Three Essays on Strategic and Organizational Uses of Information in Marketing

by

Florian Zettelmeyer

M.Sc. in Economics
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Author ................................................................. Sloan School of Management
June 14, 1996

Certified by .......................................................... Birger Wernerfelt
Professor of Management Science
Thesis Supervisor

Accepted by ..........................................................

Birger Wernerfelt
Chairman, Department Committee

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Abstract

Essay 1: Through their communications mix, firms make decisions that have implications for the ease with which consumers can search for product information. A common intuition is that firms would like to make information as easily available to consumers as possible, but choose not to because facilitating consumer search is costly to them. The main result of this paper counters this intuition – firms may not want to make consumer search easy, even if it is costless for them to do so. I identify two reasons why firms will choose to make search hard for consumers. First, firms can soften price competition by differentiating themselves on the basis of consumer search cost. Second, firms have an incentive to take advantage of consumer optimism. It is shown that the reduced cost of providing information in new electronic channels such as the Internet does not necessarily imply that firms will aid consumers in their search for information. Some of the marketing implications of this result are: The information highway may not develop into an information smorgasbord; the existence of competing stores with asymmetric search cost and prices might be more than just a transitional state from a fragmented retail environment towards only a few mass merchandisers; the fact that stores have low service might not be related only to cost pressure.

Essay 2: A popular belief is that the Internet will lead to more consumer information and lower prices. Casual empiricism seems to confirm these expectations. Many manufacturers make searching for information over the Internet very easy; for some product categories price levels of firms that sell over the Internet seem to be lower than those of their competitors in conventional outlets. This paper argues that these predicted directions ignore the following two issues: first, Internet strategies might be interdependent with strategies on other channels. Second, firm strategies might depend on the reach of the Internet. This paper shows how firm strategies may be affected by the existence and size of the Internet: firms have incentives to facilitate consumer search on the Internet, but only as long as the Internet's reach is limited. As the Internet is used by more consumers, firms' strategies on the Internet will mirror the strategies they pursue through a conventional channel. It is further shown that the profits of a
new entrant in a product category will increase with the reach of the Internet while the Internet is still relatively small. However, after customers’ use of the Internet reaches a critical mass, the benefits of the Internet to a potential entrant disappear. An extension shows that increased competitive pressures that arise from consumers with a high valuation of time can be counteracted by competing on Internet in addition to a conventional outlets. The paper suggests directions the Internet might take and derives several managerial implications.

Essay 3: It is almost an article of faith within marketing that market driven firms will do better than firms that are research driven. This paper argues that the optimality of market vs. research orientation depends on the resources (internal and cross-functional skills) of the firm’s marketing and R&D. Marketing’s cross-functional skills refer to its ability to anticipate the technical feasibility of a new product. R&D’s cross-functional skills are its ability to anticipate the technical design that consumers demand. Internal skills describe marketing’s and R&D’s ability to excel in their respective tasks. A model is presented in which a firm optimizes over the sequence in which marketing and R&D participate in a new product development process. It is shown that market orientation need not always be best. If marketing has better cross-functional skills than R&D the firm should be market driven, i.e. marketing should initiate the development process. However, if marketing’s cross-functional skills are relatively poor the firm should be research driven, i.e. R&D should initiate the process. Concurrent development is optimal if (1) the function with lower cross-functional skills also has poor internal skills or (2) neither function has clearly superior cross-functional or internal skills. The paper also models the effect of consumer preferences. If consumers are sensitive to marketing’s choices, marketing should participate late in the new product development process. Finally, it is shown that concurrent development is more attractive if late development mistakes are very costly. The paper contributes to the marketing literature by suggesting that it is not always optimal for a firm to be market driven. It shows that the optimality of market orientation depends on the firm’s resources and links a firm’s cross-functional and internal skills to that optimality.

Thesis Supervisor: Birger Wernerfelt
Title: Professor of Management Science
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I have been very fortunate to be able to complete my Ph.D. in the exceptional research environment of the marketing group at MIT. My many discussions with Drazen Prelec have expanded my interests beyond the boundaries of my core research. I am very thankful to John Hauser from whom I learned most of what I know about research and development. His relentless emphasis on getting the "model of the world" right will continue to remind me that "the world" should always be the foundation of my research.

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Dedication

To my mother, who instilled me with inquietudes.
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Essay 1

The Strategic Use of Consumer Search Cost*

* This essay has benefited greatly from many discussions with Meghan Busse, Jeromin Zettelmeyer and especially Birger Wernerfelt. In addition to them, I also thank Eric Anderson, John Hauser, and Nader Tavassoli for detailed comments. I have also benefited from comments and discussions by seminar participants at MIT, Columbia University, Carnegie-Mellon University, Harvard University, Washington University in St. Louis, University of Pennsylvania, Dartmouth College, UCLA, UC Berkeley, Stanford University, USC, New York University, and the University of Rochester.
1 Introduction

Depending on how a firm designs its communications mix it will be easier or harder for consumers to search for product information. In a traditional retail environment, the ease with which consumers can acquire product information depends on the extent to which the product can be evaluated at a retailer (shrink-wrapped package vs. display model), or the amount of technical information provided on the shelf tag or packaging. In mail-order, the ease with which consumers can acquire product information may depend on the availability of an 800 number for product inquiry, the layout of the catalog, or the amount of information, including product comparisons, provided in the catalog. Over new electronic channels, firms can influence the ease of acquiring product information by the number of on-line services in which they offer information and the amount of information they allow consumers to retrieve while on-line (e.g. only technical data or also a picture). For the rest of this paper I use "consumer search cost" to mean the time and effort that consumers incur in acquiring product information that allows them to determine how much they like a product.

It is a common intuition that firms want to make consumer search as easy as possible in order to provide value to the customer or to treat the customer as a partner. However, we frequently observe that firms don't make it as easy as possible for consumers to search. Tweeter (a music equipment specialty store chain) provides only a few stores in the Boston area instead of locating one store in each block of Boston. When I recently shopped for an amplifier, I asked the sales assistant at Lechmere (one of the region's mass merchandisers) for a summary of the technical specifications of various models. The sales assistant said that manufacturers explicitly forbade them to give out spec sheets. An obvious answer for why we observe that firms don't try to minimize consumer search cost is that it is costly for them to do so. Tweeter's inventory and real estate cost would skyrocket if they provided every city block with its own supermarket and manufacturers of music equipment would incur some cost by providing detailed printed information.

The main result of my paper is that the common intuition that firms want to make consumer search as easy as possible need not be true, even if firm costs are not a factor. I will show that there may be additional, strategic incentives to keep consumer search costs high. These reasons are of particular interest in view of the emergence of electronic shopping. Electronic

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1 These results apply to durable search goods, not to frequently purchased products.
2 "Strategic" in the sense that a firm considers the reactions of other firms and of consumers when deciding what to do.
channels such as on-line services and especially the Internet make it very cheap for firms to allow interested consumers to find out more about their products. Several business and trade magazines state:

"[On-line shopping] allows merchandisers to serve individual customers better by getting information on relevant products and services to them exactly when they need them." (American Demographics, September 1994)

"It's a low cost distribution system. You don't need thousands of stores and you don't need thousands of pieces of inventory in each location." (Business Week, July 26th, 1994)

"I am a small businessman, but the Internet gives me access to the world for next to no cost." (Canadian Business, December 1994)

"Call it just-in-time retailing. In essence, it means making retailing more efficient by reducing the length of time, the quantity of inventory and the number of middlemen that stand between a product and a consumer." (Forbes, May 24th, 1994)

"Window browsing down Fifth Avenue on a warm night aside, most people don't go shopping to entertain themselves. They shop for information." "Stores have thrived because they were the only places consumers could go comparison shopping. The flow of information - on prices and an item's information - was imperfect." "The barriers to good information are crashing down." (Forbes, May 24th, 1994)

This paper sets out to answer two questions which are of both fundamental and applied interest. Even if it is costless for a firm to make consumer search cost very low, will it want to do so? How does competition affect a firm's decision to make search cheap for consumers?

I show that there are situations in which firms will not want to facilitate consumer search. I highlight two reasons for this: (1) firms can soften price competition by differentiating themselves on the basis of consumer search cost, and (2) firms can take advantage of consumer optimism. The first reason is a result of the analysis of a duopoly, in which I show that firms can avoid ruinous price competition by using search cost in order to differentiate themselves. There exists an equilibrium in which one firm offers a high price but makes search easy while the other firm makes search costly to consumers but offers a lower price. The second reason can be shown in the simple case of a monopoly (but also holds in a duopoly). I show that higher search costs can yield higher profits if consumers are sufficiently confident that they will like a product, even before they have precise knowledge of the product's value to them. Cheap search would allow some of these consumers to avoid buying a product they would not buy had they searched. These two reasons suggest that the mere fact that electronic channels allow cheap

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3 In relation to the firm's marginal cost of production. See sections 2 and 3 for an explanation.
communication with consumers does not imply that firms will use this ability to lower consumer search costs.

The role of marketing communications includes both informing and persuading consumers (e.g. Rothschild, 1987, p.4; Kotler, 1994, p. 598, 615). Much of a firm’s communications mix is geared towards persuasion and a large stream of marketing research analyzes various aspects of persuasion. In this paper, however, I want to concentrate on the information aspect of the communications mix\(^5\). While some literature portrays consumers as passive recipients of communication, I will look at consumers as active searchers for information, although they incur costs in this process. Like Wernerfelt (1994a, 1996) I deal with efficiency in marketing communication but focus on identifying situations where firms strategically communicate inefficiently.

There is a large literature on search and search costs. In this paper, consumers make conscious decisions about the amount of information for which they search and consequently the level of search cost they incur (Bettman, 1979; Shugan, 1980; Meyer, 1982; Ratchford, 1982; Hagerty and Aaker, 1984; Johnson and Payne, 1985; Hauser and Wernerfelt, 1990; Hauser, Urban, Weinberg, 1993).

Other literature deals with sources of information that consumers use,\(^6\) consumers' search activities,\(^7\) the amount of search undertaken,\(^8\) and the benefits of consumer search.\(^9\) The eco-

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4 This second reason suggests that a firm might have incentives to behave in a way that could be considered "unethical," namely "fooling" consumers into purchasing. However, these incentives are mitigated by repeat purchasing, or return policies. We should nevertheless be aware that there are many circumstances where these incentives come into play: for example, if there is a turnover of consumers in the market or if consumers forget due to long purchase intervals. For a longer discussion please see page 42.

5 Information might become increasingly important on new electronic communication channels such as the Internet. Louis Rosetto, editor-in-chief and publisher of Wired: "A better approach [than an unsolicited sales pitch] is to use sponsorship of content to connect to the community, then prospect discreetly by inviting users into interactive dialogue. This method turns the traditional advertising ratio of 90% persuasion and 10% information on its head, and encourages users ultimately to become clients." (Rosetto, 1994)

6 See, for example, Katona and Muller, 1954; Udell 1966; Bennett and Mandell, 1969; Kiel and Layton, 1981.


9 See Punj and Staelin, 1983; Ratchford and Srinivasan, 1993.
nomics literature addresses search incentives\textsuperscript{10}, the effect of search costs on prices\textsuperscript{11}, and the effects of various sources of uncertainty\textsuperscript{12}. I will add to this literature by considering firms' strategic decisions about the level of search cost that consumers have to incur if they decide to search. Daughety and Reinganum (1991), and DeGraba (1995) also consider firm decisions that impact consumer search cost but focus on product availability.\textsuperscript{13} A paper by Gerstner, Hess, and Chu (1993) presents a result in the context of "demarketing" with the same flavor as my finding that firms might not want to facilitate consumer search even if it is costless to do so.

There are alternative explanations that lead to a result that firms purposefully limit the information they make available to consumers. If there is potential for information overload, for example, firms might want to limit the amount of information to which consumers are exposed (Jacoby, Speller, and Berning (1974), Jacoby, Speller, and Kohn (1974), Staelin and Payne (1976), Jacoby (1984)). Information overload suggests that the amount of information consumers have affects their ease of search; I will abstract from this issue by modeling firms as influencing consumers' search cost directly. Another explanation for why firms might limit the information they make available to consumers is that they can reduce competition by making direct price and feature comparisons more difficult (Shugan (1989)).

The remainder of the paper proceeds as follows. Section 2 goes through a detailed example that gives the intuition behind the model I develop in this paper and the results I obtain. Section 3 introduces and solves a model for a monopoly and one consumer segment. Section 4 extends the monopoly model to two segments of consumers. Section 5 presents and solves a model for a duopoly. Section 6 discusses the results and section 7 concludes the paper.

\textsuperscript{11} Samuelson and Zhang (1992) show that as search costs decrease exogenously, price may increase.
\textsuperscript{12} See Akerlof, 1970; Telser, 1973; Rothschild, 1974; Marvel, 1976; Alcaly, 1976; Dahlby and West, 1986; Hey and McKenna, 1981; Bond, 1982.
\textsuperscript{13} Daughety and Reinganum (1991) show that firms might want to limit availability of products to increase consumer search cost because this raises the price that can be supported in equilibrium.

In a paper explaining excess demand, DeGraba (1995) obtains a result resembling my monopoly result, namely that monopolists prefer selling to uninformed consumers. In his model, however, customers purchase while uninformed because no products remain available if consumers wait until they are informed.
2 Illustrative example

Although it abstracts somewhat from the characteristics of a particular product market, an example can quite simply illustrate the main argument of this paper.

Imagine the market for a durable search good. Consumers do not know the category very well, and thus do not have a clear idea of the benefits they might derive from the product. The only way to find out what the product is really worth to these consumers is to try it out thoroughly. Before trying, consumers only know that the benefits they will derive from the product would make them willing to pay something between $0 and $4000.\textsuperscript{14}

Suppose the products are sold by retailers whom we will think of as captive, such that the level of service they provide to potential customers is controlled by the company whose products they sell. The company can make it very hard for consumers to thoroughly "test drive" a product by providing only a few floor models or by otherwise limiting how much consumers can learn about it. They can also make it very easy by providing many floor models, or otherwise helping consumers obtain information easily.

In this scenario, a firm that produces a product has to make at least two important choices: at what price to sell the product and how easy to make it for consumers to evaluate the product. Recall that the purpose of this paper is to show that there are reasons other than costs for firms not to facilitate consumer search. In order to highlight these reasons I will choose a situation where one would think that firms should make consumer search as easy as possible. Therefore I assume that it does \textit{not} make a significant cost difference to the firm how easily consumers can evaluate its product.

If there are two firms offering products in the product category in question and a consumer really wants to know how much she likes each of them, she will have to spend some time evaluating the two products. Suppose that by learning about the product of one firm the consumer also learns something about the category and thereby about the product of the other firm. This says that if the consumer realizes that one product would be worth $3000 to her, there is a high chance (let's say 80%) that the other product will also be worth $3000 to her. With 20% probability she will find that the other product is either more or less valuable to her.

\textsuperscript{14} More precisely: the willingness to pay is uniformly distributed between $0 and $4000. The assumption about uniformly distributed "tastes" is purely for convenience. This example could also be made with, for example, a normal distribution. The actual numerical values in the later part of this example, however, would change.
Finally, suppose that it costs a firm $1200 to produce one additional product, i.e. the firm will never sell at a price below this number.

Within this example, let me present the two reasons why firms may not want to facilitate consumer search, even if it is costless to them.

(1) Firms can soften price competition by differentiating on consumer search cost

Let us look at a competitive situation where two firms offer products and assume first that firms do not change the ease with which consumers can evaluate their respective products as frequently as they change their prices.¹⁵

For this example one can show that in equilibrium one firm ("firm 1") will charge a price of $1510 and set evaluation cost¹⁶ close or equal to $0 and the other firm ("firm 2") will charge a lower price of about $1420 but set much higher evaluation cost of $167. In this equilibrium both firms make strictly positive profits. For example, firm 1 could sell its product over mail order with additional showrooms in which consumers can easily evaluate firm 1's product, whereas firm 2 is a pure mail order business, thus making it harder for consumers to search.

By setting search cost at different levels, firms can soften price competition. In our equilibrium all consumers will evaluate the product at firm 1 because search at firm 1 is virtually costless. Say a consumer were standing in the showroom of firm 1. She has the option to buy at firm 1 for $1510 under full knowledge of how much she likes the product, to buy at firm 2 (say, mail order only) for the lower price $1420 but with some uncertainty about how much she will like the product, or to incur search cost of $167 to find out exactly how much she likes the product at firm 2. Her decision will obviously depend on how much she likes the product in the show room of firm 1.

If she likes the product a lot, she will be unwilling to take the risk that the product at the mail order company will not be as good for her and hence will buy at firm 1. If she finds the product in the show room reasonably good but not great, she does not really want to pay the price premium since there is a good chance that the show room product will not be better than the mail order product. Hence she buys from firm 2 at a lower price. If she does not like the product at all, she definitely does not want to buy the product in the show room and is also unsure whether she likes any product in the category. Hence, before deciding whether to buy at all, she wants to make sure that she will like the product and therefore searches for the prod-

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¹⁵ This means that firm are committed to a level of consumer search cost when competing on price.

¹⁶ I will use "evaluation cost" and "search cost" interchangeably. "Evaluation cost of $x" means that consumers have to exert the equivalent of $x in time and effort to evaluate the product.
uct of firm 2. Firm 2's search cost of $167 is low enough to convince those (but only those) consumers who would otherwise not buy at all to evaluate firm 2's product.

The key to firms' ability to make profits in the above scenario is setting different evaluation costs, which makes consumer demand continuous in prices, and thus reduces competitive pressure. Suppose firms had set search cost symmetrically. If both firms were to make it prohibitively costly for consumers to evaluate their products, consumers would not have information that would allow them to prefer one product over the other. Therefore the only criterion on which consumers could base their decision would be price, thus making demand very price sensitive. This would lead to vigorous price competition between the two firms, eating away profits. If both firms make it easy for consumers to evaluate their products, 80% of consumers find out that they are equally satisfied with either product and thus decide solely on price. The existence of such a big segment of consumers that cares only about price has the same effect as if no consumer had evaluated the product, namely vigorous price competition.

If consumer search costs can be adjusted as frequently as price in response to the other firm's action, firms will compete away all profits and charge prices of $1200 (equal to marginal cost) and set evaluation cost to $0. The difference between this case and the previous one is that while competing on price, a firm can now steal away market share from the other firm by making it easy for consumers to find out that its product is equally satisfactory at a lower price. By making search costless, firm 2 could make 80% of consumers realize that they are equally happy with either product. Since firm 2 is charging the lower price, these consumers would then buy at firm 2 since for them price is the only difference between the products. The existence of such a big segment of consumers that cares only about price leads to vigorous price competition.

(2) Firms can take advantage of consumer optimism

This argument can be made more easily than the previous one since it also occurs in a monopolistic setting. Suppose that only one firm is offering a product and that it faces no competition. One can show that in equilibrium the firm will ask for a price of $2000 and set evaluation cost to at least $500. This is saying that the firm will choose to set strictly positive search cost, even if the provision of information to consumers is costless to the firm.

17 Recall that with a probability of 80% the willingness to pay for one product is identical to the willingness to pay for the other product. This example could also be made with many other (but not all possible) percentages.

18 This is the unique pure strategy equilibrium for the parameter values chosen in this example (see page 31).
What is the intuition behind this equilibrium? Recall that without searching consumers only know that they are willing to pay something between $0 and $4000. The expected value of the product to a consumer is thus $2000 (assuming a consumer that is risk neutral and whose willingness to pay stems from a uniform distribution). Hence, without evaluating the product, a consumer will be willing to pay up to $2000 for it. If consumers were to search, a portion of them would find out that the product is worth less to them than it costs and they would pass on the purchase. Hence, as long as the price is below $2000 the firm wants to prevent consumers from searching. To accomplish this goal the firm needs to set evaluation cost of at least $500. At a price of $2000 every consumer buys and the firm makes sizeable profits of $800 from each consumer. If the firm wanted to set a higher price it would have to permit search since consumers would no longer be willing to buy based merely on expected value. It would thereby lose at least half of its customers, a loss in demand for which higher per customer profits would not compensate.\textsuperscript{19}

This strategy is optimal for the firm as long as revenue per consumer is high enough. For example, if the cost producing an additional product were increased to $1800, this strategy would no longer be optimal – the firm would instead choose to permit all consumers to search. By setting the cost of evaluation close or equal to zero and charging a price higher than $2000 the firm sells to fewer, but informed, consumers and obtains greater profits per consumer.

In the above example a firm decides to permit (by setting evaluation cost very low) or impede (by setting evaluation cost very high) consumer search depending on the firm’s production cost. However, even at low marginal cost of $1200 there are situations in which the firm would like to set evaluation cost low enough to permit at least some consumers to evaluate the product.

Call the consumers we have looked at so far “normal” consumers. Imagine that these constitute 60% of the firm’s customer base. The other 40% are high-end consumers who know that they will derive substantial benefit from the product. Suppose high-end consumers would be willing to spend somewhere between $2000 and $4000 for the product. Like normal consumers, however, they cannot narrow down this range before having tested the product for several hours.

I find that in equilibrium the firm will ask for a price of $2828 and will adjust evaluation cost to the equivalent of $172 in time and effort for all consumers. Unlike the one segment

\textsuperscript{19} This suggests that a firm might have incentives to behave in a way that might be considered “unethically” by “fooling” consumers into purchasing. For a discussion please see page 42.
model in which search cost could be set at any level that prohibits search, the addition of a second segment of consumers leads to an evaluation cost that is bounded from below and above.

The intuition for this result is as follows. Recall that a normal consumer will be willing to pay up to $2000 if she can’t evaluate the product. Analogously, for high-end consumers this ex ante willingness to pay is $3000. The firm could continue setting its price at $2000. At that price, however, the firm would price very low in comparison to high-end consumers’ willingness to pay. Since a significant portion of consumers are high-end consumers the firm will want to price closer to $3000; however, at a higher price normal consumers will abandon their intention to purchase. By setting search cost low, the firm can convince the normal consumers to evaluate the machine rather than leaving the market altogether. Although low enough to allow normal consumers to search, the search cost should still prevent high-end consumers from searching since they are willing to buy without evaluation at any price below $3000. The price $2828 is the highest price at which we can find a level of search cost (namely $172) at which high-end consumers are still willing to purchase without evaluating,\(^{20}\) while normal consumers are still willing to evaluate rather than dropping out of the market.

This concludes the example and highlights the main results of the paper.

First, firms in competition — as long as they can commit to maintain search cost unchanged in response to a competitor’s price — can make use of search cost in order to differentiate themselves and thus ease price competition. This can lead to a situation where one firm offers a high price but makes search easy while the other firm prevents most consumers from searching but offers a lower price.

Second, this last result does not hold if firms cannot commit to maintain search cost constant while they compete in prices. In such a case, firms simply use consumer search cost as one more variable with which they can compete.

Third, in certain situations a firm does not choose to facilitate consumer search, even if providing information to consumers is completely costless. The firm thereby induces consumers to buy based on expected value rather than to search.

Fourth, by adding an additional consumer segment, search costs no longer always take on extreme values but become bounded. This explains the existence of some search in equilibrium even with strictly positive search cost.

\(^{20}\) The expected utility of searching net of search cost is $344. To make high-end consumers indifferent between buying immediately (expected value $3000-$2828=$172) and searching the firm sets search cost at $344-$172=$172.
In the succeeding sections, I derive these results formally from a more general model. I begin with the simplest case (monopoly), then add a second segment, and finally model a duopoly. While this progression aids logical explication, it means that the results will be derived in an order slightly different from the presentation above.
3 Monopoly with one consumer segment

3.1 Model

Assume consumers are interested in purchasing one unit of a product in a particular product category. Consumers know the price $p$ of the product offered by the monopolist. Consumers do not know ex ante the gross utility $u$ they will derive from the product. They are, however, aware of the distribution from which their reservation prices are drawn. Consumers' willingness to pay is distributed uniformly between 0 and 1 ($u \sim U[0, 1]$). A consumer can find out her reservation price $u$ by evaluating the firm's product. In doing so she incurs search cost $s$.

Consumers facing a purchase decision can either buy, search or decide not to buy at all. If they have searched and learned their reservation prices they then have the option to buy or to pass, i.e. searching allows consumers to ensure that they purchase only when their willingness to pay exceeds price.

![Figure 1: Consumer decisions](image)

In stage 2 consumers know their reservation prices, hence their payoffs are given by:

<table>
<thead>
<tr>
<th>Action</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy</td>
<td>$-s + u - p$</td>
</tr>
<tr>
<td>pass</td>
<td>$-s$</td>
</tr>
</tbody>
</table>

\[21\] I will use "reservation price" or "willingness to pay" synonymously with $u$. 

- 23 -
Since reservation prices $u$ are distributed uniformly between 0 and 1, the density function of $u$ is $f(u) = 1$. In stage 1, i.e. before search, the expected utility for each of a consumer's choices are as follows:

<table>
<thead>
<tr>
<th>Action</th>
<th>Expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy</td>
<td>$\int_0^1 u , du - p$</td>
</tr>
<tr>
<td>search</td>
<td>$- s + \int_p^1 (u - p) , du$</td>
</tr>
<tr>
<td>pass:</td>
<td>0</td>
</tr>
</tbody>
</table>

With these utilities I can describe consumers' best response functions to any price-search cost pair $(p, s)$. Notice that every consumer has the same expected utility. Hence, given $(p, s)$, if any consumer prefers a given action, all consumers will prefer that action.

The monopolist has two choice variables, price $p$ and search cost $s$. The monopolist maximization problem is given by:

$$\max_{p,s} \ (p - c) D(p, s)$$

(1)

where $c$ is marginal cost and $D(p, s)$ is the fraction of consumers who buy. Given consumer behavior, this fraction is as follows:

$$D(p, s) = \begin{cases} 
1 & \text{if consumers buy} \\
1 - p & \text{if consumers search} \\
0 & \text{if consumers pass}
\end{cases}$$

(2)

3.2 Solution

From table 2 we can derive the conditions on $p$ and $s$ for which buy, search, or pass is consumers' best response: it is optimal to buy rather than to search\footnote{Note that in this monopoly section "search" as a consumer strategy means: search and buy if $u \geq p$, otherwise pass.} iff $s \geq p^2/2$, to buy rather than
to pass iff \( p \leq 1/2 \) and to search rather than to pass iff \( s \leq (1 - p)^2 / 2 \). By capping these thresholds in the firm's strategy space \((p, s)\) we obtain the best response regions shown in figure 2:

![Figure 2: Firm strategy space and consumer best response](image)

In figure 2 the regions of consumers' best responses for \((p, s)\) are separated by dashed lines. For example, at a price of 0.2 and search cost of 0.05 all consumers will want to buy immediately without searching. Although the value of information that could be obtained by searching is lower than the search cost, the price is so low that consumers' priors regarding their reservation prices significantly exceed the price the monopolist charges. Maintaining the same level of search cost but increasing price to 0.6 makes it worthwhile for consumers to take the more cautious route of searching before buying. A substantial increase in search cost from \((p, s) = (0.6, 0.05)\) makes consumers drop out of the market entirely, since their expected reservation price of 1/2 is lower than \( p \).

In order to determine the equilibria of this game let us narrow down the price-search cost equilibrium candidates. Consider a point in region I. Clearly any \((p, s)\) in region I in which \( p = p_a \) will be weakly more profitable for the firm than any other point in region I since demand remains unchanged up to \( p_a \). In region II any \((p, s)\) with \( p = p_a \) again dominates since for \( p > p_a \) no consumer remains in the market. In region III consumers search, meaning
that a higher price will lower the quantity demanded. Since firms can't raise the price to the upper boundary with impunity, interior points are also candidates for equilibria.

I can now specify the game's equilibria:

Proposition 1: Let $F$ be the cumulative distribution of consumers on $[0, 1]$. Then the following characterizes the game's pure strategy equilibria:

For $c$ such that $F(c) \leq \sqrt{2} - 1$

\[ p^* = \frac{1}{2}, s^* > \frac{1}{8}, \text{all consumers buy in stage 1} \]

For $c$ such that $F(c) > \sqrt{2} - 1$

\[ p^* = \frac{(1 + c)}{2}, s^* < \frac{(1 - c)^2}{8}, \text{all consumers search in stage 1} \]

Proof of proposition 1: see appendix

Suppose the firm has low marginal cost $c$. Then there are many consumers whose reservation prices are larger than marginal cost, i.e. $1 - F(c)$ is large (or $F(c)$ is small). This allows the firm to set price well above marginal cost and still price below consumers' expected reservation price. In equilibrium, the firm will choose to price at the highest price that consumers are still willing to pay based on their expected reservation price and will set search cost high enough to prevent search (e.g. $(p_d, s_d)$ in figure 2). With marginal cost below most consumers' reservation prices, profits reaped from each customer can be large and it will not pay for the firm to charge a higher price but lose a large portion of demand by allowing consumers to search. If fewer consumers have reservation prices that are above marginal cost, the firm can't charge a price that yields significant per unit profits without exceeding consumers' expected reservation price. The firm thus chooses to permit search (e.g. $(p_b, s_b)$ in figure 2).

I can now state the main result of this section:

Corollary 1: There exists a region of the parameter space for which a monopolist will choose to set strictly positive search cost, even if the provision of information to consumers is costless to the monopolist.

Corollary 1 makes use of the fact that in the profit function of the monopolist (equation 1) there are no costs associated with lowering consumer search cost $s$. 

- 26 -
4 Monopoly with two consumer segments

In the equilibrium characterized in proposition 1 search costs for low \( R(c) \) are not bounded from above. If the firm wants to inhibit search it will set search cost to any level above \( 1/8 \) (e.g. \( \infty \)). For high \( R(c) \) the firm will set search cost to any level below \( (1 - c)^2/2 \) (e.g. 0). These extreme values result from consumers who (before searching) behave as if they were homogeneous. By introducing a second segment of consumers whose reservation prices overlap with those of the first segment, we get equilibria in which search costs are bounded from above and in which there is always at least some search.

4.1 Model

Consider the addition of a second segment of consumers, who differ from the first segment in the distribution of their willingness to pay. Call the added segment the “high” (h) segment and the existing segment the “normal” (n) segment. The high segment’s willingness to pay is distributed uniformly between \( 1/2 \) and \( 1 \) (\( h \sim U[1/2, 1] \)). The normal segment’s willingness to pay is identical to that of consumers in the first case (\( n \sim U[0, 1] \)). A proportion \( \gamma \) of consumers are in the high segment, and \( (1 - \gamma) \) in the normal segment.

In stage 2 the high segment’s payoffs are the same as the normal segment’s (given in table 1 on page 23). In stage 1 the high segment’s expected utilities are as follows:

<table>
<thead>
<tr>
<th>Action</th>
<th>Expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy</td>
<td>( \int_0^1 2u du - \rho )</td>
</tr>
<tr>
<td>search</td>
<td>( -s + \int_\rho^1 2(u - \rho) du )</td>
</tr>
<tr>
<td>pass</td>
<td>0</td>
</tr>
</tbody>
</table>

Within each segment every consumer has the same expected utility. Therefore, if any consumer in the high segment prefers to search, all \( \gamma \) consumers will want to search; similarly for the normal segment.
The monopolist's maximization problem is given by:

$$\max_{p, s} (p - c)(\gamma D^h(p, s) + (1 - \gamma) D^n(p, s))$$  \hspace{1cm} (3)

where \(c\) is again marginal cost, \(D^h(p, s)\) and \(D^n(p, s)\) are the fractions of consumers within the high and the normal segments respectively who buy, and \(\gamma\) and \((1 - \gamma)\) are the sizes of each segment in the two segment case. As a result of consumer behavior, this fraction for segment \(k\) is as follows (\(k = n, h\)):

$$L^k(p, s) = \begin{cases} 
1 & \text{if consumers in segment } k \text{ buy} \\
1 - p & \text{if consumers in segment } k \text{ search} \\
0 & \text{if consumers in segment } k \text{ pass}
\end{cases}$$  \hspace{1cm} (4)

Equation (2) does not say that \(D^h(p, s) = D^n(p, s)\); it is well possible, for example, that the high segment searches while the normal segment passes.

4.2 Solution

From table 2 on page 24 and table 3 on page 27 we can derive, as we did for the one segment case, the conditions on \(p\) and \(s\) for which buy, search, or pass is the best response for each consumer segment. High segment consumers prefer to buy rather than to search iff \(s \geq (p - 1/2)^2\), to buy rather than to pass iff \(p \leq 3/4\), and to search rather than to pass iff \(s \leq (1 - p)^2\). Normal segment consumers' optimal response is stated in the previous section. In figure 3 I add the best response of the high segment to that of the normal segment (shown in figure 2):
The regions of consumers' best responses for \((p, s)\) are separated by dashed (normal segment) and solid (high segment) lines. The intuition of figure 3 is the same as for figure 2. For example, at a price of 0.2 and search cost of 0.05 all consumers in both segments will want to buy immediately without searching. Maintaining the same level of search cost but increasing price to 0.6 makes it worthwhile for normal segment consumers to take the more cautious route of searching before buying, while high segment consumers still have so much expected surplus from the purchase that they will buy without prior search.

As in the one segment case any \((p, s)\) in region 1 (region I in figure 2) in which \(p = p_a\) will be weakly more profitable for the firm than any other point in region 1 since demand remains unchanged up to \(p_a\). However, the existence of the high segment now adds another equilibrium candidate: in region 2 the firm will prefer any \((p, s)\) in which \(p = p_c\) since demand remains unchanged up to \(p_c\). In region 3 any \((p, s)\) with \(p = p_c\) again dominates since for \(p > p_c\) no consumer remains in the market. In regions 4, 5, or 6 (similar to region III in figure 2) at least one type of consumer searches, meaning that for at least one segment of consumers a higher price will lower the quantity demanded. Since firms can't raise the price to the upper boundary without losing demand, interior points could also be equilibria.
Define the following regions of the parameter space:

Region \( A = \left\{ (c, \gamma) \left| c < \frac{2 - \gamma - \sqrt{2}}{\sqrt{2}(1 - \gamma)}, c \geq 0, \gamma \geq 0 \right. \right\} \) (5)

Region \( B = \left\{ (c, \gamma) \left| c \geq \frac{2 - \gamma - \sqrt{2}}{\sqrt{2}(1 - \gamma)}, c \leq \frac{2 - 2\sqrt{2} + \gamma}{2(\sqrt{2}(1 - \gamma) + 2\gamma)}, \right. \right\} \) (6)

\[ c \leq \frac{\sqrt{2}\gamma^2(2 - \sqrt{2}) + 2\gamma(3\sqrt{2} - 4) - (1 - \sqrt{2})(1 - \gamma)}{1 + \gamma}, \quad c \geq 0 \]

Region \( C = \left\{ (c, \gamma) \left| c \leq 1 + \frac{\sqrt{3}\gamma^2 - \gamma}{1 - \gamma}, c > \frac{2 - 2\sqrt{2} + \gamma}{2(\sqrt{2}(1 - \gamma) + 2\gamma)}, \gamma \leq 1 \right. \right\} \) (7)

Region \( D = \left\{ (c, \gamma) \left| c > \frac{\sqrt{2}\gamma^2(2 - \sqrt{2}) + 2\gamma(3\sqrt{2} - 4) - (1 - \sqrt{2})(1 - \gamma)}{1 + \gamma}, \right. \right\} \) (8)

\[ c > 1 + \frac{\sqrt{3}\gamma^2 - \gamma}{1 - \gamma}, \quad 0 \leq \gamma \leq 1 \]

I can now characterize the equilibria, which are also presented graphically in figure 4:

Proposition 2: The following characterizes the game’s pure strategy equilibria:

For \( (c, \gamma) \in A : p = \frac{1}{2}, s > \frac{1}{8}, \) high and normal segment: buy

For \( (c, \gamma) \in B : p = \frac{1}{\sqrt{2}}, s = \frac{3}{4} - \frac{1}{\sqrt{2}}, \) high segment: buy, normal segment: search

For \( (c, \gamma) \in C : p = \frac{3}{4}, s > \frac{1}{16}, \) high segment: buy, normal segment: pass

For \( (c, \gamma) \in D : p = \frac{(1 + c)}{2}, s < \frac{(1 - c)^2}{2}, \) high and normal segment: search

Proof of proposition 2: see appendix.

Note that \( (c, \gamma) \) describe the entire parameter space. \( A \) through \( D \) are illustrated in figure 4.
Proposition 2 reduces to proposition 1 for $\gamma = 0$, i.e. if all consumers are in the normal segment.

The intuition behind these equilibria is as follows: At low marginal cost $c$ (i.e. low $R(c)$) and a low proportion $\gamma$ of high valuation consumers (region $A$) we get the same result as for low $R(c)$ in the one segment case. The firm will choose to price at the highest price that the normal segment is still willing to pay based on its expected reservation price and will set search cost high enough to prevent search (e.g. $(p_a, s_a)$ in figure 3). At a medium level of marginal cost and similar segment sizes (region $B$) the firm will want to price at $p^* = 1/\sqrt{2}$ and $s^* = 3/4 - 1/\sqrt{2}$. In figure 3 this corresponds to $(p_b, s_b)$. Search costs are set to support the highest price at which high segment consumers are still willing to buy without searching while normal segment consumers are still willing to search rather than dropping out of the market. This reflects the fact that the firm cares sufficiently about the normal segment that it does not want to lose it (hence it permits it to search) while the high segment is large enough that the firm does not want to forfeit profits by charging only $p = 1/2$. As the high segment becomes very large (region $C$) the firm is willing to forfeit its profits from the normal segment in exchange for getting all the demand from the high segment at the highest price the high segment is willing to pay based on its prior reservation price (e.g. $(p_c, s_c)$ in figure 3). As marginal cost
becomes very high (region $D$) the firm has to charge a high price in order to maintain positive profits (e.g. a point in region 5 above $p_1$ in figure 3) and consequently has to permit search in order to get any demand. This is analogous to the one segment result for high $R(c)$.

In region $B$ we get the result I mentioned at the beginning of this section:

**Corollary 2:** There exists a region of the parameter space (region $B$) for which a monopolist will strictly prefer to set finite, strictly positive search cost and for which some consumers will search, even if the provision of information to consumers is costless to the monopolist.

## 5 Competition

### 5.1 Model

To illustrate the effects of competition I introduce a second firm. The two firms’ products are substitutes.

There is one segment of consumers\(^{23}\) with reservation prices distributed uniformly between 0 and 1. Consumers’ willingness to pay for firm 1’s and firm 2’s product are $u_1$ and $u_2$ respectively. These are identically but not necessarily independently distributed. For simplicity I assume the following correlation structure between reservation prices $u_1$ and $u_2$: if a consumer searches at firm $i$ and learns about her reservation price $u_i$ then with probability $q$ her reservation price at firm $j$ is identical to the reservation price she just learned, i.e. $Pr(u_j = u_i) = q$. With probability $1 - q$ the reservation price at firm $j$ is different\(^{24}\) in which case her best estimate about her reservation price at firm $j$ is her prior.

The consumers’ possible actions are illustrated in figure 5:

---

\(^{23}\) Adding a second segment does not add to the results in duopoly.

\(^{24}\) By continuity of the distribution of reservation prices "different" means the same as "independent of the reservation price of firm $i$."
A consumer has the option to buy from either firm without any search, to search either firm or to pass altogether. If she has decided to search at firm $i$ she knows her reservation price $u_i$ at that firm as well as her reservation price $u_j$ at firm $j$ with probability $q$. With this knowledge she can decide to buy at either firm, to pass or to search at firm $j$. If the consumer searches at firm $j$ she has full information, i.e. she knows $u_i$ and $u_j$ and decides to buy from one of the firms or to pass.

To determine consumers' optimal responses to both firms' price-search cost offerings $(p_1, s_1)$ and $(p_2, s_2)$ let us consider a consumer's expected utility at each stage. In stage 3 a consumer knows both reservation prices, hence her overall payoff for each action is given by:

**Table 4: Consumer utilities in stage 3**

<table>
<thead>
<tr>
<th>Action</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy at firm 1</td>
<td>$-s_1 - s_2 + u_1 - p_1$</td>
</tr>
<tr>
<td>buy at firm 2</td>
<td>$-s_1 - s_2 + u_2 - p_2$</td>
</tr>
<tr>
<td>pass</td>
<td>$-s_1 - s_2$</td>
</tr>
</tbody>
</table>
In stage 2 the consumer has only searched one firm, say firm 1\textsuperscript{25}. Her expected utilities then are:

<table>
<thead>
<tr>
<th>Action</th>
<th>Expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy at firm 1:</td>
<td>$-s_1 + u_1 - p_1$</td>
</tr>
<tr>
<td>buy at firm 2:</td>
<td>$-s_1 + q u_1 + (1 - q) \int_0^1 u_2 du_2 - p_2$</td>
</tr>
<tr>
<td>search at firm 2</td>
<td>$-s_1 - s_2 + q \cdot \max{0, u_1 - p_1, u_1 - p_2} + (1 - q) \cdot \left( \int_0^{(u_1 - p_1 + p_2)} \max{0, u_1 - p_1} du_2 + \int_{(u_1 - p_1 + p_2)}^1 \max{0, u_2 - p_2} du_2 \right)$</td>
</tr>
<tr>
<td>pass:</td>
<td>$-s_1$</td>
</tr>
</tbody>
</table>

Since at stage 2 the consumer has searched firm 1, she is certain about her payoff if she buys from firm 1. She also knows that $Pr(u_1 = u_2) = q$ and $Pr(u_1 \neq u_2) = 1 - q$ so that she forms her expected value from purchasing at firm 2 as the weighted average from her known reservation price at firm 1 and her prior expected reservation price at firm 2. If the consumer decides to do a stage 2 search for her reservation price at firm 2 she has to pay $s_2$ in addition to the search cost $s_1$ already incurred. With probability $q$ (when $u_1 = u_2$) the benefit of her stage 3 decision is the maximum of passing, buying at firm 1, and buying at firm 2, where the payoffs of each decision are already known. With probability $(1 - q)$ (when $u_1 \neq u_2$) her expected utility is the expected value of each choice for those values of $u_2$ for which the respective choice is optimal.

\textsuperscript{25} Throughout this paper and without loss of generality I will assume that if there is search at all, firm 1 will be the first firm to be searched.
In stage 1 the consumer only knows the distribution of her reservation prices but not their realizations. Her expected payoffs at this stage are:

**Table 6: Expected consumer utilities in stage 1**

<table>
<thead>
<tr>
<th>Action</th>
<th>Expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy at firm i:</td>
<td>$\int_0^1 u_i d\mu_i - p_i$</td>
</tr>
<tr>
<td>search at firm i</td>
<td>$-s_i + E_u \left[ \max \left{ 0, u_i - p_i, qu_i + (1-q) \int_0^1 u_j d\mu_j - p_j \right} \right]$</td>
</tr>
<tr>
<td></td>
<td>$-s_j + q \cdot \max \left{ 0, u_i - p_i, u_i - p_j \right} + (1-q) \cdot \left( \int_0^{(u_i - p_i + p_j)} \max \left{ 0, u_i - p_i \right} d\mu_j + \int_{(u_i - p_i + p_j)}^1 \max \left{ 0, u_j - p_j \right} d\mu_j \right} \right]$</td>
</tr>
<tr>
<td>pass:</td>
<td>0</td>
</tr>
</tbody>
</table>

If a consumer decides to buy without any search, her expected utility from the purchase is just her expected reservation price (i.e. $1/2$) minus the purchase price. If she decides not to purchase at all her utility will be 0.

The interesting case occurs if the consumer decides to search at one of the firms. She then incurs search cost $s_i$ and gains the option in stage 2 to choose between passing, buying at firm $i$ under full knowledge of her reservation price, buying at firm $j$ under partial knowledge of her reservation price $u_j$, or continuing to search at firm $j$.

The firms have two choice variables, price and search cost. I will look at two versions of the game. In one game I assume that firms set search cost and prices sequentially and know each other's search cost at the time they compete in prices. The game is called "with commitment" since I assume that in the (second step$^{26}$) price game firms remain committed to the level of search cost that they chose in the first step. In the game without commitment I assume that firms set search cost and prices simultaneously (or if they set them sequentially are not able to commit to first step search cost).

---

$^{26}$ In this paper "stage" refers to consumers' decision tree and "step" to the overall game.
In the game with commitment firms’ second and first step maximization problems are:

\[ \text{2nd step: } \max_{p_i} (p_i - c) D_i(p_i, s_i, p_j, s_j) \]  
\[ \text{1st step: } \max_{s_i} (p_i(s_i, s_j) - c) D_i(p_i(s_i, s_j), p_j(s_i, s_j)) \]  

In the game without commitment the firms’ maximization problem becomes:

\[ \max_{p_i, s_i} (p_i - c) D_i(p_i, s_i, p_j, s_j) \]

As in the monopoly case \( c \) stands for marginal cost and \( D_i(*) \) is demand for firm \( i \)’s product. For different prices and search costs there are many different forms that firms’ demands can take; I will therefore refrain from specifying fully contingent demands at this point but refer the reader to the appendix.

5.2 Solution

In this section I will first comment on a crucial aspect of consumer demand and then proceed to presenting the results of competition with and without commitment.

5.2.1 Characteristics of consumer demand

From the expected utilities in table 4 on page 33 through table 6 on page 35 we can derive a best response of consumers at each stage of the game and hence derive consumer demand for each pair of price and search cost.

A familiar result from the industrial organization literature is that if firms compete on the basis of price alone, demand will be discrete; i.e., all consumers will buy from the firm with the lowest price. This is undesirable because it leads to a result of marginal cost pricing and zero profits (Bertrand equilibrium). In much of the literature, this result is avoided by distributing consumers along a Hotelling line and positioning firms away from each other geographically or in product space. This differentiation makes the goods no longer perfect substitutes, and demand becomes continuous in price.
The model in this paper has a similar outcome, but it is a result of the information structure. If there were no search, demand would be discrete in the first stage of the game, in which consumers' willingness to pay at both firms are unknown and the only information available is price. However, as consumers realize the outcomes of search, they formulate their utilities as conditional expected values which are continuous in price, which implies that demand is continuous in price.

Suppose for a moment that consumers had to decide between purchasing at firms 1 and 2 in stage 1. Their expected utilities from purchases at firm 1 and firm 2 are \( 1/2 - p_1 \) and \( 1/2 - p_2 \) respectively. Irrespective of their true reservation price (because it is unknown to them at this stage) consumers will choose to purchase from the firm with the lower price, meaning that the firm with the lower price will get the entire demand. This is the classic Bertrand result.

Now suppose that consumers have searched for firm 1's product and are consequently at stage 2 (see table 5). Consumers now know their reservation price at firm 1 and know that their reservation price at firm 2 corresponds to that at firm 1 with probability \( q \). A consumer's utility from buying at firm 1 thus is \( u_1 - p_1 \) and the expected utility from buying at firm 2 is \( q u_1 + (1 - q) 1/2 - p_2 \). This means that buying at firm 1 is preferred to buying at firm 2 for \( u_1 - p_1 > q u_1 + (1 - q) 1/2 - p_2 \), or after solving for \( u_1 \), \( u_1 > 1/2 - (p_1 - p_2)/(1 - q) \). Since the proportion of realizations of \( u_1 \) that fall above \( 1/2 - (p_1 - p_2)/(1 - q) \) will change continuously with \( p_1 \) and \( p_2 \), demand is now a continuous function of prices.

