A Cavity-Stabilized Diode Laser for Dipole Trapping of Ytterbium

by

David Levonian

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of
Master of Engineering in Electrical Engineering and Computer Science
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Abstract

Bad-cavity lasers using a gain medium with a narrower linewidth than the laser cavity have the potential to achieve very narrow linewidths and extremely long coherence times. Such lasers could serve as active frequency standards or enable very-long-baseline interferometric telescopes at optical frequencies. The $6s6p^3P_0$ to $6s^2S_0$ ground state transition in $^{171}$Yb is a promising candidate for the gain medium of a bad-cavity laser due to its 44 mHz linewidth. For ytterbium to be used efficiently as a gain medium, its inhomogeneous broadening must be suppressed to a level lower than the linewidth of its gain transition.

In this thesis, I design, implement, and characterize an optical lattice trap for ytterbium atoms. The trap consists of a diode laser which is frequency stabilized to an adjustable-length cavity where the ytterbium atoms are trapped. The length of this cavity is then locked by comparison of the laser frequency to a stable reference cavity. The resulting standing wave has high enough intensity that the recoil energy of the gain transition is smaller than the energy spacing between motional modes of the trapped atoms. This situation is known as the Lamb-Dicke regime and means that there is an absence of recoil broadening. The large spacing between motional modes of the trap also enables sideband resolved cooling of the atoms, which allows cooling to temperatures of 3 $\mu$K, near the ground state of the trapping potential. Additionally, if the wavelength of the optical lattice is chosen to be at the magic wavelength for ytterbium, where the relative AC Stark shift for the two levels of the gain transition is zero to first order, there is no broadening due to varying intensity in the trap. Since the Doppler effect, recoil broadening and the AC Stark shift are the main sources of inhomogeneous broadening, this trapping scheme is expected to suppress inhomogeneous broadening to a level of 1 Hz.

Thesis Supervisor: Vladan Vuletić
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Chapter 1

Introduction

Atomic clocks are the most precise time-keeping instruments in the world [1][2][3]. They rely on the fact that the frequency of an electron oscillating between two atomic orbitals does not vary between atoms of a particular isotope and is constant in time. If an atom at rest in the lab frame is shielded from external electric and magnetic fields and collisions with other atoms, the only thing that can perturb the frequency of its electronic transitions is the relativistic effect of gravity $^1$ [4].

Electron oscillations between atomic orbitals do not last indefinitely. The oscillation lifetime is determined by how fast the electrons radiate away their energy as electromagnetic waves, which is in turn determined by how much the center of the electron cloud moves during the oscillation. Large movement of charge during the oscillation produces higher amplitude waves which radiate energy away faster. Since a quickly decaying oscillation produces a broader Fourier-transformed frequency peak, atomic clocks usually use electronic transitions with long lifetimes. Alkali-earth-like elements like ytterbium and mercury have dipole-forbidden transitions between their excited and ground states. Electron transitions between these states have near zero movement of the center of charge and take a long time to decay. One transition in ytterbium takes $3.6$ seconds to decay which corresponds to a frequency linewidth of $44$ mHz $^5$. Since the frequency of the transition is $519$ THz, this corresponds to a

$^1$Atomic clocks on GPS satellites are compensated for this effect, as well as relativistic effects from their orbital velocity.
timekeeping uncertainty of $10^{-18}$ after 100 seconds of averaging \(^2\) for a single atom with a lifetime-limited interrogation time.

In order to turn electron oscillations between atomic energy levels into a usable time-keeping signal, atomic clocks tune a very stable clock laser to the clock transition frequency by searching for the frequency that is the most effective at exciting an electron to the excited clock state. To measure how many atoms have been excited by the laser, clock experiments usually use another laser to excite electrons from the upper state to another, shorter-lived excited state and then measure the fluorescence as they decay back to the ground state. Over multiple experimental cycles, the clock laser can be tuned to produce the greatest fluorescence signal. The clock laser can then be used to stabilize an optical frequency comb, which in turn produces a very stable signal at frequencies in the MHz, suitable for conventional electronics [1].

The linewidth of the clock laser has a big effect on the performance of the atomic clock. If we tried to center a laser that produces frequencies over a 100 Hz range on the 44 mHz wide transition in Yb, there would be very little contrast between the laser frequency centered on the clock transition and the laser frequency centered 1 Hz away from the clock transition [6]. In this case, we would have to average over many probing cycles to achieve a precision limited by the linewidth of the transition, not the laser.

Conventional lasers have been produced with frequency linewidths as narrow as 40 mHz [9][10]. However, the linewidth of conventional lasers is fundamentally limited by the Schawlow-Townes limit [11]

$$\Delta \nu = \frac{h \nu \kappa^2}{4\pi P_{\text{out}}}$$

where \(\kappa\) is the cavity loss rate, \(h \nu\) is the photon energy, \(\Delta \nu\) is the full-width at half maximum (FWHM) of the laser line and \(P_{\text{out}}\) is the power output of the laser. Since we cannot increase the power of lasers indefinitely, the only improvement can come from having a narrower cavity linewidth. While great strides have been made in cavity

\(^2\)To put this in perspective, a clock with this precision would gain or lose at most one second in the lifetime of the universe.
Table 1.1: A comparison of the linewidths of cavities, atoms, and lasers. In cavities, only wavelengths that fit evenly in the cavity resonate and are transmitted. [7] The reflectivity of the mirrors determines how precise the fit needs to be and thus the linewidth of the transmission peaks. An atomic transition emits a wave that exponentially tapers off. In the frequency domain this creates a Lorentzian lineshape whose width is inversely proportional to the lifetime of the emission (homogenous broadening.) A laser is made up of many radiating atoms at slightly different frequencies. Together, they emit light with a Gaussian lineshape (inhomogeneous broadening) [8].
stability, cavities have limitations of their own, such as Brownian motion of their mirrors [12]. Other clever tricks like locking the laser to a passive reference cavity with feedback can produce narrower lasers, but are still ultimately limited by the linewidth of optical cavities.

Therefore, to exploit narrow linewidth atomic transitions for more accurate time-keeping, we need to develop narrower linewidth sources of light. Bad-cavity lasers are one potential source. In conventional lasers, atoms in the gain medium emit photons one at a time into an oscillating electric field in the laser cavity. The emission events happen quickly compared to the rate at which photons leak out of the cavity, so the phase of the laser beam is determined by the phase of the electric field inside the cavity.

In bad-cavity lasers, all the atoms in the gain medium oscillate at once, in phase with each other, in a phenomenon known as superradiance \(^3\). Emission of photons happens very slowly compared to the rate at which photons leak out of the cavity. This means there are on average very few photons circulating in the cavity and the phase of the emitted light is mostly determined by the collective phase of the emitting atoms. In this bad-cavity limit, the linewidth of the laser is determined by the linewidth of the atomic transition powering it, rather than the quality of the cavity [13][14]. The linewidth of the emitted light depends on the linewidth of the atomic transition of the gain medium, not the linewidth of the cavity.

A superradiant laser using a narrow linewidth atomic transition would produce narrow linewidth light free of drifts caused by a changing laser cavity. This absolute stability means it could be used as an active frequency reference, becoming both the atomic reference and clock laser of an atomic clock. More conventionally, such a narrow linewidth laser would allow atomic clocks to reach higher precisions with less averaging time. The ultimate goal of our lab is to build a superradiant laser with a gain medium of ytterbium atoms to realize these time keeping advantages.

\(^3\)From now on the terms superradiant laser and bad-cavity laser will be used interchangeably.
Figure 1-1: Superradiant vs Conventional Laser: a) In a conventional laser, the phase of the wave is stored in the electric field of the cavity and is sensitive to mirror vibrations. b) In a superradiant laser, the phase of the laser is stored in the phase of the gain medium, making the laser less sensitive to mirror vibration [15].
1.1 Technical Challenges

In a superradiant laser, the coherence between the atoms of the gain medium is maintained by their interaction with photons in the cavity. For superradiant collective emission to occur, photons need a high probability of interacting with several atoms before leaving the cavity. A factor $C$ known as the cooperativity compares the atom-photon interaction rate with the rate at which photons leave the cavity or are scattered incoherently by atoms.

$$C \equiv \frac{g^2}{\kappa \gamma}$$

where $g$ is the single photon Rabi frequency, $\kappa$ is the cavity loss rate and $\gamma$ is the inverse lifetime of the atomic state. If $NC > 1$, where $N$ is the number of atoms in the gain medium, then photons emitted by each atom are more likely to encounter other atoms than they are to leave the cavity, and superradiant lasing can occur. In addition to this condition, the gain medium of a bad-cavity laser must be spectrally narrower than its cavity $\gamma \ll \kappa$. The long lifetime of the transition in Yb atoms guarantees that $\gamma$ is small. $\kappa$ depends primarily on the mirrors of the laser cavity: higher reflectivity mirrors mean longer photon lifetime which reduces $\kappa$. $g$ describes the rate of interaction between photons and atoms. In order to make $g$ large, we need light to be highly focused on the atoms. Our experimental cavity\(^4\) is designed with highly reflective mirrors and a fundamental mode that focuses down to a narrow waist where it can strongly interact with atoms. These geometric factors, together with the long lifetime of the ytterbium transition, make superradiant lasing possible.

These conditions are not sufficient for superradiance, however; the interaction of all the atoms with the cavity photons must be identical to support superradiant lasing. If the energy levels of some atoms are slightly shifted, then their electrons will oscillate at different rates. This will destroy the coherence of the collective atomic state - a condition necessary for superradiant lasing. Processes that reduce the degree to which all the atoms radiate collectively are called inhomogeneous broadening. There are two primary causes of inhomogeneous broadening in our experiment. One is the AC Stark

\(^4\)I will also refer to the superradiant laser cavity as the science cavity.
shift, the change of atomic orbitals’ energies in the presence of light and consequent impact on the frequency of transitions. If different atoms of the gain medium are exposed to different intensities of light, then the energy levels of the clock transition will vary and different atoms will radiate at different frequencies. The second cause of inhomogeneous broadening is the Doppler shift. Atom velocity can vary due to finite temperature or due to recoil from emitting photons. If some atoms are moving at different velocities then they will emit different frequencies of light, destroying the collective effects. This increase in linewidth is known as Doppler broadening.

To prevent Doppler broadening, we would like to trap atoms in a place where the laser cavity mode is most intense and prevent their relative movement. The most effective way to tightly confine atoms in a small space is with an optical lattice of counter-propagating laser beams. This presents a conundrum: we would like to trap atoms with lasers to prevent Doppler broadening, but exposing them to light will cause AC Stark shifts. Fortunately, there is a wavelength of light, known as the magic wavelength, which shifts the two atomic energy levels of the clock transition by the same amount. An optical trap operating at this wavelength can trap atoms and prevent Doppler broadening without causing Stark shifts in the atoms.

1.2 Thesis Contributions

My master’s thesis is the implementation of an optical trap for ytterbium atoms at the magic wavelength. By bouncing laser light between the two high-reflectivity mirrors of the superradiant laser cavity, we form an optical lattice. Atoms are attracted to the intensity maxima of this beam of light. Atoms are attracted to the anti-nodes of the standing wave, which form pancake shaped potential wells, defined by the extent of the beam in the radial dimensions and the nodes and anti-nodes of the beam in the axial direction. The sharp potential wells in the axial direction lead to large energy separation between quantized motional modes of the trapped atoms. If the difference

\(^{5}\)i.e. the first order shifts in intensity will be the same.

\(^{6}\)The science cavity mirrors are designed to be highly reflective both at the magic wavelength of 759 nm and the clock transition wavelength of 578 nm.
between these modes is larger than the kinetic energy of recoil from emitting a photon, the atoms will be unable to recoil, a condition known as the Lamb-Dicke regime. An additional cooling laser can create velocity-dependent slowing of the atoms, which cool atoms down, eliminating the Doppler shift due to thermal velocity. The cooling laser is turned off during the superradiance experiment to prevent AC Stark shifts.

The main challenge in creating an optical lattice in a cavity is control of the laser’s frequency. Although lasers emit primarily one frequency of light, the frequency drifts and jumps on a time scale of microseconds. In addition to causing AC Stark shifts in the atoms, these deviations cause the laser to drift in and out of resonance with the optical cavity, changing the intensity of the optical lattice, and exciting atoms to nonzero velocities. While the cooling laser is on, these excited atoms are quickly cooled back down, but during the main experiment, the excited atoms can cause Doppler broadening and interfere with superradiant lasing.

The Pound-Drever-Hall (PDH) frequency lock is a way to keep a laser’s frequency stable [16]. It does this by using feedback to keep the laser close to the resonant frequency of a stable optical cavity. If light shines on the cavity at the frequency of a resonant mode, it bounces between the mirrors such that it is transmitted. Otherwise the light is reflected. If the mirrors are highly reflective, the range of frequencies that are transmitted through the cavity is very small, which makes it useful for keeping a laser’s frequency very stable.

