Essays in Banking Theory and Corporate Finance

by

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Abstract

This dissertation presents three essays in banking theory and corporate finance. The main thesis in the first essay is that increases in liquidity of secondary markets constitute a simple and consistent explanation for several of the trends observed in the banking industry in the past several years. In the model of Chapter 1 banks perform the functions of liquidity provision and monitoring of investments and I investigate the consequences of liquidity increases in the secondary markets in which banks trade. The model rationalizes securitization as a device against adverse selection in the loan market and generates as implications a more risky investment policy for banks as liquidity of secondary markets improve and a negative correlation between variance of liquidity shocks and the riskiness of a bank's portfolio.

In Chapter 2 I present a model of competition in the banking industry based upon the interplay of two factors: the level of capitalization of banks and their ability to monitor different types of projects (i.e. expertise.) In a moral hazard setting with limited liability, banks have to receive some rents to induce them to perform their monitoring services diligently. The size of those rents is decreasing both in banks’ expertise and in the amount of capital that they are able to commit to the project. This leads to a trade-off between capital and expertise that is used to investigate how shocks to the supply of bank capital and to interest rates affect competition and monitoring efficiency in the banking sector.

Finally Chapter 3 develops a learning model to analyze the maturity structure of debt and its evolution on time. The maturity choice is between short-term (one period debt) and long-term (two period debt.) While short-term debt promptly incorporates relevant information that affects the continuation decision of real investments it also induces a signal jamming effect that can disrupt the production process of firms. A dynamic version of the model allows the analysis of the evolution of financial maturity. In the dynamic version, growth prospects and firm age are naturally related to the choice of financial maturity.

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0.1 Preface

In the last two decades, economists have studied how financial considerations interact with actual decisions over real investments to determine the allocation of resources in capital markets. Since the celebrated paper by Modigliani and Miller (1958) many situations and institutions have been analyzed in the search for a general theory of corporate finance. However the main issue in capital markets still requires further investigation: how do people with ideas but without money interact with people with money and no ideas to allocate capital in productive investments? This dissertation can be seen as a modest contribution to the intellectual effort to construct such theory of corporate finance.

The first two chapters deal with institutions, financial intermediaries, and the third with contracts, and in particular with the issue of duration of financial contracts. In addition to the theoretical nature of the research, this dissertation attempts to shed light on relevant issues recently reported by the empirical literature in financial economics.

In Chapter 1, by integrating the basic building blocks of the theoretical literature in financial intermediation, I build a framework that encompasses several of the observed trends in the U.S. financial intermediation industry (see Boyd and Gertler (1993)). One of the central results of that chapter is that the liquidity and the profitability of investments can have opposite implications in the compositions of portfolios of the financial intermediaries.

Chapter 2 provides a model of competition for the banking industry based on the interaction between the level of capitalization of banks and the technological capabilities that they have to perform the monitoring function of investors. This question is of particular interest due to the process of financial deregulation that has occurred in the U.S. financial industry in recent years. The relevance of having financiers who also have expertise in business is shown in Berger et al. (1995). Among other things they show that the size of financier and the size of the borrower are empirically positively correlated, which suggests that there are technological complementarities and a matching aspect in lender borrower relationships.
Finally Chapter 3, although more conceptual in nature, provides testable implications on maturity of financial contracts which can be compared with empirical regularities about maturity that have been documented by financial literature (see Barclay and Smith (1995)).

Since each of the essays includes its own introduction I will stop any further discussion in this preface. I simply hope that the reader will find these three essays as useful tools to understand a bit the intricate relationships between financial and real aspects of investments and how these intricacies manifest themselves in the real world.
Bibliography


Chapter 1

On the Effects of Liquidity Changes in Secondary Bank Markets

1.1 Introduction

The modern literature on financial intermediation has assigned two main functions to banks:\(^1\)\(^2\) monitoring of investment and liquidity provision.\(^3\) For example Freixas and Rochet (1995) argue that “a bank is an institution whose current operations consist in making loans and supplying deposits.”\(^4\) These banks functions are what any financial system must accomplish. According to Merton (1992), “a well developed smooth functioning financial system facilitates the efficient life-cycle allocation of household consumption and the efficient allocation of physical capital to its most productive use in the business sector.” The

\(^1\)See Bhattacharya and Thakor (1993) for a survey of the modern literature in financial intermediation. 

\(^2\)Throughout the paper I will use the terms “banks”, “banking industry” and “financial intermediaries” interchangeably.

\(^3\)According to the American law, these two functions are what define a commercial bank. The 1970 amendment to the Bank Holding Company Act of 1956 defines a commercial bank as an institution that “(1) accepts deposits that the depositor has a legal right to withdraw on demand and (2) engages in the business of making commercial loans.” See Sinkey (1992) page 44 for details.

\(^4\)See page 2 of Freixas and Rochet (1995).
economic literature has explored both functions in detail. For instance, Diamond (1984), Ramakrishnan and Thakor (1984) and Boyd and Prescott (1986) are three of the main early papers that focus on the role of bankers as monitors, and Bryant (1980) and Diamond and Dybvig (1983) have formalized the liquidity provision function, focusing on the implications of deposit contracts.

This chapter investigates the consequences of the fact that banks are liquidity providers that themselves interact in illiquid markets. I present a model that builds on two of the basic models in the literature of financial intermediation: Diamond and Dybvig (1983) and Diamond (1984). Diamond and Dybvig (1983) solve the problem of optimal liquidity provision in an economy with exogenous liquidity shocks. Diamond (1984) relies on monitoring costs as a source of illiquidity and investigates the optimal contract among financiers to minimize the illiquidity cost. In my model, banks, with similar features to the ones in the previous two articles, interact in a market. The interbank market is affected by asymmetries in information that make the assets traded in the market endogenously illiquid.\textsuperscript{5} The idea is to analyze how the optimal provision of liquidity is altered by changes in the illiquidity of secondary bank markets, taking into account that the functioning of these secondary markets is influenced by the informational characteristics of the assets that banks produce.

The model features both moral hazard and adverse selection. Moral hazard exists because the outcome of some of the investment cannot be freely observed by outsiders. Adverse selection arises because after monitoring, monitors have better information and also incentives to transfer projects with low return to uninformed agents. Uninformed agents have to incur a cost of re-monitoring projects that depends on the number of traded projects but not on the quality of the projects. As a consequence the cost of obtaining liquidity in the market depends negatively on the quality of the loans transacted in the interbank market.

Empirically the purpose of the paper is to understand recent events in the banking industry. Boyd and Gertler (1993) identify the following trends in the banking industry:

1. More risky policies on the asset side (more loans and less cash and reserves).

\textsuperscript{5}For a model in which asset trading is motivated by regulatory reasons, see Carlstrom and Samolyk (1995).
2. Decline in the importance of checkable deposits on the liability side.

3. Growth of off-balance sheet activities, increase in securitization and elimination of high quality assets from the balance sheet.

4. Positive correlation between size and risky activities undertaken.

5. More competition.

6. Substantial increase in loan sales among banks.\(^6\)

The main thesis of this paper is that new or more efficient (liquid) secondary markets is affecting fundamentally the behavior of banks.\(^7\) In particular the following implications arise:

1. A more liquid secondary market of loans among banks increases the volume of transactions in that market (#6 above).

2. The securitization process can be understood as a reaction to a problem of adverse selection that plagues the secondary market for loans (#3 above).

3. The increase in liquidity in the secondary market changes the composition of bank portfolios: banks invest more in risky assets and less in riskless ones (#1 above).

4. There exists a positive relationship between bank size and the riskiness of the bank portfolio (#4 above).

To be sure, considerations of banking capital and regulatory changes are left out of the picture and have a role explaining those changes as well. For example, the empirical

---

\(^6\)For instance, in the 1980s, commercial and industrial loan sales among banks grew tremendously: from $26.7 billion in the second quarter of 1983 to a peak of $290.9 billion in the third quarter of 1989. The growth in traditional loan markets has been accompanied by the development of new markets. Loans of longer maturities and loans representing riskier firms are now traded among banks. See Gorton and Pennachi (1990) for details.

\(^7\)The development of new markets and new market practices has been spurred by regulatory and technological changes. Commercial banks investments in information technologies increased nearly fourfold in the eighties. In the nineties they have spent about $15 billion annually in hardware and software combined. These heavy investments have created new banking services and have increased the productivity in the services already provided. (See Steiner and Teixeira (1990)).
study by Berger et al. (1995) notes such important changes in the banking industry as the
progressive dominance of the market by the larger banks, the increase of competition for
deposits, the progressive substitution of banks with alternative sources of financing, and
the massive introduction of electronic interfaces. None of those changes is incompatible
with the analysis I perform here. What makes the analysis in this paper interesting is its
parsimony: a simple hypothesis an increase in liquidity of secondary markets is able to
produce several of the recently observed trends.

The organization of the paper is the following. In section 2 the model is developed. In
section 3 the model is analyzed and the main results of the paper are presented. Section 4
concludes. Some proofs and other analytical details are in the appendix.

1.2 The model

The building blocks of the model are variations of two representative papers of the mod-
ern literature on banking theory: Diamond and Dybvig (1983) and Diamond (1984). To
integrate both models some modifications have to be made. The purpose of these modifi-
cations is to create a coherent framework where banks’ assets and liabilities can be jointly
analyzed.\footnote{After the model is presented I discuss the basic differences and simplifications made with respect to
those articles.}

1.2.1 Activity of a particular bank

I consider an economy formed by several units (cities) that are themselves constituted by the
classes of agents described below. I am looking for a model of meaningful bank interaction
in secondary markets. Cities are an artifact used to model from first principles the existence
of different banks.

In this subsection I describe the interaction among agents that belong to the same city.
In the next subsection I consider the interaction across different cities.
Timing

The action in the economy occurs in three periods:

- In $t = 0$ investments are made. Investment consists of a portfolio choice between a safe and a risky asset. The characteristics of these assets are discussed below.

- In $t = 1$ consumers suffer a liquidity shock. Some liquidation of assets can be made.\(^9\)

- In $t = 2$ investments pay off. Contracts are enforced and the economy ends.

<table>
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<th>$t=0$</th>
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<td>Investment</td>
<td>Liquidity Shock</td>
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Timing of the model

Agents

The following classes of agents exist in each city:

A. Consumers

There is a continuum of consumers of measure 1. They are endowed with one unit of good in the first period $t = 0$ and are subject to a liquidity shock that makes their tastes at $t = 1$ uncertain at $t = 0$. Their preferences can be represented as:

$$V(c^i_1, c^i_2, \theta^i) = \begin{cases} U(c^i_1) & i f \quad \theta^i = \theta^e \\ U(c^i_2) & i f \quad \theta^i = \theta^i \\
\end{cases} \quad (1.1)$$

The variable $c^i_n$ represents the consumption of agent $i$ in period $n$. $U(\cdot)$ is a utility function with the usual properties: $U'(\cdot) > 0$ and $U''(\cdot) < 0$, and $\lim_{c \to 0^+} U'(c) = +\infty$.

\(^9\)The characteristics of liquidation will depend on the nature of the assets and the availability of markets.
The parameter \( \theta^i \) represents a liquidity shock suffered by consumer \( i \) before \( t = 1 \). I assume that \( \theta^i \) is distributed as follows:

\[
\theta^i = \begin{cases} 
\theta^i_e & \text{with} \quad \text{prob} = \frac{1}{2} \\
\theta^i_l & \text{with} \quad \text{prob} = \frac{1}{2}
\end{cases}
\]  
(1.2)

For an *early* consumer (one who has liquidity needs at \( t = 1 \)) the variable \( \theta^i \) takes the value \( \theta^i_e \). For a *late* consumer \( \theta^i = \theta^i_l \).

Aggregate consumption depends on the joint distribution of the variables representing the liquidity shocks of individual consumers. Let \( \Phi \) represent the measure of agents with liquidity needs. \( \Phi \) is distributed as:

\[
\Phi = \begin{cases} 
s & \text{with} \quad \text{prob} = \frac{1}{2} \\
1 - s & \text{with} \quad \text{prob} = \frac{1}{2}
\end{cases}
\]  
(1.3)

where \( s \in (\frac{1}{2}, 1) \).

This is the simplest distribution that generates aggregate uncertainty in consumer demand.

To simplify contracting I make Assumption 1:

**Assumption 1** *The individual shocks, \( \theta_i \), are public information.*

Assumption 1 allows consumers in the same city to write contracts on the variables \( \theta^i \).\(^{10}\)

**B. Entrepreneurs**

There is a continuum of measure \( g \) of entrepreneurs \( (g > 1) \). Entrepreneurs do not have any endowment of the consumption good but have access to a risky investment opportunity that is described below. They are risk neutral and have a zero reservation utility.\(^{11,12}\)

---

\(^{10}\)Later the optimal contract to provide liquidity to consumers is obtained. Because of Assumption 1, incentive compatibility constraints can be ignored in the optimal liquidity provision contract.

\(^{11}\)It will be clear later that the utility functions of entrepreneurs do not play a decisive role in the analysis.

\(^{12}\)To solve contractual problems, an entity that I refer to as a *bank* will endogenously arise. Given that this entity will be only a contractual agreement without any additional technological capability, I have chosen not to present it as a different class of agents in the economy.
Investment possibilities

Two kinds of investments are possible:

A. Safe Investments

Safe investments return $R$ units per unit of investment, irrespectively when they are liquidated. Without loss of generality I make:

Assumption 2 Safe investments are storage technologies: $R = 1$.

What is important about safe investments is not their profitability, but that there is no loss from early liquidation.

B. Risky Investments

Risky investments, available to the entrepreneurs require one unit in $t = 0$ and give $\hat{y}$ units in $t = 2$. If liquidated in $t = 1$ these investments do not produce anything.

The outcome of each risky investment is characterized by the following distribution:

$$\hat{y} = \begin{cases} y_g & \text{with probability } \mu \\ y_b & \text{with probability } (1 - \mu) \end{cases}$$

(1.4)

where $y_g > y_b$.

Assumption 3 The return of investment projects of different entrepreneurs are independent.

For future reference define the expected gross profitability of a risky investment:

$$B \equiv E(\hat{y}) = y_g \cdot \mu + y_b \cdot (1 - \mu)$$

(1.5)

Remark 1 The risk of a consumer's investment can be separated into two components:

- "Maturity"risk: due to the shock in the liquidity needs of consumers. Given the correlation in consumers' liquidity needs, maturity risk is partly non-diversifiable.
• "Intrinsic" risk. The outcome of the project at $t = 2$ is unknown. Assumption 3 makes this risk diversifiable.

Remark 2 Both sources of risk will play a role in the results and in the institutions that endogenously arise in the model. Maturity risk opens the possibility of beneficial trades after the liquidity shock occurs. Intrinsic risk generates an adverse selection problem between the original financier and any new investor as explained below. This problem precludes a smooth functioning of the market where financiers trade the loans.

Remark 3 Throughout the paper I refer to diversification in several instances. If the funds to be invested in the risky projects are large enough relative to the amount that any of the individual projects requires, then, by Assumption 3, the average result of the portfolio is obtained almost surely.¹³ For instance, if an amount $v$ is invested in risky projects, I will treat the outcome of that investment as equal to $v \cdot B$ with certainty.

Informational Structure and Monitoring Technology

Risky investment require monitoring to verify the actual outcome of the investment. The following informational assumptions precise the nature of the information asymmetry between entrepreneurs and financiers.¹⁴ These assumptions will shape the functioning of institutions analyzed in the next subsection.

A. Assumptions before monitoring is performed

At the time of the investment ($t = 0$) no asymmetry of information between financiers and entrepreneurs exists. Once the investment is made and before monitoring is possible, the entrepreneur in charge of the investment learns what the result of the investment will be. Any other agent has to incur a monitoring cost to observe (and collect) the returns from the investment project.

Assumption 4 Without monitoring it is impossible for anyone other than the entrepreneur to collect anything from the investment project.

¹³Judd (1985) gives the precise mathematical conditions to justify this diversification result.
¹⁴Financiers in this economy will be the consumers or someone who represents them, see below.
Note that Assumption 4 implies that to collect even $y_b$, the amount corresponding to the worst possible payoff, a monitoring cost has to be incurred.\footnote{An alternative set-up would be to have a zero return in the support of the probability distribution that characterizes the investment projects. Without any monitoring, it will always be claimed that the result of the investment was zero, and to recover something, monitoring will have to be performed. Another interpretation is that monitoring enables the monitor to transform the result of the investment into consumption goods. Without the technology, the payoff of the investment is useless.}

B. Assumptions about monitoring technology

Monitoring is crucial in the analysis that follows. I consider that the following two monitoring technologies are available:

- "Early" monitoring which must be done before $t = 1$ and costs $k_0$ per project.
- "Late" monitoring which can be done after $t = 1$ and costs $k_1$ per project.

I assume the following relationship among monitoring technologies:

**Assumption 5** Early monitoring is cheaper:

$$k_0 < k_1$$ (1.6)

Assumption 5 captures the idea that the original financier of a project has an advantage in knowing about the project development compared to an agent that buys into the project at a later stage.

Both monitoring technologies assume a constant cost per project. Hence, the monitoring cost of a risky portfolio increases proportionally with the number (measure) of projects monitored. For instance, if late monitoring occurs over a measure $z$ of projects the total monitoring cost incurred would be $z \cdot k_1$.

**Assumption 6** Early monitoring is free:

$$k_0 = 0$$ (1.7)
Assumption 6 is made for simplicity. It asserts that the original financier of the project does not need to dedicate any resources to find out what the result of the project was. This simplifies matters because now early monitoring will always be performed. In general this will be the case as long as early monitoring is cheap enough relative to late monitoring.

C. Assumptions after monitoring is performed

Assumption 7 *Information learned in the monitoring process cannot be transferred.*

Assumption 7 is also made for simplicity. As long as there is a cost of transferring the monitored information the same qualitative results will hold. Note that because information cannot be transferred, an asymmetry of information will exist between an early monitor and any agent that wants to acquire the rights over the project from the early monitor. This information asymmetry is crucial for understanding the institutions and practices that arise in the marketplace.

It is now possible to relate the expected profitability of the different classes of projects:

**Assumption 8 In expectation, risky assets are more profitable than safe ones.**

\[
y_g \cdot \mu + y_b \cdot (1 - \mu) > 1
\]  

(1.8)

Note that risky investments do not dominate safe ones because of the uncertainty over the timing of consumption.

The optimal contract as a bank

Consumers have to solve two problems in this economy. One is the allocation of resources between risky and riskless investment opportunities. The other is the proper arrangement

---

\^16\text{Without Assumption 6 the inequality that should hold is:}

\[
y_g \cdot \mu + y_b \cdot (1 - \mu) > 1 + k_0
\]
among them to perform the monitoring activity. In this subsection I argue that a contractual arrangement among consumers that resembles a bank constitutes an efficient way of solving both problems.

The arguments to justify a bank as the optimal arrangement are the same as those in Diamond and Dybvig (1983) and Diamond (1984). First, as in Diamond and Dybvig (1983), agents would like to get insurance for the eventuality that they may suffer a bad liquidity shock in \( t = 1 \). As a consequence, at \( t = 0 \), they will form a coalition (a big contract) to insure each other. The larger the coalition of agents that constitutes the insurance agreement, the less this coalition is subject to unfavorable draws of their members. This suggests a coalition that includes all the consumers as an optimal contractual arrangement. This coalition of agents will choose the aggregate investment portfolio and will determine the liquidation policy and payoffs to its members depending on the aggregate liquidity shock.\(^{17}\)

Secondly, as in Diamond (1984), to avoid the duplication of resources in the monitoring activity, it is efficient to delegate all the monitoring activity to a single agent.\(^{18}\) The question of the incentives of the delegated monitor arises. However, again by diversification, if the set of projects that this delegated agent has to monitor is very large then the agency costs becomes negligible and the problem of monitoring the delegated monitor vanishes.

I will not give the full analytical details of the previous considerations. In particular I will not go through the details of the incentive contract for the delegated monitor. The purpose of this work is not to provide a theory for the existence of banks but to consider the implications of the interaction of the two main functions that characterize a bank. From now on, I simply assume that cities are organized in a big contract that I call \textit{bank}. In subsection 2.2 the analysis of the interaction among cities will be carried out as the interaction of these

\(^{17}\)Only coalitions large enough to diversify fully will produce the maximum level of insurance for its members. In this setup, with a continuum of agents, any coalition of positive measure will produce a perfectly diversified outcome. The variable that determines the liquidity needs of the coalition will be the same (up to scale) as the variable that determines the liquidity needs of the city as a whole. Without loss of generality for the analysis that follows, I assume that all the agents in a city form a single coalition.

\(^{18}\)A caveat is that in Assumption 6, it was stated that \( k_0 = 0 \). This means that no duplication costs occurs in the monitoring activity. When late monitoring has to be performed, the monitoring cost \( k_1 \) is strictly positive and duplication costs are therefore exists.
banks (one per city).

Banks provide liquidity to their members but own intrinsically illiquid assets. I will explore how the illiquidity of the assets changes with the structure of the secondary market.

Remark 4 Relationship between the model and its building blocks.

As stated before, this model combines two classic models in the banking literature, Diamond and Dybvig (1983) and Diamond (1984). With respect to the model of Diamond and Dybvig (1983), three features have been altered: observability of individual shocks, introduction of aggregate risk and inability to liquidate the risky technology early. First, in Diamond and Dybvig (1983) shocks (and consumption) of individual agents were not observable. In the model built here, shocks are observable. The unobservability of shocks generated the possibility of bank runs; I have chosen to abstract from those considerations to concentrate on the liquidity implications. Second, in the basic model in Diamond and Dybvig (1983) there was no aggregate uncertainty: the liquidity needs in \( t = 1 \) could be perfectly foreseen in \( t = 0 \) and only bank panics prevented that demands for funds are different from expected. Here the aggregate uncertainty plays an important role because it generates an extra demand for liquidity, essential for understanding banks' interactions. Third, in Diamond and Dybvig (1983) there was no proper portfolio choice. Having the possibility of early liquidation without penalty, the risky asset dominated the riskless one, and all initial endowments were allocated to it.

I use the informational structure of Diamond (1984) to conclude that there will be a delegated monitor in the economy. However three technical details have been changed: there is a continuum of projects, those who invest in the bank are risk averse and finally the risky investment has two possible outcomes. First, the continuum of projects permits diversification results without using limit arguments. This turns out to be very convenient for the analysis of the interaction among banks. Second, risk aversion generates a demand for liquidity but, by diversification, it does not interfere with the optimal monitoring arrangement among agents. Third, the two outcome distribution simplifies greatly the analysis of bank interaction.
1.2.2 The interaction among banks

Because of the aggregate uncertainty banks provide insurance against liquidity shocks. Interaction with other banks can improve the insurance. To analyze this inter-bank insurance, it is necessary to describe the uncertainty across cities.

Uncertainty across cities

There is a continuum of cities of size $N$. Cities are ex-ante identical. What differentiates them is the realization of the variable, $\Phi$, that represents the aggregate liquidity shock of their inhabitants (consumers).

I analyze the simplest case that makes interactions between banks worthwhile.\textsuperscript{19} Half of the cities (a measure $N/2$ of them) suffer an early liquidity shock (a fraction $s > \frac{1}{2}$ of each city population are early consumers) and the other half of the cities does not suffer that shock (a fraction $1 - s$ of each city’s population become early consumers.) For simplicity, I will make the following rather extreme assumption:

Assumption 9 Banks only interact after the liquidity shock has occurred (at $t = 1$).

Without Assumption 9 contracts that resemble a big bank (that groups cities) would emerge as optimal contractual arrangements. A big bank would provide perfect insurance for all the consumers in the economy because there is no aggregate uncertainty.

By Assumption 9, contracts between banks are severely limited. Only transfer of rights over loans in banks’ portfolios will occur. More complicated contracts such as issuing bank equity or debt are also excluded by Assumption 9.\textsuperscript{20}

An alternative rationale for precluding the existence of big banks can be found in regulatory restrictions. For example, the U.S. banking industry has been traditionally subject to both geographical limits and antitrust restrictions. Still, regulatory limitations provide

\textsuperscript{19}This setup is similar to that in Bhattacharya and Gale (1987).

\textsuperscript{20}There could be other ways of modelling these contractual limitations. For instance, this setup could be enriched by introducing an additional factor only observable by banks that affects the value of its portfolio (and therefore its solvency). In that case the fact that a bank wants to obtain liquidity could mean either that it suffered a liquidity shock or a solvency shock. This would eliminate some of the contracts excluded by Assumption 9 in this setup and would make the analysis more cumbersome.
at most a partial answer to the question of why there is not a single bank (or by the same token, a single firm) in the economy. A deeper answer is beyond the scope of this paper.

**The secondary loan market**

Consider a secondary loan market that allows banks to sell part of the illiquid projects in their portfolios in case of a bad liquidity shock. Because the nature of the risky investments, one can deduce that there will be transaction costs in this market. An agent buying a loan representing a risky investment has to incur a monitoring cost to collect any amount from the investor. This duplication cost is a transaction cost that reduces the liquidity of loans.\(^{21}\) But this is not the end of the story. The original financier, having monitored the project early on, has an informational advantage over potential buyers leading to an adverse selection problem which also increases the illiquidity of the secondary loan market.

**Remark 5** *Illiquidity in the secondary loan market occurs for the following reasons:*

- **Moral Hazard** because buyers have to re-monitor the investment projects that they buy at a cost \(k_1\) per unit.

- **Adverse Selection** because sellers always have an incentive to sell a portfolio of bad projects. The main consequence is that for any amount of liquidity to be raised from the market, sellers have to sell a larger portfolio than they would under perfect information.

Summing up, any seller that participates in this secondary loan market suffers a liquidity cost \(LC\) equivalent to the product of the late monitoring cost parameter \(k_1\) and the size of the traded portfolio \(M\):

\[
LC \equiv k_1 \times M. \tag{1.9}
\]

Next, I analyze this liquidity cost in the presence of different market regimes and institutions.

\(^{21}\text{Note that any transmission costs among parties such as legal fees will introduce an illiquidity cost in the market.}\)
The functioning of the secondary loan market

Consider the situation of a seller in the secondary loan market. Having invested an amount of resources in riskless and risky assets of \((x, 1-x)\) respectively, the seller's portfolio has the following characteristics:

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th></th>
<th>VALUE (if kept)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe Investments</td>
<td>(x)</td>
<td>(x)</td>
</tr>
<tr>
<td>Good Risky Investments</td>
<td>((1-x) \cdot \mu)</td>
<td>((1-x) \cdot \mu \cdot y_g)</td>
</tr>
<tr>
<td>Bad Risky Investments</td>
<td>((1-x) \cdot (1-\mu))</td>
<td>((1-x) \cdot (1-\mu) \cdot y_b)</td>
</tr>
</tbody>
</table>

I consider three different forms of organization of the secondary loan market (three regimes.)

Regime 1: Regular Market

In the Regular Market regime (RM) sellers choose which items are for sale and buyers limit themselves to accept or to refuse the supplied items. In this model, due to the composition of sellers’ portfolios and due to the inability of buyers to distinguish among loans, sellers cannot commit to sell good loans and therefore cannot avoid that buyers value the loans on sale as representing bad risky investments.\(^{22}\) This is costly for sellers because they have to liquidate more projects and buyers incur a monitoring cost proportional to the number of projects.

Regime 2: Securitization

The Securitization regime (S) is one way to reduce the information problem. If buyers could randomly pick loans from sellers’ portfolios, then both buyers and sellers would be in a symmetric information position. Furthermore, when a buyer gets a sizable part of the

\(^{22}\) To be precise the first loans put on sale will be priced as a bad one. Only if a quantity larger than the share of bad loans in the original portfolio \((1-x) \cdot (1-\mu)\) is on sale will the market begin to value loans as successful ones.
portfolio of a seller, then by diversification, the buyer's portfolio becomes diversified and earns the average return of the seller's portfolio.\textsuperscript{23}

**Regime 3: Cherry Picking**

It is frequently observed that in liquidation procedures the more valuable items owned by a seller are the first to be sold. This corresponds to Cherry Picking in which the good projects are the first to be traded. Cherry Picking requires that buyers are able to identify the quality of sellers' projects. Any device or institution that helps to identify projects would be valuable (e.g. brokerage activities.) Note that even though buyers know exactly the quality of loans, they have to incur a monitoring cost to collect from the entrepreneur in \( t = 2 \). This collection cost generates illiquidity also in this regime.

**Market regimes and informational requirements**

The three regimes that have been described demand different amount of information from buyers. In the Regular Market regime, buyers only need to have general knowledge of the conditions of the economy (the percentage of bad projects in the economy and the size of the portfolio of the seller). In the Securitization regime buyers must additionally know sellers' portfolio. Buyers have to pick from the "right" choice set when choosing the loans (the choice set that guarantees the average profitability). Finally in the Cherry Picking regime, buyers have to distinguish the quality of the projects.