Suppose finally that consumers decide to continue searching. At stage 3 (see table 4) they know both reservation prices, \( u_1 \) and \( u_2 \). A proportion \( q \) of consumers find that \( u_1 = u_2 \), the rest will find that \( u_1 \neq u_2 \). Those consumers with identical reservation prices across firms will buy from the firm with the lower price. Consumers with different reservation prices will buy at firm 1 as long as \( u_1 - p_1 \geq u_2 - p_2 \Leftrightarrow u_1 \geq u_2 - p_2 + p_1 \) (i.e. consumers get higher surplus from firm 1). Overall, the firms' demand will be continuous where \( p_i > p_j \) (because the demand for \( 1 - q \) of the consumers is continuous in price), with a discrete jump at \( p_1 = p_2 \) (because the \( q \) consumers with equal reservation prices switch firms at this price).

5.2.2 Competition with commitment

Let us now look at the two step game in which firms face the demands I sketched out above. Firms first set search cost and then compete in prices. I solve for a subgame perfect equilibrium in the standard way: I first calculate the 2nd step price equilibrium \( p_1^*(s_1, s_2), p_2^*(s_1, s_2) \) as a function of search cost and then calculate equilibrium search cost \( (s_1^*, s_2^*) \).
Define the following prices:

\[ p_1^* = (-2q^3 + (4c + 10)q^2 - (13 + 8c)q + 2 + 4c + [4 - 8c + 16c^2] \]
\[ (8 + 104c - 64c^2)q + (5 - 224c + 96c^2)q^2 + (-16 + 176c - 64c^2)q^3 \]
\[ (20 - 56c + 16c^2)q^4 + (-16 + 8c)q^5 + 4q^6 \]^{1/2}/(12(1 - q)^2) \] (12)

\[ p_2^* = (-8q^3 + (16c + 28)q^2 - (31 + 32c)q + 8 + 16c + [4 - 8c + 16c^2] \]
\[ (8 + 104c - 64c^2)q + (5 - 224c + 96c^2)q^2 + (-16 + 176c - 64c^2)q^3 \]
\[ (20 - 56c + 16c^2)q^4 + (-16 + 8c)q^5 + 4q^6 \]^{1/2}/(24(1 - q)^2) \] (13)

I can now state the main result of this section:

**Proposition 3:** There exist parameters \( c, q \) for which the following characterizes a subgame perfect equilibrium:

**Firms:** firm 1 charges a higher price than firm 2, \( p_1^* > p_2^* \), but sets lower search cost, \( s_1^* = 0 < \frac{(1-q)}{2}(1-p_2^*)^2 = s_2^* \). Both firms make positive profits \( \Pi_1, \Pi_2 > 0 \).

**Consumers:** All search firm 1 at stage 1. At stage 2 there is positive demand for firm 1 and firm 2. No consumers pass at stage 2 but all who did not buy from firm 1 or firm 2 continue searching at firm 2. At stage 3 there is positive demand for firm 2 and no demand for firm 1.

**Proof of proposition 3:** see appendix.

The significance of this proposition is that firms can use search cost strategically in order to soften price competition; this result depends fundamentally on firms' ability to commit to different levels of search cost before they compete in prices. In equilibrium one firm will make search very easy but charge a higher price and the other firm will make search costly but offer a lower price.

The intuition behind this proposition is as follows. In equilibrium firm 1 makes search costless. As a consequence all consumers will choose to begin by searching firm 1.\(^{27}\) Consumers with high \( u_1 \) will choose to buy at firm 1 and those with lower reservation prices will consider

\(^{27}\) Actually there is a whole interval of search cost \( s_1 \) that would sustain the equilibrium. I only require that \( s_1 \) is low enough that all consumers search firm 1 in stage 1.
buying at firm 2. For consumers with the lowest reservation prices, it will be better to pass altogether than to buy at either firm; however, the search cost that firm 2 sets is chosen so that those consumers who prefer passing to buying at either firm are indifferent between passing and searching at firm 2. For these consumers the expected value of search is 0, and thus they are just willing to continue to search at firm 2 instead of passing. This way all consumers who might have left the market in stage 2 now search for their reservation prices at firm 2; for some of them it will then be beneficial to purchase at firm 2. Since the expected value of searching at firm 2 is weakly lower than that of buying at firm 1 for \( u_1 \geq p_1 \), any consumer for whom \( u_1 \geq p_1 \) will have purchased already at stage 2; firm 1 sells to no additional consumers in stage 3.

Why don't firms 1 and 2 have an incentive to deviate? Fix firm 1's search cost at \( s_1 = 0 \) and consider first firm 2. If firm 2 sets higher search cost those consumers that now buy from either firm at stage 2 are unaffected. Only those consumers that so far continued searching (and of whom some then bought at firm 2) will now cease searching and pass at stage 2. Clearly, firm 2 cannot gain from such a deviation.

Suppose now that firm 2 lowers search cost. Initially consumers who bought in stage 2 from firm 1 are unaffected (i.e. they prefer buying from firm 1 to searching at firm 2). However, some of the consumers that so far bought at firm 2 in stage 2 are added to the searchers. Allowing these consumers to get informed about their reservation prices can only hurt firm 2 because some will detect that they should not buy (although they would have bought if they had not been given the information). As firm 2 lowers search cost further some consumers that so far bought at firm 1 will also begin to search. In stage 3 some of these consumers will buy from firm 1 but also some from firm 2. Although firm 2 gets some consumers that had been buying at firm 1, this does not make up for the loss in demand from those consumers that formerly bought at firm 2 and now search instead.

At very low search cost \( s_2 \) and under the assumption that firm 2 charges the lower price, there is a large proportion of consumers from both segments who have searched both firms and have found that they have the same reservation prices at both firms. These consumers will buy at whichever firm offers the lower price. This creates incentive for firm 1 to undercut \( p_2 \) by \( \varepsilon \) and capture these consumers. The result is Bertrand price competition with profits \( \Pi_1 = \Pi_2 = 0 \).

Now fix \( s_2 = (1 - q)(1 - p_2)^2 / 2 \) and consider firm 1's incentives to deviate. Demand is discrete in \( s_1 \) (in contrast to \( s_2 \)) since \( s_1 \) influences consumers' first stage decision, which is a decision made before any individuals know their reservation prices. Thus in response to a giv-
en $s_1$, either all or no consumers will search and a change in $s_1$ will either have no effect, or will change the behavior of all consumers. Recall that in equilibrium all consumers search. If $s_1$ is high enough to impede consumers’ search, but is still below firm 2’s search cost, consumers don’t know their reservation prices at either firm because firm 2’s search cost also discourages first stage search. As discussed at the beginning of section 5.2.1 consumers will all choose to purchase from the firm with the lower price. This will result in Bertrand competition and zero profits. Firm 1 thus also has no incentive to deviate. If firm 1 impedes search, but consumers are willing to search at firm 2 in stage 1, it is similar to firms 1 and 2 swapping equilibrium roles. Since firm 2’s profits in equilibrium are lower than firm 1’s profits, firm 1 has no incentive to reverse roles with firm 2.

5.2.3 Competition without commitment

In contrast to competition with commitment, now suppose firms simultaneously choose search cost and prices. This enhances firm $j$’s ability to deviate from any price equilibrium derived under the assumption of a particular $(s_p, s_j)$:

*Proposition 4: The equilibrium proposed in proposition 3 cannot be sustained if firms choose search cost and prices simultaneously or if they are not able to commit to first step search cost. Instead, for the parameters of proposition 3 there will not exist an equilibrium in pure strategies.*

Recall that the price equilibrium in proposition 3 with $p_1^* > p_2^*$ could be sustained because consumers’ partial information in stage 2 led to demand continuous in $(p_1, p_2)$. Firm 1 was not interested in undercutting firm 2’s price by $\varepsilon$ because the potential gain in demand was minimal and firm 2’s choice in price was optimal given that its search cost was $s_2^*$. If however, firm 2 were able to change the information structure of consumers and inform them of their reservation prices, there would suddenly be a large proportion of consumers that knew that their reservation prices at both firms were identical and who then would choose the firm with the lower price, namely firm 2. We would then be at a situation where $s_1 = s_2 = 0$ and firms mix over prices.
6 Discussion

The principal use of consumer search cost as a strategic variable varies by market structure and is summarized in Table 7.

Table 7: The role of search cost

<table>
<thead>
<tr>
<th>Market Structure</th>
<th>Equilibrium ((p, s))</th>
<th>Role of search cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoly</td>
<td>(p^* &gt; c)</td>
<td>Do not permit low types to identify themselves</td>
</tr>
<tr>
<td></td>
<td>(0 &lt; s^*) ((&lt; \infty) for 2 segments)</td>
<td></td>
</tr>
<tr>
<td>Competition with commitment</td>
<td>(p_i^* &gt; p_j^* &gt; c)</td>
<td>Ease price competition.</td>
</tr>
<tr>
<td></td>
<td>(0 = s_i^* &lt; s_j^*) ((&lt; \infty))</td>
<td></td>
</tr>
<tr>
<td>Competition without commitment</td>
<td>(p_i^* = p_j^* = c)</td>
<td>Compete on a second dimension.</td>
</tr>
<tr>
<td></td>
<td>(s_i^* = s_j^* = 0)</td>
<td></td>
</tr>
</tbody>
</table>

In monopoly consumer search cost enhances the monopoly power of the firm by fine tuning the information of each consumer segment. If setting consumer search cost is more sticky than setting prices, i.e. firms can change prices more frequently than they can change consumer search cost, they can make use of search cost to differentiate themselves so that price competition becomes less severe. If firms can change search cost as frequently as prices, we have simply introduced another variable with which they can compete in Bertrand competition.

Note that we obtain positive profits in the equilibrium with commitment without a Hotelling type structure that avoids Bertrand competition through product or geographic differentiation\(^{28}\) (see Hotelling (1929), D’Aspremont et al. (1979), Salop (1979); Moorthy (1988), Economides (1989), Gerstner, Hess, and Chu (1993)). Other than the fact that for some consumers reservation prices are independently distributed, in my model both firms distinguish themselves only by their strategic variables. We can find a similar structure in Kreps and Scheinkman (1983) in which they show that a Cournot outcome results from price competition if firms have quantity constraints. In their model firms commit to a maximum level of produc-

\(^{28}\) Other models achieve price dispersion through inflation and menu costs so that there is a need for (costly) price adjustments (Bénabou (1988)).
tion quantity and thereby prevent one firm from serving the entire market. This leads to prices above marginal cost and positive profits.

It is a standard argument that a firm's ability to sell to consumers by "fooling" them (as happens to some consumers in my models) cannot be sustained if there is repeat purchasing, money back guarantees, or competitive pressure (see Wernerfelt, 1994b). However, there are circumstances under which firms can sustain strategies in which they sell to consumers that would not have bought if they had searched. For example, turnover of consumers in the market will lead to some consumers that are uninformed. Long purchase intervals and consequently forgetful consumers might also lead to uninformed consumers. The endowment effect might prevent consumers from wanting to return a product, even if they would have decided against it had they searched. Consumers might not be motivated to admit a wrong purchase decision. Consumers might have gone through changes in taste. New models of a product might have been introduced. Finally, the information revealed through search might not be the same as would be revealed though product usage. As an example of this last point consider a computer user with modest needs. If she had been sold a Pentium but she only needed a 386 she would be perfectly happy with the Pentium until she found out that the 386 would have done just as well. This information, however, is not revealed by owning a Pentium but only through search. While such practices should not necessarily be endorsed as normative recommendations, it is important to recognize that such incentives will exist for a firm in choosing a communication mix.

The model in this paper assumes that the reduction of consumer search cost is costless to the firm. The purpose of this assumption is to isolate reasons other than cost for why firms might not want to facilitate consumer search. This assumption does not generally hold in the real world. Depending on the channel, it can be very costly to fairly cheap to make it easy for consumers to determine their reservation prices. My assumption, however, is biased against my results, i.e. if my results hold under the assumption of no cost to the firm, they will also hold if it is costly for firms to lower consumer search cost.

The main contribution of this paper is (a) to show that reduced cost of information provision does not necessarily mean that firms will help consumers in their search for information, and (b) to show the ways in which firms can make use of strategically setting consumer search cost, namely to soften price competition and to take advantage of consumer optimism.
My results have a number of normative and descriptive implications for various areas in marketing:

- The information highway may not develop into an information smorgasbord. Applied to emerging electronic channels my results predict that the popular expectation of an information access paradise will not necessarily emerge. Firms might choose to purposefully limit the information available or try to differentiate themselves with different information provision strategies. However, we could also find that it is difficult to commit to a given search cost on electronic channels and that firms enter intense price competition because they can adjust consumer search cost so quickly.

- The existence of competing stores with asymmetric search cost and prices might be more than just a transitional state from a fragmented retail environment towards only a few mass merchandisers. Consider a retailing example involving a mass merchandiser and a chain of specialty music equipment stores. If one enters a mass merchandiser to purchase a pair of speakers it will be hard to find a salesperson, the salesperson may be uninformative and most likely there will be no listening room to test the speakers. In other words, it will be hard to determine one's reservation price for any given speakers. In contrast, at the specialty music equipment store with more locations, knowledgeable salespeople, and listening rooms, a consumer can more easily determine how much she is willing to pay for a set of speakers. My result suggests that this situation could occur in equilibrium and might prevent the two companies from entering intense price competition.

- Firms can use the existence of multiple channels to ease price competition. Suppose that there are some channels on which search is easier than other channels. Then we might conclude that several channels with differing ease of searching might exist in the long run. Firms that compete with each other across channel borders instead of within channels could use the precommitment to different search cost to avoid stiff price competition. This might explain why a firm like L.L. Bean, for example, has not extended beyond mail order and a few outlet stores.

- The fact that some stores have low service might not be related only to cost pressure. Low service need not be a sign of bad marketing communication, but in fact could be profit maximizing. Sales assistants that are less than experts could be the result of a con-

29 Although mass merchandisers typically carry a more low end product line there is typically overlap in the equipment sold.
scious choice of communications mix, and not the inability of the firm to find good staff at moderate salaries. The same might apply to understaffed showrooms or even low store density. Applied to a different channel, it might be optimal for mail order firms to produce catalogs that are purposely low on informational content.

- If firms have an incentive to set consumer search cost high, firms may underinvest in technology that aids information provision and dissemination. “Underinvest” is meant in comparison to a situation with efficient marketing communication (see Wernerfelt, 1996). Consider the example of in-store computers that can easily bring up a wealth of product information. Firms might consciously choose not to install or develop such devices. Underinvestment might also be the cause of firms’ reluctance to allow consumers to evaluate cars through extended test-drives. Even if it were costless for dealers to provide test vehicles, they might not want to do so.

- The emergence of channels that allow for cheap provision of information will not lead to obsolescence of information providers such as “Consumers Reports.” If, as my results suggest, at least some firms remain reluctant to inform in detail about their products, there will still be a consumer need for third party product evaluations.

- Emerging electronic channels with cheap information provision will not necessarily lead firms to “overload” consumers with massive information provision (see references in the introduction). However, it is very well possible that information overload will result from an increase in “bad” information so that services that summarize and filter information might become very important.

7 Concluding remarks

This paper presents four simple models in which firms set not only price but also consumer search cost. Setting consumer search cost is costless to firms. Consumers know the price of goods but are uncertain about their willingness to pay for the goods in question. By searching at a firm they learn about their willingness to pay at that firm and (in competition) receive a noisy signal about their willingness to pay at the competing firm.

It is shown that firms might not want to facilitate consumer search, even if doing so is costless to firms. There are at least two reasons for this: First, firms can soften price competition by differentiating on the basis of consumer search cost. Firms can only succeed in preventing ruinous price competition if they can precommit to a level of search cost, i.e. if search costs are
stickier than prices. Second, firms have an incentive to take advantage of consumer optimism. In equilibrium there are situations in which a monopolist will set strictly positive search cost, preventing some consumers from searching and, as a consequence, extracting surplus from those consumers who under full information would have chosen not to buy. This shows that the reduced cost of providing information in new electronic channels such as the Internet does not necessarily imply that firms will help consumers in their search for information.

In future research I plan to analyze the dynamics of setting consumer search cost in a more general framework by considering competition between firms that have a presence on multiple channels. This will help shed light on possible interactions between channels and firms’ search cost strategies.

This paper contributes to the marketing literature by showing the ways in which firms can strategically use consumer search cost. It also points out that reduced cost of information provision might not induce firms to help consumers search. From a theoretical point of view, this paper suggests that treating search cost as an endogenous and not an exogenous variable can have a profound impact on model results and predictions. These results have many empirical implications for existing channels. They also caution against possibly naive predictions of marketing communication in new emerging channels.
References


Appendix

Proof of proposition 1:

Proposition 1 results as a special case ($\gamma = 0$) from proposition 2. See the proof of proposition 2.

Proof of proposition 2:

Consumer strategies:

From tables 1, 2, and 3 we can derive the conditions on $(p, s)$ for which buy, search, or pass is the best response for each consumer segment. High segment consumers prefer to buy rather than to search iff $s \geq (p - 1/2)^2$, to buy rather than to pass iff $p \leq 3/4$, and to search rather than to pass iff $s \leq (1-p)^2$. Normal segment consumers' optimal response is to buy rather than to search iff $s \geq p^2/2$, to buy rather than to pass if $p \leq 1/2$ and to search rather than to pass iff $s \leq (1-p)^2/2$. The resulting best response regions are shown in figure 3.

Firm strategies:

Any other $(p, s)$ than the ones listed in table 8 are clearly dominated

<table>
<thead>
<tr>
<th>Case</th>
<th>Price at</th>
<th>Set search cost such that:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>High segment response</td>
</tr>
<tr>
<td>A</td>
<td>$p = 1/2, (p_a)$</td>
<td>buy</td>
</tr>
<tr>
<td>B</td>
<td>$1/2 &lt; p \leq 1/\sqrt{2}$</td>
<td>buy</td>
</tr>
<tr>
<td>C</td>
<td>$p = 3/4, (p_c)$</td>
<td>buy</td>
</tr>
<tr>
<td>D</td>
<td>$p &gt; 1/\sqrt{2}$</td>
<td>search</td>
</tr>
</tbody>
</table>

Any $(p, s)$ in region 1 (all regions refer to figure 3) in which $p = p_a = 1/2$ will be weakly more profitable for the firm than any other point in region 1 since demand remains unchanged up to $p_a$. Any $(p, s)$ in region 4 for $p \leq p_a$ is dominated by a point in region 1 in which
\( p = p_a \) since permitting search to the normal segment reduces demand. In region 2 the firm will prefer any \((p, s)\) in which \( p = p_c = 3/4 \). Region 6 will always be dominated by either region 2 or region 5. This leaves us with cases \( A, B, C, D \).

The firm’s profit function for each of these cases is:

**Case A**: \( \Pi_A = \left( \frac{1}{2} - c \right) \) \hspace{1cm} (14)

**Case B**: \( \Pi_B = (p_B - c)(\gamma + (1 - \gamma)(1 - p_B)) \) \hspace{1cm} (15)

**Case C**: \( \Pi_C = \left( \frac{3}{4} - c \right)\gamma \) \hspace{1cm} (16)

**Case D**: \( \Pi_D = (p_D - c)(\gamma 2(1 - p_D) + (1 - \gamma)(1 - p_D)) \) \hspace{1cm} (17)

From first order conditions we get \( p_B = c/2 + 1/(2(1 - \gamma)) \) and \( p_D = (1 + c)/2 \). Second order conditions hold. Recall that we require \( p_B \leq 1/\sqrt{2} \Leftrightarrow c \leq \sqrt{2} - 1/(1 - \gamma) \). Comparing profits from case \( A \) and \( B \) (inserting \( p_B \)) we get that \( \Pi_A \geq \Pi_B \Leftrightarrow c \leq (\sqrt{2} - 2\gamma - 1)/(1 - \gamma) \). Since \( (\sqrt{2} - 2\gamma - 1)/(1 - \gamma) \geq \sqrt{2} - 1/(1 - \gamma) \) for all \( \gamma \in [0, 1] \), \( \Pi_A \geq \Pi_B \) whenever \( p_B \leq 1/\sqrt{2} \). Therefore, whenever case \( B \) is optimal, \( p_B = 1/\sqrt{2} \).

I recalculate the condition under which \( \Pi_A \geq \Pi_B \) (with \( p_B = 1/\sqrt{2} \)) and also specify the remaining conditions under which one or the other case is optimal.

\[ \Pi_A > \Pi_B \Leftrightarrow c < \frac{2 - \gamma - \sqrt{2}}{\sqrt{2}(1 - \gamma)} \] \hspace{1cm} (18)

\[ \Pi_A > \Pi_C \Leftrightarrow c < \frac{2 - 3\gamma}{4(1 - \gamma)} \] \hspace{1cm} (19)

\[ \Pi_A > \Pi_D \Leftrightarrow c < \frac{\sqrt{2} - 2\gamma + \gamma - 1}{1 - \gamma} \] \hspace{1cm} (20)

\[ \Pi_B \geq \Pi_C \Leftrightarrow c \leq \frac{2 - 2\sqrt{2} + \gamma}{2(\sqrt{2}(1 - \gamma) + 2\gamma)} \] \hspace{1cm} (21)
\[ \Pi_B \geq \Pi_C \iff c \leq \frac{\sqrt{2} \gamma^2 (2 - \sqrt{2}) + 2 \gamma (3 \sqrt{2} - 4) - (1 - \sqrt{2})(1 - \gamma)}{1 + \gamma} \]  \hspace{1cm} (22)

\[ \Pi_C \geq \Pi_D \iff c \leq 1 + \frac{\sqrt{3} \gamma^2 - \gamma}{1 - \gamma} \]  \hspace{1cm} (23)

Since the boundaries defined in equations (19) and (20) lie above the boundary that separates case \( A \) and \( B \), case \( B \) dominates case \( A \) whenever case \( C, D \) dominate case \( A \).

Regions \( A, B, C, D \) as defined in equations (5) through (8) follow immediately from the remaining equations (18), (21), (22), and (23). Since these regions are defined as those \((c, \gamma)\) for which profit is highest, prices \( p_A = 1/2, p_B = 1/\sqrt{2}, p_C = 3/4 \), and \( p_D = (1 + c)/2 \) are equilibrium prices. By concavity of the profit functions they are unique. The search cost stated in the proposition, \( s_A > 1/8, s_B = 3/4 - 1/\sqrt{2}, s_C > 1/16, s_D < (1 - c)^2/2 \) are chosen to support consumer behavior as stated in table 8. They follow immediately from consumer strategies on page 51.

\[ \text{Q.E.D.} \]

Proof of proposition 3:

Overview and procedure

In this proof I will show that the claims in proposition 3 hold for a certain set of parameter values, namely \( c = 3/10, q = 4/5 \). There are many points for which I could show the same, however, deriving and proving the proposed equilibrium for a general set of parameter values proves to be very hard, that is why proposition 3 is formulated as an existence proof.

I solve the game backwards.

(a) I first characterize consumer behavior as a function of prices and search costs. This turns out to be quite complicated because demand is nondifferentiable in search costs at several points. As a result there are many search costs and price regions, for each of which consumer demand takes on a different form. Much of this proof is concerned with ruling out regions that cannot lead to equilibria of the overall game.

The proof begins by detailing consumers' expected utility (part 1) and its properties and then setting up conditions that rule out pathological regions of prices and search costs (part 2). It is described how to calculate consumer demand for a given set of prices and search costs (part 3). Then I describe the nondifferentiabilities of demand in search costs in form of inequality conditions on search costs and prices (part 4). The sequence of points at which demand be-
comes nondifferentiable as search costs decrease is determined next (part 5). This rules out many regions of search costs as potential equilibrium candidates.

(b) I next propose an equilibrium of the overall game. Price equilibria are calculated for search costs in those regions that have not been ruled out in (a) and to which either firm might want to deviate from the proposed equilibrium (parts 7 and 8).

(c) In the final step it is established that the proposed equilibrium is indeed the search cost equilibrium of the overall game (part 9).

1. Consumers’ expected utilities

Much of the proof is in preparation of showing that firm 2 does not have incentives to deviate from the proposed equilibrium. In checking deviations, I will fix \( s_1 = 0 \), i.e. all consumers will search at firm 1 and get to stage 2 (see figure 5). In the following I will therefore discuss consumer and firm decisions in stage 2 and stage 3 in some detail.

Stage 2 consumer utilities are shown in table 5 on page 34. To analyze consumers’ decisions at stage 2 I will look at the expected utilities net of \( s_1 \).\(^1\) Solving out the integrals and writing the utility of continuing search without the maximum operator, I obtain:

<table>
<thead>
<tr>
<th>Action</th>
<th>Expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy at firm 1:</td>
<td>( b_1 \equiv u_1 - p_1 )</td>
</tr>
<tr>
<td>buy at firm 2:</td>
<td>( b_2 \equiv u_1 + \frac{1 - q}{2} - p_2 )</td>
</tr>
</tbody>
</table>

---

\(^1\) Since \( s_1 \) is sunk when stage 2 is reached is has no effect on stage 2 decisions.
Table 9: Expected consumer utilities in stage 2

<table>
<thead>
<tr>
<th>Action</th>
<th>Expected utility</th>
</tr>
</thead>
</table>
| search at firm 2 for $p_1 \geq p_2$ | $s = -s_2 + \frac{(1-q)\cdot(1-p_2)^2}{2}$ for $u_1 \leq p_2$  
$s = -s_2 + q(u_1 - p_2) + \frac{(1-q)\cdot(1-p_2)^2}{2}$ for $p_2 < u_1 < p_1$  
$s = -s_2 + q(u_1 - p_2) + (1-q)\left((u_1 - p_1 + p_2)(u_1 - p_1) + \frac{(1-p_2)^2 - (u_1 - p_1)^2}{2}\right)$ for $p_1 \leq u_1 \leq 1$ |
| search at firm 2 for $p_1 < p_2$ | $s = -s_2 + \frac{(1-q)\cdot(1-p_2)^2}{2}$ for $u_1 \leq p_1$  
$s = -s_2 + q(u_1 - p_2) + (1-q)\left((u_1 - p_1 + p_2)(u_1 - p_1) + \frac{(1-p_2)^2 - (u_1 - p_1)^2}{2}\right)$ for $p_1 \leq u_1 \leq 1$ |
| pass: | $-s_1$ |

In the following I will use “buy1” for buying at firm 1, “buy2” for buying at firm 2, “search” for continuing search at firm 2 and “pass” for passing.

Call consumers with $u_1 \leq p_2$ “d” consumers (for down), consumers with $p_2 < u_1 < p_1$ “m” consumers (for middle), and consumers with $p_1 \leq u_1 \leq 1$ “u” consumers (for up). Since the expected utility of search is different for each of these consumers, the $s_2$ at which d, m, or u consumers will prefer to continue searching over other options, will be different.

Figure 6 shows the expected utilities for d, m, u consumers net of $s_1$ and under the assumption that $p_1 \geq p_2$. For $p_1 < p_2$ the middle region drops out, otherwise the graph would be identical. The thick lines are the (expected) utilities of buy 1, $b_1(u_1)$, and buy 2, $b_2(u_1)$. The dashed lines are the expected utilities of search $s(u_1, s_2)$. For each of d, m, u consumers respectively, I have drawn $s(u_1, s_2)$ for two or three levels of $s_2$.  

- 55 -  
(Appendix Chapter 1)
2. Some conditions and properties

Figure 6 incorporates three conditions that will turn out to hold for $c = 3/10$ and $q = 4/5$. Iff condition A1 holds, buy1 will not be dominated by buy2. The condition ensures that the intersection of $b_1$ and $b_2$ lies to the right of $p_1$.

$$p_1 < \frac{1}{2} + \frac{p_1 - p_2}{1-q} \quad (A1)$$

Iff Condition A2 holds, buy2 will not be dominated by buy1. The condition ensures that the intersection of $b_1$ and $b_2$ lies to the left of 1.

$$1 > \frac{1}{2} + \frac{p_1 - p_2}{1-q} \quad (A2)$$

Finally, iff condition A3 holds, at least some $d$ consumers will prefer buy2 to pass. The condition ensures that the $u_1$ that solves $b_2(u_1) = 0$ lies to the left of $p_2$.

$$p_2 < \frac{1}{2} \quad (A3)$$

Note a couple of crucial properties of the expected utilities of search:
(P1) For \( d \) consumers \( \frac{\partial s}{\partial u_1} = 0 \).

Proof: by inspection

(P2) For \( m \) consumers \( \frac{\partial s}{\partial u_1} = \frac{\partial b_2}{\partial u_1} \).

Proof: by inspection

(P3) For \( u \) consumers \( \frac{\partial b_2}{\partial u_1} < \frac{\partial s}{\partial u_1} < \frac{\partial b_1}{\partial u_1} \).

Proof: \( \frac{\partial b_2}{\partial u_1} = q < \frac{\partial s}{\partial u_1} = q + (1 - q)u_1 < \frac{\partial b_1}{\partial u_1} = 1 \)

P1 says that for \( d \) consumers the expected utility of continuing search in stage 2 is independent of their realized \( u_1 \), i.e. is parallel to the x-axis in figure 6. P2 says that for \( m \) consumers the slope of the expected utility of continuing search in stage 2 is identical to the slope of buy2, i.e. is parallel to buy2 in figure 6. P3 says that for \( u \) consumers the slope of the expected utility of continuing search in stage 2 is smaller than that of buy1 but larger than that of buy2. This means that if there exists a \( u \) consumer that prefers buy1 to buy2 that chooses search rather than buy1, there must exists a \( u \) consumer that prefers buy2 to buy1 and that chooses search rather than buy2.

3. Derivation of consumer demand

Derivation of Stage 2 demands

The fraction of consumers choosing buy1, buy2, search, or pass in stage 2 can be derived from table 9 by calculating the \( u_1 \) for which the expected utility of each action is higher than that of all other actions. The resulting demands will depend on \( s_2 \) since \( s_2 \) determines how attractive it is to continue searching.

Derivation of Stage 3 demands

The fraction of consumers choosing buy1, buy2, or pass in stage 3 can be derived as follows. Suppose that consumers with \( u_1 \in [b_1, b_2] \) have chosen to search at firm 2 in stage 2. Then a fraction \( q \) of them will learn that \( u_1 = u_2 \). They will choose to purchase at the firm with lower price as long as the price is lower than their willingness to pay.
A fraction \((1 - q)\) will learn that \(u_1 \neq u_2\). For these consumers \(u_2 \in [0, 1]\). Consumers with \(u_1 < p_1\) will never buy at firm 1. In stage 3 they will therefore only decide between buy2 and pass and choose to buy2 iff \(u_2 \geq p_2\). Consumers with \(u_1 \geq p_1\) that have searched both firms will prefer firm \(i\) if \(u_i - p_i \geq u_j - p_j\). The fraction of consumers choosing to buy from firm 1 and 2 respectively is given by the following expressions.

Suppose \(p_2 - p_1 < 1 - b_2\).

\[
\text{Firm 1: } \int_{b_1}^{b_2} \int_{u_1 - p_1 + p_2}^{1} \frac{1}{b_2 - b_1} \, du_2 \, du_1
\]

\[
(b_1 - p_1 + p_2) b_2
\]

\[
\text{Firm 2: } \int_{0}^{b_1} \int_{u_1 - p_1 + p_2}^{b_2} \frac{1}{b_2 - b_1} \, du_1 \, du_2 + \int_{b_1}^{b_2} \int_{(u_1 - p_1 + p_2)(u_2 - p_2 + p_1)}^{1} \frac{1}{b_2 - b_1} \, du_1 \, du_2
\]

Suppose \(p_2 - p_1 \geq 1 - b_2\).

\[
\text{Firm 1: } \int_{b_1}^{1} \int_{(1 - p_2 + p_1)}^{1} \frac{1}{b_2 - b_1} \, du_1 \, du_2
\]

\[
(b_1 - p_1 + p_2) b_2
\]

\[
\text{Firm 2: } \int_{0}^{b_1} \int_{u_1 - p_1 + p_2}^{1} \frac{1}{b_2 - b_1} \, du_1 \, du_2 + \int_{b_1}^{(1 - p_2 + p_1)} \int_{(u_1 - p_1 + p_2)(u_2 - p_2 + p_1)}^{1} \frac{1}{b_2 - b_1} \, du_1 \, du_2
\]

4. Characterization of nondifferentiable and discontinuous points

Assuming A1-A3 and with P1-P3 I can analyze the effect of \(s_2\) on consumer choice. Consider first \(d\) consumers. For very high \(s_2\) some consumers will pass and some will buy at firm 2. As \(s_2\) becomes smaller, eventually \(s \geq 0\). Then those consumer that so far passed will now search. The search cost \(s_2\) at which this happens is given in condition D1.

\[
s_2 \leq (1 - q) \frac{(1 - p_2)^2}{2} \quad \text{(D1)}
\]
As \( s_2 \) falls further, some of those consumers that chose buy2 will now prefer search. Condition D2 states those \( s_2 \) at which all \( d \) consumers search for \( p_1 \geq p_2 \) and D3 for \( p_1 < p_2 \).

\[
s_2 < (1 - q)\frac{p_2^2}{2} \tag{D2}
\]

\[
s_2 < (1 - q)\frac{p_2^2}{2} + q(p_2 - p_1) \tag{D3}
\]

Consider now \( m \) consumers. By construction, pass will always be preferred to buy1. Further, by A4, buy1 will always be preferred to pass. By P2 we know that if any \( m \) consumer chooses to search all consumers will search. Condition M1 states the \( s_2 \) under which \( m \) consumers will search. Note that M1=D2.

\[
s_2 < (1 - q)\frac{p_2^2}{2} \tag{M1}
\]

Consider now \( u \) consumers. Through P3 we know that - as \( s_2 \) falls - the first \( u_1 \) at which consumers will prefer searching to any other option will be given by \( b_1(u_1) = b_2(u_1) \). Those \( s_2 \) for which there is no search are stated in condition U3. With falling search cost, there will be \( s_2 \) at which all consumers prefer search to buy1 (condition U1) and \( s_2 \) at which all consumers will prefer search to buy2 (condition U2).

Condition U1 results from equating \( s(u_1) \) and \( b_1(u_1) \) at \( u_1 = 1 \).

\[
s_2 < (p_1 - p_2) - \frac{(1 - q)}{2}(p_1 - p_2)^2 \tag{U1}
\]

Condition U2 results from equating \( s(u_1) \) and \( b_2(u_1) \) at \( u_1 = p_1 \).

\[
s_2 < \frac{(1 - q)}{2}(p_2(2p_1 - p_2)) \tag{U2}
\]

Condition U3 results from equating \( s(u_1) \) and \( b_1(u_1) \) as defined by \( b_1(u_1) = b_2(u_1) \).

\[
s_2 > \frac{(1 - q + (p_1 - p_2)(4 - 2q))(1 - q + (p_1 - p_2)2q)}{8(1 - q)} \tag{U3}
\]
5. The sequence of nondifferentiable points (Lemma 1)

Assume \( s_1 = 0 \). Then, starting with \( s_2 = \infty \), as firm 2 decreases search cost, consumer search will increase such that the following sets of conditions hold in sequence:

<table>
<thead>
<tr>
<th>Set</th>
<th>Conditions hold</th>
<th>Conditions don't hold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>U3</td>
<td>D1, D2, D3, M1, U1, U2</td>
</tr>
<tr>
<td>2</td>
<td>U3, D1</td>
<td>D2, D3, M1, U1, U2</td>
</tr>
<tr>
<td>3</td>
<td>D1</td>
<td>D2, D3, M1, U1, U2, U3</td>
</tr>
<tr>
<td>3a</td>
<td>D1, D3</td>
<td>D2, M1, U1, U2, U3</td>
</tr>
<tr>
<td>4</td>
<td>D1, D3, U1</td>
<td>D2, M1, U2, U3</td>
</tr>
<tr>
<td>5</td>
<td>D1, D2, D3, M1, U1</td>
<td>U2, U3</td>
</tr>
<tr>
<td>6</td>
<td>D1, D2, D3, M1, U1, U2</td>
<td>U3</td>
</tr>
</tbody>
</table>

**Proof:**

I will show that any other combination leads to a contradiction. I first list the possible sequences for different \( p_1, p_2 \) and then show that each sequence leads to a contradiction. As a shorthand for table 10 I introduce the notation (with \( p_1 \geq p_2 \)): D1—\( \neg U3 \) — U1 — D2 — U2. This lists the sequence in which condition start holding as search cost decrease starting at infinity.

**Assume** \( p_1 \geq p_2 \).

**Assume** \( p_1 \leq 1/2 \).

From the definition of the conditions we can immediately conclude that D2 implies D1 and U1, U1 implies D1. By construction of U1, U2, U3 we know that U1 and U2 both independently imply that U3 does not hold. This leaves the following possible sequences:

1. D1—\( \neg U3 \) — U1 — U2 — D2
2. D1—\( \neg U3 \) — U2 — U1 — D2
3. \( \neg U3 \) — D1— U2 — U1 — D2
4. \( \neg U3 \) — D1— U1 — U2 — D2
5. \( \neg U3 \) — D1— U1 — D2 — U2

**Assume** \( p_1 > 1/2 \).

**Assume** \( p_2 \leq 1/2 \).
From the definition of the conditions we can immediately conclude that $D_2$ implies $D_1$ and $U_1$. By construction of $U_1$, $U_2$, $U_3$ we know that $U_1$ and $U_2$ both independently imply that $U_3$ does not hold. This leaves the following possible sequences:

$$\begin{align*}
D_1 & - \neg U_3 - U_1 - U_2 - D_2 & [6] \\
D_1 & - \neg U_3 - U_2 - U_1 - D_2 & [7] \\
\neg U_3 & - D_1 - U_2 - U_1 - D_2 & [8] \\
\neg U_3 & - D_1 - U_1 - U_2 - D_2 & [9] \\
\neg U_3 & - D_1 - U_1 - D_2 - U_2 & [10] \\
\neg U_3 & - U_2 - U_1 - D_1 - D_2 & [11] \\
\neg U_3 & - U_1 - U_2 - D_1 - D_2 & [12] \\
\neg U_3 & - U_1 - D_1 - U_2 - D_2 & [13] \\
\neg U_3 & - U_1 - D_1 - D_2 - U_2 & [14]
\end{align*}$$

Assume $p_2 > 1/2$.

From the definition of the conditions we can immediately conclude that $D_2$ implies $D_1$ and $U_1$, $D_1$ implies $U_1$. By construction of $U_1$, $U_2$, $U_3$ we know that $U_1$ and $U_2$ both independently imply that $U_3$ does not hold. This leaves the following possible sequences:

$$\begin{align*}
\neg U_3 & - U_2 - U_1 - D_1 - D_2 & [15] \\
\neg U_3 & - U_1 - U_2 - D_1 - D_2 & [16] \\
\neg U_3 & - U_1 - D_1 - U_2 - D_2 & [17] \\
\neg U_3 & - U_1 - D_1 - D_2 - U_2 & [18]
\end{align*}$$

Assume $p_1 < p_2$.

From the definition of the conditions we can immediately conclude that $U_2$ implies $D_3$ and $U_1$, $U_1$ implies $D_3$, $D_3$ implies $D_1$. By construction of $U_1$, $U_2$, $U_3$ we know that $U_1$ and $U_2$ both independently imply that $U_3$ does not hold. This leaves the following possible sequences:

$$\begin{align*}
\neg U_3 & - D_1 - D_3 - U_1 - U_2 & [19] \\
D_1 & - \neg U_3 - D_3 - U_1 - U_2 & [20] \\
D_1 & - D_3 - \neg U_3 - U_1 - U_2 & [21]
\end{align*}$$

Sequences [1] and [2] cannot hold since all real $p_1$ that solve the first order conditions lie above $1/2$, thus contradicting the assumption.

Sequence [3] does not hold since for the price equilibrium ($p_1^* = 0.375$, $p_2^* = 0.35$) at the highest $s_2$ for which $U_3$ is violated, also $D_1$ holds. This also excludes sequences [4] and [5].

Sequences [6] - [10] cannot hold since there is no price equilibrium with $p_1 > 1/2$ and $p_2 \leq 1/2$. 

Sequences [11] - [14] cannot hold since there is no price equilibrium so that \( p_1 \leq 1/2 \) and \( p_2 \leq 1/2 \) for which \( -U3 \) does not imply D1.

Sequences [15] - [18] cannot hold since there is no price equilibrium so that \( p_1 > 1/2 \) and \( p_2 > 1/2 \).

Sequence [19] does not hold since there is no price equilibrium for which \( p_1 < p_2 \).

Sequence [20] and [21] do not hold since we know from case C2 that a situation for which only D1 and U3 hold violates \( p_1 < p_2 \). Q.E.D.

6. Derivation of the proposed equilibrium

Assume that firm 1 has set \( s_1 = 0 \) and firm 2 has set \( s_2 = (1 - q)(1 - p_2)^2/2 \). Then all consumers will search at firm 1 and we are now at stage 2. Assume that \( p_1 \geq p_2 \). Further assume that A1, A2, A3, D1, and U3 hold and D2, M1, U1, U2 do not hold. Then the fractions of consumers that will choose each possible action at stage 2 are as following:

<table>
<thead>
<tr>
<th>Consumers</th>
<th>Action</th>
<th>Fraction of ( d, m, u ) consumers respectively</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u ) consumers</td>
<td>buy 1:</td>
<td>( \frac{1}{2} \frac{p_1 - p_2}{1 - q} )</td>
</tr>
<tr>
<td></td>
<td>buy 2:</td>
<td>( \frac{1}{2} + \frac{p_1 - p_2}{1 - q} - p_2 )</td>
</tr>
<tr>
<td></td>
<td>search</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>pass:</td>
<td>0</td>
</tr>
<tr>
<td>( m ) consumers</td>
<td>buy 1:</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>buy 2:</td>
<td>( p_1 - p_2 )</td>
</tr>
<tr>
<td></td>
<td>search</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>pass:</td>
<td>0</td>
</tr>
<tr>
<td>Consumers</td>
<td>Action</td>
<td>Fraction of $d, m, u$ consumers respectively</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>$d$</td>
<td>buy 1:</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>buy 2:</td>
<td>$p_1 - \frac{1}{2} + \frac{1 - 2p_1}{q}$</td>
</tr>
<tr>
<td></td>
<td>search</td>
<td>$\frac{1}{2} - \frac{1 - 2p_1}{q}$</td>
</tr>
<tr>
<td></td>
<td>pass:</td>
<td>0</td>
</tr>
</tbody>
</table>

These follow immediately from the assumptions by calculating cutoffs between the various expected utilities.

No $m, u$ consumers reach stage 3. $(1 - q)$ $d$ consumers that make it to stage 3 are not interested in purchasing at firm 1 since $d$ consumers by definition have reservation prices below $p_1$. The fraction $(q)$ of $d$ consumers that finds out that $u_1 = u_2$ will all pass since for $d$ consumers $u_1 < p_2 < p_1$. Of the remaining $1 - q$ consumers none purchase at firm 1 since $d$ consumers by definition have reservation prices below $p_1$. Hence, of those who searched both firms a fraction $(1 - q)(1 - p_2)$ purchases at firm 2 and none purchase at firm 1.

The firms’ profit functions follow directly from equation (9) and by constructing overall demands from table 11 and the previous paragraph.

\begin{align*}
\Pi_1 &= (p_1 - c) \left( 1 + \frac{(1 - q)/2 + p_1 - p_2}{q - 1} \right) \tag{28} \\
\Pi_2 &= (p_2 - c) \left( \frac{(1 - q)/2 - p_2)(1 - (1 - q)(1 - p_2))}{q} + \frac{(1/2)(1 - q) + p_1 - p_2}{q - 1} \right) \tag{29}
\end{align*}

Solving first order conditions and selecting those (unique) $p_1^*$ and $p_2^*$ for which second order conditions hold we obtain after simple but tedious algebra:

\begin{align*}
p_1^* &= (-2q^3 + (4c + 10)q^2 - (13 + 8c)q + 2 + 4c + [4 - 8c + 16c^2)q + (8 + 104c - 64c^2)q + (5 - 224c + 96c^2)]q^2 + (-16 + 176c - 64c^2)q^3 (30) \\
&\quad (20 - 56c + 16c^2)q^4 + (-16 + 8c)q^5 + 4q^6]^{1/2}/(12(1 - q)^2)
\end{align*}
\[ p_2^* = (-8q^3 + (16c + 28)q^2 - (31 + 32c)q + 8 + 16c + [4 - 8c + 16c^2]
(8 + 104c - 64c^2)q + (5 - 224c + 96c^2)q^2 + (-16 + 176c - 64c^2)q^3
(20 - 56c + 16c^2)q^4 + (-16 + 8c)q^5 + 4q^6)^{1/2})/(24(1 - q)^2) \] (31)

With \( \epsilon = 3/10 \) and \( q = 4/5 \) these prices are:

\[ p_1^* = 0.3773 \quad p_2^* = 0.3546 \] (32)

All assumptions hold, namely that A1, A2, A3, D1, and U3 hold, D2, M1, U1, and U2 do not hold, and that \( p_1 \geq p_2 \). Therefore (32), together with demands in table 11 characterize the equilibrium of the second step price game, given first step search cost \( s_1 = 0 \) and \( s_2 = (1 - q)(1 - p_2)^2/2 \). Profits are:

\[ \Pi_1^* = 0.02989 \quad \Pi_2^* = 0.01837 \] (33)

7. Equilibrium proof: deviations of firm 2

Lemma 1 determines the sequence with which \( d, m, u \), \( (d, u) \) consumers for \( p_2 \leq p_1 \), \( (p_2 > p_1) \) will start searching as firm 2 decreases or increases search cost deviating from its proposed equilibrium search cost. In set 1 there is no search. In set 2, those, but only those \( d \) consumers who so far passed now search at firm 2; this is the set of conditions that hold in the proposed equilibrium. In set 3 some \( d \) consumers that so far bought from firm 2 will search; some \( u \) consumers that so far bought from firm 1 or firm 2 will search. In set 4 some \( d \) consumers that so far bought from firm 2 will search; all \( u \) consumers that bought from firm 1 search, some that bought at firm 2 search. In set 5 all \( d \) consumers and \( m \) consumers search; all \( u \) consumers that bought from firm 1 search, some that bought at firm 2 search. In set 6 all \( d, m, u \) consumers search.