Predecessors of the PDH locking scheme rely on measuring the intensity of the transmitted light through the cavity to measure the frequency of the laser [17]. If the laser frequency moves, then the intensity of the transmitted light changes, and corrective action can be taken. However, if the power of the laser fluctuates, the system may be fooled into thinking the laser frequency has moved. This method also has a bandwidth limited by the linewidth of the cavity: the frequency is only corrected once the amount of light in the cavity has diminished sufficiently to be detected.

The PDH lock avoids these sources of error by using the phase of the light instead of its intensity. The laser is first sent through a modulator which creates two additional frequencies of light, equally spaced above and below the main laser frequency. When
the main laser frequency on resonance, the two side frequencies are reflected back with opposite phases. The reflected light is collected and then mixed back to the central frequency by another modulator and cancel each other out, because they have opposite phases. Even a slight change in the main laser frequency changes the phases of the side frequencies and stops them from canceling. This signal is used as feedback to adjust the laser. This makes PDH locking at once extremely sensitive to frequency fluctuations and immune to intensity fluctuations, because when the laser is on resonance the error signal is zero, regardless of intensity. The PDH lock bandwidth also exceeds the cavity linewidth, as it looks at the reflected signal, which is a combination of light from the cavity and laser light. Frequency drifts immediately cause detectable interference between the two.

My optical lattice laser uses two separate PDH locks to reduce inhomogeneous broadening due to the trapping laser. The first PDH lock keeps the laser resonant with the mode of the superradiant laser cavity where the ytterbium atoms are trapped. This way, there are no changes in the intensity of the optical lattice due to drifts in laser frequency. The length of this cavity is adjustable by piezoelectric stacks to allow greater flexibility in the experiment. Because the resonant frequency of the superradiant laser cavity can vary, I use a second PDH lock to a reference cavity to keep the optical lattice at the magic wavelength. The error signal of the second PDH lock is fed back to the piezos of the superradiant laser cavity. If the resonant mode of the laser cavity drifts from the magic wavelength, the first PDH lock pulls the laser with it. The second PDH lock detects this change in laser frequency and corrects the cavity length.

In my masters thesis, I first analyze the requirements for an optical lattice trap in a superradiance experiment. I then design and build the trapping laser and the two PDH locks needed to keep it at the magic wavelength and on resonance with the superradiant laser cavity. Finally, I measure its performance by comparing it to a separate, low-drift laser in our lab, and show that it meets the requirements of absolute and short-term stability.
Chapter 2

Superradiance

A laser has two essential elements: a gain medium, which amplifies light through stimulated emission, and a cavity, which confines the laser light. The effects of the gain medium and cavity both depend on the frequency of the light. For example, a typical cavity might consist of two partially reflective mirrors, which have a frequency response of

\[ F(\omega) = \left| \frac{1}{1 - Re^{\frac{\omega}{\omega_{FSR}}}} \right|^2 \]

where \( R \) is the mirror reflectivity, \( \omega \) is the frequency of incident light, and \( \omega_{FSR} \) is the free spectral range of the cavity. Likewise, a typical gain medium might be an excited atomic gas. In the case of a homogeneously broadened gain medium, the frequency response would be

\[ G(\omega) = \frac{g}{1 + \left( \frac{\omega - \omega_0}{\gamma} \right)^2} \]

where \( g \) is the gain at the peak gain frequency \( \omega_0 \) and \( \gamma \) is the linewidth of the gain transition.

In conventional lasers, the cavity has a narrower frequency response than the gain medium. As a result, the cavity determines the lasing frequency and linewidth. This means that fluctuations in cavity length caused by vibrations affect the laser’s frequency. In precision measurement applications, it is important for the laser’s frequency to be as independent as possible from the environment.
A bad-cavity laser has a gain medium with a linewidth narrower than the linewidth of its cavity. This can be achieved with long lifetime transitions (and correspondingly narrow gains) in rare-earth-like elements like ytterbium and mercury. A laser using these atoms as its gain medium lases at the linewidth of the atoms not the cavity. Also, since the lasing frequency depends on the frequency of the atomic transition, it is relatively insensitive to changes in cavity length [18][19][20].

In this chapter, we first derive the narrower linewidth enjoyed by a bad-cavity laser classically. Then we treat the gain medium using quantum mechanics to find the transient behavior of the superradiant laser. The long lifetime of the transition means that the atoms all undergo stimulated emission, driven by their collective field. Unlike a conventional laser, where atoms emit photons quickly and independently from one another, the atoms of the superradiant laser radiate in sync with each other. Since the power of the light is determined by the square of the electric field, the synchronous radiation of the atoms boosts the power of the laser enormously. This is fortunate, as the long lifetime of the lasing transition would otherwise cause it to radiate very weakly. Following the theoretical treatment of superradiance, we describe the implementation of a superradiant laser with ytterbium atoms and calculate the characteristics of the optical lattice trap needed for the experiment.

2.1 Theory of Superradiance

In a conventional laser, the electric field inside the cavity can be described with phasor notation as

\[ E = E_0 e^{i\omega t} \]

where \( E_0 \) is the magnitude of the cavity field, which depends on the square root of the intracavity intensity, and \( \omega \) is the laser frequency. Spontaneous emission events give kicks to this phasor, adding small random vectors to it (see figure 2-1.) When these random vectors are parallel to the main field vector, they manifest as intensity fluctuations. Intensity fluctuations are damped out as the laser gain saturates, quickly returning the intensity to its equilibrium value. Spontaneous emission vectors
Figure 2-1: Spontaneous emission of photons causes phase diffusion in the laser [11]. \( \beta \) is the electric field phasor, normalized by the single-photon cavity field.

perpendicular to the main vector manifest as fluctuations in the phase of the laser. There is no corrective mechanism that counteracts these fluctuations so they cause diffusion of the phase of the laser over time. In the frequency domain, this phase diffusion becomes the finite linewidth of the laser.

The formula for this linewidth, originally derived by Schawlow and Townes [21] and simplified in a paper by Henry [11], is quite simple. The degree of phase diffusion in a laser is simply the rate of spontaneous emission events divided by the length of the main field vector, which corresponds to the number of intracavity photons and multiplied by one half to account for the fraction of spontaneous emissions that influence intensity instead of phase.

In steady state, the spontaneous emission events balance out the loss of photons through the facets of the laser so the linewidth is the ratio of the cavity loss to the number of circulating photons:

\[
\Delta \omega = \frac{\kappa}{2I_0}
\]

We can rewrite \(^1\) the linewidth in terms of the power output of the laser and the cavity

\[^1\text{Using } P_{out} = \kappa I h \nu\]
loss rate as
\[ \Delta \nu = \frac{h \nu \kappa^2}{4\pi P_{\text{out}}} \]

As expected, a narrower cavity linewidth leads to a small laser linewidth.

### 2.1.1 Classical Treatment of the Bad-cavity Laser

A bad-cavity laser is a laser where the lifetime \( \gamma \) of individual atoms in the gain medium is longer than the lifetime \( \kappa \) of photons in the laser cavity. In other words:

\[ \gamma \ll \kappa \]

The effect of this inequality on the laser linewidth can be interpreted in two equivalent ways. Examining the gain medium in the frequency domain, we see that it is very narrow. Using the Kramers-Kronig relations, we can see that the highly-peaked gain leads to a very slow group velocity near its peak. This slow group velocity means that small fluctuations in the field envelope of the laser from spontaneous emission events arrive at the end facet of the laser at a much slower rate. Therefore, the emitted light field sees less phase kicks than it would otherwise.

An equivalent interpretation is that the long lifetime of the induced polarization of the gain medium means that the polarization of the gain medium is no longer proportionate to the instantaneous electric field. These memory effects can be inserted into the wave equation for the field inside the laser:

\[
\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{c^2 \epsilon_0} \left( \frac{\partial^2}{\partial t^2} [P_{\text{ind}} + P_{sp}] \right)
\]

where \( P_{\text{ind}} \) is the polarization induced by the electric field and \( P_{sp} \) is the polarization from spontaneous emission events. The left hand side of the equation can be elucidated by using

\[ E = \hat{E}(z,t)e^{i\omega t} \]
the slowly-varying envelope approximation for the electric field in the laser.

\[ \text{LHS} = \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \left[ -\omega_L^2 E + 2i\omega \frac{\partial E}{\partial t} \right] \]

The right hand side can be simplified using the identity

\[ \frac{\partial P}{\partial t} = -i\omega L P + \frac{\partial(\omega \chi)}{\partial \omega} \bigg|_{\omega=\omega_L} \frac{\partial E}{\partial t} e^{-i\omega t} \]

Which is derived in [22]. Dropping the \( L \) prefix on omega, \( \text{RHS} = \frac{-1}{c^2} \left[ \omega^2 \chi E + 2i\omega(\chi + \omega \frac{\partial \chi}{\partial \omega}) \frac{\partial E}{\partial t} \right] - \frac{\omega^2}{c^2 \varepsilon_0} P_{sp} \]

and equating the two sides,

\[ \frac{\partial^2 E}{\partial z^2} + \left(1 + \chi\right) \frac{\omega^2}{c^2} E(r, t) + \frac{2i\omega}{c^2} \left(1 + \chi + \frac{1}{2} \frac{\partial \chi}{\partial \omega} \right) \frac{\partial E}{\partial t} = -\frac{\omega^2}{c^2 \varepsilon_0} P_{sp} \]

\( \frac{\partial \chi}{\partial \omega} \) can be redefined in terms of the group index of refraction:

\[ n_{gr} = n + \omega \frac{\partial n}{\partial \omega} \]

\[ \frac{\partial \chi}{\partial \omega} = \frac{2n(n_{gr} - n)}{\omega} \]

Inserting this into the wave equation, considering only waves traveling in the positive \( z \) direction, and dividing through by a factor of \( 2i\hat{k} \) we find:

\[ \frac{\partial E}{\partial z} + \frac{n_{gr}}{c} \frac{\partial E}{\partial t} - \frac{1}{2} \frac{\partial^2 E}{\partial t^2} = \frac{i\omega}{2c\varepsilon_0 n} P_{sp} \]

This equation describes changes in the envelope of the electric field propagating outward with speed \( v_{gr} \). This differs from the laser considered in the Schawlow-Townes equation in that fluctuations propagate at the phase velocity, near \( c \), in a conventional laser. The phase kicks that had previously happened with a frequency of \( \kappa \) now happen with a frequency \( \kappa / n_{gr} \). Therefore, the linewidth in the bad-cavity laser is reduced by
a factor of $n_{gr}^2$. In a homogeneously broadened gain medium, the imaginary part of the dielectric susceptibility has a Lorentzian frequency response:

$$
\chi = \chi' + i\chi''
$$

$$
\chi'' = \frac{g}{1 + (\frac{\omega - \omega_0}{\gamma})^2}
$$

$$
\chi' = \frac{g(\frac{\omega - \omega_0}{\gamma})}{1 + (\frac{\omega - \omega_0}{\gamma})^2}
$$

Where $g$ is the gain per length in the laser. This gives

$$
\frac{dn}{d\omega} \bigg|_{\omega=\omega_0} = \frac{g}{2n\gamma}
$$

Given that $n \approx 1$:

$$
n_{gr} = 1 + \frac{g\omega}{2\gamma} = 1 + \frac{\kappa}{2\gamma} = \frac{\gamma}{\gamma + \frac{1}{2}\kappa}
$$

Therefore, the conventional Schawlow-Townes formula is modified to

$$
\Delta \nu = \frac{h\nu}{4\pi P_{out}} \left( \frac{\gamma\kappa}{\gamma + \frac{1}{2}\kappa} \right)^2
$$

For $\gamma \ll \kappa$ the cavity linewidth is replaced by the gain medium linewidth:

$$
\Delta \nu \approx \frac{h\nu}{4\pi P_{out}} \frac{\gamma^2}{\gamma + \frac{1}{2}\kappa}
$$

According to this classical interpretation, a laser using the clock transition in ytterbium can have a linewidth narrower than the good-cavity Schawlow-Townes limit. To achieve this linewidth, however, several other experimental conditions need to be met. First, frequency broadening of the gain profile due to anything but the lifetime of the atoms needs to be suppressed to a level lower than $\gamma = 44\text{mHz}$. This so-called inhomogeneous broadening includes variable shifts in the clock transition due to atoms being exposed to different intensities of the optical lattice. Secondly, the cavity resonance frequency needs to be less than a linewidth away from the gain transition. Otherwise, photons
emitted by the atoms will not be confined by the cavity.

