PROPERTIES OF WATER FROST FORMED AT CRYOGENIC TEMPERATURES

by

Irving Brazinsky

B.Ch.E., Cooper Union (1958)
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ABSTRACT

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Submitted to the Department of Chemical Engineering on July 6, 1967, in partial fulfillment of the requirements for the degree of Doctor of Science.

The primary purpose of this research was to determine the thickness, density, and thermal conductivity of water frost which deposited from a humidified stream of nitrogen onto a short copper plate, maintained at about $-315^\circ$F. The plate formed part of the underside of a long rectangular duct and was mounted a distance of 72 equivalent duct diameters from the entrance. The duct aspect ratio was 20:1.

The independent parameters in this study were the gas stream Reynolds Number, the humidity, and the time elapsed since the start of frosting. Frost density and thickness were obtained as functions of time by making a series of runs, each run at the same Reynolds Number and humidity, but for a different length of time. The frost density and thickness were measured at the end of each of these runs. The average thermal conductivity of the frost layer was obtained by measuring the heat flux through the frost, the plate temperature and the temperature of the frost surface. Average conductivities were correlated with frost density.

Point or local frost thermal conductivities were obtained by graphical differentiation of the measured temperature profile in the frost. This profile was established from measurements of the temperature at the frost surface, the copper plate temperature, temperatures within the frost and the heat flux through the frost. These local conductivities were correlated with frost density, using temperature as a parameter.

Data were taken at Reynolds Numbers of about 5,600, 9,300 and 14,500. For the two lower Reynolds Numbers, only humidities of 23 and 37 grains water/lb dry gas were employed.
At a Reynolds Number of 14,700 several humidities ranging between 23 and 37 grains water/lb dry gas were used. The gas pressure over the plate was essentially atmospheric.

Frost density and thickness increased continuously throughout a run. At a given humidity, higher Reynolds Numbers resulted in denser frosts, but these frosts had slightly lower thicknesses. Humidity did not affect frost density, but lower humidities resulted in significantly smaller thicknesses.

The average thermal conductivity of the frost layer was found to increase with density. In particular, thermal conductivity varied from 0.025 to 0.042 BTU/hr-ft²·°F/ft over the density range 0.055 to 0.13 gm/cc. Point frost thermal conductivity was found to increase both with increasing density and increasing temperature.

In all runs, the heat flux decreased from a high rate, initially, to a lower but constant value. The magnitude of this constant rate increased with increasing Reynolds Number but decreased with increasing humidity. In addition, the temperature difference across the frost became substantially constant during the steady heat transfer period.

Heat fluxes reached steady values because the increase in thermal resistance caused by the increasing frost thickness was balanced by the continuously increasing frost thermal conductivity. A frost of constant thermal resistance, therefore, resulted.

The increasing conductivity was due to the continuous densification of the frost. The densification was caused by the internal diffusion of water vapor which resulted from the temperature gradient within the frost layer.

Microscopic examination of the frost revealed that it was made up of irregular particles having linear dimensions ranging from 20 to 50 microns.

Thesis Supervisor: Robert C. Reid
Title: Professor of Chemical Engineering

Thesis Supervisor: P. L. Thibaut Brian
Title: Professor of Chemical Engineering
Professor E. Neal Hartley
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Dear Professor Hartley:

In accordance with the regulations of the Faculty, I herewith submit a thesis, entitled "Properties of Water Frost Formed at Cryogenic Temperatures," in partial fulfillment of the requirements for the degree of Doctor of Science in Chemical Engineering at the Massachusetts Institute of Technology.

Respectfully submitted,

Signature Redacted

Irving Brazinsky
Acknowledgement

I wish to express my gratitude and appreciation to the various faculty members and graduate students of the Chemical Engineering Department for their many helpful suggestions during the course of this research. I am particularly grateful to my thesis supervisors, Professors R. C. Reid and P. L. T. Brian, for their advice, encouragement and guidance throughout the entire investigation. Their suggestions regarding the internal diffusion model, and their advice on the fluid dynamic and low temperature problems involved, were invaluable.

Although it is impossible to individually mention all those from whom help was received, I am nonetheless appreciative of their aid. A special note of thanks is due to Professor M. E. Weber, who is presently in the Chemical Engineering Department at McGill University. He supervised my work while I was a research assistant and served later as a member of my thesis committee. Professor Weber was the source of numerous valuable ideas, many of which were incorporated directly in my work. The other members of my thesis committee, Professors C. M. Mohr and W. D. Kingery, also made significant contributions. In addition, fellow graduate students Stephan Paradis, Shafik Sadek and Yatish Shah made many pertinent, valuable suggestions. In particular, computer calculations, based on the internal diffusion model, made by Mr. Shah were extremely helpful.
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SUMMARY

Introduction

Frost growth on a very cold surface (about \(-315^\circ F\)) suddenly exposed to a warm humid gas stream (dew point of about \(32^\circ F\)) is a complex phenomenon, many aspects of which still cannot be described in a quantitative manner. It appears that, initially, there is an extraordinarily rapid nucleation of particles within the gas phase boundary layer. These nuclei probably grow by diffusion of water vapor and by collisional processes. The nucleated particles are quite small and most likely follow the motion of the fluctuating eddies in the turbulent gas stream. Some particles deposit on the wall; others are apparently swept away from the cold surface and sublime. The initial deposit is an irregular patchy frost and could have resulted from the formation of a few micro-islands of frost which acted as traps for additional ice particles.

During this initial phase, temperatures in the frost are probably not significantly different from the cold surface temperature. However, as the islands or clumps of frost grow and the valleys between these islands "fill in", a layer of frost, which now has a significant insulative effect, builds up. During this period the frost surface becomes more uniform in texture, and its temperature probably rises rapidly. Eventually, the temperature of the surface approaches, but never reaches, the bulk gas dew point.
During the frost formation period, the mechanism of frosting apparently changes from nucleation, with particle transfer, to a molecular transport of water vapor. Bar- ring any erosion of particles from the surface, the rate of the latter mechanism would be expected to be governed by the usual mass transfer laws.

This thesis presents a description of the physical properties and structure of water frost which deposits onto a cold plate by the postulated processes described qualitatively in the preceding paragraphs.

Motivation

Two areas where data on frost properties are of particular interest are industrial low temperature heat exchangers and cryogenic storage tanks for space vehicles.

In plants producing liquid oxygen and nitrogen, frost accumulates on the walls of many heat exchangers. After a short period of time, the deposit must be removed to maintain a reasonable pressure drop and rate of heat transfer. Knowledge of frost properties, such as thermal conductivity and density, would enable one to design and operate these heat exchangers in a more efficient manner.

The frost which forms on uninsulated liquid oxygen storage tanks used for space vehicles is generally undesirable because of the additional weight which the vehicle must carry aloft. The deposit, however, can also be beneficial since it acts as insulation and, therefore, decreases
liquid oxygen boil-off. Both liquid oxygen boil-off and additional frost weight could be estimated from deposition rate studies if values of the frost properties were also known.

**Previous Studies**

A number of investigators have studied simultaneous heat and mass transfer from a humid gas stream to a cold surface under frosting conditions. Most were primarily concerned with over-all heat and mass transfer rates and did not focus their attention on the properties of the frost. A study of the frost properties, however, is absolutely necessary as the rates of heat and mass transfer are ultimately determined by the values of these properties.

Frost properties vary considerably, and are in general dependent on the cold surface temperature and also on whether the frost is deposited under conditions of natural or forced convection. In natural convection the frost is usually structurally weak, especially if formed at low temperatures. It generally has a low density (in the range of 0.01 to 0.05 gm/cc for cryogenic frosts) and a low thermal conductivity (40). On the other hand, forced convection frosts, especially those formed at high temperatures, have densities ranging from 0.3 to 0.5 gm/cc and are characterized by relatively high conductivities (15).

In all types of frosting processes, it has been generally noted that, at the start of a given experiment,
the heat flux is large but, as frost accumulates, the heat flux decreases and approaches a more-or-less constant value. This steady heat flux varies somewhat with the humidity and gas phase Reynolds Number, but is surprisingly time independent, even though the mass and thickness of frost still continue to increase during this period (40, 69, 72, 87, 88).

Except for some data for very low density, natural convection frosts (40) and for forced convection layered frosts, i.e., consisting of alternate layers of solid ice and porous frost (69, 81), there are no reported studies of properties of frosts formed at temperatures near -300°F. As previously mentioned, properties of frosts formed in high temperature, forced convection experiments have also been reported (15). In addition, properties of "natural snow" have been measured.

**Objectives**

It was the primary purpose of this research to determine the thickness, density and thermal conductivity of water frost which deposits from a humidified stream of nitrogen onto a cold copper plate. The plate was maintained at about -315°F. Frost densities and thicknesses were to be obtained as functions of Reynolds Number, gas phase humidity and time elapsed since the start of frosting. Conductivity was to be correlated with frost density and temperature. In addition, the frost structure was to be examined under a microscope.
Range of Independent Parameters

Frost deposition studies were made at Reynolds Numbers of 5,600, 9,300 and 14,500. The Reynolds Number used in this investigation is based on flow between infinitely wide parallel plates. Gas stream humidities ranged from 23 to 37 grains water/lb dry gas. This corresponds to a dew point range of 29 to 40°F. (It should be noted that values of humidity expressed as grains water/lb dry gas when divided by 7000 give values in lb water/lb dry gas.) Gas pressure over the plate was essentially atmospheric.

Frost densities and thicknesses were obtained as functions of time by making a series of runs, each run at the same Reynolds Number and gas stream humidity but for a different length of time. The thickness and bulk frost density were measured at the end of each of these runs. Deposition periods ranged from 34 to 123 minutes. For all runs, the gas stream temperature was about 75°F, and the cold plate temperature ranged from -314 to -318°F.
Apparatus

1. Over-all Flow Sheet

A diagram of the flow system is shown in Figure 1. The most important part of the equipment was the cold plate onto which the frost deposited. This short plate, which formed part of the underside of a long rectangular duct, was mounted a distance of 72 equivalent duct diameters from the duct entrance. A blower recirculated nitrogen gas through the system. The nitrogen leaving the blower was divided into two streams. One stream was dried in a silica gel drier; the other was saturated with water vapor in a packed tower. The two streams when re-mixed produced a gas with the desired humidity. The temperature of the gas was controlled by varying the power to an electrically heated section of pipe located downstream of the tower-drier combination. After leaving the heater the nitrogen flow was metered. The gas, at a particular temperature (i.e., at about room temperature) and at a given humidity, then flowed through nine feet of duct before it passed over the cold plate.

2. Copper Plate and Associated Boil-off Calorimeter

(a) Description of Equipment

The plate onto which the frost deposited was 8.9 inches long, 14.0 inches wide and 1/4 of an inch thick. It consisted of inner and outer sections separated
by a 0.020-inch thick epoxy ring. The inner circular section was 2.235 inches in diameter and served as the heat transfer test section. The primary purpose of the epoxy ring was to reduce heat transfer between the inner and the outer sections. This ring also bonded the two sections of the plate together and supported the weight of the inner stainless steel cylinder, the inner section of the copper plate and the 17 equilibrator rods, which were silver soldered to the underside of this inner section. A detailed sketch of the plate arrangement is shown in Figure 2. Ninety-one copper equilibrator rods were silver soldered to the outer, annular section. The rods provided a low thermal resistance path between the copper plate and the liquid nitrogen in the calorimeter and thus maintained the plate at a temperature within several degrees of the normal boiling point of liquid nitrogen.

An outer brass cylinder filled with liquid nitrogen served as a heat transfer guard chamber for the inner stainless steel test vessel. Part of the outer section of the copper plate overhung this brass chamber, but all 91 rods were contained within the brass chamber.

(b) **Calibration of Calorimeter**

The heat flux through the test section of the copper plate (i.e., the inner section) was determined by measuring the boil-off rate of liquid nitrogen from the inner stainless steel chamber. In order to ascertain
whether the heat flux calculated from the boil-off measurement corresponded to the true value of the heat flux, the calorimeter was calibrated.

Heat was conducted into the copper plate through a thin layer of cork located on top of the plate. From temperature measurements of the upper and lower surfaces of the cork and the known cork thermal conductivity, the heat flux into the copper plate was determined. The source of heat was an electrical heater, which was well insulated. Substantially all of the electrical energy produced by the heater passed into the copper plate.

A comparison between the heat flux calculated from the measured liquid nitrogen boil-off and the known heat input is shown in Table 1. The agreement is seen to be quite good at all values of the heat flux. Thus, for all frosting runs, the heat flux determined from the liquid nitrogen boil-off measurement was taken as the true value of the heat flux into the calorimeter.

3. Rectangular Duct

The rectangular duct was 12 feet long, 15 inches wide and 3/4 of an inch high. Nine feet of warm duct (i.e., at room temperature) preceded the cold 8.9-inch long copper plate. A large aspect ratio (width/height) was used in order to simulate the case of flow between infinitely wide parallel plates. The long length of warm
duct preceding the cold copper plate ensured that the turbulent velocity profile was fully developed before the gas passed over the plate.

The top of the duct, just above the plate, was made of \( \frac{1}{2} \) inch thick Lucite. This Lucite section was easily removed, allowing the frost to be sampled at the end of each run. The movable thermocouple probe, used to measure frost surface temperature, was mounted in this Lucite section (see Figure 2). The inside of the duct was lined with aluminum foil in order to minimize radiant heat flux from the top of the warm duct to the frost surface.

**Measurements**

1. **Frost Density and Thickness**

At the end of each run, the thickness was measured with a depth micrometer and the density calculated by sampling and weighing a known volume of frost. Over the downstream two-thirds of the copper plate area, there was no significant variation with position in either the frost thickness or density. Thus the frost covering the test section (inner section of copper plate) was considered as a representative portion, and this portion was utilized for frost thermal conductivity measurements. The frost densities and thicknesses reported in this thesis are the "downstream values".
2. Frost Thermal Conductivities

The average thermal conductivity of the frost layer was calculated from Eq. (1) using measurements of the heat flux through that portion of the frost covering the inner section of the copper plate, the temperature difference across the frost, and the frost thickness.

\[
\frac{q_T}{A} = \frac{k_{f,T} \Delta T}{\delta} \quad (1)
\]

\(q_T/A\) is the measured heat flux, \(\Delta T\) the temperature difference across the frost, \(\delta\) the frost thickness, and \(k_{f,T}\) the average thermal conductivity of the frost layer.\(\text{Av.}\).

Two temperatures within the frost were measured with 0.003 inch diameter copper-constantan thermocouples. The measuring junctions of these probes were located at known heights above the plate. An "end-of-run" frost temperature profile was obtained from the temperatures measured by the two stationary thermocouples, the measured frost surface temperature and the measured temperature of the copper plate. Thermal conductivities at different temperatures, designated as point conductivities, were calculated from Eq. (2).

\[
\frac{q_T}{A} = k_{f,T} \left( \frac{dT}{dy} \right) \quad (2)
\]
The gradient, \( \frac{dT}{dy} \), was obtained at a particular location by graphically differentiating (i.e., by eye) the frost temperature profile.

3. Cold Room Measurements to Determine Frost Microstructure and Density Gradient

A frost sample was taken from the plate at the end of each run and examined in a \(-40^\circ F\) cold room.

To determine whether a density gradient through the frost was present, a portion of the frost sample was sliced horizontally into two halves with a sharp razor. The thickness of both halves was measured with a depth micrometer. Densities were obtained by sampling and weighing known volumes from each half. The frost was also viewed under a microscope. These observations allowed one to estimate the sizes and shapes of the ice particles which made up the frost.

Results

It is naturally most desirable to measure the thermal conductivity of the frost under steady state conditions. In frosting situations it is impossible to obtain a true steady state situation since frost density and thickness are continuously increasing. Even so, a steady heat transfer rate is generally established. The temperature difference across the frost also becomes substantially constant during this period of constant heat transfer. Though this
is not a true steady state situation, it is more desirable to measure the frost conductivity during this period than during the transient period when heat flux through the frost is continuously decreasing and frost surface temperature is rapidly increasing. Accurate heat flux measurements were, therefore, made only during the constant heat transfer rate period.

1. **Initial Phases of Frosting Process**

   At the start of a run (i.e., before any significant quantity of frost had deposited onto the plate), a fog was observed in the gas space immediately adjacent to the plate. The first deposits of frost consisted of a series of isolated clumps. During the run the series of clumps were transformed into a smooth frost. The time required for this to occur varied with the gas phase Reynolds Number, but seemed to be independent of humidity. The relationship is shown in Table 2.

2. **Quantitative Results**

   (a) **Frost Density and Thickness**

   Frost densities are shown as a function of time, with Reynolds Numbers as parameter, in Table 3 and Figure 3. The shaded points in Figure 3 are for a humidity of 37 grains water/lb dry gas (dew point of 40°F); the lower humidity points are unshaded. At Reynolds Numbers of 5,600 and 9,300 all low humidity points correspond to
a water vapor ratio of about 23 grains water/lb. dry gas (dew point of $29^\circ F$). At a Reynolds Number of 14,500 the low humidity points correspond to water vapor ratios ranging from 23 to 31 grains water/lb. dry gas.

The frost thickness as a function of time with Reynolds Number as parameter is shown in Figure 4. The shaded points are for a humidity of about 37 grains water/lb. dry gas; the unshaded points are for a humidity of about 23 grains water/lb. dry gas. Measured thicknesses for runs performed at a Reynolds Number of 14,500 and at low humidities are not shown as each thickness corresponds to a different humidity.

Frost density and thickness continuously increased throughout the run. Higher Reynolds Numbers resulted in frosts of greater densities but slightly lower thicknesses. Over the humidity range involved, the frost density was independent of humidity. Lower humidities, however, resulted in smaller frost thicknesses.

The reported frost densities are actually average densities as they were measured using the total thickness of the frost layer. However, measurements on the upper and lower halves of the frost indicated that the density gradient through the frost was insignificant. Thus, for the purposes of the present thesis, the frost was considered to be of uniform density throughout its depth.
(b) **Heat Flux Through Frost**

The values of the quasi-steady state heat transfer rates depend on the Reynolds Number and gas phase humidity as shown in Figure 5. The heat flux increased with increasing Reynolds Number, but decreased with increasing gas phase humidity.

(c) **Frost Thermal Conductivity**

In Figure 6, the average conductivities of the frost layer are plotted as a function of frost density. Over the density range of 0.05 to 0.13 gm/cc, thermal conductivity increased with frost density. The various points in Figure 6 correspond to frosts which were formed at different Reynolds Numbers and humidities. In plotting average thermal conductivity as a function of density only, it was implicitly assumed that a given value of density implied a given frost structure, and therefore that thermal conductivity was a unique function of density. This is equivalent to assuming that two frosts of the same density, formed at different Reynolds Number and/or different humidities, nonetheless have the same structures. Actually, some of the scatter in the data shown in Figure 6 may indeed be caused by structural differences of frosts of the same density. No definite trend, however, was observed.

Point thermal conductivity as a function of density and temperature is shown in Figure 7. The data are
extrapolated to zero density, i.e., to the pure gas. At any given density, the frost thermal conductivity shows a marked increase with temperature. For a given temperature, thermal conductivity increases with density, the rate of this increase being greater at the higher temperatures. In plotting Figure 7, it was again assumed that a unique relationship exists between frost structure and frost density.

As mentioned earlier, frost temperature profiles were differentiated to obtain point frost thermal conductivities. A typical profile is shown in Figure 8 for a run made at a Reynolds Number of 14,400 and a humidity of 37.7 grains water/lb dry gas. The temperature at position zero is the copper plate temperature; whereas the temperature at $y = 0.222$ inches, the frost thickness, is the temperature of the surface.

Discussion of Results

1. **Effects of Internal Diffusion on Frost Particles**

   The diffusion of water vapor from the frost surface into the interior is a significant factor in the determination of frost properties. This internal diffusion is caused by the water vapor concentration gradient which results from the temperature gradient within the frost.

   Internal diffusion causes significant changes in the sizes and shapes of the ice particles contained in the frost. As the frost was transformed from a series of
scattered clumps into a macroscopically smooth frost which grew and densified, the ice particles which were approximately spherical in shape, and had diameters ranging from about 5 to 10 microns, were transformed to much larger particles having linear dimensions ranging from about 20 to 50 microns. In addition, the transformed particles were no longer approximately spherical but exhibited corners and edges. These characteristics are typical of particles contained in a snow layer which has been subjected to a significant internal water vapor diffusion (77).

2. Description of Frosting Process

As previously mentioned, the frost surface temperature was quite low during the early phases of a run. Hence water vapor diffusion within the frost was insignificant at this time. However, as the frost thickness increased, the surface temperature rise and the heat flux decrease occurred quite rapidly. Very shortly thereafter the frost surface temperature reached a value at which internal water vapor diffusion became quite significant, with the result that frost density increased substantially. Frost thickness still continued to increase because of the continuous deposition of mass.

Due to the very high sensitivity of internal diffusion to the frost surface temperature, at the higher temperatures, a very small increase in the surface temperature results in a large increase in the internal water
vapor diffusion rate. The density (and hence the thermal conductivity of the frost), therefore, increased to a point where the heat flux fell off only very slightly, and the surface temperature rose only very slightly as additional frost was deposited. The frost surface temperature, therefore, never reached $32^\circ\text{F}$, or the gas phase dew point, whichever was lower.

During the constant heat transfer period, the frost thermal resistance was essentially constant as the increasing thickness was balanced by the increasing thermal conductivity which resulted from the continuous frost densification. It is highly unlikely that these two effects (i.e., thickness and density increase) balanced each other merely by chance. Certainly a more reasonable explanation is that heat flux and frost surface temperature are stabilized by a very high-gain negative feed-back effect. In other words, any large disturbance in the frost environment, i.e., in the gas phase, will not substantially disturb the surface temperature or the heat transfer rate during the quasi-steady state period. For example, it is assumed that some disturbance in the gas phase causes the heat transfer rate to increase, thus tending to overheat the frost surface. An increased surface temperature, however, would result in an increased rate of water vapor diffusion away from the frost surface and into the interior. This would result in a greater rate of conductivity increase than would have been the case had the surface temperature
not been raised. The additional conductivity increase, however, would tend to reduce the surface temperature. It can therefore be concluded that if a disturbance which tends to cause a significant rise in the surface temperature occurs, a subsequent series of processes will ensue which will tend to bring the surface temperature back down to about its former level.

Based upon the physical concepts described above, a simplified mathematical model has been developed for predicting frost layer densification rates and other frost properties. This model will presently be considered.

3. Model for Predicting Frost Properties

The model used to predict the variation of frost properties with time is based both on experimental data and on two equations. One equation is an expression of the energy balance between the gas phase and the frost layer. The other is a material balance relating the rate of densification of the frost to the rate of internal diffusion of water vapor. The experimental data used are thermal conductivities at various densities and temperatures (Figure 7) as well as curves of the mass of frost accumulated as a function of time. These latter curves were obtained by multiplying the measured values of frost density and thickness and extrapolating the curve through these points to the origin. In addition, the model used experimental heat transfer coefficients. These coefficients were determined at the
end of each run from measurements of the heat flux and the frost surface temperature.

(a) Energy Balance

The energy balance expressed mathematically is

$$h_g(T_g - T_s) + \frac{N}{A} \lambda = \frac{k_{f,T} (T_s - T_p)}{\delta}$$

Equation (3) is strictly valid only for a true steady state situation. As previously mentioned, the frosting process, even at long times, is not a true steady state situation but transient effects within the growing frost layer are insignificant, and Eq. (3) can be used with negligible error.

The magnitude of the term \(\frac{N}{A} \lambda\) is usually about 10 to 20 per cent of the magnitude of \(h_g(T_g - T_s)\). \(\frac{N}{A} \lambda\) was therefore not neglected in the calculations.

(b) Equation for Rate of Densification of Frost

As previously mentioned, frost densification is caused by the internal diffusion of water vapor, which results from the water vapor concentration gradient through the frost layer. Based on this internal diffusion
mechanism, an expression for the rate of frost densification can be derived. This derivation is outlined below.

The density of the frost at any instant is given by

\[ \rho_f = \frac{m_f}{\delta} \]  

(4)

Differentiation of Eq. (4) with respect to time yields

\[ 5 \frac{d\rho_f}{d\delta} + \rho_f \frac{d\delta}{d\delta} = \frac{dm_f}{d\delta} \]  

(5)

The term \( \frac{dm_f}{d\delta} \) represents the rate of mass accumulation of frost. The term \( \rho_f \frac{d\delta}{d\delta} \) reflects the increase of mass on the plate resulting from the continuously increasing frost thickness. The term \( 5 \frac{d\rho_f}{d\delta} \), on the other hand, represents the rate of mass accumulation resulting from the frost densification. Since the water vapor flux causing the frost densification originates at the frost surface, the following equation may be written:

\[ 5 \frac{d\rho_f}{d\delta} = \frac{M_v D_{1-2}(c_T)}{\tau(1 - Y)} (1 - \frac{\rho_f}{\rho_1})(\frac{dT}{dy})_s (\frac{dy}{dT})_s \]  

(6)

where \( (dT/dy)_s \) is the temperature gradient at the frost surface, \( \rho_f \) is the density of the frost, and \( \rho_1 \) is the density of solid ice. By substituting the Clausius-Clapeyron equation and the ideal gas law into Eq. (6),
the final expression for the rate of frost densification (Eq. 7) is derived.

\[
\frac{dp_f}{d\Theta} = \frac{1}{\tau} \left( \frac{\beta}{\sigma} \right) (1 - \frac{\rho_f}{\rho_1})(\frac{dT}{dy}) \tag{7}
\]

where

\[
\beta = \frac{D_{1-2} \rho_g (M_v)^2 \lambda y^*}{RT^2 \lambda_g} \tag{8}
\]

\(D_{1-2}\) is the ordinary diffusivity of water vapor in nitrogen as Knudsen diffusion within the pores is negligible compared with bulk diffusion.

(c) Method of Calculating Frost Properties

The method of calculating frost properties as a function of time consists of the simultaneous solution of Eqs. (3) and (7) by an iteration procedure. For each case (i.e., at a particular Reynolds Number and humidity), the calculation was started at a point in time corresponding to five minutes after the commencement of frosting. For the first five minutes thereafter, i.e., from \(\Theta = 5\) to \(\Theta = 10\), one minute time intervals were chosen. From \(\Theta = 10\) to \(\Theta = 15\), 2 1/2 minute intervals were used. After \(\Theta = 15\) minutes, and until the end of the run, the time interval employed was five minutes. It should also be noted that for each case the heat transfer coefficient
was assumed to be constant at all times. Finally, the tortuosity of the unconsolidated frost was taken to be 1.1 (73).

In order to start the calculation, an initial value of frost thickness (i.e., at $\Theta = 5$ minutes) has to be assumed. For all cases a value of 0.04 inches was arbitrarily chosen. Actually, values of frost density, frost thickness, heat flux and frost surface temperature at long times are essentially independent of the assumed initial frost thickness. This can be seen by comparing the dashed and solid curves of Figures 9, 10 and 11. The dashed curves were calculated using an initial frost thickness of 0.08 inches, whereas the solid curves are for an initial frost thickness of 0.04 inches. Initial densities were obtained by dividing the mass of frost on the plate at $\Theta = 5$ minutes by the assumed initial thickness. Mass of frost on the plate at $\Theta = 5$ minutes was read from the extrapolated mass accumulation vs. time curve.

Figures 9, 10 and 11 are for a Reynolds Number of 5,600 and a humidity of 37 grains water/lb dry gas. For this Reynolds Number and humidity, a frost thickness of 0.04 inches corresponds to a density of 0.024 gm/cc, whereas a thickness of 0.08 inches corresponds to a density of 0.012 gm/cc. Similar calculations at a higher Reynolds Number lead to the same conclusion, i.e., that theoretical values of frost properties at long times are
independent of the assumed initial thickness values. Of course, it cannot be stated definitely that this conclusion is valid outside the range of Reynolds Numbers and initial thickness values utilized in the present thesis.

Even though time intervals as large as 5 minutes were chosen for most of the steps in the calculation, the numerical solution is nonetheless convergent. This was confirmed by Shah (78) who repeated some of the calculations on an IBM 7094 computer using one minute time intervals throughout. His results are essentially the same as those of the present thesis, which were obtained by slide rule calculation.

4. Comparison of Data with Predictions based on Model

The experimental data for a Reynolds Number of 5,600 and a humidity of 37 grains water/lb dry gas are compared with the predictions of the model in Figures 9, 10 and 11. In this case, as in the other cases, the theoretical densification curve lies above the experimental curve, whereas the theoretical thickness values lie below the data. It should be noted, however, that at long times the experimental and theoretical densification rates are approximately equal. The same is also true for the rate of increase of frost thickness. A possible explanation for the divergence between the theoretical and experimental curves is that during the early phases of a run the model overestimates
the densification rate and underestimates the rate of thickness increase. However, in view of the agreement between the "long-time" theoretical and experimental rates (i.e., the rates of densification and thickness increase), it can be concluded that the model is a reasonable representation of the actual physical processes which occur within the frosting system.

A comparison between theoretical values of the heat flux and experimental data are shown in Figure 10. Theoretical and experimental frost surface temperatures are compared in Figure 11. Figures 10 and 11 are also for a Reynolds Number of 5,600 and a gas phase humidity of 37 grains water/lb dry gas.

It is seen from Figure 11 that the measured surface temperatures are somewhat higher than the theoretical values. This is not only true for this case but for all the cases studied. In general, the difference between the experimental and theoretical surface temperatures ranged from 5 to 15°F. The measured temperatures may have been too high because of incomplete burial of the thermocouple bead below the frost surface.

From Figure 10 it is observed that the measured heat fluxes are somewhat lower than the theoretical values. It is likely that the theoretical heat fluxes are too large, because the heat transfer coefficients used in calculating these heat fluxes are too high. As previously mentioned these coefficients were determined
from surface temperature measurements. If the measured surface temperature is 10 to 15 degrees too high, this results in a heat transfer coefficient which is 20 to 30% greater than the true value.

Use of a heat transfer coefficient which is larger than the true value results in calculated frost densities which are too high and calculated frost thicknesses which are too low. In fact, calculations have shown that if the heat transfer coefficient is reduced by about 20% theoretical densities and thicknesses show much better agreement with the data than is depicted in Figures 12 and 13.

5. Dependence of Frost Behavior on Reynolds Number

Gas phase heat transfer coefficients are, naturally, greater at the higher Reynolds Numbers. Thus at equivalent times and humidities, rates of heat transfer from the gas to the frost surface tend to be higher at the larger Reynolds Numbers. These increased heat transfer rates tend to overheat the frost surface, i.e., to make the frost surface temperatures significantly higher at a higher Reynolds Number. However, the higher surface temperature results in a greater frost densification rate. The higher Reynolds Number frosts, therefore, have greater densities and hence increased conductivities. The increased conductivities, however, tend to maintain the surface temperature at a lower level, and overheating of the frost
surface is not expected to be a large effect. This expectation is borne out by the data, which indicate that temperatures of the frost surface during the quasi-steady state period are not significantly affected by the gas stream Reynolds Number. As previously mentioned, theoretical frost surface temperatures do show some dependence on the gas phase Reynolds Number (see Table 4), but this effect is not very large.

The data also show that at equivalent times and humidities, higher Reynolds Numbers result in very slightly lower frost thicknesses. Some investigators (69) who observed the same effect based their explanations on the fact that higher velocity gas streams have greater shearing action. This explanation is of dubious value for the experiments of the present thesis, as the shear strength of the frost is about $2 \frac{1}{2}$ orders of magnitude greater than the highest value of the wall shear stress.

6. **Effect of Humidity on Frost Behavior**

At higher humidities the rates of transfer of mass from the gas to the frost surface tend to be greater because of the larger concentration driving forces. The increased mass transfer rate tends to raise the fugacity of the frost surface or, in other words, to increase the surface temperature. (In fact, the data show that frost surface temperatures are higher at the higher humidities.) However, as previously mentioned, higher surface temperatures
result in increased rates of diffusion of water vapor from the surface into the frost interior. Interestingly enough, these higher internal diffusion rates do not lead to higher frost densities as frost thicknesses increase significantly with increasing humidity. The increased mass of water vapor diffusing into the frost, therefore, has a greater thickness across which to spread. It should be noted that the data indicate that frost density is essentially independent of gas phase humidity.

As just mentioned, lower humidities result in lower frost surface temperatures. Since heat transfer coefficients are not affected by gas phase humidity, it would therefore be expected that heat transfer rates would increase as the humidity is lowered. This is indeed seen to be the case (see Figure 5).

Alternatively, the variation of heat transfer rate with gas phase humidity may be explained by focusing attention on the frost. The measured "end-of-run" surface temperatures of the low humidity frost were, in general, about 10°F lower than for the frosts formed at the higher humidities. The temperature difference across the high and low humidity frosts therefore differed only by about 3%. However, at equivalent times, the thicknesses of high and low humidity frosts differed by as much as 20%. Thus, despite the slightly lower temperature driving force across the lower humidity frost, the smaller thermal resistances of frosts formed at lower humidities resulted in significantly higher heat fluxes.
7. Model Predictions Regarding Effects of Reynolds Numbers and Humidity

Theoretical values of density and thickness as functions of time with Reynolds Number and humidity as parameters are shown in Figures 12 and 13, respectively. The experimental data were plotted in Figures 3 and 4, but are shown again in Figures 12 and 13. It is seen that, in general, the experimentally observed trends can be predicted from the internal diffusion model. Several exceptions, however, should be noted. The first concerns the frost surface temperature. The model predicts that the constant value of the surface temperature decreases with decreasing humidity and with decreasing Reynolds Number. The data show that the surface temperature does decrease with gas phase humidity, but it is approximately independent of the Reynolds Number, at least within the accuracy of the methods used to measure surface temperature. Reasons for this discrepancy have already been presented. (See Table 4 for a comparison between experimental and theoretical surface temperatures.)

The second exception involves the prediction of the effect of humidity on frost density. As previously
mentioned the data indicate that frost density is essentially independent of humidity. However, it is seen in Figure 12 that theoretical densities are dependent on humidity, the effect being somewhat greater at a Reynolds Number of 9,300 than at 5,600. This discrepancy may be due to the use of heat transfer coefficients which are significantly larger than the true values, or it may actually be due to defects in the model itself.

Another exception involves frost thickness. Experimental frost thicknesses decrease with decreasing humidity. This is exactly the trend predicted by the internal diffusion model. However, experimental thicknesses decrease slightly with increasing Reynolds Number. The internal diffusion model predicts the opposite (see Figure 13). Actually the discrepancy is small as theoretical frost thicknesses are very nearly equal at equivalent times and humidities. The same is true of experimental thicknesses.

Despite the discrepancies mentioned in the preceding paragraphs, the internal diffusion model is nonetheless successful in predicting most of the experimentally observed trends. It therefore seems likely that the model is indeed a reasonably good representation of the physical processes which occur within the frosting system.

8. Thermal Resistance of the Frost

Experimental and theoretical values of frost thermal resistance for the three different Reynolds Numbers and
The two humidities are presented in Table 5. The experimental values were obtained by dividing the measured frost thickness by the product of the measured conductivity and test section area. Theoretical thermal resistances were obtained from theoretical conductivities and thicknesses. The differences between theoretical and experimental resistances reflect the discrepancies between the measured frost surface temperatures and those calculated from the internal diffusion model.

9. Thermal Conductivities

(a) Average Conductivity of Frost Layer

The thermal conductivities shown in Figure 6 were calculated using the measured temperature differences across the frost layer and therefore represent average values. There is a considerable variation of frost thermal conductivity with position because of the large temperature gradient within the frost.

The contribution to the frost thermal conductivity resulting from radiant heat transfer between the warmer and colder particles is negligible. In addition, gas flow through the frost makes a negligible contribution to the thermal conductivity of the frost layer. However, vapor diffusion through the frost, arising from the temperature gradient, does contribute slightly. The contribution, however, made by this mechanism (i.e., by
evaporative transport) is only about 5% of the conductivity of the frost layer. Thus, the major contribution to the frost thermal conductivity is made by conduction through the ice gas matrix.

Thermal conductivities increase with frost density because of the higher fractions of solids present at the larger densities. Since the conductivity of the solid is greater than that of the gas, frosts with higher solids fractions have greater thermal conductivities. It is also possible that conductivities increase with density, because contacts between the ice particles are better in the higher density frosts.

(b) Point Thermal Conductivity

The measured frost thermal conductivities shown in Figure 7 are point values, i.e., each value is applicable at a particular position or temperature within the frost. At all positions within the frost, gas flow through the frost and radiant heat transfer make negligible contributions to the point thermal conductivity. At locations within the frost having temperatures lower than -10°F, evaporative transport makes a negligible contribution to the point thermal conductivity. However, evaporative transport becomes significant at higher temperatures. For example, at 20°F the contribution is about 37% of the total gas phase conductivity. The
addition to the point frost conductivity resulting from the greater gas conductivity is dependent on the real structure of the frost. This contribution usually cannot be calculated exactly as the true frost structure generally cannot be mathematically characterized. However, by assuming various bracketing structural models, it is calculated that at a temperature of 20°F evaporative transport increases the point frost thermal conductivity by about 10 to 15%.

At any given temperature, the local frost thermal conductivity increases with density. As is the case for the conductivity of the entire frost layer, higher frost densities imply larger conductivity values because of the increased solids fraction, and also possibly because of the better contacts between the ice particles.

At equivalent densities thermal conductivities are greater at higher frost temperatures for two reasons. One, it is believed that at the higher frost temperatures the contact areas between the various particles are greater than at lower temperatures since at higher temperatures the processes of constructive and destructive metamorphism (56) are more active and therefore result in additional deposition of water vapor in particle contact areas.
In addition, even if particle contact areas were not greater at the higher frost temperatures, thermal conductivities would still increase with temperature as the conductivity of the gas phase increases with increasing temperature. At the high frost porosities encountered in this thesis, the gas phase makes a substantial contribution to the total frost thermal conductivity. Thus, despite the fact that ice thermal conductivity decreases with increasing temperature, the frost thermal conductivity nonetheless increases with increasing temperature.

(c) Comparison of Measured Conductivities with Snow Data

In Figure 14 the 20°F cryogenic frost data are compared with natural snow data obtained from Refs. 43 and 48. The frost data are bracketed by the curves representing the different sets of snow data. The differences between the measured snow conductivities of various investigators is often quite large. These differences can probably be attributed to structural differences of natural snow layers having the same density.

(d) Comparison of Data with Parallel Slab Model

The effect of assumed frost structure on thermal conductivity values can be seen from Figure 15. The upper line was calculated by assuming that the frost structure consisted of an array of parallel slabs of ice and that
the heat flow was parallel to this array. The lower line was calculated by assuming that the heat flow was perpendicular to the array of ice slabs.

It is thus seen from Figure 15 that at the same density values thermal conductivities can be significantly different. A comparison between the -145°F cryogenic frost data and the calculations of the parallel slab model is also shown in Figure 15. For the case in which the heat flow is parallel to the slabs of ice, the calculated conductivities are higher than the data. When heat flow is perpendicular to the ice slabs, the data were higher than the predictions of the model. Clearly the assumption of heat flow parallel to the ice slabs overestimates the contact between ice particles. On the other hand, the contacts between ice particles in the frost are better than represented by the case of heat flow perpendicular to the ice slabs.

At -145°F the actual contacts between particles are better represented by the lower curve. As frost temperature increases, however, the data recede from the lower curve and approach the upper curve.

(e) Proposed Model: Simple Cubic Array of Cubes of Ice Connected by Ice Rods

An assumed structural model developed in this thesis showed close agreement with actual thermal conductivity data. The assumed model is a symmetrical matrix of ice cubes
interconnected by ice rods. The rods are square prisms (see Figures 16A and 16B). As is the case for other structural models, this structure does not represent the true physical situation. However, a desirable characteristic of the model is that the dimensions of the rods may be taken as a measure of the actual contact between the ice particles.

An equation for calculating frost thermal conductivity is derived by considering a unit cube of the heterogeneous structure (see Figure 16B). \( x \) is the linear dimension of the ice cube, and \( z \) is the width of the square ice rod. Actually \( x \) and \( z \) are dimensionless, as they represent, respectively, the ratio of the ice cube length to unity and the ratio of the rod width to unity. It is obvious that the actual size of the unit cube does not affect the computed value of the thermal conductivity.

From Figure 16B it is seen that \( (1 - \varepsilon) \), the solids fraction, is given by

\[
(1 - \varepsilon) = x^3 + 3z^2(1 - x)
\]  

(9)

\( (1 - \varepsilon) \) is related to the densities of the ice, gas and frost by

\[
(1 - \varepsilon) = \frac{(\rho_f - \rho_g)}{\rho_i - \rho_g}
\]  

(10)
The expression for the thermal conductivity was derived by considering the unit cube to consist of individual thermal resistances in series. These resistances are shown as separate zones in Figure 16-B. Adding the individual resistances in this manner is equivalent to assuming that the thermal conductivity of the heterogeneous structure is infinite in a direction perpendicular to the heat flow. The final expression for the thermal conductivity is:

\[
\frac{1}{k_f} = \frac{(1 - x)}{k_g,e(1 - x^2) + k_1z^2} + \frac{(x - z)}{k_g,e(1 - x^2) + k_1x^2} + \frac{z}{2k_1z(1 - x) + k_1x^2 + k_g,e \left[1 - x^2 - 2z(1 - x)\right]} \tag{11}
\]

In using this equation to calculate frost thermal conductivities, an adjustable constant, namely the ratio of the cube length to the width of the ice rod, was evaluated from the experimental data. This ratio is a function of temperature. A particular value of the adjustable constant was obtained by matching the thermal conductivity calculated from Eq. (11), with a value on the experimental curve. The specific procedure was as follows. At a given temperature the density of the frost at a solids fraction of 0.07 was evaluated from Eq. (10). A value of \(x\) was then assumed,
and z was calculated from Eq. (9). These values of x and z were substituted into Eq. (11) and a value of $k_f$ computed. The computed conductivity was compared with the value on the experimental curve corresponding to a solids fraction of 0.07. The trial and error procedure was repeated until the theoretical and experimental values agreed. Values of $x/z$ generated in this manner are shown in Figure 17.

At a particular temperature, the value of $x/z$ computed as described above is assumed to be valid for all frost densities. Thus, at any given temperature, the frost thermal conductivity can be calculated as a function of density from Eqs. (9), (10) and (11), and the appropriate value of $x/z$. More specifically, the value of $(1-E)$ at a given density was calculated from Eq. (10). x and z were then calculated from Eq. (9) using the known value of the ratio $x/z$. Finally, Eq. (11) was used to evaluate the thermal conductivity. In this manner the thermal conductivity density relationship was obtained at various desired values of temperature. The calculated theoretical values are the shaded points in Figure 7. At all temperatures there is fair agreement between theoretical points and the experimental data. This agreement tends to indicate that over the density range involved the contacts between the ice particles in the frost are reasonably well represented by the dimensions of the interconnecting ice rods.
(f) Use of Cube and Rod Model to Predict Frost Thermal Conductivities

One must exercise care in using the theoretical curves of Figure 7 to predict values of the frost thermal conductivity for other frosting situations. These curves are strictly valid for frosts having the same structural characteristics as frosts formed in the present investigation. It has previously been noted that snows of the same density can have significantly different structures. It is possible that all frosts formed at cryogenic temperatures have substantially the same structure. However, there also exists the possibility that the structures of frosts formed, for example, under natural convection conditions may be different from those of frosts formed in forced convection. In addition, frost structure may also be strongly dependent on the specific geometry of the frosting system. If structures of various cryogenic frosts are indeed different than the structures of frosts of this thesis, then the values of $x/z$ shown in Figure 17 are not applicable.

It is, however, interesting to note that Holten's (40) measured values of frost thermal conductivity, at a temperature of $-215^\circ F$, fall near the curve which represents the $-215^\circ F$ data of the present thesis (see Figure 18). This agreement provides some indication that the structures of low temperature, natural convection frosts are similar to the structures of cryogenic frosts formed under conditions of forced convection.
Conclusions

(1) Frost density and thickness increase continuously with time.

(2) At equivalent times frost density increases with increasing Reynolds Number but is substantially independent of humidity.

(3) Frost thickness, at equivalent times, decreases slightly with increasing Reynolds Number but increases significantly with increasing humidity.

(4) Point frost thermal conductivity increases with increasing temperature and density. Average frost layer conductivity, therefore, also increases with density.

(5) The temperature difference across the frost rises rapidly, at first, but becomes substantially constant after awhile.

(6) The heat flux through the frost decreases initially, but eventually attains a steady value.

