ABSTRACT

The radial position of a toroidal plasma in the Alcator tokamak was measured and controlled. A multi-port, X-ray detector was designed and built to observe the soft X-ray emission from several points across the plasma cross section. The data was reduced to determine plasma in/out position as a function of time. The vertical field circuitry of Alcator was interfaced to a controllable, high current power supply. Electronics were designed and built to permit real-time programming of the vertical-field coil current through this power supply. The radial position of the toroidal plasma was then regulated by programming the vertical field intensity during the plasma shot as a function of time. Experimental evidence indicated a dramatic improvement in plasma stability was obtained. Suggestions for instrumentation to enable feedback operation are made based on these results.
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II INTRODUCTION

Current research in the tokamak approach to nuclear fusion is concerned with improving the confinement of thermonuclear plasmas. In the case of a toroidal plasma, certain conditions must be met in order to achieve confinement and maintain stability. One important stability condition is that the magnetic and kinetic forces pushing the plasma ring outward be precisely balanced by magnetic forces directed inward. Experiments performed on the Alcator tokamak at M.I.T. from September 1975 through January 1976 indicate that a high degree of positional stability can be obtained with proper control of a vertical magnetic field. In addition, refinements of the previous vertical field system of Alcator achieved dramatic improvements in plasma confinement.

The Alcator tokamak is constructed of a donut-shaped, or toroidal, vacuum chamber of minor radius: $A = 12.5 \text{ cm}$, and major radius: $R = 54 \text{ cm}$. This chamber is surrounded by a Bitter-type magnet which is capable of producing magnetic fields of: $B = 100 \text{ kilogauss}$ in the pulsed mode. The plasma consists of a toroidal current through hydrogen gas contained in the vacuum chamber. This current is produced by an E.M.F. developed around the torus by a transformer. The transformer is
constructed of a large coil in the center of the tokamak. Before each plasma shot, it is charged up to a current of about 10 ka. The E.M.F. is developed by the collapsing magnetic field caused by discharging the coil through a resistor. The plasma is raised to temperatures of tens of millions of degrees centigrade by the ohmic heating of the plasma current. The current also produces a poloidal magnetic field :B_p. See Figure 1.

The plasma column will tend to expand outward (i.e., along the major radius) due to the kinetic pressure of the plasma, and the effect of the toroidal and poloidal magnetic fields. Qualitatively, the poloidal magnetic field pushes the plasma outward because any current ring tends to expand in order to minimize magnetic energy. The plasma kinetic pressure also pushes the plasma outward because of a small radial force component, due to the curvature of the column, of the toroidal pressure. Because of the plasma current, a vertical magnetic field, B_v, will exert a force inwards, on the plasma. However, the expansion forces must be calculated explicitly, in order to apply the proper magnetic field to establish equilibrium.

Gross plasma equilibrium requires that all forces acting on the plasma column be balanced. Not only must the net force pushing the ring outward be zero, but also there can be no net force acting to expand the column
The Alcator Tokamak

Bitter Magnet
Vacuum Chamber
Plasma
Ohmic Transformer

Figure 1
itself. The object of this first section will be to calculate the net force acting radially on the column, and set it equal to zero. This requirement imposes boundary conditions on the equations describing the in/out behavior of the plasma ring. To determine these conditions, consider a long cylindrical plasma column (See Figure 2), given a magnetic field \( B = B_z \) and a current density \( \mathbf{j} = j_z \). From electromagnetic field theory we know:

1) \( \nabla P = \mathbf{j} \times \mathbf{B} \) where \( P \) is the pressure.

In cylindrical coordinates, \( \nabla P = \frac{\partial P}{\partial R} \hat{r} + \frac{\partial P}{\partial \theta} \hat{\theta} + \frac{\partial P}{\partial Z} \hat{Z} \)

However, the \( \frac{\partial P}{\partial \theta} \) and \( \frac{\partial P}{\partial Z} \) components are zero because of symmetry. We are only concerned with the equation.

2) \( \frac{\partial P}{\partial R} \hat{r} = (j_{\theta} B_z - j_z B_{\theta}) \hat{r} \)

\( j_{\theta} \) can be calculated using Maxwell's law:

3) \( \mu_0 \mathbf{j} = \nabla \times \mathbf{B} \)

\[ j_{\theta} = -\left( \frac{\partial P}{\partial R} \right) \frac{1}{\mu_0} \]

\( j_z \) can be calculated from Ampere's law:

4) \( \oint \mathbf{B} \cdot dR = \oint \mathbf{j} \cdot dA \)

Evaluate equation 4 to obtain an expression for \( j_z \).
Cylindrical Plasma Column

Figure 2
\[ 2\pi R B_\theta = 2\pi \int_0^{R_1} j_z R dR \]

Solve for \( j_z \) by calculating the integral and rearranging terms.

5) \( j_z = \frac{1}{R} \frac{d}{dR} RB_\theta \)

Substitute the expressions for \( j_\theta \) and \( j_z \) into equation 2. The current has been eliminated and we obtain an equation involving only the pressure and magnetic field.

6) \( \frac{dP}{dR} = -\frac{1}{\mu_0} B_z \frac{dB_z}{dR} - \frac{1}{\mu_0} \frac{B_\theta}{R} \frac{d}{dR} (RB_\theta) \)

Rearrange terms and simplify.

\[
\frac{d}{dR} \left[ P + \frac{1}{\mu_0} \frac{B_z^2}{2} \right] = -\frac{1}{\mu_0} \frac{B_\theta}{R} \frac{d}{dR} (RB_\theta)
\]

7) \( R^2 \frac{d}{dR} \left[ P + \frac{1}{\mu_0} \frac{B_z^2}{2} \right] = -\frac{1}{2\mu_0} \frac{d}{dR} (RB_\theta)^2 \)

Equation 7 can now be integrated to remove the differentials. The left hand side must be integrated by parts.

8) \( A^2 \left[ P + \frac{B_z^2}{2\mu_0} \right] - \int_0^A \left[ P + \frac{B_z^2}{2} \right] 2R dR = -\frac{1}{2\mu_0} (AB_\theta)^2 \)

The remaining integral is just the definition for the average of the term \( P + B_z^2/2 \) over the plasma cross section.
9) \[ A^2 \left[ P + \frac{B_z^2}{2 \mu_0} \right] - \left[ \bar{P} + \frac{\bar{B}_z^2}{2 \mu_0} \right] A^2 = - \frac{1}{2 \mu_0} A^2 \Theta^2 \]

Simplify equation 9.

10) \[ \left( P + \frac{B_z^2}{2 \mu_0} + \frac{\Theta^2}{2 \mu_0} \right) A = \bar{P} + \frac{\bar{B}_z^2}{2 \mu_0} \]

Equation 10 must be met if the plasma is to remain stable in the R direction. It can further be reduced if we assume that \( P(A) = 0 \).

11) \[ \left( \frac{B_z^2}{2 \mu_0} + \frac{\Theta^2}{2 \mu_0} \right) A = \bar{P} + \frac{\bar{B}_z^2}{2 \mu_0} \]

For future reference, equation 11 will be termed the "equilibrium equation." It imposes restrictions on the magnetic field and plasma pressure for equilibrium to be maintained.

The preceding derivation can also be performed using the Maxwellian Stress Tensor. This approach yields the more general solution to cylindrical plasma stability:

12) \[ \frac{d}{dr} \left( P + \frac{B^2}{2 \mu_0} \right) = - \frac{\Theta^2}{r \mu_0} \]

The next step in the development of tokamak plasma stability is to use the Stress Tensor to evaluate toroidal column equilibrium. The Maxwellian Stress Tensor is defined as:
\[ T_{\alpha\beta} = \left( E_{\alpha}E_{\beta} + B_{\alpha}B_{\beta} - \frac{1}{2} \left( B_{\alpha}E_{\beta} + E_{\alpha}B_{\beta} \right) \delta_{\alpha\beta} \right) \]

However, there are no significant electric fields in the plasma column. The tensor simplifies to the magnetic Stress Tensor.

\[ T_{\alpha\beta}^{\text{mag}} = \frac{1}{\mu_0} \left( B_{\alpha}B_{\beta} - \frac{1}{2} B^{2} \delta_{\alpha\beta} \right) \]

\[ T_{\alpha\beta}^{\text{mag}} \equiv T_{\text{mag}} \]

We now define the Total Stress Tensor.

13) \[ \bar{T} \equiv \bar{T}^{\text{mag}} - \bar{P}^{\delta} \]

The force density acting on the plasma particles can be written as the divergence of the Total Stress Tensor. Force density is the force per unit volume. Thus the net force acting on a particular volume of the plasma is the volume integral of the divergence of the Stress Tensor.

