Abstract

In this thesis, I evaluated the theoretical and practical performance of a Geiger mode avalanche photodiode (GM-APD, or Geiger mode APD) array for use in optical communications systems. I designed and implemented a test pattern generator, capable of producing waveforms with OOK and PPM modulations at a variety of symbol rates and duty cycles. Using this pattern generator, I tested the receiver properties over a wide range of data rates and illumination levels, and was able to demonstrate the capability for error-free communications in both modulation formats, up to 78.8 Mbps at a sensitivity of 1.8 photons per bit for OOK, and 34.9 Mbps at 2 photons per bit for PPM.

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Chapter 1

Introduction

1.1 Underwater Communication

Underwater communication is important for many applications across a variety of fields, from military to scientific to commercial. Reliable and high-rate links are needed for applications such as interfacing with underwater sensor networks, communication between autonomous vehicles, and networking underwater devices. While underwater fibers have been enormously successful at networking around the globe, many applications cannot use a physical connection between devices, giving a need for communication systems which can work directly over the underwater channel.

Acoustic underwater communication systems have a long history, arising from research on sonar early in the 20th century. Because acoustic waves transmit very well under water, such systems are able to operate at extremely long ranges, on the order of dozens of miles. However, acoustic systems are naturally limited in available bandwidth due to the relatively small range of audio spectrum available. Additionally, acoustic systems can experience extreme Doppler shifts, channel fading, multipath propagation, and time variance, all of which make maintaining communication difficult (particularly in shallow water) \[1\].
1.2 Underwater Optical Channel

Optical communication systems can offer extremely high data rate links (in excess of a Gbit/s) by operating at much higher carrier frequencies, and by using bandwidth in a regime where it is much more plentifully available. Such systems have been used extensively in free-space applications, one of the shining examples being the recent Lunar Laser Communication Demonstration (LLCD) performed by NASA and Lincoln Laboratory in 2013 [2]. However, the underwater channel offers a particular challenge for optical systems, as electromagnetic radiation is absorbed much more readily than in air. Frequencies in the radio band, which can work over enormously long ranges for space or telecommunications applications, are absorbed over the order of centimeters in water, making them wholly unsuitable in that domain. Figure 1-1 shows the absorption spectrum of water, as compiled from several studies; most wavelengths of light have similarly poor extinction lengths, with a few peaks in the IR wavelengths corresponding to specific molecular absorption points. However, there is a window in the visible wavelengths, centered around blue-green wavelengths near 500 nm. Although water absorption still severely limits transmission of signals, the presence of such a window with considerable bandwidth offers the possibility of extremely high data rate communications, leveraging approaches developed for free-space optical communication.

A variety of underwater optical systems have been explored, primarily over the past 10 to 15 years. Hanson and Radic in 2008 demonstrated the capability for 1 Gbit/s communication over a 2-meter long pipe in the laboratory, with simulation results suggesting the possibility of similarly high data rates over distances of tens of meters [4]. Current commercial systems boast data rates of up to 250 Mbps at a range of 200 meters [5].
1.3 APDs and APD Arrays

Avalanche photodiodes, or APDs, are a class of extremely sensitive semiconductor photodetectors [6]. When a relatively small negative bias is applied, APDs operate in linear mode, in which the electrical response is proportional to the incoming optical power. Linear APDs are widely used in optical communications systems, including for underwater applications. Applying a larger negative bias to the APD puts it into Geiger mode, in which the response becomes non-linear. Incident photons while the APD is in this state trigger a chain reaction, resulting in a large sudden spike in voltage which can be read out as a digital pulse. This pulse can be timed to high precision, making it suitable for use in a photon counting system. Another benefit of Geiger-mode APDs is that they do not require cryogenic cooling, as is required for other high-sensitivity systems like superconducting nanowire detectors [7].

The main drawback of a Geiger-mode APD system is its potentially high dark count rate. Because the APDs are biased to the point of complete breakdown from a single photon, they are also susceptible to spurious fires from numerous sources, including crosstalk from other detectors in the same array. Additionally, after a
successful detection, the detector must be quenched and reset through a specific pattern of voltage biases, which leaves the APD out of operation for a short time period after recording a fire, ranging from 10’s of nanoseconds to microseconds. This limits the number of counts a single APD can record, and thus its sensitivity; a linear APD is limited instead by the speed at which its output voltage can be read, and high-speed ADCs are readily available.

Geiger-mode APDs can be combined into arrays with large numbers of detectors, allowing for scaling of dynamic range with relatively little overhead on space and power. These help to offset some, though not necessarily all, of the general issues with Geiger-mode APDs. While an array of Geiger-mode APDs does have an overall higher rate of dark counts than a single detector, this is more than compensated for by the extra detectors. A sufficiently large APD array could potentially provide a high-sensitivity detection system with extremely fine-grained timing resolution, which could work across a variety of modulation formats. While the use of Geiger-mode APD arrays has been successful in applications such as 3D imaging \[8\], the use of silicon Geiger-mode APD arrays for undersea applications has not been well-explored.

1.4 Thesis Structure

My thesis work at MIT Lincoln Laboratory was focused on exploring the potential application of a Geiger-mode APD array for communications, by designing and performing small-scale experiments with a custom Geiger-mode APD receiver array developed at Lincoln Laboratory. Chapter 2 discusses the array’s operation, and the limitations on the array’s noise floor and count rate ceiling. Chapter 3 discusses the setup of the communications experiments performed, the test transmitter developed for carrying out these experiments, and both a theoretical and experimental analysis of the communications results. Chapter 4 examines directions for possible future work based on this Geiger-mode APD array technology, and summarizes the main results.
Chapter 2

Receiver Hardware

The experiments for this thesis were performed using a Geiger-mode APD array developed by Group 87 at MIT Lincoln Laboratory, consisting of 1024 individual photodiodes arranged in a 32x32 grid. This detector array is backed by a custom readout integrated circuit, or ROIC, which takes care of resetting each of the detectors after they are triggered, determining exactly when each detector fired, and all of the other maintenance tasks required to keep each of the detectors running.