(7) A constant heat flux is reached because the increasing frost thickness is balanced by the increasing thermal conductivity resulting from the continuous frost densification. These two compensating effects lead to a frost of constant thermal resistance.
Table 1

Calorimeter Calibration, Comparison of Known Heat Fluxes with Values Obtained from Liquid Nitrogen Boil-Off

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Known Heat Flux</th>
<th>Heat Flux from Liquid Nitrogen Boil-Off Measurement</th>
<th>Error in Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BTU/min-ft(^2)</td>
<td>BTU/min-ft(^2)</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>10.05</td>
<td>10.82</td>
<td>+ 7.66 %</td>
</tr>
<tr>
<td>37</td>
<td>2.80</td>
<td>2.78</td>
<td>- 0.71 %</td>
</tr>
<tr>
<td>38</td>
<td>5.53</td>
<td>5.60</td>
<td>+ 1.27 %</td>
</tr>
<tr>
<td>39</td>
<td>14.21</td>
<td>14.30</td>
<td>+ 0.62 %</td>
</tr>
<tr>
<td>42</td>
<td>10.60</td>
<td>10.40</td>
<td>- 1.85 %</td>
</tr>
</tbody>
</table>

Notes: 1) Area of test section = 0.0277 square feet.
2) Runs 36 and 42 were made at about the same conditions.
Table 2

Effect of Reynolds Number on Length of Time Required to Form Smooth Frost

<table>
<thead>
<tr>
<th>Reynolds Number</th>
<th>Time to Form Smooth Frost (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,600</td>
<td>about 40</td>
</tr>
<tr>
<td>9,300</td>
<td>25 to 30</td>
</tr>
<tr>
<td>14,500</td>
<td>10 to 15</td>
</tr>
</tbody>
</table>
Table 3

Frost Densities and Thicknesses

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Reynolds Number</th>
<th>Humidity</th>
<th>Time After Start of Frosting (minutes)</th>
<th>Density gm/cc</th>
<th>Thickness (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>14,500</td>
<td>37.7</td>
<td>33.7</td>
<td>0.0809</td>
<td>0.156</td>
</tr>
<tr>
<td>34</td>
<td>14,500</td>
<td>37.5</td>
<td>51.5</td>
<td>0.0914</td>
<td>0.201</td>
</tr>
<tr>
<td>44</td>
<td>14,400</td>
<td>37.7</td>
<td>69.8</td>
<td>0.111</td>
<td>0.222</td>
</tr>
<tr>
<td>40</td>
<td>14,500</td>
<td>36.7</td>
<td>108.3</td>
<td>0.128</td>
<td>0.243</td>
</tr>
<tr>
<td>48</td>
<td>9,240</td>
<td>36.1</td>
<td>41.3</td>
<td>0.0608</td>
<td>0.188</td>
</tr>
<tr>
<td>47</td>
<td>9,240</td>
<td>37.9</td>
<td>74.3</td>
<td>0.0643</td>
<td>0.243</td>
</tr>
<tr>
<td>66</td>
<td>9,450</td>
<td>37.6</td>
<td>74.3</td>
<td>0.0637</td>
<td>0.241</td>
</tr>
<tr>
<td>49</td>
<td>9,350</td>
<td>38.2</td>
<td>89.0</td>
<td>0.0790</td>
<td>0.252</td>
</tr>
<tr>
<td>41</td>
<td>9,300</td>
<td>37.7</td>
<td>106.3</td>
<td>0.0918</td>
<td>0.272</td>
</tr>
<tr>
<td>59</td>
<td>5,570</td>
<td>36.2</td>
<td>66.0</td>
<td>0.0545</td>
<td>0.229</td>
</tr>
<tr>
<td>50</td>
<td>5,640</td>
<td>38.0</td>
<td>98.2</td>
<td>0.0638</td>
<td>0.274</td>
</tr>
<tr>
<td>51</td>
<td>5,600</td>
<td>37.6</td>
<td>120.0</td>
<td>0.0742</td>
<td>0.289</td>
</tr>
<tr>
<td>53</td>
<td>14,600</td>
<td>22.9</td>
<td>60.5</td>
<td>0.109</td>
<td>0.161</td>
</tr>
<tr>
<td>54</td>
<td>14,750</td>
<td>28.0</td>
<td>42.5</td>
<td>0.0882</td>
<td>0.143</td>
</tr>
<tr>
<td>69</td>
<td>14,750</td>
<td>29.5</td>
<td>39.3</td>
<td>0.0818</td>
<td>0.155</td>
</tr>
<tr>
<td>55</td>
<td>14,750</td>
<td>30.9</td>
<td>89.3</td>
<td>0.126</td>
<td>0.200</td>
</tr>
<tr>
<td>46</td>
<td>14,650</td>
<td>30.9</td>
<td>89.3</td>
<td>0.122</td>
<td>0.206</td>
</tr>
<tr>
<td>Run Number</td>
<td>Reynolds Number</td>
<td>Humidity grains water/ lb dry gas</td>
<td>Time After Start of Frosting (minutes)</td>
<td>Density gm/cc</td>
<td>Thickness (inches)</td>
</tr>
<tr>
<td>------------</td>
<td>----------------</td>
<td>----------------------------------</td>
<td>--------------------------------------</td>
<td>--------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>61</td>
<td>9,300</td>
<td>22.9</td>
<td>75.0</td>
<td>0.0638</td>
<td>0.175</td>
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<tr>
<td>62</td>
<td>9,240</td>
<td>22.2</td>
<td>91.5</td>
<td>0.0800</td>
<td>0.197</td>
</tr>
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<td>63</td>
<td>9,200</td>
<td>22.7</td>
<td>123.0</td>
<td>0.0961</td>
<td>0.210</td>
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<tr>
<td>65</td>
<td>5,680</td>
<td>23.4</td>
<td>76.3</td>
<td>0.0588</td>
<td>0.188</td>
</tr>
<tr>
<td>60</td>
<td>5,570</td>
<td>23.0</td>
<td>110.0</td>
<td>0.0691</td>
<td>0.220</td>
</tr>
</tbody>
</table>

**NOTE:**

1) Runs 47 and 66 are duplicates.

2) Runs 46 and 55 are duplicates.
<table>
<thead>
<tr>
<th>Re</th>
<th>Humidity</th>
<th>Steady Value of</th>
<th>End-of-Run Value of</th>
<th>(T_s)<em>{exp} - (T_s)</em>{theoret}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>grains water</td>
<td>(T_s)_{exp} (°F)</td>
<td>(T_s)_{theoret} (°F)</td>
<td></td>
</tr>
<tr>
<td>14,500</td>
<td>37.4</td>
<td>+24</td>
<td>+19.5</td>
<td>4.5</td>
</tr>
<tr>
<td>9,300</td>
<td>37.5</td>
<td>+25</td>
<td>+10</td>
<td>15</td>
</tr>
<tr>
<td>5,600</td>
<td>37.3</td>
<td>+20</td>
<td>+7</td>
<td>13</td>
</tr>
<tr>
<td>9,300</td>
<td>22.6</td>
<td>+12</td>
<td>+3</td>
<td>9</td>
</tr>
<tr>
<td>5,600</td>
<td>23.3</td>
<td>+12</td>
<td>-0.5</td>
<td>12.5</td>
</tr>
</tbody>
</table>
Table 5

Comparison of Experimental and Theoretical Thermal Resistances

<table>
<thead>
<tr>
<th>Humidity</th>
<th>Reynolds Number</th>
<th>Experimental $R_f$ (hr - °F)</th>
<th>Theoretical $R_f$ (hr - °F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>grains water</td>
<td>lb dry gas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>14,500</td>
<td>17.1</td>
<td>16.2</td>
</tr>
<tr>
<td>9,300</td>
<td>23.9</td>
<td>18.4</td>
<td></td>
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<tr>
<td>5,600</td>
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</tr>
<tr>
<td>23</td>
<td>9,300</td>
<td>20.9</td>
<td>18.5</td>
</tr>
<tr>
<td>5,600</td>
<td>23.9</td>
<td>20.2</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: These thermal resistances are end-of-run, or essentially steady values.
Figure 1 Flowsheet of Equipment
Figure 2  Copper Plate and Associated Boil-off Calorimeter
Figure 3  Frost Density as a Function of Time

Figure 4  Frost Thickness as a Function of Time
Figure 5: Quasi-Steady State Heat Flux as a Function of Reynolds Number and Humidity

Figure 6: Average Thermal Conductivity of the Frost Layer as a Function of Frost Density
Calculated Cube and Rod Model Points are Shaded
Data Points are Unshaded

Least Squares Line Through Data
Least Squares Line Extrapolated to Zero Density
Theoretical Lines

Data:
\[ T = -215^\circ F \]
\[ T = -145^\circ F \]
\[ T = -80^\circ F \]
\[ T = 0^\circ F \]
\[ T = 20^\circ F \]

Note: Thermal Conductivities at Zero Density, i.e. for Pure Gas, Include Effects of Evaporative Transport

Frost Thermal Conductivity as a Function of Density
with Temperature as a Parameter

Figure 7
Re = 14,400

\[ \frac{\text{grains water}}{\text{lb dry gas}} \] = 37.7

Gas Phase dew Point \( \approx 40^\circ\text{F} \)

Figure 8  End of Run Temperature Profile in Frost
Reynolds Number = 5,600

\[ H = 37 \text{ grains water per lb dry gas} \]

**Figure 9-A** Frost Density as a Function of Time
Reynolds Number = 5,600

\[ H = 37 \text{ grains water per lb dry gas} \]

Figure 9-B  Frost Thickness as a Function of Time
Figure 10: Total Heat Flux Through Frost as a Function of Time
Reynolds Number = 5,600

Data

Dashed Curve: Assumed Initial Thickness of 0.08 inches
Solid Curve: Assumed Initial Thickness of 0.04 inches

Figure 11 Surface Temperature of Frost as a Function of Time
Figure 12  Frost Density as a Function of Time with Reynolds Number and Humidity as Parameters
Figure 13: Frost Thickness as a Function of Time with Reynolds Number and Humidity as Parameters.

Curves were calculated assuming an initial thickness of 0.04 inches.

Shaded Points: High Humidity
Unshaded Points: Low Humidity

- Shaded Points: High Humidity
- Unshaded Points: Low Humidity

- Re = 5,600
- Re = 9,300
- Re = 14,500

H = 37 grains water/lb dry gas
H = 23

Figure 13: Frost Thickness as a Function of Time with Reynolds Number and Humidity as Parameters.
Figure 14: Comparison of Frost Thermal Conductivity Data at 20°F with Natural Snow Data

Curve 1: Represents data of Kuzmin (48) and Artsybashev & Yushakov (49).
Curve 2: Represents data of Jonsson (43).

Figure 15: Thermal Conductivity as a Function of Density; Comparison of Data with Parallel Slab Model at a Temperature of -145°F
Note: Though it is not shown in the diagram, rods extend from all six faces of each cube.

Figure 16A Assumed Frost Structure: Cube and Rod Model

Figure 16B Representative Unit Cube for Cube and Rod Model
Figure 17  x/z as a Function of Temperature for Cube and Rod Model

Figure 18  Frost Thermal Conductivity as a Function of Density at a Temperature of -215 °F, Comparison with Data of Holten.
CHAPTER I

Introduction

When a humid gas stream flows past a cold surface, a layer of frost will form if the surface temperature is below the gas phase dew point (and, of course, below 32°F). In fact, if the temperature of the cold surface is low enough, ice particles will nucleate in the gas phase. Generally, some of these particles will be deposited onto the surface, and a frost layer will be built up by a particle transport mechanism. On the other hand, if the cold-surface temperature is sufficiently high, but still below the gas phase dew point, then the frost layer will form solely as a result of the molecular and turbulent diffusion of water vapor from the gas phase.

1. Practical Background

One of the areas in which knowledge of frost properties is important is low temperature heat exchangers. In numerous cases, heat exchangers must operate under conditions where the wall is colder than the solidification temperature of one of the components of the gas phase. In general, this component is water. Frost forms on the walls of such heat exchangers and, after a short period of time, becomes an appreciable portion of the resistance to heat transfer. Knowledge of the frost thermal conductivity and
thickness would enable the designer to estimate the decrease in heat exchanger efficiency due to the insulating effect of the frost.

Another area in which knowledge of frost properties is important is space research. The frost which forms on a liquid oxygen rocket tank, before the vehicle is launched, is generally undesirable because of the additional weight which the vehicle must carry aloft. In fact, efforts have been made to prevent this frost formation (71). The deposit, however, can also be beneficial since it acts as insulation and decreases liquid oxygen boil-off. Both liquid oxygen boil-off and additional frost weight could be estimated if values of the frost properties were known.

The mechanism of frost formation on a rocket tank is not the same as in a low temperature heat exchanger. This is true whether frosting on the tank is occurring in a natural convection atmosphere or as a result of a wind blowing past the vertical tank. In the latter case the mechanism of frosting actually varies along the circumference of the cylindrical tank. The frost on the part of the cylinder facing the wind is of a distinctly different character than the frost on the downstream face (63). The experimental results of the present research are seemingly more applicable to heat exchangers than to rocket tanks, but a better understanding of frost behavior will enable one to explain many different types of frosting situations.
2. Objectives

It was the primary purpose of this research to determine the thickness, density and thermal conductivity of water frost which was deposited from a humidified stream of nitrogen onto a copper plate. The plate was maintained at about $-315^\circ F$. Frost density and thickness were to be obtained as functions of Reynolds Number, gas phase humidity and time elapsed since the start of frosting. Thermal conductivity was to be correlated with frost density and temperature. (It should be noted that the Reynolds Number used in this investigation is based on flow between infinitely wide parallel plates.) In addition, it was desired to examine the frost structure under a microscope. Based on microscopic observation, an idealized model of the frost microstructure was to be formulated. Values of thermal conductivity calculated from this model were to be compared with experimental conductivities.

3. Choice of Frosting System

Essentially three different systems were considered:

1) a rotating cylinder in a stagnant gas medium
2) a stationary or slowly rotating cylinder mounted in a wind tunnel, and
3) a short flat plate that formed part of the underside of a long rectangular duct and was mounted relatively far downstream from the duct entrance.
For theoretical reasons, a rotating cylinder mounted in a stagnant gas appeared to be a very attractive choice. Unfortunately, it would have been impossible to control the conditions of the bulk gas phase in such a system. Ice particles which nucleated in the gas stream and were not deposited onto the cylinder would have been thrown into the bulk gas. Control of water vapor concentration and gas phase temperature would, therefore, have been impossible. For the latter reason alone, this system had to be rejected. Another objection to the use of a rotating cylinder was the difficulty involved in measuring the surface temperature of the rotating frost layer.

The second choice was also rejected. In this case the frost layer would not have been of a uniform character around the cylinder circumference. Use of a slowly rotating cylinder would probably have resulted in a more uniform frost, but, as mentioned above, measurement of the surface temperature of the rotating frost layer would have been difficult.

The system finally chosen was alternative number three: a short flat copper plate which formed part of the underside of a rectangular duct. This system met most of the requirements listed in Appendix H. The relative ease of construction, as compared with other systems, also made the short flat plate an attractive choice. A more
detailed discussion of the choice of frosting systems is presented in Appendix H.

4. Previous Work

An understanding of various phases of the frosting process can be gained by examining the results of previous investigations. The mechanism of frost deposition was generally found to be dependent on the conditions of the investigation: i.e., the deposition mechanism depended on whether the experiments were performed at high (about 0°F) or low (about -300°F) temperatures or under conditions of natural or forced convection. The specific effects of these different parameters will be described in detail in the review of previous studies which follows. (The terms high and low temperatures refer to the temperature of the frosting surface and not of the gas phase. The gas phase temperatures generally ranged between 0 and 100°F, usually being around room temperature.)

A. Initial Phases of a Run

1. Particle Formation in Gas Phase

There is sufficient evidence to indicate that for the early phases of a run the frosting mechanism is different for high and low temperature investigations. In low temperature, natural convection studies Holten (40), Barron (7), and Ruccia and Mohr (72) observed that ice particles formed in the gas phase boundary layer.
Whitehurst (88) also observed particle formation in the gas phase in his -100°F runs, but did not observe particle formation in his 0°F experiments. Ice particles in the gas phase have not been observed in any of the high temperature experiments (10,14,29,37,84).

Owen (65) and Friedlander and Johnstone (31) have shown that turbulence is often the primary factor responsible for transporting particles from the gas phase to a wall. In the absence of turbulence, a very large percentage of particles nucleated in the gas phase probably never reach the wall. In the natural convection investigations of Refs. 7, 40,72, where turbulence presumably was not significant, the authors observed that the particles which formed in the gas phase seemed to be swept past the cold surface by the natural convection current. Some particles probably reached the wall by mechanisms other than turbulence (see Appendix A), but a large percentage of the particles were apparently not deposited. In these experiments it is likely that mass also deposited onto the frost layer by turbulent and molecular diffusion of water vapor from the gas phase. The diffusion rate, however, was apparently much lower under these circumstances than it would have been had particles been absent.

When turbulence in the gas phase is significant, it seems likely that a much higher percentage of the particles would reach the cold wall than in the absence of turbulence.
There is evidence, however, which indicates that even in
the presence of gas phase turbulence many of the particles
do not deposit, but are swept past the wall by the bulk
motion of the flowing gas. For example, Richards et al.
(69), in a low temperature turbulent flow investigation,
formed frost on a stationary horizontal cylinder. Parti-
cles were observed to form in the gas phase boundary layer.
It appeared that many of these particles were deposited
onto the cylinder, but that others were swept past the
cylindrical surface by the motion of the gas stream.

In another turbulent flow, low temperature study,
Weber formed frost on the inside wall of a long pipe.
Based on calculations, he concluded that during the early
phases of a run particle formation in the bulk gas was
quite extensive and that many of these particles never
deposited onto the wall. In Weber's experiments particles
formed in the gas bulk, because the long length of pipe super-
cooled the gas stream. In the investigations of Refs. 7,
40, 69, 72 and 88, the bulk gas phase remained at approxi-
mately room temperature. Particles therefore formed only
in the thermal boundary layer. Generally, when ice particle
formation was significant anywhere in the gas phase
(boundary layer and bulk or just boundary layer), the total
rates of frost deposition (i.e., particle transfer plus dif-
fusion) were significantly lower than would be predicted from
standard mass transfer correlations such as the Sherwood
Gilliland equation (85), the Chilton-Coburn analogy (79) or the Lewis relation (79).

As previously mentioned, in natural (10, 29, 84) or forced convection (8, 13, 14) high temperature studies, particle formation in the gas phase was not observed to occur. Apparently frost deposition occurred solely by the diffusion of water vapor from the gas phase to the frost layer. Condensation occurred at the cold surface instead of in the gas phase.

Johnstone, Kelley and McKinley (44) have derived criteria for predicting whether fogging will occur in a gas stream which is in the process of cooling and which contains a condensable component. The observation that ice particles form in the low temperature experiments, but do not form at high temperatures, is in agreement with these criteria.

The criteria derived by Johnstone et al. are usually presented in the form of curves of water vapor concentration versus the gas phase dry bulb temperature, with surface temperature as a parameter. For example, the curves shown in Figure I-1 are applicable to the nitrogen-water system. If the point representing the gas stream condition lies above the curve for the appropriate surface temperature, then heterogeneous nucleation in the gas phase is possible. On the other hand, if the point lies below the curve, then nucleation is impossible. These fog formation limits were
developed from the stipulation that at the wall \( \frac{dT_d}{dr} = \frac{dT}{dr} \), and from the usual expression for the rates of heat and mass transfer from a fluid phase to a surface.

(2) Heat Transfer in Early Phases

(a) High Temperatures

In high temperature runs, the gas side heat transfer coefficient generally increased immediately after the start of frosting. This phenomenon was observed in both natural convection (76, 84, 88) and forced convection (8, 13, 37, 38) investigations. It is likely that the increased surface roughness, caused by the initial deposits of frost, resulted in the higher coefficient. In the early phases of a run, the gas side heat transfer coefficient was controlling, and therefore the total heat transfer rate also showed an increase.

Schropp (76), Stoecker (84) and Han et al. (37) formed frost on the outside of a cylinder. In general, if a cylinder is below a certain critical radius, given by Eq. I-1 (55), it is possible to increase the heat transfer by adding insulation. (In these experiments the frost may be viewed as the insulation.) In the experiments of

\[ r_c = \frac{k_f T}{\frac{Av}{h_g}} \]  

(I-1)
Refs. 37, 76 and 84, however, the cylinder diameter was larger than the critical diameter calculated from Eq. I-1. The addition of insulation to the cylinders used by Schropp (76), Stoecker (84) or Han et al. (37) should therefore have resulted in a decrease in heat transfer rate, rather than an increase. The initial increase in the heat transfer rate observed in these three investigations is, therefore, probably attributable to the increased surface roughness caused by the initial deposits of frost.

It is interesting to note that Chung and Algren (14), in forced convection, high temperature experiments, did not observe an initial increase in the heat transfer rate. During the initial phases of their experiments, heat transfer decreased continuously because of the insulating effect of the accumulating frost layer and after about an hour reached a constant value. In three of the high temperature forced convection experiments (8, 13, 38) which involved an initial increase in the heat transfer coefficient, frosting occurred in the inside of a conduit. Chung and Algren, on the other hand, frosted onto a horizontal cylinder mounted in a wind tunnel. One might be led to believe that the geometrical differences were responsible for the discrepancy. This, however, does not seem to be the case as an initial increase in the heat transfer rate was observed by Han et al. (37), who used a system similar to that of Chung's and Algren's. Thus,
at present, there is no explanation for Chung's and Algren's observations. It should be noted that in the high temperature investigations described above, the heat transfer rate through the frost layer generally decreased after the initial increase.

(b) **Low Temperatures**

An initial increase in heat transfer rate was not observed in any of the low temperature runs. Heat flux decreased continuously from the frost free value until a constant rate was attained. None of the low temperature investigators presented reasons for their failure to observe this initial increase.

B. **Heat Transfer in Latter Phases of a Run**

The attainment of a steady heat transfer rate during the course of a frosting experiment is a common occurrence. It has been observed to occur in forced (8,14,45) and natural (76,88) convection experiments performed at high temperatures, and in low temperature, forced (69,87) and natural (40,72) convection studies.

There exists sufficient evidence to indicate that this constant heat transfer rate period is not a true steady state. For example, in a low temperature, forced convection study, Weber (87) noted that mass continued to be deposited onto the frost layer after the constant heat transfer rate period was reached. Although he could not actually
see the frost layer, he concluded from his data that the continuous frost accumulation within the pipe resulted in continuous increases in both the frost thickness and frost density.

In another low temperature forced convection study, Ruccia and Mohr (72) observed that frost thickness increased continuously during the constant heat transfer period. These investigators did not measure frost density.

Dibbern (24) frosted benzene from a nitrogen-benzene gas mixture onto the inside wall of a pipe. Heat fluxes were not measured, but frost density and thickness were observed to increase continuously.

In high temperature, natural convection investigations, both Schropp (76) and Whitehurst (88) observed that during the constant heat transfer period the frost continuously densified and increased in thickness. Schropp's frosting surface was a horizontal cylinder, whereas Whitehurst used a vertical flat plate. Thus, based on the results of Refs. 24, 72, 76, 87 and 88, it can be concluded that the constant heat transfer period is not a true steady state because of the continuously changing properties of the frost.

In general, the temperature difference across the frost was found to be substantially constant during the steady heat transfer period (45,69). Most investigators (for example, Refs. 14, 45, 81 and 87) therefore attributed
the attainment of a constant heat transfer rate to two simultaneously occurring phenomena: namely, a continuously increasing frost thickness and continuous frost densification. More specifically, the tendency toward an increasing frost thermal resistance caused by the increasing thickness was balanced by the increasing frost layer thermal conductivity which resulted from the continuous frost densification. A frost of constant thermal resistance therefore resulted.

There is sufficient data to indicate that the increased thermal conductivity resulted from the continuous densification of the frost. The relationship between conductivity and density will be discussed in detail later in the thesis.

The continuous densification of the frost is due to the diffusion of water vapor from the "warm" frost surface into the interior. The driving force for this internal diffusion is the equilibrium water vapor concentration gradient which results from the temperature gradient within the frost. Though it cannot be stated definitely that for thin (about 1/4 of an inch) cryogenic frost layers the gas within the pores is saturated at the local ice temperature, it should be noted that this has been found to be the case for relatively thick layers of natural snow (5).
(1) Factors Which Influence the Magnitude of
the Heat Transfer Rate

In natural convection experiments, the magnitude of the constant heat transfer rate was generally found to be dependent on the gas phase humidity. In forced convection experiments this rate depended not only on the humidity, but also on the gas velocity (or Reynolds Number).

(2) Natural Convection, Low Plate Temperature Investigations

Holten (40) formed frost on a 5 3/8 inch diameter aluminum sphere maintained full of liquid oxygen. Overall heat transfer rates, frost thickness, and mass of frost deposited were measured as functions of time. The gas phase humidity was constant throughout a run, but varied from run to run. The bulk gas temperature was ambient throughout.

In the early stages of frosting, the heat transfer rate decreased continuously, but as the run progressed a constant rate was attained. The initial decrease in the heat transfer rate was due to the insulating effect of the accumulating frost. The value of the steady heat flux was found to be a very weak function of the gas phase humidity.

Ruccia and Mohr (72) studied frosting under natural and forced convection conditions. The frosting surface
was an 18 inch diameter cylindrical tank maintained near the normal boiling temperature of liquid oxygen (i.e., near \(-297^\circ°F\)). The bulk gas was at ambient temperature. For their natural convection runs, the tank was located in an enclosure. Discussion will be presently restricted to their natural convection results.

Heat flux and frost thicknesses were measured at various intervals of time. Ruccia and Mohr observed, as have most other investigators who have studied frosting, that the heat flux continuously decreased during the early part of a run until a constant heat transfer period was reached. The value of the steady heat flux decreased markedly with increased ambient humidity. These results are contrary to those of Holten, who observed that the steady heat transfer rate was substantially independent of the ambient humidity. The reasons for the discrepancy are, at present, unknown.

(3) Natural Convection-High Temperatures

In high temperature, natural convection experiments (10,29,76), the effects of humidity on heat transfer have generally not been investigated. Whitehurst's work, however, is an exception. Based on limited measurements, Whitehurst concluded that lower gas phase humidities resulted in frosts with lower thermal resistances. Hence, at the lower humidities the heat flux through the frost was greater.
(4) Forced Convection - Low Temperatures

In forced convection, as in natural convection studies, heat flux during the early stages of a run decreased continuously until a steady value was reached. The investigations of Ruccia and Mohr (72), Richards et al. (69) and Weber (87) are the only three known low temperature, forced convection studies.

The frosting system employed by Richards et al. was a 2 inch diameter horizontal cylinder mounted in a low-speed wind tunnel. The surface of the cylinder was maintained at $-320^\circ F$ by the circulation of liquid nitrogen through the cylinder interior. Gas stream velocities ranged from 7.3 to 81 ft/sec. and humidities from 17 to 320 grains water/lb dry gas. The gas temperature was constant throughout each run, but varied from run to run. A temperature range of 40 to 100$^\circ F$ was employed.

It was impossible to compare the results of Richards' high humidity runs (i.e., over 50 grains water/lb dry gas) with the results of the other low temperature, forced convection work (72, 87). During these high humidity runs, Richards et al. observed that the surface temperature of the growing frost layer reached 32$^\circ F$ early in the run. As a result, liquid water condensed on the frost surface and subsequently froze into a thin layer of ice. The thermal conductivity of the entire frost layer was thus raised, and the surface temperature was consequently lowered.
Because of the lowering of the surface temperature, porous frost deposited onto the ice layer until the frost surface temperature once again reached 32°F. The above described cycle then recurred. The over-all frost layer, therefore, consisted of alternate layers of porous frost and solid ice. Alternate layer frosts did not form during Richards' low humidity runs, because the surface temperature never reached 32°F. For the high humidity runs, heat transfer through the frost decreased continuously during the early phases of the run. During the latter phases of a run, when alternate layers of ice and porous frost were forming, the heat transfer rate never became truly constant, but oscillated about a mean value.

For the lower humidity runs, the heat flux versus time curves were smooth rather than oscillatory. In the early stages of a run made at a low humidity, heat flux decreased continuously until a steady value was reached. Frost continued to accumulate during the constant heat transfer period. In addition, the temperature difference across the frost did not vary significantly during this period. The constant heat transfer rate, therefore, implied that the frost thermal resistance became constant.

Richards et al. observed that below humidities of 50 grains water/lb dry gas the constant heat flux increased with decreasing humidity. However, at gas phase humidities greater than 50 grains water/lb dry gas, i.e., in
experiments where alternately layered frosts formed, the steady heat transfer rate increased with increasing humidity. Steady heat flux as a function of gas phase humidity, with gas velocity as a parameter, is shown in Figure I-2. At both high and low gas phase humidities, Richards et al. observed that quasi-steady state heat flux increased with increasing gas velocity.

Weber (87) formed frost on the inside of a 2 1/2 foot long 3 3/4 inch diameter copper pipe. Inlet gas temperatures ranged from 55 to 80°F and inlet humidities from 3 to 44 grains water/lb dry gas. The pipe wall was maintained near the boiling temperature of liquid nitrogen. The long length of pipe caused the gas stream humidity and temperature to change significantly as the gas flowed through the pipe. Weber measured heat transfer rates as a function of time and position along the pipe. He found that, at all positions, the heat transfer rate dropped during the initial stages of the run, but reached a constant value during the latter stages. Weber also determined the variation with time of an apparent over-all heat transfer coefficient $\bar{U}_a$. In effect, the value of $\bar{U}_a$ is a measure of the average heat flux through the pipe (i.e., average with respect to position, not with time). Weber observed that values of the over-all heat transfer coefficient increased with both increasing Reynolds Number and decreasing gas phase humidity. These results are in qualitative agreement with the low humidity data of Richards et al.
As previously mentioned, Ruccia and Mohr (72) used the same frosting system for their forced and natural convection experiments. For the forced convection runs, the cylinder was located in the slip stream of a propellor. Gas velocities ranged from 17.6 to 41.2 feet/sec. and humidities from 59 to 103 grains water/lb dry gas. In their forced convection studies, they found, as expected, that the heat flux through the frost reached a steady value during the course of a run. The magnitude of these steady values were greater at the higher gas stream velocities. Surprisingly, however, the value of the steady heat flux was found to be independent of humidity. In view of the fact that other investigators (69,87) have found that quasi-steady state heat flux is dependent on the gas phase humidity, it seems likely that Ruccia's and Mohr's results, pertaining to the effect of humidity, are inaccurate. In fact, Ruccia and Mohr state that they believe heat flux should be dependent on ambient humidity, but that they were unable to observe any dependence between these two variables.

(5) Forced Convection - High Temperatures

In high temperature, as in low temperature runs, the steady heat transfer rate was generally found to increase with increasing Reynolds Number. This was observed by Han, Andrichak and Barron (37) and Chung and Algren (14) in separate studies involving the deposition
of frost from a gas stream onto a horizontal cylinder. It was also observed in experiments in which frost was deposited onto a flat surface contained in the inside of a conduit (8,13). Kamei (45), who frosted on the inner tube of an annulus, and Sugawara (14), who formed frost on a flat plate mounted in a wind tunnel, also observed higher heat transfer rates at higher Reynolds Numbers.

The effect of humidity on the steady heat transfer rate was investigated by Chung and Algren (14). They found, as was observed in the forced convection, low temperature experiments, that higher gas phase humidities resulted in lower heat fluxes.

(6) Reasons for Observed Variation of Steady Heat Flux with Reynolds Number

Higher gas velocities (or Reynolds Numbers) have been found to result in higher density frosts. In addition, frosts with slightly decreased thicknesses were observed to form at higher Reynolds Numbers (37,45). Since higher densities imply greater conductivities, frosts formed at higher Reynolds Numbers have lower thermal resistances. The smaller thermal resistances resulted in higher heat transfer rates as the temperature difference across the frost during the constant heat transfer period was generally observed to be substantially independent of Reynolds Number (69).
Explanation for Variation of Constant Heat Transfer Rate with Humidity

A possible explanation for the observed effect of humidity on heat transfer rate is suggested by the natural convection results of Ruccia and Mohr (72) and the forced convection results of Richards et al. (69). No definite explanation, however, can be stated because density measurements are not presented in references (72) and (69). However, if it is postulated that frost density is approximately independent of humidity, then higher humidities would lead to lower heat fluxes since the frosts formed at the higher humidities would have greater thermal resistances.

The postulation that frost densities are approximately independent of humidity is corroborated by the results of the present thesis. The explanation presented above does not apply, of course, to those situations where the frosts are alternately layered.

C. Mass Transfer

Generally, for low temperature investigations the actual frost accumulation rates are significantly lower than mass transfer rates based on coefficients computed from the standard correlations. For high temperature investigations, on the other hand, theoretical (i.e.,
calculated from the correlations) and experimental rates have generally been found to show good agreement (14).

(1) Natural Convection - High Temperatures

Whitehurst performed experiments at plate temperatures ranging from \(-100^\circ F\) to \(+20^\circ F\). At the lower temperatures, large chunks of frost often flaked off from the plate. At the higher plate temperatures, however, the frost adhered sufficiently well so that flaking was no problem. For those experiments in which flaking from the plate did not occur, Whitehurst observed that experimental mass accumulation rates were in approximate agreement with theoretical rates. The theoretical rates were based on mass transfer coefficients computed from a standard correlation used in natural convection situations.

(2) Natural Convection - Low Temperatures

Holten (40) observed that the mass of frost actually accumulating onto the cold aluminum sphere was about 25% of that calculated from the Lewis relation. Ruccia and Mohr noted that the total mass accumulated on a cylindrical tank, filled with liquid oxygen, was between 20 and 40% of the theoretical mass transferred. These authors used the Chilton-Coburn analogy to evaluate the theoretical mass transferred.
In Barron's experiments, the actual mass transfer rate was an order of magnitude lower than the theoretical value. Barron's frosting system was a vertical flat plate maintained at about -310°F and located in a large chamber with a controlled environment.

It is interesting to note that Weber (87) has shown that the standard correlations for forced convection heat transfer are applicable at cryogenic temperatures. Thus, even at low temperatures it is reasonable to expect that if mass transfer were occurring solely by the diffusion of water vapor in the absence of ice particles, then experimental mass transfer coefficients would agree with values calculated from the standard correlations. Actually, if theoretical and experimental mass transfer rates are to be equal, it is not only necessary that no particles form in the gas phase but also that the erosion of particles from the frost surface be negligible.

It has previously been indicated that in the early phases of natural convection, low temperature runs (7, 40, 72) particle formation occurs in the gas phase and that some of these particles may be transferred to the wall (see Appendix A). In addition to particle transport, water vapor may diffuse from the gas phase to the frost layer. As mentioned earlier, however, it is reasonable to expect that this diffusive vapor flux is lower in the presence of particles than in their absence.
The decreased vapor flux is probably not due to any reduction in the diffusion coefficient as the effect of the fog on the diffusivity of water vapor in nitrogen would be small. The reduced diffusive vapor flux can probably be attributed to the depletion of water vapor within the boundary layer due to particle formation. This depletion resulted in a reduced water vapor concentration gradient and hence in a lower diffusive flux. It should be noted again that in the low temperature, natural convection studies mentioned above (7, 40, 72), particles did not form in the bulk gas phase, because this phase never became subcooled. Particle formation occurred solely within the boundary layer.

The above remarks apply not only to the initial phases of a run but also to the latter stages since the frost surface temperature probably never rose above -40°F in any of the low temperature, natural convection runs mentioned above. According to the fog formation criteria developed by Johnstone, Kelley, and McKinley (44), this low surface temperature practically ensured that gas phase particle nucleation occurred for the entire run. Thus experimental accumulation rates are expected to be lower than theoretical rates at all times throughout the run.

(3) Forced Convection - High Temperatures

Chung and Algren found close agreement between their measured frost accumulation rates and theoretical
rates which were computed using a mass transfer coefficient obtained from the Sherwood-Gilliland equation (85). The close agreement is not surprising since in high plate temperature experiments ice particles do not form in the gas phase, and frost accumulates solely by the molecular and turbulent diffusion of water vapor. In addition, the shear strength of Chung's and Algren's frost must have been sufficiently high to prevent any significant erosion of particles from the frost surface.

(4) Forced Convection - Low Temperatures

The results of Richards et al. (69) show that for forced convection, low temperature runs, agreement between experimental and theoretical frost deposition rates is much better than for low temperature experiments carried out under conditions of natural convection. Richards et al. calculated their theoretical rates utilizing mass transfer coefficients based on the Lewis relationship. Their theoretical and experimental results are compared in Figure I-3. (The comparison between Richards' experimental and theoretical rates is discussed more fully in Chapter V.)

There are probably two reasons for the better agreement between experiment and theory for the forced convection runs than for the natural convection runs. First, in the initial stages of low temperature runs, ice particles form
in the gas phase boundary layer. For forced convection runs, however, a higher percentage of particles is transferred to the wall than in the natural convection runs because of the greater turbulence intensities.

The second reason is as follows. In natural convection runs, as stated earlier, it appears that the surface temperature was always low enough so that frost only accumulated by particle transport. In the forced convection runs, the frost surface attains a relatively high temperature early in the run. Thus, for the better part of a run, frost accumulates by turbulent and molecular diffusion rather than by particle transport.

D. Frost Density

It has generally been observed that the variables which significantly affect the frost density are the temperature of the frosting surface, the gas stream Reynolds Number (or gas velocity), the gas phase humidity and the time elapsed since the start of frosting.

(1) Effect of Reynolds Number (or Gas Velocity) on Frost Density

(a) High Temperatures

In high temperature studies, Coles (15) observed that increased gas velocities resulted in frosts of higher densities (see Figure I-4). Kamei (45), who also
studied frost formation at high temperatures, observed the same effect.

Ice particles did not form in the gas phase during these high temperature runs; hence, frost accumulation occurred solely by turbulent and molecular diffusion. Neither Coles nor Kamei attempted to explain the observed frost behavior. The theoretical results of the present thesis, however, have provided some insight into frost density behavior for situations in which frosts form by diffusion.

Coles formed his frost on a flat horizontal plate mounted in a wind tunnel. The plate temperature was maintained at about -20°C. Kamei's system differed from the one used by Coles. Kamei formed frost on the inner surface of an annulus. This inner surface was cooled by circulating brine.

(b) **Low Temperatures**

As previously mentioned Weber (87) did not measure frost density directly. However, from his heat transfer, mass transfer, and pressure drop measurements, he concluded that at higher Reynolds Numbers the frost deposits had greater densities. Richards (59) et al. also concluded that higher gas velocities resulted in denser frosts. (Interestingly, this was found to be true for his low humidity runs as well as for those experiments which involved the formation of liquid water on the frost surface.)
In investigations which involve particle formation in the gas stream, for example Refs. 69 and 87, frost density variation with Reynolds Number is usually explained as follows. Some of the ice particles which nucleate in the gas phase are thrown onto the plate by the fluctuating velocity component normal to the main gas flow. Since the fluctuating velocity component increases with increased Reynolds Number, the impact velocity of the particle with the frost layer is greater at the higher Reynolds Numbers. Greater impact velocities cause the particles to pack more tightly and hence a frost of higher density results.

The statement that higher particle impact velocities lead to more tightly packed arrays has not been proven as far as the author of the present thesis is aware. One, however, would expect this to be the case. Interestingly, however, Langmuir (51) and Macklin (53) showed that, when supercooled water droplets were projected onto a solid surface and froze, the resulting rime densities increased with increased droplet impact velocity. Higher density rimes were formed, because higher impact velocities resulted in greater spreading of the liquid droplets before freezing. The increased spreading caused more interstitial space to be filled with liquid water.

It should be noted that an explanation for frost density behavior based on particle impact velocities is not applicable to frosts formed at high temperatures, since
particles do not form in the gas phase at these higher "cold surface" temperatures.

As previously indicated, in low temperature frosting experiments (69,87), it appeared that there was a shift in the deposition mechanism as the temperature of the frost surface approached the gas phase dew point. An explanation for frost density behavior based on particle transport would, therefore, only be applicable during the early phases of the frosting process. During the latter stages, frost presumably would be depositing by diffusion, and the above explanation for frost density behavior would no longer apply.

(2) Effect of Humidity on Frost Density

Measurements of the effect of humidity on frost density have not been made over wide ranges of humidity. However, it can be concluded from Richards' data (69) that the alternately layered frosts, which formed at high gas phase humidities (i.e., over 50 grains water/lb dry gas), were of higher density than the frosts which formed at the lower humidities where liquid water did not form on the frost surface. The higher densities are probably due to the inclusion of slabs of solid ice within the frost.

When the gas phase humidity is sufficiently low to ensure that the frosts are not alternately layered, then the effect of humidity on density cannot be predicted from the results of work done prior to the present thesis. In
the present thesis, however, a model is developed which enables one to predict the effect of gas phase humidity on frost density.

(3) **Effect of Time Elapsed Since Start of Frosting on Frost Density**

As mentioned earlier the temperature gradient within the frost causes water vapor to diffuse from the frost surface and deposit in the frost interior. This internal deposition leads to a continuous densification of the frost layer. Variation of frost density with time has been measured by Han, Andrichak and Barron (37). Their results are shown in Figure 1-5. In their experiments, frost was deposited onto a cylinder from a humid gas stream flowing past the cylinder. In all runs the surface of the cylinder was maintained at 20°C, the gas stream temperature at 85°C and the gas stream humidity at 98.6 grains water/lb dry gas. The data shown in Figure 1-3 are for a Reynolds Number of 5,690. It is reasonable to assume that the high gas phase humidities resulted in alternately layered frosts. Han et al. did not actually examine the frost structure, but the postulation of alternately layered frosts seems reasonable as liquid water was observed to form on the frost surface. Thus in the experiments of Ref. 37, the observed frost densification with time can therefore be attributed to two factors: (1) the seepage of liquid
water into the frost and the subsequent freezing of this liquid, and (2) the internal diffusion of water vapor which resulted in further deposition of mass in the frost interstices.

Frost densification with time was also observed in other high temperature work (29,45,70). In Kamei's work (45), the frost surface temperature never reached 32°F, and it is reasonable to conclude that densification occurred solely by internal diffusion.

Rische (70) measured the density of benzene frosts which formed on the inside of a tube. At the higher Reynolds Numbers and after long periods of time, the density of the benzene frost approached that of solid benzene.

Feniger's (29) frosting surface was a horizontal cylinder. The frost surface temperature never reached 32°F, and frost densification was due solely to the internal diffusion of water vapor.

The phenomenon of frost densification is apparently not restricted to high temperature experiments. As previously mentioned, Weber did not measure frost density directly, but his heat and mass transfer data indicated that frost density increased continuously with time.

(4) Effect of Plate Temperature
(a) Natural Convection
Lower surface temperatures generally resulted in lower frost densities, regardless of whether the
experiments were carried out under conditions of natural or forced convection. In low temperature, natural convection experiments, Holten obtained frost densities in the range 0.025 to 0.04 gm/cc, whereas Barron's frost densities varied from 0.008 to 0.04 gm/cc. At equivalent times, these values of frost density were considerably lower than values obtained by Stoecker (84) and Feniger (29) in high temperature natural convection experiments. These latter two investigators obtained frost densities in the range 0.2 to 0.4 gm/cc.

In general, both initial and "long time" frost densities are greater at the higher plate temperatures. It is reasonable to assume that initial values of density are higher, because frost formation, at these higher temperatures, occurs by the diffusion of vapor, rather than by particle transport. At long times higher temperature frosts are denser because of the higher rates of internal diffusion of water vapor. These increased diffusion rates result from the higher temperatures within the frost.

Feniger (29) actually measured the effect of plate temperature on frost density. He found, as expected, that higher plate temperatures resulted in denser frosts.

(b) **Forced Convection**

Typical density values of frosts formed under conditions of high plate temperature and forced convection are shown in Figure I-4. These are the data obtained by Coles (15).
Besides the data of Richards et al., frost densities have not been measured under conditions of forced convection and low plate temperature. Richards' data are, however, not of general applicability because of the formation of alternate layers of ice and porous frost.

Thus, for forced convection experiments, no comparison based on previous work can be made between frosts formed at high and low plate temperatures. It would seen reasonable to expect, however, that frost densities would be lower at the lower temperatures. (This expectation is substantiated by the results of the present thesis.)

E. Thermal Conductivity of Heterogeneous Materials

Frost is a two-phase heterogeneous medium. The solid phase consists of an array of ice particles and the pore phase of gas (usually air or nitrogen). The thermal conductivities of both the ice and gas are temperature dependent, and, therefore, the frost conductivity is a function of temperature. In highly porous frosts, such as are involved in the present thesis, the conductivity of the gas is very important in determining the thermal conductivity of the heterogeneous frost.

There are other mechanisms besides conduction through the solid-gas matrix which can cause significant heat flow through the frost layer. These mechanisms, therefore, give rise to additional components of the over-all frost thermal
conductivity. (See Appendix C for a detailed discussion.) The most important of these mechanisms is evaporative transport, which refers to the transport of enthalpy resulting from the diffusion of water vapor within the frost. Temperature alone does not determine frost conductivity. The frost structure is also important as the conductivities of the ice and the gas are significantly different.

Throughout the literature, frost properties are generally presented as a function of density at a particular temperature or over a small temperature range. (15,25,45,56) If all frosts consisted of a particular type of structure (for example, a simple cubic array of ice spheres), then temperature and frost density would be sufficient to specify frost thermal conductivity. However, since frosts of the same density may have different structures, frost thermal conductivity, or for that matter other properties such as frost shear strength, or tensile strength, do not depend on temperature and density alone. This point will be illustrated by considering the thermal conductivity data for natural snows.

(1) Snow Thermal Conductivity

Natural snows often possess structures which are similar to those of cryogenic frosts. Actually, published values of snow thermal conductivities (see, for example, Ref. 56) are more numerous than published conductivities of
high temperature or cryogenic frosts. Snow conductivities are often plotted only as functions of density. The snow conductivity data obtained from Refs. 23 and 25 are shown in Figure I-6. Figure V-18 shows snow data obtained from Refs. 43 and 48. It is obvious that significant disagreement exists between the various experimental values. A reasonable explanation for the divergence is that snows of equivalent density may have significantly different structures.