Some of the interpretations of the results have to do with the intensity of the information requirements in the regimes, along with the possibilities of obtaining more cheaply the relevant information for asset trading if new technologies develop. To facilitate comparisons, I will model each regime as requiring a fixed cost to acquire the necessary information.

Let \( CR_i \) (where \( i = RM, S \) and \( CP \)) represent the informational cost of the market regime, the expense that seller must incur for his portfolio to be traded.

\textsuperscript{23}I refer to this process as securitization because it resembles the securitization procedure in real economies. The securitization process in financial markets essentially consists in pooling a homogeneous class of assets and selling participation in the pool. Shares give the average return of the pool, which is normally public information.
Assumption 10  Market regimes can be ordered according to their informational cost:

\[ CR_{RM} < CR_S < CR_{CP} \]  \hspace{1cm} (1.10)

I will also make the following assumption about the sensitivity of this market regime to the information technology:

Assumption 11  Markets regime can be ordered in terms of their sensitivity with respect to improvements in information technologies, represented by \( g \):

\[ \left| \frac{\partial CR_{RM}}{\partial g} \right| < \left| \frac{\partial CR_S}{\partial g} \right| < \left| \frac{\partial CR_{CP}}{\partial g} \right| \]  \hspace{1cm} (1.11)

Assumptions 10 and 11 capture the idea that, while more sophisticated market regimes are more costly, the progress in information technologies benefits them more.

1.3  Analysis of the model

To solve the model several elements have to be worked out: first, the contract that consumers in each city sign with their bank; second the contract that banks as financiers sign with entrepreneurs; and finally, the contractual arrangements that transfer loans among banks.

The problem is simplified by considering a contractual structure between financier and entrepreneurs that assures that what financiers obtain from the projects of entrepreneurs is independent of the rest of the elements of the model.

I assume that financiers keep all the payoffs that investments produce \( \hat{y} \) net of monitoring costs. This can be justified on several grounds. On the one hand, the measure of entrepreneurs in each city is larger than the measure of funds: the total supply of funds is smaller than the total demand and therefore supply determines the price.\(^{24}\) On the other hand altering this feature does not changes matter sustantively. As long as what the

\(^{24}\)Alternatively one could think that entrepreneurs compete \( \hat{a} \) la Bertrand or that the financiers have monopoly power.
financier keeps from the projects is independent of the number of projects, the analysis performed here is valid.

1.3.1 A benchmark: the “autarky” case

As a benchmark I analyze the case of “autarky” (a situation without secondary loan market.) In autarky each bank provides the best possible insurance to its constituents without relying on any external market. Autarky may occur because regulations expressly prohibit transferring loans from originators to secondary financiers, because the organization of secondary markets is too costly, or simply because the illiquidity of the market makes transactions among the agents undesirable.

Let $c_t^x$ represent the consumption in period $t$ if the liquidity shock of the city took the value $^25x$ and let $L_x$ the total amount of liquidation that the bank made if the liquidity shock takes the value $x$. It is assumed that those procedures from liquidation are proportionally divided among consumers. $^26$ Let $C \equiv \{c_1^s, c_2^s, c_1^{1-s}, c_2^{1-s}\}$ and $L \equiv \{L_s, L_{1-s}\}$. The optimal policy for the bank can be characterized as the solution to the following program:

$$\max_{C, x, L} \frac{1}{2} \left[ sU(c_1^s) + (1 - s)U(c_2^s) \right] + \frac{1}{2} \left[ (1 - s)U(c_1^{1-s}) + sU(c_2^{1-s}) \right]$$

(1.12)

subject to:

$$s \cdot c_1^s = L_s$$

(1.13)

$$(1 - s) \cdot c_2^s = (1 - x) \cdot B + x - L_s$$

(1.14)

$$(1 - s) \cdot c_1^{1-s} = L_{1-s}$$

(1.15)

$$s \cdot c_2^{1-s} = (1 - x) \cdot B + x - L_{1-s}$$

(1.16)

$^25$Recall that the liquidity shock of each city is defined as the fraction of consumers that are early consumers.

$^26$Given the concavity in the utility function it is evident that any optimal contract will satisfy this property.
\begin{align*}
0 \leq x \leq 1 & \quad (1.17) \\
0 \leq L_s \leq x & \quad (1.18) \\
0 \leq L_{1-s} \leq x & \quad (1.19)
\end{align*}

where \( B \) is the average profitability of the risky investments.

To characterize the solution of this problem it is useful to consider the following lemmas:

Lemma 1 \textit{There is a positive amount of resources invested both in riskless and risky assets (i.e. (1.17) does not bind):}

\[ x^* \in (0, 1) \quad (1.20) \]

Proof. See appendix.

Lemma 2 \textit{The optimal liquidation policy for the bank is the following:}

- \textit{If there is a bad liquidity shock (\( \Phi = s \)) then the riskless asset is totally consumed at \( t = 1 \):}

\[ L_s^* = x^* \quad (1.21) \]

- \textit{If there is a good liquidity shock (\( \Phi = 1 - s \)) then it is optimal to save part of the riskless asset to increase consumption at \( t = 2 \).}

\[ L_{1-s}^* < x^* \quad (1.22) \]

Proof. See appendix.

The trade-off is clear. By increasing investment in riskless asset the bank can provide more liquidity in bad times but will also be less profitable in good times.

Using Lemmas 1 and 2 the problem of the bank can be simplified. Substituting (1.21) and ignoring the non-binding restrictions results in the following:

\[ \max_{x} sU \left( \frac{x}{s} \right) + (1 - s)U \left( \frac{(1-x)B}{(1-s)} \right) + U(B - (B - 1)x) \quad (1.23) \]
The solution to 1.23 is characterized in the following proposition:

**Proposition 3** The optimal portfolio of the bank in the case of autarky is characterized by the following condition:

\[
U' \left( \frac{x^*}{s} \right) - B \cdot U' \left( \frac{1 - x^*}{s(B - 1)} \right) - (B - 1) \cdot U' (B - (B - 1)x^*) = 0
\]  

(1.24)

**Proof.** (1.24) is the first order condition of the program (1.23).

The optimal policy of the bank is therefore:

\[
S^*(x, L_s, L_{1-s}) = (x^*, x^*, [(1 - s)B - (B - 1)x^*])
\]  

(1.25)

where \(x^*\) is implicitly defined in expression (1.24).

In order to understand some of the effects that occur when a secondary market opens I analyze one particular aspect of the solution to the autarky problem. Consider the following negative result:

**Proposition 4** Increases in the average profitability of risky projects, \(B\), do not necessarily produce an increase in investment in the risky portfolio \((1 - x^*)\).

**Proof** Differentiating (1.24) yields an ambiguous sign. See the appendix for details.

Note that Proposition 4 refers to the effects of the expected profitability \(B\) on investment in the risky portfolio \((1 - x^*)\) and not to changes in consumption or utility. Proposition 4 suggests that the rise of more profitable but highly illiquid investments does not guarantee that larger amounts of resources are going to be dedicated to them. Banks provide a costly insurance (in terms of consumption) to the consumers and the curvature of their utility functions is essential to design the optimal insurance contract. If consumers are highly risk averse then it is optimal to reduce the investment on the risky asset as the profitability increase and the opposite is true if consumers are moderately risk averse. As we will see, this conclusion contrasts with changes that increase the liquidity of the risky investments.
In that case irrespectively of the utility function of the agents the effect is unambiguous: more resources will be invested in liquid investments.

1.3.2 Analysis of the model: investment and liquidation

I will solve the model in two steps. First I solve the problem for a bank in \( t = 0 \), taking as given the price for its assets in the secondary loan market. Next I analyze the price formation that arises endogenously from different regimes.

Portfolio choice of a bank: the choice between liquidity and profitability

Consider the problem of a bank with the characteristics described in Section 2. The bank has to choose between the following two assets: a riskless one (as before) and a risky asset that gives the average payoff \( B \) but with certain (exogenous) liquidation costs after early liquidation is made. Table 2 presents the yields of the assets per unit of investment:

<table>
<thead>
<tr>
<th></th>
<th>RISKLESS ASSET</th>
<th>RISKY ASSET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield per unit in ( t = 1 )</td>
<td>( w_a )</td>
<td>( \frac{w_b}{h} )</td>
</tr>
<tr>
<td>Yield per unit in ( t = 2 )</td>
<td>( 1 - w_a )</td>
<td>( B \cdot (1 - w_b) )</td>
</tr>
</tbody>
</table>

where \( w_a, w_b \in [0,1] \) represent the amount of the share of the assets that are liquidated in the first period. Risky assets can only be liquidated at a discount \( h > 1 \).

From now on I assume that liquidation costs of the risky asset are such that both assets are demanded by the bank:

**Assumption 12** The expected profitability of risky assets is smaller than the discount for early liquidation:

\[ B < h \]  

(1.26)

If Assumption 12 is not satisfied then the portfolio of the bank would consist only of risky asset. With Assumption 12, both assets will be held.

The program that characterizes the bank portfolio choice when early liquidation is possible (where again \( C = \{ c_1^i, c_2^i, c_1^{i-s}, c_2^{i-s} \} \) and \( L = \{ L_s, L_{1-s} \} \)) is.
\[
\text{Max}_{C^s, x, L} \frac{1}{2} [sU(c^s_1) + (1 - s)U(c^s_2)] + \frac{1}{2} [(1 - s)U(c^{1-s}_1) + sU(c^{1-s}_2)]
\]  \hspace{1cm} (1.27)

subject to:

\[
s \cdot c^s_1 = L_s
\]  \hspace{1cm} (1.28)

\[
(1 - s) \cdot c^s_2 = (1 - x) \cdot B + F(L_s, x)
\]  \hspace{1cm} (1.29)

\[
(1 - s) \cdot c^{1-s}_1 = L_{1-s}
\]  \hspace{1cm} (1.30)

\[
s \cdot c^{1-s}_2 = (1 - x) \cdot B + F(L_{1-s}, x)
\]  \hspace{1cm} (1.31)

\[
0 \leq x \leq 1
\]  \hspace{1cm} (1.32)

\[
0 \leq L_s \leq x + \frac{B}{h} (1 - x)
\]  \hspace{1cm} (1.33)

\[
0 \leq L_{1-s} \leq x + \frac{B}{h} (1 - x)
\]  \hspace{1cm} (1.34)

The function \(F(L, x)\) is the liquidation function. \(F(L, x)\) gives the resources lost at \(t = 2\) if the original portfolio of the bank was \((x, 1 - x)\) and \(L\) was liquidated at \(t = 1\). \(F(L, x)\) takes the following form:

\[
F(L, x) = \begin{cases} 
  x - L & \text{if } x \geq L \\
  h(x - L) & \text{if } x < L 
\end{cases}
\]  \hspace{1cm} (1.35)

Note that \(F(L, x)\) is nondifferentiable at points where \(L = x\). The reason is that while early liquidation implies a cost, keeping liquid resources an extra period does not produce a profit.

Comparing programs (1.12) and (1.27) two differences can be observed. The possibility of liquidation makes the constraints (1.18) and (1.19) of program (1.12) less demanding. In autarky, total liquidations \(L_s\) and \(L_{1-s}\), could not go beyond \(x\), the liquid resources of the bank. With costly liquidation, consumption at \(t = 1\) can go beyond the own liquid resources \(x\) up to an amount \((x + \frac{R}{h}(1 - x))\).

As in the autarky case, the following two lemmas help to characterize the optimal bank
policy:

**Lemma 5** The inequality constraints (1.32), (1.33) and (1.34) do not bind in the optimal solution.

**Proof.** See Appendix.

**Lemma 6** The optimal liquidation policy of the bank has the following form:

\[
L_s^* > x^* \quad (1.36)
\]

\[
L_{1-s}^* < x^* \quad (1.37)
\]

**Proof.** See appendix.

Lemma 6 asserts that only in case of a bad liquidity shock will the bank liquidate any loans. If the good liquidity shock occurs then liquid resources are simply maintained one more period.

Imposing the form of the liquidation policy obtained from Lemma 6, the program to solve becomes:

\[
\max_{x,L_s,L_{1-s}} \left( sU \left( \frac{L_s}{s} \right) + (1-s)U \left( \frac{(1-x)B - (L_s - x)h}{1-s} \right) \right) + \left( (1-s)U \left( \frac{L_{1-s}}{1-s} \right) + sU \left( \frac{(1-x)B + x - L_{1-s}}{s} \right) \right) \quad (1.38)
\]

To characterize the optimal liquidation policy consider the following proposition:

**Proposition 7** The optimal policy of the bank is characterized by the following set of equations:

\[
U' \left( \frac{L_s^*}{s} \right) - h \cdot U' \left( \frac{(1-x^*) \cdot B - (L_s^* - x^*) \cdot h}{1-s} \right) = 0 \quad (1.39)
\]

\[
U' \left( \frac{L_{1-s}^*}{1-s} \right) - U' \left( \frac{(1-x^*)B + x^* - L_{1-s}^*}{s} \right) = 0 \quad (1.40)
\]
\[(h - B)U' \left( \frac{(1 - x^*)B - (L_s^* - x^*) \cdot h}{1 - s} \right) - (B - 1)U' \left( \frac{(1 - x^*)B + x^* - L_{1-s}^*}{s} \right) = 0 \] (1.41)

**Proof.** Differentiate in program (1.38). ■

 totalement differentiating the three first order conditions yields the following comparative static results that relate the optimal solution to the liquidity cost:

**Proposition 8** An increase in the cost of illiquidity, \(h\), in the market affects the optimal policy of banks in the following way:

\[
\begin{align*}
\frac{dx^*}{dh} &> 0 \quad (1.42) \\
\frac{dL_s^*}{dh} &< 0 \quad (1.43) \\
\frac{dL_{1-s}^*}{dh} &< 0 \quad (1.44)
\end{align*}
\]

**Proof.** See appendix.

Note the contrast between this result, the relation between liquidity and investment, and the result in the previous subsection that related profitability and investment. The sign of \(\frac{dx^*}{dh}\) is unambiguously positive. It is striking that, at least for certain utility functions, the effects of profitability and liquidity on investment are exactly the opposite: more liquidity is what guarantees more investment in risky assets in the economy. The reason has to do with the insurance that banks provide to consumers. While increases in liquidity reduces the cost of risky assets and increase consumption in the “bad” state of nature (when the liquidation occurs) and therefore the only optimal policy would be to increase the investment in risky assets, increases in profitability increase the consumption in the “good” state. In that case a reduction in investment to provide more consumption in the “bad” state (liquidation state) might be the optimal reaction. This optimal reaction generates less variable consumption profiles that share the benefits of more profitable investments in both “bad” and “good” states.
Proposition 8 is one of the central results of the paper and will be used in the next subsection to compare market regimes.

**Analysis of the interbank market: general equilibrium considerations.**

The analysis of price formation in the secondary market is performed under the assumption that transactions are "fair": what is obtained in exchange for the loans is what buyers will get from them in $t = 2$; which is the payoff of the projects net of re-monitoring cost.\footnote{As explained before, different regimes of the market will produce different valuations for the transaction depending on the transacted loans.}

This price formation makes sense only if the amount of liquid resources in the market is larger than the value of the loans that sellers are willing to sell. Otherwise, competitive pressures would reduce the price of loans (increasing the illiquidity of the market). Bhattacharya and Gale (1986) analyzed a similar situation with a market that provides liquidity for banks subject to liquidity shocks. They focused on the free rider problem that appears because banks try to rely on the liquidity of the others and avoid having enough liquid resources of their own. However, in this model, obtaining liquidity from the market is costly due to the duplication in monitoring cost, so it is cheaper for banks to rely at least in part on their own liquidity. The resulting increase in liquid reserves guarantees enough supply of liquidity for banks in need and smooths the functioning of the interbank market.

Summarizing, the proposed price formation can be sustained if the individual liquidation policy of the banks satisfies the following two conditions:

**Condition 1** Banks demand funds only if they suffer the bad liquidity shock; otherwise they have excess liquid funds in their portfolios: $L_s^* - x^* > 0$ and $x^* - L_{1-s}^* > 0$.

**Condition 2** The demand of liquid funds by a bank with liquidity needs is smaller than the investment that a bank makes in liquid funds: $L_s^* - x^* \leq x^* - L_{1-s}^*$.

Only if these two conditions are satisfied will the solution of the problem of the individual bank be compatible with general equilibrium considerations based on the proposed price
formation. These two conditions have to be checked after the problem of the individual bank is solved.  

1.3.3 Different market regimes: investment in different regimes

We now compare the different secondary loan market regimes and consider its repercussion in the investment activities of banks. First notice that different market regimes have different liquidation functions. Those liquidation functions depend on the order in which the assets are traded in different regimes and are explicit derived in the appendix.

The main result regarding liquidation functions is the following:

**Proposition 9** If the uncertainty in the economy is not too big and the outcomes of the risky projects not too diverse, then the regimes for the secondary loan market can be ordered in terms of the size of the portfolio of risky assets: in the RM regime the investment in the safe asset is the biggest, in the S regime it is smaller and in the CP regime it is the smallest:

\[ x_{RM}^* > x_{S}^* > x_{CP}^* \]  

(1.45)

**Proof.** See the appendix.

It is possible to prove the following comparative statics result related to the evolution of the economy inside each regime:

**Proposition 10** In the same conditions that in Proposition 9, within each regime, investment in risky projects is inversely related to monitoring cost:

\[ \frac{dx^*}{dk_1} > 0 \]  

(1.46)

**Proof.** See appendix.

It is interesting that the ordering of the investment in risky projects is directly related to the monitoring cost \((k_1)\) and to the particular regime the loan market is in. Both of these

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28 These conditions are satisfied if the liquidation cost of the market is large enough. See the appendix for details.
factors can be related to information technologies and to how changes in those information technologies produce changes in regime. Recall that according to Assumptions 10 and 11, while regimes more demanding in information (Cherry Picking and Securitization) are more costly, they also benefit more from technological progress in information technologies. When the efficiency of technologies to gather information reaches a certain level, a switch among market regimes will be observed.\textsuperscript{29}

It is interesting that the evolution observed in banking portfolios resembles the simple evolution as technological improvements diffuse here: volume in the loan market has increased, securitization has increased and the banking industry has undertaken riskier behavior in recent years. The attractive feature is that here, by endogenizing the functioning of the secondary loan market and by proposing a simple explanation for the evolution, several of these stylized facts can be generated.

1.3.4 Another result: heterogenous banks’ size and portfolio risk

One of the main features of the banking industry in the U.S. has been the heterogeneous behavior of banks depending on their size. In general small banks have adopted more conservative asset positions than large banks. This difference in behavior may arise either because of technological differences among banks or because of some kind of regulatory bias, or both.\textsuperscript{30}

Here I consider another dimension of heterogeneity among banks: the variance of the liquidity shock that their depositors suffer. Note that the variance of the liquidity shock $\Phi$ is:

$$Var(\Phi) = \left(s - \frac{1}{2}\right)^2$$

(1.47)

$Var(\Phi)$ increases with $s$, since $s > \frac{1}{2}$. It is easy to prove the following proposition:

\textsuperscript{29}An easy way to formalize this in the model could be by the choice of a discrete variable to represent the different regimes that the seller can be in case of necessity of liquidity. The bank would compute the expected utility under each of the regimes.

\textsuperscript{30}For some authors (e.g. Boyd and Gertler (1993)) this heterogeneity in behavior is a consequence of the "too big to fail" doctrine. Big banks take more risk simply because in very bad states they will be bailed out.
Proposition 11 The variance of the liquidity shock is negatively correlated with the riskiness of its portfolio:

\[ \frac{dx^*}{ds} > 0 \]  \hspace{1cm} (1.48)

Proof: See appendix.

The implication of proposition 11 is clear. Heterogeneous behavior of banks will arise when banks suffer shocks of different variance; riskier policies are to be expected from banks with less variable liquidity shocks.

To link the heterogeneity in variance with heterogeneity in size requires assuming that larger banks, due to diversification, have less variable liquidity shocks. Then heterogeneous behavior among banks of different size results. More diversified banks are able to predict liquidity more accurately, so they can afford to have riskier portfolios than less diversified banks.

1.4 Concluding Remarks

The main thesis of the paper is that increases in liquidity in secondary markets constitutes a simple and consistent explanation of several trends observed recently in the banking industry. To investigate this claim, I integrated two of the central models in banking theory: Diamond and Dybvig (1983) and Diamond (1984). Adapting and combining them yields a model of a bank with illiquid assets that has to provide liquidity to their depositors. I then use the model to analyze the effects of changes in the illiquidity and practices of secondary markets on the optimal behavior of banks.

The model is able to replicate several empirical tendencies through a simple comparative statics analysis. More liquidity (or lower transaction costs) encourage investment in projects susceptible to liquidation and encourage the adoption of market mechanisms that require a more intensive use of information (i.e. securitization). Differences in variance of shocks that banks face also yield differences in bank investment.

The paper has shown the importance of market liquidity and the distinct effect of profitability. While increases in liquidity or profitability benefit consumers, it is only liquidity
that induces an unambiguous increase in investment; for certain utility functions, increases in profitability might reduce the total investment in risky assets.

Finally this paper is part of a broader research effort oriented to analyzing recent changes in the banking industry. While here liquidity and trading of banking assets have been the focus, a companion paper (Almazán (1996)) has focused on alternative phenomena also relevant to the bank industry: the interaction between bank capital and bank expertise in a competitive environment. From different perspectives, both papers attempt to rationalize the changes that we have witnessed recently in the banking industry.
Bibliography


Chapter 2

A Model of Competition in Banking: Bank Capital vs. Expertise

2.1 Introduction

The importance of bank capital has been debated in the economic literature as well as in the regulatory arena. The conventional wisdom is that without enough capital, banks do not have the incentive to manage external funds properly.\(^1\) Regulators also emphasize that a bank's own capital serves as a cushion against solvency problems.\(^2\) A third perspective, comes from the lending view of monetary policy which stresses balance sheet effects. If some firms rely exclusively upon bank credit, changes in balance sheets of banks will affect the supply of loans and have real consequences for investment.\(^3\)

\(^1\)See Berger et al. (1995) for a review of the reasons why adequate capitalization of the banking industry is important.

\(^2\)The U.S. regulators have used the CAMEL system as an early warning system to identify problems in banks. Capital adequacy is one of the elements used in that system. See Dewatripont and Tirole (1994) p. 66 for details.

\(^3\)See Kashyap and Stein (1995) for an extensive treatment of the lending channel of monetary policy and Bernanke and Lown (1991) for an empirical investigation of of the effects of capital crunch in the 1990-91 recession in the U.S..
In this paper I consider another aspect of bank capital: the effects that bank capital may have on competition in the banking industry. My model is based upon the interplay of two factors: the level of capitalization of individual banks and their varying capabilities of monitoring different types of businesses (expertise). My main purpose is to build a framework for investigating the role played by net worth in the banking industry and the effects that shocks in financial conditions may have on the competition in that industry.

The empirical motivation of the paper stems from the recent period of turbulence in the banking industry. Institutional changes are transforming the banking industry and redefining its boundaries. The growth of the commercial paper market, the growth of non-bank intermediation and the explosion in the securitization of loans have reduced the demand for bank credit. Also regulatory interventions have altered the supply of bank capital. For example, in the last few years, the Fed has lowered the reserve requirements for banks twice. The interpretation in the paper is that these structural changes can be taken, at least in the short run, as exogenous shocks that have induced a relative expansion of bank capital.

While the importance of bank capital has often been considered in the literature, the role of bank expertise is seldom mentioned. In this paper I focus explicitly on the role of expertise. I associate expertise with the efficiency of monitoring a particular technology. The model exploits the idea that scarcity of capital may preclude some banks from taking full advantage of their monitoring expertise.

The assumption that monitoring expertise varies across lines of business seems realis-

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4 See Boyd and Gertler (1993) for a description of those changes.
5 In December 1990 and April 1993 the Fed lowered the reserve requirements for the banks.
6 Not all changes have contributed to the expansion of bank capital. For example Bizer (1993) argues that regulators are using stricter criteria to evaluate the quality of bank loans, thereby raising the capital requirements of banks.
7 Unger (1992) has argued that those changes are producing overcapacity in the banking industry that can be deduced from the poor returns to lending in the 1980's and from the excessive lending costs observed in that period.
8 An exception is Holmstrom (1993) who refers to expertise as a determinant of the boundary between firms and banks. While banks invest in general knowledge firms tend to concentrate their investments in the acquisition of highly specialized technological knowledge.

From a different perspective the economic literature has stressed the importance of institutions based on peer monitoring to overcome asymmetric information problems in the absence of collateral (see for example Stiglitz (1990)).
tic. For example, Steiner and Teixeira (1990) refer to the fundamentally different skills that consumer lending and corporate lending require. Corporate lending requires credit analysis skills and relationship management skills to provide low-cost funding and service. Consumer lending, on the other hand, requires skills in mass processing of information. It is common, therefore, that banks specialize in different forms of lending. That banks (in part for regulatory reasons) tend to concentrate their portfolios either by region, industry or demographic strata is a form of specialization.

To analyze the interaction between debt capacity and expertise requires a model where both bank capital and expertise play relevant economic roles. Building upon the work of Holmstrom and Tirole (1994), I consider a model with three classes of agents: investors, entrepreneurs and bankers. The model features a double moral hazard problem. Firstly, there is moral hazard between investors and entrepreneurs that can be alleviated using monitoring services provided by a bank. Secondly there is moral hazard in the provision of monitoring services by bankers. I focus on this last problem and on how the two dimensions in which banks differ, capital and expertise, would affect the magnitude of the moral hazard problem in the provision of monitoring services.

It is well known that when limited liability exists in a moral hazard situation, the agents, for incentive reasons, have to enjoy informational rents that may create a transferability problem that precludes net present value investment to be undertaken. The rents also induce competition among agents to appropriate them. In the model, banks compete for informational rents offering capital and expertise. Banks with less expertise must offer more capital. Capital and expertise are substitutes that ameliorate the transferability problem. Without scarce bank capital, banking expertise alone would determine the match between banks and firms. One of the central points in the paper is that capital scarcity interferes with efficient monitoring in the credit market.

In addition to showing how debt capacity problems in the intermediary sector can preclude the full exploitation of technological expertise, it will also be shown how the efficiency of the banking sector varies with the amount and distribution of bank capital. Common assertions about increased efficiency due to an increase in capital have to be qualified and
depend very much on the initial conditions of the system. It is true that when the amount of bank capital increases in the system (or when the interest rate falls) borrowers are better off but this is not equivalent to an improvement in the efficiency of monitoring. For an unambiguous efficiency gain, the shock has to favor the more debt-constrained banks, or the less-constrained banks have to be in a situation where further increases in capital do not translate into more aggressive competition.

Although the model is highly stylized, it is possible to use it to study some policy measures related to the banking industry. The effect of deposit ceilings, of price interventions on loans, and forced lending policies, among others, are addressed in Section 5. Particular attention is paid to the question of financial liberalization. It is shown that the state of financial development of the economy is crucial in evaluating the different measures that constitute a financial liberalization program. Policy measures that have an expansionary effect on investment may not be appropriate for a financially developed economy because they contribute to the mismatch between banks and firms.

The model also permits an interpretation of the development process. According to the analysis, a well developed economy is characterized not only by the existence of business opportunities but also by the existence of a well informed financial sector able to provide financial assistance. More developed economies are able to invest in more specialized projects not only for technological reasons but also because the stock of informed capital is larger. Investment in the economy goes to less mainstream business as the shortage of informed capital is reduced. Policies that help to create or accelerate the formation of informed capital would also accelerate the development process in the economy.

This paper is related to two main strands in the literature of financial intermediation. One includes papers where bank capital plays an important role, e.g. Bernanke and Gertler (1987) and Holmstrom and Tirole (1994). The other strand relates to models of spatial competition: Hotelling (1929) and Salop (1979). Spatial models had been used before to analyze the banking industry, e.g. Repullo (1990), but in a context where asymmetric information is not present. These models view banks as classical firms (that buy deposits, and sell loans) rather than as monitoring agents in imperfect capital markets as here.
The paper is organized as follows: in Section 2 the basic model is developed. In Section 3 the model is analyzed. In Section 4 changes in bank capital and in the interest rate are investigated. Section 5 considers the policy implications. Section 6 concludes. Proofs and some technical derivations are relegated to the appendix.

2.2 The model

2.2.1 Agents

There are three kinds of agents in this economy: investors, entrepreneurs and bankers.

A. Investors

There is a continuum of risk neutral investors with an aggregate endowment $N$, large enough to avoid any shortage of investor capital. $N$ is also referred to as uninformed capital.

