Energy Methods for Analyzing Drag and Inertia in Cycling Kinematics

by

Emma Marie Steinhardt

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Bachelor of Science in Mechanical Engineering at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2016

© Massachusetts Institute of Technology 2016. All rights reserved.

Signature redacted

Author ..................................................

Department of Mechanical Engineering

May 6, 2016

Signature redacted

Certified by .........................

Anette Hosoi
Professor of Mechanical Engineering
Thesis Supervisor

Signature redacted

Accepted by ..............................

Anette Hosoi
Professor of Mechanical Engineering
Undergraduate Officer
Energy Methods for Analyzing Drag and Inertia in Cycling

Kinematics

by

Emma Marie Steinhardt

Submitted to the Department of Mechanical Engineering on May 6, 2016, in partial fulfillment of the requirements for the degree of Bachelor of Science in Mechanical Engineering

Abstract

A model was developed for measuring the drag and effects of inertia for a cyclist during a race. Professional cyclist data from the Tour de France was acquired for several athletes. The data contained elevation, distance, velocity, and power as a function of time. Rolling resistance, drag, inertial energy, and potential energy were then evaluated. An integral energy equation relating these terms to input power was developed. This is much more stable numerically than differential equations in the power and force equations. This formula gave excellent agreement with the theoretical assumption that inertial effects are negligible. Additionally, the measured drag agreed with wind tunnel results. This work is the first to extract drag data from a cyclist during actual race conditions. In the future, this evaluation of drag variation coupled with energy equations could lead to optimizing cycling strategy.

Thesis Supervisor: Anette Hosoi
Title: Professor of Mechanical Engineering
Acknowledgments

I’d like to thank my father Dr. Allan Steinhardt for his help navigating the immense amount of data I acquired. Special thanks to my friend Devin Neal for writing a Python script to obtain the data from TrainingPeaks. This thesis wouldn’t be possible without the generous support of Dr. Ernesto Martinez and Kopin. Lastly, I’d like to thank my advisor Professor Anette Hosoi for her guidance and knowledge.
## Contents

1 Introduction .......................... 9
   1.1 Motivation ....................... 9
   1.2 Inertia Effects .................. 10
   1.3 Drag ............................ 11
       1.3.1 Drag Factor ................ 11

2 Kinematic Model ..................... 13
   2.1 Force Balance .................... 13
   2.2 Power Equations ................. 14

3 Optimal Velocity with Negligible Acceleration 15
   3.1 Work Equation ................... 15
   3.2 Solving for Velocity ............. 16

4 Limitations of Constant Velocity .......... 19
   4.1 Critical Power Concept .......... 19
   4.2 Optimization with Critical Power Concept 20

5 Numerical Analysis of Inertial Effects .. 21

6 Calculating Drag Factor ................. 25
   6.1 Power Equation .................. 25
   6.2 Cross Correlation Method ....... 26
   6.3 Energy Method .................. 27
7 Calculating Inertial Effects

8 Results

  8.1 Limitations .............................................. 34
  8.2 Mick Rogers Summary ...................................... 34
  8.3 Selected Results .......................................... 37

9 Conclusion

A MATLAB: Percent Inertia Plot Code 45

B MATLAB: Calculate Drag Factor 49

C MATLAB: Calculate Percent Inertia 51

Bibliography 53
Chapter 1

Introduction

This thesis explores the effects of inertia for a cyclist during the course of a race using data from the Tour de France in 2014 [16]. Changes in the drag during a race for an individual cyclist will also be evaluated. Before analyzing the data, a kinematic model for cyclist will be derived and discussed in chapter 2.

After deriving the kinematics, the optimal velocity for a cyclist with negligible inertial force will be explored in chapter 3. It is found that the optimal strategy when ignoring the effects of acceleration is to ride at a constant velocity. This strategy should work in theory, but there are limits to the human bioenergetics model. Chapter 4 discusses current human power models and their limitations on the constant velocity strategy.

The effects of acceleration on cycling power will be discussed numerically in chapter 5. After this, the methods for determining the drag factor and inertial affects will be discussed in chapter 6 and 7. The data analysis and resulting plots is in chapter 8. This thesis will then conclude with a conclusion on the findings and what future work needs to be done in chapter 9.

1.1 Motivation

Mathematical analysis and optimization of long duration cycling races is a big topic of interest in today’s technological age. We now have easy access to live power
data in races and teams are able to update their strategy in real time based off this information. Numerous pieces of literature have sought to model a cyclist's kinematics and bioenergetics, but none have explored a method for optimizing a cyclist's performance in real time.

The equations of motion for a cyclist and the optimal velocity have been derived and discussed in previous literature [2, 5, 8]. A common assumption made is that the acceleration of the cyclist, and the resultant resistive inertia force, is small compared to the other resistive forces. This assumption has been made theoretically [8], but has yet to be explored experimentally.