Many of the investigators did not give the temperatures of their snow samples, but the widest variation was probably no more than 20°F. These small temperature differences are insufficient to completely account for the large differences in the conductivity values. Because of evaporative transport, the differences in temperature, however, may be partly responsible for the large scatter. The contribution made to the over-all thermal conductivity by evaporative transport is both significant and a strong function of temperature in this temperature range. Despite the large scatter of the data, it can, however, be concluded that conductivity increases with snow density.

(2) Measurements of Frost Thermal Conductivity

(a) Coles

As previously indicated, Coles (15) measured frost thermal conductivity as a function of density over a density
range of 0.38 to 0.47 gm/cc. His data are shown in Figure I-6 along with the snow data of van Dusen (25) and Devaux (23). It is seen that thermal conductivity increases with frost density.

Coles ran his experiment until the observed frost thickness became constant. At this point, Coles assumed that the frost surface temperature was equal to the gas phase dew point. Thermal conductivities were computed from values of the temperature difference across the frost, the heat flux through the frost, and the frost thickness.

Coles' assumption that the frost surface temperature became equal to the gas phase dew point is questionable. If indeed these two temperatures were equal, then no water vapor from the gas phase would be reaching the frost surface, and the frost layer would shrink because of the internal diffusion of water vapor. It is, therefore, likely that the frost thickness became constant because the rate of internal diffusion of water vapor into the frost layer became equal to the rate of deposition onto this layer from the gas phase. Deposition from the gas onto the surface only occurs if a concentration driving force between the gas phase and the frost surface is present. A concentration driving force, however, implies a temperature difference, and one is, therefore, led to conclude that the frost surface temperature was lower than the gas phase dew point. It seems likely, therefore, that in Coles' experiments the
internal diffusion of water vapor actually led to a continuous densification of the frost. In addition, the thickness probably increased slowly though the heat transfer rate and surface temperature may have remained constant. Coles apparently did not observe the continuous frost densification or slowly increasing thickness because these changes may have occurred too slowly.

(b) Holten (40)

Though the primary emphasis of Holten's research was on obtaining heat and mass transfer rates, he also determined frost thermal conductivities. Values of conductivity were obtained from the calculated temperature differences across the frost, and measured values of the heat flux and frost thicknesses. Conductivity values are shown in Figure I-7. Holten observed that average frost layer thermal conductivities are strongly dependent on the average frost temperature.

(c) Barron (7)

As previously mentioned, Barron's frosting system was a vertical flat plate. For some of his runs, he used a 10 inch high plate, whereas for others the height of the plate was only 4 inches. The bulk temperature of the gas was ambient.
Barron calculated values of thermal conductivity from his measured heat fluxes, frost thicknesses and temperature differences across the frost. A plot of his thermal conductivity values as a function of average frost temperature is shown in Figure I-7. (Holten's thermal conductivity values are also shown on this plot.) In Barron's work mean frost temperature varied from -310 to -250°F, whereas in Holten's experiments the average temperature of the frost ranged from -230 to -180°F.

Barron found that the relationship between frost conductivity and mean frost temperature was dependent on the gas phase humidity. However, he observed that he could combine all of his conductivity data into one plot if he correlated thermal conductivity with density. This plot is shown in Figure I-8. The anomalous feature of Barron's results is the decrease of thermal conductivity with increase in density over the frost density range 0.008 to 0.04 gm/cc. As an explanation for this phenomenon, Barron postulated that the water vapor flux through the 0.008 gm/cm frost was greater than through the 0.04 gm/cc frost and this vapor flux was the primary component of the frost thermal conductivity. His explanation is questionable as the slight decrease in frost porosity could not have been responsible for the large decrease in the frost conductivity. An increase in frost density from 0.008 to 0.04 gm/cc involves a decrease in porosity from 0.99 to 0.96. Taking any
possible change in tortuosity of this porous frost into account (73), the decrease in frost porosity from 0.99 to 0.96 still only involves a decrease of about 3% in the diffusion coefficient. If the vapor flux is the primary component of the heat flux, as Barron claims, then the thermal conductivity should decrease by only 3%; but it decreases by about 72%.

There are two possible explanations for the trends observed in Barron's conductivity data. First, the measured surface temperatures may have been too low. Secondly, it is possible that Barron's very low density frosts (i.e., the frosts having densities of approximately 0.025 gm/cc) were not integral frosts but a scattering of isolated clumps or islands. The local heat flux through any particular clump was much lower than the total heat flux through the entire series of clumps. Barron probably used the surface temperature of an individual frost clump and the total heat flux through the entire series of clumps to calculate thermal conductivities. His calculated values would therefore be much greater than the true frost thermal conductivity.

As a larger fraction of the plate became covered with the frost clumps, the total heat transfer decreased significantly. This presumably resulted in a large decrease in the apparent value of the thermal conductivity, just as Barron observed.
Whitehurst was primarily concerned with measuring rates of heat and mass transfer to a cold vertical plate under conditions of natural convection. The plate was contained in a large chamber, and gas phase temperature varied from 32 to 108°F. Throughout a run, the air in the chamber was maintained at constant temperature and humidity. The plate, which was 18 inches wide, 24 inches high and 2 inches thick, was double sided, and liquid coolant flowed in the space between the two sides. As previously mentioned plate temperatures ranged from -100°F to +20°F.

Whitehurst measured heat transfer rates, frost surface temperatures, frost thicknesses and total frost accumulation on the plate. He used these measurements to calculate values of thermal conductivity and density. Though the conductivity values show considerable scatter, it is still possible to discern the trends in his data. In general, frost conductivity increases with increasing density and increasing frost temperature.

F. Frost Microstructure

The frost microstructure was not studied in any of the frosting experiments described above. The microstructure of natural snows, however, has been studied quite extensively. The results of these microscopic observations are described in Appendix B.
Figure I-1  Fog Formation Limits at One Atmosphere
Figure I-2  Effect of Humidity on Steady State Heat Flux as Determined by Richards, Edmonds and Jacobs (69)
Note: Experimental Mass Flux is Equal to Frost which Actually Accumulates on the Cylinder

Figure 1-3  Mass Flow as a Function of Free Stream Velocity. Data from Ref. (69)
Figure I-4  Density of Frost as a Function of Gas Stream Velocity, Data of Coles (15)
Figure I-5  Variation of Frost Density with Time, Data of Han et al. (37)
Figure I-6 Thermal Conductivity of Frost as a Function of Density

- Data of Coles (11)
- Devaux (23)
- Van Dusen (25)
Figure I-7 Frost Thermal Conductivity as a Function of Mean Frost Temperature. Data of Barron (7) and Holten (40)
Figure I-8  Frost Thermal Conductivity as a Function of Frost Density. Data of Barron (7)
CHAPTER II

Apparatus

1. Flow Sheet

A diagram of the flow system is shown in Figure II-1. The most important feature of the equipment was the cold plate onto which the frost deposited. This short plate, which formed part of the underside of a long rectangular duct, was mounted a distance of 72 equivalent duct diameters from the duct entrance. A blower recirculated nitrogen gas through the system. The nitrogen leaving the blower was divided into two streams. One stream was dried in a silica gel drier; the other was saturated with water vapor in a packed tower. The two streams when remixed produced a gas with the desired humidity. The temperature of the gas was controlled by varying the power to an electrically heated section of pipe located downstream of the tower-drier combination. The gas flow was metered after it left the heater.

Before entering the duct, the nitrogen flowed through a large box containing two copper screens and then through a converging nozzle. The gas, which was now at the desired temperature (i.e., at about room temperature) and at the desired humidity, flowed through nine feet of duct before it passed over the cold plate. The nitrogen leaving the duct flowed into a "mixing chamber" where its temperature
was measured. From here the gas flowed back to the blower inlet. The Reynolds Number was varied by appropriately setting the flow control valves.

Before each run the system was purged with dry nitrogen from a high pressure cylinder. During a run, dry nitrogen was continuously charged to the low pressure side of the blower in order to compensate for gas leakage out of the recirculating system.

The piping for the system consisted of copper tubing. The tubing in the main recirculation path of the gas was 3 inches in diameter. Smaller diameter tubing, however, was employed for some of the parallel and bypass gas flow lines. The instrumentation used in this system is described in Appendix G.

2. Copper Plate and Associated Boil-off Calorimeter

The copper plate onto which the frost deposited was 8.9 inches long, 14.0 inches wide and 1/4 of an inch thick. The plate actually consisted of two sections: an inner section which served as the heat transfer test section and an outer annular "ring" (see Figure II-3). The inner section was 2.235 inches in diameter.

A 0.02 inch thick epoxy ring separated the two sections of the plate. The primary purpose of the epoxy ring was to reduce heat conduction from the outer annular section into the inner section. This ring also bonded the two
sections of the plate together and supported the weight of the stainless steel liquid nitrogen calorimeter vessel, the inner section of copper plate, and the 17 copper equilibrator rods silver soldered to the bottom of this inner section. A detailed sketch of the plate arrangement is shown in Figure II-2. Ninety-one copper equilibrator rods were silver soldered to the bottom of the outer annular section. (Before soldering the rods to the plate, each rod was inserted in a hole, 1/4 of an inch in diameter by 1/8 inch deep.) The rods provided a low thermal resistance path between the copper plate and the liquid nitrogen in the calorimeter, thus maintaining the plate at a temperature within several degrees of the normal boiling point of liquid nitrogen.

An outer brass cylinder filled with liquid nitrogen served as a heat transfer guard chamber for the inner stainless steel test vessel. The outer annular section of the copper plate overhung this brass chamber, but all 91 rods were contained within the brass chamber (see Figure II-2). The brass chamber fit into a groove, 1/8 inch deep, machined in the outer copper plate. Epoxy resin was used to seal this connection. The entire calorimeter assembly could be cranked up and down.

Two strips of teflon were bonded flush with the surface of the copper plate, one at the leading edge and the other at the trailing edge (see Figure II-3). Each strip was 1 inch wide, 14 inches long and 1/2 inch thick. The
purpose of the teflon pieces was two-fold. First, they were used to insulate the copper plate from the masonite bottom of the duct; and secondly they provided a smooth transition surface between the duct bottom and the copper plate.

In addition to the teflon pieces, two strips of cork were epoxied onto the sides of the copper plate. Each strip was 11 inches long, 1 inch wide and 1 inch thick. Lucite strips, each 11 inches long, 3/4 inch high and 1/2 inch thick, were in turn epoxied onto these cork pieces. When the copper plate was cranked up flush with the bottom of the duct, the Lucite strips formed part of the sides of this duct (see Figure II-2). The frost growth was viewed through these Lucite windows.

The brass chamber was insulated with four layers of fiberglas aircraft insulation. Each layer was separated by aluminum foil to reduce radiation to the chamber. The bottom of the overhanging portion of copper plate was also insulated with fiberglas.

Four lines were silver soldered to the brass cylinder (see Figure II-2). Lines I and II provided continuous ventilation of the liquid nitrogen boil-off. Line III served as the liquid inlet, and line IV was the liquid nitrogen overflow line. After each run the liquid nitrogen was drained from the calorimeter through line III.
3. **Rectangular Duct**

The rectangular duct was 12 feet long, 15 inches wide and \(\frac{3}{4}\) of an inch high. Nine feet of warm duct (i.e., at room temperature) preceded the cold 8.9 inch long copper plate. This long upstream length of warm duct ensured that the turbulent velocity profile in the gas passing over the cold plate was fully developed.

A large aspect ratio (width/height = 20) was used in order to simulate the case of flow between infinitely wide parallel plates. In fact, it was estimated that at a distance from the duct centerline equal to seven eighths of the duct width, the gas velocity was 98% of the centerline velocity. This estimate was made using the method presented in Ref. 83. It should also be noted that the Reynolds Number was constant during a run. The reduction of the clearance between the plate and the top of the duct by the growing frost layer does not affect the Reynolds Number for flow between infinitely wide parallel plates.

The top of the duct above the plate was made of one-half inch thick Lucite. This Lucite section was easily removed, allowing the frost to be sampled at the end of each run. The movable thermocouple probe, which was used to measure the frost surface temperature, was mounted in this Lucite section (see Figure II-2). The entire inside of the duct was lined with aluminum foil in order to minimize radiant heat transfer to the frost surface.
The top and bottom of the duct were made of 1/4 inch thick masonite, whereas the sides were made of 1/2 inch thick strips of Lucite. The copper plate was cranked up flush with the bottom of the duct before each run. During the cool-down period, i.e., when the calorimeter assembly was being filled with liquid nitrogen, the copper plate and the bottom of the duct were maintained flush with each other by continuously cranking up the plate. The continuous cranking compensated for the contraction of the calorimeter assembly. The duct floor and copper plate remained flush throughout the run.

4. Humidity Control System

A drier and packed tower arranged in parallel were used to control the gas stream humidity. (see Figure II-1). The desired humidity was obtained by controlling the quantities of gas flowing through the packed tower and drier, and subsequently remixing these separate streams. The gas flowing through the packed tower was saturated at the temperature of the water circulating through the tower. The gas flowing through the drier was dried to a dew point of about 20°F. Obviously, with this arrangement, humidities only as low as 15 grains water/lb.dry gas (i.e., corresponding to a dew point of 20°F) could be obtained. The highest humidity obtainable was determined by the temperature of the water circulating through the packed column. This water was supplied to the column from the building high
pressure main. Its temperature varied from $40^\circ F$ in the winter to $75^\circ F$ during the summer. Since the highest gas phase humidity employed in this thesis was about 37 grains water/lb dry gas (dew point $= 40^\circ F$), the gradual variation of water temperature over the seasons presented no problems.

A. Packed Tower

The packed tower, which was made of galvanized sheet iron, was 4 feet high by 2 feet in diameter and was packed with 3 feet of 1 inch Raschig rings. The rings were supported by a 2 foot diameter perforated plate attached to the wall of the cylindrical tank. Water was sprayed onto the packing from a coiled 1/2 inch copper tube whose end was closed off and which contained holes on its underside. The coil had four turns, and the water was supplied to it from a high pressure water main.

The outlet for the liquid leaving the packed column was located at the bottom of the tower. The exiting liquid flowed to a drain in the building. During operation of the tower, a liquid seal of about 3 inches was maintained over the liquid outlet in order to prevent gas from escaping by this route. The water level was viewed through a sight glass located near the bottom of the tower. The water flow rate through the tower and the level of the liquid seal were maintained constant throughout a run.

Gas entered the packed column through a distributor located below the bottom of the perforated plate. The gas
distributor was a 3 inch diameter copper tube with about thirty 1/4 inch holes located on its upward side. The liquid level was always below the gas distributor.

Gas exited at the top of the tower. A baffle located 2 inches in front of the outlet prevented gas from jetting into the outlet tube. Glass wool and a filter cloth trapped entrained water droplets and large dust particles which would have been carried along with the exiting gas stream.

The ratio of the liquid flow rate to the rate of gas flow was always sufficiently large to ensure that the gas stream became saturated at all gas flow rates. The inside of the tower was painted in order to prevent corrosion.

B. Drier

The drier was 4 feet high by 1 3/4 feet in diameter, and was made of galvanized sheet iron. It was packed to a height of 2 feet with 6-12 mesh silica gel particles. The gel weight was supported by a perforated plate fastened to the walls of the drier. A filter cloth, which lay on top of the plate, cleaned the gas leaving the drier. Epoxy resin bridged the gap between the tank walls and the edge of the perforated plate. The cloth filter and this epoxy bridge prevented silica gel particles from being blown out of the tank. A second filter cloth, which cleaned the gas before it contacted the gel, was located atop a perforated plate, mounted near the top of the drier.
The top of the bed of gel was viewed through an Eisenglas window. Some "telltale" silica gel was located atop the bed. The approximate moisture content of the bed could, therefore, be ascertained by noting the color of the "telltale" gel.

Moist gas entered the top of the drier through a gas distributor, which was a short section of 3 inch copper tubing closed off at each end. The distributor contained about thirty 1/4 inch holes. With this arrangement, gas was distributed over the entire cross section of the drier. The gas exited at the bottom of the drier. A baffle 2 inches away from the gas outlet prevented gas from jetting into the outlet.

Heat is evolved when water vapor is adsorbed by silica gel. In fact, if the temperature of the bed is allowed to rise significantly, water vapor adsorption capacity is drastically reduced. The bed of silica gel was therefore cooled with water, which flowed through two concentrically coiled copper tubes imbedded in the gel. With this system the bed temperature was maintained at about 70°F.

The silica gel was regenerated periodically (i.e., after several runs). The regeneration procedure consisted, essentially, of maintaining the gel at a temperature of about 350°F for approximately 8 hours while dry gas was being bled through the bed. The gel was maintained at this temperature by high pressure steam which flowed through the imbedded
copper tubing coils and by an electrical heater wrapped around the outside of the drier. The drier was insulated from the surroundings during the regeneration period only. The insulation was removed before each run.

5. **Gas Temperature Control**

The remixed gas stream from the tower-drier parallel combination flowed through an electrically heated section of pipe. As previously mentioned, the temperature of the gas leaving the heated section was controlled by adjusting the electrical power input to the heater.

The heated section was a 4 foot length of 3 inch diameter DWV copper tubing, and the heating element consisted of 49 turns of No. 14 gauge chromel-A resistance heating wire. The heating wire was separated from the bare copper tube by a 1/8 inch layer of asbestos cloth. The heated section was surrounded by 1 inch of fiberglas insulation in order to prevent significant heat loss to the ambient.

6. **Flow Control and Measurement**

The gas rate through the rectangular duct was adjusted by setting the flow control valves to the appropriate positions. The valve settings required for obtaining the desired flow rate were determined before each run.

The flow rate was measured with an orifice meter. Details of the calibration and construction are presented in Appendices I and G, respectively.
7. **Duct Inlet and Outlet Chambers**

Before the gas entered the duct, it flowed through a wooden box and then into a converging nozzle made of galvanized sheet iron. The dimensions of the box were 2 feet long by 2 feet wide by 1 foot high. Two 50 mesh copper wire screens were mounted in the box in order to eliminate vortices present in the gas flow. Gas entered this box from a 3 inch diameter copper tube epoxied into the box inlet. A 4 inch diameter baffle, containing about twenty-five 1/4 inch holes, was located about an inch from the inlet. This baffle helped distribute the gas flow around the box cross section.

The purpose of the nozzle was to provide the gas with a uniform velocity profile at the entrance to the duct. The 9 feet of warm duct preceding the cold copper plate were then sufficient for the velocity profile to reach its fully developed condition. After leaving the duct the gas flowed through a diverging nozzle and into a mixing chamber. The construction of this outlet mixing chamber was similar to that of the inlet chamber, but was of smaller size. The purpose of this outlet chamber was to mix the gas leaving the duct so that the bulk average temperature could be measured. This gas was mixed by copper turnings and by a baffle located about 1 inch from the chamber outlet. A thermocouple located in the mixing chamber was used to measure the temperature.
The nitrogen flowed out of the mixing chamber into a 4 inch diameter galvanized pipe epoxied to the wooden box. This section of galvanized pipe returned the gas to the blower inlet.
Figure II-1  Flowsheet of Equipment
Figure II-2  Copper Plate and Associated Boil-off Calorimeter
Note: 2.235" is the Diameter of the Copper Plate Inner Section Excluding the Epoxy Ring

Figure II-3  Top View of Copper Plate
1. Preliminaries Before a Run

Before each run was started, it was necessary to determine the appropriate settings for the flow control valves in order that the gas entering the duct have the desired humidity and flow rate. The pre-run procedure for finding these appropriate valve settings is described below.

Gas was circulated through the system, but none flowed through the packed column. The dew point of the gas leaving the drier was measured and the gas flow then shut off. This dew point value was used subsequently in determining the gas stream split between the packed tower and drier that would be employed during the actual run. Water was then allowed to flow through the packed tower. (No gas flowed through the packed tower at this time.) The temperature of the water was measured. The humidity of the gas stream which would leave the packed tower was then calculated, as it was known that the tower saturated the gas stream at the water temperature.

From a knowledge of the humidities of the gas streams which would leave the drier and packed tower, the quantities of gas which were to be sent to each of these pieces of equipment were calculated. A non-frosting run at the desired gas flow rate was then made, i.e., the copper plate
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was maintained at room temperature. The flow control valves were adjusted so that the desired quantities of gas flowed through the drier and packed tower. If the resultant gas phase humidity was not equal to the desired value, then the settings of the flow control valves were changed slightly until the humidity attained the proper value. The total gas flow rate was maintained constant throughout this preliminary procedure. Once the settings of the flow control valves were determined during these preliminaries, it was generally not necessary to change them during the actual run.

A second step in the preliminary (pre-run) procedure involved adjusting the temperature of the gas entering the duct. This was accomplished by changing the electrical power input to the heated section of pipe until the gas attained the desired temperature. The variac setting determined during this preliminary procedure was the same one used during the actual run.

2. **Startup**

Once the preliminaries, described above, were completed, the over-all system was purged with dry nitrogen gas. After about 20 minutes of purging, liquid nitrogen was supplied continuously to the calorimeter from a 25 liter dewar. The liquid filled the inner and outer cylinders of the calorimeter and then flowed out through the overflow line (see Figure II-2). This overflow was collected in an empty dewar.
(During the run, the liquid overflow rate was maintained as constant as possible.) The run was not started, i.e., the blower was not turned on, until the temperatures of the inner and outer sections of the copper plate became approximately equal to $-320^\circ F$.

The 20 channel recorder, used to record the various temperatures throughout the system, was started during the calorimeter cool-down period. The starting of the recorder, however, did not correspond to time zero. Time zero corresponded to the actual commencement of frosting as will be explained below.

Before the run actually started, i.e., before the gas was allowed to flow over the cold copper plate, the variac in the heater circuit was set to the desired value and the heater turned on. The blower was started and the gate valve on the blower was quickly opened. The opening of the blower gate valve corresponded to the actual start of the run and hence to time zero.

3. **Procedure During the Actual Run**

During the run the measurements described below were taken at various time intervals.

(1) The liquid nitrogen boil-off rate from the inner cylinder was measured with a calibrated rotameter. This reading was checked with a wet test meter hooked up in series with the rotameter. The boil-off measurement was used to calculate the heat flux into the heat transfer test
section (inner cylinder of the calorimeter). Under certain conditions neglect of the superheating of the boiled-off vapor which occurs within the calorimeter leads to serious errors in the heat flux measurement (42). For the experiments of the present thesis, calculations indicated that appreciable superheating of the vapor did not occur within the calorimeter vessel itself. Thus this type of error was avoided in these experiments. The boiled-off vapor, however, did become superheated as it flowed from the calorimeter to the measuring rotameter. In fact, the vapor was at room temperature before it entered the rotameter. The superheating of the vapor in the outflow line did not, of course, affect the accuracy of the heat flux measurement.

(2) The surface temperature of the frost was measured with a "movable" probe. This measurement was usually made near the end of a run. In making the measurement the probe assembly was lowered towards the frost surface. The millivoltage output of this thermocouple was recorded on a continuous Sanborn recorder. (See Figure III-1 for a sketch of a typical millivoltage versus time trace.) The output millivoltage of the thermocouple was also continually observed by the operator, and the occurrence of the second "break" (point C of Figure III-1) in the millivoltage versus time curve indicated that the thermocouple had contacted the surface of the frost.
The establishment of contact between the thermocouple junction and the frost surface was corroborated by visualization through the Lucite "windows". After contact the thermocouple assembly was lowered slightly. The thermocouple millivoltage output, however, remained constant during the post-contact lowering (period DE of Figure III-1), because the junction did not penetrate farther into the frost. Instead, the thermocouple wire was just bent slightly. The junction was then allowed to remain in contact with the surface for about a minute before the thermocouple was retracted (period EF of Figure III-1). The output millivoltage remained substantially constant during this period. This constant value was taken as the frost surface temperature.

In general, care had to be exercised to prevent the thermocouple wire from causing any disturbance to the frost surface. On occasion, when the thermocouple was retracted from the surface, large chunks were torn out of the frost layer. When this occurred the run had to be discontinued.

This type of accident was generally prevented by lowering the thermocouple wire towards the frost surface slowly. The wire would, therefore, cool significantly before contact. When a warm wire contacts the frost, localized melting results. The molten frost subsequently refreezes and bonds other parts of the frost to the wire. Retraction of the wire, of course, results in large chunks of frost being ripped out.
(3) The readings of the thermocouples attached to the copper plate and of the stationary thermocouple within the frost were recorded periodically on the 20 channel recorder. Temperatures of the gas at various points in the system were recorded similarly. Thus, no special effort had to be made during a run to obtain these temperatures.

(4) The pressures at various points in the system were measured periodically. All pressures were measured with manometers.

During the course of a run, it was generally necessary to switch liquid nitrogen supply Dewars as the first Dewar usually "ran out". The switch was accomplished by shutting the valve on the liquid nitrogen inlet line, lifting the liquid transfer tube out of the first Dewar and inserting it into the second one. After the new connection was made leak tight, the valve on the liquid line was then re-opened, allowing liquid nitrogen to flow again from the supply Dewar to the calorimeter.

4. Shutdown

At a predetermined time, the gas flow was shut by shutting the blower switch. The following operations were then performed, as quickly as possible, in the order listed below.

(1) The Lucite section of the duct (i.e., the short section just above the copper plate) was removed from the
rest of the duct. Frost samples, at varying distances from the plate leading edge, were taken with glass vials of known cross sections.

(2) After step (1) was completed, the frost thickness at each of the vial sample locations was determined with a depth micrometer.

(3) In order to examine the frost structure and to determine whether a density gradient through the frost was present, a sample of frost was taken from the plate at the end of each run. This sample was subsequently examined in the -40°F cold room. The frost sample was taken from the plate with a cold spatula (at about -320°F) which had just been immersed in liquid nitrogen. The sample was placed immediately at the bottom of a small container (container temperature was about -320°F). The container was then closed and lowered into a Dewar containing liquid nitrogen.

(4) The frost was then cleared away from the areas adjacent to the stationary thermocouple without disturbing these probes. The height above the plate of each of these stationary thermocouples was measured with a depth micrometer.

(5) The frost sample container, while still immersed in the Dewar of liquid nitrogen, was carried to the -40°F cold room. It is believed that this method of handling the frost sample ensured that the structure of the frost did not change during transit to the cold room.
In the cold room the frost was viewed under a microscope. Magnifications of 20X, 100X and 200X were used. These observations allowed one to estimate the sizes and shapes of the ice particles which make up the frost.

To determine whether a density gradient through the frost was present, a portion of the frost sample was sliced horizontally into two halves with a razor. The density of each half was determined in the following manner. The thickness of each half was measured with a depth micrometer. Samples for weighing were then taken from each half with vials of known cross sections.

(6) The vials containing the frost collected in steps (1) and (5) were weighed. (By the time the vials were weighed, the frost had melted to liquid water.) Actual frost weight was determined by subtracting the weight of the dry vial from the total vial weight. Dry weight had been determined before the start of the run. Densities were calculated from the measured frost weights and the volumes of the frost samples collected.
Figure III-1 Sketch of Typical Trace of Movable Thermocouple Probe while Taking a Surface Temperature Measurement
CHAPTER IV

Results

1. Initial Phases of a Run

At the start of a run (i.e., before any significant quantity of frost had deposited onto the plate), a fog was observed in the gas space immediately adjacent to the plate. After about 15 seconds, a thin layer of frost (about ten thousandths of an inch thick) formed over the entire plate. Clumps, or islands, of frost were then observed to form. Initially, the heights of these individual clumps increased. Before long, however, the spaces between them became filled in, and the frost became planar over the entire plate. The planarity propagated from the leading edge to the trailing edge of the plate. In other words, the frost on the upstream part of the plate became macroscopically smooth and planar before the frost on the downstream part did.

While the frost planarity was propagating down the plate, particles were observed in the lower layers of the gas phase over the clumps. The gas over the smooth frost appeared to be clear of frost particles.

The time required for the frost on the entire plate to become macroscopically smooth varied with the Reynolds Number, but seemed to be independent of gas phase humidity. The relationship between the time required and the Reynolds
Number is shown in Table IV-1 and in Figure IV-1. The time required decreased with increased Reynolds Number.

2. Microscopic Observations

As described in Chapter III, frost sampled from the cold copper plate was carried to a -40°F cold room to observe the frost microstructure. In the cold room a thin vertical slice of frost was cut away from the main sample with a razor. The frost cross section was examined by viewing this thin slice under a microscope.

In a macroscopically smooth frost, the individual particles were observed to be irregular in shape. In fact, many of the ice particles possessed edges and corners. The linear dimensions of these particles ranged from 20 to 50 microns. Some cylindrically shaped particles were also observed to be present. A typical cylindrical particle was 100 microns long by 20 microns in diameter.

Particle shape and sizes in a smooth frost were apparently not significantly affected by gas phase Reynolds Number, humidity, or time elapsed since the start of frost- ing. It is possible, however, that the microstructure of a planar frost did change with these variables, but these changes were relatively small and could not be discerned by the methods employed in this thesis.

The microstructure of the clumps formed in a run carried out at a Reynolds Number of 9,250 and a gas phase humidity of 37.7 grains water/lb dry gas was also examined. The
particles were much more spherical than those in a planar frost. In addition, the particle diameters ranged from 5 to 10 microns, values which are significantly lower than the sizes of particles in a macroscopically smooth frost.

A photomicrograph of the clump structure is shown in Figure IV-2a. In Figure IV-2b the microstructure of a macroscopically smooth frost is shown. The difference in size and shape of the individual particles in the clumps and in the planar frost can be noted. Figures IV-2c and IV-2d are sketches of the structures shown in Figures IV-2a and IV-2b respectively. These sketches show particle shape more clearly than do the photographs.

The density of individual clumps of frost formed at a Reynolds Number of 9,250 and a humidity of 37.7 grains water/lb dry gas was found to be about 0.02 gm/cc. The density measurement was made 17 minutes after the start of frosting, and on clumps located on the heat transfer test section (inner section of copper plate).

3. Quantitative Results

A. Frost Density and Thickness

Frost density and thickness varied with distance from the plate leading edge. Over the downstream two-thirds area of the plate, however, there was no significant variation with position in either the frost thickness or density. The density and conductivity values reported in
this thesis, therefore, pertain to the frost on the back two-thirds of the plate. (It should be noted that the heat transfer test section is located on this downstream two-thirds portion of the copper plate.)

Frost density and thickness as functions of the various combinations of independent parameters are shown in Table IV-2. The frost density plotted as a function of time, with Reynolds Number as parameter, is shown in Figure IV-3. The shaded points are for a humidity of about 37 grains water/lb dry gas (dew point of 40°F); the lower humidity points are unshaded. At Reynolds Numbers of 5,600 and 9,500, the unshaded points correspond to a humidity of about 23 grains water/lb dry gas (dew point of 29°F). At a Reynolds Number of 14,500 the unshaded points correspond to humidities ranging from 23 to 31 grains water/lb dry gas. Errors involved in the measurement of frost density are discussed in Appendix L.

The frost thickness as a function of time with Reynolds Number as parameter is shown in Figure IV-4. The shaded points are for a humidity of about 37 grains water/lb dry gas; the unshaded points are for a humidity of about 23 grains water/lb dry gas. Measured thicknesses for runs performed at a Reynolds Number of about 14,500 and at low humidities are not shown as each thickness corresponds to a different humidity. There is no variation of frost properties with pressure to report since the pressure of the gas over the plate was essentially atmospheric for all runs.
Frost density and thickness continuously increased with time throughout the run. At equivalent times, higher Reynolds Numbers resulted in frosts of greater densities but slightly lower thicknesses. Over the humidity range involved, the frost density was independent of humidity. Lower humidities, however, resulted in smaller frost thicknesses.

It should be noted that the density referred to in the preceding paragraphs is the bulk frost density. However, based on measurements of the upper and lower halves of the frost, as described in Chapter III, it was concluded that the density gradient through the frost was insignificant. Thus, for the purposes of the present thesis, the frost was considered to be of uniform density throughout its depth.

It should also be noted that the values of the humidity shown in Table IV-2, and referred to in the various figures, are the humidities in the gas stream immediately above the leading edge of the plate. The change in gas stream humidity, however, is small as the gas passes over the plate. For example, at a Reynolds Number of 14,500 and a humidity of 37 grains water/lb dry gas, the change in the gas stream humidity between the leading edge of the plate and the center of the inner heat transfer test section was about 3%. The maximum change in gas stream humidity between these two points was about 5%. This occurred in runs performed at a Reynolds Number of 5,600 and a gas phase
humidity of 23 grains water/lb dry gas. The above stated estimates of the humidity changes were obtained from measurements of the mass of frost which accumulated on the front part of the plate.

B. Heat Transfer Through Frost

At the start of a run, the rate of heat transfer from the gas to the bare copper surface was relatively high, but as the run progressed the heat flux decreased continuously due to the insulating effect of the accumulating frost. During each run, the heat flux eventually attained a constant value. For all runs, the constant heat transfer period was reached about 15 minutes after the frost became macroscopically smooth.

The period of constant heat transfer is naturally not a true steady state as frost conductivity and thickness increase during this period. It is, however, more desirable to measure thermal conductivities during this period than during the transient heat transfer period when heat transfer rates are dropping quickly, and the temperature of the frost surface is rapidly increasing. Therefore, accurate heat flux measurements were made only during the constant heat transfer period.

The magnitude of the steady heat transfer rate is dependent both on the gas phase humidity and Reynolds Number. These constant rates, as a function of various combinations of the independent parameters, are presented in Table IV-3.
The rates are also plotted as a function of Reynolds Number for a humidity of 37 grains water/lb dry gas in Figure IV-5. Steady heat flux as a function of Reynolds Number for a humidity of 23 grains water/lb dry gas is also shown in Figure IV-5. Heat flux as a function of humidity for a Reynolds Number of 14,500 is shown in Figure IV-6. In general, heat flux increased with increased Reynolds Number, but decreased with increased gas phase humidity.

C. Frost Thermal Conductivity

(1) Average Conductivity of the Frost Layer

As previously mentioned, thermal conductivities were measured only during the quasi-steady state heat transfer period. The average conductivity of the frost layer was calculated from Eq. IV-1 below, using the measured heat flux, the measured value of the frost surface temperature and the measured temperature of the copper plate.

\[
\frac{q}{A} = \frac{k}{A_{v}} \frac{f}{T} \frac{T_{s} - T_{p}}{h}
\]

Conductivity values are tabulated in Table IV-4. In Figure IV-7, the average frost conductivity is plotted as a function of frost density. (The straight line shown in this diagram was obtained by the method of least squares.) Over the density range 0.05 to 0.13 gm/cc, thermal conductivity increases with frost
density. The various points in Figure IV-7 correspond to frosts which were formed at different Reynolds Numbers and humidities. The measured surface temperatures of these frosts ranged from 10 to 25°F. In plotting thermal conductivity as a function of density only, it was implicitly assumed that a given value of density implied a given frost structure, and therefore that the average thermal conductivity of the frost layer was a unique function of density. This is equivalent to assuming that two frosts of the same density, formed at different Reynolds Numbers and/or different humidities, nonetheless have the same structure. Actually some of the scatter in the data, shown in Figure IV-7, may indeed be caused by structural differences of frosts of the same density. No definite trend, however, was observed. (It should be noted that the scatter may also be partly due to the different surface temperatures.) Errors involved in the measurement of conductivity are discussed in Appendix L.

Equation IV-1 is rigorously valid only for a true steady state situation. As mentioned earlier, frost formation, even at long times, is not a true steady state process because of the continuously increasing frost density and thickness. However, since transient heat effects within the frost layer are negligible, Eq. IV-1 can validly be applied during the quasi-steady state period.
(2) Local or Point Frost Thermal Conductivity

In order to obtain frost thermal conductivity as a function of density over a wide temperature range, temperatures within the frost as well as at the frost interfaces were measured. More specifically, temperatures within the frost were measured at two different vertical positions with 0.003 inch diameter copper-constantan thermocouples. The measuring junctions of these probes were located at known heights above the plate. The stationary probe readings and the measured frost interface temperatures (i.e., frost-gas and frost-plate interfaces) were used to obtain an end-of-a-run temperature profile through the frost for each run. A typical temperature profile is shown in Figure IV-8. In particular, Figure IV-8 is an end-of-a-run profile for a run made at a Reynolds Number of 14,400 and a humidity of 37.7 grains water/lb dry gas. The run lasted for 69.8 minutes.

The temperature gradient, \( \frac{dT}{dy} \), was obtained at any particular location by graphical differentiation (i.e., by eye) of the frost temperature profile. These values of the gradient and the measured heat fluxes were used to calculate the local thermal conductivity from Eq. IV-2.

\[
\frac{q_T}{A} = k_{f,T} \left( \frac{dT}{dy} \right) \tag{IV-2}
\]
As is the case for Eq. IV-1, Eq. IV-2 can validly be applied during the quasi-steady state period.

Frost thermal conductivities were generated over a temperature range of -215 to +20°F and a density range of 0.05 to 0.13 gm/cc. The values are plotted in Figure IV-9. (The straight lines through the data were calculated by the method of least squares.) At any given density, thermal conductivity showed a marked increase with temperature. For a given temperature, thermal conductivity increased with density, the rate of this increase being greater at the higher temperatures. In plotting Figure IV-9, as was the case for Figure IV-7, it was implicitly assumed that at a given temperature a particular frost structure was implied by a given value of frost density.
Table IV-1

Effect of Reynolds Number on Length of Time Required to Form Smooth Frost

<table>
<thead>
<tr>
<th>Reynolds Number</th>
<th>Time to Form Smooth Frost (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,6000</td>
<td>about 40</td>
</tr>
<tr>
<td>9,300</td>
<td>25 to 30</td>
</tr>
<tr>
<td>14,500</td>
<td>10 to 15</td>
</tr>
<tr>
<td>Run Number</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>------------</td>
<td>----------------</td>
</tr>
<tr>
<td>43</td>
<td>14,500</td>
</tr>
<tr>
<td>34</td>
<td>14,500</td>
</tr>
<tr>
<td>44</td>
<td>14,400</td>
</tr>
<tr>
<td>40</td>
<td>14,500</td>
</tr>
<tr>
<td>48</td>
<td>9,240</td>
</tr>
<tr>
<td>47</td>
<td>9,240</td>
</tr>
<tr>
<td>66</td>
<td>9,450</td>
</tr>
<tr>
<td>49</td>
<td>9,350</td>
</tr>
<tr>
<td>41</td>
<td>9,300</td>
</tr>
<tr>
<td>59</td>
<td>5,570</td>
</tr>
<tr>
<td>50</td>
<td>5,640</td>
</tr>
<tr>
<td>51</td>
<td>5,600</td>
</tr>
<tr>
<td>53</td>
<td>14,600</td>
</tr>
<tr>
<td>54</td>
<td>14,750</td>
</tr>
<tr>
<td>69</td>
<td>14,750</td>
</tr>
<tr>
<td>55</td>
<td>14,750</td>
</tr>
<tr>
<td>46</td>
<td>14,650</td>
</tr>
</tbody>
</table>
Table IV-2 (Cont.)

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Reynolds Number</th>
<th>Humidity</th>
<th>Time After Start of Frosting</th>
<th>Density gm/cc</th>
<th>Thickness inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>61</td>
<td>9,300</td>
<td>22.9</td>
<td>75.0</td>
<td>0.0638</td>
<td>0.175</td>
</tr>
<tr>
<td>62</td>
<td>9,240</td>
<td>22.2</td>
<td>91.5</td>
<td>0.0800</td>
<td>0.197</td>
</tr>
<tr>
<td>63</td>
<td>9,200</td>
<td>22.7</td>
<td>123.0</td>
<td>0.0961</td>
<td>0.210</td>
</tr>
<tr>
<td>65</td>
<td>5,680</td>
<td>23.4</td>
<td>76.3</td>
<td>0.0588</td>
<td>0.188</td>
</tr>
<tr>
<td>60</td>
<td>5,570</td>
<td>23.0</td>
<td>110.0</td>
<td>0.0691</td>
<td>0.220</td>
</tr>
</tbody>
</table>

**NOTE:**

1) Runs 47 and 66 are duplicates.

2) Runs 46 and 55 are duplicates.
Table IV-3

Measured Quasi-Steady State Heat Fluxes

<table>
<thead>
<tr>
<th>Humidity</th>
<th>Reynolds Number</th>
<th>Quasi-Steady State Heat Flux</th>
</tr>
</thead>
<tbody>
<tr>
<td>grains water lb dry gas</td>
<td></td>
<td>BTU/hr-ft²</td>
</tr>
<tr>
<td>37</td>
<td>14,500</td>
<td>709</td>
</tr>
<tr>
<td>9,300</td>
<td>517</td>
<td></td>
</tr>
<tr>
<td>5,600</td>
<td>456</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>9,300</td>
<td>567</td>
</tr>
<tr>
<td>5,600</td>
<td>495</td>
<td></td>
</tr>
</tbody>
</table>
Table IV-4

Average Frost Conductivities

<table>
<thead>
<tr>
<th>( \rho_f ) (gm/cc)</th>
<th>( k_{f,T} ) (BTU/hr-ft(^2)-0°F/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0545</td>
<td>0.0260</td>
</tr>
<tr>
<td>0.0588</td>
<td>0.0236</td>
</tr>
<tr>
<td>0.0608</td>
<td>0.0238</td>
</tr>
<tr>
<td>0.0637</td>
<td>0.0302</td>
</tr>
<tr>
<td>0.0638</td>
<td>0.0311</td>
</tr>
<tr>
<td>0.0638</td>
<td>0.0258</td>
</tr>
<tr>
<td>0.0643</td>
<td>0.0314</td>
</tr>
<tr>
<td>0.0691</td>
<td>0.0279</td>
</tr>
<tr>
<td>0.0742</td>
<td>0.0321</td>
</tr>
<tr>
<td>0.0790</td>
<td>0.0317</td>
</tr>
<tr>
<td>0.0800</td>
<td>0.0280</td>
</tr>
<tr>
<td>0.0809</td>
<td>0.0280</td>
</tr>
<tr>
<td>0.0818</td>
<td>0.0295</td>
</tr>
<tr>
<td>0.0882</td>
<td>0.0278</td>
</tr>
<tr>
<td>0.0914</td>
<td>0.0344</td>
</tr>
<tr>
<td>0.0918</td>
<td>0.0341</td>
</tr>
<tr>
<td>0.0961</td>
<td>0.0301</td>
</tr>
<tr>
<td>0.109</td>
<td>0.0348</td>
</tr>
<tr>
<td>0.111</td>
<td>0.0398</td>
</tr>
<tr>
<td>0.122</td>
<td>0.0389</td>
</tr>
<tr>
<td>0.126</td>
<td>0.0375</td>
</tr>
<tr>
<td>0.128</td>
<td>0.0415</td>
</tr>
</tbody>
</table>
Figure IV-1  Length of Time Required for Smooth Frost to Form as a Function of Reynolds Number
Figure IV - 2A  Photomicrograph of Particles in Frost Clumps

Scale: 100 microns

Figure IV - 2B  Photomicrograph of Particles in a Macroscopically Smooth Frost

Scale: 100 microns
Figure IV-2C  Sketch of Typical Particles Observed in Frost Clumps

Figure IV-2D  Sketch of Typical Particles Seen in a Macroscopically Smooth Frost
Figure IV-3  Frost Density as a Function of Time
Figure IV-4 Frost Thickness as a Function of Time

Shaded Points: High Humidity
Unshaded Points: Low Humidity

- Δ Re = 14,500
- ○ Re = 9,300
- □ Re = 5,600

Frost Thickness (inches)

Time (minutes)
Figure IV-5 Quasi-Steady State Heat Flux as a Function of Reynolds Number and Humidity
Figure IV-6  Constant Heat Flux as a Function of Humidity for a Reynolds Number of About 14,700
Average Frost Temperature $\approx -145^\circ F$

Note: Temperature Varies with Depth

Figure IV-7 Average Thermal Conductivity of the Frost Layer as a Function of Frost Density
Figure IV-8 End of Run Temperature Profile in Frost

Re = 14,400

Grains water

H = 37.7 grains water / lb dry gas

Gas Phase dew Point ≈ 40°F
Figure IV-9  Frost Thermal Conductivity as a Function of Density, with Temperature as a Parameter
CHAPTER V

Discussion of Results

1. Initial Phases of a Run

A. Fog Formation in Gas

At the start of each run the fog observed in the gas space, adjacent to the plate, resulted from ice particle nucleation. Ice particles formed because the gas in the boundary layer was cooled rapidly and hence became saturated. Application of the criteria for fog formation derived by Johnstone, Kelley and McKinley (44) indicate that, for the experiments of the present thesis, fog will definitely form in the gas phase boundary layer during the early phases of frosting. (A detailed discussion of particle nucleation is given in Ref. 1.)

As the gas passed over the plate, variations of temperature or water vapor concentration in its bulk were small. The gas bulk, therefore, always remained unsaturated. It is not known whether the particles "nucleated-out" as liquid or solid, but it seems quite likely, in view of the low temperatures in the boundary layer, that the particles were actually solid when they struck the plate. Examination of the frost microstructure tended to confirm this hypothesis. The microstructure of the cryogenic frosts obtained in this thesis are not characteristic of the microstructure of rimes (51,53),
which are formed when supercooled liquid droplets strike a solid surface, spread and then freeze.