2.1.2 Quantum Treatment of Transient Superradiance

In order to realize a steady-state superradiant laser, we need to make sure our cavity resonance frequency is centered on the gain transition. To search for this frequency, we need to use the transient behavior of the superradiant laser. If the atoms in the gain medium start in an excited state and the cavity is in resonance with the gain transition, they will emit a pulse considerably more intense than we would classically expect, scaling with the number of atoms squared, $N^2$, rather than $N$. This behavior is a signature of superradiance. To demonstrate why this is the case, we first consider two particles with two states each that can interact with the electromagnetic field via a dipole operator which describes the transitions between excited and ground state due to emission or absorption of photons:

$$\vec{d} = \sum_{i=1,2} |\uparrow_i\rangle \langle \downarrow_i| - |\downarrow_i\rangle \langle \uparrow_i|$$

If we look at the eigenstates of the angular momentum $\sigma_z$ in one direction and the total angular momentum operator

$$J^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$$

where the $\sigma$ operators are the sums of the Pauli spin matrices for the two particles, we find that a basis of four states, three of which have $J^2 = 1$:

$$|1\rangle = |\uparrow\uparrow\rangle, |0\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle), |-1\rangle = |\downarrow\downarrow\rangle$$

These are the triplet states. The fourth state has $J^2 = 0$ and is called the singlet state:

$$\frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$$
If we examine the action of the dipole operator on the triplet states we see that it couples them together in ladder:

\[ \langle 1|\vec{d}|0\rangle = \sqrt{2}, \langle 0|\vec{d}|−1\rangle = \sqrt{2} \]

The coupling between these states a factor of \( \sqrt{2} \) stronger than the dipole coupling between the excited and ground states of a single particle. The singlet state does not couple to any other states through the dipole operator, so it is incapable of radiating photons. However, if both particles start in the excited state, they will radiate two photons faster than two independent excited particles would.

This is the simplest example of superradiance; multiple particles collectively interacting with the electromagnetic field have a basis of states, some of which radiate faster than independent particles (superradiant states), others of which do not radiate at all (subradiant states) [23].

We can extend this result, by considering the Hamiltonian of N atoms interacting with a cavity mode:

\[ H = \hbar \Omega \sum_{j=1}^{n} \sigma_j^z - E \cdot \hat{d} \]

\[ \hat{d} = \sum_{j=1}^{n} i\sigma_j^y \]

where \( \sigma_i \) are the Pauli operators for the pseudospin of the clock transition, \( \hbar \Omega \) is the energy difference between the excited and ground states, \( E \) is the electric field at the atoms and \( \hat{d} \) is the collective dipole moment of the atoms. Since \( J^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \) commutes with H, we can use the formalism for addition of angular momentum to define eigenstates of \( J^2 \) with eigenvalues \( r(r + 1) \) where

\[ |m| \leq r \leq \frac{1}{2}n \]

and \( m \) is the total angular momentum. These states have a degeneracy of

\[ \frac{n!(2r + 1)}{(\frac{1}{2}n + r + 1)!(\frac{1}{2} - r)!} \]
Figure 2-2: Dicke states \[23\]. The superradiant lasing transitions are the transitions between the latter of states where \(L=N/2\).

\[
\langle r, m | i \sigma_y | r', m' \rangle = 0 \quad \text{unless} \quad r = r', m = m' \pm 1.
\]

These states form ladders with field interaction matrix elements of

\[
\langle r, m | i \sigma_y | g, m \mp 1 \rangle = \frac{1}{2} \sqrt{(r \pm m)(r \pm m + 1)}
\]

The spontaneous radiation rates are given by the square of the matrix elements:

\[
I = NC\gamma(r + m)(r - m + 1)
\]

The mean time for the entire superradiant pulse is simply the sum of the mean emission times of each transition

\[
\bar{T} = \frac{1}{NC\gamma} + \ldots + \frac{4}{N^2C\gamma} + \ldots + \frac{1}{NC\gamma}
\]

This harmonic sum can be approximated as

\[
\bar{T} \approx \frac{\ln(N)}{NC\gamma}
\]

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Figure 2-3: Optical transition frequencies and linewidths of ytterbium. The clock transition is 578 nm. The cooling transitions are 399 and 556nm.[24]

Assuming optimal loading into the trap, there should be 10,000 atoms in the optical lattice. $C = 0.1$ and $\gamma = 44\text{mHz}$. This gives a superradiant pulse duration of 250ms.

In order to observe superradiant pulses, the rate at which the phase of individual atoms decays, known as $T_2$ must be longer than the length of the superradiant pulses.

### 2.2 Experimental Implementation

Ytterbium is an excellent element for implementing a superradiant laser. It has two transitions that are good for optical cooling\(^2\) at 555.8 nm and 398.9 nm. It also has a very narrow transition from the $6s6p^3P_0$ sublevel to the ground state at 578.4 nm, which has a small dipole moment, so it radiates very slowly.

\(^2\)i.e. they are fairly broad in frequency and the electrons quickly relax back exclusively to the ground state.
2.2.1 Trapping

In our experiment, we use an electrically heated sample of ytterbium to produce a beam of atoms. We then slow and trap these atoms in a magneto-optical trap (MOT), first with a combination of 399 nm and 556 nm light, which can catch atoms with a greater spread of velocities, and then at 556 nm which is a narrow transition more suitable for cooling already-trapped atoms to a lower temperature. [25] Once the atoms are in the second MOT they have a temperature of $30 \, \mu K$. From the second MOT, they are loaded into a red-detuned optical lattice which is produced by a standing wave between the mirrors that will be used for the superradiant laser.

The superradiant laser cavity must have room for the initial magneto-optical trap while also having regions where atoms will have large coupling to the field mode. Since enhanced coupling is achieved by confining the cavity mode to a smaller space to produce high fields, these two requirements seem to be at odds with one another. However, several cavity designs can achieve high coupling between atoms and cavity modes without being too small. For example, a confocal cavity can be large, but concentrate the cavity field at its focus to a region several wavelengths across. Confocal cavities can be difficult to align however, so we use an asymmetric design with one highly curved micromirror, etched with a CO$_2$ laser and one larger, flatter mirror. This design is easier to align, but preserves a region of intense cavity field near the smaller mirror that allows high coupling to atoms.

To create the conditions for superradiant lasing, we need to trap the atoms in a potential created by optical dipole forces from light at the magic wavelength. We can then use sideband-resolved Doppler cooling to cool these atoms into the motional ground state of the potential. To use the atoms as the gain medium for a superradiant laser, all this must occur in an optical cavity with a sufficiently high coupling constant. One way to trap the atoms in a laser cavity and make the cooling and trapping process easier is to use the science cavity to make the dipole trap as well.
2.2.2 Cooling

In this case, we couple our trapping laser to a fundamental mode of the science cavity, forming a standing wave inside the cavity. Since our trapping laser is red-detuned, atoms are attracted to regions of high average field intensity. We can model these forces with a trapping potential that is linear with the intensity of the trapping light [26]:

\[ U_{\text{dipole}} = \frac{\alpha I}{2\varepsilon_0 c} = 2 \times 10^{-8} \frac{eV\mu m^2}{W} I \]

When atoms are near the minima of the trap, the potential formed by this standing wave of light can be approximated by a series of anisotropic harmonic oscillators with associated frequencies, \( \omega_x, \omega_y, \omega_z \), the trapping frequencies in each dimension. The trap frequency in a dimension \( x \) is determined by the curvature near the minimum of...
the trapping potential and the mass of the atom.

\[ \omega_x = \sqrt{\frac{\partial^2 U}{\partial x^2} \frac{1}{m}} \]

When the optical lattice confines atoms very tightly, effects from the quantization of motion states become apparent. If the spacing of the energies of motional modes in the trap is larger than the kinetic energy gained by an atom emitting a photon, the atoms enter a regime known as the Lamb-Dicke regime.

\[ \Delta E = \hbar \omega_z \ll \frac{(\hbar k_c)^2}{2m} \]

where k is the momentum of the emitted photon. Atoms in the lower motional states are unable recoil when they emit photons, meaning that they emit photons only at the frequency of the clock transition. Higher motional modes of the trap have increasingly less energy separation, so atoms must be cooled to sufficiently low motional modes to be in the Lamb-Dicke regime.

The atoms are trapped 500 \( \mu m \) from the waist of a gaussian beam standing wave with a 10 \( \mu m \) focus. We can approximate the trap potential for atoms sitting in its minimum as a harmonic potential in three dimensions. The two radial dimensions of confinement are defined by the gaussian intensity profile of the beam. The axial dimension of confinement is provided by the nodes and anti-nodes of the standing wave. The potential is

\[ U = -U_{dipole} \cos^2(kz) \left[ 1 - 2 \left( \frac{r}{w_0} \right)^2 - \left( \frac{z}{z_R} \right)^2 \right] \]

where k is the wavenumber of the trapping light, \( w_0 \) is the radius of the beam and \( z_R \) is the Rayleigh range, essentially the length over which intensity drops off from the focus. Since the confinement in the axial direction is on the length scale of half a wavelength, approximately 400 nm, it is much tighter than the radial confinement. This means that the energy spacing between axial motional modes is much larger than radial motional modes. The Lamb-Dicke regime for axial motion can be achieved with much
Figure 2-5: By red detuning our cooling laser by the trap frequency $\omega_z$, we can push the atom into lower energy motional states of the trapping potential [28].

lower intensities than for the radial dimensions. Fortunately, the superradiant light will only be propagating along the axis of the cavity, so the constraints on intensity are somewhat relaxed.

$$\hbar \omega_z \ll \left( \frac{\hbar k_c}{2m} \right)^2$$

Prior to being loaded into the dipole trap, the atoms are cooled in a magneto-optical-trap which uses the $6s6p^3P_1$ to ground green transition at 555.8 nm. This Doppler cooling is able to cool to [27]:

$$kT = \frac{h \Gamma}{2}$$

Once the atoms are in the trap, we can apply sideband-resolved cooling to cool them further [27]. Sideband cooling light is red-detuned from the atomic transition by the trapping frequency. Since this light is detuned by exactly the trapping frequency, every photon that interacts with an atom knocks it into a lower motional state and carries away one motional quantum of energy. Sideband cooling is only possible if the
trapping frequency is greater than the linewidth of the cooling transition:

\[ \Gamma_c \ll \omega_z \]

Sideband resolved heating is limited to cooling the atoms to an average motional mode of

\[ \bar{n} \approx \left( \frac{\Gamma_c}{\omega_z} \right)^2 \]

We must keep the trapped atoms in a low motional state for the entire experiment or they will cause Doppler shifts and possibly be heated into motional modes with insufficient energy spacing to be in the Lamb-Dicke regime.

### 2.3 Requirements for the Optical Lattice

There are three main requirements for the lattice laser in the superradiance experiment: the optical lattice must not cause dephasing due to relative Stark shift between the clock states on the timescale of the superradiant pulse, the atoms must remain at a low enough temperature to minimize Doppler shift and the atoms must remain in the Lamb-Dicke regime so they lase only at the clock frequency.

In order to minimize the relative Stark shift of the clock states, the laser must remain near the magic wavelength. How near depends on the distribution of optical lattice intensities seen by the atoms, the duration of the superradiant pulse, and the degree to which dephasing attenuates the superradiant pulse. For the laser to remain near the magic wavelength, the stability of the reference cavity must be high and the laser’s linewidth compared to the reference cavity must be small.

To keep the atoms in the Lamb-Dicke regime throughout the duration of the superradiant pulse, we need to make the optical trap sufficiently tightly confining and minimize the heating rate of atoms in the dipole trap. When these atoms are loaded into the optical lattice, they are cooled by red-detuned Doppler cooling beams. However, these beams are turned off at the start of the experiment. Therefore, the heating rate of the optical lattice must be low enough such that the temperature
of the atoms does not increase from the initial laser-cooled temperature out of the Lambe-Dicke regime during the experiment. Additionally, the intensity of the optical lattice needs to be high enough that the motional ground state and first couple excited states have greater energy spacing than the recoil energy.

Since the frequency of the axial motional modes in the optical lattice is 245 kHz and the recoil energy of the clock transition is

\[
\frac{(h \times 578\text{nm})^2}{2(171\text{AMU})} = 1.44^{-11}\text{eV}
\]

which corresponds to 21.9 kHz, the depth of the optical lattice is sufficient to put the atoms far into the Lamb-Dicke regime as they are cooled near the motional ground state.

### 2.3.1 Heating

There are two mechanisms by which an optical lattice heats atoms. One is pointing noise, or the shifting of the center of the trap potential. This is would be caused by the dimensions of the optical cavity changing. Therefore, the lock of the cavity length must be good enough to reduce this heating. The second optical lattice heating mechanism is intensity fluctuation. Intensity fluctuations are caused either by the intensity of the trap laser fluctuating or by the frequency of the trap laser moving away from the resonance frequency of the cavity.

The strictness of the heating requirements also depends on the ratio of the initial temperature to the maximum temperature where the atoms remain in the Lamb-Dicke regime. The initial temperature is controlled by the quality of the magneto-optical trap the atoms are loaded from, which is outside the scope of this thesis. The maximum Lamb-Dicke temperature, however, is determined primarily by how deep the potential of the optical trap is. The more powerful the optical lattice, the less stringent the requirements on heating are.