The ROIC contains an on-board memory which can buffer the time-of-fire information from the APD array, allowing readout to a PC for external processing. The address of each of these fires within the memory uniquely determines the position of the detector and the frame number in which it fired, which can be used during readout to build up a complete time- and position-stamped map of all pixel fires.

2.1 ROIC Clocking

The major unit of time for the ROIC is the frame, which consists of 1024 individual time bins. The length of each of these bins is determined by the ROIC’s clock, which can be set either to an internally generated 311 MHz clock or an external source. For all of my experiments, the frequency was kept to 311 MHz to match this internal frequency, as the array had received significant prior testing at this frequency; however, an external clock generator was used instead of the internal clock
as discussed in Section 3.1. Each element of the array can signal exactly one count per frame.

Elements of the array are reset individually, but all resets occur on a quarter-frame clock, running 4 times as fast as the overall frame clock. A minimum reset time can also be specified (as a number of quarter-frames), to ensure that a single element is not triggered, reset, and re-triggered within a single frame. The setting of the reset time is discussed below, in section 2.2.2.

The ROIC firmware also allows the setting of a switch for using the Vernier bit in timing of photon arrivals. While this mode is enabled, the array’s timing resolution is effectively doubled, with arrival events clocked on a 622 MHz clock instead of the 311 MHz base clock frequency, and the frame clock which drives the resetting of triggered elements is also doubled. This lowers the minimum reset time for a single element from 1024 311 MHz clock cycles to 1024 622 MHz clock cycles, or 1.65 μs. The Vernier-enabled mode was used for all of my tests.

2.2 Receiver Limitations

The nature of the detector array imposes limitations for both very weak signals (in the form of a noise floor) and very strong signals (from blocking loss), both of which have an impact on the maximum performance which can be expected from the array as a receiver. As discussed in 1.3, because Geiger-mode APDs are biased in the breakdown region, they frequently register dark counts, spurious fires triggered by noise within the system. The noise floor is raised significantly by particular detectors within the array which consistently register dark counts, far above the rate of the rest of the array; this can be largely mitigated by identifying those detectors and disabling them, as described in section 2.2.1. The signal ceiling is limited primarily by the number of available detectors, and how quickly they are able to reset to register new detections while having all triggering events recorded. While this ceiling is fixed for a particular size array, it can be quantified and used to determine the range of optical powers the array can reasonably handle for doing communications. This ceiling is examined in
Because the array used for testing was not a mass-production system, but rather a research-grade prototype, some irregularities were identified which needed to be corrected when using it as a communications receiver. In particular, due to minute differences in the individual detectors, the entire array was not able to be stably operated in Geiger mode at once. An operating voltage of -30.6 V was found to provide the best performance for testing; however, at this voltage, many pixels in the array would be stuck in a state of continuous fire, triggering another fire count immediately after being reset by the ROIC firmware. These always-on pixels, which at times were up to 20% of the array, made an enormous impact on the sensitivity of the array, and under most light conditions tested completely overwhelmed the non-noise portion of the incoming signal. Additionally, many of these always-on detectors caused spillover to neighboring detectors, often triggering a fire on them as well through cross-talk. This level of dark-count noise made communications completely impossible, and the problematic detectors needed to be disabled through the ROIC firmware. These issues were dealt with by generating a mask of the most problematic detectors through post-processing a no-light data collection, and then disabling those detectors through the ROIC firmware. Figure 2-1 shows a typical distribution of the counts from one of these dark count collections. A threshold was selected shortly above the “knee” of the distribution, where the number of dark counts began to rise rapidly; in the example distribution shown, this would be at around 5000 counts, out of 80,000 total chances to fire over the capture.

For later tests, a standardized mask was adopted across data collections, to keep the array properties constant while changing data rates and illumination. While initially a new mask was generated for each data set, each of these masks showed little variation from day to day, and the standard was adopted for simplicity and consistency. Rather than the one-level generation described above, this standardized mask was generated in two stages. First, a mask was generated using a higher threshold,
Figure 2-1: Cumulative distribution of dark counts for a typical unmasked collection. The spatial distribution of always-on pixels is random, as can be seen in the top grids.
which disabled only the most problematic detectors (those which were very close to 100% fire rate). Then, an additional dark count collection was taken using that mask, and a new threshold selected. This two-stage mask generation helped identify detectors which were only problematic as a result of cross-talk and allowed them to be kept on, increasing the array sensitivity. Figure 2-2 demonstrates the results of this masking; the highest rate of dark counts after masking is about 5% of highest rate pre-masking.

A dark count collection with the array masked was also used to determine the background level of dark counts; this is of importance in determining the best performance of the non-ideal array for communications as discussed in section 3.2. This background level of dark counts was found to remain very consistent across multiple days of data collection, despite not being able to completely isolate the detector from the ambient lighting conditions in the lab. Consequently, several different dark count rates were averaged together and used as a canonical level of dark-count background.

### 2.2.2 Blocking Loss

The reset time of the ROIC firmware imposes an upper bound on the number of photons the array is able to detect in a given amount of time; if additional photons are incident on a particular APD before it is re-armed, it will simply fail to fire, and the count will not be registered. For low light levels, this is not problematic, as photons are distributed around the array, and the probability of consecutive photons hitting exactly the same detector element is quite low. This is especially true as the beam profile becomes more flatly distributed around the array as would be expected from a transmission beam starting a long way away from the detector and going through diffraction. However, as more photons are incident on the detector, there becomes a higher probability of a detection event being missed because a detector is not reset in time to be re-triggered. This can limit the array’s sensitivity, especially if particular detectors have more incident photons than the array-wide time average, due to either spatial or temporal clustering.