For the present thesis, calculations indicate that the major mode of transfer of ice particles to the plate is the gas stream turbulence (see Appendix A). Other mechanisms mentioned in Appendix A, however, may also be responsible for a small percentage of the particles transferred. The transfer of these ice particles from the gas phase to the plate resulted in the formation of individual clumps or islands of frost, which appeared to form over the entire plate at about the same time. Initially, the heights of the individual clumps increased as more particles were thrown onto them. Before long, however, the valleys between the clumps filled in and a smooth frost was formed. The mechanism whereby these valleys "filled in" is not known.

As the frost "planarity" propagated down the plate, particles were observed in the gas space over the clumps, but not in the space over the smooth frost. The observed cloud of particles over the clumps may have been due to erosional effects.\textsuperscript{(58,82)} Despite the possible significance of erosion, there was a net deposition of mass onto the sections of the copper plate which contained the clumps.

Particles were not observed in the gas space over the smooth frost. If the particles observed over the clumps are
due to erosional effects, a possible explanation is that bonds between particles were stronger in the smooth frost than in the frost clumps.

An alternate explanation for this observation is that the gas adjacent to the scattered islands of frost may have been at a significantly lower temperature than the gas adjacent to the smooth frost. Ice particle nucleation in the gas space next to the clumps would, therefore, have been much more extensive than in the gas adjacent to the planar frost.

Employing the criteria of Ref. (44), it is concluded that for the 23 grains water/lb dry gas runs frost accumulation occurs solely by the molecular and turbulent diffusion of water vapor from the gas phase only after the frost surface attains a temperature of about -5°F. For the runs performed at a humidity of 37 grains water/lb dry gas, frost accumulation solely by diffusion does not occur until after the surface reaches a temperature of about +5°F.

B. Microscopic Observations

As previously mentioned, the particles contained in the frost clumps were approximately spherical and had diameters ranging from 5 to 10 microns. The sizes of the particles which actually struck the plate are unknown, but their diameters were probably less than 5 microns as these particles probably grew somewhat while they were still on the plate, before being sampled for microscopic observation.
The highly irregular particles, which comprise a smooth frost and which possess corners and edges, are very similar to particles contained in an old snow layer (77). Snow particles are irregular because they have been subjected to an internal diffusion of water vapor. It is this internal diffusion which also causes the particles in a cryogenic frost layer to undergo changes in shape and size. More specifically, ice particles which were more-or-less spherical (diameters ranging from 5-10 microns) were transformed to much larger irregular particles (linear dimensions ranging from 20-50 microns). The edges and corners possessed by these irregular particles are typical characteristics of ice particles in a snow layer which has been subjected to an internal diffusion of water vapor (77). An ice particle within the frost layer never develops into a regular symmetrical crystal such as the newly fallen snow crystal shown in Figure B-1, probably because of the interference of the surrounding particles.

C. Densification of Clumps

It has previously been indicated in Chapter IV that the density of individual frost clumps, 17 minutes after the inception of frosting, is about 0.02 gm/cc. Under these same conditions of Reynolds Number and humidity (i.e., Re = 9,250 and H = 37.7 grains water/lb dry gas), the density of the frost just as it becomes smooth is about
0.045 gm/cc. This smooth frost value was obtained by extrapolating the appropriate experimental curve of Figure IV-3 back to \( \theta = 25 \) minutes.

Frost particle formation in the gas phase during the period 17-25 minutes was probably still significant. Therefore, during this period, probably, both internal diffusion and particle transfer are responsible for the frost densification. Frost can be densified by particle transfer because the turbulent eddies in the gas stream "kick" the particles into the frost interstices as well as onto the surface. The mechanism of internal diffusion has already been described (see Chapter I).

Actually, it has been estimated that ice particles form in the gas phase for about 10 to 15 minutes after the frost layer becomes macroscopically smooth. Thus particle transfer may be partly responsible for frost layer densification and increasing frost thickness during this period also.

2. Frost Behavior as a Function of Time

Frost density and thickness as functions of time, Reynolds Number and humidity are shown in Figures IV-3 and IV-4. In all cases, mass continuously deposited onto the plate throughout the run. As can be seen from Figures IV-3 and IV-4, the continuous mass deposition resulted in a continuous increase in both the frost density and in the frost thickness.
Based solely on the experimental fact that frost deposited continuously throughout a run, it can be concluded that a concentration driving force between the gas phase and the frost surface was always present. Thus the frost surface temperature always had to be lower than the gas phase dew point. This was verified experimentally. When the gas phase dew point was 40°F (humidity of 37 grains water/lb dry gas), the measured value of the surface temperature during the quasi-steady state heat transfer period ranged from 20 to 25°F. At a gas phase dew point of 29°F (humidity equal to 23 grains water/lb dry gas), the measured frost surface temperature during the constant heat transfer period was about 12°F. Actually, it is the mechanism of internal diffusion which prevents the frost surface temperature from ever reaching either 32°F or the gas phase dew point, whichever is lower.

The attainment of a constant surface temperature and a constant heat flux can probably best be explained by considering the frosting process from its inception.

As previously mentioned, the frost surface temperature was quite low during the early phases of a run. Hence water vapor diffusion within the frost was insignificant at this time. However, as the frost thickness increased, the surface temperature rise and the heat flux decrease occurred quite rapidly. Very shortly thereafter the frost surface temperature reached a value at which internal water
vapor diffusion became quite significant, with the result that frost density increased substantially. Frost thickness still continued to increase during this period because of the continuous deposition of mass.

Due to the very high sensitivity of internal diffusion to the frost surface temperature, at the higher temperatures, a very small increase in the surface temperature results in a large increase in the internal water vapor diffusion rate. The density (and hence the thermal conductivity of the frost), therefore, increased to a point where the heat flux fell off only very slightly, and the surface temperature rose only very slightly as additional frost was deposited.

During the constant heat transfer period, the frost thermal resistance was essentially constant as the increasing thickness was balanced by the increasing thermal conductivity which resulted from the continuous frost densification. It is highly unlikely that these two effects (i.e., thickness and density increase) balanced each other merely by chance. Certainly a more reasonable explanation is that heat flux and frost surface temperature are stabilized by a very high-gain negative feed-back effect. In other words, any large disturbance in the frost environment, i.e., in the gas phase, will not substantially disturb the surface temperature or the heat transfer rate during the quasi-steady state period. For
example, it is assumed that some disturbance in the gas phase causes the heat transfer rate to increase, thus tending to overheat the frost surface. An increased surface temperature, however, would result in an increased rate of water vapor diffusion away from the frost surface and into the interior. This would result in a greater rate of conductivity increase than would have been the case had the surface temperature not been raised. The additional conductivity increase, however, would tend to reduce the surface temperature. It can therefore be concluded that if a disturbance which tends to cause a significant rise in the surface temperature occurs, a subsequent series of processes will ensue which will tend to bring the surface temperature back down to about its former level.

Based upon the physical concepts described in the preceding paragraphs, a simplified physical model was postulated. This model can be used for predicting the variation of frost properties with time.

3. **Model for Predicting Frost Properties**

The model used to predict the variation of frost properties with time is based both on experimental data and on two equations. One equation is an expression of the energy balance between the gas phase and the frost layer. The other is a material balance relating the rate
of densification of the frost to the rate of internal diffusion of water vapor. The experimental data used are thermal conductivities at various densities and temperatures (Figure IV-9) as well as the curves of mass of frost accumulated as a function of time. These latter curves were obtained by multiplying the measured values of frost density and thickness, and extrapolating the curve through these points to the origin (see Figure V-16).

The model also used experimental heat transfer coefficients. These coefficients were determined at the end of each run from measurements of the heat flux and the frost surface temperature.

A priori it would appear that the usual equation for the mass transfer rate, i.e.,

\[ N = k' A(c_1 - c*) \] (V-1)

\( \text{could} \) have been used instead of the experimental mass accumulation curve. The mass transfer coefficient \( k' \) \( \text{could} \) have been obtained from one of the standard correlations. For example, the Lewis relation \( k'_g = h_g/c_s \) (or \( k'_g = h_g/c_p \) since \( c_p \approx c_s \) \( \text{since} \)) could have been utilized to generate the frost accumulation versus time curve. However, there exists sufficient evidence in the literature (40,69,72,87) to indicate that when ice particles nucleate in the gas phase, frost accumulation rates are significantly lower.
than would be the case if frost were accumulating solely by molecular and turbulent diffusion. This effect is more pronounced in natural convection experiments (40,72), but is apparently also significant in forced convection (69,87). Equation V-1 is, therefore, not expected to be valid during the early phases of a run. In fact, it has been estimated that for the present thesis initial (i.e., at time zero) mass transfer rates calculated from Equation V-1 are about twice as large as the experimental values. Thus in the experiments of the present thesis a curve of frost accumulation as a function of time calculated from Equation V-1 would lie above the experimental curve.

Barring erosion of particles from the frost surface, Equation V-1 is expected to be valid during the latter phases of a run, since frost, apparently, accumulates by molecular and turbulent diffusion during this period. Theoretical and experimental end-of-run values of mass accumulated on the plate are compared in Table V-1. Comparison between experiment and theory for a Reynolds Number of 5,600 and a humidity of 23 grains water/lb dry gas is shown in Figure V-1. The theoretical frost accumulation curve was obtained by starting the calculation at a given point on the experimental curve. The time corresponding to each initial point was such that after this time frost accumulation occurred by molecular and turbulent diffusion. (Initial points for each case are shown in Table V-1.)
As just mentioned, mass transfer rates were calculated from Equation V-1. Mass transfer coefficients were obtained from the Lewis relationship, using experimental values of the heat transfer coefficient. $c^*$, the equilibrium water vapor concentration at the frost surface, was evaluated at the measured surface temperature. The theoretical mass transfer rate is constant (as is the experimental rate) during the latter phases of the run as $k_g'$ and $T_s$ do not vary with time. Thus mass of frost, on the plate at any time after the chosen initial time, is easily calculated.

It is seen from Table V-1 that theoretical masses are always greater than experimental ones. This suggests the presence of an erosional effect. However, as will be seen later such an explanation is of dubious value for the experiments of the present thesis. Actually, the agreement between the experimental and theoretical values is considered sufficiently close to conclude that during the latter phases of a run frost accumulates primarily by molecular and turbulent diffusion.

Theoretical mass transfer rates are also compared with approximate, experimental rates in Table V-1. The quoted experimental rates (see Table V-1), which are end of run or steady values, are very approximate as the mass accumulation data points scatter somewhat. The determination of the true slope of the line through these points is, therefore, not a very accurate procedure.
A. Energy Balance

The energy balance was obtained by noting that the heat transfer from the gas phase to the frost surface, plus the heat liberated by the water vapor condensing at the frost surface, are equal to the heat flux through the frost layer. (Radiation from the warm duct to the frost surface was calculated to be negligible.) Expressed mathematically the energy balance is:

\[
h_g(T_g - T_s) + \left(\frac{N}{A}\right)\lambda = \frac{k_{f,T}(T_s - T_p)}{A_{V}} \tag{V-2}
\]

Equation V-2 is rigorously valid only for a true steady state situation. However, Eq. V-2 can be used in this situation because transient heat effects within the frost layer are negligible.

The magnitude of the term \(\left(\frac{N}{A}\right)\lambda\) is usually as much as 10-20% of the magnitude of \(h_g(T_g - T_s)\). In calculating the variation of frost properties with time, the term \(\left(\frac{N}{A}\right)\lambda\) was therefore not neglected.

B. Frost Densification Equation

The detailed derivation of the equation used for calculating the rate of frost densification is presented in Appendix F. The derivation will be outlined below.

The density of the frost at any instant during the run is given by
\[ \rho_f = \frac{m_f}{\delta} \quad (V-3) \]

Differentiation of Eq. V-3 with respect to time yields

\[ \delta \frac{d\rho_f}{d\theta} + \rho_f \frac{d\delta}{d\theta} = \frac{dm_f}{d\theta} \quad (V-4) \]

The term \( \frac{dm_f}{d\theta} \) is equal to the rate of transfer of mass from the gas phase to the frost surface. The term \( \rho_f \frac{d\delta}{d\theta} \) represents the rate of accumulation of the mass, which is responsible for the frost thickness increase. The term \( \delta \frac{d\rho_f}{d\theta} \), on the other hand, represents the rate of accumulation of mass resulting from the frost densification. Since the frost densification is caused by the water vapor which diffuses from the frost surface into the interior, the rate of densification is set equal to the rate of diffusion of water vapor away from the frost surface:

\[ \delta \frac{d\rho_f}{d\theta} = \frac{1}{\tau} \frac{M_v}{(1 - Y)} D_{1-2} (c_T)(1 - \frac{\rho_f}{\rho_1})(\frac{dT}{dy})_s (\frac{dy}{dT})_s \quad (V-5) \]

where \((dT/dy)_s\) is the temperature gradient at the frost surface, \(\rho_f\) is the density of the frost, and \(\rho_1\) is the density of solid ice. By substituting the Clausius-Clapeyron equation and the ideal gas law into Eq. V-5, the final expression for the rate of frost densification (Eq. V-6), is derived:
\[
\frac{dp_f}{d\Theta} = \frac{1}{t} \left( \frac{\beta}{\rho} \right) (1 - \frac{\rho_f}{\rho_1})(\frac{dT}{dy})_s \tag{V-6}
\]

where

\[
\beta = \frac{D_{l-2} \rho_g (M_v)^2 \lambda y^*}{RT^2 M_g} \tag{V-7}
\]

It was estimated that Knudsen diffusion in the pores is negligible compared with bulk diffusion. Thus \(D_{l-2}\) is the ordinary diffusivity of water vapor in nitrogen.

C. Method of Calculating Frost Properties

The details of the method for calculating frost density as a function of time, frost thickness as a function of time, frost surface temperature as a function of time, and heat flux through the frost as a function of time are presented in Appendix K. Essentially, the method consists of the simultaneous solution of Eqs. V-2 and V-6 by an iteration procedure. For each case (i.e., at a given Reynolds Number and humidity), the calculation was started at a point in time corresponding to five minutes after the commencement of frosting. For the first five minutes thereafter, i.e., from \(\Theta = 5\) to \(\Theta = 10\), one minute time intervals were chosen. From \(\Theta = 10\) to \(\Theta = 15\), 2 \(\frac{1}{2}\) minute intervals were used. After \(\Theta = 15\) minutes, and until the end of the run, the time interval employed was 5 minutes.
It should also be noted that for each case the heat transfer coefficient was assumed to be constant at all times. Finally, the tortuosity of the unconsolidated frost was taken to be 1.1 (73).

In order to start the calculation, an initial value of frost thickness (i.e., at $\theta = 5$ minutes) had to be assumed. For all cases a value of 0.04 inches was arbitrarily chosen. Actually, values of frost density, frost thickness, heat flux and frost surface temperature at long times are essentially independent of the assumed initial frost thickness. This can be seen by comparing the dashed and solid curves of Figures V-2 through V-9. The dashed curves were calculated using an initial frost thickness of 0.08 inches, whereas the solid curves are for an initial frost thickness of 0.04 inches. Initial densities were obtained by dividing the mass of frost on the plate at $\theta = 5$ minutes by the assumed initial thickness. Mass of frost on the plate at $\theta = 5$ minutes was read from the appropriate mass accumulation versus time curve (see Figure V-16).

Figures V-2 through V-5 are for a Reynolds Number of 5,600 and a humidity of 37 grains water/lb dry gas. For this Reynolds Number and humidity, a frost thickness of 0.04 inches corresponds to a density of 0.024 gm/cc, whereas a thickness of 0.08 inches corresponds to a density of 0.012 gm/cc. Figures V-6 through V-9 are for a Reynolds
Number of 14,500 and a humidity of 37 grains water/lb dry gas. In this case a frost thickness of 0.04 inches corresponds to a density of 0.053 gm/cc, whereas a thickness of 0.08 inches corresponds to a density of 0.0265 gm/cc.

Actually it cannot be asserted unequivocally that the above conclusion (i.e., that frost properties at "long times" are essentially independent of assumed initial thicknesses) is valid outside the range of Reynolds Numbers and initial thickness values utilized in the present thesis.

Even though time intervals as large as 5 minutes were chosen for most of the steps in the calculation, the numerical solution is nonetheless convergent. This was confirmed by Shah (78) who repeated some of the calculations on an IBM 7094 computer using one minute time intervals throughout. His results are essentially the same as those of the present thesis, which were obtained by slide-rule calculation.

4. **Frost Properties**

A. **Comparison of Data with Predictions Based on Model**

The experimental data for a Reynolds Number of 5,600 and a humidity of 37 grains water/lb dry gas are compared with the predictions of the model in Figures V-2 through V-5. In this case, as in the other cases, the theoretical densification curve lies above the experimental curve, whereas the theoretical thickness values lie below the data.
At long times, however, the experimental and theoretical densification rates are approximately equal. The same is also true for the rate of increase of frost thickness.

A possible explanation for the divergence between the theoretical and experimental curves is that during the early phases of a run the model overestimates the densification rate and underestimates the rate of thickness increase. It should be noted that the internal diffusion model is not truly applicable during the period when frost is accumulating primarily by particle transport. The model is strictly applicable to those situations wherein frost is accumulating solely by the molecular and turbulent diffusion of water vapor from the gas phase. Thus the divergence between the experimental and theoretical curves may be partly due to the failure of the model to adequately describe the physical situation throughout the entire run. Nonetheless, in view of the agreement between the long-time theoretical and experimental rates (i.e., rates of densification and thickness increase), it can be concluded that the internal diffusion model is a reasonably accurate representation of the physical processes which occur within the frosting system.

For the case of \( \text{Re} = 5,600 \) and \( H = 37 \) grains water/lb dry gas, the theoretical heat flux is compared with experimental data in Figure V-4. Theoretical and experimental frost surface temperatures are compared in Figure V-5.
It is seen from Figure V-5 that the measured surface temperatures are somewhat higher than the theoretical values. This is not only true for this case but for all the cases studied. In general, the difference between the experimental and theoretical surface temperatures ranged from 5 to $15^\circ\text{F}$. It is possible that these differences are due to defects in the model. It is also possible that the measured temperatures were too high because of incomplete burial of the thermocouple bead below the frost surface.

From Figure V-4 it is observed that the measured heat fluxes are somewhat lower than the theoretical values. It is possible that the theoretical heat fluxes are too large, because the heat transfer coefficients used in calculating these heat fluxes are too high. As previously mentioned these coefficients were determined from surface temperature measurements. If the measured surface temperature is 10 to 15 degrees too high, this results in a heat transfer coefficient which is 20 to 30% greater than the true value.

Use of a heat transfer coefficient which is larger than the true value results in calculated frost densities which are too high and calculated frost thicknesses which are too low. In fact, calculations have shown that if the heat transfer coefficient is reduced by about 20%, theoretical densities and thicknesses show much better agreement with the data than is depicted in Figures V-2 and V-3.
The curves of theoretical frost properties as a function of time for the 14,500 Reynolds Number and 37 grains water/lb dry gas humidity case exhibit the same trends as do the curves for the case described in the preceding paragraphs (see Figures V-6 through V-9). At the higher Reynolds Number, however, better agreement is obtained between experiment and theory. For example, the measured steady value of the surface temperature is only 5 degrees higher than the theoretical end-of-run value (see Figure V-9). Actually, the better agreement between the theoretical and experimental surface temperatures is perhaps the key to the closer match between theory and experiment for this higher Reynolds Number case.

Figure V-7 is a comparison between experimental and theoretical frost thicknesses. Near the end of the run (i.e., at $\theta = 108$ minutes), experimental and theoretical values are almost equal, but the theoretical rate of thickness increase is substantially greater than the experimental rate. The reason for this anomaly is not known.

B. Dependence of Frost Behavior on Reynolds Number

If frosts were formed solely by particle transfer, then it would certainly be reasonable to expect higher Reynolds Numbers to result in frosts of greater densities for reasons presented earlier. This apparently is the case for the early parts of a run when frost accumulates primarily by particle transfer (see Figure IV-3).
As mentioned previously, in high temperature experiments (15), it has been shown that frost accumulates solely by the diffusion of water vapor from the gas phase to the frost surface. In such experiments higher Reynolds Numbers result in frosts of greater densities. There is no apparent fluid dynamical explanation available for this phenomenon (i.e., an explanation similar to the one presented for frosts formed by particle transfer).

The mechanism of formation of a cryogenic frost in its latter stages is similar to that for high temperature frosts. The variation of density of a cryogenic frost during the latter phases of a run can, therefore, not be explained in terms of fluid dynamics. A possible explanation can, however, be based on the postulate that the frost is reacting to the thermal forces imposed upon it by the gas phase.

Gas phase heat transfer coefficients are, naturally, greater at the higher Reynolds Numbers. Thus at equivalent times and humidities, rates of heat transfer from the gas to the frost surface tend to be higher at the larger Reynolds Numbers. These increased heat transfer rates tend to overheat the frost surface, i.e., to make the frost surface temperature significantly higher at a higher Reynolds Number. However, the higher surface temperature results in a greater frost densification rate. The higher Reynolds Number frosts, therefore, have greater
densities and hence increased conductivities. The increased conductivities, however, tend to maintain the surface temperature at a lower level, and overheating of the frost surface is not expected to be a large effect. This expectation is borne out by the data, which indicate that temperatures of frost surface during the quasi-steady state period are not significantly affected by the gas stream Reynolds Number. As previously mentioned, theoretical frost surface temperatures do show some dependence on the gas phase Reynolds Number (see Table V-3), but this effect is not very large.

The data also show that at equivalent times and humidities, higher Reynolds Numbers resulted in slightly lower frost thicknesses. Richards et al. (69) also observed this phenomenon for those runs in which liquid water did not form on the frost surface. In Kamei's experiments (45), higher gas stream velocities also resulted in smaller frost layers. This phenomenon has been attributed to the greater shearing action of the higher velocity streams. While it is true that the higher density frosts, which are formed at the higher Reynolds Numbers, have greater shear strengths than lower density frosts, it is implied in Ref. 69 that the greater shearing action of the higher velocity gas stream overrides this consideration and causes the higher Reynolds Number frosts to have smaller thicknesses.

It should be noted, however, that for the experiments of the present thesis the above explanation does not appear
to be applicable. The highest wall shear strength encountered in the present experiments was about $3 \times 10^{-5}$ psi. Based on data presented in Refs. 5 and 11, it was estimated that the shear strength of the frost was about $10^{-2}$ psi. Thus, it would appear that the shear strength at the wall is several orders of magnitude lower than would be required to shear off some of the frost.

However, it should also be noted that the value of $10^{-2}$ psi for the shear strength of the frost is applicable for smooth frosts whose surface temperatures are approximately $20^\circ F$. It is reasonable to expect that frosts whose surface temperatures are significantly lower, and which consist of a series of scattered islands or clumps, have shear strengths which are appreciably below $10^{-2}$ psi. It is then possible that during the early phases of a run gas shear does have a significant effect on the frost thickness. Initial frost thicknesses would then be lower at the higher gas stream velocities. Though it may be true that in the latter phases of a run the gas velocity does not cause any frost to be sheared from the surface, the initial lower thicknesses of frosts formed at higher velocities result, even at long times, in the higher velocity frosts having smaller thicknesses.

C. **Effect of Humidity on Frost Behavior**

At equivalent times and Reynolds Numbers, densities of frosts obtained at the higher humidities are about the
same as those obtained at the lower humidities (see Figure IV-3). From these results, however, it would be erroneous to conclude that over wide ranges of humidity, gas phase water content has no effect on frost density. For example, it has been found that at humidities significantly greater than 37 grains water/lb dry gas, liquid water condenses on the frost surface and results in the formation of frosts consisting of alternate layers of ice and porous frost (69). Such frosts have much higher densities than frosts formed at lower gas phase humidities and equivalent Reynolds Numbers.

For situations in which gas phase humidity is lower than 23 grains water/lb dry gas and wherein frost is formed solely by diffusion, no prediction based on fluid dynamical reasoning can be made about the variation of frost density with humidity. However, if frosts were formed solely by particle transfer, then it would be reasonable to expect that humidity would have no effect on frost density since humidity does not change the intensity of the turbulence in the gas stream. The data of the present thesis seem to corroborate this explanation as the densities of frosts formed during the initial phases of frosting are apparently independent of humidity. These "initial" frost density values were obtained by extrapolating measured frost densities to low times.

The variation of cryogenic frost density with humidity at long times cannot be explained by a fluid dynamical
model, for reasons previously discussed. A more plausible explanation is presented below.

At higher humidities the rates of transfer of mass from the gas to the frost surface tend to be greater because of the larger concentration driving forces. The increased mass transfer rate tends to raise the fugacity of the frost surface, or, in other words, to increase the surface temperature. (In fact, the data show that frost surface temperatures are higher at the higher humidities.) However, as previously mentioned, higher surface temperatures result in increased rates of diffusion of water vapor from the surface into the frost interior. Interestingly enough, these higher internal diffusion rates do not lead to higher frost densities as frost thicknesses increase significantly with increasing humidity. The increased mass of water vapor diffusing into the frost, therefore, has a greater thickness across which to spread. As a matter of fact, the data show that frost density is essentially independent of gas phase humidity.

As just mentioned lower humidities result in lower frost surface temperatures. Since heat transfer coefficients are not affected by gas phase humidity, it would, therefore, be expected that heat transfer rates would increase as the humidity is lowered. This is indeed seen to be the case (see Figure IV-5).

Alternatively, the variation of heat transfer rate with gas phase humidity may be explained by focusing
attention on the frost. The measured "end-of-run" surface temperatures of the low humidity frosts were, in general, about $10^\circ\text{F}$ lower than for the frosts formed at the higher humidities. The temperature difference across the high and low humidity frosts therefore differed only by about 3%. However, at equivalent times, the thicknesses of high and low humidity frosts differed by as much as 20%. Thus, despite the slightly lower temperature driving force across the lower humidity frost, the smaller thermal resistances of frosts formed at lower humidities resulted in significantly higher heat fluxes.

D. Model Predictions Regarding Effects of Reynolds Number and Humidity

Theoretical values of density, thickness, heat flux and surface temperature as functions of time with Reynolds Number and humidity as parameters are shown in Figures V-10 through V-13. Experimental and theoretical values of density and thickness are compared in Figures V-10 and V-11. Experimental and theoretical values of heat flux and surface temperature are compared in Tables V-2 and V-3 respectively. In general, the experimentally observed trends can be predicted from the internal diffusion model. Several exceptions, however, should be noted. The first concerns the frost surface temperature. Theory predicts that the frost surface temperature should increase slightly during the latter phases of a run. Experimental frost
surface temperatures, however, show no trend with time
during these later stages. In addition, the model pre-
dicts that the end-of-run value of the surface temperature
decreases with decreasing humidity and with decreasing
Reynolds Number. The data show that the surface tempera-
ture does decrease with gas phase humidity, but is
approximately independent of the Reynolds Number, at least
within the accuracy of the methods used to measure surface
temperature. Reasons for this discrepancy have already
been presented.

Another exception involves the prediction of the
effect of humidity on frost density. As previously
mentioned the data indicate that frost density is essentially
independent of humidity. However, it is seen in Figure IV-10
that theoretical densities are dependent on humidity, the
effect being somewhat greater at a Reynolds Number of
9,300 than at 5,600. This discrepancy may be due to the
use of heat transfer coefficients which are significantly
larger than the true values, or it may actually be due to
defects in the model itself.

Another exception involves frost thickness. Experi-
mental frost thicknesses decrease with decreasing humidity.
This is exactly the trend predicted by the internal dif-
fusion model. However, experimental thicknesses decrease
slightly with increasing Reynolds Number. The internal
diffusion model predicts the opposite (see Figure V-10).
Actually, the discrepancy is small as theoretical frost thicknesses are very nearly equal at equivalent times and humidities. The same is true of experimental thicknesses.

Finally, it is seen from Figure V-12 that the model predicts that the heat flux decreases very slightly during the latter phases of a run. The data merely indicate that the heat flux becomes essentially constant during this period. In addition, the model predicts that steady heat fluxes should increase with increasing Reynolds Number. This is in agreement with the data. As previously mentioned, however, the data indicate that steady heat flux values increase with decreasing humidity. For a Reynolds Number of 5,600, this observation is in agreement with the predictions of the model (see Figure V-12). However, at a Reynolds Number of 9,300, theory predicts that the constant heat flux values at the two different humidities are very nearly equal. The discrepancy may possibly be due to the use of heat transfer coefficients which are significantly larger than the true values.

Despite the discrepancies mentioned in the preceding paragraphs, the internal diffusion model is nonetheless successful in predicting most of the experimentally observed trends. It therefore seems likely that the model is indeed a reasonably good representation of the physical processes which occur within the frosting system.
5. **Thermal Resistance of Frost**

A. **Comparison Between Experimental and Theoretical Values**

Experimental and theoretical values of frost thermal resistance for the three different Reynolds Numbers and the two humidities are presented in Table V-4. The experimental values were obtained by dividing the measured frost thickness by the product of the measured conductivity and test section area. Theoretical thermal resistances were obtained from theoretical conductivities and thicknesses. The differences between theoretical and experimental resistances reflect the discrepancies between the measured frost surface temperatures and those calculated from the internal diffusion model.

B. **Stability of a Constant Thermal Resistance**

Experiments have shown that the frost thermal resistance becomes constant during the course of a run. It will be demonstrated mathematically that a constant thermal resistance is a stable situation, i.e., once the thermal resistance becomes constant it tends to remain constant. Consider the heat balance between the gas phase and the frost layer.

\[
h_g A(T_g - T_s) + N\lambda = \frac{k_{f,T} A(T_s - T_p)}{\text{Av.}} \quad (V-8)
\]
Now \( R_g = 1/(h_g A) \) and \( R_f = \delta/k_f, T \) A, where \( R_g \) and \( R_f \) are the thermal resistances of the gas and frost respectively.

Thus

\[
\frac{T_g - T_s}{R_g} + N \lambda = \frac{T_s - T_p}{R_f} \tag{V-9}
\]

Algebraic manipulation of Eq. V-9 results in Eq. V-10 shown below.

\[
\frac{(T_g - T_s) + N \lambda R_g}{(T_s - T_p)} = \frac{R_g}{R_f} \tag{V-10}
\]

It is now assumed that, during the constant heat transfer period, a small perturbation causes the value of the frost thermal resistance to rise. This means that the left side of Eq. V-10 has to decrease. This can only occur if the value of \( T_s \) increases. An increase in the value of \( T_s \) results in a decrease in the left hand term of Eq. V-10, because the percentage decrease in \( \left( \frac{T_g - T_s}{T_s - T_p} \right) \) is much larger than the percentage increase in \( \frac{N \lambda R_g}{T_s - T_p} \). (It is noted, of course, that both \( T_g - T_s \) and \( N \lambda R_g \) decrease with increasing \( T_s \). \( R_g \), \( T_p \), and \( T_g \) are constants.)

The higher surface temperature now causes the frost to densify more rapidly because of the increased rate of water vapor diffusion from the frost surface into the
interior. The frost thermal resistance will thus decrease. It can, therefore, be concluded that if a small perturba-
tion causes the frost resistance to rise, a series of events will occur which will force the thermal resistance
to decrease. The final conclusion then is that a constant frost thermal resistance is a stable situation.

6. Initial Heat Fluxes

As mentioned earlier, heat fluxes during the transient period (i.e., when $q_T$ was continuously decreasing) were not measured. However, values of $(q_c + q_{cond})_{initial}/A$, the total heat flux at time zero, were calculated from Eq. V-11.

$$\frac{(q_c + q_{cond})_{initial}}{A} = h_g(T_g - T_s) + k_g'(c_1 - c^*)\lambda \quad (V-11)$$

The second term on the right hand side, of course, repre-
sents the contribution to the heat flux made by the con-
densation of water vapor. Heat fluxes calculated from Eq. V-11 are shown in Figure V-14 along with the measured quasi-steady state values. The steady heat fluxes range from 17 to 24% of the initial values.

The heat transfer coefficients used in the calculation of the initial heat fluxes were obtained from Ref. 87. The mass transfer coefficients were calculated from the Lewis relation. It should be noted that these coefficients
are not fully developed values. In the experiments of the present thesis, the heat transfer test section was only three equivalent diameters downstream from the point at which heat transfer began (i.e., the leading edge of the plate). Therefore heat transfer coefficients corresponding to this downstream position were obtained from Ref. 87.

The experimental heat transfer coefficients of the present thesis and the values of $h_g$ obtained from Ref. 87 are shown in Figure V-15. There is significant disagreement between the two sets of coefficients. The prime reason for the divergence is probably the different geometries involved. The coefficients from Ref. 87 presented in Figure V-15 are frost free values. Thus in Weber's non-frosting experiments a thermal boundary layer develops in the gas stream, but a velocity boundary layer does not. In the experiments of the present thesis, heat transfer coefficients were measured in the presence of a frost layer. Thus a thermal as well as a velocity boundary layer was present. The author is not aware of any literature values of heat transfer coefficients which were obtained with the same geometrical and fluid dynamical system as in the present thesis (i.e., a gas with a fully developed velocity profile which flows over a shape similar to that of a frost layer).

7. **Comparison of Observed Heat Flux Variation with Results of Previous Studies**

The variation of heat flux with Reynolds Number
observed in the present thesis was very similar to that observed in previous studies. In two different high temperature forced convection studies, both Sugawara (14) and Chung and Algren (14) observed that the steady heat flux values increased with increasing Reynolds Number.

In the low temperature forced convection work of Richards et al. (69), it was observed that the value of the quasi-steady state heat flux increased with increasing gas phase Reynolds Number. This conclusion was valid for those runs in which the gas phase humidity was sufficiently low so that no liquid water formed on the frost surface as well as for those cases in which the frost was made up of alternate layers of porous frost and solid ice. Richards et al. also observed that quasi-steady state heat flux increased with decreasing humidity for humidities below 50 grains water/lb dry gas. Their curves of heat flux as a function of humidity with gas velocity as parameter are shown in Figure I-2. The trends observed in the present thesis with regard to the dependence of heat flux on humidity are in agreement with the trends Richards observed in his low humidity runs. In Richards' experiments, the value of the steady heat flux increased with increasing humidity at humidities greater than 50 grains water/lb dry gas. An explanation for this phenomenon was given in Chapter I.

The results of the present thesis are also in agreement with the trends observed in Ref. 87. Weber (87) observed that higher Reynolds Numbers resulted in greater values
of the constant heat flux. He also noted that lower gas phase humidities resulted in higher heat transfer rates. In none of Weber's runs did liquid water condense on the frost surface and result in the formation of an alternately layered frost.

8. **Mass Transfer**

Experimental frost accumulations have already been compared with accumulations calculated from the Lewis relation. It can be seen from Figure V-16 that mass accumulation rates increase with increasing Reynolds Number and humidity.

A. **Effect of Reynolds Number on Mass Transfer Rates**

If frosts were deposited solely by particle transfer, it would be reasonable to expect that higher Reynolds Numbers would result in larger mass transfer rates. This would be due to the greater intensity of turbulence present in the higher Reynolds Number streams.

For those cases in which frosts are formed solely by molecular and turbulent diffusion, it would also be expected that higher Reynolds Numbers would result in higher rates of mass deposition. This expectation is based on two factors: first, the increase in mass transfer coefficients with increasing Reynolds Numbers and, secondly, the experimentally observed fact that Reynolds Numbers does not significantly affect the frost surface temperature.

If the theoretical variation of frost surface temperature with Reynolds Number were just considered, then it is
not obvious a priori that higher Reynolds Numbers would lead to increased mass transfer rates. In fact, theory indicates that frost surface temperatures are lower at the lower Reynolds Numbers, and therefore the concentration driving force between the gas phase and the frost surface is greater than at the higher Reynolds Numbers. However, the slight reduction in the surface temperature resulting from the decreased Reynolds Number does not lower the concentration driving force sufficiently to cause lower Reynolds Numbers to result in larger rates of mass deposition.

B. Effect of Humidity on Mass Transfer Rates

When frost accumulates solely by particle transfer, it is difficult to state beforehand just what effect humidity will have on the deposition rate. It is certainly reasonable to expect that humidity would not affect the gas phase turbulence intensity. However, it is believed that a higher humidity would result in a larger particle concentration driving force between the gas phase and the frost surface since nucleation theory predicts (1) that nucleation rates of particles are greater at the higher humidities. Thus rates of deposition would be greater at the higher humidities because of the increased particle concentration driving force (31).

It is reasonable to expect that when frost is being deposited primarily by diffusion, rates of deposition would be higher at the higher humidities. This would occur
because the water vapor concentration driving force would be greater at the higher humidities. It should be noted, however, that experiments showed the frost surface temperatures to be lower at the lower humidities. The equilibrium water vapor concentrations at the surface, however, are not sufficiently lowered to offset the greater water vapor concentrations present in the bulk at the higher humidities. Thus when frosts are deposited by diffusion, higher humidities do result in greater rates.

C. Comparison of Results with Low Temperature-Natural Convection Studies

(1) Early Phases of a Run

As has already been stated, in natural convection, low temperature studies, a huge discrepancy exists between experimental deposition rates and those calculated from the Lewis relation. This discrepancy has generally been attributed to the formation of ice particles in the boundary layer. It is interesting to note that in the present studies the experimental transfer rate, during the period when frost was accumulating by particle transfer, was roughly 50% of the rate predicted from the Lewis relation. This estimate was arrived at by taking the slope of the mass accumulation curve. Based on this very rough estimate, it is postulated that the discrepancy between the theoretical (i.e., those calculated from the Lewis relation) and the experimental rates is less for the experiments of the present thesis than for the low
temperature, natural convection experiments (7,40,72). The reason for the apparently smaller discrepancy is the presence of gas stream turbulence in the experiments of the present thesis. The turbulence transports to the wall many of the particles which otherwise would have been swept past the plate by the moving gas. In the natural convection experiments turbulence, presumably, was not significant.

(2) Latter Phases of a Run

The agreement between the experimental and theoretical rates for the latter stages of a run is much better for the runs of the present thesis than for the low temperature, natural convection studies. In the latter stages of a run (of the present thesis), mass transfer coefficients can, theoretically, be calculated from the Lewis relation as frost accumulates by molecular and turbulent diffusion during this period. For low temperature, natural convection runs, however, it has previously been indicated that the frost surface temperature always remains sufficiently low so that particles form in the gas phase boundary layer throughout the run. Under such conditions, of course, experimental rates of mass accumulation are expected to be lower than rates computed from a standard mass transfer analogy such as the Lewis relation.

D. Comparison of Results with Low Temperature, Forced Convection Studies

In general, it appears that the agreement between
experimental mass transfer rates and end-of-run "theoretical rates" is better for the experiments of the present thesis than in Richards' work. Richards also calculated his theoretical rates from the Lewis relationship. This better agreement may be due to geometrical differences, but no definite conclusion can be reached at the present time.

Another possible explanation for the greater discrepancy in Richards' results is presented below. Richards' experimental rates shown in Figure I-3 are average rates for the entire run. The variation of experimental mass deposition rate with time in Richards' experiments is not known. Thus, a comparison between average experimental and end-of-run theoretical rates as shown in Figure I-3 is not really a fair test. Richards should have compared his end-of-run theoretical mass transfer rates with his end-of-run experimental rates, rather than with his average experimental values.

For the experiments of the present thesis, the end-of-run experimental mass transfer rates are known approximately. The comparison between these experimental rates and the end-of-run theoretical rates (i.e., calculated from the Lewis relation) has already been shown (see Table V-1). (Note that for the experiments of the present thesis, the gas velocity corresponding to the highest Reynolds Number employed is about 17 ft/sec.)

Mass transfer to the relatively short (i.e., about 8.9 inches) growing frost layer of the present thesis is not a fully developed flow or heat and mass transfer situation. A velocity boundary layer forms in the gas phase over the
frost even though the velocity profile in the gas approaching the frost is a fully developed one. In addition, mass transfer starts at the leading edge of the frost layer, and there is not sufficient flow length over the frost for the mass transfer coefficient and the water vapor concentration profiles to reach their fully developed values.

In this type of situation, the Sherwood-Gilliland equation, for example, could not be used to calculate mass transfer coefficients. This is an empirical correlation which was formulated using fully developed values of mass transfer coefficients. The Lewis relationship, on the other hand, is an analogy between heat and mass transfer and is therefore approximately valid in the entrance region (30). Thus, to obtain correct mass transfer coefficients from the Lewis relation, the appropriate value of the "entrance" heat transfer coefficient has to be known.

As mentioned in many standard texts, the Lewis relation is most accurate for the air-water system. It is, therefore, considered reasonably accurate to use it for the nitrogen-water system employed in the present thesis.

E. Comparison with High Temperature-Forced Convection Studies

Chung and Algren (14) evaluated theoretical mass transfer rates using a correlation for mass transfer coefficients applicable to flow past single cylinders. As previously mentioned, these investigators found that the theoretical mass transfer rates showed good agreement with their
experimental values of frost accumulation rates. This would be expected as the studies were conducted at relatively high temperatures, thus ensuring that deposition occurred solely by the molecular and turbulent diffusion of water vapor from the gas phase to the frost surface.

Actually, in Chung's and Algren's runs there seemed to be somewhat better agreement between the experimental and theoretical rates than for the runs of the present thesis. It is doubtful that these differences were due to erosional effects. It has previously been indicated that in the latter phases of a run (i.e., of the present thesis), the shear stress was several orders of magnitude lower than the shear strength of the frost. It is also unlikely that the geometrical differences were responsible for this slightly better agreement. Thus, at present no conclusions can be drawn regarding the slightly better agreement obtained by Chung and Algren.

9. Thermal Conductivity of Frost

Frost is a heterogeneous material. The solid phase is an array of ice particles, and the pore phase consists of gas. As previously mentioned, frost thermal conductivity is dependent only on the frost structure and the temperature. Since frost density does not necessarily imply a particular structure, frosts at the same temperature and density can have significantly different values of conductivity. For the purposes of the present thesis, however, it was assumed, despite any possible shortcomings of this assumption, that a given value of frost density implied a particular structure regardless of the conditions under which the frost was formed. This is tantamount to assuming that a
property such as frost thermal conductivity depends solely on temperature and density. Experimental values of point or local frost thermal conductivity were therefore plotted as a function of frost density with temperature as a parameter (see Figure IV-9). The scatter in the data may possibly be due to small structural differences of frosts of the same density.

The average thermal conductivities of the frost layer, shown in Figure IV-7, were obtained using the measured temperature differences across the layer. These values represent average conductivities as there is a considerable variation of thermal conductivity with position because of the large temperature gradient within the frost. The temperature differences across the frost samples, whose conductivities are shown in Figure IV-7, are substantially the same. However, it should be noted, as previously indicated, that the measured surface temperatures of the frosts ranged from $10^\circ$ to $25^\circ$F. The scatter in the data may be partly due to the different surface temperatures. However, the more plausible explanation is that structural differences of frosts having equivalent densities are primarily responsible for the scatter.

It should be noted that heat also flows through the frost by mechanisms other than conduction through the solid gas network. Thus the values of thermal conductivity shown in Figures IV-7 and IV-9 include contributions made by other mechanisms. These mechanisms are:
(1) radiation between the ice particles in the frost
(2) gas flow through the frost perpendicular to the
direction of heat transfer
(3) enthalpy transport by the diffusion of water vapor
through the frost. This diffusion may be due either to
the concentration gradient, resulting from the temperature
gradient, or to thermal diffusion. The transport of enthalpy
resulting from the ordinary concentration gradient is
denoted as evaporative transport.

A. **Effect of Radiation**

Radiation from the warmer frost particles to the colder
ones constitutes a component of the total or over-all frost
thermal conductivity. The contribution made by inter-
particle radiation was calculated using Eq. V-12, which was
presented by Damköhler. (17) (See Appendix C.)

\[ k_r = \frac{0.685}{100} \epsilon D_p \left( \frac{T}{100} \right) e_H \] (V-12)

(In experiments involving heat transfer through a packed
bed, Polack (67) found that the contribution of radiation
to the bed conductivity was in fair agreement with the pre-
dictions of the Damköhler equation.)

For the experiments of the present thesis, it was
calculated from the Damköhler equation that the component
of thermal conductivity due to radiation was less than 2%
of the lowest measured value of the point frost thermal conductivity (see Appendix C). It should be noted that the values substituted into Eq. V-12 made the computed value of the contribution to the thermal conductivity by inter-particle radiation a maximum. Since this maximum contribution to the over-all thermal conductivity was small, it was ignored.

The above described result regarding the internal radiant heat transfer is in agreement with the conclusion of van der Held (86), who demonstrated that for particulate insulators the radiation heat transfer becomes significant only at temperatures above 212°F and is negligible below 32°F.

B. Effect of Gas Flow Through Frost

The gas stream flowing past the frost undergoes a pressure drop because of expansion and contraction of the stream and also because of skin friction. A pressure difference across the frost is thus created. Since the frost is quite porous, this pressure difference causes gas to flow through the frost itself. The gas flow increases the over-all frost thermal conductivity by increasing the effective thermal conductivity of the gas phase.