Therefore, the maximum allowable time constant for an exponential heating rate is
\[ \Gamma_{\text{lattice}} = \frac{1}{t_{\text{pulse}}} \ln \left( \frac{T_{\text{Lamb-Dicke}}}{T_{\text{initial}}} \right) = (250\text{ms})^{-1} \]

To give ourselves a safety buffer, we increase this constraint to a time constant of 1 second.

Intensity noise caused by relative drift of the cavity resonance and the laser frequency causes changes in the trapping potential of the form

\[ H = \frac{p^2}{2M} + \frac{1}{2} M \omega_{tr}^2 [1 + \epsilon(t)]^2 x^2 \]

Using time-dependent perturbation theory \cite{29}\cite{30}, the heating rate is

\[ \Gamma_\epsilon = \frac{\omega_{tr}}{4} S_\epsilon (2\omega_{tr}) \]

where \( S \) is the one-sided power spectral density of the intensity noise

\[ S_\epsilon (\omega) = \frac{2}{\pi} \int_0^\infty d\tau \cos(\omega \tau) \langle \epsilon(t)\epsilon(t+\tau) \rangle \]

To achieve the heating rate required to stay in the Lamb-Dicke regime, the laser intensity fluctuation of the trap is constrained to a level of \( 10^{-11} Hz^{-\frac{1}{2}} \) at twice the trap frequency of 245 kHz. We can measure intensity fluctuations in the cavity by measuring the intensity fluctuations of the transmitted light from the cavity. The ratio of the fluctuations to intensity of this light will be equal to the ratio of the light inside the cavity.

Fluctuation of the trap center of the form

\[ H = \frac{p^2}{2M} + \frac{1}{2} M \omega_{tr}^2 [x - \epsilon(t)]^2 \]

can also cause a heating rate\cite{29} of

\[ \Gamma_x = \frac{\omega_{tr}}{4} S_\epsilon (\omega_{tr}) \frac{\langle x \rangle^2}{\langle x \rangle^2} \]

Relative motion of the cavity mirrors will cause much larger fluctuations in intensity.

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of the optical trap than it will shifts of the trap center. For example, a shift of a thousandth of a wavelength in the relative position of the mirrors, a mere nanometer of change, will cause the light in the cavity to drop by a factor of 3. Common mode motion of the mirrors, however, can cause shifts in the trap center without changing the cavity resonance.

We have done little to estimate the effect of such motion so far, but a general estimate can be arrived at by assuming that mechanical noise in the chamber follows a 1/f distribution. In the case of the stable 556 nm laser, we do not observe significant fluctuations in transmitted power when the laser is aligned with the cavity resonance. Since vibration at low frequency is significicantly smaller than a wavelength, we arrive at an upper bound of

\[ S_x(245kHz) = 10^{-24} \frac{m^2}{s} \]

at the relevant trap frequencies. Position noise at this level would cause a heating rate of \(1s^{-1}\), which is dwarfed by heating due to intensity noise.

### 2.3.2 Magic Wavelength

Ideally, the lattice laser would be at the magic wavelength, where the relative Stark shift between the clock states is zero. If the wavelength of the laser shifts away from the magic wavelength, then there will be an intensity dependent change in the energy spacing between the clock levels. This spread of energy levels will cause a spread of frequencies between atoms during the experiment, known as inhomogenous broadening. If the spread of frequencies is greater than the atomic linewidth, the linewidth of the superradiant laser will increase drastically. Additionally, inhomogenous broadening on the order of the linewidth of the atomic transition will extinguish any superradiant pulses.

Assuming the atoms are confined near the center of the trap, the intensity difference among them could be as much as a Watt. Previous measurement \([5]\) of the atomic
polarizability of the states of the clock transition revealed a shift of

\[ \frac{d\alpha}{d\nu_{\text{magic}}} = -221 \text{mHz/}E_r \text{GHz} \]

where \( E_r \) is the lattice depth in terms of the recoil energy at the lattice frequency. Using our lattice depth of \( 6.05 \times 10^{-11} eV \) and a recoil energy of

\[ E_r = \frac{(h k_{\text{magic}})^2}{2(171 AMU)} \]

\[ 1 Hz \left( \frac{d\alpha}{d\nu_{\text{magic}}} \frac{U_{\text{dip}}}{E_r} \right)^{-1} = 1 MHz \]

To achieve this level of absolute stability requires a temperature-stabilized reference cavity and a laser with a linewidth on the order of 1 MHz.
Chapter 3

Laser Construction

In order to create an optical lattice in the science cavity at the magic wavelength of the clock transition, we need to be able to maintain the laser at the magic wavelength and keep it in resonance with the cavity. This implies two important properties. First, the laser frequency must stay tuned to a resonant frequency of the cavity. This task requires excellent short term stability, as any frequency drift from the cavity resonance will manifest as intensity fluctuations in the optical lattice. Intensity fluctuations in the optical lattice heat the atoms, eventually giving them enough kinetic energy to escape the trapping potential.

In addition to short term stability between the laser and cavity resonance, both frequencies need to be kept close to the magic wavelength throughout the experiment. Ideally, we could determine the magic wavelength exactly once, and be able to return the lattice laser to that frequency on demand. This requires long term frequency stability. Since the science cavity needs to change length to conduct different parts of the experiment, it is difficult to use it as a fixed frequency reference. Instead, we use a separate reference cavity to stabilize the frequency of the laser in the long term.

In order to do this, the lattice laser is split and sent along two paths. One path is compared to the stable reference cavity and one path is sent to the science cavity. Since the cavity resonances of the reference cavity don’t necessarily match the magic wavelength, an electrooptic modulator on the path to the reference cavity creates a sideband and adjustable distance from the main laser frequency that can be compared
to the cavity resonance mode.

Since the science cavity length is controlled by piezoelectrics, it changes relatively slowly. The laser, on the other hand, can adjust its frequency very quickly. Oscillations of the intensity of the optical lattice will heat atoms if they are twice the trapping frequency ($\approx 490$ kHz.) Since changes in laser frequency are converted into intensity fluctuations by the cavity, we need to suppress frequency fluctuations out to at least 500 kHz. On the other hand, tandem drift of the optical lattice and the science cavity is less detrimental, as it does not affect the dipole trap and will not cause Stark shift broadening large enough to quench the superradiance as long as the total frequency drift does not exceed $\approx 1$ MHz. Since the drift between the science cavity and the laser needs to be controlled more tightly than the drift between the laser frequency and the magic wavelength, it makes sense to move the laser frequency to match the science cavity resonances and adjust the science cavity length to keep this resonance at the magic wavelength.

To achieve this, we use the PDH error signal generated from the reflected lattice laser at the science cavity to correct the laser frequency directly, locking the frequency of the laser to the resonance of the science cavity, and then use the PDH error from the lattice laser at the reference cavity to correct the science cavity length. There are two main negative feedback loops involved in the optical lattice system. One loop measures the frequency of the laser in relation to the science cavity with the PDH method and corrects by changing the optical feedback path piezo and the current to the laser. Another loop monitors the lattice laser frequency (now offset by some amount) relative to the reference cavity and corrects by adjusting the lengths of the science cavity with piezos.

### 3.1 Laser Diode and Optics Configuration

A Photodigm 760 nm distributed Bragg reflector diode laser produces the light for the optical lattice. This diode consists of a AlGaAs double well gain medium with a Fabry-Pérot cavity formed by two Bragg reflectors, one of which is partially transmissive.
Figure 3-1: The schematic of the entire lattice trapping system.

The body of the laser also contains a thermoelectric cooling element for temperature control and a thermistor for measuring the diode temperature. ¹

The light leaves the laser diode traveling at a broad range of solid angles. A collimating lens, held at a fixed distance from the laser by optics rails, focuses this into a collimated beam. Because the laser gain medium is horizontally wide but vertically narrow, the laser far field, which is essentially a Fourier transform of the intensity profile at the output of the laser chip, is wider in one direction than the other. In order to match this beam to the symmetric Gaussian profile of a fiber mode, we use an anamorphic prism pair to narrow the broader dimension of the collimated laser beam without changing its curvature.

A beam splitter splits off some of the collimated, symmetric beam for optical feedback. After the beam splitter, the laser passes through an isolator which attenuates any light passing through it in the wrong direction by 60dB, blocking any back reflections that might make the laser unstable. After the isolator, the laser beam passes through a half-wave plate and polarization beam splitter. The PBS ensures the laser light that is coupled into the output fiber is purely linearly polarized and matches with one of the axes of the polarization preserving fiber. The half wave plate allows adjustment of the laser output power. Finally the laser beam is coupled into a

¹[31] is an excellent reference for all parts of the construction and frequency stabilization of diode lasers.
Figure 3-2: **Pane A:** The lattice laser enclosure: 1 is a fiber coupler which couples the laser into a fiber that runs to the splitting station. 2 is a polarizing beam splitter (PBS) that guarantees the laser polarization is along one of the axes of the polarization preserving fiber. 3 is a half wave plate which, along with the PBS, allows adjustment of output power. 5 is an optical isolator that uses Faraday rotation.  

**Pane B:** The splitting station: 1 is a fiber coupler. 2 and 3 and 4 and 5 are pairs of half wave plates and PBS that allow adjustable splitting of the laser power. 6,7,8 and 9,10,13 are half wave plate, mirror, and fiber coupler combinations which send light to the reference cavity and wavemeter. 14 is an AOM that allows rapid on-off switching of light sent to the cavity. 16 is the EOM for the reference cavity PDH lock.  

**Pane C:** Close up of the inner lattice laser enclosure: 1 is the laser and collimating lens. 2 is the anamorphic prism pair. 3 is a 90-10 beam splitter for the optical feedback. 10 is a photodiode to measure the power of the optical feedback. 4 is a PBS to guarantee the light is polarized along an axis of the polarization maintaining fiber. 5 is an iris to minimize back reflection from the fiber coupler, 6. 7 is the foam encasing the feedback fiber. 9 is the piezo actuated feedback mirror. 11 is the RF input to the laser. At far left is a RF low pass filter.
polarization-preserving, single-mode fiber.

This entire laser structure is mounted on a half-inch thick aluminum optics bread-
board which sits on a layer of sorbothane rubber on the main experimental optics table. The laser breadboard is enclosed in a nearly air-tight acrylic box which is mounted on the optics table and has no contact with the laser breadboard. The acrylic box prevents air currents and sound waves from perturbing the laser. The cables that supply the laser current, TEC current and piezo voltage as well as the fiber optic that carries the laser light out all pass through a small hole in the acrylic box. All of the cables pass through a strain reliever both inside and before entering the box, reflecting vibrations they may be carrying.

For further vibration isolation, all parts to the laser prior to the optical isolator are mounted on another aluminum breadboard inside the laser box. This breadboard has another layer of sorbothane under it and another acrylic box around it to further isolate it from noise. Finally, the feedback fiber is coiled inside a block of packing foam to further isolate it from vibration.

The laser travels through the output fiber to the splitting station, shown in Pane B of figure 3-2, where the light is split and sent to the science cavity, reference cavity, and wavemeter. The splitting is done with two pairs of polarizing beam splitters and half wave plates. This allows the power splitting to be adjusted. Like the laser itself, the splitting station is mounted on a half inch aluminum breadboard which rests on a sorbothane pad. The entire splitting station is enclosed in an acrylic housing.

The optical path to the science cavity passes through an acousto-optic modulator (AOM) which allows fast on-off switching of the light to the cavity. The output fiber is aligned so that light from the first order mode of the AOM is coupled. This means that light only reaches the science cavity when RF is applied to the AOM. An 80 MHz signal for the AOM is produced by an AD9959 direct digital synthesizer and amplified to 20dBm by RF amplifiers. By carefully adjusting the angle of the AOM and the RF power supplied, I achieve 80% coupling efficiency of the first order mode.

The optical path to the reference cavity passes through an EOM which is used to create the offset frequency between the science and reference cavities as well as
the sidebands for the PDH lock. The EOM is mounted inside the splitting station to reduce the fiber length between the PBS and the EOM. This is beneficial because the performance of the EOM depends heavily on the purity of polarization of incoming light. Although the PBS isolates exactly linear polarization and the EOM input fiber is polarization preserving, vibration of the fiber can rotate the polarization, degrading the EOM performance.

3.2 Output Coupling

To efficiently couple light from the laser diode to the output fiber, I carefully chose the positions of the collimating lens, the anamorphic prism pair, and the optical isolator. To choose the position of the collimating lens, I placed a beam profiler in the far field of the laser (approximately 30 mm from the end facet) and observed the beam profile in real time as I adjusted the lens position. I adjusted the lens position for the smallest possible beam size while also minimizing diffraction peaks. In the end, I was able to achieve a beam size of 0.5 mm with only a first order diffraction peak, probably due to insufficient numerical aperture of the collimating lens in the wider beam dimension.

Following a similar procedure, I carefully adjusted the position of the anamorphic prism pair while observing the beam profile. I adjusted the position and angle of the APP to make the dimensions of the beam equal to each other, while trying to minimize diffraction peaks. While I was able to make the dimensions of the beam equal overall, some asymmetry remained, probably due to the asymmetric geometry of the laser diode end facet.

