Microgrid Operation Strategy for Improved Recovery and Inertial Response After Large Disturbances

by

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Submitted to the
Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2016

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The electric grid is one of the major achievements of human kind. In the last hundred years it has grown from small clusters of generation and loads, into large networks containing millions of elements and spanning entire continents. Recently, the increasing deployment of distributed generators (DGs) has triggered a grid transformation from a rigid to a flexible and de-centralized structure. Microgrids are an essential element in this transformation because by grouping DGs and loads into controllable units, they can provide a coordinated response to maximize their impact on the grid.

Microgrids are inherently different from the larger grid. This thesis shows how by challenging the paradigm of constant frequency and voltage operation, a new strategy can be implemented to achieve an improved response after large disturbances without compromising safety. Large disturbances are commonly encountered in the grid and disrupt the power balance that is required for a reliable operation. If the imbalance is large enough and the proper actions are not taken, then a blackout will occur, affecting millions of people and creating a severe economic impact.

To demonstrate the advantages of the proposed operation strategy, two large disturbances are studied: a fault in the distribution network that creates a reactive power imbalance due to induction motor stalling, and a sudden change in generation or consumption that leads to a real power imbalance. In the first part, a framework is created to study fault events and then used to describe a fault recovery strategy that expands the stability region of the system. In the second part, the proposed operation strategy is presented as a new control technique that allows energy to be extracted from the induction motors in the system to achieve an inertial response and provide frequency regulation. All the results are validated using a microgrid experimental set-up that was built as part of this thesis.
Acknowledgments

Five years ago when I decided to pursue a Ph.D. at MIT, I had a rough idea of what the process was going to entail, but I never imagined that I was going to join this fantastic community with so many brilliant and enthusiastic people. I owe my gratitude to all of them who helped me in this process and made it one of the best experiences of my life.

My deepest gratitude goes to my advisor Prof. James L. Kirtley for his constant support and patience in getting this work done. He was always prompt to provide suggestions and discuss ideas, but also allowed me to explore the field myself so I could build confidence as a researcher. I could not imagine a better way to go through this process. My thesis committee members, Prof. Konstantin Turitsyn and Prof. Steven B. Leeb were also instrumental in accomplishing this goal. They were always encouraging and open to discuss ideas ranging from Lyapunov functions to phasor measurement units. I would also like to thank Prof. David J. Perreault from whom I had the privilege to take two classes that were fundamental in my development as an electrical engineer.

I want to give special thanks to my friends and colleagues: Mike Po-Hsu Huang, Richard Zhang, Arijit Banerjee and Colm O’Rourke. All of them were a source of unlimited support and great ideas, and I can not imagine how I would have finished this work without their involvement. Many thanks as well to my colleagues from Masdar Institute of Science and Technology: Prof. Mohamed Elmoursi, Prof. Jimmy Peng, Prof. Mohamed Al-Hosani and Salish Maharjan, and to the people that worked in the microgrid experimental set-up at some point during the last five years: Edwin Fonkwe, Ben Thipok Rakamnouyt, Mohammad Qasim, Michael Zieve, Jared Monin, Gavin Darcy, and Nelson Wang.

Perhaps one of the best decisions made during the Ph.D. was to be part of the Laboratory for Electromagnetic and Electronic Systems (LEES), a large space in the basement of building 10 where the most creative ideas come to fruition. Thanks to all my colleagues and friends from LEES from whom I learned significantly and enjoyed so many joyful moments: John Donnal, Juan Santiago, Wardah Inam, Kendall Nowocin, Yiou He, Jin Kim, Matt Angle, Peter Lindahl, Jiankang Wang, Sam Gunter, Rakesh Kumar, Manuel Gutierrez, Jin Moon, Anas Al-Bastami, Alex Jurkov, Minjie Chen, Lisa Fernandez, Alex Hanson, Seungbum Lim and
Dave Otten. It was fantastic to know all of you and I am sure everyone is on the path to a fulfilling career.

Two other groups were also of significant importance for me during the last five years. First, my sincere gratitude to Mac McQuown and Craig Wooster from the Stone Edge Farm Microgrid project where I had the pleasure to be an intern, an experience that represented an inflection point in my professional career. And second, to the MIT Energy Club were I participated in many roles and where I met people that were key in my entrepreneurial adventures, in particular, Bessma Aljarbou, Albert Chan and Aly El-tayeb.

When looking back at the many hours of hard work and frequent frustrations that one encounters during the doctoral studies, it is hard to imagine that I would have been able to go through it without the constant love and support of my wife Andrea, who inspired and motivated me throughout the process. My parents Jorge and Elia, and my sister Andrea, were also a source of love and inspiration that pushed me to keep working hard all these years. Also important was my “elected” family at MIT formed by my friends that I met early during the Ph.D. (or even before) and with whom I keep a positive and nurturing friendship: Francisco Morocz, Sarah Spencer, Leon Valdes, Nik Nair and Gabriel Sanchez. And finally to the great community of Spanish House La Casa, where I had the privilege of serving as GRT for the last three years, and that created a home away from home.

I can not finish this section without expressing my gratitude to my sponsors CONACYT, Masdar and ABB-SAC as without their financial support this would not had happened.
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Chapter 1

Introduction

1.1 Background and Motivation

The electric grid is one of the major achievements of human kind. It started as dispersed clusters of electrical generators and loads, and transformed, over several decades, into a large and complex system with millions of interconnected elements spanning countries and, in some cases, entire continents [1, 2, 3]. Considering the size and complexity of the system, and the numerous events that can lead to a catastrophic failure, it is notable that continuous and nearly instantaneous power balance between generation and consumption can be achieved in a secure and reliable way. Such a feat is achieved by a careful study of the system dynamics under the many phenomena involved in its operation, ranging from electromagnetic effects occurring in a few milliseconds to monthly demand patterns [1, 4, 5]. Studying all these aspects simultaneously is unfeasible, but the concept of model order reduction, which focuses the analysis in one aspect while simplifying the rest, has proved to be a valuable tool and has led to well-known stability assessment techniques. Those techniques are usually performed depending on the system variable that might experience an instability, being the electrical angle, frequency or voltage, and the magnitude of the disturbance leading to linear small-signal or non-linear analysis [1, 4, 5].

Power system stability is a topic that has been widely studied in the past, but in recent years the operational structure of the grid has changed creating new challenges. Not only have end customers become active participants with the use of distributed generation (DG) [6], but also the idea to subdivide the network into smaller controllable entities called microgrids has gained strength [7, 8, 9]. Multiple conditions have triggered this transformation [2]: (a) Climate change has been recognized as a major threat and ambitious targets for carbon emissions reductions have been set worldwide [10] leading to increased penetration of
renewables; (b) solar and wind energy prices have been in steady decline for decades [11] making them competitive with traditional sources and, due to their dispersed nature, have been adopted by end energy users in the form of distributed generators; and (c) large efforts have been done to promote grid modernization by institutions such as Department of Energy (DOE) which has recognized it as one of the key milestones for a reliable energy future [12].

Due to the previous reasons, distributed generation, often in form of microgrids, is expected to become more common in the next years, which will increase the renewable penetration and create more resilient and efficient networks [6, 13, 14]. Numerous operational and control strategies for microgrids have been suggested in the past. For example, previous work presented in [15, 16, 17, 18, 19] explored methods for transitions between grid-connected and islanded operation, hierarchical control structures, and power sharing techniques, among many other topics. In this dissertation, a new operation strategy for microgrids intended for emergency situations is presented. It consists in allowing the voltage and frequency to settle at an off-nominal value for some period of time after a large disturbance. This strategy, named here “Flexible Voltage and Frequency” (FVF), challenges the conventional paradigm of power systems that require these parameters to be always close to their nominal values. Such limitation is required in large power plants to avoid resonances in turbines that can occur when they operate far from their nominal operation point [20], but the same is not true in microgrids where most of the sources connect to the system through inverters. In the two parts that comprise this dissertation, this strategy will be shown to have beneficial effects on the voltage recovery after a fault in the distribution network, and on the frequency stability of systems with low inertia.

The first part of this thesis deals with an example of a large disturbance typically seen in distribution networks: a fault in one or more of the lines causing a low voltage for several seconds and risking a voltage instability that can lead to a blackout. Power grid blackouts are events with large human and economic cost [21] and extensive efforts are continuously done to minimize their occurrence. Several techniques such as enhanced equal area criterion [1] and energy methods [22] have been developed to assess stability and design protections, but they are aimed at large power systems and the analysis focuses on the dynamics of synchronous generators. Considering that around 80% of power outages are caused by disturbances in the distribution network [3], understanding the impact of faults near the loads is essential for a proper stability assessment. In these cases, the load characteristics play a critical role in assessing the voltage behavior [23, 24]. Although in some analyses a static load representation given a power-voltage relationship can be enough to capture the desired phenomena [4], there are cases in which a dynamic model of the load is necessary. An example of such cases is the so-called fault-induced delayed voltage recovery (FIDVR) which has been recognized as a common contingency, and it can lead to load shedding due under-voltage protection.
or, even worse, to voltage collapse [25, 26, 27, 28, 29]. This has been observed in numerous cases such as, for instance, in the Arizona Public Services network in 2003 when a fault and subsequent FIDVR left more than 90,000 customers without electricity [25]. It is well known that FIDVR is caused by induction motor (IM) loads stalling [25] so that a proper dynamic model of these loads, which represent more than 45% of the total electricity consumption [30], is required for an assessment of this contingency at its root cause. This thesis demonstrates that by using the “Flexible Voltage and Frequency” strategy, the maximum duration of a fault that the system can withstand before causing FIDVR or voltage collapse can be significantly increased.

The second part of the dissertation extends the application of FVF operation to the frequency stability arena. The proposed concept consists in allowing frequency to vary inside a microgrid to extract some of the energy stored in the rotating mass of induction motors to provide an inertial response and help in the frequency regulation of the system where it connects. Frequency stability problems arise in small power systems and microgrids where there is not much inertia because only a small fraction of the power is created by generators possessing large rotating mass. A reduced inertia leads to larger under-frequency excursions when sudden changes of load or generation occur. This issue has become relevant in recent years as the penetration of renewable energy sources, that interface the system through inverters, has increased. These sources contribute to the total electricity generation but add no inertia and, due to their intermittent nature, cause rapid changes in the system input power. Although inverters are able to provide frequency response by adjusting their output to changes in system frequency, that typically adds cost as some form of energy storage is needed. By using the FVF strategy, the energy stored in induction motors can be extracted controllably to provide the desired inertial and frequency response.

1.2 Literature Review

To properly discuss the contributions presented in the next chapters, a literature review is done for each of the two parts that constitute this thesis.

1.2.1 Voltage Recovery After a Fault

A fault in an electrical network causes a voltage sag in one or more lines of the loads and may lead to an unstable condition. IM loads are of particular interest because they represent more than 45% of all worldwide electricity consumption [30] and have been identified as the leading cause of fault induced delayed voltage recovery (FIDVR) in distribution systems by numerous studies [25, 26, 27, 28, 31]. FIDVR occurs when motors stall causing them to draw large currents which depresses the voltage around them. In some cases, protections will eventually disconnect some of the motors. However, their action is slow, typically in the
order of seconds [32], so they can not prevent FIDVR [25]. Besides the IM loads, distribution networks contain "composite" loads, i.e. a combination of loads with a particular power-voltage relationship [4]. If that relationship is quadratic, then the corresponding ZIP load is constructed with a constant impedance (Z), a constant current (I) and a constant power (P) load.

Previous studies have described techniques to estimate the critical clearing time (CCT) of systems with large IM load penetration using time-domain simulations [33, 34, 35, 36] or analytical techniques [37, 38, 39]. The latter approach is preferred because simulations tend to be time consuming and hard to generalize to scenarios with different parameters and operating conditions. The analytical methods presented in [37, 38] give equations to directly calculate the CCT but only accounted for systems with one IM. In real power systems, a large number of IM loads are connected to a network with some impedance that causes coupling effects. In [39], the authors did account for coupling by presenting an analysis and a CCT estimation algorithm for a system with several induction generators. However, their estimation was based on the assumption that the system is unstable if the net torque of one machine is negative at the fault clearing time, which is a conservative approach and can lead to large errors.

An approach common to all previous analytical studies [38, 37, 39] was to assume that the IM speed decay during the fault is caused only by a reduced electrical torque that leads to a torque imbalance. It will be shown that, immediately after the fault, there is also a transient electromagnetic phase that has an impact on the machine speed. In this transient phase, part of the motor mechanical energy is used to dissipate the trapped flux in the magnetizing inductance, causing a rapid deceleration. This electromagnetic transition is named "braking mode" in this thesis. Neglecting this effect results in an over-estimation of the CCT which depends on the system parameters and operating conditions. A 5-10% error in the prediction can be observed in the reported results of [38, 39], but the error can be as large as 20% in low inertia motors which have been recognized as being more prone to stall [25]. The results obtained in this dissertation demonstrate that the CCT estimation error is significantly reduced if braking mode is accounted for.

Coupled systems with several IM loads are more complex and stability assessment requires the knowledge of the stability region. A stable equilibrium point (SEP), such as the one the system is operating in before the fault, has an associated stability region which is the collection of operation points within a state space which are asymptotically stable. In this context, asymptotically stable points are those for which all the IMs will return to normal operation once the fault is cleared, allowing the system to recover nominal conditions without the need of load shedding. The boundary of that region is the stability boundary or separatrix. The CCT can be computed as the precise time during the fault-on trajectory when the system states, given by the motors' speed, cross the stability boundary. Although there is no analytical expression for the stability boundary of a non-linear system, in this dissertation a study of a prototypical two-machine system is done
to reveal some of its characteristics and its relationship with voltage and current as system parameters vary. This study is presented in the second chapter and it is validated using an experimental set-up and time domain simulations for a case study with numerous IMs. Although [39] presented a good starting point in the visualization of these systems, the figures presented here identify for the first time the stability boundary of these systems and its relationship to the electrical parameters.

In the third chapter, an analysis of the stability assessment of these systems is presented. The analysis is divided in three parts: IM rotor speed stability, source current stability and voltage stability. The first refers to the ability of all IM loads in the system to recover after a fault and return the system to a SEP. The idea of studying rotor speed instability as a new concept was proposed in [40] for induction generators, while IM stalling has typically been accounted for as part of voltage stability studies [1, 4], usually without much detail on the load dynamics. The decision to study the rotor speed stability separately from its effect on voltage is two-fold: (a) when a motor stalls, even if voltage stays within acceptable limits, protections can be activated, risking the disconnection of large areas and severe impact on the end user; and (b) the stability analysis can be posed as a dynamical system which is a first step for direct stability assessment analysis. For rotor speed stability, this thesis proposes a method that includes the calculation of a lower and upper bound for the CCT, and an approximation that correctly tracks its behavior as the system parameters change.

For large power systems constructed with synchronous generators, several techniques such as the ones presented in [22, 41] have been developed to calculate the CCT using direct methods for stability assessment, typically employing energy-like functions. In distribution networks and microgrid systems, and in general when considering IM loads, the same techniques will give extremely conservative bounds due to the dissipative nature of the system, arising from the need of a rotor resistance to generate electric torque and the small X/R ratio of the transmission lines. The mathematical formulation that serves as a first step to obtain a direct method for these systems is presented in Appendix B.

The second stability assessment deals with the source current limitation. If during the recovery period, the current from the system sources exceeds rated levels for a long enough period of time, the sources can trip on over-current and affect a large area. Finally, the last assessment corresponds to the voltage stability of the system. It is done by performing a reactive power balance analysis at each bus of the network with and without stalled motors, in order to identify if the operation point is within the acceptable voltage limits and if there is risk of voltage collapse. This technique has been described before as the $dQ/dv$ criterion [4]. It was observed that when a few small motors stall, there is no risk of voltage instability and the network can just wait for the IMs to be disconnected or reduce their mechanical torque to recover nominal conditions. The concern arises when either large motors or a large number of small motors stall, because there is significant risk of operating the system outside the acceptable voltage band. Moreover, if there is a large penetration...
of constant power loads in the system, then there is a possibility of voltage collapse.

After the analysis of fault trajectories and stability assessment techniques are presented in Chapters 2 and 3, a fault-recovery strategy for microgrids is proposed in Chapter 4. The incorporation of microgrids into distribution systems has been proposed in recent decades as a way to increase the system resiliency [7, 8]. Microgrids that are able to power their loads when the main grid is down can increase the energy availability if they are capable of transitioning from a grid connected to an islanded mode in any situation. During planned events, this process can be achieve seamlessly [15, 16] by disconnecting the grid before the electricity interruption. However, when the cause is a large disturbance in the distribution network such as a fault, then islanding is performed after the microgrid has been disturbed to a point from which it might not be able to recover without shedding some load.

Many have previously studied the behavior of microgrids after faults and have proposed control strategies to ride through or clear it by islanding [18, 34, 35, 42, 43, 44]. In [18, 42, 43], the analysis focused on fast electromagnetic transients caused by different types of faults and they proposed control techniques for DGs to mitigate the disturbance effect and expedite the voltage recovery. Their analysis, however, accounted only for static loads. Other work did account for IM loads in their analysis, as for example [25, 35] where several control techniques were presented for the grid operator or individual DGs.

The operation strategies that have been proposed to improve a system response after a fault consist essentially in using DGs or FACTS to provide reactive power support [34, 35, 45, 46]. This is useful in the context of IM stalling and FIDVR because by providing reactive power, the voltage at the IM load terminal is increased leading to larger electrical torque. However, these techniques depend on the network characteristics, and its impact on voltage stability can be limited in cases where the impedance is large or the source is current limited. Variations of the concept have also been presented, as for example in [34], where the q-axis component of the current was given priority during the fault-recovery process of an inverter-based microgrid with IM loads. From a more general perspective, [25] proposed a variety of techniques to avoid FIDVR including solutions at the grid level such as quicker clearing of faults, network reconfigurations and addition of capacitors, as well as unit level solutions such as faster breakers for low inertia motors, control for the IM mechanical port and use of motor drives in more IM loads, all of which add cost to the system or require coordination in large geographical areas.

Chapter 4 presents a novel approach to the problem which can be employed as a complement to previously proposed strategies. The main idea is based on the concept of “Flexible Voltage and Frequency” operation and consists in changing the inverters' set-points to low frequency and high voltage values during a fault, so that the stability region of the system with high penetration of IM loads can be expanded and conditions that would be unstable can be recovered. This strategy can be pre-programmed in the DGs controllers as
the default response when a fault is detected so that it does not require information exchange between the sources regardless if they operate in droop or master-slave control architectures.

The proposed strategy also benefits the distribution network where microgrids are connected. By islanding the microgrids, implementing the fault-recovery strategy and avoiding IM stalling, the currents flowing through the network are reduced, leading to increased voltages and faster IM recovery. After conditions are back to nominal, the microgrids can reconnect to the network returning the system to the pre-fault SEP. Using experimental results and time domain simulations, the strategy is shown to yield an improved response of the system after a fault.

1.2.2 Frequency and Inertial Response

Chapter 4 demonstrates that the concept of “Flexible Voltage and Frequency” operation can be applied to improve the stability of a system after a fault. The fifth chapter extends the same concept to a different application: providing frequency support. Maintaining the electrical frequency within a tight range is one of the fundamental operational principles of power systems [47, 48, 49]. A frequency far from its nominal value poses a risk to power plants [20], can destabilize a system [48] and might lead to the disconnection of DGs and loads [50]. Several standards such as [51] and [52] have been put in place to establish limits within which all equipment should be able to operate, and large power systems might follow even stricter grid codes so that they rarely operate beyond a half a percentage point from nominal. To achieve such precise control it is necessary to maintain balance between power generation and consumption at all times, which requires the implementation of control actions at different levels [1, 4, 47, 53]: (a) the first level is the inertia and primary control, also called frequency or governor response, which adjusts the source output when a change in frequency is detected; (b) above that level, there is a secondary control which involves central dispatch by an automatic generation control (AGC); and (c) finally a tertiary control which is related to the management of the spinning and non-spinning reserves in the system.

The work on this thesis focuses on the first level of control that involves the inertia and the governor response of sources. The total system inertia determines the rate of change of frequency immediately after disturbances such as large changes in load and generation, and low inertia will lead to larger under- and over-frequency excursions [49] risking the activation of protections. To prevent this, regulations are typically implemented [53] to force generators in some power plants to provide primary frequency control. This topic has gained significant attention lately as renewable deployment into the grid has increased which removes conventional generators and reduces the total inertia of the system. Renewables such as solar photovoltaics (PV) and wind turbines interface with the system through power electronic converters that do not have an
intrinsic relationship between output power and frequency. In the case of PV, its lack of dispatchability and stored energy complicates its use for primary frequency regulation. However, techniques have been suggested to curtail its output when there is an excess of generation [54] and to operate it below the maximum power point [55] essentially sacrificing efficiency but gaining dispatchability. Techniques to use the kinetic energy in wind turbines blades to provide frequency response have also been proposed in several studies [49, 56, 57] leading to a concept commonly known as synthetic or virtual inertia. The main idea in all those techniques consists in either slowing down the wind turbine to get extra power at the output or speeding it up to reduce the output and store energy. The same concept of synthetic inertia can be obtained with energy storage systems such as batteries or flywheels that connect to the network through inverters, but these elements significantly increase the cost of the systems.

An alternative method to provide frequency regulation is through demand response, which consists in intentionally disconnecting loads or reducing their demand when consumption is larger than generation, and doing the opposite when there is excess of generation [58, 59]. Demand response can be implemented by a centralized hierarchical control structure [60] where a load manager sends signals to a diversity of loads to adjust their consumption, or using a distributed approach [61] where only local measurements are needed. Microgrids open new opportunities for demand side management as they are systems that integrate renewables with other distributed generators, loads and energy storage devices in a single controllable entity [8] so that more information is available locally. An element with rotating mass typically found in microgrids and distribution networks is the induction motor load. As mentioned before, these loads are used in a variety of applications and represent around 45% of all worldwide electricity consumption [30]. IM loads contribute to the total inertia of the system because their consumption changes with the electrical frequency. That property is useful, but by simply following the grid frequency there is no way to manipulate or shape the power consumed by the motors in order to use their stored energy in a controllable manner. In Chapter 5, the concept of "Flexible Voltage and Frequency" operation is used and posed as a control technique for microgrids to provide inertial and frequency response by controlling the stored kinetic energy in the IM loads. This strategy is implemented without affecting the mechanical output torque of the motors and is intended for microgrids that connect to the network through an asynchronous link, a transformer that decouples the frequencies at both ports. This can be achieved with several existing technologies such as variable frequency or solid state transformers. With this strategy, the microgrid electrical frequency is actively controlled within a specified band, so energy can be taken from or given to the IM rotor. The results are shown to give the expected inertia and frequency response and they are validated numerically and experimentally.
1.3 Contributions

The work presented in this thesis contributes to the state of the art by presenting a novel operational strategy for microgrids in which active control of voltage and frequency is used for an improved response after a large disturbance. The particular contributions are described next, in the context of the disturbance under study:

1. In the field of voltage instabilities due to reactive power imbalance caused by induction motor stalling, the contributions are two:
   (a) The creation of a new and complete framework for stability assessment of systems with induction motor loads. It includes details related to electromagnetic effects, stability regions, current limits of inverters, effect of network elements, and the relationship of voltage instabilities and IM stalling.
   (b) The proposition of a fault-recovery strategy to increase the critical clearing time and expedite recovery of systems with induction motors by controlling their voltage and frequency.

2. In the field of frequency instabilities due to real power imbalance in low inertia networks, the contribution is:
   (a) The proposition and development of a technique to extract energy from IM loads to provide an inertial response and frequency regulation

In developing the work presented in this dissertation, many insights were obtained that led to interesting new concepts that also contribute to the state of the art on these fields. In particular, the following are noted:

- The concept of a “braking mode” transition that occurs immediately after an induction motor experiences a voltage sag and that dissipates part of the rotor kinetic energy in the machine resistances using the trapped flux in the magnetizing inductance as excitation.
- The identification of the exact stability boundary of a two motor system, the saddle-node bifurcation that occurs when coupling is large, and the effect of the system parameters on the stability region.
- The computation of the maximum current boundary and the effect of stability region contraction because of source regulation beyond that boundary.
- The concept of cascade recovery of induction motors that occur in coupled systems.
- The method to relate induction motor stalling and voltage instabilities using reactive power balance analysis.
- The creation of performance maps for the fault-recovery strategy that help in the selection of control parameters for its proper implementation.

- The concept of shaping the power consumed by induction motors by controlling frequency to use them as energy storage devices.

1.4 Thesis Organization

As mentioned above, the thesis is organized in two parts, the first dealing with voltage stability due to induction motor stalling after a fault and comprising the next three chapters, and the second dealing with frequency stability in low inertia systems developed in the fifth chapter. Voltage stability issues are studied in more detail because the problem required a new framework for its study, while the problem of frequency stability, as dealt with here, has been studied in detail in the past (as for example in [4]). Both parts are linked together by the concept of “Flexible Voltage and Frequency” operation which is used to improve the system response after a large disturbance.

The four chapters that comprise the thesis are the following:

- Chapter 2 analyzes the behavior during and after a fault of a system with multiple induction motor loads. It includes a calculation of the electromagnetic and electromechanical transients that shape the fault-on trajectory, the identification of the stability region of the post-fault system, and a study of the impact of different system elements.

- Chapter 3 builds on the insights from Chapter 2 to propose a stability assessment method for systems with arbitrary number of motors. It includes a characterization of the stability region to provide bounds for the critical clearing time, the inclusion of current calculation to assess the possibility of source tripping, and a link from induction motor stalling to voltage instability.

- Chapter 4 poses the FVF operation as a fault-recovery strategy for the systems studied in the previous chapters and shows that the stability region of the post-fault system can be significantly expanded. An analysis of its impact on IM stalling and voltage stability is assessed, and guidelines are given for the selection of important parameters.

- Chapter 5 extends the FVF operation to the frequency stability domain and poses it as a control technique. With it, microgrids can provide inertia and frequency response. All the concepts required for its implementation are explained and guidelines for the selection of parameters is given.
All the results shown in these four chapters are validated experimentally. A description of the experimental set up, the model used for simulations, and the mathematical analysis for direct stability assessment are all described in appendices. The three appendices of this dissertation are:

- Appendix A presents the details of the microgrid experimental set-up used to validate some of the concepts presented in the dissertation. The experimental set up has many more capabilities than the ones used for this work and some of its features such as its control architecture and its modularity are noteworthy.

