DEVELOPMENT OF A
NITINOL HEAT ENGINE

by
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SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF
BACHELOR OF SCIENCE
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
May, 1980
A Nitinol engine has been proposed for the utilization of low grade thermal energy. A continuous band of Nitinol wrapped between tightly engaged rollers produces torque by thermal cycling of the Nitinol. Engine performance was analyzed for a simple configuration using four rollers. A model engine with a band thickness of 0.024" was constructed. Projected power output for the engine was 7.4 watts, efficiency was 12%. Due to mechanical problems, the model is not operational at this time.
Acknowledgements

The author wishes to express his sincere gratitude to Y.T. Li, without who's ingenuity, as well as patience and understanding, this project would not have been possible.
# Nomenclature

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<td>$R_1$</td>
<td>Small Roller Diameter</td>
<td>in</td>
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<td>$R_2$</td>
<td>Large Roller Diameter</td>
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<tr>
<td>$t$</td>
<td>Nitinol band thickness</td>
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<td>$w$</td>
<td>Nitinol band width</td>
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<td>$E$</td>
<td>Young's Modulus Nitinol (temp. Ms)</td>
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<td>$\gamma$</td>
<td>Kinematic Viscosity</td>
<td>ft²/hr</td>
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I. Introduction

In recent times, decreasing availability of petroleum products has made alternative sources of energy increasingly attractive. High fuel costs have caused previously economically unfavorable sources to be looked upon more favorably. There is presently a vast resource to be found in the form of low grade thermal energy.

A. Types of Energy Available

1. Solar Power is one area receiving intense interest at the present time. Present schemes to utilize the available power include the direct conversion of sunlight to electrical power (photovoltaics). Heat from the sun can be used directly for the heating of residential homes, water heating, cooking, or the desalination of sea water. Still another option for the utilization of solar energy is to use the sun's heat to operate a Rankine cycle, or some other form of heat engine to produce mechanical power. This mechanical energy may be used directly, or easily converted to electrical power using a conventional generator. Ammonia is the most likely working fluid for a Rankine cycle used in such an application.

2. OTEC. OTEC, or Ocean Thermal Energy Conversion, is another potential source of useful low grade thermal energy. In the tropics, surface sea water is heated by the sun to between 75 and 85°F year round. In the deep sea, 3000 to 4000 feet beneath the surface, temperatures are not very much above freezing. This temperature difference makes OTEC possible.
Most present plans to utilize this temperature gradient concentrate on the use of a Rankine cycle based on ammonia vapor as the working fluid. Due to the low temperature difference between the hot and cold water, the thermodynamic efficiency would necessarily be very low, on the order of 8%. Ammonia was chosen as the working fluid because it has a thermal conductivity of about three times that of freon. Water is not suitable for this application because of its low vapor pressure at the temperatures which are available. Low efficiency means that a huge amount of heat has to be exchanged through a very small temperature differential to get an appreciable amount of net useful mechanical work from the cycle. This means the flow rate of ammonia must be very large, and that the heat exchangers for evaporating and condensing the ammonia must be huge. The presence of sea water and ammonia pose a substantial corrosion problem for the heat exchangers. This, coupled with the exceedingly large size required makes the cost of the heat exchangers roughly equivalent to the fuel cost in conventional power plants. For this reason, Ocean Thermal Energy Conversion is not practical at this time. As fuel costs rise in the future, the economic conditions might become favorable to the construction of an OTEC system based on an ammonia Rankine cycle.

3. **Bottoming Cycles.** In conventional fossil fueled and nuclear power plants, waste heat is rejected to a low temperature reservoir, such as a body of water or the atmosphere. The temperature of this heat rejection is high enough in some of these
plants that another Rankine or alternative thermal cycle could operate using the waste heat from the condensor of the conventional power plant as a high temperature reservoir. In this manner, overall plant efficiency and power output could be increased, with no additional cost of fuel. Convention-research is along the lines of an ammonia based Rankine cycle, as suggested for use in OTEC.