14) \[ \bar{F} = \iiint_{\text{volume}} \nabla \cdot \bar{T} \, dv \]

Equation 14 can be converted to a surface integral by the divergence theorem.
Toroidal Plasma Column
Figure 3
15) \[ \mathbf{F} = \int_{\text{surface}} \mathbf{T} \cdot \mathbf{n} \, dA \]

The physical interpretation of \( \mathbf{T} \cdot \mathbf{n} \) is that it is the force per unit area acting on the surface. By taking the component acting perpendicular to each surface element and integrating over the entire surface, one obtains the total force exerted on the volume. We define the force per unit area, \( \mathbf{f} \), as:

16) \[ \mathbf{f} = \mathbf{T} \cdot \mathbf{n} \quad \text{where} \]

\[ \mathbf{F} = \int_{\text{surface}} \mathbf{f} \, dA \]

Using equation 16, it is then possible to calculate the forces acting on a toroidal plasma column. The object will be to determine the net force acting in the radial (R) direction. Figure 3 is a diagram of the toroidal plasma. The procedure used will be to calculate the force per area acting on all the surfaces of the volume element \( dV \), and integrating over the entire surface. Finally, the radial component of the total force will be extracted from these values. Figure 4 is a diagram of the individual volume element \( dV \). Refer to these figures for the coordinates used in the following derivation. The first step will be to calculate the tensile force density \( f^{(1)} \) acting on \( dV \) as shown.
Volume Element $dV$

Figure 4
Using the Total Stress Tensor:

17) \( \overline{f}^{(1)} = \overline{n} \cdot \overline{T} = \overline{\phi} \cdot \frac{\overline{BB}}{\mu_0} - \frac{\overline{B}^2}{2\mu_0} \overline{\delta} - \overline{P} \overline{\delta} \)

Calculate the individual components:

18) \( \overline{f}^{(1)} = \left( \frac{\overline{B}_\phi}{\mu_0} \right) \left( B_\phi \overline{\phi} + B_\rho \overline{\rho} + B_\theta \overline{\theta} \right) - \frac{\overline{B}^2}{2\mu_0} \overline{\delta} - P \overline{\delta} \)

Equation 18 can be simplified by symmetry arguments.

Because of top/bottom symmetry, there is no net radial or theta component to \( \overline{f}^{(1)} \).

\( \frac{\overline{B}_\theta}{\mu_0} \overline{\rho} = \frac{\overline{B}_\phi}{\mu_0} \overline{\phi} = \frac{\overline{B}^2}{2\mu_0} = 0 \)

Equation 18 becomes:

19) \( \overline{f}^{(1)} = \left( \frac{\overline{B}_\phi^2}{2\mu_0} - \frac{\overline{B}_\theta^2}{2\mu_0} - P \right) \overline{\phi} \)

Integrate over the entire surface in order to obtain the net tensile force acting on the element.

20) \( F^{(1)} = \int \overline{f}^{(1)} \, ds \)

21) \( F^{(1)} = \int \left( \frac{\overline{B}_\phi^2}{2\mu_0} - \frac{\overline{B}_\theta^2}{2\mu_0} - P \right) \, dA = \pi A^2 \left( \frac{\overline{B}_\phi^2}{2\mu_0} - \frac{\overline{B}_\theta^2}{2\mu_0} - \overline{P} \right) \)

The integral is evaluated by defining \( \overline{B}_\phi, \overline{B}_\theta, \) and \( \overline{P} \) as the average of \( B_\phi, B_\theta, \) and \( P \) over the entire surface. \( \pi A^2 \) is just the total surface area.
The net radial force due to the two tensile force vectors \( \vec{F}^{(1)} \) from the geometry of the torus is:

\[
22) \quad F_{R}^{(1)} = -2F^{(1)} \sin \frac{d\phi}{2}
\]

Simplify and substitute for \( F^{(1)} \).

\[
F_{R}^{(1)} = -F^{(1)}d\phi
\]

\[
23) \quad F_{R}^{(1)} = -d\phi \pi A^{2}\left(\frac{B^{2}}{\mu_{0}} - \frac{B^{2}}{\mu_{0}} - \frac{P}{2}\right)
\]

The next step is to calculate the force per area acting on the \( \hat{\rho} \) surface. Again using the Stress Tensor and the same procedure as before, the force per area, \( \vec{f}^{(2)} \), is:

\[
24) \quad \vec{f}^{(2)} = \hat{\rho} \cdot \left( \frac{\vec{B}}{\mu_{0}} - \frac{B^{2}}{\mu_{0}} \hat{\rho} - \frac{B^{2}}{\mu_{0}} \hat{\sigma} - \frac{P}{2} \hat{\rho} \right)
\]

Expand equation 24 into the individual components.

\[
25) \quad \vec{f}^{(2)} = \frac{B^{2}}{\mu_{0}} \hat{\rho} - \frac{B^{2}}{\mu_{0}} \hat{\rho} - \frac{B^{2}}{\mu_{0}} \hat{\rho} - \frac{B^{2}}{\mu_{0}} \hat{\rho} + \frac{B^{2}B_{\mu}}{\mu_{0}} \hat{\sigma} + \frac{B^{2}B_{\mu}}{\mu_{0}} \hat{\sigma}
\]

Equation 25 can also be simplified using simple physical arguments. Start with the vector identity:

\[
26) \quad \vec{B} \cdot \hat{j} \times \vec{B} = 0
\]
Substitute equation 1 into equation 26 for \( j \times B \).

1) \[ \vec{\nabla} \rho = j \times B \]

27) \[ B \cdot \vec{\nabla} \rho = 0 \]

On the surface \( \rho = \lambda \), \( \vec{\nabla} \rho \) must be entirely in the radial direction because of the circular symmetry of the column. If \( \vec{\nabla} \rho \) is entirely radial, then \( B \cdot \vec{\nabla} \rho = 0 \) implies that there is no radial component of \( \vec{B} \): \( \vec{B} \rho = 0 \). The surface \( \rho = \lambda \) is defined as a magnetic flux surface. Setting all the terms involving \( \vec{B} \rho \) to zero in equation 25, we obtain:

28) \[ \vec{f}^{(2)} = - \frac{B^2}{2\mu_0} \hat{\rho} - \frac{B^2}{2\mu_0} \hat{\rho} - \rho \hat{\rho} \]

Equation 28 can be further simplified if we again impose the constraint that \( \rho(\lambda) = 0 \). This does not affect the calculation of the total force because the integral is performed on the surface \( \rho = \lambda \).

29) \[ \vec{f}^{(2)} = - \hat{\rho} \left( \frac{B^2}{2\mu_0} + \frac{B^2}{2\mu_0} \right) \]

The total force acting on the cylindrical \( \rho \) surface of \( dV \) in the major radial \( \hat{R} \) direction is:

30) \[ F_r^{(2)} = \int (\vec{f}^{(2)} \cdot \hat{R}) dA \]
\( P_r^{(2)} = \int (f^{(2)} \cos \theta) \, dA \)

From the geometry of the volume element, a surface element on \( dV \) is seen to be:

\[ dA = (rd\theta)(Ad\phi) \]

Substitute this expression for \( dA \), \([R + A \cos \theta] \) for \( r \), and the expression for \( f^{(2)} \) that was derived earlier.

\[
31) \quad P_r^{(2)} = -d\phi \int_0^{2\pi} A[R + A \cos \theta] \cos \left( \frac{B \theta}{2\mu_\theta} \right) + \\
\quad + \frac{B^2 \theta}{2\mu_\theta} \bigg| A \, d\theta
\]

The total radial force acting on the \( \hat{i} \rho \) surface is found by integrating equation 31. In order to simplify this procedure, we will break up the integral and solve the \( B\theta^2/2 \) and \( B^2/2 \) integrals separately.

Start with the integral involving \( B\theta^2/2\mu_\theta \) only.

\[
32) \quad P_r^{(2)} = -d\phi \int_0^{2\pi} A[R + A \cos \theta] \cos \theta \left( B\theta^2/2\mu_\theta \right) \bigg| A \, d\theta
\]

In order to evaluate this integral, it is first necessary to calculate \( B\theta \). For a toroidal magnet such as Alcator, to a first approximation the toroidal field is:

\[
B\theta = \frac{B\phi_0}{1 + \frac{A}{R} \cos \theta} \approx B\phi_0 \left( 1 - \frac{A}{R} \cos \theta \right)
\]
Ignoring higher order terms, $B_\phi^2$ can be written as:

$$B_\phi^2 = B_\phi^2 (1 - \frac{2A}{R} \cos \Theta)$$

Substitute this equation into equation 32 for $B_\phi^2$.

33) \[ F_r^{(2)} = -d\phi \int_0^{2\pi} A \left[ R + A \cos \Theta \right] \cos \Theta \left[ \frac{B_\phi^2}{2 \mu_0} \right] \left( 1 - \frac{2A}{R} \cos \Theta \right) d\Theta \]

Factor out an $R$ and simplify.

\[ F_r^{(2)} = -d\phi A R \frac{B_\phi^2}{2 \mu_0} \int_0^{2\pi} d\Theta \left( 1 + \frac{A}{R} \cos \Theta \right) \left( 1 - \frac{2A}{R} \cos \Theta \right) \cos \Theta \]

34) \[ F_r^{(2)} = -d\phi A R \frac{B_\phi^2}{2 \mu_0} \int_0^{2\pi} d\Theta \left( \cos \Theta - \frac{A}{R} \cos^2 \Theta - \frac{2A^2}{R^2} \cos^3 \Theta \right) \]

Equation 34 can be directly integrated to yield the radial force due to the toroidal field.

35) \[ F_r^{(2)} = d\phi \pi A^2 \frac{B_\phi^2}{2 \mu_0} \]

I have so far calculated the radial pressure exerted on the plasma ring due to the tensile force on the $\hat{\phi}$ surface and the toroidal magnetic field pressure on the $\hat{\rho}$ surface. The only calculation that remains is to determine the radial force exerted by the poloidal magnetic field on the $\hat{\rho}$ surface.