- Appendix B presents the problem of IM stalling formulated in three forms: (a) Port-Hamiltonian, (b) Brayton-Moser and (c) Contraction Theory

- Appendix C describes the model used to generate the results in the first part of the thesis.
Chapter 2

Analysis of Fault Events in Systems with IM Loads

2.1 Introduction

In this chapter, the framework to study systems with induction motors is introduced. It is done by presenting an analysis of the electromagnetic and electromechanical dynamics that the system is subject to during and after a fault. The chapter is organized as follows:

- Section 2 presents a detailed description of the scenario under study including the system construction, the phases that it goes through during the fault event, a description of the models used to study it, and the simplifications done at each phase.

- Section 3 describes the behavior of the system while the fault persists, a transition that it is called the fault-on trajectory. It includes the analysis of the electromagnetic and electromechanical effects that take place.

- Section 4 presents an analysis on the post-fault system stability region defining important concepts such as the equilibrium points, the nullclines, the stability boundary and the maximum current boundary. A study is also done on the effect of different parameters in the stability regions.

- Section 5 closes the chapter by validating the concepts described above using simulations and experimental results.
2.2 Scenario Description

The generic system under study is shown in Fig. 2.1. It contains one source at the slack bus which could be a substation, a connection point to a larger grid, or a distributed generator operating in "Grid-forming" or "Master" mode. This source sets the voltage and frequency of the system and connects with other elements through a network that is parametrized by its branch-node incidence matrix and the resistances and inductances of each branch. Since the study focuses on distribution networks and microgrids, the assumption is made that branch capacitance is negligible. At each node of the system, a generic bus constructed by an IM load, a ZIP load and a DG is connected. The DGs in these buses are operated in "grid-following" or "slave" mode, although a generalization to a droop controlled system can be made by adjusting the sources' output to changes in voltage and frequency.

![Network Diagram](image)

Figure 2.1: Schematic view of the system under study that connects a voltage source with buses containing DGs, ZIP loads and IM loads through a network.

The first part of the thesis, that spans this and the next two chapters, studies the system behavior after a fault occurs somewhere in the network. In transient stability analysis of power systems, the fault process spanning from normal operation to system collapse or recovery is studied by dividing the event in the three phases [22] that are shown in Fig. 2.2. Although other references such as [1] use different names for these phases, they are essentially the same, and in this dissertation the framework presented in the figure is used.

![Phase Diagram](image)

Figure 2.2: Diagram of the three phases used to study a fault event.
The details of each phase are the following:

- **Pre-fault phase:** It corresponds to the conditions before the disturbance, when the system operates in a stable equilibrium point with all motors rotating at a speed determined by their mechanical torques but close to the nominal value.

- **Fault-on phase:** It is related to the period of time while a fault persists. During this phase, most buses experience a voltage sag causing the IMs to have a reduced voltage at their terminals which leads to low electrical torque and a speed decay.

- **Post-fault phase:** It refers to the system behavior after the fault is cleared. In this period, the bus voltages and the IMs' electric torque increase, but the low rotational speed of some of the motors causes large real and reactive power to be demanded. Timing is important while studying the behavior of the system during this period. If the fault clearing occurs before the critical clearing time (CCT), then all the IMs will go back to their SEP after some time. This recovery is usually fast enough so that there is no risk of activating under-voltage or over-current protections. If clearing occurs after the CCT then some of the IMs will stall, leading to a depressed voltage, large currents and the need for some action to take the system back to the SEP.

Between pre-fault and post-fault operation, the IM loads experience both electromagnetic and electromechanical transitions that describe their dynamics through the process. Those transitions have different time constants so that a model order reduction can be done to study them separately, leading to different models depending on the effect being investigated:

- For electromagnetic effects, the magnetic flux dynamics are of primary interest so that the full model of the IMs is required.

- For electromechanical effects, the focus is on the mechanical states. All electrical quantities such as fluxes inside the machine are smooth, have much faster dynamics and can be considered almost instantaneous which leads to a quasi-steady state first order model.

The electromagnetic analysis can be performed at each motor independently because the only required information is the voltage at the terminals. The full model to study them uses five states, namely the d- and q-axis stator fluxes, the d- and q-axis rotor fluxes, and the rotational speed.

The electromechanical analysis uses only the IMs rotational speed to describe the system operation. However, the model has to consider the coupling between all the motors that arises due to the network impedance and the sources' characteristics. To explain the coupling effect consider, for instance, a scenario
with two IM loads connected close to each other but located towards the end of a feeder line so that there is significant impedance between them and the stiff voltage source. If one of the motors has a significant power consumption, then it depresses the voltage and leads to a reduced electrical torque for the other IM. The system dynamics including this coupling effect can be described by a set of first-order models, one for each IM:

\[
\frac{d\omega_{mi}}{dt} = \frac{1}{2H_i} (T_{ei}(\omega_m) - T_{mi}(\omega_{mi}) - F_i\omega_{mi})
\]  

(2.1)

where \(\omega_{mi}\) is the motor rotational speed, \(H_i\) the per unit inertia given in seconds, \(F\) is the friction coefficient, \(T_{ei}\) the electrical torque and \(T_{mi}\) the mechanical torque. Coupling can be seen in Eq. 2.1 because the electrical torque is a function of the rotational speed of all the machines in the system \(\omega_m = [\omega_{m1} \omega_{m2} \cdots \omega_{mN}]^T\). To compute the electrical torque for a given operation point (specified by \(\omega_m\)), the entire network has to be solved to compute the bus voltages and the currents going into the IMs. This calculation can be done assuming that there is no change in the rotational speed while the voltages and currents settle, which is the essence of the quasi-static approximation. Knowing the current and voltage at the terminals of each IM, the electric torque can be computed as:

\[
T_{ei} = \frac{r_{ri}}{s_i} \mid i_{ri} \mid^2
\]

(2.2)

where \(r_r\) is the per unit rotor resistance, \(s_i = \omega_e - \omega_r\) is the motor slip, \(i_r\) is the per unit rotor current, \(\omega_e\) the per unit electrical frequency, and \(\omega_r\) the per unit rotational speed.

### 2.3 Fault-on Trajectories

During the analysis of the fault-on phase, the main interest is on the fault-on trajectories, which refer to the path along the state space that the system will travel while the fault persists. The behavior of an IM during a fault can be described by two phenomena: (a) An electromagnetic transient phase identified here as "braking mode" in which the IM dissipates part of its kinetic energy while the trapped flux in the internal inductances decays, and (b) a mechanical torque that is larger than the electric torque.

Previous work such as [26, 37, 38, 39] have investigated the effect of the torque imbalance, but they neglected the electrical transient phase. Its inclusion results in a more accurate estimation of the IMs' speeds during the fault followed by a better prediction of the system behavior after the fault is cleared. The scenario presented Fig. 2.3 illustrates the importance of this concept. It shows a three-phase machine with constant mechanical torque operating at a SEP when a fault occurs at \(t = 0\). For simplicity in the explanation, the scenario considers a three-phase fault that results in zero voltage at the IM terminals and
that is not cleared within the shown time period. The solid blue line corresponds to the full state model simulation result and shows the rapid deceleration that occurs immediately after the fault and the linear decay due to torque imbalance that follows it. The dashed red line indicates the estimated speed decay if only the torque imbalance is accounted for, which leads to a significant error. By defining the speed after braking mode (identified by a black square in the figure), and assuming that the speed decay due to torque imbalance starts there, then the speed error is made negligible as seen with the black dashed line. This effect becomes particularly important in IMs with low inertia.

The same two phenomena occur in single phase motors in essentially the same way. The only difference is that in single phase motors the dissipated energy during braking mode depends on the point along the voltage waveform when the fault occurs. This is illustrated in Fig. 2.4 where three time domain simulations of the full state model of a single phase motor are presented for faults occurring at the peak of voltage ($\theta=90^\circ$), at the voltage zero crossing ($\theta = 0$), and at midpoint between them ($\theta=45^\circ$). The maximum speed decay corresponds to faults at the voltage zero crossing because that is when the peak of magnetizing flux occurs, so that the braking mode transition last longer.
2.3.1 Electromagnetic Transients

2.3.1.1 Induction Motor Models

The dynamics of a three-phase induction motor with a balanced supply are given by the following non-linear equations:

\[
\frac{d}{dt} \begin{bmatrix}
\frac{1}{\omega_0} \psi_{ds} \\
\frac{1}{\omega_0} \psi_{qs} \\
\frac{1}{\omega_0} \psi_{dr} \\
\frac{1}{\omega_0} \psi_{qr} \\
\omega_m
\end{bmatrix} = \begin{bmatrix}
u_{ds} + \omega_e \psi_{qs} - r_s i_{ds} \\
u_{qs} - \omega_e \psi_{ds} - r_s i_{qs} \\
(\omega_e - \omega_m) \psi_{qr} - r_r i_{dr} \\
- (\omega_e - \omega_m) \psi_{dr} - r_r i_{qr} \\
\frac{1}{2H} (T_e - T_m - F \omega_m)
\end{bmatrix}
\]  

(2.3)

where: \( \psi_s = \{\psi_{ds}, \psi_{qs}\} \) are the stator magnetic fluxes in d-q reference frame, \( \psi_r = \{\psi_{dr}, \psi_{qr}\} \) are the rotor magnetic fluxes also in d-q reference frame, \( \omega_m \) is the mechanical rotor speed, \( \omega_e \) is the electrical frequency, \( r_s \) the stator resistance, \( r_r \) the rotor resistance, \( i_s = \{i_{ds}, i_{qs}\} \) the stator currents, \( i_r = \{i_{dr}, i_{qr}\} \) the rotor currents, \( F \) the friction coefficient, and \( \omega_0 \) the nominal electrical frequency. The model is presented in normalized values so that all quantities are given in per-unit (except \( \omega_0 \)). Care should be taken in calculations that pertain to the entire network to avoid adding quantities in different bases. This is explained in Appendix C that describes the mathematical model. Notice also that, in per unit quantities, the number of poles does not appear in the equations, but it is used to normalize the mechanical speed \( \omega_m \).
To study the electromagnetic transients, the model is reduced by assuming that they are fast enough that the mechanical state does not change significantly. This might be seen as an odd assumption given that it is actually the rotational speed decay during these transitions that is of interest. However, the speed decay is not expected to be more than 15% of the nominal, so that even if the assumption leads to an error, it still provides a better estimation than just ignoring the effect altogether, as it will be shown in Section 2.5.1.

The advantage of this model order reduction is that, if mechanical speed is constant, then the dynamics are linear and linear system analytical tools can be used.

To start the analysis, the model is first expressed using currents as the dynamic states instead of fluxes. This is for convenience in the calculation of dissipated energy as it will be shown below. Assuming that the inductors do not saturate, there is a linear relationship between flux and current so that the transformation is simple. Performing small signal analysis around the SEP in Eq. 2.3 gives:

$$ \frac{dE}{dt} i = \tilde{A}i + \tilde{B} $$

with

$$ E = \frac{1}{\omega_0} \begin{bmatrix} l_s & l_m \\ l_m & l_r \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} $$

$$ \tilde{A} = \begin{bmatrix} -r_s & x_s & 0 & x_m \\ -x_s & -r_s & -x_m & 0 \\ 0 & s \cdot x_m & -r_r & s \cdot x_r \\ -s \cdot x_m & 0 & -s \cdot x_r & -r_r \end{bmatrix} $$

$$ \tilde{B} = \begin{bmatrix} \epsilon \cdot v_{ds} & \epsilon \cdot v_{qs} & 0 & 0 \end{bmatrix}^T $$

$$ i = \begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr} \end{bmatrix}^T $$

where $l_s, l_r, l_m$ are the per unit stator, rotor and mutual inductances, $x_s = \omega_0 l_s, x_r = \omega_0 l_r$ and $x_m = \omega_0 l_m$ are the corresponding per unit reactances, $\epsilon$ is the magnitude of voltage during the sag, and $s = (\omega_e - \omega_r) / \omega_e$ is the slip. Let $A = E^{-1} \tilde{A}$ and $B = E^{-1} \tilde{B}$, then the equations can be written in the regular ODE form with currents as states:

$$ \frac{d}{dt} i = Ai + B $$

41
Single phase motors can also be written in this form, although the dynamics depend on the technique used to generate the rotating magnetic field required to produce starting torque. The two most common implementations are: (a) capacitor-start, (b) capacitor-start-run. In both cases, a capacitor is connected in series with the auxiliary winding to obtain a voltage in quadrature with the main one, although, in the capacitor-start configuration, it is disconnected after the motor has reached a certain speed.

In general, the model for single phase motors is presented in stator reference frame with the \( \alpha \)-axis representing the main winding and the \( \beta \)-axis the auxiliary winding resulting in:

\[
\frac{d}{dt} \begin{bmatrix} (1/\omega_m) \psi_{\beta s} \\ (1/\omega_m) \psi_{\alpha s} \\ (1/\omega_m) \psi_{\beta r} \\ (1/\omega_m) \psi_{\alpha r} \\ \omega_m \end{bmatrix} = \begin{bmatrix} u_{\beta s} - r_s i_{\beta s} \\ u_{\alpha s} - r_s i_{\alpha s} \\ n \cdot \omega_m \psi_{\alpha r} - r_R i_{\beta r} \\ -n^{-1} \cdot \omega_m \psi_{\beta r} - r_R i_{\alpha r} \\ 1/2H (T_e - T_m - F_\omega_m) \end{bmatrix}
\]

where \( r_s, r_r \) are the stator and rotor resistances as seen in the main winding, \( r_s, r_R \) are the stator and rotor resistances as seen in the auxiliary winding, and \( n \) is the turns ratio between auxiliary and main windings.

In the capacitor-start implementation, when the capacitor is disconnected, the constraint \( i_{\beta s} = 0 \) has to be set as an algebraic constraint on the equations. This eliminates one of the dynamic equations by relating two states as \( \psi_{\beta s} = (L_{ms}/L_R) \psi_{\beta r} \). The linearized equations in terms of currents that result from assuming constant mechanical speed take the form:

\[
E = \frac{1}{\omega_0} \begin{bmatrix} l_s & 0 & l_{ms} \\ 0 & l_R & 0 \\ l_{ms} & 0 & l_r \end{bmatrix}
\]

\[
\hat{A} = \begin{bmatrix} -r_s & 0 & 0 \\ n \cdot \omega_m l_{ms} & -r_R & n \cdot \omega_m l_r \\ 0 & n^{-1} \cdot \omega_m l_R & -r_r \end{bmatrix}
\]

\[
\hat{B} = \begin{bmatrix} \epsilon v_{\alpha s} & 0 & 0 \end{bmatrix}^T
\]

\[
i = \begin{bmatrix} i_{\alpha s} & i_{\beta r} & i_{\alpha r} \end{bmatrix}^T
\]

where \( l_s, l_r \) and \( l_{ms} \) are the per unit stator, rotor and mutual inductance of the main winding, and \( l_R \) is the per unit rotor inductance of the auxiliary winding.
In the capacitor-start-run case, the capacitor value is reduced but there is still some current in the auxiliary winding. For simplicity, instead of including the dynamics of the capacitor charge, it is assumed that it only creates a voltage in quadrature at the auxiliary winding. With these assumptions, the reference frame can be rotated along with the electrical frequency to write the equations in dq reference frame obtaining a set of equations equivalent to that in Eq. 2.3. The main difference, in this case, is that the d and q-axis impedances are not necessarily equal. The matrices for the capacitor-start-run single phase motor are:

\[
E = \frac{1}{\omega_0} \begin{bmatrix}
    l_S & l_{ms} \\
    l_s & l_{ms} \\
    l_{ms} & l_R \\
    l_{ms} & l_r
\end{bmatrix}
\]  

(2.15)

\[
\tilde{A} = \begin{bmatrix}
-\tau_S & x_S & 0 & x_{ms} \\
-x_s & -\tau_S & -x_{ms} & 0 \\
0 & s \cdot x_{ms} & -\tau_R & s \cdot x_R \\
-s \cdot x_{mr} & 0 & -s \cdot x_r & -\tau_r
\end{bmatrix}
\]

(2.16)

\[
\tilde{B} = \begin{bmatrix}
    \epsilon_{uds} & \epsilon_{uqs} & 0 & 0
\end{bmatrix}^T
\]

(2.17)

with \( l_S \) and \( l_{ms} \) are the stator and mutual per unit inductances in the auxiliary winding. The models can be written as in Eq. 2.9 by making \( A = E^{-1} \tilde{A} \) and \( B = E^{-1} \tilde{B} \).

### 2.3.1.2 Speed After Braking Mode

During the braking mode transition, the IM dissipates energy by using the trapped flux in its magnetizing inductance as the excitation. The energy dissipated in the internal resistances is taken from the rotor mechanical kinetic energy, so that the speed decreases rapidly as the stator flux decays. This dissipated energy can be calculated using the dynamics presented in Eq. 2.9. For the fault-on speed trajectory to consider this transition, the “speed after braking mode” is defined. It is the rotor speed that results if braking mode occurs instantaneously. It is calculated by subtracting the dissipated energy from the rotor kinetic energy resulting in:

\[
\omega_{mf} = \left( \omega_{m0}^2 - \frac{E_d}{H} \right)^{\frac{1}{2}}
\]

(2.18)
where \( \omega_{m0} \) is the speed at the pre-fault SEP, and \( E_d \) is the per unit energy dissipated during the transition. The dissipated energy can be calculated as:

\[
E_d = \int_0^{t_d} \mathbf{i}^T(t) \mathbf{R} \mathbf{i}(t) \, dt
\]

(2.19)

where \( \mathbf{R} = \text{diag} \left\{ r_s, r_s, r_r, r_r \right\} \) is a diagonal matrix constructed with the stator \( (r_s) \) and rotor \( (r_r) \) resistances, and \( t_d \) is the time when the braking mode transition finishes. Immediately after the fault, the currents rise leading to large dissipation, but then they quickly decay as the magnetic flux in the machine inductances is dissipated. That behavior of the current can be obtained by integrating Eq. 2.9 to get:

\[
\mathbf{i}(t) = e^{A^T} (\mathbf{i}_0 - \mathbf{i}_f) + \mathbf{i}_f
\]

(2.20)

where \( \mathbf{i}_0 \) is the current vector while operating in the pre-fault SEP and \( \mathbf{i}_f = -A^{-1} \mathbf{B} \) is the final current vectors after all the machine flux has decayed. For mathematical convenience, Eq. 2.19 can be rewritten to take the integration to \( t \to \infty \):

\[
E_d = \int_0^\infty \mathbf{i}^T \mathbf{R} \mathbf{i} \, dt - \int_0^\infty \mathbf{i}_f^T \mathbf{R} \mathbf{i}_f \, dt
\]

(2.21)

Although the fault-on phase only lasts until the fault is cleared, most dissipation takes place during the first few milliseconds so that the error in modifying Eq. 2.19 to Eq. 2.21 is negligible. Solving the integrals from Eq. 2.21, the following expression is obtained:

\[
E_d = (\mathbf{i}_0 - \mathbf{i}_f)^T \left[ \int_0^\infty e^{A^T t} \mathbf{R} e^{A t} \, dt \right] (\mathbf{i}_0 - \mathbf{i}_f) - \mathbf{i}_f^T \mathbf{R} A^{-1} (\mathbf{i}_0 - \mathbf{i}_f) - (\mathbf{i}_0 - \mathbf{i}_f)^T [A^T]^{-1} \mathbf{R} \mathbf{i}_f
\]

(2.22)

The integral in brackets is computed by defining:

\[
\mathbf{P} = \int_0^\infty e^{A^T t} \mathbf{R} e^{A t} \, dt
\]

(2.23)

and solving for \( \mathbf{P} \) using the Lyapunov equation:

\[
A^T \mathbf{P} + \mathbf{P} A + \mathbf{R} = 0
\]

(2.24)

In the case of unbalanced faults, the previous calculations have to be implemented twice for the positive and negative sequence systems. In each case, the matrices \( A, B, \mathbf{i}_0 \) and \( \mathbf{i}_f \) will be different because the
sign of the electrical frequency $\omega_e$ has to be changed (which affect the reactances and slip) and the voltage sag magnitude $\varepsilon$ will be different. Let $\varepsilon_+$ and $\varepsilon_-$ be the voltage sags in the positive and negative sequence systems that results from an unbalanced fault. Given a voltage sag in the abc lines, they can be calculated as:

\begin{align}
\varepsilon_+ &= \frac{1}{3} (\varepsilon_a + \varepsilon_b + \varepsilon_c) \tag{2.25} \\
\varepsilon_- &= \frac{1}{3} \left[ (\varepsilon_a - \frac{\varepsilon_b}{2} - \frac{\varepsilon_c}{2})^2 + \left( \frac{\sqrt{3}}{2} \varepsilon_b - \frac{\sqrt{3}}{2} \varepsilon_c \right)^2 \right]^{\frac{1}{2}} \tag{2.26}
\end{align}

The value of the total dissipated energy $E_d$ that can be used in Eq. 2.18 is the addition of the dissipated energies in the positive and negative sequence systems.

### 2.3.1.3 Calculation of Peak Stator Current

Another useful quantity that can be calculated for this transition is the peak stator current that occurs during the fault (i.e. the maximum value of $i_s = \sqrt{i_{ds}^2 + i_{qs}^2}$ encountered while the dynamics from Eq.2.9 have not reach steady state). For simplicity, the maximum of $i_s^2$ is calculated instead as it allows the system to be constructed as a quadratic form and gives an equivalent result (the maximum of $i_s^2$ occurs at the same time as the maximum of $i_s$). The calculation starts by defining:

\begin{align}
y = i^\top M i = i_{ds}^2 + i_{qs}^2 \tag{2.27}
\end{align}

\begin{align}
M = \begin{bmatrix}
1 & 0 \\
0 & 0 \\
\end{bmatrix} \otimes \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix} \tag{2.28}
\end{align}

Then by performing differentiation in time, the extrema of $y$ can be calculated as the value of current ($i_m$) where the derivative is zero. This results in:

\begin{align}
\frac{dy}{dt} \bigg|_{i_m} = \frac{di^\top}{dt} \bigg|_{i_m} M i_m + i_m^\top M \frac{di}{dt} \bigg|_{i_m} = 0 \tag{2.29}
\end{align}

which after algebraic manipulations can be written as a quadratic form:

\begin{align}
i_m^\top (A^\top M + MA) i_m + B^\top M i_m + c_m^\top MB = 0 \tag{2.30}
\end{align}

The value of $i_m$ can be found using numerical solvers. The calculation of this current was presented here as it is useful in the design of protection, but it is not used further in this dissertation as it is not relevant for
rotor speed calculations.

2.3.2 Electromechanical Transient

2.3.2.1 Electrical Torque During the Fault

After the braking mode transition has finished, the IM speed will continue to decay due to a load torque that is larger than the electrical. This happens because the mechanical torque is independent of voltage and it will continue to act during the fault-on phase, while the voltage sag will cause a significant reduction of the electric torque.

The calculation of the total electric torque during an unbalanced fault \( T_{ei}(f) \) can be computed as the addition of the positive and negative sequence torques. The presence of a negative sequence component leads to an oscillatory behavior at twice electrical frequency but its amplitude is typically small and it decays rapidly, so that the average electrical torque is used for CCT calculation. The average electrical torque during the fault is computed as:

\[
T_{ei}(f)_{\text{avg}} = (\epsilon^+_2 + \epsilon^-_2) T_{ei}(\omega_m)
\]  

where \( T_{ei} \) is the pre-fault electrical torque, and \( \epsilon_+ \), \( \epsilon_- \) are the voltage sags for the positive and negative sequence systems as defined in Eq. 2.25.

2.3.2.2 Voltage Sag Calculation in Network

As seen in Eq.2.7 and Eq.2.31, the values of \( \epsilon_+ \) and \( \epsilon_- \) are critical for the correct analysis of the fault-on trajectories. The location of the fault in the network has a large impact on the voltage sag that the buses will experience. If the fault is close to the voltage source then most of the buses will be affected with a low voltage, while for faults near the end of the feeder the impact will be localized in a region close to the disturbance. These scenarios are depicted in Fig. 2.5 with a simplified feeder, showing the voltages due to a fault towards the end of the line and then close to the voltage source. The regions where the voltage dips below 0.5 pu are indicated in red.

To calculate the fault-on voltages, the network has to be solved with the fault connected to one of the buses. In most cases, the fault will not cause the voltage to reduce to zero in all three lines, so that the calculation has to be made for the positive and negative sequence systems that result from the disturbed voltages in the abc reference frame. This will result in coefficients \( \epsilon_+ \) and \( \epsilon_- \) for each bus.
2.3.3 Trajectory Construction

Due to the decoupled analysis done for electromagnetic and electromechanical effects in the fault-on phase, in a N-dimensional state space, the trajectory will be depicted by a sudden jump of the speeds representing the braking mode transition, followed by a smooth speed decay due to torque imbalance. Integrating the dynamics given by Eq. 2.1, the speed of each motor during the fault-on phase as a function of time is given by the equation:

$$
\omega_{mi}(t) = \omega_{nfi} + \frac{1}{2H_i} \int_0^t \left( \tau_{ei} - T_{mi} \right) dt
$$

(2.32)

where \( \omega_{nfi} \) is the speed after braking mode for that particular machine and friction has been included in the mechanical torque. The importance of assuming the initial speeds to be the speed after braking mode (\( \omega_{nfi} \)) and not the pre-fault SEP speed (\( \omega_{m0} \)) is illustrated in Fig. 2.6 where the speed of two motors is shown (with each speed plotted on one axis) to observe its evolution during the fault-on phase. This is the two machine version of Fig. 2.3. The full state model simulation is represented by the solid blue line that starts at \( \omega_{m0} \) and reaches point B after some time. If the speed decay due to torque imbalance is assumed to start at \( \omega_{m0} \) then the model predicts the system to be at point A instead of B which results in a significant error. If the torque imbalance starts from \( \omega_{nf} \) then point B is correctly predicted and the error is negligible. In this case, it is not possible to see how fast the braking mode transition is, but the assumption of an instantaneous jump from \( \omega_{m0} \) and \( \omega_{nf} \) yields a good approximation.

2.4 Post-Fault Stability Region Characterization

After the fault is cleared, the system will start operating in the post-fault phase. The location of the operation point in the phase plane at the clearing time is critical to assess the post-fault system behavior; in particular, its location with respect to the stability region of the post-fault SEP. The stability region is the
collection of state values that are asymptotically stable, meaning that they have bounded trajectories and converge to the SEP as $t \to \infty$. If at the clearing time the IMs' operation point is inside the stability region, then the IMs are guaranteed to converge to the SEP and no instabilities will occur. Otherwise, at least one of the motors will stall and some action will be needed to return to the system to the SEP. The stability region is an invariant set [22], which means that any trajectory starting inside the region will stay inside and, therefore, only the operation point at the moment when the fault is cleared needs to be evaluated to assess stability. However, it is often difficult to get an analytical expression for the stability boundary of non-linear systems and approximation methods have to be employed. This section provides a characterization of the stability boundary of the post-fault system to gain insights for the development of those approximations (which are presented in Chapter 3).