Finally, I adjusted the optical isolator while measuring the power of the beam. After I had optimized for maximum beam power, I measured the beam profile. The isolator blocked the extra diffraction peaks of the beam, leaving a mostly Gaussian profile. The laser diode to output fiber coupling efficiency was 45%, comparable to other diode lasers in our lab.
Figure 3-3: Lattice laser beam profiles with distances in microns. The beam profile before the collimator is very broad, as the laser is diverging rapidly from the small end facet. The laser profile after the collimating lens is narrower, but still much broader in one dimension than the other. The anamorphic prism pair corrects the difference in dimensions at the cost of some small diffraction fringes. The narrow isolator blocks these additional fringes, making the profile symmetric and Gaussian.

### 3.3 Intrinsic Linewidth

A laser’s frequency can vary over a wide variety of timescales. Very fast variation (i.e. faster than the relevant time scale, in this case, the residence time of light in the science cavity) contributes to the laser’s linewidth. Frequency variation slower than the cavity residence time manifests as jitter - drift of the laser frequency. Another way of putting this is that the laser frequency rapidly varying over a range of frequencies is equivalent to the laser emitting all frequencies in the range simultaneously.

One way to quantify the variation in the laser’s frequency is with the frequency spectral noise density, defined by the Fourier transform of the auto-correlation function of its frequency.

\[ S_\nu(\omega) = \int d\tau e^{i\omega\tau} \Gamma_\nu(\tau) \]

\[ \Gamma_\nu(\tau) = \langle \nu(t)\nu(t-\tau) \rangle \]

One relatively simple way to measure the spectral noise density is to lock the laser
to the side of a transmission fringe of a low finesse cavity with a very low bandwidth feedback loop. Since the bandwidth of the lock is low, the spectral noise density is only affected at very low frequencies. Meanwhile, the transmission function of the low finesse cavity, effectively converts frequency fluctuations into amplitude fluctuations, which can then be measured by a photodiode.

To measure the free-running spectral noise density of the lattice laser, I locked a Thorlabs SA200 Fabry-Pérot cavity, which has a finesse of roughly 200, to the laser. For my feedback actuator, I used the piezo-adjustable cavity, with a feedback bandwidth of 10 Hz. The resulting power fluctuations of the light transmission were measured with a DC-coupled APD with a bandwidth of 20MHz and fed into a spectrum analyzer. The resulting spectrum is shown in figure 3-4.

To keep the laser’s frequency equal to the cavity resonance frequency, we can compare the laser frequency to the resonance of the cavity and use electronic feedback to correct the laser’s frequency. The maximum speed at which the feedback can correct the laser determines the bandwidth of frequencies over which the feedback works. If the feedback bandwidth is less than the cavity finesse, the laser’s linewidth will not be modified, but laser jitter will be suppressed and the laser frequency will be centered at the cavity resonance. If the feedback bandwidth is greater than the interaction frequency, then the linewidth of the laser will be narrowed. Greater feedback bandwidth leads to smaller linewidth until the bandwidth reaches a saturation frequency $f_c \approx 1.78h_0$ determined by the intrinsic level of noise in the laser $h_0$. At this point the feedback has narrowed the laser linewidth as much as possible and mechanical noise in the reference cavity is the primary contributor to the laser linewidth.

The linewidth of the Photodigm laser is 3 MHz in the absence of feedback. Therefore, to narrow the linewidth of the laser we would need electronic feedback with a bandwidth of at least 3 MHz. This poses several technical challenges. The first is that the active electronics that don’t add phase delay above 1 MHz are expensive and complicated. The bigger challenge, however, is that the effect of laser diode current modulation changes past 1 MHz. At lower frequencies, additional injection current heats the laser diode, causing the gain medium to expand slightly and increasing the
wavelength of emitted light. At higher frequencies, injected current does not have time to heat the diode, so the primary mechanism of action is an increase in the index of refraction caused by higher electron density. This causes the emitted wavelength to be shorter. It is difficult to design feedback around these conflicting effects.

3.4 Optical Feedback

Fortunately, we can use optical feedback to narrow the laser’s linewidth to 1 MHz and simplify the design of the electrical feedback [32]. To remove the higher frequency noise, we use a technique known as optical feedback. Inserting a beam splitter in the laser’s path we split off some of the light and send it down an optical fiber. At the end of the optical fiber the light is reflected back along the same path by a mirror whose position can be changed very precisely by a piezoelectric stack. This light then reenters the laser. The photons of the returning beam, which were emitted hundreds of nanoseconds prior, stimulate more emission in the laser’s gain medium. These returning photons guarantee that the laser will emit a frequency very close to the one it was emitting hundreds of nanoseconds ago.

This optical feedback suppresses quick changes in the laser’s frequency, comple-
menting the electronic feedback. The optical feedback alone does not completely stabilize the laser as slow drifts are essentially imperceptible at the time scale of the optical feedback. Furthermore, mechanical vibrations can move the feedback mirror, changing the phase of the returning light. This pulls the laser’s frequency away from its stabilized value. To counteract this, we need a PDH lock to a reference cavity. As the laser’s frequency moves, the error signal form the PDH lock can be used to change the position of the piezoelectric stack under the feedback mirror.

The effect of the optical feedback also depends on its power relative to the laser’s intensity [33]. There are four different feedback regimes available to this laser. In regime I, with no feedback, the laser operates at its natural linewidth. In regime II, with feedback powers -45dB below the laser output power, the laser linewidth is narrowed, but it modehops often. To avoid the modehopping, I put the laser in regime III, from -45 to -39 dB feedback. Regime III has laser narrowing and no modehops. If the feedback power is increased above -39 dB, the laser undergoes coherence collapse, producing light across a broad range of frequencies. I use a photodiode at the feedback beamsplitter to tune the amount of feedback power to put the laser in regime III.
3.5 Current Controller

The current controller is modeled after a very popular design first used by Libbrecht and Hall [34]. The design uses a small sense resistor to sense the current flowing through an IRF5305 power transistor and control it through a feedback loop. The input to the feedback loop is a knob potentiometer which allows adjustment of the laser current. The power supply is filtered with a variety of capacitors that remove high frequency noise and held at a steady DC value by a linear regulator. A temperature stabilized Zener diode feeds one terminal of the current potentiometer. (See figure 3-6)

This guarantees that the current is both of free of high frequency noise and has low long-term drift, which increases experimental reproducibility. Downstream from the power transistor, modulation current is added to the current output. The voltage across a second sense resistor is measured and amplified by an instrumentation amplifier and output at a separate current monitor port for diagnostic purposes.

The modulation path for the current controller starts at the Mod In port, which is buffered with an instrumentation amplifier. A knob potentiometer on the case adds a DC offset to the modulation input and serves as a fine adjustment for the current. The modulation signal is then split. One path is divided by an adjustable modulation gain potentiometer buffered by and instrumentation amplifier and then injected into
the laser diode current. The other path is buffered by an op amp and sent through an adjustable low pass filter and then output at the piezo port. This signal then travels to a high voltage amplifier which drives the optical feedback piezo.

To measure the noise performance of the current controller, I sent the current through a dummy laser diode consisting of three 1N4001 power diodes in series with a 10 Ω sense resistor. The diodes mimic the IV characteristics of the laser diode and the sense resistor converts current noise into a voltage signal. Analyzing this signal with a spectrum analyzer gives a good measure of the current noise at different frequencies (see figure 3-8.) Aside from the initial bulge, which is caused by the limited bandwidth of the spectrum analyzer, the spectrum is below $1 \text{nA}\sqrt{\text{Hz}^{-1}}$ rms. Since the sensitivity of the laser diode to current is about $1 \text{MHz/\mu A}$, the current limited linewidth is 1 kHz.

### 3.6 Temperature Controller

Both laser diodes and optical reference cavities are highly sensitive to temperature. For example, the wavelength emitted by the optical lattice laser changes by 1 GHz
Figure 3-8: The noise spectrum of the current controller for the optical lattice laser, measured across a dummy laser diode.

Figure 3-9: The flow of heat between the environment, thermoelectric cooler, and environment

K⁻¹. Room temperature, even in a well controlled laboratory fluctuates by up to a degree Kelvin over a period of a day. In order to stabilize the laser frequency to the desired tolerance, we must use some form of feedback to control the temperature.

For feedback control, we use a thermistor attached to the laser diode to measure the laser temperature and correct this temperature by varying the current through a thermoelectric cooling (TEC) element that moves heat from the laser to a heatsink. Both the laser and the thermoelectric element have finite heat capacity and the laser has thermal contact with the room as well as the TEC.

This system of heat flows can be modeled as a circuit as in figure 3-9. These heat flows can be modeled as a lowpass filter with a time constant $\tau \approx 10s$. The transfer function from TEC input to temperature is

$$A_{\text{diode}} = \frac{A_0}{1 + s\tau}$$
where $A_0$ is the TEC gain and $s = j\omega$. We can add feedback with gain $\beta$, modifying the transfer function

$$A_{feedback} = \frac{A_0}{1 + \beta A_0} \frac{1}{1 + \frac{s\tau}{1 + \beta A_0}}$$

Increasing $\beta$ decreases the impact of fluctuations in the environment on the diode temperature. However, if we increase $\beta$ enough, we will eventually be limited by the gain-bandwidth product of our feedback electronics or the nonlinearity of our gain. We can control this by using integral gain. This adds a second time constant to the system and gives a transfer function of

$$A_{feedback} = \frac{A_0}{\tau_1 \tau_2} \frac{1}{s^2 + s \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) + \frac{1 + \beta A_0}{\tau_1 \tau_2}}$$

This has poles

$$2s = -\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) \pm \sqrt{\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)^2 - \frac{4\beta A_0}{\tau_1 \tau_2}}$$

If $\beta$ is too large these poles will have imaginary components, causing oscillations:

$$\beta > \sqrt{\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)^2 \cdot \frac{\tau_1 \tau_2}{4A_0}}$$

This imposes a spacing between the two poles $^2$

$$\frac{\tau_1}{\tau_2} < \frac{2\beta A_0}{2\beta A_0}$$

If we solve the original differential equation, we see that the time required for the system to settle within $\pm\Delta$ of a new set point is

$$\Delta = 2\tau_2 \ln\left(\frac{1}{\Delta}\right)$$

for $\tau_1 \gg \tau_2$. Since we have one time constant that is fixed by the thermal properties of the laser diode, we would like to decrease the time constant set by our integrator as much as we can without inducing oscillations. In other words, we would like to

$^2$Technically we need only satisfy $\frac{\tau_1}{\tau_2} < 4\beta A_0$ for real poles but we build in a 50% safety margin.
turn up the integral gain of our controller to reduce the impact of the environment, but we past a certain gain we induce oscillations.

Yet another useful way to look at the issue is to examine the phase lag of the TEC-controller loop at the unity gain point, where the feedback gain is one. If the loop phase lag exceeds $\pi$ at any frequency below this point, then self-reinforcing oscillations will develop. The phase margin is the difference of the actual loop phase lag from $\pi$ at the unity gain point.

We can quantitatively examine the effects of different integral gain on the phase margin. For a given ratio $\alpha$ of the time

$$\frac{\tau_1}{\tau_2} = \alpha \beta$$

$$\phi_m = \pi - \arctan\left(\frac{f_{0dB}}{f_1}\right) - \arctan\left(\frac{f_{0dB}}{f_2}\right)$$

Since

$$\left(\frac{f_{0dB}}{f_1}\right) = \beta \gg 1$$

we can set

$$\arctan\left(\frac{f_{0dB}}{f_1}\right) = \frac{\pi}{2}$$

$$\phi_m = \frac{\pi}{2} - \arctan\left(\frac{\beta f_1}{\alpha \beta f_1}\right) = \arctan(\alpha)$$

So we can improve the noise rejection of the system but past a certain point this will be at the expense of the phase margin.

To implement this control system in circuit form, we first need to measure the temperature of the laser. We use a constant current source in the Howland Current pump topology to pull supply current through the laser thermistor (see figure 3-11.) The negative voltage across the thermistor is then linearly related to its resistance. This voltage is then compared to the voltage across a resistor in the controller box with a constant current flowing through it. A switch allows comparison to different resistances for gross temperature adjustment. The output of this comparison is then fed to the proportional/integral gain stage.
Figure 3-10: A Bode plot of the loop gain of the TEC controller system. [35]

Figure 3-11: The temperature controller uses a constant current supply of the Howland Current Pump topology to pull current through the laser thermistor. The resulting negative voltage is added to the voltage drop of a resistor in the controller box with a summing op amp.
The PI controller is detailed in figure 3-12. Two potentiometers allow the adjustment of the proportional and total gain. A diode ensures the TEC is never sent a signal with reverse polarity. A switch on the box allows the user to short the integral gain capacitor. This is useful during gross temperature adjustment when the integrator might otherwise see a large error signal and over correct.