B. Entrepreneurs

There is a continuum of entrepreneurs, also risk neutral, who possess no capital. Each entrepreneur has a project or idea that requires $I$ units of funds to initiate and generates a stochastic payoff that may take one of two values: $R > 0$, or 0.

The probability of a positive payoff depends on an unobservable action taken by the entrepreneur. If the action $a_h$ (alternately $a_l$) is undertaken, the probability of success is $p_h$ ($p_l$), where $p_h > p_l$. A moral hazard problem arises because the entrepreneur enjoys a private benefit $B$ ($B > 0$) from choosing $a_l$ rather than $a_h$.

Table 1 summarizes actions, probabilities and private benefits.

\[
\begin{array}{|c|c|c|}
\hline
\text{Action} & a_h & a_l \\
\hline
\text{Probability of success} & p_h & p_l \\
\hline
\text{Private Benefit} & 0 & B \\
\hline
\end{array}
\]

There is also a safe technology that yields a gross return of $\gamma$ per unit of investment. The existence of this safe asset simplifies the analysis by fixing the cost of uninformed capital at $\gamma$. 

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To motivate the demand for banking I introduce the following parameter restrictions.

**Assumption 13** The project has a positive net present value only if \( a_h \) is chosen:

\[
p_h \cdot R > I \cdot \gamma > p_l \cdot R + B
\]

**Assumption 14** There exists no feasible contract between an entrepreneur and an investor that would allow any investment project to be undertaken:

\[
p_h \cdot \left( R - \frac{B}{\Delta p} \right) < I \cdot \gamma
\]

where \( \Delta p = p_h - p_l \).

The moral hazard problem (unobservable action and private benefit \( B \)) implies that the entrepreneur has to be guaranteed at least \( R / \Delta p \) in case of success. If \( R_e \) represents the amount that the entrepreneur keeps for himself in the event of success, then in order for \( a_h \) to be individually rational for the entrepreneur, the inequality \( p_h \cdot R_e \geq p_l \cdot R_e + B \) must be satisfied.

Assumptions 13 and 14 imply that if the economy contains only investors and entrepreneurs, no project is financed. This creates a demand for banking.

C. Bankers

There are \( L \) risk neutral bankers. The amount of capital possessed by banker \( l \), is \( k_l \). The endowment of two different banks can be different. Each banker also has access to a monitoring technology. The monitoring technology is such that, by incurring a private cost \( C \), a banker can reduce the private benefit of an entrepreneur from \( B \) to \( b \).

**Assumption 15** Gross of the monitoring cost, under bank monitoring, there exists a feasible investment contract between the entrepreneur and the investor:

\[
p_h \cdot \left( R - \frac{b}{\Delta p} \right) > I \cdot \gamma
\]
Note that without taking into account the costs related to the participation of the bank in the project, Assumption 15 would permit the financing of the project. For example a sharing rule that gives $R_e = \frac{b}{\Delta P}$ to the entrepreneur would leave $p_h \cdot (R - \frac{b}{\Delta P}) > I \cdot \gamma$ to the financier, and make incentive compatible for him to participate in the project.

2.2.2 The monitoring technology

A central feature of my model is that intermediaries have different kinds of expertise. To be able to monitor a project, a banker has to be informed about the line of business of the project. The less the monitor knows about a line of business, the more costly it is to monitor projects in that business.

To capture this aspect of financial intermediation in the simplest way possible, I assume that projects differ along a single dimension related to their technology. I assume that technologies can be represented as points on a line segment of length 1. Similar technologies are closer to each other on the segment. Bank expertise is similarly represented by a point on the technological segment (0,1). The private resources spent by a bank on monitoring increase with the distance between the bank and the project. In this framework, each project has a natural monitor, namely the banker closest to it.

For convenience, I assume that banks are located at the end of the unit segment. This simplifies computations while maintaining the basic ingredient required for the analysis: the fact that banks are heterogeneous in their ability to monitor different projects. I assume that the cost of monitoring is linear in the distance between the bank and the project to be monitored.\(^9\) The aggregate capital of banks\(^10\) located at the end point $i \in \{0, 1\}$ is $K_i$. In the analysis that follows,\(^11\) only the aggregate amount of capital at each of the banking centers will matter.\(^12\)

---

\(^9\)The fact that the cost of monitoring increases linearly only simplifies computations. In fact, any increasing function will suffice to obtain the same qualitative results.

\(^10\)Although in this paper I refer to the intermediary sector as two banking centers or two banks, the strict interpretation will depend on the strategic nature of the relationship between bankers and entrepreneurs. This has to be with the assumption of perfect competition in the bank behavior that I will use. See the beginning of Section 3 for a justification for the strategic behavior.

\(^11\)See Subsection 2.3 for a justification.

\(^12\)Although I have motivated the spatial model in terms of technological distance, it could just as easily
Figure 2-1: Locations.

Figure 2-1 describes the spatial features of the economy. In order to monitor the project located at $x$, a bank at 0 would incur in a private cost of $c \cdot x$ while a bank at 1 would incur in a cost of $(1 - x) \cdot c$. Thus, projects to the left of the point $\frac{1}{2}$ are efficiently monitored by the banks at 0, while the projects to the right of $\frac{1}{2}$ are efficiently monitored by the banks at 1.

2.2.3 Stochastic structure of the economy

The basic assumption\textsuperscript{13} is that projects monitored by the same bank are perfectly correlated.

**Assumption 16** Let $\theta_i$ be the variable that determines the success of the projects monitored by bank $I$. $\theta_i$ is distributed according to a uniform distribution in $[0, 1]$. If $\theta_i > p_h$, then all the projects fail; if $p_i \leq \theta_i \leq p_h$, then a project succeeds if and only if the action $a_h$ is taken; and if $\theta_i \leq p_i$, then all of the projects succeed unconditionally.

This assumption simplifies the analysis by implying that only the aggregate amount of capital in each banking center (and not the distribution of capital among its members) matters. This happens because, as shown in Lemma 13 below, banks have to finance part of the investment with their own capital $k_I$. The assumption guarantees that the amount that a bank finances in a project do not depend on other projects financed by the same bank. Without perfect correlation, the stochastic relationship among project returns would have to be considered in the optimal financial contract. In that case, the interaction among

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\textsuperscript{13}This assumption is also made in Holmstrom and Tirole (1994).
bank capital, size and the joint distribution of projects would determine the structure of the banking industry.

On the other hand, Assumption 16 implies that the size of a particular bank, measured by its own capital, determines only the scale of its operations. No other technological difference exists among banks of different size. The cost of intermediation does not depend on the size of the bank. Summing up, Assumption 16 guarantees constant returns to scale in the banking industry even when the agency problems for the banks are considered.

Although perfect correlation is extreme, its main implication for the analysis, namely constant returns to scale, is not rejected by the data.\textsuperscript{14,15}

If we were to move to the other extreme and assume independence as in Diamond (1984), then a bank could overcome the moral hazard problem by monitoring a sufficiently large number of projects. The banking sector would become a natural monopoly for informational reasons and its capital would not play any role.

Such conclusions are clearly counterfactual. There are many banks in the economy and the importance of bank capital is evident. One simple way to reconcile the analysis with these facts is to assume a certain degree of correlation among the projects that a bank handles. I chose to consider the extreme case of perfect correlation for its tractability.

### 2.3 Analysis of the model

An equilibrium in this setting will determine the projects that are undertaken and the source of their financing. To simplify matters I suppress strategic behavior by individual bankers. This could be justified by assuming a continuum of bankers. Alternatively, one could give all the bargaining power to the entrepreneurs, in which case banks have no strategic opportunities. Under either interpretation, the model can be solved as one of perfect competition with one scarce production factor, capital.

\textsuperscript{14}Empirically there exists some consensus that scale economies exist only up to a very modest size ($100 million). For larger sizes some studies find slight economies and others slight diseconomies. For a recent study, see Boyd and Runkle (1993).

\textsuperscript{15}Berger and Humphrey (1991) have shown that the degree of efficiency in the banking industry is not related to the size of the institution.
The optimal contract that each entrepreneur will propose specifies the share of the investment that bankers and investors will finance and the share of the return for each of the financiers in the event that the project succeeds. Note that the unobservability of both $b$ and $C$ prevents contracts to be written on those variables (as well as contracts that specify which actions entrepreneur and banks have to take.)

For expository reasons and to illustrate the main forces operating in the model, I first consider a simplified version with only one banking center located at one extreme of the segment.

2.3.1 Analysis of the model with one banking center

Consider the optimal contract that an entrepreneur at a distance $x$ units from the banking center will offer to his eventual financiers. This contract solves the following problem:

$$\max_{S, P} R_e(x)$$

subject to the constraints:  \(^{16}\)

$$I_f(x) + I_b(x) = I \tag{2.5}$$

$$R_e(x) + R_b(x) + R_f(x) \leq R \tag{2.6}$$

$$p_h \cdot R_f(x) \geq I_f(x) \cdot \gamma \tag{2.7}$$

$$p_h \cdot R_b(x) - C(x) \geq I_b(x) \cdot \beta \tag{2.8}$$

$$R_e(x) \geq \frac{b}{\Delta p} \tag{2.9}$$

$$R_b(x) \geq \frac{C(x)}{\Delta p} \tag{2.10}$$

Here $S = (I_d(x), I_f(x))$ and $P = (R_b(x), R_f(x), R_e(x))$ are respectively the vector of investments in the project and the vector of shares in the payoff if the project is successful. \(^{17}\) $\beta$ is an endogenous variable representing the return on bank capital. The equilibrium value

\(^{16}\)All the variables are non-negative unless otherwise specified.

\(^{17}\)Throughout the paper, $f$ refers to investors (financiers), $e$ refers to entrepreneurs and $b$ to bankers.
of $\beta$ is derived below.

The interpretation of the constraints is straightforward: (2.5) says that the investment has to be shared between the two possible financiers. (2.6) describes the sharing rule. (2.7) is the participation constraint for investors. (2.8) is the participation constraint for banks. (2.9) is the incentive compatibility constraint for the entrepreneur and (2.10) is the incentive compatibility constraint for banks.

The following lemmas characterize the optimal contract:

**Lemma 12** A project is financed if and only if the monitoring cost of the project is less than or equal to $C^*$, where

$$C^* = \frac{R - \frac{\gamma}{p_h} - \frac{b}{\Delta p}}{\left(\frac{1}{\Delta p} - \frac{\gamma}{p_h \beta \Delta p}\right)}$$

(2.11)

**Proof** See appendix.

**Result 1** The cutoff cost $C^*$ increases with the net present value of the projects $(R - \frac{\gamma}{p_h})$ and decreases with the prices of capital ($\beta$ and $\gamma$) and the size of the private benefit ($b$).

**Lemma 13** The optimal financial contract satisfies the following:

$$R^*_b(x) = \frac{C(x)}{\Delta p}$$

(2.12)

$$I^*_b(x) = \frac{p_l \cdot C(x)}{\beta \cdot \Delta p}$$

(2.13)

$$R^*_f(x) = \left(1 - \frac{p_l \cdot C(x)}{\beta \cdot \Delta p}\right) \cdot \frac{\gamma}{p_h}$$

(2.14)

$$I^*_f(x) = I - \frac{p_l \cdot C(x)}{\beta \cdot \Delta p}$$

(2.15)

**Proof** See appendix.

According to (2.13), the optimal policy for the entrepreneur is to reduce the participation of the bank to the minimum level that makes monitoring credible, $I^*_b(x)$. The remaining investment is financed with cheaper uninformed capital, $I^*_f(x)$.$^{18}$ Investors are willing to participate in the project given the presence of a sufficient amount of informed capital.

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$^{18}$It is natural to assume that $\beta \geq \gamma$. In equilibrium, this will be the case (see below).
Although all projects require the participation of a bank, projects differ in the monitoring cost and consequently in the amount that entrepreneurs have to pay to banks. Any bank has to be guaranteed an ex-post payment that is increasing in its monitoring cost. As shown below (see equation (2.16)) projects with larger costs also guarantee larger net profits to banks. Banks will compete more strongly for projects that give larger profits. The competition for projects occurs through the capital that banks are willing to invest in the projects. The market price of bank capital, $\beta$, determines the ratio between the share of returns that the bank keeps and the share of the investment that the bank finances.\(^{19}\)

Note that there are two reasons why a project with positive net present value may not be financed. The first is that even with bank monitoring there is still a too severe agency problem. The second is that banks have to be paid rents in addition to being compensated for their monitoring costs. This rent is a consequence of the scarcity of bank capital and can be computed as:

$$
W(x) = p_h \cdot R_b(x) - \gamma \cdot I_b(x) - C(x) = \left( \frac{\beta - \gamma}{\beta} \right) \frac{p_I \cdot C(x)}{\Delta p} \quad (2.16)
$$

**Result 2** The rent that a bank obtains from monitoring a project located at $x$, $W(x)$, increases with the cost of bank capital, $\beta$, and with the monitoring cost that the bank has to incur, $C(x)$.

**Determination of the cost of bank capital and the total volume of investment**

To close the model we must compute the equilibrium cost of bank capital, $\beta^*$ and characterize the marginal project that is financed $m^*$.\(^{20}\) To do so the following two equations have to be solved:

$$
\frac{C(m^*)}{\Delta p} + \frac{b}{\Delta p} + \frac{(I - I_b(m^*)) \cdot \gamma}{p_h} = R \quad (2.17)
$$

and

\(^{19}\)In equilibrium, $\beta$ will be the same in every project. See the following subsection.

\(^{20}\)The marginal project is the more distant project that is financed in the economy. It is characterized by the binding of the restriction (2.9).
\[ \int_{0}^{m^*} I_b^*(x)dx = K_0 \]  

(2.18)

where \( I_b^*(x) \) is the optimal amount of bank financing that an entrepreneur located at \( x \) would demand. Its value in equilibrium comes from solution of the program (2.4), considering the equilibrium cost of bank capital \( \beta^* \):

\[ I_b^*(x) = \frac{p_l \cdot C(x)}{\beta^* \Delta_p} \]  

(2.19)

Replacing (2.19) into equations (2.17) and (2.18), the solution of the following system of equations characterizes the equilibrium:

\[ \frac{C(m^*)}{\Delta_p} + \frac{b}{\Delta_p} + \frac{(I - \frac{p_l \cdot C(m^*)}{\beta^* \Delta_p}) \cdot \gamma}{p_h} = R \]  

(2.20)

\[ \int_{0}^{m^*} \frac{p_l \cdot C(x)}{\beta^* \Delta_p}dx = K_0 \]  

(2.21)

Equations (2.20) and (2.21), although have to be simultaneously solved, are better interpreted separately. For a given \( \beta \), equation (2.20) determines the marginal project that is financed. Equation (2.20) represents the sharing of the profits for the marginal project. Note that the marginal project is characterized by the fact that the entrepreneur keeps the minimum share that is consistent with incentive compatibility, \( \left( \frac{b}{\Delta_p} \right) \). Equation (2.21) is the resource constraint for bank capital. Its interpretation has to do with the competitive nature of the model. Bank capital is completely employed because the allocation of projects to bankers depends on the capital that they are committing to the projects. For a given \( m \), \( \beta \) will adjust to employ all informed capital on the projects.

**Remark 6** This is a competitive model, and therefore its equilibrium has the usual properties of a competitive equilibrium. Banks take as given the price of bank capital, \( \beta \), and consequently maximize their expected payoff by using all the capital to finance projects. Entrepreneurs maximize their expected payoff using the finance scheme deduced in Lemma 13. Compared with the usual competitive equilibrium the only non-standard feature is that incentive compatibility constraints also affect agents’ decisions.
Using the assumption that the monitoring cost is linear in distance, we get $m^*$ and $\beta^*$:

$$m^* = \frac{(R - \frac{b}{\Delta p} - \frac{c\gamma}{p_n}) + \sqrt{(R - \frac{b}{\Delta p} - \frac{c\gamma}{p_n})^2 + \frac{8}{p_n \Delta p} K_0}}{\frac{2}{\Delta p}}$$  \hspace{1cm} (2.22)

$$\beta^* = \frac{p_l \cdot c \cdot (m^*)^2}{2 \Delta p \cdot K_0}$$  \hspace{1cm} (2.23)

The following results (derived in the appendix) give the main comparative statics.

**Result 3** The distance of the marginal project to the bank $m^*$ increases with the net present value of the projects $(R - \frac{c\gamma}{p_n})$, and with the amount of bank capital $K_0$ and decreases with the monitoring cost $c$, the interest rate $\gamma$ and the size of the private benefit $b$.

**Result 4** The cost of capital $\beta^*$ increases with the net present value of the projects $(R - \frac{c\gamma}{p_n})$ and decreases with the bank capital $K_0$, the interest rate $\gamma$, the monitoring cost $c$, and the size of the private benefit $b$.

Figure 2-2 is a graphical summary of the equilibrium. The use of bank capital is linearly increasing with the distance from the bank to the project. Equation $I_b(x) = \frac{p_l c}{\beta p_n \Delta p} x$ characterizes the segment $OF$. Projects to the right of $m^*$ are not feasible given $\beta^*$ because the sharing rule (2.6) cannot be satisfied.

The solution $(\beta^*, m^*)$ is valid only if $\beta^* \geq \gamma$ because bankers can always obtain the return of uninformed capital $\gamma$. The reason why $\beta^* > \gamma$ is possible even though perfect
competition among banks exists, is that bank capital can be scarce. If bank capital is not scarce (in the sense defined below) then $\beta^* = \gamma$ and banks gain no rents any more.

The precise quantity of bank capital that makes the restriction $\beta^* \geq \gamma$ binding can be found by substituting into (2.22) for the marginal project $m^1$ that will be financed when $\beta^* = \gamma$. The marginal project $m^1$ can be obtained from equation (2.24):

$$\frac{c \cdot m^1}{\Delta p} + \frac{b}{\Delta p} + \left(1 - \frac{p_t \cdot c \cdot m^1}{p_h \cdot \Delta p \cdot \gamma}\right) \cdot \gamma = R$$

(2.24)

Solving explicitly for $m^1$:

$$m^1 = \frac{(R - \frac{L \cdot \gamma}{p_h} - \frac{b}{\Delta p})}{\left(1 - \frac{p_t}{p_h}\right) \cdot \frac{c}{\Delta p}}.$$  

(2.25)

The level of capital that attains this value is:

$$K_{m^1} = \frac{p_t \cdot \left(1 - \frac{L \cdot \gamma}{p_h} - \frac{b}{\Delta p}\right)^2}{2 \cdot \left(1 - \frac{p_t}{p_h}\right) \frac{c}{\Delta p}}$$

(2.26)

**Result 5** The equilibrium cost of bank capital $\beta^*$ and the equilibrium marginal project $m^*$ with a banking sector are: $\beta^* = \max\{\gamma, \beta^*\}$ and $m^* = \min\{m^*, m^1\}$.

Figure 2-3 presents graphically the effect of an increase in bank capital. As capital increases the segment $OF$ becomes longer and rotates upwards to $OF'$. This goes on until $OF''$ where the slope reaches the value $\alpha = \frac{p_t \cdot c}{\Delta p \cdot \gamma}$. At that point $m^1$ is reached and additional bank capital would not expand the universe of projects that are financed.

Finally, it is worthy noting that banks in this economy could help to finance projects even if they lacked any capital of their own. Banks alter the allocation in this economy through two channels: by providing a monitoring technology and by providing capital. Even without capital banks would play a crucial role as monitors. The marginal project if banks lack capital is:

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21 This occurs when $\beta = \gamma$.  

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Figure 2.3: Effects of capital changes

\[ \frac{c \cdot m_0}{\Delta p} + \frac{b}{p_h} + \frac{I \cdot \gamma}{p_h} = R \]

or explicitly:

\[ m_0 = \left( R - \frac{I \cdot \gamma}{p_h} - \frac{b}{\Delta p} \right) \frac{\Delta p}{c} \quad (2.27) \]

The following remark summarizes the effects that banks produce in this economy.

**Remark 7** Without banks no project would be financed in this economy. Adding a monitoring technology without bank capital expands investment from 0 to \( m_0 \). With bank capital investment expands from \( m_0 \) to \( m^s \). The maximum amount of investment, \( m_1 \), occurs when bank capital is not scarce at all.

### 2.3.2 Analysis of the model with two banking centers

I now solve the model with two banking centers located at the two extremes of the unit segment. I assume the following:

**Assumption 17** The center located at 0 is better capitalized: \( K_0 > K_1 \).

**Assumption 18** \( m_1 \geq \frac{1}{1+\Delta p} \).

**Assumption 19** \( m_0 < \frac{1}{2} \).
Assumption 17 introduces the basic asymmetry in the model. Assumptions 18 and 19 are technical assumptions. Assumption 18 guarantees that banks in one center can finance projects beyond the midpoint if they have enough capital. The value $\frac{1}{1+\Delta p}$ has to do with the form in which competition occurs (and with the linearity of monitoring costs) and will be discussed below when the solution of the model is computed.\textsuperscript{22,23} Assumption 19 has to do with the behavior of the system without any capital. I argued that bank capital is not essential for financing some projects as long as banks are willing to monitor. Assumption 19 guarantees that without any capital some of the projects cannot be financed and therefore, that additional bank capital in a situation of severe scarcity\textsuperscript{24} increases the investment in the economy.

**Definition 14** The market share of the center at 0, $y$, is the ratio of the projects that center 0 finances over the total projects that are financed in the economy.

**Definition 15** The market share $y = s^* = \frac{1}{2}$ has the property that minimizes the cost of monitoring for the economy. I will call $s^*$ the efficient market share.

In an equilibrium of the economy it is evident that projects that are financed by the center at 0 and those that are financed by the center at 1 are separate intervals. For that reason, the solution to the model features four regimes depending on the amounts of capital that the centers have.

\textsuperscript{22}Note that $m^1 \in \left(\frac{1}{2}, 1\right)$.

\textsuperscript{23}These last two assumptions could be reexpressed in terms of the original parameters of the model. In particular an alternative formulation would be to restrict the parameter that describes the monitoring technology to lie within the bounds implicit in the definitions of $m^0$ and $m^1$:

\[
\frac{c}{R - \frac{L^2}{p_n} - \frac{L}{\Delta p}} \in \left[2\Delta p, \frac{1+\Delta p}{\Delta p} p_h\right]
\]

\textsuperscript{24}This corresponds to the regime 1 below.
Regime 1: Scarce capital in both centers, insufficient to finance all investment projects

The analysis of Regime 1 is identical to the analysis with one banking center because banking capital is so scarce that the two centers do not compete. Assumption 19 guarantees that for very low levels of capital, this case is relevant.

This regime is illustrated in Figure 2-4. In Figure 2-4 the projects between 0 and \( m \) are financed by banks at 0 and the projects between \( m' \) and 1 by banks at 1. The projects between \( m \) and \( m' \) are not financed. The computations of \( \beta^*_0 \) and \( \beta^*_1 \) and the marginal projects proceed along the same lines as with one banking center and are therefore omitted.

Regime 2: Scarce capital in both centers, but sufficient to finance all the projects in the economy

This is the first regime with interaction between the two centers. To analyze this interaction it is necessary to analyze the trade-off that an entrepreneur faces when he decides which bank to use.

Consider an entrepreneur located at \( x > \frac{1}{2} \), and assume that banks at 0 and 1 are willing to finance the project, offering respectively the amounts \( I_0 \) and \( I_1 \) of capital. Note that the entrepreneur has to offer either \( \frac{c x}{\Delta p} \) (of his payoff) to banks located at 0 or \( \frac{c (1-x)}{\Delta p} \) to banks located at 1 for monitoring to be credible and to be able to attract the rest of the capital needed for the investment in the project. The entrepreneur will choose its natural financier
(the bank at 1) provided that:

\[ I_0 \cdot \gamma - \frac{c \cdot x}{\Delta p} p_h \leq I_1 \cdot \gamma - \frac{c \cdot (1 - x)}{\Delta p} p_h \]

or after rearranging:

\[ (2x - 1) \cdot \frac{c \cdot p_h}{\Delta p} \geq (I_0 - I_1) \cdot \gamma \]

(2.28)

The left hand side of inequality (2.28) reflects the cost differences between the banks (including the monitoring cost and the rent that banks must be paid). The right hand side is the difference in cost of raising capital from external sources.

Since payments to banks are fixed by incentive compatibility reasons, banks compete by offering capital to the firm. Banks with more capital are able to increase the capital they offer while banks with more expertise have the natural advantage of being able to reduce the ex-post amount of resources to which they can credibly commit to. This trade-off between capital and expertise drives the results of the model.

I compute now the equilibrium in Regime 2.

**Definition 16** An equilibrium is the triple \((y^*, \beta^*_0, \beta^*_1)\) where \(y^*\) is the equilibrium share that separates projects financed by banks at 0 of projects financed by banks at 1, and \(\beta^*_0\) (resp. \(\beta^*_1\)) is the equilibrium cost of capital of banks at 0 (resp. at 1).

The equilibrium satisfies the following conditions:

1. Entrepreneurs who finance from banks at 0, (resp. at 1) use an amount of bank capital according to the expression (2.29) (resp. (2.30)):

\[ I_b^0(x) = \frac{p_1 \cdot c}{\beta^*_0 \cdot \Delta p} x \]  

(2.29)

\[ I_b^1(x) = \frac{p_1 \cdot c}{\beta^*_1 \cdot \Delta p} (1 - x) \]  

(2.30)

Differently from the case with only one bank center, entrepreneurs now have to compare the financial conditions offered by the two different banking centers. This is equivalent to solve the program (2.4) with respect to both centers and comparing
the expected payoff from alternative finance sources. Entrepreneurs will be divided among those who finance from banks at 0 (equation (2.29)) and those who finance from banks at 1 (equation (2.30). The foundations for (2.29) and (2.30) are the same that those for the solution of the program (2.4) in the previous subsection.

2. In equilibrium, there is a unique entrepreneur that is indifferent between both centers.\textsuperscript{25} Let \( R_e(x, n) \) be what an entrepreneur with a project located at \( x \) keeps after borrowing from a bank located at \( n \). In that case:

\[
R_e(x, 0) = R - \frac{c \cdot x}{\Delta p} - \frac{(I - I_b^0(x)) \cdot \gamma}{p_h} \tag{2.31}
\]

\[
R_e(x, 1) = R - \frac{c \cdot (1 - x)}{\Delta p} - \frac{(I - I_b^1(x)) \cdot \gamma}{p_h} \tag{2.32}
\]

The equilibrium market share is obtained by finding the location of the indifferent entrepreneur:

\[
R_e(y^*, 0) = R_e(y^*, 1) \tag{2.33}
\]

3. Capital from both centers have to be completely employed. Idle capital is incompatible with the fact that banks are earning rents from capital use.

\[
\int_0^{y^*} I_b^0(x) dx = K_0 \tag{2.34}
\]

\[
\int_{y^*}^1 I_b^1(x) dx = K_1 \tag{2.35}
\]

Equations (2.29), (2.30), (2.33), (2.34), and (2.35), characterize the equilibrium. After simple manipulations of them, it is possible to relate the equilibrium market share \( y^* \), with the amounts of capital \( (K_0, K_1) \) in the banking centers:

\[
\frac{2K_0}{p_h \cdot y^*} - \frac{2K_1}{p_h(1 - y^*)} = \frac{(2y^* - 1) \cdot c}{\Delta p \cdot \gamma} \tag{2.36}
\]

\textsuperscript{25}There is an extra technical condition that guarantees uniqueness (and that is used also in the next two regimes). The condition has to do with the decisions of entrepreneurs in case of indifference. I assume that the entrepreneur, if indifferent, goes to the closest bank center.
what allows to obtain $y^*$. Given $y^*$ the equilibrium costs of bank capital are:

$$
\beta_0^* = \frac{\int_0^y (\frac{c+\alpha_h}{\Delta p} - c) x \, dx}{K_0} \tag{2.37}
$$

and

$$
\beta_1^* = \frac{\int_y^1 (\frac{c+\alpha_h}{\Delta p} - c)(1-x) \, dx}{K_1} \tag{2.38}
$$

The following result regarding market shares can be stated.

**Result 6** The banking center with more capital (located at 0) finances too many projects relative to first best:

$$
y^* > s^* = \frac{1}{2}. \tag{2.39}
$$

**Proof** See appendix.

Regarding the cost of bank capital, we have:

**Result 7** In equilibrium $\beta_0^* < \beta_1^*$.

**Proof** See appendix.

Equations (2.36), (2.37) and (2.38) hold as long as the costs of capital satisfy the inequality: $\beta_1^* > \beta_0^* > \gamma$. Conditions on $K_0$ and $K_1$ that imply these inequalities are given in Result 10.

A graphical representation of the equilibrium is given in Figure 2-5. Note that $y^*$, the equilibrium market share is to the right of the efficient point $\frac{1}{2}$. Banks at 0 have more capital and therefore can compete away some projects that "naturally belong" to banks at 1, (in the sense that these projects would be monitored more efficiently by banks at 1).