2.4.1 Equilibrium Points and Nullclines

The first step in the stability boundary characterization is to find the equilibrium points and the nullclines. The equilibrium points are found by equating all differential equations in Eq. 2.1 to zero. In general, there will be more than one equilibrium point depending on the system characteristics and operating conditions. The equilibrium points are classified by the number of stable eigenvalues that a linearized system around them has. SEPs have only eigenvalues with negative real part, unstable equilibrium points (UEPs) have only eigenvalues with positive real part, and a type-k (or saddle) equilibrium points have k eigenvalues with
positive real part.

In a one machine system such as the one studied in [37, 38], the electrical and mechanical torques can intersect at different speeds. In this dissertation only cases in which there can be at most 2 intersection points are considered. This condition is presented in Fig. 2.7, where the electrical and mechanical torques of an IM are shown. The mechanical torque is shown as almost constant for simplicity, but it has to be lower initially for the motor to be able to start. This is not an unreasonable assumption in loads such as compressors that build pressure over time leading to an increased mechanical torque once they have been running for some time. These type of loads, particularly in A/C applications, are the ones identified to cause FIDVR [25].

In the figure, the commonly called critical speed of an IM load (identified with a dashed vertical line) is an UEP and the normal operation point at low slip is a SEP. The point of maximum electrical torque is usually referred as a bifurcation point, where the SEP and UEP meet if torque increases or voltage decreases. In this case, the stability region of the IM is given by any value of speed above the critical speed because any fault cleared before crossing that boundary will lead to a condition with more electrical than mechanical torque, so that the motor will accelerate and recover nominal operation. Faults cleared after this value will cause the motor to stall as the electrical torque will not be enough to overcome the mechanical and acceleration will not be possible.

The nullclines are the values of $\omega_m$ that make one of the derivatives in Eq. 2.1 equal to zero. These are the operation points in which the electrical and mechanical torques are equal for exactly one motor. In
decoupled systems, where there is no interaction between the motors, the nullclines form an N-dimensional hyper-cube and each machine has an associated critical speed that can be used to assess stability. However, if there is impedance in the source and between the IMs, then the nullclines bend with their actual shape being affected by the system parameters and mechanical torques. This can be illustrated using the prototypical two bus system shown in Fig. 2.8. It is convenient to use a system with two motors because the dynamics can be represented in a phase plane for which many techniques exist and solutions for the stability region can be obtained exactly. Although these advantages require a simplification of the system, insights about the characteristics of more general systems can be obtained such as the effect of the line impedances, the impact of DGs and ZIP loads, and control methods for the voltage source.

![Figure 2.8](image)

Figure 2.8: Schematic view of the prototypical two machine system used for analysis.

The nullclines for the prototypical system, given a set of system and IM parameters, are shown in Fig. 2.9. They were calculated by computing the electric torque for each motor in the phase plane (i.e. calculate $T_{ei}(\omega_m)$ given an operation point $\omega_m$) and then finding the points where it is equal to the mechanical torque for that machine (i.e. $T_{mi}(\omega_m)$). Alternatively, an analytical expression can be calculated by making one of the equations in 2.1 equal to zero and then solving for one motor speed as a function of the others, to obtain an expression such as:

$$\omega_m = f(\omega_2, \ldots, \omega_N)$$

In the two motor case, this expression represents lines in the phase plane which coincide with the nullclines shown in Fig. 2.7. This analytical method, however, is hard to extend to system with large number of motors and usually leads to complicated mathematical expressions. The numerical approach was used to obtain the results presented in this dissertation.

By their definition, the nullclines serve as boundaries for regions in the state space where the net torques of the motors ($T_e - T_m$) have constant sign. As expected, the equilibrium points are located at the intersection of the nullclines because the differential equations for both motors are zero at those points. In the shown case, there are 4 equilibrium points: a SEP, a UEP and two saddle points.
Figure 2.9: Post-fault equilibrium points and nullclines of the two motors in the prototypical system. The nullclines separate regions where the net torques have constant sign.

The nullclines' shape depends on the mechanical torques and interesting phenomena can be observed as they vary. An example is presented in Fig. 2.10 that shows the nullclines for three values of mechanical torque on the motor in node 2 (IM 2). As torque increases, a saddle-node bifurcation occurs (in the figure it happens at \( T_{m2} = 0.885 \) pu), leaving the system with only two equilibrium points: a SEP and a saddle. This bifurcation will affect some of the system characteristics but the techniques to find the stability boundary presented in the next section can still be applied. In an N machine system there can be up to \( 2^N \) equilibrium points, although some of them might have disappeared in bifurcations such as the one just described, or might not occur within the window \( 0 < \omega_m < \omega_{m0} \).

### 2.4.2 Stability Boundary

Although the nullclines provide some information about the system, they do not determine the stability boundary. The exception are systems where the IMs are decoupled and the stability boundary is given by the nullcline closer to the SEP along the fault-on trajectory. In coupled systems, even if a machine seems unstable because it has a negative net torque at the fault clearing operation point, it might still be within the stability region. This occurs because as the other motors recover their nominal speed, they reduce their current demand which leads to an increased voltage and electrical torque.

The concepts required to find a stability boundary in a phase plane are explained in Theorem 3.5 from [22] which specifies that the stable manifold of the saddle points is the stability boundary and it is tangent...
Figure 2.10: Effect on the nullclines of increasing the mechanical torque of the induction motor located at bus 2. A saddle-node bifurcation occurs when the torque exceeds a certain threshold ($T_m = 0.885$ pu in this example).

to their stable eigenvector. Given this, the stability boundary of the prototypical two machine system can be found with the following procedure:

1. Identify the equilibrium points of the system to obtain a plot such as the one in Fig. 2.9. This is done finding the points $\omega_{me}$ that solve the set of equations:

$$T_{ei}(\omega_{me}) = T_{mi}(\omega_{me}) \forall i$$

(2.34)

2. Linearize the system around the equilibrium points and identify the saddle points (the ones with one stable and one unstable eigenvalue). This is done by computing the Jacobian at a given equilibrium point as:

$$A = \begin{bmatrix}
\frac{1}{2H_1} \left( \frac{\partial T_{e1}}{\partial \omega_{m1}} \bigg|_{\omega_{me}} - \frac{\partial T_{m1}}{\partial \omega_{m1}} \bigg|_{\omega_{me}} \right) & \frac{1}{2H_1} \left( \frac{\partial T_{e1}}{\partial \omega_{m2}} \bigg|_{\omega_{me}} - \frac{\partial T_{m1}}{\partial \omega_{m2}} \bigg|_{\omega_{me}} \right) \\
\frac{1}{2H_2} \left( \frac{\partial T_{e2}}{\partial \omega_{m1}} \bigg|_{\omega_{me}} - \frac{\partial T_{m2}}{\partial \omega_{m1}} \bigg|_{\omega_{me}} \right) & \frac{1}{2H_2} \left( \frac{\partial T_{e2}}{\partial \omega_{m2}} \bigg|_{\omega_{me}} - \frac{\partial T_{m2}}{\partial \omega_{m2}} \bigg|_{\omega_{me}} \right)
\end{bmatrix}$$

(2.35)

and then computing its eigenvalues.

3. Find the eigenvector corresponding to the stable eigenvalue of the saddle points. This results in vectors at each saddle point as illustrated in Fig. 2.11.

4. Perform an integration backwards in time for the post-fault system dynamics starting close to the saddle points and with a perturbation along the eigenvector found above.
The trajectory found in the last step is the stability boundary. The boundary for the two motor system is represented by the black solid line in Fig. 2.12 and 2.13 for the cases where the saddle-node bifurcation occurred and where it did not, respectively. As expected, this boundary intersects the saddle equilibrium points and the UEP. To demonstrate that this boundary correctly separates stable from unstable trajectories, two time domain simulations using a full model of the system are also included in both figures. Notice that in the two cases, the system can recover from faults cleared after one of the nullclines has been crossed (i.e. cleared when $T_{ei} < T_{mi}$ for one of the machines) where one might think that the system would not be able to return to the SEP (as assumed by [39]). This phenomena can be thought of as a cascade recovery process of the induction motors. The difference between crossing the first nullcline and crossing the stability boundary can be significant, especially in systems where there is no UEP due to a saddle-node bifurcation as the one in Fig. 2.13.

For systems of more than two machines, the concepts described above can be applied but there is no simple way to visualize them as the nullclines and the stability boundary are surfaces ($N = 3$) or hypersurfaces ($N > 3$). Although the trajectories found with the procedure described above will be on the stability boundary, they are only a few of the many trajectories on that surface. To illustrate the complexity, the nullclines and stability boundary for a three IM load system are presented in Fig. 2.14, along with four different potential fault-on trajectories to help on the visualization of the three dimensional shapes. Instead of using backward integration, the boundary in Fig. 2.14 was found by sampling different points in the space, performing a forward integration and checking if the trajectory converged to the SEP or not. Notice that in...
Figure 2.12: Stability boundary of a post-fault system where the saddle-node bifurcation did not occur. Stable and unstable trajectories are superimposed to demonstrate that the boundary correctly separates the two regions.

Figure 2.13: Stability boundary of a post-fault system where the saddle-node bifurcation occurred, with stable and unstable trajectories superimposed.
specific cases the stability boundary can be located after two of the nullclines, meaning that the system is stable even if two of the three IMs have larger mechanical than electrical torque when the fault is cleared. This again is a cascade recovery of IM loads.

The effect of cascade recovery is caused mainly by the impedances in the system which include the transmission lines, transformers and inverter filters. If the system under study has a large impedance between the IM loads and the voltage source, then the stability boundary will contract and bend. To observe this effect, consider the four stability boundaries shown in Fig. 2.15 that correspond to the prototypical system with $Z_2 = 0$ and increasing values of $Z_1$ starting from zero. In the decoupled case, the nullclines form a square which determine the stability boundary of the system. Cascade recovery is not observed in this case and the nullcline of each motor determines when it will become unstable. As the transmission line impedance increases, coupling is observed by the bending of the stability boundary and the contraction of the stability region.

2.4.3 Maximum Source Current Boundary

The previous analysis assumed that the source can provide any current demanded by the loads. However, this will not be the case in systems such as microgrids powered through inverters where the sources are current limited. This leads to an alternative stability region and additional coupling between the motors. Two responses from the inverters are considered when they reach their current limit:
Figure 2.15: Effect of increasing the transmission line impedance $Z_1$ on the stability boundary and the SEP of the prototypical system.

1. The output can be shut-off to protect the device in which case the system is lost when maximum current is reached. Although the inverters can provide a current significantly larger than the nominal for a few cycles, this is not likely to affect the boundary as IM recovery takes several seconds.

2. The output voltage can be reduced to maintain current at its maximum allowed steady state value. This stems from the fact that inverters are typically current controlled, and they can ride-through the fault with a technique such as the ones presented in [62].

In the first case, the system is lost at the maximum current boundary because any point after that will trip the inverter protections. Boundaries of constant current (in per unit) for a source with a rating of twice the total rating of the motors are shown in Fig. 2.16 superimposed on the nullclines and the stability boundary of the system. In this case, the fault-on trajectory will cross the 1 pu current boundary before the stability boundary, so the critical time to avoid losing the system might be significantly lower than the one given by motor stalling. This maximum current boundary depends on the source power rating as well as other loads, DGs and voltage support elements present in the network.

In the second case, the stability boundary is contracted as the source reduces its output voltage to avoid over-current during the fault recovery process. The process to find this contracted boundary is the same as in the regular case, but the change in the source output voltage has to be accounted for in operation points beyond the maximum current boundary. In general, a given operation point $\omega_m$ is properly solved if one of the following two conditions is true for the voltage source in the system:

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• The voltage is at its nominal value $v_b = v_{nom}$ and current is below the maximum $i_b < i_{max}$.

• The voltage is below or equal to the nominal $v_b \leq v_{nom}$ and current is at the maximum $i_b = i_{max}$.

If none of these two conditions is true, then the solution found for the system is not correct as the voltage source is not in a valid operation point. The following two conditions can then be used to construct an algorithm:

• If voltage is at its nominal value $v_b = v_{nom}$ and current is above the maximum $i_b > i_{max}$ then the output voltage is too high.

• If voltage is below its nominal value $v_b < v_{nom}$ and current is below its maximum $i_b < i_{max}$ then the output voltage is too low.

Using an appropriate algorithm to determine the correct voltage should not lead to a case not considered above.

An example of the contracted stability region is shown in Fig. 2.17. The effect of current limitation creates additional coupling between the IMs in the system because a large current demand from one motor will cause the source to intentionally regulate its output voltage causing all electrical torques to reduce. The contracted region is still larger than the region bounded by the maximum current limit, meaning that there are some operation points that can return to the SEP even if the system voltage has to be temporarily reduced. These insights imply that regulating the inverter output voltage should be a preferred response.
compared to shutting it off when the maximum current is reached. This could be extended to a comparison between master-slave and droop control strategies for the system. In a master-slave approach, the voltage source has to take on all the current required for the IM loads to recover which might cause an over-current tripping. Although secondary control could dispatch other DGs to help in the recovery, it might not be fast enough to prevent protections from activating. A droop control system reduces the voltage and contracts the stability boundary, but the current is shared among all the sources so that the maximum current boundary will not be encountered.

2.4.4 Effect of ZIP Loads DGs

The presence of additional elements in the system such as ZIP loads and DGs will impact the location of both the stability boundary and the maximum current boundary of the voltage source. Fig. 2.18 shows these boundaries for a two machine system with a constant power load of various power factor angles connected next to one of the motors. As expected, both unity and lagging (inductive) loads move the boundaries closer to the SEP leading to reduced critical clearing times. On the contrary, leading (capacitive) loads expand the stability region and move the current boundary farther away from the SEP.

The impact of DGs on the boundaries can also be analyzed using this framework. Consider, for example, the case shown in Fig. 2.19 where a current controlled DG injects power at unity power factor to the network. Since the motors draw mainly reactive power when operating at low speeds, the stability boundary does not
move significantly. However, the DG helps in relieving the current demand causing the maximum current boundary to move away from the SEP.

2.5 Validation

This section presents a validation of the concepts presented above. First, a CCT estimation of a single IM load system is done to demonstrate the importance of accounting for braking mode. Then, an experimental validation of a two machine system is performed to observe the fault-on and post-fault trajectories and check if the stability boundary correctly separates stable and unstable cases. Finally, results of a case study using the IEEE 34 Node Test Feeder are presented.

2.5.1 Improved CCT Estimation in Single IM Load System

To demonstrate the importance of accounting for braking mode in the CCT calculation, a single machine system is used. There is no loss of generality in doing this simplification, as braking mode can be calculated at each machine independently. Moreover, single machine systems have the advantage that the CCT can be calculated exactly by computing the critical speed using analytical expressions such as the ones proposed in [37, 38]. Accounting for braking mode does not change the critical speed but it does change the time that the IM takes to get there during the fault-on trajectory.

The CCT with and without braking mode is compared in Table 2.1 along with the one obtained using...
Figure 2.19: Effect of current source DG operating at unity power factor on the stability boundary of the SEP and the maximum current boundary. The DG is located at bus 2 of the prototypical system.

time-domain simulations. These results demonstrate that the CCT error is significantly reduced when braking mode is accounted for. The error of ignoring braking mode increases as the per-unit inertia decreases, so that low inertia machines that are prone to stall might have a CCT much lower than expected when using previously proposed methods. The correction proposed in this paper makes the error much smaller.

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>$H=0.5$ sec</th>
<th>$H=0.3$ sec</th>
<th>$H=0.1$ sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without braking mode</td>
<td>0.416 s</td>
<td>0.250 s</td>
<td>0.083 s</td>
</tr>
<tr>
<td>With braking mode</td>
<td>0.392 s</td>
<td>0.230 s</td>
<td>0.061 s</td>
</tr>
<tr>
<td>Full model simulation</td>
<td>0.388 s</td>
<td>0.221 s</td>
<td>0.056 s</td>
</tr>
</tbody>
</table>

Table 2.1: Comparison of CCT obtained with a full model simulation and with estimations using the quasi-static approximation with and without accounting for the braking mode transition.

2.5.2 Experimental Validation for Two IM Loads System

Most of the concepts described in Section 2.4 refer to systems with two or more IM loads. To validate those concepts, the microgrid experimental set-up described in Appendix A was used. For this particular series of experiments, the set-up was arranged as shown in Fig. 2.20, where two 1 HP IMs were connected to a stiff grid ($3\phi - 4L$, 208 VLL, 60 Hz) through a line impedance of $Z_I = 0.5\Omega + j4.5\Omega$. Mechanical torque was controlled by coupling the IM with a DC generator whose output armature current was regulated by a DC/DC converter. The output of the DC/DC converter was connected to a resistor. To create a fault, an instant-on switching solid state relay connected a short circuit at the load end of the line impedance. After a controllable period of time, the solid state relay was opened and the supply from the grid restored.
2.5.2.1 Speed After Braking Mode

The experiments were first used to validate the analytical calculation of the speed after braking mode described in Section 2.3.1.2. This is shown in Fig. 2.21 where the speeds of the IMs before and during the fault are presented. In the pre-fault phase, the motors were operating in steady state at a slip determined by their mechanical torque. At \( t = 0 \) the solid state relay created a three-phase short circuit close to the IMs' terminals starting the fault-on phase. Both the sudden speed drop caused by the braking mode transition and the speed decay due to torque imbalance are clearly observable in the results. The linear decay in speed (after the braking mode transition has finished) was caused by both a constant mechanical torque maintained through the experiment and a fault causing the voltage to drop near zero during the fault. The theoretical values of speed after braking mode \( (\omega_{mf}) \) calculated with Eq. 2.22 are \( \omega_{mf1} = 0.909 \text{ pu} \) and \( \omega_{mf2} = 0.931 \text{ pu} \) which closely match the experimental results.

As explained in Section 2.3.1, the braking mode transition is caused by large currents at the terminals of the IM that cause energy to be dissipated. Those currents were measured experimentally at the moment of the fault and are shown in Fig. 2.22. The large peak in current lasting several milliseconds is responsible for the rapid deceleration.

2.5.2.2 Stability Region

Although the fault-on trajectories were studied at each machine independently, the post-fault behavior exhibits the coupling in the system. The fault-on and post-fault trajectories for various clearing times are shown in Fig. 2.23, along with the nullclines and the theoretical stability boundary calculated by

Figure 2.20: Schematic view of the experimental microgrid configuration used to validate the concepts described in this chapter.
Figure 2.21: Experimental measurement of the rotational speed of two motors with a fault occurring at \( t = 0 \). The braking mode transition and the speed decay due to torque imbalance can be observed in the plots.

Figure 2.22: Stator currents measured during the braking mode transition. The sudden peak is caused by a three phase fault occurring close to the motor terminals at \( t = 0 \).
backwards integration from the type-1 EP (notice that the type-1 EP in this case occurs outside the window $\omega_m < \omega_{m0}$ but the technique can still be applied). Torques $T_{m1} = 0.9$ pu and $T_{m2} = 1.0$ pu were used leading to a system where the saddle-node bifurcation has occurred as seen by the lack of UEP in the phase plane. All trajectories cleared before the theoretical stability boundary were stable, and those cleared after led to IM stalling.

In this case, the region where both machines have positive net torque is small and the fault-on trajectory leaves it rapidly (i.e. the first nullcline of the post-fault system is crossed early during the fault-on trajectory). The nullcline of the motor labeled IM 2 is the first one to be crossed, which means that the cascade recovery effect is required for it not to stall. This is clearly visible in the results as, even in stable cases, the IM 2 continues its deceleration after the fault has been cleared until it crosses back its nullcline and enters the region of positive net torque. As expected, the post-fault trajectories have zero slope when they cross the nullclines because the time derivative of IM 2 speed is zero there (i.e. $\frac{d\omega_m}{dt} = 0$ as per nullcline definition). The CCT in this experiment was 0.35 sec.

### 2.5.3 Multiple Machine Case Study

Real distribution systems and microgrids contain a large number of coupled motor loads. To study the dynamics of Eq. 2.1 in a more realistic scenario, the IEEE 34 Node Test Feeder system shown in Fig. 2.24 was used, with an IM and a constant impedance load connected at each node except the first one where the
voltage source is. This scenario could represent, for example, a distribution network with bus 1 being the substation, or a microgrid with a single point of connection to the grid (such as a university campus) or with a single power source (such as a community). In this case study, DGs were not included in the system, but that could be easily done in the model as described in Appendix C.

Figure 2.24: Single line layout of the IEEE 34 Node Test Feeder with the buses renumbered for convenience. A constant impedance load and an induction motor are placed at each bus except the first one where the voltage source is located.

For simplicity, all the IM loads were modeled with the same parameters, except for those in buses 2, 14, 28 and 32, which were considered to be machines with 5 times the power rating. Per unit resistances and inertia were scaled as $r_s \propto P_{nom}^{-1/3}$, $r_r \propto P_{nom}^{-1/3}$, $H \propto P_{nom}^{1/3}$, and the other per-unit machine parameters left unchanged. All machines were operated at a constant mechanical torque $T_m = 0.95 \pm 0.05$ pu, where the 0.05 was selected randomly in order to introduce some variability. IM at bus 2 was further overloaded to $T_m = 1.2$ pu. A three phase fault was introduced at bus 2 very close to the substation, so that all IMs see a voltage very close to zero during the fault.

As mentioned before, with more than 2 IMs there is no simple way to represent the system trajectories and time-domain simulations are required. Figs. 2.25 to 2.27 show the time-domain behavior of all the motors at three important timings:

- In Fig. 2.25 the fault was cleared at the time along the fault-on trajectory when the first post-fault nullcline was crossed. In this case, all motors quickly accelerate back to the SEP and there are no signs of instability.

- In Fig. 2.26 the fault was cleared 2 ms before the CCT (and 34 ms after the previous case). Since the system is still inside the stability boundary, all motors recover, but some of them continue decelerating after the fault is cleared. The cascade recovery effect is required for some IMs to return to their SEP.
In Fig. 2.27 the fault was cleared 2 ms after the CCT (4 ms after the previous case). Since the system has now left the stability region, at least one motor should stall. In the figure, it is clear that even when some IMs show cascade recovery, other had a rotational speed so low that even when all other motors have recovered, they still stall.

2.6 Conclusions

This chapter presented an analysis and characterization of the fault-on and post-fault dynamics that occur in systems with induction motor loads. Some interesting concepts were developed in detail such as the concept of braking mode that quantifies the electromagnetic transitions that occur immediately after the fault, the construction of fault-on trajectories for balanced and unbalanced faults, the computation of the stability boundary and the maximum current boundary for a prototypical two machine system, and the boundary’s characterization with respect to some system parameters. Most of the concepts were validated using time-domain simulations and experimental measurements.
Figure 2.26: Time domain simulation results showing the speed of the 33 motors when the fault is cleared 2 ms before the CCT.

Figure 2.27: Time domain simulation results showing the speed of the 33 motors when the fault is cleared 2 ms after the CCT.
Chapter 3

System Stability Assessment

3.1 Introduction

The previous chapter presented an analysis of the dynamics involved during the fault-on and post-fault phases of a fault event, as well as the characterization of important boundaries that impact the system behavior. This chapter extends the analysis and presents a systematic way to assess the stability of such systems.

Stability analysis of power systems is usually divided in categories depending on the phenomenon under study [1]. The instabilities that can occur in the system presented in Fig. 2.1 are three:

- **Speed Stability:** A system is said to be speed stable if none of the IM loads stall. If some motors stall, then some action is required to return to the SEP and avoid the risk of shedding large number of loads.

- **Source Current Stability:** A system is source current stable if, at a given operation point, the output current from the voltage source is below its maximum limit. If that is not the case, then the inverter can trip on over-current and there is a risk of a blackout. For inverters that regulate their output voltage when they reach their maximum capability, this instability should not be encountered.

- **Voltage Stability:** A system is voltage stable if the voltage at all the buses is within the acceptable limits in post-fault steady state operation. As it will be seen, IM stalling might cause some voltages to get trapped at unacceptable low values or even trigger a voltage collapse.

These three conditions are considered in this chapter and methods to assess them are presented. Many of the insights obtained in the previous chapter are used for the explanations. The chapter is organized as follows:
Section 2 describes with more detail the instabilities listed above to understand the causes and effects of each one, and introduces the process to study them systematically in the context of fault events.

Section 3 studies speed instabilities, introduces the concept of nullcline crossing times, and presents method to compute the CCT bounds and an estimation.

Section 4 studies source current instabilities accounting for details such as time-current curves of inverters and inrush currents in motors.

Section 5 presents the framework for voltage stability assessment by linking induction motor stalling and a reactive power analysis technique.

Section 6 describes a case study used to test the above methodology in a multi-machine system.

3.2 Instabilities Description

3.2.1 Rotor Speed Instability

Rotor speed stability implies that all motors return to their nominal rotational speed after the disturbance, i.e. that no motor stalls after the fault. The concept of “rotor speed stability” was first introduced in [40] to be included as a subtopic of voltage stability. Although originally intended for induction generators, its use is suitable in this case to differentiate between methods that determine if IMs will stall and methods detecting voltage issues that arise from IM stalling.

As done in the previous chapter, throughout this chapter and the next, the concept of critical clearing time (CCT) is defined in the context of rotor stability. In other words, CCT is defined as the moment during the fault-on trajectory when the system leaves the stability region of the SEP. If clearing occurs after that time, then at least one of the motors will stall. This CCT definition gives a boundary between rotor speed stable and unstable conditions, which applies even for systems with large number of motors. This concept was studied in the previous chapter and the shape of the stability region is shown in Fig. 3.1 for a prototypical two machine system.

Rotor instability of a motor with low power rating might not be a concern for the network because its stalled current consumption might not be large enough cause a significant voltage dip. However, even if only small motors stall, problems could arise in three ways: (a) Protections can be tripped due to continuous current demand larger than nominal causing a large number of loads to be disconnected; (b) a large number of these small motors can stall at the same time causing significant voltage sags; and (c) even a small voltage drop can be enough to cause other motors to stall when their mechanical load increases. Due to these
reasons, rotor instability of any IM requires some action to be taken, such as tripping that motor or reducing its mechanical load, in order to return the system to the SEP.

