficiency.

The rotor of the induction motor is supplied with alternating voltage at an adjustable frequency which can be controlled in phase and in magnitude. Varying the magnitude of the voltage varies the speed, while varying the phase adjusts the power factor of the motor. To secure an alternating voltage which may be controlled in this way, an a.c. commutator motor is required, neutralized for armature reaction. Other apparatus is also necessary such as exciters, transformers, rheostats and a control panel, making the cost of the installation rather excessive.

The correction of power factor is so slight that the device would never be used as corrective apparatus per se, therefore it is considered unimportant and will not be discussed in detail.
ADVANTAGES AND DISADVANTAGES OF THE DIFFERENT TYPES OF SYNCHRONOUS APPARATUS

I. Synchronous Motor and Synchronous Condenser

Advantages:

1. The most economical means of securing power factor correction, since the motor can also be used to carry a load.

2. Especially suitable for mills, etc., that purchase power from public utility companies.

Disadvantages:

1. Sufficient field to hold the motor in synchronism under load.

2. Costly if low speed.

3. Requires attention.

II. Synchronous Condenser

Advantages

1. The best and most flexible apparatus for power factor correction and voltage regulation.

2. Can be built for high speed and hence lower cost per KVA.

3. Especially adapted for use at the end of long transmission lines.
Disadvantages

1. More expensive than the synchronous motor of same rating.
2. Special foundations are required, because of high speed.
3. Requires some attention.

III. Synchronous Converters

Advantages

1. High efficiency for power factors as low as 90%.

Disadvantages

1. Very narrow range of operation for purposes of power factor correction.
2. Requires an attendant.
3. Subject to overheating and low efficiency with low power factors.

IV. Static Condensers

Advantages

1. Very cheap in small sizes.
2. No special location or foundations are required.
3. High efficiency at all loads.
4. Requires practically no attention.
5. No moving parts.

6. Especially suited to installations of small capacity with fixed inductive load.

Disadvantages

1. Not flexible

2. No lagging correction is obtainable.

3. The cost is high in large sizes as compared with a synchronous condenser.

4. Very inflammable.

V. Phase Advancers.

Advantages

1. Economical for large machines.

2. Speed variation above and below synchronism.

Disadvantages

1. Can only be used with wound rotor induction motors.

2. The corrective effect applies to only one condition of load on the motor.

3. It is unsuitable for motors that are frequently started and stopped, or that are reversible.
INDUSTRIAL APPLICATION OF SYNCHRONOUS MOTORS
AS A MEANS OF CORRECTING POWER FACTOR.

The increasing tendency of power companies to penalize consumers for low power factor has stimulated keen interest in the use of synchronous motors for industrial purposes. This movement is becoming so universal that even though it does not fall exactly within the technical sphere of power factor correction, it has a distinct relation to the subject and hence will be treated here.

In the days when low power factor was not such a burning question, textile mills, factories, machine shops, and other large users of power, naturally turned to the induction motor as the simplest, strongest and most flexible means of driving machinery. The result was a lavish use of induction motors without any particular thought or care as to the economies which might be effected by proper selection. The main idea was to furnish adequate power and even double capacity if necessary, in order to eliminate the possibility of breakdowns and delays. The effect of this attitude is at once apparent; mills and factories operated motors at about one-half of their rated capacity and hence
brought down the power factor to an excessively low value.

With the ever-increasing demand for electric power this slip-shod method of applying motors cannot be tolerated because the expense is prohibitive. The first step in the right direction has been to properly select the motor for the load it is to carry, but even this does not suit the demands of the present. It is now necessary to cut down reactive currents to give power factors well within the limits of 80%.

The most satisfactory method for obtaining a high power factor in an industrial plant is by substituting synchronous for induction motors wherever it is practicable.

The use of synchronous motors has been restricted in the past for several reasons, namely: their low starting torque, the cost of making them in small sizes, their constant speed characteristic, and their inability to carry heavy overloads. Most of these disadvantages have been overcome so that they are supplanting induction motors entirely in several fields of application.
A comparison of the characteristics of synchronous and induction motors will perhaps show more clearly than anything else, the advantages and limitations of each in the industrial field.

**CAPACITY AND SIZE:** The upper limit of capacity for both types of motors has not been reached. At present the largest machines are in the neighborhood of 30,000 KVA. However, these large sizes come only within specialized fields, the induction motor for roll mill drives and ship propulsion, and the synchronous motor for voltage regulation and power factor correction of long, high voltage transmission lines.