Additionally, the effects of drag during the course of a road race have yet to be explored during the duration of actual races. Aerodynamic drag in cycling is the main resistive force to motion. It has been found that for speeds greater than 14 m/s, the drag accounts for 90% of the total resistive forces [1, 3]. Because of its high influence, understanding the drag leads to understanding a cyclist's kinematics.

1.2 Inertia Effects

Inertia is the resistance to a change in motion of an object. The principle of inertia states that an object will stay in its current state of motion unless acted upon by an external force [11]. Newton's second law of motion states that the sum of the external forces acting on an object is equal to the inertial force, mass times acceleration. If an object is accelerating, then there must be an external force acting on it.

For a cyclist, this relates to the force required to accelerate. Anytime a cyclist accelerates, there is a resistive inertia force they must overcome. Even without inputting any power, the cyclist can still accelerate due to potential energy. Gravity propels the cyclist downhill, causing the cyclist to accelerate and experience an inertial force without any input.

Including inertia into the power equations for cycling creates a differential equation. Including the differential equation when solving for the optimal power output produces a transcendental equation with no closed-form solution. Neglecting the
effects of inertia eliminates this issue and can lead to closed-form solutions.

1.3 Drag

The drag force \( F_D \) is proportional to the flow velocity squared \( u^2 \) and depends on the air density \( \rho \), effective frontal area \( A \), and drag coefficient \( C_D \).

\[
F_D = \frac{1}{2} \rho C_D A u^2 \tag{1.1}
\]

The effective frontal area \( A \) for a cyclist is the most essential parameter [3]. It depends on the body height, position on the bicycle, and cycling equipment. Methods to measure frontal area include image processing and computer models. These methods are not very easy to use and makes it very undesirable to measure. Additionally, there is a multitude of positions a cyclist can be in during a race.

The drag coefficient itself is dependent on the air flow, shape, and position of the cyclist. Wind tunnels are the most common way for evaluating the drag coefficient. Due to the varying nature of a cyclist’s position in a race, a wind tunnel can’t properly analyze how the cyclist’s drag changes during a race. The only other current methods for evaluating the drag coefficient involve a cyclist going at constant velocity and looking at the power meter data. Any variation in velocity would produce a noisy signal and variation in the drag making it difficult to measure.

1.3.1 Drag Factor

One way to analyze the drag coefficient and effective frontal area for a cyclist is to look at the drag factor. The drag factor \( c \) is defined by the terms in front of the velocity in the force equation.

\[
c = \frac{1}{2} \rho C_D A \tag{1.2}
\]

For road racing, all the cyclists start and ride simultaneously. It’s common in these types of races for a cyclist to follow closely behind another rider (draft) in
order to reduce their drag. This practice gives variation to the drag factor of the
cyclist. Additionally, a cyclist can change their drag factor depending on the way
they position themselves. Going downhill, cyclists will often reposition themselves in
a more aerodynamic position.

This variation in the drag factor has yet to be evaluated during the course of a
race. Optimization papers often look at time trial races where a cyclist rides solo
making the assumption of constant drag factor plausible. For road races, however,
the change in drag via drafting is a key component in strategy. Finding a way to
evaluate a cyclist’s drag factor without a wind tunnel and in real time can thus
provide valuable information about the kinematics of the cyclist.
Chapter 2

Kinematic Model

This chapter serves to identify the system of the cyclist and define the governing equations.

2.1 Force Balance

It has been found that the frictional loses in wheel bearings is small compared to other losses [15]. Thus, the resistance forces acting on the cyclist that produce noticeable losses are the rolling resistance, drag force, and gravitational force. A simplified free body diagram of a cyclist riding up a hill is shown in figure 1.

![Free Body Diagram](image)

Figure 2-1: A propulsion force $F_m$ is balanced by the drag force $D$, the rolling resistance $F_R$, and the gravitational force $F_g$. The cyclist is traveling up an incline of angle $\theta$. 
The force balance in the direction of the cyclist's motion results in:

\[ F_{\text{net}} = F_{\text{in}} - D - F_R - F_g \sin \theta \] (2.1)

The drag force for the cyclist is \( D = cv^2 \), where \( c \) is the drag factor (in kg m\(^{-1}\)). The gravitational force is \( F_g = mg \). The rolling resistance is normalized to \( F_R = mg\mu \). The slope of the hill \( \sin \theta \) is often referred to as the grade and will be referred to as \( G \). This results in:

\[ F_{\text{net}} = F_{\text{in}} - cv^2 - mg\mu + mgG \] (2.2)

Newton's Second Law of motion states that \( F_{\text{net}} = ma \) where \( m \) is the mass of the object in motion and \( a \) is the acceleration. In order for an object to accelerate, it must overcome this inertial force. Using the fact that acceleration is the derivative of velocity with respect to time, we can solve for \( F_{\text{in}} \) in terms of velocity and grade:

\[ F_{\text{in}} = cv^2 + mg\mu + mgG + m\frac{dv}{dt} \] (2.3)