Note that for all sets but set 3 optimal search cost must be set to obtain a boundary between the sets. This is because within each set, the marginal profit off each consumer that starts searching is equal. In set 3, however, an interior solution is possible.
For \( p_1 \geq p_2 \) this means that I have to check the price equilibria for the following cases:

<table>
<thead>
<tr>
<th>Case</th>
<th>( s_2 = )</th>
<th>( d ) consumers</th>
<th>( m ) consumers</th>
<th>( u ) consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>( \infty )</td>
<td>no search</td>
<td>no search</td>
<td>no search</td>
</tr>
<tr>
<td>C2</td>
<td>solves ( s_d(s_2) = 0 )</td>
<td>search only by those who so far passed</td>
<td>no search</td>
<td>no search</td>
</tr>
<tr>
<td>C3</td>
<td>solves ( s_u(s_2, u_1) = b_2(u_1) ) at ( u_1 ) that solves ( b_1(u_1) = b_2(u_1) )</td>
<td>additional some who bought at firm 2 search (given by ( s_2 ) )</td>
<td>no search</td>
<td>no search</td>
</tr>
<tr>
<td>C4</td>
<td>internal point between ( s_2 ) as defined in C3 and C5</td>
<td>additional some who bought at firm 2 search (given by ( s_2 ) )</td>
<td>no search</td>
<td>search from some who bought at firm 1 or firm 2 (given by ( s_2 ) )</td>
</tr>
<tr>
<td>C5</td>
<td>solves ( s_u(s_2, u_1) = b_2(u_1) ) at ( u_1 = p_1 )</td>
<td>additional some who bought at firm 2 search (given by ( s_2 ) )</td>
<td>no search</td>
<td>all who bought at firm 2 search; some who bought at firm 1 search (given by ( s_2 ) )</td>
</tr>
<tr>
<td>C6-1</td>
<td>solves ( s_m(s_2, u_1) = b_2(u_1) )</td>
<td>all search</td>
<td>no search</td>
<td>all who bought at firm 2 search; some who bought at firm 1 search (given by ( s_2 ) )</td>
</tr>
<tr>
<td>C6-2</td>
<td>solves ( s_m(s_2, u_1) = b_2(u_1) )</td>
<td>all search</td>
<td>all search</td>
<td>all who bought at firm 2 search; some who bought at firm 1 search (given by ( s_2 ) )</td>
</tr>
<tr>
<td>C7</td>
<td>solves ( s_u(s_2, u_1) = b_1(u_1) ) at ( u_1 = 1 )</td>
<td>all search</td>
<td>all search</td>
<td>all search</td>
</tr>
</tbody>
</table>

If I can show that equilibrium prices in C2 are higher that in any other case, I have shown that firm 2 does not have an incentive to deviate.
I can immediately exclude two dominated cases. Case C1 will always be dominated by case C2 since in case 2 firm 2 can increase demand at the same price. Case C6-2 will be dominated by case C6-1 since firm 2's demand will strictly decrease as consumers decide to search rather than to buy at firm 2 at stage 2.

Case C1:
Dominated by case 2

Case C2:
This is the equilibrium case derived on page 64.

Case C3:
Demands can be readily derived from table 12 and consumer utilities in table 9 as we did in the derivation of the proposed equilibrium. Substituting \( c = 3/10 \) and \( q = 8/10 \) we obtain profit functions:

\[
\Pi_1 = \left( p_1 - \frac{3}{10} \right) \left( \frac{1}{2} + 5p_2 - 5p_1 \right)
\]

\[
\Pi_2 = \left( p_1 - \frac{3}{10} \right) \left( \frac{17}{32} - \frac{53}{8} p_2 + \frac{45}{8} p_1 - 6p_1 p_2 + 3p_1^2 + \frac{23}{8} p_2^2 \right.
\]

\[
- \frac{1}{4} \left( p_2^2 - \frac{13}{10} p_2 + \frac{24}{5} p_1 p_2 + \frac{12}{5} p_1^2 + \frac{23}{10} p_2^2 + \frac{1}{20} \right)(1 - p_2)
\]

Solving first order conditions and selecting those \( p_1^* \) and \( p_2^* \) for which \( p_1, p_2 \in [0, 1] \) and for which second order conditions hold we obtain:

\[
p_1^* = 0.3757 \quad p_2^* = 0.3515
\]

All assumptions hold, namely that A1, A2, A3, and U3 hold, D1, D2, M1, U1, and U2 do not hold, and that \( p_1 \geq p_2 \). Therefore (36), together with corresponding consumer demands characterize the equilibrium of the second step price game, given first step search cost \( s_1 = 0 \) and \( s_2 = 0.03854 \). Profits are:

\[
\Pi_1^* = 0.02868 \quad \Pi_2^* \approx 0.0177
\]

Firm 2 profits are less than in case C2, hence firm 2 has no incentive to deviate from the equilibrium actions characterized in C2.
Case C4:
Claim: Any optimal $s_2$ is defined either by the boundary to case C3 or by the boundary to case C5.

Proof:

\[
\frac{\partial^2 \Pi_2}{\partial s_2^2} = (10p_1 - 3) \left( \frac{X(p_1 - p_2 + 4) + Y(p_1 - p_2)}{2XY} \right)
\]

where

\[
X = (10s_2 - 2p_1p_2 + p_1^2 + p_2^2)^{3/2}
\]

\[
Y = (10s_2 - 2p_1p_2 + p_1^2 + p_2^2 + 10(p_2 - p_1))^ {3/2}
\]

Clearly for all $p > c = 3/10$, \( \frac{\partial^2 \Pi_2}{\partial s_2^2} > 0 \), i.e. \( \Pi_2 \) is convex in \( s_2 \). Therefore any \( s_2 \) that maximizes \( \Pi_2 \) must be at extreme values, i.e. at the boundaries to case C3 or C5.  

Q.E.D.

Case C5:

Demands can be readily derived from table 12 and consumer utilities in table 9 as we did in the derivation of the proposed equilibrium. Substituting \( c = 3/10 \) and \( q = 8/10 \) we obtain profit functions:

\[
\Pi_1 = \frac{(10p_1 - 3)}{-100} (12(p_1 - p_2) - 1 - p_1^2 + p_1^4 + 2p_1^2p_2 - 2p_1^3 + \sqrt{10(p_2 - p_1) + p_1^2(-8 + 2(p_2 - p_1)))})
\]

\[
\Pi_2 = \frac{(10p_2 - 3)}{-200} (-24p_1 + 2p_1^2 + 6p_1^4 - 6p_1^2 - 4p_1^2 - 4p_1p_2 + 8p_2^2 - 40p_2 - p_1p_2^2 + p_2^3 - 18 + \sqrt{10(p_2 - p_1) + p_1^2(16 - 4(p_2 + p_1)))})
\]

Solving first order conditions and selecting those \( p_1^* \) and \( p_2^* \) for which \( p_1, p_2 \subset [0, 1] \) and for which second order conditions hold we obtain:

\[
p_1^* = 0.326 \quad p_2^* = 0.3248
\]

All assumptions hold, namely that A1, A2, A3, D1, and U1 hold, D2, M1, U2, and U3 do not hold, and that \( p_1 \geq p_2 \). We might therefore suspect that (41) describe equilibrium prices. However, firm 1 has incentives to deviate from \( p_1^* \) to \( \varepsilon \) undercut \( p_2^* \). By doing so it gains a portion
\( q \) or those \( u \) consumers that searched both firms. The profits associated with the prices in (41) are:

\[
\Pi_1^* = 0.00896 \quad \Pi_2^* = 0.0093
\] (42)

Firm 1's profits by charging \( p_1 = p_2^* - \epsilon \) are \( \Pi_1^P = 0.01589 \). By \( \epsilon \) undercutting firm 1's price, firm 2 can gain those consumers back. This does, however, not result in a Bertrand equilibrium since \( \Pi_1^*(\epsilon, 0) > \Pi_1^*(0, 0) = 0 \). No pure strategy equilibrium exists. Thus, firm 2 has no incentive to deviate from the equilibrium actions characterized in C2.

**Case 6-1:**

Demands can be readily derived from table 12 and consumer utilities in table 9 as we did in the derivation of the proposed equilibrium. Substituting \( \epsilon = 3/10 \) and \( q = 8/10 \) we obtain profit functions:

\[
\Pi_1 = \frac{(10p_1 - 3)}{100} (12(p_2 - p_1) + 1 + (8 + 2(p_1 - 2p_2))
\]

\[
(10(p_2 - p_1) + 2p_2^2 + p_1^2 - 2p_1p_2) + 2(p_1^2 + p_2^2) - 4p_1p_2)
\] (43)

\[
\Pi_2 = \frac{(10p_2 - 3)}{-100} (20p_2 - 12p_1 - 9 + (8 + 2(p_1 - 2p_2))
\]

\[
(10(p_2 - p_1) + 2p_2^2 + p_1^2 - 2p_1p_2) + 2(p_1^2 + p_2^2) - 2p_1p_2)
\] (44)

Solving first order conditions and selecting those \( p_1^* \) and \( p_2^* \) for which \( p_1, p_2 \in [0, 1] \) and for which second order conditions hold we obtain:

\[
p_1^* = 0.3246 \quad p_2^* = 0.3237
\] (45)

All assumptions hold, namely that A1, A2, A3, D1, D2, M1, and U1 hold, U2, and U3 do not hold, and that \( p_1 \geq p_2 \). We might therefore suspect that (45) describe equilibrium prices. However, firm 1 has incentives to deviate from \( p_1^* \) to \( \epsilon \) undercut \( p_2^* \). By doing so it gains a portion \( q \) or those \( u \) consumers that searched both firms. The profits associated with the prices in (45) are:

\[
\Pi_1^* = 0.0085 \quad \Pi_2^* = 0.00885
\] (46)

- 68 - (Appendix Chapter 1)
Firm 1's profits by charging \( p_1 = p_2^* - \varepsilon \) are \( \Pi_1^* = 0.01518 \). By \( \varepsilon \) undercutting firm 1's price, firm 2 can gain those consumers back. This does, however, not result in a Bertrand equilibrium since \( \Pi_1^*(\varepsilon, 0) > \Pi_1^*(0, 0) = 0 \). No pure strategy equilibrium exists. Thus, firm 2 has no incentive to deviate from the equilibrium actions characterized in C2.

**Case C6-2:**
Dominated by case C6-1.

**Case C7:**
Demands can be readily derived from table 12 and consumer utilities in table 9 as we did in the derivation of the proposed equilibrium. Substituting \( c = 3/10 \) and \( q = 8/10 \) we obtain profit functions:

\[
\Pi_1 = \frac{1}{5} \left( p_1 - \frac{3}{10} \right) \left( 1 - p_1 \right) \left( p_2 - \frac{P_1}{2} + \frac{1}{2} \right) \tag{47}
\]

\[
\Pi_2 = \left( p_2 - \frac{3}{10} \right) \left( p_2 \left( 1 - p_2 \right) / 5 \right) + \left( p_1 - p_2 \right) \left( 1 - p_2 \right) / 5 + \left( 1 - p_1 \right) \left( 9/10 + p_1 / 10 - p_1 / 5 \right) \tag{48}
\]

Solving first order conditions and selecting those \( p_1^* \) and \( p_2^* \) for which \( p_1, p_2 \in [0, 1] \) and for which second order conditions hold we obtain:

\[
p_1^* = 0.61375 \quad p_2^* = 0.6425 \tag{49}
\]

This violates \( p_1 \geq p_2 \). Hence case C7 can't be a valid equilibrium of the price game.

For \( p_1 < p_2 \) I have to check the price equilibria for the following cases:

<table>
<thead>
<tr>
<th>Case</th>
<th>( s_2 = )</th>
<th>Consumer search behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( d ) consumers</td>
<td>( a ) consumers</td>
</tr>
<tr>
<td>C1a</td>
<td>( \infty )</td>
<td>no search</td>
</tr>
<tr>
<td>C2a</td>
<td>solves ( s_d(s_2) = 0 )</td>
<td>search only by those who so far passed</td>
</tr>
</tbody>
</table>

- 69 - (Appendix Chapter 1)
Table 13: Cases to check for incentives to deviate of firm 2 ($p_1 < p_2$)

<table>
<thead>
<tr>
<th>Case</th>
<th>$s_2 = $</th>
<th>Consumer search behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$d$ consumers</td>
</tr>
<tr>
<td>C3a</td>
<td>solves</td>
<td>additional some who bought at firm 2 search (given by $s_2$)</td>
</tr>
<tr>
<td></td>
<td>$b_u(s_2, u_1) = b_2(u_1)$ at $u_1$ that solves $b_1(u_1) = b_2(u_1)$</td>
<td></td>
</tr>
<tr>
<td>C4a</td>
<td>internal point between $s_2$ as defined in C3 and C5</td>
<td>additional some who bought at firm 2 search (given by $s_2$)</td>
</tr>
<tr>
<td>C5a</td>
<td>solves</td>
<td>all search</td>
</tr>
<tr>
<td></td>
<td>$b_u(s_2, u_1) = b_2(u_1)$ at $u_1 = p_1$</td>
<td></td>
</tr>
<tr>
<td>C6a</td>
<td>solves</td>
<td>all search</td>
</tr>
<tr>
<td></td>
<td>$b_u(s_2, u_1) = b_1(u_1)$ at $u_1 = 1$</td>
<td></td>
</tr>
</tbody>
</table>

Case C1a:

Demands can be readily derived from table 13 and consumer utilities in table 9 as we did in the derivation of the proposed equilibrium. Substituting $c = 3/10$ and $q = 8/10$ we obtain profit functions:

$$
\Pi_1 = \left( p_1 - \frac{3}{10} \right) \left( \frac{1}{2} + 5p_2 - 5p_1 \right)
$$

(50)

$$
\Pi_2 = \left( p_2 - \frac{3}{10} \right) \left( \frac{5}{8} - \frac{25}{4}p_2 + 5p_1 \right)
$$

(51)

Solving first order conditions and selecting those $p_1^*$ and $p_2^*$ for which $p_1, p_2 \in [0, 1]$ and for which second order conditions hold we obtain:

$$
p_1^* = 0.375 \quad p_2^* = 0.35
$$

(52)

Since the assumption $p_1 < p_2$ is violated, this cannot be an equilibrium of the price game.
Case 2a:
Profit functions are identical to case 2. Since in case 2, \( p_1^* \geq p_2^* \), the assumption \( p_1 < p_2 \) is violated.

Case 3a:
Profit functions are identical to case 3. Since in case 3 \( p_1^* \geq p_2^* \), the assumption \( p_1 < p_2 \) is violated.

Case 4a:
Profit functions are identical to case 4. Hence any optimal \( s_2 \) is defined either by the boundary to case C3a or C5a.

Case 5a:
Notice first that condition U1 implies condition D3. This means that from this case on all \( u_1 < p_1 \) search. Assume that \( p_2 - p_1 < 1 - b_2 \) (see the derivation of stage 3 demand on page 57). Then demands can be readily derived from table 12 and consumer utilities in table 9 as we did in the derivation of the proposed equilibrium. Substituting \( c = 3/10 \) and \( q = 8/10 \) we obtain profit functions:

\[
\Pi_1 = \frac{(10p_1 - 3)}{-100} (-12p_2 + 20p_1 - 2p_1^2 + 2p_1p_2 + -9 + 2(p_2 - p_1)\sqrt{10(p_2 - p_1) + p_1^2})
\]

\[
\Pi_2 = \frac{(10p_2 - 3)}{-200} (40(p_2 - p_1) + 4p_1^2 + 8p_2^2 + p_2^3 - 8p_1p_2 - p_1p_2^2 - 2 + 4(1 - p_2)\sqrt{10(p_2 - p_1) + p_1^2})
\]

Solving first order conditions we notice that there are no \( p_1^* \) such that \( p_1 > 0 \). Hence there is no equilibrium such that \( p_2 - p_1 < 1 - b_2 \).

Assume now that \( p_2 - p_1 \geq 1 - b_2 \). Solving first order conditions and selecting those \( p_1^* \) and \( p_2^* \) for which \( p_1, p_2 \in [0, 1] \) we obtain:

\[
p_1^* = 0.6425 \quad p_2^* = 0.6138
\]

Since the assumption \( p_1 < p_2 \) is violated, this cannot be the equilibrium of the price game.
Case 6a:

Demands can be readily derived from table 12 and consumer utilities in table 9 as we did in the derivation of the proposed equilibrium. Substituting \( c = 3/10 \) and \( q = 8/10 \) we obtain profit functions:

\[
\Pi_1 = \frac{(10p_1 - 3)}{-100} (-9 + 10p_1 - 2p_2 + 2p_1p_2 - p_1^2)
\]  
(56)

\[
\Pi_2 = \frac{(10p_1 - 3)}{-100} (-1 + 2(p_2 - p_1) + p_1^2)
\]  
(57)

Solving first order conditions and selecting those \( p_1^* \) and \( p_2^* \) for which \( p_1, p_2 \in [0, 1] \) and for which second order conditions hold we obtain:

\[
p_1^* = 0.6436 \quad p_2^* = 0.6182
\]  
(58)

This violates \( p_1 < p_2 \). Hence case C7a can't be a valid equilibrium of the price game.

8. Equilibrium proof: deviations of firm 1

Given that firm 2 sets \( s_2 = (1 - q)/(1 - p_2)^2/2 \), consumers' expected value of searching at firm 1 at stage 1 is:

\[
\int_0^A 0du_1 + \int_A^B (qu_1 + \frac{(1-q)}{2} - p_2)du_1 + \int_B^I (u_1 - p_1)du_1
\]

\[
A = \frac{1}{2} + \frac{2p_2 - 1}{2q}
\]  
(59)

\[
B = \frac{1}{2} + \frac{p_1 - p_2}{1 - q}
\]

Notice that \( A, B \) are those \( u_1 \) in stage 2 that separate search from buy2 and buy2 from buy1 respectively.

Assume \( s_1 < s_2 \)

By this assumption a consumer that decides to search will always start searching at firm 1. Hence I don't have to consider the expected utility of searching at firm 2 in stage 1.
Then consumers’ stage 1 expected utilities are:

<table>
<thead>
<tr>
<th>Action</th>
<th>Expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy at firm 1:</td>
<td>$1/2 - p_1$</td>
</tr>
<tr>
<td>buy at firm 2:</td>
<td>$1/2 - p_2$</td>
</tr>
<tr>
<td>search at firm 1</td>
<td>$-s_1 + \int_A^B \left(qu_1 + \frac{1-q}{2} - p_2\right)du_1 + \int_B^c (u_1 - p_1)du_1$</td>
</tr>
<tr>
<td>pass:</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 14: Expected consumer utilities in stage 1**

Assume $p_2 < p_1$:

Then buy1 is dominated by buy2 and consumers choose between buy2, search, and pass.

Assume $p_2 < 1/2$:

Then the only effect that $s_1$ has is to determine whether consumers buy at firm 2 or search at firm 1. Since firm 1 gets no demand if consumers buy at firm 2, setting $s_1 \neq 0$ or $s_1$ large cannot be optimal. Firm 1 has no incentive to deviate from the equilibrium actions characterized in C2.

Assume $p_2 \geq 1/2$:

Then $s_1$ determines whether consumer pass or search at firm 1. Clearly search is better for firm 1. Hence firm 1 has no incentive to deviate from the equilibrium actions characterized in C2.

Assume $p_2 \geq p_1$:

Then buy2 is dominated by buy1 and consumers choose between buy1, search at firm 1, and pass.

Assume $p_1 > 1/2$:

Clearly firm 1 must allow search to obtain any demand from consumers. Hence firm 1 has no incentive to deviate from the equilibrium actions characterized in C2.

Assume $p_1 \leq 1/2$:

Then firm 1 gets all demand without allowing search, i.e. setting $s_1$ low and permitting search is bad for firm 1. Firm 1 must set $s_1$ so that the expected value of buy1 exceeds that of search
at firm 1. However, now firm 2 has incentives to $e$ undercut firm 1 because firm 2 makes 0 profits in this arrangement. The result will be $p_1 = p_2 = e$ and $\Pi_1 = \Pi_2 = 0$. Firm 1 has thus no incentive to deviate from the equilibrium actions characterized in C2.

Assume $s_1 \geq s_2$:

Then consumers will search at firm 2 first - if they search at all.

Assume $p_1 \geq p_2$:

Then buy1 is dominated by buy2 and consumers choose between buy2, search at firm 2, and pass. Assume that the expected value of buy2 exceeds the expected value of search at firm 2. Then firm 1 gets no demand at all. Firm 1 prefers to set $s_1 = 0$, i.e. firm 1 has no incentive to deviate from the equilibrium actions characterized in C2.

Assume that the expected value of search at firm 2 exceeds the expected value of buy2. Then all consumers search and we are at stage 2. Conditional on being in stage 2, however, we know that the best profits that can be achieved are those that firm 2 get in the proposed equilibrium. Since firm 1’s profits in the proposed equilibrium are higher that firm 2’s, firm 1 has no incentive to exchange roles with firm 2. Hence firm 1 will prefer to set $s_1 = 0$, i.e. firm 1 has no incentive to deviate from the equilibrium actions characterized in C2.

Assume $p_1 < p_2$:

Assume that the expected value of search at firm 2 exceeds the value of buy1. Then all consumer will search and get to stage 2. We then have the situation of our proposed equilibrium, only with exchanged roles between firms 1 and 2. Conditional on being in stage 2, however, we know that the best profits that can be achieved are those that firm 2 gets in the proposed equilibrium. Since firm 1’s profits in the proposed equilibrium are higher that firm 2’s, firm 1 has no incentive to exchange roles with firm 2. Hence it will prefer to set $s_1 = 0$, i.e. firm 1 has no incentive to deviate from the equilibrium actions characterized in C2.

Assume that the value of buy1 exceeds the expected value of search at firm 2. Then firm 1 gets all demand and firm 2’s profits are $\Pi_2 = 0$. Now firm 2 has incentives to $e$ undercut firm 1. The result is $p_1 = p_2 = e$ and $\Pi_1 = \Pi_2 = 0$. Firm 1 has thus no incentive to deviate from the equilibrium actions characterized in C2.
9. Conclusion

I have shown that neither firm 1 nor firm 2 have an incentive to deviate from the proposed equilibrium for \( c = \frac{3}{10}, \ q = \frac{2}{5} \). Consumers behave optimally.

Hence proposition 3 holds. Q.E.D.

Proof of proposition 4:

Equilibrium prices and profits from the equilibrium case C2 in the proof of proposition 3 were:

\[
p_1^* = 0.3773 \quad p_2^* = 0.3546
\]

\[
\Pi_1^* = 0.02989 \quad \Pi_2^* = 0.01837
\]  

(60)  

(61)

In contrast to the equilibrium with commitment it is now assumed that firms in the price game firms are not committed to maintaining a particular level of search cost.

The argument of the proof is as follows: The firm that charges positive search cost (firm 2) can do strictly better by increasing its price just below the one of its competitor and allowing all consumers to search at no cost. By allowing them to search at no cost, firm 2 can get the fraction \( q \) of consumers (for whom \( u_1 = u_2 \)) to purchase at firm 2. If its search cost were positive, some consumers would only be willing to buy or search at firm 2 if its price were lower than firm 1's.

Claim:

Firm 2 has an incentive to deviate from \( p_2^* = 0.3546 \) and \( s_2^* = ((1 - q)(1 - p_2^*)^2)/2 \) to \( p_2 = 0.3773 - \varepsilon \) and \( s_2 = 0 \) where \( \varepsilon \) is infinitesimally small.

Proof: Since \( s_1^* = 0 \), and if firm 2 sets \( s_2 = 0 \), then all consumer will search both firms. This means that consumer will make all purchase decisions at stage 3. Recall that in stage 3 a fraction \( q \) of consumers will have determined that \( u_1 = u_2 \). Since firm 2 prices slightly lower than firm 1 all consumer for which \( u_1 = u_2 > 0.3773 - \varepsilon \) will purchase from firm 2. Those \( (1 - q) \) consumer for which \( u_1 \neq u_2 \) will split among the firms as described in “Derivation of Stage 3 demands” on page 57. The resulting profits are:

\[
\Pi_1 = 0.0066 \quad \Pi_2 = 0.4515
\]  

(62)
Clearly firm 2 has an incentive to deviate from the equilibrium actions characterized in case C2.

Since firm 1's demand increases discretely by pricing \( \varepsilon \) below firm 2, firm 1 has an incentive to do so and firm 2 likewise. This does, however, not result in a Bertrand equilibrium since
\[
\Pi_i(\varepsilon, 0) > \Pi_i(0, 0) = 0.
\]
Hence, no pure strategy equilibrium exists.

Q.E.D.
Essay 2

Expanding to the Internet:
Pricing and Communications Strategies
When Firms Compete on Multiple Channels*

* This essay has benefited greatly from many discussions with Meghan Busse and especially Birger Wernerfelt. I also thank Erik Brynolfsson, John Hauser, Jeromin Zettelmeyer and seminar participants at MIT and Cornell University for helpful comments.
1 Introduction

The amount of information available to customers and the ease with which they can obtain that information differs across sellers and products. It is widely believed that the emergence of the Internet as a communications channel will lead to an information explosion and perhaps also to more competitive retail markets. Some of the implications of these expected developments are lower prices, and more entry into certain product categories. Business and trade magazines have stated early on:

"[On-line shopping] allows merchandisers to serve individual customers better by getting information on relevant products and services to them exactly when they need them." (American Demographics, September 1994)

"Window browsing down Fifth Avenue on a warm night aside, most people don't go shopping to entertain themselves. They shop for information." "Stores have thrived because they were the only places consumers could go comparison shopping. The flow of information - on prices and an item's information - was imperfect." "The barriers to good information are crashing down." (Forbes, May 24th, 1994)

A first look at evidence from the Internet points towards this direction. There are in fact numerous cases where firms whose main presence is on the Internet offer products at substantially reduced prices compared to firms that are considered to price competitively over retail outlets or mail order. The "Internet Shopping Network" (a subsidiary of the Home Shopping Network), for example, offers many software products significantly cheaper than the competitively priced mail order giant PC/MacWarehouse. In a comparison of seven randomly picked popular software products for the Macintosh, prices at the Internet Shopping Network were between 4 and 16% cheaper, averaging 11% cheaper than at MacWarehouse (see appendix).

This is a substantial price difference given that prices between software mail order companies rarely vary more than a few percent. There is also evidence that some firms post large quantities of information on the Internet that would be quite difficult for consumers to obtain over retail outlets. Hewlett Packard, for example, describes each of its printers over about four pages with a photo, a detailed list of features, a "value proposition," and very detailed technical specifications.

Using this type of evidence for a projection of the future direction of the Internet ignores at least two issues. First, a firm's strategy on the Internet might be affected by its business over retailing, mail order etc. Firm strategies are likely to be interdependent over different channels of information and distribution. Second, a firm's optimal pricing and communications strategy
might vary with the extent to which potential customers make use of the Internet. The reach of the Internet might matter.

The goal of this paper is to analyze in which way the existence and the size of the Internet affect the optimal pricing and communications strategies that firms might want to pursue. It takes into account that firms’ dealings on the Internet could be dependent on the strategies these firms pursue over other outlets.

The following example illustrates why the interdependence of firm strategies across sales channels and the reach of the Internet matter. Consider two competing stores in the Boston area. “Tweeter etc.” is a specialty audio equipment store chain, “Lechmere” is one of the region's mass merchandisers. In addition to many other products, Lechmere also sells audio equipment. These two stores carry similar product lines despite the fact that they pursue quite different strategies in terms of pricing and the amount of help they extend to consumers. Tweeter facilitates consumers’ search by providing listening rooms in which consumers can evaluate equipment and by having well informed sales staff. At Lechmere it is very difficult for consumers to evaluate a product. Lechmere does not allow consumers to listen to most of their audio equipment and is known for scarce and badly informed sales personnel. Tweeter charges somewhat higher prices than Lechmere. In this example I will first sketch why it is good for Tweeter and Lechmere to facilitate consumers' search to different degrees. This will explain why they can sustain different price levels and make positive profits. I will then look at a hypothetical situation in which Tweeter has established a very informative presence on the Internet and Lechmere has to decide which communications and pricing strategy to pursue in following Tweeter to the Internet. We will consider two scenarios. In the first scenario we will suppose that only 10% of interested customers that would consider purchasing at these two stores can also use the Internet. In the second scenario we will assume that the vast majority, say 80% of consumers can use the Internet in addition to Tweeter's and Lechmere's regular stores. Suppose further that the two firms compete in amplifiers.¹ Since consumers know that each firm offers the same product across sales channels, arbitrage restricts firms to charging the same price in both the regular store and the Internet.

*Competition in regular stores.*² If Lechmere has a lower price than Tweeter, what prevents consumers from simply listening to the amplifier at Tweeter, determining whether or not they

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¹ This paper does not consider within channel issues. Tweeter and Lechmere should be thought of as captive retailers that each only sell the product of one manufacturer respectively. Alternatively this can be viewed as the competition between two manufacturers.

² See essay 1.
like it and then purchasing the amplifier at Lechmere at a lower price? One important reason is that this is made difficult by manufacturers' practice of assigning different model numbers to identical products (Shugan, 1989). Although consumers can guess that the products might be similar, there is some uncertainty whether a consumer would be purchasing exactly the same product at Lechmere as the product she liked at Tweeter. Differentiation on the ease with which consumers can evaluate products at both firms' stores allows Tweeter and Lechmere to avoid intense price competition. To illustrate this point consider why it would not be optimal for the two firms to pursue symmetric strategies. If Tweeter were imitating Lechmere and making it difficult for consumers to evaluate their amplifier, consumers would have no information other than price on which to base a purchase decision. They would know of no tangible difference between the product at Lechmere and the product at Tweeter. This would make them strictly choose the cheaper of the two products and in the process initiate intense price competition between Tweeter and Lechmere. If Lechmere were imitating Tweeter and giving consumers the opportunity to precisely evaluate their amplifier, consumers would have all the information they need to decide which amplifier to buy. This leads to a problem for both Tweeter and Lechmere if there is a portion of consumers that after listening to both amplifiers decide that they can't tell the difference and therefore choose the lower priced product. Competing for these consumers will start intense price competition, similar to the case where consumers had no information at all. If consumers are only informed by Tweeter but not by Lechmere, both firms can sustain higher profits by softening price competition. Depending on how much they liked the amplifier at Tweeter, consumers will decide whether to buy at Tweeter at a higher price but knowing what they will get or to buy at Lechmere at a lower price but with a chance that the product will be different from the one at Tweeter. If consumers, after evaluating the amplifier at Tweeter, have different valuations for it, a change in prices by either Lechmere or Tweeter will induce only a few consumers to switch from one to the other store. This "insensitive" reaction of consumers' demand softens price competition because none of the stores can capture the whole market by slightly lowering prices.

*Competition on the Internet in addition to regular stores.* Suppose that 10% of consumers that consider frequenting Tweeter's and Lechmere's regular stores can use the Internet. We think of Tweeter as having built up a very informative web presence. What should Lechmere do? If Lechmere does not provide useful information about the amplifier on the Internet, it essentially replicates its strategy in the regular store for the Internet. The competition between the two firms does not change. If, on the other hand, Lechmere imitates Tweeter on the Internet by helping those 10% of consumers evaluate its amplifier, it can gain additional customers. In ad-
dition to consumers that determine that they prefer Lechmere's over Tweeter's amplifier, Lechmere's lower price will also attract those consumers that could not tell the difference between the two amplifiers. If Lechmere had imitated Tweeter through its regular store, it would have induced a price cut by Tweeter (and a subsequent price war), since too many of Tweeter's customers would have wanted to switch over to Lechmere. By informing on the Internet only, Lechmere can avoid such a competitive reaction by Tweeter. Tweeter will not lower prices because it has too much revenue to lose from the 90% of consumers that cannot evaluate Lechmere's amplifier.

Suppose now that Lechmere could help 80% of consumers evaluate its amplifier, meaning that the Internet reached 80% of consumers. Then a large number of Tweeter's customers would want to switch over to Lechmere and Tweeter would react by cutting its price and initiate a price war. As a result, Lechmere will choose not to facilitate consumers search over its Internet presence.

This example shows, first, that Lechmere's optimal Internet strategy depends on the reach of the Internet and, second, that its strategy is based on the strategies that the competing firms follow in their regular stores.

The first main result of this paper has just been illustrated in the preceding example: The amount of information provided by firms depends on the reach of the Internet. If the Internet is "small", more information is likely to be provided than through a conventional channel only (such as retailing). If the Internet is "large", there will not be more information provided than through a conventional channel only. The second main result is derived from modeling entry into a product category if competition is taking place on the Internet as well as a conventional channel: entry is made more attractive by the Internet but only as long as the Internet's reach is small. The derivation of the main results of this paper does not assume properties of the second channel that are idiosyncratic to the Internet. Most of the findings are thus generalizable to other channels as long as they can be used to selectively inform consumers. Nevertheless, the results are of particular interest to the Internet since its size is rapidly changing.

This paper contributes to the marketing literature in three areas. First, it points out possible directions the Internet might take. It predicts future developments regarding the amount of information that might be provided over the Internet, firms' pricing strategies, the profitability

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3 This wording equates "providing more information" with "facilitating consumers search". This need not always be the case (Jacoby, Spiller, and Berning (1974), Jacoby, Spiller, and Kohn (1974), Staelin and Payne (1976), Jacoby (1984)). I equate the two for expositional reasons only. Below, I will abstract from this issue by modeling firms as influencing consumers' search cost directly.
of competing on the Internet, and incentives for entry.\(^4\) Second, the paper derives implications for managers. It discusses whether conventional strategies should be duplicated onto the Internet, whether firms want to limit information, whether there are first mover advantages to being on the Internet and how long-term profits might be affected. Third, this paper makes some theoretical contributions. It introduces channels as a means of segmenting consumers by selectively informing them. It also shows that manipulating consumer information can be a powerful competitive tool. Finally, it shows that the analysis of competition between firms can be strongly affected by considering all channels over which they compete.

Other literature on marketing aspects of the Internet deals mainly with marketing implications from the way the Web changes the interaction with consumers (Chatterjee and Narasimhan, 1994; Hoffman and Novak, 1995; Hoffman, Novak and Chatterjee, 1995; Hoffman and Novak, 1996). Economists have focused on issues of access pricing and the pricing of information goods (MacKie-Mason and Varian, 1994; Varian, 1995a, 1995b, 1996; Bakos and Brynolfsson, 1996). Researchers in information technology have discussed electronic markets (Malone, Yates and Benjamin, 1987) and their effect of lowering search costs (Bakos, 1991).

While the role of marketing communications includes both informing and persuading consumers (e.g. Rothschild, 1987, p.4; Kotler, 1994, p. 598, 615), I restrict myself to the information aspect of the communications mix. I will look at consumers as active searchers for information, although they incur costs in this process. Like Wernerfelt (1994, 1996) I deal with efficiency in marketing communication but concentrate on ways in which firms can exploit multiple channels to strategically communicate inefficiently.

There is a large literature on search and search costs. In this paper, consumers make conscious decisions about the amount of information for which they search and consequently the level of search cost they incur (Bettman, 1979; Shugan, 1980; Meyer, 1982; Ratchford, 1982; Hagerty and Aaker, 1984; Johnson and Payne, 1985; Hauser and Wernerfelt, 1990; Hauser, Urban, Weinberg, 1993). Other literature deals with sources of information that consumers use, consumers’ search activities, the amount of search undertaken, and the benefits of consumer search. The economics literature addresses search incentives, the effect of search costs on prices, and the effects of various sources of uncertainty.\(^5\)

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\(^4\) There are many different characteristics of the Internet that will influence the way firms price and communicate. I concentrate on only two, first the fact that the Internet is particularly well suited as a medium of information, and second that it sets a clear dividing line as to who can use it and who cannot.

\(^5\) See essay 1 for references on these topics.
The remainder of this paper proceeds as follows. Section 2 provides the background for the base case in the preceding example by modeling competition on one channel. The core of the paper is section 3. It introduces and solves the duopoly model with a conventional channel as well as the Internet. Section 4 adds a third firm to the 2 channel model as a Stackelberg follower and analyzes this firm's incentives to enter a product category. In an extension to the model, section 5 analyzes the effect of consumers that react heterogeneously to search costs. Section 6 discusses the results of the paper and presents some possible directions for the Internet as well as implications for managers. Section 7 concludes the paper.

2 Duopoly in one channel

This section begins by explaining how consumers are modeled and points out the key characteristics of the resulting demand functions. Consumer demands are then simplified while retaining their key features. The third part of this section solves the simplified model to derive an asymmetric equilibrium for competition in one channel.

2.1 Derivation of consumer demand from first principles

This exposition closely follows essay 1. Assume consumers are interested in purchasing one unit of a product in a particular product category. Consumers know the prices $p_1, p_2$ of the products offered by firms 1 and 2 respectively. Consumers do not know ex ante the gross utility $u_i$ they will derive from the product of firm $i$. They are, however, aware of the distribution from which their reservation prices are drawn. A consumer can find out her reservation price $u_i$ at firm $i$ by evaluating the firm’s product. In doing so she incurs search cost $s_i$. Consumers’ reservation prices are distributed uniformly between 0 and 1. Consumers’ willingness to pay for firm 1’s and firm 2’s product are $u_1$ and $u_2$ respectively. These are identically but not necessarily independently distributed. For simplicity I assume the following correlation structure between reservation prices $u_1$ and $u_2$: if a consumer searches at firm $i$ and learns about her reservation price $u_i$ then with probability 1/2 her reservation price at firm $j$ is identical to the reservation price she just learned, i.e. $Pr(u_j = u_i) = 1/2$. With probability 1/2 the reservation price at firm $j$ is different in which case her best estimate about her reservation price at firm $j$ is her prior.  

---

6 I will use “reservation price” or “willingness to pay” synonymously with $u_i$.

7 By continuity of the distribution of reservation prices “different” means the same as “independent of the reservation price of firm $i$."

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The consumers' possible actions are illustrated in figure 1:

![Diagram of consumer decisions]

A consumer has the option to buy from either firm without any search, to search either firm or to pass altogether. If she has decided to search at firm $i$ she knows her reservation price $u_i$ at that firm as well as her reservation price $u_j$ at firm $j$ with probability $1/2$. With this knowledge she can decide to buy at either firm, to pass, or to search at firm $j$. If the consumer searches at firm $j$ she has full information, i.e., she knows $u_i$ and $u_j$ and decides to buy from one of the firms or to pass.

A familiar result from the industrial organization literature is that if firms compete on the basis of price alone, demand will be discrete; i.e., all consumers will buy from the firm with the lowest price.\(^8\) This is undesirable for firms because it leads to a result of marginal cost pricing and zero profits (Bertrand equilibrium). In much of the literature, this result is avoided by distributing consumers along a Hotelling line and positioning firms away from each other geo-

---

\(^8\) This model requires that there exist a discrete mass of consumers that, after having searched both firms, have identical reservation prices. However, it is not essential for the results of this paper that $Pr(u_i = u_j) = 1/2$. The applicability of the model is thus restricted to product categories where some consumers are in fact indifferent between the products once they have acquired all necessary information. One example might be audio equipment where many consumers cannot tell the difference between the sound of amplifiers of competing manufacturers.

\(^9\) This assumes that firms compete in pure strategies.
graphically or in product space. This differentiation makes the goods no longer perfect substitutes, and demand becomes continuous in price.

The model in this paper has a similar outcome, but it is a result of the information structure. If there were no search, demand would be discrete in the first stage of the game, in which consumers' willingness to pay at both firms are unknown and the only information available is price. However, as consumers realize the outcomes of search, they formulate their utilities as conditional expected values which are continuous in price, which implies that demand is continuous in price.

Suppose for a moment that consumers had to decide between purchasing at firms 1 and 2 in stage 1. Since they don't have any information that might differentiate the products, consumers' expected value for the products of firms 1 and 2 are identical. Irrespective of their true reservation price (because it is unknown to them at this stage) consumers will choose to purchase from the firm with the lower price, meaning that the firm with the lower price will get the entire demand. This is the classic Bertrand result.

Now suppose that consumers have searched for firm 1's product and are consequently at stage 2 (see figure 1). Consumers now know their reservation price at firm 1 and know that their reservation price at firm 2 corresponds to that at firm 1 with probability 1/2. Depending on a consumer's utility from buying at firm 1, $u_1$, she will either buy at firm 1, buy at firm 2, or search for the product of firm 2. The proportion of realizations of $u_1$ for which one or the other choice is optimal will change continuously with prices $p_1$ and $p_2$. Thus, demand is now a continuous function of prices.

Suppose finally that consumers decide to continue searching. At stage 3 (see figure 1) they know both reservation prices, $u_1$ and $u_2$. Half of the consumers find that they like the products of the two firms equally well, the rest will find that one is preferable to the other. Those consumers with identical reservation prices across firms will buy from the firm with the lower price. Consumers with different reservation prices will buy at firm 1 as long as they get higher surplus from firm 1. Overall, the firms' demand will be continuous where $p_i > p_j$ (because the demand for 1/2 of the consumers is continuous in price), with a discrete jump at $p_1 = p_2$ (because the 1/2 of the consumers with equal reservation prices switch firms at this price).

From this discussion we can identify the key properties of the resulting demand functions.\(^\text{10}\) The key insight is that the elasticities of demand with respect to price differ according

\(^{10}\) I don't discuss properties that are standard assumptions about demand functions ($\frac{\partial D_j}{\partial p_i} < 0$, $\frac{\partial D_j}{\partial p_j} > 0$ for example).
to consumers' information. If consumers have not searched at all (stage 1 in figure 1), the elasticity of demand will be 0 for \( p_1 \neq p_2 \) and infinity at \( p_1 = p_2 \). If they have searched one firm only (stage 2 in figure 1), there are no discontinuities in demand, but the elasticity of demand with respect to price is lower for the firm that has been searched than for the firm that has not been searched. Finally, if both firms have been searched, both firms' demand have the same, non-zero demand elasticity with respect to price for all \( p_1 \neq p_2 \) and infinite demand elasticity at \( p_1 = p_2 \).

### 2.2 Simplified consumer demand

The following simplified\(^{11}\) demand functions have the same properties as the demand functions implicit in section 2.1.

\[
D_i(z_i, z_j, \nu) = \begin{cases} 
(1 - \beta)(1/2 - (2 - z_i)p_i + (2 - z_j)p_j) & \text{for } p_i > p_j \\
\beta + (1 - \beta)(1/2 - (2 - z_i)p_i + (2 - z_j)p_j) & \text{for } p_i \leq p_j 
\end{cases}
\]  

(1)

where

\[
\begin{align*}
&\begin{align*}
&z_i = 0 \\
&z_j = 0
\end{align*} & \beta = 1 & \text{if consumers have not searched} \\
&\begin{align*}
&z_i = 1 \\
&z_j = 0
\end{align*} & \beta = 0 & \text{if consumers have searched firm } i \\
&\begin{align*}
&z_i = 1 \\
&z_j = 1
\end{align*} & \beta = 1/2 & \text{if consumers have searched both firms}
\]

In equation 1, \( \beta \) stands for the fraction of consumers that consider the products of firms \( i \) and \( j \) to be identical. These consumers have the same (expected) willingness to pay for either firm's product. Variables \( z_i \) and \( z_j \) capture whether consumers have searched at firm \( i \) and \( j \) respectively. Consumers have searched at firm \( i \) iff \( z_i = 1 \).

The main characteristics of the demand structure in the previous section are captured by varying \( z_i, z_j, \beta \) as functions of the firms that consumers have searched. First we will determine how many consumers consider the competing products to be identical and how many differ in the utility they derive from each of the products. Second, we will specify the elasticity of demand with respect to price for each of the resulting groups.

The fraction of consumers who consider the products to be identical depends on the number of firms that they have searched. If they have searched no firm, consumers consider the products of both firms identical since they have no information other than price\(^{12}\) that might dif-

\(^{11}\) The simplifications will be highlighted below.  
\(^{12}\) Note that price does not carry any signal about consumers' willingness to pay.
ferentiate the products, hence $\beta = 1$. If consumers have searched one firm only, they have no uncertainty about their willingness to pay at that firm but are only imperfectly informed about their willingness to pay at the other firm. Their choice will be determined by their individual valuation of the product at the firm they searched, the expectation of their valuation at the other firm and both firms' prices. As a result, each consumer will derive a different utility from each product, hence $\beta = 0$. If consumers have searched both firms, a fraction of them will find out that their valuations of both products are identical. The remaining consumers will have determined that they have different valuations for either product. From the probability distributions assumed in section 2.1 it then follows that $\beta = 1/2$.

The elasticity of demand for each of these three cases can be seen as follows. Consider first the case where consumers have not searched either firm. Then

$$D_i = D_i(0, 0, 1) = \begin{cases} 0 & \text{for } p_i > p_j \\ 1 & \text{for } p_i \leq p_j \end{cases}$$

Clearly the demand of such consumers is infinitely elastic with respect to price at $p_i = p_j$.

If consumers have searched firm $i$ only, their demand becomes a continuous function of price and the elasticity of both firm $i$'s and firm $j$'s demand with respect to firm $i$'s price is lower than that with respect to firm $j$'s price:

$$D_i = D_i(1, 0, 0) = 1/2 - p_i + 2p_j$$
$$D_j = D_j(0, 1, 0) = 1/2 - 2p_j + p_i$$

If consumers have searched both firms, half of the consumers decide based on which price is higher only, while the other half's demand is continuous in price.

$$D_i = D_i(1, 1, 1/2) = \begin{cases} (1/2)(1/2 - p_i + p_j) & \text{for } p_i > p_j \\ 1/2 + (1/2)(1/2 - p_i + p_j) & \text{for } p_i \leq p_j \end{cases}$$

Clearly, the elasticity of demand with respect to price is non-zero for $p_i \neq p_j$ and infinite for $p_i = p_j$. The resulting demands in equations 2 and 3 are practically identical to those implied by section 2.1. Only the function in equations 4 is a simplification since it linearizes demand.

---

13 For a discussion of the importance of $\beta$ see footnote 8 and the text on page 108.
In summary, demand depends on the specific firm and the total number of firms searched. Demand is very elastic if no firm is searched, a little elastic if only one firm is searched and fairly elastic if both firms are searched. In addition, demand is less elastic in the price of a firm that has been searched than in the price of a firm that has not been searched. These correspond exactly to the desired properties described in section 2.1.

2.3 Firms

Firms have two choice variables, price $p_i$ and search cost $s_i$. I assume that firms set search costs and prices sequentially and know each other’s search costs at the time they compete in prices. In the (second step$^{14}$) price game firms remain committed to the level of search costs that they chose in the first step.$^{15}$

For simplicity normalize marginal cost to 0. Firms’ second and first step maximization problems are:

\[ \begin{align*}
\text{2nd step: } & \max_{p_i} \Delta_i(p_i, s_i, p_j, s_j) \\
\text{1st step: } & \max_{s_i} \Delta_i(p_i^*(s_i, s_j), p_j^*(s_i, s_j))
\end{align*} \]

(5) \hspace{1cm} (6)

In the preceding equations $\Delta_i(*)$ is the total demand for firm $i$’s product. Prices should be interpreted as margins. Further assume that firms can either set search costs high ($h$) or low ($l$), $s_i \in \{l, h\}$. If a firm sets $s_i = l$ consumers will search at that firm, i.e. $z_i = 1$. If $s_i = h$ consumers will not search and $z_i = 0$.

2.4 Solution

I solve for subgame perfect equilibria in the usual way. First I will calculate step 2 price equilibria for each combination of search costs that firms 1 and 2 can choose in the first step of the game, $(s_1, s_2) \in \{(h, h), (h, l), (l, h), (l, l)\}$. Then, using the payoffs from the price subgame I calculate the subgame perfect equilibria of the overall game. Throughout this paper I restrict

---

$^{14}$ In this paper “stage” refers to consumers’ decision tree and “step” to the overall game.

$^{15}$ In essay I I consider a second version of the game in which I assume that firms set search costs and prices simultaneously (or if they set them sequentially are not able to commit to first step search costs).
myself to pure strategy equilibria in both search costs and prices. Mixed strategy equilibria in search costs don't have a clear interpretation. The important insights from this model can be obtained with pure strategies in the price game, while at the same time simplifying the analysis. Proposition 1 replicates one of the main results of essay 1 with the simplified model used in this paper. The result states that firms will want to differentiate in search costs in order to avoid intense price competition in the second step.

*Proposition 1:* \( s_i = l, p_i = 1/2, s_j = h, p_j = 1/4 \) characterizes the unique subgame perfect equilibrium in pure strategies of the overall game.