It is important to understand why two different prices for bank capital ($\beta_0^*$ and $\beta_1^*$) can coexist in equilibrium without creating arbitrage opportunities. The reason an entrepreneur does not necessarily choose the cheapest source of bank capital is simple. The quantity of bank capital, and not only its price, is relevant for the decision.
It is also important to notice that bank capital is not transferable between centers. The only way that a banker can credibly commit to monitor is by demanding a residual share of the profits of the project; if that happens then no residual share is left to pay the owner of the transferred capital. Hence there is no room to borrow and lend funds simultaneously. An interbank market, for example, would not alleviate the problem of transferring capital from the center with excess capital to the center that is capital constrained.\textsuperscript{26}

**Regime 3: Scarce capital in one center only**

If the difference between the quantities of bank capital in the two centers is large enough ($K_0$ much larger than $K_1$) then the restriction $\beta_0^* \geq \gamma$ will eventually bind.\textsuperscript{27} To compute the equilibrium (see Definition 2 above) in this regime the following conditions have to be satisfied:

1. Entrepreneurs that finance from banks at 0, (resp. at 1) use an amount of bank capital according to the expression (2.40) (resp. (2.41)):

\[
I_b^0(x) = \frac{p_1 \cdot c}{\beta_0^* \cdot \Delta p} x
\]

\[
I_b^1(x) = \frac{p_1 \cdot c}{\beta_1^* \cdot \Delta p} (1 - x)
\]

\textsuperscript{26} Moral hazard impedes the smooth working of an interbank market.

\textsuperscript{27} The precise conditions are given in Result 10.
2. In equilibrium, there is a unique entrepreneur that is indifferent between both centers. Using equations (2.31) and (2.32) and by the same argument that in Regime 2, the equilibrium market share \( y^* \) has to satisfy:

\[
R_e(y^*, 0) = R_e(y^*, 1)
\]

(2.42)

3. While capital from banks at 1 is completely employed, the amount of capital from banks at 0 is so large that its total use would imply a smaller return than \( \gamma \). Instead, bank capital at 0 is paid the minimum return \( \gamma \) and not completely used.\(^{28}\) In mathematical terms:

\[
\int_{y^*}^{1} I^*_b(x)dx = K_1
\]

(2.43)

\[
\int_{0}^{y^*} I^*_b(x)dx \leq K_0
\]

that implies:

\[
\beta^*_0 = \gamma
\]

(2.44)

In Regime 3, equations (2.40), (2.41), (2.42), (2.43), and (2.44) characterize the equilibrium.

Simple manipulations in the equilibrium conditions give the following relationship between the amount of banking capital at 1 and the equilibrium share, \( y^* \):

\[
\frac{2K_1 \cdot \gamma}{p_h \cdot (1 - y^*)} = \frac{c}{\Delta p} \left[ 1 - (2 - \frac{p_i}{p_h})y^* \right]
\]

(2.45)

Finally \( \beta^*_1 \) can be computed as before:

\[
\beta^*_1 = \frac{\int_{y^*}^{1} \frac{(c\cdot p_h - c)(1 - x)}{\Delta p} dx}{K_1}
\]

(2.46)

In terms of prices Regime 3 corresponds to \( \beta^*_1 > \beta^*_0 = \gamma \). The result regarding market shares is now:

\(^{28}\)The amounts of capital that make the restriction \( \beta^*_0 \leq \gamma \) to bindi appear in Result 10.
**Result 8** The market share is suboptimal:

\[ y^* > s^* = \frac{1}{2} \]

**Proof** See appendix.

Graphically, Regime 3 is illustrated in Figure 2-6. We see that the scarcity of capital at 1 impedes the financing of the projects over which banks at 1 have a comparative advantage. In particular, banks at 0 finances all the projects between \( \frac{1}{2} \) and \( y^* \) even though these projects naturally belong to the center at 1. The economy expends more resources on monitoring than would be required if bank capital were more symmetrically distributed.

**Regime 4: Excess capital in both centers**

In Regime 4 relative expertise alone determines the matching between the entrepreneurs and banks, because capital is so plentiful that banks receive no scarcity rents; \( \beta_0^* = \beta_1^* = \gamma \).

In this regime monitoring is efficient:

**Result 9** Without scarce bank capital, the equilibrium market share is \( y^* = s^* = \frac{1}{2} \).

**Proof** See appendix.
Graphically this case is illustrated in Figure 2-7. The equations that relate the participation of the banks in the projects are:

\[ I_b^0(x) = \frac{pl \cdot c}{\Delta p \cdot \gamma} \]  

(2.47)

and

\[ I_b^1(x) = \frac{pl \cdot c}{\Delta p \cdot \gamma} (1 - x) \]  

(2.48)

To complete the analysis of the substitution between capital and expertise consider the following remark:

**Remark 8** There is a limit to the substitution between capital and expertise due to the restriction that bank capital must earn at least \( \gamma \). To compute this limit consider the extreme case where banks at 1 has no capital and banks at 0 is offering the most favorable financing: \( \beta_0 = \gamma \). Given these conditions the entrepreneur located at \( x_h \) is indifferent between banks provided that:

\[ c \cdot x_h = c \cdot \frac{1 - x_h}{\Delta p} \]  

(2.49)

or reexpress:

\[ x_h = \frac{1}{1 + \Delta p} \]  

(2.50)

The left hand side of expression (2.49) is the cost for the entrepreneur of financing the project from a bank at 0. As \( \beta_0 = \gamma \) the bank does not earn any rent and it is paid only
for the monitoring cost that it incurs \((c \cdot x_h)\). The right hand side is the cost of using the bank at 1. The cost of using this bank as financier is the amount that has to be guaranteed by incentive compatibility considerations \(\left(\frac{c(1-x_h)}{\Delta p}\right)\) minus the amount that the bank finance ex-ante (zero because it lacks of any capital).

Therefore all the projects between \(x_h\) and 1 will be funded by banks at 1 irrespective of how bank capital is distributed among centers. Assumption 18 guarantees that this limit can indeed be reached.

Equilibrium relationships between bank capital and regimes

To describe the equilibrium relationships between bank capital and the different regimes it is necessary to consider the quantities of capital that each particular center has and not only the total capital in the banking system.

The following result describes the equilibrium relationships by partitioning the \((K_0, K_1)\) plane into four regions corresponding to the four regimes described before.

Result 10 The level of bank capital in each center determines the equilibrium in the industry. In particular, if \(K_0 > K_1\) the following four regions can be distinguished in the space \((K_0, K_1)\) (see Figure 2-8):

- **Regime 1:** If \(K_1 \leq f(K_0)\) and \(K_0 \in [0, K_0^0]\).
- **Regime 2:** If \(f(K_0) \leq K_1 \leq g(K_0)\) and \(K_0 \in [K_0^0, K_0^1]\).
- **Regime 3:** If \(g(K_0) \leq K_1 \leq K_1^2\) and \(K_0 > K_0^1\).
- **Regime 4:** If \(K_1 \geq K_1^2\).

The function \(f(.)\) relates the level of capital \(K_0\) with the minimum level of capital \(K_1\) needed to finance all projects. The function \(g(.)\) relates the a level of capital \(K_0\) with the minimum level of capital \(K_1\) that makes the restriction \(\beta_0 \geq \gamma\) exactly binding. The functions \(f(.)\) and \(g(.)\) are both continuous and decreasing. They are explicitly defined in the appendix. The values \(K_0^0, K_0^1\) and \(K_1^2\) are also explicitly derived in the appendix.
2.4 Comparative statics: changes in bank capital vs. changes in the interest rate

In this section I consider two comparative statics exercises. The first analyzes changes in the quantities of bank capital. The second analyzes changes in the interest rate (profitability of the safe alternative investment).

2.4.1 Changes in bank capital

Let bank capital in both centers suffer positive multiplicative shocks of size \((1 + dK_0)\) and \((1 + dK_1)\) respectively. I assume that after the change, the center at 0 is still better capitalized than the center at 1:

\[
K_0 \cdot (1 + dK_0) > K_1(1 + dK_1)
\]  

(2.51)

The analysis would require a separate treatment of changes inside regimes and switches between regimes. Switches between regions will occur if the changes in capital are large enough. I limit my attention to changes inside regimes.
Changes within Regime 1

An increase in banking capital allows more projects to be financed. Since projects have a positive net present value, an increase in bank capital makes the system more efficient by expanding investment. As it was shown in Result 4, the cost of bank capital decreases when capital increases; therefore both $\beta_0^*$ and $\beta_1^*$ would decrease.

Changes within Regime 2

In this regime all the projects are funded and therefore, although the financial conditions for the entrepreneurs improve, the number of projects that are financed do not change. To assess the efficiency of the banking system in terms of monitoring, the initial levels of capital and the size of the shocks have to be considered.

**Proposition 17** In Regime 2, the equilibrium market share $y^*$ increases (resp. decreases) whenever the ratio of the capital shocks satisfies $\frac{dK_0}{dK_1} > G^*(K_0, K_1)$ (resp. $\frac{dK_0}{dK_1} < G^*(K_0, K_1)$).

The cut-off value $G^*(K_0, K_1)$ is given by:

$$
G^*(K_0, K_1) = \frac{K_1}{K_0} \cdot \frac{y^*}{1-y^*} 
$$

(2.52)

where $y^*$ is the equilibrium share obtained from expression (2.36). The main property of $G^*(K_0, K_1)$ is:

$$
G^*(K_0, K_1) < 1 
$$

(2.53)

Expressed differently, proportional increases in capital make the system less efficient.

**Proof:** See appendix.

In order to increase the efficiency of the system, the relative increment of capital has to be greater at the less capitalized center at 1 than at the better capitalized center at 0. A banking system where capital is scarce and plays an important role in competition would monitor less efficiently after proportional positive shocks in the capital of its members.
Changes inside Regime 3.

Again an increase in capital has no effect on investment, while financial conditions for some of the entrepreneurs improve. However, now we have:

**Proposition 18** In Regime 3 the equilibrium market share, \( y^* \), decreases with bank capital. Expressed differently, increases in bank capital make the system more efficient.

**Proof:** See appendix.

In Regime 3 as capital increases, only banks at 1 can improve the financial terms that they offer to their clients. Banks at 0 already offer the best possible conditions. The improvement in the financial terms of banks at 1 translates into an increase in the market share of the center at 1\(^2\) and therefore an increase in the efficiency of the banking system (the new equilibrium market share is closer to the efficient one \( s^* \)).

The reason why the conclusion is different than in Regime 2 is that, in that regime, when capital increases, both centers improve their financial conditions and compete more aggressively. In Regime 2 the relative size of the shock has to be considered to assess the relative improvement in financial terms and market share.

**Changes within Regime 4**

In this regime increases in bank capital are redundant. All the projects are funded and no improvement in the financial conditions occurs \( (\beta_0^* = \beta_1^* = \gamma) \).

**Discussion**

When bank capital increases financial conditions for entrepreneurs improve. This is hardly surprising given the competitive behavior of the banking system. This improvement in financial terms can translate into an extension of the projects financed or merely into a transfer of resources to the entrepreneurs\(^{30} \). The fact that after a certain level further

\(^{29}\)In terms of the Figure 2-6 an upwards rotation of the line on the right occurs.

\(^{30}\)This may be a formalization of the "overcapacity" of the banking industry that was described in the introduction (see Unger (1992)).
increases of capital only imply more transfers is not very attractive. It is an artifact of the assumption that investment is fixed in this model. Altering this assumption would complicate the analysis but would not change the other results substantively. In a model with flexible investment better terms and more investment would result.

Although the model is static, the results of the analysis of capital shocks suggests a dynamical interpretation. Figure 2-9 describes a dynamic expansion path assuming a constant ratio of capital accumulation between the centers ($\frac{K_a}{K_1} = \alpha > 1$). Figure 2-9 relates the amount of capital in the less capitalized center at 1 with the equilibrium market share. It would correspond to the expansion path of a ray from the origin (and below the 45° line) in Figure 2-8. The cut-offs $K_{1a}$, $K_{1b}$, and $K_{1c}$ correspond to the levels of capital at 1 that switch the regions.\(^{31}\) The system would begin in a hypothetical efficiency state (in terms of monitoring cost)\(^{32}\) where capital considerations of the banking system are absent. Then it would depart from efficiency\(^{33}\) more and more until it reaches a minimum level of monitoring efficiency. After that, as the capital continues to accumulate the banking system would return to an efficient matching of monitors and investors.

\(^{31}\)The corresponding levels of capital at 0 would be $\alpha K_{1a}$, $\alpha K_{1b}$, and $\alpha K_{1c}$ respectively.

\(^{32}\)Again one has to be careful not to equate efficiency with market share. When the number of financed projects increases this is not correct.

\(^{33}\)Between 0 and $K_1^0$ and expansion of investment would be occurring. Without any capital the market share would be equal to $\frac{1}{2}$. Once differences in capitalization are introduced the system moves away from $\frac{1}{2}$.
The stage where the system is “recovering” efficiency (from $K_{1a}$ to $K_{1b}$ in Figure 2-9) admits the following interpretation. Assume that banks appear at different moments in time and that their capital increases continuously over time. The banking centers will then be at different points in their “life cycle”. The center with more capital will be “older” than the one with less capital. As long as the “younger” banking center is accumulating capital while the “older” one is not doing so, the whole system allocates its resources better and the monitoring is done more efficiently. The claim that increases in bank capital will increase the efficiency of the system may be interpreted as an acceleration of this renewal process among banking centers.

2.4.2 Changes in the interest rate

The second comparative static exercise consists of changes in the interest rate. I identify the interest rate with $\gamma$, the price of capital of the external uninformed sector. In the analysis, $\gamma$ has been treated as an exogenous parameter, representing the marginal productivity of some external technology (a safe technology).\footnote{An implicit assumption in this section is that the inequalities of assumptions 1 and 3 hold when $\gamma$ is changed.}

Before entering into an analysis of the various regimes, consider what would happen in an economy with a single banking center.

Economy with a single banking center

The analysis of a single banking center is summarized in the following proposition:

**Proposition 19** Assume an economy with a single banking center and projects distributed along the line as in subsection 3.1. As the interest rate $\gamma$ changes the following effects occur:

- If bank capital was scarce ($\beta^*>\gamma$) then $\frac{dm^*}{d\gamma} < 0$ and $\frac{d\rho^*}{d\gamma} < 0$.\footnote{This can be justified considering that, in expectation, the net present value of the projects is positive.}

- If bank capital was not scarce ($\beta^*=\gamma$) then $\frac{dm^*}{d\gamma} < 0$ and $\frac{d\rho^*}{d\gamma} > 0$.

**Proof:** See appendix.
The effect on $m^*$ is as expected. It is hardly surprising that investment in the economy falls if the interest rate increases.

The effect on $\beta^*$ shows an asymmetry depending on the scarcity of capital. If bank capital is not scarce, then by arbitrage considerations its cost has to increase with $\gamma$. If bank capital is scarce things are different. Changes in $\gamma$ do not affect the supply of bank capital, because bankers, that will be also earning scarcity rents after the change, will no transfer any capital to the external market. However the demand of bank capital decreases. With the new interest rate less projects are now feasible and less projects are financed. As bankers receive a compensation per project that does not depend on the interest rate, when less projects are financed so does the total compensation for bankers and consequently their profits.

Economy with two banking centers

Next consider the effects with two banking centers:

**Proposition 20** If there is competition between two banking centers, then an increase in the interest rate worsens the financial conditions in the market and affects the efficiency of the banking system in the following way:

- *Regime 1:* Investment is reduced and the cost of bank capital falls: $\frac{d\beta_0^*}{d\gamma} < 0$ and $\frac{d\beta_1^*}{d\gamma} < 0$.

- *Regime 2:* The banking system becomes less efficient: The market share $y^*$ moves away from 0. The effects on $\beta^*$'s are: $\frac{d\beta_0^*}{d\gamma} > 0$ and $\frac{d\beta_1^*}{d\gamma} < 0$.

- *Regime 3:* The banking system becomes more efficient. The market share $y^*$ moves towards $\frac{1}{2}$. The effects on the $\beta^*$'s are as follows.$^{36}$ $\beta_0^* = \gamma'$ and $\frac{d\beta_1^*}{d\gamma} > 0$.

- *Regime 4:* The new external interest rate determines the retribution of the capital in the banking system ($\beta_0^* = \beta_1^* = \gamma'$). The market share is still $y^* = s^* = \frac{1}{2}$.

$^{36}\gamma'$ denotes the new interest rate.
Proof: See appendix.

Formally the analysis of an increase in the interest rate is similar to the analysis of a proportional reduction in bank capital. The only difference is that arbitrage considerations affect the price of bank capital even when the restriction $\beta^s \geq \gamma$ binds.

Note also that because the amounts of bank capital have not changed, changes in the $\beta$s imply corresponding changes in profits of the centers.\textsuperscript{37} When there is interaction among centers (all regimes except 1) the center at 0 always increases profits while the effect on the center at 1 depends on the particular regime that describes the equilibrium.

In Regime 2 an asymmetric results appears. When the interest rate increases, better capitalized banks gain while less capitalized banks loose. The explanation for this result is that, when the price of external funds increases, the optimal mix of external to internal funds changes. Since the ex-post payoff to banks never changes, the capital that banks offer is more valuable to firms after $\gamma$ increases. There will be competitive pressures that favor better capitalized banks and make them to gain market share and increase their profits while the opposite occurs to the less capitalized banks.

When issues concerning bank capital are analyzed, this result shows the importance of disaggregating bank capital rather than considering the balance sheet of the banking sector as a whole. If capital is heterogeneously divided among banks, competition effects are to be relevant.

Which regime is the best characterization of the competition in the banking industry is an empirical matter in which the model has little to say. However given that this analysis has abstracted of other costs that banks may incur on, one can argue that the return on the bank capital must be, unless in gross terms, above the safe return of the economy. Accepting that then Regime 2 appears as the leading candidate to characterize the equilibrium in the industry. The implication is that poorly capitalized banks are negatively affected when external financing conditions tighten. In tight monetary conditions, relative financial strength of banks is more valuable than the relative technological expertise.

\textsuperscript{37}Although profits have to be discounted differently because the interest rate has changed.
The result obtained here complements results related to the lending channel of monetary policy (see for example in Kashyap and Stein (1993)). There it is argued that the tightening of monetary conditions affect more strongly those firms that are more bank dependent, usually small firms, because of their strong reliance on banks. Here low capitalized, highly specialized intermediaries suffer more when monetary conditions are worsened because their comparative advantage in expertise is now less valuable.\footnote{And indirectly small firms tied to them.}

A parallel result could be deduced from a comparative statics analysis on \( c \). The parameter \( c \) measures the importance of expertise for the competition for projects. Note that \( c \) enters in the expressions (2.36) and (2.45) that characterize competition among centers in regimes 2 and 3 exactly in the inverse way than \( \gamma \). While this is a particular feature of the model, it helps to highlight the effects of the trade-off between capital and expertise in the competition for projects. As bank capital and bank expertise substitute each other in bank competition, variables that affect capital have exactly the opposite effects that variables that affect expertise.

\textbf{2.5 Policy implications}

To perform a normative analysis with a very simple model is always delicate. Any policy recommendation is subject to the criticism that it is at best simplistic or at worst wrong. However even with these caveats a normative analysis is of interest. If the forces that have been identified in the model are relevant to competition in the banking system then it is important to analyze the effects of policy measures on the basic trade-off that those forces represent.

First we must consider the welfare criterion. In models like this, e.g. Bernanke and Gertler (1990), limits on wealth transfers are at the root of the problem but, because there are no external effects or any other market failures, the obtained solution is Pareto optimal. No policy measure would be unanimously preferred by all the agents. To evaluate policy measures it is necessary to use some other criterion. The obvious candidate is total surplus
maximization. One justification is that it would be unanimously accepted by all the agents ex-ante before they knew what type of agent they would be in the future. All the assessments of efficiency so far made have relied on the total surplus maximization criterion.\footnote{This criterion is more demanding than Pareto optimality. An equilibrium where surplus is maximized is a Pareto optimum as well.}

In the following subsections I treat direct transfers separately from other policies. I analyze not only whether policies help to reach an hypothetical first best but also whether they have a positive impact on the total surplus of the economy.

### 2.5.1 Direct transfer policies

Total surplus maximization leads to certain trivial but dubious recommendations. As has been argued before, distribution problems are the key to understanding this model. The problems that prevent the economy from achieving the first best (inefficient monitoring and insufficient investment) occur because neither entrepreneurs nor banks have enough resources, or because bank capital is not evenly distributed among centers. If the purpose is to maximize investment, then direct transfers of capital from investors or bankers to entrepreneurs would be desirable. Moreover, transfers of capital to those banks that are insufficiently capitalized would also improve the efficiency of the monitoring.

Although direct transfers can solve the distribution problem, in practice direct transfers are rarely used.\footnote{Transfers do not need to occur ex-ante. A policy that would implement the first best could consist of a progressive tax on the ex-post profits of entrepreneurs that used to guarantee the necessary ex-post rents to successful banks.} Most of the time observed direct transfers have to do more with fairness considerations than with the promotion of efficiency. For those reasons I concentrate the rest of the normative analysis in a different kind of policies.

### 2.5.2 Other policies

In the last twenty years liberalizing reforms in the financial system have been common in several countries.\footnote{Besides the changes in less developed countries, according to Caprio et al. (1994) the following countries have been prominent in such efforts: Australia, Denmark, France, Greece, Italy, Japan, New Zealand, Portugal, Spain, Sweden, U.K. and U.S.} Financial liberalization implies an easing or abandonment of a set of
regulations including portfolio constraints, direct-credit programs and credit and interest-rate ceilings. I focus on the following six kind of policies\textsuperscript{42} that have been extensively used:

- **Deposit ceilings\textsuperscript{43}**

  Regulation q in the U.S. is an example of deposit ceilings. Since 1933, regulation q had limited the interest rate offered on time deposits. After 1979, Congress passed the Depository Institution Deregulation and Monetary Control Act of 1980 that has removed those interest rate ceilings on deposits. In this model deposit ceilings are equivalent to restricting what can be paid to the external financiers (limits on $\gamma$).

- **Intervention on loan prices**

  Policies of subsidized credits have been extensively used in several countries. See Margaritis et al. (1994) for the experience of New Zealand in this respect before the eighties. Intervention on loan prices can be interpreted as the fixing of all $\beta$s at a different level than the market would determine.

- **Geographical limitations**

  Geographical limitations have been particularly relevant in the United States. The Geographic Branches rules have been in place since 1927, the year that the McFadden Act was passed to prohibit interstate branching.\textsuperscript{44} Interpreting the model as a model of geographical differentiation, geographical limitations correspond to preventing banks in one geographical location from lending to projects in distant locations.

\textsuperscript{42}This analysis abstracts from other public-finance considerations. For example Giovannini and de Melo (1993) consider the issue of financial liberalization (or its counterpart financial repression) and conclude that their public finance repercussion must be taken into account. These considerations are more relevant in LDC countries where they estimate that the average revenue is about 2\% of the GNP or 9\% of total government revenue.

\textsuperscript{43}From a different perspective these deposit ceilings can be the consequence of changes in the monetary conditions of the economy if the deposit contract is denominated in nominal and not in real terms.

\textsuperscript{44}The states developed further limitations in intrastate branching. At the present time most of the states (all except 13) allow branching within the state without limits. Interstate branching will be permitted with the Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994.
• Forced lending\textsuperscript{45}

A policy of forced lending can be implemented directly or indirectly. The policy can be directly implemented by excluding certain sectors from borrowing or by setting lending quotas on banks. The indirect implementation of this policy would involve subsidizing loans to certain sectors or limiting the interest rate that can be charged on certain loans. In the U.S. "Usury" regulations had been in place until the eighties. After the Depository Institutions Deregulation and Monetary Control Act of 1980 and the Depository Institutions (Garn-St. Germain) Act of 1982, the rate ceilings imposed by states on business and agricultural loans were preempted\textsuperscript{46} and state ceilings on residential mortgages were eliminated unless states re-impose ceilings.\textsuperscript{47}

• Non interest bearing reserve requirements

All countries impose reserve requirements on banks. Increasing reserves is equivalent to a reduction in the capital of the banking system. In the last few years reserve requirements for U.S. banks have been lowered twice (December 1990 and April 1993.)

• Recapitalization of banks

Recapitalizations of (generally big) banks that are in trouble have been common.\textsuperscript{48} Examples include the recapitalization of Credit Lyonnais in France with state guarantees and the nationalization of the Latin-American banks in the eighties.

The usual reason given for these actions is the "too-big-to-fail" doctrine. A big bank failure may induce, it is argued, a financial panic. Such considerations are completely absent in my model. However, the model offers an alternative rationale for recapitalizations. An

\textsuperscript{45}In the real world this has also implied forced lending to the government and to state enterprises.

\textsuperscript{46}The limits refered to loans in excess of $25,000 for three years.

\textsuperscript{47}See Kaufman (1986) for details. In other countries subsidized credit programs have been extensive. Farming protection, export promotion, mortgages financing or "strategic sector" considerations have been frequently argued. Examples are New Zealand's guidelines to trading banks regarding credit allocation by sector (referred in Margaritis et al (1994)) or Korea's subsidies to agriculture, housing or export financing (see Nam (1994)).

\textsuperscript{48}See Bovenzi and Muldoon (1990) for the U.S. case and Goodhart and Shoemaker (1993) for a worldwide comparison.
evenly capitalized financial sector expands the set of investment projects as do the direct transfers of the previous section.

Of course if recapitalizations are expected by the banks then the incentives can be perverse. A policy of indiscriminate rescues will produce an inoperative banking industry. One possibility would be to institute a policy that comes into effect only after systemic failures occur (probably based on macroeconomic shocks) but not on particular failures (probably related to misbehavior of the bank).49 Macroeconomic shocks that are no fault of banks can be met with recapitalization without causing moral hazard.

I analyze the impact of these measures separating between two different classes of economies: Regime 1 economy (economy with unfinanced projects) and Regime 2 and 3 economy (economy without unfinanced projects). Dynamically this separation can be interpreted as impacts on financially underdeveloped economies (Regime 1) and on financially developed economies (regimes 2 and 3). The main difference is that while in underdeveloped economies the problem is insufficient investment, in more financially developed economies effects on the efficiency of monitoring are also important.

Effects on financially underdeveloped economies (Regime 1)

- Deposit ceilings

Introducing deposit ceilings produces an expansion in investment. This is discussed in Proposition 19.

- Regulation of loan prices

A binding floor on loan prices will leave some capital unused and will reduce the level of investment. On the other hand a binding ceiling will produce rationing. The ceiling does not guarantee an expansion of investment and rationing will produce a misallocation of bank capital. Resources may go from projects with low monitoring costs to projects with high monitoring costs.

49How to distinguish between systemic and idiosyncratic shocks can be a delicate matter.
• Geographical limitations

Geographical limitations do not apply because there is no interaction between banking centers.

• Forced lending

Forced lending changes which firms receive funds and would increase the cost of capital for all of them. The effect on total investment is ambiguous. The reason is that if forced lending is imposed on projects close to the marginal one, there will be a first order impact on new investment and a second order impact on projects that are actually receiving funds. However, if the forced lending is on projects very far from the marginal one (and therefore very costly to monitor) the misallocation of capital may, in fact, reduce investment in the economy.

• Non interest bearing reserve requirements

The effects of reserve requirements are the same as those for a reduction in the capital of the banking sector: an increase in the price of loans, decrease in investment and a decrease in the profits of the banking industry.

• Recapitalization of banks

Assuming that recapitalizations are financed with taxes from the other agents of the economy (investors and entrepreneurs\(^{50}\)) and that the recapitalizations do not alter incentives (see above), they are equivalent to an increase of capital in the system and therefore to an expansion in investment. This is discussed in Result 3.

Effects on financially developed economies (Regimes 2 and 3)

• Deposit ceilings

\(^{50}\)Taxes that do not affect the incentives of the entrepreneurs.
The effects of deposit ceilings were considered in Proposition 20 in the previous section. Basically they produce better conditions for entrepreneurs and a better matching if both centers are capital restricted (Regime 2). The matching is worse if only one center is restricted (Regime 3).

- **Regulation of loan prices**

Regulation of loan prices produce rationing in the loan market. Its concrete effects depend on the assumptions made about the rationing scheme. Consider for example the imposition of a limit in the price of loans. This limit will affect more severely to the less capitalized center, the center at 1.\(^{51}\) To assess the effects on the equilibrium, one has to be precise about the rationing process. One possibility would be that projects very close to the bank at 1 are receiving no capital but still standing with the financier. The consequence of that would be an expansion of the market share of the center at 1. However another possibility is that the intervention of loans means that the rationing affect the projects where competition among centers can occur. This instead would produce a reduction in the market share of the sector at 1.