### 2.2 Power Equations

Power \( P \) is related to force by \( P = Fv \). Using (2.3), the power input of the cyclist is given by:

\[ P_{\text{in}} = cv^3 + mg\mu v + mgGv + m\frac{dv}{dt}v \] (2.4)

Where \( cv^3 \) is the power due to drag \( P_D \), \( mg\mu v \) is the power due to rolling resistance \( P_R \), \( mgGv \) is the power due to gravity \( P_g \), and \( m\frac{dv}{dt}v \) is the power due to inertia \( P_i \).
Chapter 3

Optimal Velocity with Negligible Acceleration

In this chapter, we will formulate a solution to the optimal velocity profile in a race with negligible acceleration. For this problem we will assume the cyclist isn’t drafting, making the drag factor constant. Additionally, we assume the cyclist as set amount of work they can expend W and the cyclist’s power has no maximum.

3.1 Work Equation

Power and work are related by $W = \int P \, dt$. The velocity $v$ is defined by $\frac{dx}{dt}$. Using the reciprocal of velocity $\frac{1}{v}$, the total work $W$ for the cyclist to complete a race of length $L$ can be given by:

$$W = \int_0^L \frac{P}{v} \, dx \quad (3.1)$$

This works out to be the same as $W = \int P \, dt$ since $dx \div \frac{dx}{dt} = dt$.

Substituting (2.4) into (3.1) and setting the acceleration $\frac{dv}{dt} = 0$ yields the following:

$$W = \int_0^L \left[ cv^2 + mg(\mu + G) \right] \, dx \quad (3.2)$$

The integral in (3.2) can be separated into two components. One component $\int_0^L mg(\mu + G) \, dx$ is independent of velocity and only dependent on the course profile.
and will be denoted as $B$. $B$ is the work required to complete a given course. Our work equation can then be written as:

$$W = B + \int_0^L cv^2 \, dx$$  \hspace{1cm} (3.3)

### 3.2 Solving for Velocity

The total time $T$ for the cyclist to complete a route of length $L$ is given by:

$$T = \int_0^T dt = \int_0^L \frac{1}{v} \, dx$$  \hspace{1cm} (3.4)

Velocity is inversely proportional to time. In order to minimize time, one must maximize their velocity. This makes sense given that the fastest person wins the race.

Using (3.3), we are given a constraint on the velocity in terms of work input $W$, course work $B$, and drag factor $c$. Denoting the constraint as $g(x) = 0$ results in:

$$g(x) = \int_0^L cv^2 \, dx + (B - W) = 0$$  \hspace{1cm} (3.5)

We thus seek to maximize $f(x) = \int_0^L v \, dx$ subject to the constant $g(x) = 0$. This problem can be solved using Lagrangian multipliers.

An extrema of $f$ can only lies on $g$ if their gradients are in the same direction. When in the same direction, the gradient of $f$ is a multiple $(-\lambda)$ of the gradient of $g$.

$$\nabla f = -\lambda \nabla g$$  \hspace{1cm} (3.6)

The multiple $\lambda$ is known as the Lagrange multiplier.

For two equal vectors, all the components must be equal. For each point $x$, this gives us:

$$\frac{\partial f}{\partial x} = -\lambda \frac{\partial g}{\partial x}$$  \hspace{1cm} (3.7)
Evaluating 3.7 with our equation for $f$ and $g$ results in:

$$v = -\lambda v^2 \quad (3.8)$$

Finally, solving for $v$ gives the following relation:

$$v = -\frac{1}{\lambda} \quad (3.9)$$

Knowing $\lambda$ is constant, it can be seen from this result that the maximum solution to $f$ subject to our constraint $g$ is to ride at constant velocity.
Chapter 4

Limitations of Constant Velocity

In chapter 3 the optimal velocity for a given energy input was found to be constant. For races of varying gradient, this produces required power input above the maximum a cyclist can output. Attempts in past research [2, 5] have sought to model the energy of the cyclist as a function of time and power expended using a bioenergetics model.

4.1 Critical Power Concept

A variety of models for human bioenergetics has been discussed and used in sports literature. The critical power model is the most widely used for modeling human bioenergetics. Critical power is defined by the constant power a person can output for a given duration. There have been variety of models have developed over the last 60 years [10] for critical power.

There are four main assumptions to the critical power concept first discussed by Hill [6, 10]:

1. There are only two components of energy supply, the aerobic and anaerobic.

2. The aerobic supply is infinite in capacity but rate limited. This limit is known as the VO_2 max.

3. The anaerobic supply is not rate limited, but is finite in capacity. This capacity is known as the anaerobic work capacity (AWC).
4. Exhaustion occurs when all the AWC is utilized.