*Proof of proposition 1: see appendix*

The intuition for this result is most easily understood by looking at consumer demand as a function of search costs. If \((s_1, s_2) = (h, h)\) consumers will search neither firm. Consumer demand will be given by equation 2. This is the classic case of Bertrand competition. Since undercutting the competitor's price by an infinitesimal amount will generate a discontinuous jump in demand, firms will compete prices down to marginal cost \((p_1 = p_2 = 0)\). If \((s_1, s_2) = (l, l)\), demand will be given by equation 4. Although demand elasticities are non-zero for all \(p_1 \neq p_2\), demand still jumps at \(p_1 = p_2\), resulting in an incentive for any firm with a higher price to undercut the competitor's price slightly in order to gain the additional demand of \(1/2\). However, in contrast to \((s_1, s_2) = (h, h)\) this case will not result in marginal cost pricing. Analogous to the familiar result from the sales promotion literature, this case has no equilibrium in pure strategies (Varian, 1980; Narasimhan, 1988; Lal, 1990; Raju, Srinivasan, and Lal, 1990; Hess and Gerstner, 1992). Setting \((s_i, s_j) = (l, h)\) manipulates consumers' information in a way that demand becomes continuous in prices. The result is a pure strategy equilibrium of the price game with positive profits. Clearly this last case is the unique subgame perfect equilibrium of the overall game.  

---

16 See page 108 for a discussion of the pure strategy restriction.
17 Any price above marginal cost will be undercut by the competitor. At marginal cost, however, a firm will want to raise its price since half of consumers don't have identical valuations for the products. Some of these consumers will buy, even although the firm that raised price above marginal cost now has the higher price. But then any price above marginal cost will again be undercut by the competitor.
18 As long as \(\beta\) is sufficiently large the result that firms will play \((s_i, s_j) = (l, h)\) instead of \((s_1, s_2) = (l, l)\) will not change by allowing mixed strategies in prices. See "Notes on mixed strategy equilibria" in the appendix for a sketch of the proof.
3 Duopoly in two channels

To illustrate how firm strategies might be affected by the Internet I introduce a second channel. I will refer to it as the “electronic channel” to contrast it to the “conventional channel”.

3.1 Firms

Firms compete in prices and the search cost that consumers have to incur if they want to search. Firm $i$ sets one price $p_i$ for both channels. This reflects that since consumers know that each firm offers the same product across both channels, arbitrage between channels would make it hard to sustain different prices on the two channels. Firms can, however, set search cost differently for both channels. Let $s_i^f$ and $s_i^c$ be the search costs that firm $i$ sets on the conventional channel and the electronic channel respectively. Search costs can be either high or low, $s_i^f, s_i^c \in \{h,l\}$.

As in the one channel case, firms compete sequentially, i.e. in a first step they set search costs $s_i^f$ and $s_i^c$. In a second step they compete in prices $p_i$. Firms’ optimization problem is analogous to equations 5 and 6 with the addition of having to optimize over a search cost variable for each of the two channels. Consumers’ demand is a function of prices and the search costs that each firm chooses to set.

3.2 Consumers

Assume that all consumers have access to the conventional channel. In addition, a fraction $\alpha$ of consumers can use the electronic channel in order to search for information and purchase the product. All consumers who can use the electronic channel also have access to the conventional channel.

<table>
<thead>
<tr>
<th>Conventional channel</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic channel</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td></td>
</tr>
</tbody>
</table>

If firm $i$ sets $s_i^f = l$ consumers will search for the product of firm $i$. If consumers have access to the electronic channel and search costs are low, $s_i^f = l$, consumers will search the electronic channel. There is no additional information gained from having searched one firm on
both channels. Consumers who do not have access to the electronic channel will only search for firm $i$'s product if $s^*_i = l$, i.e. if firm $i$'s search costs are low on the conventional channel.

Demand is given by equation 1. In contrast to the one channel case, however, not all consumers will have searched the same number of firms. If, for example, firm 1 sets $(s^*_1, s^*_2) = (l, l)$ and firm 2 sets $(s^*_1, s^*_2) = (h, l)$ there is a fraction $(1 - \alpha)$ of consumers who will have searched for their willingness to pay at one firm only. Demand will be determined by equation 3, i.e. $D_1 = 1/2 - p_1 + 2p_2$ and $D_2 = 1/2 - 2p_2 + p_1$. The remaining fraction $\alpha$ of consumers will have searched both firms. Demand for these consumers is specified by equation 4, i.e. $D_1 = (1/2)(1/2 - p_1 + p_2)$ and $D_2 = 1/2 + (1/2)(1/2 - p_2 + p_1)$ (for $p_1 > p_2$).

### 3.3 Solution

I will solve for pure strategy subgame perfect equilibria in the usual way. First I calculate the price equilibria of the second step price game for all possible first step strategies, then I determine the subgame perfect equilibria of the overall game.

In order to formulate firms' profit functions in the second step, it is necessary to determine which search cost strategies will be used in the first step. Firm $i$'s search costs come from the following set.

$$(s^*_1, s^*_2) \in \{(h, h), (l, h), (h, l), (l, l)\}$$

Note first that either $(s^*_1, s^*_2) = (l, h)$ or $(s^*_1, s^*_2) = (l, l)$ is redundant since everyone in the electronic channel can also use the conventional channel. Allowing consumers to search on the conventional channel is thus the same as allowing them to search on both channels. This leaves three strategies that we have to consider for each of the firms.
Figure 2: Relevant search cost strategies by firms

<table>
<thead>
<tr>
<th>(s_i^<em>, s_j^</em>)</th>
<th>(h, h)</th>
<th>(h, l)</th>
<th>(l, l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_i^<em>, s_j^</em>)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h, h)</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>(h, l)</td>
<td></td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>(l, l)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Strategies [(h, h), (h, l)], [(h, h), (l, l)], and [(h, l), (l, l)] don't have to be considered since they correspond to [(h, l), (h, h)], [(l, l), (h, h)], and [(l, l), (h, l)] respectively with firm names i and j exchanged. Calculating price equilibria for the six remaining cases and comparing payoffs leads to the main result of this paper.

**Proposition 2**: The set of subgame perfect equilibria in pure strategies is characterized by the following \((s_i^*, s_j^*)\), \(p_i\), \(s_j^*\), \(p_j\):

For \(\alpha = 0\): \(s_i^* = l\) \(p_i = 1/2\)
\(s_j^* = h\) \(p_j = 1/4\)

For \(0 < \alpha \leq \alpha_1^*\):
\((s_i^*, s_j^*) = (l, l)\) \(p_i = (6 - \alpha)/(6(2 - \alpha))\)
\((s_i^*, s_j^*) = (h, l)\) \(p_j = (6 + \alpha)/(6(4 - 3\alpha))\)

For \(\alpha_1^* < \alpha < \alpha_2^*\):
\((s_i^*, s_j^*) = (l, l)\) \(p_i = 1/2\)
\((s_i^*, s_j^*) = (h, h)\) \(p_j = 1/4\)

For \(\alpha_2^* < \alpha \leq 1\):
\((s_i^*, s_j^*) = (h, l)\) \(p_i = 1/6 + 1/(3\alpha)\)
\((s_i^*, s_j^*) = (h, h)\) \(p_j = -1/12 + 1/(3\alpha)\)

where \(\alpha_1^* = -24 - 21 \sqrt{3} + \frac{\sqrt{4998 + 2040\sqrt{6}}}{2} = 0.27\)

---

19 This is the one channel case, see proposition 1.
and \[ \alpha_2^* = 10 - 2\sqrt{21} = 0.83 \]

Proof of proposition 2: see appendix.

Proposition 2 says that firms’ pricing and search cost strategies depend on the fraction of consumers that, in addition to the conventional channel, can use the electronic channel.

**Figure 3: Equilibrium search costs as a function of \( \alpha \)**

<table>
<thead>
<tr>
<th>Conventional channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic channel</td>
</tr>
<tr>
<td>[(l,l),(h,l)]</td>
</tr>
<tr>
<td>[(l,l),(h,h)]</td>
</tr>
<tr>
<td>[(h,l),(h,h)]</td>
</tr>
</tbody>
</table>

\[ \alpha_1^* \approx 0.27 \quad \alpha_2^* = 0.83 \]

If \( \alpha = 0 \), i.e. there is no electronic channel, we get exactly the result stated in proposition 1. One firm sets search cost low but charges a high price (call this firm 1), the other firm sets search cost high and compensates with a lower price (call this firm 2).\(^{20}\) Firms differentiate in search costs in order to avoid price undercutting in the price game. As the electronic market comes to existence but \( \alpha \) remains small, firm 1 simply replicates its previous search cost strategy onto the electronic channel by setting \( s_1 = l \). Firm 2 will not do the same as it does on the conventional channel. It will set search cost on the electronic channel lower, \( s_2 = l \). Firm 2 has an incentive to offer low search cost because it can gain all those consumers that, after having searched both firms, realize that their willingness to pay for the product at either firm is identical.

When this strategy was applied to the conventional channel it lead to price undercutting in the price game. Why does this not happen on the electronic channel as long as \( \alpha \) is small? Facing demand as described in equation 4 creates an incentive for firm 1 to undercut firm 2’s price by \( \epsilon \) and gain those consumers for whom the products are identical. However, as long as the fraction of consumers that can be gained in that way is small, i.e. \( \alpha \) is small, firm 1’s loss in profit on the conventional channel due to the price cut is higher than the potential gain on the electronic channel.

\(^{20}\) In the graphs I retain the general formulation by referring to firms as \( i \) and \( j \). However, “firm 1” in the text will always correspond to firm \( i \) in the graph. Similarly, “firm 2” in the text will always correspond to firm \( j \) in the graph.
This ceases to hold once the size of the electronic channel exceeds \( \alpha_1^* \). In order to avoid triggering a price war, firm 2 reverts to replicating its conventional channel strategy on the electronic channel. For intermediate values of \( \alpha \), the addition of an electronic channel does not change firms' competitive interaction if compared to the one-channel setup.

As the size of the electronic channel approaches that of the conventional channel, \( \alpha > \alpha_2^* \), firm 1 chooses to deviate from its low search cost strategy on the conventional channel. Without an electronic channel this search cost strategy led to Bertrand competition. For \( \alpha > \alpha_2^* \), however, a large fraction of consumers on the conventional channel have been able to search firm 1 through the electronic channel. Since firm 1's price is higher than firm 2's price, firm 1 does not sell to any of the \((1 - \alpha)\) consumers that have not searched either firm. How can this be advantageous to firm 1? The \((1 - \alpha)\) consumers that have no information about either firm are sensitive only to whether one firm's price is lower than the other firm's price (as long as the price does not exceed consumers' reservation price). This creates an incentive for firm 2 to raise its price. Since the prices of firms 1 and 2 are strategic complements (Bulow, Geanakoplos and Klemperer, 1985), firm 1 reacts by raising \( p_1 \) and reaping more profits from the \( \alpha \) consumers (which still consider purchasing at firm 1). For \( \alpha > \alpha_2^* \) this competitive effect outweighs having lost the fraction of the \((1 - \alpha)\) consumers that bought from firm 1 when it informed them. Firm 1 effectively gives firm 2 some monopoly power in the sense that the \((1 - \alpha)\) consumers will only shop at firm 2 as long as \( p_2 < p_1 \).

An implication of proposition 1 is that the number of consumers that are informed about one or both firms’ products varies with the size of the electronic channel. Corollary 1 captures this insight.

**Corollary 1:** As \( \alpha \) increases, the total number of consumers that are informed about their willingness to pay at firms 1 and 2 is higher than in the absence of an electronic channel. For \( \alpha_1^* < \alpha < \alpha_2^* \) an equal number of consumers are informed about their willingness to pay as would be the case without an electronic channel. For \( \alpha_2^* < \alpha < 1 \) fewer consumers are informed about their willingness to pay than would be if the conventional channel were the only communications channel.

Corollary 1 says that an electronic channel will initially increase the number of informed consumers. A further increase in its size, however, will not lead to more informed consumers than in a conventional channel alone and will ultimately lead to fewer informed consumers.\(^{21}\)

\(^{21}\) For \( \alpha_2^* < \alpha < 1 \) the number of uninformed consumers decreases in \( \alpha \). For \( \alpha = 1 \) the same number of consumers as in the conventional channel alone will be informed.
Profits of the competing firms vary according to the reach of the electronic channel. Proposition 3 characterizes firm profits.

**Proposition 3:** Firm profits corresponding to the equilibria characterized in proposition 2 are:

For $\alpha = 0$  
$\Pi_i^* = 1/4 \quad \Pi_j^* = 1/8$

For $0 < \alpha \leq \alpha_1^*$  
$\Pi_i^* = \frac{(6-\alpha)^2}{72(2-\alpha)} \quad \Pi_j^* = \frac{(6+\alpha)^2}{72(4-3\alpha)}$

For $\alpha_1^* < \alpha < \alpha_2^*$  
$\Pi_i^* = 1/4 \quad \Pi_j^* = 1/8$

For $\alpha_2^* < \alpha \leq 1$  
$\Pi_i^* = \frac{(2+\alpha)^2}{36\alpha} \quad \Pi_j^* = \frac{(4-\alpha)^2}{72\alpha}$

Proof of proposition 3: see appendix

The following figure plots firms’ equilibrium profits and the corresponding search cost strategies as functions of the size of the electronic channel.

**Figure 4:** Equilibrium profits as a function of $\alpha$

Profits of both firms initially rise as the reach of the electronic channel increases. Firm 2 is gaining additional consumers by informing on the electronic channel while offering a lower price than firm 1. Firm 1 benefits from the same effect that leads to increased profits for $\alpha > \alpha_2^*$: by informing consumers on the electronic channel, firm 2 obtains some monopoly
power and can thus price higher, which benefits firm 1. Once the reach of the electronic channel exceeds $\alpha_1^*$, firm strategies, and subsequent profits, mimic the case without an electronic channel. It is only when $\alpha > \alpha_2^*$ that firm 1 can give firm 2 some monopoly power and induce higher prices and profits.

4 Incentives for entry into a product category

The purpose of this section is to analyze whether and in which way incentives to enter a product category change as the reach of the electronic market increases. I will approach this issue by extending the duopoly model in part 3 to three firms and performing comparative statics on the profits of the entrant as $\alpha$ changes. I model a sequential entry game.

4.1 Firms

In the first step the two incumbent firms (firms 1 and 2) set their search costs simultaneously. They take into account that in the second step the entrant (firm 3) will set its search costs and decide whether to enter the market. In the third step all three firms (or two firms if the third firm does not enter) simultaneously compete in prices.

As before, firms each set one price for both channels but can set different search costs for different channels. Each firm can set search costs either high or low, $s \in \{h, l\}$, $g \in \{c, e\}$. Firms' differentiation is modeled via the demand function facing each firm and will be explained in the next section.

4.2 Consumers

In order to model demand for three instead of two firms I need to extend the general demand function in equation 1 on page 87 by specifying how the level of search costs affects whether consumers have a similar expected willingness to pay for the products of each of the firms. I model demand for the case of three firms as follows.

$$D_i(s_i, s_j, s_k, \beta) \equiv \begin{cases} 
(1 - \beta)\left(\frac{1}{2} - (2 - s_i)p_i + \frac{1}{2}((2 - s_j)p_j + (2 - s_k)p_k)\right) & \text{for } p_i > p_j \\
\beta + (1 - \beta)\left(\frac{1}{2} - (2 - s_i)p_i + \frac{1}{2}((2 - s_j)p_j + (2 - s_k)p_k)\right) & \text{for } p_i \leq p_j
\end{cases} \quad (8)$$
where

\[
\begin{align*}
D_i &\equiv D_i(0, 0, 0, 1), & D_j &\equiv D_j(0, 0, 0, 1), & D_k &\equiv D_k(0, 0, 0, 1), & \text{if consumers do not search} \\
D_i &\equiv D_i(1, 0, 0, 0), & D_j &\equiv D_j(1, 0, 0, 0), & D_k &\equiv D_k(1, 0, 0, 0), & \text{if consumers search } i \text{ only} \\
D_i &\equiv D_i(1, 1, 0, \frac{1}{2}), & D_j &\equiv D_j(1, 1, 0, \frac{1}{2}), & D_k &\equiv D_k(1, 1, 0, 0), & \text{if consumers search } i \text{ and } j \\
D_i &\equiv D_i(1, 1, 1, \frac{1}{2}), & D_j &\equiv D_j(1, 1, 1, \frac{1}{2}), & D_k &\equiv D_k(1, 1, 1, \frac{1}{2}), & \text{if consumers search all firms}
\end{align*}
\]

Notice first that demand in equation 8 simply extends equation 1 by taking into account that each firm now has two competitors. If no consumers search, demand behaves identical to the duopoly case. If consumers have searched at firm \( i \) only, demand behaves again identical to the duopoly case. In addition to the standard assumption that knowing about one’s willingness to pay at one firm carries information about one’s willingness to pay at the other firms, this assumes that the consumers don’t consider products of firm \( j \) and \( k \) to be identical in expectations after they have searched \( i \). If consumers have searched two firms, demand between those two firms is modeled analogous to the duopoly case. The third firm, however, faces different demand in the sense that there are no consumers that consider the third firm’s product identical to firm 1’s or 2’s products. In the case where consumers have searched all three firms, demand is modeled analogous to the duopoly case. The addition of the third firm expands overall demand, keeping prices constant. The expansion of demand has no bearing on the results in the next section and only serves computational convenience.

4.3 Results

I solve for subgame perfect equilibria by first calculating price equilibria for all possible combinations of search costs. Let \( S_i = \{(h, h), (h, l), (l, h), (l, l)\} \) be the set of firm \( i \)'s search cost strategies. The strategy set for which price equilibria have to be computed is then given by \( S = S_1 \times S_2 \times S_3 \). Similar to section 3.3 all strategies containing \( (l, h) \), as well as duplicate strategies that can be obtained by exchanging firm subscripts can be excluded without loss of generality. Based on the payoffs from the price games, I calculate the entrant’s reaction function

\[22\] To illustrate this assumption, suppose that consumers think that the products of the two incumbents are more similar to each other than to the entrant’s product. If there is no information known about any of the products consumers will consider them equal in expectation. If, however, consumers have searched the product of one incumbent, they have learned more about the other incumbent’s product than about the entrant’s product. This leads to different posterior expectations.
in search costs. I solve for the optimal search cost strategies of the incumbents conditional on the entrant's reaction functions. The set of subgame perfect equilibria of the game are as follows.

**Proposition 4:** The set of subgame perfect equilibria in pure strategies is characterized by the following \((s^i, s^j), p^i, (s^i, s^k), p^j, (s^j, s^k), p^k:\)

For \(\alpha = 0\)

- Incumbents set \(s^i = l\) and \(s^j = h\).
- Entrant follows with \(s^k = h\).

\[
\begin{align*}
p^i_* &= 1/2 \\
p^j_* &= 1/4 \\
p^k_* &= 1/4
\end{align*}
\]

For \(0 < \alpha \leq \alpha_3^*\)

- Incumbents set \((s^i, s^j) = (l, l)\) and \((s^j, s^k) = (h, l)\).
- Entrant follows with \((s^k, s^j) = (h, h)\).

\[
\begin{align*}
p^i_* &= \frac{200 - 210\alpha + 43\alpha^2 + 2\alpha^3}{5(80 - 98\alpha + 27\alpha^2 + \alpha^3)} \\
p^j_* &= \frac{(10 + \alpha)}{(40 - 29\alpha - \alpha^2)} \\
p^k_* &= \frac{200 - 220\alpha + 52\alpha^2 + 3\alpha^3}{10(80 - 98\alpha + 27\alpha^2 + \alpha^3)}
\end{align*}
\]

For \(\alpha_3^* < \alpha < \alpha_1^*\)

- Incumbents set \((s^i, s^j) = (l, l)\) and \((s^j, s^k) = (h, l)\).
- Entrant does not enter.

\[
\begin{align*}
p^i_* &= \frac{(6 - \alpha)}{(6(2 - \alpha))} \\
p^j_* &= \frac{(6 + \alpha)}{(6(4 - 3\alpha))}
\end{align*}
\]

For \(\alpha_1^* < \alpha \leq \alpha_2^*\)

- Incumbents set \((s^i, s^j) = (l, l)\) and \((s^j, s^k) = (h, h)\).
- Entrant follows with \((s^k, s^j) = (h, h)\).

\[
\begin{align*}
p^i_* &= 1/2 \\
p^j_* &= 1/4 \\
p^k_* &= 1/4
\end{align*}
\]

For \(\alpha_2^* < \alpha < 1\)

- Incumbents set \((s^i, s^j) = (h, l)\) and \((s^j, s^k) = (h, h)\).
- Entrant does not enter.

\[
\begin{align*}
p^i_* &= \frac{1/6 + 1/(3\alpha)}{} \\
p^j_* &= \frac{-1/12 + 1/(3\alpha)}{}
\end{align*}
\]
For $\alpha = 1$, the incumbent set $(s_i^*, s_j^*) = (l, l)$ and $(s_j^*, s_k^*) = (h, h)$.

Entrant follows with $(s_k^*, s_k^*) = (h, h)$.

$$p_i^* = 1/2 \quad p_j^* = 1/4 \quad p_k^* = 1/4$$

where $\alpha_i^* = 10 - 2\sqrt{21}$ (\approx 0.835), $\alpha_j^* = 0.25$.

Proof of proposition 4: see appendix.

Equation 9 lists the search cost strategies (for three firms) for which there exist pure strategy equilibria in the price game for at least some $\alpha$.

$$[(s_i^*, s_j^*), (s_j^*, s_j^*), (s_j^*, s_k^*)] = \begin{cases} 
[(b, h), (h, h), (l, l)] & (a) \\
[(b, l), (h, h), (l, l)] & (b) \\
[(l, l), (h, h), (h, h)] & (c) \\
[(l, l), (h, h), (h, l)] & (d) \\
[(l, l), (h, l), (h, h)] & (e) 
\end{cases}$$

where $(i, j)$ are incumbent firms and $k$ is the entrant. I will now show which of the search cost strategies are dominated and then specify the optimal search cost strategies depending on the reach $\alpha$ of the electronic channel.

All search cost strategies excluded from (9) do not differentiate firms sufficiently, resulting either in the absence of pure strategy equilibria of the price game, or in ruinous price competition. From strategy profiles (a) through (e) one can construct the entrant's (firm $k$'s) best response function to first step strategies by firms $i$ and $j$.

The incumbent firms would never play according to strategy profiles (a) and (b) since these are dominated by profiles (c) and (d). Whichever firm plays $(l, l)$ has higher profits than its competitors. Since incumbent firms choose search costs before the entrant, firm $i$ will always choose search costs $(l, l)$ over $(h, h)$ or $(h, l)$. The analogous reason eliminates strategy profile (d). A firm playing $(h, l)$ (given another firm plays $(l, l)$) gains demand from consumers that consider the products of the two firms they have searched identical. That is why firm $j$ will always choose to initiate profile (e) over profile (d). This leaves us with strategy profiles (c) and (e). Similar to the duopoly case, the choice between these two equilibria depends on the size of the electronic channel. Profile (e), $[(l, l), (h, l), (h, h)]$, can only be sustained for small $\alpha$. By having offered consumers low search costs on the electronic channel, firm $j$ can gain all those consumers that – having searched firms $i$ and $j$ – realize that their willingness to pay for the
product of either firm is identical. Once firm 1 gains more by undercutting firm j’s price and gaining those consumers back than it loses from cutting its price to consumers that can only use the conventional channel, this equilibrium breaks down. The equilibrium induced by strategy profile (c) holds for all \( \alpha \) since demand is fully continuous.

If there is no electronic channel, i.e. \( \alpha = 0 \), the entrant can only avoid ruinous price competition by setting search costs high. Consumers are perfectly informed about their willingness to pay at incumbent \( i \) and partially informed about their reservation price at incumbent \( j \) and entrant \( k \). Since consumers’ demand is continuous for all \( p_i, p_j, p_k \) these equilibrium search cost strategies lead to the least competitive pressure in the price game. If incumbent \( j \) were to lower its search cost to \( s_j = l \), half of the consumers would consider firm \( i \)’s and \( j \)’s products identical, which would create an incentive for firms \( i \) and \( j \) to undercut each other’s prices. The same would apply if entrant \( k \) were to set search costs low. Finally, firm \( i \) does not have an incentive to set its search cost high since the resulting demand would trigger Bertrand competition.

With an electronic market but with a small \( \alpha \), firm \( i \) offers low search costs on the electronic channel as well. As in section 3.3, firm \( j \) will deviate from its conventional channel strategy by offering \( s_j^e = l \) on the electronic channel. That way it can gain all those consumers that, after having searched firms \( i \) and \( j \), realize that their willingness to pay for the product of either firm is identical. As mentioned before, this does not trigger a competitive reaction by firm \( i \) as long as \( \alpha \) is small. Entrant \( k \) sets high search costs on both channels. This way it can benefit from increased prices that stem from both firms \( i \) and \( j \) setting low search costs on the conventional channel. At the same time, firm \( k \) avoids competing for a discrete segment of consumers. If firm \( k \) were to adopt the same strategy as firm \( j \), a pure strategy equilibrium could only exist for \( p_j = p_k \). Since firm \( j \)’s and \( k \)’s search cost strategies lead to consumers that consider their products identical, an equal price level is unsustainable. Either firm has an incentive to lower its price by \( \varepsilon \) and gain all of those consumers. As \( \alpha > \alpha_3^* \), firm \( i \)’s gains from undercutting firm \( j \)’s price exceed its losses from charging a lower price to consumers that can only use the conventional channel.

So far we would have obtained the same equilibria if firms had chosen search costs simultaneously and not sequentially. For a small intermediate region of the parameter space, \( \alpha_3^* < \alpha < \alpha_1^* \), firm \( i \)’s and \( j \)’s first mover advantage changes the equilibrium. Although the

\[ \text{See the proof of proposition 4} \]
three firm equilibrium cannot be sustained for $s^* = l$ as long as $\alpha > \alpha_3^*$, the two incumbent firms alone can coexist without initiating a competitive reaction by firm $i$.

As $\alpha > \alpha_1^*$, $s^* = l$ leads to ruinous price competition between the two incumbents. Firm $j$ has to set $s^* = h$ to avoid triggering a competitive reaction by firm $i$. The incumbents can then no longer blockade entry. For a size of the electronic channel exceeding $\alpha_1^*$ but below $\alpha_2^*$ the subgame perfect equilibrium of the game consists of an extension of firms’ strategies from the conventional channel to the electronic channel.

For $\alpha_2^* < \alpha < 1$, firm $i$’s and $j$’s first mover advantage changes the equilibrium again. As explained in section 3.3, the incumbent firm that chose to set $(l, l)$ can now reduce the set of consumers that are perfectly informed. The firm relies solely on the electronic channel to inform consumers. As long as the electronic channel is large, enough consumers remain informed to prevent a price war. This strategy choice keeps the entrant out of the market since any strategy by the entrant would trigger ruinous price competition. Although equation 9 on page 100 lists profile (b), $((h, l), (h, h), (l, l))$, as an equilibrium strategy profile, it only holds for small $\alpha$. Notice that profile (b) is identical to (e) only with reversed firms. I explained previously why profile (e) is not an equilibrium strategy profile for large $\alpha$. Search cost actions $(h, l)$ and $(h, h)$ also initiate price wars. If search costs were not set sequentially but simultaneously, search costs strategy profile $((l, l), (h, h), (h, h))$ would be the equilibrium for all $\alpha > \alpha_3^*$.

The principal insight from this section comes from analyzing the entrant’s equilibrium profits. This will permit inferences about incentives for entry.

**Proposition 5:** Let $\Pi_k^*$ be the entrant’s equilibrium profits. Then the following is true:

If $\alpha < \alpha_3^*$, $\frac{\partial}{\partial \alpha} \Pi_k(\alpha)^* > 0$.

If $\alpha_1^* \leq \alpha < \alpha_2^*$, $\frac{\partial}{\partial \alpha} \Pi_k(\alpha)^* = 0$ and $\Pi_k^*(\alpha) = \Pi_k^*(0)$

where $\alpha_1^*$, $\alpha_2^*$ and $\alpha_3^*$ are defined in propositions 2 and 4.

Proof of proposition 5: see appendix.

The following figure plots firms’ equilibrium profits and the corresponding search cost strategies as a function of the size of the electronic channel.
For $\alpha = 0$ incumbent $j$ and entrant $k$ have the same profits, incumbent $i$ has higher profits. The entrant's profits increases with $\alpha$. The reason lies in firm $j$'s decision to set low search costs on the electronic channel. The $(1 - \alpha)$ consumers that consider firm $i$ and $j$'s products identical are sensitive only to whether one firm's price is lower than the other firm's price (as long as the price does not exceed consumers' reservation price). Similar to the case with high $\alpha$ in the duopoly case (see page 95), this creates an incentive for firm $j$ to raise its price. Since prices are strategic complements, firm $k$ can increase its price. Profits increase because the entrant increases prices at a lower rate than incumbent $j$. Once $\alpha$ surpasses $\alpha_1^*$, i.e. search cost strategies on the electronic channel are identical to those on the conventional channel, profits are same as they were in the absence of an electronic channel. For $\alpha$ between $\alpha_3^*$ and $\alpha_1^*$, as well as for large $\alpha$, firm $k$ does not enter.

I can now state the key insight of this section.

**Corollary 2:** Assume that there is some fixed cost of entry. Then entry is more likely when the size of the electronic channel is small than either the case where an electronic channel does not exist or the case where the electronic channel has a wide reach.
5 Extension: Heterogeneous disutility of search

This extension is intended to capture the effect of having some $\gamma$ consumers whose valuation of time (or disutility from search) is higher than that of other consumers. One might think of increased work hours for professionals, families in which both adults have full time jobs, or long commuting times. This extension to the model captures this notion by assuming that consumers with a high valuation of time will only opt to search (vs. making a purchase decision based on the expectation of their willingness to pay) if firms make search particularly easy.

For many products, however, there is a lower bound on how easy search through a conventional channel can be made. As an example, consider the purchase of a compact disc. A consumer that wants to find out whether she would like to purchase the new album by a certain artist has to exert a significant amount of effort to sample the songs on the CD. She has to incur transportation costs to get to the store, locate the CD, ask the sales assistant to play the CD for her and stand (maybe in a hot coat!) while listening. Although the CD store can make the sampling experience somewhat more comfortable than I have described, transportation costs are hard to lower. In contrast, search costs for sampling a CD on Internet can be made very low by providing 30 second sound bites of each song. The model explicitly accounts for the fact that (for certain goods)\(^\text{24}\) search costs can be lowered further on an electronic than on a conventional channel.

In this section of the paper I first modify firms’ strategy space, then model consumer heterogeneity in search costs, and finally point out some characteristics of the solution to this game.

5.1 Firms

Two firms compete in prices and search costs. Firm $i$ sets one price $p_i$ for both channels but can set different search costs for each channel. Recall that I denoted the search costs that firm $i$ sets on the conventional channel and the electronic channel $s_i^c$ and $s_i^e$ respectively. On the conventional channel firm $i$ has the same strategy space as in the previous model; the firm can set search costs either high or low, $s_i^c \in \{4, 4\}$. Suppose now, that the electronic channel enables firms to lower consumer search costs more than is possible on a conventional channel. In ad-

\(^{24}\) Although this might not be true for all goods sold on conventional channels, it seems reasonable to assume that search can be made easier on an electronic channel as long as consumers’ willingness to pay depends on some information that can be transmitted over electronic media. This might apply to software, books, CDs, information services etc.
dition to setting search costs high \((h)\) and low \((l)\), firms can now set search costs on the electronic channel to “very low” \((v)\), \(s_f^e \in \{h, l, v\}\). The sequence of competition between firms remains unchanged, i.e. in a first step they set search costs \(s_f^e\) and \(s_f^e\). In a second step they compete in prices \(p_f^e\).

5.2 Consumers

Similar to the model in section 3 of this paper, suppose that all consumers have access to a conventional channel. A fraction \(\alpha\) of consumers can use the electronic channel and these consumers also have access to the conventional channel. I will model consumer heterogeneity in search costs as follows. Assume that there are two types of consumers. A fraction \(\gamma\) of consumers of both the conventional and the electronic channel will only search for the product of firm \(i\) if firm \(i\) sets search costs very low, \(s_i^e = v\) (in contrast to low and high). The remaining \((1 - \gamma)\) consumers search if a firm’s search costs are either low or very low, \(s_i^e \in \{l, v\}\). Note that the fraction \(\gamma\) of consumers that have a high valuation of time or a high disutility of search will never search on the conventional channel. Firms cannot set search costs on the conventional channel low enough to induce search.

\[
\begin{array}{c|c}
\text{Conventional channel} & \gamma & (1-\gamma) \\
\hline
\text{Electronic channel} & \gamma & (1-\gamma) \\
& \alpha &
\end{array}
\]

5.3 Results

Subgame perfect equilibria for the model can be determined analogously to section 3.3. Firm \(i\)’s search costs now come from the set:

\[(s_f^e, s_f^e) \in S_i = \{(h, h), (h, l), (h, v), (l, h), (l, l), (l, v)\}\] (10)

Either \((s_f^e, s_f^e) = (l, h)\) or \((s_f^e, s_f^e) = (l, l)\) is redundant since everyone in the electronic channel can also use the conventional channel. The strategy set for which price equilibria have to be computed is then given by \(S = S_1 \times S_2\). Similar to section 3.3 all strategies containing \((l, h)\), as well as duplicate strategies that can be obtained by exchanging firm subscripts can be excluded without loss of generality.
The insight that is gained by extending the model to consumers with a low and a high valuation of time is captured in the following proposition.

**Proposition 6:** (a) \( \exists \varepsilon \text{ such that } \forall \alpha \in (0, 9/20) \text{ there is no subgame perfect equilibrium in pure strategies for } \gamma > 1 - \varepsilon. \)

(b) For \((\alpha, \gamma) \text{ such that } \alpha > 1 + (9 - 2 \sqrt{21})/\gamma \text{ where } \alpha \in (0, 1) \text{ and } \gamma \in [0, 1) \text{ there exists a subgame perfect equilibrium with strictly positive prices } p_1 > 0, p_2 > 0 \text{ and positive profits } \Pi_1 > 0, \Pi_2 > 0.

Proof of proposition 6: see appendix

The regions in \( \alpha, \gamma \) space mentioned in proposition 6 are illustrated in Figure 6.

**Figure 6: Equilibrium regions in proposition 6**

No price equilibria in pure strategies

\[ \begin{array}{c}
0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \\
0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \\
\end{array} \]

\[ \begin{align*}
\alpha & \quad \gamma \\
\gamma & \quad \gamma \\
\end{align*} \]

\[ \begin{align*}
\Pi_1, \Pi_2 & > 0 \\
p_1, p_2 & > 0 \\
\end{align*} \]

Proposition 6 states that having too many consumers with a high valuation of time, relative to the fraction of consumers that can use the electronic channel, leads to ruinous price competition. For low \( \alpha \), very few consumers with a high valuation of time will search for the products of either firm since firms cannot set the search cost on the conventional channel low enough so that these consumers will want to search. These consumers consider the products of either firm identical (in expectation) and make their purchase decisions based on price only. As long as \( \gamma \) is small, firm 1's potential gain from undercutting firm 2's price by \( \varepsilon \) and capturing demand from the \( \gamma \) consumers will be lower than the loss in profits from the \((1 - \gamma)\) consumers. For large \( \gamma \) the gain from \( \gamma \) type consumers outweighs the loss from \((1 - \gamma)\) type consumers. Why can more consumers with a high valuation of time be accommodated without triggering
ruinous price competition if the electronic channel reaches many consumers? The electronic channel allows firms to set search costs low enough that consumers with a high valuation of time will want to determine their willingness to pay at one of the firms. This changes their demand function from discontinuous at $p_1 = p_2$ (equation 2 on page 88) to continuous in all $p_1, p_2$ (equation 3 on page 88). The more consumers use the electronic channel, the more consumers with a high valuation of time can be informed by setting sufficiently low search costs. At the two extremes, if there is no electronic channel, some fraction of consumers with a high valuation of time is sufficient to induce ruinous price competition between firms. If the electronic channel is accessible by most consumers, firms can avoid ruinous price competition, no matter how large the fraction of consumers with a high valuation of time.

The main finding of this part of the paper is summarized in the following corollary of proposition 6.

**Corollary 3:** An electronic channel can relieve some of the competitive pressure that stems from consumers with a high valuation of time or a high disutility of search.

6 Discussion

By competing on multiple channels, firms can achieve finer consumer segmentation than if they competed in one channel only. This allows firms to provide selected groups of consumers with different amounts of information. Firms can exploit this segmentation to increase their market power. This is the driving force behind most results in this paper. The results suggest that a proliferation of channels will not increase but decrease competition between firms. This is somewhat similar to offering a menu of contracts to discriminate between consumers.

Except for the section considering consumers with a heterogeneous disutility of search (section 5), this paper does not assume properties of the second channel that are idiosyncratic to the Internet. The above findings should thus be generalizable to other types of channels. Since testing some of the presented implications on Internet data should be possible only in the long run, other channels such as mail order might provide a good empirical base to test some of the results of this paper.

This paper argues that entry will be blockaded for very large ($\alpha$). If one interprets the model as a repeated game, entry can still be blockaded since the incumbent firms can reduce the entrant's potential profits. Assume that firms start with the game discussed in section 4. The
incumbents set search costs first, then the entrant sets search costs and then all firms compete in prices. It seems reasonable to assume that firms would repeat the price game \( n \) times before simultaneously (re)setting search costs. This reflects that search costs can be adjusted less easily than prices. The entrant could start making positive profits when firms simultaneously reset their search costs. Depending on the number of periods in which the incumbent firms could hold down the entrant to zero profits, entry could be blockaded if the fixed cost of entry were high enough.

This paper has various limitations. Demand in the two channel case is modeled in the aggregate. However, by carefully replicating the properties of the demand functions derived from primitives in the one channel case, I hope to have retained the key features of consumer demand while greatly simplifying the model.

Throughout the paper I restrict myself to pure strategy equilibria in search costs and prices. For search costs this is not problematic. They are set in the first step of the game and are assumed observable before entering the pricing stage. Because of search costs' commitment value in this model, a mixed strategy has no easy interpretation. The restriction on pure strategies in prices corresponds to excluding promotions from firms' strategy space. It can be argued that the institutional setting of the Internet does not lend itself to a natural interpretation of mixed strategies in prices. The Internet's technical characteristics allow a firm to adjusted its price instantaneously upon the realization of the opponent's price, thus undermining the intuition that normally underlies mixed strategies.

Under certain circumstances, however, mixed strategies might still be possible: firms might implement technologies to prevent an instantaneous reaction by competitors or they might not be able to identify who their competitors are. The results of this paper will continue to hold even if mixing in prices is allowed as long as \( \beta \) is sufficiently large, meaning that there is a sizeable fraction of consumers that after searching both firms have identical reservation prices.\(^{25}\) If \( \beta \) is sufficiently large, then the profits that can be achieved from consumers searching only one firm are higher than profits when consumers search none or all of the firms, and the results of this paper hold.

As already discussed in footnote 8, this model requires that there exist a discrete mass of consumers that, after having searched both firms, have identical reservation prices. The applicability of the model is thus restricted to product categories where some consumers are in fact

\(^{25}\) See "Notes on mixed strategy equilibria" in the appendix for a sketch of the proof.
indifferent between the products once they have acquired all necessary information. One example might be audio equipment where many consumers cannot tell the difference between the sound of amplifiers of competing manufacturers.

If consumers search both firms, firms will or will not undercut each others prices depending on the size of the Internet. In the case of low $\alpha$, the firms' asymmetry on the conventional channel allows firm 2 to set low search costs on the electronic channel without triggering a competitive reaction by firm 1. Other types of (strong) asymmetries – which are not considered in this model – would have the same effect. Initiating a competitive reaction in price is only a threat if the firms' prices are relatively similar. Hence, the results in this paper are more likely to apply to close substitutes than to strongly differentiated products.

One of the implications of the model is that with increasing size of the Internet, firms' Internet strategies revert back to the strategies originally pursued on the conventional channel. Profits revert back to the profits firms made when there was only a conventional channel. Firms in the model are induced to maintain a presence on the Internet because it is costless for them to be on two vs. one channel. The latter will not hold in the real world. However, the cost of being on the Internet will be counterbalanced by the fact that the Internet will most likely increase the number of potential customers - an effect that I do not model since it would not provide additional insight.

One of the major limitation of this paper is that the reach of the Internet (\(\alpha\)) is considered to be given exogenously. Although this is most likely a reasonable assumption in the short run, there is little doubt that firm actions will influence how many consumers make use of the Internet. This model should therefore be taken for what it is, a mainly static model that analyzes the nature of competition at one moment in time.

The results of this paper suggest possible directions the Internet might take as well as some managerial implications. It is important to keep in mind that this paper only deals with a subset of all possible factors that might affect firms' information and pricing strategies. One of the effects that I do not consider in this paper are the different costs of marketing and distributing products over the Internet. While this is likely to influence the use of the Internet, I have excluded cost factors from consideration in order to highlight implications that arise from purely strategic issues in competing in search costs and prices. I first consider possible directions for the Internet and then discuss implications of managerial relevance.
Possible directions for the Internet

*Explosion of information*

- The explosion of information by vendors on the Internet might be a temporary phenomenon. The results of this paper suggest that a firm that does not facilitate consumer search on the conventional channel will provide more information on the Internet — but only as long as the Internet’s reach is limited. As the reach of the Internet increases firms are more likely to replicate their conventional channel strategies on the Internet. Since the Internet is growing very fast this suggests that the extent of the current explosion of information by vendors might be restricted to the short to medium term.

*Profits and prices*

- The overall price level need not decline by adding the Internet. The results of this paper suggest that the addition of the Internet decrease rather than increase the competitiveness of the market since consumers can be informed selectively. Consequently, prices should not be seen to decline for competitive reasons. This prediction, however, needs to be qualified. There might be numerous reasons not discussed in this paper that might in fact lead to price declines. An important reason is that the costs of providing information to consumers is most likely lower over the Internet than over a conventional channel. Another reason might be lower marginal costs of distributing some products over the Internet. An example could be the Internet Shopping Network described in the introduction of this paper.

- Firms’ profits might initially rise, even if the Internet does not increase the size of the market. This is because the Internet enhances firms’ capability to segment consumers by informing them selectively. As a result, firms’ market power increases. This allows firms to increase prices and earn higher profits. This result might hold even if the consumer base of firms is not expanded through the Internet. Once the Internet is more mainstream, firms might no longer be able to maintain higher prices and profits relative to the pre-Internet days.

*Entry*

- It is more attractive for firms to be on the Internet earlier than later. This reflects that firms might be able to make higher profits as long as the small reach of the Internet allows consumer segmentation. Firms have an incentive to take advantage of the Internet initially; as

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26 This makes the simplifying assumption that facilitating consumer search equals providing more information. See footnote 3.
the Internet becomes more mainstream the advantages might be lost. If there is a recurrent fixed cost of being on the Internet this suggests that firms might want to reconsider their presence on the Internet once advantages stemming from finer segmentation of consumers are lost.

• An initially high amount of entry into product categories may be an artifact of the early stages of the Internet. The presence of a second channel might increase the profits a firm can make from entering a product category. Selectively informing consumers gives one of the firms local monopoly power. This results in an increase in the price and profits an entrant can sustain as compared to the one channel case. As the second channel becomes larger, consumers can no longer be segmented and the entering firm's profits revert to its one channel level. Depending on firms' fixed cost of entry, this indicates that there might be more entry, but only as long as the Internet's reach is limited.

**Importance of the Internet**

• Firms might find it optimal to shift their “high service” presence to the Internet. As the Internet approaches full coverage of potential consumers it might be in firms' best interest to abandon the conventional channel as a means of facilitating consumers search. Information to consumers might then be transmitted over the Internet. This allows firms to soften competition and charge higher prices. This effect will be enhanced if providing service over the Internet has cost advantages over more traditional channels.

• The Internet can ease the competitive pressures that might arise if consumers' disutility of search increases. For certain products it will allow firms to lower consumer search cost below the level possible over conventional retail channels. This might allow firms to regain the advantage of differentiating on search costs and thus soften price competition.

**Managerial implications**

**Firms' Internet strategies**

• It might not be optimal for firms to simply duplicate their conventional strategies on the Internet. Consider, for example, a firm that is a low service retailer. As long as the reach of the Internet is limited, it might be in the retailer's best interest to change its profile on the Internet towards facilitating consumer search. The optimal Internet strategy will thus depend on the reach of the Internet as well as the competitive situation in both channels.

• Firms might want to limit the size of consumer segments that are offered low or high search costs. The “reach of the Internet” as used in this paper is not equivalent to the number of
households that could potentially use the Internet. What is of importance is the size of the
customer segment that uses the Internet to find out about a particular product category. It
was argued earlier that firm advantages of the Internet could only be sustained as long as
the Internet's reach was limited. The technology on the Internet might permit firms to ar-

tificially limit the reach of the Internet by restricting information to certain types of con-
sumers. This way the competitive advantages of the Internet might be maintained even if
many potential consumers have access to the Internet.

- Multi-product firms might want differentiated Internet strategies for each of their prod-

cuts. Depending on the segment sizes of potential consumers that use the Internet for dif-
ferent product categories, firms might want to facilitate consumer search in some and make
search hard in other product categories. Hewlett Packard, for example, might want to pro-
vide a lot of information for its printers but restrict access for its measurement equipment
(assuming that a smaller fraction of potential printer customers will use the Internet than
purchasers of measurement equipment.)

Entry

- Early entry might be more profitable than late entry. The results of this paper suggest that
the Internet will make entry into a product category more attractive than in the one chan-
nel case only if the reach of the Internet is limited. A potential entrant might consequently
try to enter a product category at the beginning stages of the Internet, rather than wait until
the Internet reaches most potential consumers.

- Firms that don't facilitate consumer search on a conventional channel might be able to gain
real first mover advantages on the Internet as long as the Internet's reach is limited. The
firm that establishes a presence on the Internet first has more flexibility in deciding which
pricing and information strategy to pursue. Faced with the choice of whether to be the one
of two firms that facilitates consumer search, a firm would choose to offer low search costs,
thus leaving the later entrant the high search cost position only. This might explain why
many firms are rushing towards a presence on the Internet.

- Firms might be able to use consumer segmentation as a means to blockade entry. The re-
results in this paper show that as the Internet approaches the reach of the conventional chan-
nel, firms can manipulate consumers' information structure in a way that leaves less room
for additional competitors. Firms create an environment that is so "competitive" that it re-
results in ruinous price competition if another firm tried to compete.
• Early entrants might overestimate long term profits. If entering firms “naively” assume that profit levels will not be affected by the reach of the Internet, they could overestimate the long term profitability of competing in the product category. The profits earned in the long run, however, are more likely to be similar to profits that could be earned in the absence of the Internet.

*Importance of the Internet*

• Being on the Internet might not be imperative for the competitive reasons considered in this paper. If being on the Internet neither expands the potential customer base nor leads to cost advantages, it will under certain conditions still be profitable for firms to use the Internet. However, these advantages could disappear as the Internet becomes more mainstream. This statement has to be qualified with the fact that the Internet may very well reduce costs and expand the customer base. For purely competitive reasons, however, firms should not automatically assume that being on the Internet will be worthwhile.

• A firm that cannot differentiate over a conventional channel because search costs outside the firm’s control are too high might be able to use the Internet to inform consumers about certain types of products.

7 Concluding Remarks

This paper analyzes in which way the existence and the size of the Internet affect the optimal pricing and communications strategies that firms might want to pursue. It takes into account that firms’ dealings on the Internet could be dependent on the strategies these firms pursue over other outlets. Incentives for entry are considered by adding a Stackelberg follower to the model. An extension analyzes the effect of consumers that react heterogeneously to search costs.