B. An Alternative Method

One alternative method for the utilization of low grade thermal energy for the production of mechanical power has been suggested by Dr. Y.T. Li of the M.I.T. Innovation Center. The solution he proposes is an engine using Nitinol as a solid state thermodynamic working medium.

1. Description of Nitinol

Nitinol is short for Nickel and Titanium Naval Ordinance Laboratory. It is composed mainly of nickel and titanium, and belongs to the class of materials known as shape-memory alloys. These materials possess the property of remembering and returning to a programmed shape, even when deformed plastically to some other shape. The alloys, once deformed at a temperature near the martensitic tranformation temperature (Ms), can be fully recovered to their original shape upon subsequent heating to a moderately higher temperature^3. In addition to the nickel(50a/o%) titanium alloy discussed in this paper, other alloys exhibiting the shape memory effect
Figure 1
Dr. Li's Engine. Four Rollers.
include systems of: In-Tl\textsuperscript{4}, Cu-Zn\textsuperscript{5}, Cu-Al\textsuperscript{6}, Ag-Cd\textsuperscript{7,8}, Ni-Al\textsuperscript{9,10}, Cu-Ni\textsuperscript{11}, Zn-Sn\textsuperscript{12}, Au-Cu-Zn\textsuperscript{13}, Cu-Zn-Si\textsuperscript{14}, Cu\textsubscript{3}-Al\textsuperscript{15}, An-Cd\textsuperscript{16}. More information on these materials may be found in the references given.

Nitinol is presently the most desirable material for use in a solid state engine for a number of reasons. It can be made with a wide variety of Ms temperatures, from -50 to +175°C, depending on composition and heat treatment. This puts it easily in the range of most sources of low grade thermal energy. Its narrow hysteresis means that the temperature differential between the hot and cold reservoirs can be small. It has a high recoverable strain of about 8%.

Fatigue properties are good: 70,000 p.s.i. for $10^7$ cycles. Its hardness and extremely good corrosion resistance make it very attractive for dealing with the corrosive sea-water environment found in OTEC applications. More specific properties of Nitinol may be found in reference 2.

2. Engine Designs. Nitinol is suitable for use in an engine because the stress required to deform it below Ms is relatively low, about 20,000 p.s.i. When it is heated above Ms, there is an internal change in stress, and the material recovers its original shape while exerting a considerable mechanical force. The result is that it takes less work to deform the Nitinol initially than the work done when it recovers from a deformed condition. In this way, a portion of
the heat of transformation is converted to useful mechanical work.

Several designs of Nitinol engines have been built in the past. They were all small models, with power outputs of a few watts maximum. Efficiency and power density were low.

Offset crank engines, field engines, and turbine engines have been built. They will not be described here, but additional information is available in reference 17. In the first two engine types mentioned, large volumes were cycled thermally. This cycling of engine components provided no useful work, and contributed to low efficiency.

3. Dr. Li's Design.

a. Description. Dr. Y.T. Li's design incorporates a continuous band of Nitinol wrapped between a cluster of rollers. This engine is designed for deriving maximum possible work from each cycling of the Nitinol. The design is optimized for power density and total engine efficiency.

A simplified version of the original design is shown in Figure 1. This engine consists of four rollers, 2 of radius \( R_2 \), and 2 of radius \( R_1 \). Nitinol, programmed to radius \( R_2 \) is wrapped around the \( R_1 \) rollers and passes between the \( 2 R_2 \) rollers. As marked in the figure, there are two definite hot sections, and two cold sections in the engine. If the thermal conductivity of the rollers themselves is low, thermal cycling of the engine parts is kept to a minimum, and only the Nitinol is cycled. Hot feedwater (temperature \( M_s \)) flows into the engine at A and A', and leaves at B and B'. As can
be seen from the direction of roller rotation (indicated by arrows), the hot feedwater moves counter to the direction of feedwater travel. This results in a maximum heat transfer from the water to the Nitinol. Direct contact of the water with the Nitinol results in a high heat transfer rate. Cold water, with temperature $M_s$, flows in $C$ and $C'$, and out at $D$ and $D'$. This is also a counterflow pattern.