4.2 Optimization with Critical Power Concept

Using this model, a cyclist riding at their maximum power can burn out in a matter of seconds. Conversely, a cyclist riding at their VO₂ max could in theory ride forever. Calculating the optimal strategy using this model involves expending the AWC strategically throughout the race.

Gordon [5] and Dahmen [2] use Morton’s critical power model [9] to optimize cycling performance for a time trial. Acceleration effects are neglected in Gordon’s paper. An exertion model is proposed by Gordon in which a cyclist adds to the exertion when riding above VO₂ max and decreases when riding below. Dahmen’s includes inertial effects in his model, and solves for power through advanced algorithm techniques that require a lot of computing time.
Chapter 5

Numerical Analysis of Inertial Effects

We're interested in the proportion of a cyclist's total power expended due to overcoming inertia. An example scenario will be used to analyze the effects of inertia numerically. Assuming a constant drag factor, a typical value of $c = 0.17 \text{ kg/m}$ will be used and a rolling resistance of $\mu = 0.003$ [7]. In this example, the mass of the cyclist and bicycle is $m = 84 \text{ kg}$.

The power due to inertia is given by the equation:

$$P_i = mv \frac{dv}{dt} \quad (5.1)$$

Input power for the cyclist is given by the equation (2.4). We can calculate the input power for a given velocity, acceleration, and grade using our values for $c$, $\mu$, and $m$. The proportion of power due to inertia is simply the inertial power $P_i$ divided by the cyclist's input power $P_i$. This expression can then be evaluated for an array of velocity and acceleration values at a given grade.

Using MATLAB, we can plot a color map of the percent inertial power at a specified grade as a function of acceleration and velocity. The code used to make this plot can be found in Appendix A. Figures 5-1a and 5-1b show the percent inertial power for a grade of 5% and 15% respectively.
Figure 5-1: Color maps of the percent inertial power for possible velocity and acceleration outputs. Contour lines specifying constant constant power input in Watts are shown in black. It can be seen that the inertial power dominates at low velocity and higher acceleration. Increasing the grade is found to reduce the effects of inertia.
Contour lines of constant power input are superimposed on the plot to show the effort required by a cyclist. Values of power input greater than 1200 W are cut off. This value was decided from data aggregated at the 2005 Tour de France by Vogt et al [14]. The highest power sustained for 15 seconds at the tour was around 1200 W for a flat stage and 1050 W for a mountain stage. Maximal mean power, of all the cyclists measured, for 15 seconds on a flat stage was 895 W and 836 W for a mountain stage. Average power overall was approximately 227 W.

At lower velocities, the dominate drag force has less influence due to its power loss scaling with velocity cubed. This leads to the inertial power loss dominating at lower velocity. At an average power output around 200 W, a cyclist accelerating at the beginning of a race will have the effects of inertia mostly dominate their resistive forces. For races like the Tour de France, this has little influence because the riders start off neutral before the beginning of the race. A neutral section in a race is when everyone is moving at a controlled moderate pace.

The power required to accelerate increases exponentially with the velocity. Accelerating at a higher velocity requires more power, leading to lower possible accelerations. Since inertia scales linearly with acceleration, the effects at high velocity become negligible for the possible power inputs. We can conclude from this that inertia will have little at high velocity.

Comparing the two plots shows that increasing the grade results in lower potential velocities from the gravitational force having a higher magnitude. The increase in magnitude of the gravitational force results in inertia having less influence. At steeper inclines, we can conclude that the effects of inertia start being negligible. This insight has practical value, since we can ignore the inertial term, thereby obtaining an equation that is now a polynomial in velocity and linear in height as opposed to a differential equation.
Chapter 6

Calculating Drag Factor

We’re interested in evaluating the drag factor over the course of a race to see how drafting plays a role. We assume the cyclist doesn’t brake due to lack of data on this and the inefficiencies in doing so. It should be noted that deviations in the drag factor can thus be contributed to braking as well. Typical values for the drag factor $c$ range from 0.1 to 0.3 kg m$^{-1}$ [7].

6.1 Power Equation

The race data acquired provides us with the distance, height, velocity, and power values for each second. This gives us $x(t), h(t), v(t),$ and $P(t)$. In order to analyze this data, we need to reformulate our model in terms of time.

Using trigonometry, the grade can be given by $rac{dh}{dx}$ where $h$ is the elevation and $x$ is the position of the cyclist. Velocity $v$ is defined by $\frac{dx}{dt}$. This relation allows us to simplify our gravitational power term:

$$P_g = mg\sin \theta v = mg \frac{dh}{dx} \frac{dx}{dt} = mg \frac{dh}{dt} \quad (6.1)$$

Substituting (6.1) into (2.4) yields the following:

$$P_m = cv^3 + mg\mu v + mg \frac{dh}{dt} + m \frac{dv}{dt}v \quad (6.2)$$
We can then solve for the drag factor directly from the power equation in (6.2) using our data. Plotting the drag factor over time using this direct method is shown in figure 6-1.