An increase in the price of loans will also produce rationing that would affect to projects close to the original equilibrium.

- **Geographical limitations**

Geographical (or regional) limitations make sense if prevent matchings that are motivated by rent seeking considerations. However, too stringent limitations could eventually prevent some projects from obtaining funds at all.

- **Forced lending**

Forced lending does not apply because in these cases the market is covered.

- **Non interest bearing reserve requirements**

\(^{51}\) In equilibrium \(\beta_0^c < \beta_1^c\); see Result 6.
Reserve requirements can be analyzed as a proportional reduction in the capital of the banking system. Again see the discussion in Proposition 1.

- Recapitalization of banks

With the same caveat that in the case of underdeveloped economies, the analysis would correspond to increases in the bank capital of the banks. (See Propositions 17 and 18).

Conclusion

The main lessons that one can draw from this section are related to financial liberalization. While real economies do not feature as sharp a separation between matching problems and investment problems as in the model, a drastic financial liberalization program may cause difficulties to undercapitalized intermediaries of the financial system. If those intermediaries play an important role in financing highly specialized investment projects, then the evaluation of a financial liberalization program has to consider the competition effects described in this paper. For example, Berger et al. (1995) show that in the U.S. banking industry size of the borrower and size of the lender are closely related. If the end of the geographical restrictions leads to the elimination of small, local banks then such deregulation will produce a contraction on lending for small business that depend on those small banks.

2.6 Concluding Remarks

This paper has presented a model of competition in the banking industry based on the interaction between bank capital and monitoring expertise. Expertise is associated with technological knowledge that allows a bank to offer its credit services less expensively. Expertise is a relative concept. Because different projects require different knowledge, the expertise of banks refers to particular projects or lines of business.

Banks considered in this economy differ in two dimensions: the amount of capital that they have and the knowledge they possess. One of the main insights of the paper is that, as banks compete for credit, their capital and expertise are substitutes. The reason for this is that, in an environment of moral hazard and limited liability, banks have to be guaranteed

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ex-post rents in order to monitor diligently and both expertise and capital affect the size of those rents.

The joint consideration of capital and expertise leads to some novel efficiency conclusions. According to our efficiency criterion the banking industry will be more efficient the more important is expertise relative to capital. This rationalizes the comparative statics results that have been obtained. Increases in bank capital or quicker mobilization of banking resources will increase the efficiency of the banking industry as long as these increments go predominantly to the more efficient but more capital constrained banking sector. On the other hand, a tightening of monetary conditions that increases the interest rate will make banking capital more valuable than before and therefore will tend to cause more inefficiencies in monitoring.

From a different perspective, the model permits a differentiation between financially developed economies (those able to provide financial assistance to all the projects of the economy) and financially underdeveloped economies. With that in mind an analysis of several policy measures was provided. In the case of underdeveloped economies, considerations of the expansion of investment dominate; while in financially developed economies, the matching aspect is also relevant.

I conclude with two observations about future research. The first observation concerns the entrepreneurial character of a bank. It may be critical to incorporate agency considerations. It is assumed that capital severely limits the expansion of the bank. This is true but it is also true that banks inside equity (equity in the hands of the managers) is small relative to outside equity. Either the analysis must be interpreted in terms of inside equity, or explicit agency considerations will have to be included to make the model more consistent. The second observation has to do with the exogeneity of location and distribution capital. One justification is to consider this as a short-run analysis. A more satisfactory treatment could require a model of capital accumulation for the banking sector. This remains a critical and challenging task on the research agenda of banking theory.
Bibliography


Chapter 3

A Model of Debt Maturity:
Determinants and Evolution

3.1 Introduction

The composition and maturity of the financing varies widely across firms. A stylized description of the financial life cycle of firms suggests several stages. A first stage in which firms are internally financed is followed by a second stage in which the financial structure is dominated by short-term closely monitored funds (bank loans), which is gradually replaced by private placements of debt\(^1\) and finally with publicly issued financial instruments. Firms move towards less monitored (external) and longer-term funds\(^2\).

The financial literature has analyzed the composition of the financial structure of firms from different perspectives. For example, Holmstrom and Tirole (1994) present a model that explains how differences in collateral lead firms to have different classes of financiers. Less collateralized firms end up borrowing from intermediaries while more collateralized firms

\(^1\)The U.S. legal criteria for a private placement is a security that is issued in the United States but exempt from registration from the Securities and Exchange Commission because it does not include any public offering. See Carey et al. (1993) for details.

\(^2\)Carey et al. (1993) describe bank loans as assets of short to intermediate-term maturities (in their sample the median maturity was nine months), private placements as assets with intermediate to long-term maturity (median maturity of nine years) and publicly issued bonds as long-term assets (median maturity of more than twelve years).
borrow from external financial markets. Bolton and Scharfstein (1996) use an incomplete contract model to focus on other issues concerning debt structure: the optimal number of creditors to borrow from, the optimal allocation of security interests among creditors and the voting rules among creditors. This paper abstracts from issues of financial composition in order to focus on maturity structure. The model developed in the paper provides a rationale for different loan maturities and proposes a pattern of evolution of financial maturity over time.

To analyze the evolution of financial maturity, the first task is to justify why the maturity of financial contracts matters at all. In a world with perfect information and complete markets, the maturity of financial contracts will not affect either the investment of firms or the selection of projects; total surplus maximization would induce a fully flexible financial policy that incorporates all the available information as it appears. In that world no reason for long term financial contracts exists. But the implications of such an ideal world, where markets see through the contract veil, changes in the presence of informational imperfections. The financial literature has offered several papers that use informational imperfections to link the financial maturity of contracts with firms' investments. Two important papers are Hart and Moore (1990) and Diamond (1991). Hart and Moore study the role of short term contracts in compensating lenders that are subject to opportunistic behavior on the part of borrowers. Borrowers, unable to commit to repay their financial obligations, always could “take the money and run.” In Hart and Moore’s model, long term contracts solve the problem of excessive liquidation but also accentuate borrower’s opportunistic behavior. Diamond (1991) builds an adverse selection model and focuses on the opportunities that different durations of contracts offer to screen among different classes of borrowers with private information about the prospect of projects (and without resources to undertake them.) The trade-off for (better informed) borrowers in Diamond’s model is between short term contracts that produce too much liquidation and long term contracts that are expensive.

The basic model in this paper is about signal jamming. The model considers the effects

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3 Other theoretical papers that analyze the effect of financial maturity are Brick and Ravid (1985), Diamond (1993), Flannery (1986), Kaie and Noe (1990), Lewis (1990) and von Thadden (1995).

4 See Holmstrom (1982), Fudenberg and Tirole (1986) and St. in (1989).
two informational imperfections acting simultaneously: the imperfect observability by financiers of certain managerial choices, and the impossibility of a complete transfer of utility among parties. In the basic model, the maturity of financial contracts is a key element that provides the right incentives in the lender-borrower relationship. Two basic contracts can be signed between borrowers and lenders: a short term contract, offering the opportunity to liquidate and incorporating relevant information as it appears; and a long term contract, that, by ignoring some information, avoids diversion of resources to distort information.

After showing why maturity matters, the second task for a theory of financial maturity, involves comparative statics. The model in this paper proposes a pattern of how the financial maturity of firms evolves over time. The basic result is that old firms tend towards longer financial maturities. The intuition is that the learning component involved in every investment decision, grows less important as time passes. Section 4 will discuss how this dynamic implication is broadly consistent with the observed dynamic patterns that have been documented in the literature.

A recent paper by Barclay and Smith (1995) points out several factors affecting the maturity structure of corporate debt. Among other things, they find that firms with fewer growth opportunities, large firms, regulated firms and firms without large potential informational asymmetries have more long term debt on their balance sheets. Note that several of these tendencies are associated more with dynamic aspects of the financial maturity than with a static composition of the balance sheet. Thus, the necessity of a theory of financial maturity to provide implications about the evolution of financial maturity over time becomes evident.

From a different perspective, the paper can be understood as a simple description of the process of investment selection that capital markets perform, and what it implies for the duration of committed funds. Markets value a fully flexible financial relationship because it most effectively incorporates information about the prospects of the investment, but they

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5The model in this paper is too stylized to distinguish between debt and equity; however since the model is built with maturity identified as the right to liquidate, it would be natural to interpret the model as describing the trade-off between short-term and long-term debt.
also consider the disruption that the learning process may produce in the outcome of the real investments.

The paper is organized as follows. In section 2 the basic model is introduced. In section 3 the basic comparative statics results are derived. Section 4 presents a dynamic extension of the model. Section 5 considers two variations of the model. In one, an economy with endogenous and stochastic liquidation values is considered. In the other, the effects of changes in the interest rates on financial maturity are analyzed. Section 6 concludes. Proofs and other technical derivations are relegated to the appendix.

3.2 The basic model

3.2.1 Description of the economy

Assume an economy with three different periods $t = 0, 1$ and 2 and one good that can be invested or stored to be consumed at $t = 2$.

Agents

There exist two kinds of risk neutral agents, lenders and borrowers, characterized as follows:

A. Lenders

Lenders (also referred to as investors) have all the endowment, $E$. That endowment can be invested in risky projects in the hands of borrowers or kept in a storage technology that returns $R$ as the gross interest rate per period. For simplicity I assume that the endowment $E$ is large enough to avoid any shortage of funds. For that reason one can assume that the opportunity cost of the endowment is $R$ per period. Lenders maximize their consumption in the second period of their lives ($t = 2$).

B. Borrowers

Borrowers (also referred to as entrepreneurs) lack any endowment. Instead, each borrower has one project (idea) that requires $I$ units of funds and that gives a stochastic payoff, $	ilde{y}(e)$, two periods after the investment.
Each borrower also enjoys a private (non-transferable) continuation rent \( C \). The continuation rent \( C \) is obtained only if the project that a particular borrower represents is not liquidated in \( t = 1 \). If the project is liquidated, then \( C \) is dissipated. I will make the following assumption about \( C \).

**Assumption 20** The continuation rent \( C \) is infinite.

The rather extreme Assumption 20 simplifies the analysis in two respects. First Assumption 20 avoids the need to consider renegotiation. If \( C \) were finite, there could be some renegotiation in certain states of nature where liquidation is convenient for lenders. Second, the fact that \( C \) is infinite constrains severely the incentive contracts that investors can offer to entrepreneurs. An infinite \( C \) makes borrowers completely insensitive to monetary incentives. The intent is to capture a fundamental conflict of interest between borrowers and lenders that is essential to the maturity choice of financial contracts. If \( C \) were finite, then more complicated contracts, including not only the maturity choice but also a financial incentive scheme, would have to be considered. However, as long as \( C \) is non-transferable, the basic trade-off about maturity choice developed below will carry over.

**Projects**

There are two main kinds of projects in the economy:

**A. Safe projects**

Safe projects consist of a storage technology with a gross return \( R > 1 \) per period. These projects are available at \( t = 0 \) as well as at \( t = 1 \). One unit of investment in a safe project gives \( R \) units of consumption good one period from the time of the investment (and therefore \( R^2 \) units two periods from the time of investment.)

**B. Risky projects**

Risky projects are in the hands of borrowers. At \( t = 0 \), the information about the characteristics of risky projects is identical for both borrowers and lenders. They both
learn about the profitability of the projects during the life of the project and only after the investment is undertaken. The actual profitability of the risky project also depends on actions taken by borrowers. Borrowers can pursue two different activities: they can either allocate their effort to the fundamentals of the project, increasing the average profitability of the projects, or they can alter the mechanism that reveals information about the prospect of the project.

**Assumption 21** Borrowers expend a total amount of effort (time) equal to 1.

Borrowers suffer no disutility by exerting effort, $e$, but they can decide how much effort to dedicate to different tasks. The effort, completely unobservable, is put into the fundamentals of the project unless an alternative occupation concerns borrowers. If this happens, borrowers will distribute their effort according to their private interests.

Risky projects take two periods to produce an stochastic outcome $\tilde{y}(e)$. The profitability of the project $\tilde{y}(e)$ depends not only on its intrinsic quality but also on the effort, $e$ dedicated to the project. At $t = 0$, the prior distribution that both borrowers and lenders have about the payoff of the projects in $t = 0$ is the following:

$$\tilde{y}(e) \sim N \left( m_0 + e, \sigma^2 \right)$$

(3.1)

After the first period, the project can be liquidated. The following assumptions describe the liquidation process:

**Assumption 22** If a project is liquidated in $t = 1$, then $L$, a fixed and known amount of consumption good, is obtained.

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6See next subsection for the actual mechanism through which agents get informed.

7This formulation resembles Holmstrom and Milgrom (1991) which analyzes how to provide incentives for an optimal allocation of time into tasks.

8Normality is proposed for analytical tractability and also because its mean and variance are independent. However, the normality assumption raises the issue of the interpreting a negative return. One possible interpretation is to assume that investors have enough funds to finance a pool of independent projects and therefore that by the law of large numbers, the probability that the whole portfolio produces a negative consumption is zero. See Judd (1985) for the conditions required to apply the law of large numbers more rigoursly.
This is the simplest assumption that can be made about liquidation. Some variants of the model will be considered in Section 5 by relaxing Assumption 22.

For simplicity I will restrict the analysis to the following case:

**Assumption 23** The present value of the liquidation procedure is less than the cost of investment:

\[ I \geq \frac{L}{R} \]  

(3.2)

Assumption 23 simply states that projects are not undertaken to be liquidated but with the hope of completing them. Assumption 23 is made for simplicity and does not change the fundamental results.

**Learning about the project: signals**

A signal about the quality of the project is received in \( t = 1 \). Signals, independent across projects, are received before the liquidation decision is made. However, signals can be altered by borrowers; if financiers use signals to condition their liquidation decisions, then borrowers will attempt to manipulate the signals. In equilibrium, although lenders are not fooled by this attempted manipulation, borrowers still put some effort to make up the signal reducing the expected profitability of the project. This is the well known *signal jamming* situation, as in Holmstrom (1982) and Stein (1989).

In this context, different maturities of financial contracts can be understood as different commitment devices to use or not use the signal to liquidate. Consider the following definitions:

**Definition 21** A short-term financing scheme is one that gives the lender the right to liquidate the project at \( t = 1 \).

**Definition 22** A long-term financing scheme is one where lenders explicitly renounce their right to an early liquidation at \( t = 1 \).\(^9\)

\(^9\)One can assume that nobody has the right to liquidate or that the right to liquidate is in the hands of the borrower.
The structure of the signal in $t = 1$ is:

$$
\tilde{s}(e) = \tilde{g}(e) + g(e) + \bar{e}
$$

(3.3)

where $\bar{e} \sim N(0, \nu^2)$. The signal can be affected by the effort allocation. According to (3.3), the signal $\tilde{s}(e)$ can be low for three reasons: because the fundamentals of the project are bad ($\tilde{g}(e)$ is low), because the “make-up” of the signal is bad ($g(e)$ is low) or simply because the random variable that affects the signal idiosyncratically $\bar{e}$ is low.

**Assumption 24** The make-up function, $g(\cdot):[0, 1] \rightarrow \mathbb{R}^+$, has the following properties:

$$
g(\cdot) > 0; \quad g'(\cdot) < 0; \quad g''(\cdot) < 0 \text{ and } \lim_{x \to 1} g'(x) = -\infty
$$

(3.4)

Assumption 24 becomes relevant below when the borrower’s effort allocation is considered.

The timing of the risky projects can be represented as follows:

<table>
<thead>
<tr>
<th>$t=0$</th>
<th>$t=1/2$</th>
<th>$t=1$</th>
<th>$t=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>Effort</td>
<td>Signal and Liquidation</td>
<td>Payoffs</td>
</tr>
</tbody>
</table>

Timing

Having presented the elements of the model, we are ready to analyze how the model works.

### 3.2.2 Analysis of the basic model

The analysis of the model consists of the design of the optimal contract between borrowers and lenders. I will assume that lenders, at $t = 0$, propose the contract to borrowers.

Lenders have to make three decisions:

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10If the problem were solved considering that the contract is proposed by borrowers, it would still be possible to analyze the maturity choice. As it will be clear later on, the qualitative results will hold but some of the details of the model, e.g. the cut-off levels computed below, would change.
1. Whether or not to invest in the project at $t = 0$.

2. What incentive scheme to offer to borrowers if the funds are committed to the project.

3. What duration of financial commitment to give to the project.

Due to Assumption 20 (borrowers enjoy an infinite and nontransferable continuation rent) the design of optimal monetary incentives for borrowers becomes irrelevant. Continuing the project is so valuable for borrowers that they only care about maximizing the probability of its continuation. Note also that no participation constraint is required for borrowers. The mere fact of getting funds for the project makes a borrower willing to participate in the contractual relationship.

Even with those simplifications, the problem remains interesting, and the choice of maturity of the initial contract still relevant. Consider what lenders, as residual claimants,\textsuperscript{11} will obtain with a short term and a long term financial scheme.

Case A: A long-term financial scheme

A long term financial contract is equivalent to the formal renunciation of the lender’s right to liquidate the project in $t = 1$. The solution of the model can be readily computed in this case. The signal is not used to decide if projects continue or not so borrowers do not care about "jamming" signals and consequently put all their effort into the project fundamentals.

Therefore the expected yield of the project at $t = 0$ is the expectation of the prior (3.1) evaluated at $e = 1$:\textsuperscript{12}

$$E(\hat{y}(1)) = m_0 + 1$$

(3.5)

The decision to invest is summarized in the following result:

**Result 11** The investment rule is:

- If $m_0 + 1 \geq I \cdot R^2$ then finance the project.

\textsuperscript{11}Lenders are not only residual claimants but also the only claimants.

\textsuperscript{12}An epsilon share of the project outcome will make borrowers to strictly prefer $e = 1$.  

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• If \( m_0 + 1 < I \cdot R^2 \) then do not finance the project (with a long term financial scheme.)

The expected surplus for the lender of financing the project with a long term contract, \( W_{LR} \), is:

\[
W_{LR} = m_0 + 1 - I \cdot R^2 \quad (3.6)
\]

**Case B: A short-term financial scheme**

The analysis is more complicated. Now the optimal behavior for the lender involves a dynamic aspect because of the possibility of liquidating at \( t = 1 \). I will solve the model backwards:

**2.2.2.a. Liquidation decision at \( t = 1 \)**

Lenders will liquidate at \( t = 1 \) if the expected profitability of the project is below the amount \( L \) that they can recover by liquidating. Let \( m(e^c, s) \) be the best conjecture that they can make about the expected profitability of the investment. \( m(e^c, s) \) depends on a conjectured level of effort from the borrower \( e^c \) and on the observed signal \( s \).

The optimal liquidation rule is summarized in the following result:

**Result 12** The optimal liquidation rule for the lender is the following cut-off rule:

- If \( m(e^c, s) \geq L \cdot R \) then continue the project.
- If \( m(e^c, s^o) < L \cdot R \) then liquidate the project.

Computing \( m(e^c, s) \) requires the analysis of the borrower's choice of effort. First, consider the following Result 13,\(^{13}\) about updating of normal distributions after observing normal signals.

**Result 13** Assume that the prior distribution about parameter \( \bar{x} \) is of the form:

\[
\bar{x} \sim N(x_0, \sigma^2) \quad (3.7)
\]

\(^{13}\)For a proof see, for example, DeGroot (1970).
with known variance $\sigma^2$. Assume that a signal about $\tilde{x}$ is observed of the form:

$$\tilde{s} = \bar{x} + \tilde{e}$$

(3.8)

where $\tilde{e}$ is independent of $\bar{x}$:

$$\tilde{e} \sim N(0, \nu^2)$$

(3.9)

If $s$ is the realization of the signal $\tilde{s}$, then the posterior distribution $\tilde{x}/s$ has the form:

$$\frac{\tilde{x}}{s} \sim N\left(\frac{\nu^2}{\nu^2 + \sigma^2 \bar{x}_0} + \frac{\sigma^2}{\nu^2 + \sigma^2 \bar{s}}, \frac{\sigma^2 \nu^2}{\nu^2 + \sigma^2}\right)$$

(3.10)

2.2.2.b. Borrower’s effort decision

Knowing how the liquidation decision is made, and because the mean of the posterior distribution is increasing in the signal, borrowers will maximize the realization of the signal. Therefore borrowers will solve:

$$\max_e E(\tilde{s}(e))$$

(3.11)

$$s.t. \quad 0 \leq e \leq 1$$

(3.12)

Remembering that $E(\tilde{s}(e)) = m_0 + e + g(e)$, the first order condition (in this case a sufficient condition) is:

$$g'(e^*) = -1$$

(3.13)

Equation (3.13) defines the effort level $e^*$ for a borrower subject to a short term financial contract. Effort $e = e^*$ maximize the expected value of the signal. Note that $e^* < 1$ that is the level of effort that maximizes the outcome of the project. In a rational expectations equilibrium the conjectured effort $e^c$ coincides with $e^*$ and therefore the equilibrium is completely characterized:

$$e^* = e^c$$

(3.14)
Hence the best conjecture after observing the signal $s$, $\mu(e^*, s)$ can be expressed as:

$$m(e^*, s) \equiv \frac{\nu^2}{\sigma^2 + \nu^2} (m_0 + e^*) + \frac{\sigma^2}{\sigma^2 + \nu^2} (s - g(e^*))$$  \hspace{1cm} (3.15)

and the optimal liquidation rule can be expressed in terms of a cut-off signal $s'$.

Result 14 The optimal liquidation rule can be expressed as a cut-off level for the observed signal, $s'$:

- If $s \geq s'$ then continue with the project.
- If $s < s'$ then liquidate the project.

where:

$$s' = g(e^*) + \frac{\sigma^2 + \nu^2}{\sigma^2} L \cdot R - \frac{\nu^2}{\sigma^2} (m_0 + e^*)$$  \hspace{1cm} (3.16)

Proof. Consider the definition of $m(e^*, s)$ and that the optimal liquidation rule implies $m(e^*, s) \geq L \cdot R$. $lacksquare$

Several comparative statics results for the liquidation cut-off $s'$ immediately follow.

Result 15 The cut-off signal $s'$ increases with the variance of the prior $\sigma^2$ and with the amount that can be obtained in the liquidation process $L$ and decreases with the variance of the random component of the signal $\nu^2$ and with the expectation of the prior $m_0$.

Proof. It follows from the expression (3.16). $lacksquare$

2.2.2.c. Expected surplus from a project financed with short term funds

Once the optimal liquidation decision has been characterized, we can compute the actual surplus when a short-term financial contract is in place. Define the following random variable:

$$\tilde{m} \equiv m(e^*, \tilde{s})$$  \hspace{1cm} (3.17)

The variable $\tilde{m}$ represents the distribution of the best conjecture about expected profitability prior to observing the signal, when the signal has the form (3.3) and the level of effort is $e^*$, the equilibrium level of effort. Consider the following lemma:
Lemma 23 The variable $\tilde{m}$ has the following distribution:

$$\tilde{m} \sim N \left( m_0 + e^*, \frac{c^4}{\sigma^2 + v^2} \right)$$  \hspace{1cm} (3.18)

**Proof.** See appendix.

Considering $\tilde{m}$ and the optimal liquidation rule, it is possible to compute the expected value of the surplus $W_{SR}$ when the short-term financial scheme is in place:

$$W_{SR} = E(\tilde{m} | \tilde{m} \geq L \cdot R) \cdot Pr(\tilde{m} \geq L \cdot R) + L \cdot R \cdot Pr(\tilde{m} < L \cdot R) - I \cdot R^2$$  \hspace{1cm} (3.19)

**Maturity decision**

The comparison between $W_{SR}$ and $W_{LR}$ determines the choice of maturity. The following four possibilities can occur:

1. The project is financed with short term funds if:

$$W_{LR} < W_{SR} \quad \text{and} \quad W_{SR} \geq 0$$  \hspace{1cm} (3.20)

2. The project is financed with long term funds if:

$$W_{LR} > W_{SR} \quad \text{and} \quad W_{LR} \geq 0$$  \hspace{1cm} (3.21)

3. The project is financed and the choice of maturity is irrelevant if:

$$W_{SR} = W_{LR} \geq 0$$  \hspace{1cm} (3.22)

4. The project is not financed when:

$$W_{SR} < 0 \quad \text{and} \quad W_{SR} < 0$$  \hspace{1cm} (3.23)
3.3 Analysis of maturity choice

This section shows how the value of a project depends on the maturity of its financing and how the relevant variables of the model affect it. It places these results into a framework where a short term contract can be seen as an option to liquidate the project. However before entering into the comparative statics analysis, I will relate several of the expressions in the previous section to the primitive parameters of the economy.

Lemma 24 \( E(\bar{m} | \bar{m} \geq L \cdot R) \) can be expressed as follows:

\[
E(\bar{m} | \bar{m} \geq L \cdot R) = (m_0 + e^*) + n \frac{\phi \left( \frac{L \cdot R - m_0 - e^*}{n} \right)}{\Phi \left( \frac{L \cdot R - m_0 - e^*}{n} \right)}
\]  

(3.24)

where

\[
n = \frac{\sigma^2}{\sqrt{\sigma^2 + \nu^2}}
\]

(3.25)

and where \( \phi(\cdot) \) represents the density function of a standard normal random variable and \( \Phi(\cdot) \) represents the corresponding distribution function.

Proof. See appendix.

Note that \( n \) is the standard deviation of \( \bar{m} \). This variable, which combines the two sources of risk in the model, plays an important role in the analysis. Its main properties are summarized in the following result:

Result 16 The standard deviation of the expected profitability of the project at \( t = 0 \), \( n \), has the following properties:

\[
\frac{d n}{d \sigma^2} > 0, \quad \frac{d n}{d \nu^2} < 0, \quad \lim_{\sigma^2 \to 0} n = 0, \quad \lim_{\nu^2 \to +\infty} n = 0, \quad \lim_{\nu^2 \to 0} n = \sigma \quad \text{and} \quad \lim_{\sigma^2 \to +\infty} n = +\infty
\]

(3.26)

Note that \( n \) increases monotonically with \( \sigma^2 \) and decreases monotonically with \( \nu^2 \).

Remark 9 From now on, the symbols \( \phi \) and \( \Phi \) will represent the standard normal density and distribution functions evaluated at \( \frac{L \cdot R - m_0 - e^*}{n} \); i.e. \( \phi \left( \frac{L \cdot R - m_0 - e^*}{n} \right) \) and \( \Phi \left( \frac{L \cdot R - m_0 - e^*}{n} \right) \)
respectively, unless otherwise stated.

Using Lemma 24, $W_{SR}$ can be reexpressed as:

$$W_{SR} = (m_0 + e^*) (1 - \Phi) + L \cdot R \cdot \Phi + n \cdot \phi - I \cdot R^2$$  \hspace{1cm} (3.27)

where $\Pr(\hat{m} < L \cdot R) = \Phi$.

### 3.3.1 Comparative statics

Having expressed $W_{SR}$ more conveniently we can compare the surplus that is obtained from short-term financing versus long-term financing.

**Result 17** The value of using a short-term financial scheme instead of a long-term financial scheme, $V$, can be expressed as:

$$V \equiv W_{SR} - W_{LR} = V_f + V_d$$  \hspace{1cm} (3.28)

where:

$$V_f \equiv (L \cdot R - m_0 - e^*) \cdot \Phi + n \cdot \phi > 0$$  \hspace{1cm} (3.29)

$$V_d \equiv e^* - 1 < 0$$  \hspace{1cm} (3.30)

The trade off is clear: a short-term financial policy produces more flexibility, $V_f > 0$, by liquidating projects with bad prospects in $t = 1$,\(^{14}\) but also produces a misallocation of resources due to the signal jamming problem, $V_d < 0$. The comparative statics on the maturity choice is reduced to compute the derivatives of expression $V$ with respect to the relevant parameters of the model.

\(^{14}\)In the appendix $V_f > 0$ is shown.
Proposition 25 Other things equal, financial maturity increases with the expected value of the project at \( t = 0 \), \( m_0 \) and decreases with the value of liquidation \( L \):

\[
\frac{dV}{dm_0} < 0 \tag{3.31}
\]

\[
\frac{dV}{dL} > 0 \tag{3.32}
\]

Proposition 26 Other things equal, financial maturity increases with the variability of the idiosyncratic component of the signal \( \nu^2 \) and decreases with the variability of the prior distribution of the project at \( t = 0 \) \( \sigma^2 \):

\[
\frac{dV}{d\nu^2} < 0 \tag{3.33}
\]

\[
\frac{dV}{d\sigma^2} > 0 \tag{3.34}
\]

Proposition 27 Assume that the "make-up" function \( g(\cdot) \) defined above can be parametrized in the following way:

\[ g(e) = k \cdot g_0(e) \tag{3.35} \]

where \( g_0(\cdot) \) inherits all the properties of \( g(\cdot) \) defined in (3.4). Then, the length of financial contracts increases with \( k \):

\[
\frac{dV}{dk} < 0 \tag{3.36}
\]

Proof See appendix.