In small sizes the induction motor has had the advantage of simplicity, ruggedness, large starting torque, low cost and the lack of direct current excitation. It is now possible to duplicate many of these desirable qualities of the induction motor in the synchronous motor with the added feature of unity power factor at all loads. At the present time the smallest standard synchronous motors have a capacity of 30 h.p. while induction motors are made in sizes as small as 1/16 h.p.
TORQUE: The starting torque of induction motors depends on the resistance of the rotor circuit. With the squirrel cage type this has a fixed value depending on the design of the machine, but with the wound rotor type, the resistance can be varied to accommodate a wide range of loads. The starting torque of a synchronous motor depends on the resistance of its damper winding. For maximum starting torque the resistance should be equal to the reactance at the impressed frequency. Since a synchronous motor starts as an induction motor, to pull into step easily, it should have a damping winding of very low resistance, but in order to start readily under load, the resistance of its damper should be high. The conditions for good starting torque are incompatible with pulling into step readily, and a compromise is, therefore, necessary. Standard synchronous motors are designed with 25% full load torque at starting, and 15% pull-in torque.

The breakdown torque of induction motors depends on the impressed voltage and the rotor and stator resistances and leakage reactances. As
ordinarily designed the breakdown torque has limits between 175% and 250% full load torque. The breakdown torque of synchronous motors depends on the terminal voltage and the field excitation. With different motors of normal design the "pull-out" torque varies between 150% and 300% full load torque, according to the particular class of service to which the motor is applied.

**SPEED** Squirrel cage induction motors are essentially constant speed machines although the speed drops off about 10% from no load to full load. The wound rotor type is suitable for variable speed. Speed control is accomplished by inserting resistance in the rotor circuit. As the name implies, synchronous motors can be operated at only one speed.

**Power Factor:** The older types of induction motors operated at power factors ranging from 60% at 1/2 full load to 80% at full load. The latest machines have power factors of 80% at 1/2 full load and 90% at full load. This increase is due to shorter air gaps which decrease the leakage reactance and hence the magnetizing current.

Synchronous motors, as previously explained, have a variable power factor which is controlled by the excitation.
EFFICIENCY: The efficiency of synchronous motors is generally higher than that of induction motors, even when operating at leading power factors as low as 0.8. Particularly is this true of the more modern synchronous motors designed for unity power factor operation, whose efficiency is practically constant from half load to full load, and is only slightly lower at one quarter load.

EFFECT OF VARIABLE VOLTAGE: Voltage higher than the rated value when impressed on an induction motor produces an abnormally large magnetizing current. This current is highly reactive and causes low power factor, overheating and low efficiency. Low voltage causes a large reduction in torque since the latter varies as the square of the voltage.

With a synchronous motor the effect of voltage variation is not nearly so detrimental to good operation. High voltage changes only the power factor and its effect may be counteracted by manipulating the excitation. Low voltage causes a slight reduction in torque and a change in the power factor which may be corrected.

DEPENDABILITY: The induction motor has the reputation of being able to stand heavy overloads and the
hardest kind of service with very little care. It is simple enough so that it is practically foolproof, which is a strong argument for its use in places where unskilled labor is employed.

On the other hand, the synchronous motor requires more care because of its d.c. excitation and starting characteristics. It is not as compact and rugged as the induction motor, but with continuous improvement in design, there is no reason why it should not equal the induction motor in strength and reliability.

COSTS: The manufacture of induction motors has been on such a large and varied scale that costs have been cut to a minimum by both large quantity production and standardization. The starting and control equipment, which is comparatively simple, is also made on the same basis, so that the net cost of an induction motor installation is small.

The synchronous motor has not been fully appreciated, and hence it has never been manufactured on an extensive scale. This feature together with the necessity for direct current excitation brings the cost considerably above that of the induction
COMPARISON OF NET RETAIL
PRICES OF AVERAGE SYNCHRONOUS AND IN-
DUCTION MOTORS. PRICE INCLUDES START-
ING COMPENSATOR AND EXCITER BUT NO
PANEL.