From the PI stage, the signal is sent to the output stage (see figure 3-13.) The output stage uses a similar design as the current controller, with the voltage from a sense resistor buffered and used as feedback for the control of a power transistor.

By tuning the proportional and total gain, I was able to optimize the controller performance. The step response is shown in figure 3-14. I traded some phase margin for quick response to changes in environment. Notice the settling time is \( \approx 1 \) second. The oscillations are only present in the step response but do not show up in normal operation.
Figure 3-13: The current supply to the thermoelectric cooling element is delivered from a 5V supply through a D45H8 transistor. The current to the transistor is adjusted by an op amp which varies the supply by comparing the voltage from a sense resistor the input from the PI module.

Figure 3-14: The response of the laser temperature controller to a change in setpoint.
3.7 Drift in absence of feedback

All of the above improvements to the laser greatly improve its passive stability. However, none of them guard against slow drift in the absolute frequency of the laser. To cancel this drift, we need the frequency lock described in the following chapters.

To see how stable the laser is, I let the laser drift for a period of 12 hours, while recording its frequency with a Advantest Q8326 wavemeter. The wavemeter has a precision of 10 MHz and I collected one data point per second with a National Instruments DAQ and LabVIEW. Pane A of figure 3-15 shows a histogram of the resulting frequency values. The $\approx 1$ GHz breadth of values is consistent with the 1 GHz/K temperature sensitivity of the laser diode and the 1 K daily fluctuation of the lab temperature.

The time series of frequency values shows clear modehops at the feedback path FSR of 100 MHz at a rate of approximately one per minute caused by relative drift of the optical feedback path and the laser center frequency. These hops cause the narrower peaks in the frequency histogram. I wrote a MATLAB script to track the drift of the optical feedback modes and the hops of the laser frequency between them. Using this script, I was able to find the center frequency of the feedback mode at every measurement point. The deviations of the laser frequency from the mode center are plotted in pane B of figure 3-15.

The breath of this histogram is still 300 MHz, even though the laser linewidth with
optical feedback is 300 kHz. This means that length of the optical feedback path can
drift around 3\(\lambda\) or 2.4 \(\mu\text{m}\) in 12 hours. Thus, to achieve long-term absolute stability
we need to correct the optical feedback path length with feedback.
To lock the laser, we need a frequency reference to compare it to. For a frequency that doesn’t correspond to the frequency of an atomic transition, the only viable reference is a high-finesse cavity. An optical cavity is a pair of partially transparent mirrors. If light of the correct wavelength and frequency shines on the cavity, it bounces off the mirrors such that it is totally transmitted, as if the cavity weren’t there. Otherwise the light is reflected. These resonant frequencies follow a simple formula:

\[ \nu_m = \frac{m c}{2L} \]

where the \( \nu_m \) are the resonance frequencies, \( m \) is an integer, \( L \) is the length of the cavity and \( c \) is the speed of light.

If the mirrors are highly reflective, the range of frequencies that are transmitted through the cavity is very small and does not vary over a timescale of seconds. A detection scheme with a high signal to noise ratio (SNR) can be used to detect deviation of the laser from this small band of frequencies and correct it.

The simplest detection scheme is to put the laser’s frequency on one side of the cavity transmission fringe and measuring the transmitted light through the cavity. If the laser frequency moves, then the intensity of the transmitted light decreases or decreases, and corrective action can be taken. However, this scheme is incapable of distinguishing laser amplitude fluctuations from frequency fluctuations. Additionally,
the lock is vulnerable to large hops in laser frequency - if the frequency jumps to the other side of the transmission fringe, the lock will push it in the wrong direction. This scheme also relies on the transmission from the cavity, which limits its bandwidth to the cavity linewidth.

The Pound-Drever-Hall frequency lock avoids these sources of error by using the light reflected from the cavity instead [16] [36]. Although the reflected light from the cavity is symmetric around resonance, its derivative is anti-symmetric. We can measure the derivative by modulating the laser frequency very quickly, back and forth, and checking what effect this has on the reflected light.

In practice, the PDH lock uses a polarizing-beam-splitter and a quarter wave plate to send the reflected light from the cavity along a different path than the incoming light. The incoming light is frequency modulated by an electro-optic modulator prior to reaching the cavity. The variations produced by this fast frequency modulation are collected by an avalanche photodiode. The variations are then multiplied by the same signal that originally modulated the laser, producing a signal proportional to the frequency derivative of the reflection from the cavity. The sign of this signal is a sensitive frequency discriminator and can be used to correct the laser.

In this chapter, I briefly discuss the properties of the two cavities the laser is PDH
locked to and the theory behind the PDH lock.

4.1 Coupling Light to Cavities

There are two factors which determine how well a laser will be coupled into an optical cavity. The first is optical impedance matching. This describes how well the wave coming from inside the cavity cancels the reflected light when the cavity is on resonance. Optical impedance is matched when the transmission of input mirror equals the losses of the mirrors. This depends mainly on the properties of the mirrors and cannot be changed after the fact.

The second factor is mode matching. This is the degree to which the intensity and phase profile of the incoming light on the cavity mirrors match the profile of the fundamental mode. If the match is imperfect, then some optical power will be coupled into other cavity modes. This is undesirable, as it makes the cavity transmission signal harder to interpret and interferes with the laser frequency lock.

I chose lenses to match the q-parameter of the beam from the fiber coupler to the q-parameter of the cavity resonance beam. This meant that the radius and curvature of the cavity mode and the incoming beam were the same. After that, I adjusted the angle of the fiber outcoupler and the mirror that reflects the mode into the cavity. There are four knobs which control the location and direction of the cavity input beam.

I used two indicators to help me align this beam to the cavity: the back-reflected beam and the transmission spectrum as the laser frequency is swept over the cavity resonance frequency. For rough alignment, I used the retroreflected beam. I adjusted the fiber coupler and mirror so the input beam was going into the center of the cavity. Then I used an index card placed near but not in the incoming beam to find the retroreflected beam. While keeping the incoming beam centered on the cavity, I adjusted the mirrors to direct the reflected beam back along the incoming beam.

When the two beams were aligned perfectly to the naked eye, I switched to aligning with the transmission of the laser frequency sweep. The transmitted light for the
cavity modes follows a Poisson distribution. Since my rough alignment was close, I saw a Poisson distribution with a small lambda factor, and it was easy to tell which mode was the fundamental by looking for a mode at lower frequency with higher transmission that all the other modes. Once I found the fundamental mode, I fine tuned the alignment to maximize its transmission while minimizing other modes. At the end of the alignment, the first higher-order mode had one tenth the transmission of the fundamental mode.

### 4.2 Reference Cavity

The reference cavity\(^1\) is constructed in the common vertical configuration [37]. The cavity spacer is made out of Corning’s ultra-low-expansion (ULE) glass with a thermal expansion coefficient of \(\alpha = 0 \pm 30\ \text{ppb K}^{-1}\) between 5 and 23 °C. Four holes for cavities are drilled through the long axis of the spacer. One of these holes is dedicated to the 759nm laser reference cavity. This cavity is 10 cm long and constructed in an asymmetric configuration with one flat mirror and one mirror with a 50 cm radius of curvature. The cavity mirrors are titanium oxide on a substrate of fused silica. The transmission plus losses are \(13212 \pm 201\ \text{ppm}\) for the flat mirror and \(221 \pm 4\ \text{ppm}\) for the curved mirror. Together, these give a finesse of 234. The free spectral range of the cavity is 1.5 GHz and the linewidth is 6.4 MHz.

The reference cavity is supported by low-thermal-conductivity stainless steel spacers and situated inside a shield made out of copper, which is temperature stabilized within 25 mK. This shield keeps the spacer in a bath of thermal radiation of constant temperature. The temperature of the ULE spacer is also measured with a thermister and temperature deviations are recorded. To prevent convective heat transfer, the entire assembly is put inside a vacuum chamber with a pressure of \(3 \times 10^{-3}\) Torr maintained by ion pump.

In addition to thermal stabilization, the reference cavity is on an optics table which is vibrationally isolated with laminar flow columns. Assuming worst case expansion

---

\(^1\)The reference cavity was constructed and characterized by André Heinz.
coefficients, a fluctuation of 25 mK leads to a shift of 390 kHz in the resonant frequency of the reference cavity. This is significantly less than the 1 MHz stability needed to prevent Stark shift broadening.

To verify the temperature stability of the reference cavity, we heat it over a range of temperatures while measuring the drift of the resonant frequency. This is done using the 556 nm cooling laser as a reference, which is locked to another reference cavity. The 556 nm reference cavity resonance has been measured to drift less than 1 MHz per year relative to atomic transitions in ytterbium, so it can be assumed to be nearly constant over the course of the measurement. To measure the cavity drift relative to the 556 nm laser, we shine the 759 nm and 556 nm lasers into the science cavity. The science cavity length is tuned to resonance with the 556 nm laser and the frequency of the 759 nm laser is adjusted so it is also in resonance. The 759 nm is

Figure 4-2: Resonant frequency of the 759 nm reference cavity compared to the ultrastable 556nm cooling laser. Graph credit to André Heinz.
split and simultaneously sent to the reference cavity through an EOM. By adjusting the RF frequency sent to the EOM, we can adjust the frequency offset of the laser sidebands until one of them is resonant with the reference cavity. As the cavity is heated, the required EOM offset changes and the changes are recorded along with the values of a thermistor on the cavity spacer. (See figure 4-2.)

The data show a slope of $-24.635\text{MHz/K} \pm 0.452$. At this stability a 25 mK temperature fluctuation would cause 600 kHz of drift, still well within the experimental requirement of 1 MHz.

### 4.3 Science Cavity

The science cavity is asymmetric with one highly curved micromirror, etched with a CO$_2$ laser and one larger, flatter mirror. This design is easier to align, but preserves a region of intense cavity field near the smaller mirror that allows high coupling to atoms. Its FSR is 5.96 GHz. For 556 nm light it has a finesse of 20,000 and a corresponding linewidth of 298 kHz. At 759 nm it has a finesse of 3,150 and a linewidth of 1.89 MHz. Its length is tunable by piezoelectric stacks under the smaller mirror.

### 4.4 Pound-Drever-Hall Locking

To find the sensitivity of the PDH method to changes in frequency, we consider the PDH system in depth. Light reflected from the reference cavity is the sum of an infinite number of bounces of light inside the cavity (see figure 4-3.)

Summing these terms lead to the closed form expression:

$$F(\omega) = \frac{E_{\text{ref}}/E_{\text{inc}} = \frac{-r_1 + r_2(r_1^2 + t_1^2)exp(i\frac{\omega}{\Delta\nu_{FSR}})}{1 - r_1r_2exp(i\frac{\omega}{\Delta\nu_{FSR}})}}$$

This forms an offset circle in the complex plane:

$$|F(\omega) - Z_0|^2 = R^2$$
Figure 4-3: The terms contributed by each light path through the cavity. Paths with more bounces contribute progressively less. On resonance, all paths interfere constructively [7].

Where:

\[
Z_0 = -\frac{r_1}{1 - r_1^2 r_2^2} [1 - R_2^2 (r_2^2 + t_1^2)]
\]

\[
R = \frac{t_1^2 r_1}{1 - r_1^2 r_2^2}
\]

Modulation of the input signal with an EOM produces a sinusoidally varying phase:

\[
E_{inc} = E_0 e^{i(\omega t + \beta \sin \Omega t)}
\]

We can expand this using Bessel functions:

\[
E_{inc} \approx E_0 [J_0(\beta) + 2iJ_1(\beta)\sin \Omega t] e^{i\omega t}
\]

If \( \beta \) is small enough, we find that the majority of the power is concentrated in the original frequency (the carrier) and two sidebands offset by \( \Omega \)

\[
= E_0 [J_0(\beta)e^{i\omega t} + J_1(\beta)e^{i(\omega + \Omega) t} - J_1(\beta)e^{i(\omega - \Omega) t}]
\]

Their relative powers are:

\[
P_c = J_0^2(\beta) P_0
\]

\[
P_s = J_1^2(\beta) P_0
\]

\[
P_c + 2P_s \approx P_0
\]
To find the reflected field we multiply the input field by the cavity reflection function:

\[ E_{ref} = E_0 [F(\omega)J_0(\beta)e^{i\omega t} + F(\omega + \Omega)J_1(\beta)e^{i(\omega+\Omega)t} - F(\omega - \Omega)J_1(\beta)e^{i(\omega-\Omega)t}] \]

To find the reflected power registered by the APD, we take the complex magnitude of the reflected field:

\[ P_{ref} = |E_{ref}|^2 = P_c|F(\omega)|^2 + P_s|F(\omega + \Omega)|^2 + F(\omega - \Omega)|^2 \]

\[ + 2\sqrt{P_cP_s}[Re[F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)]\cos \Omega t \]

\[ + Im[F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)]\sin \Omega t} \] + (terms varying at 2\Omega)

The modulation frequency is high enough that the sidebands are almost entirely reflected:

\[ F(\omega \pm \Omega) \approx -1 \]

\[ F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega) \approx -i2ImF(\omega) \]

Thus the non-DC part of the error signal is:

\[ \epsilon = -2\sqrt{P_cP_s}ImF(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega) \]

Near resonance,

\[ P_{ref} \approx 2P_s - 4\sqrt{P_cP_s}ImF(\omega)\sin \Omega t + (2\Omega \text{ terms}) \]

If we approximate the finesse as

\[ \mathcal{F} \approx \pi/(1 - r^2) \]
Figure 4-4: Oscilloscope screen captures of the PDH error signal as the laser is swept over cavity resonance. The signal is shown for several different relative phases between the modulation and demodulation signals. Pane A: relative offset 90 ° Pane B: relative offset 45 ° Pane C: relative offset 0 ° Pane D: relative offset 180 °
Then we can redefine small variations in the center frequency

\[
\frac{\omega}{\Delta \nu_{fsr}} = 2\pi N + \frac{\delta \omega}{\Delta \nu_{fsr}}
\]

And rewrite the reflection coefficient as

\[
F \approx \frac{i \delta \omega}{\pi \delta \nu}
\]

\[
\delta \nu \equiv \Delta \nu_{fsr}/F
\]

The sensitivity of the error signal to frequency change can therefore be written as D:

\[
\epsilon = D \delta f
\]

\[
D \equiv -\frac{8 \sqrt{P_c P_s}}{\Delta \nu}
\]

D, the gain off the error signal depends only the powers of the carrier and sidebands and the linewidth of the cavity. To optimize the powers of the sidebands, we note that

\[
\sqrt{P_c P_s} \approx \sqrt{\frac{P_0}{2}} \sqrt{1 - \frac{P_c}{P_0}} \frac{P_c}{P_0}
\]

The contribution from the power is maximized if \( P_c/P_0 = 0.5 \).