The engine pictured is a very simple version of Li's design, suitable for model construction. An engine for practical applications would have clusters of many rollers per band of Nitinol. Several clusters of rollers approximately 10 ft long would be assembled into an engine. Estimated power output for such a configuration is 500 hp for an engine occupying a 10 x 10 x 10' space. That engine is fully described by Li in "A Nitinol Engine For Low Grade Heat".

b. Operation of The Engine. A brief description of how the engine shown in Figure 1 would operate is contained in this section. A more detailed description and qualitative analysis follows in section II entitled Analysis.

Consider a portion of the Nitinol band located at 1 on the Figure. It is moving downwards, and is in contact with the cold water flowing from $C$ to $D'$. When the material under consideration reaches point 2, it is subject to an instantaneous change in curvature from radius $R_2$ to $R_1$. The deformation is in a direction opposite to the bands desired shape, therefore the engine does work on the Nitinol at this point. However, the band has been chilled below $M_s$ by con-
act with the cold water flowing from C' to D'. Its yield stress is low, and the work required to deform the band is not great. As the band leaves 2, and moves towards 3, it is heated by the hot water flowing from A' to B'. This raises its temperature above Ms, and results in an internal change in stress, since the materials yield stress has been raised by the transformation. Moving to point 4, another instantaneous change in curvature takes place from R₁ to R₂. This time, the deformation is in a direction such that the Nitinol recovers its programmed shape (R₂). Elastic energy stored in the Nitinol is released, and a torque is produced on R₂ which serves to rotate that roller in the direction of its motion, and keep the engine running. As the band moves to point 5, it is cooled as at point 1, and the Nitinol goes through another identical cycle while passing through the other rollers.
II. Analysis

A. Engine Torque

As mentioned in section I, the introduction, torque is developed as the band moves from a roller of radius \( R_1 \) to \( R_2 \). During this motion, the Nitinol is in the heated condition. Its yield strength is high, and all deformation is elastic. The band, of rectangular cross-section, may be modelled as an elastic beam of bending stiffness:

\[
EI = Ewt^3/12 \quad \text{(II-1)}
\]

Figure 2 shows the stress distribution in a band programmed to radius \( R_2 \). At the inflection point of band curvature, the portion of the band between the neutral axis and roller \( R_2 \) is in tension, while the other half is in compression. At the point shown in the figure, the resultant is a bending moment about the neutral axis. For an elastic beam, the bending moment divided by the bending stiffness is equal to reciprocal the radius of beam curvature:

\[
\frac{M_B}{EI} = \frac{1}{\rho} \quad \text{or} \quad M_B = \frac{EI}{\rho} \quad \text{(II-2)}
\]

In this case, the neutral axis is not straight in the undeformed condition since the band is programmed to radius \( R_2 \). Therefore, from (II-2):

\[
M_B = \frac{EI}{R_2} + \frac{EI}{R_1}
\]

\[
\Rightarrow M_B = EI(1/R_2 + 1/R_1) \quad \text{(II-3)}
\]

Combining this result with equation (II-1) gives:

\[
M_B = Ewt^3/12 \left(1/R_2 + 1/R_1\right) \quad \text{(II-4)}
\]

As shown in Figure 3, this same bending moment may be produced
Figure 2
Stress distribution in Nitinol band travelling from \( R_1 \) to \( R_2 \).
Figure 3
Couple Equivalent to Bending Moment
by a couple, with each force equal to:
\[ \frac{Ewt^3}{12} \left( \frac{1}{R_2^2} + \frac{1}{R_1^2} \right) \]
\[ \cdot \frac{2t}{2} \]
\[ = \frac{Ewt^2}{12} \left( \frac{1}{R_2^2} + \frac{1}{R_1^2} \right) \] (II-5)