THIS COMPARISON DOES NOT
APPLY TO LOW SPEED MACHINES

NET RETAIL PRICE IN DOLLARS

SYNCHRONOUS

INDUCTION (SQUIRREL CASE)

HORSE POWER + SYNCHRONOUS SPEED.
motor. The control equipment is also a trifle more elaborate and this increases the net cost. The curves, page 64, give a very clear idea of the difference in cost between synchronous and induction motor installations. Whether or not this difference in first cost can be made up by the operating economies of the synchronous motor, is a problem which can only be solved by reference to a particular application. However, if the demand for, and production of, synchronous motors reach a magnitude comparable with the present production of induction motors, it is quite possible that prices might thereby be reduced until the two types are more nearly on an equal footing.

In order to obtain an idea of the operating economy of the synchronous motor as contrasted with that of the induction motor, consider two machines of the same rating running 3500 hours per year at 90% full load. Let the power rate be five cents per kw. hour and the additional charge for reactive power be 0.75 cents per reactive KVA hour. Assume a capital charge rate of 15%. The following data are for two Westinghouse machines of the latest standard design at current prices. (May 1921).
<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Induction Motor</th>
<th>Synchronous Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>50 h.p.</td>
<td>50 h.p.</td>
</tr>
<tr>
<td>Voltage</td>
<td>440</td>
<td>440</td>
</tr>
<tr>
<td>Speed</td>
<td>1200 r.p.m.</td>
<td>1200 r.p.m.</td>
</tr>
<tr>
<td>Power factor at 90%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>full load</td>
<td>92%</td>
<td>100%</td>
</tr>
<tr>
<td>Efficiency at 90%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>full load</td>
<td>90%</td>
<td>92%</td>
</tr>
<tr>
<td>Initial cost</td>
<td>$569.0</td>
<td>$929.0 (including exciter)</td>
</tr>
</tbody>
</table>

**OPERATING EXPENSES:**

**Induction motor**

**Power costs:**

\[
KW = \frac{50 \times 0.746}{0.9} \times 3500 \times 0.05 = \$7260.0
\]

\[
RKVA = \frac{50 \times 0.746 \times 0.392}{0.9 \times 0.92} \times 3500 \times 0.0075 = 464.0
\]

Capital charge = $568 \times 0.15 = $85.50

**Total**

$7809.50

**Synchronous motor**

**Power costs**

\[
KW = \frac{50 \times 0.746}{0.92} \times 3500 \times 0.65 = 7100.0
\]

Capital charge = $929 \times 0.15 = $139.20

**Total**

$7239.20

$7809.50

$7239.20

**Difference in favor of the synchronous motor**

$570.10
This shows a saving of 7.3% for the year and is fair enough representative of the best conditions under which an induction motor would be operated. If the load had been assumed to be 80% full load or a combination of fractional loads for the year, the power factor of the induction motor would decrease and the cost of reactive power would increase. The synchronous motor would still operate at unity power factor and hence the saving would be even greater.

With machines of larger capacity the efficiencies of both will be higher, but even in the largest sizes the synchronous machine will have a margin of from 1% to 2%. The induction motor power factor does not increase appreciably with an increase in capacity so that the net effect of efficiency and power factor is to slightly reduce the difference in operating costs between the two types.

In small machines the difference in efficiency increases leaving the advantage with the synchronous motor.

The speed considered in the example given (1200 r.p.m.) is typical and shows the induction motor to the best advantage. For lower speeds
the induction motor power factor falls off considerably, while the synchronous motor efficiency increases. This will show a large saving in favor of the synchronous machine.

The reactive power rate assumed for this comparison is 15% of the power rate. This figure was reached after an analytical study of power costs as shown in the chapter on "Economics of Power Factor Correction." It is lower than the prevailing rates of the majority of central station companies using such a charge. If a larger value were used the induction motor would suffer even more by the comparison as the cost of reactive power would be increased.

Using a variable reactive power rate, the extra cost for lagging power factor would be comparatively small at 90% full load, and the induction motor would be shown to better advantage. At lower power factors the rate would be larger and the cost would increase.

In view of the foregoing comparison it can be seen that the synchronous motor is more economical than the induction motor under all con-
ditions of load, for any capacity, large or small, and with any power rate, which includes a charge for reactive power.

SUMMARY OF THE ADVANTAGES AND DISADVANTAGES OF SYNCHRONOUS AND INDUCTION MOTORS.

I. Induction motors

A. Advantages

1. Simplicity, ruggedness and adaptability.
2. Good starting torque.
3. High breakdown torque
4. Low cost.

B. Disadvantages

1. Low power factor at small loads
2. Affected by variations in voltage

II. Synchronous motors

A. Advantages

1. Adjustable power factor at all loads
2. Adaptability to slow speed drives
3. High efficiency
4. Economy

B. Disadvantages

1. Constant speed
2. D.C. excitation
3. Low starting torque
Note: the torque characteristics of the induction motor can be duplicated in the synchronous motor at small sacrifice to the other operating characteristics.