Before the effect of the gas flow on the frost thermal conductivity could be evaluated, the actual flow rate of the gas through the frost had to be estimated. The rate of
gas flow through the frost was calculated by two independent methods. In one case, the Ergun correlation \((28)\) for flow through packed beds was used. (See Appendix C for more details.) The final Ergun expression used for calculating the flow rate is

\[
\frac{\Delta P_f}{L} = \frac{\mu G_o (150)(1 - \varepsilon)^2}{g_c (D_p)^2 \rho_g \varepsilon^3} \quad \text{(V-13)}
\]

In the other method, D'Arcy's law (Eq. V-14), in conjunction with snow permeability data obtained from Ref. 4, was used.

\[
\frac{G_o}{\rho_g} = K_D \left(\frac{\Delta P_f}{L}\right) \quad \text{(V-14)}
\]

It is seen in Appendix C that the calculated flow rate, using D'Arcy's law, was found to be significantly greater than the value calculated from the Ergun correlation. Since it was desired to calculate the maximum possible effect that gas flow through the frost had on the frost conductivity, the value calculated from D'Arcy's law was used. D'Arcy's law probably yields a more reliable estimate as actual permeability data are utilized.

A method for predicting the effect of gas flow normal to the direction of heat transfer on the heat transfer through a packed bed was developed by Yoon \((92)\). (See
Appendix C for more details.) It was shown that when heat is transferred perpendicularly to the direction of a flowing gas, the modified Peclet number is equal to 11 over a wide range of Reynolds Numbers, particle sizes and solid thermal conductivities. The definition of the modified Peclet number is presented below:

\[
\text{Pe} = \frac{D_p \left( \frac{G_o}{\varepsilon} \right) c_p}{k_{f,f} - k_f} \quad (V-15)
\]

Equation V-16 was derived from Eq. (V-15) by simple algebraic manipulation.

\[
\frac{k_{f,f}}{k_g} = \frac{k_f}{k_g} + \left( \frac{1}{11} \right) Pr \left( \frac{D_p G_o}{\mu \varepsilon} \right) \quad (V-16)
\]

Since Eq. V-16 is valid over a wide range of Reynolds Numbers, particle sizes and solid thermal conductivities, this equation was considered applicable to the frosts of the present thesis.

Using Eq. V-16 and the value of the gas flow calculated from D'Arcy's law, the contribution to thermal conductivity due to gas flow through the frost was computed to be less than 0.02% of the lowest measured conductivity value. Even if the true gas flow through the frost were 100 times greater than the value calculated from D'Arcy's law (an unlikely situation), the contribution made by
this flow to the over-all frost thermal conductivity would still be only 2% of the lowest measured conductivity value.

From Figure IV-7 it can be seen that at equivalent densities frosts formed at different Reynolds Numbers have about the same value of thermal conductivity. The gas flows through the frost were greater at the higher Reynolds Numbers, but for a given density the increased rate of gas flow did not seem to affect the measured thermal conductivity values. If it is assumed that frosts of equivalent densities have the same structures, then the above facts constitute an experimental verification that the over-all frost thermal conductivity was not affected by the gas flow through the frost.

The work of Willhite, Kunii and Smith (90) corroborates the above analysis and experimental observations. These investigators found that gas flow through a packed bed of very small particles (diameters ranging from 29 to 940 microns) had no effect on the bed thermal conductivity. The conductivity was measured in a direction normal to the gas flow.

Based on the theoretical analysis and the experimental evidence described above, it was concluded that the effect of gas flow on frost thermal conductivity was insignificant.
C. Diffusion of Water Vapor Through Frost

Diffusion of water vapor from the warmer parts of the frost to the colder regions represents a transport of enthalpy and is, therefore, an additional component of the frost thermal conductivity. There are two mechanisms which give rise to this interior flux of water vapor. One, the temperature gradient across the frost leads to a large concentration gradient which results in water vapor diffusion. Secondly, water vapor is transported through the frost by the mechanism of thermal diffusion. A priori it would seem that in the lower parts of the frost, where the temperature gradient is steep and the concentration gradient small, water vapor flux due to thermal diffusion is important. This, however, is not the case. Terms proportional to the driving force gradients for the water vapor fluxes arising from these two sources (i.e., usual molecular diffusion caused by a concentration gradient, and thermal diffusion) are shown in Table V-5. The ratio of the two terms $(d\eta_1/dT)$ and $(0.175/T)\eta_1\eta_2$ is the same as the ratio of the mass fluxes. (The equation for calculating the total flux is presented in Appendix C.) As can be seen from the table, the water vapor flux caused by thermal diffusion is negligible compared with the flux due to the concentration gradient at all levels within the frost. The water vapor flux due to thermal diffusion can therefore be ignored. Actually the concentration gradient and thermal diffusion cause water vapor to flux in opposite directions. In this thesis only the magnitude of these two effects have been compared.
As previously mentioned the water vapor flux resulting from the concentration gradient is denoted as evaporative transport. Since the diffusion may be considered to occur solely in the gas phase, the additional component of thermal conductivity arising from this diffusion can be added to the ordinary thermal conductivity of the nitrogen gas. This additional component, which is due to evaporative transport, is designated as $k_{xs}$. Thus the total or effective value of the gas phase conductivity is equal to the sum of $k_{xs}$ and $k_g$, where $k_g$ is the ordinary value of gas thermal conductivity. Thus, in calculating the thermal conductivity of frost from different assumed structural models, the total conductivity of the gas phase ($k_g + k_{xs}$), rather than just the intrinsic gas conductivity ($k_g$), was used.

An expression for $k_{xs}$ can be obtained by considering the total heat flux through the pore phase at some point in the frost.

$$\frac{q}{A} = k_{g,e} \frac{dT}{dy} = k_g(\frac{dT}{dy}) + \frac{D_{1-2}}{RT} M \frac{dp^*}{dy} \lambda \quad (V-17)$$

A more detailed explanation of Eq. V-17 is presented in Appendix C. From Eq. V-17 and the definition of $k_{xs}$,

$$k_{xs} = k_{g,e} - k_g \quad (V-18)$$

Eq. V-19 is derived:
\[
k_{xs} = \frac{D_{1-2}}{RT} \left( \frac{dP^*}{dT} \right) \lambda M_v
\]  
(V-19)

Combining Eq. V-19 with the Clausius-Clapeyron (61)

\[
dP^*/dT = \frac{M_v \lambda P^*}{RT^2}
\]

the final expression for \( k_{xs} \) is obtained:

\[
k_{xs} = \frac{D_{1-2}}{R^2 T^3} \lambda^2 P^*(M_v)^2
\]  
(V-20)

The computed values of \( k_{xs} \) as a function of temperature are shown in Table V-6. At positions within the frost, where the temperature is lower than \(-10^\circ F\), evaporative transport makes a small contribution to the total gas phase thermal conductivity and hence to the point conductivity of the frost. However, evaporative transport becomes significant at higher temperatures. For example, at \(20^\circ F\) the contribution is about 37% of the total gas phase conductivity. Obviously, evaporative transport increases the over-all conductivity of the frost since it increases the total gas phase conductivity. The exact contribution made by evaporative transport to the over-all thermal conductivity of the frost is dependent on the real frost structure. This contribution usually cannot be calculated exactly as the true frost structure generally cannot be mathematically characterized.
Thus to calculate the contribution made by evaporative transport, a particular structure has to be assumed. As an illustration, suppose the frost consists of a series of parallel slabs of ice with heat flow parallel to these slabs. Then for a 20°F frost having a density of 0.09 gm/cc thermal conductivity with evaporative transport is equal to 0.145 BTU/hr-ft²-°F/ft, whereas without evaporative transport the conductivity is 0.138 BTU/hr-ft²-°F/ft. Thus, for this particular model, evaporative transport adds 5.1% to the point conductivity. However, if the frost structure is idealized to consist of a parallel array of slabs with heat flow normal to the array, then the conductivity including evaporative transport is 0.024 BTU/hr-ft²-°F/ft. Without evaporative transport the thermal conductivity is 0.015 BTU/hr-ft²-°F/ft. Thus, in this case evaporative transport increases the point conductivity by 60%.

As a further illustration, assume that the frost consists of a cubic array of ice spheres. For this model, the thermal conductivity with evaporative transport was calculated to be 0.0420 BTU/hr-ft²-°F/ft, whereas without evaporative transport conductivity is equal to 0.0293 BTU/hr-ft²-°F/ft. Thus, for this particular model, evaporative transport makes a difference of 43.5%.

The contribution of evaporative transport to the average conductivity of the frost layer can be calculated
from the curves of Figure V-17, which are applicable for a model consisting of a cubic array of ice spheres. The curve ABC represents the thermal conductivity with evaporative transport, whereas ABC' represents the thermal conductivity without evaporative transport. The average conductivity of the frost layer is given by

\[ k_{f,T}^{\text{Av.}} = \frac{\int_{T_p}^{T_s} k_{f,T} \, dT}{T_s - T_p} \]  

\[ (V-21) \]

(For the purposes of the present thesis, the terms \( k_{f,T} \) and \( k_f \) may be used interchangeably as gas flow through the frost and interparticle radiation have a negligible effect on the over-all frost conductivity.) The area under the curve ABC divided by \( (T_s - T_p) \) is equal to the average thermal conductivity with evaporative transport. On the other hand, average frost thermal conductivity without evaporative transport is equal to the area under the curve ABC' divided by \( (T_s - T_p) \). Thus conductivity with evaporative transport was found to be equal to 0.0214 BTU/hr-ft\(^2\)-\(^{\circ}\)F/ft, whereas without evaporative transport the average conductivity was found to be 0.0209 BTU/hr-ft\(^2\)-\(^{\circ}\)F/ft. Even though the effect of evaporative transport on the point conductivity is quite significant at 20\(^{\circ}\)F, this phenomenon only affects the value of the average conductivity of a frost layer having a surface temperature of 20\(^{\circ}\)F by about 4.4%.
D. Variation of Conductivity with Density

(1) Average Frost Layer Conductivity

Average frost layer conductivity is plotted as a function of density in Figure IV-7. Since it has been shown that radiation, gas flow through the frost, and evaporative transport do not affect the average thermal conductivity of the frost layer significantly, it is concluded that the major contributor to the average frost thermal conductivity is conduction through the solid gas matrix.

The thermal conductivity increases with increasing frost density, because at the higher densities larger areas of low thermal resistance paths are present. The larger contact areas are a consequence of the increased solids fraction. In addition, higher density frosts may have structures which are significantly different than lower density frosts. These different structural characteristics may also be partly responsible for the increased conductivities at the higher densities.

(2) Point Thermal Conductivity

Point thermal conductivities at different density values and with temperature as a parameter are plotted in Figure IV-9. As was true for average frost layer conductivities, higher densities result in larger values of point conductivities because of the larger areas of low
thermal resistance paths. These large areas are a consequence of the increased percentages of solid present at the higher frost densities. In addition, possible structural differences implied by the higher densities may be partly responsible for the increased conductivities.

E. Variation of Thermal Conductivity with Temperature

At equivalent densities higher frost temperatures lead to increased thermal conductivity values for three reasons. First, the contribution to the over-all thermal conductivity due to evaporative transport increases at higher temperatures. Secondly, it is believed that the contact areas between the various ice particles increase with increasing temperature. At the higher temperatures the processes of constructive and destructive metamorphism (21,47) are more active, and it seems likely that additional water vapor is transferred to the contact areas between the particles. Thirdly, the conductivity of the gas phase increases with increasing temperature and, at the high frost porosities encountered in this thesis, the gas conductivity is very important in determining the over-all frost conductivity. Thus, despite the fact that ice thermal conductivity decreases with increasing temperature, the intrinsic conductivity of the ice-gas matrix increases at the higher temperatures. Thus frost thermal conductivity would increase with increasing temperature even if increased particle contact areas and evaporative transport were not present at the higher temperatures.
F. Comparison of Measured Frost Thermal Conductivities with Data for Natural Snows

Since the temperature of the snow samples is about 20°F, these snow data should be compared only with frost data at the same temperature. In Figure V-18 the upper curve represents the snow data of Artsybashev and Yushakov (48), whereas the lower curve represents the data of Jansson (43). The measured 20°F frost conductivities are also shown in Figure V-18.

The frost data are seen to be bracketed by the curves representing the different sets of snow data. It is also seen that the divergence between the measured snow conductivities of the different investigators is quite large. As indicated earlier this divergence is probably due to the fact that there can be large structural differences between two snow samples of the same density.

G. Comparison of Data with Various Frost Models

Since cryogenic frosts consist of an array of irregular particles, it is clearly impossible to mathematically model the frost exactly. In order to calculate values of thermal conductivity, one has to assume an idealized geometrical array of regularly shaped particles. Different idealized frost structures are described below. Of course, none of them represents the real structure, but these models are nonetheless quite useful as conductivities calculated from
them often provide bracketing limits for the real values. These limits can generally be used to extend conductivity data over a range where no data exist. In fact, for some of the models described below, such as a cubic array of ice cubes, the structures cannot physically exist, i.e., the frost would collapse. A more detailed discussion of the models to be presented below is contained in Appendix D. For obvious reasons only a limited number of geometric idealizations could be considered. Reference (33) describes a number of different structures not considered below.

(1) Parallel Slabs of Ice

In texts (46) dealing with the conductivity of heterogeneous materials, two different cases are generally considered. In one case, heat conduction is parallel to the array of slabs of ice; in the other, the flow of heat is perpendicular to the array of slabs. Maximum thermal conductivity of a heterogeneous structure is attained when heat flow is parallel to the array of ice slabs, and minimum conductivity occurs when the heat flow is normal to the slabs.

In the former case individual thermal resistances of the ice and gas are in parallel; but in the latter case, these individual resistances are in series. The derivations of the equations for calculating the thermal conductivity for each of these two cases is presented in Appendix D.
For heat flow parallel to the ice slabs, the final expression is

$$k_F = \varepsilon k_{g,e} + (1 - \varepsilon)k_1 \quad (V-22)$$

When heat flow is normal to the arrangement of slabs, the final expression is

$$k_F = \frac{k_{g,e}k_1}{\varepsilon k_1 + (1 - \varepsilon)k_{g,e}} \quad (V-23)$$

(Note that the gas phase thermal conductivity is equal to the ordinary conductivity plus the contribution to the conductivity made by evaporative transport.) Obviously, in deriving Eqs. V-22 and V-23, the actual number of ice slabs or gas spaces involved is immaterial. Thus, for the parallel heat flow case, the assumed structure could have been represented either by Figure D-1A or D-1B; for perpendicular heat flow, either Figure D-2A or D-2B could have represented the structure.

The thermal conductivity predicted from the parallel slab models are shown in Figures V-19 through V-22 for temperatures of 20°, 0°, -80°, -145° and -215°F. The experimental data are also included in these figures. The upper lines correspond to the case of heat flow parallel to the ice slabs; the lower lines to the case of heat flow normal to the slabs. In the former case, the calculated
conductivities are higher than the data, but in the latter the experimental conductivities are higher than the predictions of the model. Clearly, the assumption of heat flow parallel to the ice slabs overestimates the actual contacts between ice particles in the frost. On the other hand, the actual contacts between ice particles in the frost are better than represented by the case of heat flow perpendicular to the ice slabs.

At temperatures of \(-145\,^\circ F\) and \(-215\,^\circ F\), the experimental data lie much closer to the lower curves than to the upper ones. As frost temperature increases, however, the data recede from the lower curve and approach the upper one. This tends to indicate that particle contact areas increase at the higher temperatures.

(2) **Simple Cubic Array of Non-touching Ice Cubes**

For this structure (see Figure D-3) two distinct cases have to be considered. In one case, it is assumed that the thermal conductivity in a direction perpendicular to the heat flow is zero. In the other case, it is assumed that the conductivity in this direction is infinite. The assumption of infinite sidewise conductivity implies that the isotherms are parallel planes. The over-all thermal resistance of a unit cube is therefore obtained by considering the resistances of Zones 1', 2' and 3' to be in series (see Figure D-5). The assumption of zero sidewise
conductivity, however, implies that the planes of constant heat flux are parallel. Thus the over-all thermal resistance of the unit cube is obtained by considering the thermal resistances of Zones 1 and 2 to be in parallel (see Figure D-4). The detailed derivations of the expressions for the thermal conductivity for these two different cases is presented in Appendix D. For the case of infinite side-wise conductivity, the final expression is

$$\frac{k_f}{k_{g,e}} = \frac{x^2 + \left(\frac{k_{g,e}}{k_1}\right)(1-x^2)}{(1-x)\left[x^2 + \frac{k_{g,e}}{k_1}(1-x^2)\right] + x\left(k_{g,e}/k_1\right)}$$

(V-24)

When the conductivity is assumed to be zero in the side-wise direction, the final expression is

$$\frac{k_f}{k_{g,e}} = (1-x^2) + \frac{x^2}{x\left(k_{g,e}/k_1\right) + (1-x)}$$

(V-25)

Using Eqs. V-24 and V-25, curves of thermal conductivity as a function of density can be generated. In Figures V-24 and V-25, these curves are shown for temperatures of \(-145\) and \(-215\,^\circ\text{F}\) respectively. The measured conductivities at these temperatures are also shown in these diagrams.
It should be noted that the calculated thermal conductivity values for the case of infinite sidewise conductivity are greater than the calculated values for the case of zero sidewise conductivity. Actually the true conductivity values for the assumed structure of a cubic array of ice particles lie between the values calculated for the two cases assumed above. As has previously been mentioned, when infinite sidewise conductivity is assumed, the isotherms are parallel planes and, when zero sidewise conductivity is assumed, the planes of constant heat flow are all parallel. In reality, for a frost structure consisting of a cubic array of ice cubes, both the heat flow planes and the isotherms are curved. The conductivity for this structure can, therefore, be calculated most accurately by a relaxation technique. In fact, relaxation techniques have been used to calculate the thermal conductivity of regularly packed arrays (19, 20). For the assumed structure of a cubic array of ice cubes, it was decided, however, that nothing would be gained by using a relaxation technique since the model did not correspond to reality. It was therefore considered sufficient to calculate curves which would bracket the results of a more accurate relaxation calculation.

In general, the conductivity values based on Eqs. V-24 and V-25 fall below the data. This tends to indicate that a model consisting of a cubic array of ice cubes does not adequately represent the contacts between the actual ice
particles in the frost. A conductivity versus density curve, calculated by an accurate relaxation technique, would, of course, also have fallen below the experimental data.

(3) **Simple Cubic Array of Cubes of Gas**

A model closely related to the one just described is a cubic array of gas cubes interspersed in a continuous ice network (see Figure D-6). This structure, unlike most of the structures previously considered, can exist physically. However, for this model only the case of zero sidewise conductivity will be considered. The detailed derivation of an expression for calculating the thermal conductivity of this array is given in Appendix D. The final expression is:

\[
\frac{k_f}{k_i} = 1 - (\varepsilon)^{2/3} + \frac{\varepsilon^{2/3}}{\left[\varepsilon^{1/3} \left(\frac{k_i}{k_{g,e}}\right)^{1/3} + 1 - \varepsilon^{1/3}\right]}
\]

(Eq. V-26)

Equation V-26 was derived by finding the equivalent resistance of a parallel combination of the resistances of Zones 1 and 2 (see Figure D-7).

In Figure V-26, the -145°F data are compared with the thermal conductivities calculated from Eq. V-26. The calculated conductivities fall far below the data. This most
likely means that the model over-represents the contacts between the ice particles in the real frost.

(4) Weber Model (87)

Weber assumed that the frost consisted of two phases: a pocket phase and a cluster phase. The cluster phase was made up of an array of closely packed spherical ice particles. The pocket phase consisted of spheres of gas, arranged in a cubic array within the cluster phase (see Figure D-8). The gas pockets were assumed to be an order of magnitude larger than the individual ice particles contained in the cluster phase. This type of array was chosen, because such a model could exist physically, i.e., it has a coherent structure, unlike a non-touching array of ice particles. Weber used this model to calculate frost thermal conductivity as a function of density with temperature as parameter. (A more detailed description of the Weber model is presented in Appendix D.)

In Figures V-27 and V-28 the data are compared with the conductivities calculated from the Weber model. Figures V-27 and V-28 are applicable at temperatures of -145°F and -215°F respectively. As can be seen from these figures, the thermal conductivities based on the Weber model fall below the data. The primary reason, probably, is that the assumed structure did not adequately represent the contacts between the particles in the frost. In
addition, the assumption of zero sidewise conductivity resulted in lower values than would have been calculated by a relaxation technique.

(5) Simple Cubic Array of Non-Touching Ice Spheres (Woodside Model)

Woodside assumed that the frost, or snow, structure consisted of a cubic array of spherical particles. Actually, for frost or snow with densities lower than about 0.48 gm/cc, such a structure cannot exist physically. As previously mentioned, however, physically non-realistic structures are often assumed for thermal conductivity calculations as they have been found to yield values in substantial agreement with experimental data.

Woodside assumed that the thermal conductivity in the direction perpendicular to the heat flow was infinite. This is tantamount to assuming that the isotherms are parallel planes. The final expression for \( \frac{k_f}{k_{g,e}} \) was derived by considering the resistances of Zones I and II to be in series. The resistance of Zone 1 was a parallel combination of the resistances of the solid and gas in that zone (see Figure D-11). The detailed derivation is given in Appendix D. The result is

\[
\frac{k_f}{k_{g,e}} = \left[ 1 - \left( \frac{5(1-\varepsilon)}{\pi} \right)^{1/3} \left[ 1 - \frac{(a^2-1)}{2a} \ln \left( \frac{a+1}{a-1} \right) \right] \right]^{-1}
\]

(V-27)
where

\[ a = \left\{ \frac{1}{\pi} \left[ \frac{6(1-\epsilon)}{\pi} \right]^{2/3} \right\}^{1/2} \]

and

\[ \psi = \frac{k_i - k_{g,e}}{k_{g,e}} \]

Equation V-27 was used to calculate frost thermal conductivities at various densities with temperature as a parameter. These curves are shown in Figures V-27 through V-31. In calculating the curves the total gas phase conductivity was used, i.e., the contribution to the gas phase conductivity due to evaporative transport was taken into account as was done in calculating conductivities for all of the assumed models.

Comparison of the theoretical curves with the data at temperatures of +20°, 0°, -80°, -145° and -215°F are shown in Figures V-27 through V-31. At all temperatures, with the exception of -215°F, the data are higher than the theoretical curves. These differences cannot be attributed to the assumption of infinite sidewise conductivity as this assumption results in higher conductivity values than would be calculated using a relaxation technique. One is therefore led to conclude that the differences between the theoretical curves and the data are
probably due to inadequate representation of the contacts between the ice particles in the frost.

It is interesting to note that the discrepancy between the theoretical curves and the data increases at the higher temperatures. This tends to indicate that those sections of the frost at the higher temperatures have different structural characteristics than the sections at the lower temperatures, even though the densities of these different sections are the same. It would seem that at the higher temperatures contact areas between ice particles in the frost are greater than at lower temperatures. The subject of particle contact areas as a function of temperature will be discussed in more detail when the cube and rod model is considered.

(6) Deissler Models

In calculating thermal conductivities of heterogeneous materials, Deissler used both a "simplified" and a "refined" analysis. In the simplified analysis (29), thermal conductivities were calculated for two different idealized arrays: namely, a simple cubic array of solid spheres and a square array of long cylinders. (Krupicska (49) has analyzed the conductivities of these two types of structures by a more rigorous method than Deissler's simplified analysis.) The array of spheres has a porosity of 0.475, whereas the array of cylinders has a porosity of 0.214. Deissler calculated thermal conductivities for these
two arrays at various temperatures by assuming that the conductivity was zero in the sidewise direction. The details of the derivation of the equations used for these calculations are presented in Ref. (20). The derivation is similar to that of the equations used in the Woodside model.

More specifically, Deissler generated curves of \( \frac{k_H}{k_G} \) as a function of \( \frac{k_s}{k_G} \) for the two different porosities mentioned above. Applying these two generalized curves to frost, and utilizing the known conductivities of pure ice and pure gas, four points on a conductivity versus porosity plot were established at two different temperatures, -215 and -145°F. Conductivity values at porosities ranging from 0.95 to 0.80 were, therefore, obtained by interpolation. The values are plotted in Figures V-27 and V-28. (Note that the abscissa is frost density rather than porosity.)

As previously noted, when thermal conductivities are calculated using either the assumption of zero or infinite sidewise conductivity, the calculation does not take into account the curvature of the isotherms and of the lines of constant heat flux. If this curvature is to be considered, the thermal conductivity of the heterogeneous structure should be calculated by a relaxation technique. In Deissler's "refined" conductivity calculations (19,20), the thermal conductivities of three different structures were calculated by a relaxation technique. (The method of calculating conductivities by relaxation is described in
Ref. 20.) Two of these structures are the same as employed in the simplified analysis. The third structure is a triangular array of infinitely long cylinders, which has a porosity of 0.093.

The known conductivities of pure ice and pure gas plus Deissler's "refined analysis" curves of $k_H/k_G$ as a function of $k_S/k_G$ were used to establish plots of thermal conductivity as a function of porosity. This was done for temperatures of -215 and -145°F. As in the simplified analysis, conductivity values at porosities ranging from 0.95 to 0.80 were obtained by interpolation. These interpolated values are shown in Figures V-27 and V-28.

It can be seen from Figures V-27 and V-28 that the "simplified analysis" curves lie slightly below those calculated by the "refined analysis". In fact, curves calculated, assuming infinite sidewise conductivity, would lie above the "refined analysis" curves.

From Figures V-27 and V-28, it is seen that the conductivities calculated by the "refined" and "simplified" methods are significantly lower than the data. The apparent reason, as with other proposed structures, is that Deissler's models do not adequately represent the true contacts between particles in the frost.

(7) Proposed Model: Cubes of Ice Connected by Ice Rods

An assumed structural model developed in this thesis showed close agreement with actual thermal conductivity
data. The assumed model is a symmetrical matrix of ice cubes interconnected by ice rods. The rods are square prisms (see Figure E-1). As is the case for other structural models, this structure does not represent the true physical situation. However, a desirable characteristic of this model is that the dimensions of the rods may be taken as a measure of the actual contact between the ice particles.

An equation for calculating frost thermal conductivity is derived in the same manner as for other structural models, i.e., by considering a typical unit cube of the heterogeneous structure (see Figure E-2). \( x \) is the linear dimension of the ice cube, and \( z \) is the width of the square ice rod. Actually \( x \) and \( z \) are dimensionless as they represent, respectively, the ratio of the ice cube length to unity and the ratio of the rod width to unity. It is, of course, obvious that the actual size of the unit cube does not affect the computed value of the thermal conductivity.

From Figure E-2 it is seen that \( (1 - \varepsilon) \), the solids fraction, is given by

\[
(1 - \varepsilon) = x^3 + 3z^2(1 - x) \quad (V-28)
\]

\( (1 - \varepsilon) \) is related to the densities of the ice, gas and frost by

\[
(1 - \varepsilon) = \frac{\rho_f - \rho_g}{\rho_f - \rho_g} \quad (V-29)
\]
The expression for the thermal conductivity was derived by considering the unit cube to consist of several individual thermal resistances in series. These resistances are shown as separate zones in Figure E-2. Adding the individual resistances in this manner is equivalent to assuming that the thermal conductivity of the heterogeneous structure is infinite in a direction perpendicular to the heat flow. The final expression for the thermal conductivity is

\[
\frac{1}{k_f} = \frac{(1 - x)}{k_{g,e}(1-x^2) + k_1 z^2} + \frac{(x - z)}{k_{g,e}(1-x^2) + k_1 x^2} + \frac{z}{2k_1 z(1-z) + k_1 x^2 + k_{g,e} \left[1 - x^2 - 2z(1-x)\right]}
\]

(V-30)

The detailed derivation of Eq. V-30 is given in Appendix E.

In using this equation to calculate frost thermal conductivities, an adjustable constant, namely the ratio of the cube length to the width of the ice rod, was evaluated from the experimental data. This ratio is a function of temperature. A particular value of the adjustable constant was obtained by matching the thermal conductivity calculated from Eq. V-30 with a value on the experimental curve. The specific procedure was as follows. At a given temperature the density of the frost at a solids fraction of 0.07 was
evaluated from Eq. V-29. A value of $x$ was then assumed, and $z$ was calculated from Eq. V-28. These values of $x$ and $z$ were substituted in Eq. V-30 and a value of $k_f$ computed. The computed conductivity was compared with the value on the experimental curve corresponding to a solids fraction of 0.07. The trial and error procedure was repeated until the theoretical and experimental values agreed. The values of $x/z$ generated in this manner are shown in Figure V-32.

At a particular temperature, the value of $x/z$ computed as described above is assumed to be valid for all frost densities. Thus, at any given temperature, frost thermal conductivity can be calculated as a function of density from Eqs. V-28, V-29 and V-30, and the appropriate value of $x/z$. More specifically, the value of $(1-\varepsilon)$ at a given density was calculated from Eq. V-29. $x$ and $z$ were then calculated from Eq. V-28 using the known value of $x/z$. Finally, Eq. V-30 was used to evaluate the thermal conductivity. In this manner the thermal conductivity-density relationship was obtained at various desired values of temperature. The calculated theoretical values are the shaded points in Figure V-33. At all temperatures there is fair agreement between theoretical points and experimental data. This agreement tends to indicate that over the density range involved the contacts between the ice particles in the frost are reasonably well represented by the dimensions of the interconnecting ice rods. In
Use of Cube and Rod Model to Predict Frost Thermal Conductivities

One must exercise care in using the theoretical curves of Figure V-33 to predict values of the frost thermal conductivity for other frosting situations. These curves are strictly valid for frosts having the same structural characteristics as frosts formed in the present investigation. It has previously been seen that snows of the same density can have significantly different structures. Now, it is possible that all frosts formed at cryogenic temperatures have substantially the same structure. The curve shown in Figure V-32 would then be of general applicability. However, there also exists the possibility, for example, that the structures of cryogenic frosts formed under natural convection conditions are different from those of cryogenic
frosts formed in forced convection. In addition, frost structure may also be strongly dependent on the specific geometry of the frosting system. If indeed structures of various cryogenic frosts are different than the structures of frosts of this thesis, then the curve of x/z versus temperature, shown in Figure V-32, is not of general applicability.

It is, however, interesting to note that Holten's measured values of frost thermal conductivity (40) at a temperature of \(-215^\circ F\) fall near the curve which represents the \(-215^\circ F\) data of the present thesis (see Figure V-34). This agreement provides some indication that the structures of cryogenic frosts formed under natural and forced convection conditions are similar.
Table V-1
Comparison Between Experimental and Theoretical Frost Accumulation

<table>
<thead>
<tr>
<th>Humidity</th>
<th>Re</th>
<th>Starting Point</th>
<th>Mass at End of Run</th>
<th>( \frac{(N_A)}{exp} )</th>
<th>( \frac{(N_A)}{theoret} )</th>
<th>( \frac{(N/A)}{theoret} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>grains water</td>
<td>lb dry gas</td>
<td>Time Min.</td>
<td>Mass</td>
<td>Exp. Value (read from Exp. Curve)</td>
<td>Theoret. Value</td>
<td>lb mass/hr-ft²</td>
</tr>
<tr>
<td>37</td>
<td>14,500</td>
<td>33.7</td>
<td>0.065</td>
<td>0.181</td>
<td>0.209</td>
<td>0.0945</td>
</tr>
<tr>
<td>9,300</td>
<td>41.3</td>
<td>0.0445</td>
<td>0.122</td>
<td>0.142</td>
<td>0.0675</td>
<td>0.0894</td>
</tr>
<tr>
<td>5,600</td>
<td>66.0</td>
<td>0.065</td>
<td>0.112</td>
<td>0.1405</td>
<td>0.0530</td>
<td>0.0840</td>
</tr>
<tr>
<td>23</td>
<td>9,300</td>
<td>75.0</td>
<td>0.0665</td>
<td>0.106</td>
<td>0.1095</td>
<td>0.0485</td>
</tr>
<tr>
<td>5,600</td>
<td>76.3</td>
<td>0.057</td>
<td>0.0815</td>
<td>0.0858</td>
<td>0.0430</td>
<td>0.0508</td>
</tr>
<tr>
<td>Humidity</td>
<td>Reynolds Number</td>
<td>Quasi-Steady State Heat Flux (Experimental) BTU/hr-ft(^2)</td>
<td>End-of-Run Heat Flux (Theoretical) BTU/hr-ft(^2)</td>
<td>Percent Error Theo.-Exp. x 100 Exp.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-----------------</td>
<td>----------------------------------------------------------</td>
<td>---------------------------------------------------</td>
<td>----------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>grains water lb dry gas</td>
<td>14,500</td>
<td>709</td>
<td>746</td>
<td>5.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>9,300</td>
<td>517</td>
<td>642</td>
<td>24.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5,600</td>
<td>456</td>
<td>551</td>
<td>20.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>9,300</td>
<td>567</td>
<td>630</td>
<td>11.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5,600</td>
<td>495</td>
<td>570</td>
<td>15.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table V-3

Experimental and Theoretical Surface Temperatures

<table>
<thead>
<tr>
<th>Re</th>
<th>Humidity</th>
<th>Steady Value $T_s$ of $\exp (\text{OF})$</th>
<th>End-of-Run Value of $T_s$ $\text{theoret}$ $\exp (\text{OF})$</th>
<th>$(T_s)<em>{\exp} - (T_s)</em>{\text{theoret}}$ $(\text{OF})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14,500</td>
<td>37.4</td>
<td>+24</td>
<td>+19.5</td>
<td>4.5</td>
</tr>
<tr>
<td>9,300</td>
<td>37.5</td>
<td>+25</td>
<td>+10</td>
<td>15</td>
</tr>
<tr>
<td>5,600</td>
<td>37.3</td>
<td>+20</td>
<td>+7</td>
<td>13</td>
</tr>
<tr>
<td>9,300</td>
<td>22.6</td>
<td>+12</td>
<td>+3</td>
<td>9</td>
</tr>
<tr>
<td>5,600</td>
<td>23.3</td>
<td>+12</td>
<td>-0.5</td>
<td>12.5</td>
</tr>
</tbody>
</table>
### Table V-4

**Comparison of Experimental and Theoretical Thermal Resistances**

<table>
<thead>
<tr>
<th>Humidity</th>
<th>Reynolds Number</th>
<th>Experimental $R_f$ (hr - °F)</th>
<th>Theoretical $R_f$ (hr - °F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>grains water lb dry gas</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>14,500</td>
<td>17.1</td>
<td>16.2</td>
</tr>
<tr>
<td></td>
<td>9,300</td>
<td>23.9</td>
<td>18.4</td>
</tr>
<tr>
<td></td>
<td>5,600</td>
<td>26.6</td>
<td>21.2</td>
</tr>
<tr>
<td>23</td>
<td>9,300</td>
<td>20.9</td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>5,600</td>
<td>23.9</td>
<td>20.2</td>
</tr>
</tbody>
</table>

**NOTE:** These thermal resistances are end-of-run, or essentially steady values.
Table V-5
Comparison of Thermal Diffusion with Ordinary Molecular Diffusion

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>$\frac{dn_1}{dt}$</th>
<th>$\frac{0.175 n_1 n_2}{T}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-300</td>
<td>$6.98 \times 10^{-16}$</td>
<td>$0.0734 \times 10^{-16}$</td>
</tr>
<tr>
<td>-215</td>
<td>$2.61 \times 10^{-12}$</td>
<td>$0.0194 \times 10^{-12}$</td>
</tr>
<tr>
<td>-145</td>
<td>$2.27 \times 10^{-9}$</td>
<td>$0.0132 \times 10^{-9}$</td>
</tr>
<tr>
<td>-80</td>
<td>$7.65 \times 10^{-7}$</td>
<td>$0.0346 \times 10^{-7}$</td>
</tr>
<tr>
<td>+20</td>
<td>$1.70 \times 10^{-4}$</td>
<td>$0.0124 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

$dn_1/dT$: term proportional to the driving force gradient for ordinary molecular diffusion

$\frac{0.175 n_1 n_2}{T}$: term proportional to the driving force gradient for thermal diffusion

Table V-6
Excess Thermal Conductivity as a Function of Temperature

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>$k_{xs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-215</td>
<td>negligible</td>
</tr>
<tr>
<td>-145</td>
<td>negligible</td>
</tr>
<tr>
<td>-80</td>
<td>negligible</td>
</tr>
<tr>
<td>0</td>
<td>0.0030</td>
</tr>
<tr>
<td>+20</td>
<td>0.0078</td>
</tr>
</tbody>
</table>
Reynolds Number = 5,600

$H = 23 \frac{\text{grains water}}{\text{lb dry gas}}$

- Data
- Curve Through Data
- Curve Through Data Extrapolated to Origin
- Theoretical Curve

Figure IV-1 Comparison Between Experimental and Theoretical Frost Accumulation
Reynolds Number = 5,600

\[ H = \frac{37 \text{ grains water}}{\text{lb dry gas}} \]

![Graph showing frost density as a function of time with theoretical and experimental curves.](image)

**Figure V-2** Frost Density as a Function of Time
Reynolds Number = 5,600

\[ H = \frac{37}{1 \text{ lb dry gas}} \text{ grains water} \]

Dashed Theoretical Curve: Assumed Initial Thickness of 0.08 inches

Solid Theoretical Curve: Assumed Initial Thickness of 0.04 inches

Experimental Curve

Figure V-3 Frost Thickness as a Function of Time
Data
Reynolds Number = 5,600

\[ H = \frac{37 \text{ grains water}}{\text{lb dry gas}} \]

Dashed Curve: Assumed Initial Thickness of 0.08 inches
Solid Curve: Assumed Initial Thickness of 0.04 inches

Figure V-4 Total Heat Flux Through Frost as a Function of Time
Figure V-5 Surface Temperature of Frost as a Function of Time

Reynolds Number = 5,600

\[ H = \frac{37 \text{ grains water}}{1 \text{ lb dry gas}} \]

Dashed Curve: Assumed Initial Thickness of 0.08 inches

Solid Curve: Assumed Initial Thickness of 0.04 inches

Data
Reynolds Number = 14,500

\[ \frac{H = 37 \text{ grains water}}{1 \text{ lb dry gas}} \]

○ Data

Theoretical Curves

Dashed Theoretical Curve: Assumed Initial Thickness of 0.08 inches

Solid Theoretical Curve: Assumed Initial Thickness of 0.04 inches

Figure V-6  Frost Density as a Function of Time
Reynolds Number = 14,500

\[ H = 23 \frac{\text{grains water}}{\text{lb dry gas}} \]

Experimental Curve

Dashed Theoretical Curve: Assumed Initial Thickness of 0.08 inches

Solid Theoretical Curve: Assumed Initial Thickness of 0.04 inches

Figure V-7 Frost Thickness as a Function of Time
Reynolds Number = 14,500

\[ H = \frac{37 \text{ grains water}}{\text{lb dry gas}} \]

Dashed Curve: Assumed Initial Thickness of 0.08 inches

Solid Curve: Assumed Initial Thickness of 0.04 inches

Figure V-8 Total Heat Flux Through Frost as a Function of Time
Figure V-9  Frost Surface Temperature as a Function of Time

Reynolds Number = 14,500

\[ H = \frac{37 \text{ grains water}}{\text{lb dry gas}} \]

○ Data

Dashed Curve: Assumed Initial Thickness of 0.08 inches

Solid Curve: Assumed Initial Thickness of 0.04 inches

Time (minutes)
Figure VI-10  Frost Thickness as a Function of Time with Reynolds Number and Humidity as Parameters
Figure V-11  Frost Density as a Function of Time with Reynolds Number and Humidity as Parameters
Curves were Calculated
Assuming an Initial
Thickness of 0.04 inches.

Figure V-12 Calculated Heat Flux as a Function of Time with Reynolds Number and Humidity as Parameters.
Curves were Calculated
Assuming an Initial
Thickness of 0.04 inches

Figure V-13 Calculated Frost Surface Temperature as a Function of Time with Reynolds Number and Humidity as Parameters
Figure V-14 Comparison of Calculated Initial Heat Fluxes with Measured Quasi-Steady State Values
Figure V-15  Comparison of Heat Transfer Coefficients from Reference 87 with Experimental Values
Figure IV-16  Total Mass of Frost on Plate as a Function of Time

Note: For humidities corresponding to Runs 46, 53, 54, 55 and 69, see Table IV-2

<table>
<thead>
<tr>
<th>Shaded Points:</th>
</tr>
</thead>
<tbody>
<tr>
<td>H ≈ 37 grains water</td>
</tr>
<tr>
<td>lb dry gas</td>
</tr>
</tbody>
</table>

- △ Re = 14,500
- ○ Re = 9,300
- □ Re = 5,600

Time (minutes)

Mass of Frost on Plate (lb mass/ft²)
The Calculated Curves are based on a Frost Structure Consisting of a Cubic Array of Ice Spheres (Woodside Model)

Figure V-17 Point Thermal Conductivity as a Function of Temperature for a Frost Density of 0.09gm/cc
Curve 1 Represents data of Kuzmin (48) and Artsybashev & Yushakov (48)

Curve 2 Represents data of Jansson (43)

Figure IV-18 Comparison of Frost Thermal Conductivity Data at 20°F with Natural Snow Data
Figure IV-19 Thermal Conductivity as a Function of Density; Comparison of Data with Parallel Slab Model at a Temperature of +20°F
Figure V-20 Thermal Conductivity as a Function of Density; Comparison of Data with Parallel Slab Model at a Temperature of 0°F
Figure V-21  Thermal Conductivity as a Function of Density; Comparison of Data with Parallel Slab Model at a Temperature of -80°F
Figure V-22: Thermal Conductivity as a Function of Density; Comparison of Data with Parallel Slab Model at a Temperature of -145°F
Figure X-23  Thermal Conductivity as a Function of Density; Comparison of Data with Parallel Slab Model at a Temperature at -215°F
Figure V-24 Comparison of Data with Thermal Conductivity Based on an Assumed Model of a Cubic Array of Ice Cubes, Temperature = -145°F
Figure V-25 Comparison of Data with Thermal Conductivity Based on an Assumed Model of a Cubic Array of Ice Cubes, $T = -215^\circ F$
Figure V-26  Comparison of Data with Thermal Conductivity Based on an Assumed Model of a Cubic Array of Gas Cubes (Case of Zero Sidewise Conductivity), Temperature = -145 °F
Comparison of Weber and Woodside Models with Thermal Conductivity Data at a Temperature of -145°F. Results of Deissler Analysis Also Included.

Figure V-27
Figure V-28 Comparison of Weber and Woodside Models with Thermal Conductivity Data at a Temperature of -215°F
(Results of Deissler Analysis Also Included)
Figure V-29  Comparison of Woodside Model with Thermal Conductivity Data at a Temperature at 20°F
Figure V-30 Comparison of Woodside Model with Thermal Conductivity Data at a Temperature of 0°F
Figure V-31: Comparison of Woodside Model with Thermal Conductivity Data at a Temperature of -80°F
Figure IV-32  \( x/z \) as a Function of Temperature for Cube and Rod Model
Figure V-33: Frost Thermal Conductivity as a Function of Density with Temperature as a Parameter.
Figure V-34 Frost Thermal Conductivity as a Function of Density at a Temperature of -215 °F, Comparison with Data of Holten.
CHAPTER VI

Conclusions

(1) Frost density and thickness increase continuously with time.

(2) At equivalent times frost density increases with increasing Reynolds Number, but is substantially independent of humidity.

(3) Frost thickness, at equivalent times, decreases slightly with increasing Reynolds Number, but increases significantly with increasing humidity.

(4) Point frost thermal conductivity increases with increasing temperature and density. Average frost layer conductivity, therefore, also increases with density.

(5) The temperature difference across the frost rises rapidly, at first, but becomes substantially constant after awhile.

(6) The heat flux through the frost decreases initially, but eventually attains a steady value.

(7) A constant heat flux is reached because the increasing frost thickness is balanced by the increasing thermal conductivity resulting from the continuous frost densification. These two compensating effects lead to a frost of constant thermal resistance.
The densification of the frost layer results from the diffusion of water vapor from the "warm" frost surface into the interior.

A macroscopically "smooth" frost is made up of irregularly shaped ice particles having linear dimensions ranging from 20 to 50 microns.

It is impossible to geometrically characterize the frost because of the irregular nature of the ice particles. Calculated conductivity values based on assumed frost structures are, therefore, either higher or lower than the measured values. This occurs because idealized structures either over-represent or represent, inadequately, the actual contacts between the particles in the frost.
CHAPTER VII

Recommendations

As indicated in Chapter V, the predictions of the internal diffusion model are in fair agreement with the data. The model, however, could be tested more critically if data were taken over wider ranges of the independent parameters. Specifically, then, it is recommended that:

(1) The gas phase temperature should be varied from 30 to 150°F.

(2) The gas phase humidity range 5 - 20 grains water/lb dry gas should also be investigated. (In this investigation the humidity range was 23 - 37 grains water/lb dry gas.) Humidities greater than 45 grains water/lb dry gas should be avoided because of the complications which would result from the formation of liquid water on the frost surface.

(3) The plate temperature should be varied from -320 to about -100°F, and

(4) A different condensible component should be used. A suitable choice would be carbon dioxide.

Other recommendations are listed below.

(5) If it proves feasible from a practical viewpoint, the copper plate should be continuously lowered during a run. The plate should be lowered at such a rate as to maintain the surface of the growing frost layer flush
with the bottom of the duct. The fluid dynamics then would be understood better than if the frost layer were protruding into the gas stream (as in the present thesis).