The net torque around \( C_2 \), the center of roller \( R_2 \), is:
\[ \frac{Ewt^2}{12} \left( \frac{1}{R_2^2} + \frac{1}{R_1^2} \right) \left( (R_2 + t) - R_2 \right) \]

or \[ \frac{Ewt^3}{12} \left( \frac{1}{R_2^2} + \frac{1}{R_1^2} \right) \] (II-6)

If the band and roller move together without slipping, this torque is in the direction of roller motion, and helps the roller to rotate. Even if the roller is fixed, the band will still move as long as the friction between roller and band is low enough. After the initial deflection of the band in passing from \( R_1 \) to \( R_2 \), the band is constrained by roller \( R_2 \) as well as by the tension in the band to act as a rigid body while in contact with \( R_2 \). The torque caused by the bending moment in the band itself causes it to rotate around \( C_2 \) as any rigid body would, regardless of the motion of roller \( R_2 \).

In moving from roller \( R_2 \) to \( R_1 \), the band is bent away from its programmed shape. However, during this process it is in the unheated condition and its yield stress is low. Although the torque produced is in an unfavorable direction, it is low in comparison with the favorable torque produced in moving from \( R_1 \) to \( R_2 \). Stress distribution for the process is shown in Figure 4. Besides a lowered yield stress, the Young's modulus of Nitinol is also low when cooled below the transformation temperature. As a result, the stress distribution in the Nitinol when bent as shown in the figure can be approximated as linear. For practical band thicknesses and...
Figure 4

Stress Distribution in Nitinol travelling from $R_1$ to $R_2$. 

- plastic region
- tension
- compression
- elastic region
- neutral axis
- nitinol
- $C_1$
- $C_2$
- $R_1$
- $R_2$
roller diameters, the low modulus means that the width of the plastic region is small, even if the yield stress is very low. An equation of exactly the form of (II-4) can be written to express the bending moment in the band as it travels from $R_2$ to $R_1$ in the unheated condition. $E_{\text{cold}}$ is defined as the Young's modulus of the Nitinol band in the unheated condition (temperature below the transformation temperature):

$$M_B = E_{\text{cold}} \frac{wt}{12} \left( \frac{1}{R_2} + \frac{1}{R_1} \right) \quad (\text{II}-7)$$

As a result of this internal bending moment in the band, there is a net torque causing the band to rotate as a rigid body around $C_1$, the center of $R_1$ as shown in the figure. This torque is equal to the bending moment expressed in (II-7), and is in a direction opposite to band and wheel rotation. In this manner, work is done on the band by the engine as it passes from $R_2$ to $R_1$.

The total torque produced by the engine is the sum of all individual torques on the rollers. For a 4 roller engine, as shown in Figure 1, there are 2 $R_1$ rollers and 2 $R_2$ rollers. The net torque produced is then equal to twice the torque expressed in equation (II-6) minus twice the torque in (II-7).

$$\tau_{\text{net}} = 2(Ewt^{3/12} \left( \frac{1}{R_2} + \frac{1}{R_1} \right)) +$$
$$-2(E_{\text{cold}}wt^{3/12} \left( \frac{1}{R_2} + \frac{1}{R_1} \right))$$

or:

$$\tau_{\text{net}} = (E - E_{\text{cold}}) \left( \frac{1}{R_2} + \frac{1}{R_1} \right)wt^{3/6} \quad (\text{II}-8)$$

Conveniently, all the torque produced by the entire engine may be extracted from any one roller shaft. Referring back to Figure 3, there is a net torque around $C_2$ due to the
bending moment in the Nitinol band. The band acts as a rigid body while in contact with \( R_2 \), except for the deflection where it first contacts and finally leaves the roller. If there is no external torque applied to \( C_2 \), a force of approximately

\[
\frac{Ew_2}{R_2} \left( \frac{1}{R_2} + \frac{1}{R_1} \right)
\]

at a distance \( R_2 \) from \( C_2 \) and directed perpendicular to the radial direction is required to maintain equilibrium. Such a force would be realized as a tensile force in the band, as shown in Figure 5. In this manner, the torques produced at the various rollers may show up as tensile forces on the Nitinol band. All the torque may then be extracted from one roller, as long as the band does not slip on that roller.