Referring back to equation 31, we now calculate $F_r^{(2)}$ due to $B_\phi$ only.
The same procedure as before will be repeated, thus we must first calculate $B_\theta$. The general solution for the magnetic field of a current loop of small inverse aspect ratio is:

37) \[ B_\theta = \frac{\mu_0 I}{2\pi \rho} + \left( -\frac{\mu_0 I}{4\pi R} \ln \frac{8R}{\rho} - C \left( \frac{\rho}{A} \right)^2 \right) \cos \theta \]

Since we are evaluating these integrals on the surface $\rho = A$, we need only know $B_\theta$ on that surface. Remember that $B_\phi(A) = 0$. (Refer to equation 27)

To solve equation 38 we must then take:

38) \[ B_\phi = \left( -\frac{\mu_0 I}{4\pi R} \ln \frac{8R}{\rho} - 1 \right) + C \left( \frac{A^2}{\rho^2} \right) \sin \theta \]

To simplify the writing we define $B_{\phi o} \equiv \frac{\mu_0 I}{2\pi A}$ and evaluate equation 39 on the surface $\rho = A$.

39) \[ B_\theta = \frac{\mu_0 I}{2\pi \rho} + \frac{\mu_0 I}{4\pi R} \left( -\ln \frac{8R}{\rho} - \left( \frac{A}{\rho} \right)^2 \left( \ln \frac{8R}{A} - 1 \right) \right) \cos \theta \]

Substitute equation 40 for $B_\theta$ into the force equation.
\[
F_{B\theta}^{(2)} = -d\phi \int_0^{2\pi} A \left[ R + A \cos \theta \right] \cos \left( \frac{B_{\theta}^2}{2\mu_0} \right) d\theta
\]

41) \[
F_{B\theta}^{(2)} = -d\phi A R \int_0^{2\pi} (1 + \frac{A}{R} \cos \theta) \frac{\cos \theta}{2\mu_0} \left( B_{\theta} - B_{\theta_0} \right) \cos \theta \left( \ln \frac{8R}{A} - 1/2 \right)^2
\]

All terms except those in \( \cos^2 \theta \) drop out. Equation 41 can then be simplified to:

42) \[
F_{B\theta}^{(2)} = -d\phi A R \left( \frac{B_{\theta_0}^2}{2\mu_0} \right) \int_0^{2\pi} \left[ \frac{A}{R} \cos^2 \theta - \frac{2A}{R} \cos^2 \theta \left( \ln \frac{8R}{A} - 1/2 \right) \right] d\theta
\]

The integral is evaluated directly to obtain the force on the \( \hat{\rho} \) surface due to the poloidal field.

43) \[
F_{B\theta}^{(2)} = -d\phi A R \left( \frac{B_{\theta_0}^2}{2\mu_0} \right) \ln \frac{A}{R} - \frac{2A}{R} \left( \ln \frac{8R}{A} - 1/2 \right)
\]

Collecting terms and simplifying equation 43, we obtain the final expression:

44) \[
F_{B\theta}^{(2)} = d\phi \pi A^2 \left( \frac{B_{\theta_0}^2}{\mu_0} \right) \left( \ln \frac{8R}{A} - 1 \right)
\]

Recall the expressions for the other forces acting on the plasma ring.

\[
F_r^{(1)} = \pi A^2 d\phi \left[ -\frac{B_\phi^2}{2\mu_0} + \frac{B_\theta^2}{2\mu_0} + P \right]
\]
\[ F_{B\phi}^{(2)} = \frac{1}{2} \pi A^2 d\phi \left[ \frac{B_{\phi}^2}{\mu_0} \right] \]

Take the sum of these forces to find the net force on the plasma column:

\[ F_r = \frac{1}{2} \pi A^2 d\phi \left[ \frac{B_{\phi}^2}{\mu_0} \left( \ln \frac{8R}{A} - 1 \right) - \frac{B_{\phi}^2}{\mu_0} + \frac{B_{\theta}^2}{\mu_0} + \bar{P} + \frac{\phi_0^2}{\mu_0} \right] \]

We now make use of the equilibrium equation (equation 11) that was derived earlier to simplify this equation.

\[ \frac{B_{\phi}^2}{\mu_0} + \frac{B_{\theta}^2}{\mu_0} = \bar{P} + \frac{\phi_0^2}{\mu_0} \]

Substituting equation 46 into 45, we obtain:

\[ F_r = \frac{1}{2} \pi A^2 d\phi \left[ \frac{B_{\phi}^2}{\mu_0} \left( \ln \frac{8R}{A} - 3/2 \right) + 2\bar{P} + \frac{B_{\theta}^2}{\mu_0} \right] \]

\[ F_r = \frac{1}{2} \pi A^2 d\phi \left[ \frac{B_{\phi}^2}{\mu_0} \left( \frac{2\bar{P}}{B_{\phi}^2} + \frac{B_{\theta}^2}{2B_{\theta}} + \left( \ln \frac{8R}{A} - 3/2 \right) \right) \right] \]

We have derived the total radial force acting on the plasma ring entirely in terms of the toroidal and poloidal magnetic fields and the average kinetic pressure.
Now define: \( l_i = \frac{B^2}{\Theta^2} \) and \( B_{pol} = \frac{2\mathcal{P}}{\Theta^2} \) 

Equation 47 can then be rewritten:

\[
F_r = \pi A^2 d\phi \left[ \frac{B}{\Theta} \frac{\Theta^2}{\mu_0} \right] \left[ B_{pol} + \frac{li}{2} + \ln \frac{8R}{A} - 3/2 \right]
\]

In order to maintain equilibrium, \( F_r \) must be balanced precisely with a force inward. The inward force is achieved by imposing a vertical magnetic field on the plasma current ring. The inward force will be:

\[
F_r = l_B v R d\phi
\]

Solving for \( B_v \) and substituting in equation 48 for the forces, we then can calculate the equilibrium vertical field in terms of the other magnetic fields and the kinetic pressure.

\[
B_v = \frac{\pi A^2 d\phi}{IRd\phi} \frac{B}{\Theta} \frac{\Theta^2}{\mu_0} \left[ B_{pol} + \frac{li}{2} + \ln \frac{8R}{A} - 3/2 \right]
\]

\[
B_v = \frac{2\pi A}{\mu_0 R} \left( \frac{A}{2R} \right) B \frac{\Theta^2}{\Theta} \left[ B_{pol} + \frac{li}{2} + \ln \frac{8R}{A} - 3/2 \right]
\]

Equation 51 can be simplified by recalling that:
\[ B_{\theta_0} = \frac{\mu_0 I}{2 \pi A} \quad \text{Substitute for } B_{\theta_0}^2. \]

52) \[ B_v = I \left( \frac{\mu_0}{4 \pi R} \right) \left[ \beta_{\text{pol}} + \frac{1}{2} + \ln \frac{8R}{A} - 3/2 \right] \]

The significance of equation 52 is that it describes how the vertical field must scale with other parameters. In particular, if we remove the constants:

53) \[ B_v \propto I \left[ \beta_{\text{pol}} + \frac{1}{2} \right] \]

Although the quantities \( \beta_{\text{pol}} \) and \( \frac{1}{2} \) are not constants, they do not vary over a wide range. \( B_v \) is thus primarily dependent on the plasma current \( I \), if equilibrium is maintained. The most important result of this derivation is that in the design of a vertical field system, the vertical field must scale with the plasma current. The vertical field is also dependent on other factors (These will be considered later.), but to a first approximation:

54) \[ B_v \propto I \]

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III TOKAMAK OPERATION

The plasma current, $I$, is induced by an E.M.F. developed around the torus by the transformer. The time integral of this loop voltage is defined as the ohmic drive and is measured in volt seconds. The term "ohmic" is used because the current generated heats the plasma resistively. The plasma and the ohmic drive can be modeled as a simple electronic network consisting of a resistor, inductor and a voltage source. (See Figure 5)

We now calculate the dependence of the current on the ohmic drive.

The voltages across each of the components can be expressed in terms of the currents through them:

$$V_R = IR$$

$$V_L = L \frac{dI}{dt}$$

These voltages can be related by Kirchhoff's Voltage Law:

55) $V_S = V_R + V_L$

Substitute for $V_R$ and $V_L$.

56) $V_S = IR + L \frac{dI}{dt}$
Plasma Circuit Model
Figure 5
Equation 56 is the basic differential equation describing the behavior of the plasma current. Multiply through by \( dt \) and integrate.

57) \[ \int V_S \, dt = R \int I dt + \int L dI \]

To evaluate equation 57, notice that the left-side term is merely the total number of volt seconds developed by the ohmic drive. The second term can be defined as the time average of the plasma current and the last term is the inductance times the maximum current achieved.