Figure 6-1: A plot of the drag factor vs time for Mick Rogers during stage 16. The drag factor $c$ was evaluated using the power equation (6.2). It can be seen that the resultant calculation for the drag factor is outside the expected range and fluctuates rapidly.

A cyclist’s power input and velocity is a noisy signal due to various factors. The cyclist, being human, does not perfectly couple muscular energy into the bike, drag will depend on any ambient wind, sensor readings are imprecise, and braking dissipates energy unlike inertial energy. Differentiating a noisy signal creates even more noise. We also don’t want to filter our data because we lose the important acceleration spikes from attacks.

### 6.2 Cross Correlation Method

Attempts at using data processing techniques such as cross correlation also yield noisy and unrealistic outputs when solving for $c$. Figure 6-2 shows the drag factor calculated using cross correlation.
Figure 6-2: The drag factor $c$ is evaluated by solving the power equation for $c$ and using cross correlation. It can be seen that this method yields values of $c$ outside the typical range of 0.1 to 0.3 kg m$^{-1}$.

It's evident from the results in Figure 6-2 that an alternative method is needed for evaluating the drag factor throughout the race.

### 6.3 Energy Method

Integrating a noisy signal reduces the amount of noise. Using this fact, we can reform our method for calculating $c$ by using conservation of energy. Energy $E$ is related to power by $E = \int P \, dt$. We can use this fact to reformulate our system in terms of energy, and reduce noise by integrating (6.2). This gives us:

$$
\int P_{in} \, dt = \int \left[ cv^3 + mg\mu v + mg \frac{dh}{dt} + m \frac{dv}{dt} v \right] \, dt \quad (6.3)
$$

Using the commutative property, we can separate each component in the integral and evaluate them separately. Looking at the inertial power term $P_i$ in the integral and evaluating it we get:

$$
\int m \frac{dv}{dt} v \, dt = \int mv \, dv = \frac{1}{2} mv^2 \quad (6.4)
$$
This is colloquially known as the kinetic energy. Substituting this back into (6.3) and separating the terms in the integral yields:

\[
\int P_{\text{in}} \, dt = \int cv^3 \, dt + \int mg\mu v \, dt + \int mg\frac{dh}{dt} \, dt + \frac{1}{2}mv^2
\]

(6.5)

Note that we do not evaluate \( \int mg\frac{dh}{dt} \, dt = mgh \) because we want to know the change in elevation over time as opposed to the actual height. Evaluating the integral to \( mgh \) gives gravitational energy values that are higher because it adds in the initial value. We could in theory just reset \( h(0) = 0 \), but leaving the function as is eliminates the need for manipulating data.

For a discrete function, the integral can be evaluated as a summation. At a given time \( t \) we can then write the energy equation (6.5) as the sum of previous points in time.

\[
\sum_{i=0}^{t} P_{\text{in}}(i) = \sum_{i=0}^{t} c(t)v(i)^3 + \sum_{i=0}^{t} mg\mu v(i) + \sum_{i=0}^{t} mg\frac{dh}{dt}(i) + \frac{1}{2}mv(t)^2
\]

(6.6)

The drag factor \( c \) is assumed constant in the summation over short time intervals, so we can move it on the outside and solve for \( c \). This yields the following equation:

\[
c(t) = \frac{\sum_{i=0}^{t} P_{\text{in}}(i) - \sum_{i=0}^{t} mg\mu v(i) - \sum_{i=0}^{t} mg\frac{dh}{dt}(i) - \frac{1}{2}mv(t)^2}{\sum_{i=0}^{t} v(t)^3}
\]

(6.7)

Using the results in (6.7) we will now attempt to evaluate the drag factor at a given point in the race. Using data from Mick Rogers again, we get the following output shown in figure 6-3.
Figure 6-3: The drag factor $c$ is evaluated by solving the energy equation for $c$ without any filtering. It can be seen that this method yields reasonable values of $c$ inside the typical range of 0.1 to 0.3 kg m$^{-1}$. This plot also shows Mick breaking away from the pack at 90 minutes where he is no longer drafting and thus his drag factor increases to a quasi-steady value.
Chapter 7

Calculating Inertial Effects

We should in theory be able to directly evaluate the proportion of power due to inertia \(P_i = m \frac{dv}{dt}\) with our velocity and power data. Unfortunately, our velocity and power data are noisy signals as mentioned prior. We're attempting to differentiate a noisy signal and multiply it by itself while dividing it by another noisy signal. Additionally, when the input power approaches zero the output of our equation \(\frac{P}{P_{in}}\) approaches infinity. Figure 7-1 shows the proportion of power due to inertia calculated directly.

Figure 7-1: The proportion of inertial power calculated directly plotted over time. This method produces percent values greater than 100 due to the propagation of noise. It's evident from this plot that a different method for evaluating the effects of inertia must be devised.
We learned from calculating the drag factor that evaluating the system using energy prevents the propagation of noise from giving unrealistic results. Instead of looking at the percent of power due to inertia, we will thus look at the proportion of energy.