The interpretation of Propositions 25, 26 and 27 is straightforward but interesting. A short-term financial contract in this context is nothing more than an option that the lenders have to liquidate the project earlier (a put option) at price \( L \). Proposition 25 simply asserts that if the expected value of the project with respect to the "liquidation price" is very high, then the probability that the option is exercised is low, and so is the value of the option. Proposition 25 also says that the higher the exercise price of the put option that the short term financial scheme represents, the more valuable it is to have such an option in \( t = 0 \).
Proposition 26 relates the two sources of variability in the economy to the financial maturity of the projects. Again, the value of a put option increases with the variance of the underlying asset; in this context, the higher the variability in the economy (large $\sigma^2$), the shorter the length of financial contracts. This may explain why in countries with large uncertainty, entrepreneurs have problems raising long-term "patient" capital; financiers in those economies always want to reserve the right to withdraw the funds. On the other hand, if signals are very volatile (large $\nu^2$), then their value is questionable and a tendency toward long term financing is to be expected.\textsuperscript{15,16}

Finally Proposition 27 allows an interesting economic interpretation. If it is easy to make up signals (large $k$), then one should expect more long-term financing. Firms that rely heavily on accounting information or internal evaluations will be financed with longer maturity than firms where markets or external institutions could give reliable information about their performance. The reason is that an excessive reliance on internal procedures will tend to mobilize resources to affect those measures instead of worrying more about real business profitability.\textsuperscript{17}

The connection of these propositions to the empirical tendencies observed about financial maturities is postponed until after the dynamic version of the model is discussed in Section 4.

\textsuperscript{15}This is an application of Blackwell's theorem.

\textsuperscript{16}Note the importance of the model chosen for the sign of the expression: $\frac{dV}{d\sigma^2}$. The fact that this is a model of allocation of tasks (effort) and not a model where effort is costly for the entrepreneur is crucial to obtain a non ambiguous sign for $\frac{dV}{d\sigma^2}$. In this set up the distortion in the signal is independent on its volatility. However, if effort were costly then a more volatile signal (and therefore a less informative signal) will induce less distortion on the entrepreneur and the sign of $\frac{dV}{d\sigma^2}$ will be in general ambiguous (a more volatile signal is less useful but also less distorted).

\textsuperscript{17}This last argument resembles the one by Milgrom and Roberts (1988) about influence activities in organizations.
3.3.2 The “mean-variance” characterization

As stated above, the standard deviation of the expected profitability $n$ is an important variable in the analysis of the model. Remember that $n$ was defined in (3.25) as:

$$n = \frac{\sigma^2}{\sqrt{\sigma^2 + \nu^2}}$$

In this subsection I will characterize the $(m_0, n)$ space in terms of the financial maturity of projects. This will be useful for the next section when the dynamic extension of the model is considered. Consider the following three loci:

- $A(m_0, n) = \{(m_0, n) \text{ such that } W_{SR} = W_{LR}\}$. Locus of points where the surplus of the short-term and long-term financing is the same.

- $B(m_0, n) = \{(m_0, n) \text{ such that } W_{LR} = 0\}$. Locus of points where the surplus of the long-term funding is zero.

- $C(m_0, n) = \{(m_0, n) \text{ such that } W_{SR} = 0\}$. Locus of points where the surplus of the short-term financing is zero.

Results 28, 29 and 30 characterize the three loci:

Lemma 28 The equation:

$$(L \cdot R - m_0 - e^*) \cdot \Phi + n \cdot \phi + (e^* - 1) = 0$$

characterizes $A(m_0, n)$. Graphically, $A(m_0, n)$ is an increasing and concave line:

$$\frac{dn}{dm_0} \bigg|_A > 0$$

$$\frac{d^2 n}{dm_0^2} \bigg|_A < 0$$

Lemma 29 The equation:

$$m_0 + 1 = I \cdot R^2$$

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Figure 3-1: Maturity in the mean-variance analysis

characterizes \( B(m_0, n) \). Graphically \( B(m_0, n) \), is a perpendicular line that goes through \( m_0 = I \cdot R^2 - 1 \).

Lemma 30 The equation:

\[
(L \cdot R - m_0 - e^*) \cdot \Phi + n \cdot \phi + (m_0 + e^*) = 0
\]  
(3.41)

characterizes \( C(m_0, n) \). Graphically, \( C(m_0, n) \) is a decreasing and concave line:

\[
\frac{dn}{dm_0} \bigg|_C < 0 \\
\frac{d^2n}{dm_0^2} \bigg|_C < 0
\]  
(3.42)  
(3.43)

Proof See appendix for the characterization of the three loci.

Considering the three loci, it is possible to divide the space \((m_0, n)\) in three regions (see graph (3.1)):

- Region 1. Projects belonging to Region 1 will be financed with long-term funds. Those are projects with prior mean \( m_0 \) higher than the threshold \( I \cdot R^2 - 1 \), and
with relatively low variance. For simplicity I will refer to Region 1 as high mean-low variance region.

- Region 2. Projects belonging to Region 2 will be financed with short-term funds. I will refer to Region 2 as high-variance region.

- Region 3. Projects in this region are not financed. This region contains low mean-low variance projects (low mean-low variance region).

For future reference define \( n_p \) as the variance level at which \( W_{SR} = W_{LR} = 0 \). The value \( n_p \) can be found as the solution of the equation

\[
A(I \cdot R^2 - 1, n_p) = 0
\]  
(3.44)

In the appendix, the existence of \( n_p \) is proven.

3.4 Dynamic extension: the evolution of maturity in time

The purpose of this section is to extend the basic model to a simplified dynamic context in order to develop a dynamic pattern for firms' financial structure. This pattern will be contrasted with empirical regularities observed about maturity.

3.4.1 Dynamic set-up

Projects

As in the basic model, there exist two kind of projects in this economy: safe projects, that give a gross interest rate \( R \) per period, and risky projects. However, several "generations" of risky projects coexist in the economy. Each generation of projects consists of a continuum of projects (of measure 1) that become available in the same period. A new generation of projects appears every two periods; hence if the economy begins at \( t = 0 \), there will be a new generation of projects in \( t = 0, 2, 4, 6 \ldots \).
The prior for all the risky projects in the same generation is:

\[ \tilde{y}_t(e) \sim N(m_0 + e, \sigma^2) \]  

(3.45)

**Agents**

Although projects can last for an indefinite number of periods, the agents in the economy live only two periods. Every two periods, new generations of agents, entrepreneurs and investors, are born. Entrepreneurs are randomly assigned to the projects, new and old, and investors are born with the endowment \( E \) as before.\(^{18}\)

Agents know the past history of old projects. Inheritance of knowledge is the dynamic link in this economy.\(^{19}\)

**Informational assumptions**

In addition to the informational assumptions of the basic model, I will make three new assumptions relating to the observability of the interim signal and the informational value of one period's output for future output.

**Assumption 25** Interim signals about the profitability of projects are produced independently of the duration of financial contracts. Those interim signals have the following structure:

\[ \tilde{z}_{t+1} = \tilde{y}(e_t) + g(e_t) + \varepsilon_{t+1} \]  

(3.46)

Assumption 25 asserts two things. On the one hand, according to (3.46), signals give information about the general profitability of the project \( \tilde{y} \) and only indirectly about the realization of that profitability in the next period, \( \tilde{y}_{t+2} \). On the other hand, knowledge about projects is independent of their financial past.\(^{20}\) This property will be convenient for

---

\(^{18}\)Without being very precise, I am implicitly assuming that \( E \) is big enough to guarantee funds to any project that gives expected profitability above the interest rate of the safe project \( R \).

\(^{19}\)Again, Assumption 20 is important in the dynamic set-up. If agents were infinite lived, then Assumption 20 would make the problem ill-defined. However, if \( C \) were not infinite, then the dynamic program, although very complicated, could again be introduced.

\(^{20}\)The distribution of the profitability is independent on how many times projects have been long-term or short-term financed.
the analysis of maturity.

**Assumption 26** The value of the output of the project in period $t$ constitutes a signal of the profitability of the project:

$$
\tilde{y}_{t+2}(e_t) = \bar{y}(e_t) + \tilde{\eta}_{t+2}
$$

where $\tilde{\eta}_{t+2} \sim N(0, \tau^2)$.

**Assumption 27** The sequences of variables $\tilde{e}_{t+1}$ and $\tilde{\eta}_{t+2}$, for $t = 0, 2, 4...$ are mutually independent.

Note that independence can occur only because interim signals $\tilde{s}_t$ do not inform directly about $\tilde{y}_{t+2}$. If $\tilde{e}_{t+1}$ and $\tilde{\eta}_{t+2}$ were independent of each other but autocorrelated, the inference problem would be more complicated.

### 3.4.2 Analysis of the dynamic model

The analysis of the dynamic model must consider both investments in new projects (essentially the same as in the basic model), and reinvestments by new generations of agents in old projects. Agents make those decisions by computing the expected profitability of projects in different stages of a project' life, updating the initial prior of the projects with all the signals that projects have generated during their lives.

**Definition 31** The maturity ratio $M^t_n$ is the ratio of projects of generation $n$ (born at $n$) that, at $n + t$, are still alive and funded with long term funds.

$$
M^{n+t}_n \equiv \frac{\text{measure of long-term financed projects}}{\text{measure of financed projects}}
$$

To analyze the dynamic evolution of $M^{n+t}_n$ it is necessary to specify a set of initial values for the parameters of the economy. In the next subsection I consider an economy where $M^n_n = 0$. 

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Maturity ratio after one production period \((M^{n+2}_n)\) in an economy where \(M^n_n = 0\).

Assume that the initial conditions of the economy place the new born projects in Region 2 in the graph (3.1). This requires the following two conditions to hold:

\[
WSR > 0 \\
WSR - WLR > 0
\]  
(3.49)  
(3.50)

After the first set of signals, projects are no longer homogeneous. Consider the situation of those projects after one round of production has occurred. Projects from generation \(n\) can be partitioned into two classes:

1. Projects liquidated at \(n + 1\) (a measure \(\Phi\) of them).

2. Projects that survived liquidation at \(n + 1\) (a measure \(1 - \Phi\) of them).

After one production period, projects that survive liquidation ("survivors") have produced two signals about their profitability, the signal \(s_{n+1}\) that "saves" them from liquidation, and the output after one production period \(y_{n+2}\). Considering these two signals, survivors will be divided in three subclasses:

(a) Projects that get long-term funds for the next production period. The average posterior about their performance (formed with the signal in \(n + 1\) and with observation of the output in \(n + 2\)) is above a threshold \(u_2\).

(b) Projects that get short-term funds for the next production period. The average posterior is below the threshold \(u_2\) but above another threshold \(u_1\).

(c) Project that are not reinvested. The average posterior is below \(u_1\).

Before computing the thresholds \(u_1\) and \(u_2\), define \(\sigma^2(1)\) as the variance of the posterior distribution of the projects that have survived liquidation at \(n + 1\). This variance \(\sigma^2(1)\) is deterministic and equal for all survivors:

\[
\sigma^2(1) = \frac{\sigma^2 \nu^2 \tau^2}{\sigma^2 \nu^2 + \tau^2 \nu^2 + \tau^2 \sigma^2}
\]  
(3.51)
Note that $\sigma^2(1) < \sigma^2$ (see Result 13), and therefore $n$ also diminishes:

$$n(1) = \frac{\sigma^2(1)}{\sqrt{\sigma^2(1) + \nu^2}} < n$$  \hspace{1cm} (3.52)

We are ready now to compute the thresholds:

**Definition 32** Assume that $n(1) > n_p$. Then the cut-off value that separates funded from unfunded projects, $u_1$, is defined by the solution of the following equation:

$$C(u_1, n(1)) = 0$$  \hspace{1cm} (3.53)

The cut-off value that separates long-term financed projects from short-term financed projects, $u_2$, is defined by the solution of the following equation:

$$A(u_2, n(1)) = 0$$  \hspace{1cm} (3.54)

where $C(m_0, n)$ and $A(m_0, n)$ where defined in the previous section as the locus of points where the surplus of the short-term financing is zero and the locus of points where the surplus of the short-term and long-term financing is the same respectively.

**Result 18** If $n(1) > n_p$ then $u_2 > u_1$. If $n(1) = n_p$ then $u_2 = u_1$. Finally if $n(1) < n_p$ then $u_2 < u_1$.

**Proof.** See appendix.

To compute the measure of projects in the three different categories (liquidated, short-term financed and long-term financed), it is necessary to relate the cut off levels $u_1$ and $u_2$ to the actual signals that generate them and to the probability that those signals occur.

**Remark 10** The value of the maturity ratio is greater than 0:

$$M_n^{n+2} > 0$$  \hspace{1cm} (3.55)

\footnote{In the appendix the evolution of both $\sigma(t)$ and $n(t)$ is considered.}
This illustrates the main effect in this dynamic economy.\textsuperscript{22} There is a tendency for the maturity ratio to increase due to two factors. On the one hand, a measure of short-term funded projects become long-term financed, increasing the numerator of $M_{n}^{n+2}$. On the other, the total measure of projects in the economy that are still being financed is less than one because of liquidations at $n + 1$ and failures to reinvest at $n + 2$.

Analysis of maturity in the long-run

The evolution of $M_{n}^{n+4}$ depends on the initial conditions of the economy. In the computations of the previous subsection, initial conditions were chosen such that the maturity ratio had to increase (it began at zero). It is easy to chose initial conditions that produce the opposite result (for example if all the projects receive long-term funds in the first period then $M_{n}^{n} = 1$ but $M_{n}^{n+2}$ decreases). Therefore $M_{n}^{n+4}$ is not monotonic for all initial conditions in all the periods of the economy. Nevertheless, it is possible to characterize the behavior of the maturity ratio in the long run.

Proposition 33 After $j$ production periods only the cut-off that separates long term financed projects from unfunded projects remains relevant.

$$u_{1}^{j} \geq u_{2}^{j} = I \cdot R^{2} - 1$$

(3.56)

where $u_{i}^{j}$ represents the cut-off value $i$ after $j$ periods of production. The value of $j$ is given by:

$$j = \arg \min_{d} (n(d) < n_{p})$$

(3.57)

Proof. See appendix.

The main property of the long run in this economy is this "long or out" proposition. After $j$ periods all projects financed in the economy will be long-term funded. The interpretation of Proposition 33 is straightforward. The only reason to fund projects with short-term funds was to have the option to liquidate them after learning about them. Once\textsuperscript{22}See the Appendix for an explicit computation of $M_{n}^{n+2}$.


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the project is sufficiently well-known, that option loses all value and long-term financing dominates short-term finance.

The implications of Proposition 33 are immediate.

**Corollary 34** *The maturity ratio after j periods is one:*

\[ M_{n+j}^n = 1 \] (3.58)

Note the importance of the assumptions to get this "long or out" proposition. The dynamic set-up that has been considered precludes any accumulation of assets by firms. That, together with Assumption 20 that makes monetary incentives useless produces the strong "long or out" proposition. If both are relaxed the "long or out" result may change; accumulation of assets would lead toward more internal finance that align the incentives of entrepreneurs and financiers (provided that entrepreneurs respond to monetary incentives) and can make that short term financing does not cause any distortion after all. However, controlling for the accumulation of assets, the main insight of the paper, that learning induces more long-term financing, would still hold.

### 3.4.3 Discussion

The results obtained in the dynamic version of the model complement the results obtained in Section 3. Together both sets of results are broadly consistent with those documented in the empirical paper by Barclay and Smith (1995). According to Barclay and Smith, four factors are empirically relevant to explain longer maturity in firms' debt structure: growth opportunities, size, regulation and informational asymmetries. Firms with less growth opportunities, large firms, regulated firms and firms without large potential informational asymmetries have more long term debt in their balance sheets.

Consider first the implications about growth opportunities. One can interpret projects with growth opportunities as projects with high variance of their profitability. In the model high variance is directly correlated with short maturity. One can also suppose that more regulated firms are guaranteed a less variable, higher than the market rate of return. Again,
this is consistent with the intuition of the model, that regulated firms should tend to be long term financed. Next, consider the empirical result that relates size with maturity. One can argue that size and age are correlated, due to informational problems (for example firms need to build in their collateral base to expand) or simply due to the fact that firm expansion takes time. Then, the dynamic pattern rationalizes the correlation between size and maturity that is empirically observed.\textsuperscript{23}

3.5 Further extensions

In this section I extend the model in two respects. First I consider the impact of increases in interest rates in this economy. Second I allow a richer liquidation process; the consequences of having either endogenous or random liquidation values are analyzed.

3.5.1 Interest rates and short-termism

Assume that $R$, the profitability of safe projects, changes. Assume an economy (similar to the one described in the dynamic extension) with risky projects that are heterogenous in expected profitability but homogenous in variance.

\textbf{Result 19} An increase (decrease) in the interest rate, $t$, produces two effects: it reduces (increases) total investment and decreases (increases) financial maturity.

\textbf{Proof}. See appendix.

The negative effects of a restrictive monetary policy on investment has been considered repeatedly by the economic literature. The more interesting effect obtained here has to do with the negative correlation between maturity and interest rates. High interest rates promote the existence of “speculative” capital instead of patient long term capital, which may have additional macroeconomic implications.

\textsuperscript{23}The empirical observation that large asymmetries of information explain financial maturity cannot be directly investigated with this model in which there are no asymmetries of information along the equilibrium path.
3.5.2 Endogenous liquidation values

Suppose that the liquidation value, $L$, depends on the total amount of liquidation in the economy, $L(Q)$.

**Result 20** Assume that there exists a set of projects of measure 1 in this economy and assume that $L = L(Q)$ where $Q$ is the total measure of projects that are liquidated in the economy. Different assumptions about $L'(Q)$ generate the following results about maturity:\(^{24}\)

- If $L'(Q) < 0$ then there exists a "maturity capacity" in the economy. Identical projects might be financed with different maturities.

- If $L'(Q) > 0$ then there exist "maturity waves" in the economy. We are in a multiple equilibrium situation, with all projects either long term financed or short term financed.

**Proof.** See appendix.

The first part of Result 20 resembles the argument by Shleifer and Vishny (1992). By focusing on the potential buyers of assets, they show that it is possible to derive endogenous liquidation values that are likely to be low in bad times, therefore affecting debt capacities in firms. Here the logic is similar. If everybody decides to finance projects with short term contracts and if liquidation of those projects implies a sale of physical assets, then it is likely that the price of liquidation decreases with the amount of projects to be liquidated ($L'(Q) < 0$), and therefore there is a "maturity capacity" in the industry. When everybody decides to get short term financing then it is desirable to obtain long term financing and vice versa. The optimal maturity becomes dependent on the general conditions of the market, yielding an equilibrium measure of projects that are short term financed.

The second part of Result 20 shows that introducing general equilibrium considerations into the model, in the form of a richer $L(Q)$, a range of situations may occur. If the price of liquidation increases with the wideness of the market ($L'(Q) > 0$), then multiple equilibrium ("maturity waves") may occur. The reason is that by funding a project with short-term

\(^{24}\)Other technical are required. See appendix for details.
funds an externality is created. That externality, in the form of a better (wider) short term financial market, may induce others to invest in projects that, without that more liquid market, may have been funded with long-term funds or not funded at all.

3.5.3 Random liquidation values

Assume that the liquidation value is a random variable \( \tilde{L} \) centered at \( L \):\(^{25}\)

\[
\tilde{L} \sim N(L, \sigma^2_L)
\]  
(3.59)

Assume also that the liquidation decision is made after the value of \( \tilde{L} \) is realized. The value of the project if it is short term financed, is:

\[
W^*_SR = E \left[ \max(\tilde{m}, \tilde{L} \cdot R) \right] - I \cdot R^2
\]  
(3.60)

The main implications of randomness are considered in the following result:

Result 21 The value of short term financing decreases with the covariance between \( \tilde{L} \) and \( \tilde{m}, \sigma_{LM} \) and increases with the variance of the random liquidation value \( \sigma^2_L \):

\[
\frac{dW^*_SR}{d\sigma_{LM}} < 0
\]  
(3.61)
\[
\frac{dW^*_SR}{d\sigma^2_L} > 0
\]  
(3.62)

Proof. See appendix.

Result 21 is quite interesting. Consider first (3.61). One leading case occurs when the liquidation price \( \tilde{L} \) is a function of the expected profitability of the project \( \tilde{m} \).\(^{26}\) Short term finance is less interesting due to the correlation between both variables. Short term financing is attractive to liquidate projects with bad prospects. If the outcome of the project

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\(^{25}\)Assume that the joint distribution of \( \tilde{L} \) and \( \tilde{m} \) is a bivariate normal with the specified moments and covariance \( \sigma_{LM} \).

\(^{26}\)One possible modelization is \( \tilde{L} = k \cdot \tilde{m} + \xi \) with \( k < 1 \) and \( \xi \sim N(0, \sigma^2_\xi) \) independent of \( \tilde{m} \). In that case \( \sigma^2_{LM} = k \cdot n^2 \) where \( n \) is the standard deviation of \( \tilde{m} \) that was defined in (3.25) as \( n \equiv \frac{\sigma}{\sqrt{\sigma^2 + \nu^2}} \).
is correlated with the liquidation price then liquidation is less useful. In the limit case of perfect positive correlation, short term finance is never desirable.27

The second part of Result 21 asserts that short term financing is encouraged by the variability in the liquidation procedures, $\sigma_L^2$. One interesting situation occurs when $R$, the interest rate is what induces variability in the liquidation price. Assume that $\tilde{R} \sim N(R, \sigma_R^2)$ and that $\tilde{R}$ is uncorrelated with $\tilde{m}$. For instance, assume that monetary policy is creating variability in $R$. Expression (3.62) shows that a more variable monetary policy encourages short-term funding of projects. This is a complementary effect of Result 19. Summarizing, a restrictive monetary policy, increasing $E(\tilde{R})$, or a highly variable one, increasing $\sigma_R^2$, both make short term financing more desirable.

### 3.6 Concluding Remarks

This paper presents a simple model about the choice of financial maturity. The static model uses signal jamming à la Holmstrom (1982) to build a basic trade-off between short and long term financial contracts. While short term contracts incorporate information more quickly and allow liquidation of unprofitable projects, they also create a distortion by diverting resources to unproductive activities. Resources are lost because of a conflict of interest between borrowers and lenders, since borrowers enjoy a non-transferable continuation rent if the project is not liquidated.

From a different perspective, the model considers a learning situation in capital markets. Agents learn about the profitability of investment projects through information that is revealed after projects are undertaken. When the uncertainty about projects is large or when the learning technology is efficient, there is a tendency to short term finance. Short term finance is also encouraged for projects of marginal profitability or for projects that do not lose too much from early liquidation.

In the model, a fixed amount is received after a project is liquidated, which makes financing a project short term like having a put option on the project. Standard results

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27In the example of the previous footnote, perfect correlation occurs when $\sigma_L^2 = 0$.  

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from option theory follow.

The second part of the paper is devoted to a dynamic extension of the static model. The purpose is to exploit the simple insight that when projects become better known, then learning about them becomes less important. This results in a dynamic pattern for maturity. The main insight in the dynamic extension is that maturity is negatively correlated with firms' age. In particular, short term contracts disappear after projects are old enough.

The implications of the model are broadly consistent with the empirical evidence. Barclay and Smith (1995) show that growth opportunities, size, regulation and asymmetries of information can explain empirically the maturity choice. In the model, a natural interpretation of parameters related to growth opportunities, size and regulation generates effects on maturity of the same sign that the empirical investigation by of Barclay and Smith.

Extensions of the model show that interest rates affect not only the quantity but also the maturity of finance. A richer structure for the liquidation process is also considered. Random and endogenous liquidation values are analyzed and implications toward financial maturity derived.

Finally it is interesting to compare the model developed here with the model developed in Von Thadden (1995). Von Thadden (1995) also studies the problem of the optimal financial contracting for a firm that wants to raise capital for a risky long-term investment project. He analyzes the maturity choice and in particular the influence that monitors may have on that choice (how monitors help to implement long-term profitable investment projects.) The main difference between Von Thadden (1995) and this paper (apart from technical issues on the modelization) has to do with the focus of the analysis. While Von Thadden (1995) focus on the obtention of optimal contracts and on the analysis of how those optimal contracts resemble institutions observed in practice, (e.g. priorities on bankruptcy proceedings or contracts that resemble credit lines) I, by stressing the learning aspect that any investment decision involves, consider the influence that characteristics of investment projects or the economic environment may have on the choice of financial maturity and on the dynamic pattern for financial maturity that the optimal learning induces. It is evident the complementarity of both studies to understand better the choice of financial maturity.
in firms.
Bibliography


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Appendix A

Chapter 1: Proofs

Proof of Lemma 1

The solution of this problem exists. The objective function is continuous and the choice set for the variables is closed, so by Weierstrass' Theorem the existence of an optimal solution is guaranteed.

First part: $x^* > 0$.

Assume on the contrary that the optimal solution is $S^{**} \equiv (x^*, L^*_s, L^*_{1-s}) = (0, 0, 0)$. This generates a contradiction because $S^{**}$ leads toward zero consumptions in certain states and we assumed that $\lim_{c \to 0^+} U(c) = +\infty$.

Second part: $x^* < 1$.

Assume on the contrary that $x = 1$ is one of the elements of the optimal liquidation policy. Among the policies with $x = 1$ the one that produces more utility is $S^{**} \equiv (1, s, 1-s)$. In particular note that this policy produces the following consumptions: $c_1^1 = c_2^2 = c_1^{1-s} = c_2^{1-s} = 1$. To prove that $S^{**}$ cannot be the optimal policy consider the alternative policy: $S' = (s, s, 1-s)$. $S'$ produces instead the following consumptions: $c_1^1 = c_1^{1-s} = 1$, $c_2^2 = B > 1$ and $c_2^{1-s} = B + \frac{2s-1}{s} > 1$, what contradicts the optimality of $S^{**}$.

Proof of Lemma 2

Let $S^* = (x^*, L^*_s, L^*_{1-s})$ be the optimal policy of the bank. Such policy can belong to one of the following four classes:
(A) \( L_s^* < x^* \), \( L_{1-s}^* < x^* \)
(B) \( L_s^* < x^* \), \( L_{1-s}^* = x^* \)
(C) \( L_s^* = x^* \), \( L_{1-s}^* = x^* \)
(D) \( L_s^* = x^* \), \( L_{1-s}^* < x^* \)

The strategy of the proof will be to show that policies in classes (A), (B) and (C) are incompatible with optimality, and that therefore (D) characterizes the optimal policy.

(A) \( L_s^* < x^* \), \( L_{1-s}^* < x^* \).

Keep, for any realization of \( \Phi \), part of the riskless asset for the second period: (Consider w.l.o.g. that \( L_s^* > L_{1-s}^* \); an identical argument follows if \( L_s^* \leq L_{1-s}^* \).

**Fact 1** If \( S^* \) belongs to class A, \( S^* \) is not an optimal policy.

**Proof.** Note that if \( S^* \) belongs to class A then with the following policy: \( S^{s'} = (L_s^*, L_s^*, L_{1-s}^*) \)
consumptions in the second period can be increased without reducing consumptions of the first period. This policy produces the following consumptions: \( c_1^{s'} = \frac{L_s^*}{s} = c_1^*, c_1^{1-s^{s'}} = \frac{L_{1-s}^*}{1-s} = c_1^{1-s} \) and \( c_2^{s'} = \frac{(1-L_s^*)B_s}{s} > \frac{(1-x^*)B_s + x^* - L_s^*}{s} = c_2^*, \)
\( c_2^{1-s^{s'}} = \frac{(1-L_s^*)B_s + L_s^* - L_{1-s}^*}{(1-s)} > \frac{(1-x^*)B_s + x^* - L_s^*}{(1-s)} = c_2^{1-s} \).

(B) \( L_s^* < x^* \), \( L_{1-s}^* = x^* \).

Keep part of the riskless asset if and only if \( \Phi = s \) (lots of early consumers).