3.2.2 Source Current Instability

The maximum current that the source is able to provide also defines a boundary in the state space representation of the system. The concept of "source current stability region" is used here to define all the points between this boundary and the SEP. This is shown in Fig. 3.2 for a prototypical two machine system. If the system operates outside this region, then there is a risk of triggering protections from the source leading to a system loss.
3.2.3 Voltage Instability

If an IM load stalls, it does not necessarily follow that a voltage instability will occur. For example, as explained above, if the stalled induction motor has a small power rating and therefore large stator and rotor resistances, its current demand will not be large enough to depress the voltage significantly. Voltage issues arise either by the system operating outside the allowable range given by a grid code, or the voltage collapsing by the network being unable to provide the reactive power required by the loads.

When the system operates outside specified bounds the protections act on under-voltage and disconnect the load. The bounds are typically set by grid codes or standards, and give a maximum time that a specific voltage sag can occur before triggering protections. Two examples of such standards are the IEEE 1547 [51] and NERC [52] whose voltage limits as a function of time are shown in Fig. 3.3.

The second risk of IM stalling is for the system to experience a voltage collapse. This condition occurs when the system is unable to provide the reactive power required by the loads. Several techniques exist to evaluate this condition and the $dQ/dV$ criterion described in [4] is used in this dissertation for voltage stability assessment. It consists on calculating, at a particular bus, the reactive power that is required to maintain a given voltage and then computing the corresponding load reactive power consumption. The intersection of the two curves are the voltage operation points which are stable if $dQ_s/dV < dQ_L/dV$ or unstable otherwise.

Figure 3.2: Stability region related to the source current capabilities. The maximum current boundary separates safe operation points from those where the source will trip on over-current.
3.2.4 Stability Assessment Procedure

Stability assessment of the post-fault dynamics is done following three steps:

1. Calculate the post-fault nullcline crossing times along the fault-on trajectory. This gives CCT upper and lower bounds, and determines the order in which the IM loads are expected to stall.

2. Compute the post-fault source current along the fault-on trajectory to determine if there is risk of triggering a protection.

3. Perform a reactive power balance analysis accounting for IM stalling to determine when will an unacceptable low voltage occur and if there is risk of voltage collapse.

3.3 Rotor Speed Stability Assessment

3.3.1 Nullcline Crossing Times

In a generic N machine system, the nullcline crossing times \( t_n \) are the times during the fault-on trajectory where the post-fault system nullclines are crossed. Mathematically, this requires solving a non-linear equation for each machine to find the value \( t_{ni} \) that satisfy the following relationship:

\[
T_{ei}(\omega_m(t_{ni})) = T_{mi}(\omega_m(t_{ni}))
\]  

(3.1)
where $T_{mi}$ is the mechanical torque, $T_{ei}$ is the electrical torque in the post-fault system, and the $\omega_{mi}(t)$ represents the dynamics during the fault-on phase as given by Eq. 2.23. Although several non-linear solvers could be used to find the values of $t_{ni}$, a simple process is to start at $t = 0$ at the speed after braking mode ($\omega_m(0) = \omega_{mf}$), increase time by a small time step $\delta t$ and check if the electric torque of any of the IM loads is smaller than its mechanical torque using post-fault system characteristics. In some cases, the IM speed may reach zero during the fault-on trajectory before that motor crosses its nullcline, which means that the IM will not stall. The set of values $t_{ni}$ determine which IMs are expected to stall and in what order. That information then serves as an input to the voltage stability analysis presented in Section 3.5.

### 3.3.2 Regions and Bounds

An additional advantage of obtaining the nullcline crossing times is that they provide an upper and lower bound for the time when at least one of the motors will stall. This process is computationally inexpensive compared to calculating the real CCT (given by the time when the stability boundary is crossed) which requires multiple time domain simulations. The bounds are found using the following two characteristics of the stability region:

- It contains the region where all IMs have larger electrical than mechanical torque. That is, let $A$ be the stability region, then:
  \[ A \supset \{ \omega_m \mid T_{ei} (\omega_m) > T_{mi}, \forall i \} \]  
  \[ (3.2) \]

- It excludes the region where all IMs have larger mechanical than electrical torque. Let $B$ be this region, then:
  \[ A \cap B = \emptyset \mid B = \{ \omega_m \mid T_{ei} (\omega_m) < T_{mi}, \forall i \} \]  
  \[ (3.3) \]

The first part can be proven by using a Lyapunov function $V$ of the form:

\[ V = \sum_i \frac{1}{2} J_i (\omega_{mi} - \omega_{m0i})^2 > 0 \]  
[3.4]

which is clearly positive except at the equilibrium point $\omega_m = \omega_{m0}$. Its orbital derivative is given by the function:

\[ \dot{V} = \sum_i (\omega_{mi} - \omega_{m0i}) (T_{ei} - T_{mi}) \]  
[3.5]

Notice that $\dot{V} < 0$ if $\omega_{mi} < \omega_{m0i}$ and $T_{ei} > T_{mi}$ which is true before any nullcline has been crossed.

The two regions identified above are shown graphically in Fig. 3.4 for a two machine system, but these characteristics are also true for a generic system with an arbitrary number of motors.
Figure 3.4: Identification of regions that are known to be inside and outside the real stability region of the SEP delimited by the first and the last nullcline crossed along the fault-on trajectory.

The previous concepts can be interpreted intuitively in the following way. If at the fault clearing time, all the electrical torques are larger than mechanical ones (i.e. no nullcline was crossed) then the system is guaranteed converge to the SEP as all motor accelerate and reduce their current demand; there is no effect during the recovery process that will reduce the voltage or electric torque of any motor to lead to an instability. Notice that this implies that $\frac{\partial T_e}{\partial \omega_m} > 0$ in the region of interest. Using the same reasoning, if all the mechanical torques are larger than the electrical (i.e. all nullclines were crossed) then the system is guaranteed to collapse, as no effect during the decaying process will increase the motors electric torque. The existence of these regions can be used to provide bounds on the real CCT which will be at some point between the first and the last nullcline. After all the nullcline crossing times ($t_n$) have been calculated, the upper and lower bounds are given by $t_{ub} = \max \left\{ t_{n1}, t_{n2}, \ldots t_{nN} \right\}$ and $t_{lb} = \min \left\{ t_{n1}, t_{n2}, \ldots t_{nN} \right\}$ respectively.

The concepts described above are shown in Fig. 3.5 and 3.6 for a prototypical two machine system with and without the saddle-node bifurcation having occurred. In both figures, the CCT lower bound is given when the fault-on trajectory crosses the nullcline of the motor in bus 2. In cases such as the one in Fig. 3.6 where there is no UEP due to the bifurcation, the difference between crossing the first nullcline and crossing the real stability boundary can be significant and large errors can be encountered if the cascade recovery effect is ignored.

Having upper and lower bounds is beneficial for calculating the real CCT of a system with an arbitrary number of motors. Without them, finding the CCT requires performing time domain simulations along all
Figure 3.5: Fault-on trajectory indicating the points where the lower and upper bounds are located in relationship with the real CCT.

Figure 3.6: Fault-on trajectory indicating the points where the lower and upper bounds are located. The lower bound can be located far from the real CCT in systems without UEP due to the saddle-node bifurcation.
the fault-on trajectory using an exponential search or a similar technique and checking at each point if the post-fault system converges or not to the SEP as $t \to \infty$. With the bounds, a binary search can be performed between them which significantly reduces the search time.

### 3.3.3 Effect of Inertia and Approximation

The nullclines are affected by all parameters in the system except the motors inertia ($H_i$). This is because the inverse of the inertia multiplies both the electrical and mechanical torques in each dynamic equation so that it does not affect the point where they intersect. However, inertia has a significant impact on the stability boundary as it affects the post-fault dynamics that lead to cascade recovery. The effect of inertia can be isolated by assuming an artificial scenario where the machines parameters remain constant while the inertia changes. This assumption does not represent a machine changing size, but it is rather a mathematical construct to isolate a parameter. The boundaries for three systems with different inertia ratio between the machines but otherwise identical are shown in Fig. 3.7 and 3.8. An important insight obtained from this construct is that if one inertia is much larger than the other, then the stability boundary approaches the nullcline of the lighter machine, while the heavy machine has a “critical speed” defined by its corresponding type-1 equilibrium point (for cases with UEP such as the one in Fig. 3.7) or the bifurcation point (for systems without UEP due to the saddle-node bifurcation). This last case is shown in Fig. 3.8 with the bifurcation point indicated with a B.

The described behavior of the stability boundary can be explained intuitively in the following way: if at the fault clearing time only the heavy machine has negative net torque, then the light machine can quickly accelerate leading to an increased voltage and subsequent recovery of both. However the opposite is not true. If it is the light machine the one that has negative net torque, it will decelerate to a speed that can not be recovered even if the heavier machine is slowly increasing its speed. This is an important observation because it means that light IM loads do not benefit from larger machines recovering, and they are the ones prone to stall and cause FIDVR [25]. In other words, the cascade recovery phenomenon described in Section 2.4 does not benefit the light machines as much as the heavier ones.

Given the above insights, it is possible to construct an approximate location of the CCT between the bounds. Considering that the stability boundary approaches the nullcline of the lighter machines, the following equation is proposed:

$$t_{\text{est}} = \left[ \sum_i \left( \frac{t_{\text{of}}}{H_i} \right) \right] \left[ \sum_i \left( \frac{1}{H_i} \right) \right]^{-1} \quad (3.6)$$

This is not meant to be the exact location of the boundary but rather an estimation that follows the trend
Figure 3.7: Effect of changing the ratio of the IMs inertia for the two machine system. The stability boundary is seen to approach the post-fault nullcline of the lighter machine.

Figure 3.8: Effect of changing the ratio of the IMs inertia for the two machine system without UEP due to a saddle-node bifurcation. The stability boundary is also seen to approach the post-fault nullcline of the lighter machine in this case.
Figure 3.9: Comparison of lower bound, upper bound, estimation and real CCT for a two machine system with different inertia ratios.

of the CCT as parameters vary. A comparison of this estimation with the lower bound, upper bound and exact CCT for a two machine system with increasing inertia ratio \( \frac{H_2}{H_1} \) is shown in Fig. 3.9 and for a three machine system in Fig. 3.10. The CCT estimation correctly follows the trend of the real CCT and provides better estimation than the lower bound, especially in systems where saddle-node bifurcations has occurred. The bounds and this estimation can be useful, for example, in quickly comparing the expected effect on the CCT of several locations for voltage support elements or DGs with different control techniques. This concept is studied with an example in Sec. 3.6.4.

### 3.4 Current Stability Assessment

As described in Sec. 2.4.3, the system can be lost due to inverter tripping much before the IM loads stall. To account for this, the source current needs to be calculated at the points along the fault-on trajectory given by Eq. 2.23. This is done in Fig. 3.11 and the intersection of the trajectory with the maximum current boundary, the contracted boundary due to voltage regulation, and the original boundary are indicated with points A, B and C respectively. The current calculation has to be performed using the post-fault system characteristics because it represents the current that the source would have to provide if the fault is cleared at that particular point along the trajectory. In the general case where the system has ZIP loads, DGs and the IM loads, the source current has to be calculated using power flow analysis. This is done by setting the bus with the voltage source as the slack bus and all the others as PQ buses with constant impedance loads.
in parallel. If all loads are constant impedance and the DGs are current controlled, then the network can be solved using linear circuit tools because, in the quasi-static approximation, the IMs can be regarded as constant impedance loads given a set of rotational speeds ($\omega_m$).

An alternative method to visualize the results obtained with the above calculation is to plot the current magnitude for different fault duration. This has the advantage of being applicable to system with any number of motors. Fig. 3.12 shows the same results as Fig. 3.11 in this alternative format. The figure presents two plots which are identical before the current reaches the 1 pu current value (point A occurring at around 0.09 sec). If the source does not regulate its output voltage then the current continues to rise for longer faults as shown with the blue solid line. The point where the original rotor speed stability is crossed occurs some time after (point C at around 0.2 sec) and then the current stabilizes at a plateau when both motors reach zero speed. In this case, there is a risk of tripping the source on over-current much before the rotor speed stability boundary is crossed. If the source regulates its output voltage, then there is no risk of tripping the source because the current is maintained at its maximum value by the current controller of the inverter. This is represented by the red line in the figure. However, the stable region is contracted and the stability boundary is crossed at an earlier time (point B) than the original boundary.

As described in the previous chapter, the electromagnetic transitions played an important role in the IM behavior immediately after the fault. Similar electromagnetic effects take place when the fault is cleared as all motors need to be re-magnetized before electric torque can be restored. This time, the effect on IM
Fault-on trajectory indicating the point where the system becomes source current unstable if the output voltage is not regulated (A) and the point where the system becomes rotor speed unstable if the source is regulated (B). The original stability boundary crossing point (C), corresponding to a source without current limitation, is shown for comparison.

Figure 3.12: Source current output for increasing fault duration. The blue line correspond to the case where the source is not regulated so that any fault after point A can activate an over-current protection. The red line correspond to the case where the source is regulated so that current is maintain at its maximum value by reducing voltage.
speed is minimal as the re-magnetization energy is taken from the voltage source and not the motor kinetic energy. Therefore, instead of a sudden decrease in speed, the electromagnetic effects are seen as a peak in the demanded current by the motors as shown in Fig. 3.13. Notice that the even though the two electromagnetic effects are in essence the same (they are caused by the magnetic energy in the inductances), the processes that take place are different:

- During braking mode, the magnetic energy stored in the windings is used as excitation and the motors acts as generators for a few cycles (until the excitation decays). In this case, most of the dissipated energy does not come from the magnetic energy in the inductances, but from the rotor kinetic energy.

- During the re-magnetization process, the only energy that needs to be provided is the magnetic energy in the inductances which is significantly smaller than the energy dissipated during braking mode.

An important aspect to analyze for source current stability assessment is the possibility reaching an over-current during this motor re-magnetization process. Those effects are not accounted for in the quasi-static model used to compute the current in Fig. 3.12, but they are part of the motor in-rush current during the recovery or stalling processes. In-rush current into an IM has two components: the one required for the motor re-magnetization and the one required for motor acceleration. These two components are shown in Fig. 3.14 by the blue line corresponding to the current magnitude. A typical inverter current capability limit is superimposed given by the red dashed line. As seen, inverters are typically designed to withstand a current several times larger than the rated one for a couple of cycles in order to provide the re-magnetization.
current (which is also required by the transformers and some types of loads in the network), and about two times larger than nominal for a several seconds to help with in-rush currents.

With the parameters used to generate Fig. 3.14, there is no risk of activating the inverter protections because the current magnitude is always inside the inverter current capability. However, if IMs stall, they will not accelerate back to their SEP and the large current consumption will continue until they are disconnected. In that case, there is a risk of activating the inverter protections and losing the entire system as operation will be outside the current capability limit as illustrated in Fig. 3.15.
3.5 Voltage Stability Assessment

IM stalling is problematic because it requires some action to be taken in the loads or, as just described, it might trigger protections in the system sources. However, it is their impact on the network that can lead to FIDVR or voltage collapse. These contingencies span a wide area and can affect millions of energy users.

3.5.1 Relationship between IM Stalling and Voltage

The link between the voltage magnitude at one of the buses and the stability region for IM stalling can be observed in Fig. 3.16 for a two motor system. In it, the voltage at bus 2 was calculated given an IM operation point in the phase plane. As with source current, this calculation has to be done using power flow analysis to account for constant power loads and DGs, but if only constant impedance loads and current controlled DGs are in the system then linear circuit tools can be employed.

In Fig. 3.16, when the system operates in the SEP at point A, the bus voltage is above 0.95 pu and even if it dips close to 0.85 pu during the recovery process, it is not a serious problem as it quickly returns to an acceptable value without the need of any action. However, when both motors stall, the system operates at point B causing voltage to stay around 0.8 pu until the motors are tripped. This is an example of an FIDVR event. The same is true when the induction motor at bus 1 (IM 1) stalls, taking the system to point C in which bus 2 has a voltage of around 0.86 pu, resulting in another example of an FIDVR event. That is
Figure 3.16: Lines on the phase plane where the voltage magnitude at bus 2 is constant are shown in cyan, superimposed on the stability boundary of the SEP. Points indicated as A, B, C and D represents the operation points when none, one or two motors stall.

not observed at operation point D that occurs when the induction motor in bus 2 (IM 2) stalls. In that case, the voltage stays above 0.9 pu and, although some action needs to be taken to return the motor to its SEP, the system does not experience a voltage instability.

3.5.2 Reactive Power Balance

An alternative method to visualize the relationship between IM stalling and voltage is by performing a reactive power balance analysis. Contrary to phase plane analysis, reactive power balance has the advantage that can be implemented in a system with an arbitrary number of IM loads. Many methods exist to perform such analysis, and in this dissertation the one described in [4] as the $\frac{d\Delta Q}{dV}$ criterion is used. It consists in comparing the reactive power capability of the network and the reactive power demand of the load, at a particular bus and for all voltage levels in order to find stable and unstable equilibrium points.

The network reactive power capability can be calculated analytically at a particular bus if a Thevenin equivalent circuit of the rest of the network can be computed and the resistance of the transmission lines is ignored. In that case, the reactive power capability $Q_s$ at a given voltage level $V$ is given by:

$$Q_s(V) = \left[ \left( \frac{V_{th} V}{X_{th}} \right)^2 - (P_L(V))^2 \right]^{\frac{1}{2}} - \frac{V^2}{X_{th}}$$

(3.7)

where $V_{th}$ is the Thevenin’s equivalent voltage, $X_{th}$ is the Thevenin’s equivalent reactance, and $P_L$ is the
real power consumption at the bus under study. A generalization of this equation to networks where the transmission lines resistances can not be neglected leads to an implicit equation of the form:

\[ Q_s(V) = \left[ \left( \frac{V_{th}V}{X_Q} \right)^2 - \left( \frac{X_P}{X_Q} P_L(V) \right)^2 \right]^{\frac{1}{2}} - \frac{V^2}{X_Q} \]  (3.8)

with:

\[ X_P = X_{th} - R_{th} \tan \phi \]  (3.9)

\[ X_Q = X_{th} + R_{th} \cot \phi \]  (3.10)

where \( \phi \) is the power factor angle at the node under study which depends on \( P_L \) and \( Q_S \). This expression reduces to Eq. 3.7 if the resistance is set to zero. Equation 3.8 does not work for \( \phi = 0 \) or \( \phi = \pi/2 \), and it is complicated to use. A simpler alternative is to use a power flow solver in which each node is modeled as a P-V bus with the voltage set at a certain value, and then calculate the required reactive power into the bus to sustain that voltage given a certain power level.

The result of implementing the last technique for bus 2 of the prototypical two machine system with constant impedance loads connected next to the motors is shown in Fig. 3.16. Plots for the cases when no motor stalls, when one stalls and when both stall are shown, leading to four stable voltage equilibrium points. Voltage SEPs are recognized because they are the intersection points of the supply and demand curves where \( dQ_L/dV > dQ_S/dV \). Intersections where this condition is not true are voltage UEPs (which occurs at the origin in the figure). In operation point A of the figure, none of the motors have stalled resulting in a voltage SEP above 0.9 pu. When the motors stall, the reactive power capability of the network is decreased (because of increased reactive power consumption of the IM loads in other buses) and the reactive power demand is increased, causing the stable equilibrium point to be at lower voltages. Points C and D are obtained when one of the two motors stalls, and point B when both stall; all three cases are examples of FIDVR.

Even though the previous examples of FIDVR led to a low voltage, those cases are still stable equilibrium points, meaning that voltage collapse will not occur. Voltage collapse can occur in extreme cases when the network operates near its reactive power capability limit due to the presence of other elements besides the IMs such as constant power loads and DGs. Consider, as an illustration, a case where constant power loads drawing large amounts of reactive power are connected next to the IMs. For voltages below 0.2 pu the constant power loads behave as a constant impedances leading to a lower reactive power consumption. To operate the system at an acceptable voltage, capacitors are connected on each bus, leading to a load reactive power demand with negative slope for voltages above 0.2 pu. The results of the reactive power balance
Figure 3.17: Results of the reactive power balance analysis performed at bus 2 of a prototypical system. Blue lines represent curves where the IM at bus 1 has not stalled, and red line represent the ones where it did stall. Dashed lines represent curves where the IM from bus 2 stalls. Black lines represent the load reactive power demand. All intersection points at the right are stable equilibrium points.

analysis are presented in Fig. 3.18. When none of the motors stall, the operation point is at A and the voltage is above 0.9 pu. When the IM load at bus 2 stalls, there is a sudden increase in reactive power demand which moves the operation point to C which is at a low voltage but still stable. When the IM load at bus 1 stalls, the reactive power capability of the network decreases and a bifurcation occurs causing a sudden decrease of voltage until it stabilizes at point B. This case shows an example of voltage collapse caused by IM stalling.

3.6 Evaluation

The stability assessment methods described above are now implemented in the IEEE 34 Note Test Feeder shown in Fig. 2.24 from the previous chapter. As done in Section 2.5.3, an IM and a constant impedance load are connected at each bus. Due to the absence of constant power loads, there is no risk of voltage collapse, but FIDVR caused by IM stalling can still be observed.

3.6.1 Rotor Speed Stability

Following the procedure described in Section 2.3, the fault-on trajectory can be constructed. This is the starting point for rotor speed and source current stability assessment. At each point along the trajectory, the electrical and mechanical torques of all the machines are compared to detect the nullcline crossing times.
The result of doing this calculation is portrayed in Fig. 3.19 with the assumption that the voltage source at bus 1 does not regulate its output after the maximum current is reached. In the figure, the number of the bus where the machine is located is shown in the abscissa with the corresponding nullcline crossing time in the ordinate. This representation provides the following information about the system:

- An upper and lower bound for the CCT to prevent IM stalling.
- Combined with the inertia of each machine, an approximation of the CCT between the bounds.
- The order in which the motors are expected to stall.

Using the information in Fig. 3.19, the obtained values for the CCT lower bound ($t_{lb}$), upper bound ($t_{ub}$) and estimation ($t_{est}$) are presented in Table 3.1 along with the real CCT found by performing time domain simulations ($t_{cct}$) and checking if the system converges to the SEP or not. As expected, the real CCT is between the bounds which offered the advantage of limiting the points along the fault-on trajectory in which the search for the CCT had to be conducted. Although, $t_{est}$ over-estimates the CCT, not only does it provides a better approximation that the lower or upper bounds, but it can also reduce the number of iterations required in a binary search. Consider, for instance, that the initial guess for the search is taken to be the midpoint between the bounds $\frac{1}{2}(t_{ub} + t_{lb})$, then 7 iterations would be required to find the CCT; if $t_{est}$ is used instead, then only 4 iterations are needed. Reduction in the number of iterations could be larger for higher tolerance values.
Figure 3.19: Time during the fault-on trajectory at which each nullcline is crossed \( (t_{ni}) \) for the 33 motors in the system. Lower and upper bounds, estimation and real CCT are indicated.

<table>
<thead>
<tr>
<th>Bus</th>
<th>Nullcline Crossing Time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
</tr>
<tr>
<td>6</td>
<td>0.25</td>
</tr>
<tr>
<td>9</td>
<td>0.30</td>
</tr>
<tr>
<td>12</td>
<td>0.35</td>
</tr>
<tr>
<td>15</td>
<td>0.40</td>
</tr>
<tr>
<td>18</td>
<td>0.45</td>
</tr>
<tr>
<td>21</td>
<td>0.50</td>
</tr>
<tr>
<td>24</td>
<td>0.55</td>
</tr>
<tr>
<td>27</td>
<td>0.60</td>
</tr>
<tr>
<td>30</td>
<td>0.65</td>
</tr>
<tr>
<td>33</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Table 3.1: Lower and upper bounds, estimation and actual CCT for the system from the case study.

The previous results will change if the system parameters and certain assumptions are modified. For instance, Fig. 3.19 was generated considering that the voltage source in bus 1 does not regulate its output voltage which means that there is a risk of system loss due to the over-current protections being activated. If the output voltage is regulated, then the nullcline crossing times are expected to be reduced as the stability region contracts. This can be calculated using the conditions presented in Section 2.4.3 that guarantee that current is below its maximum value or voltage is reduced to maintain current at its maximum value. The reduction in the nullcline crossing time caused by the limit of the source current to 1 pu is shown in Fig. 3.20 for a source with power rating equal to the rating of all loads at unity voltage.

### 3.6.2 Source Current Stability

Since the system does not have any constant power loads or DGs, the output current of the source can be calculated at all points along the fault-on trajectory using linear circuit techniques. The result of performing this calculation is shown in Fig. 3.21 also for a source that does not regulate its output and with a power rating equal to the rating of all the loads at unity voltage. In the figure, the solid red line correspond to the source output current in per unit which starts at around 0.5 pu at the speed after braking mode, and
quickly rises as the fault duration increases. A longer fault duration means that the operation point after the fault is farther away from the SEP so that more current is demanded. The risk of activating an over-current protection occurs at around 50 ms which is before even the first nullcline is crossed. Since the source is not regulating its voltage in this case, the dashed blue line representing the source output voltage stays at 1 pu for any fault duration.

As described in Section 2.4.3, an alternative to avoid activating an over-current protection of the source is for it to regulate its output voltage. In that case, the current is maintained at its maximum value by reducing the source output voltage. The same calculation performed above can be done accounting for this effect. The result is shown in Fig. 3.22. Now, instead for the current to increase above 1 pu, the source voltage is reduced as soon as that limit is reached. Although this avoids tripping the source, it causes a significant reduction of the nullcline crossing times, increasing the risk of motor stalling. Notice that the reduction in output voltage due to regulation can be significant in faults of long duration.

### 3.6.3 Voltage Stability

The third step in the system stability assessment is to calculate the impact that IM stalling will have on the system voltage. As mentioned before, in this case study there is no risk of voltage collapse as the system is constructed only with constant impedance loads and IMs so that the voltage UEP is at the origin. Generalization to other system can be done using the same techniques described above. Using the nullcline
Figure 3.21: Magnitude of the source output current for faults of increasing duration. The source does not regulate its output so that current increases beyond the maximum current limit.

Figure 3.22: Magnitude of the source current output for faults of increasing duration. In this case, the output voltage is reduced in order to maintain the current at its maximum value eliminating the risk of over-current tripping.
crossing times information, the voltage stability analysis described in Section 3.6.1 can be performed. It is done by calculating the reactive power capability of the network at a particular bus for an increasing number of IMs stalling. The results are shown in Fig. 3.23. As IMs stall, the voltage operation point decreases until it reaches a value of approximately 0.75 pu that occurs when all the motors have stalled. If a voltage of 0.9 pu is taken as the threshold value for under-voltage, then only 6 motors need to stall for this condition to happen.