(6) If possible, a method for measuring frost density continuously during a run should be developed. The number of runs required to obtain frost density as a function of time, for a given Reynolds Number and humidity, would be significantly reduced. It may be possible to measure the frost density continuously by determining the variation, with time, of the frost capacitance.

(7) A more accurate method for determining the frost density at several vertical positions within the frost layer should be developed. A method which could possibly be utilized is described in Appendix M.
APPENDIX A

Transfer of Particles from a Gas Stream
to a Solid Surface

The primary purpose of this appendix is to examine the various forces acting on a particle suspended in a gas stream, and to determine which mechanisms are important in causing the particles to be deposited. The concepts regarding particle deposition mechanisms developed by Owen (65), and by Friedlander and Johnstone (31), will be utilized in examining the deposition of ice particles from a gas stream onto a frost layer. These concepts are reviewed below.

1. Previous Work

Particles in a gas stream can be transferred to a wall by one or more of the following mechanisms.

(1) turbulence in the gas stream (31, 65)
(2) gravity, if deposition is from above
(3) force due to temperature gradient in the gas stream (74)
(4) electrostatic forces (31)
(5) fluid shear (65)

In their investigation Friedlander and Johnstone (31) concluded that it is gas phase turbulence which is primarily responsible for transporting the particles to the wall.
These investigators concluded that electrostatic forces were insignificant. The force on a particle due to the temperature gradient was negligible, because the experiment was carried out in an isothermal medium. A vertical tube eliminated the possibility of deposition by gravity.

To corroborate their thesis that the turbulent eddies were responsible for transporting the particles to the wall, Friedlander and Johnstone noted that no particle deposition occurred in the inlet section of the pipe where a relatively thick laminar boundary layer existed adjacent to the wall. However, after a length of pipe corresponding to \( \text{Re}_x = 100,000 \), the position at which the laminar boundary layer is expected to go turbulent (75), deposition of particles onto the pipe wall was clearly observed.

Actually the process of transferring particles by turbulence in a direction transverse to the direction of the gas stream is not completely understood. The investigations of Refs. 31 and 65 have, however, provided some insight into this phenomenon.

In both of these investigations (31, 65), it was concluded that the mechanism for transporting solid particles through a turbulent gas stream differs from the mechanism involved in molecular transport. In molecular transport it is generally postulated that molecules are transported to the edge of the laminar sublayer by the eddies in the gas stream and that the molecules then travel from the edge of
the laminar sublayer to the wall by molecular diffusion. In the case of particle transport, it is theorized that the particles are carried to the edge of the laminar sublayer by the turbulent eddies in the gas stream. (It is generally assumed that the particles faithfully follow the motion of the eddies, as the particles are usually much smaller than the scale of turbulence.) However, in contrast to molecular transport, there is no diffusive mechanism which enables the particles to travel across the laminar sublayer. Thus the velocity imparted to each of these particles at the edge of the laminar sublayer by the turbulent eddies in the gas stream must suffice to get these particles across the laminar sublayer and onto the wall. Friedlander and Johnstone (31), however, have shown that in general the Stokes law range of these particles (i.e., the distance a particle will travel through a stagnant medium under the impetus of an initial velocity imparted to it) is less than the thickness of the laminar sublayer. These particles, therefore, should never reach the wall. It is observed, however, that they do. Owen, and Friedlander and Johnstone, therefore, based their deposition mechanism on the postulate that the turbulent eddies do not end at the edge of the laminar sublayer, but extend well into this sublayer. Particles would thus be "left off" by the eddies much closer to the wall and could, therefore, traverse the remaining distance to the
wall under their own inertia. The assumption regarding the existence of turbulent eddies in the laminar sublayer is based on the work of Lin, Moulton, and Putnam (52), who actually developed an expression for the eddy diffusivity within the laminar sublayer.

For deposition from a gas stream onto a horizontal surface below, Owen (65) derived an expression which relates the quantity \[(\text{mass flux of particles due to turbulence})/(\text{mass flux of particles due to gravity plus turbulence})\] to the ratio \[(\text{terminal velocity of particles})/(\text{friction velocity of gas stream})\]. This relationship is shown graphically in Figure A-1. The data of Dawes and Slack (18), which are not shown in Figure A-1, are in agreement with Owen's theory, thus corroborating the validity of the theoretical relationship.

2. Application of Concepts Developed in Previous Work to Frosting

A. Experimental Evidence

In the present thesis, two laminar flow runs were made: one at a Reynolds Number of about 600 and the other at a Reynolds Number of about 1800. Both runs lasted about 34 minutes and were made at a humidity of about 37 grains water/lb dry gas. In each of these runs there was, essentially, no deposition of frost onto the plate. Actually, a thin film of frost, about ten thousandths of an inch, formed on the plate during
the first 15 seconds of the run. The subsequent formation of frost clumps and the "filling in" of these clumps (i.e., to form a smooth frost), however, never occurred. During the entire duration of each of these runs, a fog of frost particles was observed to be present in the gas phase.

A plausible explanation for the above-described observations is that the ice particles which nucleated in the gas were not transported to the plate, because of the absence of turbulent eddies. Other mechanisms, such as the thermal force on a particle due to the temperature gradient in the gas \((7^4)\), and gravity were apparently too weak to deposit the particles onto the plate. The thin frost layer, which formed at the very start of the run, was probably due to water vapor diffusion from the gas phase.

B. Mechanism of Deposition of Particles

For the frosting experiments, electrostatic forces and forces arising from fluid shear can be rejected immediately as being responsible for the deposition of the ice particles. It is possible, however, that in addition to turbulence, gravity and the temperature gradient in the gas stream are significant in causing particle deposition. These two latter mechanisms will be considered below in more detail.
(1) Deposition Due to Gravity

For the frosting runs of the present thesis, the quantity \( \left( \frac{N_p T_u}{N_p,gr} \right) \) was evaluated from the curve shown in Figure A-1, using the appropriate value of the ratio \( \left( V_{o,gr} / u^* \right) \). The terminal velocity of a particle due to gravity, \( V_{o,gr} \), is given by

\[
V_{o,gr} = \frac{1}{18} \left( \frac{D_p}{\mu} \right)^2 \frac{(\rho_p)g}{\mu}
\]

Thus, to calculate \( \frac{N_p T_u}{N_p,gr} \), the diameter of the ice particle which is depositing onto the plate has to be known. Unfortunately, this diameter is not known, and the calculation was, therefore, performed for two assumed particle sizes, 1 and 5 microns. As indicated earlier in Chapter IV, the sizes of the ice particles in the clumps of frost ranged from 5 to 10 microns. The sizes of the ice particles being thrown onto the plate were almost certainly smaller than these values.

Terminal velocities calculated from Eq. A-1 for particle sizes of 1 and 5 microns are respectively \( 1.5 \times 10^{-4} \) and \( 3.75 \times 10^{-3} \) feet/sec. The ratios \( \left( V_{o,gr} / u^* \right) \) and \( \frac{N_p T_u}{N_p,gr} \) for these particle sizes are shown in Table A-1. The friction velocity \( u^* \) is, of course, given by

\[
u^* = U_{Av} \sqrt{f/2}
\]
According to Owen's theory, deposition by gravity is insignificant compared with deposition by turbulence for the 1 micron particles. However, for the 5 micron particles, particularly at the lower velocities, gravity is quite important. For example, at this particle size and for a Reynolds Number of 5,600, particle flux due to turbulence is 1.74 times that due to gravity. It is interesting to note that for this same particle size and Reynolds Number, the theory developed by Friedlander and Johnstone (31) predicts that particle flux due to turbulence is about five times the flux due to gravity. If the relationship shown in Figure A-1 is assumed to be valid, then one has to conclude, in view of the experimental evidence presented earlier, that the sizes of the particles being thrown onto the plate are probably closer in size to 1 micron than to 5 microns.

(2) Deposition due to Thermal Force

Saxton and Ranz have derived an expression (Eq. A-3) for the force on a particle which results from a temperature gradient in the gas

$$ F_{Th} = -4.5\pi D_p \left( \frac{k_g}{2k_g + k_p} \right) \left( \frac{\mu^2}{\rho g T} \right) \left( \frac{dT}{dr} \right) \quad (A-3) $$

An expression for the terminal velocity of a particle, $V_{o,Th}$, resulting from the thermal force is obtained by
combining Eq. A-3 with the expression for the drag force acting on a moving particle, Eq. A-4.

\[ F_D = \frac{\pi}{4} (D_p)^2 \frac{\rho_g (V_{o, Th})^2}{2} \quad C_D \]  

(A-4)

where

\[ C_D = \frac{24}{\text{Re}_p} = \frac{24}{D_p (\rho_g)(V_{o, Th})} \frac{\mu}{\mu} \]

The final expression is

\[ V_{o, Th} = -\frac{1.5k_g}{2k_g + k_1} \frac{\partial}{\partial T} \frac{dT}{dr} \]  

(A-5)

For the frosting experiments of the present thesis, the value of \(V_{o, Th}\), as calculated from Eq. A-5, is \(1.15 \times 10^{-4}\) ft/sec. \((V_{o, Th}\) is independent of particle diameter.) At the lowest Reynolds Number employed in the present experiments, 5,600, \(V_{o, Th}/u^*\) equals \(2.44 \times 10^{-4}\). This is negligibly small and it can therefore be concluded that deposition of ice particles due to the temperature gradient in the gas is negligible compared with deposition by turbulence.
Table A-1

Comparison Between Particle Deposition Rates
Caused by Turbulence and Gravity

<table>
<thead>
<tr>
<th>Particle Size (microns)</th>
<th>Reynolds Number</th>
<th>Terminal Velocity due to Gravity (ft/sec)</th>
<th>( \frac{V_{o,gr}}{u^*} )</th>
<th>( \frac{N_{p,Tu}}{N_{p,gr}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5,600</td>
<td>1x5x10^{-4}</td>
<td>3.19x10^{-4}</td>
<td>Very large ( (N_{p,gr} \approx 0) )</td>
</tr>
<tr>
<td>14,500</td>
<td>1.5x10^{-4}</td>
<td>1.32x10^{-4}</td>
<td></td>
<td>Very large ( (N_{p,gr} \approx 0) )</td>
</tr>
<tr>
<td>5</td>
<td>5,600</td>
<td>3.75x10^{-3}</td>
<td>7.97x10^{-3}</td>
<td>1.74</td>
</tr>
<tr>
<td>14,500</td>
<td>3.75x10^{-3}</td>
<td>3.29x10^{-3}</td>
<td></td>
<td>3.76</td>
</tr>
</tbody>
</table>
Figure A-1  Ratio of Rate of Particle Deposition by Turbulence to Rate of Particle Deposition by Gravity and Turbulence.
APPENDIX B

Natural Snow, Its Structure and Properties

1. Comparison with Cryogenic Frosts

Natural snows sometimes have structures which are similar to those of cryogenic frosts. Thus a study of the properties of snow often yields information about the properties of cryogenic frosts. In this appendix the structure and properties of natural snow will be discussed and comparisons with cryogenic frosts will be made.

The microscopic observations of the present thesis revealed that cryogenic frosts are composed of irregular ice particles. Under certain meteorological conditions newly fallen snows can also consist of an array of irregular ice particles. Often, though, newly fallen snows are composed of beautiful, symmetrical ice crystals, such as the one shown in Figure B-1. The types of snow having structures most similar to those of cryogenic frosts are "old snows". Old snow refers to snow which has been on the ground for a while, but has never melted and refroze. As indicated earlier, it is the internal diffusion of water vapor which results in the similarity between particles contained in an "old snow" layer and those in a cryogenic frost.

The diffusion of water vapor into the frost interior causes frost and snow properties to be time dependent. Changes in properties caused by this mechanism are designated as constructive metamorphism (21).
Prior to a discussion of the various metamorphic processes, the meteorological factors which determine the microstructure of a newly fallen snow, and the factors which influence the behavior of particles in a cryogenic frost- ing study, will be considered.

2. Snow Crystal Shape

Bentley and Humphreys (9) and Nakaya (64) have studied the forms of ice crystals which make up a newly fallen snow layer. Nakaya concluded that the primary factor influencing ice crystal habit is the temperature at which the crystal is formed. He also concluded that the supersaturation of the environment in which the crystal grows exerts a secondary influence on the crystal shape. This will be discussed below in more detail.

The shape of an individual snow crystal which forms in the clouds may be significantly altered as it falls through the atmosphere, because it will encounter air at different temperatures and at different degrees of supersaturation. In addition, if a strong wind is present, falling snowflakes may be broken up into smaller irregular particles. It is obvious that if observations are made outdoors, correlating crystal habit with the conditions under which the crystal formed can be an extremely difficult task.

In addition to his atmospheric investigations, Nakaya performed laboratory studies in which individual snow
crystals were formed in a cold chamber with carefully controlled conditions. His results are in substantial agreement with those of Hallet and Mason (36).

Hallet and Mason studied crystal habit as a function of temperature and supersaturation by forming individual snow crystals on a thin filament suspended in a cold chamber. Their results are shown in Figure B-2. It is seen that temperature is the most important factor in determining crystal habit and that supersaturation is secondary. The basic crystal forms obtained in the various temperature ranges between 0 and -40°C are shown in Table B-1.

From the above observations it is concluded that basically there are two primary types of snow crystals: plates and prisms. Dendritic forms and needle crystals are secondary and occur only at high supersaturation levels. Actually, dendritic crystals belong to the plate family, whereas needles belong to the prism family (36).

3. **Snow Crystal Formation**

A better understanding of the variation of crystal habit with temperature can be obtained by examining the entire process of crystal growth. The formation of a snow crystal in a supersaturated cloud usually starts with the nucleation of a liquid droplet (56). There are generally an adequate number of foreign particles in the atmosphere to ensure that nucleation is heterogeneous. The droplet
usually grows to about 10 microns before freezing (56).
Actually, the entire liquid droplet does not freeze at once, but a small ice nucleus forms first within the liquid. This small ice nucleus is probably hexagonal in shape since in the water system solid crystals have a hexagonal morphology. The ice crystal grows rapidly at the expense of other liquid droplets in the clouds as the vapor pressure of solid ice is lower than that of liquid water.

During its growth, the ice crystal retains its edges and corners instead of developing into a sphere (54, 57). The corners and edges are retained, probably, because the crystal tends toward that configuration which enables it to release its latent heat and add mass as rapidly as possible. Corners and edges are more efficient than spherical surfaces for these purposes.

As previously stated, there are two basic types of hexagonal crystals in the water system: plates and prisms. The reasons for the prevalence of plates in certain temperature ranges and the prevalence of prisms at other temperatures are not at present clearly understood.

4. Particle Formation in Cryogenic Frosting Experiments

When a particle first nucleates, its radius is equal to the critical radius. This critical radius is a function of the temperature and the supersaturation level. Before the particle strikes the plate its size increases significantly as a result of growth mechanisms described in Appendix 2.
of Ref. 1. The behavior of these ice particles in the frost layer has already been described.

5. **Metamorphic Processes**

   Snow generally undergoes four types of metamorphic processes. These are:

   (1) pressure metamorphism (4)
   (2) melt metamorphism (21)
   (3) destructive metamorphism (21)
   (4) constructive metamorphism (21)

   **(A) Pressure Metamorphism**

   Pressure metamorphism is the structural change a snow layer undergoes when subjected to a heavy load. The load causes the bonds between the ice particles in the snow to either break or deform.

   The density gradient in a mass of snow having a large depth is generally the result of pressure metamorphism (2). The bottom layers obviously are compacted more than are the layers at the top and are, therefore, denser. In thin frost layers (i.e., about 1/4 of an inch thick) pressure metamorphism is of no significance.

   **(B) Melt Metamorphism**

   Melt metamorphism is the change in snow structure which results from melting and refreezing. This type of metamorphism is in general not significant in a cryogenic
frost layer since temperatures within the layers are almost always below 32°F. The experiments of Richards, Edmonds and Jacobs (69) are a notable exception. As described in Chapter I, the frost surface in many of their runs reached 32°F and consequently liquid water formed on the surface.

C. Destructive and Constructive Metamorphism

In general, structural changes within cryogenic frosts are caused by the processes of constructive and destructive metamorphism. Both of these processes involve molecular migration. The end product of destructive metamorphism in snows is an agglomeration of small, rounded grains, whereas the end product of constructive metamorphism is a mass of large, irregular particles (see Figure B-3 (50)). Both of these processes, constructive and destructive metamorphism, are more important at the higher temperatures, i.e., near the freezing point of water. Thus, in a cryogenic frost these processes are more significant in the upper portions of the frost than in the lower regions.

D. Destructive Metamorphism

Destructive metamorphism occurs because many portions of the ice particle such as corners and edges are of convex curvature. The fugacity of ice at a corner or an edge is greater than the fugacity at a flat or concave surface. Mass is thus transferred from the sharp edges and corners
to the flat and concave areas. Destructive metamorphism, therefore, has a rounding effect on sharp-edged ice particles. Unlike constructive metamorphism, destructive metamorphism can occur in the absence of a temperature gradient.

**Postulated Mechanism of Destructive Metamorphism**

The mechanism by which destructive metamorphism occurs is still open to question. Kingery (47) asserts that it is surface diffusion which is responsible for the isothermal transfer of mass from a convex ice surface to one which is concave or flat. He supported his thesis by measuring the rate of growth of the contact area between two ice spheres of the same size. Particle diameters ranged from 0.011 to 0.3 cm. Temperatures ranged from -2.2 to -25.1°C. The experimentally observed rates of growth were compared with calculations based on the assumed mechanisms of surface diffusion, vaporization-condensation, viscous creep and bulk creep (47). The rates calculated from the surface diffusion equation were in substantial agreement with the data. Thus Kingery concluded that surface diffusion was the predominant mode of material transfer to the contact areas.

Hobbs and Mason (39) have taken issue with Kingery's conclusion. They re-derived the expression for material
transport by evaporation-condensation and showed that the equation used by Kingery was in error. Using their own re-derived expression, Hobbs and Mason re-examined Kingery's data, which they assumed to be correct. They concluded that evaporation-condensation rather than surface diffusion was the mechanism responsible for the transport of material into the particle contact areas.

De Quervain (22) observed that the rate of destructive metamorphism of a dendritic crystal was several times lower in kerosene than in air. Since it seems likely that the kerosene could only suppress material transport by vaporization-condensation and not by surface diffusion, it can be concluded that the destructive metamorphism of a snow crystal in air occurs primarily by an evaporation-condensation process. This is in agreement with Hobbs' and Mason's conclusion. There is the possibility, of course, that the kerosene also suppressed the surface diffusion process and that conclusions based on de Quervain's experiments (22) are erroneous. This, however, seems unlikely.

In the process of destructive metamorphism the centers of the ice spheres do not necessarily move closer together. Thus, ideally, if a snow layer is made up of an array of touching spherical ice particles, the mass transport to the particle contact areas would not necessarily cause an increase in the frost density. The thermal conductivity would, however, increase, because the contacts between adjacent spheres have been enlarged from points to finite areas.
E. Constructive Metamorphism

As previously mentioned the process of constructive metamorphism is caused by temperature gradients within the snow layer. Water evaporates from the warmer particles and recondenses on the colder ones. This addition of mass to the "colder" ice particles changes their shape. Snow crystals which have been subjected to constructive metamorphism are shown in Figure B-3b. It is seen that the process of constructive metamorphism changes a symmetrical crystal into a highly irregular one.

Unlike destructive metamorphism, constructive metamorphism always involves an increase in density because of the deposition of water vapor in the interstices. The increase in density, of course, involves an increase in the intrinsic conductivity of the ice-gas matrix. In addition, the internal water vapor diffusion, which causes constructive metamorphism, contributes to the over-all thermal conductivity as it represents a transport of enthalpy. As previously mentioned the additional conductivity component arising from the transport of enthalpy is denoted as evaporative transport.

6. Snow Properties

In general, snow properties such as shear strength and thermal conductivity do not depend on density alone. It is obvious that two frosts of the same density (and at the
same temperature) can have vastly different shear strengths and thermal conductivities because of significantly different structures. A general meteorological observation is that newly fallen snows of the same density usually have significantly different structures. However, for higher density snows (i.e., with densities of about 0.5 gm/cc or higher) which have densified as a result of the metamorphic processes described above, equal densities usually imply similar structures. Thus it would be expected that data from various investigators on snow properties such as tensile strength and thermal conductivity would scatter more at low densities than at high densities. This is indeed the case (56) for a mechanical property such as tensile strength. In the case of thermal conductivity, however, the data from various investigators scatter as much at high densities as at the lower densities (56).

7. Effects of Gas Flow Through Snow on Snow Structure

When a wind blows through a snowpack, the shapes of the individual ice crystals are altered (77). The metamorphism of these particles is known as wind packing. Seligman (77) postulates that wind packing is a modified form of constructive metamorphism, since the wind carries moisture with it. The effects of wind packing on the frosts investigated in the present thesis are believed to be negligible. The basis for this assertion is that
the quantity of water vapor carried by the gas flowing through the frost was calculated to be negligible compared with the internal water vapor transport caused by the temperature gradient.

8. Densification of Natural Snows

As previously stated, constructive metamorphism generally involves frost densification. In addition, pressure and melt metamorphism also lead to increases in snow density. These latter two processes are often important in natural snows, but were not discussed in much detail because of their relative unimportance in cryogenic frosts.

Table B-2 lists the densities of various types of snows. Some of the types of snow listed in this table are newly fallen, whereas others have been subjected to one or more of the various metamorphic processes. To reiterate, "old snow" is snow which had undergone one or more of the metamorphic processes, except melting and refreezing.

9. Review of Important Differences and Similarities between Natural Snows and Cryogenic Frosts

Based on the results of the present thesis, the following conclusions can be drawn about the relationships between natural snow and cryogenic frosts.

(1) In general, old snows and cryogenic frosts have similar structures. The average particle size for cryogenic
frosts, however, is about an order of magnitude lower than the sizes of particles in natural snow.

(2) The average temperature level of cryogenic frosts is significantly below the average temperature of the natural snow layer. However, in the upper layers of a cryogenic frost, the temperature is about the same as in natural snow. Thus, the processes of constructive and destructive metamorphism, which are negligible at low temperatures, are important in both natural snows and cryogenic frosts.
Table B-1

Crystal Type as a Function of Temperature

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>Type of Crystal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to -5</td>
<td>plates</td>
</tr>
<tr>
<td>-4 to -9</td>
<td>prisms</td>
</tr>
<tr>
<td>-10 to -25</td>
<td>plates</td>
</tr>
<tr>
<td>-25 to -40</td>
<td>prisms</td>
</tr>
</tbody>
</table>
Table B-2

**Densities of Various Types of Natural Snows**

<table>
<thead>
<tr>
<th>Type of Snow</th>
<th>Density (gm/cc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wild Snow</td>
<td>0.01 - 0.03</td>
</tr>
<tr>
<td>Ordinary new snow, immediately after falling</td>
<td>0.05 - 0.065</td>
</tr>
<tr>
<td>Settling Snow</td>
<td>0.07 - 0.19</td>
</tr>
<tr>
<td>Settled Snow</td>
<td>0.2 - 0.3</td>
</tr>
<tr>
<td>Very slightly wind-toughened snow, immediately after falling</td>
<td>0.063 - 0.82</td>
</tr>
<tr>
<td>Average wind-toughened snow</td>
<td>0.28</td>
</tr>
<tr>
<td>New Firm Snow</td>
<td>0.4 - 0.55</td>
</tr>
<tr>
<td>Thawed Firm Snow, Spring Snow</td>
<td>0.6 - 0.7</td>
</tr>
<tr>
<td>Glacier Ice</td>
<td>0.9169</td>
</tr>
<tr>
<td>Water</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Figure B-1  A Typical Crystal in a Newly Fallen Snow
Figure B-2  The Variation of Ice-Crystal Habit with Temperature and Supersaturation of the Vapor (36)
Snow Crystals Which Have Been Subjected to Destructive Metamorphism

Snow Crystals Which Have Been Subjected to Constructive Metamorphism

NOTE: Magnification is the same for Figures B-3A and B-3B.
Mechanisms Which Contribute to the Thermal Conductivity

As mentioned earlier, heat is not only transferred through the frost by conduction through the solid gas network, but also by radiation between ice particles, forced convection through the gas in the pores, and finally by the transfer of water vapor through the interior of the frost. The contribution to the over-all frost thermal conductivity made by each of these mechanisms will be evaluated below.

1. Effect of Radiation

The warmer ice particles, located at the top of the frost layer, radiate to the colder particles located below them. The lower particles absorb part of the radiation striking them, and then re-emit radiant heat to particles at the next lower layer. The component of thermal conductivity, resulting from radiation, is included in the measured conductivity values. The value of this radiative component was calculated from the Damköhler equation presented below (17).

\[ k_r = \frac{0.684}{100} \varepsilon D_p \left( \frac{T}{100} \right) e_H \]  

(C-1)

where \( k_r \) is the contribution to the over-all frost conductivity
made by radiation between the particles. The values chosen for substitution into Eq. C-1 were those which would yield the highest possible contribution radiation could make to the frost thermal conductivity.

As previously mentioned, the sizes of the ice particles, of which frost is composed, are in the 20-50 micron range. Thus, $D_p = 50$ microns was substituted into Eq. C-1. The void fraction chosen for calculating $k_r$ was taken as 0.95, the largest frost void fraction encountered in the experiments of the present thesis. $32^0F$ was the temperature chosen for substitution into Eq. C-1 as this is the highest possible temperature that can exist within a frost layer. (In the experiments of the present thesis, the frost surface temperature never reached $32^0F$.)

Several values of frost emissivity are reported in the literature ($16, 26$). Reported values generally vary between 0.082 and 0.098. 0.098 was chosen, as this value represented the upper limit of the reported emissivities. Using the values of $E$, $D_p$, $T$ and $e_H$ presented above, the value of $k_r$ was calculated to be $0.000124$ BTU/hr-ft$^2$-F/ft. Assuming Damkohler's equation valid, this value of $k_r$ is the highest possible contribution that radiation can make to the frost thermal conductivity. Since $0.000124$ is negligible compared with $0.01$ BTU/hr-ft$^2$-F/ft, the lowest measured thermal conductivity, it is concluded that radiation makes a small contribution to the frost thermal
conductivity. Radiant heat transfer is, therefore, neglected in calculating theoretical values of conductivity.

2. **Effect of Gas Flow Through Frost**

   **A. Use of Ergun Equation to Calculate Rate of Gas Flow Through Frost**

   The Ergun correlation, which has been found to be applicable over wide ranges of gas flow rate, and for various particle shapes (28), is presented below.

   \[
   f' = 1.75 + \frac{150(1 - \varepsilon)}{D_p (G_o/\mu)} \tag{C-2}
   \]

   where \( f' \) is a modified friction factor and is defined by

   \[
   f' = \frac{\Delta P_f g_c D_p \rho_g}{L G_o^2} \frac{\varepsilon^3}{1 - \varepsilon} \tag{C-3}
   \]

   \( G_o \) is the superficial mass velocity, i.e., it is the mass flow rate divided by the total cross sectional area of the heterogeneous mass (in this case frost).

   For the conditions of the experiments of the present thesis, the value 1.75 is negligible compared with the value of the term \( 150 (1 - \varepsilon)/D_p (G_o/\mu) \). Equation C-2 therefore becomes

   \[
   f' = \frac{150(1 - \varepsilon)}{D_p (G_o/\mu)} \tag{C-4}
   \]
Combining Eqs. C-3 and C-4, an expression for the flow rate of gas through the frost is obtained:

$$\frac{\Delta P_f}{L} = \frac{\mu G_0 (150)}{\varepsilon_c D_p^2 \rho_g} \frac{(1 - \xi)^2}{\xi^3} \quad (C-5)$$

In calculating mass velocity, the values substituted into Eq. C-5 were chosen such that the calculated value of $G_0$ would be a maximum. This would yield the maximum effect that flow rate through the frost could have on the frost thermal conductivity.

The pressure drop across the frost, $\Delta P_f$, was calculated to be 0.00176 psi. This value was calculated from the standard equations for pressure drop due to expansion, contraction and skin friction. (The largest possible value of the gas velocity was used in calculating $\Delta P_f$.) The frost thickness was taken as 0.25 inches, thus leaving the top half inch of the duct height free for gas flow. Values of $D_p$ and porosity were chosen such that the value of $G_0$ calculated from Eq. C-5 was a maximum. By this method $G_0$ was calculated to be 0.00515 lb mass/hr-ft$^2$. The particle Reynolds Number ($D_p G_0 / \mu \varepsilon$) is, therefore, 0.000024. This value will be used shortly in calculating the effect of gas flow on the frost conductivity.
B. Use of D'Arcy's Law to Calculate Gas Flow Rate Through Frost

As mentioned earlier in the Discussion of Results, the flow rate through the frost was also calculated from D'Arcy's law (59) using actual snow permeability data (4). D'Arcy's law is:

\[
\frac{G_0}{\rho_g} = \frac{K_D(\Delta P_f)}{L} \quad (C-6)
\]

where \(K_D\) is the coefficient of permeability, \(L\) the length of frost through which the gas flows, and \(\Delta P_f\) the pressure drop across the frost. No direct measurements of the permeability of cryogenic frost layers are available, but these values can be estimated from published data on snow (4), as will be shown below.

Bader's snow data, which extend over the density range 0.094 to 0.138 gm/cc, were extrapolated to a density of 0.05 gm/cc. The permeability of snow at this density is 86,000 ft²/hr-psi. In addition, Bader's experimental observations revealed that snow permeability varied as the 1.63 power of the particle diameter. Thus, in calculating the permeability of a cryogenic frost layer, the data of Ref. 4 had to be corrected for particle size as the sizes of particles in snow and in cryogenic frosts differ by about an order of magnitude. Bader's data are for an effective particle diameter of 0.7 mm., however, the sizes
of the particles in a cryogenic frost range from 20 to 50 microns. The permeability ($K_D$) of a 0.05 gm/cc density cryogenic frost made up of 50 micron particles is therefore:

$$K_D = \left(\frac{0.05}{0.7}\right)^{1.63} (86,000)$$

$$K_D = 1,070 \text{ ft}^2/\text{hr-psi}$$

The temperature of the snow in Bader's experiments was about 20°F. Temperatures within a cryogenic frost vary from 25°F to about -315°F. A viscosity correction, therefore, had to be applied to the permeability value of 1070 ft²/(hr-psi), as permeability is inversely proportional to viscosity (59). The viscosity of the gas flowing through the frost was evaluated at a temperature of -315°F as this value of viscosity maximizes the permeability constant and hence the calculated flow rate through the frost. The value of the permeability corrected for viscosity is therefore

$$(1,070) \frac{\mu_{20}}{\mu_{-315}} = 2,340 \frac{\text{ft}^2}{\text{hr-psi}}$$

Substituting this value of the permeability, the $\Delta P_f$ previously calculated, i.e., 0.00176 psi, a frost length of 8.9 inches, and the density of nitrogen at room temperature into D'Arcy's law, the value of $G_o$ was calculated to be 0.408 lb mass/ft²-hr. The value of the superficial flow rate through the frost calculated from
D'Arcy's law is two orders of magnitude greater than the value of the flow rate calculated from the Ergun correlation. The value of $G_0$ calculated from D'Arcy's law was therefore used to estimate the effect of flow through frost on the frost conductivity as it was desired to compute the maximum effect that the flow could have.

C. Calculation of Effect of Gas Flow through the Frost on Frost Conductivity

The method for calculating this effect was described by Yoon (92). In his thesis Yoon (92) investigated the effect of axial gas flow on the radial thermal conductivity of a packed bed. In addition, he analyzed the various mechanisms which contribute to the bed thermal conductivity. He reached the conclusion that the over-all thermal conductivity was made up of two main contributions (assuming radiant heat transport negligible). These are the contribution to thermal conductivity by the solid-fluid-solid conduction mechanism and the contribution to over-all thermal conductivity resulting from the gas flow through the pores. Yoon showed that these two mechanisms could be considered to be in parallel, and that the first (i.e., the solid-fluid-solid series conduction mechanism) was independent of Reynolds Number. The contribution that the solid-fluid-solid conduction mechanism made to the over-all thermal conductivity could therefore be calculated by just considering a stagnant situation. The contribution to bed thermal conductivity made by
gas flow through the frost is a function of gas flow rate.

As mentioned in Chapter V, it was shown that when heat is transferred perpendicularly to the direction of the flowing gas the modified Peclet number is equal to 11 (92). Other experimentors have also observed that the Peclet number for heat transfer was 11 over wide ranges of Reynolds Numbers, particle sizes and solid conductivities.

The definition of the modified Peclet number is:

$$\frac{G}{D_p \left( \frac{c_p}{\epsilon} \right) c_p} = \frac{k_f}{k_f - k_f}$$

where $k_{f,f}$ is the thermal conductivity of the frost with gas flow through it, and $k_f$ is the thermal conductivity of the frost with stagnant gas.

By algebraic manipulation, Eq. C-7 becomes:

$$k_{f,f} = k_f + \frac{1}{Pe} D_p \left( \frac{c_p}{\epsilon} \right) c_p$$

Dividing the entire expression by $k_f$, and multiplying and dividing the second term on the right side by $\mu$, Eq. C-8 is transformed to:

$$k_{f,f} = \frac{k_f}{k_f} + \frac{1}{Pe} \frac{D_p}{\epsilon} \frac{c_p \mu}{k_f}$$

Since $Pe$ was found to be equal to 11, and $(c_p \mu/k_f) = Pr$, Eq. C-9 becomes:
\[
\frac{k_{fr}}{k_g} = \frac{k_f}{k_g} + \frac{1}{11} \text{Pr} \left( \frac{D_p G_0}{\mu \varepsilon} \right)
\]  
(C-10)

In the experiments of the present thesis, gas flow through the frost was perpendicular to the direction of the heat transfer, and Eq. C-10 is, therefore, applicable.

The particle Reynolds Number \( \left( \frac{D_p G_0}{\mu \varepsilon} \right) \) for a superficial mass velocity of 0.408 lb mass/ft\(^2\)-hr is 0.00191. Since the Prandtl number is equal to about 0.7, the value of the term \( \left[ \text{Pr}/11 \left( \frac{D_p G_0}{\mu \varepsilon} \right) \right] \) is therefore \( 1.5 \times 10^{-4} \). The lowest value of \( \frac{k_{fr}}{k_g} \) measured in the present thesis was 1.43. Thus, from Eq. C-10, it can be concluded that \( \frac{k_{fr}}{k_g} \approx \frac{k_f}{k_g} \).

3. **Effect of Diffusion Through Frost Layer**

   A. **Thermal Diffusion**

   As mentioned in the Discussion of Results, it appears that the large temperature gradient across the frost layer makes water vapor transport by thermal diffusion significant compared with transport caused by the concentration gradient. This appears to be especially true in the lower portions of the frost where the temperature gradient is large and the water vapor concentration gradient small. In this appendix the role of thermal diffusion will be evaluated.
The total flux of water vapor within the frost due to both thermal diffusion and the concentration gradient is given by Eq. C-11.

\[
\text{Total Flux} = \frac{D_{1-2}}{n_1n_2} \left( \frac{dn_1}{dy} \right) + \frac{k_T}{T} \left( \frac{dT}{dy} \right)
\]  

(C-11)

The term \((D_{1-2}/n_1n_2)(dn_1/dy)\) represents the mass flux due to the ordinary concentration driving force; the term \((D_{1-2}/n_1n_2)(k_T/T)(dT/dy)\) is the water vapor flux due to thermal diffusion. Grew and Ibbs (35) define \(k_T\), the thermal diffusion coefficient, to be equal to \(\alpha n_1n_2\), where \(\alpha\) denotes the thermal diffusion factor. Various methods for evaluating \(\alpha\) are presented in Ref. 35. In the present thesis \(\alpha\) was evaluated by assuming the molecules to be hard spheres. Calculations based on a hard sphere model result in the maximum values of \(\alpha\) and thus in the maximum value of the mass flux caused by thermal diffusion. It should be noted that for a hard sphere model \(\alpha\) is independent of temperature and a very weak function of concentration. From the graphs on page 29 of Ref. 35, it was found that \(\alpha \approx 0.175\) for very low water vapor concentrations of a nitrogen-water vapor gas mixture. This value was considered applicable at all positions within the frost.

The contribution to the total mass flux caused by thermal diffusion was evaluated by comparing the terms
\[ \frac{dn_1}{dy} \text{ with } \frac{k_T}{T} \frac{dT}{dy}. \] Comparing these two terms is essentially the same as comparing the terms \( \frac{dn_1}{dT} \) and \( \frac{0.175}{T} n_1 n_2 \). This is shown below.

From elementary calculus,

\[ \frac{dn_1}{dy} = \frac{dn_1}{dT} \left( \frac{dT}{dy} \right) \]

Now, as mentioned earlier,

\[ \frac{k_T}{T} \left( \frac{dT}{dy} \right) = \frac{0.175}{T} n_1 n_2 \left( \frac{dT}{dy} \right) \]

Comparing \( \frac{dn_1}{dT} \left( \frac{dT}{dy} \right) \) with \( \frac{0.175}{T} n_1 n_2 \frac{dT}{dy} \) is obviously the same as comparing \( \frac{dn_1}{dT} \) with \( \frac{0.175}{T} n_1 n_2 \).

Values of \( \frac{dn_1}{dT} \) and \( \frac{0.175}{T} n_1 n_2/T \) at different temperatures are shown in Table V-5. At any given temperature \( \frac{dn_1}{dT} \) was obtained from the slope of the vapor pressure curve. Values of \( \frac{0.175}{T} n_1 n_2 \) were also obtained from the vapor pressure curve. At all temperatures between -300°F and 20°F the value of \( \frac{0.175}{T} n_1 n_2 \) was less than 1.04% of \( \frac{dn_1}{dT} \). It was therefore concluded that at all positions within the frost the mass flux due to thermal diffusion is small compared with the flux due to the concentration gradient. Thus, even where the temperature gradient is large and the concentration gradient small, the effects of thermal diffusion could be ignored.
Barron (7) reached the same conclusion about the role of thermal diffusion in a cryogenic frost layer. In evaluating \( \alpha \) he employed both a Sutherland (12) force model and a hard sphere model.

B. Evaporative Transport

The essentials of the derivation of the relationship for \( k_{xs} \) (Eq. V-20) were presented in Chapter V. At this point, however, the relationship for the total heat flow through the pore phase (Eq. V-17) will be examined in more detail.

Consider a differential element of gas within the pore phase. The cross sectional area of the element is unity and its thickness is \( dy \) (see Figure C-1). There are two sources for the heat which flows into this element. First, heat is conducted into the element as a result of the temperature gradient. The expression for the heat flow by this mechanism is \( k_g A \frac{dT}{dy} \). Secondly, enthalpy is brought into the element by the diffusion water vapor. The expression for the heat (or enthalpy) flux arising from this mechanism is \( \frac{D_{1-2}}{RT} \frac{dp^*}{dy} (\lambda M_v) A \). The total heat flux into the element \( dy \) is therefore

\[
\frac{q}{A} = k_g \frac{dT}{dy} + \frac{D_{1-2}}{RT} \frac{dp^*}{dy} (\lambda) M_v
\]

which is Eq. V-17.
As previously indicated, $D_{1-2}$ is the ordinary diffusivity of water vapor in nitrogen. The small gas flow through the frost did not alter significantly this value of the diffusivity. This conclusion was reached from an analysis similar to that used for evaluating the effect of gas flow rate through the frost on the over-all frost thermal conductivity.
Figure C-1  Heat Transfer Through Differential Element of Gas in the Pores
APPENDIX D

Models for Calculating Frost Thermal Conductivity

As previously mentioned, the thermal conductivity depends on three factors: the temperature, the frost density and the frost structure. In deriving expressions for the thermal conductivity, frost structure and density were considered to be independent parameters since two frost samples of the same density can have different structures. This point will be illustrated in the actual derivation of the expressions. It will be seen that the conductivity-density relationship is dependent on the assumed structure. (As mentioned earlier, however, in plotting experimental values of thermal conductivity as a function of density, it was implicitly assumed that density and structure have a unique correspondence.)

1. Parallel Slabs of Ice

   A. Heat Flow Parallel to Ice Slabs

   Sketches of the assumed structure are shown in Figure D-1. The porosity of the structural array is denoted by $\varepsilon$. $k_{\text{g,e}}$ denotes the total gas phase conductivity, and $k_1$ the thermal conductivity of the ice. The thermal resistance of all the ice slabs in the array is given by

   \[ R_1 = \frac{L}{A_1 k_1} \quad \text{(D-1)} \]
$A_1$ is the total area of the ice slabs normal to the direction of the heat flow; $l$ is the length of both an ice slab and a slab of gas (see Figure D-1A). The thermal resistance for all the slabs of gas is

$$R_g = \frac{l}{A_g k_g e} \quad (D-2)$$

where $A_g$ is the total area of the slabs of gas normal to the direction of the heat flow. Since these two resistances are in parallel, the total resistance (or frost resistance) is given by

$$\frac{1}{R_f} = \frac{1}{R_1} + \frac{1}{R_g} \quad (D-3)$$

Combining Eqs. D-1, D-2 and D-3

$$\frac{1}{R_f} = \frac{A_1 k_1}{l} + \frac{A_g k_g e}{l}$$

Since $R_f = l/k_f A$, where $A$ is the total cross sectional area, i.e., $A = A_1 + A_g$,

$$\frac{1}{R_f} = \frac{k_f A}{l} = \frac{A_1 k_1}{l} + \frac{A_g k_g e}{l}$$

or
\[ k_f = \frac{A_1}{A} k_i + \frac{A_g}{A} k_{g,e} \]

Now \( \varepsilon = \frac{A_g}{A} \) and \((1 - \varepsilon) = \frac{A_i}{A}\). Thus,

\[ k_f = (1 - \varepsilon)k_i + \varepsilon k_{g,e} \quad (D-4) \]

B. Heat Flow Normal to Array of Ice Slabs

The expression for heat flow perpendicular to the ice slabs is derived in a similar manner. (See Figure D-2 for a sketch of the assumed structure.) As in the case for the heat flow parallel to the slabs, the number and spacing of the ice and gas slabs is immaterial.

The thermal resistance of any individual ice slab is

\[ R_i = \frac{w_i}{A_i k_i} \quad (D-5) \]

where \(A_i\) is the area normal to the heat flow of any one slab of ice. \(w_i\) can vary from slab to slab. The thermal resistance of any individual slab of gas is

\[ R_g = \frac{w_g}{A_g k_{g,e}} \quad (D-6) \]

\(w_g\) can be different for different slabs. The equivalent thermal resistance of the composite is
\[ R_{eq} = \sum \left( \frac{w_1}{A_1 k_1} \right) + \sum \left( \frac{w_g}{A_g k_{g,e}} \right) \quad (D-7) \]

Now

\[ R_{eq} = \frac{\sum w_1 + \sum w_g}{k_f A} \]

and, since \( A = A_1 = A_g \), Eq. D-7 becomes

\[ \frac{\sum w_1 + \sum w_g}{k_f} = \frac{1}{k_1} \sum w_1 + \frac{1}{k_{g,e}} \sum w_g \quad (D-8) \]

or

\[ \frac{1}{k_f} = \frac{1}{k_1} \frac{\sum w_1}{\sum w_1 + \sum w_g} + \frac{1}{k_{g,e}} \frac{\sum w_g}{\sum w_1 + \sum w_g} \]

Now

\[ \frac{\sum w_1}{\sum w_1 + \sum w_g} = (1 - \varepsilon) \quad \text{and} \quad \frac{\sum w_g}{\sum w_1 + \sum w_g} = \varepsilon \]

Thus

\[ \frac{1}{k_f} = \frac{(1 - \varepsilon)}{k_1} + \frac{\varepsilon}{k_{g,e}} \quad (D-9) \]
Inverting Eq. D-9,

\[
k_f = \frac{(k_{g,e})(k_i)}{\varepsilon k_i + (1 - \varepsilon)k_{g,e}}
\]  
(D-10)

2. Simple Cubic Array of Non-Touching Ice Cubes

As mentioned before, two distinct cases were considered: in one, thermal conductivity was assumed to be zero in a direction perpendicular to the heat flow, and in the second case the conductivity was assumed to be finite in the direction perpendicular to the flow of heat. A three dimensional view of this structure is shown in Figure D-3.