The maximum counting capacity of the array can be found by simply multiplying

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Figure 2-2: The top left image shows a typical dark count capture, before masking of always-on detectors. The top right image is the standard mask used for communications experiments, with 167 disabled detectors. The bottom image is another dark count capture, with the disable mask applied; the highest dark count rate is about 5% of the rates seen in the unmasked capture.
the number of active pixels by the maximum rate at which they can be reset and fire again. With the Vernier bit mode enabled, the minimum reset time of one full ROIC frame is 1024 bins divided by a 622 MHz clock signal, or 1.65 \( \mu s \). With the standardized pixel mask as described in the previous section disabling 167 of the 1024 detectors in the array, this gives an absolute maximum rate of \( 520 \cdot 10^6 \) counts per second. In practice, blocking loss can be observed significantly below this maximum level, as the spatial distribution of the beam is not uniform around the array, and incoming photons are not able to perfectly redistribute to use all available detectors. Figure 2-3 below shows the rate of counts found under constant illumination at various power levels, with the ROIC firmware’s minimum reset time set to several different values.

As indicated in figure 2-3, the count rates under various illuminations show three broad regions. At low powers, the total count rate is completely dominated by dark counts, which occur at a rate of roughly 4.5 Mcounts/s over the entire array. At illuminations from about -70 to -55 dBm, the number of detected photons grows roughly linearly with input power. Past that point, the number of counts the array can detect saturates, approaching the count rate at which all enabled detectors are firing continuously. The longest reset time tested, equivalent to 5 ROIC quarter-frames, left additional dead time after an APD fire and began exhibiting signs of blocking loss at lower power levels, which can be seen in the black curve’s downward tail before -55 dBm. Although the shorter reset times performed about equivalently over the linear portion of the input-to-detected graph, they offer the possibility of counts later in a frame overwriting those coming earlier in the frame. Similarly to the effects of blocking loss, these overwrites could cause burst errors in the communications signal, degrading the signal quality in the first half of the frame. These shorter reset times also did not offer any advantage in the maximum number of counts able to be detected, due to the limitation on the speed at which the ROIC can report counts; reset times of 2, 3, and 4 quarter frames all gave a maximum count rate of approximately \( 500 \cdot 10^6 \) counts per second, compared to \( 400 \cdot 10^6 \) for the longest reset time. For these reasons, the reset time for all experiments was fixed to be equal to one ROIC frame, 1.65 \( \mu s \).
Figure 2-3: Detected photons vs. input power for several different ROIC minimum reset times.
Chapter 3

Communications Experiments

These experiments were primarily designed with the goal of proof-of-concept use of the Geiger mode array as a communications receiver under laboratory conditions. Consequently, the experimental setup was in many ways simplified from how it would appear in a larger-scale commercial system. The primary clock input for the ROIC board was sourced from the same board providing the transmission test pattern, as discussed below in section 3.1, rather than performing clock recovery at the receiver board. This avoided the need for a dedicated clock feedback system, and ensured that the ROIC bins remained at the same clock speed as the transmitted pattern, although perfect alignment of the receiver time bins to the input pattern was not guaranteed due to the delay in the optical system.

Illumination of the detector array was provided through a fiber coupling, mounted a few centimeters above the array. The beam pattern was Gaussian in shape, with maximum illumination at the center of the array and pixels at the edge receiving approximately half as many photon counts as at the center. This pattern of illumination does mean that the center of the array begins to experience blocking loss before the edge, although under most input powers tested blocking loss did not significantly come into play. However, a non-uniform beam pattern is crucial for being able to distinguish the center of the incident beam for performing tracking, and the beam pattern was kept the same for the communications experiments as well, to provide consistency and to evaluate the possibility for future experiments to provide both
Two different modulation formats were used for these experiments: on-off keying (OOK) and pulse-position modulation (PPM). In OOK, bits are directly modulated as 1 or 0 by the presence or absence of transmitted signal. In PPM, signal is transmitted in exactly one of $m$ slots which make up a symbol; the choice of slot encodes $\log_2 m$ bits of information. Here PPM-$m$ is used to refer to a PPM modulation scheme in which each symbol has $m$ slots.

When discussing the rate at which the system operates, there are several important terms to define. The symbol rate refers to the rate at which OOK or PPM symbols are sent from the transmitter, given in baud. For OOK, this was always equal to either 1/2 or 1/4 the clock rate of the ROIC collection bins, which comes out to either 311 MBd or 155.5 MBd. For PPM, each slot was set to either 2 or 4 ROIC bins, the same as a single OOK symbol; however, PPM-16 requires 16 of these slots per symbol, for
a symbol rate of either 19.4 MBd or 9.7 MBd. Block repeats are discussed in detail below, and refer to layering repetitions of the input pattern over itself to aggregate more counts in demodulating a single symbol. The channel rate refers to the rate at which uncoded bits are sent over the channel, and is equal to the symbol rate, divided by the number of block repetitions, and then multiplied by the number of bits encoded by a single symbol. For OOK modulation, this multiplicative factor is 1, while for PPM-16 this factor is \( \log_2 16 \). Finally, the user rate refers to the error-free data rate which could expect to be achieved by putting the demodulated bit stream through a forward error correction (FEC) code, given in MBit/s.