B. Calculation of Roller Diameters

Several factors play an important role in selecting the roller diameters for a Nitinol engine. Among these are:

1. Efficiency of the engine
2. Nitinol fatigue
3. Heat transfer considerations

1. Efficiency. The heat required for the martensite to austenite transformation is essentially constant, regardless of the amount of stress the material is under during the transformation process. For a given speed, to maximize the power produced from each cycling of the band, it is desirable to maximize the torque output of the engine. The net engine torque is given in (II-8) as:
Figure 5

Tensile Force Produced by Torque
The two Young's modulii are properties of the Nitinol, and can not be easily altered. For a fixed cross section of the band (constant t and w), the net engine torque may be maximized by maximizing the quantity \((1/R_2^2 + 1/R_1^2)\). It is apparent that selection of a small \(R_1\) and \(R_2\) will maximize this quantity and result in a high output torque. Physically, such small roller diameters mean that the deflection of the band as it travels from roller to roller is large. However, the amount of allowable band deflection is limited by other considerations, so that very small diameters are not realizable in a practical engine. Taking the two Young's modulii as \(6E_6\) and \(2E_6\) p.s.i., for \(E\) and \(E_{\text{cold}}\) respectively, and band thickness \(t\) as 0.025", net torque per inch of band width is plotted against mean roller radius \((R_1R_2)/(R_1 + R_2)\) in Figure 6. It can be seen from this figure that efficiency must decrease with increasing mean roller radius, assuming all Nitinol is transformed completely during a cycle (heat input to engine is constant for all roller diameters).

2. Fatigue Considerations. Fatigue characteristics of Nitinol limit the minimum mean roller radius that can be used. For an elastic beam, the maximum stress produced in bending is:

\[ \sigma = -\frac{(Et/2)}{\rho} \]

or, in the case of the Nitinol heat engine:

\[ \sigma = -\frac{Et(R_1 + R_2)}{2R_1R_2} \]  \hspace{1cm} (II-9)

Tests indicate that Nitinol has a fatigue strength of 70 k.s.i.
Figure 6

Engine Torque vs. Mean Roller Radius
(Given by equation (11-8))

\[
\text{Mean Roller Radius} \quad \frac{R_1 R_2}{(1/R_1 + 1/R_2)}
\]
for more than $10^7$ cycles.$^9$ Alloys with a higher transformation temperature were found to have decreased fatigue limits than for the lower temperature ranges (Nitinol is available with transformation temperatures, depending on alloy and heat treatment, between $-50 - +175^\circ C$). For an engine operating at a maximum stress of 90 k.s.i., the allowable strain is 1.5%. For a 1.5% strain, and a Young's modulus $E = 6 \times 10^6$ p.s.i., equation (II-9) gives a value of $R_1 R_2/(R_1 + R_2)$ of:

$$90,000 = 6 \times 10^6 \times 0.025 (R_1 + R_2)/2R_1 R_2$$

$$\Rightarrow R_1 R_2/(R_1 + R_2) = 0.835$$

(for $t = 0.025''$ as in section II-A)

Looking back at Figure 6, it can be seen that such a value for the mean roller radius limits engine efficiency, and may prove to be the limiting factor on the maximum fraction of Carnot efficiency that a Nitinol engine can achieve. A typical plot of stress vs. strain for Nitinol is shown in Figure 7.$^{20}$ From this plot was taken the Young's modulus and strain at 90,000 p.s.i. stress used in the above calculation.