The first main result of this paper is that the amount of information provided by firms depends on the reach of the Internet. If the Internet is “small,” *more* information is provided than through a conventional channels only. If the Internet is “large,” there will *not* be more information provided than through a conventional channel only. The second main result is that entry into a product category is made more attractive by the Internet but only as long as the Internet’s reach is small. The third result of this paper is that the Internet can relieve some of the competitive pressure that might arise from consumers with a high disutility of search. In evaluating these results it should be noted that this paper deals only with a subset of all possible factors that might affect firms’ information and pricing strategies.
Although the results in this paper have been given in the context of the Internet, many of the results should also apply to other channels (e.g. mail order). Future research should first and foremost test some of the implications of this paper on historical data from the emergence of other channels.

This paper contributes in three areas. First, it has pointed out possible directions the Internet might take. It has predicted future developments regarding the amount of information that might be provided over the Internet, firms' pricing strategies, the profitability of competing on the Internet, and incentives for entry. Second, the paper has derived implications for managers. It has discussed the duplication of conventional strategies onto the Internet, whether firms want to limit information, whether there are first mover advantages to being on the Internet and how long-term profits might be affected. Third, this paper has made some theoretical contributions. It has introduced channels as a means of segmenting consumers by selectively informing them. It has also shown that manipulating consumer information can be a powerful competitive tool. Finally it has highlighted that the analysis of competition between firms can be strongly affected by considering all channels over which they compete.
References


Appendix

Proof of proposition 1

Proposition 1 results as a special case ($\alpha = 0$) from proposition 2. See the proof of proposition 2.

Proof of propositions 2 and 3

In this proof I first compute the price equilibria of the second step price game for all possible first step strategies, then I determine the subgame perfect equilibria of the overall game.

Preliminaries

Firm $i$’s search costs come from the following set.

$$(s_f^i, s_f^j) \in \{(h, h), (l, h), (h, l), (l, l)\} \tag{11}$$

Either $(s_f^i, s_f^j) = (l, h)$ or $(s_f^i, s_f^j) = (l, l)$ is redundant since everyone in the electronic channel can also use the conventional channel. The search cost strategy set for which price equilibria have to be computed is given by (see also figure 2 on page 93):

$$[((s_f^i, s_f^j), (s_f^k, s_f^j)) \in S = \{\{(h, h), (h, h)\}, \{(h, l), (h, h)\}, \{(h, l), (h, l)\},$$

$$\{(l, l), (h, h)\}, \{(l, l), (h, l)\}, \{(l, l), (l, l)\}\} \tag{12}$$

Strategies $\{(h, h), (h, l)\}$, $\{(h, h), (l, l)\}$, and $\{(h, l), (l, l)\}$ are redundant since they correspond to $\{(h, l), (h, h)\}$, $\{(l, l), (h, h)\}$, and $\{(l, l), (h, l)\}$ respectively with firm names $i$ and $j$ exchanged.

If consumers don’t search, firms face demand (see equation 2):

$$D_i = D_i(0, 0, 1) \equiv \begin{cases} 0 & \text{for } p_i > p_j \\ 1 & \text{for } p_i \leq p_j \end{cases}$$

If consumers search firm $i$ but not firm $j$, firms face demand (see equation 3):

$$D_i = D_i(1, 0, 0) \equiv 1/2 - p_i + 2p_j$$
$$D_j = D_j(0, 1, 0) \equiv 1/2 - 2p_j + p_i$$
If consumers search firms \( i \) and \( j \), firms face demand (see equation 4):

\[
D_i = D_i(1, 1, 1/2) = \begin{cases} 
(1/2)(1/2 - p_i + p_j) & \text{for } p_i > p_j \\
1/2 + (1/2)(1/2 - p_i + p_j) & \text{for } p_i \leq p_j 
\end{cases}
\]

Since firms compete on two channels consumers need not be homogeneous with respect to their revealed search behavior. The total demand that each firm faces is thus a linear combination of the above demand functions. For brevity, denote a firm’s search cost strategy \((h, h)\) by \(1\), \((h, l)\) by \(2\), and \((l, l)\) by \(3\). Let \(m,n\) denote that firm \(i\) is playing search cost strategy \(m\) and firm \(j\) is playing search cost strategy \(n\). \(2,1\), for example, refers to \([[s^e_i, s^e_i], (s^e_j, s^e_j)] = [(h, l), (h, h)]\). Let \(D_i^{m,n}\) be the demand that firm \(i\) faces if firms play search cost strategies \(m,n\). Then

\[
\begin{align*}
D_i^{1,1} &= D_i(0, 0, 1) \\
D_i^{2,1} &= (1 - \alpha)D_i(0, 0, 1) + \alpha D_i(0, 0, 0) \\
D_i^{2,2} &= (1 - \alpha)D_i(0, 0, 1) + \alpha D_i(1, 1, 1/2) \\
D_i^{3,1} &= D_i(0, 1, 0) \\
D_i^{3,2} &= (1 - \alpha)D_i(0, 1, 0) + \alpha D_i(1, 1, 1/2) \\
D_i^{3,3} &= D_i(1, 1, 1/2)
\end{align*}
\] (13)

Let the firms play search cost strategies \(m,n\). Then firm \(i\)’s and \(j\)’s maximization problems in the price game are given by:

\[
\begin{align*}
\max_{p_i} \Pi_i(p_i, p_j) &= p_i D_i^{m,n} \\
\max_{p_j} \Pi_j(p_i, p_j) &= p_j D_i^{m,n}
\end{align*}
\] (14)

Note that in \(\Pi_i(p_i, p_j)\) and \(\Pi_j(p_i, p_j)\), \(p_i\) is always listed before \(p_j\).

Equilibria of the second step price game

*Case 1,1:*  
Firms’ problem is given by (23) and (14). This is the classical case of Bertrand competition. Hence \(p_i^* = 0, p_j^* = 0\), \(\Pi_i^* = 0\), and \(\Pi_j^* = 0\).
Case 2.1:
Firms' problem is given by (23) and (14). Assume \( p_i \geq p_j \). Solving the system of first order conditions yields \( p_i^* = 1/6 + 1/(3\alpha) \), \( p_j^* = -1/12 + 1/(3\alpha) \), \( \Pi_i^* = (2 + \alpha)^2/(36\alpha) \), and \( \Pi_j^* = (4 - \alpha)^2/72\alpha \). For \( \alpha \leq \alpha_2 \equiv 10 - 2\sqrt{21} \), the assumption \( p_i \geq p_j \) is violated since \( \Pi_i^*(p_i^*, p_j^*) < \Pi_i^*(p_i^*, -\varepsilon, p_j^*) \). Assume \( p_i < p_j \). Solving the system of first order conditions yields \( p_i^* = (4 - \alpha)/(6\alpha) \) and \( p_j^* = 1/12 + 1/(6\alpha) \). This violates the assumption \( p_i < p_j \). Assume \( p \equiv p_i = p_j > 0 \). Since \( \Pi_i^*(p - \varepsilon, p) > \Pi_i^*(p, p) \) the assumption \( p_i = p_j \) is violated for all \( p > 0 \). Assume \( p \equiv p_i = p_j = 0 \). For \( \alpha > 0 \) there is no pure strategy equilibrium with \( p_i = p_j = 0 \) since \( \Pi_i^*(0, 0) > \Pi_i^*(0, 0) = 0 \). For \( \alpha = 0 \) the equilibrium is given by \( p_i^* = p_j^* = 0 \). Hence a pure strategy price equilibrium with positive profits exists only for \( \alpha > \alpha_2 \) and is described by \( p_i^* = 1/6 + 1/(3\alpha) \) and \( p_j^* = -1/12 + 1/(3\alpha) \) with \( \Pi_i^* = (2 + \alpha)^2/(36\alpha) \) and \( \Pi_j^* = (4 - \alpha)^2/72\alpha \).

Case 2.2:
Firms' problem is given by (23) and (14). Assume \( p_i \geq p_j \). Solving the system of first order conditions yields \( p_i^* = 1/6 + 2/(3\alpha) \) and \( p_j^* = -1/6 + 4/(3\alpha) \). This violates the assumption \( p_i \geq p_j \). Assume \( p_i < p_j \). Solving the system of first order conditions yields \( p_i^* = -1/6 + 4/(3\alpha) \) and \( p_j^* = 1/6 + 2/(3\alpha) \). This violates the assumption \( p_i < p_j \). Assume \( p \equiv p_i = p_j > 0 \). Since \( \Pi_i^*(p - \varepsilon, p) > \Pi_i^*(p, p) \) the assumption \( p_i = p_j \) is violated for all \( p > 0 \). Assume \( p \equiv p_i = p_j = 0 \). For \( \alpha > 0 \) there is no pure strategy equilibrium with \( p_i = p_j = 0 \) since \( \Pi_i^*(0, 0) > \Pi_i^*(0, 0) = 0 \). For \( \alpha = 0 \) the equilibrium is given by \( p_i^* = p_j^* = 0 \). Hence there exists no pure strategy price equilibrium with positive profits.

Case 3.1:
Firms' problem is given by (23) and (14). Solving the system of first order conditions yields \( p_i^* = 1/2 \) and \( p_j^* = 1/4 \). In solving for equilibrium prices no assumptions on an ordering of prices is needed since \( D_i^* \) is continuous in \( p_i, p_j \). The pure strategy price equilibrium is characterized by \( p_i^* = 1/2 \) and \( p_j^* = 1/4 \) with \( \Pi_i^* = 1/4 \) and \( \Pi_j^* = 1/8 \).

Case 3.2:
Firms' problem is given by (23) and (14). Assume \( p_i \geq p_j \). Solving the system of first order conditions yields \( p_i^* = (6 - \alpha)/(6(2 - \alpha)) \), \( p_j^* = (6 + \alpha)/(6(4 - 3\alpha)) \), \( \Pi_i^* = (6 - \alpha)^2/72(2 - \alpha) \), and \( \Pi_j^* = (6 + \alpha)^2/72(4 - 3\alpha) \). For

\[
\alpha > \alpha_1 \equiv -24 - 21 \frac{3}{\sqrt{2}} + \sqrt{4998 + 2040\sqrt{6}}
\]

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(Appendix Chapter 2)
the assumption \( p_i \geq p_j \) is violated since \( \Pi_i^*(p_i^*, p_j^*) < \Pi_i^*(p_j^* - \varepsilon, p_j^*) \). Assume \( p_i < p_j \).

Solving the system of first order conditions yields \( p_i^* = (6 + \alpha)/(6(2 - \alpha)) \) and \( p_j^* = (6 - \alpha)/(6(4 - 3\alpha)) \). This violates the assumption \( p_i < p_j \). Assume \( p \equiv p_i = p_j > 0 \).

Since \( \Pi_i^*(p - \varepsilon, p) > \Pi_i^*(p, p) \) the assumption \( p_i = p_j \) is violated for all \( p > 0 \). Assume \( p \equiv p_i = p_j = 0 \). For \( \alpha > 0 \) there is no pure strategy equilibrium with \( p_i = p_j = 0 \) since \( \Pi_i^*(\varepsilon, 0) > \Pi_i^*(0, 0) = 0 \). For \( \alpha = 0 \) the equilibrium is given by \( p_i^* = p_j^* = 0 \). Hence a pure strategy price equilibrium with positive profits exists only for \( \alpha < \alpha_1^* \) and is described by \( p_i^* = (6 - \alpha)/(6(2 - \alpha)) \) and \( p_j^* = (6 + \alpha)/(6(4 - 3\alpha)) \) with \( \Pi_i^* = (6 - \alpha)^2/72(2 - \alpha) \) and \( \Pi_j^* = (6 + \alpha)^2/72(4 - 3\alpha) \).

**Case 3.3:**

Firms' problem is given by (23) and (14). Assume \( p_i \geq p_j \). Solving the system of first order conditions yields \( p_i^* = 5/6 \) and \( p_j^* = 7/6 \). This violates the assumption \( p_i \geq p_j \). Assume \( p_i < p_j \). Solving the system of first order conditions yields \( p_i^* = 7/6 \) and \( p_j^* = 5/6 \). This violates the assumption \( p_i < p_j \). Assume \( p \equiv p_i = p_j > 0 \). Since \( \Pi_i^*(p - \varepsilon, p) > \Pi_i^*(p, p) \) the assumption \( p_i = p_j \) is violated for all \( p > 0 \). Assume \( p \equiv p_i = p_j = 0 \). There is no pure strategy equilibrium with \( p_i = p_j = 0 \) since \( \Pi_i^*(\varepsilon, 0) > \Pi_i^*(0, 0) = 0 \). Hence there exists no price equilibrium in pure strategies.

**Subgame perfect equilibria of the overall game**

Note that there exist no \( \alpha \) for which search cost strategies 2,1 and 3,2 can both yield pure strategy price equilibria with positive profits. Let \( \Pi_i^{m,n} \) be firm i's optimized profit if firms play search cost strategies \( m,n \).

*Assume \( \alpha = 0 \)*

Only search cost strategies 3,1 and 3,2 yield price equilibria with positive profits. Since \( \alpha = 0 \), 3,1=3,2. The equilibrium search cost strategies are thus 3,1=3,2. Equilibrium prices and profits are characterized in "Case 3.1."
Assume $0 < \alpha < \alpha_1^*$

Only search cost strategies $3, 1$ and $3, 2$ yield price equilibria with positive profits. Clearly firm $i$'s best response to strategy $1$ and $2$ by firm $j$ is $3$ since any other response results in no equilibrium or a zero profit equilibrium. Firm $j$'s best response to strategy $3$ by firm $i$ is $1$ since

\[
\Pi^3_{j, 2} - \Pi^3_{j, 1} = \frac{64 - 25\alpha + \alpha^2}{72\alpha} > 0 \quad \forall \alpha \in (0, 1).
\]  

The equilibrium search cost strategies are thus $3, 2$. Equilibrium prices and profits are characterized in “Case 3, 2.”

Assume $\alpha_1^* \leq \alpha \leq \alpha_2^*$

Only search cost strategy $3, 1$ yields a price equilibrium with positive profits. The equilibrium search cost strategies are thus $3, 1$. Equilibrium prices and profits are characterized in “Case 3, 1.”

Assume $\alpha_2^* < \alpha \leq 1$

Only search cost strategies $2, 1$ and $3, 1$ yield price equilibria with positive profits. Firm $i$'s best response to strategy $1$ by firm $j$ is $2$ since

\[
\Pi^2_{i, 1} - \Pi^3_{i, 1} = \frac{4 - 5\alpha + \alpha^2}{36\alpha} > 0 \quad \forall \alpha \in (0, 1).
\]  

Clearly firm $j$'s best response to strategy $2$ and $3$ by firm $i$ is $1$ since any other response results in no equilibrium or a zero profit equilibrium. The equilibrium search cost strategies are thus $2, 1$. Equilibrium prices and profits are characterized in “Case 2, 1.”

Assume $\alpha = 1$

Only search cost strategies $2, 1$ and $3, 1$ yield price equilibria with positive profits. Since $\alpha = 1$, $2, 1=3, 1$. The equilibrium search cost strategies are thus $2, 1=3, 1$. Equilibrium prices and profits are characterized in “Case 3, 1.”

Q.E.D.
Proof of proposition 4

In this proof I first compute the price equilibria of the second step price game for all possible first step strategies, then I determine the subgame perfect equilibrium of the sequential entry game.

Preliminaries

Let $m, n, o$ denote that firm $i$ is playing search cost strategy $m$, firm $j$ is playing search cost strategy $n$, and firm $k$ is playing search cost strategy $o$. 2, 1, 1 for example, refers to $[(s_i^f, s_j^f, s_k^f), (s_i^f, s_j^f, s_k^f), (s_i^f, s_j^f, s_k^f)] = [(h, l), (h, h), (h, h)]$. Let $S_i = \{(h, h), (h, l), (l, h), (l, l)\}$ be the set of firm $i$'s search cost strategies. Either $(s_i^f, s_j^f) = (l, h)$ or $(s_i^f, s_j^f) = (l, l)$ is redundant since everyone in the electronic channel can also use the conventional channel. The search cost strategy set for which price equilibria have to be computed is given by

$S = S_1 \times S_2 \times S_3$. Duplicate strategies that can be obtained by exchanging firm subscripts can be excluded w.l.o.g. This leaves us with search cost strategies (1,1,1), (1,1,2), (1,1,3), (2,1,1), (2,1,2), (2,1,3), (2,2,1), (2,2,2), (2,2,3), (3,1,1), (3,1,2), (3,1,3), (3,2,1), (3,2,2), (3,2,3), (3,3,1), (3,3,2), and (3,3,3). Define

$$D_i(s_i, s_j, s_k, q) = \begin{cases} 
(1-q)\left(\frac{1}{2} - (2-s_i)p_i + \frac{1}{2}(2-s_j)p_j + (2-s_k)p_k \right) & \text{for } p_i > p_j \\
q + (1-q)\left(\frac{1}{2} - (2-s_i)p_i + \frac{1}{2}(2-s_j)p_j + (2-s_k)p_k \right) & \text{for } p_i \leq p_j
\end{cases}$$

where

\[
\begin{align*}
D_i &\equiv D_i(0, 0, 0, 1), & D_j &\equiv D_j(0, 0, 0, 1), & D_k &\equiv D_k(0, 0, 0, 1), & \text{if consumers do not search} \\
D_i &\equiv D_i(1, 0, 0, 0), & D_j &\equiv D_j(1, 0, 0, 0), & D_k &\equiv D_k(1, 0, 0, 0), & \text{if consumers search i only} \\
D_i &\equiv D_i(1, 1, 0, \frac{1}{2}), & D_j &\equiv D_j(1, 1, 0, \frac{1}{2}), & D_k &\equiv D_k(1, 1, 0, 0), & \text{if consumers search i and j} \\
D_i &\equiv D_i(1, 1, 1, \frac{1}{2}), & D_j &\equiv D_j(1, 1, 1, \frac{1}{2}), & D_k &\equiv D_k(1, 1, 1, 1), & \text{if consumers search all firms}
\end{align*}
\]

Since firms compete on two channels consumers need not be homogeneous with respect to their revealed search behavior. The total demand that each firm faces is thus a linear combina-
tion of the above functions. Let $D_i^{m,n,o}$ be the demand that firm $i$ faces if firms play search cost strategies $m,n,o$. Then

\[
\begin{align*}
D_i^{1,1,1} &= D_i(0, 0, 0, 1) \quad D_i^{1,1,2} = (1 - \alpha)D_i(0, 0, 0, 1) + \alpha D_i(0, 0, 1, 0) \\
D_j^{1,1,1} &= D_j(0, 0, 0, 1) \quad D_j^{1,1,2} = (1 - \alpha)D_j(0, 0, 0, 1) + \alpha D_j(0, 0, 1, 0) \\
D_k^{1,1,1} &= D_k(0, 0, 0, 1) \quad D_k^{1,1,2} = (1 - \alpha)D_k(0, 0, 0, 1) + \alpha D_k(0, 0, 1, 0)
\end{align*}
\]

\[
\begin{align*}
D_i^{1,1,3} &= D_i(0, 0, 1, 0) \quad D_i^{2,1,1} = (1 - \alpha)D_i(0, 0, 0, 1) + \alpha D_i(1, 0, 0, 0) \\
D_j^{1,1,3} &= D_j(0, 0, 1, 0) \quad D_j^{2,1,1} = (1 - \alpha)D_j(0, 0, 0, 1) + \alpha D_j(1, 0, 0, 0) \\
D_k^{1,1,3} &= D_k(0, 0, 1, 0) \quad D_k^{2,1,1} = (1 - \alpha)D_k(0, 0, 0, 1) + \alpha D_k(1, 0, 0, 0)
\end{align*}
\]

\[
\begin{align*}
D_i^{2,1,2} &= (1 - \alpha)D_i(0, 0, 0, 1) + \alpha D_i(1, 0, 1, 1/2) \\
D_j^{2,1,2} &= (1 - \alpha)D_j(0, 0, 0, 1) + \alpha D_j(1, 0, 1, 0) \\
D_k^{2,1,2} &= (1 - \alpha)D_k(0, 0, 0, 1) + \alpha D_k(1, 0, 1, 1/2)
\end{align*}
\]

\[
\begin{align*}
D_i^{2,1,3} &= (1 - \alpha)D_i(0, 0, 1, 0) + \alpha D_i(1, 0, 1, 1/2) \\
D_j^{2,1,3} &= (1 - \alpha)D_j(0, 0, 1, 0) + \alpha D_j(1, 0, 1, 0) \\
D_k^{2,1,3} &= (1 - \alpha)D_k(0, 0, 1, 0) + \alpha D_k(1, 0, 1, 1/2)
\end{align*}
\]

\[
\begin{align*}
D_i^{2,2,1} &= (1 - \alpha)D_i(0, 0, 0, 1) + \alpha D_i(1, 1, 0, 1/2) \\
D_j^{2,2,1} &= (1 - \alpha)D_j(0, 0, 0, 1) + \alpha D_j(1, 1, 0, 1/2) \\
D_k^{2,2,1} &= (1 - \alpha)D_k(0, 0, 0, 1) + \alpha D_k(1, 1, 0, 0)
\end{align*}
\]

\[
\begin{align*}
D_i^{2,2,2} &= (1 - \alpha)D_i(0, 0, 0, 1) + \alpha D_i(1, 1, 1, 1/2) \\
D_j^{2,2,2} &= (1 - \alpha)D_j(0, 0, 0, 1) + \alpha D_j(1, 1, 1, 1/2) \\
D_k^{2,2,2} &= (1 - \alpha)D_k(0, 0, 0, 1) + \alpha D_k(1, 1, 1, 1/2)
\end{align*}
\]

\[
\begin{align*}
D_i^{2,2,4} &= (1 - \alpha)D_i(0, 0, 1, 0) + \alpha D_i(1, 1, 1, 1/2) \\
D_j^{2,2,4} &= (1 - \alpha)D_j(0, 0, 1, 0) + \alpha D_j(1, 1, 1, 1/2) \\
D_k^{2,2,4} &= (1 - \alpha)D_k(0, 0, 1, 0) + \alpha D_k(1, 1, 1, 1/2)
\end{align*}
\]
\[ D^4_{i,1,1} = D_i(1,0,0,0) \quad D^4_{i,1,2} = (1-\alpha)D_i(1,0,0,0) + \alpha D_i(1,0,1,1/2) \]
\[ D^4_{j,1,1} = D_j(1,0,0,0) \quad D^4_{j,1,2} = (1-\alpha)D_j(1,0,0,0) + \alpha D_j(1,0,1,0) \]
\[ D^4_{k,1,1} = D_k(1,0,0,0) \quad D^4_{k,1,2} = (1-\alpha)D_k(1,0,0,0) + \alpha D_k(1,0,1,1/2) \]
\[ D^4_{i,1,4} = D_i(1,0,1,1/2) \quad D^4_{i,2,1} = (1-\alpha)D_i(1,0,0,0) + \alpha D_i(1,1,0,1/2) \]
\[ D^4_{j,1,4} = D_j(1,0,1,0) \quad D^4_{j,2,1} = (1-\alpha)D_j(1,0,0,0) + \alpha D_j(1,1,0,1/2) \]
\[ D^4_{k,1,4} = D_k(1,0,1,1/2) \quad D^4_{k,2,1} = (1-\alpha)D_k(1,0,0,0) + \alpha D_k(1,1,0,0) \]
\[ D^4_{i,2,2} = (1-\alpha)D_i(1,0,0,0) + \alpha D_i(1,1,1,1/2) \]
\[ D^4_{j,2,2} = (1-\alpha)D_j(1,0,0,0) + \alpha D_j(1,1,1,1/2) \]
\[ D^4_{k,2,2} = (1-\alpha)D_k(1,0,0,0) + \alpha D_k(1,1,1,1/2) \]
\[ D^4_{i,2,4} = (1-\alpha)D_i(1,0,1,1/2) + \alpha D_i(1,1,1,1/2) \]
\[ D^4_{j,2,4} = (1-\alpha)D_j(1,0,1,0) + \alpha D_j(1,1,1,1/2) \]
\[ D^4_{k,2,4} = (1-\alpha)D_k(1,0,1,1/2) + \alpha D_k(1,1,1,1/2) \]
\[ D^4_{i,4,1} = D_i(1,0,1,1/2) \quad D^4_{i,4,2} = (1-\alpha)D_i(1,1,0,1/2) + \alpha D_i(1,1,1,1/2) \]
\[ D^4_{j,4,1} = D_j(1,0,1,1/2) \quad D^4_{j,4,2} = (1-\alpha)D_j(1,1,0,1/2) + \alpha D_j(1,1,1,1/2) \]
\[ D^4_{k,4,1} = D_k(1,0,1,0) \quad D^4_{k,4,2} = (1-\alpha)D_k(1,1,0,0) + \alpha D_k(1,1,1,1/2) \]
\[ D^4_{i,4,4} = D_i(1,1,1,1/2) \]
\[ D^4_{j,4,4} = D_j(1,1,1,1/2) \]
\[ D^4_{k,4,4} = D_k(1,1,1,1/2) \]
Let the firms play search cost strategies \( m,n,o \). Then firm \( i \)'s, \( j \)'s, and \( k \)'s maximization problems in the price game are given by:

\[
\begin{align*}
\max_{p_i} \Pi_i(p_i, p_j, p_k) & = p_iD_i^{m,n,o} \\
\max_{p_j} \Pi_j(p_i, p_j, p_k) & = p_jD_j^{m,n,o} \\
\max_{p_k} \Pi_k(p_i, p_j, p_k) & = p_kD_k^{m,n,o}
\end{align*}
\]

(18)

Note that in \( \Pi_i(p_i, p_j, p_k) \), \( \Pi_j(p_i, p_j, p_k) \), and \( \Pi_k(p_i, p_j, p_k) \) prices are always listed in the order \( p_i, p_j, p_k \).

**Equilibria of the second step price game**

**Case 1,1,1:**

Firms' problem is given by (17) and (18). This is the classical case of Bertrand competition, only with three firms. Hence \( p_i^* = 0 \), \( p_j^* = 0 \), \( p_k^* = 0 \), \( \Pi_i^* = 0 \), \( \Pi_j^* = 0 \), and \( \Pi_k^* = 0 \).

**Case 1,1,2:**

Firms' problem is given by (17) and (18). Assume \( p_j < p_i, p_k \). Solving the system of first order conditions yields \( p_i^* = (2 + 3\alpha)/(20\alpha) \), \( p_j^* = (6 - \alpha)/(20\alpha) \), and \( p_k^* = 3/10 + 1/(5\alpha) \). This violates the assumption \( p_j < p_i \). Assume \( p_k < p_i, p_j \). Solving the system of first order conditions yields \( p_i^* = (2 + 3\alpha)/(20\alpha) \), \( p_j^* = (2 + 3\alpha)/(20\alpha) \), and \( p_k^* = (6 - \alpha)/(10\alpha) \). This violates the assumption \( p_k < p_i \). Hence there exists no price equilibrium.

**Case 1,1,3:**

Firms' problem is given by (17) and (18). Solving the system of first order conditions yields \( p_i^* = 1/4 \), \( p_j^* = 1/4 \), and \( p_k^* = 1/2 \). In solving for equilibrium prices no assumptions

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1 In this and all the following cases I omit solving the price game under the assumption that two or all three of the prices are equal. Whenever there is a demand discontinuity at the point where the prices of two competing firms are equal, and these two firm's prices have been assumed to be equal, there will either be no pure strategy equilibrium or a Bertrand equilibrium will result. The reasoning is analogous to the one in the proof of proposition 2.
on an ordering of prices is needed since $D_{1}^{1.4}$ is continuous in $p_{i}, p_{j}, p_{k}$. The pure strategy price equilibrium is characterized by $p_{i}^{*} = 1/4, p_{j}^{*} = 1/4,$ and $p_{k}^{*} = 1/2$.

Case 2.1.1:

Firms' problem is given by (17) and (18). Assume $p_{i} < p_{j}, p_{k}$. Solving the system of first order conditions yields $p_{i}^{*} = (6 - \alpha)/(10\alpha), p_{j}^{*} = (2 + 3\alpha)/(20\alpha),$ and $p_{k}^{*} = (2 + 3\alpha)/(20\alpha)$. This violates the assumption $p_{i} < p_{j}$. Assume $p_{j} < p_{i}, p_{k}$. Solving the system of first order conditions yields $p_{i}^{*} = (6 - \alpha)/(20\alpha), p_{j}^{*} = (6 - \alpha)/(20\alpha),$ and $p_{k}^{*} = (2 + 3\alpha)/(20\alpha)$. This violates the assumption $p_{j} < p_{k}$. Assume $p_{k} < p_{i}, p_{j}$.

Solving the system of first order conditions yields $p_{i}^{*} = 3/10 + 1/(5\alpha), p_{j}^{*} = (2 + 3\alpha)/(20\alpha)$, and $p_{k}^{*} = (6 - \alpha)/(20\alpha)$. This violates the assumption $p_{k} < p_{j}$. Hence there exists no price equilibrium.

Case 2.1.2:

Firms' problem is given by (17) and (18). Assume $p_{i} < p_{j}, p_{k}$. Solving the system of first order conditions yields $p_{i}^{*} = (12 - \alpha)/(10\alpha), p_{j}^{*} = 3/20 + 1/(5\alpha),$ and $p_{k}^{*} = 3/10 + 2/(5\alpha)$. This violates the assumption $p_{i} < p_{j}$. Assume $p_{j} < p_{k} < p_{i}$. Solving the system of first order conditions yields $p_{i}^{*} = 1/2 + 1/(5\alpha), p_{j}^{*} = (6 + \alpha)/(20\alpha),$ and $p_{k}^{*} = 9/10 + 1/(5\alpha)$. This violates the assumption $p_{k} < p_{i}$. Assume $p_{j} < p_{i}, p_{k}$. By symmetry of firm $i$ and $k$ and the contradiction derived for $p_{j} < p_{k} < p_{i}$, this violates the assumption $p_{i} < p_{k}$. Assume $p_{k} < p_{i}, p_{j}$. Solving the system of first order conditions yields $p_{i}^{*} = 3/10 + 2/(5\alpha), p_{j}^{*} = 3/20 + 1/(5\alpha),$ and $p_{k}^{*} = (12 - \alpha)/(10\alpha)$. This violates the assumption $p_{k} < p_{j}$. Hence there exists no price equilibrium.

Case 2.1.3:

Firms' problem is given by (17) and (18). Assume $p_{i} < p_{k}$. Solving the system of first order conditions yields $p_{i}^{*} = (10 - \alpha)/(-40 + 29\alpha + \alpha^2),$ $p_{j}^{*} = (200 - 220\alpha + 52\alpha^2 + 3\alpha^3)/(10(80 - 98\alpha + 27\alpha^2 + \alpha^3)), $ and $p_{k}^{*} = (200 - 210\alpha + 43\alpha^2 + 2\alpha^3)/(5(80 - 98\alpha + 27\alpha^2 + \alpha^3))$. This violates the assumption $p_{i} < p_{k}$ for $\alpha \geq \alpha_3^{*} = 0.24998$ since $\Pi_{k}^{*}(p_{i}^{*}, p_{j}^{*}, p_{k}^{*}) < \Pi_{k}^{*}(p_{i}^{*}, p_{j}^{*}, p_{i}^{*} - \epsilon)$ for $\alpha \geq \alpha_3^{*}$. Assume $p_{k} < p_{i}$. Solving the system of first order conditions yields $p_{i}^{*} = (10 + 3\alpha)/(-40 + 29\alpha + \alpha^2),$ $p_{j}^{*} = (200 - 220\alpha + 58\alpha^2 - 3\alpha^3)/(10(80 - 98\alpha + 27\alpha^2 + \alpha^3))$, and $p_{k}^{*} = (200 - 130\alpha - 13\alpha^2 - 2\alpha^3)/(5(80 - 98\alpha + 27\alpha^2 + \alpha^3))$. This violates the assumption $p_{k} < p_{i}$. Hence a pure strategy price equilibrium exists only for $0 \leq \alpha < \alpha_3^{*}$. It is characterized by $p_{i}^{*} = (10 - \alpha)/(-40 + 29\alpha + \alpha^2)$.
\[ p_j^* = \frac{(200 - 220\alpha + 52\alpha^2 + 3\alpha^3)}{(10(80 - 98\alpha + 27\alpha^2 + \alpha^3))}, \quad \text{and} \]
\[ p_k^* = \frac{(200 - 210\alpha + 43\alpha^2 + 2\alpha^3)}{(5(80 - 98\alpha + 27\alpha^2 + \alpha^3))}. \]

Case 2.2.1:
Firms’ problem is given by (17) and (18). Assume \( p_j < p_i, p_k \). Solving the system of first order conditions yields \( p_i^* = 3/10 + 2/(5\alpha) \), \( p_j^* = -1/10 + 6/(5\alpha) \), and \( p_k^* = 3/20 + 1/(5\alpha) \). This violates the assumption \( p_j < p_i \). Assume \( p_k < p_j < p_i \). Solving the system of first order conditions yields \( p_i^* = 1/2 + 1/(5\alpha) \), \( p_j^* = 9/10 + 1/(5\alpha) \), and \( p_k^* = (6 + \alpha)/(20\alpha) \). This violates the assumption \( p_j < p_i \). Assume \( p_k < p_j < p_i \). By symmetry of firm \( i \) and \( j \) and the contradiction derived for \( p_k < p_j < p_i \), this violates the assumption \( p_i < p_j \).

Case 2.2.2:
Firms’ problem is given by (17) and (18). Assume \( p_j < p_i, p_k \). Solving the system of first order conditions yields \( p_i^* = 3/10 + 2/(5\alpha) \), \( p_j^* = -1/10 + 6/(5\alpha) \), and \( p_k^* = 3/10 + 2/(5\alpha) \). This violates the assumption \( p_j < p_i \). Assume \( p_k < p_i, p_j \). By symmetry of firm \( i, j \), and \( k \) and the contradiction derived for \( p_j < p_i, p_k \), this violates the assumption \( p_k < p_i \). Hence there exists no price equilibrium.

Case 2.2.3:
Firms’ problem is given by (17) and (18). Assume \( p_j < p_i, p_k \). Solving the system of first order conditions yields \( p_i^* = (-10 + 3\alpha)/(40 + 30\alpha) \), \( p_j^* = (-10 - \alpha)/(40 + 30\alpha) \), and \( p_k^* = (-10 + 3\alpha)/(10(-2 + \alpha)) \). This violates the assumption \( p_j < p_i \). Assume \( p_k < p_i, p_j \). Solving the system of first order conditions yields \( p_i^* = (-10 + 3\alpha)/(40 + 30\alpha) \), \( p_j^* = (-10 + 3\alpha)/(40 + 30\alpha) \), and \( p_k^* = (-10 - \alpha)/(10(-2 + \alpha)) \). This violates the assumption \( p_k < p_i \). Hence there exists no price equilibrium.

Case 3.1.1:
Firms’ problem is given by (17) and (18). Solving the system of first order conditions yields \( p_i^* = 1/2 \), \( p_j^* = 1/4 \), and \( p_k^* = 1/4 \). In solving for equilibrium prices no assumptions on an ordering of prices is needed since \( D_i \) is continuous in \( p_i, p_j, p_k \). The pure strategy price equilibrium is characterized by \( p_i^* = 1/2 \), \( p_j^* = 1/4 \), and \( p_k^* = 1/4 \).

Case 3.1.2:
Firms’ problem is given by (17) and (18). Assume \( p_i < p_k \). Solving the system of first order conditions yields \( p_i^* = (200 - 130\alpha - 13\alpha^2 - 2\alpha^3)/(5(80 - 98\alpha + 27\alpha^2 + \alpha^3)) \),
\( p_{i}^{*} = (200 - 220\alpha + 58\alpha^2 - 3\alpha^3)/(10(80 - 98\alpha + 27\alpha^2 + \alpha^3)), \) and
\( p_{k}^{*} = (-10 + 3\alpha)/(-40 + 29\alpha + \alpha^2). \) This violates the assumption \( p_{i} < p_{k} \). Assume 
\( p_{k} < p_{i} \). Solving the system of first order conditions yields
\( p_{i}^{*} = (200 - 210\alpha + 43c^2 + 2\alpha^3)/(5(80 - 98\alpha + 27\alpha^2 + \alpha^3)), \)
\( p_{j}^{*} = (200 - 220\alpha + 52\alpha^2 + 3\alpha^3)/(10(80 - 98\alpha + 27\alpha^2 + \alpha^3)), \) and
\( p_{k}^{*} = (-10 - \alpha)/(-40 + 29\alpha + \alpha^2). \) This violates the assumption \( p_{k} < p_{i} \) for \( \alpha \geq \alpha_{3}^{*} \)
since \( \Pi_{i}^{*}(p_{i}^{*}, p_{j}^{*}, p_{k}^{*}) < \Pi_{i}^{*}(p_{k}^{*} - \epsilon, p_{j}^{*}, p_{k}^{*}) \) for \( \alpha \geq \alpha_{3}^{*} \). Hence a pure strategy price equilibrium exists only for \( 0 \leq \alpha < \alpha_{3}^{*} \). It is characterized by
\( p_{i}^{*} = (200 - 210\alpha + 43\alpha^2 + 2\alpha^3)/(5(80 - 98\alpha + 27\alpha^2 + \alpha^3)), \)
\( p_{j}^{*} = (200 - 220\alpha + 52\alpha^2 + 3\alpha^3)/(10(80 - 98\alpha + 27\alpha^2 + \alpha^3)), \) and
\( p_{k}^{*} = (-10 - \alpha)/(-40 + 29\alpha + \alpha^2). \)

**Case 3.1.3:**
Firms' problem is given by (17) and (18). Assume \( p_{i} < p_{k} \). Solving the system of first order conditions yields \( p_{i}^{*} = 11/10, p_{j}^{*} = 7/20, \) and \( p_{k}^{*} = 7/10 \). This violates the assumption \( p_{i} < p_{k} \). Assume \( p_{k} < p_{i} \). By symmetry of firm \( i \) and \( k \) and the contradiction derived for \( p_{i} < p_{k} \), this violates the assumption \( p_{k} < p_{i} \). Hence there exists no price equilibrium.

**Case 3.2.1:**
Firms' problem is given by (17) and (18). Assume \( p_{i} < p_{j} \). Solving the system of first order conditions yields \( p_{i}^{*} = (200 - 130\alpha - 13\alpha^2 - 2\alpha^3)/(5(80 - 98\alpha + 27\alpha^2 + \alpha^3)), \)
\( p_{j}^{*} = (-10 + 3\alpha)/(-40 + 29\alpha + \alpha^2), \) and
\( p_{k}^{*} = (200 - 220\alpha + 58\alpha^2 - 3\alpha^3)/(10(80 - 98\alpha + 27\alpha^2 + \alpha^3)). \) This violates the assumption \( p_{i} < p_{j} \). Assume \( p_{j} < p_{i} \). Solving the system of first order conditions yields
\( p_{i}^{*} = (200 - 210\alpha + 43\alpha^2 + 2\alpha^3)/(5(80 - 98\alpha + 27\alpha^2 + \alpha^3)), \)
\( p_{j}^{*} = (-10 - \alpha)/(-40 + 29\alpha + \alpha^2), \) and
\( p_{k}^{*} = (200 - 220\alpha + 52\alpha^2 + 3\alpha^3)/(10(80 - 98\alpha + 27\alpha^2 + \alpha^3)). \) This violates the assumption \( p_{j} < p_{i} \) for \( \alpha \geq \alpha_{3}^{*} \) since \( \Pi_{i}^{*}(p_{i}^{*}, p_{j}^{*}, p_{k}^{*}) < \Pi_{i}^{*}(p_{k}^{*} - \epsilon, p_{j}^{*}, p_{k}^{*}) \) for \( \alpha \geq \alpha_{3}^{*} \). Hence a pure strategy price equilibrium exists only for \( 0 \leq \alpha < \alpha_{3}^{*} \). It is characterized by
\( p_{i}^{*} = (200 - 210\alpha + 43\alpha^2 + 2\alpha^3)/(5(80 - 98\alpha + 27\alpha^2 + \alpha^3)), \)
\( p_{j}^{*} = (-10 - \alpha)/(-40 + 29\alpha + \alpha^2), \) and
\( p_{k}^{*} = (200 - 220\alpha + 52\alpha^2 + 3\alpha^3)/(10(80 - 98\alpha + 27\alpha^2 + \alpha^3)). \)

**Case 3.2.2:**
Firms' problem is given by (17) and (18). Assume \( p_{i} < p_{j}, p_{k} \). Solving the system of first order conditions yields \( p_{i}^{*} = (10 + \alpha)/(10(2 - \alpha)), p_{i}^{*} = (-10 + 3\alpha)/(-40 + 30\alpha), \) and
\[ p_k^* = \frac{-10 + 3\alpha}{-40 + 30\alpha}. \] This violates the assumption \( p_i < p_j \). Assume \( p_k < p_i, p_k \). Solving the system of first order conditions yields
\[ p_i^* = \frac{10 - 3\alpha}{10(2 - \alpha)}, \quad p_j^* = \frac{-10 - \alpha}{-40 + 30\alpha}, \] and
\[ p_k^* = \frac{-10 + 3\alpha}{-40 + 30\alpha}. \] This violates the assumption \( p_j < p_k \). Assume \( p_k < p_i, p_j \). By symmetry of firm \( j \) and \( k \) and the contradiction derived for \( p_j < p_i, p_k \), this violates the assumption \( p_k < p_j \). Hence there exists no price equilibrium.

**Case 3.2.3:**

Firms’ problem is given by (17) and (18). Assume \( p_i < p_j, p_k \). Solving the system of first order conditions yields
\[ p_i^* = \frac{220 - 168\alpha + 3\alpha^2}{10(20 - 14\alpha - \alpha^2)}, \]
\[ p_j^* = \frac{14 - 7\alpha}{40 - 28\alpha - 2\alpha^2}, \] and
\[ p_k^* = \frac{140 - 112\alpha + 7\alpha^2}{10(20 - 14\alpha - \alpha^2)}. \] This violates the assumption \( p_i < p_k \). Assume \( p_k < p_i, p_j \). By symmetry of firm \( i \) and \( k \) and the contradiction derived for \( p_i < p_j, p_k \), this violates the assumption \( p_k < p_i \). Assume \( p_k < p_i, p_j \). By symmetry of firm \( i \) and \( k \) and the contradiction derived for \( p_j < p_k < p_i \), this violates the assumption \( p_i < p_k \). Hence there exists no price equilibrium.

**Case 3.3.1:**

Firms’ problem is given by (17) and (18). Assume \( p_j < p_i \). Solving the system of first order conditions yields
\[ p_i^* = \frac{7}{10}, \quad p_j^* = \frac{11}{10}, \quad p_k^* = \frac{7}{20}. \] This violates the assumption \( p_j < p_i \). Assume \( p_i < p_j \). By symmetry of firm \( i \) and \( j \) and the contradiction derived for \( p_j < p_i \), this violates the assumption \( p_i < p_j \). Hence there exists no price equilibrium.

**Case 3.3.2:**

Firms’ problem is given by (17) and (18). Assume \( p_j < p_i, p_k \). Solving the system of first order conditions yields
\[ p_i^* = \frac{140 - 112\alpha + 7\alpha^2}{10(20 - 14\alpha - \alpha^2)}, \]
\[ p_j^* = \frac{220 - 268\alpha + 81\alpha^2 + 2\alpha^3}{10(20 - 14\alpha - \alpha^2)}, \] and
\[ p_k^* = \frac{14 - 7\alpha}{40 - 28\alpha - 2\alpha^2}. \] This violates the assumption \( p_i < p_j \). Assume \( p_i < p_j, p_k \). By symmetry of firm \( i \) and \( j \) and the contradiction derived for \( p_j < p_i, p_k \), this violates the assumption \( p_j < p_i \). Assume \( p_k < p_j < p_i \). Solving the system of first order conditions yields
\[ p_i^* = \frac{140 - 132\alpha + 29\alpha^2 - 2\alpha^3}{10(20 - 14\alpha - \alpha^2)}, \]
\[ p_j^* = \frac{220 - 268\alpha + 81\alpha^2 + 2\alpha^3}{10(20 - 14\alpha - \alpha^2)}, \] and
\[ p_k^* = \frac{14 - 5\alpha + 2\alpha^2}{40 - 28\alpha - 2\alpha^2}. \] This violates the assumption \( p_j < p_i \). Assume
$p_i < p_j < p_k$. By symmetry of firm $i$ and $j$ and the contradiction derived for $p_i < p_j < p_k$, this violates the assumption $p_i < p_j$. Hence there exists no price equilibrium.

**Case 3,3,3:**
Firms' problem is given by (17) and (18). Assume $p_j < p_i, p_k$. Solving the system of first order conditions yields $p_i^* = 7/10, \ p_j^* = 11/10, \ p_k^* = 7/10$. This violates the assumption $p_j < p_i$. Assume $p_i < p_j, p_k$. By symmetry of firm $i, j, k$ and the contradiction derived for $p_j < p_i, p_k$, this violates the assumption $p_i < p_j$. Assume $p_k < p_i, p_j$. By symmetry of firm $i, j, k$ and the contradiction derived for $p_j < p_i, p_k$, this violates the assumption $p_k < p_i$. Hence there exists no price equilibrium.

**Subgame perfect equilibria of the overall game**
Firms $i$ and $j$ set their search cost strategies first, then firm $k$ decides on search costs. Note that only search cost strategies $(1,1,3), (2,1,3), (3,1,1), (3,1,2)$, and $(3,2,1)$ result in equilibria with positive profits in the price game for at least some $\alpha$. I first derive firms' best response functions.

**Claim:** The entrant's best response function is

$$BR_k = \begin{cases} 
3 & \text{if firms } i \text{ and } j \text{ play } 1, 1 \\
3 & \text{if firms } i \text{ and } j \text{ play } 2, 1 \text{ and } \alpha < \alpha_3^* \\
- & \text{if firms } i \text{ and } j \text{ play } 2, 1 \text{ and } \alpha \geq \alpha_3^* \\
2 & \text{if firms } i \text{ and } j \text{ play } 3, 1 \text{ and } \alpha < \alpha_3^* \\
1 & \text{if firms } i \text{ and } j \text{ play } 3, 1 \text{ and } \alpha \geq \alpha_3^* \\
- & \text{if firms } i \text{ and } j \text{ play } 3, 2 \text{ and } \alpha < \alpha_3^* \\
1 & \text{if firms } i \text{ and } j \text{ play } 3, 2 \text{ and } \alpha \geq \alpha_3^* \\
- & \text{if firms } i \text{ and } j \text{ play } 3, 3
\end{cases}$$

(19)

**Proof:** Consider the results in cases $1,1,1$ through $3,3,3$:
Firms $i, j$ play $1,1$: 1 results in Bertrand. 2 yields no equilibrium. Hence $k$ plays 3.
Firms $i, j$ play $2,1$ and $\alpha < \alpha_3^*$: 1 and 2 yield no equilibrium. Hence $k$ plays 3.
Firms $i, j$ play $2,1$ and $\alpha \geq \alpha_3^*$: 1, 2, and 3 yield no equilibrium. Hence $k$ does not enter.
Firms $i, j$ play 3,1 and $\alpha < \alpha_3^*$: 3 yields no equilibrium. Also $
abla \kappa \|_{k}^{3,1} \geq \nabla \kappa \|_{k}^{3,1,1}, \forall \alpha \in [0, 1]$. Hence firm $k$ plays 2.