The energy due to inertia is the kinetic energy $\frac{1}{2}mv^2$ from (6.4). We’re concerned with how much of the energy the cyclist puts in goes to overcoming inertia. The energy the cyclist puts in $E_in(t)$ is given by $\sum_{i=0}^{t} P_{in}(i)$ as shown in (6.6). Using this, we can calculate the proportion of energy due to inertia $E_i(t)$ by:

$$E_i(t) = \frac{1}{\sum_{i=0}^{t} P_{in}(i)} \frac{1}{2}mv^2 \quad (7.1)$$

Using (7.1) we can now attempt to plot the effects of inertia for a cyclist. Figure 7-2 shows the percent of energy due to inertia over time for the same dataset used in prior examples. It can be seen that using energy produces realistic results and the effects of inertia are very small.

Figure 7-2: The proportion of inertial energy calculated plotted over time. This method produces percent values within the range of 0 to 100. For this particular dataset, the percent of energy due to inertia is under 4 throughout the race.
Chapter 8

Results

Using the model proposed, we can analyze the effects of inertia and variation in drag for a cyclist during a race. Data from the 2014 Tour de France [16] will be used in this section to obtain preliminary results. Future work may seek to compare different cyclists at the same race to explore drafting.

Datasets were cropped to exclude neutral portions of the race. The subsequent 25 minutes were cropped after evaluating the drag factor and inertial effects due to the integration method taking time to equilibrate. Even with this cropping, the beginning of the race for the datasets evaluated showed the most variation and highest values. This is most likely due to the pelotons forming at the beginning of the race.

The percent of energy due to inertia measured at less than 5% during the entire race for data sampled. During the majority of the race, energy due to inertia accounted for less than 0.5% of the total energy. We can conclude from this that inertia plays a minor role in the cyclist’s kinematics. This observation that inertia is small has practical value, since we can ignore the inertial term, thereby obtaining an equation that is now a polynomial in velocity and linear in height as opposed to a differential equation.

It was found that the average drag factor varied for the same cyclist during different stages. Possible variations may be due to the limitations of the model as discussed in section 8.1.
8.1 Limitations

The data collected in this study, and the resultant model have limitations. Integration can’t smooth everything and future work may seek to eliminate this error with additional instrumentation. Currently, there is no wind velocity or braking data and the rolling resistance is assumed constant. Additionally, higher resolution sensors can provide more accurate results.

The model used assumes there is no ambient wind. In reality, there is usually some amount of wind affecting the rider. The velocity term in drag in actuality is the difference in velocity of the rider to wind velocity. Compact wind meters are available and be easily added to the bicycle for future analysis.

Currently there is no inclusion of braking in the data or model. Evidence in the data during downhill segments suggests that braking has an effect on the results. In these downhill segments when the cyclist isn’t pedaling, the increase in kinetic energy is less than expected. Braking data could be incorporated with a simple strain gauge attached to the brake cable.

For road races entirely on paved road, the terrain doesn’t vary much and the resultant variation in the rolling resistance is somewhat negligible. Occasionally road races include segments not on paved roads such as cobblestone or gravel, which have a different coefficient of rolling resistance. Collecting rolling resistance data prior to a race for different segments can quantify the variation. A simple piecewise set of data based on position should be sufficient.

8.2 Mick Rogers Summary

Data for ten stages of the 2014 Tour de France was procured for Mick Rogers. A graphical summary of this data can be seen in figure 8-1.

Mick’s average mean drag factor for the ten stages was found to be 0.104 kg/m with an average standard deviation of 0.005 kg/m. It’s evident from his variation in drag that a constant drag factor is insufficient for modeling. For the ten stages, the
average drag factor varied with a standard deviation of 0.026 kg/m. The variation in average drag indicates that the ambient wind plays a crucial role.

From the data spread, there appears to be no direct correlation between elevation variation (mountainous vs flat) and drag factor variation. The box plot, however, doesn't take into account if the elevation variation is due to going uphill or downhill. Stage 15 (see figure 8-3), for example, is mostly downhill in comparison to stage 16 (see figure 8-4) which is mostly uphill.

Drafting plays the most important role in the value and variation in drag factor. Having less variation in the drag factor during a stage probably indicates that the cyclist was in the same peloton for the majority of the race. For stage 3, Mick was described [16] as staying with the front group. It may help in future studies to gather information on what occurred during the race.
(a) The spread of drag factor values for Mick Rogers during the 2014 Tour de France.

(b) The spread of elevation data for Mick Rogers during the 2014 Tour de France.

(c) The spread of percent inertial energy for Mick Rogers during the 2014 Tour de France.