3. Heat transfer considerations. The heat transfer rate between the feedwater and the Nitinol band plays an important role in choosing relative roller diameters. For an engine operating between the temperature extremes of 20 and $60^\circ C$, maximum engine efficiency is limited to Carnot engine efficiency at those temperatures:

$$\eta = 1 - (20 + 273)/(40 + 273) = 12\%$$

This means that only 12% percent of the heat transferred from the hot water stream to the engine can show up as useful energy.
Figure 7

Nitinol Stress vs Strain
MECHANICAL WORK. The remaining 88% must be rejected to the cold water stream. If the coefficient of heat transfer is the same for both the hot and cold streams, the surface area of Nitinol available for heat transfer from the cold stream should be approximately 88% of the area provided for the hot water. Lower efficiency in a practical engine, due in part to Nitinol fatigue considerations, dictates an area ratio closer to unity.

C. Power and Heat Transfer Rate

1. Fluid Dynamics. The dynamics of the feedwater flow are intimately related to heat transfer to the Nitinol. As mentioned in the Introduction, a counterflow pattern is highly desirable in that it maximizes the amount of heat that can be transferred to the Nitinol for a given change in feedwater temperature across the engine. As shown in Figure 8, the flow may be modelled approximately as turbulent flow over a flat plate. The flow velocity of the water with respect to the Nitinol band is the sum of the fluid velocity in a fixed reference frame plus the velocity at which the Nitinol band is moving. The highly turbulent flow of the water in direct contact with the Nitinol insures a relatively high heat transfer rate to the Nitinol.

2. Heat Transfer Rate. The rate of heat transfer required for engine operation is dependent on engine speed. If full torque is to be produced, the entire cross section of the band must be in the transformed condition as it passes between
Figure 8
Flat plate model of heat transfer.
the rollers. Estimated energy required for the martensite to austenite transformation range from 1.6 to 5.8 cal/gm\(^{22,23}\). This is equivalent to 2.88 to 10.47 BTU/lbm. For a density of 0.24 lbm/in\(^3\) of Nitinol\(^{24}\), the mass flow rate of Nitinol, in lbm/hr, is given by:

\[ m_{\text{Nitinol}} = tw_60NR_2 \times 2\pi x.24 \]  \hspace{1cm} (II-10)

This is simply the velocity of the band multiplied by the cross sectional area of the band multiplied by the density of Nitinol.

Taking an average value of 6 BTU/lbm for the transformation energy of Nitinol gives a required heat flow rate of:

\[ \dot{q} = 6 \dot{m} \]  \hspace{1cm} (II-11)

Combining (II-10) and (II-11) gives:

\[ q = 542twNR_2 \]  \hspace{1cm} (BTU/hr) \hspace{1cm} (II-12)

For water having a specific heat of 1 BTU/lbm\(^0\)F, the flow rate of water required for a 5\(^0\)F temperature drop across the engine is:

\[ m_{H_2O} = \frac{542twNR_2}{5x1} \]  \hspace{1cm} (II-13)

Since there are two independent hot water flow paths, the total mass flow rate of water required with T Ms is:

\[ \dot{m}_{H_2O} = 217twNR_2 \]  \hspace{1cm} (II-14)

3. Power. The power output of the engine is proportional to the torque \( \tau \) given by (II-8) and the angular speed \( N \). For \( N \) in units of RPM and \( \tau \) in units of in·lbf, the power \( P \) in hp is given by:
A. General Description

A small, 4 roller model of a Nitinol engine was constructed of the configuration shown in Figure 1. Nitinol available for the engine (currently the only supply is from China) was in the form of a rectangular ribbon 0.024" thick and 0.050" wide. It was decided to make the band by spot welding 4 individual loops of the Nitinol to form a band 0.20" wide. Transformation temperature of the material was 40°C, and it was planned that the engine would operate on feedwater at temperatures of 20 and 60°C. Material selected for the engine was Plexiglass, a trade name for polymethyl-methacrylate. This material was selected for its low thermal conductivity, ease of workmanship, optical transparency, and availability. The engine was built in the form of a three layered sandwich of Plexiglass. Details are shown in Figure 9.