Firms $i, j$ play 3,1 and $\alpha \geq \alpha_3^*$: 2 and 3 yield no equilibrium. Hence $k$ plays 1.

Firms $i, j$ play 2,2: 1, 2, and 3 yield no equilibrium. Hence $k$ does not enter.

Firms $i, j$ play 3,2 and $\alpha < \alpha_3^*$: 2 and 3 yields no equilibrium. Hence firm $k$ plays 1.

Firms $i, j$ play 3,2 and $\alpha \geq \alpha_3^*$: 1, 2 and 3 yield no equilibrium. Hence $k$ does not enter.

Firms $i, j$ play 3,3: 1, 2, and 3 yield no equilibrium. Hence $k$ does not enter.

Q.E.D. (claim)

Recall that I have assumed w.l.o.g. that in any strategy pair involving 1 and 2 or 2 and 3 firm $i$ plays 2 and 3 respectively. This explains why the following best response function of firm $i$ does not include a best response to firm $j$ playing 3. Similarly, the best response function of firm $j$ does not include a best response to firm $i$ playing 1.

Claim: Firm $i$'s best response function is

$$BR_i = \begin{cases} 
3 & \text{if firms j plays 1 and } \alpha < \alpha_3^* \\
3 & \text{if firms j plays 1 and } \alpha_3^* \leq \alpha \leq \alpha_2^* \\
2 & \text{if firms j plays 1 and } \alpha > \alpha_2^* \\
3 & \text{if firms j plays 2 and } \alpha < \alpha_1^* \\
\text{any} & \text{if firms j plays 2 and } \alpha \geq \alpha_1^* 
\end{cases}$$ (20)

Proof: Consider the results in cases 1,1,1 through 3,3,3 as well as equation 19.

Firm $j$ plays 1 and $\alpha < \alpha_3^*$: If $i$ plays 1 $k$ follows with 3. If $i$ plays 2 $k$ follows with 3. If $i$ plays 3 $k$ follows with 2. Note that $\Pi_i^{3,1,1} \geq \Pi_i^{3,1,3} \geq \Pi_i^{1,1,3}, \forall \alpha \in [0, \alpha_3^*]$. Hence $i$ plays 3.

Firm $j$ plays 1 and $\alpha \geq \alpha_3^*$: If $i$ plays 1 $k$ follows with 3. If $i$ plays 2 $k$ does not enter. If $i$ plays 3 $k$ follows with 1. Note that $\Pi_i^{3,1,1} \geq \Pi_i^{1,1,3}, \forall \alpha \in [0, 1]$. From Case 2,1 in the previous proof we know that for $\alpha \leq \alpha_2^*$ there is no price equilibrium. Hence $i$ plays 3.

Firm $j$ plays 1 and $\alpha > \alpha_2^*$: If $i$ plays 1 $k$ follows with 3. If $i$ plays 2 $k$ does not enter. If $i$ plays 3 $k$ follows with 1. Note that $\Pi_i^{2,1} \geq \Pi_i^{3,1,1} \geq \Pi_i^{1,1,3}, \forall \alpha \in (\alpha_2^*, 1]$. Hence $i$ plays 2.

Firm $j$ plays 2 and $\alpha < \alpha_1^*$: Let $\alpha < \alpha_3^*$. If $i$ plays 2 $k$ does not enter. If $i$ plays 3 $k$ follows with 1. From Case 2,2 in the previous proof we know there is no price equilibrium. Let $\alpha_3^* \leq \alpha < \alpha_1^*$. If $i$ plays 2 $k$ does not enter. If $i$ plays 3 $k$ does not enter. From the previous
proof we know that there is no price equilibrium for Case 2.2. For Case 3.2, however, an equilibrium exists for $\alpha < \alpha_1^*$. Hence $i$ plays 3.

Firm $j$ plays 2 and $\alpha \geq \alpha_1^*$: If $i$ plays 2 $k$ does not enter. If $i$ plays 3 $k$ does not enter. From Case 2.2 and Case 3.2 in the previous proof we know there is no price equilibrium. Hence $i$ plays 1,2, or 3.

Q.E.D. (claim)

Claim: Firm $j$'s best response function is

$$\text{BR}_j = \begin{cases} 
1 & \text{if firms } i \text{ plays 2 and } \alpha < \alpha_3^* \text{ or } \alpha > \alpha_2^* \\
2 & \text{if firms } i \text{ plays 3 and } \alpha < \alpha_1^* \\
\text{any} & \text{if firms } i \text{ plays 2 and } \alpha_3^* \leq \alpha \leq \alpha_2^* \\
1 & \text{if firms } i \text{ plays 3 and } \alpha \geq \alpha_1^* 
\end{cases} \quad (21)$$

Proof: Consider the results in cases 1,1,1 through 3,3,3 as well as equations 19 and 20.

Firm $i$ plays 2 and $\alpha < \alpha_3^*$ or $\alpha > \alpha_2^*$: Let $\alpha < \alpha_3^*$. If $j$ plays 1 $k$ follows with 3. If $j$ plays 2 $k$ does not enter. From Case 2.2 in the previous proof we know there is no price equilibrium. Hence $i$ plays 1. Let $\alpha > \alpha_2^*$. If $j$ plays 1 $k$ does not enter. If $j$ plays 2 $k$ does not enter. From Case 2.2 in the previous proof we know there is no price equilibrium. From Case 2.1 in the previous proof we know there is a price equilibrium for $\alpha > \alpha_2^*$. Hence $i$ plays 1.

Firm $i$ plays 2 and $\alpha_3^* \leq \alpha \leq \alpha_2^*$: If $j$ plays 1 $k$ does not enter. If $j$ plays 2 $k$ does not enter. From Case 2.2 in the previous proof we know there is no price equilibrium. From Case 2.1 in the previous proof we know there is no price equilibrium for $\alpha \leq \alpha_2^*$. Hence $i$ plays 1,2, or 3.

Firm $i$ plays 3 and $\alpha < \alpha_1^*$: Let $\alpha < \alpha_3^*$. If $j$ plays 1 $k$ follows with 2. If $j$ plays 2 $k$ follows with 1. If $j$ plays 3 $k$ does not enter. Note that $\Pi_{j}^{3,1,2} > \Pi_{j}^{3,1,1}, \forall \alpha \in [0, \alpha_3^*)$. From Case 3,3 in the previous proof we know there is no price equilibrium. Let $\alpha_3^* \leq \alpha < \alpha_1^*$. If $j$ plays 1 $k$ follows with 1. If $j$ plays 2 $k$ does not enter. If $j$ plays 3 $k$ does not enter. From Case 3,3 in the previous proof we know there is no price equilibrium. For Case 3,2, however, an equilibrium exists for $\alpha < \alpha_1^*$. Also $\Pi_{j}^{3,1,2} > \Pi_{j}^{3,1,1}, \forall \alpha \in (\alpha_3^*, \alpha_1^*)$. Hence $j$ plays 2.

Firm $i$ plays 3 and $\alpha \geq \alpha_1^*$: If $j$ plays 1 $k$ follows with 1. If $j$ plays 2 $k$ does not enter. If $j$ plays 3 $k$ does not enter. From Case 3,3 and Case 3,2 in the previous proof we know there is no price equilibrium. Hence $j$ plays 1.

Q.E.D. (claim)
The subgame perfect equilibria of the game result directly from the best response functions in equations 19, 20, and 21.
Assume \( \alpha = 0 \). Since \( \alpha = 0, 3,2,1=3,1,1 \). The equilibrium search cost strategies are thus \( 3,2,1=3,1,1 \). Equilibrium prices are characterized in Case 3,1,1.
Assume \( 0 < \alpha < \alpha_3^* \). The equilibrium search cost strategies are \( 3,2,1 \). Equilibrium prices are characterized in Case 3,2,1.
For \( \alpha_3^* \leq \alpha \leq \alpha_2^* \). The equilibrium search cost strategies are \( 3,1,1 \). Equilibrium prices are characterized in Case 3,1,1.
For \( \alpha_2^* < \alpha < 1 \). The equilibrium search cost strategies are \( 2,1 \). Firm \( k \) does not enter. Equilibrium prices are characterized in Case 2,1.
Assume \( \alpha = 1 \). The equilibrium search cost strategies are \( 3,1,1 \). Equilibrium prices are characterized in Case 3,1,1.
Q.E.D.

Proof of proposition 5
Assume \( \alpha < \alpha_3^* \). The search cost equilibrium is then \( 3,2,1 \). From Case 3,2,1 we can compute
\[
\Pi_k^*(\alpha) = \frac{(200 - 220\alpha + 52\alpha_2 + 3\alpha_3^2)^2}{50(80 - 98\alpha + 27\alpha^2 + \alpha_3^2)^2}
\]
Differentiating with respect to \( \alpha \) yields
\[
\frac{d}{d\alpha}\Pi_k^*(\alpha) = (200 - 220\alpha + 52\alpha_2 + 3\alpha_3^2)((-220 + 104\alpha + 9\alpha^2) + (-98 + 54\alpha + 3\alpha^2)(200 - 220\alpha + 52\alpha_2 + 3\alpha_3^2))/25(80 - 98\alpha + 27\alpha^2 + \alpha_3^2)^3
\]
There are three real roots from solving \( \frac{d}{d\alpha}\Pi_k^*(\alpha) = 0 \), \( \alpha_1 \approx -20.98, \alpha_2 \approx 1.44, \alpha_3 \approx 2.21 \).
Hence \( \frac{d}{d\alpha}\Pi_k^*(\alpha) \) does not change sign for \( \alpha \in [0, \alpha_3^* ] \). Evaluating \( \frac{d}{d\alpha}\Pi_k^*(\alpha) \) at \( \alpha = 1/8 \) yields \( \frac{d}{d\alpha}\Pi_k^*(1/2) = 0.037 > 0 \). Hence \( \frac{d}{d\alpha}\Pi_k^*(\alpha) > 0 \) for \( \alpha \in [0, \alpha_3^* ] \).
Assume $\alpha_3^* \leq \alpha \leq \alpha_2^*$. The search cost equilibrium is then $3, 1, 1$. From Case $3, 1, 1$ we can compute $\Pi_k^*(\alpha) = 1/8$. Differentiating with respect to $\alpha$ yields $\frac{d}{d\alpha}\Pi_k^*(\alpha) = 0$. Also clearly $\Pi_k^*(\alpha) = 1/8 = \Pi_k^*(0)$.

Q.E.D.

Proof of proposition 6

Let $S_i = \{(h, h), (h, l), (h, v), (l, l), (l, v)\}$ be the set of firm $i$'s relevant search cost strategies. The search cost strategy set for which price equilibria have to be computed is given by $S = S_1 \times S_2$. Duplicate strategies that can be obtained by exchanging firm subscripts can be excluded w.l.o.g. For brevity, denote a firm's search cost strategy $(h, h)$ by 1, $(h, l)$ by 2, $(h, v)$ by 3, $(l, l)$ by 4, and $(l, v)$ by 5. This leaves us with search cost strategies $\bar{S} = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5)\}$. For later use define $\bar{S}^a = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$ and $\bar{S}^b = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)\}$ where $\bar{S} = \bar{S}^a \cup \bar{S}^b$.

The demand functions that firms face are given by

$$D_i(z_i, z_j, q) = \begin{cases} (1 - q)(1/2 - (2 - z_i)p_i + (2 - z_j)p_j) & \text{for } p_i > p_j \\ q + (1 - q)(1/2 - (2 - z_j)p_j + (2 - z_i)p_i) & \text{for } p_i \leq p_j \end{cases}$$  (22)

where

$$\begin{cases} z_i = 0 \quad z_j = 0 \quad q = 1 & \text{if consumers have not searched} \\ z_i = 1 \quad z_j = 0 \quad q = 0 & \text{if consumers have searched firm i} \\ z_i = 1 \quad z_j = 1 \quad q = 1/2 & \text{if consumers have searched both firms} \end{cases}$$

Since firms compete on two channels consumers need not be homogeneous with respect to their revealed search behavior. The total demand that each firm faces is thus a linear combination of the above functions. Let $D_i^{m, n}$ be the demand that firm $i$ faces if firms play search cost strategies $m, n$. Then
\[ D_{i}^{1,1} = D_i(0, 0, 1) \]
\[ D_{i}^{2,1} = (1 - \alpha (1 - \gamma)) D_i(0, 0, 1) + \alpha (1 - \gamma) D_i(1, 0, 0) \]
\[ D_{i}^{3,1} = (1 - \alpha) D_i(0, 0, 1) + \alpha D_i(1, 0, 0) \]
\[ D_{i}^{3,2} = (1 - \alpha) D_i(0, 0, 1) + \alpha \gamma D_i(1, 0, 0) + \alpha (1 - \gamma) D_i(1, 1, 1/2) \]
\[ D_{i}^{3,3} = (1 - \alpha) D_i(0, 0, 1) + \alpha D_i(1, 1, 1/2) \]
\[ D_{i}^{4,1} = \gamma D_i(0, 0, 1) + (1 - \gamma) D_i(1, 0, 0) \]
\[ D_{i}^{4,2} = \gamma D_i(0, 0, 1) + (1 - \gamma)(1 - \alpha) D_i(1, 0, 0) + \alpha (1 - \gamma) D_i(1, 1, 1/2) \]
\[ D_{i}^{4,3} = \gamma (1 - \alpha) D_i(0, 0, 1) + (1 - \gamma)(1 - \alpha) D_i(1, 0, 0) + \alpha \gamma D_i(0, 1, 0) + \alpha (1 - \gamma) D_i(1, 1, 1/2) \]
\[ D_{i}^{4,4} = \gamma D_i(0, 0, 1) + (1 - \gamma) D_i(1, 1, 1/2) \]
\[ D_{i}^{5,1} = \gamma (1 - \alpha) D_i(0, 0, 1) + (1 - \gamma + \alpha \gamma) D_i(1, 0, 0) \]
\[ D_{i}^{5,2} = \gamma (1 - \alpha) D_i(0, 0, 1) + (1 - \alpha - \gamma + 2 \alpha \gamma) D_i(1, 0, 0) + \alpha (1 - \gamma) D_i(1, 1, 1/2) \]
\[ D_{i}^{5,3} = \gamma (1 - \alpha) D_i(0, 0, 1) + (1 - \gamma)(1 - \alpha) D_i(1, 0, 0) + \alpha D_i(1, 1, 1/2) \]
\[ D_{i}^{5,4} = \gamma (1 - \alpha) D_i(0, 0, 1) + \alpha \gamma D_i(1, 0, 0) + (1 - \gamma) D_i(1, 1, 1/2) \]
\[ D_{i}^{5,5} = \gamma (1 - \alpha) D_i(0, 0, 1) + (1 - \gamma(1 - \alpha)) D_i(1, 1, 1/2) \]

Let

\[ p_{i}^{*} = \arg \max_{p_{j}} \Pi_{i}^{m,n}(p_{i}, p_{j}) \]

\[ p_{j}^{*} = \arg \max_{p_{j}} \Pi_{j}^{m,n}(p_{i}, p_{j}) \]

**Proof of proposition 6a:**

It is a necessary condition for the existence of a pure strategy equilibrium that \( \Pi_{i}(p_{i}^{*}, p_{j}^{*}) \geq \Pi_{i}(p_{j}^{*} - \kappa, p_{j}^{*}), \kappa > 0 \). Because of the definition of \( D_{i}(0, 0, 1) \) and \( D_{i}(1, 1, 1/2) \), demand functions \( D_{i}^{m,n}(p_{i}, p_{j}) \) can be separated into a part that is continuous
in \( p_i, p_j \) and a discrete part that the firm with the lower prices faces. Denote these with 
\( D_{i,c}^{m,n}(p_i, p_j) \) and \( D_{i,d}^{m,n}(p_i, p_j) \) respectively. We can now write 

\[
\Pi_i(p_i^*, p_j^*) = p_i^* D_{i,c}^{m,n}(p_i^*, p_j^*) \geq (p_j^* - \kappa) D_{i,c}^{m,n}(p_j^* - \kappa, p_j^*) = \Pi_i(p_j^* - \kappa, p_j^*) = \Pi_i(p_j^* - \kappa, p_j^*) \quad (25)
\]

\[
\Leftrightarrow p_i^* D_{i,c}^{m,n}(p_i^*, p_j^*) \geq (p_j^* - \kappa) (D_{i,c}^{m,n}(p_j^* - \kappa, p_j^*) + D_{i,d}^{m,n}(p_j^* - \kappa, p_j^*))
\]

\[
\Leftrightarrow D_{i,d}^{m,n}(p_j^* - \kappa, p_j^*) \leq \frac{p_i^*}{(p_j^* - \kappa)} D_{i,c}^{m,n}(p_i^*, p_j^*) - D_{i,c}^{m,n}(p_j^* - \kappa, p_j^*)
\]

Considering that \( D_{i}^{m,n} \) is also a function of \((\alpha, \gamma)\) we can rewrite the necessary condition for the existence of a pure strategy equilibrium given parameters \((\alpha, \gamma)\) as 

\[
h^{m,n}(\alpha, \gamma) \leq f^{m,n}(\alpha, \gamma) \quad (26)
\]

where 

\[
h^{m,n}(\alpha, \gamma) \equiv D_{i,d}^{m,n}(p_j^* - \kappa, p_j^*)
\]

\[
f^{m,n}(\alpha, \gamma) \equiv \frac{p_i^*}{(p_j^* - \kappa)} D_{i,c}^{m,n}(p_i^*, p_j^*) - D_{i,c}^{m,n}(p_j^* - \kappa, p_j^*) \quad (27)
\]

**Claim 1**

Assume \( p_i > p_j \). Then \( \forall (m, n) \in \bar{S}^a \), \( p_i^* \leq p_j^* \). Assume \( p_i < p_j \). Then \( \forall (m, n) \in \bar{S}^a \), \( p_i^* \geq p_j^* \). Assume \( p_i = p_j > 0 \). Then \( \forall (m, n) \in \bar{S}^a \), \( p_i^* \neq p_j^* \).

**Claim 2**

\( f^{m,n}(\alpha, \gamma) \) is continuous in \((\alpha, \gamma)\) for all \( m, n \in \bar{S}^b \).

**Claim 3**

\( h^{m,n}(\alpha, \gamma) \) is continuous in \((\alpha, \gamma)\) for all \( m, n \in \bar{S}^b \).

**Claim 4**

For each \( m, n \in \bar{S}^b \) \( \exists \alpha^{m,n}, \eta \) such that \( \lim_{\gamma \to 1} f^{m,n}(\alpha, \gamma) < h^{m,n}(\alpha, 1 - \eta) \), \( \forall \alpha < \alpha^{m,n} \).
Claim 5

For all \( m, n \in \mathbb{Z}^b \), \( \exists \varepsilon > 0 \) such that \( f^{m,n}(\alpha, 1 - \varepsilon) < h^{m,n}(\alpha, 1 - \varepsilon) \), \( \forall \alpha < \min \{ \alpha^{m,n} \} \), \( m, n \in \mathbb{Z}^b \).

Proof of claim 1

Let \((m, n) = (1, 1)\). Firms' problem is given by (23) and (24). This is the classical case of Bertrand competition. Hence \( p_i^* = 0 \), \( p_j^* = 0 \), \( \Pi_i^* = 0 \), and \( \Pi_j^* = 0 \).

Let \((m, n) = (2, 2)\). Firms' problem is given by (23) and (24). Assume \( p_i \geq p_j \). Solving the system of first order conditions yields \( p_i^* = (4 + \alpha(1 - \gamma))/(6\alpha(1 - \gamma)) \) and \( p_j^* = (8 - \alpha(1 - \gamma))/(6\alpha(1 - \gamma)) \). This violates the assumption \( p_i \geq p_j \), \( \forall (\alpha \in (0, 1], \gamma \in [0, 1]) \). Assume \( p_i < p_j \). By symmetry of firm \( i \) and \( j \), \( p_i^*, p_j^* \) violates the assumption \( p_i < p_j \). Assume \( p \equiv p_i = p_j > 0 \). Since \( \Pi_i^*(p - \varepsilon, p) > \Pi_i^*(p, p) \) the assumption \( p_i = p_j \) is violated for all \( p > 0 \).

Let \((m, n) = (3, 3)\). Firms' problem is given by (23) and (24). Assume \( p_i \geq p_j \). Solving the system of first order conditions yields \( p_i^* = (4 + \alpha)/(6\alpha) \) and \( p_j^* = (8 - \alpha)/(6\alpha) \). This violates the assumption \( p_i \geq p_j \), \( \forall (\alpha \in (0, 1], \gamma \in [0, 1]) \). Assume \( p_i < p_j \). By symmetry of firm \( i \) and \( j \), \( p_i^*, p_j^* \) violates the assumption \( p_i < p_j \). Assume \( p \equiv p_i = p_j > 0 \). Since \( \Pi_i^*(p - \varepsilon, p) > \Pi_i^*(p, p) \) the assumption \( p_i = p_j \) is violated for all \( p > 0 \).

Let \((m, n) = (4, 4)\). Firms' problem is given by (23) and (24). Assume \( p_i \geq p_j \). Solving the system of first order conditions yields \( p_i^* = (5 - \gamma)/(6(1 - \gamma)) \) and \( p_j^* = (7 + \gamma)/(6(1 - \gamma)) \). This violates the assumption \( p_i \geq p_j \), \( \forall (\alpha \in (0, 1], \gamma \in [0, 1]) \). Assume \( p_i < p_j \). By symmetry of firm \( i \) and \( j \), \( p_i^*, p_j^* \) violates the assumption \( p_i < p_j \). Assume \( p \equiv p_i = p_j > 0 \). Since \( \Pi_i^*(p - \varepsilon, p) > \Pi_i^*(p, p) \) the assumption \( p_i = p_j \) is violated for all \( p > 0 \).

Let \((m, n) = (5, 5)\). Firms' problem is given by (23) and (24). Assume \( p_i \geq p_j \). Solving the system of first order conditions yields \( p_i^* = (5 - \gamma + \alpha\gamma)/(6(1 - \gamma + \alpha\gamma)) \) and \( p_j^* = (7 + \gamma - \alpha\gamma)/(6(1 - \gamma + \alpha\gamma)) \). This violates the assumption \( p_i \geq p_j \), \( \forall (\alpha \in (0, 1], \gamma \in [0, 1]) \). Assume \( p_i < p_j \). By symmetry of firm \( i \) and \( j \), \( p_i^*, p_j^* \) violates the assumption \( p_i < p_j \). Assume \( p \equiv p_i = p_j > 0 \). Since \( \Pi_i^*(p - \varepsilon, p) > \Pi_i^*(p, p) \) the assumption \( p_i = p_j \) is violated for all \( p > 0 \).

Q.E.D. (claim)
Proof of claim 2:

From (22), (23), (24), and (27) we can derive \( f^m(n(\alpha, \gamma)). \) Assume \( p_i \geq p_j \). Then

\[
f^2.1(\alpha, \gamma) = \frac{3\alpha^2(1-\gamma)^2}{4(4-\alpha + \alpha \gamma)}
\]

\[
f^3.1(\alpha, \gamma) = \frac{3\alpha^2}{4(4-\alpha)}
\]

\[
f^3.2(\alpha, \gamma) = \frac{(-2 + \alpha + 2\gamma + 3\alpha \gamma + 2\alpha \gamma^2)^2}{3(1+\gamma)(1+3\gamma)(8-\alpha)(1-\gamma)}
\]

\[
f^4.1(\alpha, \gamma) = \frac{(3(1-\gamma))^2}{4(3+\gamma)}
\]

\[
f^4.2(\alpha, \gamma) = \frac{(-6 + 9\alpha - 2\alpha^2 + 6\gamma - 7\alpha \gamma + 2\alpha^2\gamma)^2}{3(\alpha - 2)(-4 + 3\alpha)(6 + \alpha + 2\gamma - \alpha \gamma)}
\]

\[
f^4.3(\alpha, \gamma) = \frac{(-6 + 9\alpha - 2\alpha^2 + 12\gamma - 16\alpha \gamma + 11\alpha^2 \gamma + 6\gamma^2 + 19\alpha \gamma^2 - 15\alpha^2 \gamma^2)^2}{3(6 + \alpha + 2\gamma - 3\alpha \gamma)(4 - 3\alpha - 4\gamma + 5\alpha \gamma)(2 - \alpha - 2\gamma + 5\alpha \gamma)}
\]

\[
f^5.1(\alpha, \gamma) = \frac{3(1-\gamma + \alpha \gamma)^2}{4(3+\gamma - \alpha \gamma)}
\]

\[
f^5.2(\alpha, \gamma) = \frac{(-6 + 9\alpha - 2\alpha^2 + 12\gamma - 28\alpha \gamma + 11\alpha^2 \gamma - 6\gamma^2 + 19\alpha \gamma^2 - 15\alpha^2 \gamma^2)^2}{3(6 + \alpha + 2\gamma - 3\alpha \gamma)(4 - 3\alpha - 4\gamma + 7\alpha \gamma)(2 - \alpha - 2\gamma + 3\alpha \gamma)}
\]

\[
f^5.3(\alpha, \gamma) = \frac{(-6 + 9\alpha - 2\alpha^2 + 12\gamma - 19\alpha \gamma + 7\alpha^2 \gamma - 6\gamma^2 + 12\alpha \gamma^2 - 6\alpha^2 \gamma^2)^2}{3(6 + \alpha + 2\gamma - 2\alpha \gamma)(4 - 3\alpha - 4\gamma + 4\alpha \gamma)(2 - \alpha - 2\gamma + 2\alpha \gamma)}
\]

\[
f^5.4(\alpha, \gamma) = \frac{(1 - 5\alpha \gamma - \gamma^2 + 5\alpha \gamma^2 - 6\alpha^2 \gamma^2)^2}{3(7 + \gamma - 2\alpha \gamma)(1 - \gamma + 2\alpha \gamma)(1 - \gamma + 4\alpha \gamma)}
\]
Assume $p_i < p_j$. Then

\[ f^{2,1}(\alpha, \gamma) = \frac{3((\alpha \gamma - \alpha)^2 - 4)}{4(4 - \alpha + \alpha \gamma)} \]

\[ f^{3,1}(\alpha, \gamma) = \frac{3(\alpha^2 - 4)}{4(4 - \alpha)} \]

\[ f^{3,2}(\alpha, \gamma) = \frac{(2 - \alpha + 10\gamma - 2\alpha \gamma^2)(2\alpha + 10\gamma + 3\alpha \gamma + \alpha \gamma^2)}{3(1 + \gamma)(1 + 3\gamma)\alpha(1 + \gamma) - 8} \]

\[ f^{4,1}(\alpha, \gamma) = \frac{3(\gamma - 3)(1 + \gamma)}{4(3 + \gamma)} \]

\[ f^{4,2}(\alpha, \gamma) = \frac{(-6 + 3\alpha + 2\alpha^2 - 6\gamma + 7\alpha \gamma - 2\alpha^2\gamma)}{(18 + 15\alpha - \alpha^2 + 6\gamma - 5\alpha \gamma + \alpha^2\gamma)} \]

\[ f^{4,3}(\alpha, \gamma) = \frac{((-6 + 3\alpha + 2\alpha^2 + 16\alpha \gamma - 11\alpha^2\gamma + 6\gamma^2 - 19\alpha \gamma^2 + 15\alpha^2\gamma^2)}{(18 - 15\alpha + \alpha^2 - 24\gamma + 44\alpha \gamma - 13\alpha^2 \gamma + 6\gamma^2 - 29\alpha \gamma^2 + 30\alpha^2 \gamma^2)} \]

\[ f^{5,1}(\alpha, \gamma) = \frac{3(-1 - \gamma + \alpha \gamma)(3 - \gamma + \alpha \gamma)}{4(3 + \gamma - \alpha \gamma)} \]

\[ f^{5,2}(\alpha, \gamma) = \frac{((-6 + 3\alpha + 2\alpha^2 + 4\alpha \gamma - 11\alpha^2 \gamma + 6\gamma^2 - 19\alpha \gamma^2 + 15\alpha^2 \gamma^2)}{(18 - 15\alpha + \alpha^2 - 24\gamma + 44\alpha \gamma - 7\alpha^2 \gamma + 6\gamma^2 - 17\alpha \gamma^2 + 12\alpha^2 \gamma^2)} \]

\[ f^{5,3}(\alpha, \gamma) = \frac{((-6 + 3\alpha + 2\alpha^2 + 7\alpha \gamma - 11\alpha^2 \gamma + 6\gamma^2 - 12\alpha \gamma^2 + 6\alpha^2 \gamma^2)}{(18 - 15\alpha + \alpha^2 - 24\gamma + 29\alpha \gamma - 5\alpha^2 \gamma + 6\gamma^2 - 12\alpha \gamma^2 + 6\alpha^2 \gamma^2)} \]

\[ f^{5,4}(\alpha, \gamma) = \frac{((-4 - 6\gamma + 19\alpha \gamma + 2\gamma^2 - 7\alpha \gamma^2 + 6\alpha^2 \gamma^2)}{(-1 - 7\alpha \gamma + \gamma^2 - 5\alpha \gamma^2 + 6\alpha^2 \gamma^2)} \]

\[ \times \frac{(3(7 + \gamma - 2\alpha \gamma)(1 - \gamma + 2\alpha \gamma)(1 - \gamma + 4\alpha \gamma))}{(3(7 + \gamma - 2\alpha \gamma)(1 - \gamma + 2\alpha \gamma)(1 - \gamma + 4\alpha \gamma))} \]
All \( f^{m,n}(\alpha, \gamma) \) are of the form \( f^{m,n}(\alpha, \gamma) = \frac{g(\alpha, \gamma)}{k(\alpha, \gamma)} \) where \( g(\alpha, \gamma) \) and \( k(\alpha, \gamma) \) are continuous functions in \( \alpha, \gamma \). Since \( k(\alpha, \gamma) \neq 0 \ \forall (\alpha \in (0, 1], \gamma \in [0, 1]) \), \( f^{m,n}(\alpha, \gamma) \) is continuous for all \( \alpha \in (0, 1], \gamma \in [0, 1] \). Q.E.D. (claim)

**Proof of claim 3:**

From (22) and (23) we can derive \( h^{m,n}(\alpha, \gamma) \):

\[
\begin{align*}
  h^{2,1}(\alpha, \gamma) &= 1 - \alpha(1 - \gamma) \\
  h^{3,1}(\alpha, \gamma) &= 1 - \alpha \\
  h^{3,2}(\alpha, \gamma) &= 1 - \alpha + \alpha(1 - \gamma)/2 \\
  h^{4,1}(\alpha, \gamma) &= \gamma \\
  h^{4,2}(\alpha, \gamma) &= \alpha(1 - \gamma)/2 + \gamma \\
  h^{4,3}(\alpha, \gamma) &= (1 - \alpha)\gamma + \alpha(1 - \gamma)/2 \\
  h^{5,1}(\alpha, \gamma) &= (1 - \alpha)\gamma \\
  h^{5,2}(\alpha, \gamma) &= (1 - \alpha)\gamma + \alpha(1 - \gamma)/2 \\
  h^{5,3}(\alpha, \gamma) &= (1 - \alpha)\gamma + \alpha/2 \\
  h^{5,4}(\alpha, \gamma) &= (1 - \alpha)\gamma + (1 - \gamma)/2
\end{align*}
\]

Clearly \( h^{m,n}(\alpha, \gamma) \) is continuous \( \forall (\alpha \in [0, 1], \gamma \in [0, 1]) \). Q.E.D. (claim)
Proof of claim 4:

Let \( \eta^{2,1} = 1/10 \). Assume \( p_1 \geq p_2 \). Then

\[
\lim_{\gamma \to 1} f^{2,1}(\alpha, \gamma) = 0 < 1 - \frac{\alpha}{10} = h^{2,1}(\alpha, 1 - \eta^{2,1}) \quad \forall \alpha < 1. \] Assume \( p_1 < p_2 \). Then

\[
\lim_{\gamma \to 1} f^{2,1}(\alpha, \gamma) = \frac{2}{3} < 1 - \frac{\alpha}{10} = h^{2,1}(\alpha, 1 - \eta^{2,1}) \quad \forall \alpha < 1. \] Let \( \alpha^{2,1} \equiv 1 \).

Let \( \eta^{3,1} = 1/10 \). Assume \( p_1 \geq p_2 \). Then

\[
\lim_{\gamma \to 1} f^{3,1}(\alpha, \gamma) = \frac{3\alpha^2}{4(4 - \alpha)} < 1 - \alpha = h^{3,1}(\alpha, 1 - \eta^{3,1}) \quad \forall \alpha < (20 - 4\sqrt{21})/2. \] Assume

\( p_1 < p_2 \) and \( \alpha \leq 2/5 \). Then \( \lim_{\gamma \to 1} f^{3,1}(\alpha, \gamma) = \frac{(2 + \alpha)}{3} < 1 - \alpha = h^{3,1}(\alpha, 1 - \eta^{3,1}) \quad \forall \alpha < 1. \)

Let \( \alpha^{3,1} \equiv (20 - 4\sqrt{21})/2 \).

Let \( \eta^{3,2} = 0 \). Assume \( p_1 \geq p_2 \). Then

\[
\lim_{\gamma \to 1} f^{3,2}(\alpha, \gamma) = \frac{3\alpha^2}{4(2 - \alpha)} < 1 - \alpha = h^{3,2}(\alpha, 1 - \eta^{3,2}) \quad \forall \alpha < (20 - 4\sqrt{21})/2. \] Assume

\( p_1 < p_2 \) and \( \alpha \leq 2/5 \). Then \( \lim_{\gamma \to 1} f^{3,2}(\alpha, \gamma) = \frac{(2 + \alpha)}{3} < 1 - \alpha = h^{3,2}(\alpha, 1 - \eta^{3,2}) \quad \forall \alpha < 1. \)

Let \( \alpha^{3,2} \equiv (20 - 4\sqrt{21})/2 \).

Let \( \eta^{4,1} = 1/10 \). Assume \( p_1 \geq p_2 \). Then

\[
\lim_{\gamma \to 1} f^{4,1}(\alpha, \gamma) = \frac{3(1 - \gamma)^2}{4(3 + \gamma)} < \gamma = h^{4,1}(\alpha, 1 - \eta^{4,1}) \quad \forall \alpha < 1. \] Assume \( p_1 < p_2 \). Then

\[
\lim_{\gamma \to 1} f^{4,1}(\alpha, \gamma) = \frac{2}{3} < \gamma = h^{4,1}(\alpha, 1 - \eta^{4,1}) \quad \forall \alpha < 1. \] Let \( \alpha^{4,1} \equiv 1 \).
Let $\eta^{4, 2} = 1/10$. Assume $p_1 \geq p_2$. Then

$$\lim_{\gamma \to 1} f^{4, 2}(\alpha, \gamma) = \frac{\alpha^2}{6(2 - \alpha)(4 - 3\alpha)} < \frac{9}{10} + \frac{\alpha}{20} = h^{4, 2}(\alpha, 1 - \eta^{4, 2}) \quad \forall \alpha < 1.$$ Assume $p_1 < p_2$. Then

$$\lim_{\gamma \to 1} f^{4, 2}(\alpha, \gamma) = \frac{4(1 - \alpha)}{3(2 - \alpha)} < \frac{9}{10} + \frac{\alpha}{20} = h^{4, 2}(\alpha, 1 - \eta^{4, 2}) \quad \forall \alpha < 1.$$ Let $\alpha^{4, 2} \equiv 1$.

Assume $\alpha < 3/2$. Let $\eta^{4, 3} = 1/10$. Assume $p_1 \geq p_2$. Then

$$\lim_{\gamma \to 1} f^{4, 3}(\alpha, \gamma) = \frac{(12\alpha - 6\alpha^2)^2}{24(8 - 2\alpha)\alpha^2} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{20} = h^{4, 3}(\alpha, 1 - \eta^{4, 3}) \quad \forall \alpha < 1.$$ Assume $p_1 < p_2$ and $\alpha \leq 2/5$. Then

$$\lim_{\gamma \to 1} f^{4, 3}(\alpha, \gamma) = \frac{7\alpha - 4}{12} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{20} = h^{4, 3}(\alpha, 1 - \eta^{4, 3}) \quad \forall \alpha < 1.$$ Assume $p_1 < p_2$ and $\alpha > 2/5$. Then

$$\lim_{\gamma \to 1} f^{4, 3}(\alpha, \gamma) = \frac{9\alpha^2}{4(4 - \alpha)} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{20} = h^{4, 3}(\alpha, 1 - \eta^{4, 3}) \quad \forall \alpha < (\sqrt{3865} - 43)/28.$$

Let $\alpha^{4, 3} \equiv (13 - \sqrt{145})/2$.

Assume $\alpha \geq 3/2$. Let $\eta^{4, 3} = 0$. Assume $p_1 \geq p_2$. Then

$$\lim_{\gamma \to 1} f^{4, 3}(\alpha, \gamma) = \frac{(12\alpha - 6\alpha^2)^2}{24(8 - 2\alpha)\alpha^2} < 1 - \alpha = h^{4, 3}(\alpha, 1 - \eta^{4, 3}) \quad \forall \alpha < (8 - 4\sqrt{3})/2$$

which contradicts the assumption $\alpha \geq 3/2$. Assume $p_1 < p_2$. Then

$$\lim_{\gamma \to 1} f^{4, 3}(\alpha, \gamma) = \frac{7\alpha - 4}{12} < 1 - \alpha = h^{4, 3}(\alpha, 1 - \eta^{4, 3}) \quad \forall \alpha < 1.$$ Let

$\alpha^{4, 3} \equiv \alpha^{4, 3} \equiv (13 - \sqrt{145})/2$.

Let $\eta^{5, 1} = 1/10$. Assume $p_1 \geq p_2$. Then

$$\lim_{\gamma \to 1} f^{5, 1}(\alpha, \gamma) = \frac{3\alpha^2}{4(4 - \alpha)} < \frac{9(1 - \alpha)}{10} = h^{5, 1}(\alpha, 1 - \eta^{5, 1}) \quad \forall \alpha < 5^{5, 1} = (30 - 2\sqrt{201})/2.$$ Assume $p_1 < p_2$ and $\alpha \leq 2/5$. Then

$$\lim_{\gamma \to 1} f^{5, 1}(\alpha, \gamma) = \frac{2 + \alpha}{3} < \frac{9(1 - \alpha)}{10} = h^{5, 1}(\alpha, 1 - \eta^{5, 1}) \quad \forall \alpha < 1.$$ Assume $p_1 < p_2$ and
\[ \alpha > 2/5. \text{ Then } \lim_{\gamma \to 1} f^{5.1}(\alpha, \gamma) = \frac{3(4 - \alpha^2)}{4(\alpha - 4)} < \frac{9(1 - \alpha)}{10} = h^{5.1}(\alpha, 1 - \eta^{5,1}) \forall \alpha < 1. \text{ Let } \\
\alpha^{5.1} = (30 - 2\sqrt{201})/2. \]

Assume \( \alpha < 3/2 \). Let \( \eta_1^{5.2} = 1/10 \). Assume \( p_1 \geq p_2 \). Then

\[ \lim_{\gamma \to 1} f^{5.2}(\alpha, \gamma) = \frac{3\alpha^2}{2(8 - 2\alpha)} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{20} = h^{5.2}(\alpha, 1 - \eta_1^{5.2}) \forall \alpha < 1. \text{ Assume } p_1 < p_2 \text{ and } \alpha \leq 2/5. \text{ Then } \\
\lim_{\gamma \to 1} f^{5.2}(\alpha, \gamma) = \frac{2 + \alpha}{3} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{20} = h^{5.2}(\alpha, 1 - \eta_1^{5.2}) \forall \alpha < 1. \text{ Assume } p_1 < p_2 \text{ and } \alpha > 2/5. \text{ Then } \\
\lim_{\gamma \to 1} f^{5.2}(\alpha, \gamma) = \frac{3(4 - \alpha^2)}{4(\alpha - 4)} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{20} = h^{5.2}(\alpha, 1 - \eta_1^{5.2}) \forall \alpha < 1. \text{ Assume } p_1 < p_2 \text{ and } \alpha > 2/5. \text{ Then } \\
\lim_{\gamma \to 1} f^{5.2}(\alpha, \gamma) = \frac{(12\alpha - 6\alpha^2)^2}{24(8 - 2\alpha)^2} < 1 - \alpha = h^{5.2}(\alpha, 1 - \eta_2^{5.2}) \forall \alpha < 1. \text{ Assume } p_1 < p_2 \text{ and } \\
\alpha > 2/5. \text{ Then } \lim_{\gamma \to 1} f^{5.2}(\alpha, \gamma) = \frac{3(4 - \alpha^2)}{4(\alpha - 4)} < 1 - \alpha = h^{5.2}(\alpha, 1 - \eta_2^{5.2}) \forall \alpha < 1. \text{ Let } \\
\alpha^{5.2} = \min\{\alpha_1^{5.2}, \alpha_2^{5.2}\} = (43 - \sqrt{1705})/2. \]

Let \( \eta^{5.3} = 1/10 \). Assume \( p_1 \geq p_2 \). Then

\[ \lim_{\gamma \to 1} f^{5.3}(\alpha, \gamma) = \frac{(2\alpha - \alpha^2)^2}{3(8 - \alpha)^2} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{2} = h^{5.3}(\alpha, 1 - \eta^{5.3}) \forall \alpha < 1. \text{ Assume } \\
p_1 < p_2 \text{ and } \alpha \leq 4/5. \text{ Then } \lim_{\gamma \to 1} f^{5.3}(\alpha, \gamma) = \frac{\alpha}{4} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{2} = h^{5.3}(\alpha, 1 - \eta^{5.3}) \forall \alpha < 1. \text{ Assume } p_1 < p_2 \text{ and } \alpha > 4/5. \text{ Then } \\
\lim_{\gamma \to 1} f^{5.3}(\alpha, \gamma) = -\frac{\alpha}{4} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{2} = h^{5.3}(\alpha, 1 - \eta^{5.3}) \forall \alpha < 1. \text{ Assume } p_1 < p_2 \text{ and } \alpha > 4/5. \text{ Then } \\
\]
\[
\lim_{\gamma \to 1} f^{5.3}(\alpha, \gamma) = \frac{2(1 + \alpha)(\alpha - 2)}{24 - 3\alpha} < \frac{9(1 - \alpha)}{10} + \frac{\alpha}{2} = h^{5.3}(\alpha, 1 - \eta^{5.3}) \quad \forall \alpha < 1. \quad \text{Let} \quad \alpha^{5.3} \equiv 1.
\]

Assume \(\alpha < 1/2\). Let \(\eta^5_1 = 1/10\). Assume \(p_1 \geq p_2\). Then

\[
\lim_{\gamma \to 1} f^{5.4}(\alpha, \gamma) = \frac{3\alpha^2}{2(8 - 2\alpha)} < \frac{9(1 - \alpha)}{10} + \frac{1}{20} = h^{5.4}(\alpha, 1 - \eta^5_1) \quad \forall \alpha < \alpha^{5.4} \equiv (91 - \sqrt{7369})/6. \quad \text{Assume} \ p_1 < p_2 \quad \text{and} \quad \alpha \leq 2/5. \quad \text{Then}
\]

\[
\lim_{\gamma \to 1} f^{5.4}(\alpha, \gamma) = \frac{\alpha^2}{3} < \frac{9(1 - \alpha)}{10} + \frac{1}{20} = h^{5.4}(\alpha, 1 - \eta^5_1) \quad \forall \alpha < 1. \quad \text{Assume} \ p_1 < p_2 \quad \text{and} \quad \alpha > 2/5. \quad \text{Then}
\]

\[
\lim_{\gamma \to 1} f^{5.4}(\alpha, \gamma) = \frac{3(4 - \alpha^2)}{4(\alpha - 4)} < \frac{9(1 - \alpha)}{10} + \frac{1}{20} = h^{5.4}(\alpha, 1 - \eta^5_1) \quad \forall \alpha < 1.
\]

Assume \(\alpha \geq 1/2\). Let \(\eta^5_2 = 0\). Assume \(p_1 \geq p_2\). Then

\[
\lim_{\gamma \to 1} f^{5.4}(\alpha, \gamma) = \frac{3\alpha^2}{2(8 - 2\alpha)} < 1 - \alpha = h^{5.4}(\alpha, 1 - \eta^5_2) \quad \forall \alpha < \alpha^{5.4} \equiv (20 - 4\sqrt{21})/2.
\]

Assume \(p_1 < p_2\). Then

\[
\lim_{\gamma \to 1} f^{5.4}(\alpha, \gamma) = \frac{3(4 - \alpha^2)}{4(\alpha - 4)} < \frac{9(1 - \alpha)}{10} + \frac{1}{20} = h^{5.4}(\alpha, 1 - \eta^5_1) \quad \forall \alpha < 1. \quad \text{Let} \quad \alpha^{5.4} = \min\{\alpha^{5.4}_1, \alpha^{5.4}_2\} = (20 - 4\sqrt{21})/2. \quad \text{Q.E.D. (claim)}
\]

Proof of claim 5:

Let

\[
\bar{S}^{b_1} = \left\{ (m, n) \mid (m, n) \in S \cap \left\{ (m, n) \mid \frac{\partial}{\partial \alpha} h^{m,n}(\alpha, \gamma) \geq 0, \forall \alpha \in (0, \min\{\alpha^m, n\}) \right\} \right\}
\]

\[
= \{ (2, 1), (3, 1), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4) \}
\]

Assume \(\varepsilon < \min\{\eta^m, n\}\), \((n, m) \in \bar{S}^{b_1}\). Since \(\forall (n, m) \in \bar{S}^{b_1}\) it holds that

\[
\frac{\partial}{\partial \alpha} h^{m,n}(\alpha, \gamma) \geq 0,
\]

\[
h^{m,n}(\alpha, 1 - \varepsilon) > h^{m,n}(\alpha, 1 - \eta^m, n) . \tag{28}
\]
From equation 28, claims 2, 3 and the definition of continuity, \( \exists \varepsilon > 0 \) such that
\[
f^{m,n}(\alpha, 1 - \varepsilon) < h^{m,n}(\alpha, 1 - \eta^{m,n}) < h^{m,n}(\alpha, 1 - \varepsilon), \quad \forall \alpha < \min \{ \alpha^{m,n} \} \quad \text{where} \quad (n, m) \in \bar{S}_b^1.
\]
Let
\[
\bar{S}_b^2 = \left\{ (m, n) \mid (m, n) \in \bar{S} \cap \left\{ (m, n) \mid \frac{\partial}{\partial \alpha} h^{m,n}(\alpha, \gamma) < 0, \forall \alpha \in [0, 1] \right\} \right\}
= \{(3, 2)\}
\]
Since for all \((n, m) \in \bar{S}_b^2\) it holds that \(\frac{\partial}{\partial \gamma} h^{m,n}(\alpha, \gamma) < 0\),
\[
h^{m,n}(\alpha, 1 - \varepsilon) > h^{m,n}(\alpha, 1).
\] (29)

From equation 29, claims 2, 3 and the definition of continuity, \( \exists \varepsilon > 0 \) such that
\[
f^{m,n}(\alpha, 1 - \varepsilon) < h^{m,n}(\alpha, 1) < h^{m,n}(\alpha, 1 - \varepsilon), \quad \forall \alpha < \min \{ \alpha^{m,n} \} \quad \text{where} \quad (n, m) \in \bar{S}_b^2.
\]
Since \( \bar{S}^b = \bar{S}_b^1 \cup \bar{S}_b^2 \), it is shown that for all \(n, m \in \bar{S}^b\), \( \exists \varepsilon > 0 \) such that
\[
f^{m,n}(\alpha, 1 - \varepsilon) < h^{m,n}(\alpha, 1 - \varepsilon), \quad \forall \alpha < \min \{ \alpha^{m,n} \}, \quad m, n \in \bar{S}^b.
\]
Q.E.D. (claim)

By claim 1 no \((m, n) \in \bar{S}^a\) result in pure strategy price equilibria with positive profits. Claim 5 shows that \( \forall \varepsilon \) such that \( \forall \alpha < \min \{ \alpha^{m,n} \}, \quad m, n \in \bar{S}^b \) the necessary condition (26) for the existence of a pure strategy price equilibrium with positive profits is violated. Hence, there exists no pure strategy price equilibria with positive profits for any \( \alpha < \min \{ \alpha^{m,n} \} \) where \((m, n) \in \bar{S} = \bar{S}^a \cup \bar{S}^b\). To conclude the proof note that
\[
\min \{ \alpha^{m,n} \} = \left(13 - \sqrt{145}\right)/2 = 0.48 > 9/20.
\]
Q.E.D.