APPLICATIONS OF SYNCHRONOUS MOTORS

Synchronous motors have been successfully applied for driving the following:

1. Motor generator sets
2. Frequency converters
3. Air compressors
4. Ammonia compressors
5. Jordans
6. Pulp grinders
7. Stone crushers
8. Centrifugal pumps
9. Plunger pumps
10. Screw pumps
11. Blowers
12. Fans
13. Conveyors
14. Tube mills
15. Flour mills
16. Cement mills
17. Line shafting
18. Steel and copper rolls

The above list applies to fields where the synchronous motor is rapidly supplanting the induction motor. There are countless other instances of a specialized character where the synchronous motor is applicable which will not be mentioned here.
FUTURE DEVELOPMENTS IN USE OF SYNCHRONOUS MOTORS

As to what the future holds forth for the synchronous motor one field of vast proportions has already been opened up. This is electric ship propulsion.

Low power factor is a serious consideration in this field. In large installation, that is, 3000, h.p. and over, it is unsatisfactory to use direct current because of lower transmission efficiency, higher cost of apparatus and lack of flexibility in operation. All the development therefore, has been made with alternating current of frequencies from 33 to 60 cycles and voltages from 1200 to 5000 volts. Induction motors have been used because of their constant speed characteristics, ease of control and reversal, and general all round flexibility. Recently, however, the synchronous motor has been applied to this service with very satisfactory results.

Cargo ship service requires low propeller speeds, 100 r.p.m. being the usual value. The turbo-alternator may run at high speed, but at 50 cycles this requires a 30 pole motor, and an induction motor with a large number of poles has an
inherently low power factor. A typical installation of this type has a power factor of 81\% at full speed, and the power factor will be somewhat less at normal cruising speed. Obviously, the general economic equations cannot be applied to this case, but there are other factors which make synchronous motors so desirable that the question of power factor is of somewhat secondary importance.

A typical installation using a synchronous motor is the S.S. "Cuba," the equipment being designed and installed by the General Electric Co. Operating at unity power factor, the motor and generator can be made lighter, and it has been found that even with the added exciter capacity the cost of the complete installation is from 1 1/4\% to 2\% less than for the induction motor drive. An equally important item is the ease with which repairs can be made on the motor; a salient pole can be removed and any part of the stator winding becomes easily accessible, while in repairing an induction motor the entire rotor must be taken out, which is a very difficult operation on board ship. A third advantage is the large air gap possible in synchronous
apparatus. The small air gap necessary in an induction motor limits the clearance and restricts the bearing design. An increase in the air gap would make a very high leakage reactance and lower the power factor, which would decrease the efficiency.

Exciter capacity is kept within reasonable limits by using a three wire generator or a 230v. machine with balancer, the generator and motor fields being on different branches. During reversal, large currents are never required in both fields at the same time so the load is shifted from one side to the other of the exciter.

It might be of interest to discuss a particular installation, showing the flexibility of the synchronous motor.

The synchronous motor used in the "Cuba" has an amortisseur winding and is started as an induction motor without field at reduced generator voltage. The reversal when running at normal speed ahead is accomplished as follows: Both motor and generator field circuits are opened and the motor connections reversed when no current is flowing. Then normal field is thrown on the motor which,
being driven by the propeller, and electrically connected to the turbo-alternator, acts as a generator and stops almost immediately. The motor field is then opened, generator field closed through a resistance, and the motor brought up to speed in the reverse direction as an induction motor at reduced voltage. Field is then thrown on the motor and it pulls into step. These operations are performed very satisfactorily on the "Cuba" and the limitations of the machinery are not reached at less than the safe load on the shaft and propeller. All these switching operations can be made at a distance from the motor and in a ship having boilers and engine rooms amidships, there is no auxiliary equipment such as resistors, etc, in the motor compartment.