Fatigue considerations, as described in section (II-B) dictated an $\frac{R_1 R_2}{(R_1 + R_2)}$ of 0.835 for a ribbon thickness $t = 0.025"$ and a maximum allowable strain of 1.5%. The relative diameters of the rollers were chosen to give approximately the correct

$$P = \frac{T \cdot N}{396,000}$$

(II-15)
Figure 9
Engine Model
ratio of hot and cold heat transfer areas, as outlined in section (II-B-3). Details of this calculation are presented in Appendix B. As shown in the figure, hot and cold water flow through the engine in a direction counter to roller rotation. Water enters the engine at the locations marked "hot in" and "cold in" and leaves the back of the engine, as shown in the side view of Figure 10. An output shaft was provided on one of the small rollers to extract power from the engine, as well as for starting it.

The rollers can be seen sandwiched between the outer layers of plexiglass with the Nitinol riding on their rim in the cross section shown in Figure 10. The small wheels were provided with a grooved rim to prevent the Nitinol from running off the wheel edges. Brass bolts were used to hold the three layers of the engine together, while cork gasket material was used in between the layers to prevent leakage.

A tensioning device, located at the top of the engine, was used to maintain tension in the band and prevent it from sliding on the wheels.

B. Problems

After programming the Nitinol to radius \( R_2 = 3" \) by heating to \( 1300^\circ F \) in a muffle furnace, the Nitinol was welded to form loops. Four individual loops were formed by wrapping pieces of Nitinol around an aluminum form (a 1/4" disc of 6.771" diameter), lapping the ends, and spot welding them together. The welds were then ground down to approximately the cross section of the ribbon itself. Embrittlement of the material around the weld area caused the ribbons to frequently
tensioner (not shown Fig. 9)

aluminum hub

brass shaft

Plexiglass wheel

Nitinol in hot water

brass shaft hot/cold out

cork gasket

brass bolt

Figure 10

Engine Side View
break. Alternate joining methods, such as silver soldering proved unsatisfactory. Finally, the Four ribbons were held tightly together by wrapping 0.001" stainless steel ribbon around the band at \( \frac{1}{2} \)" intervals and spot welding.

2. Tracking Problems. The original large wheels of Figures 9 and 10 had a 0.020" clearance between their faces and the engine sides. During wheel rotation, a ribbon would creep to one side, tilt slightly, deflect the wheel and wedge itself between the wheel and engine side. This would result in the ribbon running completely off the wheel rim. In an attempt to correct this problem, wider wheels, with a face clearance of 0.005" were made. One of these wheels is shown in Figure 11. Grooves were turned in both faces of the wheels to accommodate a number of 3/16" diameter copper-plated steel balls. These balls were intended to serve as a means of preventing the wheels from contacting the engine sides, while precisely locating them. Although these wider wheels did solve the original problem of keeping the band on track, they created a new problem. The rolling of the balls in their grooves at such a large distance from the wheel centers raised friction to such high levels that the engine could not operate.

At this point, the cork gasket stock was removed from between the Plexiglass layers. The original large wheels were reinstalled, this time with new aluminum hubs. The hubs were turned down on a lathe to form 1/16" diameter shafts. Small ball bearings were mounted on these shafts. Spacing between the bearings was large to precisely locate the wheels.
Figure 11
Large Wheel With Balls in Place
Knobs were provided on the shaft ends for starting the engine. Provisions were made for tensioning the Nitinol band by sliding the two large wheels towards one another. This allowed the shafts to be removed from the two small wheels, cutting down on friction even further. Tension in the Nitinol and pressure from the large wheels served to keep the small wheels in the correct positions. A rubber gasket material was then applied between the layers. Correct gap was held between the wheels and outer layers while the rubber cured. The outer Plexiglass was greased, to permit separation from the gasket, and engine disassembly.

With friction reduced to a minimum, the engine turned freely, even while cold water was circulated through it. However, when hot water was introduced, the Plexiglass was distorted to a point where the engine did not turn. At the time of this writing, that problem has not been solved.