58) \[ \text{Volt-Seconds} = RI_T + LI_{\text{max}} \]

Experimentally, the resistive term is very small. The measured resistive voltage around the loop is about 1 volt. Since the ohmic drive lasts a fraction of a second, \( T \) is very small, and \( RT \) is small compared to \( LI_{\text{max}} \). Equation 58 can be approximated:

59) \[ \text{Volt-Seconds} \approx LI_{\text{max}} \]

The significance of equation 59 is that to a first approximation the plasma current is proportional to the number of volt seconds generated by the ohmic drive transformer.
IV DESCRIPTION OF PREVIOUS SYSTEM

The Alcator tokamak is considered a remarkable engineering achievement because it has been able to scale the vertical field with the plasma current. It does this by coupling the vertical field coils to the ohmic drive circuitry. The vertical field coil current is made proportional to the volt seconds developed by the transformer. Figure 6 is a schematic of the Alcator magnet drive circuitry. With the vacuum breaker closed, the A.R.U. power supply slowly builds a current in the ohmic drive transformer coils. After a predetermined current has been built up, the power supply is turned off and the vacuum breaker opens up. The opening of the breaker is called commutation. The breaker contacts are protected from arcing by discharging a capacitor bank, in parallel with it, at commutation time. When the vacuum contacts are opened, the current flowing through the ohmic transformer coil is forced through a 1.5 ohm resistor. As a result of the low L/R time of this network, the current is brought to zero rapidly. The diminished current causes the magnetic field to collapse which then generates the E.M.F. around the torus. The technique of storing the ohmic drive energy in the magnetic field of the transformer is termed inductive storage.
From elementary considerations we know that the total number of volt seconds is proportional to the total magnetic flux developed. Therefore, the ohmic drive is directly dependent on the maximum transformer current prior to commutation. The proof is simple; start with the following definitions:

\[
\text{Volt-Seconds} \equiv \int E \cdot dt \\
\phi \equiv \int B \cdot dA
\]

Multiply Faraday's law by \( dt \) and integrate:

\[
\int E \cdot dt = \int d\phi
\]

60) Volt-Seconds = \( \phi = \int B \cdot dA \propto I_{\text{transformer}} \)

The vertical field coil current is also proportional to the maximum ohmic transformer current. About 450 micro-seconds after commutation, the vertical field igniton fires, shunting the vertical field coils across the 1.5 ohm resistor. The isolation igniton, which connects the opposite side of the vertical field coil system to ground, self-fires due to the high voltage switched across transformer \( T_1 \) by the vertical field igniton. The enormous voltage drop developed across this resistor builds up the vertical field current very quickly. Qualitatively,
the vertical field is dependent on the peak transformer current because the voltage drop across the 1.5 ohm resistor is proportional to the peak current, and the vertical field coil current is proportional to the voltage drop. After commutation and after the vertical field ignitron has fired, the Alcator circuitry can be modeled as the simple network shown in Figure 7. For this first analysis, ignore \( R_1 \), because it is small. The network equations for the circuit in Figure 7 are:

61) \( I = I_{L1} + I_{L2} \) and

62) \( V_L = -V_R \)

Substituting in the voltage-current constraints for these quantities, equations 61 and 62 become:

63) \( I_L = \int_0^{t'} \frac{V_L}{L} \, dt \)

64) \( I = \int_0^{t'} \left( \frac{V_L}{L_1} + \frac{V_L}{L_2} \right) \, dt \)

65) \( V_L = -IR \)

Substitute equation 64 into 63, simplify, and differentiate with respect to time.
Alcator Magnet Drive Circuit Model
Figure 7

Alcator Magnet Drive Reduced Circuit Model
Figure 8
The solution to this differential equation is:

\[ I = I_0 e^{\left(-\frac{R(L_2 + L_1)}{L_2L_1}\right)t} \]

where \( I_0 \) is the ohmic transformer current prior to commutation. The voltage drop across \( L_2 \) and then the vertical field coil current can now be calculated by substituting the expression for \( I \) into equations 64 and 63.

\[ V_L = -RI_0 e^{\left(-\frac{R(L_2 + L_1)}{L_2L_1}\right)t} \]

Equation 70 can be evaluated if we make the approximation that \( t' = \infty \). This is a reasonable assumption because the time constant:

\[ \tau = \frac{L_2L_1}{R(L_2 + L_1)} \]

is very small compared to the time scale of the plasma shot. Using the measured values of \( L_2, L_1 \) and \( R, \tau \) is calculated to be: \( \tau = 2.7 \) ms. Integrating equation 70 and evaluating at zero and infinity:
After a brief time equal to several time constants, $\tau$, the voltage across $R$, is very nearly zero, and this component can be ignored. The network reduces to the model shown in Figure 8. We now consider the behavior of the vertical field coil current, including the affect of $R_1$. The circuit of Figure 8 is just a resistor and inductor in series.

The equation describing the current through the vertical field coils, $I_{L2}$, can be determined by inspection.

\[ I_{L2} = I_{L2}^{'} \cdot e^{-t/\tau} \]

where \[ \tau = \frac{L_1 + L_2}{R} \]

is calculated to be: $\tau \approx 500$ ms. Notice that this value is much larger than the time constant for the first network. The final equation describing $I_{L2}$ is just:

\[ I_{L2} = \left( \frac{-R_1 t}{L_1 + L_2} \right) e^{\frac{-R_1 t}{L_1 + L_2}} \]
The significance of equation 74 is that it proves that vertical field coil current is directly proportional to the maximum ohmic transformer current prior to commutation.

75) \( \frac{I_{L2}}{I_0} \)

Another important result of equation 74 is that it specifies the time constant of the decay of the vertical field with time.

The operation of the Alcator vertical field system can now be understood more clearly. Recall several of the equations of the preceding derivation:

59) \( I_{\text{plasma}} \propto \text{Volt-Seconds} \)

60) \( \text{Volt-Seconds} \propto I_0 \quad I_0 \equiv I_{\text{transformer max.}} \)

65) \( I_0 \propto I_{L2} \)

76) \( I_{L2} \propto B_v \)

These relationships clearly indicate that on Alcator:

77) \( I_{\text{plasma}} \propto B_v \)

Equation 77 is identical to equation 54 which was derived as a criterion for toroidal plasma stability.
The success of the Alcator vertical field system is that it behaves according to this relationship. Another advantage of the Alcator system is that the vertical field is built up very quickly after commutation. Positional equilibrium is achieved from the beginning of the plasma pulse. Experimental evidence indicates that the vertical field system does work extremely well.
V LIMITATIONS TO THE THEORY

This derivation has demonstrated the strengths of the Alcator vertical field system; however, it also reveals the weaknesses. In particular, many approximations were made which should now be reconsidered. These approximations constitute deviations of the actual toroidal plasma and vertical field behavior from the theory. The most significant contribution to this error was made when $\beta_{\text{pol}}$ and li/2 were assumed to be constant, in order to arrive at equation 54. The original equation was:

$$53) \quad B_v \propto I\left(\beta_{\text{pol}} + \text{li}/2\right)$$

In fact $\beta_{\text{pol}}$ and li/2 are not constants. $\beta_{\text{pol}}$ and li/2 depend on both the density and the plasma current, as well as other plasma characteristics. The variation of these terms with plasma parameters introduces a small, but significant error into the approximation

$$54) \quad B_v \propto I$$

Another error was incorporated into the derivation, when the resistive term in equation 58 was ignored. We began with:
58) Volt-Seconds = \( \text{RT} + L \text{I}_{\text{max}} \)

and it was approximated as:

59) Volt-Seconds \( \propto \text{I}_{\text{max}} \)

The quantities \( \beta_{\text{pol}} \) and \( \text{l}/2 \) depend on plasma parameters that vary throughout the plasma shot. The effect of the omission of these terms on the accuracy of the theory is to introduce a time dependent error. Another time dependent error is due to the difference between the decay times of the plasma current and the vertical field. Although the Alcator vertical field system may initially achieve positional equilibrium, the relationship \( B_v \propto I_p \) will not be maintained throughout the plasma shot. The decay time constant of the vertical field current, which was calculated to be \( T \sim 500 \text{ ms} \), is many times larger than the decay time constant of the plasma current.

The elimination of these inaccuracies in vertical field stabilization should conceivably improve plasma equilibrium. The purpose of this thesis research was to investigate methods for refining the present Alcator vertical field system so that these effects could be compensated for. The results should then improve toroidal plasma confinement.
VI CONCEPTUAL DESIGN OF A NEW VERTICAL FIELD SYSTEM

The design of a new vertical field system for Alcator should include the advantages of the old system and eliminate its weaknesses. The advantages that should be maintained are: 1) approximate scaling of the vertical field with plasma current and 2) a fast build-up of the vertical field after commutation. The new system should also have the additional capability of perturbing the vertical field coil current to compensate for deviation in plasma position from equilibrium. These deviations are due to the inaccuracies in the existing vertical field system just discussed. However, because of the time dependency and complexity of the error terms, it would be unreasonable to predict or measure them and adjust the vertical field accordingly.

The best system would be one that would detect the plasma position and generate a signal indicating its departure from equilibrium. This error signal would then be used to modulate the vertical field coil current sufficiently to re-establish equilibrium. Ideally, the plasma would be approximately positioned by a system similar to the existing one and accurately positioned by this type of feedback mechanism. The design of this vertical field system resolves into three distinct
problems: 1) the problem of plasma detection, 2) the problem of vertical field coil current control, and 3) the problem of processing the position data to modulate the coil current controller and thus close the feedback loop. The first two of these problems were solved by our research efforts. We also indicate how the third may be solved, based on our results.
VII THE PROBLEM OF DETECTION

The conventional method for measuring plasma in/out position on tokamaks is to use pick-up coils that detect changes in magnetic flux. By winding opposing coils on both the inside and outside of the torus, an E.M.F. can be measured corresponding to the relative change in magnetic flux between the inside and outside. The relative change in magnetic flux is proportional to the change in position of the plasma. The voltage signal developed is integrated to obtain a signal which is then corrected by dividing it by the plasma current. This compensated signal indicates the plasma position as a function of time, but it is only relative position. Since the signal is integrated, an undetermined constant is introduced into the position signal. The plasma position is known only relative to a particular point, but the position of this point is unknown. Clearly, this technique is not sufficiently accurate for determining position.