Proof of proposition 6b:

Define
\[
I^j = \{h | \Pi_1^{h,j}, \Pi_2^{h,j} > 0, \forall h \in I\} \quad I = \{2, 3, 4, 5\}
\]
\[
J^j = \{k | \Pi_1^{j,k}, \Pi_2^{j,k} > 0, \forall k \in J\} \quad J = \{1, 2, 3, 4\}
\] (30)
Claim 6

Search cost strategies $(5, 1)$ yield a $p_1 > 0, p_2 > 0$ and $\Pi_1 > 0, \Pi_2 > 0$ for

$$\alpha > \left(1 + (9 - 2\sqrt{21})/\gamma\right).$$

Proof of claim 6:

Let $(m, n) = (5, 1)$. Assume $p_1 > p_2$. From (23) and (24) we get

$$p_1^* = \frac{3 - \gamma + \alpha \gamma}{12(1 - \gamma + \alpha \gamma)} \text{ and } p_2^* = \frac{3 + \gamma - \alpha \gamma}{24(1 - \gamma + \alpha \gamma)}.$$

$p_1^* > p_2^*$ clearly holds. Also $\Pi_i^{(1)}(p_1^*, p_2^*) > \Pi_i^{(1)}(p_2^* - \kappa, p_2^*), \kappa > 0$ iff

$$\alpha > \left(1 + (9 - 2\sqrt{21})/\gamma\right).$$

Q.E.D. (claim)

Claim 7

Let $\Pi_i^{(m, n)}$ be firm $i$'s optimized equilibrium profits if firms $i, j$ play search cost strategies $(m, n)$. Assume that $\Pi_1^{(m, n)}, \Pi_2^{(m, n)} > 0$, $\forall (m, n) \in \mathbb{R}^b$. Then the following statements about the ordering of payoffs from the price game are true $\forall (0 < \alpha, \gamma < 1)$.

One of the two inequalities holds:

(a) $\Pi_1^{(2, 1)} > \Pi_1^{(3, 1)} > \Pi_1^{(4, 1)} > \Pi_1^{(5, 1)}$, (b) $\Pi_1^{(2, 1)} > \Pi_1^{(4, 1)} > \Pi_1^{(3, 1)} > \Pi_1^{(5, 1)}$.

One of the two inequalities holds:

(a) $\Pi_1^{(3, 2)} > \Pi_1^{(4, 2)} > \Pi_1^{(5, 2)}$, (b) $\Pi_1^{(4, 2)} > \Pi_1^{(3, 2)} > \Pi_1^{(5, 2)}$.

It holds that $\Pi_1^{(5, 3)} > \Pi_1^{(4, 3)}$.

It holds that $\Pi_2^{(3, 2)} > \Pi_2^{(3, 1)}$.

One of the two inequalities hold:

(a) $\Pi_2^{(4, 2)} > \Pi_2^{(4, 3)} > \Pi_2^{(4, 1)}$, (b) $\Pi_2^{(4, 2)} > \Pi_2^{(4, 1)} > \Pi_2^{(4, 3)}$.

One of the two inequalities hold:

(a) $\Pi_2^{(5, 4)} > \Pi_2^{(5, 3)} > \Pi_2^{(5, 2)} > \Pi_2^{(5, 1)}$, (b) $\Pi_2^{(5, 4)} > \Pi_2^{(5, 4)} > \Pi_2^{(5, 2)} > \Pi_2^{(5, 1)}$.

Proof of claim 7:

Notice that the demand functions in (23) on page 135 all share the form
\[ D_{i}^{m,n} = xD_i(0, 0, 1) + yD_i(1, 0, 0) + zD_i(0, 1, 0) + tD_i(1, 1, 1/2) \] where

\[ 0 \leq x, y, z, t \leq 1 \text{ and } x + y + z + t = 1. \]

Let

\[ \Pi_i(x, y, z, t) = p_i(xD_i(0, 0, 1) + yD_i(1, 0, 0) + zD_i(0, 1, 0) + tD_i(1, 1, 1/2)) \] (31)

and denote the optimized profits of firm \( i \) by \( \Pi_i^*(x, y, z, t) \).

From first order conditions we can derive:

\[ \Pi_i^*(a, 1-a, 0, 0) = \frac{(6(1-a) + 4a)^2}{288(1-a)} \] (32)

\[ \Pi_i^*(a, 1-a-b, 0, b) = \frac{(4a + 6(1-a-b) + 5b)^2}{144(2(1-a-b) + b)} \] (33)

\[ \Pi_i^*(b, c, a, 1-a-b-c) = \frac{(6a + 4b + 5(1-a-b-c) + 6c)^2}{144(1 + 3a-b+c)} \] (34)

Note that

\[ \frac{\partial}{\partial a} \Pi_i^*(a, 1-a, 0, 0) = \frac{3 + 2a - a^2}{72(1-a)^2} > 0 \quad a \in [0, 1] \] (35)

\[ \frac{\partial}{\partial a} \Pi_i^*(a, 1-a-b, 0, b) = \frac{12 + 8a - 4a^2 + 4b - 4ab - b^2}{72(2-a-b)^2} > 0 \quad a, b \in [0, 1] \] (36)

\[ \frac{\partial}{\partial a} \Pi_i^*(b, c, a, 1-a-b-c) = \frac{-65 + 2a + 3a^2 + 18b - 2ab - b^2 - 18c + 2ac + 2bc - c^2}{144(1 + 3a-b+c)^2} < 0 \quad a, b, c \in [0, 1] \] (37)

Note from (23) and (31) that

\[ \Pi_i^{2,1} = \Pi_i(1-\alpha(1-\gamma), \alpha(1-\gamma), 0, 0) \]
\[ \Pi_i^{3,1} = \Pi_i(1-\alpha, \alpha, 0, 0) \]
\[ \Pi_i^{4,1} = \Pi_i(\gamma, 1-\gamma, 0, 0) \]
\[ \Pi_i^{5,1} = \Pi_i(\gamma(1-\alpha), 1-\gamma(1-\alpha), 0, 0) \] (38)
From (35) and (38) it is immediate that

\[ \Pi^{(2,1)*} > \Pi^{(3,1)*} > \Pi^{(4,1)*} > \Pi^{(5,1)*} \iff 1 - \alpha > \gamma, \ 0 < \alpha, \gamma < 1 \]

\[ \Pi^{(2,1)*} > \Pi^{(4,1)*} > \Pi^{(3,1)*} > \Pi^{(5,1)*} \iff 1 - \alpha \leq \gamma, \ 0 < \alpha, \gamma < 1 \]  

(39)

Note from (23) and (31) that

\[ \Pi^{3,2}_1 = \Pi_1(1 - \alpha, \alpha \gamma, 0, \alpha(1 - \gamma)) \]

\[ \Pi^{4,2}_1 = \Pi_1(\gamma, (1 - \alpha)(1 - \gamma), 0, \alpha(1 - \gamma)) \]  

(40)

\[ \Pi^{5,2}_1 = \Pi_1(\gamma(1 - \alpha), 1 - \alpha - \gamma + 2\alpha \gamma, 0, \alpha(1 - \gamma)) \]

From (36) and (38) it is immediate that

\[ \Pi^{(3,2)*} > \Pi^{(4,2)*} > \Pi^{(5,2)*} \iff 1 - \alpha > \gamma, \ 0 < \alpha, \gamma < 1 \]

\[ \Pi^{(4,2)*} > \Pi^{(3,2)*} > \Pi^{(5,2)*} \iff 1 - \alpha \leq \gamma, \ 0 < \alpha, \gamma < 1 \]  

(41)

Note from (23) and (31) that

\[ \Pi^{4,3}_1 = \Pi_1(\gamma(1 - \alpha), (1 - \alpha)(1 - \gamma), \alpha \gamma, \alpha(1 - \gamma)) \]

\[ \Pi^{5,3}_1 = \Pi_1(\gamma(1 - \alpha), (1 - \alpha)(1 - \gamma), 0, \alpha) \]  

(42)

From (37) and (38) it is immediate that

\[ \Pi^{(5,3)*} > \Pi^{(4,3)*} \iff 0 < \alpha, \gamma < 1 \]  

(43)

From first order conditions we can derive:

\[ \Pi_2*(b, a, 0, 1 - a - b) = \frac{(6a + 7(1 - a - b) + 8b)^2}{144(1 + 3a - b)} \]  

(44)

\[ \Pi_2*(a, b, 1 - a - b - c, c) = \frac{(8a + 6b + 6(1 - a - b - c) + 7c)^2}{144(4b + 2(1 - a - b - c) + c)} \]  

(45)

\[ \Pi_2*(b, 1 - a - b, 0, a) = \frac{(7a + 6(1 - a - b) + 8b)^2}{144(a + 4(1 - a - b))} \]  

(46)

Note that
\[ \frac{\partial}{\partial a} \Pi_2^*(b, a, 0, 1 - a - b) = -\frac{161 + 2a + 3a^2 - 30b - 2ab - b^2}{144(1 + 3a - b)^2} < 0 \quad a, b \in [0, 1] \quad (47) \]

\[ \frac{\partial}{\partial a} \Pi_2^*(a, b, 1 - a - b - c, c) = -\frac{(10 - 2a + 4b - c)(6 + 2a + c)}{72(2(a - b - 1) + c)^2} > 0 \quad a, b, c \in [0, 1] \quad (48) \]

\[ \frac{\partial}{\partial a} \Pi_2^*(b, 1 - a - b, 0, a) = \frac{156 + 8a - 3a^2 + 40b - 8ab - 4b^2}{144(4 - 3a - 4b)^2} > 0 \quad a, b \in [0, 1] \quad (49) \]

Note from (23) and (31) that

\[ \Pi_2^{3, 1} = \Pi_2(1 - \alpha, \alpha, 0, 0) \]
\[ \Pi_2^{3, 2} = \Pi_2(1 - \alpha, \alpha \gamma, 0, \alpha(1 - \gamma)) \quad (50) \]

From (47) and (50) it is immediate that

\[ \Pi_1^{(3, 2)*} > \Pi_1^{(3, 1)*} \iff 0 < \alpha, \gamma < 1 \quad (51) \]

Note from (23) and (31) that

\[ \Pi_2^{4, 1} = \Pi_2(\gamma, 1 - \gamma, 0, 0) \]
\[ \Pi_2^{4, 2} = \Pi_2(\gamma, (1 - \alpha)(1 - \gamma), 0, \alpha(1 - \gamma)) \]
\[ \Pi_2^{4, 3} = \Pi_2(\gamma(1 - \alpha), (1 - \alpha)(1 - \gamma), \alpha \gamma, \alpha(1 - \gamma)) \quad (52) \]

From (48) and (52) it is immediate that

\[ \Pi_1^{(4, 2)*} > \Pi_1^{(4, 3)*} \iff 0 < \alpha, \gamma < 1 \quad (53) \]

From (49) and (52) it is immediate that

\[ \Pi_1^{(4, 2)*} > \Pi_1^{(4, 1)*} \iff 0 < \alpha, \gamma < 1 \quad (54) \]

Note from (23) and (31) that
\[ \Pi_2^{5.1} = \Pi_2(\gamma(1 - \alpha), 1 - \gamma(1 - \alpha), 0, 0) \]
\[ \Pi_2^{5.2} = \Pi_2(\gamma(1 - \alpha), 1 - \alpha - \gamma + 2\alpha\gamma, 0, \alpha(1 - \gamma)) \]
\[ \Pi_2^{5.3} = \Pi_2(\gamma(1 - \alpha), (1 - \alpha)(1 - \gamma), 0, \alpha) \]
\[ \Pi_2^{5.4} = \Pi_2(\gamma(1 - \alpha), \alpha\gamma, 0, 1 - \gamma) \]  

(55)

From (49) and (55) it is immediate that

\[ \Pi_2^{(5, 4)*} > \Pi_2^{(5, 3)*} > \Pi_2^{(5, 2)*} > \Pi_2^{(5, 1)*} \Leftrightarrow 1 - \alpha > \gamma, 0 < \alpha, \gamma < 1 \]
\[ \Pi_2^{(5, 3)*} > \Pi_2^{(5, 4)*} > \Pi_2^{(5, 2)*} > \Pi_2^{(5, 1)*} \Leftrightarrow 1 - \alpha \leq \gamma, 0 < \alpha, \gamma < 1 \]  

(56)

Q.E.D. (claim)

**Claim 8**

If \(|I| = 1, i \in I_i\) and \(|J| = 1, j \in J^i\) then \((i, j)\) is a Nash Equilibrium.

**Proof of claim 8:**

Given that firm 2 plays \(j\), \(\Pi_2^{i, j} > 0 = \Pi_1^{h, j}, \forall h \in I_i\). Given that firm 2 plays \(j\),

\[ \Pi_2^{i, j} > 0 = \Pi_2^{i, k}, \forall k \in J \setminus j. \] Hence neither firm has an incentive to deviate.  

Q.E.D. (claim)

**Claim 9**

If \(|I| \leq 1, \forall k \in J \setminus j\) and \(|J| \leq 1, \forall h \in I_i\) then the game has a Nash Equilibrium.

**Proof of claim 9:**

All strategy pairs that yield positive profits contain strategies \(i\) or \(j\). Let \(h^* = \arg\max_{h \in I_i} \Pi_2^{h, j}\).  

Assume \(h^* \neq i\). By the definition of \(h^*\) firm 1 does not deviate. Since

\[ \Pi_2^{h^*, k} < \Pi_2^{h^*, i}, \forall k \in J \setminus j \] firm 2 does not deviate. Thus \((h^*, j)\) is a Nash Equilibrium. Assume \(h^* = i\). Let \(k^* = \arg\max_{k \in I} \Pi_1^{i, k}\). Assume \(k^* \neq j\). Firm 2 has no incentive to deviate.

Since \(\Pi_1^{i, k^*} < \Pi_1^{i, k^*}, \forall h \in I_i\) firm 1 does no deviate. Thus \((i, k^*)\) is a Nash Equilibrium. As-
sume \( k^* = j \). Then \((h^*, k^*)\) is a Nash Equilibrium since \( h^* = \arg\max_{h \in I} \Pi_{1}^{h, j} \) and

\[
k^* = \arg\max_{k \in I} \Pi_{1}^{i, k}.
\]

Q.E.D. (claim)

**Claim 10**

If \( \exists i \) such that \( |J| = 1, j^* \in J^i \) then one of the following two statements is true.

(a) \((i, j^*)\) is a Nash Equilibrium.

(b) Any Nash Equilibrium of the game also lies in the game where \( |J| = 0 \).

If \( \exists j \) such that \( |I| = 1, i^* \in I^j \) then one of the following statements is true.

(a) \((i^*, j)\) is a Nash Equilibrium.

(b) Any Nash Equilibrium of the game also lies in the game where \( |I| = 0 \).

**Proof of claim 10:**

Assume \( \exists i \) such that \( |J| = 1, j^* \in J^i \). Let \( h^* = \arg\max_{h \in I} \Pi_{1}^{h, j} \). Assume \( h^* = i \). Given that firm 2 plays \( j^* \) firm 1 has no incentive to deviate. Given that firm 1 plays \( i \), since

\[\Pi_{2}^{j^*} < \Pi_{2}^{j}, \; \forall k \in J \setminus j^* \],

firm 2 has no incentive to deviate. Hence \((i, j^*)\) is a Nash Equilibrium. Assume \( h^* \neq i \). Given that firm 2 plays \( j^* \) firm 1 will always deviate from \( i \) to \( h^* \). Hence \((i, j^*)\) is not a Nash Equilibrium. Hence all Nash Equilibria of the game will also be in the game where \( \Pi_{1}^{i, j^*} = \Pi_{1}^{i, j^*} = 0 \).

Assume \( \exists j \) such that \( |I| = 1, i^* \in I^j \). Proof analogous.

Q.E.D. (claim)
Claim 11

Consider the following game where $\Pi_1^{(m, n)*}, \Pi_2^{(m, n)*} \geq 0, \forall (m, n) \in \mathbb{S}.$

<table>
<thead>
<tr>
<th>Firm 2's strategy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Firm 1's strategy</td>
<td>$\Pi_1^{(1, 1)<em>}, \Pi_2^{(1, 1)</em>}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\Pi_1^{(2, 1)<em>}, \Pi_2^{(2, 1)</em>}$</td>
<td>$\Pi_1^{(2, 2)<em>}, \Pi_2^{(2, 2)</em>}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\Pi_1^{(3, 1)<em>}, \Pi_2^{(3, 1)</em>}$</td>
<td>$\Pi_1^{(3, 2)<em>}, \Pi_2^{(3, 2)</em>}$</td>
<td>$\Pi_1^{(3, 3)<em>}, \Pi_2^{(3, 3)</em>}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\Pi_1^{(4, 1)<em>}, \Pi_2^{(4, 1)</em>}$</td>
<td>$\Pi_1^{(4, 2)<em>}, \Pi_2^{(4, 2)</em>}$</td>
<td>$\Pi_1^{(4, 3)<em>}, \Pi_2^{(4, 3)</em>}$</td>
<td>$\Pi_1^{(4, 4)<em>}, \Pi_2^{(4, 4)</em>}$</td>
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<tr>
<td>5</td>
<td>$\Pi_1^{(5, 1)<em>}, \Pi_2^{(5, 1)</em>}$</td>
<td>$\Pi_1^{(5, 2)<em>}, \Pi_2^{(5, 2)</em>}$</td>
<td>$\Pi_1^{(5, 3)<em>}, \Pi_2^{(5, 3)</em>}$</td>
<td>$\Pi_1^{(5, 4)<em>}, \Pi_2^{(5, 4)</em>}$</td>
<td>$\Pi_1^{(5, 5)<em>}, \Pi_2^{(5, 5)</em>}$</td>
</tr>
</tbody>
</table>

Assume that one of the following holds:

(a) $|I_1| = |I_2| = 2$ $i_1, i_2 \in I_1 = I_2$ and $|J_1| = |J_2| = 2$ $j_1, j_2 \in J_1 = J_2.$

(b) $|I_1| = |I_2| = |I_3| = 3$ $i_1, i_2 \in I_1 = I_2 = I_3$ and $|J_1| = |J_2| = 2$ $j_1, j_2, j_3 \in J_1 = J_2.$

(c) $|I_1| = |I_2| = 2$ $i_1, i_2, i_3 \in I_1 = I_2 = I_3$ and $|J_1| = |J_2| = 2$ $j_1, j_2 \in J_1 = J_2.$

Then the game has a Nash Equilibrium.

Proof of claim 11

Under the assumption that $\Pi_1^{(m, n)*}, \Pi_2^{(m, n)*} > 0, \forall (m, n) \in \mathbb{S}^b,$ we can use claim 7 to establish that depending on $(\alpha, \gamma)$ four different search cost games can result. The payoffs in games A, B, C, D specify the ordering in optimized profits. A payoff $\Pi_1^{(m, n)*} = 0$ denotes that the price equilibrium is either Bertrand or that there exists no price equilibrium in pure strategies. From claim 1 we know that the strategies $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$ do not
result in pure strategy equilibria with positive profits. I omit payoffs for which $\Pi_i^m,n = 0$ as well as payoffs that can be generated by exchanging $(m, n)$ to $(n, m)$.

\begin{tabular}{|c|c|c|c|c|c|}
\hline
\textbf{Game A} & \textbf{Firm 2's strategy} & 1 & 2 & 3 & 4 & 5 \\
\hline
Firm 1's strategy & 1 & & & & & \\
& 2 & 4,1 & & & & \\
& 3 & 3,1 & 3,2 & & & \\
& 4 & 2,1 & 2,3 & 1,2 & & \\
& 5 & 1,1 & 1,2 & 2,3 & 1,4 & \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|}
\hline
\textbf{Game B} & \textbf{Firm 2's strategy} & 1 & 2 & 3 & 4 & 5 \\
\hline
Firm 1's strategy & 1 & & & & & \\
& 2 & 4,1 & & & & \\
& 3 & 2,1 & 2,2 & & & \\
& 4 & 3,1 & 3,3 & 1,2 & & \\
& 5 & 1,1 & 1,2 & 2,4 & 1,3 & \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|}
\hline
\textbf{Game C} & \textbf{Firm 2's strategy} & 1 & 2 & 3 & 4 & 5 \\
\hline
Firm 1's strategy & 1 & & & & & \\
& 2 & 4,1 & & & & \\
& 3 & 3,1 & 3,2 & & & \\
& 4 & 2,2 & 2,3 & 1,1 & & \\
& 5 & 1,1 & 1,2 & 2,3 & 1,4 & \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|}
\hline
\textbf{Game D} & \textbf{Firm 2's strategy} & 1 & 2 & 3 & 4 & 5 \\
\hline
Firm 1's strategy & 1 & & & & & \\
& 2 & 4,1 & & & & \\
& 3 & 2,1 & 2,2 & & & \\
& 4 & 3,2 & 3,3 & 1,1 & & \\
& 5 & 1,1 & 1,2 & 2,4 & 1,3 & \\
\hline
\end{tabular}

From claim 6 we know that $\Pi_1^{5,1}, \Pi_2^{5,1} > 0$ for all relevant $(\alpha, \gamma)$. Hence any subset of positive payoffs has to include $(m, n) = (5, 1)$.

I go through all possibilities of setting $\Pi_1^{m,n} = \Pi_2^{m,n} = 0$ in accordance with the conditions laid out in this claim and determine whether the resulting games have a Nash Equilibrium.

Let $\Pi_1^{m,n}, \Pi_2^{m,n} > 0$, $\forall (m, n) \in \{(4, 1), (4, 2), (5, 1), (5, 2)\}$ and $\Pi_1^{m,n} = \Pi_2^{m,n} = 0$, $\forall (m, n) \notin \{(4, 1), (4, 2), (5, 1), (5, 2)\}$. Clearly in games A, B, C, and D, $(4, 2)$ is a Nash Equilibrium.

Let $\Pi_1^{m,n}, \Pi_2^{m,n} > 0$, $\forall (m, n) \in \{(4, 1), (4, 3), (5, 1), (5, 3)\}$ and $\Pi_1^{m,n} = \Pi_2^{m,n} = 0$, $\forall (m, n) \notin \{(4, 1), (4, 3), (5, 1), (5, 3)\}$. Clearly in games A, B, C, and D, $(5, 3)$ is a Nash Equilibrium.
Let $\Pi_1^{m,n}, \Pi_2^{m,n} > 0, \forall (m, n) \in \{(3, 1), (3, 2), (5, 1), (5, 2)\}$ and $\Pi_1^{m,n} = \Pi_2^{m,n} = 0, \forall (m, n) \notin \{(3, 1), (3, 2), (5, 1), (5, 2)\}$. Clearly in games A,B,C, and D, (3, 2) is a Nash Equilibrium.

Let $\Pi_1^{m,n}, \Pi_2^{m,n} > 0, \forall (m, n) \in \{(4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$ and $\Pi_1^{m,n} = \Pi_2^{m,n} = 0, \forall (m, n) \notin \{(4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$. Clearly in games A,B,C, and D, (4, 2) is a Nash Equilibrium.

Let $\Pi_1^{m,n}, \Pi_2^{m,n} > 0, \forall (m, n) \in \{(3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (5, 2)\}$ and $\Pi_1^{m,n} = \Pi_2^{m,n} = 0, \forall (m, n) \notin \{(3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (5, 2)\}$. Clearly in games A and C, (3, 2) is a Nash Equilibrium and in games B and D, (4, 2) is a Nash Equilibrium.

Q.E.D. (claim)

From claims 1 and 6 we know that the first step search cost subgame can be represented in this normal form game where $\Pi_1^{(m,n)*}, \Pi_2^{(m,n)*} \geq 0, \forall (m, n) \in \tilde{s}^U(5, 1)$ and $\Pi_1^{(m,n)*}, \Pi_2^{(m,n)*} > 0$. I omit all “0” payoffs.

<table>
<thead>
<tr>
<th>Firm 1's strategy</th>
<th>Firm 2's strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\Pi_2^{(2,1)<em>}$, $\Pi_2^{(2,1)</em>}$</td>
</tr>
<tr>
<td>3</td>
<td>$\Pi_2^{(3,1)<em>}$, $\Pi_2^{(3,1)</em>}$</td>
</tr>
<tr>
<td>4</td>
<td>$\Pi_2^{(4,1)<em>}$, $\Pi_2^{(4,1)</em>}$</td>
</tr>
<tr>
<td>5</td>
<td>$\Pi_2^{(5,1)<em>}$, $\Pi_2^{(5,1)</em>}$</td>
</tr>
</tbody>
</table>

Pick any subset of positive $\Pi_1^{(m,n)*}, \Pi_2^{(m,n)*}$. Then clearly either claims 8, 9, 10, or 11 applies. Assume claim 8 applies. As claim 8 proves, then there exists a Nash Equilibrium. Assume claim 9 applies. As claim 9 proves, then there exists a Nash Equilibrium. Assume claim 11 applies. As claim 11 proves, then there exists a Nash Equilibrium. Assume claim 10 applies. As claim 10 proves, then *either* there exists a Nash Equilibrium *or* we can look at the game where
\[ |I| = 0 \text{ or } |J| = 0. \] Clearly either claims 8, 9, 10, or 11 apply to the new game. If follows that the above game with \( \Pi^*_1(m, n), \Pi^*_2(m, n) \geq 0, \forall (m, n) \in \mathbb{N}^b \setminus (5, 1) \) always has a pure strategy Nash Equilibrium with positive profits for \( \alpha > (1 + (9 - 2\sqrt{21})/\gamma), 0 < \alpha, \gamma < 1. \) From proposition 2 we know that there is always has a subgame perfect equilibrium with positive profits for \( \gamma = 0. \) Hence the overall game always has a subgame perfect equilibrium with positive profits for \( \alpha > (1 + (9 - 2\sqrt{21})/\gamma), 0 < \alpha < 1, 0 \leq \gamma < 1. \) Q.E.D.

**Notes on mixed strategy equilibria**

Consider the model in sections 2.2 and 2.3. From proposition 1 we know that 
\( s_i = l, p_i = 1/4, s_j = h, p_j = 1/4 \) characterizes the unique subgame perfect equilibrium in pure strategies. The corresponding equilibrium profits are \( \Pi^*_1 = 1/4, \Pi^*_2 = 1/8. \)

If one allows mixing in prices, a possible candidate for a subgame perfect equilibrium is \( s_i = l, s_j = l \) where firms play a mixed strategy in prices. As noted in footnote 8 on page 85 it is not essential for the results in this paper that in the demand functions given by \( s_i = l, s_j = l \) the parameter \( \beta \) be set to 1/2. Hence consider the generalized version of the demand in equation 4.

\[
D_i \equiv \begin{cases} 
(1 - \beta)(1/2 - p_i + p_j) & \text{for } p_i > p_j \\
\beta + (1 - \beta)(1/2 - p_i + p_j) & \text{for } p_i \leq p_j 
\end{cases}
\]

Firms' profits are then given by

\[
\Pi_i = \begin{cases} 
p_i(1 - \beta)(1/2 - p_i + p_j) & \text{for } p_i > p_j \\
p_i(\beta + (1 - \beta)(1/2 - p_i + p_j)) & \text{for } p_i \leq p_j
\end{cases}
\] (57)

Denote profits resulting from a mixed strategy equilibrium with \( \Pi^*_1(\beta), \Pi^*_2(\beta). \)

**Claim:** \( \exists \beta^* \text{ such that } \forall \beta > \beta^*, \Pi^*_1(\beta) < 1/4, \Pi^*_2(\beta) < 1/8. \)

**Sketch of proof:** Suppose \( \beta = 1. \) Then the unique equilibrium of the price game is given by \( p_1^* = p_2^* = 0 \) with \( \Pi^*_1(1) = \Pi^*_2(1) = 0. \) Suppose \( \beta = 1 - \epsilon. \) The equilibrium corre-
spondence is continuous in $\beta$ since $\Pi_1'(\beta)$, $\Pi_2'(\beta)$ are continuous in $\beta$. Hence $\exists \epsilon^*$ such that $\forall \epsilon < \epsilon^*, \Pi_1'(1 - \epsilon) < 1/4, \Pi_2'(1 - \epsilon) < 1/8$. Let $\beta^* = 1 - \epsilon^*$. \[ \text{Q.E.D.} \]

Hence for sufficiently large $\beta$, firms will prefer $s_i = l$, $s_j = h$ to $s_i = l$, $s_j = l$, even if firms can play mixed strategies in prices.

**Price comparisons (4/26/96)**

<table>
<thead>
<tr>
<th>Product</th>
<th>Internet Shopping Network</th>
<th>MacWarehouse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Now Utilities 5.01</td>
<td>$75.43</td>
<td>$89.95</td>
</tr>
<tr>
<td>Retrospect 3.0</td>
<td>$136.87</td>
<td>$149.00</td>
</tr>
<tr>
<td>Quickeys 3.01</td>
<td>$83.59</td>
<td>$99.00</td>
</tr>
<tr>
<td>Adobe Persuasion 3.0</td>
<td>$250.08</td>
<td>$259.95</td>
</tr>
<tr>
<td>Framemaker 5.1</td>
<td>$556.45</td>
<td>$599.95</td>
</tr>
<tr>
<td>Microsoft FoxPro 3.0</td>
<td>$426.73</td>
<td>$489.95</td>
</tr>
<tr>
<td>Eudora Pro</td>
<td>$52.72</td>
<td>$59.95</td>
</tr>
</tbody>
</table>
Essay 3

On the Optimality of Market Orientation

* My interest in the problems covered in this essay stems from my work with John Hauser and the many insightful discussions I have had with him. This essay has benefited greatly from discussions with Meghan Busse, Albert Wenger, and especially John Hauser and Birger Wernerfelt. I also thank Jose Silva and seminar participants at MIT for helpful comments. Financial support from ICRMOT is gratefully acknowledged.
1 Introduction

It is almost an article of faith within marketing that market driven firms will do better than firms that are research driven. George Day described the advantages of being market driven by stating that: “Rewards are based on measurable improvements in customer satisfaction and retention, employees are empowered to resolve customer problems without approvals, recruiting is based on customer problem-solving skill ...” (Day, 1994, p41). Research driven firms, the argument goes, will be poorly guided by market considerations and not learn as effectively from their environment. Their performance is expected to be worse than that of market oriented firms. The normative implication of this is that firms should modify their organizational processes to become more market oriented. This advice is taking hold in practice. In interviews with 43 Chief Technical Officers, CEOs and researchers at 10 research intensive organizations, Zettelmeyer and Hauser (1995) found that most of the firms were modifying their new product development processes with the explicit goal of enhancing their market orientation.\(^1\) Interestingly, however, we also found that some firms were adjusting their new product development processes to decrease their market and increase their research orientation. These situations were indicative:

The main R&D lab of a very successful telecommunications company is 80% funded by the corporation. As a result, most projects are initiated by R&D and not by the marketing departments of the business units. An executive at the company told us that the companies main input for R&D is not marketing but the strategic vision of the company. He gave as one of the reasons for being research driven: “If one wants to have more customer focus in R&D, the customer has to know where he wants to go” (paraphrased).

At a major oil company’s research division the majority of R&D projects are proposed by R&D and then approved by the business units, even though 80% of the funding comes from business units.

At a large chemical company researchers are not instructed what to work on but are supposed to come up with concepts based on the strategic orientation of the company.

This paper analyzes whether all firms should follow the advice to become market oriented. In particular I look at how a firm’s resources (its relative strengths in marketing and R&D) influence the decision (Wernerfelt, 1984). A model is presented in which a firm optimizes over the sequence of marketing and R&D’s participation in a new product development process. In a

\(^1\) See also Hauser and Zettelmeyer (1996). A review is in Griffin and Hauser (1996).
market driven scenario, marketing initiates the development of the new product and then hands off the project to R&D. In the research driven scenario R&D starts and marketing completes the new product development process. In a third scenario, concurrent development, marketing and R&D initiate the development process simultaneously. The difference between these three scenarios is that whichever function (marketing or R&D) trails in the development process can make use of valuable information that was learned while the other function did its part of the development effort.

This paper identifies conditions under which firms are best off being market driven, being research driven, or performing concurrent development. These conditions are expressed in terms of marketing’s and R&D’s cross-functional and internal skills. Marketing’s cross-functional skills refer to its ability to anticipate the technical feasibility of a new product. R&D’s cross-functional skills are its ability to anticipate the technical design that consumers demand. Internal skills describe each function’s ability to achieve what they identify as their respective goals.

The results show that a firm should not always try to be market driven. Being market driven is optimal if marketing’s cross-functional skills are very good compared to R&D’s. Then marketing has a fairly precise idea what is technically feasible, even if it does not receive further information by R&D. R&D, in contrast, can benefit greatly from any information that marketing might make known to it since it has problems anticipating customer needs. By letting marketing move first, R&D will receive valuable information while marketing will not be too affected by its lack of information relating to R&D. Conversely, if marketing’s cross-functional skills are poor, the firm will prefer to be research driven. By initiating development, R&D can furnish marketing with useful information while still being able to determine fairly well what technical design consumers demand. Concurrent development is optimal if neither R&D’s nor marketing’s cross-functional skills are clearly superior to each other. This reflects that information from both marketing and R&D is revealed early on. Both functions can then use this information in subsequent stages of the product’s development. Only if either marketing or R&D could benefit particularly from the other’s information will it be optimal to do all the development work of one function first, i.e. be market driven or research driven.

In academic marketing, many authors have advocated firms’ market orientation (see Day, 1994 for a review). This paper draws its motivation from this literature but builds on the literature on resource dependence for an explanation (Wernerfelt, 1995 highlights the stream of literature on the resource-based view of the firm). Methodologically it most resembles work on
organizational design (Rotemberg & Saloner, 1995 and Athey & Schmutzler, 1995 are examples).

The rest of this paper proceeds as follows. Section 2 presents the basic model and links a firm's resources to its decision regarding market orientation. Section 3 extends the model to consider the effect of consumer preferences on the firm's optimal decision. Section 4 allows for the possibility that initial mistakes in R&D and in marketing can have a large impact on the profitability of the new product. It is analyzed whether this makes concurrent development more or less attractive. Section 5 discusses results and highlights some empirical implications. Section 6 concludes the paper.

2 A resource based model of market orientation

2.1 Model idea

Suppose a firm wants to develop a new product. For simplicity we think of the two functions of the firm that are involved in the development of the new product as research and development (R& D) and marketing. The firm has to decide among three choices. (1) Should marketing do its part of the product development process before R&D (market driven)? (2) Should R&D do its part of the product development process before marketing (research driven)? (3) Should marketing and R&D perform product development concurrently? To model this, suppose that each function performs two tasks. Marketing's tasks are $M_1, M_2$ and R&D's tasks are $R_1, R_2$. The firm's three choices can be written as follows.

\[(1) \text{marketing initiates} \quad (2) \text{R&D initiates} \quad (3) \text{concurrent development} \]

\[
\begin{array}{c|c}
\text{period 1} & \text{period 2} \\
\hline
M_1, M_2 & R_1, R_2 \\
\end{array}
\quad
\begin{array}{c|c}
\text{period 1} & \text{period 2} \\
\hline
R_1, R_2 & M_1, M_2 \\
\end{array}
\quad
\begin{array}{c|c}
\text{period 1} & \text{period 2} \\
\hline
M_1, R_1 & M_2, R_2 \\
\end{array}
\]

The difference between these three sequences is that tasks in the second period can be performed in light of information that is revealed through first period tasks. Which of these three approaches is best (market driven, research driven, or concurrent development), will depend on how much second period tasks can benefit from information revealed after the first period. The usefulness of information will depend on the functions' "cross-functional" and "internal" skills.

\[\text{2 The argument does not depend on the definition of marketing and R&D as functions. One can also consider them a set of processes.}\]
2.2 Model interpretation

To give this model some context one should think of marketing as determining the customers' needs and planning the marketing strategy. R&D uncovers the feasibility of design alternatives and executes the product design. I will conceptualize these activities as a two step process. In a first step, R&D and marketing (possibly jointly) determine what it is they want to do. In a second step, marketing and R&D attempt to implement their respective plans. One can speak of this two step process as “determining the targets” in the first step and “trying to meet the targets” in the second step.

Suppose that R&D and marketing knew exactly what to target as well as how to achieve those targets with certainty. Then the firm would not have much of an optimization problem (except perhaps for reasons of budgetary constraints). This paper considers a situation where R&D and marketing (i) are not quite certain what their targets are and (ii) cannot achieve their respective target for sure, even if they knew precisely what these targets were. R&D’s and marketing’s resources are expressed as the degree of certainty with which they can predict their respective targets (cross-functional skills) and the precision with which they can actually achieve them (internal skills). I assume that during the firm's period of decision making, its resources are fixed. The firm has to decide whether to let marketing initiate the product development process (market driven), R&D initiate the product development process (research driven), or whether both should concurrently initiate product development. The key idea of the model is that each alternative results in information that is useful to either R&D, marketing or both. Suppose that, for example, marketing initiates the new product development process. It determines customers’ needs and plans a marketing strategy. This allows R&D to better determine which design alternatives should be tested and what product design to execute. The downside is that marketing has to determine, for example, customers’ needs without knowing the feasibility of design alternatives (since R&D starts after marketing has finished).

This argument is analogous for the case where R&D starts the product development process. Marketing benefits by knowing what R&D can technically achieve and what the executed product design is. This allows marketing to design, for example, a marketing strategy with more certainty. R&D, on the other hand, is uncertain about where to start testing the feasibility of

---

3 This terminology was introduced by Wernerfelt (1984). Prahalad and Hamel (1990) refer to it as “competency” and it is elsewhere known as “capability”.

4 In the long run, resources will certainly be a choice variable of the firm. However, this amounts to an investment problem that is outside the scope of this paper.
design alternatives or which product design to execute, since it has not received any marketing information.

In concurrent development, marketing determines customers' needs at the same time as R&D uncovers the feasibility of design alternatives. They then continue by simultaneously executing the product design and planning a marketing strategy. The latter development can benefit from information that was gathered in the earlier work. Which of these three approaches is best (market driven, research driven, or concurrent development), will depend on the degree of certainty with which the two functions can predict their respective targets (cross-functional skills) and come close to achieving them (internal skills).

2.3 A basic two period model

Target value formulation

In line with the above ideas I propose a simple target value model. In determining a target, R&D and marketing establish what it is they should do. In trying to meet the target, marketing and R&D attempt to implement their respective plans. The profits of the firm decrease as R&D and marketing deviate from their targets. The basic formulation of the profit function is

\[
\Pi = - \sum \text{('realized choice' - 'target')}^2
\]

(1)

Marketing's and
R&D's activities

In the following sections I first discuss how the targets are determined. Then I model how marketing and R&D try to meet the targets and what their realized choices are. Finally I show how the order in which the firm sequences marketing's and R&D's activities can influence the new product development process.

Determinants of targets

To allow for the comparison of different product development sequences, each function is responsible for two activities. Let \( \tilde{M}_1^* \) and \( \tilde{M}_2^* \) be marketing's, and \( \tilde{R}_1^* \) and \( \tilde{R}_2^* \) be R&D's targets. One should think of \( \tilde{M}_1^* \) as targets relating to those tasks that are needed to determine the customers' needs. Targets for those tasks required to design a marketing strategy are represented by \( \tilde{M}_2^* \). The symbol \( \tilde{R}_1^* \) stands for targets for activities that should uncover the feasibility of design alternatives and \( \tilde{R}_2^* \) represents targets of tasks that are needed to execute the product design.\(^5\)
Targets are uncertain. R&D's and marketing's uncertainty about their targets is due to a lack of information that can only be resolved by performing some of the development work. Let \( m_1 \) be information that marketing learns from determining the customers' needs. Similarly, \( m_2 \) is information that marketing learns while designing a marketing strategy. \( r_1 \) is information that R&D gets while uncovering the feasibility of design alternatives, and \( r_2 \) is information learned from executing the product design. \(^6\)

Table 1 summarizes which of this information is beneficial to either R&D or marketing in determining what their targets are.

<table>
<thead>
<tr>
<th>Target</th>
<th>Information useful in determining target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{M}_1^* ): (determine the customers' needs)</td>
<td>( r_1 )</td>
</tr>
<tr>
<td>( \tilde{M}_2^* ): (design a marketing strategy)</td>
<td>( m_1, r_1, r_2 )</td>
</tr>
<tr>
<td>( \tilde{R}_1^* ): (uncover the feasibility of design alternatives)</td>
<td>( m_1 )</td>
</tr>
<tr>
<td>( \tilde{R}_2^* ): (execute the product design)</td>
<td>( r_1, m_1, m_2 )</td>
</tr>
</tbody>
</table>

When compiling the targets of tasks required to determine the customers' needs (\( \tilde{M}_1^* \)), it will be useful to know which basic design alternatives are feasible (\( r_1 \)). This way the firm can decide to focus on markets that can most likely be served by the new product (see the example in footnotes 5 and 6). The decision on how to design a marketing strategy (\( \tilde{M}_2^* \)) will be aided by

---

5 Consider an illustrative example of a computer hardware company that has to determine what range of processor speeds should be offered on the next generation of computers.

**Marketing:**
Task: “Run a conjoint analysis to determine the minimum speed of computers for home use.” The corresponding target \( \tilde{M}_1^* \) is the range of clock speeds to test.

Task: “Perform a conjoint, cluster, and discriminant analysis to decide on the range in clock speeds that are needed to price discriminate among home and business users.” The corresponding target \( \tilde{M}_2^* \) is the range of clock speeds to consider in this analysis.

**R&D:**
Task: “Test various versions of the processor on an existing (cheap) motherboard design to determine the fastest clock speed that can be used on this motherboard.” The corresponding target \( \tilde{R}_1^* \) is the range of clock speeds to test on the existing motherboard.

Task: “Build a collection of motherboards to accommodate processors of various clock speeds.” The corresponding target \( \tilde{R}_2^* \) is the range of clock speeds for which to design motherboards.

(All the targets in this example lie along the same dimension. In many applications, however, tasks have targets that measure very different things. The model can be easily adapted to such diverse targets by defining a norm over these targets.)
information about the customers' needs ($m_1$), the feasibility of design alternatives ($r_1$), as well as the product design ($r_2$). All three pieces of information will clearly be useful in determining the target. If information about the customers' needs ($m_1$) were known at the time of planning activities to uncover the feasibility of design alternatives ($R_1^*$), various alternatives could be excluded ahead of time because they would not satisfy the customers' needs. Finally, in determining how to execute the product design ($R_2^*$), the firm will benefit from information about the feasibility of design alternatives ($r_1$), the customers' needs ($m_1$), as well as the marketing strategy ($m_2$). The first two pieces of information will clearly be useful. Knowing about the marketing plan will highlight design issues that are of importance in marketing the product.

This formulation explicitly takes into account that R&D's and marketing's tasks are not independent from each other. R&D's targets can only be determined with input from marketing and marketing's targets can only be determined with input from R&D.

I will model the four pieces of information as random variables. For simplicity, assume that these random variables $\tilde{m}_1, \tilde{m}_2, \tilde{r}_1, \tilde{r}_2$ are all mean centered around 1, i.e. $E[\tilde{m}_1] = E[\tilde{m}_2] = E[\tilde{r}_1] = E[\tilde{r}_2] = 1$. The targets are simply the average of the “useful information” in table 1.

\begin{align}
\tilde{M}_1^* &= \tilde{r}_1 \\
\tilde{M}_2^* &= (\tilde{m}_1 + \tilde{r}_1 + \tilde{r}_2)/3 \\
R_1^* &= \tilde{m}_1 \\
R_2^* &= (\tilde{r}_1 + \tilde{m}_1 + \tilde{m}_2)/3
\end{align}

(2)

Since each of $\tilde{m}_1$, $\tilde{m}_2$, $\tilde{r}_1$, and $\tilde{r}_2$ has an expected value of 1 and enter the targets additively, the expected target value for each of the four choice variables equals 1.

\begin{align}
E[\tilde{M}_1^*] = E[\tilde{M}_2^*] = E[R_1^*] = E[R_2^*] = 1.
\end{align}

(3)

Trying to meet the targets

Marketing's choice variables are $M_1$ and $M_2$ where $M_1$ stands for determining the customers' needs, and $M_2$ for planning the marketing strategy. R&D chooses $R_1$ and $R_2$ where $R_1$}

---

6 Examples are:

$m_1$: "Consumers' minimum needs in terms of clock speed." (e.g. 60 mhz)

$m_2$: "The spread of clock speeds that is required to price discriminate between home and business users." (e.g. 60-150 mhz)

$r_1$: "The fastest clock speed for which a processor can be incorporated into the old (cheap) motherboard design." (e.g. 90 mhz)

$r_2$: "The fastest processor for which a motherboard can be engineered." (e.g 150 mhz)
stands for uncovering the feasibility of design alternatives, and $R_2$ represents executing the product design. To model that neither marketing nor R&D are perfect at the above tasks, I introduce random variables $\tilde{s}_M$ and $\tilde{s}_R$. These random variables express the likely errors associated with attempting to fulfill a target. Suppose that marketing sets $M_1 = \bar{M}_1$. Then marketing's choice results in $\tilde{s}_M \bar{M}_1$. Similarly, if marketing chooses $M_2 = \bar{M}_2$, the result is $\tilde{s}_M \bar{M}_2$. The same applies to R&D's choices. The result of choosing $\bar{R}_1$ and $\bar{R}_2$ is $\tilde{s}_R \bar{R}_1$ and $\tilde{s}_R \bar{R}_2$ respectively.

I will assume that R&D learns the realization of $\tilde{s}_R$ after it has determined the feasibility of design alternatives ($\bar{R}_1$). This means that R&D learns about itself in the early stages of its work. Similarly, marketing learns the realization of $\tilde{s}_M$ after identifying customer's needs ($\bar{M}_1$).

I normalize the expected value of the random variables $\tilde{s}_M$ and $\tilde{s}_R$ to 1.\(^7\)

$$E[\tilde{s}_M] = E[\tilde{s}_R] = 1.$$  \hspace{1cm} (4)

I can now fully describe the profit function:

$$\Pi(M_1, M_2, R_1, R_2) = -\sum_{j \in \{1, 2\}} (\tilde{s}_M M_j - \bar{M}_j^*)^2 - \sum_{k \in \{1, 2\}} (\tilde{s}_R R_k - \bar{R}_k^*)^2$$  \hspace{1cm} (5)

The structure of the new product development process

In the second period the firm can make use of information that it learned in the choice process during the first period. R&D and marketing can thus make second period decisions with greater certainty since they know more about their respective targets. Depending on the sequence in which new product development was performed, R&D and marketing also learn about the effect their efforts will have towards accomplishing their respective goals. The exact information that is available to R&D and marketing depends on the sequence in which the new prod-

\[^7\] In section 3 I allow for the possibility that either marketing's or R&D's mistakes are more damaging to the firm.
uct development process takes place. The following table summarizes the information that is revealed after the first period.