3. Theoretical Performance. Expected power output and efficiency for the model are calculated in Appendix B.

For:  
- band width \( w = 0.200" \)
- thickness \( t = 0.025" \)
- \( \dot{m}_{H_2O} = 25,200 \text{in}^3/\text{hr} \) (hot side)
- transformation energy = 6 BTU/lbm (Nitinol)

Engine power predicted is:

\[
P = 7.4 \text{watts}
\]

Efficiency is:

\[
\eta = 1.22\%
\]

Temperature change used in the calculations was 5\(^\circ\)F across the engine.
IV Conclusions

Mechanical problems have thus far prevented the model from operating. With a very small ratio of band width over wheel diameter, friction and viscous drag on the wheels become very significant losses. The problems encountered in the model have not been due to error in theory, but problems associated with trying to create a crude working model using Nitinol in a shape other than optimal for the application. The author is confident, however, that the problems can be overcome, and a successful model will be made to run.

Analysis indicates that this type of engine could be a viable solution to a very real need: the effective utilization of low grade thermal energy sources. Since the energy required to run the Nitinol engine is free, the real challenge is to keep material and maintenance costs low enough so that the Nitinol engine is economically attractive as a source of useful power.
References


15. Nagasawa, A. Ibid.


20. Nee., Ibid. p. 44.
References (cont.)


22. Warlimont, H., Ibid,


Ideally, the area for heat transfer on the cold side should equal the area on the hot side multiplied by one minus the engine efficiency. According to Figure 12, this means:

\[ L_c = L_h (1 - \eta) \]

From geometrical considerations, it can be seen that:

\[ L_c = R_1(2\pi - 2\cos^{-1} \frac{a}{R_1 + R_2}) \]
\[ L_h = R_2(2\sin^{-1} \frac{a}{R_1 + R_2}) \]

For an estimated efficiency of 8%:

\[ (.92)R_2(2\sin^{-1} \frac{a}{R_1 + R_2}) = R_1(2\pi - 2\cos^{-1} \frac{a}{R_1 + R_2}) \]
\[ \Rightarrow R_2/.92R_1 = \pi/\sin^{-1}(a/R_1 + R_2) - \cos^{-1}\left(\frac{a}{R_2 + R_1}\right) \]

Although this is not solvable explicitly for \( a \) as a function of \( R_1 \) and \( R_2 \), this equation is useful in selecting values by an iterative process. Fatigue considerations in section II on theory gave an \( R_1R_2/(R_1 + R_2) = .835 \). This gives the following values for the three important dimensions:

\[ R_1 = 1.375\text{in} \]
\[ R_2 = 3\text{in} \]
\[ a = 1.5\text{in} \]
Figure 12

Critical Engine Dimensions
Appendix B  Engine Performance

For:  \( w = 0.20 \) in
\( t = 0.025 \) in
\( R_1 = 1.374 \) in
\( R_2 = 3.0 \) in

Equation (II-8) gives a net engine torque of:

\[
\tau = (3E7 - 6E6)(1/3 + 1/1.374)^{1/3} \times 0.20 \times 0.025^3 / 6
\]

\( \Rightarrow \tau = 14 \) in·lbf

For a hot water flow rate of 7in³/sec (908 lbm/hr), equation (II-14) gives RPM(N):

\[
908 = 217 \times 0.025 \times 0.20 \times N \times 3
\]

\( \Rightarrow N = 279 \) RPM

Power given by equation (II-15) is:

\[
P = \frac{14 \times 279}{396,000}
\]

\( \Rightarrow P = 0.01 \times E^{-3} \text{hp} = 7.4 \text{ watts} \)

Heat transfer to the Nitinol at temperature \( T_s \) is given by (II-12) as:

\[
\dot{q} = 542 \times 0.025 \times 2N \times 3
\]

\( \Rightarrow \dot{q} = 2200 \text{ BTU/hr} = 600 \text{ watts} \)

Engine Efficiency = \( \frac{7.4}{600} = 1.2\% \)