Another method for position detection, suggested by Dr. Ronald R. Parker, utilizes the X-ray emissions from plasmas. Since a plasma produces soft X-rays dependent on both the temperature and density, the plasma can be "seen" with proportional counters. Several of these detectors are positioned on the tokamak, spaced equally
across the vacuum chamber cross section. Each proportional counter receives radiation from one particular segment of the chamber. Long collimating tubes leading from the torus to the detector insure that the segments of the plasma cross section, that the X-ray counters look at, do not overlap. Figure 9 is a diagram of this arrangement.

The output of each proportional counter corresponds to the intensity of the plasma soft X-ray emission from that segment of the cross section. Therefore, each detector output is an indication of the plasma density and temperature at that point. By monitoring these signal levels, the plasma location within the vacuum chamber can be determined. The counters are also spaced closely enough so that several counters receive radiation from the plasma column simultaneously. An approximate profile of plasma density and temperature and its position in the vacuum chamber is obtained by interpolating these signal levels. A shift in the position of the plasma column results in a change in the output levels of the proportional counters. The measured plasma profile will also shift position to reflect this change.

The instrumentation for position measurement consists of the proportional counters, a preamplifier stage and the Alcator data system. This data system is unique
X-Ray Detector Apparatus
Figure 9

- Detectors
- Brass Box
- Collimators
- Vacuum Chamber
- Plasma
because it stores signal wave forms on a magnetic drum and replays them on oscilloscopes. The wave forms have a maximum bandwidth of 300 kHz and a maximum length of 300 ms. With this apparatus, the X-ray emission from any segment of the plasma cross section can be known for any point in time in the plasma shot.

The preamplifiers and proportional counters are housed in a box machined from 1/4" brass plate. The seams of the box, except for the top cover, are soldered together so the box is, as nearly possible, a perfect conducting enclosure. The high conductivity of brass and the thickness of the metal insures that the enclosed electronics are shielded from quickly varying magnetic fields. This precaution was taken to prevent the very powerful fields from the Alcator magnet from introducing noise into the system.

The brass box is mounted on an access port on top of the tokamak. The flange supports the top ends of the collimating tubes. Each collimating tube is capped with a beryllium "window" which is mounted in a flange. The beryllium is a sheet .001 inches thick and serves to pass X-rays while maintaining the integrity of the vacuum system. Each proportional counter, mounted in the brass box, is aligned with one of the beryllium windows. Although the access port has six windows and six views of the chamber
cross section, only four were found to have any appreciable X-ray flux. Thus the brass box only contains four propor-
tional counters.

The type of proportional counter selected for this application was the L.N.D. model #452B. The counter was specified to be filled with P-10 gas at one atmosphere, which is particularly sensitive to the X-ray energy range of interest (1-10 kev) and also has a very high amplification. This counter is an aluminum cylinder 4" long and 1" in diameter. The X-ray window is a 1/2" in diameter, .001 inch thick beryllium sheet. The X-ray entrance port is located on the side of the tube. On the opposite side of the tube from this window is an exit window of the same characteristics. The purpose of this configuration is to minimize the hard X-ray interference with the soft X-ray signal. The soft X-rays, being of lower energy, will generally be stopped by the P-10 gas in the tube and generate an electronic pulse. However, the majority of the hard X-ray flux passes through the gas and both bery-
lium windows, without interacting. Without the exit port, much of this radiation would flouresce on the inside wall of the counter. The flouresced X-rays would be of lower energy and more likely to be stopped by the P-10 gas. Entrance and exit holes were drilled in the brass box, aligned with the beryllium windows of the counters,
to permit the passage of X-ray flux. This design was suggested by Dr. Heikki Helava.

The proportional counter can be modeled electronically as an almost ideal current source. Each time an X-ray photon interacts with the gas, a current pulse proportional to the energy of the X-ray is output. However, the plasma position is indicated by the total X-ray energy flux received by the detectors at any one time. The output of a counter, corresponding to a changing X-ray flux, is a time varying direct current signal. In order to amplify a direct current, the proportional counter must be coupled directly, not capacitively, to the preamplifier stage. Since the preamplifier electronics are at roughly ground potential, the output of the proportional counter (the center wire) must also be at ground. This configuration requires that, to properly bias the proportional counter, the case must be at a high negative potential, approximately 1800 volts, relative to ground. The counters are, therefore, located on insulating bakelite slabs to prevent the cases from shorting to ground through the brass box. The bakelite slabs were drilled and tapped so the counters could be screwed into them. These assemblies were mounted with screws on a plate in the brass box. Holes were drilled in this plate to allow for ample clearance between the proportional counters and
the metal plate. The mounting screw holes in the bake-
lite slabs were milled out as slits so the assembly could
be rotated. When all the detectors were installed on the
plate, they were rotated to align the beryllium windows
with the holes in the box.

The preamplifier stage was designed to meet require-
ments imposed by the output of the X-ray detectors. Since
proportional counters are current sources, a current to
voltage amplifier must be used. This amplifier must also
have a very high gain because the currents from the counters
are very small. A low impedance output is also important
to minimize noise, and to drive the input to the data sys-
tem without distortion. Finally, the amplifier should
have a wide bandwidth because of the high speed of the
X-ray photon pulses. The ideal circuit to meet these
requirements seems to be the inverting feedback configura-
tion using an operational amplifier. The schematic dia-
gram is shown in Figure 10.

Assuming an ideal operational amplifier, the output
voltage of the circuit in Figure 10 equals the input cur-
rent times the feedback resistance. As can be seen from
the circuit equations, the relation:

\[ V_o = -RI_i \]

will be maintained only if \( I_R = I_i \).

Consider an op amp with gain \( k \); then, for a voltage
on the inverting input:

* Although the position measurement does not require resolving
individual photon pulses, we plan to use the detectors for
other experiments.
X-Ray Detector Amplifier Circuit
Figure 10
79)  \( V_o = -kV_i \)

If the input impedance of the op amp is much higher than the feedback resistor \( R \):

80)  \( I_R = I_i \)

The voltage drop across \( R \) must be:

81)  \( V_o - V_i = I_R R = I_i R \)

Now substitute equation 79 for \( V_i \).

82)  \( V_o = -RI_i \)

A resistor value of 20 megohms was chosen so the output voltage would be on the order of 1 volt for the currents expected from the counters. Although the circuit is simple, the choice of operational amplifier is critical. For the circuit to work properly, the input resistance of the op amp must be at least a factor of 10 greater than the feedback resistance. Otherwise the current from the proportional counter will flow into the input of the op amp and not through the feedback resistor.

Only one variety of commercial operational amplifier has an input impedance even close to the 200 megohm value required: This op amp has a Field Effect Transistor input
and has an impedance of $10^{10}$ ohms. The particular type chosen was the National Semiconductor LH0042. The LH0042 also has a wide bandwidth ($\sim 1$ MHz) and a gain of $1.5 \times 10^5$. It is also cheap.

The preamplifiers were constructed on a printed circuit board which was then mounted in the brass box along with the proportional counters. Jacks were provided on the side of the box to connect the preamplifier outputs to the data system and power to the op amps and the proportional counters. In the extremely high magnetic field environment of Alcator, great care was taken to eliminate any potential ground loops. All the signal cables were twisted pairs balanced to ground in shielded wire. The shields were broken between the preamplifiers and the data system so that no ground currents could flow. The high voltage power supply for the proportional counters was connected with high voltage coaxial cable, but it was left floating. The only connection to ground was through the low voltage power supply which provided power for the op amps. This supply was grounded to the Instrumentation ground of the Magnet Laboratory, and the ground was also connected to the brass box.
VIII EXPERIMENTAL RESULTS: DETECTION

With this experimental apparatus, we were able to measure the X-ray flux from the Alcator plasma and determine the position of the column. Three types of experiments were carried out. The first was merely engineering: We found out which tokamak operating regimes provided sufficient soft X-ray radiation to make a position measurement. Both the total signal level and the signal to noise ratio were tested. Secondly, methods for evaluating the X-ray data were developed, and their accuracy tested. After several attempts, a reliable and accurate method was found and proven. Finally, knowing how to calculate the position, the behavior of the plasma column was observed under different operating conditions.

The results of the first set of experiments indicate that the use of X-rays for position measurements is restricted to particular regimes in Alcator. Specifically, in the low density regime, the total signal flux is barely detectable, and the signal to noise ratio deteriorates. Even in the high density regime, the over-all X-ray flux was so low that only three proportional counters provided any reasonable data for position measurement. These three detectors were: the one directly over the center of the vacuum chamber and the two immediately adjacent to it. These two are spaced 4 centimeters inside and 4 centimeters
outside of the center. There is also a fourth counter spaced 8 centimeters outside from the center, but it rarely provided position data. This fourth detector was generally used as a measure of hard X-ray interference with the other detectors.

Figure 11 is a photograph of the three position detector outputs, and the plasma current, as displayed by the data system. This data corresponds to a low density plasma (shot #19 on November 4, 1975). Notice that none of the three X-ray signals shows anything but noise pulses.* It would be impossible to determine plasma position from this information. Contrast Figure 11 with Figure 12, which is a high density plasma (shot #35 on October 24, 1975). Here the noise pulses are small in comparison with the total X-ray flux and a position calculation can easily be made. Our results indicate that usable information is still present at moderate densities in Alcator, on the order of $1.2 \times 10^{14}$ particles/cm$^3$. This is sufficiently low to permit the use of X-rays except at the lowest part of the density range.