3.1 Test Transmitter

To generate waveforms for communications testing of the array, I developed a pseudo-random bit sequence (PRBS) transmitter, using the GTX high-speed transceivers on a Xilinx Virtex 5 FPGA development board. I used a pre-existing 32-bit parallel PRBS IP core, the output of which is then passed through a PPM encoder and then to the input of the GTX transmitter block. The GTX transmitter output was passed through RocketIO electrical outputs to an optical modulator, which gates light from a 780 nm fiber-coupled laser. Figure 3-2 shows the overall architecture of the system, up until the GTX transmitter. Switching between operating modes of the transmitter is controlled through DIP switches on the development board.

The operating frequency of the transmitter is controlled using an on-board MAX3674 clock generation chip, with its frequency set through more DIP switches. A standard operating frequency of 311 MHz was selected to match the ROIC’s onboard generated clock frequency. The 311 MHz reference clock from the transmitter board was also piped to the receiver board, for easier clock synchronization during initial testing. For a field test, the full experimental setup would also include clock recovery and phase alignment circuitry. GTX transceivers on the Virtex 5 FPGA platform operate at an internal frequency of 10 times the reference clock, but the inputs are duplicated such that the output bit clock operates at 2 times the reference frequency, or 622 MHz in
Figure 3-2: Block diagram of test transmitter.
typical operation.

The transmitter can also be configured for either NRZ or RZ transmission, with adjustable duty cycles of 80, 60, or 40 percent on for 1-bits. Multiple PRBS generators operating at different frequencies are included to allow seamless switching between OOK and PPM-16. For both modulation settings (PPM and OOK), the input unmodulated bit sequence is the same. Because the length of the test pattern is odd, the same number of OOK and PPM-16 symbols are needed for encoding the full pattern, although the PPM sequence requires 16 times as many ROIC bins to repeat because of the size of the symbols.

For lower data rates, such as to accommodate aggregating multiple bins at the receiver end to give better symbol differentiation, the transmitter can be reconfigured to divide down the output rate by a factor of 2 or 4. This is done by further dividing down the clocks used to drive the PRBS generator and PPM encoder, and duplicating each bit more times on the GTX block input. While other clock divisions would be possible to do with the same hardware, they would require large reconfiguration of the transmitter design, in particular interfacing with the GTX’s 20-bit internal data bus, and thus they were not included in this design. The table in figure 3-3 shows the uncoded data rates of the transmitter in its different modes of operation, with the reference clock from the transmission board’s clock generator set to 311 MHz. Figure 3-2 shows a simplified block diagram of the PRBS transmitter system. Figure 3-4 shows the optical output pattern from OOK modulation at several duty cycles and symbol rates, while figure 3-5 shows the optical output pattern from PPM modulation with a 60% duty cycle at the three available base symbol rates. In both of the figures with traces, the modulator output shown is inverted; these traces were taken from the modulator’s electrical drive signal, where the higher voltage corresponds to the modulator being set to block output.

The 80% duty cycle traces in figure 3-4 demonstrate why that duty cycle was ultimately abandoned for testing, and why the 60% duty cycle was kept as the standard for the communications experiments. Although the most prominent peak in the waveform occurs within a single OOK symbol duration, the modulator is not able
<table>
<thead>
<tr>
<th>Modulation</th>
<th>1 slot/clock</th>
<th>2 slots/clock</th>
<th>4 slots/clock</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOK</td>
<td>622 MBd</td>
<td>311 MBd</td>
<td>155.5 MBd</td>
</tr>
<tr>
<td>PPM-16</td>
<td>38.9 MBd</td>
<td>19.4 MBd</td>
<td>9.7 MBd</td>
</tr>
</tbody>
</table>

Figure 3-3: Symbol rates used on PRBS generator used for communications test patterns.

To cleanly return to zero within a single symbol duration, and consequently the extinction ratio between adjacent “on” and “off” symbols is extremely poor. While this issue still exists to a small degree in the 60% duty cycle case, it is far less pronounced, and at a symbol rate of 155.5 MBd there is clear return to the baseline level between adjacent symbols. The top row of traces, taken with a symbol rate of 622 MBd, show the same issue; even at a duty cycle of 40% (not pictured), the modulator was not able to respond quickly enough to return to the baseline level between adjacent “on” symbols. Preliminary tests with a commercial BER tester were unable to lock onto the signal at any light level, suggesting high levels of inter-symbol interference.

The PRBS generator was designed to be easily reconfigurable for a longer test pattern, by swapping out the $2^7 - 1$ pattern for an IP core with a longer repeat time, several of which existed in-house. However, $2^7 - 1$ was deemed to be sufficient for communications benchmarking, and using a relatively short pattern was advantageous for the modifications being made in post-processing on the benchmark data. In particular, this allowed post-processing results with block repeats added, while having enough symbols in the collections (limited by the ROIC’s on-board memory) to draw meaningful results. Block repeating, or aggregating received counts over multiple repetitions of the input pattern, allows for the simulation of a larger and more sensitive array, or equivalently a longer symbol duration.

### 3.2 Theoretical Communications Analysis

For a photon-counting detector such as a Geiger-mode APD, the distribution of photons typically follows a Poisson distribution. This distribution is defined completely by a mean number of counts $\lambda$; given this mean, the probability of detecting exactly
Figure 3-4: Electrical drive from modulator for OOK modulation, at various symbol rates and duty cycles. The left column is an 80% duty cycle, the right column is 60%. From top to bottom, the rows are at symbol rates of 622 MBd, 311 MBd, and 155.5 MBd.
Figure 3-5: Electrical drive from modulator for PPM modulation, with a 60% duty cycle at 39.4 MBd, 19.7 MBd, and 9.8 MBd.
The number of photons during a particular symbol duration is given by \( \frac{\lambda^k e^{-\lambda}}{k!} \). For modulation formats such as PPM and OOK, in which each slot is either in an “on” or “off” position, these two possibilities will have separate associated means, and thus different distributions of expected counts. For the discussions below, \( f(k; \lambda) \) denotes the probability of detecting exactly \( k \) photons in a particular slot from a Poisson distribution with mean \( \lambda \).