3.6.4 Using CCT Estimation

Finally, to demonstrate the usefulness of having bounds and an approximate value for the CCT, a situation is studied in which active voltage support elements operating in droop mode are to be added to the system. Their location should be selected to obtain the larger increase in the CCT of the system. However, the number of combinations in which “n” voltage support elements can be placed in any of the 33 buses grows rapidly. If the real CCT is computed with accuracy for every case using time domain simulations with a starting point selected by an exponential search along the fault-on trajectory, then the procedure is extremely time consuming. The computation of the bounds and the estimated CCT is much faster as the nullcline crossing times do not require time domain simulations of the post-fault system. Even if the real CCT computation is required, performing a binary search between the bounds is more efficient than performing an exponential search along the trajectory. Using the same computer, calculating the real CCT by performing
an exponential search along the fault-on trajectory took 8.5 times more than computing the bounds and the estimation. Computing the CCT using binary search between the bounds took just 4 times more. Using the estimation or the bounds, maps such as the one shown in Fig. 3.24 can be quickly generated to quantify the expected CCT increase when the voltage support elements are located at different points along the network.

3.7 Conclusions

This chapter built on the tools and insights obtained in the previous chapter and presented a stability assessment procedure for systems with IMs. The procedure accounts for three phenomena that can cause problems in the system: IM stalling, source protections activating on over-current, and voltage issues due to reactive power imbalances. Each effect was studied in detail using a two machine prototypical system and then employed in a case study of a 33 machine system.
Chapter 4

Fault Recovery Strategy

4.1 Introduction

This fourth chapter presents the fault recovery strategy for systems with IM loads using the "Flexible Voltage and Frequency" operation introduced in Chapter 1. With this strategy, the recovery of a microgrid or a distribution network after a fault is enhanced by temporarily setting voltage and frequency at off-nominal values. The description, analysis and quantification of the strategy effects is done using the framework to study fault events and assess stability presented in the previous two chapters.

The chapter is divided in the following sections:

- Section 2 describes the strategy and the main concepts required for its implementation.
- Section 3 uses the tools from the previous chapters to quantify the effect of the strategy in the stability regions of the system.
- Section 4 presents a methodology to select important control parameters.
- Section 5 validates the strategy at the microgrid level using the experimental set-up presented in Appendix A.
- Section 6 studies the strategy in the context of distribution networks through time domain simulations of a case study using the IEEE 34 Node Test Feeder.
4.2 Fault-Recovery Strategy Description

The previous chapters showed that a fault somewhere in the network causes IM loads to undergo transitions that may cause contingencies that affect large areas such as FIDVR or blackouts. The goal of the fault recovery strategy is to expand the stability region of the stable equilibrium point to avoid such contingencies by increasing the motors electric torque at low speeds. This avoids stalling and improves the cascade recovery effect by expediting the IMs re-acceleration back to nominal speed. To achieve this goal, the voltage and frequency set-points of the sources are modified during the fault-on phase to follow the shapes shown in Fig. 4.1.

![Frequency and voltage wave-forms to implement the fault recovery strategy.](image)

Figure 4.1: Frequency and voltage wave-forms to implement the fault recovery strategy.

The strategy is implemented in a three step algorithm:

1. When the fault is detected, the frequency and voltage set-points are modified to $f_{\text{mod}}$ and $v_{\text{mod}}$, where $f_{\text{mod}}$ is lower than the nominal frequency, but the value of $v_{\text{mod}}$ can change depending on the microgrid loads characteristics.

2. The modified set-points are maintained for a time interval of $\Delta t_1$ that starts when the fault is cleared.

3. The set-points are ramped to nominal frequency ($f_0$) and voltage ($v_0$) during a time interval of $\Delta t_2$ that starts immediately after $\Delta t_1$.

Controlling the frequency and voltage of an IM supply is the basic operation principle of a motor drive. In this case, however, the values are modified for the entire microgrid where several IMs, DGs and other
types of loads interact. This limits the range of values that can be selected for voltage and frequency, the
time intervals that can be maintained, and requires the analysis of the system to be done including all the
elements.

The strategy is mainly intended for implementation in inverter-based microgrids while they operate
disconnected from the main grid. In those cases, the inverters ability to operate at off-nominal voltage and
frequency, and to modify those variables almost immediately, makes them ideal sources for the strategy
implementation. Microgrids with diesel generators do not posses those characteristics and limitations exist
on the implementation as it is discussed later in this chapter. The use of the strategy is also limited in large
distribution networks because frequency can be controlled only in the special cases where the feeder connects
to the system through an asynchronous link such as a variable frequency or a solid state transformer. In
the general case, frequency will be set by the transmission system and it can not be adjusted as desired.
However, even in those cases, a distribution system can benefit from the strategy if it contains inverter-based
microgrids that can be disconnected from the network during a fault, so that they implement the strategy
to recover their IM loads and then reconnect.

4.3 Effects on Stability

Implementing the strategy by modifying the voltage and frequency to recover from a fault brings two im-
portant benefits to the system:

1. It expands the rotor speed stability region by increasing the electric torque at low speeds.

2. It enhances the reactive power capability of the network by reducing the impedance between the loads
   and the source, and by increasing the voltage.

In both cases, the stability of the system is improved as it is described next.

4.3.1 Rotor Speed Stability Enhancement

The analysis starts with a single motor system for simplicity. Reducing frequency and increasing voltage
during the fault-on phase creates a new torque-speed curve for the motor in the post-fault system, as shown
in Fig. 4.2. The black dashed line represents the original post-fault electric torque of the machine when
operating at 1 pu of voltage and frequency. The intersection of this line with the mechanical torque at
high speeds determines the desired SEP, as that is where the IM should be operating as \( t \to \infty \), and the
intersection at low speeds gives the original critical speed or UEP. By operating the system at 1.1 pu of
voltage and 0.95 pu of frequency, the electric torque given by the solid black line is obtained. With these
conditions, there is a new SEP which is where the system will operate if the modified values are kept and, more importantly, there is a new critical speed located farther away from the SEP. This represents an expansion of the stability region as identified in the figure.

![Diagram](image)

**Figure 4.2**: Expansion of stability region for a single IM load seen as a shift of the torque-speed curve when frequency is reduced and voltage increased.

The effect just described is not unique to a single machine system. Reducing frequency and increasing voltage in a system with several IMs causes the electric torque of all the motors to increase at low speed. The two IM load case is presented in Fig. 4.3 where frequency was reduced to 0.95 pu, while maintaining voltage at nominal. In the figure, it was assumed that the voltage source regulated its output for points beyond the maximum current boundary so that a contraction of the stability region is seen (the maximum current boundary is not shown for clarity). The stability boundary of the post-fault SEP with the system operating at nominal frequency is given by the dashed black line, while the boundary of the expanded region caused by the reduced frequency is given by the solid black line. To validate the effect of the strategy on the recovery of IM loads, the results of two time domain simulations of the full system model are superimposed in the same figure. The red line represents the trajectory of the system when the strategy is not implemented and for a fault cleared after the original stability boundary, so that one of the IMs stalls. If the strategy is implemented, the same fault leads to recovery as seen in the trajectory given by the blue line. As these simulations were done for illustration, the frequency was not ramped to nominal (i.e. $\Delta t_1 \to \infty$) so that in the trajectory ends at a modified SEP. This SEP, however, is inside the stability region of the desired SEP and even if the frequency is changed abruptly to nominal (i.e. even if $\Delta t_2 \to 0$) the system will be stable. It
is clear then, that reducing frequency can stabilize an unstable scenario.

In Fig. 4.3 the electrical frequency of the system was reduced but the source voltage was maintained constant. This was done to isolate the impact of each variable and observe their effects independently. The value of $v_{\text{mod}}$ also has a significant impact on the stability boundary, even in cases where the source voltage is regulated beyond the maximum current boundary. The effect of modifying voltage between 0.9 pu and 1.1 pu while maintaining frequency at 0.95 pu is shown in Fig. 4.4. Both the stability boundary for IM stalling and the maximum source current boundary are shifted as a consequence of changing $v_{\text{mod}}$. Since the UEP is located beyond the maximum current boundary, where the voltage is regulated, it is not affected by changing the voltage set-points. However, even with the UEP staying at the same place, the stability region is expanded as the saddle points move farther away from the SEP. The effect of $v_{\text{mod}}$ on the maximum current boundary depends on the other type of loads present in the system. In Fig. 4.4 the plots were obtained connecting a constant impedance load next to the IMs so that the current boundary moves closer to the SEP as $v_{\text{mod}}$ increases (as expected by the constant impedance loads requiring more current at larger voltages). However, the current boundary behavior will change if constant power loads are connected next to the IM loads instead. Therefore, if there is a risk of activating the source protections at the current boundary, then it might be advisable to implement the strategy changing only the frequency. If the source output is regulated, then modifying voltage is also beneficial.
4.3.2 Voltage Stability Enhancement

An additional effect of implementing the strategy is that the reactive power capability of the network is increased. Using the techniques described in Section 3.5, the modified voltage and frequency are seen as an upwards shift of the inverted parabola representing the reactive power “source” curve seen in Fig. 3.17 of the previous chapter. This is caused both by a larger voltage at the source and a reduced impedance between the source and the loads. The reactive power “demand” curve also increases as the reduced frequency decreases the machine reactances leading to a larger consumption. The shift on the curves caused by the strategy implementation are shown in Fig. 4.5. In this case, the operation point moves from A to B avoiding FIDVR even when motors stall. The overall benefits of the strategy are two-fold: (a) it increases the voltage stable equilibrium point which might take the system above the 0.9 pu threshold, and (b) it prevents voltage collapse in heavily loaded systems by avoiding the bifurcation that leaves the system without equilibrium points. An unintended consequence of reducing frequency is that capacitors in the system will reduce their reactive power injection. This effect, however, is compensated by the increased voltage and it represents a secondary effect.
3.5 - Expanded network reactive power capability

3 - reactive power

2.5

'1.5

1 / Load reactive power demand increases slightly

0.5 - increases slightly

0.5

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1

Voltage [pu]

Figure 4.5: Effect of the reduced frequency and increased voltage on the network reactive power capability. The inverter parabola is shifted upwards as the nominal voltage is higher, and the impedance between the source and load is reduced.

4.4 Strategy Implementation

4.4.1 Selection of Modified Frequency ($f_{\text{mod}}$) and Voltage ($u_{\text{mod}}$)

Operating the system at off-nominal frequency and voltage is possible, but care must be taken to always operate the devices in a safe operation point. The motors saturation characteristics, the presence of other loads, and transformers maximum volt-second capabilities, limit the range of frequency and voltage that can be safely implemented with the strategy. In this dissertation, it is suggested for $f_{\text{mod}}$ and $u_{\text{mod}}$ to follow a standard for distributed generators so that all equipment should be designed to comply with them, guaranteeing that any operation point that results from the strategy will be safe. Examples of such standards are IEEE 1547 [51] and NERC [52] which recommend that DGs should be able to operate at a frequency of 0.95 pu for up to 300 seconds and a voltage of 1.1 pu continuously. In this thesis, these standards are followed, but the benefits of operating outside this range are explored. The results presented above in Figs. 4.3, 4.4 and 4.5 to show the expansion of the stability boundary, used a minimum frequency and maximum voltage in compliance with these standards.

In general, it will be desired to minimize the value of frequency during the strategy implementation but, as explained above, the selection of $u_{\text{mod}}$ depends on the other types of loads in the system. If the system sources shutdown their output after the maximum current boundary is reached, then the value of $u_{\text{mod}}$ has to be selected by characterizing the power-voltage relationship of the load as done in the construction of
"composite loads". If the total load has a large constant impedance component, then voltage should be left at nominal; if the constant power component dominates, then $v_{\text{mod}}$ should be increased to its maximum safe value. In cases where sources regulate their output or are significantly over-sized, then maximizing voltage is also recommended.

4.4.2 Selection of Time Intervals ($\Delta t_1$ and $\Delta t_2$)

The parameters $\Delta t_1$ and $\Delta t_2$ determine the time intervals when frequency and voltage are controlled to be off-nominal, and they play a critical role in the implementation of the strategy. Consider, for instance, that both $\Delta t_1 \to 0$ and $\Delta t_2 \to 0$ which means that the modified set-points are maintained for just a brief period before returning to nominal. In that case, the IMs do not have time to accelerate back to a speed within the stability region of the original SEP before being caught by the boundary that is contracting back to its original position. When the IMs operation point is caught by the contracting boundary, the system would be left outside the stable region and some IMs will stall. The system CCT will then be the same as if no strategy was implemented. On the other hand, if $\Delta t_1 \to \infty$ or $\Delta t_2 \to \infty$ then the CCT is given by the stability boundary obtained using the modified voltage and frequency, but the original SEP will not be recovered which is not the desired result. For $\Delta t_1$ and $\Delta t_2$ values in between these two extreme cases a performance metric $\eta \in [0, 1]$ is defined as:

$$\eta = \frac{t_{c}^{(r)} - t_{c}}{t_{c}^{(r)} - t_{c}}$$

(4.1)
where \( t_c^{(r)} \) is the CCT considering the impact of finite values of \( \Delta t_1 \) and \( \Delta t_2 \), \( t_c \) is the CCT as calculated with a post-fault system having nominal voltage and frequency values, and \( t'_c \) is the CCT with a post-fault system having modified values. The real benefit of implementing the strategy is not \( t'_c \) but \( t_c^{(r)} \).

In a generic system where there is coupling between the motors, it is not possible to find an analytical expression between the time intervals and \( t_c^{(r)} \), and numerical methods are needed. In the case of a single machine or a weakly coupled system (where each machine can be studied independently) an analytical method can be obtained. Both cases are explained next.

### 4.4.2.1 Single Machine or Weakly Coupled Systems

In a weakly coupled system or whenever the machines can be studied separately, the following procedure can be implemented. For a given pair of values \( \Delta t_1 \) and \( \Delta t_2 \), the value of \( t_c^{(r)} \) is calculated in two steps:

1. Find the motor speed \( \omega_{\text{int}} \) for which there is a valid value of \( \Delta t_2 \) that satisfies the relationship:

   \[
   \frac{1}{2H} \left[ T_e(\omega_{\text{int}}) - T_m(\omega_{\text{int}}) \right] = \frac{2\pi(f_0 - f_{\text{min}})/p}{\Delta t_2} \tag{4.2}
   \]

   This value has to be bounded from above to the original critical speed \( \omega_c \). This constraint guarantees that given a starting speed \( (\omega_{\text{int}}) \) the contraction of the stability boundary is not faster than the acceleration of the motor, meaning that the operation point is always inside the stability region.

2. Calculate the minimum motor speed \( \omega_r \) for which the following relationship is satisfied

   \[
   2H \int_{\omega_r}^{\omega_{\text{int}}} \frac{d\omega}{T_e(\omega) - T_m(\omega)} = \Delta t_1 \tag{4.3}
   \]

   This relationship guarantees that given a fault clearing speed \( (\omega_r) \) there is enough time for the motor to accelerate to a new speed \( (\omega_{\text{int}}) \) which was shown to be stable in the previous step.

The result of implementing the two steps in a single machine system is shown in Fig. 4.7 where a contour map of \( \eta \) is displayed for various values of \( \Delta t_1 \) and \( \Delta t_2 \). Motor and system parameters were taken from the experimental set-up to validate the map (see Section 4.5). Three interesting features can be highlighted from the figure:

- For a given value of \( \eta \), selecting a value of \( \Delta t_2 = 0 \) gives the minimum time between fault clearance and return to nominal conditions. Although this should be stable in most cases (as the modified SEP is typically inside the original stability region), a sudden step in frequency might cause abrupt transitions in the system and some slew rate is recommended.
• There is a minimum value of $\Delta t_2$ below which it ceases to have any effect in the CCT (in Fig. 4.7 this occurs around 0.85 secs). That value is determined by the derivative $\frac{d\omega_c}{dt}$ evaluated at the critical speed $\omega_c$ of the system with nominal frequency. If $\Delta t_2$ is smaller than the minimum value, then the contraction of the SEP stability region is so fast that the IM operation point will always be left outside.

• The value of $\eta = 1$ is never reached because $t_c'$ is an equilibrium point itself. If the fault is cleared exactly at that instant then the torques are balanced and there is no acceleration. Although $\eta = 1$ is not achievable, as $\Delta t_1 \to \infty$ and/or $\Delta t_2 \to \infty$, $\eta$ asymptotically approaches 1.

Figure 4.7: Performance map of a single machine system showing the value of the metric $\eta$ for a given pair of parameters $\Delta t_1$ and $\Delta t_2$.

### 4.4.2.2 Multiple Machine Systems

For coupled systems, time domain simulations are needed to compute $t_c^{(r)}$ and $\eta$ because there is no analytical expression for the stability boundary or how it contracts as voltage and frequency ramp back to nominal. However, a similar map of $\eta$ as a function of $\Delta t_1$ and $\Delta t_2$ can be generated for those cases. The main idea is to ensure that the operation point is always inside the stability region even as it contracts. Consider the phase plane in Fig. 4.8 which shows a zoom into the fault clearing moment of the stable trajectory presented in Fig. 4.3. All the important instances are labeled in the figure, with $t_c$ occurring at the intersection of the fault-on trajectory and the original boundary, and $t_c'$ at the intersection with the expanded boundary. $t_c^{(r)}$ is the moment when the fault is cleared which is initially stable. The modified values of voltage and frequency are maintained from $t_c^{(r)}$ to $t_0$ (i.e. $\Delta t_1 = t_0 - t_c^{(r)}$). Then, a race starts between the operation point moving along the trajectory given by the blue line and the contracting boundary represented by black and gray lines.
in the figure. Six snapshots of the stability boundary and the operation point are shown and labeled $t_0$ to $t_5$ (i.e. $\Delta t_2 = t_5 - t_0$). In the shown case, the contraction of the boundary almost reaches the operation points at times $t_3$, $t_4$ and $t_5$ but the system manages to stay within the stability region so that the SEP is recovered. For these $\Delta t_1$ and $\Delta t_2$, the shown value of $t_c^{(r)}$ represents the farthest point along the fault-on trajectory that is stable and results in a value of $\eta = 0.57$. This means that if either $\Delta t_1$ and $\Delta t_2$ were slightly smaller, then the stability region contraction would catch up with the operation point and leave it outside the stable region, leading to an unstable condition. Notice that if $\Delta t_1 = t_5 - t_c^{(r)}$ (i.e. the modified set points are maintained until the operation point is inside the stability region of the original SEP) then $\Delta t_2$ has no effect and stability is guaranteed. However, that implies that $\eta$ could be larger as a fault cleared after that value of $t_c^{(r)}$ can still be stable.

The performance map for a two machine system will have different characteristics than the one for a single machine presented in Fig. 4.7. Using the concepts described above, the performance map for a two motor system using $f_{mod} = 0.95$, $v_{mod} = 1.1$ is shown in Fig. 4.9. Two of the characteristics of the map for a single machine described in Section 4.4.2.1 are still observed, namely, that for a given $\eta$ the minimum time to obtain the desired SEP is achieved by making $\Delta t_2 = 0$, and that the value of $\eta = 1$ is only approached asymptotically as $\Delta t_1 \to \infty$ or $\Delta t_2 \to \infty$. The other characteristic, that set a threshold value below which $\Delta t_2$ has no effect, is not observed in the multi-machine case because the operation point is now moving in a two dimensional space so that there is no single value of $\Delta t_2$ that guarantees that the system will be caught

![Figure 4.8: Snapshots of the fault-recovery process and the stability boundary returning to its original position in a two machine system. After the fault is cleared, a race starts between the operation point and the boundary.](image)
by the stability boundary. As seen in the map, with the system parameters used, even if $\Delta t_1$ and $\Delta t_2$ are left only for 1 second and then ramped to nominal in another second, a performance of approximately $\eta = 0.99$ can be obtained.

Figure 4.9: Values of the performance metric $\eta$ of a two machine system for various values of $\Delta t_1$ and $\Delta t_2$.

4.4.3 Considerations Regarding Microgrid Control Architecture

The above descriptions assumed that the system can adjust its voltage and frequency as desired, which is a proper assumption in systems with a single power source or where all except one of the sources act in grid-following mode with a fixed output. In the last case, the system is said to be operating in Master-Slave control architecture where the master source sets the voltage and frequency of the system (also called "grid-forming" or slack source) and the rest adjusts to any change. With that control scheme, the strategy needs only to be implemented in the master source for it to be effective. Microgrids can also operate in droop control mode, in which the system frequency and voltage are used as shared signals in order to achieve real and reactive power sharing. With that control scheme, there is no direct control on the voltage and frequency, but the strategy can still be implemented by adjusting some of the set-points on the droop characteristics. Consider, for instance, the typical P-f droop implementation for two sources shown on the left side of Fig. 4.10. Given a total power consumption $P_L$, the operation frequency is set as the unique point where $P_1 + P_2 = P_L$. If the frequency set-point at zero power is lowered as shown in the right side of the figure, then the operation frequency will also be reduced even though its exact value is set by the total load demand. The same result can be achieved using a Q-V droop for modifying voltage and achieving reactive power sharing.
4.4.4 Considerations Regarding Synchronous-based Sources

As mentioned above, the fault-recovery strategy is meant to be implemented in systems with inverter-based sources to take advantage of their ability to rapidly adjust voltage and frequency. If the system also has sources that interface to the network through synchronous machines, such as diesel generators, then the frequency is set by the rotational speed of the machine and it will exhibit a dynamic behavior that complicates the strategy implementation. Consider, as an illustration, the waveform shown in Fig. 4.11 that presents the dynamics of the rotational speed of a diesel generator throughout a fault event of 500 ms. The fault occurs at $t = 0$ causing a sudden deceleration due to the braking mode transition (for the same reasons it happened in the IM loads case) but then the generator start accelerating as the mechanical input power continues while the electrical output is close to zero. The generator acceleration stops (or decreases) when the fault is detected and the engine power is set to a minimum. In the figure, this is seen as a plateau in the speed that starts at around 100 ms, but the process might take longer depending on the engine control characteristics. During the time while the fault is on, the frequency reference is changed (shown in a dashed black line), but the lack of output power prevents the generator from slowing down. When the fault is cleared, the output power is restored allowing the generator to move towards its new speed reference but it does it in about 0.5 seconds, so that the change is not immediate.
The voltage of the system also exhibit some dynamics that depend on the exciter and AVR control characteristics. The behavior of the generator output voltage (which sets the voltage of the system) is shown in Fig. 4.12 for the same fault studied above. During the fault-on period that starts at \( t = 0 \) the voltage reference (shown by a dashed black line) is increased so that when the fault is cleared, the AVR takes the system to the new voltage level but does it with some dynamics due to the resistance and inductance of the field winding. Even though it takes some time for the voltage to set in its new value, it is still faster than the speed dynamics. The details of the required changes in the governor and exciter controllers to implement the strategy with synchronous sources is left as future work.
4.5 Microgrid Level Validation

The functionality of the fault-recovery strategy described above was validated in an inverter-based microgrid system using the experimental set-up described in Appendix A.

4.5.1 Scenario Description

The microgrid system used for this validation has constant impedance and IM loads, and an inverter-based source, all of them connected to the distribution network through an interconnection switch. The inverter is large enough to power all the loads simultaneously in steady state. For the study, the system goes through the three phases of a fault event described in Chapter 2:

- **Pre-fault phase:** Before the disturbance, the microgrid is in steady state operation and connected to the distribution network. The inverter operates in grid-following mode so that it injects real and/or reactive power to the system.

- **Fault-on phase:** A fault causes a severe voltage sag at microgrid PCC. In this validation, the voltage is taken all the way down to zero which implies that the fault occurred close to the microgrid interconnection point.

- **Post-fault phase:** For faults lasting less than 160 ms [51], the microgrid should stay connected to the
network, but the following study assumes that regardless of the fault duration, the operator decides to disconnect and island the microgrid.

4.5.2 Experimental Set-up

The validation of the fault-recovery strategy was done using the experimental set-up arranged as shown in Fig. 4.13. The two IM loads were rated at 1 HP and connected in the pre-fault phase to the MIT power system \((3\phi - 4L, 208 \text{VLL}, 60 \text{Hz})\) through a line impedance. Mechanical torque was controlled by coupling the IM with a DC generator whose output armature current was regulated by a DC/DC converter. During the fault-on state, a solid state relay connected a short circuit at the indicated point. After an adjustable period of time, the interconnection switch was opened clearing the fault and islanding the microgrid. In the post-fault state, a 2kVA inverter, connected through an LCL filter and a line impedance (representing a transformer), acted as the voltage source of the system.

![Figure 4.13: Schematic view of the microgrid experimental configuration to test the fault recovery strategy.](image)

4.5.3 Results

The validation was done twice, first using the system with a single IM load and then using the two IM loads. Results and discussion are presented next.

4.5.3.1 Experimental One Machine System

For the single motor case, the rotational speed as a function of time is presented in Fig. 4.14 for five different cases. The dark red line labeled as “No strategy” represents the base case where the fault-recovery strategy
was not implemented and the motor was tried to be recovered at 1 pu of voltage and frequency, but eventually stalled as islanding occurred after the critical speed. As the modified frequency changed, the motor entered the stability region of the post-fault SEP and stalling was avoided. Even though at $f_{\text{mod}} = 59$ Hz (0.98 pu) the motor still stalled (shown in red line), when $f_{\text{mod}}$ crossed the 58Hz boundary recovery was observed (cyan and blue lines). Recovery was expedited significantly when electrical frequency was reduced to 40Hz (0.67pu) for only 0.3 seconds before ramping back to 60 Hz as shown in the dark blue line, but this condition is outside the standards specifications so that there would be a risk of affecting some other loads and DGs in the system. However, the 40Hz case is a demonstration that if the safe operation region for microgrids can be expanded, then a significant improvement on fault recovery can be obtained.

![Figure 4.14: Experimental results of motor speed during and after a fault with the post-fault system operating at different frequencies for a brief time. When the frequency is reduced to 58 Hz or lower, an unstable condition is made stable.](image)

In each of the plots from Fig. 4.14 the values for $\Delta t_1$ and $\Delta t_2$ were different depending on the modified frequency that was used. The actual increase in the critical clearing time depends on these parameters, as it was shown in the performance map of Fig. 4.7. To validate such map, experimental measurements were done on two cross sections of it for constant $\Delta t_2$. The analytical values of $\eta$ as a function of the time interval $\Delta t_1$ (for two fixed values of $\Delta t_2$) are shown in Fig. 4.15 by the blue and red solid lines. A sequence of tests were done to find bounds on the value of $\eta$, by fixing $\Delta t_1$ and $\Delta t_2$ and changing the clearing time until close enough stable and unstable cases were found, representing the lower and upper bounds respectively. These are indicated in the figure as error bars with their midpoint highlighted by an asterisk. In all cases, not only does the analytical $\eta$ falls inside the bounds but it is also close to their midpoint.
4.5.3.2 Experimental Two Machine System

The second series of tests to validate the fault-recovery strategy were done with two IM loads connected at the PCC. In this case, coupling effects can be observed and the system can be analyzed using a phase plane. The base case in which the strategy was not implemented is presented in Fig. 4.16. This is similar to the results presented in Fig. 2.23 from Chapter 2, although in this case, during the post-fault phase, the microgrid is powered with an inverter instead of the grid. As before, all faults cleared before the theoretical stability boundary converged to the post-fault SEP while the ones cleared after caused the motors to stall.