We can experimentally determine the ratio of frequency to voltage produced by the DC discriminator, by sweeping the laser frequency over the cavity resonance and examining the resulting waveform. Using the frequencies of the sidebands as a reference, we can look at the frequency extent of the linear slope of the PDH signal and its height in voltage to determine the DC gain of the PDH lock.

We can determine the finesse and linewidth of the cavity in two ways: either by ring-down measurement or by looking at the cavity transmission fringe as the laser is swept across the cavity resonance. Using the latter method we determine the cavity linewidth to be 1.9 MHz. From this we can determine the full transfer function of the cavity.
The previous derivation assumed that the laser frequency was static, when deriving the response of the PDH lock. High frequency deviations of the laser frequency produce a different response however. If the laser frequency moves fast enough that the light still in the cavity cannot adjust, the reflected signal is just the carrier frequency and the new laser frequency mixed together:

\[ E_{ref} = rE_0[J_1(\beta)e^{i(\omega_0+\Omega)t+\phi(t)} - J_1(\beta)e^{i(\omega_0-\Omega)t+\phi(t)} + \sqrt{1-r^2}e^{i\omega_0t+\int_0^{T_c} dt\phi(t)}] \]

Then, taking the reflected power and only keeping terms that depend on the deviation frequency:

\[ P_{ref} = |E_{ref}|^2 \propto \sin \Omega t \sin \left[ \phi(t) \int_0^{T_c} dt\phi(t) \right] \]

\[ \epsilon = P_{ref}\sin \Omega t \propto \phi(t) \int_0^{T_c} dt\phi(t) \]

Thus, we find that the response of the PDH lock to frequency deviations falls off as \( 1/\omega \) after the cavity linewidth. This is convenient, as we would like our loop gain to decrease at high frequencies.
Chapter 5

Laser Feedback

In addition to the features of the optical lattice laser that contribute to its passive stability, we apply active stabilization to further narrow its linewidth. At the heart of this stabilization is the science cavity. By comparing the laser’s frequency to the frequency of the modes of the science cavity with the PDH lock we can detect any deviation between the two frequencies. This error signal can then be used to correct the laser’s frequency through several complementary methods. In this chapter, I describe my implementation of the PDH lock and the performance of my system.

5.1 Feedback

The basic principle of the feedback loop is shown in 5-1. The deviation of the laser frequency from some setpoint is detected and used to correct the laser frequency. There is a time lag between when the laser’s frequency deviates and when the feedback loop tries to correct, caused by the finite time needed to detect the frequency change, send it to the laser, and then take corrective action. This delay causes increasing phase lag for signals of higher frequencies.

\[ \theta_{lag} = \omega \tau_{lag} \]
Figure 5-1: A schematic of the components of the laser feedback loop and the various sources of noise. The PDH lock serves as a frequency discriminator and feeds back to the laser through the optical feedback piezo as well as the current (not shown) [38]. Noise can enter at the laser, through vibration in the optical feedback path, at the cavity, through shot noise at the photodiode, or through the feedback loop via electrical noise or interference.

Similarly, poles in the response of the electronics of the feedback loop, caused by resistance and capacitance, add 90° of phase lag per RC pole.

If this phase delay becomes greater than 180° then the feedback signal will exacerbate the error it is trying to correct. Therefore, the gain of the feedback system must be less than 1 at and above the frequencies with more than 180° phase lag, hopefully with some margin to spare. In control theory, this margin of stability is quantified by the phase margin, the number of degrees from 180° the feedback is at the unity gain frequency. At frequencies below the unity gain point, we can apply feedback to reduce noise in the system.

Greater feedback bandwidth leads to smaller linewidth until the bandwidth reaches a saturation frequency $f_c \approx 1.78h_0$ determined by the intrinsic level of noise in the laser $h_0$. At this point the feedback has narrowed the laser linewidth as much as possible and mechanical noise in the reference cavity is the primary contributor to the laser linewidth.

5-2 shows that the optical feedback leads to a suppression of noise at frequencies above the free spectral range 1 MHz. At low frequencies near the range suppressed by
Figure 5-2: The noise spectrum of laser light transmitted through a Fabry-Perot cavity. The cavity is locked loosely to the laser, so the laser’s free-running frequency fluctuations are translated into intensity noise. The $S_\nu = \omega/1.78$ line that determines the critical lock bandwidth is plotted in red.
the lock, we see a marked increase in noise. This low frequency noise follows a 1/f distribution and is ubiquitous in physical systems. In between these two features, the laser noise plateaus to a value of $h_0 = -70dB\sqrt{Hz^{-1}}$.

There are two goals for the lattice laser stabilization: to prevent frequency fluctuations at the specific resonant frequencies of the optical lattice, and to narrow its overall linewidth. The relationship between the spectral density of frequency noise and the lineshape of the laser is complicated, but Thomann et al [39], have developed a heuristic that states that the linewidth is approximately equal to the frequency of intersection between the noise spectrum and the line:

$$S_\nu = \omega/1.78$$

Greater feedback bandwidth leads to smaller linewidth until the bandwidth reaches a saturation frequency $f_c = 1.78h_0$. At this point the feedback has narrowed the laser linewidth as much as possible and mechanical noise in the reference cavity is the primary contributor to the laser linewidth. Using the frequency noise spectral density observed in chapter 3, we can determine $f_c \approx 1MHz$. Therefore, we gain nothing by suppressing noise above 1 MHz until we suppress noise below 1 MHz by 30 dB.

The main barrier to this suppression is the 1/f noise at low frequencies. To reduce this noise, we would like our feedback gain to be large at low frequencies. Simultaneously, we would like the unity gain of our lock to be above 1 MHz. To prevent oscillations we must also have a phase lag substantially less than 180°. As in the case of the temperature controller, this condition is equivalent to requiring that the magnitude of the transfer function cross the unity gain point with a slope of less than -20dB/decade. Therefore, we optimize all the components of our loop to have minimum phase lag and maximum bandwidth.

### 5.2 Frequency Discriminator

Ideally, we want the performance of the lock to be limited only by the performance of the laser and the properties of the cavity. This means that we would like laser noise
to be suppressed up to the maximum bandwidth allowed by the current modulation of the laser. We would also like the noise in the lock to be as low as the laser’s shot noise will allow. Therefore, we need to pick the elements of the feedback loop with care.

The bandwidth of the PDH frequency discriminator is determined by the linewidth of the cavity. In our case, the linewidth is 1.9 MHz, well above the modulation frequencies possible with the laser. Furthermore, the -20dB/decade roll off of the PDH response does not add undo phase lag. However, factors like residual amplitude modulation from the EOM, etalon effects in the PDH optics and noise in the RF electronics of the lock can degrade the SNR of the frequency discriminator and broaden the linewidth of the locked laser.

Residual amplitude modulation (RAM) is an undesirable effect in EOMs that causes the EOM to amplitude modulate light rather than frequency modulating it. RAM can be caused by a number of factors, but the dominant source in the lattice laser lock is polarization. Light polarized perpendicular to the normal input polarization is amplitude modulated. If all optical components were ideal, the input light would be entirely of the correct polarization. However, polarization preserving fibers are not perfect and can sometimes rotate the polarization of input light when they are mechanically perturbed. Amplitude modulation on light in the PDH lock gives a flat offset to the error signal. For slowly varying RAM this could be corrected, but varying RAM due to mechanical vibrations is detrimental to the lock. Although I initially had trouble with RAM, I was able to greatly reduce it by placing the EOM for the PDH lock inside the vibrationally isolated splitting system and having as little fiber between it and the PBS as possible.

Etalon effects are variations in laser intensity due to interference between light reflected from the surfaces of optical elements in the PDH lock. Variations in laser intensity normally don’t effect the PDH lock, but the etalon interference acts differently on the two PDH sidebands, causing an offset in the error signal. All optical components are anti-reflection coated to prevent this interference, but I rotated all components with a surface perpendicular to the laser beam by small angles, so that their reflections wouldn’t interfere.
Figure 5-3: The capacitively coupled APD circuit. The 100 nF and 100pF capacitors filter the HV supply. The 100kΩ resistor quenches the APD by dropping the voltage if it draws too much current. The 10 nF capacitor couples the signal out. The Hamamatsu S9073 has a capacitance of 3 pF giving the whole circuit a bandwidth of 900 MHz, assuming a 50Ω output load.
To prevent noise in the RF components of our PDH lock from spoiling our SNR, we must choose components judiciously. To detect the reflected signal from the cavity, we use a capacitively coupled avalanche photodiode. This has the advantage of blocking any DC components of the reflected signal, while retaining a large AC bandwidth. With a 50 Ω impedance at the output the circuit bandwidth is 900 MHz. To verify that the APD is shot-noise limited, we shine varying intensities of laser light on the diode and examine the output with a spectrum analyzer. The resulting noise is flat out to the bandwidth of the diode circuit and follows the formula for shot noise:

\[
S_{\text{shot}} = G \sqrt{2qP_0R}
\]

where \(G\) is the transimpedance of the circuit in volts per amp, \(q\) is the charge of an electron, \(P_0\) is the incident power, and \(R\) is the response of the diode in amps per watt. At 1.5 mW of input power this corresponds to \(5 \times 10^{-10}\) W/Hz. The slope of the PDH frequency discriminator is 500mV/MHz giving an SNR of 93dB. Ideally this would allow the lock to achieve a linewidth 10 mHz. However, noise introduced by feedback electronics and the bandwidth limits of the laser modulation make this impossible to achieve in practice.

We amplify the signal from the APD with Minicircuits ZFL-1000LN+ RF amplifiers, which are specified to have a gain of 20 dB and a noise figure \(^1\) of 2.9 dB. By trial and error, cascading two of these amplifiers gives the best SNR of the error signal.

After the amplifiers, the signal is fed into the RF port of a Minicircuits ZPM-2-S+ mixer. The local oscillator signal for this mixer is produced by an AD9959 direct digital synthesizer. The phase of the LO signal must be adjusted to maximize the slope off the linear part of the error signal. The easiest way to do this is to find the phase that makes the center of the error signal flat and then shift by an additional 90°.

---

\(^1\)The noise figure is how much the component degrades the SNR.
<table>
<thead>
<tr>
<th>Component</th>
<th>Gain</th>
<th>Bandwidth</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLP-2.5+ Low Pass Filter</td>
<td>0dB</td>
<td>flat group delay to 1 MHz</td>
<td>NA</td>
</tr>
<tr>
<td>ZMSCJ-2-2 Splitter w/ π phase shift</td>
<td>-3dB</td>
<td>10 MHz</td>
<td>NA</td>
</tr>
<tr>
<td>ZFSC2-372-S+ Power Splitter</td>
<td>-0.6dB</td>
<td>3.7 GHz</td>
<td>NA</td>
</tr>
<tr>
<td>ZFL-500LN+ Low Noise Amplifier</td>
<td>24 dB</td>
<td>200 MHz</td>
<td>2.9dB</td>
</tr>
<tr>
<td>ZFL-1000LN+ Low Noise Amplifier</td>
<td>20 dB</td>
<td>1000 MHz</td>
<td>2.9dB</td>
</tr>
<tr>
<td>S9073 Si APD</td>
<td>50</td>
<td>900 MHz</td>
<td>0.28</td>
</tr>
<tr>
<td>ZFM-2-S+ Mixer</td>
<td>-5.7dB</td>
<td>1-1000 MHz LO/RF</td>
<td>NA</td>
</tr>
</tbody>
</table>

Table 5.1: The signal to noise ratios, bandwidth, and gain of the RF components involved in the laser PDH lock.