The success of this installation and its economic advantages has led the engineers to recommend the use of synchronous motors for cargo ship propulsion, and to predict that within fifteen years, every cargo ship built will be equipped in this manner. Better engineering in power layouts will undoubtedly bring about an increased use of synchronous
motors in the future. The demand for them will make it worth while to the manufacturers to devote more attention to their design and increase the number of stock sizes. There is no electrical reason why small sizes, 5 to 10 h.p., are not possible, and it is reasonable to suppose that the development of a line of motors will include these small sizes. The much lauded individual drive has proved neither entirely satisfactory nor very economical. The tendency now is to use "group drive," a combination of several machines run by one motor by means of line shafting. There is one feature of this system which makes it especially suitable for synchronous motors and that is, slow speed. Particularly is this true in textile and rubber mills where reduction gears and large pulley ratios have always been necessary with induction motors. The chief difficulty when using line shafting is the large starting torque required. This can be eliminated by using a clutch, making synchronous motors entirely satisfactory. Another way of solving the problem is by installing a synchronous motor with excess capacity in order to
handle the heavy starting load and to correct the power factor of the miscellaneous induction motors which cannot be grouped.

If "individual drive" is necessary it is possible to install small synchronous motors with a central motor generator set to supply the d.c. excitation. This system would require greater skill in starting the machines (or semi-automatic control for the d.c. fields) than with induction motors, but the economies might be well worth the extra investment. Recent developments in small capacity mercury arc rectifiers suggest their use in connection with this problem. They could be used to take the place of the motor generator set. They are simple, compact, inexpensive and ideally suited for use with motors of from 5 to 25 h.p.

In brief, the synchronous motor is just at the beginning of its development. The prejudices which have held it back in the past are being overcome rapidly because of its established reputation for economy. It is reasonable to predict that it will share evenly with the induction motor the industrial power of the future.
POWER FACTOR OF TRANSMISSION LINES

The power factor of high voltage transmission lines is of great importance, and is a much more complex subject than the power factor of distribution systems. It is most desirable to have good regulation on long lines and the synchronous apparatus is installed primarily to improve the regulation, the correction of power factor being of secondary value. However, improving the regulation improves the power factor, therefore the efficiency of the entire system is increased from every standpoint, and synchronous condensers are now embodied in the design of all long lines using voltages over 110,000 volts.

When there is no load at the receiver end of a long line, the capacity current or "charging current," so called, may be of such a value that one generating unit at the power house cannot supply it, and great difficulty is experienced in energizing a long line which is open at the receiver end. Furthermore, this charging current, which leads the voltage, causes a rise in voltage at the receiver end, due to its effect through the inductance of the line. At
no load, therefore, it is necessary to under-excite the synchronous apparatus, causing its current to lag the voltage, and neutralizing the effect of the charging current.

Under load conditions, the synchronous apparatus must be adjusted to correct the lagging power factor of the load and thereby maintain the correct voltage. In this case it will take a leading current. No other form of corrective apparatus can be adjusted to take either a leading or a lagging current, therefore the synchronous condenser is the only type which can be used for this service.

A complete discussion of this subject is beyond the scope of this paper, the intention being to make a survey of power factor correction solely, not its connection with voltage regulation. The size of the problem has been indicated and a few specific results have been found, but it is impossible to cover such a large subject in all of its phases in so brief a paper.
APPENDIX A

Data secured from Public Document 35, Commonwealth of Massachusetts, report of Edison Electric Illuminating Co. of Boston, for year ending June 30, 1919.

Operating Income:

<table>
<thead>
<tr>
<th></th>
<th>(No KWH)</th>
<th>(Amount)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commercial Lighting</td>
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<tr>
<td></td>
<td>7.992</td>
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<tr>
<td>Public Lighting</td>
<td>20,342,234</td>
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<tr>
<td></td>
<td>837,683.35</td>
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</tr>
<tr>
<td></td>
<td>4.118</td>
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<tr>
<td>Power</td>
<td>123,111,023</td>
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<tr>
<td></td>
<td>3,229,729.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.63</td>
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</tr>
</tbody>
</table>

Expense of Manufacture:

<table>
<thead>
<tr>
<th></th>
<th>$/KWH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>0.703</td>
</tr>
<tr>
<td>Rentals</td>
<td>0.017</td>
</tr>
<tr>
<td>Oil &amp; waste</td>
<td>0.002</td>
</tr>
<tr>
<td>Water</td>
<td>0.009</td>
</tr>
<tr>
<td>Wages</td>
<td>0.247</td>
</tr>
<tr>
<td>Tools</td>
<td>0.020</td>
</tr>
<tr>
<td>Station repairs</td>
<td>0.008</td>
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<tr>
<td>Steam repairs</td>
<td>0.081</td>
</tr>
<tr>
<td>Electric repairs</td>
<td>0.057</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>1.144</td>
</tr>
</tbody>
</table>