A. Zero Sidewise Conductivity

For this case the division of a typical unit cube of the heterogeneous structure into thermal resistance zones is shown in Figure D-4. The shaded portion is the cube of ice. The remainder of the cube is gas. An expression for the thermal conductivity of this array, assuming zero sidewise conductivity, is derived by considering the thermal resistances of Zones 1 and 2 to be in parallel. Zone 1 is a cube with a square hole in it, whereas Zone 2 is a plug consisting of gas and solid in series.
For Zone 1

Thermal Resistance = \frac{1}{(1-x^2)k_{g,e}} \quad (D-11)

For Zone 2

Thermal Resistance = \frac{x}{x^2k_1} + \frac{1-x}{x^2k_{l,g,e}} \quad (D-12)

or

Thermal Resistance = \frac{xk_{g,e} + k_1(1-x)}{x^2k_1k_{g,e}} \quad (D-13)

Adding the parallel resistances of Zones 1 and 2,

\frac{1}{\left[\left(\text{length of a unit cube}\right)\right]} = (1-x^2)k_{g,e} + \frac{x^2k_1k_{g,e}}{xk_{g,e} + (1-x)k_1}

By algebraic manipulation

\frac{k_f}{k_{g,e}} = (1-x^2) + \frac{x^2}{x(k_{g,e}/k_1) + (1-x)} \quad (D-14)
B. Infinite Sidewise Conductivity

An expression for the case of infinite sidewise conductivity is derived in a similar manner. The equivalent thermal resistance of a typical unit cube is derived by considering Zones 1', 2' and 3' to be in parallel. The combined thermal resistance of Zones 1' and 3' is (see Figure D-5)

\[
\frac{1-x}{2} \frac{1}{k_{g,e}(1)} + \frac{1-x}{2} \frac{1}{k_{g,e}} = \frac{1-x}{k_{g,e}} \quad (D-15)
\]

For Zone 2'

\[
\text{Thermal Resistance} = \frac{1}{\frac{k_{i} x^2}{x} + \frac{k_{g,e}(1-x^2)}{x}}
\]

or

\[
\text{Resistance} = \frac{x}{k_{i} x^2 + k_{g,e}(1-x^2)} \quad (D-16)
\]

Combining Eqs. D-15 and D-16,
Total Resistance of Series Composite of Zones 1', 2' and 3' 

\[ R = \frac{1-x}{k_{g,e}} + \frac{x}{k_i x^2 + k_{g,e}(1-x^2)} \]

Length of a Unit Cube 

\[ \frac{k_f(\text{unit cube})}{k_f(1)} = \frac{(1)}{k_f(1)} = \frac{1-x}{k_{g,e}} + \frac{x}{k_i x^2 + k_{g,e}(1-x^2)} \]

By algebraic manipulation, Eq. (D-17) becomes

\[ \frac{k_f}{k_{g,e}} = \frac{x^2 + \frac{k_{g,e}}{k_i} (1 - x^2)}{(1-x) \left[ x^2 + \frac{k_{g,e}}{k_i} (1-x^2) \right] + x \frac{k_{g,e}}{k_i}} \]

(D-18)

3. Simple Cubic Array of Cubes of Gas

Unlike a simple cubic array of ice cubes, a structure consisting of a cubic array of gas cubes can physically exist.

For this structure (see Figure D-6), only the case of zero sidewise conductivity will be considered. The detailed derivation of the thermal conductivity expression is given in Ref. 62. The important features of the derivation will
be presented below. A typical unit cube of this heterogeneous array is shown in Figure D-7.

The thermal conductivity of this array can be obtained by considering the resistances of Zones 1 and 2 to be in parallel. Thus the derivation would proceed in exactly the same manner as for the case of a cubic array of ice cubes having zero sidewise conductivity. The final equation would then be the same as Eq. D-14, except \( k_1 \) would be replaced by \( k_{g,e} k_{g,e} \) by \( k_1 \) and \( x \) by \( \varepsilon^{1/3} \), where \( \varepsilon \) is the porosity of the structure. \( \varepsilon^{1/3} \) is, of course, the linear dimension of the ice cube. The final equation, therefore, is

\[
\frac{k_f}{k_1} = (1 - \varepsilon^{2/3}) + \frac{\varepsilon^{2/3}}{\varepsilon^{1/3} \left( \frac{k_i}{k_{g,e}} \right) + (1 - \varepsilon^{1/3})}
\]

(D-19)

4. **Simple Cubic Array of Large Gas Spheres Interspersed in an Ice-Gas Matrix (Weber Model) (87)**

A. **Material Balance**

Equation D-20 can be written for the cluster-phase since it consists of closely packed spheres (see Figure D-8).
\[
\frac{\rho_c - \rho_g}{\rho_i - \rho_g} = 0.74 \quad (D-20)
\]

Since \( \rho_i \gg \rho_g \), Eq. D-20 reduces to

\[
\frac{\rho_c}{\rho_i} = 0.74 \quad (D-21)
\]

Thus the cluster phase density is always equal to about 0.74 \( \rho_i \).

The volume fraction of the pocket phase, i.e., of the large spherical gas bubbles, is defined as \( \varepsilon' \) and is given by

\[
\varepsilon' = 1 - \left( \frac{\rho_f - \rho_g}{\rho_c - \rho_g} \right) \quad (D-22)
\]

Since \( \rho_c \gg \rho_g \), Eq. D-22 reduces to

\[
\frac{\rho_f}{\rho_c} = 1 - \varepsilon' \quad (D-23)
\]

Both phases are continuous when \( \varepsilon' > 0.524 \). At this value of \( \varepsilon' \) the gas pockets intersect at a point. For \( \varepsilon' > 0.965 \) the dense phase becomes discontinuous, and the structure collapses. Therefore, according to this model, minimum frost density is 0.024 gm/cc. In actuality, snow densities as low as 0.013 gm/cc have been observed (32).
B. Thermal Conductivities

In calculating thermal conductivities, Weber assumed zero conductivity in a sidewise direction. The heat flux could therefore be written as an integration of series conduction paths over a unit area normal to the heat flow (see Figure D-9).

\[ q = -k_f \frac{T_2 - T_1}{L_{po} + L_c} = -\int \frac{T_2 - T_1}{L_{po} \frac{k_{po}}{k_c} + L_c \frac{k_c}{k_{po}}} \, dA \quad (D-24) \]

Noting that \((L_{po} + L_c)\) and \(k_c\) are constants, Eq. D-24 can be re-arranged to

\[ \frac{k_f}{k_c} = \int \frac{1 + L_c/L_{po}}{k_c \frac{k_{po}}{k_c} + L_c \frac{k_c}{L_{po}}} \, dA \quad (D-25) \]

Thus the ratio of the frost conductivity to the cluster phase conductivity is determined by the ratio of the cluster and pocket conduction path lengths, and the ratio of the conductivities of the two phases. The relationship between \(L_{po}/L_c\) and \(A\) is, of course, determined by the geometry of the structure. Weber obtained an expression for the conductivity of the composite material (pockets and cluster), \(k_f\), by analytically integrating Eq. D-25 for \(\varepsilon' \ll 0.524\).

The result is:
\[ \frac{k_f}{k_c} = 1 - \frac{\pi}{4} \left( \frac{6 \varepsilon'}{\pi} \right)^{2/3} \left\{ 1 - \frac{2}{\gamma} \left[ 1 - \ln \left( \frac{1+\gamma}{\gamma} \right) \right] \right\} \]  
\hspace{\stretch{1}} (D-26)

where
\[ \gamma = \left( \frac{k_c}{k_{po}} - 1 \right) \left( \frac{6 \varepsilon'}{\pi} \right)^{1/3} \]  
\hspace{\stretch{1}} (D-27)

For \( \varepsilon' \) greater than 0.524, i.e., where the contact areas between the gas pockets are greater than points, Eq. D-25 was integrated numerically by Simpson's rule. Using the results of his numerical integration, and Eq. D-26 shown above, Weber generated curves of frost thermal conductivity as a function of density with temperature as a parameter. Actually, in generating these curves, Weber required values of \( k_c \) as a function of temperature. He obtained these values from Ref. 92. In this reference (i.e., Ref. 92), a curve of \( \frac{k_c}{k_G} \) versus \( \frac{k_s}{k_G} \) was generated by integrating Eq. D-28 shown below.

\[ \frac{k_c}{k_s} = \int \frac{1 + \frac{L_s}{L_G}}{\frac{k_s}{k_G} + \frac{L_s}{L_G}} \, dA \]  
\hspace{\stretch{1}} (D-28)

5. Simple Cubic Array of Ice Spheres (Woodside Model (91))

As mentioned earlier in the Discussion of Results, Woodside assumed that the heterogeneous structure consisted of a simple
cubic array of ice spheres and that conductivity was infinite in a sidewise direction. (See Figure D-10 for a three dimensional view of the structure.) The derivation of an expression for calculating the thermal conductivity of this array is presented in Ref. 91. The essential features of the derivation will be presented below.

In deriving the expression for the conductivity, Woodside chose as a representative unit cube that portion of the heterogeneous structure which contained one-eighth of an ice sphere rather than one which contained the entire sphere of ice. Woodside's typical unit cube is shown in Figure D-11. Obviously the expression for the thermal conductivity does not depend on whether the chosen unit cube contains one-eighth of the ice sphere or the entire ice sphere.

The thermal conductivity of the composite was obtained by finding the equivalent thermal resistance of Zones I and II. The resistances of these zones are in series.

From a material balance it can be shown that \( (1 - \varepsilon) \), the solids fraction, is \( \left[ \frac{(\rho_f - \rho_g)}{(\rho_i - \rho_g)} \right] \). From Figure D-11 it can be seen that \( (1 - \varepsilon) \) is also equal to one-eighth the volume of the sphere of ice divided by unity (the volume of the typical unit cube); i.e.,

\[
(1 - \varepsilon) = \frac{1}{8} \left( \frac{4}{3} \pi R_s^3 \right) = \frac{\pi R_s^3}{6} \quad \text{(D-29)}
\]
The thermal resistance of Zone I consists of the parallel combination of the solid resistance and the resistance of the adjacent gas. At any given values of $x$, the thermal resistance of a differential element of gas $dz_s$ is

$$dz_s \frac{\pi r^2}{(1 - \frac{\pi r_s^2}{4})k_{g,e}}$$

At this same position the thermal resistance of a differential element of solid is

$$dz_s \frac{\pi R^2}{\frac{\pi R_s^2}{4}k_1}$$

Thus the reciprocal of the equivalent thermal resistance of the differential slice in Zone I is

$$\frac{1}{\text{Resistance of } dz_s} = \frac{1}{dz_s \sqrt{\frac{\pi r^2}{4}k_1}} + \frac{1}{dz_s \sqrt{\frac{\pi r_s^2}{4}k_{g,e}}} \quad \text{(D-30)}$$

or
The resistance of Zone II is

\[ R_{\text{zone II}} = \frac{1 - R_s}{k_{g,e}} \]

Since Zones I and II are in series, the equivalent resistance of the unit cube is

\[ R_{\text{eq}} = \frac{1 - R_s}{k_{g,e}} + \int_0^{R_s} \frac{dz_s}{k (\frac{\pi r^2}{4}) + k_{g,e} (1 - \frac{\pi r^2}{4})} \]

or, since \( R_{\text{eq}} = \frac{1}{(1/k_f)} \)

\[ (1/k_f) = \frac{1 - R_s}{k_{g,e}} + \int_0^R \frac{dz_s}{k (\frac{\pi r^2}{4}) + k_{g,e} (1 - \frac{\pi r^2}{4})} \]

The variable \( r_s \) is eliminated by use of the equality
This leaves $x$ as the only variable under the integral.

Equation D-33 thus becomes

$$
\frac{1}{k_f} = \frac{1 - R_s}{k_{g,e}} \int_0^R dz_s \frac{dz_s}{\left( (k_1 - k_{g,e}) \frac{\pi}{4} R_s^2 + k_{g,e} \right) - (k_1 - k_{g,e}) \frac{\pi}{4} z^2}
$$

(D-34)

After integration, substitution of $\left[ \frac{6(1-\varepsilon)}{\pi} \right]^{1/3}$ for $R_s$, and appropriate algebraic manipulation, Eq. D-35 is derived. (For the sake of brevity, the details of the integration and of the algebraic manipulation have not been shown.)

$$
k_f/k_{g,e} = \left\{ 1 - \left( \frac{6(1-\varepsilon)}{\pi} \right)^{1/3} \left[ 1 - \left( \frac{a^2}{2a} \right) \ln\left( \frac{a+1}{a-1} \right) \right] \right\}^{-1}
$$

(D-35)

where

$$
a = \left\{ 1 + \frac{4}{\gamma \pi \left[ \frac{6(1-\varepsilon)}{\pi} \right]^{2/3}} \right\}^{1/2}
$$

and

$$
\gamma = \frac{k_1 - k_{g,e}}{k_{g,e}}
$$
Figure D-1A  Heat Flow Parallel to an Array of Ice Slabs
Figure D-1B  Heat Flow Parallel to Two Ice Slabs

Note: In Deriving Equation D-4 the Structure in Either Figure D-1A or D-1B Could have been Considered.
Figure D-2A  Heat Flow Normal to Parallel Array of Ice Slabs
Figure D-28  Heat Flow Normal to two Ice Slabs

Note: In Deriving Equation D-10 the Structure in Either Figure D-2A or D-2B Could have been Considered
Figure D-3  Three Dimensional View of a Structure Consisting of a Simple Cubic Array of Ice Cubes
Figure D-4  A Unit Cube of the Heterogeneous Structure Consisting of a Simple Cubic Array of Non-Touching Ice Cubes. (Case of Zero Sidewise Thermal Conductivity)
Figure D-5  A Unit Cube of the Heterogeneous Structure Consisting of a Simple Cubic Array of Non-Touching Ice Cubes (Case of Infinite Sidewise Thermal Conductivity)
Figure D-6  Three Dimensional View of a Structure Consisting of a Simple Array of Non-Touching Cubes of Gas Interspersed in an Ice Matrix (Case of Zero Sidewise Thermal Conductivity)
Figure D-7  A Unit Cube of the Heterogeneous Structure Consisting of a Simple Cubic Array of Non-Touching Gas Cubes Interspersed in an Ice Matrix (Case of Zero Sidewise Thermal Conductivity)
Figure D-8  Weber Model

Figure D-9  Heat Conduction in Weber Model
Figure D-10  Three Dimensional View of Structure Consisting of a Simple Cubic Array of Non-Touching Ice Spheres

Figure D-11  Unit Cube of Heterogeneous Structure Consisting of a Simple Cubic Array of Non-Touching Ice Spheres
APPENDIX E

Proposed Frost Model: Cubes of Ice Interconnected by Ice Rods

1. Material Balance

\[ 1 - \varepsilon = \frac{\text{volume of solid in a unit cube}}{\text{volume of unit cube}} \]

\[ = \text{volume fraction of a solid ice in frost} \]

Thus

\[ 1 - \varepsilon = \frac{x^3}{\text{volume of ice cube}} + 6\left[\frac{1-x}{2}(z^2)\right] \]

\[ \text{total volume of six ice rods} \]

or

\[ 1 - \varepsilon = x^3 + 3z^2(1-x) \]

2. Equation for Frost Thermal Conductivity

Three dimensional and planar views of the frost structure are shown in Figures E-1 and E-2 respectively.

A representative unit cube of the heterogeneous structure is shown in Figure E-3. Only the case of infinite sidewise thermal conductivity will be considered. Thus the equivalent thermal resistance of the unit cube is obtained by considering
the resistances of Zones 1, 2, 3, 4 and 5 to be in series.

The length of the ice cube is \( x \). The rods are square prisms with cross sectional area equal to \( z^2 \). It should be noted that the resistances of Zones 1 and 5 are identical and so are the resistances of Zones 2 and 4. The combined thermal resistance of Zones 1 and 5 is

\[
\text{Resistance} = \frac{1 - x}{k_{g,e}(1 - z^2) + k_1 z^2} \quad (E-1)
\]

The combined thermal resistance of Zones 2 and 4 is

\[
\text{Resistance} = \frac{(x - z)}{k_{g,e}(1 - x^2) + k_1 x^2} \quad (E-2)
\]

For Zone 3 the thermal resistance is

\[
\text{Resistance} = \frac{z}{2k_1 z(1 - x) + k_1 x^2 + k_{g,e} \left[ 1 - x^2 - 2z(1-x) \right]} \quad (E-3)
\]

The combined thermal resistance is obtained by adding Eqs. E-1, E-2 and E-3 as these resistances are in series. Thus,

\[
\text{Total resistance of unit cube} = \frac{1}{k_f \left\{ \frac{\text{(thickness)}}{k_f \cdot \text{(Area)}} \right\}} = 1
\]
Thus,

\[
\frac{1}{k_f} = \frac{(1 - x)}{k_{g,e}(1 - z^2) + k_1 z^2} + \frac{(x - z)}{k_{g,e}(1 - x^2) + k_1 x^2}
\]

\[
+ \frac{z}{2k_1 z(1-x) + k_1 x^2 + k_{g,e} \left[1-x^2-2z(1-x)\right]} \quad (E-4)
\]
Note: Though it is not shown in this diagram, rods extend from all six faces of each cube.

Figure E-1 Assumed Frost Structure: Cube and Rod Model
Figure E-2  Representative Unit Cube for Cube and Rod Model
Figure E-3  Planar View of a Representative Unit Cube
APPENDIX F

Frost Densification with Time

In this appendix an expression for predicting rates of densification with time will be derived. The procedure is similar to the method used by Weber (87).

As indicated in the Discussion of Results, Eq. F-1, presented below, is obtained by setting the term \( \frac{\delta\rho_f}{\delta\Theta} \) equal to the rate of water vapor which diffuses from the frost surface into the frost interior.

\[
5 \frac{\delta \rho_f}{\delta \Theta} = \frac{(M_v) D_{1-2} c_T}{\tau (1 - Y)} (1 - \frac{\rho_f}{\rho_i}) (\frac{dV}{dy})_s
\]  

(The tortuosity for this unconsolidated medium was assumed to be 1.1.)

From elementary calculus

\[
\left(\frac{dV}{dy}\right)_s = \left(\frac{dT}{dy}\right)_s \cdot \left(\frac{dV}{dT}\right)_s
\]

Thus, combining Eqs. F-1 and F-2, Eq. F-3 is derived:

\[
5 \frac{\delta \rho_f}{\delta \Theta} = \frac{(M_v) D_{1-2} c_T}{\tau (1 - Y)} (1 - \frac{\rho_f}{\rho_i}) (\frac{dV}{dT})_s (\frac{dT}{dy})_s
\]

If it is assumed that at the frost surface the gas within the pores is saturated at the temperature of the surface, Eq. F-3 becomes:
\[ \frac{5}{d\theta} \frac{d\rho_f}{d\theta} = \frac{(M_v)D_{1-2}c_T}{\tau(1 - \gamma^*)} (1 - \frac{\rho_f}{\rho_1}) \frac{dy^*}{dT} \frac{dT}{dy} \] (F-4)

By combining Eq. F-4 with the Clausius-Clapeyron equation--Eq. F-5--(61), Eq. F-6 is derived:

\[ \frac{d\gamma^*}{dT} = \frac{\gamma^*}{RT^2} \lambda M_v \] (F-5)

\[ \frac{5}{d\theta} \frac{d\rho_f}{d\theta} = \frac{D_{1-2}c_T}{\tau RT^2} (1 - \frac{\rho_f}{\rho_1}) \frac{\gamma^*}{RT^2} \lambda (M_v)^2 \frac{dT}{dy} \] (F-6)

Even at the relatively high temperatures of the frost surface (i.e., about 20°F), \( \gamma^* \) is negligible compared with 1. Thus Eq. F-6 becomes

\[ \frac{5}{d\theta} \frac{d\rho_f}{d\theta} = \frac{D_{1-2}c_T}{\tau RT^2} (1 - \frac{\rho_f}{\rho_1}) \frac{\gamma^*}{RT^2} \lambda (M_v)^2 \frac{dT}{dy} \] (F-7)

Now, replacing \( c_T \) with \( \rho_g/M_g \), Eq. F-7 becomes

\[ \frac{5}{d\theta} \frac{d\rho_f}{d\theta} = \frac{D_{1-2} \rho_g}{\tau RT^2} \frac{M_v^2}{M_g} \lambda \frac{\gamma^*}{RT^2} (1 - \frac{\rho_f}{\rho_1}) \frac{dT}{dy} \]

or

\[ \frac{5}{d\theta} \frac{d\rho_f}{d\theta} = \frac{\beta(1 - \frac{\rho_f}{\rho_1}) \frac{dT}{dy}}{\tau(1 - \frac{\rho_f}{\rho_1})} \] (F-8)
where, of course,

\[ \beta = \frac{D_{1-2} \rho_g (M_v^2) \lambda y^*}{RT^2 M_g} \]  \quad (F-9)

Thus, at any given pressure \( \beta \) is a function of temperature (i.e., neglecting any dependence of \( D_{1-2} \) on concentration). Figures F-1A, F-1B and F-1C show \( \beta \) as a function of temperature for a pressure of one atmosphere.

In calculating values of \( \beta \), the value of \( D_{1-2} \) chosen was, as mentioned in Chapter V, the ordinary value of the diffusivity of water vapor in nitrogen. \( D_{1-2} \) was evaluated at the different temperatures using the method presented in Ref. 68.
Figure F-IA  $\beta$ as a Function of Temperature; High Temperature Range
Figure F-1B  $\beta$ as a Function of Temperature; Intermediate Temperature Range
Figure F-1C $\beta$ as a Function of Temperature; Low Temperature Range
APPENDIX G

Instrumentation

1. Temperature Measurements

All temperature measurements were made with copper-constantan thermocouples. The size of the thermocouple wire depended on the application.

A. Gas Temperatures

Measurements of the gas temperatures were made at various locations throughout the gas flow system. Generally, the purpose of measuring these temperatures was to ascertain whether the various components of the system were functioning properly. With the exception of the temperature of the gas above the copper plate, gas phase temperatures were not reported in this thesis. The gas temperature above the copper plate was measured with a movable thermocouple probe—the same probe used to measure the temperature of the frost surface.

Table G-1 lists the locations of the various thermocouples used to measure temperatures in the recirculating gas stream. All of these thermocouples, with the exception of the movable probe, were made of No. 24 gauge copper constantan wire (diameter = 0.05 inches).

Errors from various sources can arise when a thermocouple is used to measure the temperature of a gas
stream (27). For example, at high gas velocities the true gas temperature is significantly different from the stagnation temperature. The measured temperature lies somewhere between these two values. In this investigation the gas velocities at all points in the system were sufficiently low to ensure that the difference between the true gas temperature and the stagnation temperature was negligible.

Another possible source of error in the measurement of the gas phase temperature is heat conduction along the wires to the measuring junction. This error was made negligible by installing a sufficiently long length of wire in the gas stream. Errors due to radiation from the walls to the thermocouple beads were negligible because of the small temperature differences involved.

B. Temperature Measurements on Copper Plate

The thermocouples located on the copper plate, and at other points within the calorimeter, were made of No. 24 gauge copper constantan wire. Two thermocouples were silver soldered to the underside of the inner section of the copper plate. One thermocouple was used to check the reading of the other. Thermocouples were also silver soldered to the underside of the outer section of the copper plate.
C. Calorimeter Temperatures

The temperature of the gas space in the outer chamber was measured with a No. 24 gauge, copper constantan thermocouple. During normal operation of the calorimeter, this temperature was usually within a couple of degrees of the liquid temperature. (The liquid nitrogen in the guard chamber was essentially at the normal boiling point.) However, a drop of 3 to 4 inches in the liquid nitrogen level would result in a rise of about 20°F in the gas space temperature. The level of the liquid nitrogen within the guard chamber would drop on occasion because of an insufficient flow of liquid nitrogen from the supply Dewar. The gas space temperature rise was therefore caused by the increased heat leak into this space, which resulted from the drop in the liquid level. Actually, overheating of the gas space rarely occurred during a run as the liquid nitrogen level was carefully monitored.

D. Determination of Liquid Level in Guard Chamber

Three thermocouples, each attached at a different level on the outside of the brass chamber, were used to determine the location of the gas-liquid nitrogen interface. (All of these thermocouples were made of No. 24 gauge copper constantan wire.) Aluminum epoxy resin was used to attach the probes to the brass chamber. The
relatively high thermal conductivity of this adhesive ensured good thermal contact between the probes and the brass. The three junctions were located respectively at the liquid nitrogen overflow line, one-half inch above this line and one-half inch below it. When the liquid level in the outer chamber was at the overflow line, both $T_{\text{lower}}$ and $T_{\text{middle}}$ read liquid nitrogen temperature (-320.5°F); the reading of $T_{\text{upper}}$ was several degrees higher. When the level dropped to a position about one-half way between the middle and lower thermocouple locations, $T_{\text{lower}}$ still read liquid nitrogen temperature, but $T_{\text{middle}}$ and $T_{\text{upper}}$ read temperatures several degrees above -320.5°F. If the level rose high enough, all three thermocouples would read liquid nitrogen temperature.

The liquid level was controlled by adjusting the valves so that a small steady stream of liquid nitrogen flowed out of the overflow line. In practice it proved quite difficult to maintain this small steady flow continuously, and the liquid level occasionally dropped below the overflow line. As soon as this occurred, more liquid nitrogen was flowed to the calorimeter, and the liquid level returned to its former position. Heat flux measurements were made only when the liquid nitrogen level had been at the overflow line position for a sufficiently long duration, thus ensuring that the calorimeter was operating under steady state conditions.
E. Temperature Measurements in Frost Interior

Temperatures in the frost interior were measured with No. 38 gauge (wire diameter = 0.003 inches) thermocouples located at fixed distances above the cold copper plate. The exact distances between the copper plate and the wires were determined at the end of each run. The wires were parallel to the plate and gas flowed in a direction transverse to the length of the wire.

When the plate reached a temperature of about -320°F during the cool-down period, each of the stationary thermocouple wires was drawn taut. The wires were maintained in this condition by the insertion of thin tapered wooden probes into the small holes drilled in the lucite sides. The upper and lower thermocouples were generally about 0.14 and 0.06 inches, respectively, above the plate. The distance between each thermocouple and the plate surface could be varied over a range of 0.03 inches. The junction of each thermocouple was a lap joint about 1/8 of an inch wide. The joint was about 0.007 inches thick at its thickest part and had a thickness of 0.003 inches at its thinnest.

F. Probe for Measurement of Frost Surface Temperature

A movable thermocouple probe was used to measure the surface temperature of the frost. The probe was attached to a depth gauge micrometer. This probe, which could therefore move up and down in a vertical plane, consisted
essentially of a 0.005 inch diameter horizontal wire
strung between two vertical rods (see Figure II-2).

The measuring junction, which was about 0.011 inches
thick at its thickest and 0.005 inches thick at its thin-
nest, was located in the middle of the horizontal wire.
The method employed for obtaining a frost surface tempera-
ture measurement was described in Chapter III.

During those portions of a run when the frost sur-
face temperature was not being measured, the probe assembly
(i.e., item 2 of Figure II-2) was retracted completely into
a slit in the one-half inch thick lucite section located
over the copper plate. Thus, protrusions into the gas
stream were kept to a minimum.

G. Errors in Surface Temperature Measurement

A possible source of error in the measurement of
the frost surface temperature is conduction to the junction
along the thermocouple wires. This error was made negligible
by taking two precautions: first, thin thermocouple wire
was used; secondly, when a surface temperature measurement
was being made, long lengths of both the copper and con-
stantan wires extending from the junction contacted the
frost surface. This ensured long isothermal paths on
either side of the junction and hence conduction to the
bead was minimized.

Another possible source of error is radiant heat
transfer from the warm duct to the bead in contact with
the surface. The magnitude of this error cannot be estimated accurately as the contact thermal resistance between the thermocouple junction and the frost itself is unknown. The error was reduced by taking two precautions. First, when making a surface temperature measurement, the thermocouple junction was allowed to remain in contact with the frost surface for a relatively long length of time (about one minute). Thus, presumably, a substantial portion of the bead was covered by the growing frost. Secondly, radiant heat transfer from the warm inside of the duct to the colder thermocouple junction was minimized by lining the inside of the duct, including the lucite section immediately above the copper plate, with aluminum foil.

The magnitude of the error caused by gas flow over the thermocouple bead in contact with the frost surface also cannot be estimated accurately, since the thermal contact resistance between bead and frost is unknown. Again, this error was reduced by allowing a significant portion of the thermocouple bead to become covered with frost when a surface temperature measurement was being made. Errors caused by both radiation and gas flow over the thermocouple bead would result in thermocouple readings which are too high.

As mentioned earlier, the movable thermocouple probe also measured the temperature of the gas stream over the
copper plate. The gas flow past the probe did not cause a significant error in the gas stream temperature measurement.

H. Instrumentation for Recording Temperature Measurements

The millivoltage output of the movable probe was recorded on one of the channels of a two-channel, continuous Sanborn recorder. A 20 channel Leeds and Northrup recorder was used to record the output millivolts of the stationary thermocouples within the frost. Each channel on the 20 point recorder printed every six seconds; thus the time for a complete cycle was two minutes. Each stationary thermocouple was hooked up to five channels; its output millivoltage was therefore printed every 24 seconds.

The temperatures at the various locations in the recirculating gas stream were also read with the 20 channel recorder. Each thermocouple was connected to one channel. The temperatures of the various sections of the copper plate were also read on the 20 channel recorder. The maximum millivoltage which could be measured by the 20 channel recorder was five millivolts. If the thermocouples attached to the copper plate had been referenced to crushed ice baths, their outputs would have been about -5.4 millivolts. These thermocouples were therefore referenced to a dry ice
acetone bath. Their outputs were, therefore, about -2.7 millivolts, values which could be read on the recorder.

Actually, the readings of the thermocouples which were attached to the brass cylinder and which were used to determine the liquid nitrogen level were greater than five millivolts. These thermocouples, however, could be referenced to a crushed ice bath as they were read with a portable precision potentiometer, an instrument which has a total range of 0 to 80.5 millivolts.

All of the other thermocouples (i.e., with the exception, of course, of those attached to the copper plate) were referenced to crushed ice baths. The readings of all of these thermocouples, including those of the stationary probes buried in the frost, never exceeded five millivolts.

2. Pressure Measurements

Pressures at all points in the recirculating gas, with the exception of the pressure immediately downstream of the blower, were measured with water-filled manometers. A mercury-filled manometer measured the pressure downstream of the blower. Pressures at the various locations in the recirculating gas stream were used merely to check that the various parts of the equipment were functioning properly. These pressure values are not reported in this
thesis. It should be noted, though, that the pressure in the gas stream over the copper plate was essentially atmospheric for the three different Reynolds Numbers.

Pressures in the inner and outer cylinders of the calorimeter were also measured with water-filled manometers. These measured pressures are not reported in this thesis, but were used to determine the location of the liquid nitrogen level in the inner, stainless steel cylinder.

3. Flow Measurement

As mentioned earlier the flow rate was measured with an orifice meter. The orifice hole was 1.397 inches in diameter. The meter was constructed according to the ASME code described in Ref. 3. Radius taps were used for each orifice meter, i.e., the centers of the upstream and downstream tap holes were located exactly at distances of one tube diameter and 1/2 tube diameter from the upstream and downstream faces of the orifice plate respectively. The tap lines were 1/4 inch copper tubes soldered into the 1/4 inch tap holes.

For maximum accuracy the orifice hole was positioned so that it was concentric with the tube opening. To further ensure accurate flow rate determination the lengths of straight pipe upstream and downstream of the orifice plate were made longer than the recommended minimum requirements (66).
Despite all the precautions taken in constructing the orifice meter, it was still decided to calibrate. (See Appendix I for details of the calibration procedure and the results obtained.)

4. Measurement of Humidity

The humidity of the recirculating gas was measured a short distance upstream of the copper plate by analyzing a sample drawn from the flowing stream. To be more specific the humidity was measured by determining the quantity of water vapor contained in a measured volume of gas sample. A schematic of the humidity measuring equipment is shown in Figure G-1. The gas sample was drawn through the tube of drierite (anhydrous calcium sulphate) and the wet test meter by a water aspirator pump. The flow rate of the gas sample was insignificant compared with the flow rate of the recirculating gas.

The volume of gas bled off for analysis was measured with a wet test meter. The weight of water vapor contained in this gas sample was determined by weighing the tube of drierite before and after the sample was passed through it. This method of measuring gas stream humidity is quite reliable as the tube of drierite substantially adsorbs all of the moisture contained in the gas sample passing through it. The reliability of the method was checked by measuring the humidity of a gas stream which had been saturated with water vapor at a known temperature in a
packed tower. "Checks" were performed at several humidities. In all cases the humidity measured by the method described above was within 3% of the true humidity. The true humidity was taken as that corresponding to saturation at the temperature of the water flowing through the packed column.

During a frosting run, the humidity of the recirculating gas stream was measured at four or five different time intervals. This was done in order to determine whether gas stream humidity was varying during a run. The measured humidity variation during a run was generally within $\pm$ 2% of the value given in column 3 of Table IV-2. The humidity variation was usually random with time, i.e., it did not show a consistent trend during the course of a run.

It should be noted that the observed humidity variation and the error in the humidity measurement were approximately equal. Thus it is probable that the observed random humidity variation was due to errors in the measurement of the humidity rather than to variations in the performance of the drier and packed tower system.
Table G-1

Locations of Thermocouples Used to Measure Temperatures in Recirculating Gas Stream

1) In outlet mixing chamber, i.e., immediately after rectangular duct

2) In inlet chamber, i.e., immediately upstream of rectangular duct

3) Immediately downstream of electrical heater

4) Immediately upstream of electrical heater

5) Over copper plate
Gas Stream

Clamp (Open when Taking Sample)
Tygon Tubing
Copper Tubing
Tube of Drierite (Anhydrous Calcium Sulphate)

Note:
A Water Aspirator Pump Draws the Gas Sample Through the Tube of Drierite and the Wet Test Meter

Figure G-1  Schematic of Equipment for Taking Gas Sample for Humidity Analysis
Choosing a Frosting System

In this appendix the reasons for choosing a flat plate frosting system will be reviewed. In addition, the relative merits of some alternative systems will be discussed.

The primary requisite of the system is that the experimental results be of general applicability. In frosting situations such a system is difficult, if not impossible, to construct, because it appears that the type of frost formed is often dependent on the specific geometry. The system chosen should then be one which would facilitate an understanding of the frosting process. With this general criterion in mind, more specific criteria are listed below.

(1) It should be possible to observe the frost as it forms. This will enable the observer to determine whether the frost consists of ridges and/or ripples (6), or is reasonably smooth and planar.

(2) It should also be possible to make the following measurements at various intervals during a run: (a) frost thickness and frost surface temperature, (b) temperatures at known locations within the frost, and (c) heat flux through the frost.
The test section should be constructed so that at the end of each run it is relatively easy to take a frost sample for density and microscopic analysis.

With these criteria in mind, several alternative systems were visualized. As mentioned in Chapter I, these were: (1) a short flat plate that formed part of the underside of a long rectangular duct and was mounted relatively far downstream in the duct, (2) a rotating cylinder in a stagnant gas medium, and (3) a stationary or rotating cylinder mounted in a wind tunnel. If a rotating cylinder were chosen, the rotation should be negligible compared with the motion of the gas flowing past the cylinder.

1. Rotating Cylinder Mounted in a Stagnant Gas

At first glance a rotating cylinder mounted in a stagnant gas appeared very attractive as this system simulates a fully developed heat, mass and momentum transfer situation \(^{(80)}\). In other words the rotating cylinder represents the situation at a point relatively far downstream in a closed conduit. However, as mentioned in Chapter I, this system had to be rejected, because it would have been impossible to control the conditions of the bulk gas. In addition, a rotating cylinder was not used because of the difficulty that would be involved in measuring the frost surface temperature. If a thermocouple were used,
this probe would probably shear away the upper portions of the frost layer upon contact with the rotating frost surface.

2. Stationary or Rotating Cylinder in a Wind Tunnel

This system would be arranged so that the velocity profile of the gas approaching the cylinder is uniform. As previously mentioned, however, the primary disadvantage of such a system is that the frosting mechanism would vary along the circumference of the cylinder and lead to a frost of a non-uniform character. In fact, in experiments where humid gas flowed normal to a vertical cylinder containing liquid oxygen, Mohr (63) observed just such an effect.

In the early phases of experiments employing a stationary cylindrical system, frost would accumulate on the front face of the cylinder by direct impact of the particles; at the top and bottom of the cylinder particles would be transferred to the surface by the turbulent eddies. In the back the gas flow will have separated and particles would be transported by the vortex motion of the gas and by eddies. Even if particles did not form in the gas phase, the variation of heat and mass transfer coefficients along the circumference would probably still result in the variation of frost properties around the periphery.
As previously mentioned, the use of a slowly rotating cylinder instead of a stationary one would probably have resulted in a more uniform frost. The deposited frost, however, would probably represent some sort of average between the types of frosts formed under conditions of turbulence, direct impact, or in the presence of vortices. The results would certainly be difficult to interpret. Thus this system also had to be rejected.

3. Short Flat Plate in a Rectangular Duct

As stated in Chapter I, the short flat plate system was the one finally chosen. The plate was mounted relatively far downstream in the duct so that the gas stream attained a fully developed velocity profile before passing over the frost layer. The velocity profile of the gas, immediately upstream of the frost, remained unchanged with time. The growing frost layer, however, caused the velocity of the gas stream over the plate to change during the run. As mentioned earlier, though, the variation in velocity did not alter the gas stream Reynolds Number.
APPENDIX I
Calibration of Orifice Meter

The orifice meter was only calibrated over 25% of its pressure range because of pressure limitations of the calibrating instrument. This pressure range covered about one-half of the instrument flow range.

The calibrating device was a large gasometer. Constant flow rates were produced by the displacement of gas from an inverted vessel which fell with constant velocity through a tank containing water. Gas flows up to about 60 (std. ft\(^3\))/(min) were produced.

Agreement to within 2% was obtained between the actual flow rates determined by measuring the displacement of the gasometer (i.e., the true flow rates) and the "calculated" rates. The "calculated" rates were obtained by substituting the measured pressure drop across the orifice into the standard orifice meter equation, (3). The error values at the various flow rates did not exhibit any trend with flow rate.

Because of the excellent agreement between the true and the "calculated" flow rates, it was decided that the standard orifice meter equation, (3), rather than a special calibration curve, would be used to determine gas flow for the various frosting runs. This method ensured an accuracy
of at least 2%, which is sufficient for the present thesis. The errors in the reported Reynolds Number values are approximately the same as the errors involved in the measurement of flow rate.
APPENDIX J

Calibration of Calorimeter

The details of the calorimeter construction and operation have been presented in Chapters II and III. When heat flux is measured in this manner, errors can arise from various sources. For example, one possible source of error is the entrainment of liquid droplets in the calorimeter boil-off, and the subsequent vaporization of these droplets in the inner cylinder outlet line (see Figure II-2). In addition, as indicated in Chapter III, another source of error is the superheating of the boiled-off nitrogen vapor in the inner stainless steel calorimeter vessel. Thus in order to have confidence in the heat flux measurements the calorimeter had to be calibrated.

Essentially, the calibration was accomplished by conducting a known heat flux into the copper plate through a thin layer of cork located atop the plate. This known heat flux was compared with the value obtained from the measured liquid nitrogen boil-off. All calibration runs were made with the calorimeter operating at steady state.

The source of heat was an electrical heater, which was well insulated. The electrical energy output of the heater was varied by changing the voltage. Substantially all of the electrical energy produced by the heater passed into the copper plate.
The details of the calibration "set-up" are shown in Figure J-1. The electrical heater was sandwiched between two different slabs of cork of known thermal conductivities. The top slab was 4 inches thick. The thickness of the lower slab depended on the heat flux (i.e., the thickness employed was such that the top surface of this lower slab was at about room temperature). The energy output from the heater was determined by measuring the current and the voltage. (Direct current was employed for all calibration runs.) The heat flux through each piece of cork was calculated from the measured temperature difference across each cork slab and the known cork thermal conductivities.

Theoretically, the sum of the heat flux through the upper and lower slabs of cork should equal the total energy output of the heater (i.e., neglecting end losses from the cork and heater). The cork heat balance for the various calibration runs is shown in Table J-1. The system was designed such that substantially all of the energy output of the heater flowed through the thin, bottom slab of cork.

The heat flux calculated from the boil-off measurement was compared with the known heat flux (i.e., the calculated value of the flux through the bottom cork slab). This comparison is shown in Table J-2. The close agreement
indicates that measurement of the liquid nitrogen boil-off is a reliable method for obtaining the true heat flux into the calorimeter. In the worst case, the heat flux values disagreed by 7.9%.

No calibration curve for the calorimeter was established as the agreement between known heat fluxes and values determined from the boil-off measurements was quite good. The reported heat flux values for the various frosting runs were, therefore, determined solely from the liquid nitrogen boil-off measurement.

1. **Electrical Heater**

The heating element was No. 22 gauge Advance-Ideal resistance heating wire and was wound in a number of horizontal loops. These loops were all located in the same plane and were sandwiched between two 1/8 inch copper plates. The copper plates were cemented to each other and the space between the two plates was filled with asbestos to minimize heat losses.

The measured electrical resistance of the heater was 14.05 ohms. This value was essentially independent of wire temperature or applied voltage over the range of these conditions used in the calibration runs.
2. Cork Thermal Conductivities

Obviously it was important to have accurate values of the cork conductivities as these values were used to calculate the "known" heat fluxes. Cork conductivity data were obtained from three different sources (34, 41, 89). These references furnished data over a wide range of temperature and cork density. The agreement between the data from these sources was quite good.

The cork data indicated that thermal conductivity is a strong function of density. The densities of the various pieces of cork employed in the calibration experiments were, therefore, measured, and these values used to obtain the appropriate conductivities.

3. Measurements of Liquid Nitrogen Boil-off

For Runs 36, 37, 38 and 39, the liquid nitrogen boil-off vapor was measured with a rotameter only. For Run 42, however, the rotameter reading was checked with a wet test meter connected in series. The readings of these two instruments agreed to within 3%. (The rotameter reading is the one shown in Table J-2.) In general, for frosting runs the rotameter reading was checked with a wet test meter.
Table J-1

Comparison of Generated Electrical Heat with the Total Heat Flux Through the Top and Bottom Cork Slabs

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Electrical Heat Generated BTU/min-ft²</th>
<th>Heat Flux Through Top Cork Slab BTU/min-ft²</th>
<th>Heat Flux Through Bottom Cork Slab BTU/min-ft²</th>
<th>Percentage Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>11.0</td>
<td>0.383</td>
<td>10.05</td>
<td>+5.14%</td>
</tr>
<tr>
<td>37</td>
<td>2.9</td>
<td>0.017</td>
<td>2.80</td>
<td>+2.86</td>
</tr>
<tr>
<td>38</td>
<td>5.6</td>
<td>0.058</td>
<td>5.53</td>
<td>+2.14</td>
</tr>
<tr>
<td>39</td>
<td>14.7</td>
<td>0.107</td>
<td>14.21</td>
<td>+2.60</td>
</tr>
<tr>
<td>42</td>
<td>10.7</td>
<td>0.067</td>
<td>10.60</td>
<td>+0.308</td>
</tr>
</tbody>
</table>

Note: 1) Area of test section = 0.0277 ft².
2) Runs 36 and 42 were made at about the same conditions.
Table J-2

Calorimeter Calibration, Comparison of Known Heat Fluxes with Values Obtained from Liquid Nitrogen Boil-Off

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Known Heat Flux BTU/min-ft(^2)</th>
<th>Heat Flux From Liquid Nitrogen Boil-Off Measurement BTU/min-ft(^2)</th>
<th>Error in Measurement (Boil-Off) - Known Flux Known Flux x 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>10.05</td>
<td>10.82</td>
<td>+ 7.66 %</td>
</tr>
<tr>
<td>37</td>
<td>2.80</td>
<td>2.78</td>
<td>- 0.71 %</td>
</tr>
<tr>
<td>38</td>
<td>5.53</td>
<td>5.60</td>
<td>+ 1.27 %</td>
</tr>
<tr>
<td>39</td>
<td>14.21</td>
<td>14.30</td>
<td>+ 0.62 %</td>
</tr>
<tr>
<td>42</td>
<td>10.60</td>
<td>10.40</td>
<td>- 1.85 %</td>
</tr>
</tbody>
</table>
Electrical Wires

Thick Slab of Cork
Copper Plate Heater

Copper Plate

Electrical Wires

Thin Slab of Cork

Calorimeter Assembly

Note: For More Details of the Calorimeter Assembly, See Figure II-2

Figure J-1 Arrangement for Calibration of Calorimeter
APPENDIX K

Calculations for Internal Diffusion Model

As mentioned earlier, curves of frost density, frost thickness, frost surface temperature and heat flux through the frost as functions of time were calculated by solving Eqs. V-2 and V-6 simultaneously. An iteration procedure had to be used, and the time interval chosen was 5 minutes from $\Theta = 15$ minutes until the end of the run. From $\Theta = 5$ minutes until $\Theta = 10$ minutes, one minute time intervals were chosen, whereas $2 \frac{1}{2}$ minute intervals were employed from $\Theta = 10$ minutes to $\Theta = 15$ minutes. The details of the method used will be described below. Numbers from an actual calculation will be used in order to illustrate the method.

1. Sample Calculation for a Typical Time Interval

As an illustration, consider the time interval 95 to 100 minutes for the 5,600 Reynolds Number, 23 grains water/lb dry gas humidity case. The results of the calculation for this time period are shown in Table K-1.

A trial and error procedure was used to perform the calculation for each time interval. The initial trial value of $\rho_f$ at $\Theta = 100$ minutes was obtained by multiplying the densification rate for $\Theta = 95$ minutes by 5 minutes and adding the product to the final density value for
\( \theta = 95 \) minutes. The final density value for \( \theta = 95 \) minutes is 0.0797 \text{ gm/cc} (see Table K-1). The initial value of frost density for \( \theta = 100 \) minutes is, therefore, 
\[ 0.0797 + 5(0.000459) \text{ or } 0.0820 \text{ gm/cc}. \] (Frost densities shown in Table K-1 are final values for each increment; initial trial values are not shown.) The frost thickness for \( \theta = 100 \) minutes was then obtained by dividing the mass of frost accumulated on the plate after 100 minutes by 0.0820 \text{ gm/cc}. Frost mass at \( \theta = 100 \) minutes was obtained from the 5,600 Reynolds Number, 23 grains water/lb dry gas humidity curve of Figure V-16.