If the fault recovery strategy is implemented, then the stability region is expanded and unstable trajectories can be made stable. This is presented in Fig. 4.17 for $f_{\text{mod}} = 0.95$ pu and $v_{\text{mod}} = 1.1$ pu. The boundary of the theoretical stability region is indicated with the solid black line, which can be compared with the original boundary shown in the black dashed line. The trajectory shown with a red line represents an unstable condition where the strategy was not implemented and the fault was cleared after the original stability boundary. The trajectory given by the blue line demonstrates that, by implementing the strategy, a fault that is cleared after the previous unstable case can be made stable as it is within the expanded stability region.

The stability region can be expanded significantly more if the modified frequency is reduced to $f_{\text{mod}} = 0.67$ pu for a brief period of time. In that case, the stability region contains the entire phase plane in the region of interest so that rotor speed stability is guaranteed. Example of trajectories for this scenario are shown in Fig. 4.18. Faults cleared much after the stability boundaries of the previous cases are still...
Figure 4.16: Stable and unstable trajectories obtained experimentally for a two machine system without the implementation of the strategy. All faults cleared before the theoretical stability boundary returned to the SEP, while those cleared after caused IMs to stall. Nullclines are shown in red and blue dashed lines.

Figure 4.17: Experimental validation of stability region expansion. A stable trajectory obtained by implementing the strategy with $f_{mod} = 0.95$ pu and $v_{mod} = 1.1$ pu is shown in blue. The fault is cleared outside the original stability region but inside the expanded one. An unstable trajectory is shown for a case where the strategy was not implemented.
Figure 4.18: Stable trajectories obtained by implementing the strategy with $f_{\text{med}} = 0.67\text{pu}$ and $v_{\text{med}} = 1.1\text{pu}$ for faults cleared much after the original CCT. In this case, the entire phase plane in the shown window is inside the expanded stability region.

recovered, although they are seen to converge to a modified SEP before the frequency is ramped to nominal. This modified SEP is within the stability region of the original SEP so no instability will occur. Even if that was not the case, if the ramp of voltage and frequency to nominal (i.e. $\Delta t_2$) is slow enough, then the operation point should always be stable (a condition that would be indicated in the performance map).

4.6 Distribution System Study

Having proved the effects of the strategy experimentally, a numerical study is now performed at the feeder level considering a system with a larger number of IM loads.

4.6.1 System Description

For the study at this level, the system is shown schematically in Fig. 4.19. In it, several of the microgrids described above are connected to a network with a voltage source that could be a substation or an interconnection point to a larger system. While the microgrids are connected to the grid, their DGs operate in slave mode and, due to the network impedances, coupling effects are expected. If a microgrid island, then the coupling of its elements is only local and they have no effect on the larger distribution network.
As in the microgrid level study, the fault in this case is studied in three phases:

- **Pre-fault phase:** Before the fault, all the microgrids operate in grid connected mode and are seen as a PQ bus with constant impedances in parallel.

- **Fault-on phase:** A fault somewhere in the network occurs so that the voltage is reduced until the fault is cleared. During this period, the microgrids can ride through the fault or island.

- **Post-fault phase:** The fault in a distribution network is typically cleared by re-configuring the network and disconnecting the branches that contain the fault. In this study, however, it is assumed that the fault is cleared without reconfiguration. This simplification is not significant as most of the analysis is done in the post-fault system regardless of which form it takes.

The impact of the fault recovery strategy in distribution systems was evaluated by performing numerical simulations on the IEEE 34 Node Test Feeder with a constant impedance and an IM load at each bus. DGs were also connected at specific locations to allow for microgrids to be formed and operate on their own.

### 4.6.2 Strategies Comparison

The strategies used to test the functionality of the fault-recovery strategy were divided in two groups:
1. Fault-ride through cases

- **Case 1a:** The DGs were used to provide reactive power, and voltage and frequency were kept at nominal values.

- **Case 1b:** The DGs were used to provide reactive power, and the fault recovery strategy was implemented in the entire system.

2. Islanding cases

- **Case 2a:** Microgrids were islanded during the fault and the entire system was operated at nominal voltage and frequency.

- **Case 2b:** Microgrids were islanded during the fault and the fault recovery strategy was implemented in them. The rest of the system was operated at nominal voltage and frequency.

The first two cases correspond to situations in which the system maintains its topology during the fault event and, when used, the strategy is implemented in all the buses. If the system represents a feeder, case 1b could be realized if the substation connects to other networks through an asynchronous link such a solid state or variable frequency transformers, so that voltage and frequency can be manipulated. The advantage of comparing cases by implementing the strategy in all the buses is that its effect can be observed directly.

An alternative approach is to implement the strategy only when the microgrids of the distribution network operate islanded. This is done in the last two cases, where some buses in the feeder form microgrids, as shown in Fig. 4.20, and the fault recovery strategy can be used in them. For microgrids to be able to operate on their own, a DG large enough to power all the loads inside was connected at the far end of each. The buses that are not part of a microgrid always operate at nominal values of voltage and frequency so that no special interface to the transmission system is needed at the substation.
4.6.2.1 Fault-ride Through Results

As described above, the first two cases deal with a system in which all the elements ride through the fault. The CCT was found following the method described in Sec. 3.3 which includes finding the bounds and then using time domain simulations. The obtained results are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Case 1a</th>
<th>Case 1b</th>
</tr>
</thead>
<tbody>
<tr>
<td>190 ms</td>
<td>253 ms</td>
</tr>
</tbody>
</table>

Table 4.1: CCT for cases where microgrids ride through the fault. In case 1a the strategy was not implemented, and in case 1b it was used in the entire system.

The speed of all the motors is shown in Fig. 4.21 for case 1a with a fault cleared at 210 ms. Since the fault is cleared after the CCT, the system is unstable and some IMs stall. This unstable condition leads to FIDVR as it can be seen in Fig. 4.22 that presents the voltages at all the buses before and after the fault. As seen, even though all buses had voltages above 0.9 pu before the fault, IM stalling causes some of them to experience a voltage below the threshold that will be maintained until the stalled motors are disconnected from the system.
By implementing the strategy with $f_{\text{mod}} = 0.95 \, \text{pu}$, $v_{\text{mod}} = 1.1 \, \text{pu}$, $\Delta t_1 = 1 \, \text{sec}$ and $\Delta t_2 = 1 \, \text{sec}$, the unstable condition from above can be made stable. The value of $\eta$ for the pair $\Delta t_1, \Delta t_2$ can be calculated to be 0.94, so that a fault cleared at 210 ms leaves the system inside the expanded stability region and without risk of being caught by the contracting boundary. The speed of all motors is shown in Fig. 4.23 where it is
seen that all of them recover their nominal speed. The figure also shows that, as expected, the system first settles at a modified SEP (seen as a plateau in the speeds) before the set-points are ramped back to nominal and the original SEP is recovered. The evolution of voltage at all the buses is presented in Fig. 4.24 and demonstrates that FIDVR is avoided as all voltages return to their pre-fault value after a few seconds.

Figure 4.23: Time domain simulation results showing the speed of the 33 motors for case 1b where the fault recovery strategy is implemented. All motors recover their SEP after staying in a modified SEP for a brief period.

Figure 4.24: Time domain simulation results showing the voltage at the 33 buses for case 1b where all motors and voltages are recovered.
4.6.2.2 Islanding Results

In the cases where the microgrids island during the fault, the strategy leads to an improved recovery after a fault, as seen in the CCT for cases 2a and 2b that are shown in Table 4.2. There are two inherent advantages of islanding the microgrids with respect to the previous cases: (a) The distance between the voltage source and the loads is reduced, and (b) voltages are increased at the distribution network as the power consumption decreases. These two advantages can be obtained without islanding if the DGs are controlled to make the power flow equal to zero at the interconnection between the microgrid and the distribution network. However, that scenario would require the fault recovery strategy to be implemented in the entire system because, without islanding, the voltage and frequency at the microgrids cannot be changed independently. Islanding the microgrids avoids this restriction and the strategy can be implemented by the microgrid sources while the substation operates at nominal frequency.

<table>
<thead>
<tr>
<th>Case 2a</th>
<th>Case 2b</th>
</tr>
</thead>
<tbody>
<tr>
<td>262 ms</td>
<td>327 ms</td>
</tr>
</tbody>
</table>

Table 4.2: CCT for the cases where the system was divided in microgrids. In case 2a all buses operated at nominal voltage and frequency, while in case 2b the strategy was implemented inside the microgrids.

The speed of all the IMs for Case 2a is presented in Fig. 4.25 for a fault cleared at 300 ms which is after the CCT so many of the motors are observed to stall. With these system conditions, the only motors that stall are those inside a microgrid. If this was not the case and some motors in the distribution network also stalled, then the fault-recovery strategy would not have any impact as it only affects elements inside microgrids. This would be seen as a performance map with \( \eta \rightarrow 0 \) for all values of \( \Delta t_1 \) and \( \Delta t_2 \).

By implementing the strategy inside the microgrids with \( f_{\text{mod}} = 0.95 \text{ pu} \), \( v_{\text{mod}} = 1.1 \text{ pu} \), \( \Delta t_1 = 1 \text{ sec} \) and \( \Delta t_2 = 1 \text{ sec} \), the behavior of this system is greatly improved as shown in Fig. 4.26 where all the motors return to the SEP. Notice that the motors not belonging to any microgrid reach the desired SEP fast, while the ones in microgrids settle first to a modified SEP before being ramped up to nominal.

4.7 Conclusions

This chapter described a recovery strategy that modifies the voltage and frequency of a system after a fault. The effect on the system stability was analyzed, and a methodology to select important parameters for its implementation was described. Results showed that using the strategy leads to a significant increase on the system CCT and faster recovery to nominal conditions. Validation was done both experimentally using the microgrid test-bed described in Appendix A, and through simulations of a case study using the IEEE 34 Test Node Feeder.
Figure 4.25: Time domain simulation results showing the speed of the 33 motors for case 2a when the microgrids island but the fault recovery strategy is not used. The fault was cleared after the CCT leading to IM stalling.

Figure 4.26: Time domain simulation results showing the speed of the 33 motors for case 2b where the microgrids island and fault recovery strategy is implemented inside them. All motors recover their SEP after the fault.
Chapter 5

Inertial and Frequency Response

5.1 Introduction

This fifth chapter of the dissertation extends the concept of "Flexible Voltage and Frequency" operation to the frequency stability domain. Instead of using it to avoid IM stalling after a fault, the concept is posed as a control technique to use the kinetic energy in the IM loads for frequency regulation and inertial response. As described in Chapter 1, maintaining frequency inside a tight range is essential for a safe operation of power systems, and it is becoming challenging to do so as more renewable sources, that posses no inertia, are added to the grid. The results presented in this chapter demonstrate that microgrids with an asynchronous connection to the distribution network can make sudden changes of load or generation to have an inertial response, and help the grid frequency regulation without the need of energy storage devices.

This chapter is organized in the following sections:

- Section 2 introduces the topic of frequency stability and presents basic concepts involved in its study.
- Section 3 describes the operation strategy and explains its use to extract energy stored in the IM loads in a controllable manner.
- Section 4 poses the strategy as a control technique that can be implemented in the asynchronous link that connects the microgrid to a larger system.
- Section 5 presents simulation results for two case studies to demonstrate that, with the strategy, a microgrid can be used to provide frequency regulation and an inertial response to sudden changes occurring inside it.
- Section 6 validates the results experimentally using the microgrid test-bed described in Appendix A.
5.2 Frequency Stability

When studying frequency stability, it is common to look at the frequency behavior following a large disturbance. A typical response is presented in Fig. 5.1 for a low inertia network experiencing a sudden increase in load at $t = 0$. The power imbalance immediately after the load connection causes a deceleration of the synchronous generators in power plants (or sources like diesel generators) leading to an under-frequency excursion. The reduced frequency is maintained until the controllers on the prime movers increase the input power and accelerate the generators back to their nominal speed.

This behavior can be explained by looking at the mechanical dynamics of a synchronous machine given by the equation:

$$j \frac{d\omega_m}{dt} = \frac{P_m - P_e}{\omega_m}$$  \hspace{1cm} (5.1)

where $J$ is the inertia, $\omega_m$ the rotational speed, $P_e$ the output electrical power, and $P_m$ the input mechanical power. From this, it becomes clear that given an imbalance between the mechanical and the electrical power, a system with lower inertia will lead to a larger change in $\omega_m$. This is also seen in Fig. 5.1 where the frequency behavior is presented for generators with the same control parameters but different inertias. The typical frequency control techniques described in Chapter 1 are implemented by adjusting the mechanical power $P_m$ as a function of frequency deviations from its nominal value (primary control) and by external
commands (secondary control). Aggregating the dynamics of all synchronous machines in a system and their respective controllers is usually a complicated task and leads to complex dynamic phenomena such as inter-area oscillations which are out of the scope of this study.

In large systems where deviations after a disturbance are a concern, frequency should always be maintained inside a region bounded by what is typically called the "trumpet" characteristic [4]. This is the equivalent of the maximum frequency deviations defined in Fig. 4.6 for distributed generators. If the frequency leaves the region contained within the trumpet characteristic, then there is a risk of tripping more generating units and causing a blackout in a large area. As renewable integration into the grid continues, inertia is reduced and the total power input into the system becomes intermittent, so that not only does the frequency nadir (point C in Fig. 5.1) becomes lower but large disturbances become more common.

5.3 Strategy Description

The goal of most frequency response techniques is to help the generators to maintain the frequency within the allowed range and to restore it quickly to nominal. This can be done by adding more inertia to the system through synchronous-based sources (such as diesel generators), using solar PV to provide frequency response through curtailment [54] or operation below its maximum power [55], controlling the inverters of energy storage devices and wind turbines to react to frequency deviations [49, 56, 57, 63], or adjusting the energy consumption through demand response schemes [58, 59, 60, 61]. The strategy proposed in this chapter uses the energy stored in the IM loads to provide frequency and inertial response. It is a combination of a demand side management and a virtual inertia approach as it deals with the power consumption of loads but does it by using their energy stored.

5.3.1 Transitions between Torque-speed Curves

The main concept behind the operation strategy is the ability of induction motors to act as generators and provide power for some time. Taking energy back from a motor is typically employed in regenerative braking applications where the presence of variable speed drives provides full controllability of the supply. In microgrids, where other elements share the same voltage and frequency, the safe operation range is limited. However, by adjusting frequency within the range given by standards such as [51, 52], it is possible to extract energy and to shape the power to the IM loads. To do that, the strategy creates controllable transitions between torque-speed curves by varying the system electrical frequency. Examples of such transitions are shown in Fig. 5.2. Consider, as an illustration, the transition between points A-B-C that occurs when the frequency is reduced from 63 to 50Hz, with a slew rate faster than the mechanical time constant of the motor.
Figure 5.2: Transitions between operation points in different torque-speed curves triggered by changes in electrical frequency.

The system starts in steady state operation at point A, when the sudden decrease in frequency modifies the torque-speed characteristic causing the system to reach point B. At that point, the IM has a negative slip and it essentially behaves as a generator until the rotor speed reduces and reaches steady state operation point at C. Although there is a negative peak in electric torque, the rotational speed of the motor decreases monotonically from A to C. The reverse transition is also possible: starting from C, the frequency can be increased to 63 Hz causing a rise in the power consumption while the motor accelerates to point A. These transitions are non-linear and show a different behavior whether the frequency is decreased or increased. Although the transitions were shown for a constant mechanical torque and heavily loaded conditions, the concept also applies for different mechanical torque characteristics and if the machines are lightly loaded.

5.3.2 Equivalent Energy Storage

As a result of the above transitions, microgrids can controllably extract energy from the IM loads. The total energy exchange is given by the difference in the IM kinetic energy:

$$ E_i = H_i \cdot P_{\text{nom},i} \cdot (\omega_{\text{max}}^2 - \omega_{\text{min}}^2) \quad (5.2) $$

where $H_i$ is the per unit inertia of the IM and the mechanical load, $P_{\text{nom}}$ is the nominal power of the IM, and $\omega_{\text{max}}, \omega_{\text{min}}$ are the maximum and minimum rotational speeds (in p.u.). To maintain the microgrid within a safe range, maximum and minimum frequencies of 1.05 and 0.95 were used as benchmarks. However,
explorations of the strategy functionality with looser bounds is also done for illustration and to provide an argument for a potential extension of the safe frequency range in microgrids. As it was mentioned above, inverter-based microgrids could be able to operate at a wider range of frequencies than large power plants, as there are less risks associated with operation far from nominal.

Although the energy that can be extracted from a single motors is not large, microgrids usually contain a large number of motors and their energy can be added to obtain the total stored energy in the system as:

\[ E_t = \sum_i E_i \]  
(5.3)

Given that the change in rotational speed is approximately proportional to the change in electrical frequency, the total energy is a sum of the motors rated power weighted by their per-unit inertia. Typical values of per-unit inertia in motors for different applications can range from 0.1 sec in air conditioners to more than 2 sec in large pumps found in industrial settings [64].

### 5.3.3 Power Dynamics

Another important characteristic of the torque-speed transitions is their capability to shape the power given or taken from the induction motors. This concept is analyzed by constructing a model of the IM defining the electrical frequency as an input and power as the output. Since the total output is just the addition of the individual motors output, the analysis is first done in a single machine system.

#### 5.3.3.1 Single Machine Model

In Chapter 1, the dynamics of an IM were presented using magnetic fluxes as the states. The same model is used in this chapter, but with the electrical frequency as an input rather than a constant. The other inputs are, as before, the voltage and mechanical torque, while power is the output of interest. The system dynamics and output can be written as:

\[
\begin{align*}
\frac{d}{dt} & \begin{bmatrix}
(1/\omega_0) \psi_{ds} \\
(1/\omega_0) \psi_{qs} \\
(1/\omega_0) \psi_{dr} \\
(1/\omega_0) \psi_{qr}
\end{bmatrix} \\
\begin{bmatrix}
\omega_m
\end{bmatrix}
\end{align*}
\begin{bmatrix}
-r_s i_{ds} \\
-r_s i_{qs} \\
-\omega_m \psi_{qr} - r_r i_{dr} \\
\omega_m \psi_{dr} - r_r i_{qr}
\end{bmatrix}
+ \begin{bmatrix}
\psi_{qs} \\
\psi_{ds} \\
\psi_{qr} \\
-\psi_{ds}
\end{bmatrix}
\begin{bmatrix}
\omega_e + 0 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
-1/2 \frac{1}{H} T_m
\end{bmatrix}
\]

\[ P_{im} = v_{ds} i_{ds} + v_{qs} i_{qs} \]  
(5.5)
where $i_{ds}$ and $i_{qs}$ are a linear combinations of the states $\psi = \begin{bmatrix} \psi_{ds} & \psi_{qs} & \psi_{dr} & \psi_{qr} \end{bmatrix}^T$.

Since the transitions occur rapidly, the terminal voltage and mechanical torque of all the motors can be assumed to be constant, leaving the electrical frequency as the only input to the system for the next study.

The power response obtained by simulations of the full model can be observed in Fig. 5.3 for three different electrical frequency wave-forms. The blue line presents the response of the power consumed by the IM load when the electrical frequency is subject to a step change of 1 Hz. Its shape resembles a high pass filter response where there is a sudden initial rise followed by an exponential decay as the output returns to steady state. The shape of this response is changed if the frequency input is modified with a slew rate. The red line presents the output when the frequency ramps 1 Hz in $\Delta T = 0.3$ seconds, and the yellow line when it is done in $\Delta T = 0.6$ seconds. Although in all three cases the stored energy is approximately the same (i.e. the area below the curve is equal), the shape of power can be modified significantly depending on the way the electrical frequency is controlled.

The previous results were calculated using time domain simulations of the non-linear model presented in Eqs. 5.4 and 5.5. Similar results can be obtained if the system is linearized around an operation point. This approximation is valid as long as the electrical frequency variations are maintained close to the nominal value, which is one of the assumptions made above. When compared with the non-linear model, the linear approximation yields an almost identical response as shown in Fig. 5.4. In this figure, the frequency was controlled to extract energy from an induction motor for 0.4 seconds at an almost constant rate by lowering
5.3.3.2 Multiple Motors Model

For its implementation in microgrids, the power response of a system with multiple induction motors needs to be calculated. In such a system, each motor has the set of equations presented in Eq. 5.4 and the total power is the addition of power from individual motors resulting in:

\[ P_{\text{tot}} = v_{ds} \sum_k i_{ds,k} + v_{qs} \sum_k i_{qs,k} \]  \hspace{1cm} (5.6)

The response of the total power of a multi-motor system is presented in Fig. 5.5 for a 1 Hz step occurring at \( t = 0 \). The results were obtained by performing time domain simulations of five motors with power ratings of 1, 2, 4, 8 and 16 kVA, and scaling some parameters as \( r_s \propto P_{\text{nom}}^{-1/3} \), \( r_t \propto P_{\text{nom}}^{-1/3} \) and \( H \propto P_{\text{nom}}^{1/3} \) while leaving per unit reactances constant. As expected, the small machines have a minimum effect on the total power response and most of the contribution comes from the large machines.

The idea of linearizing the system around an operation point can also be applied to systems with multiple machines. Given that the output is a linear combination of the states, each motor can be linearized independently and then aggregated to construct the full system. The procedure to do that is presented next.
Figure 5.5: Response of the power demanded by multiple induction motors to a 1 Hz step change of electrical frequency. The total power is the addition of all the individual motors power, although small machines have minimum effect.

Start by linearizing a single motor to obtain a system of equations with the form:

\[
\dot{x}_i = A_i x_i + B_i \omega_e \tag{5.7}
\]

\[
P_i = C_i x_i \tag{5.8}
\]

where \(x_i = \begin{bmatrix} \psi_{ds,i} & \psi_{qs,i} & \psi_{dr,i} & \psi_{qr,i} & \omega_{im,i} \end{bmatrix}^T\) are the states of that particular motor. Then the total system dynamics are constructed as:

\[
\dot{x} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{bmatrix} x + \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix} \omega_e \tag{5.9}
\]

\[
P_{tot} = \begin{bmatrix} C_1 & C_2 & \cdots & C_N \end{bmatrix} x \tag{5.10}
\]

where \(x = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix}^T\) is a collection of all states. Using Eq. 5.9 and Eq. 5.10, linear tool can be used in a system with an arbitrary number of induction motors for analysis and control design.
5.4 Strategy Implementation

5.4.1 Prototypical System

For the strategy to be used, the electrical frequency needs to be controlled, so that it is intended for inverter-based microgrids that are either islanded or connected to the network through an asynchronous link. A prototypical structure of such system is shown in Fig. 5.6, and contains inverter-based sources, induction motors and ZIP loads. The asynchronous link that connects the system to a larger network is a two port element that not only changes the voltage level but also can also adjust the electrical frequency. These asynchronous links can be realized with variable frequency or solid state transformers. The former type has been used in a variety of locations such as the 100MVA Texas-Mexico and the Quebec-NE 100MVA interconnections. The latter type is typically used in smaller installations and has been recognized as a critical element for the future of the grid [65] due to the multiple benefits it brings such as better fault-ride through capabilities and improved power quality, among others. Since microgrids are the aim of this study, the asynchronous link is assumed to be a solid state transformer.

![Diagram](image)

Figure 5.6: Schematic view of the prototypical microgrid connecting to a larger system through an asynchronous link.

The two ports of the asynchronous link are controlled differently. The one that connects to the grid behaves as a PQ bus, while the one that connects to the microgrid is controlled as a grid-forming inverter setting voltage and frequency (although these values can be variable to allow for droop control operation). Except for filter elements and DC link capacitors, there is no energy storage, so that there must be an almost instantaneous power balance between both ports, which is achieved by a proper control of the DC link voltage.
5.4.2 Control Technique

To implement the strategy, a control technique has to be designed. This section presents an example of a controller, but it is not the only solution and other controllers could be designed to achieve similar results or for different goals. This is discussed in the future work section of next chapter.

The control implementation can be divided in two parts: (a) Control for frequency response to sudden changes occurring outside the microgrid, and (b) control for inertial response to sudden changes occurring inside the microgrid.

5.4.2.1 Control for Frequency Response

The control for sudden changes outside the microgrid has to be triggered by an input from the grid side. In demand response applications, that input can be a signal from a central energy management system or a local measurement of frequency. For this application, the system reaction has to be fast so that local measurements are preferred. The intrinsic time delay in central controlled architectures reduces the impact of using the strategy. The controller at the asynchronous link sees the microgrid as an energy storage device capable of providing a large peak of power for a brief period of time (similar to fast acting energy storage devices such as flywheels and super-capacitors). The block diagram of the control structure is shown in Fig. 5.7 and resembles that of a governor used in synchronous machines with the error in grid frequency as the input and power as the output. However, instead of the low-pass filter typically used in governors, the proposed controller uses a series connection of a constant gain and a filter with a bode plot as shown in Fig. 5.8 which resembles a resonant controller but with poles on the real axis. The reason for selecting this controller is that by properly choosing the low cutoff and high cutoff frequencies, noise immunity is achieved while commanding actions on the system only for frequency changes that are sufficiently fast to require support.
5.4.2.2 Control for Inertial Response

The second application of the strategy is to provide an inertial response to sudden changes occurring inside the microgrid. In this case, the asynchronous link is controlled so that any sudden change in the power through it triggers a change in the electrical frequency of the system. This is shown schematically in Fig. 5.9. Since most of the loads do not change their real power consumption as a function of frequency, it can be assumed that the outputs $P_{ld}$ and $P_{dg}$ from the two upper blocks are constant except for exogenous disturbances that are the ones to compensate with the controller. A generic relationship between $P_{ld}$, $P_{dg}$ and frequency can also be incorporated if desired, to account for non-IM frequency dependent loads or droop control operation. The output from the IM loads ($P_{lm}$) is naturally sensitive to changes in frequency as demonstrated in the previous section.