### 3.2.1 OOK

To demodulate an OOK signal, a static threshold is set, against which the number of photons detected in a symbol duration is compared. Any number of photons greater than the threshold is demodulated as a 1, and any number less than the threshold is demodulated as a 0. The mean number of photons in a dark count symbol is denoted here by \( \lambda_D \), while the number of photons in a “signal” symbols is denoted by \( \lambda_S \). Because spurious counts also occur during “signal” symbols, \( \lambda_S \) will always be greater than \( \lambda_D \). To optimize the bit error rate, the threshold should be set to the point at which the two Poisson distributions with these means intersect. The lowest integer number of counts which should be demodulated as a 1 is the first \( n \) for which \( f(n; \lambda_S) > f(n; \lambda_D) \) and \( f(n - 1; \lambda_S) < f(n - 1; \lambda_D) \).

When performing OOK demodulation with no background noise, the optimal strategy will always be to decode anything with 0 counts as 0, and anything with positive counts as a 1. However, when background is factored in, the optimal decoding threshold will change as a function of the input power. This can be seen in the theory vs. experiment curves in Section 3.3, as the theoretical BER curve is not a single smooth curve, but rather several distinct curves spliced together, with a discontinuous derivative at the joining point. These discontinuities mark where the optimal threshold value changes, as the input power becomes sufficiently high that the point at which the 0-bit and 1-bit Poisson curves intersect moves to the right past an integer.
3.2.2 PPM

While there are multiple possible PPM demodulation schemes, for the theoretical analysis here, it is assumed that each individual symbol is separately demodulated, and that the result of the demodulation for each symbol is the bit sequence corresponding to the PPM slot with the highest number of counts. In the event of a tie for the highest number of counts, one of the tied slots is selected randomly with uniform probability. For a successful symbol demodulation, then, the “signal” slot must either contain the unique highest number of photons of all slots within that symbol (unambiguous case), or be the joint-highest with one or more other bins and then be selected (ambiguous case). The decision process described below is similar to the theoretical calculations presented in [9], although the procedure is slightly different with the APDs operating in Geiger rather than linear mode.

In the equations below, $s$ denotes the number of photons detected in the “signal” slot. Similarly, $d$ denotes the number of photons detected in an arbitrary non-signal slot. The probability of a particular symbol having an unambiguously correct demodulation is given by:

$$\sum_{s=1}^{\infty} f(s; \lambda_S) \cdot \left( \sum_{d=0}^{s-1} f(d; \lambda_D) \right)^{m-1}$$

(3.1)

The right-hand term is simply the constraint that the non-signal slots must have fewer than $s$ counts, the number of counts in the signal slot.

For the ambiguous case, the probability of correct decoding can be calculated similarly, although the equation becomes more complex. This case requires at least one (and up to $m-1$) of the non-signal slots must have the same number of detected photons as the signal slot. All non-signal slots not included in this tie must have fewer than $s$ photons detected. Applying these constraints gives the following:

$$\sum_{s=0}^{\infty} f(s; \lambda_S) \sum_{j=1}^{m-1} \frac{1}{j+1} \binom{m-1}{j} \cdot (f(s; \lambda_d)^j \cdot \left( \sum_{d=0}^{s-1} f(d; \lambda_D) \right)^{m-1-j})$$

(3.2)

Here, $j$ denotes the number of non-signal slots which have the same number of received photons as the signal slot. The sum of these two terms gives the total
probability of a correct demodulation, so the probability of a symbol error (that is, an incorrect demodulation) can be found by subtracting both of these terms from 1.

\[
SER = 1 - \sum_{s=1}^{\infty} f(s; \lambda_S) \cdot \left( \sum_{d=0}^{s-1} f(d; \lambda_D) \right)^{m-1} \\
- \sum_{s=0}^{\infty} f(s; \lambda_S) \sum_{j=1}^{m-1} \frac{1}{j+1} \binom{m-1}{j} \cdot (f(s; \lambda_D)^j \cdot \left( \sum_{d=0}^{s-1} f(d; \lambda_D) \right)^{m-1-j})
\]

(3.3)

### 3.2.3 Error Correction Capacity

Given a channel bit error rate \( p \), the maximum error-free data rate of the channel can be approximated as the channel data rate, multiplied by:

\[
1 + p \cdot \log_2 p + (1 - p) \cdot \log_2 (1 - p)
\]

This is the Shannon capacity of the channel; many varieties of modern FEC code are able to approach this limit. This quantity therefore gives an approximation for the rate of the FEC code that would be needed for encoding the bits being sent over the channel, such that error-free communication could occur. As an example, using \( p = 3 \cdot 10^{-2} \) gives a value of 0.8056, suggesting that if the uncoded data has a bit error rate of 3\%, then a rate .8 (or 4/5) code would be an appropriate choice of encoding. This approximation is used below in discussion of appropriate choices of error correction code rates.

### 3.3 Communications Results

For both modulation schemes, data sets of 100,000 ROIC frames were collected, corresponding to about 1/6 of a second worth of data. This was close to the maximum capacity of the ROIC board’s on-board memory, and provided enough repetitions of the test pattern to draw meaningful conclusions about the data; at the maximum data rate tested, these 100,000 frames contain over 400,000 repetitions of the input.
pattern, and over 50 million symbols. The input power level on the array was varied between -70 and -55 dBm, as measured from the tip of the fiber shining onto the detector array. This corresponds to the region on figure [2-3] where the ROIC showed a linear response to input power, with the outermost data points lying just in the background-dominated and saturation regions. The results below are discussed in terms of detected signal photons, rather than a direct measurement of power, as the exact quantum efficiency of the detector is not known, and measurement in terms of detected photons allows for the best comparison to existing systems. As discussed in the theory section above, and in the discussion on receiver limitations in section 2.2, the level of background dark counts was assumed to be constant for the purposes of these tests. Verification of this assumption is discussed in the section on OOK demodulation below.