### 5.3 Lockbox Design and Optimization

Once we have the highest SNR frequency discriminator, we can think about the best way to use this to correct the laser’s frequency. There are three mechanisms by which we can adjust the laser’s frequency: the optical feedback path, slow current feedback, and RF current feedback. The piezo actuated mirror at the end of the optical feedback path can be used to change the length of the path changing the lasers frequency. Within the modes defined by the optical feedback, the lasing frequency can be adjusted with current. We distinguish between slow and fast current feedback because the mechanisms are different. Slow current changes change the temperature of the laser, influencing the lasing frequency. More current causes the lasing frequency to be smaller. At high frequencies current modulation changes the number of charge carriers in the gain medium, changing the index of refraction. More current leads the lasing frequency to move higher. The effect of high frequency current feedback is actually the opposite of the other two avenues of feedback.

Due to the different effects of and frequency responses of different mechanisms of laser feedback, we feed the error to each one with a different gain and with different frequency compensation. First we split off some of the PDH signal the fast current feedback. Since these frequencies are beyond the bandwidth of most active components, we use only passive elements to filter this signal and then feed it directly to the laser diode through a coupling capacitor.
The rest of the error signal is sent to a circuit called a lockbox, which uses it to adjust the DC current to the laser and the voltage of the feedback piezo. Ideally, we would like the action of the slow feedback to have a high gain at low frequencies to counteract the laser noise. However, we would also like the slope of the loop transfer function to be -20 dB/decade at the unity gain point, to prevent the phase from passing $180^\circ$. Since the response of the PDH lock has a $1/f$ dependence at frequencies above the cavity linewidth, a $1/f$ loop gain at all frequencies would produce an overall dependence of $1/f^2$ at high frequencies and cause instability. Therefore, we implement a proportional-integral controller to control the laser DC current and piezo. The response of the PI controller is $1/f$ at low frequencies but it bottoms out to a uniform proportional gain at a frequency lower than the unity gain frequency. This way there can be extremely high gain at low frequencies that falls quickly to the unity gain point at -40dB/decade, turning into a -20 db/decade slope just before the unity gain point.

The lockbox receives the PDH error signal through the photodiode port, low pass filters it, and amplifies it by an adjustable factor. It is then buffered and has an adjustable error offset added to it. The filtered, scaled, and offset signal is then sent to the proportional and integral gain controller as well as being buffered and sent out the error out.

In between the PI controller and the error signal input, a switch allows a dithering signal to be swapped for the error signal. This locking switch allows the frequency of the laser to be swept back and forth to find the PDH lock point and adjust the laser. Once the lockbox has the appropriate output and error offset, the box can be switched into lock mode to frequency lock the laser to the cavity.

The PI controller has two potentiometers that control the total gain and the proportional gain, similar to the temperature controller. The output signal also has an adjustable additive offset for fine tuning the laser frequency prior to locking.

Laser frequency noise comes from several different sources. Temperature variations, RF noise, and vibration of the optical feedback path can all cause frequency drift. It is difficult to distinguish from just the PDH error signal whether frequency fluctuations are due to temperature fluctuations and other processes intrinsic to the laser or
Figure 5-4: The final loop gain of the PDH lock, with contributions from individual components [38]. The integrating gain of the lockbox and the frequency response of the feedback piezo lead to a -40dB/decade slope at low frequencies. At around 1 kHz, the slow current feedback becomes the more powerful feedback path, with a -20dB/decade slope caused by the lockbox integrator. At the cavity linewidth, the response of the PDH lock begins to decrease at -20dB/decade. The lockbox proportional gain is tuned to overtake the integral gain around this point, maintaining an even -20dB/decade slope. As the heating effect of current modulation decreases, the ferrite bead in the laser mount also takes effect, dropping the gain of slow current feedback below the fast RF feedback path. The RF feedback path follows the current modulation gain of the laser all the way down to the unity gain point.
whether they are caused by movement in the feedback path. One way to deal with this ambiguity is to split fluctuations in the error signal by frequency. Fast deviations (>10 Hz) are assumed to come from the laser itself and are corrected by changing the laser current. Slow fluctuations (<10 Hz) are attributed to changes in the optical feedback path and are corrected by moving the feedback mirror.

To separate the fast current feedback and the DC current feedback, we build two different paths into the laser mount. The DC path travels through an inductor before being fed into the diode, low-pass filtering the input. The RF path travels through a capacitor before being fed into the diode, high-pass filtering the input and blocking DC. To further protect the diode from DC in the RF path, a DC-block made by Minicircuits is placed in the RF path, external to the laser mount.

The low frequency part of the signal passes through the lockbox, where it is filtered and modulated, before being fed to the current controller. The current controller adds the feedback signal to the laser current and passes the piezo voltage to a rescaler, which amplifies and adds a bias to bring the piezo voltage to an operating point on its nonlinear response curve. The piezo signal is then amplified roughly 100x by a high
Figure 5-6: The proportional-integral gain module of the lockbox.
Figure 5-7: The transfer function from the input of the lockbox to the error out and output ports.
Figure 5-8: The transfer function from the DC current port of the laser diode to the laser diode voltage and the transfer function from the RF current port of the laser mount to the laser diode voltage. A fit for the DC port is plotted with a pole at 250 kHz and a zero at 400MHz. The zero is likely due to parasitic capacitance to ground through the laser case. A fit for the RF port is plotted with a zero at 50 kHz, a pole at 2.5 MHz, and a zero at 400 MHz. The high-frequency pole and zero are likely due to the capacitance of the laser diode and parasitic capacitance of the case, respectively.
voltage amplifier. Because most low frequency drift in the laser is due to mechanical drift of the feedback path, it might be detrimental to feedback to both the current and piezo at extremely low frequencies. To correct this, I modified the current controller to be AC-coupled at frequencies 3 Hz and below.

5.4 Piezo Feedback

The piezo feedback path is passes through the lockbox, and then to the current controller, where the modulation signal is low pass filtered. The reason for this is that the piezo has mechanical resonances in the low kHz which cause massive phase lag. This would cause instability in the lock if the gain of this feedback path was not artificially suppressed at high frequencies. By mounting a smaller mirror on the feedback piezo, I was able to push the resonant frequency of the piezo to higher frequencies. This allowed me to relax the low-pass filter cutoff in the current controller without causing oscillations.

The HV amplifier buffers the piezo signal with an instrumentation amplifier and send it to an output stage.

The output stage uses a current value from a 2kΩ sense resistor to feedback the
Figure 5-10: The input buffer of the HV amplifier that drive the piezo

Figure 5-11: The feedback loop for the output of the HV amplifier
current to a TIP31C transistor that modulates the current pulled from the 250 V supply through a 50\(k\Omega\) resistor. The output port outputs the voltage across this resistor. The transfer function of the whole piezo feedback path is shown in figure 5-12.

5.5 Initial Locking

In order to lock the laser, we first need the laser’s frequency to be near a cavity resonance. For this purpose, the laser has two modes of operation, dither and lock. In ‘dither’ mode, a sawtooth signal is fed to the piezo and current which sweep the laser back and forth across a controlled range of frequencies. Looking at the error signal over this range of frequencies, we can choose set-points that will bring the laser close to the desired locking frequency and switches to ‘lock’ mode.

In lock mode, the laser moves to the set-points chosen. If the error signal deviates from the set-point, the electronics automatically adjust the laser current until the error signal returns to the set-point. This allows the electronics to automatically keep the laser close to some desired frequency. Once the laser is locked with slow current and feedback piezo feedback, we can optimize the fast RF current feedback.
5.6 RF Feedback

For the fast current feedback, we avoid interposing any active elements between the error signal and the laser diode - most op-amps would add too much phase delay to the RF signal. The RF current feedback is taken directly from the output of the PDH lock, fed through a $\pi$ phase shifter, a lead-lag filter to prevent oscillations, attenuators, and then fed into the AC-coupled port of the laser. The pi phase shifter inverts the error signal to account for the fact that it has an opposite effect on the laser frequency from the other feedback mechanisms.

The lead-lag filter is a network of resistors and capacitors that has a zero and a pole in its frequency response. Its purpose is to boost the phase at frequencies near the unity gain point, increasing the phase margin. It does this at the expense of faster increase in phase lag at higher frequencies, but since the gain is much less than one at these frequencies, this does not effect the system too much. The overall gain of this part of the feedback loop is controlled by the RF attenuators in the feedback path. I empirically adjusted the amount of attenuation by examining the amount of noise in the lock at each level.

5.7 Lock Performance

The performance of the lock can be determined by looking at light transmitted through the cavity. The ratio between total intensity and intensity noise of the transmitted light is the same for the transmitted light as it is inside the cavity. Therefore, I placed a DC-coupled APD at the cavity output and measure both the total light intensity and the spectrum of the transmitted intensity. A sample of such spectrum is shown in figure 5-14

A complementary measure of the deviation of the laser frequency from the cavity resonance at different frequencies is the spectral noise density of the PDH error signal itself. By multiplying this noise by the PDH discriminator transfer function, we can establish another measure of the performance of the lock at different frequencies.
Figure 5-13: The lead-lag filter Bode plot. The filter gives the loop phase lead at frequencies near the unity gain point, preventing oscillations, at the cost of phase lag at higher, irrelevant frequencies. Green circles are collected data. Blue line is a fit of a lead-lag filter with a zero at 350 kHz and a pole at 3.1 MHz.

Finally, to measure the performance of the double lock system, we can tune the frequency offset between the reference and science cavities so that the science cavity is resonant with the 556 nm laser as well as the 759 nm laser. Since the 556 nm laser has a linewidth of 10 kHz and is locked to a reference cavity with 1Hz/s absolute stability, the transmission noise of the 556 nm laser through the science cavity can be used as a measure of how absolutely stable the double lock is.
Figure 5-14: The spectral noise density of the relative intensity noise of the 759 nm light transmitted through the science cavity while the laser is locked.
Figure 5-15: The spectral noise density of the PDH error signal while the laser is locked to the cavity, multiplied by the gain of the PDH frequency discriminator.
Figure 5-16: The spectral noise density of the relative intensity noise of the 556 nm transmitted through the doubly-locked science cavity.
Chapter 6

Summary and Outlook

In my thesis, I designed, implemented and characterized an optical lattice trap for a ytterbium gain medium for a superradiant laser. The three main design requirements were that the lattice allow the atoms to be trapped in the Lamb-Dicke regime, remain in the motional ground state of the trap, and have inhomogeneous broadening due to the AC Stark shift below the homogenous atomic linewidth.

To trap atoms in the motional ground state, we loaded the atoms into the lattice from a magneto-optical-trap at 30°C. To enable sideband cooling, the lattice potential had to be at least 1000 E_r deep, necessitating 3W of circulating optical power. Furthermore, the lattice had to be capable of maintaining the atoms in the motional ground state for up to 1 second without any cooling lasers, so that the superradiance experiment could be performed without excessive AC Stark shift. This put limitations on the maximum acceptable heating rate of the lattice and therefore limited the frequency noise present in the trap laser to $10^{-10}\sqrt{Hz^{-1}}$ and below in the 400-500 kHz range.

By using a diode laser of the distributed Bragg reflector configuration and further stabilizing it with optical, electrical, and thermal feedback, I created a laser with $3 \times 10^{-10}\sqrt{Hz^{-1}}$ frequency stability from 300-500 kHz, capable of supplying 3 mW to the science cavity. Since the science cavity has a finesse of 3000 at 759 nm, this corresponds to 3 W. This corresponds to a lattice depth of 1000 recoil energies. Cooling light on the 556 nm transition can cool the atoms to a mean motional mode of $n =$
1.2. The heating constant from frequency noise is $900 \text{ ms}^{-1}$. This means that in a one second long experimental sequence as much as 20 percent of the atoms could be in the second motional mode of the lattice. Overall, this could lead to Doppler broadening of $12 \text{ mHz}$.

To satisfy the requirement of minimizing inhomogeneous broadening due to the AC Stark shift, I took advantage of an ultrastable reference cavity as an absolute frequency reference. Locking the laser frequency to the reference cavity, I was able to achieve an absolute frequency stability of $600 \text{ kHz}$. Given that the trapping depth seen by the atoms varied from 1200-700 recoil energies, this variation could cause inhomogeneous broadening of up to $500 \text{ mHz}$.

The combined inhomogeneous broadening due to variable Stark shift and Doppler broadening is then $1 \text{ Hz}$. This is below the threshold of $4 \text{ Hz}$ for superradiant pulses on the clock transition. Therefore, my optical lattice trap meets the requirements for trapping ytterbium atoms as a gain medium for a superradiant laser.
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