Adding for current purchased Expense of distribution 1.461
Expense of management 0.531
Expense of management 0.491
Expense of Taxes miscellaneous

Total cost of mfr.,
Dist., & Purchase

Net Earnings:
Income from other sources
Interest
Dividends
Capital charges, de-preciation, etc.
Balances

To determine the cost of power, other items must be added to the cost of operation to obtain commercial cost:

Operating cost
Dividends
Capital charges
Balances
Interest

TOTAL

At a recent meeting of the Edison Illuminating Societies in Philadelphia, it was the consensus of opinion that 75% of the investment cost is effected by reactive kVA, or, in other words, was chargeable to the electric part of the plant. The cost of fuel was estimated to be from one-third to one-half the total cost at the switchboard. This fuel cost includes main prime movers and auxiliaries. On the basis that, with a constant voltage system, the in-
stallation of electric machinery is determined by amperes and not by watts, the following fractions of the total power cost will be assumed proportional to the KVA, while the rest of the cost will be assumed proportional to the KW load.

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost (¢/KWH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2 wages</td>
<td>0.123</td>
</tr>
<tr>
<td>1/2 tools</td>
<td>0.010</td>
</tr>
<tr>
<td>1/2 station repairs</td>
<td>0.004</td>
</tr>
<tr>
<td>Electric repairs</td>
<td>0.057</td>
</tr>
<tr>
<td>Distribution Expense</td>
<td>0.531</td>
</tr>
<tr>
<td>3/4 capital charges</td>
<td>0.126</td>
</tr>
<tr>
<td>3/4 management</td>
<td>0.370</td>
</tr>
</tbody>
</table>

TOTAL chargeable to electric plant = 1.221 = R_{EP}

Balance chargeable to steam plant = 3.588 = R_{SP}

TOTAL = 4.809 = R_P

Distribution of Boston Edison Load on Maximum Demand Basis

29.0% 30000 KW induction motors on 550 volts or less
Average power factor about 60%.

34.0% 35000 KW Direct current

5.8% 6000 KW Street lighting

2.91% 3000 KW Rotaries for street railways

28.27% 29000 KW High tension customers. Power factors of 80% or greater due to use of synchronous machinery,
100%  103000 KW Maximum demand for year ending
June 30, 1919.
Yearly load factor about 40%. Daily load factor
is sometimes as high as 55%

Cost data secured from the Westinghouse Elec-
& Mfg. Co. for standard sizes of synchronous motors
shows that the average price from 50 KVA to 300 KVA is
about $9.00 per KVA including exciter. An allowance
of 25% of the cost is made for installation and start-
ing equipment, bringing the total cost up to $11.25
per KVA.

The lowest leading power factor of synchron-
ous apparatus is assumed to be 4.5%, which is a
reasonable value for sizes below 1000 KVA. For
small sizes this value will be too low, but, as is
shown in the discussion, a 15% variation in this fac-
tor makes a very small difference.

Capital charge of 20% is assumed large
purposely to allow for an adequate return on the
capital invested in other places. This covers in-
terest, depreciation and insurance.
The load is assumed to be connected continuously, i.e., for 8760 hours per year. The value of 7000 hours per year used for corrective apparatus allows for a 5-hour shut-down each night, which will cover the light load period of the average central station company.
BIBLIOGRAPHY - POWER FACTOR CORRECTION

Correction of power factor with industrial motors.

Power Factor Adjustments by the use of Synchronous motors.

Power Factor Correction and its Effect.

Power Factor Correction on Distrib. Systems.

Synchronous Motors.
Power 51:850-1 May 25, '20

Correction of power factor with static Condensers.

Control of power Factor on Transmission Lines.

Power Factor Correction by Synchronous Motors.

Electrostatic Condensers.
V.E. Goodwin, A.I.E.E. Jour Nov. 1920, p. 980.

Points to Consider in Adjusting for Consumer's power Factor.
Elec. World 77:663 March 19, 1921.

Continental Practice in Elec. Driving of Textile Factories
W.D. Fox, Electrician 86:185-8 Feb. 4, 1921.

Power Applications to Cotton-finishing Plants.
Power Factor Corrective Apparatus.

Constant Voltage Transmission.
H.B. Dwight.

Textbooks and manufacturers' Bulletins.