Figure 8-1: Box plots of the data spread for stage data of Mick Rogers. On each box, the central red mark indicates the median. The data inside the blue box represents the interquartile range. Whiskers span 1.5 times the interquartile range. Outliers are in red marked with a + sign.
8.3 Selected Results

(a) A color map plot of Mick Rogers’ drag factor measured in kg/m during stage 11.

(b) A color map plot of the percent of Mick Rogers’ energy due to inertia during stage 11.

Figure 8-2: Mick’s drag factor varied the most on this stage out of the ten stages evaluated. His drag factor had a standard deviation of 0.008 kg/m during this stage. The elevation in this stage also varied a lot, with steep uphill and downhill sections.
Figure 8-3: It was a rainy windy day during stage 15. Mick’s drag factor had little variation (standard deviation of 0.003 kg/m) during this stage. This was probably due to the lack of attacks and shifting in the peloton due to the treacherous weather.
(a) A color map plot of Mick Rogers' drag factor measured in kg/m during stage 16.

(b) A color map plot of the percent of Mick Rogers' energy due to inertia during stage 16.

Figure 8-4: Mick established a break away from the pack at 100 minutes and went on to win the stage. His drag factor on average was higher than in other stages due to the lack of drafting as a solo rider in the front. Mick's drag factor is shown to increase after his breakaway, indicating he is no longer drafting. The percent of Mick's energy due to inertia is highest while he was establishing his breakaway at the beginning of the race.
Simon Clarke Stage 5 2014

(a) A color map plot of Simon Clarke’s drag factor measured in kg/m during stage 5.

(b) A color map plot of the percent of Simon Clarke’s energy due to inertia during stage 5.

Figure 8-5: Simon was part of the breakaway group from the start of stage 5. This stage had treacherous conditions and cobblestone sections [16]. As the race progressed, Clarke’s drag factor was shown to increase. This may be due to the breakaway pack spreading out and shifting.
Figure 8-6: Simon made the breakaway group at the start of the stage. To stay in the group at the end of the stage, Simon had to produce over 800 W. The drag factor during this stage for Simon stayed fairly constant after 150 minutes. It’s likely that the breakaway group stayed in the same formation for the second half of the stage.
(a) A color map plot of Mikel Nieve’s drag factor measured in kg/m during stage 13.

(b) A color map plot of the percent of Mikel Nieve’s energy due to inertia during stage 13.

Figure 8-7: The end of stage 13 included a category 1 climb up the Col de Palaquit. Mikel’s drag during the final climb was higher than earlier parts of the stage. It’s possible that the peloton became segmented during the final climb, resulting in Mikel drafting less. The percent of Mikel’s energy due to inertia during the final climb was found close to 0% of his total energy. This result agrees with the numerical analysis done in chapter 7.
Chapter 9

Conclusion

In this thesis, a new model was proposed for evaluating the drag factor and the effects of inertia using energy methods. The model takes an input of power, elevation, and velocity as a function of time for a cyclist. Looking at the energy of system in lieu of power required integration. This integration helped to smooth the data without any filtering that would eliminate important features.

Data from the 2014 Tour de France was used to test the model and obtain preliminary results. It was found with the data tested that the inertia accounts for less than 5% of the energy during the entire race with a majority of it being under 0.5%. The drag factor varied during the races with an average mean of 0.104 kg/m and standard deviation of 0.005 kg/m for Mick Rogers.

Results of this study concluded that the effects of inertia are negligible when compared to other sources of energy loss. The drag factor was found to vary enough in the Tour de France that its insufficient to model it as constant. Drafting, as expected, plays an important role in races.

Future work may seek to include braking and wind data. The inclusion of braking and wind will provide more accurate results for the drag factor and provide less variation in the average drag factor during different races for the same cyclist.

Using the consumption energy model proposed in this thesis in conjunction with an bioenergetics supply model can lead to a powerful optimization tool for cyclists. This thesis is the first to provide a model for evaluating the drag factor during a race,
and future work may seek to explore the correlation of drag to different variables.
Appendix A

MATLAB: Percent Inertia Plot Code

%% create velocity and acceleration vectors

v = .01*(1:2800)'; % velocity vector
a = .01*(0:1000)'; % acceleration vector
n = length(v); % length of velocity vector
n_a = length(a); % length of acceleration vector
w = [0, 400, 800, 1200]; % contour lines of constant power to label

%% Assign constants

G = .15; % grade
m = 84; % mass of cyclists and bike (kg)
c = 0.17; % drag factor (kg/m)
g = 9.807; % acceleration due to gravity (m^2/s)
mu = 0.003; % rolling resistance coefficient

%% evaluate total power and percent inertial power

% blank matrices to assign values
P_i = ones(n_a,n); % inertial power
Ptot = ones(n_a,n); % cyclist input power (total power)
for i = 1:n_a  % for loop for calculating power at each index
    for j = 1:n
        Ptot(i,j) = c*v(j)^3 + m*g*(mu+G)*v(j) + m*a(i)*v(j);
        P_i(i,j) = (m*a(i)*v(j))/Ptot(i,j);
    end
end