3.3.1 Block Repetition

To simulate the results from performing the same test with a slower data clock, multiple repetitions of the received pattern were layered on top of one another, and counts across the repetitions were aggregated to perform demodulation. This scheme provides results equivalent to those which could be expected by reducing the data clock on the input and aggregating counts over additional time bins on the output, but without needing to desync the generated clock from the clock used in the data pathway, and without requiring additional redundant data collection. While aggregating counts over multiple repetitions does increase the number of signal photons available per symbol, it also does increase the total dark count background present for each symbol, which for OOK modulation requires raising the decoding threshold, in addition to reducing the effective symbol rate.

3.3.2 Background Counts

Because of the nature of the testing, an exact measurement of background counts for any particular data set is impossible to count, as it cannot be determined whether
any particular detection event was actually triggered by the input signal or was spontaneous. Instead, background counts were assumed to occur with the same frequency as seen in the complete darkness tests, with signal counts added on top of those. As shown in figure 3-6 for tests in which the signal light level was not high enough to encounter blocking loss in the array, this provided a good estimate for the background count level, with counts in 0 symbols closely matching a Poisson distribution with a mean of the assumed dark count rate. When blocking loss became a large enough effect to distort the distribution of signal counts, the distribution of dark counts among non-signal slots also became heavily distorted.

3.3.3 OOK Modulation

Although the 311 MHz master clock was shared between the test pattern generator and the receiver array on the ROIC, there is still potential for a mismatch in alignment between the transmitted pattern and the received pattern, from the time needed to propagate the optical signal through the modulator and fiber. For instance, it would not be safe to assume that the repeated PRBS pattern begins a cycle directly at the start of the first frame recorded; it is not only possible but highly probable that the first symbol received is from somewhere in the middle of the PRBS sequence. Further, it is not even guaranteed that the first bin recorded corresponds to the first bin of a symbol which spans multiple ROIC bins, as is the case for all of the symbol rates considered for further testing. In finding the bit error rates reported below, all reasonable offsets in both the ROIC bins within a single symbol and symbols within the full PRBS sequence were considered, and the reported error rates represent the minimum of these.

Distributions of Bin Counts

The offsets used in finding the optimal BER (symbol offset within pattern, and ROIC bin offset within symbol) were recorded, and used to generate a distribution of detected photon counts for symbols now known to be 1 or 0. Figure 3-6 shows an
Figure 3-6: Distribution of counts for symbols corresponding to 0 in input pattern, for OOK modulation at 311 MBd, input power of -57 dBm, and no block repeats. The blue histogram indicates experimental data; the red stars is the proportion predicted by a Poisson distribution of the same mean.

example of this distribution for the 0 bits in a particular input, while figure 3-7 shows the count distribution for the 1 bits in that same data set. The blue bars are a histogram of the number of detected photons in each symbol; the red stars overlaid correspond to a Poisson distribution with the same mean. This same process can be repeated for all combinations of input power, symbol rate, and block repeats. For input powers well within the linear region, both the 0 bit and 1 bit histograms consistently matched up well with the theoretical predictions. However, as the input power rose to the saturation-limited region, the experimental curves began to match up much less well, with high-count symbols overrepresented in the histogram.
Figure 3-7: Distribution of counts for symbols corresponding to 1 in input pattern, for OOK modulation at 311 MBd, input power of -57 dBm, and no block repeats. The blue histogram indicates experimental data; the red stars is the proportion predicted by a Poisson distribution of the same mean.
Bit Error Rates

Figure 3-8 gives an example of the relationship between input power and symbol error rate, given a fixed channel rate and number of block repetitions. The solid lines show the theoretical best bit error rates, with the measured signal level. The lower blue line represents shows the bit error rate assuming no background noise, while the red line assumes the same level of background counts as measured in no-signal captures. For lower input powers, the shape of the experimental curve mirrors the shape of the red theory curve; at the highest input powers measured, a clear uptick in BER can be observed. This is due to blocking loss from the array beginning to approach its detection capacity; the highest input power levels being tested were very close to the boundary discussed in 2.2.2 between the linear response and saturation regions. As the signal level approaches saturation, a particularly strongly detected symbol earlier in a ROIC frame can block out detections later in the same frame, leading to errors in the later symbol. In this example, the minimal BER is reached at around 10 detected photons per bit, or a detection rate of $260 \cdot 10^6$ photons per second. This is very close to the count rate at the cutoff between the linear and saturation regions from figure 2-3, and indeed collections at other symbol rates and numbers of block repetitions also showed saturation effects at a similar overall count level (though not at the same number of photons per bit).

Figures 3-9 and 3-10 show the BER performance over a range of input powers and block repeats for OOK modulation, at 311 MBd and 155.5 MBd respectively. As the number of block repeats increases, the curves shift lower and to the right, indicating a greater number of signal photons per symbol and a lower BER, as expected.

Similar to figure 3-8, we can observe the onset of blocking loss at a count rate of between 250 and 300 Mcounts per second, most evident in the black triangles in figure 3-9, which correspond to data at a symbol rate of 311 MBd and 10 block repetitions.

The dotted lines in figures 3-9 and 3-10 represent the error correction thresholds discussed in section 3.2.3. Three particular thresholds of interest were chosen, corresponding to the error correction capabilities of Shannon-capacity codes at rates 1/2,
Figure 3-8: BER for OOK modulation at 155.5 MBd, with 6 block repeats.
Figure 3.9: BER for OOK modulation at 311 Mbd.