In addition to the lack of soft X-ray flux, there is greater hard X-ray interference in the low density regime. One of the principle reasons for hard X-ray production in tokamaks is "runaway" electrons. These are relativistic electrons that cease to collide with the plasma particles.

* Here noise pulses refers to hard X-Ray interference.
Figure 11

Plasma Current
Inside Detector
Center Detector
Outside Detector

Figure 12

Plasma Current
Inside Detector
Center Detector
Outside Detector
and, therefore, attain very large velocities. Because they build up so much energy when they finally do collide with the chamber walls, hard X-rays are generated. Runaway electrons are more likely in a low density plasma because the probability for a collision is less, and, therefore, the chance for attaining relativistic velocity is greater.

The effect of hard X-ray interference is shown in Figures 13 and 14. Figure 13 is the data from a high density plasma, \( N_e = 2070 \times 10^{11} \text{ particles/cm}^3 \) (shot #40 on November 24, 1975). The picture shows the In/Out coil, second outside detector, center detector, and hard X-ray traces. Second outside detector (S.O.D.) refers to the proportional counter spaced 8 centimeters out from the center. Since it rarely provides position information, it can be used to calibrate the hard X-ray (H.X.R.) interference that the other proportional counters receive. Especially in the low density regime, the signal from this detector will be due to hard X-rays. Notice that the H.X.R. level was very low, and the S.O.D. signal was virtually zero for this shot.

Compare Figure 13 to Figure 14. Figure 14 is a low density shot, and again the S.O.D. and H.X.R. traces are shown. The hard X-ray signal has increased dramatically and the second outside detector signal has become significant. Notice also that the S.O.D. signal seems to follow
the H.X.R. trace. Figure 14 is data from shot #8 on November 1, 1975 (the density was not measured).

The second set of experiments evaluated various methods for calculating the plasma position from the X-ray data. The results were tested by comparing them against the position measurement given by the compensated In/Out coils. As discussed earlier, these coils measured the change in magnetic flux between the inside and outside of the torus. The technique is accurate except that the signal is integrated so that it is only an indication of relative position. However, if the plasma column position were known accurately, by some other means, at just one particular time during the shot, then this point could be used to calibrate the In/Out coil signal. This is precisely the technique we used.

Regardless of the evaluation method used, the X-ray data always specified that the plasma is centered when the signals from the outside and inside detectors are identical. Since all the data is recorded simultaneously by the data system, the point in time at which the inside and outside detector signals are equal specifies the value of the In/Out coil signal corresponding to the plasma being in the center of the chamber. The plasma position for the entire shot can then be plotted from the In/Out coil data and using this point as a reference.
Once the plasma position is known, the various techniques for calculating position from the X-ray data can be tested by comparing the results to the known position. The method that proved most accurate, modeled the plasma X-ray profile as a gaussian. Although the actual profile is probably not a gaussian, it is similar to it in certain key respects. First, both the gaussian and the actual X-ray profile attain maximum at the center; both go to zero away from the center; and both are symmetrical. Assuming a gaussian profile, the X-ray radiation is given for any point on the plasma by:

\[ A = A_0(t) e^{-y^2/\gamma^2} \]

\(A_0(t)\) is an amplitude function which is dependent on time only. \(y\) is the distance from the center of the plasma and \(\gamma\) is the size of the gaussian. The object of this derivation will be to calculate the displacement of the plasma from the center, given the X-ray flux levels from the three detectors and this model. In order to be of value, the displacement function must be independent of both \(A_0(t)\) and the size of the gaussian.

Define the following symbols:

\(X\) \(=\) distance of plasma center away from vacuum chamber center. Positive \(X\)-direction is outside
\( \Delta X \) \text{ distance of the three X-ray proportional counters from each other}

\( A_I \) \text{ Normalized amplitude of inside detector}

\( A_C \) \text{ Normalized amplitude of center detector}

\( A_T \) \text{ Normalized amplitude of outside detector}

Express the amplitudes of the three detectors in terms of the deviation of the plasma from center, \( X \), and the spacing of the counters, \( \Delta X \), using the model given by equation 83.

\[
84) \quad A_I = A_0(t) e^{-\frac{(\Delta X + X)^2}{\delta^2}}
\]

\[
85) \quad A_C = A_0(t) e^{-\frac{X^2}{\delta^2}}
\]

\[
86) \quad A_T = A_0(t) e^{-\frac{(\Delta X - X)^2}{\delta^2}}
\]

Take the natural log of these three expressions in order to eliminate the exponent. Rearranging terms and simplifying yields:

\[
87) \quad \ln A_I = \ln A_0(t) - \frac{1}{\delta^2} (\Delta X^2 + 2X\Delta X + X^2)
\]

\[
88) \quad \ln A_C = \ln A_0(t) - \frac{1}{\delta^2} (X^2)
\]

\[
89) \quad \ln A_T = \ln A_0(t) - \frac{1}{\delta^2} (\Delta X^2 - 2X\Delta X + X^2)
\]
Now eliminate all terms except those in $X$ by subtracting equation 89 from equation 87.

90) $\ln A_I - \ln A_T = \left(-\frac{1}{\gamma^2}\right) 4X \Delta X$

Equation 90 gives the position as a function of the two amplitudes, $A_I$ and $A_T$, but it also contains the unknown size factor $\gamma$. Now solve for $\gamma$ and divide it out of equation 90. Eliminate all terms except those in $\Delta X$, with the following equation:

91) $\ln A_I + \ln A_T - 2\ln A_C = -\frac{2}{\gamma^2} \Delta X^2$

Divide equation 90 by equation 91 to eliminate $\gamma$:

92) $\frac{\ln A_I - \ln A_T}{\ln A_I + \ln A_T - 2\ln A_C} = \left(-\frac{1}{\gamma^2}\right) (4X \Delta X) \left(-\frac{\gamma^2}{2\Delta X^2}\right)$

Finally, solve for $X$ in terms of the measured X-ray signals and the spacing between counters: $\Delta X = 4$ cm

93) $X = 2 \left[ \frac{\ln A_I - \ln A_T}{\ln A_I + \ln A_T - 2\ln A_C} \right]$ centimeters

Figure 15 is a plot of the position calculated using equation 93 and X-ray data (from shot #40 on November 24, 1975). The "actual" position is plotted as well, using the In/Out coil data, having been calibrated as discussed earlier. The agreement is remarkable. The slight deviation near the end of the shot is due to three inaccuracies. The first is that the In/Out coils seem to be reliable only when the plasma moves slowly. As soon as rapid changes in position
occur, the In/Out coil signal no longer reflects these shifts accurately. This limitation was discerned whenever the plasma became unstable and made violent shifts in position. The In/Out coil signal was barely affected by these dislocations. Another error was incorporated into the results as the data was reduced. When the signal traces became very small, they were read off as zero. Since the model is based on the gaussian, which goes to zero only at infinity, the calculations were upset by this approximation. The final cause for inaccuracy is the fact that the gaussian does not describe the profile of the plasma precisely. In order to guarantee the reliability of these results, these data were taken on over 500 plasma shots and many were reduced to the form of Figure 15.

The final stage of experimentation consisted of using this model to evaluate how the plasma position behaved in different regimes. Of principal interest was how the plasma position varied with time and how it varied with density. These tests are the culmination of all the engineering and experimentation performed with the soft X-rays so far.

Three representative shots were chosen from the data, having different densities. The positions were calculated and plotted as a function of time. The three shots are:
shot #24 on November 24, 1975, density: \( N_e = 1134 \times 10^{11} \)
Plasma Position
Figure 15

Position in Centimeters

Time in Milliseconds
particles/cm$^3$; shot #40 on November 24, 1975, density $N_e = 2070 \times 10^{11}$ particles/cm$^3$; and shot #56 on November 25, 1975, density $N_e = 4314 \times 10^{11}$ particles/cm$^3$.

Pictures of the X-ray traces from the data system are shown in Figures 16, 17, and 18 for the three shots respectively. The calculated positions as a function of time are shown in Figure 19.

Two important conclusions can be drawn from the results in Figure 19. The first is that a crude measurement of $B_{pol}$ can be made. Recalling equation 83:

$$83) \quad B_v \propto \frac{I_p}{2} \left[ B_{pol} + \frac{11}{2} \right]$$

Since the Alcator vertical field system scales with the plasma current, and assuming the term $11/2$ is constant, then any changes in overall plasma position must be due to the $B_{pol}$ term. As the density is increased, the plasma pressure also increases and $B_{pol}$ increases. The effect of a larger $B_{pol}$ is that a larger $B_v$ is required to maintain equilibrium. Since the vertical field on Alcator is fixed, the increased $B_{pol}$ term forces the plasma to move outward against a vertical magnetic field that is slightly too small. The results in Figure 19 provide a verification to the theory.
Figure 18
Plasma Position
Figure 19

Position In Centimeters

Ne = 4314
Ne = 2070
Ne = 1134

Time In Milliseconds
In addition, the curves in Figure 19 demonstrate that the decay times of the vertical field and the plasma current are different. Since the vertical field decays slower than the plasma current, it becomes too strong and the plasma is driven into the inner wall. The plasma is then extinguished by this complete loss of equilibrium. These results clearly show that the deviations in plasma equilibrium result from the inaccuracies predicted by theory: the $\beta_{\text{pol}}$ term and the differences in decay time constants. With an appropriate but minor modification of the vertical field current, these deviations could be corrected. We have proven a method for position detection, and the first problem is solved.
IX  THE PROBLEM OF CONTROL

The new vertical field system will behave essentially as the old, except that it must be capable of modifying the current to compensate for the deviations in plasma position. An examination of the results in the previous section reveals that the changes in current will be small and, therefore, have to be precise. Also, since the plasma shifts position quickly, the new system must be capable of responding within 10 ms. These requirements are difficult to meet primarily because the controller must be able to handle roughly 3 kA of current in order to generate the fields necessary. Another problem is the fact that it drives the vertical field coils, which are highly inductive loads. We also put two more restrictions on the new design: 1) it build up vertical field current quickly after commutation and 2) it maintain the relationship of vertical field roughly proportional to plasma current. These were the advantages of the first system and should not be sacrificed in the second.