**Fact 2** If \( S^* \) belongs to class B, \( S^* \) is not an optimal policy.

**Proof.** If \( (x^*, L_s^*, L_{1-s}^*) \) is optimal and \( L_s^* < x^* \), \( L_{1-s}^* = x^* \) then by the optimality of the liquidation policies the following relationships hold:

\[
s \cdot U' \left( \frac{L_s^*}{s} \right) = (1-s) \cdot U' \left( \frac{(1-x^*)B_s + x^* - L_s^*}{(1-s)} \right) \tag{A.1}
\]

\[
(1-s) \cdot U' \left( \frac{x^*}{(1-s)} \right) \geq s \cdot U' \left( \frac{(1-x^*)B_s}{(1-s)} \right) \tag{A.2}
\]

but:

\[
\frac{L_s^*}{s} < \frac{x^*}{1-s} \iff U' \left( \frac{L_s^*}{s} \right) > U' \left( \frac{x^*}{1-s} \right)
\]
what implies:

\[ s \cdot U' \left( \frac{L_s^*}{s} \right) > (1 - s) \cdot U' \left( \frac{x^*}{1 - s} \right) \] (A.3)

On the other hand:

\[ \frac{(1 - x^*)B + x^* - L_s^*}{1 - s} < \frac{(1 - x^*)B}{s} \]

what implies:

\[ U' \left( \frac{(1 - x^*)B + x^* - L_s^*}{1 - s} \right) < U' \left( \frac{(1 - x^*)B}{s} \right) \] (A.4)

and therefore

\[ (1 - s) \cdot U' \left( \frac{(1 - x^*)B + x^* - L_s^*}{1 - s} \right) < s \cdot U' \left( \frac{(1 - x^*)B}{s} \right) \] (A.5)

but expressions (A.1), (A.2), (A.3) and (A.5) are inconsistent with each other. (A.3) and (A.2) imply that:

\[ s \cdot U' \left( \frac{L_s^*}{s} \right) > s \cdot U' \left( \frac{(1 - x^*)B}{s} \right) \]

and (A.1) and (A.5) imply that:

\[ s \cdot U' \left( \frac{L_s^*}{s} \right) \leq s \cdot U' \left( \frac{(1 - x^*)B}{s} \right) \]

what is a contradiction.

(C) \( L_s^* = x^* \), \( L_{1-s}^* = x^* \).

The aggregate consumption is exactly \( x^* \) no matter what the realization of the variable \( \Phi \) is.

To prove that this cannot be an optimal policy consider the following:

\textbf{Fact 3} \textit{If the optimal solution belongs to class C then:}

\[ x^* = \frac{B}{B + 1} \] (A.6)

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Proof It is enough to solve the following problem:

$$\max_x sU \left( \frac{x}{s} \right) + (1-s)U \left( \frac{(1-x)B}{(1-s)} \right) + (1-s)U \left( \frac{x}{(1-s)} \right) + sU \left( \frac{(1-x)B}{s} \right)$$ \hspace{1cm} (A.7)

that has as a first order condition:

$$U' \left( \frac{x^*}{s} \right) + U' \left( \frac{x^*}{(1-s)} \right) = U' \left( \frac{(1-x^*)B}{(1-s)} \right) + U' \left( \frac{(1-x^*)B}{s} \right)$$ \hspace{1cm} (A.8)

But this implies that:

$$\frac{x^*}{s} + \frac{x^*}{(1-s)} = \frac{(1-x^*)B}{(1-s)} + \frac{(1-x^*)B}{s}$$

that gives (A.6). ■

If $x^* = L^*_s = L^*_{1-s} = \frac{B}{B+1}$ is the solution then after any realization of the variable $\Phi$ then to reduce the liquidation of the riskless asset cannot increase the utility. Consider the case that $\Phi = 1-s$ (few agents in the economy with liquidity needs.) In that case the optimality of the policy requires that the solution of the problem:

$$\max_\epsilon (1-s)U \left( \frac{x^* - \epsilon}{1-s} \right) + sU \left( \frac{(1-x^*)B + \epsilon}{s} \right)$$

subject to:

$$\epsilon \geq 0$$

is reached at $\epsilon^* = 0$. In terms of the first order condition this means that:

$$-U' \left( \frac{x^* - \epsilon}{1-s} \right) + U' \left( \frac{(1-x^*)B + \epsilon}{s} \right) \bigg|_{\epsilon=0} \geq 0$$

substituting and using the concavity of the utility function this implies:

$$\frac{B}{(B+1)(1-s)} \leq \frac{B}{(B+1)s}$$

that is a contradiction. ■
Conclusion: the optimal solution in the autarchy case is of the form:

\[ L_s^* = x^*, \ L_{1-s}^* < x^* \]

Proof of Proposition 4

Totally differentiating (1.24) we obtain:

\[
\frac{dx^*}{dB} = \frac{\frac{B}{1-s}(1 - x^*)U''\left(\frac{1-x^*}{1-s}\right)^B + U'(B - (B - 1)x^*) + (B - 1)(1 - x^*)U''(B - (B - 1)x^*)}{\frac{1}{s}U''(\frac{x^*}{s}) + \frac{B^2}{1-s}U''\left(\frac{1-x^*}{1-s}\right) + (B - 1)^2U''(B - (B - 1)x^*)}
\]

(A.9)

in which while the denominator has a clear sign (negative), the numerator does not.

Proof of Lemma 5

First note that a solution always exists. The objective function is continuous and the choice set for the variables is closed, so by Weierstrass' Theorem the existence of an optimal solution is guaranteed. Second, the strategy of the proof is to proceed by contradiction. I will propose other (supposedly) optimal policies that bind the constraints to find an improvement over them.

Fact 4 The optimal consumptions in all states have to be positive:

\[ c_t^n > 0 \quad n = s, \ 1-s \ and \ t = 1, \ 2 \]

Proof. This is a consequence of the assumption: \( \lim_{c \to 0} U'(c) = +\infty \).

Fact 5 In the optimal policy, liquidation over the amount of liquid resources only occurs for an immediate consumption.

Proof. Given the assumption \( h < 1 \) any liquidation above is suboptimal.

First part: \( 0 < x < 1 \).

Assume \( x = 0 \) and assume that the optimal policy is \( S^* = (0, L_s^*, L_{1-s}^*) \). By Fact 4 \( L_s^* > 0 \) and \( L_{1-s}^* > 0 \). Assume also, w.l.o.g., that \( L_s^* \geq L_{1-s}^* \) (the argument will be the
symmetric one for \( L_s^* \geq L_{1-s}^* \). Consider the alternative policy: \( S^{**} = (L_{1-s}^*, L_s^*, L_{1-s}^*) \). The claim is that \( S^{**} \) is an improvement over \( S^* \). The reason is that consumptions at \( t = 1 \) do not change and consumptions at \( t = 2 \) increase due to the fact that \( B < h \).

Assume \( x = 1 \) and assume that the optimal policy is \( S^{***} = (1, L_s^*, L_{1-s}^*) \). By Facts 4 and 5, \( L_s^* < 1 \) and \( L_{1-s}^* < 1 \). Assume w.l.o.g., that \( L_s^* \geq L_{1-s}^* \) (the argument will be the symmetric one for \( L_s^* \geq L_{1-s}^* \)). Consider the alternative policy: \( S^{**} = (L_{1-s}^*, L_s^*, L_{1-s}^*) \). The claim now is that \( S^{**} \) is an improvement over \( S^{***} \). The reason is that consumptions at \( t = 1 \) do not change and consumptions at \( t = 2 \) increase due to the fact that \( B > 1 \).

Second part: \( 0 < L_s^* < x^* + \frac{B}{h}(1 - x^*) \)

Easy, otherwise, either Fact 4 or Fact 5 would be violated.

Third part: \( 0 < L_s^* < x^* + \frac{B}{h}(1 - x^*) \)

Also easy, as in the previous case, otherwise, either Fact 4 or Fact 5 would be violated.

Proof of Lemma 6

The following nine possibilities for the optimal solution exist:

1) \( L_s^* > x^*; L_{1-s}^* > x^* \)
2) \( L_s^* > x^*; L_{1-s}^* = x^* \)
3) \( L_s^* > x^*; L_{1-s}^* < x^* \)
4) \( L_s^* = x^*; L_{1-s}^* > x^* \)
5) \( L_s^* = x^*; L_{1-s}^* = x^* \)
6) \( L_s^* = x^*; L_{1-s}^* < x^* \)
7) \( L_s^* < x^*; L_{1-s}^* > x^* \)
8) \( L_s^* < x^*; L_{1-s}^* = x^* \)
9) \( L_s^* < x^*; L_{1-s}^* < x^* \)

Note that liquidation is costly, therefore liquidations must have the purpose of immediate consumption. If that is the case, then by concavity of the utility function any optimal policy must satisfy that:

\[
L_s^* \geq L_{1-s}^* \tag{A.10}
\]

what leaves us with the following six possibilities:

1) \( L_s^* > x^*; L_{1-s}^* > x^* \)
2) \( L_s^* > x^*; L_{1-s}^* = x^* \)
3) \( L_s^* > x^*; L_{1-s}^* < x^* \)
5) \( L_s^* = x^*; L_{1-s}^* = x^* \)
6) \( L_s^* = x^*; L_{1-s}^* < x^* \)
9) \( L_s^* < x^*; L_{1-s}^* < x^* \)

The strategy would be to exclude the possibilities 1), 2), 5), 6) and 9) and therefore to obtain 3) by elimination.

Exclusion of 1) \( L_s^* > x^*, L_{1-s}^* > x^* \).
Consider that \( S^*_1 = (x^*, L^*_s, L^*_{1-s}) \) were an optimal policy (and therefore that \( L^*_s > L^*_{1-s} \)) and consider alternatively the policy \( S'^*_1 = (L^*_{1-s}, L^*_s, L^*_{1-s}) \). This alternative policy, \( S'^*_1 \), keeps the same consumption at \( t = 1 \) and increases the consumptions at \( t = 2 \), what contradicts the fact that \( S^*_1 \) was an optimal policy.

**Exclusion of 2) \( L^*_s > x^*, L^*_{1-s} = x^* \).**

This policy is incompatible with Proposition 7 below. The comparative statics derived in Proposition 7 (that are valid independently of the fact that \( L^*_{1-s} = x^* \)) shows that \( x^* \) and \( L^*_s \) move in opposite directions, therefore if they where equal in there would exist optimal policies with inequalities \( L^*_{1-s} < x^* \) and \( L^*_{1-s} > x^* \) when the parameter \( h \) is perturbed. But that is a contradiction with the fact that the optimal policy cannot be such that: \( L^*_{1-s} > x^* \).

**Exclusion of 5) \( L^*_s = x^*, L^*_{1-s} = x^* \).**

By the same argument that in lemma 2, part c (see above) in that case the optimal policy would be: \( x^* = \frac{B}{B+1} \). But again same deviation than above contradicts the optimality of the policy.

**Exclusion of 6) \( L^*_s = x^*, L^*_{1-s} < x^* \).**

This policy is incompatible with Proposition 7 below. The argument is similar to the one made to exclude 2. Proposition 7 shows that \( x^* \) and \( L^*_s \) move in opposite directions. If they where equal then, there would exist optimal policies where \( L^*_s < x^* \) and \( L^*_s > x^* \). That is a contradiction with the fact that the optimal policy cannot be such that: \( L^*_s < x^* \).

**Exclusion of 9) \( L^*_s < x^*, L^*_{1-s} < x^* \).**

Again consider that \( S^*_0 = (x^*, L^*_s, L^*_{1-s}) \) were the optimal policy, and consider the alternative policy: \( S'^*_0 = (L^*_s, L^*_s, L^*_{1-s}) \). Note that \( S'^*_0 \) keeps the same consumptions at \( t = 1 \) that the optimal policy and increases both consumptions at \( t = 2 \), what is a contradiction with the optimality of \( S'^*_0 \).

**Proof of Proposition 7**

From (1.40) can be obtained the following expression for \( L^*_{1-s} \):

\[
L^*_{1-s} = (1 - s) - (1 - s)(B - 1)x^* \tag{A.11}
\]
that can be substituted in (1.41) giving (A.13) below. The new system of equations to be
differentiated is therefore:

\[ U' \left( \frac{L_s^*}{s} \right) - hU' \left( \frac{(1-x)B - h(L_s^* - x)}{1-s} \right) = 0 \]  

(A.12)

and:

\[ (h-B)U' \left( \frac{(1-x)B - h(L_s^* - x)}{1-s} \right) - (B-1)U' \left( \frac{(B - (1-s) - s(B-1)x^*)}{s} \right) = 0 \]  

(A.13)

Totally differentiating the system:

\[ \left( \frac{1}{s}U_1' + \frac{h}{1-s}U_2'' \right) dL_s^* - \left( \frac{(h-B) \cdot h}{1-s} U_2'' \right) dx^* = \left( U_2' - \frac{h}{1-s}(L_s^* - x)U_2'' \right) dh \]

\[ \left( \frac{(h-B) \cdot h}{1-s} U_2'' \right) dL_s^* - \left( \frac{(h-B)^2}{1-s} U_2'' + (B-1)^2 U_3'' \right) dx^* = \left( U_2' - \frac{(h-B)}{1-s}(L_s^* - x)U_2'' \right) dh \]

where:

\[ U_1 \equiv U \left( \frac{L_s^*}{s} \right) \]  

(A.14)

\[ U_2 \equiv U \left( \frac{(1-x)B - h(L_s^* - x^*)}{1-s} \right) \]  

(A.15)

and

\[ U_3 \equiv U \left( \frac{(B - (1-s) - s(B-1)x^*)}{s} \right) \]  

(A.16)

Note that:

\[ \frac{dx^*}{dh} = \begin{vmatrix}
\frac{1}{s}U_1'' + \frac{h^2}{1-s}U_2'' & U_2' - \frac{h}{1-s}(L_s^* - x^*)U_2'' \\
\frac{h(h-B)}{1-s}U_2'' & U_2' - \frac{h-B}{1-s}(L_s^* - x^*)U_2'' \\
\frac{1}{s}U_1'' + \frac{h^2}{1-s}U_2'' & \frac{(h-B)h}{1-s}U_2'' \\
\frac{h(h-B)}{1-s}U_2'' & \frac{(h-B)^2}{1-s}U_2'' + (B-1)^2 U_3''
\end{vmatrix} \]  

(A.17)
Define $D$ as the denominator of (A.17). $D$ can be expressed as:

$$D = -\frac{1}{s}U_1'' \cdot \left( \frac{(h-B)^2}{1-s}U_2'' + (B-1)^2 \cdot U_3'' \right) - \frac{h^2}{1-s}(B-1)^2U_2'' \cdot U_3'' < 0$$  \hspace{1cm} (A.18)$$

Define $N \left[ \frac{dx^*}{dh} \right]$ as the numerator of (A.17). $N \left[ \frac{dx^*}{dh} \right]$ can be expressed as:

$$N \left[ \frac{dx^*}{dh} \right] = \frac{1}{s}U_1'' \cdot U_2' - \frac{(h-B)}{s \cdot (1-s)}(L_s^* - x^*)U_2'' \cdot U_1'' + \frac{B}{1-s}U_2'' \cdot U_2' < 0$$  \hspace{1cm} (A.19)$$

Therefore:

$$\frac{dx^*}{dh} = \frac{N \left[ \frac{dx^*}{dh} \right]}{D} > 0$$  \hspace{1cm} (A.20)$$

The expression that characterizes $L_s^*$ is:

$$\frac{dL_s^*}{dh} = \begin{vmatrix}
U_2' - \frac{h}{1-s}(L_s^* - x^*)U_2'' & -\frac{(h-B)}{1-s}U_2'' \\
U_2' - \frac{h-B}{1-s}(L_s^* - x^*)U_2'' & -\left( \frac{(h-B)^2}{1-s}U_2'' + (B-1)^2U_3'' \right)
\end{vmatrix}$$  \hspace{1cm} (A.21)$$

Define $N \left[ \frac{dL_s^*}{dh} \right]$ as the numerator of A.21. $N \left[ \frac{dL_s^*}{dh} \right]$ can be expressed as:

$$N \left[ \frac{dL_s^*}{dh} \right] = G + H + I > 0$$  \hspace{1cm} (A.22)$$

where

$$G \equiv -U_2' \cdot \left( \frac{(h-B)^2}{1-s}U_2'' + (B-1)^2U_3'' \right) > 0$$  \hspace{1cm} (A.23)$$

$$H \equiv \frac{h}{1-s}(L_s^* - x^*)(B-1)^2U_2'' \cdot U_3'' > 0$$  \hspace{1cm} (A.24)$$

$$I \equiv -\frac{(h-B)^2h}{1-s}U_2''U_2' > 0$$  \hspace{1cm} (A.25)$$
therefore:
\[
\frac{dL^*_s}{dh} = N \left[ \frac{dL^*_s}{dh} \right] < 0
\]  \hspace{1cm} (A.26)

Finally to get:
\[
\frac{dL^*_{1-s}}{dh} < 0
\]

it is enough to use the relationship: \( L^*_{1-s} = (1 - s) - (1 - s)(B - 1)x^* \) combined with expression (A.20). \( \blacksquare \)

**Lemma 35 (Verification of conditions for general equilibrium compatibility)** If the liquidity cost \( h \) is big enough, then \( L^*_s - x^* > 0 \) and \( x^* - L^*_{1-s} > 0 \) (Condition 1) and \( L^*_s - x^* \leq x^* - L^*_{1-s} \) (Condition 2) are verified.

**Proof:** I will use a limit argument. In the autarchy case (\( h \rightarrow \infty \)) we had (See lemma 2) that \( L^*_s - x^* = 0 \) and \( x^* - L^*_{1-s} > 0 \), by continuity if \( h \) is big enough and using the comparative statics result of proposition 7 we have that \( L^*_s - x^* = 0 \) while still \( x^* - L^*_{1-s} > 0 \) (Condition 1). To check condition 2, again using a limit argument, when \( h \rightarrow \infty \), then \( L^*_s - x^* \rightarrow 0 \) and \( x^* - L^*_{1-s} > 0 \) and therefore condition 2 holds.

By continuity there exists \( h^0 \) such that if \( h > h^0 \) both conditions hold. \( \blacksquare \)

**Proof of Proposition 9**

The first thing to define the liquidation function in the different regimes.

**Regular Market:**

In this regime bad loans are the first to be sold. Only when the amount of loans \((1 - \mu)\) has been sold the good loans go to the market:

\[
F_{RM}(L, x) = \left\{ \begin{array}{ll}
   x - L & \text{if } x \geq L \\
   \frac{y_b}{y_b - k_1} (x - L) & \text{if } x < L \leq L' \\
   \frac{y_b}{y_b - k_1} (1 - x) (1 - \mu) (y_b - k_1) + \frac{y_b}{y_b - k_1} (x - L) & \text{if } L' < L < L''
\end{array} \right.
\]  \hspace{1cm} (A.27)

where \( L' \equiv x + (1 - x) (1 - \mu) (y_b - k_1) \) and \( L'' \equiv x + (1 - x) (B - k_1) \). Note that the liquidation function is nondifferentiable at two sets of points: points of the form \( L = x \) and
points of the form \( L = L' \). The first change of regime occurs when \( L = x \); more liquid resources than the own reserves are required. The second, \( L = L' \), occurs when a change in the quality of the loans on sale is considered. The form of the coefficient that constitute the liquidity cost \( \frac{y_b}{y_b - k_1} \) also deserves attention. It can be deduced from the definition of liquidation function. Remember that a liquidation function gives the cost in terms of \( t = 2 \) consumption of one unit of liquid funds at \( t = 1 \). By renouncing to a bad loan tomorrow \( (y_b) \) it is obtained \( y_b - k_1 \) funds today. This is why the liquidity cost has the proposed form: \( \frac{y_b}{y_b - k_1} \).

**Securitization:**

In this case the liquid:tion function is the following:

\[
F_S(L, x) \equiv \begin{cases} 
    x - L & \text{if } x \geq L \\
    \frac{B}{B - k_1} (x - L) & \text{if } x < L
\end{cases}
\]  

(A.28)

Note that by the form that securitization occurs everything is evaluated at the average loan.

**Cherry Picking:**

Finally Cherry Picking regime also features two sets of nondifferentiable points:

\[
F_{CP}(L, x) \equiv \begin{cases} 
    x - L & \text{if } x \geq L \\
    \frac{y_g}{y_g - k_1} (x - L) & \text{if } x < L \leq L'''
\end{cases}
\]  

(A.29)

where \( L''' \equiv x \div (1 - x) \cdot \mu \cdot (y_g - k_1) \) and \( L'' \) was defined above.

Once defined the liquidations functions consider the following ordering among liquidation costs:

\[
\frac{y_g}{y_g - k_1} < \frac{B}{B - k_1} < \frac{y_b}{y_b - k_1}
\]  

(A.30)

Expression (A.30) identifies the liquidation cost up to the amount in which no change
in the quality of the project has occurred. In other words this liquidation costs are valid as long as the amount of liquidation belongs to the second branch of the liquidation functions. Sufficient conditions for that are:

\[
\begin{align*}
    s & \rightarrow \frac{1}{2} \quad \text{(A.31)} \\
    y_b & \rightarrow y_g \quad \text{(A.32)}
\end{align*}
\]

To see why conditions (A.31) and (A.32) are sufficient consider the following lemma:

**Lemma 36** Consider an auxiliary program of the form 1.27. Assume also that \( s = \frac{1}{2} \). The optimal solution of that program is such that:

\[
x^* = L^*_s = L^*_{1-s} = \frac{B}{B-1}
\]

**Proof.** Easy, no uncertainty exists, therefore it is optimal to make \( x^* \) noncontingent.

**Lemma 37** By continuity, if \( s \rightarrow \frac{1}{2} \) then the optimal solution of a program of the form 1.27 verifies the following inequality:

\[
L^*_s - x^* < \delta
\]

for any \( \delta \).

**Proof.** Simply by continuity.

Assume without loss of generality that \( \mu < \frac{1}{2} \). Consider the following choice for \( \delta \):

\[
\delta^* = (1 - x^*_A) \cdot \mu \cdot (y_b - k_1)
\]

where \( x^*_A \) is the optimal choice for \( x^* \) in an autarky problem. Consider the following proposition:
Lemma 38 Consider an auxiliary program of the form 1.27. For \( s \) close enough to \( \frac{1}{2} \), and \( y_b \rightarrow y_g \) the optimal solution of that program verifies:

\[
L^*_s - x^* < \delta^*
\]  

(A.36)

Proof. Lemma 38 gives sufficient conditions to guarantee that the optimal solution with nonlinear constrains is the same than the optimal solution with linear ones. Condition (A.31) makes sure that the optimal solution belongs to the first branch on the liquidation function and condition (A.32) makes sure that changing policy to end up in the second branch, while having a cost (the previous policy is a local optimum), the benefit is bounded.

Proof of Proposition 10

Easy. Simply note that as long as we are in the second branch of the liquidation function (again this must be checked using above conditions):

\[
\frac{dx^*}{dk_1} = \frac{dx^*}{dh} \cdot \frac{dh}{dk_1}
\]  

(A.37)

but by Proposition 7: \( \frac{dx^*}{dh} > 0 \) and by definition \( h \equiv \frac{a}{a-k_1} \) where \( a \) is a parameter that changes in the regime. In that case:

\[
\frac{dh}{dk_1} = \frac{a}{(a-k_1)^2} > 0
\]

because \( a > k_1 \) ■

Proof of Proposition 11

Again to prove this proposition it is enough with differentiate completely the system of first order conditions (1.40) and (1.41) with respect to \( s \).

\[
\left( \frac{1}{s} U''_1 + \frac{h}{1-s} U''_2 \right) dL^*_s - \left( \frac{h-B}{1-s} U''_2 \right) dx^*
\]  

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\[
\begin{align*}
&= \left( \frac{L_s^*}{s^2} U_1'' + \frac{h[(1-x^*)B - h(L_s^*-x)]}{(1-s)^2} U_2'' \right) ds \\
\text{and} \\
&= \left( \frac{(h-B) \cdot h}{1-s} U_2'' \right) dL_s^* - \left( \frac{(h-B)^2}{1-s} U_2'' + (B-1)^2 U_3'' \right) dx^* \\
&= \left( \frac{-(h-B) [(1-x^*)B - h(L_s^*-x)]}{(1-s)^2} U_2'' - \frac{(B-1)^2}{s^2} U_3'' \right) ds
\end{align*}
\]

where again:

\[
U_1 \equiv U \left( \frac{L_s^*}{s} \right) \\
U_2 \equiv U \left( \frac{(1-x^*)B - h(L_s^*-x^*)}{1-s} \right)
\]

and

\[
U_3 \equiv U \left( \frac{(B-(1-s)) - s(B-1)x^*}{s} \right)
\]

Note that:

\[
\frac{dx^*}{ds} = \begin{vmatrix}
\frac{1}{s} U_1'' + \frac{h^2}{1-s} U_2'' & \frac{L_s^*}{s^2} U_1'' + \frac{h[(1-x^*)B - h(L_s^*-x)]}{(1-s)^2} U_2'' \\
\frac{h(h-B)}{1-s} U_2'' & -(h-B)[(1-x^*)B - h(L_s^*-x)] U_2'' - \frac{(B-1)^2}{s^2} U_3'' \\
\frac{1}{s} U_1'' + \frac{h^2}{1-s} U_2'' & \frac{h(h-B)}{1-s} U_2'' - \frac{(h-B)^2}{1-s} U_2'' + (B-1)^2 U_3''
\end{vmatrix} \\
\text{(A.38)}
\]

while the denominator, developed in Proposition 7 is negative, the numerator of \( \frac{dx^*}{ds} \) have the following expression:

\[
N \left[ \frac{dx^*}{ds} \right] = \begin{vmatrix}
(-) & (-) \\
(-) & (+)
\end{vmatrix} < 0 \quad \text{(A.39)}
\]

and therefore:

\[
\frac{dx^*}{ds} > 0 \quad \text{(A.40)}
\]
Appendix B

Chapter 2: Proofs

Lemmas 12 and 13.

To obtain the optimal values $R^*_6(x)$, $R^*_j(x)$, $I^*_6(x)$ and $I^*_j(x)$ and the value of the objective function $R^*_e(x)$, it is sufficient to solve the linear system that form the restrictions (2.5), (2.6), (2.7), (2.8) and (2.10) imposing that they are satisfied with equality. If the values found verify the restriction (2.9) then the solution is found, otherwise the choice set is empty and the problem does not have solution (the project will not be funded).

The marginal project that is funded, satisfies (2.9) with equality:

$$\frac{C(x)}{\Delta p} + \frac{b}{\Delta p} + \frac{(I - I_b(x))\gamma}{p_h} = R$$

(B.1)

substituting $I_b(x)$ for the optimal value $I^*_b(x)$ gives the cut-off $C^*$ in the text. ■

Results 3 and 4

I will limit myself to the comparative effects that are not obvious from the expressions that characterize $m^*$ and $\beta^*$.