Detected Signal Photons per Symbol

Symbol error rate

OOK Performance with 311 Mbd Symbol Rate

4/5 Rate code BER threshold
1/2 Rate code BER threshold
10 block reps
5 block reps
1 block rep
Figure 3-10: BER for OOK modulation at 155.5 MBd.
4/5, and 9/10. These three code rates are all used in the DVB-S2 standard, and between them provide a representative sample of common code rates which would be seen in telecommunications systems. At a symbol rate of 311 MBd, OOK modulation is not able to pass any of these decoding thresholds without the use of block repetitions. However, with block repetitions, the error rate can be pushed below the decoding thresholds for both the rate 1/2 and 4/5 codes, although the tradeoff in both symbol rate and photons per bit required to reach the error threshold for 4/5 make that impractical. Three different operating points of particular note were found, with 4, 5, and 6 block repetitions. At 4 block repetitions, for a channel rate of 77.8 Mbit/s, the 1/2 rate error decoding threshold was passed with a signal strength of 3.25 detected photons per bit. With 5 repetitions, this threshold could be improved to 2.20 photons per bit, and with 6 repetitions, it could be improved again to 1.98 photons per bit, although the extra block repetitions lead to lower channel rates of 62.2 Mbit/s and 51.8 Mbit/s respectively. This gives a maximum user data rate of 38.9 Mbit/s.

With a symbol rate of 155.5 MBd, a BER below the cutoff for a rate 1/2 code could be reached with no block repetitions at a level of 1.78 photons per bit, yielding a user data rate of 78.8 Mbit/s. This operating point can be seen at the right end of the data points shown as red circles in figure 3-10. This was the lowest photons per bit required across all OOK data which passed at least one of the error correction thresholds, and across both modulation formats was the highest user data rate achieved. Figure 3-10 also shows that with block repetitions, these data were able to pass the lower-rate error thresholds, and achieve symbol error rates as low as $10^{-4}$. However, because of the block repetitions, this operating points require significantly more photons per bit and a much lower user data rate, even with the higher-rate FEC codes.

### 3.3.4 PPM Modulation

Similar to the method used for OOK modulation described above, the best symbol error rate for PPM modulation was determined by testing all possible alignments of the detected sequence of counts with respect to the ROIC frame and the input
pattern, as this is the same function which would be performed by a clock recovery
loop in a real-world end-to-end communications system. As described in section \[3.1\]
the test pattern used for PPM modulation was the same as for OOK, but with bits in
the pattern divided into groups of 4 and encoded as PPM symbols. Over a period of
127 symbols, this gives a nearly flat distribution of symbols and a realistic distribution
of the number of PPM slots between adjacent symbols, making this modulated PRBS
pattern useful for PPM testing.

The nature of PPM modulation dictates that the symbol error rate specifies an
upper bound on the bit error rate. For symbols which are correctly demodulated,
then all $\log_2 M$ bits (for PPM-16, 4) are correctly identified. For symbols which are
incorrectly demodulated, we can still expect that on average, about half of the bits
for that symbol will have been correctly identified as well, simply from the chance of
having incorrectly guessed a symbol which shares some bits. The exact proportion of
correct bits from incorrect demodulations depends on several factors, including the
nature of the errors leading to the incorrect demodulation and whether the ‘signal’
slot for the PPM encoding is Gray-coded or directly translated from binary, and the
discrepancy between bit error rate and symbol error rate is not explored further here.
However, this upper bound does give some information about the code rates which
would be viable for forward error correction, as with OOK, and the data collected
show that PPM encoding is viable for a variety of measured data rates.

Figure \[3-11\] shows a sample of one of the symbol error rate curves collected for
PPM modulation, in this case with a symbol rate of 9.7 MBd and no block repeating.
Similar to the OOK results above, the experimental curve closely follows the shape of
the theoretical curve, before blocking loss causes a sharp drop in BER performance
as more of the detectors are spending more time waiting to be reset. As with OOK
demodulation above, the background count level is assumed to be equivalent to the
measured background with no incoming signal, and the signal count level is based on
the measured counts over the collection period with background counts subtracted
out. The blue line again represents the theoretical performance with no background,
while the red line indicates theoretical best performance including background.
Figure 3-11: SER for PPM modulation at 9.7 MBd. From top to bottom, the three dashed lines show the FEC error correction cutoffs for rate 1/2, 4/5, and 9/10 codes.
As in the OOK results, the PPM results clearly demonstrate the effect of blocking loss, as evidenced by the symbol error rate decreasing with incoming signal power, and however, the number of detected photons before the array exhibits blocking loss is dramatically lower with PPM encoding. To again use figure 3-11 as an example, at a symbol rate of 9.7 MBd, the lowest SER was achieved at a detection level of 5.7 photons per bit. This corresponds to a total count rate of $220 \cdot 10^6$ counts per second, as compared to count rates of 260 to $300 \cdot 10^6$ counts per second with OOK. However, this still leaves room for a greater number of photons per bit before blocking losses occur, as PPM signals require a far lower proportion of the slots to contain signal, and thus the same average power is concentrated into fewer slots with higher power in each of those slots. This allowed the PPM collections to reach a much lower error floor than OOK. With higher block repetitions, completely error-free runs were seen, although the sample size of the data collections available was not sufficient to firmly conclude that the channel was capable of completely error-free operation. For these cases, while there is insufficient data to conclude that the communication is truly error-free, we can give an approximation of the true symbol error rate as the inverse of the number of symbols collected, multiplied by 10, on the order of $10^{-5}$; this can be taken as a reasonable estimate of the error floor of the system, based on the data available.