The values of \( T_s \) and \( k_{f,T}^{\text{Av.}} \) were obtained by a graphical technique. Essentially, Eq. V-2 was solved simultaneously with the curve representing integrated values of frost thermal conductivity as a function of frost surface temperature. The method for calculating integrated frost conductivity as a function of frost surface temperature will be described later.

Curve 3 of Figure K-1 is a plot of Eq. V-2 for a \( T_p \) of -317°F, a \( T_g \) of 73°F, an \( h_g \) of 7.24 BTU/hr-ft\(^2\)-°F and a \( \delta \) of 0.176 inches. These values of \( T_p, T_g \) and \( h_g \) are applicable for the entire 5,600 Reynolds Number, 23 grains water/lb dry gas humidity run, whereas a \( \delta \) of 0.187 inches is, of course, only applicable at 100 minutes. Curve 2 is a plot of the integrated thermal conductivity as a function of surface temperature for a density of...
0.0820 gm/cc. The point of intersection of these two curves represents the frost surface temperature and the average frost thermal conductivity. These values are -0.5°F and 0.0268 BTU/hr-ft²°F/ft respectively. The heat flux through the frost was then calculated, from Eq. K-1 below, to be 580 BTU/hr-ft²°F.

\[
\frac{q_f}{A} = \frac{k_f (T_s - T_p)}{Av.} \tag{K-1}
\]

As previously mentioned, the use of Eq. K-1 for this process, which is not a true steady state process, only introduces negligible error.

The value of the temperature gradient at the frost surface, \(\left(\frac{dT}{dy}\right)_s\), was then calculated from Eq. K-2 shown below.

\[
\frac{q_f}{A} = \left(\frac{dT}{dy}\right)_s \tag{K-2}
\]

The value of \(\frac{q_f}{A}\) substituted into Eq. K-2 was, of course, 580 BTU/hr-ft². The value \(k_f\) was found from the point thermal conductivity data of the present thesis. At a density of 0.0820 gm/cc and a temperature of -0.5°F, \(k_f\) is equal to 0.060 BTU/hr-ft²°F/ft. Using these values of \(\frac{q_f}{A}\) and \(k_f\), \(\left(\frac{dT}{dy}\right)_s\) was calculated to be 9,660 °F/ft.
From Figure F-1A, which is a plot of $\beta$ as a function of temperature, it was found that at $T_s = -0.5^\circ\text{F}$, $\beta$ is equal to $2.95 \times 10^{-6}$ lb/hr-ft-$^\circ\text{F}$. After adjusting the values of $\beta$, $\delta$, $(dT/dy)_s$ and $\rho_f$ listed above to a consistent set of dimensions, these values were substituted into Eq. K-3, which is identical to Eq. V-6.

\[
\frac{d\rho_f}{dt} = \frac{1}{\tau} \frac{\beta}{\delta} (1 - \frac{\rho_f}{\rho_1}) (\frac{dT}{dy})_s
\]  

\( (K-3) \)

\( d\rho_f/d\theta \) was calculated to be equal to 0.000431 gm/cc min. The average value of the densification rate is therefore

\[
\frac{0.000451 + 0.000431}{2} \text{ or } 0.00045 \text{ gm/cc min}
\]

The new value of the frost density at $\theta = 100$ minutes is then $0.0797 + 5(0.00222)$ or 0.0819. This is very close to the initial density value for $\theta = 100$ minutes (0.0820 gm/cc), and it was, therefore, not necessary to repeat the calculation for this time interval. In general, the second trial value of the frost density was very close to the initial value, and it was, therefore, usually not necessary to repeat the calculation for any of the time intervals. The above procedure was repeated for any given time increment when the initial and final frost density values for that
increment were substantially different. Two trials were sometimes required for the time intervals near the start of a run.

2. Start of the Calculation for a Particular Case

As mentioned earlier, the iteration procedure was started 5 minutes after the inception of frosting. In order to start the calculation, a value of thickness or density at time equal to 5 minutes had to be arbitrarily chosen. A thickness of 0.04 inches was chosen for all cases. (It has previously been shown that "long time" values of frost density, frost thickness, heat flux and frost surface temperature are relatively insensitive to the assumed "initial thickness".) The density at time = 5 minutes was calculated from the "initial thickness" values (0.04 inches) and the value of the mass accumulated at \( \Theta = 5 \) minutes. The value of the mass accumulated at \( \Theta = 5 \) minutes was obtained from the appropriate curve of Figure V-16. Actually the curves through the data were extrapolated to the origin. Values of \( T_s, \frac{k_f T}{A}, \frac{q_T}{A}, (dT/dy)_s, \beta \) and \( dp_f/d\Theta \) at \( \Theta = 5 \) minutes were calculated in the same manner as described above for the 95-100 minute interval. The initial value of \( \rho_f \) for \( \Theta = 6 \) minutes was calculated from the following equation.

\[
(p_f)_{initial} = \rho_f \text{ at 5 min.} + (1)(\frac{dp_f}{d\Theta})\text{at 5 min.}
\]
The values of $\delta$, $T_s$, $k_{f,T}$, $q_{T/A}$, $(dT/dy)_s$, $\beta$, $d\rho_f/d\theta_{AV}$, and $(\rho_f)_{final}$ at $\theta = 6$ minutes were also calculated as described above. The calculational procedure was then repeated at $\theta = 6$ minutes until two successive values of frost density were substantially the same (i.e., usually within 0.0002 gm/cc of each other). Thus, for any given case, the only density value which was not calculated by a trial and error procedure (even if only one trial was necessary) was the density at $\theta = 5$ minutes.

3. Calculation of Integrated (Average) Frost Thermal Conductivities

The curves of integrated or average frost thermal conductivity as a function of density for surface temperatures of +20, 0, and -20°F were obtained from the experimentally determined curves of point frost thermal conductivity (see Figure V-33).

The experimental data shown in Figure V-33 were cross plotted to obtain $k_{f,T}$ as a function of $T$ with $\rho_f$ as parameter. An integrated value of conductivity at any particular value of frost density (for example, 0.07 gm/cc) and at a surface temperature of +20°F was obtained by integrating under the 0.07 gm/cc curve from $T_p$ to +20°F. For a surface temperature of 0°F (and a density of 0.07 gm/cc), the appropriate value of the integrated conductivity was obtained by finding the area under the 0.07 gm/cc curve,
which was bounded on the left by the vertical line \( T = T_p \) and on the right by the vertical line \( T = 0^\circ F \). The value of the integrated thermal conductivity corresponding to a frost surface temperature of \(-20^\circ\) was obtained in a similar manner. Values of integrated conductivities at another density were obtained by finding the areas under the curve representing the appropriate density. In this manner curves of \( k_{f,T,Av.} \) as a function of density for surface temperatures of \(+20, -20\) and \(0^\circ F\) were obtained. Curve \(2\) of Figure K-1 was obtained from the \(+20, -20\) and \(0^\circ F\) curves by reading the appropriate conductivity values at a density of \(0.0820 \text{ gm/cc.}\)
Table K-1

Results of Calculation Based on the Internal Diffusion Model for a Typical Time Interval

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>$\delta$ (inches)</th>
<th>$T_s$ ($^\circ$F)</th>
<th>$k_{f,T}$</th>
<th>$q_T$</th>
<th>$(\frac{dT}{dy})_s$</th>
<th>$\beta$</th>
<th>$\frac{dp_f}{d\theta}$</th>
<th>$(\frac{dp_f}{d\theta})_{Av.}$</th>
<th>$\rho_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>0.172</td>
<td>-0.5</td>
<td>0.0264</td>
<td>585</td>
<td>10,000</td>
<td>2.95x10^{-6}</td>
<td>0.000459</td>
<td>0.0797</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.176</td>
<td>-0.5</td>
<td>0.0268</td>
<td>500</td>
<td>9,660</td>
<td>2.95x10^{-6}</td>
<td>0.000431</td>
<td>0.0819</td>
<td></td>
</tr>
</tbody>
</table>

Note: The calculation within this time interval was not sufficiently sensitive to indicate any slight change in surface temperature as this temperature was changing slowly. At $\theta = 125$ the surface temperature was calculated to be $+1.0^\circ$F.
Figure K-1  Average Frost Layer Conductivity as a Function of Surface Temperature
1. Frost Density

Frost density values reported in this thesis are actually indirectly measured quantities, since they were calculated from the measured weight of the collected frost sample and the measured sample thickness. Thus the error in the density measurement is related to the errors involved in weighing the frost sample and in measuring the frost thickness. The error in the density measurement, $\Delta \rho_f$, can be estimated from Eq. L-1 (60) shown below.

$$\Delta \rho_f = \frac{\partial \rho_f}{\partial m_{f,v}} \Delta m_{f,v} + \frac{\partial \rho_f}{\partial A_v} \Delta A_v + \frac{\partial \rho_f}{\partial \delta} \Delta \delta \quad (L-1)$$

Since $\rho_f = m_{f,v}/A_v$, Eq. L-1 reduces to

$$\Delta \rho_f = \frac{1}{A_v \delta} \Delta m_{f,v} - \frac{m_{f,v}}{\delta} \Delta A_v - \frac{m_{f,v}}{A_v \delta^2} \Delta \delta \quad (L-2)$$

Actually it is desired to calculate the maximum error inherent in the density measurement. Thus the magnitude of the three terms on the right-hand side of Eq. L-2 are added to each other. The equation actually used to evaluate $\Delta \rho_f$ is, therefore,
The results of Run 47 will be used to illustrate the method of evaluating $\Delta \rho_f$. For this run

$$A_v = 0.866 \text{ cm}^2$$
$$\delta = 0.618 \text{ cm}$$

and

$$m_{f,v} = 0.0344 \text{ gms}$$

Thus

$$\rho_f = \frac{m_{f,v}}{(\delta)A_v} = \frac{0.0344}{(0.618)(0.866)} = 0.0643$$

It was estimated that in general the weight of the collected frost sample could be determined to the nearest 0.002 gms. This limitation on the accuracy was due to loss of material during the collection process. It was also estimated that the frost thickness could be measured accurately to the nearest 0.005 inch (or 0.0127 cm). The primary cause of error in this measurement is the difficulty involved in determining the actual location of the frost surface.

The diameter of the vial used to collect the frost sample could be estimated to the nearest 0.002 inches (0.00508 cm). From Eq. L-4 shown below

$$\Delta \rho_f = \frac{1}{A_v} \Delta m_{f,v} + \frac{m_{f,v}}{A_v} \frac{1}{2} + \frac{m_{f,v}}{(\delta)^2} \Delta \delta$$  (L-3)
it was calculated that this uncertainty in the diameter implied an error of 0.00836 cm² in the vial cross sectional area. Thus, from the preceding discussion, the values of \( \Delta A_v, \Delta m_{f,v}, \) and \( \Delta \delta \) chosen for this calculation were:

\[
\Delta A_v = 0.00836 \text{ cm}
\]

\[
\Delta m_{f,v} = 0.002 \text{ gms}
\]

and

\[
\Delta \delta = 0.0127 \text{ cm}
\]

It should be noted that these error values are not restricted to Run 47 but are applicable to all the runs.

Substituting into Eq. L-3 the values of \( \Delta A_v, \Delta m_{f,v}, \) and \( \Delta \delta \) listed above, and the values of \( A, m_{f,v}, \) and \( \delta \) shown earlier, \( \Delta \rho_f \) was found to be 0.005678. Since \( \rho_f \) for Run 47 is equal to 0.0643 gms/cc, the maximum percentage error in the density measurement is 0.005678/0.0643 x 100 or \( +8.83\% \)

As indicated in Chapter IV, Runs 47 and 66 are duplicate runs. The densities measured in these two runs agree to 0.935%, a considerably smaller separation than the estimated error, \( \pm 8.83\% \). Actually such a discrepancy is expected as \( \pm 8.83\% \) is the maximum error involved in the density measurement. The calculation of this error by Eq. L-3 does not allow for compensating
errors. The true magnitude of the error is most likely lower than 8.83%. On the other hand, the true error is probably greater than indicated by the results of Runs 47 and 66. A truer estimate of the actual error could, of course, have been obtained if Run 47 were repeated several times instead of only once.

The procedure described above will also be used to determine the maximum error inherent in the density measurement of Run 55. For this run

\[
A_v = 0.866 \text{ cm}^2
\]
\[
\delta = 0.554 \text{ cm}
\]
\[
m_{f,v} = 0.0605 \text{ gms}
\]

and as before

\[
\Delta A_v = 0.00836 \text{ cm}^2
\]
\[
\Delta m_{f,v} = 0.002 \text{ gms}
\]

and

\[
\Delta \delta = 0.0127 \text{ cm}
\]

Substituting these values into Eq. L-3, \(\Delta \rho_f\) was calculated to be 0.0082 gm/cc. For Run 55, \(\rho_f = 0.126\), thus the maximum error is ±6.57%. As previously indicated, Runs 55 and 46 are duplicates. The difference between the measured densities for these two runs is 3.18%. As
for Runs 47 and 66, the agreement between the measured densities is much better than indicated by the estimated error.

2. Thermal Conductivities

A. Average Frost Layer Conductivity

The method for determining the error in the conductivity measurement is essentially the same as that used for determining $\Delta \rho_F$.

The average frost layer conductivity is given by

$$k_{f,T}^{Av} = \frac{q_T^5}{A(T_S - T_p)} \quad (L-5)$$

The error in the conductivity measurement is given by

$$\Delta k_{f,T}^{Av} = \left( \frac{q_T}{A(T_S - T_p)} \right) \Delta \delta + \left( \frac{\delta}{A(T_S - T_p)} \right) q_T \Delta T + \frac{\delta q_T}{A(T_S - T_p)} \Delta A \quad (L-6)$$

The combination of Eqs. L-5 and L-6 yields Eq. L-7 shown below.

$$\Delta k_{f,T}^{Av} = \frac{q_T}{A(T_S - T_p)} \Delta \delta + \frac{\delta}{A(T_S - T_p)} q_T \Delta T + \frac{\delta q_T}{A^2(T_S - T_p)} \\ \quad + \left( \frac{\Delta T_S + \Delta T_p}{A(T_S - T_p)} \right) \frac{\delta q_T}{A(T_S - T_p)^2} \quad (L-7)$$
The signs of all terms have been taken as positive as was done in estimating $\Delta p_f$.

Due to thermocouple error, the plate temperature could only be measured to about the nearest $2^\circ F$. The measurement of the surface temperature, on the other hand, may involve an error as large as $10^\circ F$. The reasons for an error of such magnitude have been previously discussed in Chapter V.

Based on the calorimeter calibration results, it was estimated that the maximum error in the heat flux measurement was 0.5 BTU/hr. In addition, the uncertainty with which the area of the heat transfer test section is known is about 0.0001228 ft$^2$. This value of $\Delta A$ was obtained from Eq. L-8, using a value of $\Delta D$ equal to 0.000416 ft (i.e., 0.005 inches).

$$\Delta A = \frac{D_m}{2} \Delta D \quad (L-8)$$

The diameter of the inner copper plate was actually known more accurately than to the nearest five thousandths of an inch. However, the heat transfer test section area also included one-half the area of the epoxy ring, as it was assumed that the heat fluxing into the inner half of the epoxy ring boiled off liquid nitrogen in the inner stainless steel chamber, whereas the heat fluxing into the outer half of the epoxy ring boiled off liquid nitrogen.
in the outer chamber. It is the difficulty involved in determining half the thickness of the epoxy ring which leads to an error estimate as large as 0.005 inches. The diameter of the actual copper inner plate could be determined to the nearest 0.002 inches.

As mentioned earlier, the error in the frost thickness measurement is five-thousandths of an inch or 0.000416 ft. Based on the above discussion, the errors involved in the various measured quantities were taken as follows:

\[
\Delta q_T = 0.5 \text{ BTU/hr}
\]
\[
\Delta T_p = 2^\circ F
\]
\[
\Delta T_s = 10^\circ F
\]
\[
\Delta s = 0.000416 \text{ ft}
\]
and
\[
\Delta A = 0.0001228 \text{ ft}^2
\]

Again, Run 47 will be used to illustrate the calculation of \( \Delta k_{f,T} \). For this run

\[
\frac{q_T}{A} = 14.7 \text{ BTU/hr}
\]
\[
A = 0.0277 \text{ ft}^2
\]
\[
s = 0.02025 \text{ ft}
\]
\[
T_s = 25^\circ F
\]
\[
T_p = -317^\circ F
\]
Substituting these values into Eq. L-7, $\Delta k_{f,T}^{\text{Av.}}$ was calculated to be 0.00297. The measured $k_{f,T}^{\text{Av.}}$ for Run 47 is 0.0314 and, therefore, the estimated error is ± 9.5%. The measured difference between the average conductivities measured in Runs 47 and 66 is 3.82%.

For Run 55 the values of $\Delta q_T$, $\Delta T_p$, $\Delta T_s$, $\Delta \delta$ and $\Delta \Lambda$ are the same as for Run 47. However, for Run 55

$$q_T = 20.9 \text{ BTU/hr}$$

$$\delta = 0.01818 \text{ ft}$$

$$T_s = 23^\circ\text{F}$$

$$T_p = -313^\circ\text{F}$$

Using these values $\Delta k_{f,T}^{\text{Av.}}$ was calculated to be 0.00355. Since the measured value of $k_{f,T}^{\text{Av.}}$ is 0.0375, the maximum error is ± 9.4%. The difference between the measured average frost conductivities for Runs 55 and 46 is 3.73%.

B. Point Thermal Conductivities

The point thermal conductivities shown in Figures IV-9 and V-33 were obtained by graphical differentiation (i.e., by eye) of the measured frost temperature profiles. The accuracy of this differentiation procedure was tested for a typical run by formulating a cubic equation which passed through the measured frost interior and interface temperatures. Temperature gradients were obtained by analytically differentiating this equation.
Run 44 was chosen as a typical run. The measured frost temperature profile for this run is shown in Figure IV-8. Conductivities calculated by analytical and graphical differentiation at temperatures of -215, -145 and -80°F are compared in Table L-1. The agreement is quite good, thus indicating that the graphical differentiation procedure, used in the present thesis, for obtaining point conductivities is satisfactory.
Table L-1

Comparison of Point Thermal Conductivities
Obtained by Graphical and Analytical Differentiation
of the Measured Frost Temperature Profile

<table>
<thead>
<tr>
<th>Temperature</th>
<th>$k_{f,T}$ Obtained by Graphical Differentiation</th>
<th>$k_{f,T}$ Obtained by Analytical Differentiation</th>
<th>$%$ Error $(\text{Graphical-Analytical}) \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-215</td>
<td>0.0184</td>
<td>0.0188</td>
<td>-1.075</td>
</tr>
<tr>
<td>-145</td>
<td>0.0285</td>
<td>0.0272</td>
<td>-4.78</td>
</tr>
<tr>
<td>-80</td>
<td>0.0481</td>
<td>0.0486</td>
<td>+1.03</td>
</tr>
</tbody>
</table>
APPENDIX M

Density Gradient in Frost

It should be emphasized that in the present thesis it was assumed that there is no density gradient in the frost. This assumption is based on the results of density measurements on the upper and lower halves of the frost. These measurements will be presented in the latter part of the appendix.

If it is assumed that the gas in the pores is saturated with water vapor at the local ice temperature, and that the transfer of mass within the frost occurs only by vapor diffusion, then a density gradient within the frost is inevitable. At the higher temperatures within the frost, i.e., at about 0°F, the vapor pressure of ice is relatively high and, therefore, the vapor flux in these warmer sections is also large. However, at the bottom of the frost, where temperatures are close to -300°F, the vapor flux is very small. Thus, the warmer parts of the frost densify significantly because of the increased vapor flux in these sections, whereas the colder sections hardly densify since they do not receive a significant supply of water vapor. It should be pointed out, however, that in a real frost the above assumption may not be completely valid. The gas within the pores may be saturated with water vapor at the local ice temperature, but ice particles
may be nucleating in the pores and transfer of these particles might be occurring as a result of the temperature gradient. If indeed this were the case, then a density gradient within the frost would not necessarily develop. In calculating the density gradient in this appendix, however, it was assumed that the transfer of mass occurs solely by vapor diffusion. It was also assumed that the gas in the pores was everywhere saturated at the local ice temperature.

1. Calculation of Density as a Function of Position in Frost

The measured frost temperature profiles were utilized to calculate the density gradient in the frost. In addition, to start the calculation it had to be assumed that at some time, relatively early in the run, the frost was of uniform density. For example, for the low humidity, intermediate Reynolds Number runs, it was assumed that the frost was still of uniform density 75 minutes after the start of frosting. The uniform frost density was taken as the measured bulk density at this time, i.e., 0.0638 gm/cc. It seems reasonable to postulate that the frost is of uniform density in the early phases of a run, as the deposition of particles into a frost layer would not necessarily cause a density gradient to develop.

The two equations required to calculate the development of the density gradient within the frost, M-1 and M-2, are presented below.
\[
\frac{N_f}{A} = \frac{\rho_f}{\tau} (1 - \frac{\rho_f}{\rho_l}) \frac{dT}{dy} \tag{M-1}
\]

and

\[
\frac{d\rho_f}{dy} = \frac{d(N_f/A)}{dy} \tag{M-2}
\]

(Note that \( \rho_f \) is the bulk or average of frost layer density, whereas \( \rho_{f,y} \) is the local or point value of density.)

Equation M-1, which is an expression for the rate of diffusion of water vapor within the frost, is applicable at all positions. The derivation of this equation has been presented earlier. Equation M-2 is easily derived from a material balance at a position in the frost. Essentially, this equation states that the rate of change of the diffusion flux with position, at a point within the frost, is equal to the rate of change of density with time at that point.

Equation M-1, and the measured frost temperature profiles at different times, were used to calculate mass flux within the frost as a function of position and time. Graphical differentiation of these flux profiles yielded rates of densification at various points within the frost layer (see Eq. M-2). These computed densification rates were then used to calculate local frost density as a function
of time and position. Density profiles calculated in this manner are shown in Figure M-1, for a Reynolds Number of about 9,300 and a humidity of about 23 grains water/lb dry gas.

It is important to note that the frost density profiles shown in Figure M-1 are not necessarily the true ones. As indicated previously, the validity of these profiles is subject to the following assumptions: (1) the frost was of uniform density 75 minutes after the start of frosting; (2) the gas in the pores was everywhere saturated at the local ice temperature; and (3) densification occurred solely by the transfer and deposition of water in vapor form.

In addition, calculating the frost density profile is not an accurate procedure, primarily for two reasons. First, experimental data have to be differentiated twice in order to obtain local frost densification rates. Secondly, small errors in the measured frost surface temperatures can lead to very significant errors in the calculation of the local mass fluxes as the vapor pressure is a strong function of temperature in this temperature range.

2. Measured Density Values of Frost Sections

The density gradient in the frost was determined by measuring the density of the upper and lower halves of the frost by the method described in Chapter III. The most reliable results are presented in Table M-1.
3. Comparison of Measurements with Calculations

The sectional density measurements of Runs 62 and 63 are compared with the calculated profiles. Actually, to make a comparison the calculated profiles have to be integrated over the appropriate range of positions within the frost, as the measured sectional densities are integrated or average values. A comparison of the measured sectional densities, and those determined from the calculated profiles, is shown in Table M-2. It is seen that the calculated profile indicates the density gradient to be much steeper than would be concluded from the measurements. In fact, the most reasonable conclusion one can draw from the measured sectional densities is that frost density does not vary with depth.

It should be noted that the measured thicknesses of the upper and lower sections of the frost do not add up to the total measured frost thicknesses which are listed in the last column of Table IV-2. This occurred because the sectional density measurements were often made on two different portions of a given frost sample.

4. Conclusions and Recommendations

The possibility of a density gradient in the frost layer is still an unresolved matter. The matter can only be resolved after more accurate sectional density measurements
are made. A possible method for obtaining more accurate measurements is described below.

A thin walled tube, accurately notched with thin grooves every fifty thousandths of an inch, could possibly be used. The tube should probably be about 3/8 of an inch in diameter and about 1 inch long. In the cold room, this tube would be inserted into the frost sample and then withdrawn, thus collecting a portion of the sample. The tube would then be cut at every groove. Each section of the tube, with the frost adhering to it, would then be placed in a separate container. Each container plus its contents would then be weighed. After drying, each container and the fifty thousandths inch ring would be weighed again. The frost density of each thin frost section would then be calculated from the measured weight of the sectional sample, the known cross section of the tube, and the known height (fifty thousandths).

The material of which the tube is to be made will have to be determined by preliminary experimentation. The material should be such that it can be cut easily at a temperature of -40°F, and to which frost will adhere.
Table M-1

**Measured Densities of Upper and Lower Frost Sections**

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Density of Upper Section (gm/cc)</th>
<th>Thickness of Upper Section (inches)</th>
<th>Density of Lower Section (gm/cc)</th>
<th>Thickness of Lower Section (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>0.124</td>
<td>0.158</td>
<td>0.116</td>
<td>0.077</td>
</tr>
<tr>
<td>62</td>
<td>0.082</td>
<td>0.104</td>
<td>0.0785</td>
<td>0.119</td>
</tr>
<tr>
<td>63</td>
<td>0.0955</td>
<td>0.106</td>
<td>0.0966</td>
<td>0.138</td>
</tr>
<tr>
<td>69</td>
<td>0.0827</td>
<td>0.079</td>
<td>0.0790</td>
<td>0.073</td>
</tr>
<tr>
<td>Run Number</td>
<td>Location</td>
<td>Measured Sectional Density (gm/cc)</td>
<td>Measured Sectional Density (gm/cc)</td>
<td>Measured Average (or Bulk) Density</td>
</tr>
<tr>
<td>------------</td>
<td>-----------</td>
<td>-----------------------------------</td>
<td>-----------------------------------</td>
<td>-----------------------------------</td>
</tr>
<tr>
<td>62</td>
<td>upper 0.104&quot;</td>
<td>0.082</td>
<td>1.025</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>lower 0.119&quot;</td>
<td>0.0785</td>
<td>0.981</td>
<td>0.067</td>
</tr>
<tr>
<td>63</td>
<td>upper 0.106&quot;</td>
<td>0.0955</td>
<td>0.994</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>lower 0.138&quot;</td>
<td>0.0966</td>
<td>1.005</td>
<td>0.072</td>
</tr>
</tbody>
</table>

Notes:
1. Density of lower section for Run 62 is computed from \( \int_0^{0.119} \rho_{f,y} \, dy \).
   Sectional density for Run 63 is calculated in same manner.
2. Average density for Run 62 is computed from \( \int_0^6 \rho_{f,y} \, dy \).
3. Measured average densities are listed in the fifth column of Table IV-2.
Re = 9,300

$H = 23 \text{ grains water} \over \text{lb dry gas}$

Run 61, $\theta = 75\text{ minutes}$

Run 62, $\theta = 91.5\text{ minutes}$

Run 63, $\theta = 123\text{ minutes}$

Figure M-1 Computed Values of Frost Density as a Function of Position in the Frost
### Additional Data and Calculated Conductivities

#### Table N-1

**Temperature as a Function of Position in Frost**

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Position in Frost, i.e., Distance from Copper Plate (inches)</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>0.060 0.131</td>
<td>-105 9</td>
</tr>
<tr>
<td>34</td>
<td>0.066 0.142</td>
<td>-107 10</td>
</tr>
<tr>
<td>44</td>
<td>0.063 0.133</td>
<td>-120 40</td>
</tr>
<tr>
<td>40</td>
<td>0.058 0.144</td>
<td>-133 36</td>
</tr>
<tr>
<td>48</td>
<td>0.046 0.140</td>
<td>-142 18</td>
</tr>
<tr>
<td>47</td>
<td>0.066 0.126</td>
<td>-139 66</td>
</tr>
<tr>
<td>66</td>
<td>0.114</td>
<td>- 79</td>
</tr>
<tr>
<td>49</td>
<td>0.046 0.120</td>
<td>-187 85</td>
</tr>
<tr>
<td>41</td>
<td>0.052 0.135</td>
<td>-195 88</td>
</tr>
<tr>
<td>59</td>
<td>0.100</td>
<td>-115</td>
</tr>
<tr>
<td>50</td>
<td>0.120</td>
<td>-115</td>
</tr>
<tr>
<td>51</td>
<td>0.135</td>
<td>-120</td>
</tr>
<tr>
<td>53</td>
<td>0.044 0.115</td>
<td>-104 18</td>
</tr>
<tr>
<td>54</td>
<td>0.066 0.130</td>
<td>-64  8</td>
</tr>
<tr>
<td>69</td>
<td>0.113</td>
<td>- 17</td>
</tr>
<tr>
<td>55</td>
<td>0.061 0.103</td>
<td>-102 55</td>
</tr>
<tr>
<td>46</td>
<td>0.048 0.109</td>
<td>-130 52</td>
</tr>
<tr>
<td>61</td>
<td>0.124</td>
<td>- 39</td>
</tr>
<tr>
<td>62</td>
<td>0.116</td>
<td>- 58</td>
</tr>
<tr>
<td>63</td>
<td>0.116</td>
<td>- 79</td>
</tr>
<tr>
<td>65</td>
<td>0.100</td>
<td>- 85</td>
</tr>
</tbody>
</table>

**Note:** The temperatures presented in this table are end-of-run values.
Table N-2

Gas Temperatures, Plate Temperatures and Heat Fluxes

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Plate Temperature (OF)</th>
<th>Steady or End-of-Run Heat Flux (BTU/hr-ft²)</th>
<th>Gas Temperature (OF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>314</td>
<td>694</td>
<td>75</td>
</tr>
<tr>
<td>40</td>
<td>312</td>
<td>694</td>
<td>75</td>
</tr>
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<td>43</td>
<td>314</td>
<td>722</td>
<td>70</td>
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<td>44</td>
<td>314</td>
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<td>75</td>
</tr>
<tr>
<td>41</td>
<td>316</td>
<td>510</td>
<td>75</td>
</tr>
<tr>
<td>47</td>
<td>317</td>
<td>530</td>
<td>75</td>
</tr>
<tr>
<td>48</td>
<td>314</td>
<td>515</td>
<td>71</td>
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<td>49</td>
<td>317</td>
<td>515</td>
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<td>66</td>
<td>316</td>
<td>513</td>
<td>72</td>
</tr>
<tr>
<td>50</td>
<td>317</td>
<td>459</td>
<td>77</td>
</tr>
<tr>
<td>51</td>
<td>318</td>
<td>450</td>
<td>72</td>
</tr>
<tr>
<td>59</td>
<td>318</td>
<td>459</td>
<td>73</td>
</tr>
<tr>
<td>53</td>
<td>310</td>
<td>838</td>
<td>80</td>
</tr>
<tr>
<td>54</td>
<td>312</td>
<td>775</td>
<td>79</td>
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<tr>
<td>55</td>
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<td>755</td>
<td>75</td>
</tr>
<tr>
<td>69</td>
<td>315</td>
<td>758</td>
<td>79</td>
</tr>
<tr>
<td>46</td>
<td>313</td>
<td>758</td>
<td>79</td>
</tr>
<tr>
<td>61</td>
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<td>579</td>
<td>75</td>
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<tr>
<td>62</td>
<td>315</td>
<td>560</td>
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<td>316</td>
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<td>77</td>
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<td>60</td>
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<td>495</td>
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</tr>
<tr>
<td>65</td>
<td>317</td>
<td>494</td>
<td>73</td>
</tr>
</tbody>
</table>

Table N-3

Measured Surface Temperatures

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Time (Min)</th>
<th>Surface Temperature (OF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>58</td>
<td>23</td>
</tr>
<tr>
<td>44</td>
<td>69</td>
<td>25</td>
</tr>
<tr>
<td>47</td>
<td>55.5</td>
<td>28</td>
</tr>
<tr>
<td>49</td>
<td>58.5</td>
<td>25</td>
</tr>
<tr>
<td>66</td>
<td>74</td>
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</tr>
<tr>
<td>50</td>
<td>81</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
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<td>23</td>
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<tr>
<td>51</td>
<td>57</td>
<td>20</td>
</tr>
<tr>
<td>51</td>
<td>119</td>
<td>18</td>
</tr>
<tr>
<td>53</td>
<td>49</td>
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<td>55</td>
<td>57</td>
<td>23</td>
</tr>
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<td>69</td>
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<td>20</td>
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<td>60</td>
<td>107</td>
<td>13</td>
</tr>
</tbody>
</table>

Note: In calculating conductivities the average measured surface temperature for a particular case (i.e., at a given Reynolds Number and humidity) was used.
Table N-4
Calculated Point Conductivities

<table>
<thead>
<tr>
<th>Temperature = -215°F</th>
<th>Temperature = -145°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>( \rho_f )</td>
</tr>
<tr>
<td>No.</td>
<td>gm/cc</td>
</tr>
<tr>
<td>34</td>
<td>0.0914</td>
</tr>
<tr>
<td>40</td>
<td>0.128</td>
</tr>
<tr>
<td>43</td>
<td>0.0809</td>
</tr>
<tr>
<td>44</td>
<td>0.111</td>
</tr>
<tr>
<td>41</td>
<td>0.0918</td>
</tr>
<tr>
<td>47</td>
<td>0.0643</td>
</tr>
<tr>
<td>48</td>
<td>0.0608</td>
</tr>
<tr>
<td>49</td>
<td>0.0790</td>
</tr>
<tr>
<td>59</td>
<td>0.0545</td>
</tr>
<tr>
<td>53</td>
<td>0.129</td>
</tr>
<tr>
<td>54</td>
<td>0.0882</td>
</tr>
<tr>
<td>55</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature = -80°F</th>
<th>Temperature = 0°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>( \rho_f )</td>
</tr>
<tr>
<td>No.</td>
<td>gm/cc</td>
</tr>
<tr>
<td>34</td>
<td>0.0914</td>
</tr>
<tr>
<td>40</td>
<td>0.128</td>
</tr>
<tr>
<td>43</td>
<td>0.0809</td>
</tr>
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<td>44</td>
<td>0.111</td>
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<tr>
<td>41</td>
<td>0.0918</td>
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<td>47</td>
<td>0.0643</td>
</tr>
<tr>
<td>48</td>
<td>0.0608</td>
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<td>0.0790</td>
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<td>0.0618</td>
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<tr>
<td>59</td>
<td>0.0545</td>
</tr>
<tr>
<td>65</td>
<td>0.0568</td>
</tr>
<tr>
<td>61</td>
<td>0.0618</td>
</tr>
<tr>
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<td>0.0109</td>
</tr>
<tr>
<td>54</td>
<td>0.0882</td>
</tr>
<tr>
<td>55</td>
<td>0.126</td>
</tr>
<tr>
<td>69</td>
<td>0.0818</td>
</tr>
<tr>
<td>62</td>
<td>0.0800</td>
</tr>
<tr>
<td>63</td>
<td>0.0961</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature = +20°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
</tr>
<tr>
<td>No.</td>
</tr>
<tr>
<td>34</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>43</td>
</tr>
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<tr>
<td>41</td>
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<tr>
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</tr>
<tr>
<td>55</td>
</tr>
<tr>
<td>69</td>
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</tbody>
</table>
### Table of Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area for heat and mass transfer</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Total ice slab area for heat transfer. Parallel slab model</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Total gas slab area for heat transfer. Parallel slab model</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Cross sectional area of vial used for taking frost sample for density analysis</td>
</tr>
<tr>
<td>a</td>
<td>Factor in Equation V-27. Defined in Chapter 5, Section 9G-5</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag coefficient on particle</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Concentration of water vapor in gas phase</td>
</tr>
<tr>
<td>$c^*$</td>
<td>Equilibrium concentration of water vapor</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Heat capacity of dry nitrogen gas</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Heat capacity of humid gas ($c_s \approx c_p + 0.46$)</td>
</tr>
<tr>
<td>$c_T$</td>
<td>Total concentration of gas, i.e., gas density</td>
</tr>
<tr>
<td>$D_{1-2}$</td>
<td>Diffusivity of water vapor in nitrogen</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Diameter of ice particle</td>
</tr>
<tr>
<td>$D_v$</td>
<td>Diameter of vial used for taking frost sample</td>
</tr>
<tr>
<td>$e_H$</td>
<td>Emissivity of frost</td>
</tr>
<tr>
<td>$F_D$</td>
<td>Drag force on a particle</td>
</tr>
<tr>
<td>$F_{Th}$</td>
<td>Thermal force on a particle due to temperature gradient in gas</td>
</tr>
<tr>
<td>f</td>
<td>Friction factor</td>
</tr>
<tr>
<td>f'</td>
<td>Modified friction factor, defined by Equation C-2</td>
</tr>
<tr>
<td>$G_o$</td>
<td>Superficial mass velocity</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$g_c$</td>
<td>Gravitational constant</td>
</tr>
<tr>
<td>$H$</td>
<td>Humidity of gas phase</td>
</tr>
<tr>
<td>$h_g$</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>$K_D$</td>
<td>Permeability coefficient of frost</td>
</tr>
<tr>
<td>$k_c$</td>
<td>Thermal conductivity of cluster phase</td>
</tr>
<tr>
<td>$k_f$</td>
<td>Thermal conductivity of frost; sometimes denoted as point frost thermal conductivity. Excludes effects of interparticle radiation and gas flow through frost, but includes effects of evaporative transport</td>
</tr>
<tr>
<td>$k_{f,T}$</td>
<td>Total or over-all frost thermal conductivity; sometimes denoted as point frost thermal conductivity. $k_{f,T}$ is the measured value. Includes effects of interparticle radiation, gas flow through frost, and evaporative transport. (For this thesis $k_f$ is essentially equal to $k_{f,T}$)</td>
</tr>
<tr>
<td>$k_{f,T,Av}$</td>
<td>Average thermal conductivity of the frost layer</td>
</tr>
<tr>
<td>$k_{f,f}$</td>
<td>Thermal conductivity of frost. Same as $k_f$, but includes effects of gas flow through frost.</td>
</tr>
<tr>
<td>$k_g$</td>
<td>Ordinary thermal conductivity of nitrogen gas</td>
</tr>
<tr>
<td>$k_{g,e}$</td>
<td>Total nitrogen gas thermal conductivity, i.e., includes effects of evaporative transport</td>
</tr>
<tr>
<td>$k_G$</td>
<td>Thermal conductivity of gas</td>
</tr>
<tr>
<td>$k'_g$</td>
<td>Mass transfer coefficient</td>
</tr>
<tr>
<td>$k_H$</td>
<td>Thermal conductivity of heterogeneous materials in general, Deissler model</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Thermal conductivity of solid ice</td>
</tr>
<tr>
<td>$k_{po}$</td>
<td>Thermal conductivity of pocket phase, Weber model</td>
</tr>
<tr>
<td>$k_r$</td>
<td>Contribution made by radiation to over-all frost thermal conductivity</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Thermal conductivity of solid material in a heterogeneous structure</td>
</tr>
</tbody>
</table>
\( k_T \) Thermal diffusion coefficient

\( k_{xs} \) Excess thermal conductivity of gas phase due to evaporative transport

\( L \) Length of frost through which gas flows

\( L_c \) Heat conduction length of cluster phase, Weber model

\( L_G \) Heat conduction length of gas in cluster phase, Weber model

\( L_{po} \) Heat conduction length of pocket phase, Weber model

\( L_s \) Heat conduction length of solid in cluster phase, Weber model

\( \ell \) Length of a slab of ice or gas. Parallel slab model

\( M_g \) Molecular weight of total gas, nitrogen plus water vapor

\( M_v \) Molecular weight of water vapor

\( m_f \) Mass of frost, per unit area, on plate at any instant

\( m_{f,v} \) Mass of frost per unit area collected in vial

\( N \) Rate of mass transfer of water vapor from gas phase to frost surface

\( N_f \) Rate of transport of water vapor within frost

\( N_{p,gr} \) Rate of transport of particles from gas to frost due to gravity

\( N_{p,Tu} \) Rate of transport of particles from gas to frost due to turbulence

\( n_1 \) Mole fraction of water vapor in gas contained in pores of frost

\( n_2 \) Mole fraction of nitrogen in gas contained in pores of frost

\( \Delta P_f \) Pressure drop across frost. Pressure loss due to expansion and contraction of the gas stream, and to skin friction
$Pe$  
Peclet number

$p^*$  
Vapor pressure of water vapor in equilibrium with ice at local ice temperature

$q$  
Rate of heat transfer through frost. Includes heat transferred by conduction and evaporative transport, but not by radiation between the particles or by convection arising from gas flow through the frost

$q_c$  
Heat transferred by convection from gas bulk to frost surface

$q_{cond}$  
Rate of liberation of heat at frost surface by water vapor condensing on the surface

$q_T$  
Total rate of heat transfer through the frost. Sum of heat transferred by conduction, evaporative transport, radiation between particles, and by convection arising from the gas flow through the frost. Also equal to total heat transfer from gas phase to frost surface

$R$  
Universal gas constant

$R_{eq}$  
Equivalent thermal resistance

$R_f$  
Thermal resistance of frost

$R_g$  
Thermal resistance of gas

$R_i$  
Thermal resistance of solid ice

$R_s$  
Radius of an ice sphere. Woodside model

$R_{zone\ 1}$  
Thermal resistance of Zone 1

$R_{zone\ 2}$  
Thermal resistance of Zone 2

$R_{zone\ 3}$  
Thermal resistance of Zone 3

$R_{zone\ I}$  
Thermal resistance of Zone I. Woodside model

$R_{zone\ II}$  
Thermal resistance of Zone II. Woodside model

$Re$  
Reynolds Number

$Re_p$  
Particle Reynolds Number

$Re_x$  
Length Reynolds Number
\( r \) Radial distance from center of pipe or distance from horizontal centerline of duct

\( r_c \) Critical radius of cylinder

\( r_s \) Radial distance from center of ice sphere. Woodside model

\( S_c \) Schmidt Number

\( S_h \) Sherwood Number

\( T \) Temperature

\( T_1 \) Refers to the temperature at some location in the cluster phase. Weber model

\( T_2 \) Refers to the temperature at another location in the cluster phase. Weber model

\( T_d \) Gas phase dew point

\( T_g \) Gas phase temperature

\( T_p \) Copper plate temperature

\( T_s \) Frost surface temperature

\( \Delta T \) Temperature difference across frost layer

\( U_{Av} \) Average velocity of gas in duct

\( u^* \) Friction velocity

\( V_{o,gr} \) Terminal velocity of a particle due to gravity

\( V_{o,th} \) Terminal velocity of a particle due to the temperature gradient in the gas

\( w_g \) Thickness of a slab of gas. Parallel slab model

\( w_i \) Thickness of an ice slab. Parallel slab model

\( x \) Linear dimension of ice cube

\( y \) Mole fraction of water vapor in gas

\( y^* \) Mole fraction of water vapor in gas which is in equilibrium with ice at local ice temperature

\( y \) Vertical position in frost, i.e., vertical distance from frost-plate interface
$z$  Width of ice rod. Cube and rod model

$z_s$  Position within a unit cube. Woodside model

**Greek**

$\alpha$  Thermal diffusion factor

$\beta$  Diffusion function, defined by Equation V-7

$\delta$  Thickness of frost layer

$\epsilon$  Porosity of frost

$\epsilon'$  Volume fraction of pocket phase, Weber model

$\gamma$  Defined by Equation D-27, Weber model

$\theta$  Time

$\lambda$  Latent heat of sublimation of ice

$\mu$  Viscosity

$\mu_{20}$  Viscosity at 20°F

$\mu_{-315}$  Viscosity at -315°F

$\nu$  Kinematic viscosity

$\pi$  $\pi_1 = 3.14$

$\rho_f$  Average or bulk density of frost

$\rho_g$  Density of gas

$\rho_p$  Density of ice particle

$\rho_i$  Density of solid ice

$\tau$  Tortuosity of frost, taken as 1.1

$\psi$  Defined as $\frac{k_i - k_{g,e}}{k_{g,e}}$
Additional Information about Subscripts

$g$ refers to gas, either in main stream or pores of frost

1 refers to solid ice

$p$ refers to particle, except for $T_p$, where it refers to copper plate

$s$ refers to frost surface

1 denotes water vapor as a component

2 denotes nitrogen gas as a component
APPENDIX P

Bibliography


Biographical Note

The author was born in New York City on October 27, 1936. He attended elementary school in Brooklyn, New York, and was graduated from Stuyvesant High School in Manhattan in June, 1954. He then enrolled as a freshman at Cooper Union the following September. After graduation from Cooper Union in June, 1958, he attended Lehigh University from September, 1958, to October, 1959. The degree of M.S. in Chemical Engineering from Lehigh was awarded in October, 1960. The following September he enrolled at M.I.T. as a full-time graduate student to start work toward a doctorate degree.

In addition to several summer positions, the author was employed from November, 1959, to September, 1961, as a research engineer by the Lewis Research Center of the National Aeronautics and Space Administration in Cleveland, Ohio. While at NASA he published the following reports.


The author has been employed since August, 1966, with the Polaroid Corporation in Waltham, Massachusetts. Upon
graduation, he will continue his employment with that company.

The author is also a member of A.I.Ch.E. and Sigma Xi. In June, 1959, he married the former Rosalie Seligson of New York City. They have a son, Howard Robert.