%% cut off values and find max velocity and acceleration

% blank matrices to be filled with values
Pi = zeros(n_a,n);  % i, j or a,v
Pt = zeros(n_a,n);

% initialize max values
amax = 0;
vmax = 0;

for i = 1:n_a
    for j = 1:n
        if Ptot(i,j) < 1210  % used 1210 so 1200 contour line shows
            Pi(i,j) = Pi_0(i,j);
            Pt(i,j) = Ptot(i,j);
            if a(i) > amax
                amax = a(i);  % change a max if a(i) is greater
            end
            if v(j) > vmax
                vmax = v(j);  % change a max if v(i) is greater
            end
        else
            % set indices to NaN if Ptot is greater than 1210
            Pi(i,j) = NaN;
            Pt(i,j) = NaN;
        end
    end
end
% round amax and vmax to next integer
amax = ceil(amax);
vmax = ceil(vmax);

%% create plot

% create title string based off G value
s1 = 'Grade = ';
s2 = num2str(G*100);
s3 = '%' s = strcat({s1},{s2},{s3});

% create new figure, specify size and colormap
fig = figure;
set(fig, 'Position', [100, 100, 600, 400]);
colormap(fig,'jet'); % specify colormap style
ax = gca;

% create color contour plot
contour(v,a,100*Pi);
pcolor(v,a,100*Pi);
shading interp;

% add constant power contour plot
hold on;
[C,h] = contour(v,a,Pt,'k','LineWidth',1.5);
clabel(C, h, w,'labels-spacing', 165);
clabel(C,h,'FontName','Ariel','FontSize',12,'FontWeight','Bold');
hold off;

% make colorbar and format
caxis([0 100]); % specify colorbar range
colorbar % create colorbar
cb = colorbar('peer',ax);
cb.LineWidth = 1.5;
title(cb,'% Inertia','FontSize',14,'Fontweight', 'bold');
axis([0,vmax,0,amax]); % set axis maximum

% add labels and specify format
set(gca,'FontSize',14,'FontName','Ariel');
title(s,'FontSize',16,'FontName','Ariel','Fontweight','bold');
xlabel('Velocity (m/s)','FontSize',16,'FontName','Ariel','Fontweight','bold');
ylabel('Acceleration (m/s^2)','FontSize',16,'FontName','Ariel','Fontweight','bold');
set(gca,'Layer','top');
ax.XAxis.Color = 'black';
ax.YAxis.Color = 'black';
ax.BoxStyle = 'full';
ax.LineWidth = 1.5;
Appendix B

MATLAB: Calculate Drag Factor

% M: (t,d,h,P,v)
% energy method for calculating c

function [c] = calc_c(M,mass)

cut = 1500; % number of samples to cut at end

% create vectors from matrix
t = M(:,1); % time vector
P = M(:,4); % power vector
h = M(:,3); % elevation vector
v = M(:,5); % velocity vector
dh = [0; diff(h)]; % dh/dt

s = t(20)-t(19); % calculate sample spacing

mu = .003; % rolling resistance coefficient

g = 9.81; % acceleration due to gravity

n = length(P); % length of vectors
% create vectors for energy terms
PE = ones(n,1); % potential energy
KE = ones(n,1); % kinetic energy
RR = ones(n,1); % rolling resistance energy loss
E_in = ones(n,1); % input energy
v3 = ones(n,1); % sum of velocity cubed

% calculate energy terms at each index
for i = 1:n
    PE(i) = s*mass*g*sum(dh(1:i));
    KE(i) = .5*mass*v(i).^2;
    RR(i) = s*sum(mass*g*mu*v(1:i));
    E_in(i) = s*sum(P(1:i));
    v3(i) = s*sum(v(1:i).^3).^(-1);
end

% calculate c
C = (-PE - KE - RR + E_in).*v3;

% cut out beginning indices
for i = 1:cut
    C(i) = NaN;
end
Appendix C

MATLAB: Calculate Percent Inertia

% M: t, d, h, P, v, a, G

% percent of energy that is kinetic aka inertial

function [i_E] = calc_in(M, mass)

cut = 1500; % number of samples to cut at end

% create vectors from matrix
 t = M(:,1);  % time vector
 P = M(:,4);  % power vector
 v = M(:,5);  % velocity vector

s = t(20)-t(19);  % calculate sample spacing

n = length(P);  % length of vectors

% create vectors for energy terms
 KE = ones(n,1);  % kinetic energy
 E_in = ones(n,1);  % input energy
% calculate energy terms at each index
for i = 1:n
    KE(i) = .5*mass*v(i).^2;
    E_in(i) = s*sum(P(1:i));
end

% calculate percent of inertial energy
i_E = 100*KE./E_in;

% cut out beginning indices
for i = 1:cut
    i_E(i) = NaN;
end
Bibliography