In this application, the input to the controller is the total power through the asynchronous link given by $P_{tot} = P_{lm} + P_{ld} + P_{dg}$. The signal is then used with a filter with the same bode plot shown in Fig. 5.8 followed by a constant gain. The low cut-off frequency is required to avoid the system to react to slow changes in the total power where the inertial response is not needed, and the high cut-off frequency avoids the measurement noise to interfere with the controller operation.
5.5 Simulation Results

The strategy using the previous control implementation was validated using time domain simulations in the two applications described above: (a) for disturbances outside the microgrid where it provides frequency response, and (b) for disturbances inside the microgrid where sudden changes in load or generation are given an inertial response. In all cases, the microgrid electrical frequency was limited between the bounds set by the IEEE 1547 standard [51].

5.5.1 Disturbances Outside Microgrid

The first case study demonstrates that the control described in the previous section can be used for frequency support of a low inertia grid in order to decrease the frequency excursions after a disturbance. The controller was implemented in the asynchronous links of three microgrids that connect to an island system as shown in Fig. 5.10. All three microgrids were rated at a total of 100 kVA with IMs representing 40% of the total load. The system was powered by a 500 kVA gas-turbine generator that served as the only source. In the simulations, the prime mover dynamics were ignored except for the inertia, as they are not a concern in the time frame of interest.
When one of the large PQ loads not belonging to any microgrid is suddenly connected, the system frequency decreases. This is shown in Fig. 5.11 with the disturbance occurring at $t = 0$. The red dashed line presents the frequency behavior when the control strategy is not implemented, meaning that the microgrids continued acting as PQ loads during and after the disturbance. The blue solid line presents the response using the strategy, demonstrating that by controlling the microgrids frequency and extracting energy stored in their induction motors, the under-frequency excursion can be reduced by about 15%.

Figure 5.10: Single line layout of the power system used in the case study. Three microgrids with IM loads connected through asynchronous links were used to help in frequency regulation.

Figure 5.11: Reduction in the under frequency excursion achieved by using the microgrids to provide frequency response with the energy in their induction motor loads.
5.5.2 Disturbances Inside Microgrids

The second application of the strategy is to provide an inertial response to changes occurring inside the microgrid. In the prototypical system, all of the DG sources are based on inverters so that their output can be lost in a few milliseconds. Such a situation can cause a microgrid to increase its power demand sharply which will cause an under-frequency excursion in the larger grid. With the proposed control, this sudden change can be given an inertial response using the energy in the induction motors to shape the power. The time domain simulation results are shown in Fig. 5.12. For simplicity in the explanation, the system was simulated with a single DG and two IM loads, but other loads could also be included without changing the main result. The power consumed by the IM loads is shown in the blue dashed lines and the output power from the DGs in the red dashed line. The power imported from the grid is shown in green and it is the difference between the IM loads consumption and the output of the DG. At \( t = 0 \) the DG output is suddenly lost and reduces to zero. The controller at the asynchronous link reacts by lowering the system frequency. This shapes the IM loads power consumption to compensate the lost power briefly and then return to the new SEP with an exponential decay. The power imported from the grid instead of increasing sharply exhibits an inertial behavior. This result is obtained without affecting the mechanical torques and with the IMs speed smoothly changing from one value to another.

Figure 5.12: Inertial behavior achieved after a sudden loss of generation inside a microgrid done by shaping the power consumed by the IM loads.
5.6 Experimental Validation

To validate some of the results presented above, the experimental set-up described in Appendix A is used. Due to the nature of the experimental set-up, it was only possible to validate the strategy implementation for sudden changes inside the microgrid.

5.6.1 Experimental Set-up Configuration

The configuration of the experimental set up used for these tests is shown in Fig. 5.13. It used two 1 HP induction motors with a controllable mechanical load, 1kVA RLC loads, and a 2kVA inverter that represented the microgrid-side port of the asynchronous links. Power to the inverter was provided from a DC supply so that there was no grid-side port. Since the focus was on the behavior inside the microgrid, to obtain the desired results there was no need for the grid port.

![Figure 5.13: Microgrid experimental configuration used to test the operation strategy and proposed controllers.](image)

5.6.2 Transitions between Torque-speed Curves

To validate the concept of the transitions between the torque-speed curves shown in Fig. 5.2, the microgrid was first run with only 1 IM load powered with the inverter. The IM mechanical torque was maintained constant at $T_m = 2.0 \text{Nm}$ during the test to simplify the visualization. The frequency was then changed rapidly between 65Hz and 55Hz and the machine electrical torque calculated from the power consumption (accounting for the losses in the IM resistors). The obtained transitions are presented in Fig. 5.14. The machine correctly settles at the steady state point corresponding to the intersection between the mechanical and electrical torques for both frequencies. Immediately after the fault, it transitions from one torque-speed curve to the other and then monotonically reduces or increases its speed until reaching the new steady state.
5.6.3 Inertial Response

The second test was done to demonstrate that it is possible to provide an inertial response from sudden changes inside the microgrid. The sudden changes were created by connecting and disconnecting the RLC loads at the microgrid PCC. The inverter operated as the slack bus of the system, so that it was able to measure the total power and implement the control technique described above. The resulting power delivered by the inverter with and without the strategy is presented in Fig. 5.15 with the sudden connection of the RLC loads occurring at $t = 0$. As expected, the power provided by the inverter when the strategy was not implemented exhibits a sharp increase immediately after the load connection as seen in the blue solid line. When the controller was used, it reacted to the disturbance by adjusting the electrical frequency to provide the inertial behavior that can be observed in red line of the figure.

An inertial response can also be obtained when the load is suddenly disconnected. This case is presented in Fig. 5.16 with the blue line representing the transition without the strategy and showing a sudden change in power, while the transition with the controller given by the red line exhibits an inertial behavior.
Figure 5.15: Experimental result showing the inertial behavior obtained by implementing the strategy in an otherwise sudden increase in load.

Figure 5.16: Experimental result showing the inertial behavior obtained by implementing the strategy in an otherwise sudden decrease in load.
5.7 Conclusions

An operation strategy for microgrids to provide frequency regulation and an inertial response was proposed. It consists in using the kinetic energy in the IM loads by actively controlling the microgrid electrical frequency inside a safe band. The strategy was demonstrated to be useful in providing frequency regulation to the network where the microgrids are connected to, and to give an inertial response to any sudden change occurring inside them. The operation of the strategy and a suggested implementation were validated using numerical simulations and an experimental microgrid.
Chapter 6

Conclusions and Future Work

6.1 Conclusions

Microgrids bring numerous benefits to utilities and end energy users, and are a key component in the grid modernization process. Their characteristics are inherently different than the larger grid. This thesis challenged the paradigm of always having to operate the voltage and frequency close to nominal, and proposed a new operation strategy, named "Flexible Voltage and Frequency", that significantly improves the response of a microgrid after a large disturbance and helps the system in which it is connected.

The dissertation was divided in two parts, each one focusing on a specific large disturbance:

1. The first part focused on a fault in the distribution network that causes IM loads to stall and leads to FIDVR or voltage collapse.

2. The second part studied a change of load or generation in low inertia networks that causes large frequency excursions.

For the first part, a detailed study of the disturbance was performed before analyzing the impact of the operation strategy. That was the focus of Chapter 2 and 3 that not only provided an analysis of fault events in distribution systems and microgrids, but also presented a framework to assess their stability. Unlike most previous work, the proposed framework approached the scenario accounting for the dynamics of the IM loads, including both electromagnetic and electromechanical transitions, to then incorporate important effects such as current limitation in the sources and the impact on reactive power balance. The creation of this framework to study faults in distribution networks and microgrids is one of the main contributions of this thesis.
After presenting the analysis of the disturbance, the effect of the operation strategy was studied. In Chapter 4 it was shown that by operating a system at a low frequency and high voltage after a fault, the stability region was expanded and the reactive power capability of the network was increased. In the same chapter, a method to implement the strategy and select the important parameters was described. The validity of the results was successfully tested using an experimental microgrid, and the impact in a larger system was demonstrated using time domain simulations. The application of the new operation strategy in the fault recovery of systems with IM loads is also a major contribution of this dissertation.

The second part of the thesis, presented in Chapter 5, extended the use of the "Flexible Voltage and Frequency" operation to the frequency stability domain. It was demonstrated that by varying a microgrid electrical frequency, the energy stored in the induction motor loads can be used to provide an inertial response to otherwise sudden changes, and to participate in the frequency regulation of the grid where it connects. A potential implementation of the strategy was then presented by suggesting a control technique, and the impact was validated using numerical simulations and an experimental microgrid. The use of the energy stored in induction motors to enhance the frequency stability of systems is the third major contribution of this thesis.

Throughout the development of this work, many new insights were obtained and they constitute important observations to the field of power system stability in the contexts of the two large disturbances studied. For instance, the thesis presented for the first time the effect of different parameters in the stability boundary of a system with coupled IM loads and identified phenomena such as cascade recovery and stability region contractions. All this was possible by analyzing the problem at its root cause. Two important lessons can be extracted from the experience. First, that by analyzing a problem at its root cause provides insights into its most important characteristics; and second, that challenging existing paradigms opens new paths in a field that might seem tough for innovation.

6.2 Suggestions for Future Work

As with most dissertations, there is always the possibility of extending the concepts and ideas that were studied. This section presents a list of suggestions that have been recognized as promising for future work.

For the first part of the thesis, the possibility of implementing a direct method for transient stability assessment is a promising idea. Although having bounds on the CCT is useful, being able to evaluate a single function and determine if the system will be stable or not could be useful for microgrid operators. Appendix B presents a first step in that direction by providing the formulation of the coupled non-linear dynamic system in the framework of contraction theory, port-Hamiltonian and Brayton-Moser, but more
work is needed in order to extract useful results from them. Another important extension of this work is the inclusion of uncertainty in the analysis framework. Although stability assessment should be performed for the worst case scenario in order to get a conservative bound, including uncertainty in parameters and operating conditions would make it easier to use by system and microgrid operators in the design of protections.

Regarding the fault-recovery strategy presented in Chapter 4, its implementation in hybrid microgrids that contain both synchronous machines and inverter-based sources has to be further developed. Although most DGs today are interfaced through inverters, some microgrids still use diesel generators in their energy mix. As discussed in the chapter itself, a proper controller in the synchronous-based source can adjust the rotational speed, but their response tend to be slow, limiting the impact of the strategy. Exploring ways, for example, in which these sources can be operated as slaves to use their energy during their deceleration to further improve the response could be an interesting path.

In the second part of the thesis, there are also several suggestions to extend the work. An intriguing idea is the possibility of implementing the control strategy in a larger area, such as a feeder, so that the electrical frequency controller can be used as a “knob” for an “analog” demand response. In that case, if power is suddenly needed, the “knob” can be turned down reducing the electrical frequency with a particular waveform to shape the power to the IM loads as desired. This implementation would benefit from the numerous IM loads that tend to be operating continuously in a feeder, and, as shown, there is no need to take the system outside the safe frequency range to obtain some benefits. Another possible extension is to control the system voltage along with its frequency so that power can be shaped using two variables. Although this was thought during the development of this work, the caveat is that voltage affects the consumption of most of the loads and the operation of all DGs in the system. Finally, the last suggestion pertaining the second part of the thesis concerns the control design. As it was mentioned in Chapter 5, the presented controller is just one of the many implementations that can be used for the operation strategy. Studying possible implementations with non-linear controllers that can shape the power to the IMs in any desired way could be a powerful tool for system operators.
Appendix A

Experimental Microgrid

The microgrid experimental set-up used to test the models and control strategies throughout the dissertation also constitutes an important contribution. Its most important features are presented in this appendix.

The microgrid was designed to have three important characteristics:

- **Modularity**: The elements are replicable and their integration into the system does not require a redesign of the hardware infrastructure or the control architecture.

- **Flexibility**: All aspects of the system regarding control, communications and monitoring, are modifiable so that any configuration or control technique can be tested.

- **Upgradability**: It is possible to upgrade most of the control elements without the need to redesign the hardware or software of other components.

To achieve these characteristics, the system was built by creating subsystems that are simple to replicate and that represent typical elements found in microgrids such as diesel generators, inverters or induction motor loads. The subsystems are generic so that, with the same topology, different elements can be emulated with only software changes.

Another important characteristic of this set-up is its capability to perform tests where the microgrid is subject to a fault. This was achieved by designing and placing the proper protections to avoid affecting the larger power system.

### A.1 Control Architecture

The microgrid was designed with a hierarchical control architecture [17], where decisions are made both at the system level through a central controller (for both secondary and tertiary control) and at the subsystem
level through distributed controllers (for primary control). A schematic view of this control architecture is shown in Fig. A.1. The interaction between the two upper levels of decision making is done through a communication network. Below the subsystem level lies the device level that contains the power processing devices. This level tends to be simplified in microgrid studies, but it is of major importance to build a safe and robust experimental system.

![Control Architecture Diagram]

Figure A.1: Control architecture implemented in the experimental microgrid.

### A.1.1 System Level Control

The system level control was designed to supervise the microgrid, command actions to its elements and interact with external entities. It is the “central nervous system” of the microgrid where most decisions are made and with access to all parts of the system. The tasks performed by this control level are the following:

- Interact with external elements such as utilities or other microgrids
- Present an interface for a user interaction with the microgrid.
- Monitor the state of the system by receiving, processing and displaying information from different points around the network.
- Control the topology of the system by opening or closing relays either manually or following a pre-programmed sequence.
- Dispatch and perform secondary control of the sources by sending commands such as start/stop and power set-points.
- Adjust certain control parameters of the subsystems in real-time.
- Create a synchronizing clock for phasor measurements.
- Collect and record data from experiments.
A schematic view of the different components that constitute this control level is presented in Fig. A.2.

Figure A.2: Schematic view of system level control comprised by an HMI, a central controller and a relay controller.

A user can operate the microgrid by interacting with a human-machine interface, an implementation of which is shown in Fig. A.2. The main goal of this interface was to provide an intuitive way for the user to command actions such as changing the topology, synchronizing elements, or dispatching sources, among many others. It was programmed in Labview and both manual actions or pre-programmed sequences can be used to obtain a particular result.

Figure A.3: Front panel of an implementation of the user interface showing some of the options to operate and monitor the system.

The interface interacts with two controllers in parallel: (a) the central controller of the microgrid, and (b) a relay controller. In future versions of this set-up, the relay control should be included inside the central controller in order to reduce the components count.

The central controller can be regarded as the “inner brain” of the microgrid. It contains a digital signal processor, RAM and EEPROM memory, USB interface, RS-485 interface and a clock output. All elements
were placed on a PCB as shown in Fig. A.4. The USB connects the controller to a computer with the user interface, while the RS-485 transceiver connects it to a communication network that spans the entire system. The clock output is used as a synchronization signal. The relay controller was implemented using a NI DAQ-6008 and some circuitry required to raise the DAQ output to the 24V required by the relays.

Figure A.4: Photograph of the constructed and implemented microgrid central controller.

A.1.2 Communication Network

The communication link between the central controller and the subsystems is the spinal cord of the microgrid. It was constructed following the RS-485 standard. The main advantage of this standard against other possible implementations, such as Ethernet, is that an RS-485 network can be constructed in a bus or party-line topology while others require a star connection. Using this topology, and taken advantage of modern RS-485 transceivers, a fast communication network operating at 17.5MHz was created. A schematic view of it is shown in Fig. A.5. With this structure, subsystems are not limited to receive information from the central controller but can also communicate and listen from other subsystems and monitoring units in the network, opening the possibility of testing distributed control schemes.

The communication network is operated by default using a polling technique, where the central controller calls each element and waits for its response. Other techniques such as token ring can also be implemented.
One of the key aspects in the construction of this communication network was the proper handling of grounds. Since the distance between the central controller, the subsystems and the monitoring units can be significant, it is not possible to connect all of them to local ground without causing problems such as ground loops, that prevent the correct operation of the system and limit its modularity. To avoid them, isolated RS-485 transceivers were used, so that their network-side was connected to a ground shared among all elements while the subsystem-side connected to local ground. This is shown schematically in Fig. A.6 and resulted in a robust implementation with few limitations on the physical size or the number of elements that can be connected to the microgrid.

Figure A.5: Schematic representation of the communication network that connects the central controller with all the subsystems and monitoring units in a bus topology.

Figure A.6: Connection of grounds in the different systems interconnected by the communication network. By using isolated RS-485 transceivers at each board, it is possible to avoid ground loops and construct a reliable system.
A.1.3 Subsystem Level Control

Just as the spinal cord in the body, the communication network reaches all the subsystems in the microgrid. At each one, it is received by an isolated RS-485 transceiver at a distributed controller. These controllers serve as bridges between the system level decision making and the local control that is required to operate the subsystem. A schematic representation of the subsystem level control is shown in Fig. A.7 and a photograph of its implementation in Fig. A.8. The distributed controllers perform feedback control algorithms and signal processing techniques that are required to make the power source operate. It also has a USB port that is useful while debugging a certain code or when a human interface is desired for an individual subsystem.

![Schematic View of Subsystem Level Control](image)

**Figure A.7:** Schematic view of the subsystem level control that serves as a bridge between the power sources and the system level control.

![Photograph of Distributed Controller](image)

**Figure A.8:** Photograph of one of the distributed controllers that were constructed and installed.

A.1.4 Device Level Control

The exchange of information between the devices and the subsystem level is an important aspect for the microgrid safe and reliable operation. As shown in the diagram of Fig. A.7, this exchange includes commands from the controller to the power conversion devices in the form of PWM signals, and analog signals from sensors back to the controller. The commands were connected directly to the MOSFET drivers, but the analog signals required special consideration to avoid corrupting the wave-forms. This was particularly important in some subsystems where the sensors required for close loop control were located far from the...
Figure A.9: Analog link between a monitoring unit and the subsystem controller used to safely send analog signals through long distances and noisy environments.

To achieve noise immunity and a proper ground connection, all signals were sent using the structure shown in Fig. A.9. This is a robust construction that allows signals to maintain their integrity even in noisy environments and over long distances. It is achieved by avoiding the creation of ground loops and exposing the wires only to common mode noise which can be easily rejected at the differential receiver in the distributed controller.

A.2 Monitoring

One of the key aspects for a proper microgrid operation was the monitoring system. A monitoring unit (MU) with voltage and current sensing capabilities was designed for two purposes: (a) To process those signals and transmit their values through the communication network, and (b) to create a robust analog signal for close loop control at the distributed controllers (achieved with the structure shown in Fig. A.9). A schematic view of the monitoring unit showing these two functionalities is presented in Fig. A.10. This diagram also shows the reception of the reference clock for phasor measurement synchronization (so that it can be used as a PMU), and the USB port that is used for user interfaces designed for the MUs. Six of these devices were constructed and installed in the system. A picture of a MU is shown in Fig. A.11.
A.3 Microgrid Subsystems

For the experimental set-up, four subsystems, representing the most typical elements found in microgrids, were developed. They can be replicated to increase the microgrid total power and its number of elements, and they are flexible so that different control techniques can be tested. The developed subsystems are:

1. Synchronous machine source
2. Inverter based source

3. Constant impedance load

4. Induction Motor load

**A.3.1 Synchronous Machine Source**

The schematic view of a subsystem that interfaces the microgrid with synchronous generator is shown in Fig. A.10. The input mechanical torque to the generator is created using a DC motor and it is given by the equation:

\[ T_m = G I_f I_a \]  

(A.1)

where \( T_m \) is the motor mechanical torque, \( G \) is a constant, \( I_f \) is the motor field winding current, and \( I_a \) is the motor armature current. Since the motor field winding is connected to a constant supply, \( I_f \) is also a constant, and torque can be controlled with the armature current. This was done with a DC/DC converter that received commands from the distributed controller. With this implementation, a diesel engine, a gas turbine, or any other prime mover can be emulated by programming the desired dynamics in the distributed controller.

![Schematic representation of the elements and their relationship in the synchronous source subsystem.](image)

Figure A.12: Schematic representation of the elements and their relationship in the synchronous source subsystem.

The generator terminal voltage is adjusted by controlling the voltage at its field winding (identified as "rotor winding" in the schematic above). This was done using a second DC/DC converter which also takes
PWM signals from the distributed controller. The output of this converter can be programmed to emulate the dynamics of any type of exciter or AVR.

To perform closed loop control, the values of voltages, currents and frequency at the terminals are measured with a monitoring unit (MU). The signals from the sensors are sent to the controller using a differential analog link as shown in Fig. A.9. To measure the generator speed, a quadrature encoder was tied to its shaft and its output connected directly to the distributed controller.

The synchronous machine output was tied to the Microgrid PCC through a relay controlled by the System Level Control.

A.3.2 Inverter-based Source

The block diagram of the source that interfaces the microgrid through an inverter is shown in Fig. A.13.

A DC Power Supply serves as the energy source of this subsystem and it connects to both a six switch H-bridge converter and an auxiliary resistive load. The auxiliary load allows bidirectional power to flow through the converter, for applications where it is desired to emulate an energy storage element, for example. The switches in the H-bridge are controlled by the distributed controller using a space vector pulse width modulation (SVPWM) technique to convert the DC voltage into AC. Sensors in the filter and in a MU located at the output measure the voltages and currents required for close loop operation. As in the previous subsystem, a relay is used to tie the inverter to the microgrid PCC.
A.3.3 Constant Impedance Load

The constant impedance loads were used to create different scenarios of source loading and of power exchange between the microgrid and the external network. A block diagram of this subsystem implementation is shown in Fig. A.14. This subsystem was designed to allow the microgrid user to connect resistive, inductive and capacitive loads in any desired combination.

![Diagram of Constant Impedance Load Subsystem]

Figure A.14: Schematic representation of the elements and their relationship in the constant impedance load subsystem.

A.3.4 Induction Motor Load

Induction motors were the second type of load included in the microgrid and they were used extensively in this thesis. The block diagram of the IM load subsystem is shown in Fig. A.15.

![Diagram of Induction Motor Load Subsystem]

Figure A.15: Schematic representation of the elements and their relationship in the induction motor load subsystem.
The shaft of a squirrel cage IM is coupled with a DC machine which acted as a generator and provided a controllable torque. A DC machine torque is given by Eq. A.1 and it can be controlled with the armature current, given that \( G \) and \( I_f \) are constant in this configuration. This is done with a DC/DC converter, a distributed controller and an auxiliary load where the energy is dissipated. Using the connection to the communication network and the system level control, the motor torque can be easily modified by the user.

A.4 Fault Capability

As mentioned above, one of the most important characteristics of this set-up is its ability to create faults to study the microgrid transient behavior experimentally. This feature was used extensively in the dissertation. The faults are created using a relay that is triggered by the System Level Control. To achieve precise control on the fault duration, it is necessary to use an instant-on solid state relay. Mechanical relays have delays in the order of milliseconds, which can significantly change the result of a transient stability experiment, and zero-crossing relays can only create the faults at specific instants of the voltage wave-form which limits the flexibility of the experiments.

To protect the grid from these faults, three large inductors were connected between the microgrid PCC and the grid interconnection point. They were designed to limit the fault current from the grid side to 25 Amps, while still exposing the microgrid to the desired fault conditions.

A.5 List of Components and Parameters

The most important components that constitute the microgrid are the listed below. The list is not intended to be comprehensive as the elements and specific part numbers are expected to change in the future.

1. System Level Control
   (a) A computer with Labview and USB connection.
   (b) A central controller described in Section A.1.1.
   (c) Relay controller including a DAQ NI-6008 and a signal conditioning board.

2. Monitoring Units boards as described in Section A.2.

3. Synchronous Machine Source subsystem
   (a) 1 HP wound-rotor machine (parameters shown in Table A.1)
   (b) 1.5 HP DC motor (parameters shown in Table A.2)
(c) 2 DC/DC buck converters rated at 2.5 kW.
(d) 1 distributed controller.
(e) 3 power supplies (which were shared with other elements).
(f) 1 quadrature encoder.
(g) 1 monitoring unit.
(h) 1 mechanical relay.

4. Inverter base source subsystem

(a) 1 three phase inverter rated at 2.5 kW.
(b) Auxiliary load with resistors rated at a minimum of 1 kW.
(c) LCL Filter (parameters given in Table A.3).
(d) 1 distributed controller.
(e) 1 monitoring unit.
(f) 1 mechanical relay.

5. Constant impedance load subsystem

(a) 2 resistive load banks each rated at 750W.
(b) 1 capacitive load rated at 650 VAr (40 μF per phase).
(c) 1 inductive load rated at 1.05 kVAr (108 mH per phase).
(d) 4 mechanical relays

6. Induction motor load subsystem

(a) 1 HP squirrel cage induction motor (parameters given in Table A.4)
(b) 1.5 HP DC motor
(c) 1 DC/DC buck converter rated at 2.5 kW
(d) 1 auxiliary resistive load rated at a minimum of 1 kW.
(e) 1 distributed controller.
(f) 1 mechanical relay.
Table A.1: 1 HP wound-rotor machine parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance</td>
<td>1.9 Ω</td>
</tr>
<tr>
<td>Field resistance (reflected on stator)</td>
<td>7.51 Ω</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>235 mH</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>6.3 mH</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>6.3 mH</td>
</tr>
<tr>
<td>Inertia (generator + DC motor)</td>
<td>0.035 kg·m²</td>
</tr>
<tr>
<td>Viscous friction coefficient (generator + DC motor)</td>
<td>0.0025 Nm/rad/sec</td>
</tr>
<tr>
<td>Coulomb friction torque (generator + DC motor)</td>
<td>0.1267 Nm</td>
</tr>
</tbody>
</table>

Table A.2: 1.5 HP DC motor parameters. Mechanical parameters are given in combination with the induction motor and the wounded rotor machine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field winding resistance</td>
<td>275 Ω</td>
</tr>
<tr>
<td>Armature winding resistance</td>
<td>2.78 Ω</td>
</tr>
<tr>
<td>Armature winding inductance</td>
<td>28 mH</td>
</tr>
<tr>
<td>Mutual inductance (G or Lm)</td>
<td>1.92 H</td>
</tr>
</tbody>
</table>

Table A.3: LCL filter parameters.

<table>
<thead>
<tr>
<th>Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance</td>
<td>2.35Ω</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>1.67Ω</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>200 mH</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>8 mH</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>8 mH</td>
</tr>
<tr>
<td>Inertia (IM + DC machine)</td>
<td>0.029 kg·m²</td>
</tr>
<tr>
<td>Viscous friction coefficient (IM + DC machine)</td>
<td>0.002 Nm/rad/sec</td>
</tr>
<tr>
<td>Coulomb friction torque (IM + DC machine)</td>
<td>0.1150 Nm</td>
</tr>
</tbody>
</table>

Table A.4: 1 HP squirrel cage induction motor parameters.

A.6 Photographs of Experimental Microgrid

The following are some photographs of the experimental microgrid.
Figure A.16: Photograph of the microgrid central controller and the user interface.

Figure A.17: Photograph of the microgrid PCC and relays used to change the topology.
Figure A.18: Photograph of the synchronous machine source implementation.