In addition to exemplifying the typical shape of the curve seen across all rates, with the symbol error rate markedly rising at the saturation point, the data shown in figure 3-11 also represent the best performance for PPM modulation among all of the tests carried out. As with OOK, there is a tradeoff between user data rate and the number of photons per bit needed to achieve that rate. Figure 3-11 contains three particular operating points of interest, in the points which lie on or adjacent to the FEC cutoff lines. At a symbol rate of 9.7 MBd, to achieve symbol error rate below the 1/2 rate code cutoff requires 1.00 detected photons per bit, and yields a user data rate of 19.4 Mbit/s. To improve to the error threshold of a rate 4/5 code requires moving up to 1.45 photons per bit, for a user data rate of 31.0 Mbit/s. Finally, for a rate 9/10 code and a data rate of 34.9 Mbit/s requires 2.05 detected photons per
bit. The 19.4 MBd tests were unable to reach the 1/2 rate code correction threshold without block repeats, pushing both the photons required per bit up and the effective channel rate down to less favorable levels than the slower channel rate. Similarly, the collections which were able to achieve a symbol error rate below the 4/5 rate cutoff performed worse on both of these metrics, with a minimum of 2.5 photons per bit required at a channel rate of 19.4 Mbit/s, for a user data rate of 15.5 Mbit/s.
Chapter 4

Conclusions

This thesis demonstrates the potential for using a Geiger mode APD array as the primary receiver in an optical communications system. Successful receipt and demodulation of an input signal was demonstrated over a wide range of data rates and input powers, and at channel data rates of 30 Mbit/s and above (enough bandwidth for applications such as streaming high-definition video), symbol error rates of less than $10^{-4}$ were achieved, a low enough error floor for a wide variety of FEC codes to function.

For OOK, a maximum user data rate of 78.8 Mbit/s was achieved, with a symbol rate of 155.5 MBd, an input power of 1.78 photons per bit, and a BER suitable for a rate 1/2 FEC code. At a channel rate of 311 MBd, operating points were achieved with user data rates of 38.9, 31.1, and 25.9 Mbit/s, with input powers of 3.25, 2.20, and 1.98 photons per bit. With PPM, user data rates of 34.9, 31.0, and 19.4 Mbit/s were achieved, with corresponding input powers of 2.05, 1.45, and 1.00 photons per bit. For both OOK and PPM modulations, using the slowest of the available channel rates was significantly better for achieving both a higher user data rate and a lower required input power. Further study with lower base symbol rates, which could be done with a redesign of the test transmitter, could yield higher potential data rates, and would be an interesting topic for future work in this area.
4.1 Future Work

4.1.1 450 nm Detector

The system used for this project operated at 780 nm, in the near-infrared. While this wavelength is suitable for free-space optical communications (and indeed is used in many such systems currently in operation), as discussed in the introduction, it has significant absorption underwater, and thus would be unsuitable for underwater use. A replacement detector array specifically designed for a blue-green wavelength of around 450 nm is in development, but was not available at the time of this project. This replacement will use exactly the same ROIC and underlying firmware, but the detectors themselves and the optics will be replaced to be optimized for the new wavelength. Once this updated detector is available, it would be of value to replicate the results from this thesis, to ensure that the new detector has the same potential as the current 780 nm system. Following that, tests under less idealized conditions would help establish the system’s potential, including testing through an actual water channel, which was not practical for the system available for this thesis due to the high absorption of 780 nm light.

4.1.2 Real-Time Communications

In order to determine the best possible conditions for demodulation, all of the processing for this thesis took place in post-processing, with data sets that had been collected in full and buffered off the detector’s memory with effectively unlimited time for processing. In order to run a real-world communications system, though, this processing would need to take place in real time, with demodulation and decoding happening directly on a data stream.

4.1.3 Pointing and Tracking

Experiments involving pointing and tracking were originally part of the proposed work for this thesis; unfortunately, due to time constraints, they were not able to
be included in the work presented here. However, as previously mentioned, an APD array such as this one has a great deal of potential for use in a tracking system. The spatial information on the number of fires at each pixel location is already available, and the firmware could be modified to process that information in real time, giving a centroid value as an estimate for the beam’s position. This centroid information could be fed back to the transmitter over a low rate out-of-band system, and the transmitter beam could then be adjusted to properly center on the primary detector.

One of the challenges in adapting the ROIC firmware for tracking would be in properly accounting for the anomalous always-on pixels. For communications, the only result of these pixels is a loss in efficiency, as they must be turned off to prevent background noise from overwhelming the detected signal. However, having many detectors turned off, in a random arrangement around the detector array, could have serious consequences for the performance when used for tracking. If the detectors in question are clustered toward a specific part of the array, this can induce a systemic bias into the perceived center of the tracking beam, and feeding back this skewed value to the transmitter would lead to incorrect adjustments. As an extreme example, if the upper right quadrant contained all of the disabled detectors (which for the mask used for communications here would require turning off over half of the detectors in the quadrant), the perceived center of the tracking beam would be permanently biased toward the lower left quadrant, causing overcorrection in steering the signal toward the part of the array with the worst sensitivity. Although this type of defect would be much less likely to occur in a consumer-grade rather than a research-grade system, it is still worth considering possible approaches to solving this problem.

One possible scheme to account for this asymmetry would be to duplicate the mask of disabled detectors in a symmetric pattern about the center, such that if a detector is disabled in one quadrant, the corresponding detector in the other three quadrants is also disabled. Figure 4-1 shows an example of such a mask, based on the standardized mask used in this thesis. While this type of mask could greatly reduce the sensitivity of the array, even halving the number of available detectors should provide sufficient sensitivity to perform tracking; many tracking systems currently in
use have far fewer elements.

Alternative schemes for performing tracking with an array that may not have all elements available would have to involve either interpolating the missing values based on the surrounding counts, or assigning different weights to the pixels for which the counts are known in order to account for the missing information. These would require much more complex firmware, but would allow for better usage of the array’s detection sensitivity.
Bibliography