Totally differentiating the expressions that characterize the equilibrium (equations (2.17) and (2.18)) gives the following:
a) $m^*$ with respect to $\gamma$:

$$
\frac{c}{\Delta p} \frac{dm^*}{d\gamma} + \left( \frac{I}{p_h} - \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h} \right) d\gamma - \left( \frac{c}{\Delta p} \cdot \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) dm^* + \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2 \cdot p_h} d\beta^* = 0
$$

therefore:

$$
\frac{dm^*}{d\gamma} = \frac{- \frac{c}{\Delta p} \cdot \left( 1 - \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) \cdot \frac{p_i \cdot c \cdot (m^*)^2}{2\Delta p \cdot (\beta^*)^2} - \left( \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h} \cdot \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2} \right)}{\left( - \frac{c}{\Delta p} \cdot \left( 1 - \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) \cdot \frac{p_i \cdot c \cdot (m^*)^2}{2\Delta p \cdot (\beta^*)^2} - \left( \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h} \cdot \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2} \right) \right)} = \begin{cases} (+) & < 0 \end{cases}
$$

(B.2)

b) $\beta^*$ with respect to $K_0$:

$$
\frac{c}{\Delta p} \frac{dm^*}{d\gamma} - \left( \frac{c}{\Delta p} \cdot \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) dm^* + \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2 \cdot p_h} d\beta^* = 0
$$

therefore:

$$
\frac{d\beta^*}{dK_0} = \frac{\frac{c}{\Delta p} \cdot \left( 1 - \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) \cdot \frac{p_i \cdot c \cdot (m^*)^2}{2\Delta p \cdot (\beta^*)^2} - \left( \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h} \cdot \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2} \right)}{\left( - \frac{c}{\Delta p} \cdot \left( 1 - \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) \cdot \frac{p_i \cdot c \cdot (m^*)^2}{2\Delta p \cdot (\beta^*)^2} - \left( \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h} \cdot \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2} \right) \right)} = \begin{cases} (+) & < 0 \end{cases}
$$

(B.3)

c) $\beta^*$ with respect to $c$:

$$
\frac{c}{\Delta p} \frac{dm^*}{dc} + \frac{m^*}{\Delta p} \frac{dc}{dc} - \left( \frac{m^*}{\Delta p} \cdot \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) dc - \left( \frac{c}{\Delta p} \cdot \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) dm^* + \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2 \cdot p_h} d\beta^* = 0
$$

therefore:

$$
\frac{d\beta^*}{dc} = \frac{- \frac{c}{\Delta p} \cdot \left( 1 - \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) \cdot \frac{p_i \cdot c \cdot (m^*)^2}{2\Delta p \cdot (\beta^*)^2} + m^* \cdot \left( 1 - \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) \cdot \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot \beta^*}}{\left( - \frac{c}{\Delta p} \cdot \left( 1 - \frac{p_i \cdot \gamma}{p_h \cdot \beta^*} \right) \cdot \frac{p_i \cdot c \cdot (m^*)^2}{2\Delta p \cdot (\beta^*)^2} - \left( \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h} \cdot \frac{p_i \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2} \right) \right)} = \begin{cases} (+) & < 0 \end{cases}
$$

(B.4)
Result 6

Note that the expression (2.36) can be expressed as a function of $x$:

$$
\Phi(y) = \frac{2K_0}{p_h \cdot y} - \frac{2K_1}{p_h \cdot (1 - y)} - \frac{(2y - 1) \cdot c}{\Delta \rho \cdot \gamma}
$$

(B.5)

this expression is continuous in $(0, 1)$, increasing, and verifies that $\lim_{y \to 0} \Phi(y) = +\infty$ and that $\lim_{y \to 1} \Phi(y) = -\infty$. By Bolzano's theorem, there exists a point in which its value is zero. To prove that such point is bigger than $\frac{1}{2}$ it is enough to show that the value of the expression at $\frac{1}{2}$ is positive:

$$
\Phi\left(\frac{1}{2}\right) = \frac{4(K_0 - K_1)}{p_h} > 0 \quad \blacksquare
$$

(B.6)

Result 7

We have established in Result 6 that $y^* > \frac{1}{2}$. Therefore the project at $\frac{1}{2}$ satisfies that:

$$
-\frac{c \cdot \frac{1}{2}}{\Delta \rho} + \frac{c \cdot p_1 \cdot \gamma \cdot \frac{1}{2}}{p_h \beta_0} > \frac{c \cdot (1 - \frac{1}{2})}{\Delta \rho} + \frac{c \cdot p_1 \cdot \gamma \cdot \frac{1}{2}}{p_h \beta_1^*}
$$

(B.7)

that directly implies that $\beta_1^* > \beta_0^*$. $\blacksquare$

Result 8

If $y^* = \frac{1}{2}$ were the equilibrium market share then the level of capital of the center at 1 would be equal to: $K_1 = \frac{c p_1}{8 \cdot \gamma \cdot \Delta \rho}$. But for the system to be characterized by Regime 3 the capital in center at 1 must be strictly smaller than that quantity (see the constructions of bounds below). Smaller quantities of capital at 1 imply larger equilibrium shares. Therefore $y^* > \frac{1}{2}$. $\blacksquare$

Result 9

The entrepreneur is the residual claimant of the profits of the project. When both sources of capital have the same cost, profits can be expressed as follows:

$$
R_e^*(x) = R - \frac{I \cdot \gamma}{p_h} - C(x)
$$

(B.8)
To maximize these profits, entrepreneurs will make the monitoring costs as small as possible what occurs by looking for funds in the closest bank. ■

Result 10

\textbf{a) Derivation of } f(.)

Let \((K_0, K_1)\) be a pair of capitals that are just enough to finance all the projects of the economy. If that happens consider the marginal projects that both centers:

\[ m^{0\ast} = \left( R - \frac{b}{\Delta p} - \frac{L\gamma}{p_h} \right) + \sqrt{\left( R - \frac{b}{\Delta p} - \frac{L\gamma}{p_h} \right)^2 + 8 \frac{c \gamma}{p_h \Delta p} K_0} \left( \frac{2 \varepsilon}{\Delta p} \right) \]  
\label{eq:B.9}

and:

\[ 1 - m^{1\ast} = \left( R - \frac{b}{\Delta p} - \frac{L\gamma}{p_h} \right) + \sqrt{\left( R - \frac{b}{\Delta p} - \frac{L\gamma}{p_h} \right)^2 + 8 \frac{c \gamma}{p_h \Delta p} K_1} \left( \frac{2 \varepsilon}{\Delta p} \right) \]  
\label{eq:B.10}

be defined as in expression (2.22). If the market is just covered when:

\[ m^{0\ast} = m^{1\ast} \]  
\label{eq:B.11}

Therefore the following expression defines the function \( f(.) \):

\[ K_1 = \frac{\left( 2 \frac{c}{\Delta p} - 2 \left( R - \frac{b}{\Delta p} - \frac{L\gamma}{p_h} \right) - \sqrt{\left( R - \frac{b}{\Delta p} - \frac{L\gamma}{p_h} \right)^2 + 8 \frac{c \gamma}{p_h \Delta p} K_0} \right)^2}{\frac{p_h \Delta p}{\delta c \gamma}} \]  
\label{eq:B.12}

\textbf{b) Derivation of } g(.)

The function \( g(.) \) can be constructed using the following three expressions: equation (2.45), the constraint \( \beta_0 = \gamma \) and the definition of \( \beta_0 = \frac{\int_0^{y^*} \left( \frac{c p_h}{\Delta p} - c \right) \, dz}{K_0} \).

From the last two expressions:

\[ K_0 \cdot \gamma = \frac{p_1 \cdot c \cdot (y^*)^2}{2 \gamma \Delta p} \]  
\label{eq:B.13}
or explicitly:

\[ y^* = \sqrt{\frac{2 \cdot K_0 \cdot \gamma^2 \Delta p}{p_i \cdot c}} \]  

(B.14)

that can be substituted in equation (2.45) to find a relation between \( K_1 \) and \( K_0 \). To obtain \( g(.) \) it is enough to solve for \( K_1 \).

c) Cut-offs:

\( K_1^2 \) coincide with the area of the triangle \((0, \frac{1}{2}, f_0^0(\frac{1}{2}))\) in Figure 2-7:

\[ K_1^2 = \frac{c \cdot p_l}{\gamma \cdot 8 \Delta p} \]  

(B.15)

\( K_0^0 \) can be obtained as \( f^{-1}(0) \), solving the following equation:

\[
2 \frac{c}{\Delta p} - 2 \left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right) - \sqrt{\left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right)^2 + \frac{c \cdot \gamma}{p_h \cdot \Delta p} K_0^0} = \left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right)
\]

or explicitly:

\[
K_0^0 = \frac{9c \cdot p_h + p_h \cdot \Delta p}{2 \Delta p \cdot \gamma} \left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right) - \frac{9p_h}{2\gamma} \left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right)
\]  

(B.16)

Finally \( K_0^1 \) can be constructed as:

\[
K_0^1 = \int_0^{\frac{1}{1+\Delta p}} \frac{p_l \cdot c}{2 \Delta p \cdot \gamma} dx = \frac{p_l \cdot c}{4 \Delta p \cdot \gamma} \cdot \left( \frac{1}{1 + \Delta p} \right)^2
\]  

(B.17)

Proposition 17

First consider \( G^*(K_0, K_1) \):

\[
\frac{2K_0}{p_h \cdot y^*} - \frac{2K_1}{p_h \cdot (1 - y^*)} = \frac{(2y^* - 1) \cdot c}{\Delta p \cdot \gamma}
\]

Now assume that the shocks are such that: \( K_0 = K_0(1 + dK_0) \) and \( K_1 = K_1(1 + dK_1) \). Substituting them in expression (2.36) it is obtained the following equation:

\[
\frac{2K_0(1 + dK_0)}{p_h \cdot y^*} - \frac{2K_1(1 + dK_1)}{p_h \cdot (1 - y^*)} = \frac{(2y^* - 1) \cdot c}{\Delta p \cdot \gamma}
\]  

(B.18)
The equilibrium of the system does not changes as long as the right hand side of new has not changed with respect to (2.36). This occurs provided that:

\[
\frac{dK_0}{dK_1} = \frac{K_1}{K_0} \cdot \frac{y^*}{(1 - y^*)} \tag{B.19}
\]

and therefore that ratio defines \( G^*(K_0, K_1) \):

\[
G^*(K_0, K_1) = \frac{K_1}{K_0} \cdot \frac{y^*(K_0, K_1)}{(1 - y^*(K_0, K_1))} \tag{B.20}
\]

If \( \frac{dK_0}{dK_1} > \frac{K_0}{K_1} \cdot \frac{y^*}{1 - y^*} \) then \( y^* \) has to increase to preserve the equilibrium; the opposite occurs if the inequality is reversed.

To show that \( G^*(K_0, K_1) < 1 \) note that in expression (2.36):

\[
\frac{2K_0}{p_h \cdot y^*} - \frac{2K_1}{p_h \cdot (1 - y^*)} > 0 \tag{B.21}
\]

which directly implies the result. \( \blacksquare \)

Proposition 18

By differentiating in expression (2.45) we obtain the following:

\[
\frac{dy^*}{dK_1} = -\frac{c}{\Delta p} \left( 2 - \frac{p_h}{p_h} \right) + \frac{2\gamma K_1}{(1 - y^*)^2 p_h} < 0 \tag{B.22}
\]

what proves the result. \( \blacksquare \)

Proposition 19

The scarcity case was proven in result 3 above.

By arbitrage considerations the cost of capital \( \beta \) moves with \( \gamma \). To obtain the result on \( m \) it is enough to inspect equation (2.25). \( \blacksquare \)

Proposition 20

Let us consider the four cases:

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Case 1 is identical to the case on proposition 3 (scarcity case).

Case 2: It is enough to show the market share has increased. By differentiating in expression (2.36) it is obtained the following expression:

\[
\frac{dy^*}{d\gamma} = \frac{(2y^*-1)c}{\Delta p \cdot \gamma} \frac{(2y^*-1)c}{\Delta p + \frac{2\gamma K_0}{p_h} + \frac{2\gamma K_1}{p_h(1-y^*)}} > 0 \tag{B.23}
\]

Case 3: \( \beta_0^* \), by arbitrage considerations, adjusts to the new level of the interest rate \( \gamma' \).
To show the increase in \( \beta_1^* \) is enough to show that the market share has fallen. Differentiating now in the expression (2.45) it is obtained:

\[
\frac{dy^*}{d\gamma} = -\frac{2K_1}{(1-y^*)p_h} < 0 \tag{B.24}
\]

Case 4: Simply by arbitrage considerations \( \beta_0^* \), and \( \beta_1^* \) adjust to the value \( \gamma' \).
Appendix C

Chapter 3: Proofs

Lemma 23

$m$ is defined as:

$$m = \frac{\nu^2}{\sigma^2 + \nu^2} (m_0 + e^*) + \frac{\sigma^2}{\sigma^2 + \nu^2} (\bar{s} - g(e^*))$$ (C.1)

but after an independent normal signal normal distributions update as follows:

$$E(m) = E \left[ \frac{\nu^2}{\sigma^2 + \nu^2} (m_0 + e^*) + \frac{\sigma^2}{\sigma^2 + \nu^2} \bar{s} \right] = (\mu_0 + e^*)$$ (C.2)

and:

$$\text{Var}(m) = \left( \frac{\sigma^2}{\sigma^2 + \nu^2} \right)^2 \text{Var}(\bar{s}) = \left( \frac{\sigma^2}{\sigma^2 + \nu^2} \right)^2 (\sigma^2 + \nu^2) = \frac{\sigma^4}{\sigma^2 + \nu^2}$$ (C.3)

Using the fact that both $\bar{e}$ and $\bar{y}(e)$ are normal:

$$m \sim N \left( m_0 + e^*, \frac{\sigma^4}{\sigma^2 + \nu^2} \right) \blacksquare$$

Lemma 24

To deduce 3.24 consider the following:

$$E(m| m \geq L \cdot R) = E \left( \frac{\nu^2}{\sigma^2 + \nu^2} (m_0 + e^*) + \frac{\sigma^2}{\sigma^2 + \nu^2} (\bar{s} - g(e^*)) \middle| \bar{s} \geq s' \right)$$ (C.4)
To compute $E((\tilde{s} - g(e^*))|\tilde{s} \geq s')$ consider the expression of the mean of a truncated normal variable.

**Fact 6** If $\tilde{x} \sim N(\mu, \sigma^2)$ then

$$E(\tilde{x}|\tilde{x} > a) = \mu + \sigma \frac{\phi\left(\frac{a-\mu}{\sigma}\right)}{1 - \Phi\left(\frac{a-\mu}{\sigma}\right)} \quad (C.5)$$

Using the previous fact and noting that $\tilde{s} - g(e^*) \sim N(m_0 + e^*, \sigma^2 + \nu^2)$, we have that:

$$E((\tilde{s} - g(e^*))|\tilde{s} > s') = m_0 + e^* + \left(\sqrt{\sigma^2 + \nu^2}\right) \frac{\phi\left(\frac{s'-(m_0+e^*)}{\sqrt{\sigma^2 + \nu^2}}\right)}{1 - \Phi\left(\frac{s'-(m_0+e^*)}{\sqrt{\sigma^2 + \nu^2}}\right)} \quad (C.6)$$

and therefore substituting in (C.4) we have:

$$E(\tilde{m}|\tilde{m} \geq L \cdot R) = (m_0 + e^*) + n \frac{\phi\left(\frac{n}{n} R - m_0 - e^*\right)}{1 - \Phi\left(\frac{n}{n} R - m_0 - e^*\right)} \quad \blacksquare$$

Check that $V_f > 0$

$V_f > 0$ is by definition true if:

$$(L \cdot R - m_0 - e^*) \cdot \Phi + n \cdot \phi > 0 \quad (C.7)$$

Define the following variable:

$$z \equiv -\frac{L \cdot R - m_0 - e^*}{n} \quad (C.8)$$

then $V_f > 0$ when:

$$-z \cdot n \cdot \Phi(-z) + n \cdot \phi(-z) > 0 \quad (C.9)$$

or reexpressing:

$$\frac{\phi(z)}{1 - \Phi(z)} > z \quad (C.10)$$

that is a property of the normal distribution. (See Green (1993) and references herein for the proof of that property.) \( \blacksquare \)

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Proof of Proposition 25

Taking the corresponding derivatives:

\[
\frac{dV}{dm_0} = -\Phi - \left(\frac{L - m_0 - e^*}{n}\right) \cdot \phi - \phi' = -\Phi < 0 \tag{C.11}
\]

\[
\frac{dV}{dL} = \Phi + \left(\frac{L - m_0 - e^*}{n}\right) \cdot \phi + \phi' = \Phi > 0 \tag{C.12}
\]

Proof of Proposition 26

\[
\frac{dV}{dv^2} = \frac{dV}{dn} \cdot \frac{dn}{dv^2} \tag{C.13}
\]

but:

\[
\frac{dV}{dn} = \left(1 - \left(\frac{L - m_0 - e^*}{n}\right)^2\right) \cdot \phi - \left(\frac{L - m_0 - e^*}{n}\right) \cdot \phi' \tag{C.14}
\]

given that by normality: \(\frac{\phi'(x)}{\phi(x)} = -x\), the previous expression results:

\[
\frac{dV}{dn} = \phi > 0 \tag{C.15}
\]

also:

\[
\frac{dn}{dv^2} = -\frac{\sigma^2}{2\sqrt{(\sigma^2 + \nu^2)^3}} < 0 \tag{C.16}
\]

therefore:

\[
\frac{dV}{dv^2} < 0 \tag{C.17}
\]

On the other hand:

\[
\frac{dV}{d\sigma^2} = \frac{dV}{dn} \cdot \frac{dn}{d\sigma^2} \tag{C.18}
\]

and:

\[
\frac{dn}{d\sigma^2} = \frac{\sqrt{\sigma^2 + \nu^2} - \frac{1}{2\sqrt{\sigma^2 + \nu^2}}}{\sigma^2 + \nu^2} = \frac{\frac{1}{2}\sigma^2 + \nu^2}{\sqrt{(\sigma^2 + \nu^2)^3}} > 0 \tag{C.19}
\]
\[ \frac{dV}{d\sigma^2} > 0 \]

Proof of Proposition 27

To prove that \( \frac{dv'}{dk} < 0 \), it is enough to show that \( \frac{de^*}{dk} < 0 \). Totally differentiating the first order condition we have (3.13):

\[ \frac{de^*}{dk} = - \frac{g_0'(e^*)}{k \cdot g_0''(e^*)} < 0 \]  \hspace{1cm} (C.20)

Proof of Lemma 28

 Totally differentiation of \( A(m_0,n) \) gives:

\[ \frac{dn}{dm_0} \bigg|_A = \frac{-\Phi - \left( \frac{LR-m_0-e^*}{n} \right) \phi - \phi'}{\left( \frac{LR-m_0-e^*}{n} \right)^2 \phi + \phi - \left( \frac{LR-m_0-e^*}{n} \right) \phi'} \]  \hspace{1cm} (C.21)

But note that the normal distribution satisfies:

\[ x \cdot \phi = -\phi' \]  \hspace{1cm} (C.22)

therefore the previous expression can be simplified:

\[ \frac{dn}{dm_0} \bigg|_A = \frac{\Phi}{\phi} > 0 \]  \hspace{1cm} (C.23)

Now consider the second derivative:

\[ \frac{d^2n}{dm_0^2} \bigg|_A = \frac{d}{dm_0} \left( \frac{dn}{dm_0} \right) = \frac{-\frac{1}{n} \phi^2 - (\Phi \cdot \frac{1}{n} \phi')}{\phi^2} = \frac{1}{n \cdot \phi^2} \left( -\phi^2 + \Phi \cdot \phi' \right) \]  \hspace{1cm} (C.24)

Again, as in the normal distribution (C.22) is verified then we can express:

\[ \frac{d^2n}{dm_0^2} \bigg|_A = - \frac{\phi}{n \cdot \phi^2} \left( \phi + \left( \frac{LR-m_0-e^*}{n} \right) \Phi \right) \]  \hspace{1cm} (C.25)
but by C.10 we know that $\phi + \left(\frac{L \cdot R - m_0 - e^*}{n}\right) \Phi > 0$. Hence:

$$\left.\frac{d^2n}{dm_0^2}\right|_A < 0$$

Proof of Lemma 30

Total differentiation of $C(m_0, n)$ gives:

$$\left.\frac{dn}{dm_0}\right|_C = \frac{-\Phi - \left(\frac{L \cdot R - m_0 - e^*}{n}\right) \phi - \phi' + 1}{\left(\frac{L \cdot R - m_0 - e^*}{n}\right)^2 \phi + \phi - \left(\frac{L \cdot R - m_0 - e^*}{n}\right) \phi'}$$  \hspace{1cm} (C.26)

Simplifying gives:

$$\left.\frac{dn}{dm_0}\right|_C = -\frac{1 - \Phi}{\phi} < 0$$ \hspace{1cm} (C.27)

Now consider the second derivative:

$$\left.\frac{d^2n}{dm_0^2}\right|_C = \frac{d}{dm_0}\left(\frac{dn}{dm_0}\right) = -\frac{1}{n} \phi^2 - \left((-1 + \Phi) \cdot \frac{1}{n} \phi'\right) < 0$$ \hspace{1cm} (C.28)

Again, as in the normal distribution (C.22) is verified then we can express:

$$\left.\frac{d^2n}{dm_0^2}\right|_C = -\frac{\phi}{n \cdot \phi^2} \left(\phi + \left(\frac{L \cdot R - m_0 - e^*}{n}\right) \Phi\right)$$ \hspace{1cm} (C.29)

but by (C.10) we know that $\phi + \left(\frac{L \cdot R - m_0 - e^*}{n}\right) \Phi > 0$. Hence:

$$\left.\frac{d^2n}{dm_0^2}\right|_C < 0$$

Existence of $n_p$

The equation: $A(I \cdot R^2 - 1, n_p) = 0$ can be expressed as:

$$\left(L \cdot R - I \cdot R^2 - 1 - e^*\right) \cdot \Phi + n \cdot \phi + (e^* - 1) = 0$$ \hspace{1cm} (C.30)
Define \( V_p \equiv (L \cdot R - I \cdot R^2 - 1 - e^*) \cdot \Phi + n \cdot \phi + (e^* - 1) \). To prove the existence of \( n_p \), remind that \( V_p \) is continuous and increasing in \( n \) (it was proven that \( \frac{dV}{dn} > 0 \)). Hence to prove existence using Bolzano’s theorem, it is enough to show that: \( \lim_{n \to +0} V_p < 0 \) and that \( \lim_{n \to +\infty} V_p > 0 \). Note that:

\[
\lim_{n \to +0} V_p = e^* - 1 < 0
\]  

(C.31)

and also that:

\[
\lim_{n \to +\infty} V_p = +\infty > 0
\]  

(C.32)

**Evolution of \( \sigma^2(t) \) and \( n(t) \)**

Define the precision, \( P(\bar{x}) \), of a random variable as the inverse of its variance.

\[
P(\bar{x}) = \frac{1}{Var(\bar{x})}
\]  

(C.33)

Let \( \alpha, \beta \) and \( \gamma \) be the respective precisions of \( \bar{y}, \bar{e} \) and \( \bar{\eta} \). Therefore: \( \alpha = \frac{1}{\sigma^2}, \beta = \frac{1}{\nu^2} \) and \( \gamma = \frac{1}{\tau^2} \). By using the normal updating rule (see 13) and expressing the result in terms of precisions we have:

\[
\alpha(t) = \alpha + t(\beta + \gamma) \quad \forall t = 0, 1, 2...
\]  

(C.34)

In other words, the variance of the posterior increases linearly with the production period. In the limit it goes to infinity (the variance of the posterior distribution goes to zero):

\[
\sigma^2(t) = \frac{1}{\alpha + t(\beta + \gamma)}
\]  

(C.35)

To derive the sequence for \( n(t) \) it is enough to apply the relationship:

\[
n(t) = \frac{\sigma^2(t)}{\sqrt{\sigma^2(t) + \nu^2}}
\]  

(C.36)

**Proof of Result 18**

Consider \( n(1) > n_p \) and define \( m_0^A \) as \( m_0 \) such that \( A(m_0^A, n(1)) = 0 \) and \( m_0^C \) as \( m_0 \) such that \( C(m_0^C, n(1)) = 0 \). Remember also that by construction: \( A(I \cdot R - 1, n_p) = C(I \cdot R - 1, n_p) \),
and also that \( \frac{d \mu_A}{dn} \bigg|_A > 0 > \frac{d \mu_C}{dn} \bigg|_C \). Therefore we have that:

\[
m_0^A = u_2 > u_1 = m_0^C \quad \blacksquare
\] (C.37)

The argument is the same for other signs of the relationship.

**Computation of \( M_n^{n+2} \)**

To compute \( M_n^{n+2} \) we have to compute:

\[
N \equiv \Pr(\hat{s}_{n+1} > s' \text{ and } \hat{y}_{n+2} > y'(s')) = \Pr(\hat{y}_{n+2} > y'(s') | \hat{s}_{n+1} > s') \quad \text{(C.38)}
\]

where \( s' \) is defined as in Result 3 and \( y'(s') \) verifies the following equation:

\[
\frac{\nu^2 \tau^2}{D} (m_0 + e^*) + \frac{\sigma^2 \tau^2}{D} (s' - g(e^*)) + \frac{\sigma^2 \nu^2}{D} y' = u_1 \quad \text{(C.39)}
\]

where \( D \equiv \sigma^2 \nu^2 + \sigma^2 \tau^2 + \nu^2 \tau^2 \) and \( u_1 \) was defined in Result 18. Note that \( y' \) is a linear function of \( s' \). The left hand side of equation (C.39) represents the expected profitability of a project financed with short term funds after receiving one signal and one observation of the output. Hence, \( N \) is computed as:

\[
N = \int_{s'}^{+\infty} \frac{\Pr(\hat{y}_{n+2} > y'(s))}{\Pr(\hat{s}_{n+1} > s')} dF(s) \quad \text{(C.40)}
\]

or explicitly as:

\[
N = \int_{s'}^{+\infty} \frac{1 - \Phi \left( \frac{\sqrt{\nu^2 + \tau^2}}{\sqrt{\sigma^2 + \nu^2}} \left( \frac{y' - s - g(e^*)}{\sqrt{\sigma^2 + \nu^2}} \right) \right)}{1 - \Phi \left( \frac{L - R - m_0 - e^*}{n \sigma^2} \right)} \cdot \frac{1}{\sqrt{\sigma^2 + \nu^2}} \cdot \phi \left( \frac{s - g(e^*) - m_0 - e^*}{\sqrt{\sigma^2 + \nu^2}} \right) ds \quad \text{(C.41)}
\]

**Proof of Proposition 33**

To find \( j \) simply solve the following equation:

\[
n_p = n(j) \quad \text{(C.42)}
\]

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where \( n(j) \) is given by (C.36) and take the closest larger integer if \( j \) is not an integer itself. This equation also have a solution because \( n_p \) is a fixed positive number and \( n(1) > n_p \) by assumption and \( \lim_{t \to +\infty} n(t) = 0 \).

Next consider the evolution of the cut-offs. Note the following facts:

\[
\begin{align*}
\frac{du_1}{dn} &< 0 \\
\frac{du_2}{dn} &> 0 \\
u_1(n_p) &= u_2(n_p)
\end{align*}
\]

(C.43) \hspace{1cm} (C.44) \hspace{1cm} (C.45)

therefore as \( n(j) \leq n_p \) then \( u_1(n(j)) \geq u_2(n(j)) \). But this implies that \( W_{SR} < W_{LR} \), and therefore that the only relevant cut-off is \( u_2(n(j)) \). And remember that \( u_2(n(j)) \) solves \( B(u_2, n) = 0 \) that for every \( n \) is \( u_2 = I \cdot R^2 - 1 \). ■

Proof of Result 19

Note that \( \frac{dW_{SR}}{dR} < 0 \), \( \frac{dW_{LR}}{dR} = -2 \cdot I \cdot R < 0 \) and also that: \( \frac{dV}{dR} = \frac{dV}{dL} > 0 \). (See Result 25).

It is immediate to check that \( \frac{dW_{SR}}{dR} < 0 \):

\[
\frac{dW_{SR}}{dR} = L \cdot \Phi - 2 \cdot I \cdot R < 0
\]

(C.46)

Therefore the proposition immediately follows. ■

Proof of Result 20

Sufficient conditions for these results are simply:

\[
\lim_{Q \to 0} L(Q) = +\infty \quad \text{and} \quad \lim_{Q \to 1} L(Q) = -\infty
\]

(C.47)

if \( L'(Q) < 0 \), and

\[
\lim_{Q \to 0} L(Q) = -\infty \quad \text{and} \quad \lim_{Q \to 1} L(Q) = +\infty
\]

(C.48)

if \( L'(Q) > 0 \).

With those conditions Result 20 follows immediately from Result 25. ■
Proof of Result 21

Consider the expectation of the maximum of two normal random variables:

\[ \theta \equiv E[\max(\tilde{x}, \tilde{y})] \tag{C.49} \]

where \( \tilde{x} \sim N(\mu_x, \sigma_x^2), \tilde{y} \sim N(\mu_y, \sigma_y^2) \) with \( \sigma_{xy} \) as the covariance between \( \tilde{x} \) and \( \tilde{y} \).

Consider first the case of independence: \( \sigma_{xy} = 0 \).

**Property 1.** If the variables \( \tilde{x} \) and \( \tilde{y} \) are independent then \( \theta \) increases with \( \sigma_x^2 \) or \( \sigma_y^2 \).

**Proof.** Using the law of iterated expectations:

\[ E[\max(\tilde{x}, \tilde{y})] = E_y[E[\max(\tilde{x}, y)|y]] \tag{C.50} \]

and:

\[ E[\max(\tilde{x}, y)] = E[\tilde{x}|\tilde{x} > y] \cdot Pr(\tilde{x} > y) + y \cdot Pr(\tilde{x} < y) \tag{C.51} \]

but this was considered in Result 26 where it was shown that that expectation increases with the variance. Therefore if \( \frac{dE[\max(\tilde{x}, y)]}{dy} \) is greater than zero for every value of \( y \) then \( \frac{d\theta}{dy} > 0 \) follows. The argument is the same for \( \frac{d\theta}{dx} > 0 \).

Consider now the case of variables \( \tilde{x}, \tilde{y} \) with the same mean:

**Property 2** If \( \mu_x = \mu_y = \mu \) then there is a close form solution for \( \theta \):

\[ \theta = \mu + \sqrt{\frac{\sigma_x^2 + \sigma_y^2 - 2\sigma_{xy}}{2\pi}} \tag{C.52} \]

**Proof.** See Afonja (1972).

Note that in this case the impact of \( \sigma_x^2, \sigma_y^2 \) and \( \sigma_{xy} \) are immediately determined from the inspection of (C.52).

Consider the general case, different means and correlation.

**Property 3** In general, the value of \( \theta \) increases with \( \sigma_x^2, \sigma_y^2 \) and decreases with \( \sigma_{xy} \).
Proof. See Afonja (1972) and references therein. □