The actual regulation of the vertical field current was accomplished with a new power supply, purchased from the Transrex Corporation. The Transrex (T-rex) power supply has a maximum output of 5 kA and 650 volts; however, this is in the pulsed mode only. The T-rex is unique be-
cause it is entirely programmable. The output of the supply is modulated by an input voltage. The output will follow the input voltage wave form, so that any shape output pulse is attainable. The input has a voltage range of 0-10 volts and draws only a few milliamperes. Either current or voltage programming is available and can be selected from the front panel. In the current programming mode, the T-rex will alter its output voltage to maintain whatever output current is specified by the input voltage. The larger the difference between the actual and specified currents, the larger the voltage the T-rex will exert. The output current is regulated to an accuracy of a few amperes, and is controlled by a signal that can be generated with low-power electronics.

There are several factors that govern the response time of the T-rex to a change in programming signal. The first is that the electronics of the supply cannot respond to input voltage transitions faster than a few volts per millisecond. However, this is far faster than the time scale required for plasma stabilization, which is on the order of 10 ms. The T-rex also has a stability control, which regulates how fast it will alter its output. On the fastest settings the supply will respond in no more than 5-8 milliseconds, which is still within range. The final limitation is that the Transrex drives an inductive
load. Since it has a maximum output voltage of 650 volts, the fastest current slew rate driving the entire vertical field system is:

\[ \frac{dI}{dt} = \frac{V}{L} = 55 \text{ amps/ms} \]

This speed is just fast enough for plasma stabilization.

The Transrex power supply satisfied most of the requirements of the new vertical field system. It is programmable; it can control current accurately; it has a sufficiently fast response; and it could be programmed so that \( B_0 \propto I \). However, because its maximum output voltage is only 650 volts, it could not build up the vertical field current quickly enough after commutation. It also could not start to build up the field early, because then its pulse would be so long that it could not safely dissipate all the power. The problem of the vertical field system control is not so simple that the Transrex could be merely connected in series with the vertical field coils.

An elegant solution was conceived by Dr. Ronald R. Parker. The Transrex was connected to the vertical field coils as shown in Figure 20. The advantage to this system is that the original vertical field circuitry is entirely preserved. The Transrex and the switching ignit on to connect it were merely added on. The new vertical fields system follows the same sequence of operations as before. After commutation the current is built up very rapidly to
Vertical Field System
Figure 20

A.R.U.

MECH. BREAKER

VACUUM BREAKER

FS-

12A
13A
11A
12
13
T-REX

15Ω

T1

ISOLATION

T-REX
the values specified by $B_v \propto I$. The difference in the new system is that at commutation, the T-rex is programmed for $\approx 2.5$ kA of current. It immediately applies 650 volts across its output since no current is flowing through it. Nothing happens because the T-rex ignitron is off. Approximately 40 ms after commutation this ignitron is fired, and the T-rex is shunted across the isolation ignitron. The vertical field current is diverted away from the isolation ignitron through the power supply. As soon as the current through the ignitron reaches zero, it shuts off. Once the T-rex has commutated off the isolation ignitron, the entire vertical field current can be controlled by the programming signal. This technique satisfied all the requirements mentioned for a refined vertical field system. It also has the advantage of being capable of operating without the T-rex and being used exactly like the original system.

In order to build this circuit, the T-rex supply had to be modified. It was originally constructed as a negative ground supply, but as can be seen from Figure 20, the positive output must be at ground potential. The change from negative to positive ground required that the bus-work inside the supply be reworked. In addition, the electronics that regulate the output had to be reversed to reflect the change in polarity. Because of the basic symmetry of the circuit, these changes were performed easily.
The power supply was tested and worked without any problems.

The control instrumentation for the T-rex consisted of two circuits. The first was a programmer, which was capable of producing any shape wave form desired, at any amplitude between 0 and 12 volts, for any length pulse between 50 ms and 1 second. The circuit output consists of 10 sequential square-wave pulses. The amplitude of each pulse can be varied from 0 to 12 volts by adjusting a potentiometer. By appropriately setting all 10 "pots," both the amplitude and overall shape of the 10 pulse sequence can be controlled. The length of all the individual pulses can be varied with another pot, so that the length of the composite pulse can be adjusted.

The programmer circuit is shown in Figure 21. The sequence of operation begins when a start pulse triggers the first one shot. When this one shot times out, it triggers the next one, and the process continues until the circuit has stepped through all 10. All of the one shots produce the same length pulse. The output voltage of each is then divided by a potentiometer. The pulses, which have varying amplitudes depending on the "pot" settings, are finally summed and the composite signal is amplified. The effect is to time multiplex the individual pulses into a single complex pulse. In addition, the time constants of all the one shots can be varied simultaneously by adjusting $R_1$. 

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Programmer Circuit Schematic
Figure 21
The output of the programmer is interfaced to the Transrex power supply through an isolation circuit. Isolation is provided in order to prevent ground loops. In the high current, and hence high magnetic field environment of Alcator, circuits with ground loops would be particularly susceptible to the E.M.F.'s developed by rapidly changing fields. Because the T-rex program signal is so critical to vertical field operation, extra care must be taken to prevent interference resulting from a ground loop. Both ground and common mode isolation is provided by the circuit in Figure 22. The programmer output pulse is coupled to the T-rex through an optical coupler. An optical coupler consists of a discreet Light Emitting Diode and a discreet Phototransistor. An input signal to the L.E.D. causes it to shine on the phototransistor. The phototransistor then conducts because of the light. Although the signal is transmitted from the input to the output of the O.C., the electrical resistance between the terminals is virtually infinite.

The isolation circuit also has a few transistors to amplify the output of the optical coupler. The T-rex electronics power supply (+15 volts) is used to power this half of the circuitry. Finally, filtering is provided to slow down the transition times of the programming pulses. The electronics of the T-rex responds poorly if the slew
Isolation Circuit Schematic
Figure 22
rate of the input pulse is greater than a few volts per millisecond. The filtering is provided in the isolation circuit so that no input signal can exceed this rate.

Six control functions are required to activate the Transrex in the new vertical field system. These pulses (or relay contact closures) are provided by the sequencer, which controls the entire operation of Alcator as well. Fifty seconds before the plasma shot, the sequencer signals to close the main power starter. This breaker connects 4160 volts to the primary coils of the T-rex control circuitry. Then, at commutation, the programmer circuit is pulsed and it brings the T-rex to full output voltage. Roughly 40 ms later the T-rex ignitron is fired, connecting the vertical field coils to the power supply. The vertical field current is now controlled by the programmer pulse. Immediately after the shot the power supply control circuitry is turned off, which shuts down the system. Finally, a few seconds later, the 4160 volts is disconnected by the starter. During the plasma shot, the vertical field current is linearly dependent on the programmer output voltage, as shown in Figure 23. This instrumentation permits complete specification of the vertical field coil current as a function of time throughout the plasma shot.
Figure 23
X EXPERIMENTAL RESULTS: CONTROL

We were able to perform only one full day's experimentation after the new vertical field system was completed. On the second day the Alcator vacuum chamber broke vacuum and further work had to be discontinued. From the data taken, however, two results became clear. First, from an engineering standpoint, the new vertical field circuitry was tested and the system proven as a viable scheme. Secondly, we demonstrated a facile control of the plasma position, which indicates that improvement of the confinement should be possible.

The major problem with the new vertical field system was considered to be the danger of applying an excessive voltage to the T-rex power supply. The vertical field coils, in order to build up the current quickly, are shunted across a very high voltage (on the order of 15 kV). If this voltage, or any large fraction of it, were inadvertently connected across the T-rex, the Silicon Controlled Rectifiers in the power supply would be destroyed. These semiconductors break down if any voltage over 700 volts is applied, and are very expensive to replace (on the order of $20K). The switching of the new system was designed so that no large voltage could appear across the T-rex, but this had to be tested. The switching is very critical;
at commutation a very large voltage spike appears across the isolation ignitron. Only 40 ms later the Transrex is switched across this ignitron. However, our results indicate that this switching was accomplished safely and reliably.

Another potential problem with the new vertical field system could have developed if the isolation ignitron would not commutate off. An ignitron will turn off, and stay off, if the current through it is brought to zero for a sufficiently long time. If the current is not maintained at zero sufficiently long, or, conceivably, if there were a large voltage spike soon after shutting off, the ignitron might turn back on. If this ignitron were to restrike, the T-rex would be shorted, and no control of the vertical field would be possible. Although many plasma shots were fired, there was not a single incidence when the isolation ignitron did not commutate off and remain off.