**Table 2: Information revealed after the first period**

<table>
<thead>
<tr>
<th>Period 1 choices</th>
<th>Information revealed</th>
<th>Period 2 choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marketing initiates “Market driven”</td>
<td>$M_1$: determine customer needs</td>
<td>$m_1, m_2, s_M$</td>
</tr>
<tr>
<td></td>
<td>$M_2$: plan marketing strategy</td>
<td></td>
</tr>
<tr>
<td>R&amp;D initiates “Research driven”</td>
<td>$R_1$: uncover feasibility of design alternatives</td>
<td>$r_1, r_2, s_R$</td>
</tr>
<tr>
<td></td>
<td>$R_2$: execute the product design</td>
<td></td>
</tr>
<tr>
<td>Concurrent development</td>
<td>$M_1$: determine customer needs</td>
<td>$m_1, s_M, s_R$</td>
</tr>
<tr>
<td></td>
<td>$R_1$: uncover feasibility of design alternatives</td>
<td></td>
</tr>
</tbody>
</table>

The attractiveness of each new product development sequence varies with the value of the revealed information for the second period choices. If, for example, the variance on the $\hat{m}$'s is large compared to the variance on the $\hat{r}$'s, marketing will profit less from knowing what R&D revealed (R&D initiates) than R&D will profit from the information that was revealed while marketing chose (marketing initiates).\(^8\) R&D's and marketing's resources can be expressed as the degree of certainty with which they can predict their respective targets (“cross-functional skills”) and come close to achieving them (“internal skills”). We can model R&D’s cross-functional skills with the variances of $\hat{m}_1$ and $\hat{m}_2$. The smaller the variances, the better R&D's cross-functional skills; R&D knows within a small range what the effect of market requirements on its targets are. The internal skills of R&D can be modeled with the variance on $s_R$. The smaller the variance, the better R&D is at achieving the targeted design, i.e. it has higher

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\(^8\) Within the example from footnotes 5 and 6 this would be the case if, for example, marketing included some former chip engineers while R&D discouraged its researchers from spending any of their time on tasks other than research (such as talking to customers).

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internal skills. Similarly, a smaller variance of $\tilde{r}_1$ and $\tilde{r}_2$ means that marketing has better cross-functional skills while a lower variance of $\tilde{s}_M$ expresses better internal skills.

For simplicity, assume that $\text{Var}[\tilde{r}_1] = \text{Var}[\tilde{r}_2]$ and $\text{Var}[\tilde{m}_1] = \text{Var}[\tilde{m}_2]$. For the purposes of this paper only the relative and not the absolute skill levels of R&D and marketing are of importance. Hence, I let $\text{Var}[\tilde{r}_j] = 1 - \text{Var}[\tilde{m}_j], \; j \in \{1, 2\}$ and $\text{Var}[\tilde{s}_R] = 1 - \text{Var}[\tilde{s}_M]$. These properties are embedded in the following simple distributions.

\[
\begin{align*}
\tilde{r}_j & \sim g_{r}(x) = \frac{1}{2} & & x \in \{1 - \omega, 1 + \omega\} \\
\tilde{m}_j & \sim g_{m}(x) = \frac{1}{2} & & x \in \{1 - (1 - \omega), 1 + (1 - \omega)\} \\
\tilde{s}_R & \sim g_{s}(x) = \frac{1}{2} & & x \in \{1 - v, 1 + v\} \\
\tilde{s}_M & \sim g_{s}(x) = \frac{1}{2} & & x \in \{1 - (1 - v), 1 + (1 - v)\}
\end{align*}
\]  

(6)

The parameter $\omega$ describes the relative cross-functional skills of marketing compared to R&D. For example, a high $\omega$ implies that marketing's cross-functional skills are good compared to R&D's. The parameter $v$ captures the relative internal skills of marketing as compared to R&D's. A high $v$ implies that marketing's internal skills are good compared to R&D's.

In summary, a firm engages in a new product development process in which marketing determines customers' needs and plans a marketing strategy and R&D explores the feasibility of design alternatives and executes the product design. R&D and marketing first determine what their targets are and then try to match those targets. Both the targets and the outcomes of choices are uncertain. The firm performs its development work in two periods. Information is learned after having done some of the development work in the first period. The firm can use this information to make better choices in the second period. By choosing the sequence in which development work is performed, the firm determines which uncertainty to resolve. The resources of the firm are modeled as cross-functional and internal R&D and marketing skills. They are operationalized as the variances of the random variables that affect the target (cross-functional skills) and the targeting ability (internal skills).

---

9 For example, R&D's internal skills could refer to the ability to shield high frequencies in testing motherboards with processors of different clock speeds. The better they can shield high frequencies, the better they will be able perform tests on the entire range of clock speeds they plan to test.

10 In section 4 I allow for the possibility that mistakes that are made early in the new product development process are more (less) damaging than late mistakes.
2.4 Results

The firm makes two types of choices. First, it decides on the sequence in which to perform the product development process. It can decide to have marketing initiate the process (the sequence being \( \{ M_1, M_2 \} \), \( \{ R_1, R_2 \} \)), it can let R&D initiate the process, (the sequence being \( \{ R_1, R_2 \} \), \( \{ M_1, M_2 \} \)) or it can ask marketing and R&D to begin the product development process concurrently (resulting in the sequence \( \{ M_1, R_1 \} \), \( \{ M_2, R_2 \} \)). Second, the firm’s marketing and R&D functions set the optimal levels of \( M_1, M_2 \) and \( R_1, R_2 \).\(^{11}\)

I will calculate the firm’s optimal choices by determining the optimal \( M_1 \) through \( R_2 \) for each of the three sequences and then comparing the resulting profits. In trying to meet the second period targets, the firm’s choices can condition on the information learned after the first period. Expanding the firm’s profit function from equation 5 and substituting equation 2 for R&D’s and marketing’s targets we get:

\[
\Pi(M_1, M_2, R_1, R_2) = -(\bar{s}_M M_1 - \tilde{r}_1)^2 - (\bar{s}_M M_2 - (\bar{m}_1 + \tilde{r}_1 + \tilde{r}_2)/3)^2 \\
- (\bar{s}_R R_1 - \bar{m}_1)^2 - (\bar{s}_R R_2 - (\tilde{r}_1 + \bar{m}_1 + \bar{m}_2)/3)^2
\]

(7)

The firm’s decision problem depends on the sequence of the development process. If marketing initiates the new product development process, i.e. if the firm is market driven, its first and second period decision problem will be:\(^{12}\)

\[
\text{Period 1} \quad \max_{M_1, M_2} E[\Pi(M_1, M_2, R_1, R_2)]
\]

\[
\text{Period 2} \quad \max_{R_1, R_2} E[\Pi(M_1, M_2, R_1, R_2)|m_1, m_2, s_M]
\]

If R&D initiates the process, i.e. the firm is research driven, the decision problem becomes:

\[
\text{Period 1} \quad \max_{R_1, R_2} E[\Pi(M_1, M_2, R_1, R_2)]
\]

\[
\text{Period 2} \quad \max_{M_1, M_2} E[\Pi(M_1, M_2, R_1, R_2)|r_1, r_2, s_R]
\]

\(^{11}\) This is equivalent to the firm making these choices. I don’t consider contractual or strategic issues between the firm and marketing and R&D.

\(^{12}\) In the first period the firm takes into account that it will set second period choice variables with more information.
Finally, if R&D and marketing do concurrent development, the firm solves:

\[
\begin{align*}
\text{Period 1} & \quad \max_{M_1, R_1} E[\Pi(M_1, M_2, R_1, R_2)] \\
\text{Period 2} & \quad \max_{M_2, R_2} E[\Pi(M_1, M_2, R_1, R_2)|m_1, r_1, s_M, s_R]
\end{align*}
\]

(10)

After solving the problems in (8), (9), and (10) the firm will choose the sequence of the development process with the highest profits. Proposition 1 specifies the optimality of the three sequences.

Define the following regions for the parameter space:

\[
A = \{ (\omega, v) | \omega^2 + v^2 - 2\omega v (\omega + v) + 2(1 - 2\omega)(1 - v) > 0, \\
(2 - 2v + v^2)((7 - 4v^2)\omega^2 - 2(9 - 2v^2)\omega) - \\
v^2(31 - 40v + 20v^2) + 9 > 0 \}
\]

\[
B = \{ (\omega, v) | (2 - 2v + v^2)((7 - 4v^2)\omega^2 - 2(9 - 2v^2)\omega) - \\
v^2(31 - 40v + 20v^2) + 9 < 0, \\
(1 + v^2)((3 + 8v - 4v^2)\omega^2 + (2 - 2v + v^2)\omega) + \\
2v(1 - v)(11 - 10(v - v^2)) - 13 < 0 \}
\]

\[
C = \{ (\omega, v) | ((1 + v^2)((3 + 8v - 4v^2)\omega^2 + (2 - 2v + v^2)\omega) + \\
2v(1 - v)(11 - 10(v - v^2)) - 13 > 0, \\
\omega^2 + v^2 - 2\omega v (\omega + v) + 2(1 - 2\omega)(1 - v) < 0) \}
\]

**Proposition 1:** Let \( \Pi^*_M, \Pi^*_{R&D}, \) and \( \Pi^*_C \) be maximized profits given \( \{M_1, M_2\}, \{R_1, R_2\} \) (marketing initiates), \( \{R_1, R_2\}, \{M_1, M_2\} \) (R&D initiates), and \( \{M_1, R_1\}, \{M_2, R_2\} \) (concurrent development) respectively.

Then:

\( \Pi^*_M > \Pi^*_{R&D}, \Pi^*_C \), i.e. \( \{M_1, M_2\}, \{R_1, R_2\} \) is best iff \( (\omega, v) \in A \).

\( \Pi^*_C > \Pi^*_M, \Pi^*_{R&D} \), i.e. \( \{M_1, R_1\}, \{M_2, R_2\} \) is best iff \( (\omega, v) \in B \).

\( \Pi^*_{R&D} > \Pi^*_M, \Pi^*_C \), i.e. \( \{R_1, R_2\}, \{M_1, M_2\} \) is best iff \( (\omega, v) \in C \).

where \( \omega, v \in [0, 1] \).

Proof of proposition 1: see appendix.
Recall that a high $\omega$ means that marketing has good cross-functional skills relative to R&D. Similarly, a high $\nu$ implies strong internal skills by marketing relative to R&D. By looking at profits along these two types of skills, I can state the main result of this paper.

**Corollary 1:** Suppose that marketing's cross-functional skills are good relative to R&D's. Then, if R&D's internal skills are poor relative to marketing's, concurrent development is optimal. Otherwise it is optimal to be market oriented.

Suppose that R&D's cross-functional skills are good relative to marketing's. Then, if marketing's internal skills are poor relative to R&D's, concurrent development is optimal. Otherwise it is optimal to be research oriented.

This result can be illustrated by graphing the optimal regions in $\omega$, $\nu$ space.

**Figure 1:** Optimal strategies according to parameter space

The intuition for the result in corollary 1 is as follows. Fix some intermediate level of internal skills $\nu$ and consider variations in cross-functional skills. If marketing's cross-functional skills are high compared to R&D's (low $\omega$), marketing should initiate the development process. If R&D's cross-functional skills are high relative to marketing's (high $\omega$), R&D should initiate development. If neither of them are clearly better, development should be carried out concurrently.
Consider the case where marketing's cross-functional skills are better (low \( \omega \)). Then marketing has a fairly precise idea what its target is, even without information from R&D. R&D, in contrast, can benefit greatly from any information from marketing since R&D's poor cross-functional skills translate into a high variance of its targets. By letting marketing move first, R&D will receive valuable information after the first period while marketing will not be too affected by its lack of information relating to R&D.

Similar reasoning can be applied if R&D's cross-functional skills are good relative to marketing's (high \( \omega \)). By initiating development, R&D can furnish marketing with useful information while still being able to determine the target relatively precisely.

If neither R&D's nor marketing's cross-functional skills stand out, concurrent development becomes optimal. The reason is that concurrent development does not only reveal information regarding the targets \((m_1, r_1)\) but also information \((s_M, s_R)\) that is useful in improving both functions' targeting abilities at a time where this information can still be used. Although \(s_M\) is also revealed if, for example, marketing chooses first, it does not help R&D to improve its ability to target. Similarly, if R&D moves first, \(s_R\) is known after the first period but is not useful to marketing's subsequent decisions. This advantage of concurrent development remains as long as neither marketing nor R&D can gain significantly from the target information \((m_1, m_2\) or \(r_1, r_2)\) that the other function reveals by choosing first.

Consider now variations in relative internal skills. Fix first a low \(\omega\), i.e. high cross-functional skills by marketing and low cross-functional skills by R&D. If R&D's internal skills are also poor relative to marketing's (high \(\nu\)), it is optimal to do concurrent development. By having R&D do some initial work concurrently with marketing, the information \((s_R)\) allows R&D to improve its targeting in the second period, thus compensating for its bad internal skills. This benefit will more than make up for the fact that if marketing had chosen first, more information about the target would have been known to R&D, since R&D's imprecise targeting would destroy any benefits it might have gotten from knowing what the target is. If R&D's internal skills are average to good, the additional information \((m_1, m_2)\) about its targets is more important than knowing about \(s_R\) since R&D is already good along that dimension. Thus, the firm will want marketing to initiate the development process.

Fix now a high \(\omega\), i.e. marketing has low cross-functional skills and R&D has high cross-functional skills. By an argument analogous to the low \(\omega\) case, concurrent development will be better than "R&D initiates" only if marketing's internal skills are low. Concurrent development helps marketing's second period ability to target since marketing's first period activity helped learn \(s_M\).
If marketing's internal skills are not poor, it will profit more from knowing about the target \((r_1, r_2)\) than from knowing about its targeting ability \(s_M\). It will thus be optimal to have R&D initiate the development process.

In summary, if marketing has better cross-functional skills than R&D the firm should be market oriented, i.e. marketing should initiate the development process. However, if marketing's cross-functional skills are relatively poor the firm should be research oriented, i.e. R&D should initiate the process. Concurrent development is optimal if the function with lower cross-functional skills also has poor internal skills, or if neither function has clearly superior cross-functional or internal skills.

### 3 The role of consumers' preferences

The last section argued that a firm's optimal orientation depends on the cross-functional and internal skills of marketing and R&D. The firm's market orientation can also be affected by the preferences of consumers. The preferences considered here relate to the tasks that marketing and R&D perform in the course of the new product development process. For marketing these are identifying the customers' needs and designing a marketing strategy. For the purposes of this section I combine these by saying that consumers have preferences over the fit of the new product with their needs. R&D's activities consist of exploring the technical feasibility of design alternative and executing the product design. I combine these by saying that consumers have preferences over the design quality of the new product.

#### 3.1 Model

One can model the importance that consumers assign to the fit of the new product with their needs by specifying the penalty on profits that marketing's deviations from its targets entail. Similarly for R&D, I model the impact that target deviations have on profits. This can be easily done by expanding the basic profit function in equation 5. Since only the relative importance of design quality vs. fit matter, I weigh deviation by R&D with \(\theta\) and deviations by marketing with \((1 - \theta)\).

\[
\Pi(M_1, M_2, R_1, R_2) = -\theta \sum_{k \in \{1, 2\}} (\tilde{s}_R R_k - \tilde{R}_k^*)^2 - (1 - \theta) \sum_{j \in \{1, 2\}} (\tilde{s}_M M_j - \tilde{M}_j^*)^2
\]

This formulation describes the relative negative impact that deviations in design quality and fit have on profits. If \(\theta\) is small, consumers care mostly about marketing's choices. This means
that a deviation from marketing targets \((M_j^*)\) will reduce profits more than a deviation from R&D targets \((R_j^*)\). If \(\theta\) is large, consumers care mostly about R&D's choices.

### 3.2 Results

The firm's problems in the market driven and research driven cases are described by (8) and (9) on page 169 where profits are given by equation 11. I calculate the firm's profits by determining the optimal \(M\)'s and \(R\)'s for each of the two sequences. In choosing the optimal levels of design and fit, the firm's second period choices can condition on the information learned after the first period. In order to determine the effect of consumers' preferences on the optimality of market orientation, I calculate how maximized profits for the two sequences (marketing initiates, R&D initiates) change with \(\theta\). Proposition 2 captures the main result.

**Proposition 2**: Let \(\Pi_M^*\) and \(\Pi_{R&D}^*\) be the maximized profits given \(\{M_1, M_2\}, \{R_1, R_2\}\) (marketing initiates) and \(\{R_1, R_2\}, \{M_1, M_2\}\) (R&D initiates). Then the following holds:

\[
\frac{\partial}{\partial \theta} (\Pi_M^* - \Pi_{R&D}^*) > 0, \forall \{\omega, v\}
\]

**Proof of proposition 2**: see appendix.

Proposition 2 states that market orientation becomes more attractive as consumers care more about design quality. The intuition for this result is as follows. If consumers care a lot about design quality this is equivalent to saying that the firm will be penalized if it does not get the design quality right. This means that the firm will want to choose the design quality of the new product with as much information as possible. R&D will want to wait until marketing has provided it with all the information needed to exactly determine what design quality consumers need, i.e. marketing will be the first to choose. Analogously, if consumers care very much about fit with their needs, it is best for marketing to resolve all technical uncertainty before trying to target customer needs. In this case R&D will want to choose first.\(^{13}\)

\(^{13}\) Consider the example from footnotes 5 and 6. Suppose we had a low \(\theta\), i.e. mistakes by marketing were worse for the firm that mistakes by R&D. This would be the case if, for example, consumers could be very well price discriminated against while motherboard components other than the processor would not differ considerably in price with different processor speeds. Then there would be large opportunity costs from making a mistake in the portfolio of clock speeds. Hence, marketing could benefit greatly from knowing the range of clock speeds that motherboards can technically support. A lack of marketing information, however, will not increase the costs of motherboards significantly.
It seems counterintuitive that a firm should be less market driven if consumers care very much about marketing's choices. In interpreting this result it is important to keep in mind that if marketing chooses later as opposed to earlier this does not mean that its choices are less important. It means that marketing can do its job with better information, which translates into fewer errors in finding a fit with customers' needs. Statements about market versus research orientation do not reflect the importance of markets vs. technologies but stress who can benefit more from information about the market or the technological possibilities.

4 The impact of early mistakes

We have assumed so far that the costs of deviating from design or fit targets are independent of the moment in time in which these deviations occur. Clearly, there can be situations where it is more costly to make mistakes early on. Fine tuning a technical specification that is based on the incorrect technology choice will not make up for the fact that the technology is not appropriate for the product. Similarly, if marketing starts with the wrong premises about customer needs, this could do more harm than minor mispecified details in marketing's final stages.

4.1 Model

The impact of early mistakes relates to the initial choice variables $M_1$, $R_1$ of marketing and R&D respectively. One can model the effect of early mistakes by specifying a penalty on profits from target deviations for $M_1$ and $R_1$. I take the basic profit function from equation 5 on page 166 and assign weights that determine the impact of deviations for $M_1$, $R_1$ as compared to deviations for $M_2$, $R_2$. For simplicity I assume that the impact of early mistakes will be the same for marketing and R&D.

$$\Pi(M_1, M_2, R_1, R_2) = -\gamma[(\tilde{s}_M M_1 - \tilde{M}_1^*)^2 + (\tilde{s}_R R_1 - \tilde{R}_1^*)^2]$$

$$- (1 - \gamma)[(\tilde{s}_M M_2 - \tilde{M}_2^*)^2 + (\tilde{s}_R R_2 - \tilde{R}_2^*)^2]$$

If $\gamma$ is small, early deviations or mistakes don't matter very much if compared to late mistakes. If $\gamma$ is large, profits will be affected much more strongly by an early mistake of R&D or marketing than if these mistakes occur later.
4.2 Results

Since I have assumed that the impact of early mistakes will be the same for marketing and R&D, I will focus on the relative attractiveness of having marketing initiate development vs. doing concurrent development. The same conclusions holds for the comparison of R&D and concurrent development.

The firm's problems in the market driven and concurrent development cases are described by (8) and (10) on page 170 where profits are given by equation 12. I calculate the firm's profits by determining the optimal M's and R's for each of the two sequences. In choosing the optimal M's and R's, the firm's second period choices can again condition on the information learned after the first period. In order to determine the effect of early mistakes on the optimality of market orientation, I calculate how maximized profits for the three sequences change with \( \gamma \). Proposition 3 captures the main result.

**Proposition 3:** Let \( \Pi^*_M \), \( \Pi^*_{R&D} \), and \( \Pi^*_C \) be maximized profits given \( \{M_1, M_2\}, \{R_1, R_2\} \) (marketing initiates), \( \{R_1, R_2\}, \{M_1, M_2\} \) (R&D initiates), and \( \{M_1, R_1\}, \{M_2, R_2\} \) (concurrent development) respectively. Then the following holds:

\[
\frac{\partial}{\partial \gamma}(\Pi^*_M - \Pi^*_C) > 0, \ \forall \{\omega, \nu\}
\]

\[
\frac{\partial}{\partial \gamma}(\Pi^*_{R&D} - \Pi^*_C) > 0, \ \forall \{\omega, \nu\}
\]

Proof of proposition 3: see appendix.

Proposition 3 states that market orientation (and R&D orientation) becomes more attractive compared to concurrent development as mistakes in initial choices become more costly. The intuition is simple: if a firm chooses to do concurrent development, both function's initial choices \((M_1, R_1)\) are made in the first period. This means that all information is revealed at a time where the potentially costly choices have already been made. By being market driven, the firm can postpone setting one of the initial choice variables until the second period. Thus, the firm can explore the feasibility of design alternatives while taking advantage of information that was revealed after the first period.
5 Discussion

This paper uses a very simple mechanism to compare the advantages of being market driven, being research driven or performing concurrent development. The mechanism solely relies on the information that has been identified in earlier periods of development and on the informational effect on subsequent choices. I abstract from contractual and strategic issues. While this is also a limitation of the paper, it highlights that one can demonstrate the resource dependence of market orientation without using complicated mechanisms.

Resources, i.e. cross-functional and internal skills are modeled as variances. An alternative approach is to use a revenue - cost formulation for profits and to interpret skills as a scaling parameter on costs. This formulation, however, does not have the natural interpretation of first determining what targets are and then trying to achieve them.

The paper interprets a low variance of a function's targets as that function having good cross-functional skills. For example, if the variance on marketing's target is low, it was said that marketing knew well what R&D's technical possibilities were. An alternative interpretation is that cross-functional information is not important to marketing. This interpretation might be appropriate if the new product development is technically straightforward.

The result in section 3.2 states that a research orientation becomes more attractive as consumers care less about design quality and more about fit with their needs. If a firm is research driven this means that marketing tries to meet customers' needs after R&D has executed the product design. It seems counterintuitive that a firm should be less market driven if consumers care very much about marketing's choices. As mentioned before, statements about market versus research orientation do not reflect the importance of markets vs. technologies but emphasize who can benefit more from information about the market or the technological possibilities.

Cross-functional development's main difference compared to market or research oriented development is that second period choices can condition on useful information about both functions' ability to target. Cross-functional development reveals strictly more information than either market orientation or research orientation. This makes cross-functional development the optimal sequence unless either R&D's or marketing's cross-functional skills are very unequal (see figure 1 on page 171). Then there is a large benefit to having the function with good cross-functional skills initiate development. This will provide the unskilled function with information needed to identify its target better. However, if a function's cross-functional as well as its internal skills are bad, concurrent development becomes optimal again. The benefit of knowing the target (by initiating with marketing or R&D) are destroyed by the function's bad
targeting ability (which concurrent development would improve). Cross-functional skills and internal skills are thus complementary.

This paper shows that a firm's optimal market orientation depends on its resource endowment. This result should not be surprising given that other fundamental firm decisions are critically linked to resources. "[...] the organization should base its mission on its distinctive competencies [a.k.a. resources]" (Kotler, 1994, p. 36). Nevertheless, it has been argued frequently that firms should be market oriented (Day, 1994; Jaworski and Kohli, 1992), or that concurrent development is always optimal. As Wernerfelt concludes in *The Resource-Based View of the Firm: Ten Years After* (1995): "Basing strategies on the differences between firms should be automatic, rather than noteworthy."

This paper has various limitations. First, it does not consider strategic issues between marketing and R&D or the contractual relationships that may exist between the firm and these two functions. While I think that these are important issues, they would make it harder to identify whether the results are attributable to resource or strategic issues. Second, I collapse all aspects of being market oriented or market driven into a firm's decisions to initiate a new product development process with marketing's choices. While there are certainly other aspects of market orientation, this should be an important one: does a technology push (research orientation) or a market pull (market orientation) dominate? Finally, I only consider two period decision problems. I exogenously exclude shorter or longer development processes. While this can be defended, it should ideally have come from within the model through the inclusion of discounting.

The results of this paper imply several testable hypotheses.\(^\text{14}\)

Firms are more likely to be market driven if marketing's cross-functional skills are good relative to R&D's.

Concurrent development is more likely if neither R&D's nor marketing's cross-functional skills are clearly better than those of the other function respectively.

Cross-functional development is more likely if one function has not only poor cross-functional skills but also poor internal skills compared to the other function.

Firms are more likely to be market driven if consumers care more about design than they care about fit.

Firms are more likely to be market driven if mistakes in the early work by R&D and marketing are costly compared to later mistakes.

---

\(^{14}\) I use "market driven", "research driven", "cross-functional development", "cross-functional skills", and "internal skills" as implied by the earlier sections of this paper.
While testing these hypothesis might require a large scale survey, the operationalization of "cross-functional skills", "internal skills", and the order of the new product development process should be fairly straightforward.

6 Concluding remarks

This paper analyzes the resource dependence of a firm's optimal market orientation. A model is presented in which a firm optimizes over the sequence in which marketing and R&D participate in a new product development process. In a market driven scenario, marketing initiates the development of the new product and then hands off the project to R&D. In the research driven scenario R&D starts and marketing completes the new product development process. Concurrent development refers to the simultaneous initiation of the development by marketing and R&D. These three scenarios differ in the points in time at which information about targets and information that helps R&D and marketing to target better become known. The main result of this paper is that a firm's optimal market orientation depends on its resources. Being market driven is optimal if marketing's cross-functional skills are very good compared to R&D's. If marketing's cross-functional skills are poor, the firm will prefer to be research driven. Concurrent development is optimal if neither R&D's nor marketing's cross-functional skills are clearly superior to each other or if one function has not only poor cross-functional skills but also poor internal skills. The paper shows further that market orientation is more attractive if consumers care more about design quality than they care about fit and if costs from early mistakes are more costly than those from later mistakes.

Further theoretical research should consider contractual relationships within the firm and strategic issues between marketing and R&D. Also, the relationship between various business units and a central R&D department should be modeled. A particularly interesting extension is to analyze firms' optimal investment strategies in cross-functional as well as internal skills. Empirical research might focus on testing the hypotheses presented in this paper.

The paper contributes to the marketing literature by suggesting that it is not always optimal for a firm to be market driven. It shows that the optimality of market orientation depends on the firm's resources and links a firm's cross-functional and internal skills to its optimal market orientation. The paper contributes towards a more differentiated view of the advantages of being market driven.
References


Appendix

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{M}_1 ) *</td>
<td>Task target that marketing should perform with choice variable ( M_1 ).</td>
</tr>
<tr>
<td>( \tilde{M}_2 ) *</td>
<td>Task target that marketing should perform with choice variable ( M_2 ).</td>
</tr>
<tr>
<td>( \tilde{R}_1 ) *</td>
<td>Task target that R&amp;D should perform with choice variable ( R_1 ).</td>
</tr>
<tr>
<td>( \tilde{R}_2 ) *</td>
<td>Task target that R&amp;D should perform with choice variable ( R_2 ).</td>
</tr>
<tr>
<td>( \tilde{m}_1 )</td>
<td>Random variable expressing uncertainty about customer needs. Revealed after the first period if ( M_1 ) is chosen in the first period. Information is useful for determining the targets.</td>
</tr>
<tr>
<td>( \tilde{m}_2 )</td>
<td>Random variable expressing uncertainty about the marketing strategy. Revealed after the first period if ( M_2 ) is chosen in the first period. Information is useful for determining the targets.</td>
</tr>
<tr>
<td>( \tilde{r}_1 )</td>
<td>Random variable expressing uncertainty about the feasibility of design alternatives. Revealed after the first period if ( R_1 ) is chosen in the first period. Information is useful for determining the targets.</td>
</tr>
<tr>
<td>( \tilde{r}_2 )</td>
<td>Random variable expressing uncertainty about the execution of the product design. Revealed after the first period if ( R_2 ) is chosen in the first period. Information is useful for determining the targets.</td>
</tr>
<tr>
<td>( \tilde{s}_M )</td>
<td>Random variable expressing uncertainty associated with attempting to fulfill targets ( \tilde{M}_1 ) * and ( \tilde{M}_2 ) * . Revealed after the first period if ( M_1 ) is chosen in the first period. Information useful for marketing's targeting.</td>
</tr>
<tr>
<td>( \tilde{s}_R )</td>
<td>Random variable expressing uncertainty associated with attempting to fulfill targets ( \tilde{R}_1 ) * and ( \tilde{R}_2 ) * . Revealed after the first period if ( R_1 ) is chosen in the first period. Information useful for R&amp;D's targeting.</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Parameter of the distributions of ( \tilde{m}_1 ), ( \tilde{m}_2 ), ( \tilde{r}_1 ), and ( \tilde{r}_2 ). Describes relative R&amp;D vs. marketing crossfunctional skills. Low ( \omega ) means low R&amp;D compared to marketing's crossfunctional skills.</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Parameter of the distributions of ( \tilde{s}_M ), ( \tilde{s}_R ). Describes relative R&amp;D vs. marketing internal skills. Low ( \nu ) means low R&amp;D compared to marketing's internal skills.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Weight on the quadratic terms of the profit function in section 3. Describes the relative impact on profits of deviations in R&amp;D’s vs. marketing’s choices. Small $\theta$ means profits are mostly affected by marketing’s choice variables.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Weight on the quadratic terms of the profit function in section 4. Describes the relative impact on profits of deviations in first vs. second period choices. Small $\gamma$ means deviations from targets are more costly in the second period.</td>
</tr>
</tbody>
</table>

**Proof of proposition 1**

Recall that profits are given by equation 7 on page 169.

$$\Pi(M_1, M_2, R_1, R_2) = -(\tilde{s}_M M_1 - \tilde{r}_1)^2 - (\tilde{s}_M M_2 - (\tilde{m}_1 + \tilde{r}_1 + \tilde{r}_2) / 3)^2$$

$$-(\tilde{s}_R R_1 - \tilde{m}_1)^2 - (\tilde{s}_R R_2 - (\tilde{r}_1 + \tilde{m}_1 + \tilde{m}_2) / 3)^2$$

The distributions of the random variables are given by (6) on page 168.

$$\tilde{r}_j \sim g_x(x) = 1/2 \quad x \in \{1 - \omega, 1 + \omega\}$$

$$\tilde{m}_j \sim g_x(x) = 1/2 \quad x \in \{1 - (1 - \omega), 1 + (1 - \omega)\}$$

$$\tilde{s}_j \sim g_x(x) = 1/2 \quad x \in \{1 - \nu, 1 + \nu\}$$

$$\tilde{s}_M \sim g_x(x) = 1/2 \quad x \in \{1 - (1 - \nu), 1 + (1 - \nu)\}$$

This proof first derives optimal firm profits for the three sequences $\{M_1, M_2\}$, $\{R_1, R_2\}$, $\{R_1, R_2\}$, $\{M_1, M_2\}$, and $\{M_1, R_1\}$, $\{M_2, R_2\}$ and then determines conditions that order the resulting profits.

**Case 1: $\{M_1, M_2\}, \{R_1, R_2\}$**

The firm’s problem is described by equation 8 on page 169.

$$\text{Period 1} \quad \max_{M_1, M_2} E[\Pi(M_1, M_2, R_1, R_2)]$$

$$\text{Period 2} \quad \max_{R_1, R_2} E[\Pi(M_1, M_2, R_1, R_2) | m_1, m_2, s_M]$$

Since all cross derivatives of $E[\Pi]$ are zero, the firm’s problem is time separable. From first order conditions for the first period maximization we obtain $M_1 = 1/(2 - 2\nu + \nu^2)$ and $M_2 = 1/(2 - 2\nu + \nu^2)$. First order conditions in the second period yield
\[ R_1 = (1 + m_1)/(1 + \nu^2) \text{ and } R_2 = (3 + m_1 + m_2)/(1 + \nu^2). \] Second order conditions in both periods clearly hold. We can now calculate the maximized expected profits given the sequence \( \{M_1, M_2\}, \{R_1, R_2\} \):

\[
\Pi^*_M = E[\Pi(M_1, M_2, R_1, R_2)] = \frac{-20 + (4\omega - 26\omega^2)(1 - \nu) + 24\omega(1 - \omega)(\nu^4 - 2\nu^3) + 38\nu - 97\nu^2 + \nu^3(-97 + 50\omega - 61\omega^2) + 96\nu^3 - 48\nu^4}{9(1 + \nu^2)(2 - 2\nu + \nu^2)}
\]  

(13)

**Case 2: \{R_1, R_2\}, \{M_1, M_2\}**

The firm's problem is described by equation 9 on page 169.

\[
\begin{align*}
\text{Period 1} & \quad \max_{R_1, R_2} E[\Pi(M_1, M_2, R_1, R_2)] \\
\text{Period 2} & \quad \max_{M_1, M_2} E[\Pi(M_1, M_2, R_1, R_2)|r_1, r_2, s_R]
\end{align*}
\]

Since all cross derivatives of \( E[\Pi] \) are zero, the firm's problem is time separable. From first order conditions for the first period maximization we obtain \( R_1 = 1/(1 + \nu^2) \) and \( R_2 = 1/(1 + \nu^2) \). First order conditions in the second period yield \( M_1 = (1 + r_1)/(2 - 2\nu + \nu^2) \) and \( M_2 = (3 + r_1 + r_2)/(3(2 - 2\nu + \nu^2)) \). Second order conditions in both periods clearly hold. We can now calculate the maximized expected profits given the sequence \( \{R_1, R_2\}, \{M_1, M_2\} \):

\[
\Pi^*_{R&D} = E[\Pi(M_1, M_2, R_1, R_2)] = \frac{-42 + 60\omega - 108\nu^2 + 96\nu^3 - 48\nu^4 + \omega(-37 + 48\nu - 61\nu^2 + 48\nu^3 - 24\nu^4) + \omega(48 - 48\nu + 72\nu^2 - 48\nu^3 + 24\nu^4)}{9(1 + \nu^2)(2 - 2\nu + \nu^2)}
\]  

(14)

**Case 3: \{M_1, R_1\}, \{M_2, R_2\}**

The firm's problem is described by equation 10 on page 170.

\[
\begin{align*}
\text{Period 1} & \quad \max_{M_1, R_1} E[\Pi(M_1, M_2, R_1, R_2)] \\
\text{Period 2} & \quad \max_{M_2, R_2} E[\Pi(M_1, M_2, R_1, R_2)|m_1, r_1, s_M, s_R]
\end{align*}
\]

Since all cross derivatives of \( E[\Pi] \) are zero, the firm's problem is time separable. From first order conditions for the first period maximization we obtain \( R_1 = 1/(1 + \nu^2) \) and
\( M_1 = 1/(2 - 2v + v^2) \). First order conditions in the second period yield \( R_2 = (3 + m_1 + r_1)/6 \) and \( M_2 = (3 + m_1 + r_1)/6 \). Second order conditions in both periods clearly hold. We can now calculate the maximized expected profits given the sequence \( \{M_1, R_1\}, \{M_2, R_2\} \):

\[
\Pi_C^* = E[\Pi(M_1, M_2, R_1, R_2)] = \\
\frac{-29 + 38v + 66v^2 + 56v^3 - 28v^4 - 20(\omega - \omega^2)(-2 + 2v - 3v^2 + 2v^3 - v^4)}{9(1 + v^4)(2 - 2v + v^2)}
\]

(15)

Simple algebraic manipulation of the differences between \( \Pi_M^*, \Pi_{R&D}^*, \) and \( \Pi_C^* \) results in

\[
\Pi_M^* > \Pi_{R&D}^* \iff \omega^2 + v^2 - 2\omega v (\omega + v) + 2(1 - 2\omega)(1 - v) \geq 0
\]

\[
\Pi_M^* > \Pi_C^* \iff (2 - 2v + v^2)((7 - 4v^2)\omega^2 - 2(9 - 2v^2)\omega - v^2(31 - 40v + 20v^2)) + 9 \geq 0
\]

\[
\Pi_{R&D}^* > \Pi_C^* \iff (1 + v^2)((3 + 8v - 4v^2)\omega^2 + (2 - 2v + v^2)\omega + 2v(1 - v)(11 - 10(\omega - \omega^2)) - 13 \geq 0
\]

Q.E.D.

**Proof of proposition 2**

Profits are given by equation 11 on page 173. After substituting equation 7, we obtain:

\[
\Pi(M_1, M_2, R_1, R_2) = -\theta((\bar{s}_R R_1 - \bar{m}_1)^2 + (\bar{s}_R R_2 - (\bar{r}_1 + \bar{m}_1 + \bar{m}_2)/3)^2)
- (1 - \theta)((\bar{s}_M M_1 - \bar{r}_1)^2 + (\bar{s}_M M_2 - (\bar{m}_1 + \bar{r}_1 + \bar{r}_2)/3)^2)
\]

This proof first derives optimal firm profits for the two sequences \( \{M_1, M_2\}, \{R_1, R_2\} \), \( \{R_1, R_2\}, \{M_1, M_2\} \) and then shows that \( \frac{\partial}{\partial \bar{r}} (\Pi_{R&D}^* - \Pi_M^*) < 0 \).

**Case 1: \{M_1, M_2\}, \{R_1, R_2\}**

As before the firm’s problem is described by equation 8. Again, the firm’s problem is time separable. From first order conditions for the first period maximization we obtain \( M_1 = 1/(2 - 2v + v^2) \) and \( M_2 = 1/(2 - 2v + v^2) \). First order conditions in the second period yield \( R_1 = (1 + m_1)/(1 + v^2) \) and \( R_2 = (3 + m_1 + m_2)/(1 + v^2) \). Second order conditions in both periods clearly hold. Let \( \Pi_M^* = E[\Pi(M_1, M_2, R_1, R_2)] \) be the maximized expected profits given the sequence \( \{M_1, M_2\}, \{R_1, R_2\} \).
Case 2: \( \{ R_1, R_2 \}, \{ M_1, M_2 \} \)

The firm's problem is described by equation 9 and is time separable. From first order conditions for the first period maximization we obtain \( R_1 = 1/(1 + v^2) \) and \( R_2 = 1/(1 + v^2) \). First order conditions in the second period yield \( M_1 = (1 + r_1)/(2 - 2v + v^2) \) and \( M_2 = (3 + r_1 + r_2)/(3(2 - 2v + v^2)) \). Second order conditions in both periods clearly hold. Let \( \Pi_{R&D}^* = E[\Pi(M_1, M_2, R_1, R_2)] \) be the maximized expected profits given the sequence \( \{R_1, R_2\}, \{M_1, M_2\} \).

After some simplification one obtains

\[
\Pi_{R&D}^* - \Pi_M^* = \frac{11(\omega^2 - r(2 - 4\omega + 3\omega^2) + 2r\omega(1 - \omega)^2 + v^2(r(2\omega - 1) + \omega^2(1 - 2r)))}{9(1 + v^2)(2 - 2v + v^2)}
\]  

(16)

Differentiating this difference with respect to \( r \) yields

\[
\frac{\partial}{\partial r}(\Pi_{R&D}^* - \Pi_M^*) = \frac{-2 + 4\omega - 3\omega^2 + 2v(1 - \omega)^2 + v^2(-1 + 2\omega - 2\omega^2)}{9(1 + v^2)(2 - 2v + v^2)}
\]

(17)

The denominator is clearly always positive. Hence I only have to show that the numerator is negative for all \( \{\omega, v\} \in [0, 1] \). Let \( N \) be the numerator of equation 17. Since

\[
\frac{\partial^2 N}{\partial v^2} = -2 + 4\omega - 4\omega^2 < 0 \text{ for all } \omega \in [0, 1],
\]

\( N \) is concave in \( v \) and there exists a unique \( v^*(\omega) \) that maximizes \( N \). Hence

\[
N(\omega, v^*(\omega)) < 0, \forall \omega \in [0, 1] \Rightarrow N(\omega, v) < 0, \forall \{\omega, v\} \in [0, 1]
\]

(18)

From solving \( \frac{\partial N}{\partial v} = 0 \) one obtains \( v^* = \frac{(1 - \omega)^2}{1 - 2\omega + 2\omega^2} \). This results in

\[
N(\omega, v^*(\omega)) = \frac{(-1 + 4\omega - 9\omega^2 + 10\omega^3 - 5\omega^4)}{1 - 2\omega + 2\omega^2}
\]

(19)

The denominator of \( N(\omega, v^*(\omega)) \) is clearly always positive for all \( \omega \in [0, 1] \). Hence I only need to show that the numerator of \( N(\omega, v^*(\omega)) \) is negative for all \( \omega \in [0, 1] \). I go through the same steps as for the numerator of equation 17. Let \( K \) be the numerator of equation 19.
Since $\frac{\partial^2 K}{\partial \omega^2} = -18 + 60\omega - 60\omega^2 < 0$ for all $\omega \in [0, 1]$, $K$ is concave in $\omega$ and there exists a unique $\omega^*$ that maximizes $K$. Hence

$$K(\omega^*) < 0 \Rightarrow K(\omega) < 0, \forall \omega \in [0, 1] \quad (20)$$

From solving $\frac{\partial K}{\partial \omega} = 0$ one obtains only one real root $\omega^* = 1/2$. This results in

$$K(\omega^*) = -\frac{5}{16} < 0 \quad (21)$$

Since $K$ is negative for all $\forall \omega \in [0, 1]$, $N$ is negative for all $\{\omega, \nu\} \in [0, 1]$. Hence

$$\frac{\partial}{\partial r}(\Pi^*_{R&D} - \Pi^*_{M}) < 0 \quad \forall(\{\omega, \nu\} \in [0, 1]) \quad (22)$$

Q.E.D.

**Proof of proposition 3**

Profits are given by equation 12 on page 175. After substituting equation 2 we obtain:

$$\Pi(M_1, M_2, R_1, R_2) = -\gamma((s_1 M_1 - \bar{r}_1) \cdots (s_R R_1 \cdots s_R R_2 \cdots s_R R_2)^{-\gamma} \cdot ((s_M M_2 - (m_1 + \bar{r}_1 + \bar{r}_2)/3)^2 + (s_R R_2 - (\bar{r}_1 + m_1 + m_2)/3)^2)$$

I will show that $\frac{\partial}{\partial r}(\Pi^*_{R&D} - \Pi^*_{C}) > 0$. The proof for $\frac{\partial}{\partial r}(\Pi^*_{M} - \Pi^*_{C}) > 0$ is analogous. This proof proceeds in the same way as the previous one. It first derives optimized profits for the two sequences $\{R_1, R_2\}, \{M_1, M_2\}, \{M_1, R_1\}, \{M_2, R_2\}$ and then shows that $\frac{\partial}{\partial r}(\Pi^*_{R&D} - \Pi^*_{C}) > 0$.

**Case 1: $\{R_1, R_2\}, \{M_1, M_2\}$**

The firm's problem is described by equation 9 and is time separable. From first order conditions for the first period maximization we obtain $R_1 = 1/(1 + \nu^2)$ and $R_2 = 1/(1 + \nu^2)$. First order conditions in the second period yield $M_1 = (1 + r_1)/(2 - 2\nu + \nu^2)$ and $M_2 = (3 + r_1 + r_2)/(3(2 - 2\nu + \nu^2))$. Second order conditions in both periods clearly
hold. Let \( \Pi_{R\&D} \equiv E[\Pi(M_1, M_2, R_1, R_2)] \) be the maximized expected profits given the sequence \( \{R_1, R_2\}, \{M_1, M_2\} \).

**Case 2: \( \{M_1, R_1\}, \{M_2, R_2\} \)**

The firm's problem is described by equation 10 and is time separable. From first order conditions for the first period maximization we obtain \( R_1 = 1/(1+v^2) \) and \( M_1 = 1/(2-2v+v^2) \). First order conditions in the second period yield \( R_2 = (3+r_1+m_1)/6 \) and \( M_2 = (3+r_1+m_1)/6 \). Second order conditions in both periods clearly hold. Let \( \Pi_C^* \equiv E[\Pi(M_1, M_2, R_1, R_2)] \) be the maximized expected profits given the sequence \( \{M_1, R_1\}, \{M_2, R_2\} \).

Differentiating \( \Pi_{R\&D}^* - \Pi_C^* \) with respect to \( \gamma \) yields

\[
\frac{\partial}{\partial \gamma} (\Pi_{R\&D}^* - \Pi_C^*) = \frac{27 - 36\omega + 22\omega^2 + 2v(-18 + 18\omega - 11\omega^2) + v^2(49 - 26\omega + 19\omega^2) + 8v^3(-5 + \omega - \omega^2) + 4v^4(5 - \omega + \omega^2)}{9(1+v^2)(2-2v+v^2)}
\]

The denominator is clearly always positive. Hence I only have to show that the numerator is positive for all \( \{\omega, v\} \in [0, 1] \). Let \( N \) be the numerator of equation 23. Since

\[
\frac{\partial^2 N}{\partial \omega^2} = 44 - 44v + 38v^2 - 16v^3 + 8v^4 > 0 \text{ for all } v \in [0, 1], N \text{ is convex in } \omega \text{ and there exists a unique } \omega^*(v) \text{ that minimizes } N. \text{ Hence}
\]

\[
N(\omega^*(v), v) > 0, \forall v \in [0, 1] \Rightarrow N(\omega, v) > 0, \forall \{\omega, v\} \in [0, 1]
\]

From solving \( \frac{\partial N}{\partial \omega} = 0 \) one obtains \( \omega^* = \frac{9 + 2v^2}{11 + 4v^2} \). This results in

\[
N(\omega^*(v), v) = \frac{(135 - 234v + 494v^2 - 512v^3 + 372v^4 - 152v^5 + 76v^6)}{11 + 4v^2}
\]

The denominator of \( N(\omega^*(v), v) \) is clearly always positive for all \( v \in [0, 1] \). Hence I only need to show that the numerator of \( N(\omega^*(v), v) \) is positive for all \( v \in [0, 1] \). Let \( K \) be the numerator of equation 25. Solving (numerically) for the roots of \( K \) reveals that all six roots lie in complex numbers. Hence there is no real \( v \) for which \( K = 0 \). Evaluating \( K \) at \( v = 1/2 \)

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yields $K(1/2) = 1555/6 > 0$. Since $K$ does not change sign for any $\nu \in [0, 1]$, this implies $K > 0$, $\forall \nu \in [0, 1]$. By (24) and (25) it follows that $N(\omega, \nu) > 0$, $\forall \{\omega, \nu\} \in [0, 1]$. Hence

$$\frac{\partial}{\partial \gamma} (\Pi^{\ast}_{R&D} - \Pi^{\ast}_{C}) > 0 \quad \forall\{\omega, \nu\} \in [0, 1]$$

(26)

Q.E.D.