Figure A.19: Photograph of the inverter based source implementation.
Figure A.20: Photograph of the constant impedance load implementation.

Figure A.21: Photograph of the induction motor load implementation
Appendix B

Formulations

One of the suggestions for future work given in Chapter 6 was to construct a direct method to determine the IM rotor speed stability after a fault. Direct methods for transient stability assessment are useful because they can determine if the system is stable or not by evaluating a Lyapunov function and without the need of any post-fault time domain simulations [22]. However, finding a useful Lyapunov function is complicated. This appendix presents the first step required in such quest by formulating the problem in three ways:

- Using a port-Hamiltonian structure
- Using a Brayton-Moser structure
- Using contraction theory

By framing the problem in its proper structure, analysis can be done using standard techniques to then obtain conditions for stability using direct methods.

B.1 Port-Hamiltonian

Port-Hamiltonian (PH) is an extension of the Hamiltonian representation of system dynamics. In it, the system is modeled with some internal dynamic energy storage components and ports that interact with external elements [66, 67]. This representation is useful for analysis and control design because it explicitly writes the damping terms and their dependence with the energy function. In port-Hamiltonian models, the
dynamics are written in the following form:

\[
\dot{x} = (J(x) - R(x)) [\mathcal{H}]^\top + g(x)u \\
y = g^\top(x) [\mathcal{H}]^\top
\]  \hspace{1cm}(B.1)

where \(x\) is the state vector, \(u\) the ports' input and \(y\) the ports' output. The product \(uy\) defines the power transfer between the system and the external elements so that it has units of power. \(\mathcal{H}\) is the Hamiltonian, equal to the energy content of the system, and \(\nabla \mathcal{H}\) gives the co-energy variables. \(J = -J^\top\) is called the interconnection matrix, \(R = R^\top > 0\) the damping matrix and \(g(x)\) the port connection matrix.

In the next sections, the dynamics of a system with multiple induction motors coupled through line impedances are written in the form given in Eq. B.1. For simplicity, this is done first for a single induction motor.

**B.1.1 Single Induction Motor**

Let the motors states be defined as:

\[
x = \begin{bmatrix} \lambda_s^\top & \lambda_r^\top & p \end{bmatrix}^\top
\]  \hspace{1cm}(B.2)

where \(\lambda_s = \begin{bmatrix} \lambda_{sd} & \lambda_{sq} \end{bmatrix}^\top\) are the stator magnetic flux linkages in d-q reference frame, \(\lambda_r = \begin{bmatrix} \lambda_{rd} & \lambda_{rq} \end{bmatrix}^\top\) are the rotor flux linkages, and \(p = J\omega_m\) is the rotor momentum. The IM dynamic equations are:

\[
\dot{\lambda}_s = -R_s i_s - \omega M \lambda_s + v_s \\
\dot{\lambda}_r = -R_r i_r - (\omega - p \cdot \omega_m) M \lambda_r \\
\dot{p} = T_e - T_m - F\omega_m
\]  \hspace{1cm}(B.3)

where \(v_s = \begin{bmatrix} v_d & v_q \end{bmatrix}^\top\) is the terminal voltage, \(i_s = \begin{bmatrix} i_{sd} & i_{sq} \end{bmatrix}^\top\) the stator currents, \(i_r = \begin{bmatrix} i_{rd} & i_{rq} \end{bmatrix}^\top\) the rotor currents, \(\omega\) the electrical frequency, \(\omega_m\) the rotational speed, \(p\) the number of poles, \(T_m\) the mechanical torque, \(F\) the friction coefficient, \(T_e = \lambda_r^\top M i_r\) the electric torque, and \(M\) is a skew-symmetric matrix of the form:

\[
M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}
\]  \hspace{1cm}(B.4)

The Hamiltonian \(\mathcal{H}\) is the addition of the magnetic energy stored in the inductances and the kinetic
energy of the rotor. To calculate it, first define an inductance matrix as:

\[
L_{im} = \begin{bmatrix}
L_s & L_m \\
L_m & L_r
\end{bmatrix} \otimes I_{2 \times 2}
\]  

(B.5)

where \(L_s, L_r\) and \(L_m\) are the motor stator, rotor and mutual inductances respectively. The Hamiltonian is then:

\[
\mathcal{H} = \frac{1}{2} \lambda^T L_{im}^{-1} \lambda + \frac{1}{2} p^T J
\]  

(B.6)

The gradient of the Hamiltonian gives the co-energy vector:

\[
\nabla \mathcal{H} = \begin{bmatrix}
i_s & i_r & \omega_m
\end{bmatrix}
\]  

(B.7)

Writing the dynamics of Eq. B.3 in port-Hamiltonian form results in:

\[
\dot{x} = \begin{bmatrix}
-L_s \omega M & 0 & 0 \\
0 & -L_r \omega M & M \lambda_r \\
0 & \lambda_r M & 0
\end{bmatrix}
- \begin{bmatrix}
R_s \\
R_r \\
F
\end{bmatrix} \nabla \mathcal{H}^T + \begin{bmatrix}
I & 0 \\
0 & 0 \\
0 & -1
\end{bmatrix} \begin{bmatrix}
v_b \\
T_m
\end{bmatrix}
\]  

(B.8)

Notice that matrix \(J(x)\) interconnects the electrical and mechanical subsystems through the back voltage and the electric torque as seen in the schematic view of Fig. B.1. It also couples the \(d\) and \(q\) axes through the “speed voltage” that arises from the rotating reference frame.

Figure B.1: Schematic view of the interconnection between the electrical subsystem (\(\Sigma_\sigma\)) and the mechanical subsystem (\(\Sigma_m\)).

The inputs to the system are given by the mechanical torque at the rotor shaft and the source voltage connected to the stator. The corresponding outputs are the rotational speed (\(\omega_m\)) and the stator current.
Notice that these input-output pairs represent the power transfer between the voltage source and the mechanical loads.

### B.1.2 General System Construction

Consider now the generic system shown in Fig. B.2. It consists of a network with impedances connecting "N" induction motors and resistive loads (required to avoid creating an index-2 DAE system\(^1\)). To construct the port-Hamiltonian model, start by defining a state vector as:

\[
x = \begin{bmatrix} x_{\text{net}} \\ x_{\text{ims}} \end{bmatrix}
\]

(\text{B.9})

where \(x_{\text{net}} = \lambda_1 = \begin{bmatrix} \lambda_{i1}^+ & \lambda_{i2}^- & \cdots & \lambda_{i\text{im}}^- \end{bmatrix}^T\) contains the magnetic flux linkages all the branch inductances in d-q reference frame, and \(x_{\text{ims}}\) are the states required to describe each of the \(N\) induction motors in the system. For simplicity in the matrix construction, the order of the motors' states is defined as \(x_{\text{ims}} = \begin{bmatrix} \lambda_{ss} & \lambda_{rs} & p \end{bmatrix}^T\) with \(\lambda_{ss}\) being all the motors' stator fluxes, \(\lambda_{rs}\) all the rotor fluxes, and \(p\) all the momentums.

There are two electrical and \(N\) mechanical ports in this generic system. In the electrical ports, the inputs are given by the d-q voltages \(v_s = \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix}\) of a source connected at bus 1 while the outputs correspond to the current through the branch connecting the source to the network (\(i_{t11}\)). The mechanical ports' inputs are the load torques at every machine and their corresponding speeds are the outputs.

\(^1\)An index-2 DAE is one in which the algebraic variables (the bus voltages in this case) do not appear in the algebraic constraint (Kirchhoff current law). By adding a resistor, the algebraic constraint contains the algebraic variable so it is an index-1 DAE and it can be back-substituted into the dynamic equations. Adding a capacitor will convert the algebraic constant into a dynamic equation.
The energy content of the system is given by the Hamiltonian $\mathcal{H}$:

$$\mathcal{H} = \frac{1}{2} \lambda_{\text{u}}^\top L_{\text{net}}^{-1} \lambda_{\text{u}} + \frac{1}{2} \lambda_{\text{im}}^\top L_{\text{im}}^{-1} \lambda_{\text{im}} + \frac{1}{2} p^\top J_m^{-1} p$$

(B.10)

where $L_{\text{net}} = \text{diag}\left\{ L_{\text{u1}}, L_{\text{u2}}, \ldots, L_{\text{utm}} \right\}$ are the network branches inductances and the motor inertias form a vector $J_m = \text{diag}\left\{ J_1, J_2, \ldots, J_N \right\}$ (not to be confused with matrix $J(x)$ of the port-Hamiltonian formulation). Notice that since we reorder the states of $\lambda_{\text{im}}$, the matrix $L_{\text{im}}$ will have rows and columns rearranged compared to the one in Eq. B.5. As expected, the co-energy variables are the currents and rotational speeds:

$$\nabla \mathcal{H} = \begin{bmatrix} i_t & i_{\text{us}} & i_{\text{sr}} & \omega_{\text{ms}} \end{bmatrix}$$

(B.11)

The matrices $J(x)$ and $R$ depend on the network topology. Let $A$ be the branch-node incidence matrix including all transmission lines and buses (except the first one where the voltage source is), and let $R_L = \text{diag}\left\{ R_{L1}, R_{L2}, \ldots, R_{LN} \right\}$, $R_{tl} = \text{diag}\left\{ R_{tl1}, R_{tl2}, \ldots, R_{tln} \right\}$, etc. The matrices for the PH formulation are then given as:

$$J(x) = \begin{bmatrix} -\omega_{\text{tl}} & 0 & 0 & 0 \\ 0 & -\omega_{\text{sl}} & -\omega_{\text{sm}} & 0 \\ 0 & -\omega_{\text{lm}} & -\omega_{\text{lr}} & A \\ 0 & 0 & -A^\top & 0 \end{bmatrix}$$

(B.12)

$$R = \begin{bmatrix} A^\top R_A^L A & -A^\top R_L & 0 & 0 \\ -R_A^L A & R_L & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} R_{tl} \\ R_{sa} \\ R_r \\ F \end{bmatrix}$$

(B.13)

$$g(x) = \begin{bmatrix} I_{2 \times 2} & 0 \\ 0 & 0 \\ 0 & -I_{N \times N} \end{bmatrix}$$

(B.14)

where matrix $A$ was defined:

$$A = \text{diag}\left\{ M\lambda_{r1}, M\lambda_{r2}, \ldots, M\lambda_{rN} \right\}$$

(B.15)
**B.1.3 Application to Prototypical Two Machine System**

The above formulation can be applied to the prototypical two machine studied in Chapters 2, 3 and 4 by defining its branch-node incidence matrix (ignoring the first bus) as:

$$
\Lambda = \begin{bmatrix}
1 & -1 \\
0 & 1 
\end{bmatrix}
$$

The equations are too long to be written here, but they can be easily programmed into the model presented in Appendix C. Several sample regions found with this model are presented below.

- Hamiltonian ($\mathcal{H}$): At each point on the phase plane between $\omega_{mi} \in [0, 1]$ the model computes the source voltage ($v_{sr}$) and current ($i_{sr}$), the motor terminal voltage ($v_a$), stator current ($i_a$), rotor voltage ($v_r$) and rotor current ($i_r$), and $\mathcal{H}$ can be computed from them. The levels of constant Hamiltonian are given in Fig. B.3.

- Input power ($v_a \cdot i_{i1}$): The levels of constant input power delivered by the voltage source are given in Fig. B.4.

- Net power ($v_a \cdot i_{i1} - \sum_i T_m(\omega_{mi})$): The net power into the system is the difference between the power delivered by the voltage source and the output power at the mechanical ports. Levels of constant net power are given in Fig. B.5.
Figure B.4: Constant input power contour lines in a prototypical two motor system superimposed on the stability boundary.

Figure B.5: Constant net power contour lines in a prototypical two motor system superimposed on the stability boundary.
B.2 Brayton-Moser Formulation

Although the port-Hamiltonian formulation is useful in some system, it has some limitations in cases with continuous dissipation (or more precisely when there is dissipation while the system operates in the SEP). An alternative method is to use a power based approach as originally proposed in [68, 69] and applied to electrical and electromechanical system is a series of papers [67, 70, 71, 72, 73]. This method is called Brayton-Moser (BM) and has the following advantages:

- It is based on currents and voltages, instead of magnetic fluxes and capacitor charges.
- It naturally includes non-energetic elements like sources and resistors, which are complicated to include into PH systems.
- In the system under study with induction motors, it avoids the need to define potential energy as $E = \int T_m d\theta = T_m \theta$ which is problematic because $\theta$ is unbounded and increases with time. In terms of power, the equivalent expression is a function of an input and a state: $P = T_m \omega_m$.

It is possible to convert a PH formulation into a BM formulation by following the procedures described in [72] (the process is not presented here). The main concept behind the BM formulation is the mixed-potential function $P(v, i)$ that allows the dynamics to be written as:

\[- \frac{d}{dt} \nabla_v \mathcal{H}^* = \nabla_v P \]
\[\frac{d}{dt} \nabla_i \mathcal{H}^* = \nabla_i P \]

where $\mathcal{H}^*$ is the co-Hamiltonian in terms of voltages and currents, $\nabla_v$ is the derivative with respect to voltages, and $\nabla_i$ the derivative with respect to currents. If the system of equations can be written in this form, then there are techniques to guarantee stability in certain regions as described in [71]. This is left as future work.

B.3 Contraction Theory Formulation

Contraction theory formulates the problem from a different perspective by defining regions where the Jacobian is negative definite to then guarantee stability in a ball around a SEP [74]. The dynamics of the system are:

\[ \frac{d\omega_{mi}}{dt} = \frac{1}{2H} (T_b(\omega_m) - T_m(\omega_{mi})) \]
Figure B.6: Torque-speed curve of an IM load indicating the stability region and the contraction region.

so that the Jacobian is:

\[
A = \begin{bmatrix}
\frac{1}{H_1} \left( \frac{\partial T_{e1}}{\partial \omega_{m1}} - \frac{\partial T_{m1}}{\partial \omega_{m1}} \right) & \frac{1}{H_1} \frac{\partial T_{e1}}{\partial \omega_{m2}} & \cdots & \frac{1}{H_1} \frac{\partial T_{e1}}{\partial \omega_{mN}} \\
\frac{1}{H_2} \frac{\partial T_{e2}}{\partial \omega_{m1}} & \frac{1}{H_2} \left( \frac{\partial T_{e2}}{\partial \omega_{m2}} - \frac{\partial T_{m2}}{\partial \omega_{m2}} \right) & \cdots & \frac{1}{H_2} \frac{\partial T_{e2}}{\partial \omega_{mN}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{H_N} \frac{\partial T_{eN}}{\partial \omega_{m1}} & \frac{1}{H_N} \frac{\partial T_{eN}}{\partial \omega_{m2}} & \cdots & \frac{1}{H_N} \left( \frac{\partial T_{eN}}{\partial \omega_{mN}} - \frac{\partial T_{mN}}{\partial \omega_{mN}} \right)
\end{bmatrix}
\]

B.3.1 Single Machine System

For a single machine system with constant mechanical torque, the contraction region is given by all the points where the derivative of the electric torque is negative. This is shown in Fig. B.6. The gradient around the SEP is negative until the bifurcation point, which marks the end of the contraction region. Notice that if the mechanical torque is assumed to be constant, then the contraction region is independent from it and the system is guaranteed to be stable regardless of the torque load level. If the mechanical torque has a positive derivative with respect to \( \omega_m \) (as shown in Fig.B.6) then the contraction region will expand even further.

B.3.2 Two Machine System

The same insights can not be obtained directly in a system with a larger number of machines. For the two motor system, the region where the Jacobian is negative definite is shown in Fig. B.7 on a phase plane that indicates the real stability boundary. The contraction region was calculated by sampling the
plane, computing the Jacobian and using its eigenvalues to check if it is negative definite. This process is computationally inexpensive, particularly if done along the fault-on trajectory, as shown in Fig. B.8, to identify the point where the contraction region is crossed. However, unless the eigenvalues are purely real, the contraction region can not be guaranteed to be inside the stability region. This is because by just computing the Jacobian there is no way to determine if the second order terms could make the system unstable. Contraction theory can only guarantee stability for a ball around the SEP that is tangent to the contraction region boundary [74]. There are methods to expand this contraction region by defining metrics, but that is left as for future work.
Appendix C

Model Description and Code

The model used to generate the figures in Chapters 2, 3 and 4 is briefly explained in this appendix. A schematic view of the relationship between the files that constitute it is shown in Fig. C.1. The inputs to the model are three files that specify the system and machine parameters. That information is packaged by a main file which call functions from a repository, process their output and displays the results. The repository is constructed by nine different functions that perform the calculations required to model the system and obtain important quantities. Each file is explained next.

![Figure C.1: Functional relationship between files in the model](image)

C.1 Input Files

The three input files that determine the system and machine parameters are the following:
• The file “bus.csv” contains the information of the buses in the system with the following format in each row:

\[
\text{bus} = \left[ \begin{array}{cccc}
\text{bus number}, & \text{pre-fault bus type}, & \text{post-fault bus type}, & \ldots \\
\ldots & \text{x-position}, & \text{y-position}, & \text{microgrid number} \\
\end{array} \right]
\]

The “bus number” is an identifier and has to increase by one in each row, the “x-” and “y- positions” are used in the construction of a graph of the network, the “microgrid number” specifies which microgrid that bus part of (write 0 if the bus is not part of a microgrid), and the “bus type”, given for the pre-fault and post-fault scenarios, can be one of the following:

- Type “0”, always located at bus one, represents the main voltage source of the system.
- Type “1” is a bus that contains a IM load with a constant impedance load in parallel.
- Type “2” is a bus with three elements in parallel: IM load, a constant impedance load and a DG operating in slave mode.
- Type “3” is a bus with three elements in parallel: IM load, a constant impedance load and a DG operating in master mode. This type should only be selected when a section of the network is separated into a microgrid.

• The file “net.csv” contains the information of the branches in the system with the following format in each row:

\[
\text{net} = \left[ \begin{array}{cccc}
\text{from bus}, & \text{to bus}, & \text{pre-fault resistance}, & \ldots \\
\ldots & \text{pre-fault reactance}, & \text{post-fault resistance}, & \text{post-fault reactance} \\
\end{array} \right]
\]

All the buses of the network have to be connected together through a branch. If islanding of a section is desired, it has to be done by specifying a large resistance and reactance at that branch. Reactances and resistances are given in Ohms (Ω) and for nominal frequency. The larger bus specified in this file has to correspond to the larger bus from the file “bus.csv” otherwise an error will be indicated.

• The file “params.csv” contains information about the machine parameters and the constant impedance loads in per unit basis of each bus. The bus power base is given by the induction motor power rating.
The parameters are given as:

\[
\text{params} = \begin{bmatrix}
\text{bus number} & P_{\text{nom}} & r_s & r_r & l_s & l_{lr} & l_m & H & F & z_L
\end{bmatrix}
\]

where "bus number" is the location identifier, \(P_{\text{nom}}\) is the motor nominal power, \(r_s\) the stator resistance, \(r_r\) the rotor resistance, \(l_s\) the stator leakage inductance, \(l_{lr}\) the rotor leakage inductance, \(l_m\) the mutual inductance, \(H\) the per unit inertia, \(F\) the friction coefficient, and \(z_L\) the constant impedance load in parallel.

C.2 Repository Functions

There are nine functions in the repository that perform different calculation. The following seven interface with the main file:

1. "dynamics.m": This function is used to calculate equilibrium points by posing the state derivatives of the quasi-static model:

\[
\frac{d\omega}{dt} = \frac{1}{2H_i} \left( T_{ei}(\omega_m) - T_{mi}(\omega_m) \right)
\]

- Input: Operation point (\(\omega_m\)), control inputs, and system and machine parameters.
- Output: Derivative of rotational speed for all motors \(d\omega_m/dt\).

2. "dynamics_neg.m": This function does the same as "dynamics.m" but changes the sign of the output. It is used to perform backwards integration.

- Input: Operation point (\(\omega_m\)), control inputs, and system and machine parameters.
- Output: Negative of derivative of rotational speed for all motors \(-d\omega_m/dt\).

3. "dynamics_sim.m": Similar to the functions above, but accounting for time-varying control parameters. This function is used for time domain simulations of scenarios that use the fault recovery strategy.

- Input: Operation point (\(\omega_m\)), time-varying control inputs, and system and machine parameters.
- Output: Derivative of rotational speed for all motors \(d\omega_m/dt\).
4. "grad.m": This method calculates the Jacobian at a given equilibrium point ($\omega_m$) using the equation:

$$A = \begin{bmatrix}
\frac{1}{2H_1} \left( \frac{\partial T_{a1}}{\partial \omega_{m1}} - \frac{\partial T_{a1}}{\partial \omega_{m2}} \right) & \frac{1}{H_1} \frac{\partial T_{a1}}{\partial \omega_{m2}} & \cdots & \frac{1}{H_1} \frac{\partial T_{a1}}{\partial \omega_{mN}} \\
\frac{1}{H_2} \frac{\partial T_{a2}}{\partial \omega_{m1}} & \frac{1}{2H_2} \left( \frac{\partial T_{a2}}{\partial \omega_{m2}} - \frac{\partial T_{a2}}{\partial \omega_{m3}} \right) & \cdots & \frac{1}{H_2} \frac{\partial T_{a2}}{\partial \omega_{mN}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{H_N} \frac{\partial T_{aN}}{\partial \omega_{m1}} & \frac{1}{H_N} \frac{\partial T_{aN}}{\partial \omega_{m2}} & \cdots & \frac{1}{H_N} \left( \frac{\partial T_{aN}}{\partial \omega_{mN}} - \frac{\partial T_{aN}}{\partial \omega_{mN}} \right)
\end{bmatrix}
$$

The derivatives are calculated by perturbing the system along the different directions.

- **Input:** Operation point ($\omega_m$), control inputs, and system and machines parameters.
- **Output:** Jacobian ($A$) as calculated above.

5. "voltage.m" calls MatPower to solve the network using power flow analysis and calculate the reactive power consumption at a particular bus given a voltage value. To pose the problem for MatPower, the bus under study is defined as a P-V bus, the bus with the voltage source as the Slack bus, and all the others as P-Q buses.

- **Input:** System and machines parameters
- **Output:** Power flow analysis solution including reactive power consumption at each bus.

6. "init.m" computes the state values of the full model at the stable equilibrium point. These constitute the vector $i_0$ used to compute the dissipated energy during the braking mode transition.

- **Input:** Stable equilibrium point ($\omega_{m0}$), control inputs, and system and machines parameters.
- **Output:** Currents at the SEP $i_0 = [i_{sd0} \ i_{sq0} \ i_{rd0} \ i_{rq0}]^T$

7. "CCT.m" calculates the critical clearing time of a system by forward integration using the method described in Chapter 3. It includes the calculation of the nullcline crossing times and a binary search between the bounds using time domain simulations.

- **Input:** Speed after braking mode ($\omega_{mf}$), system and machine parameters.
- **Output:** Critical clearing time, upper bound, lower bound, and approximation.

There are two functions that carry out most of the calculations and which are always called by one of the functions described above. These are:

1. "$Te_{-\text{vect.m}}$" calculates the electric torque ($T_e$), the stator voltage and current ($v_s$, $i_s$), and the rotor voltage and current ($v_r$, $i_r$) for all motors in the system. The steps to achieve this are the following:
(a) Calculate the equivalent impedance for each motor in its own base given the operation point \( \omega_m \).

(b) Transform that impedance to the system base.

(c) For linear systems call model “circuit.m” and for non-linear cases call MatPower to obtain the voltages and currents at each bus.

(d) Calculate the stator current from the bus current and transform it to each machine base.

(e) Calculate the rotor voltage and current in machine base quantities.

2. “circuit.m” is used in linear circuits and constitutes the main piece of the model. It poses the network in a set of linear equations that are solved to find current and voltages at each bus. The equations are written in the following form:

\[
\begin{bmatrix}
M & 0 \\
A^T & Z
\end{bmatrix}
\begin{bmatrix}
v_{bus} \\
i_{bt}
\end{bmatrix}
= 
\begin{bmatrix}
I \\
0
\end{bmatrix}
\]

where \( A \) is the branch-node incidence matrix, \( v_{bus} \) is a vector containing all the bus voltages, \( i_{bt} \) is a vector containing the branch currents, and \( i_{bs} \) a vector with the currents into each bus. The matrices \( M \) and \( Z \) are constructed as follows:

- \( M \) is a matrix with a row for each voltage source in the system. In each row, a “1” is placed in the column corresponding to the bus where the voltage source is located and all the other elements are zero. With this matrix, a set of equations are constructed as:

\[
v_{bi} = 1
\]

- \( Z = \text{diag} \left\{ Z_{t1}, \ldots, Z_{tn}, Z_{L1}, \ldots, Z_{LN} \right\} \) is a diagonal matrix constructed with all the branch and load impedances to yield a set of equations of the form:

\[
v_a - v_b - Z_{ab} \cdot i_{ab} = 0
\]

C.3 Main File

The information from the three input files is collected by the procedure shown with the name “Main.m” in Fig. C.1. In this procedure, the information is packaged in several vectors that serve as inputs to the functions in the repository, and then the output from those functions is processed for display. This main file performs the following sequence of operations:
1. Reads the parameters from the three files described above.

2. Prepares vectors for parameter exchange with functions.

3. Calculates the stable equilibrium point using function “dynamics.m” and a non-linear solver. Validation that the SEP was found correctly is done using function “grad.m” that calculates the gradient and then checking the Jacobian eigenvalues.

4. Computes the voltage sag at each bus given the fault characteristics and location.

5. Computes the speed after braking mode using function “init.m” to compute $i_0$ and then following the procedure described in Chapter 2.

6. Determines the fault-on trajectory using the procedure explained in Chapter 2.

7. Calculates the unstable equilibrium point using function “dynamics.m” and a non-linear solver. The UEP should have only eigenvalues with positive real part.

8. For two motor system only:

   (a) Calculates the saddle equilibrium points using function “dynamics.m”. These points should have one eigenvalue with positive real part and one with negative real part.

   (b) Calculates the eigenvector corresponding to the stable eigenvalue of each saddle point using function “grad.m”.

   (c) Performs backwards integration using function “dynamics_neg.m” starting close to the saddle point and with a perturbation along the eigenvector found above.

   (d) Determines the nullclines by computing the electric torque for each machines at all points in the phase plane and then checking where are they equal to the mechanical torque.

9. For all cases, calculates the critical clearing time, bounds and approximation using function “CCT.m”. The function can be set to calculate the bounds and approximation only to avoid making the step extremely time consuming in system with large number of motors.

10. Performs time domain simulations with time-varying control parameters using function “dynamics_sim.m”.

11. Performs the reactive power balance analysis of the system using function “voltage.m”.

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Bibliography


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