The second set of tests performed measured the response of plasma column position to changes in vertical field programming. In particular, we wished to prevent the plasma from crashing into the inside wall at the end of each shot. It is this catastrophic loss of equilibrium that extinguishes the plasma long before it would otherwise have decayed out. These tests were performed in the low density regime, so that the X-rays could not be used for a position
measurement, and we had to rely on the in/out coil. If this final instability could be corrected, then both the speed of response and the versatility of the new system would be demonstrated.

The results were dramatic: the plasmas became so long that the data system could no longer display the entire pulse. Critical data had to be displayed on storage oscilloscopes, which could accommodate the longer pulses, but were restricted to only a couple of channels. The position, plasma current, and X-rays were all measured. The vertical field current was also monitored and compared to the programming pulse. Finally, the output of the Transrex was recorded so that we could see that it was switched properly. In order to run a comparison, a few shots were made using the old vertical field system. Because of the versatility of Dr. Parker's design, Alcator could be run under the old system merely by not activating the Transrex control functions.

Figures 24 and 25 depict the plasma data for shot #2 on January 16, 1976, the only day the new system was in operation. This shot was made using the old system. The time scale on these pictures is 50 ms/division and all the data was taken off the data system because the pulse was relatively short (roughly 200 ms). The main field was set to 45 kA and the O.H. current built up to 9.09 kA before commutation. The plasma current, in/out position,
HXR, loop voltage and positional X-ray traces are shown. These are marked on the figures. Figure 26 is the engineering data for shot #2. The currents for the main field, ohmic drive and vertical field are all shown. Of interest is the vertical field trace, which is indicated in the figure. The vertical field current is shown independently in Figure 27. Notice that it reaches a peak very quickly and then decays slowly. The time scale on this data is 5 seconds for the entire sweep. The rapid build up of current, of course, occurs at commutation.

An examination of Figure 25 reveals the typical behavior of the plasma position during a shot. The plasma begins roughly in equilibrium, moves outward slightly, then collapses inward and is extinguished. These results are consistent with Figures 15 and 19, which are the plots of plasma positions from the X-ray data. The object of programming the vertical field was to reduce it sufficiently so that the plasma would not be extinguished.

Figures 28 and 29 are the plasma data from one of the early attempts at maintaining stability (shot #10 on January 16, 1976). Shot #10 used the new vertical field system in which the vertical field was programmed. Already the plasma length has exceeded 300 ms, so that the data system has truncated the latter part of the shot. The plasma current and in/out position are displayed fully in
Vertical Field Current

Figure 26

Vertical Field Current

Figure 27

Current

2.5 kA

Time

-87-
Figure 28

Plasma Current

Loop Voltage

HXR

Figure 29

In/Out Position

Inside Detector

Center Detector

Outside Detector
Figure 30, which is data from a storage oscilloscope. In this case the plasma expanded outward at the end of the shot and extinguished against the opposite wall as before. We had overcompensated for the decay of the plasma current, and as the vertical field was collapsed, the plasma pushed outward. A comparison of this shot with the normal behavior of the position, demonstrates clearly that we were able to drastically shift the plasma column. Although positional stability was not maintained, shot #10 proves that it is possible with the new system.

The effect of vertical field programming can be seen in Figure 31. Compare the vertical field current trace in this picture to the one for shot #2. Both currents rise to the same level at commutation, but in shot #10 the current is brought to zero much faster than it decayed in shot #2. The programming pulse for this vertical field current profile is shown in Figure 32. We tuned the vertical field by adjusting the levels of the various "steps" of this pulse.

The Transrex output voltage for this shot is shown in Figure 33. The time scale is 10 ms/division and the voltage scale is 500 volts per division. This voltage was measured across the isolation ignitron. The oscilloscope was triggered by the high voltage spike that occurs at commutation and can be seen at the beginning of the trace. Approximately 50 ms later the T-rex is switched in and begins to reduce the
Program Pulse

Figure 32

Transrex Voltage Output

Figure 33
vertical field current. The step-like changes in output voltage correspond to the step-like changes of the programming pulse. Data, the same as Figure 33, was taken for every shot to ensure that the Transrex was connected at the proper time.

By adjusting the controls of the programmer, we were able to tune the vertical field current to obtain much improved results over any previous plasma shot. Shot #28 on January 16, 1976 was an all-time record for Alcator! Figure 34 shows the current and in/out traces from the storage oscilloscope. The plasma pulse lasted 650 ms, which is better than 3 times as long as the pulses obtained under identical conditions with the old system (refer to shot #2)! Shot #28 is also more than 200 ms longer than any shot Alcator ever achieved under any conditions at any time. The program pulse and vertical field current for this shot are shown in Figures 35 and 36. The current, X-ray, loop voltage, and position data for the first 300 ms of the pulse are shown in Figures 37 and 38. These pictures were taken from the data system. The results of the experimentation of January 16, 1976, and of shot #28 in particular, demonstrate the incomparable improvements that can be obtained with even crude control of the vertical field. However, the vertical field system can be further refined. Notice that in shot #28, the plasma decays out slowly, and doesn't crash into either side of the chamber.
Figure 34
Figure 35

Program Pulse

Figure 36

Vertical Field Current
XI  CONCLUSION

The preliminary results obtained with the new vertical field system are impressive, but it is still very crude. The vertical field current was specified for each shot by adjusting the program pulse based on the position results of the previous shot. Using this successive approximation technique, the vertical field could be controlled to establish excellent equilibrium. The obvious disadvantage to this technique is that many shots are required to tune the system. Also, if any changes in operating parameters are made, the entire process has to be repeated. The solution is to add real time feedback, which is the third phase of designing a new vertical field system.

The feedback instrumentation would measure the plasma positions, and modify the vertical field in order to maintain plasma equilibrium. Specifically, the feedback circuit would process the outputs of the X-ray detectors and generate a signal corresponding to the plasma position off center. This signal would be added to the original programming pulse. The composite program signal controls the output of the TransreX, and hence would adjust the vertical field to compensate for the plasma deviation off center.
One of the significant results of our experiments is that the instrumentation for X-ray detection and position control was built and tested at a high level of sophistication. All that remains to close the feedback loop is to insert an interface circuit between the X-ray detectors and the control circuitry. This circuit would calculate the position, off center, of the plasma from the X-ray data, and modify the program pulse to compensate.

The results of the X-ray experiments indicate that the plasma position can be calculated accurately from the X-ray data. Using the formula:

\[ X = 2 \frac{\ln A_T - \ln A_I}{\ln A_T + \ln A_I - 2\ln A_C} \]

the position of the plasma was determined as a function of time. This formula can be calculated in real time by an analog computer. The analog computer would be constructed of simple operational amplifier circuits. Operational amplifier configurations of just a few components are capable of adding, subtracting, dividing and taking the natural log of signals. The interface circuit for the feedback loop would consist of this kind of analog computer, and a circuit to sum the output with the program pulse.
With this kind of feedback capability, the vertical field system could maintain equilibrium without needing many shots to tune. Any errors in programming would be compensated for by the feedback network. In addition, if the composite pulse were recorded on a storage oscilloscope, reprogramming would be made simple. The program pulse could be adjusted to match the composite pulse for the previous shot. The feedback system would also permit changes in other operating parameters without affecting equilibrium. Positional stability would be virtually guaranteed for the duration of the plasma pulse, for all plasma shots, under nearly all conditions.
I wish to express my deepest respect and affection for the Alcator staff. It has been a great pleasure, and a learning experience, to have worked with them. In no other area of my life have I ever witnessed a single group of people as intelligent, competent, friendly and dedicated. This project could not have been even contemplated without their willing assistance. I wish to mention a few names in particular.

Sam Callogero and Jimmy Maher helped in rebuilding the vertical field circuitry. They would build or change anything on a moment’s notice. They also helped run the machine from down in the cell. Jimmy Maher also took the data on the switching of the Transrex.

Bob Childs built the arc-extinguisher for the A.R.U. starter. This starter was not working properly under the load, and without it we could not have realized the vertical field system as it is today.

Heikki Helava offered very important suggestions for X-ray detectors. Dr. Helava is an expert in X-ray physics and his help was invaluable.

Edward Thiebault built the brass box for the X-ray detectors. His work was magnificent; there were many stringent requirements put on the design because of the
harsh environment of Alcator, and Ed not only met them, but produced a piece of precision gear as well. It is rare to find men who are as much of a craftsman as he.

Jamie Replogle typed this entire thesis, proofread, corrected my mistakes and deciphered my writing. She also did this on a moment's notice -- for someone who was desperate to get it completed on time.

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Frank Silva and Charlie Park are responsible for keeping Alcator on the air. They are geniuses at finding mistakes and solving problems. They also double and triple checked everything I did, and made sure that I was safe in the high-voltage environment of the Alcator anti-transformer room. I am probably still alive due to their caring. They spent long nights and hard days putting this system together. Not only is most of the work theirs, but most of the inspiration, and most of the solutions to problems. I cannot cite specifically what they did -- they did everything.

The conceptual design of the vertical field system is the creation of Dr. Ronald R. Parker. He also taught me the theory in the introduction of this thesis. Ron Parker
is a fine man to work for: he solved all the problems I couldn't, allowed me to solve all the problems I could, allowed me more responsibility than I deserved, was not mad when I really screwed up, and was a whole lot of fun to work for. As the leader of the Alcator group, he has earned the respect of the entire staff.

Finally, I would like to thank Professor Rainer Weiss and Dr. Robert Taylor for getting me started in it all . . .

--- Mark M. Pickrell